Title: Lost in Translation

Date: May 01, 2008 04:40 PM

URL: http://pirsa.org/08050018

Abstract: We consider the question of forward and backward translation between measurement-based quantum computing, called patterns, and quantum circuit computation. It is known that the class of patterns with a particular properties, having flow, is in one-to-one correspondence with quantum circuits. However we show that a more general class of patterns, those having generalised flow, will sometime translate to imaginary circuits, quantum circuits with time-like curves. Extending this approach, we first present the semantics of quantum circuits with time-like curves in terms of post-selection quantum computing and then characterise the class of curves with unitary or completely-positive semantic. Finally we present the re-write rules for opening the loops to transform an imaginary circuit to a normal circuit and discuss the connection between time-like curves and depth complexity.

Lost in Translation

Elham Kashefi

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joint work with

Sofia Coppola

Motivation

How can we obtain a parallel algorithm for a given task ?

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How can we obtain a parallel algorithm for a given task ?

- Minimising quantum depth is a way around decoherence.
- New results in complexity theory.
- **Jozsa Conjecture**. Any polynomial-time quantum algorithm can be implemented with only $O(\log(n))$ quantum layers interspersed with polynomial-time classical computations.
- New measure for comparing quantum models.

but what if QC is شبیه پذیر rizaphibahS

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BQP $\stackrel{?}{=}$ BPP

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- Quantum computing is intrinsically parallel $NC \subset BPP$

but what if QC is شبیه پذیر rizaphibahS

BQP ?= BPP

- Quantum computing is intrinsically parallel   NC ⊂ BPP

- New method for parallelising classical algorithm   NC = BPP

Where to start?

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A very quantum model of computing where entanglement is the main ingredient.

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Measurement-based Quantum Computing

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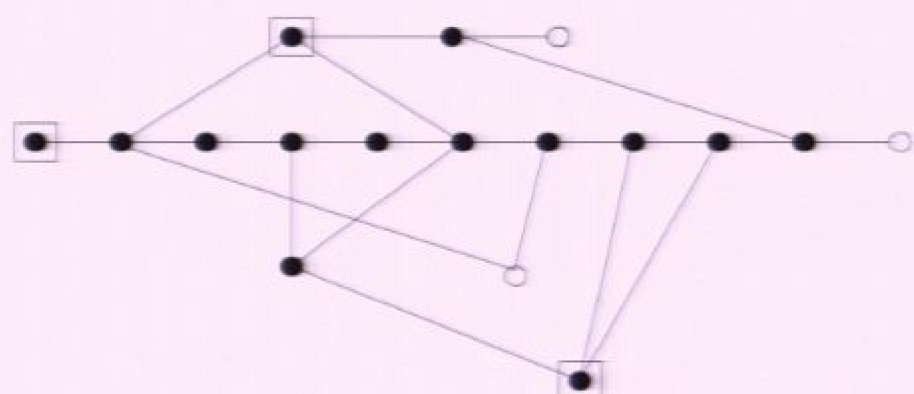
PROJECTION

Measurement-based Quantum Computing

Projection-based QC

- Initial entangled state (graph state)
- 1-qubit Projections



Projection-based QC

- Universal for QC
- PostBQP = PP
- Any computation has **constant depth**

Constant Depth

- Preparation depth for any graph state is $\Delta(G)$ or $\Delta(G)+1$
- Pick your favourite universal resource with constant degree
- Do all the projections at once

Parallel Algorithm for any Unitary

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$$U = \frac{1}{C} \sum_{\mathbf{x} \in \{0,1\}^V} e^{iQ(\mathbf{x})} \, |\mathbf{x}_O\rangle\langle\mathbf{x}_I|$$

$U : \mathcal{H}_2^{\otimes I} \to \mathcal{H}_2^{\otimes O}$, *where* Q *is a real-valued quadratic form on* $\mathbf{x}$, *and* $C \in \mathbb{C}$

arXiv:0801.2461, N. de Beaudrap, V. Danos, EK, M. Roetteler

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$$Q(\mathbf{x}) = \sum_{\{u,v\} \subseteq V} \theta_{uv} x_u x_v$$

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Road Map

Road Map

- From Projection-based QC to MBQC

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- **From Projection-based QC to MBQC**
  - Determinism in MBQC
  - Flow and generalised flow construction

V. Danos and EK Phys. Rev. A 2006
D. Browne, EK, M. Mhala and S. Perdrix, New Journal of Physics 2007
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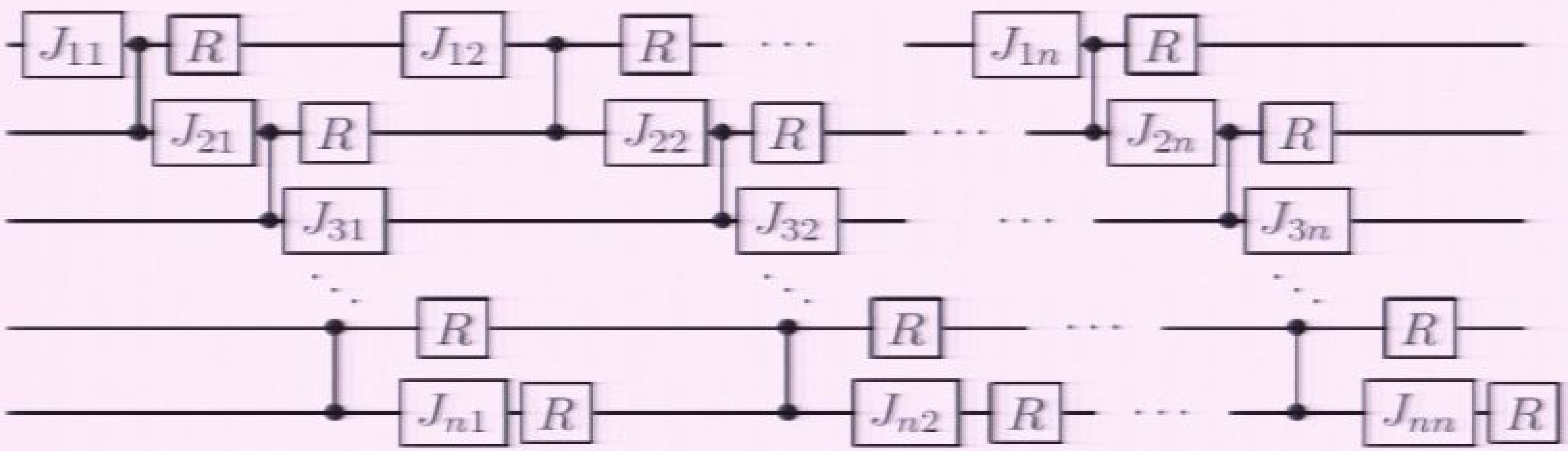
- **From Quantum Circuit to MBQC**
    - Automated technique for parallelising quantum circuits
    - Structural characterisation of pattern with depth D
    - Logarithmic depth separation

A. Broadbent and EK, arXiv:0704.1736

Magical Clifford



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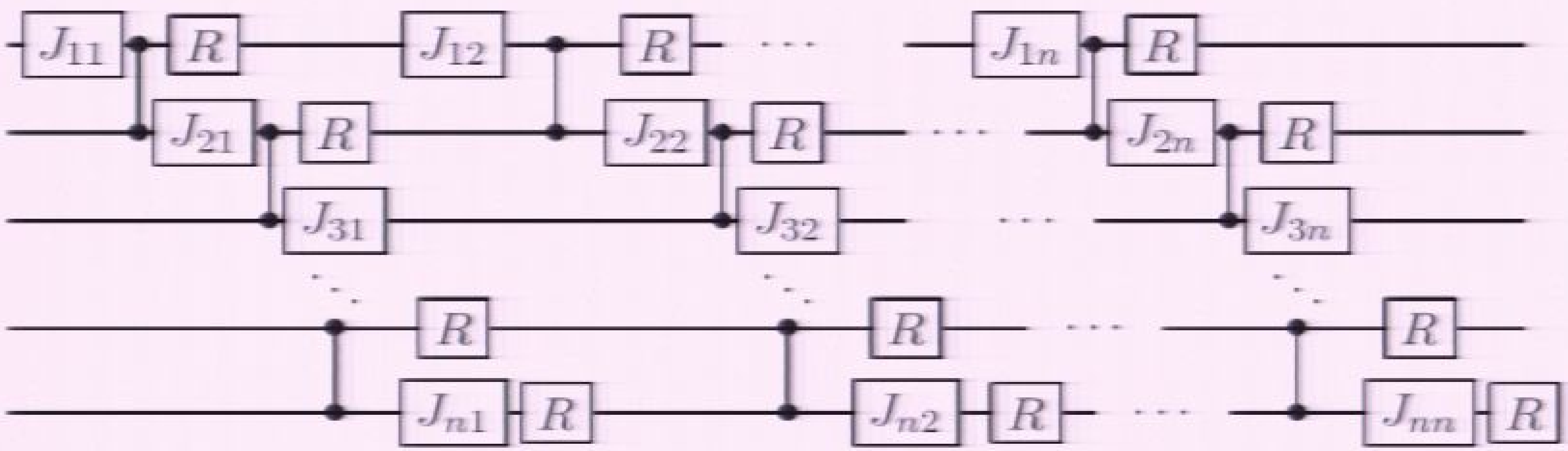

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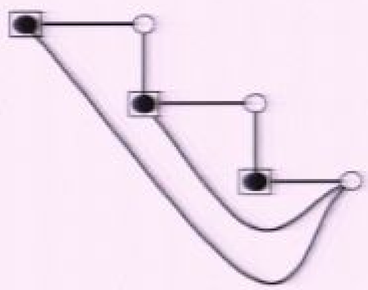
MBQC vs QCircuit



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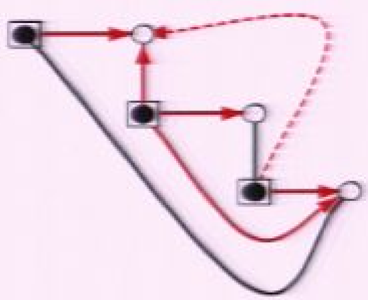

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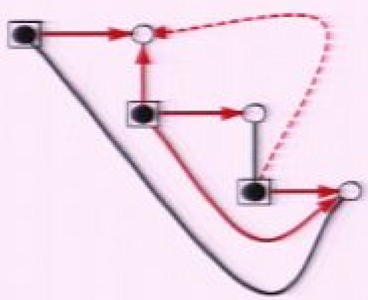

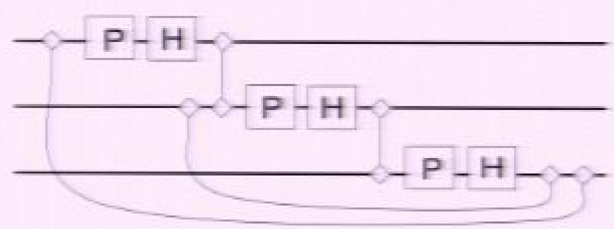

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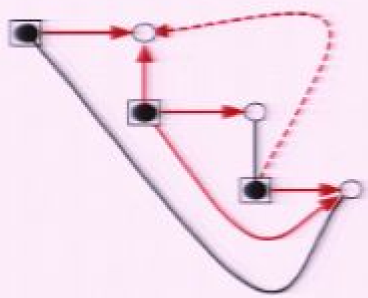

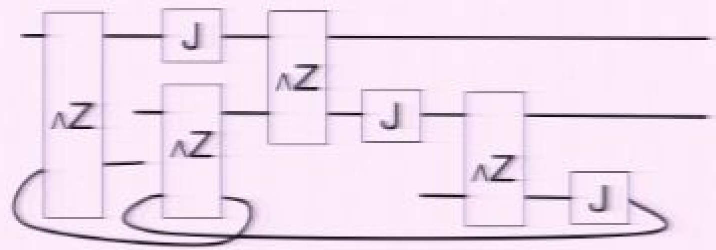

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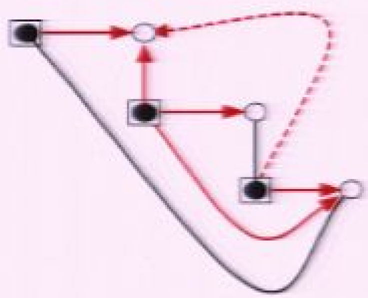

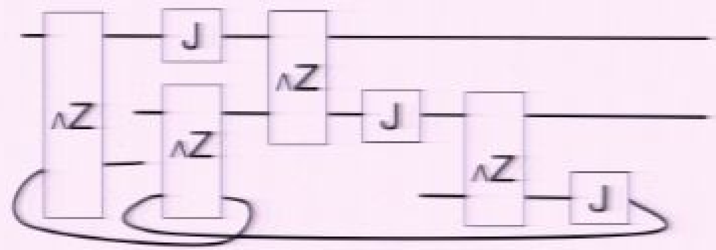

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- **From Quantum Circuit with Time-like curves to MBQC**

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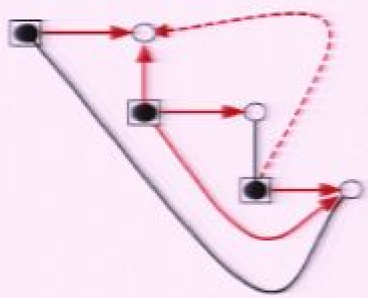

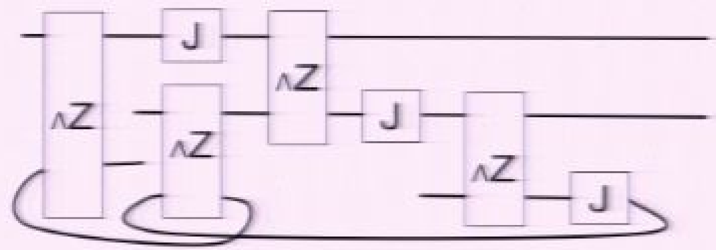

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➡ Semantics
➡ Characterisation Results
➡ Opening Loops

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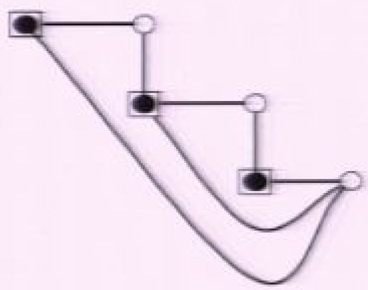

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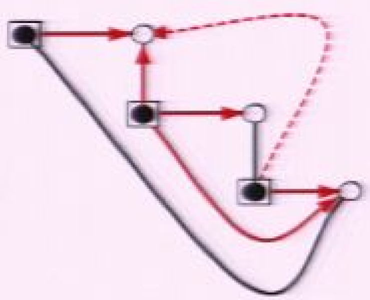

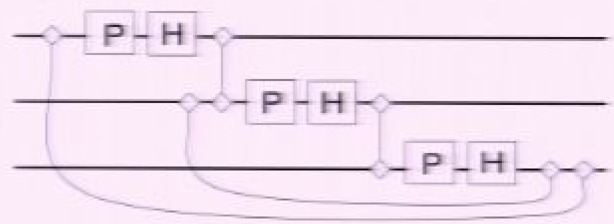

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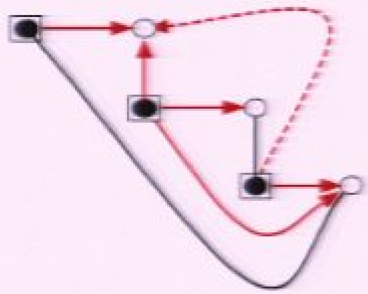

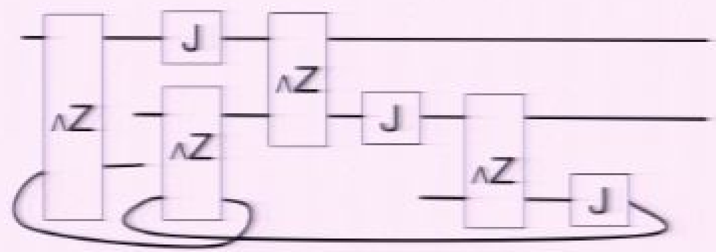

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Notation

- N_i prepares qubit in $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
- M_i^α projects qubit onto basis states $\frac{1}{\sqrt{2}}(|0\rangle \pm e^{i\alpha}|1\rangle)$
  (measurement outcome is $s_i = 0, 1$)
- E_{ij} creates entanglement
- Local Pauli corrections X_i, Z_i
- **Feed forward:** C_i^s $M_i^{(-1)^s\alpha}$ $M_i^{\alpha+s\pi}$

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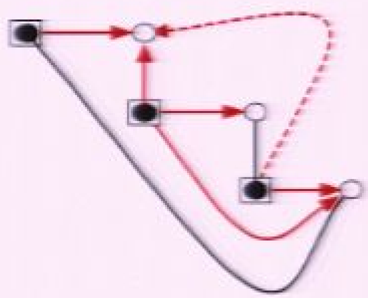

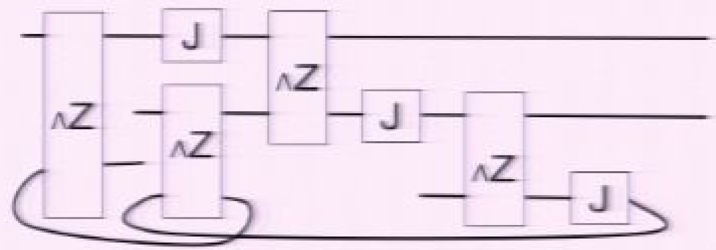

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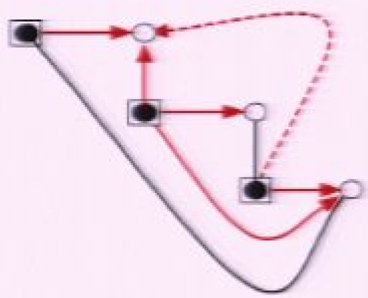

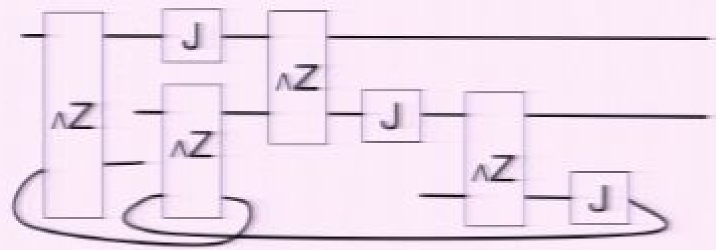

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From Projection to Measurement

$$P_i^{|+\alpha\rangle} =$$

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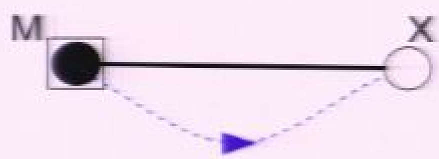

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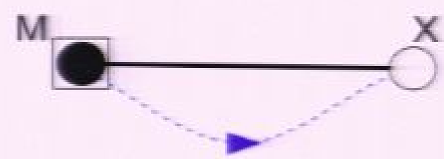

$$P_i^{|+\alpha\rangle} E_{ij} = M_i^{\alpha} \; X_j^{s_i} \; E_{ij} \; X_j^{s_i}$$

From Noise to Determinism

A pattern is **deterministic** if all the branches are the same.

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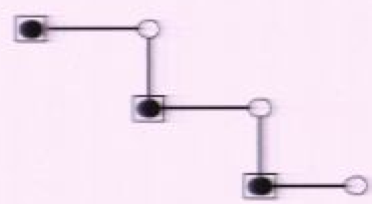
A necessary and sufficient condition for determinism based on **geometry** of entanglement is given by flow

Flow

Definition. An entanglement graph (G, I, O) has flow if there exists a map $f : O^c \rightarrow I^c$ and a partial order $\preceq$ over qubits

— (i) $x \sim f(x)$
— (ii) $x \preceq f(x)$
— (iii) for all $y \sim f(x)$, we have $x \preceq y$



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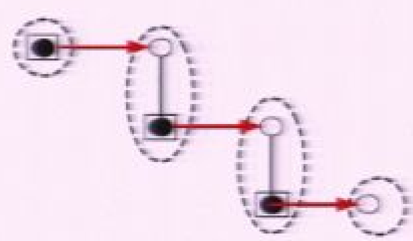

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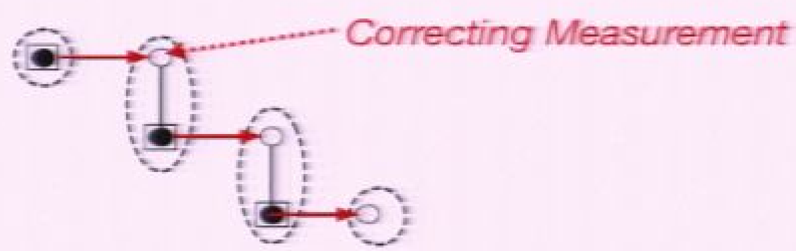

Flow

Theorem. A pattern with flow is uniformly and strongly deterministic.

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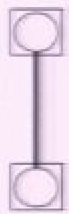
Patterns with flow → Unitary embedding

From Pattern to Circuit



$$\wedge 3 := E_{12}$$

V. Danos, EK, P. Panangaden, Phys Rev. A. , 2006

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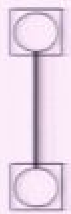
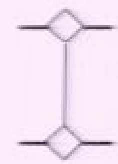


$$\wedge\mathfrak{Z} := E_{12}$$



$$\mathfrak{J}(\alpha) := X_2^{s_1} M_1^{-\alpha} E_{12}$$

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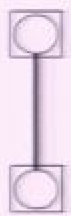

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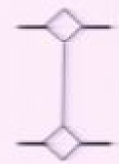

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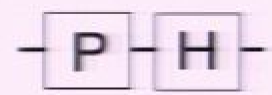

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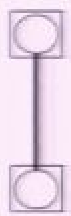

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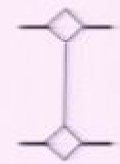

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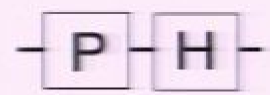

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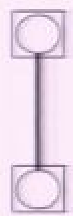

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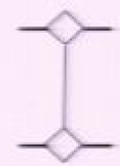

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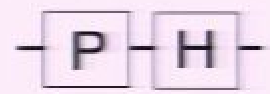

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From Pattern to Circuit

Star Pattern: $X_2^{s_1} M_1^{\alpha} E_{12} E_{13} \cdots E_{1n}$

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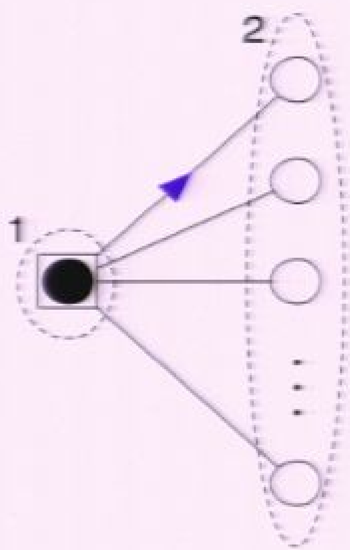

From Pattern to Circuit

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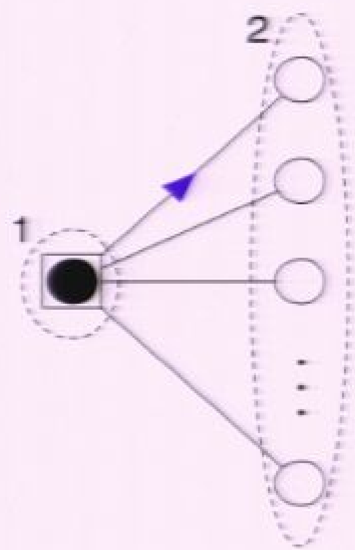
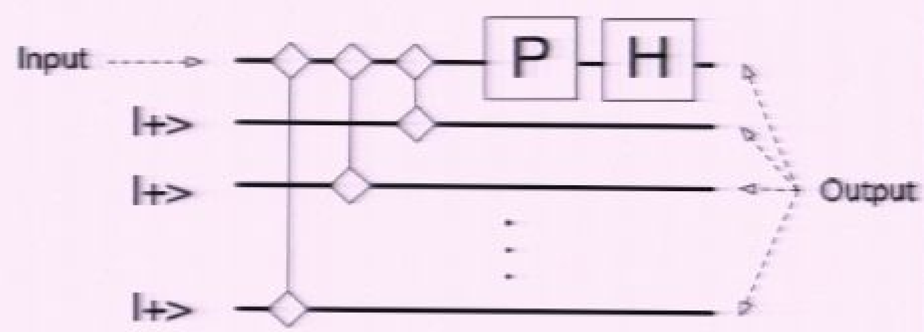


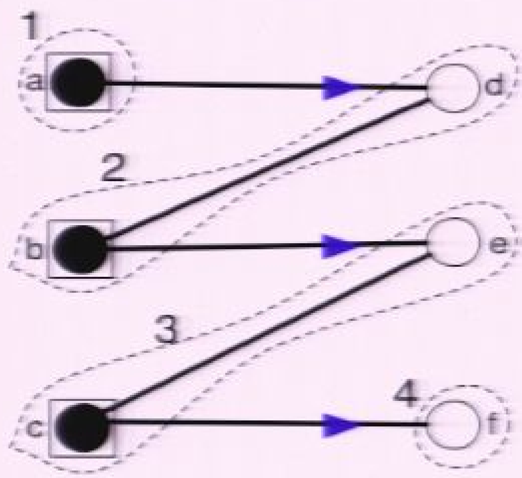


Star Decomposition

Theorem. Every pattern such that the underlying graph state has flow can be decomposed into star patterns.

Patterns with flow ⟷ Quantum Circuit

Star Decomposition



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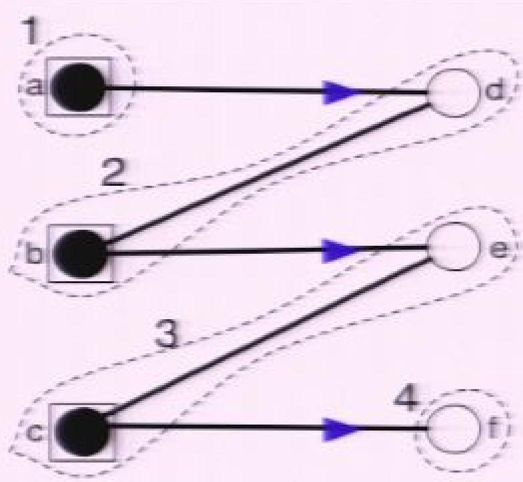

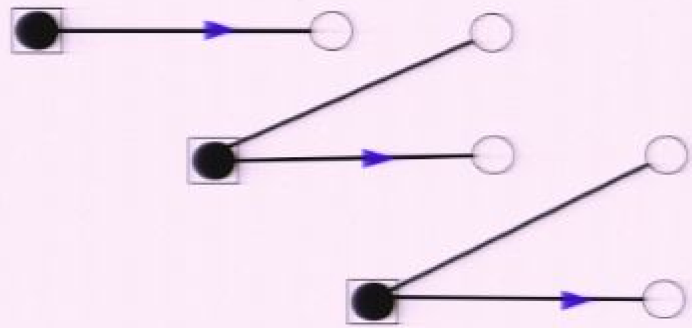

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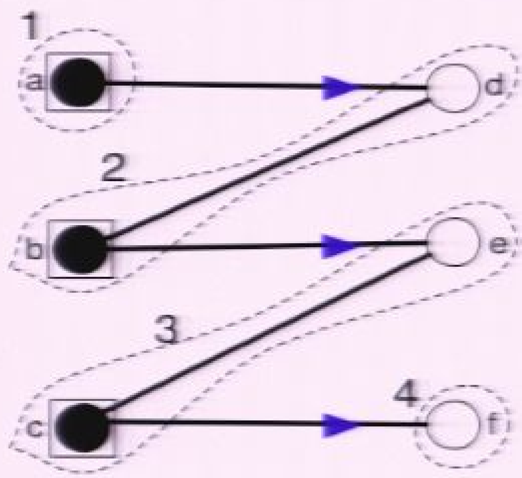
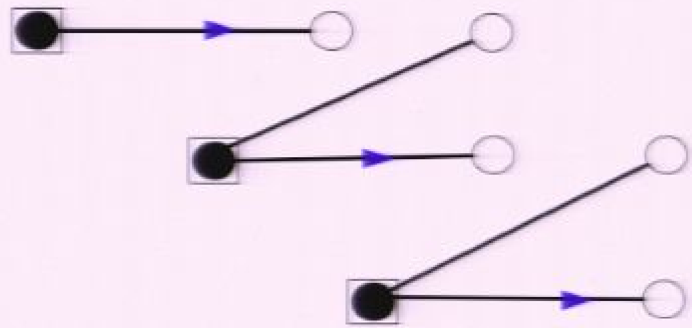
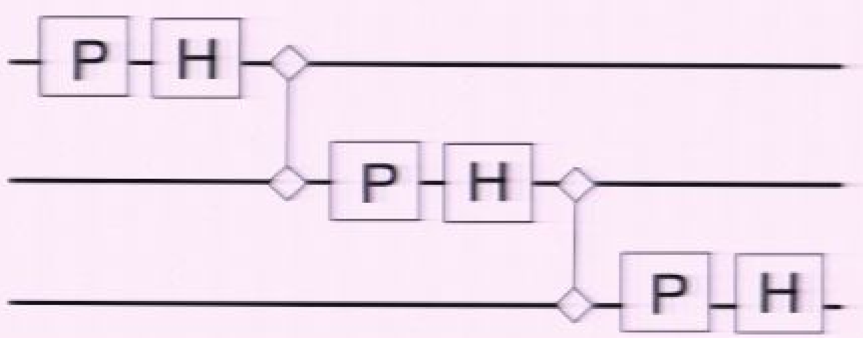
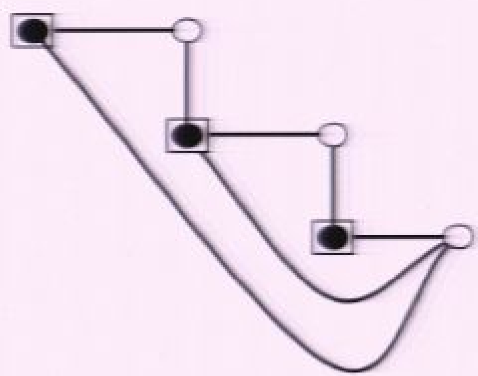






Generalised Flow

Correcting with a **set** of qubits instead of one qubit.



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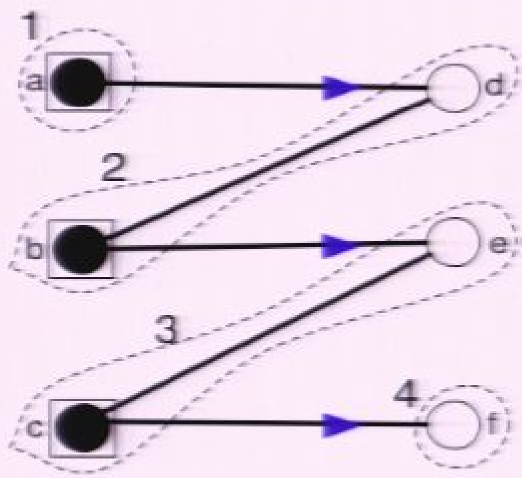

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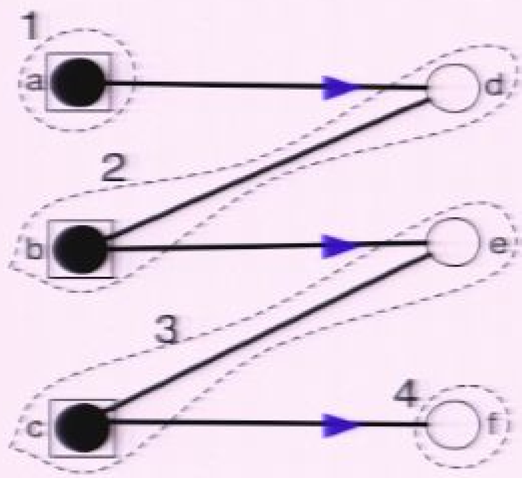

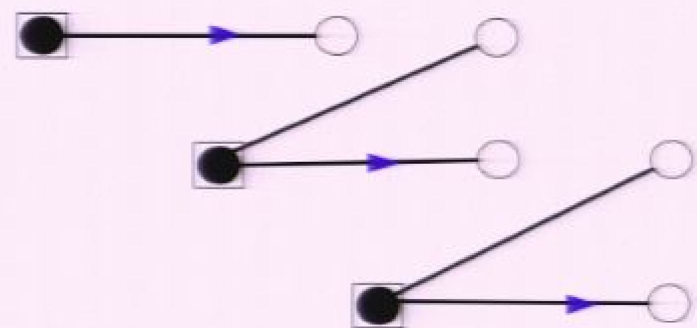

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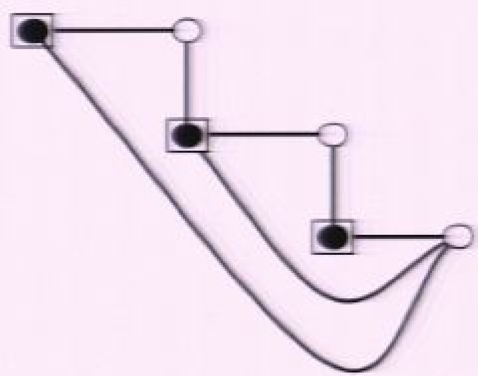

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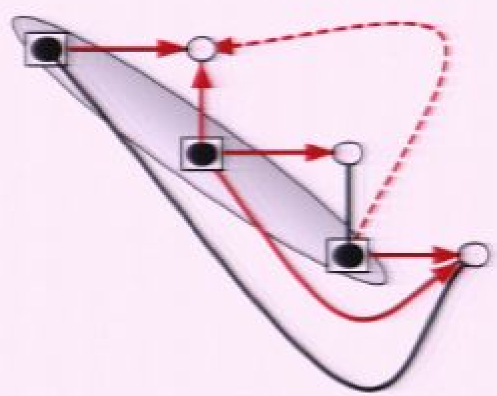

Generalised Flow

Definition. An entanglement graph (G, I, O) has generalised flow if there exists a map $f : O^c \longrightarrow \mathcal{P}^{I^c}$ and a partial order $\leq$ over qubits

— (i) $i \notin g(i)$ and $i \in \mathrm{Odd}(g(i))$,
— (ii) if $j \in g(i)$ and $i \neq j$ then $i < j$,
— (iii) if $j \leq i$ and $i \neq j$ then $j \notin \mathrm{Odd}(g(i))$

$\mathrm{Odd}(K) = \{u \, , \, |N_G(u) \cap K| = 1 \mod 2\}$

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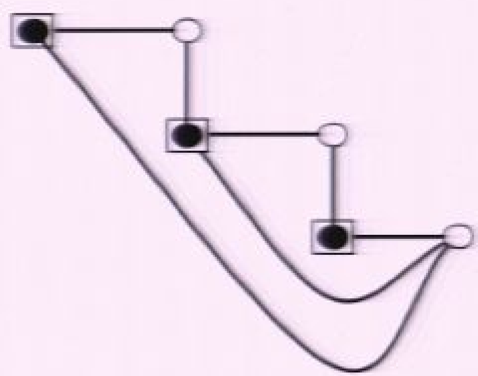

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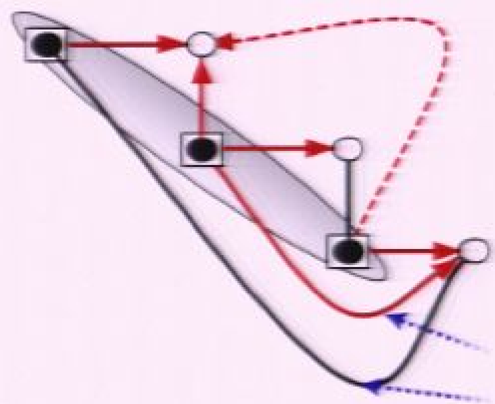

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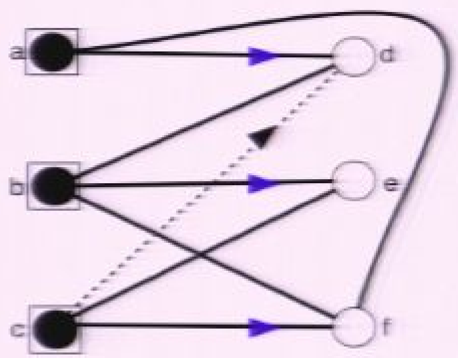
Theorem. A pattern is strongly and step-wise deterministic if and only if its graph has a generalised flow.

Star Decomposition



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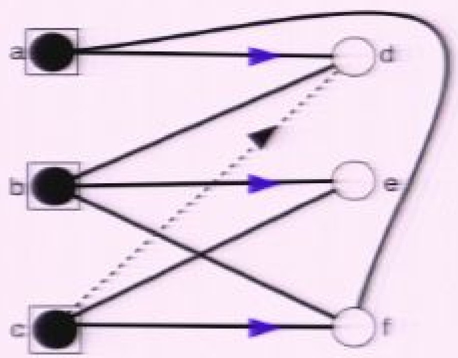

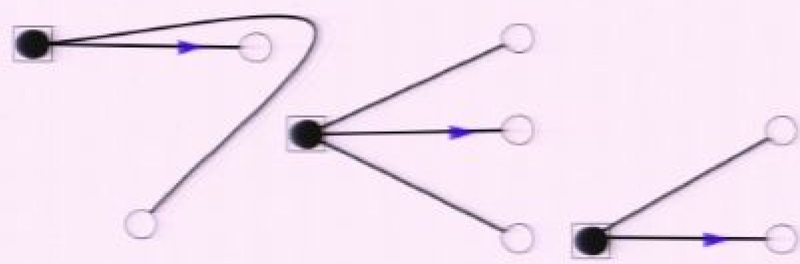

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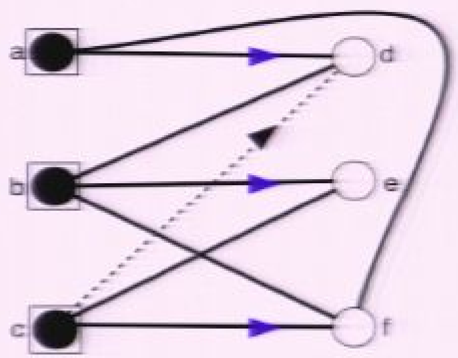

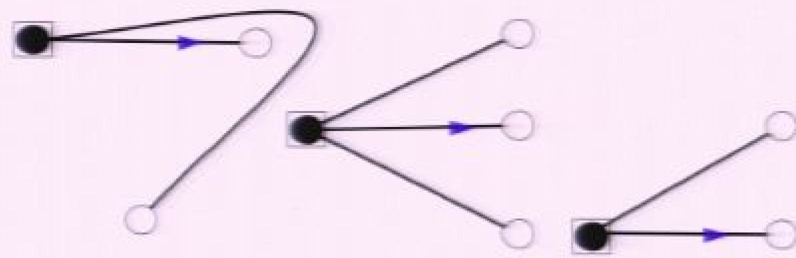

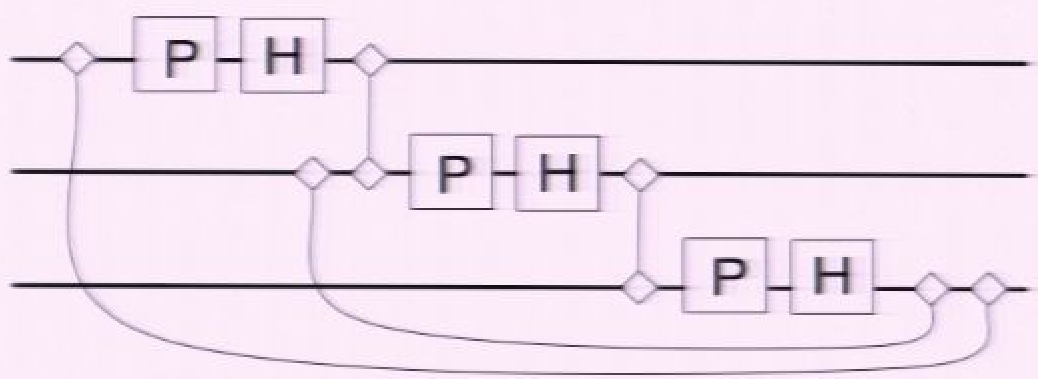

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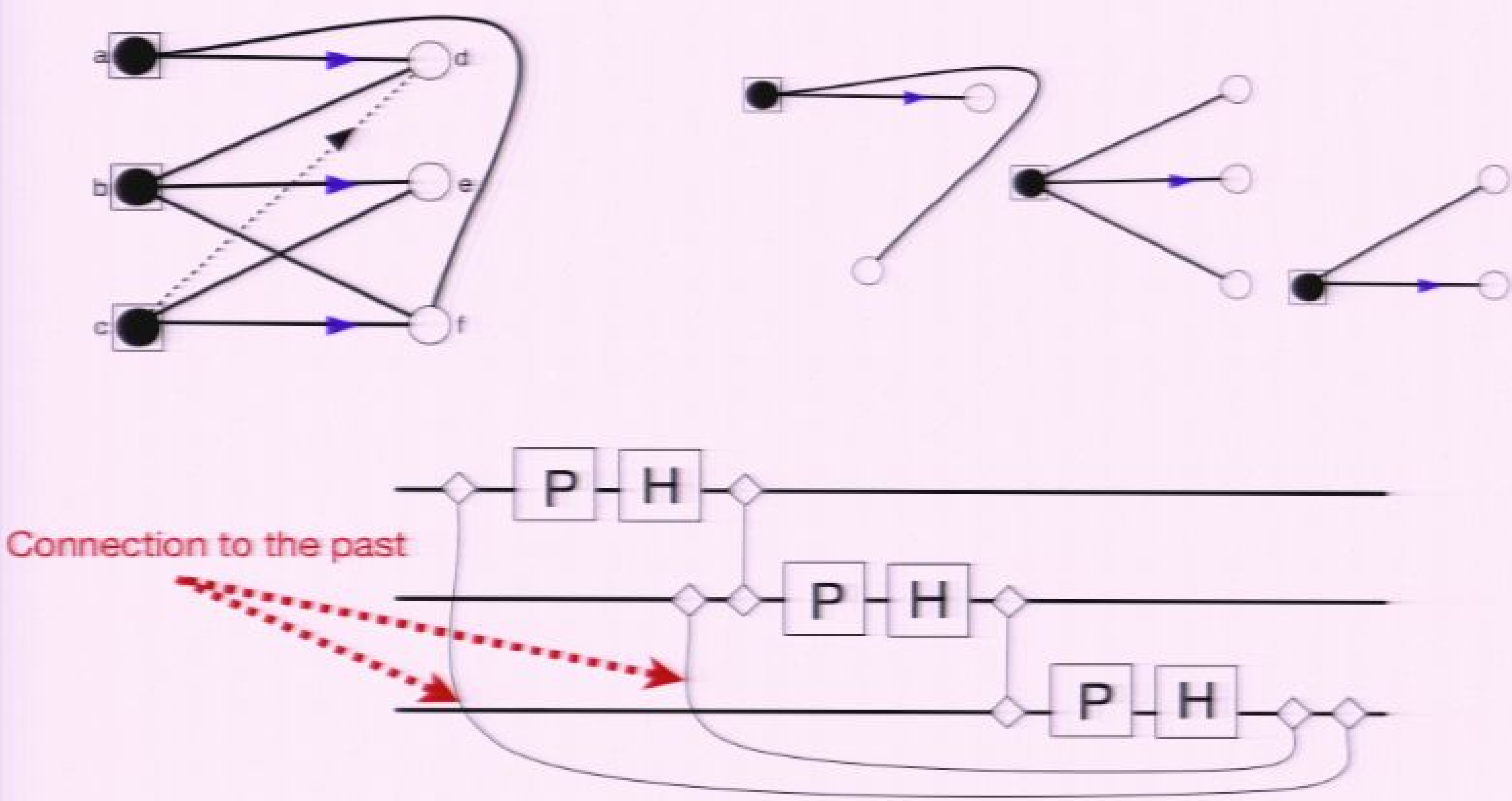

So what

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Patterns with gflow ⟷ Cyclic Quantum Circuit

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Observation. There exists a subclass of cyclic circuits implementing unitary operator !

Characterisation of Good Loops

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Theorem. Any loop can be traversed back to the future via a new path.

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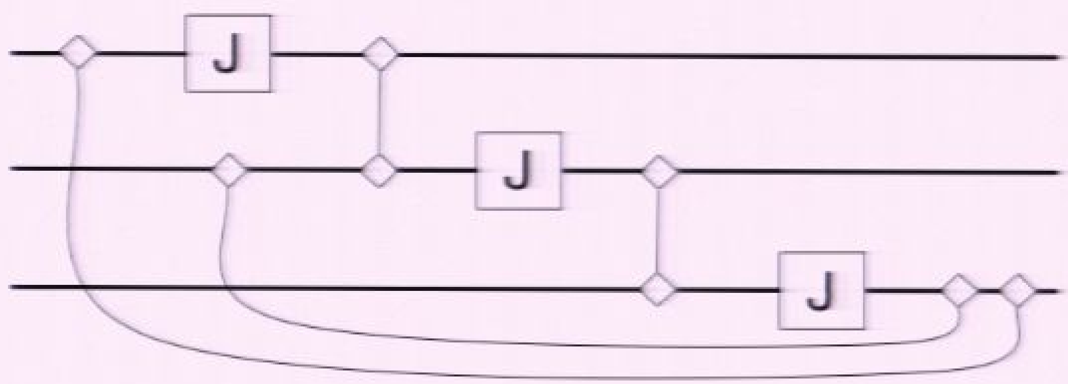

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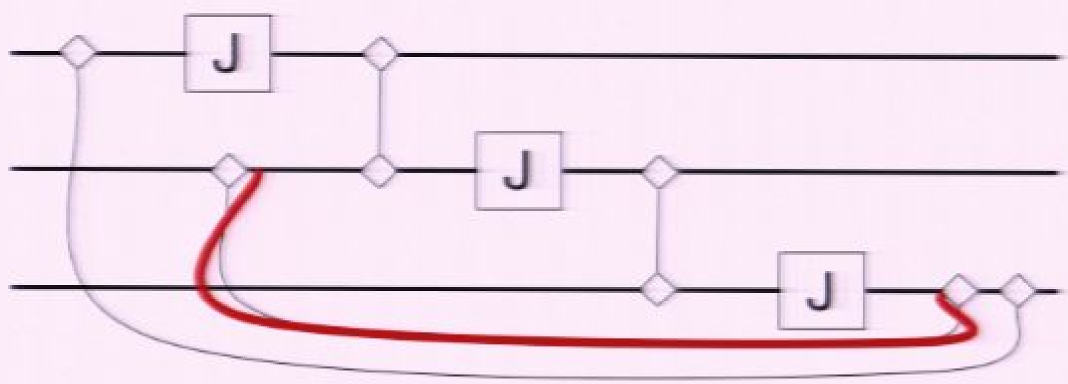

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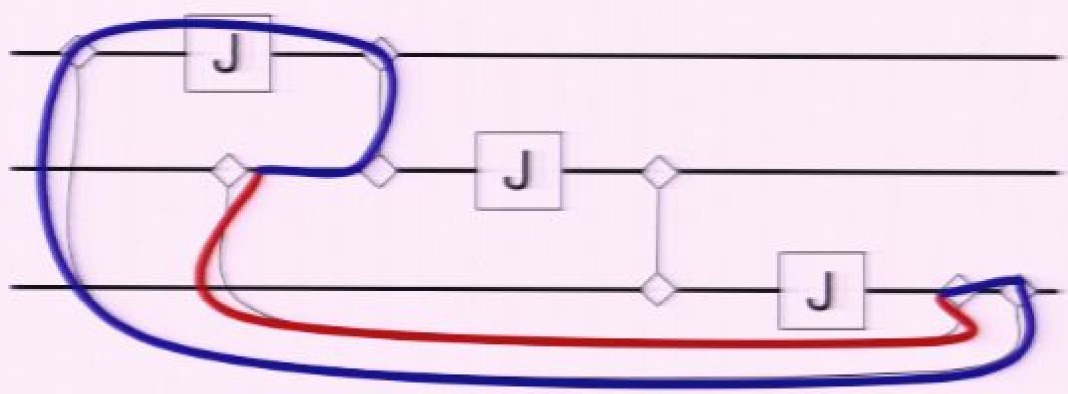
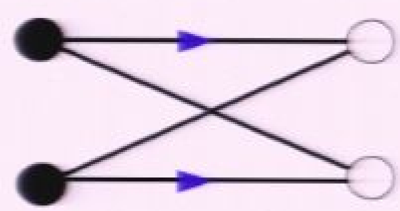


Idea of Proof

Characterising acceptable loop in generalised Flow

Vicious Cycle. A closed path with no two consecutive non-flow edges.



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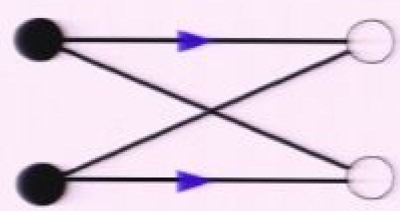
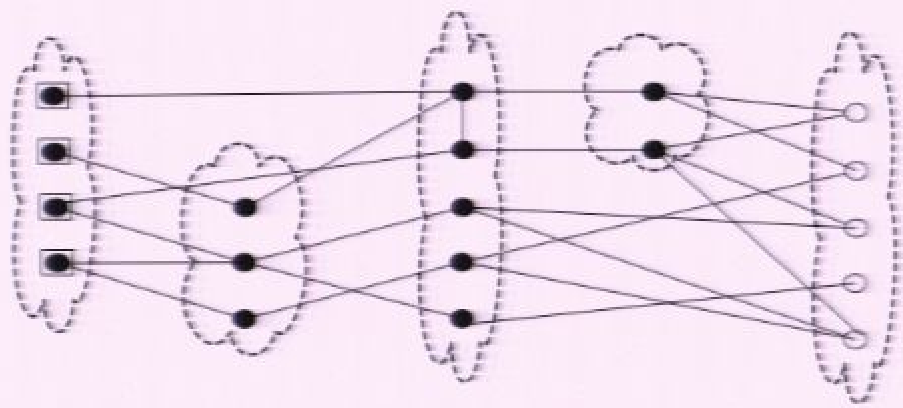


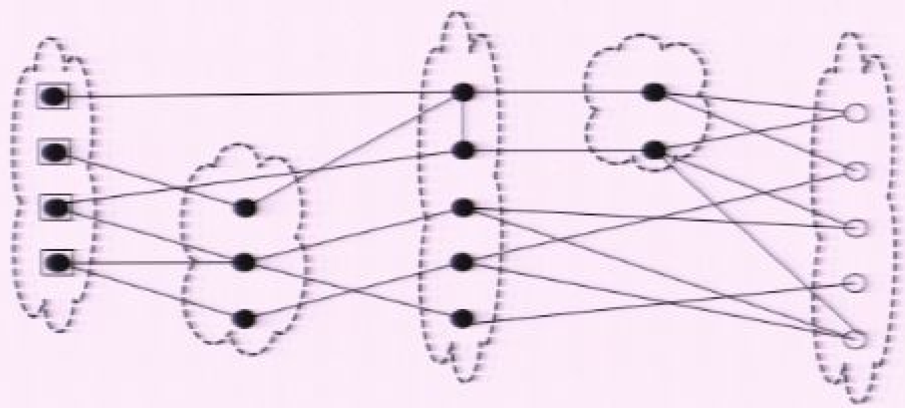
Lemma. Any gflow leads to a flow with possible vicious cycles.

Maximally Delayed gflow



M. Mhala and S. Perdrix, ICALP 2008

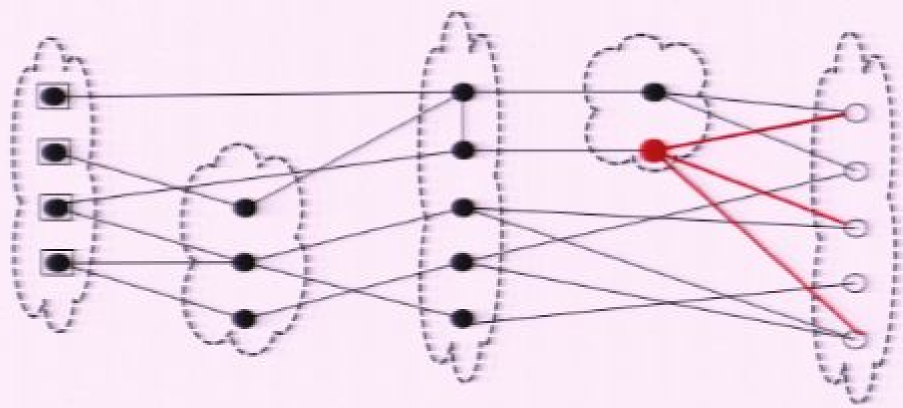
Maximally Delayed gflow



Lemma. Any vertices at level *i* is the unique odd neighbours of a set in level *i*+1.

M. Mhala and S. Perdrix, ICALP 2008

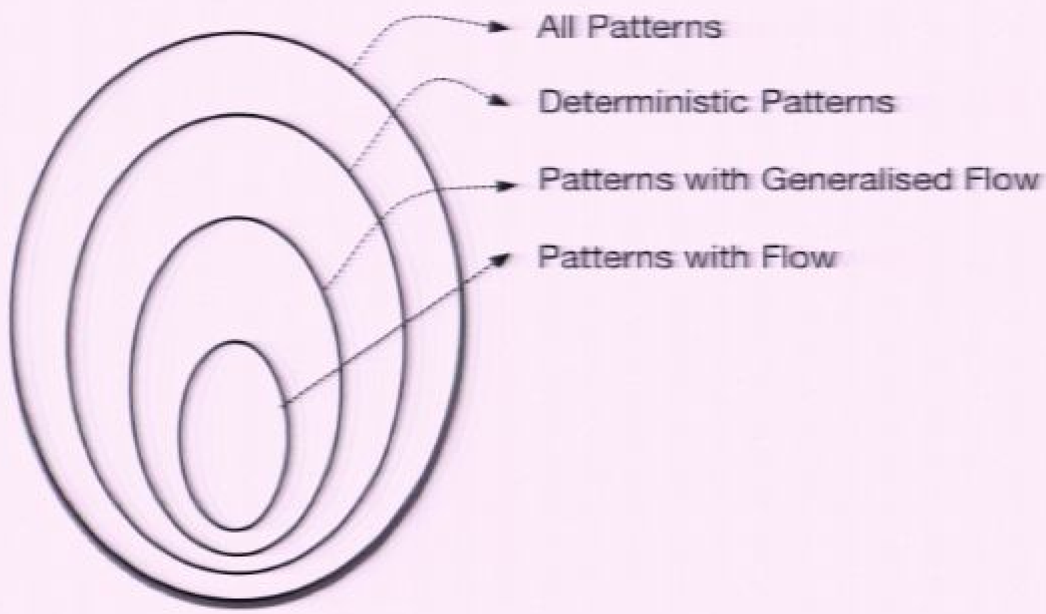
Maximally Delayed gflow



Lemma. There exists a matching between layers.

Lemma. Any connection to path should return back to the future

Summary- *MBQC vs Qcircuit*



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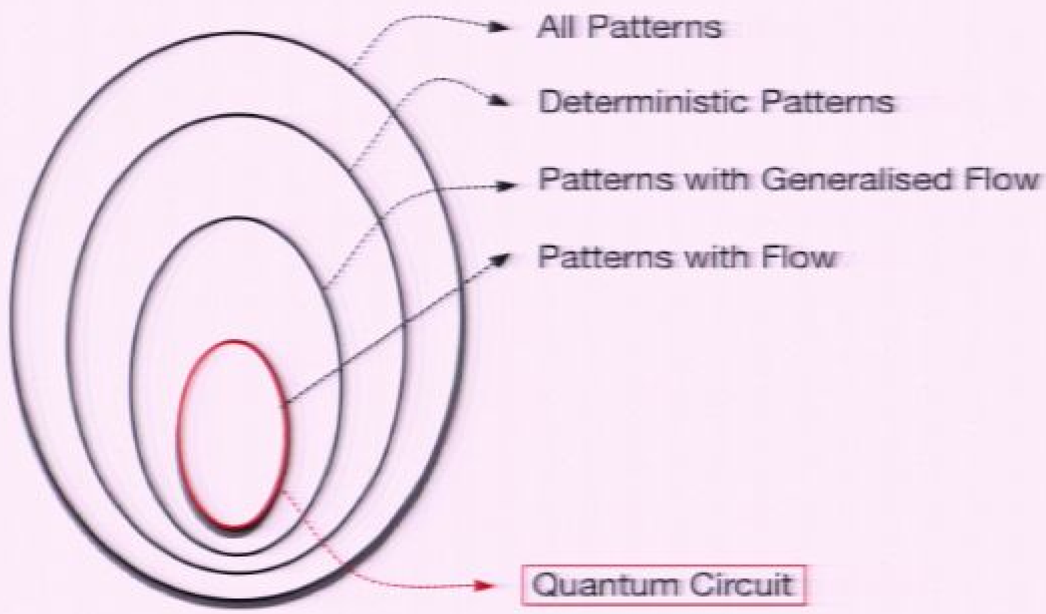

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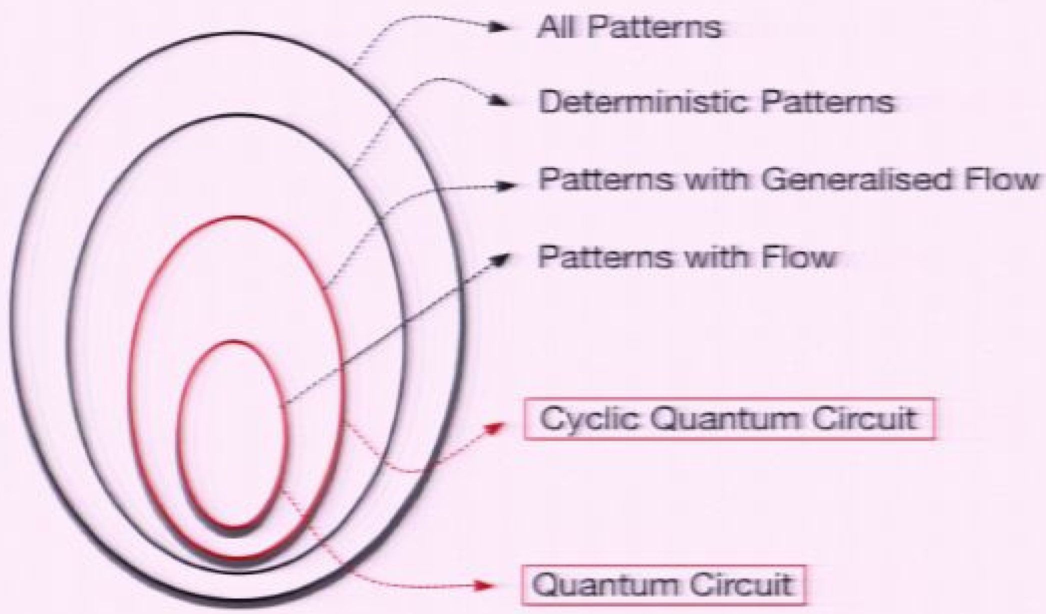

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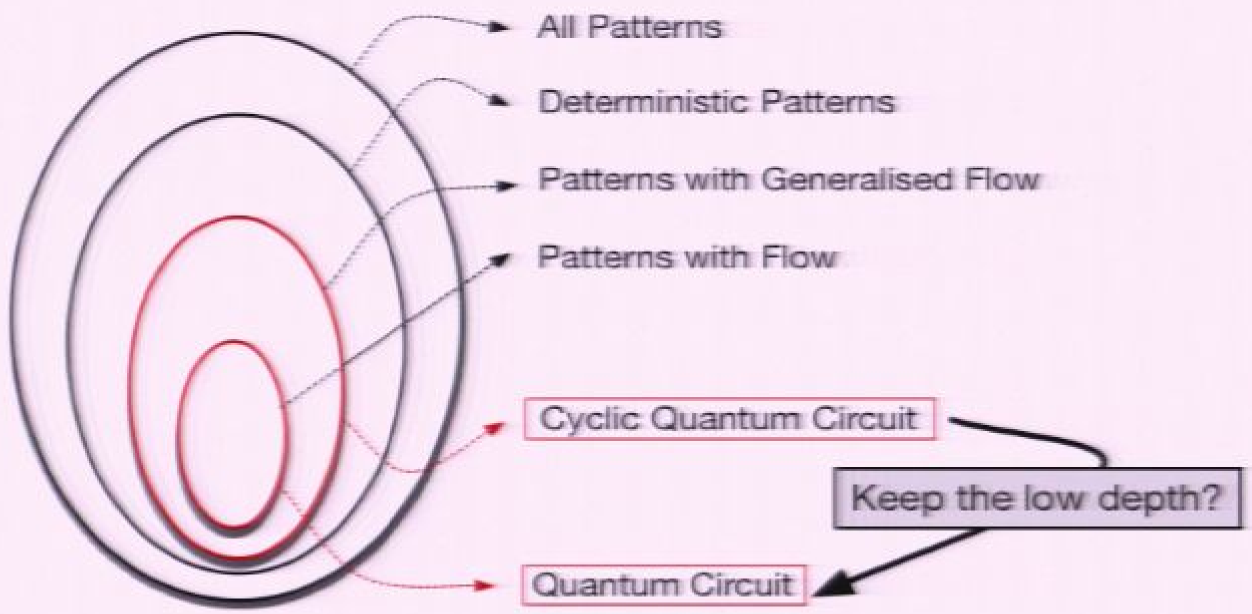

Topological Rewriting Rules

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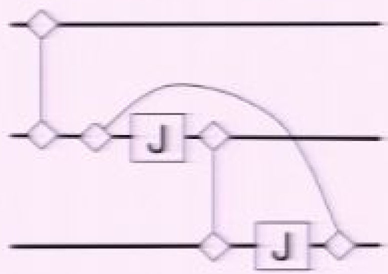

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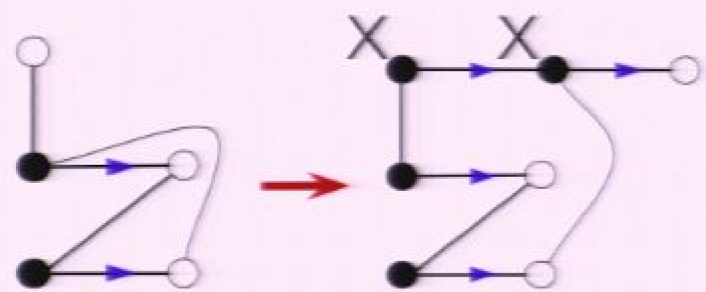
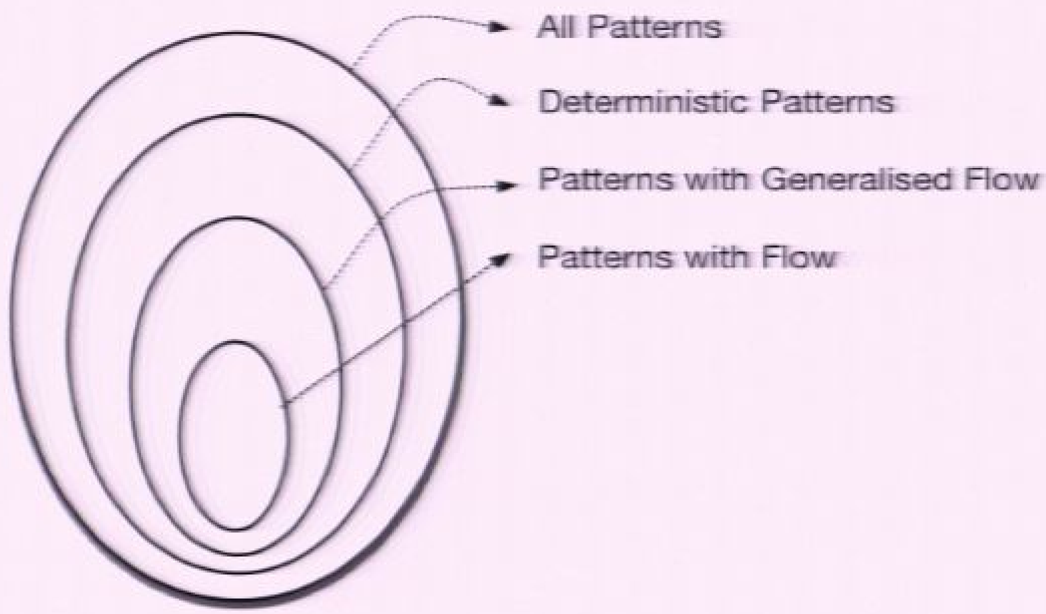


What next

- Full set of rewriting rules for opening loops
- Exact trade off in terms of depth
- Other type of determinism vs quantum circuits
- And also other road map, from PBQC to NMBQC and TQC

Summary- *MBQC vs Qcircuit*



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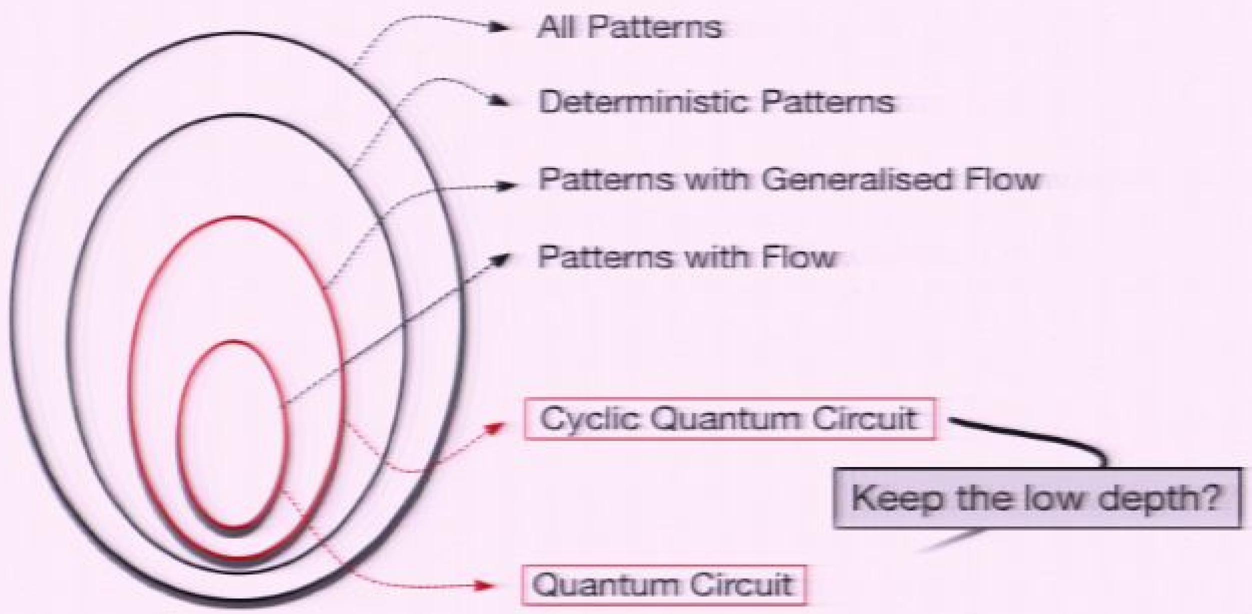

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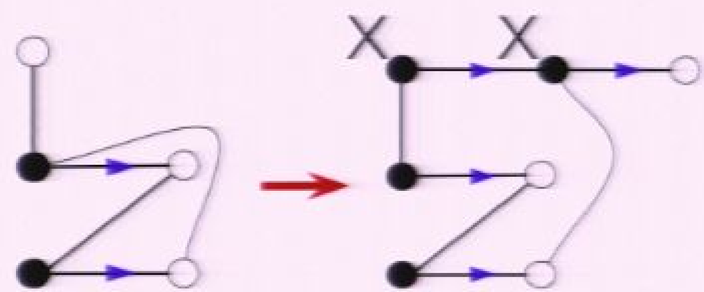

Topological Rewriting Rules

From Pattern to Circuit

Star Pattern: $X_2^{s_1} M_1^{\alpha} E_{12} E_{13} \cdots E_{1n}$

From Projection to Measurement

$$P_i^{|+_\alpha\rangle} = M_i^\alpha Z_i^{s_i}$$

Anachro

MBQC vs QCircuit