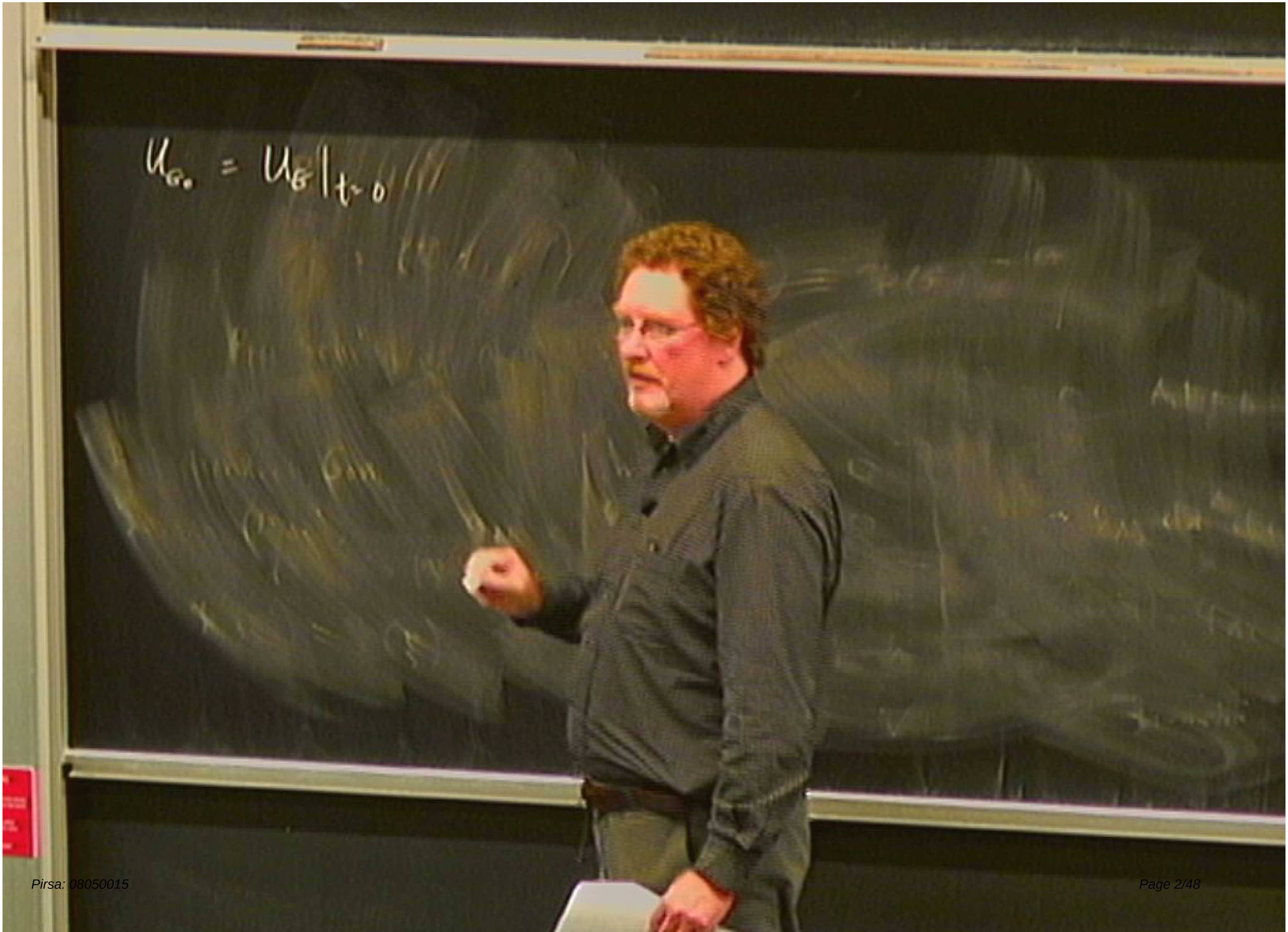
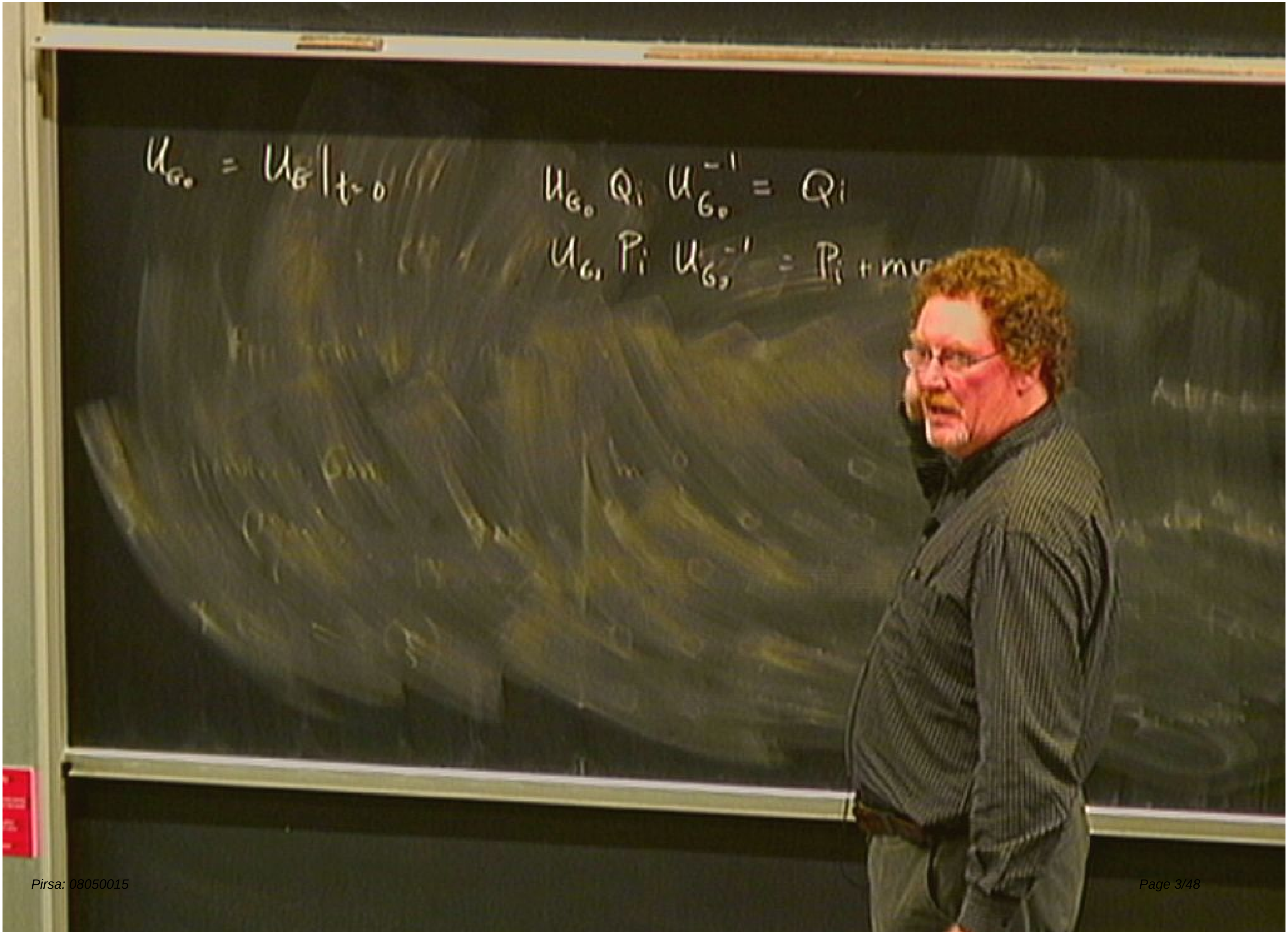
Title: Symmetry Principles in Physics - Lecture 6B

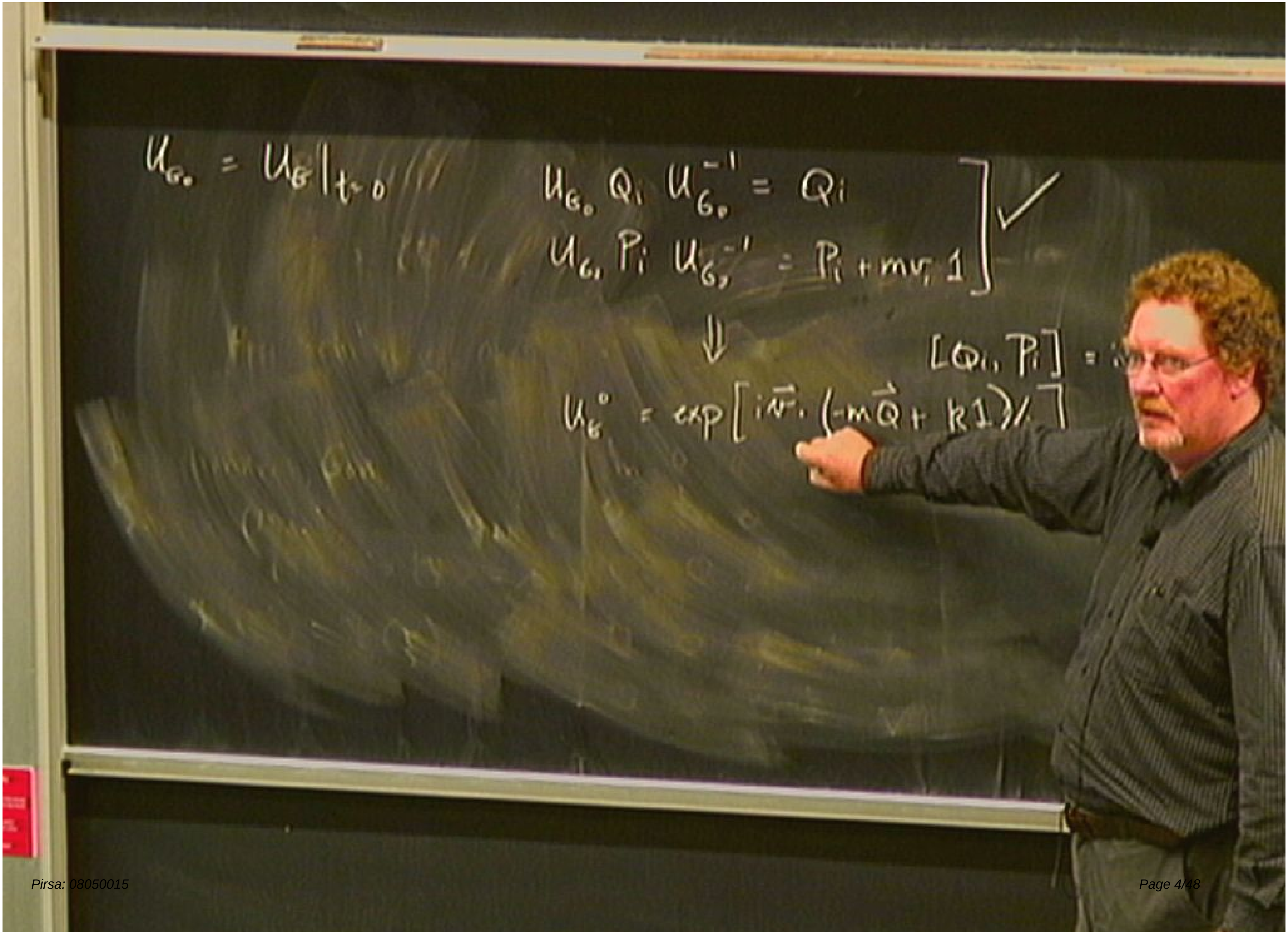
Date: May 26, 2008 12:00 PM

URL: http://pirsa.org/08050015

Abstract:


$U_{G_0} = U_G|_{t=0}$


$U_{G_0} = U_G|_{t=0}$
$U_{G_0} Q_i U_{G_0}^{-1} = Q_i$
$U_{G_0} P_i U_{G_0}^{-1} = P_i + mv$



U_{G_0} = U_G |_{t=0}
U_{G_0} Q_i U_{G_0}^{-1} = Q_i
U_{G_0} P_i U_{G_0}^{-1} = P_i + m v_i 1
⇓
[Q_i, P_i] =
U_G^0 = exp[i v · (-m Q + k 1)/

(i) All canonical operators transform invariantly

$$H = \frac{(P-A)^2}{2m} + V$$

EEL

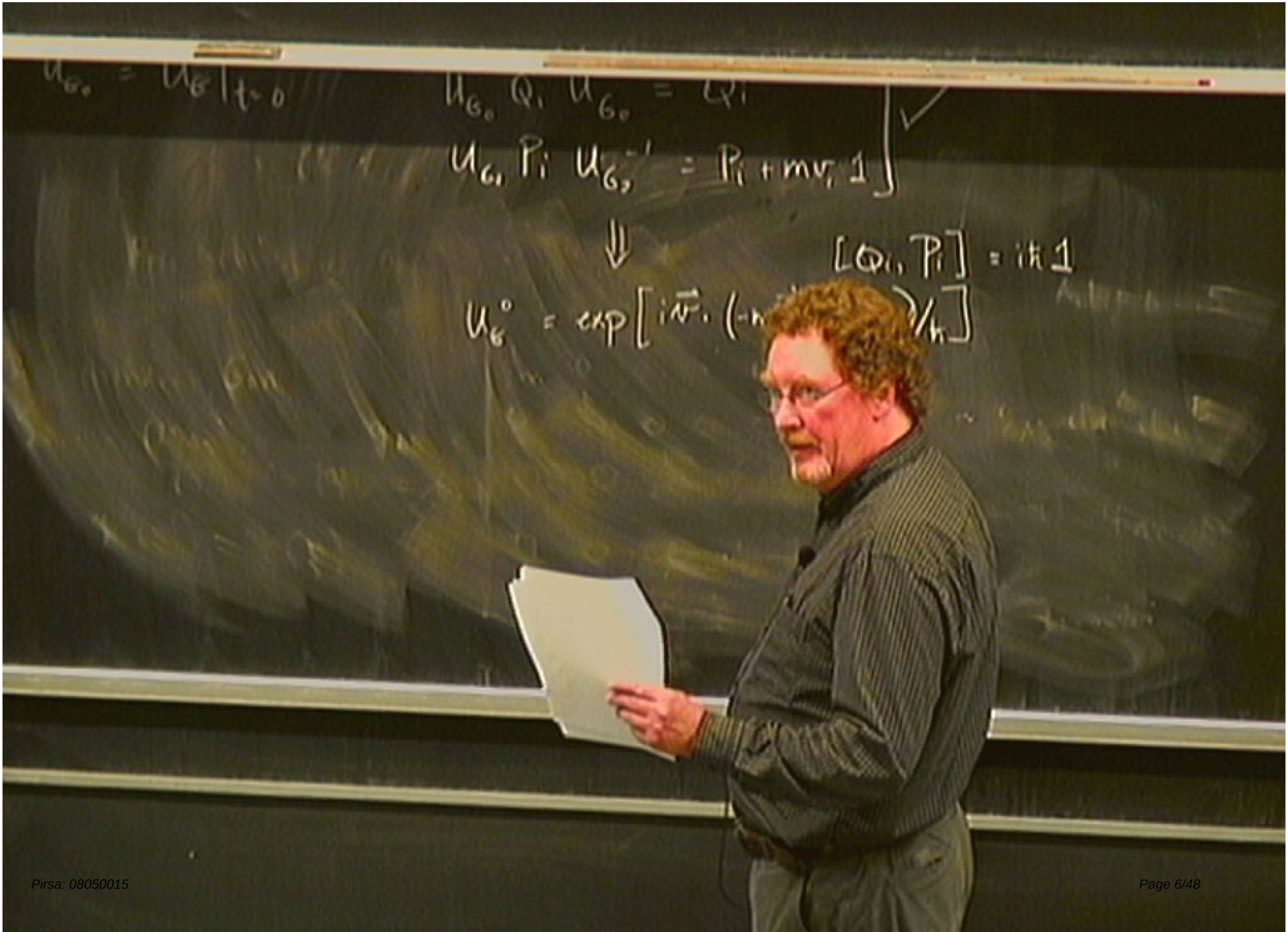
$$Q_i' = Q_i \,;\quad P_i' = P_i \,\ldots\ldots$$

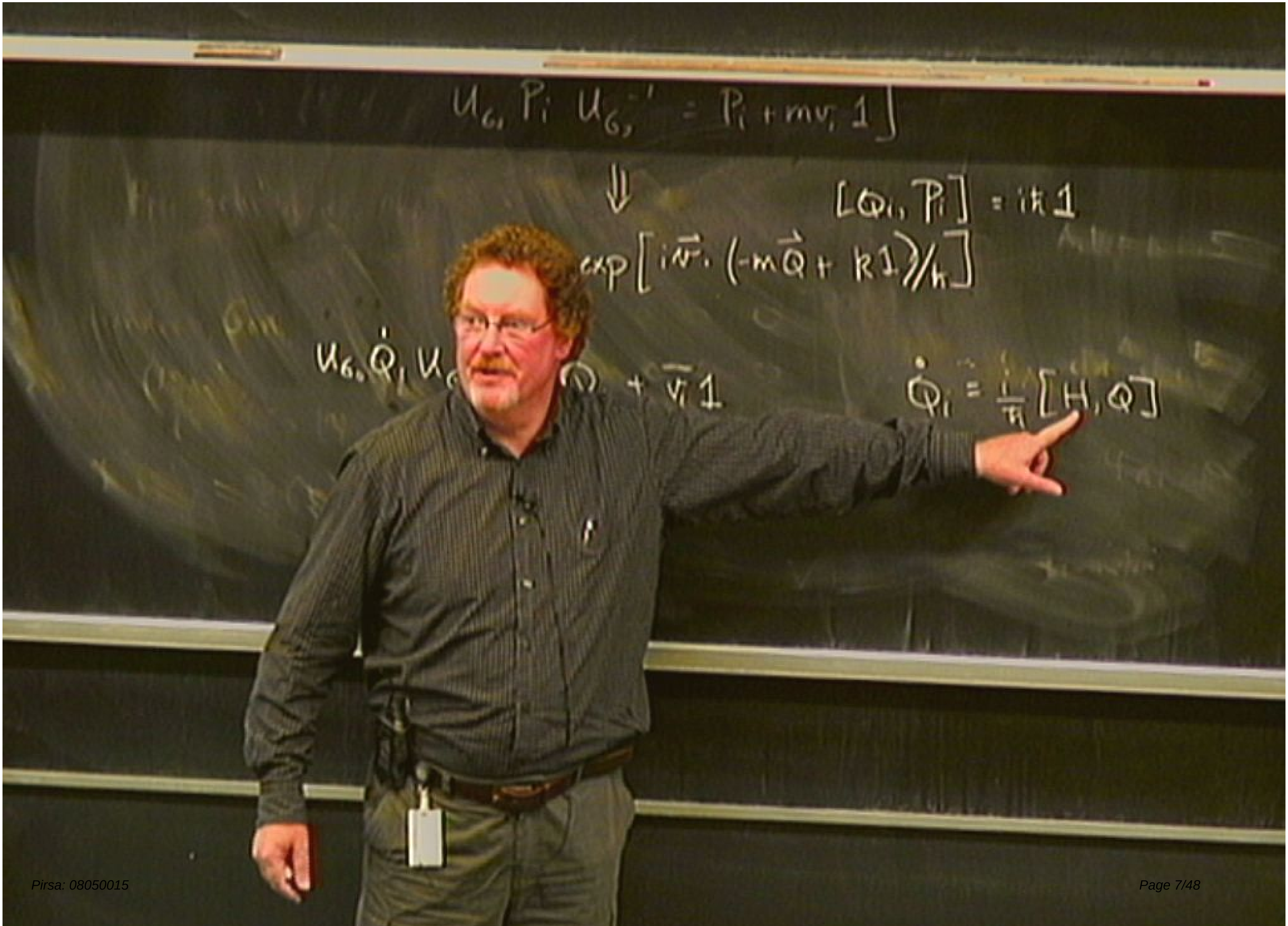
(ii) $$H' = U_G H U_G^{-1} + i\hbar \frac{\partial U_G}{\partial t} U_G^{-1} = \frac{(P - U_G A U_G^{-1})^2}{2m} + U_G(V - \vec{v}\cdot\vec{A})U_G$$

(iii) $$\dot{Q}' \equiv \frac{i}{\hbar}[H', Q']$$ $$\left[\dot{Q}_i' = \frac{1}{m}(\vec{P} - U_G \vec{A} U_G^{-1}) \neq \dot{Q}_i\right]$$

(iv) $$\dot{P}_i' = U_G \dot{P}_i U_G^{-1} \qquad \ddot{Q}_i' = U_G \ddot{Q}_i U_G^{-1}$$

$$L_z = Q_x P_y - Q_y P_x$$


U_{G_0} = U_G|_{t=0}
U_{G_0} Q_i U_{G_0}^{-1} = Q_i
U_{G_0} P_i U_{G_0}^{-1} = P_i + m v_i 1
[Q_i, P_i] = iħ 1

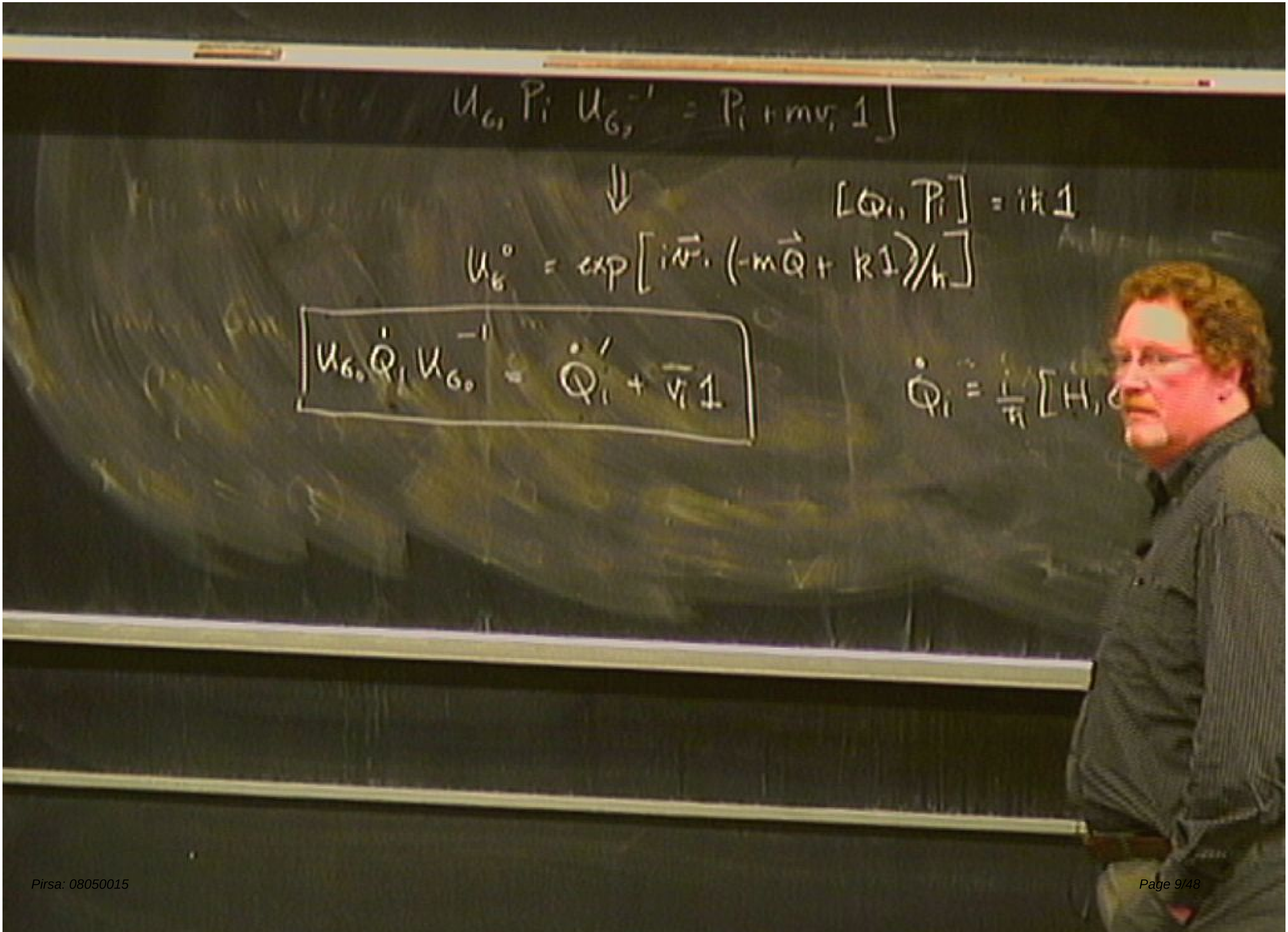

$U_{G_1} P_i U_{G_1}^{-1} = P_i + m v_i 1$
$[Q_i, P_i] = i\hbar 1$
$\exp[i \vec{v} \cdot (-m\vec{Q} + k1)/\hbar]$
$\dot{Q}_i = \frac{1}{\hbar}[H, Q]$

$$U_{G_1} P_i U_{G_1}^{-1} = P_i + m v_i 1$$

$$\Downarrow \qquad [Q_i, P_i] = i\hbar 1$$

$$U_G^0 = \exp\left[i\vec{v}\cdot\left(-m\vec{Q} + k1\right)/\hbar\right]$$

$$U_{G_0} \dot{Q}_i U_{G_0}^{-1} = \dot{Q}_i + \bar{v}_i 1 \qquad \dot{Q}_i = \frac{i}{\hbar}[H, Q]$$


$U_{G_1} P_i U_{G_1}^{-1} = P_i + mv_i 1$
$\Downarrow$
$[Q_i, P_i] = i\hbar 1$
$U_G^0 = \exp[i\vec{v} \cdot (-m\vec{Q} + k1)/\hbar]$
$U_{G_0} \dot{Q}_i U_{G_0}^{-1} = \dot{Q}_i' + v_i 1$
$\dot{Q}_i = \frac{i}{\hbar}[H,$

$$U_{G_1} P_i U_{G_1}^{-1} = P_i + m v_i 1$$

$$\Downarrow$$

$$[Q_i, P_j] = i\hbar 1$$

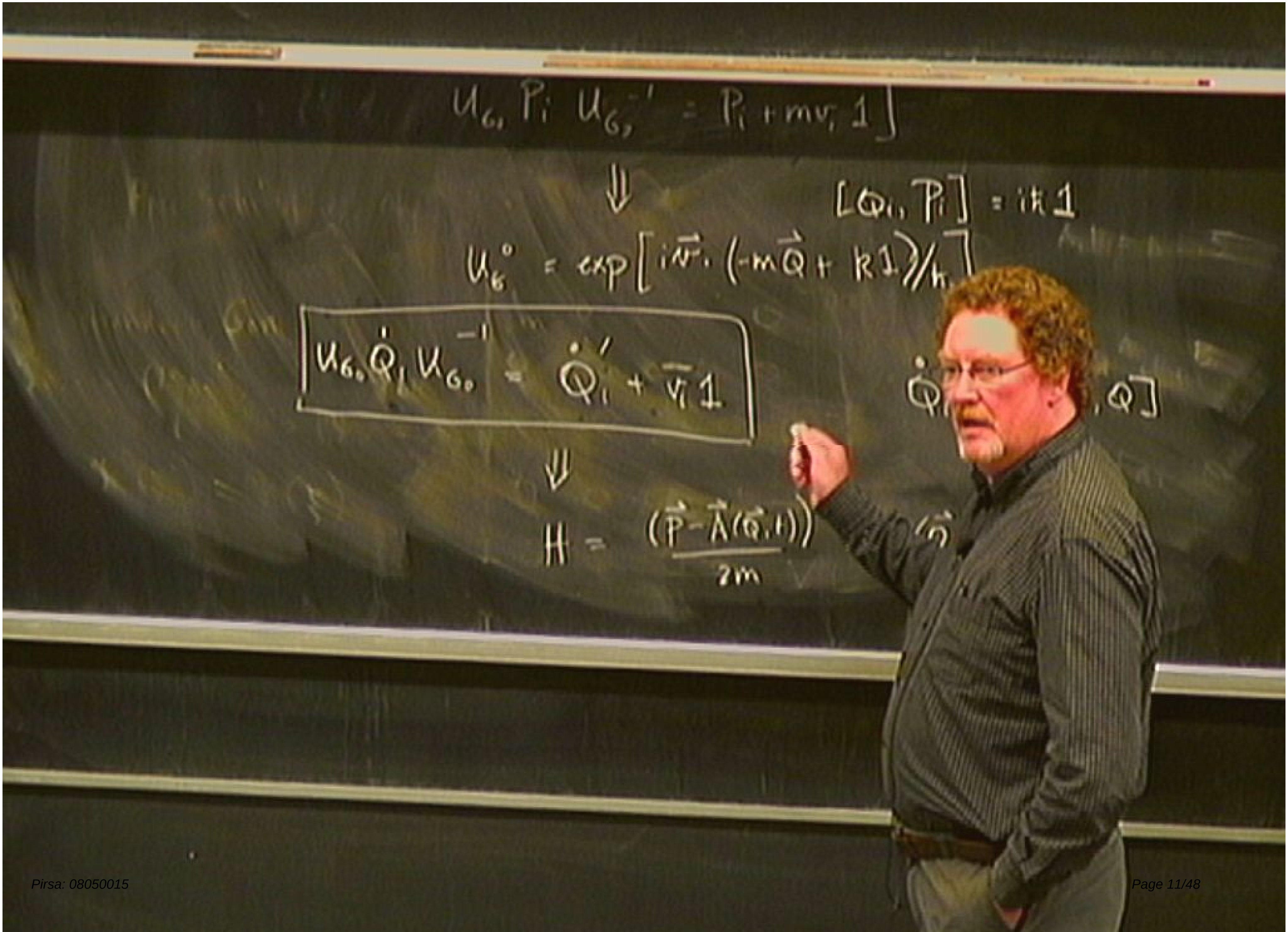
$$U_G^0 = \exp\left[i\vec{v}\cdot(-m\vec{Q} + k1)/\hbar\right]$$

$$\boxed{U_{G_0} \dot{Q}_i U_{G_0}^{-1} = \dot{Q}_i' + \bar{v}_i 1}$$

$$\dot{Q}_i = \frac{i}{\hbar}[H, Q]$$

$$\Downarrow$$

$$H = \frac{(\vec{P} - \vec{A}(\vec{Q},t))^2}{2m} + V(\vec{Q},t)$$


$U_{G_1} P_i U_{G_1}^{-1} = P_i + m v_i 1$
$[Q_i, P_i] = i\hbar 1$
$U_{G_0} \dot{Q}_i U_{G_0}^{-1} = \dot{Q}_i' + \bar{v}_i 1$
$H = \frac{(\vec{P} - \vec{A}(\vec{Q}, t))^2}{2m}$

$$U_{G_1} P_i U_{G_1}^{-1} = P_i + m v_i 1$$

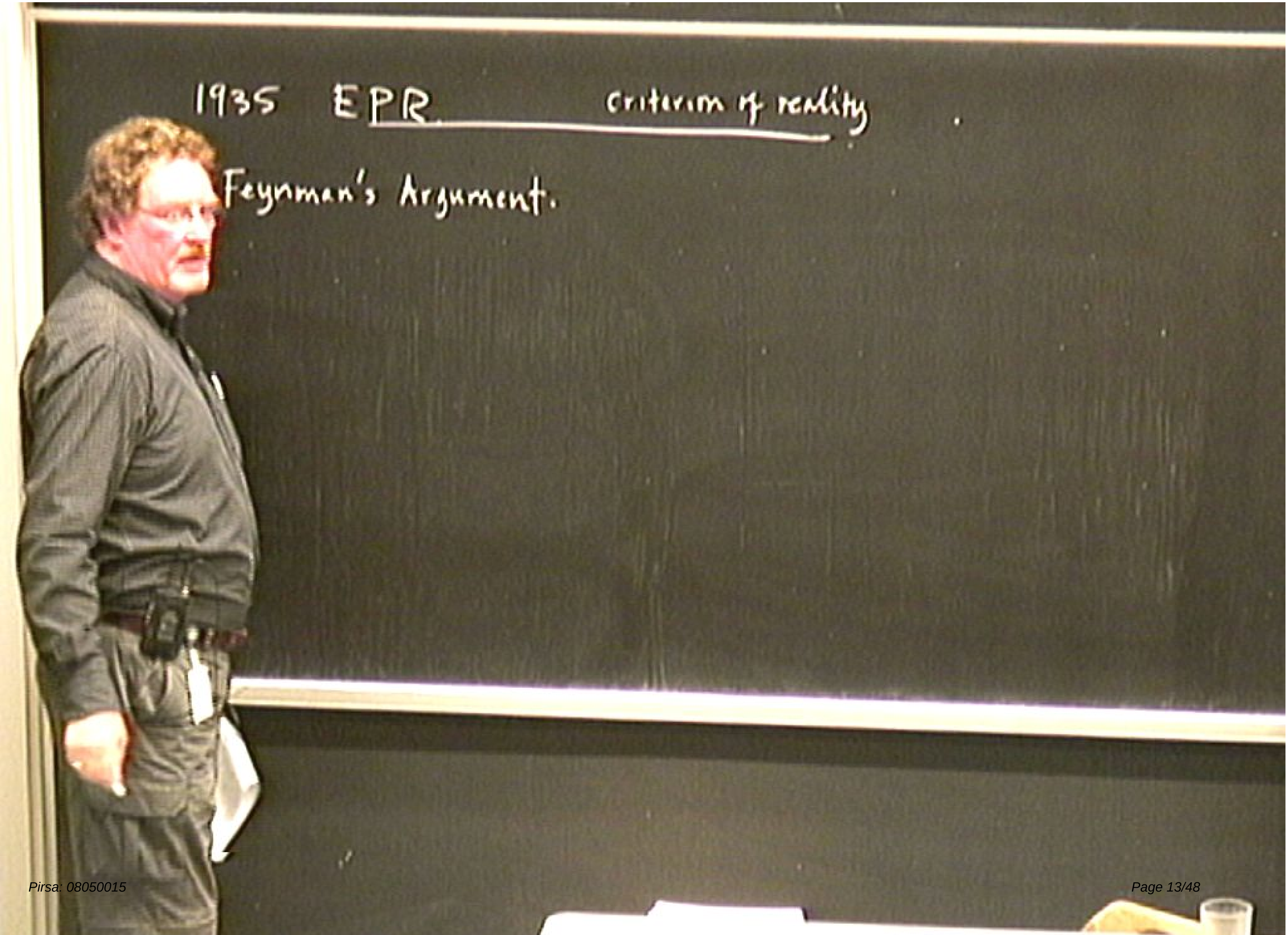
$$\Downarrow \qquad [Q_i, P_i] = i\hbar 1$$

$$U_G^0 = \exp\left[i\vec{v}\cdot(-m\vec{Q} + \vec{k}1)/\hbar\right]$$

$$\boxed{U_{G_0} \dot{Q}_i U_{G_0}^{-1} = \dot{Q}_i' + \bar{v}_i 1} \qquad \dot{Q}_i = \frac{i}{\hbar}[H, Q]$$

$$\Downarrow$$

$$H = \frac{(\vec{P} - \vec{A}(\vec{Q},t))^2}{2m} + V(\vec{Q},t)$$


1935 EPR
criterion of reality
Feynman's Argument.

1935 EPR criterium of reality

Feynman's Argument. Dyson AJP (1990).

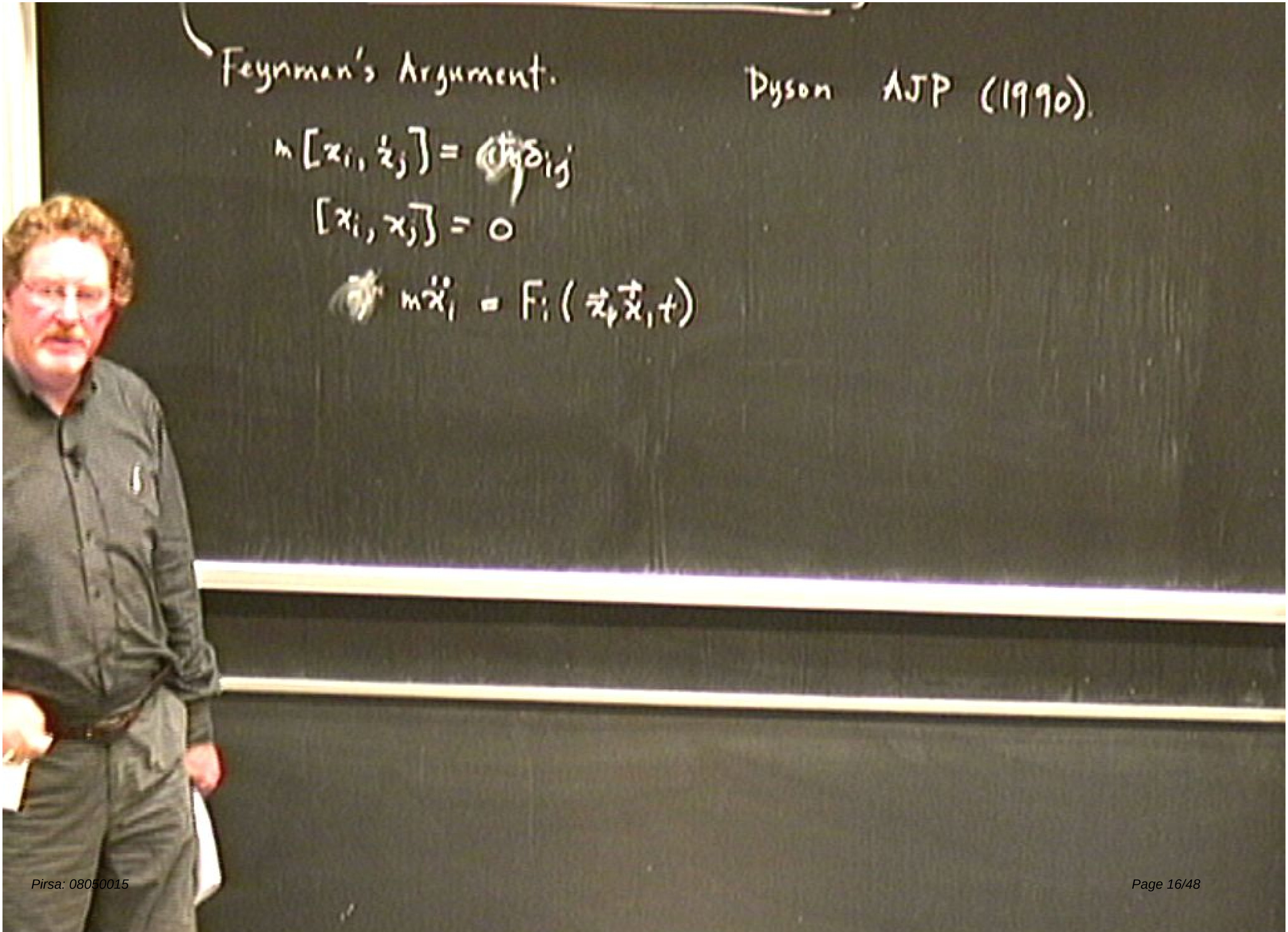
1935 EPR criterium of reality

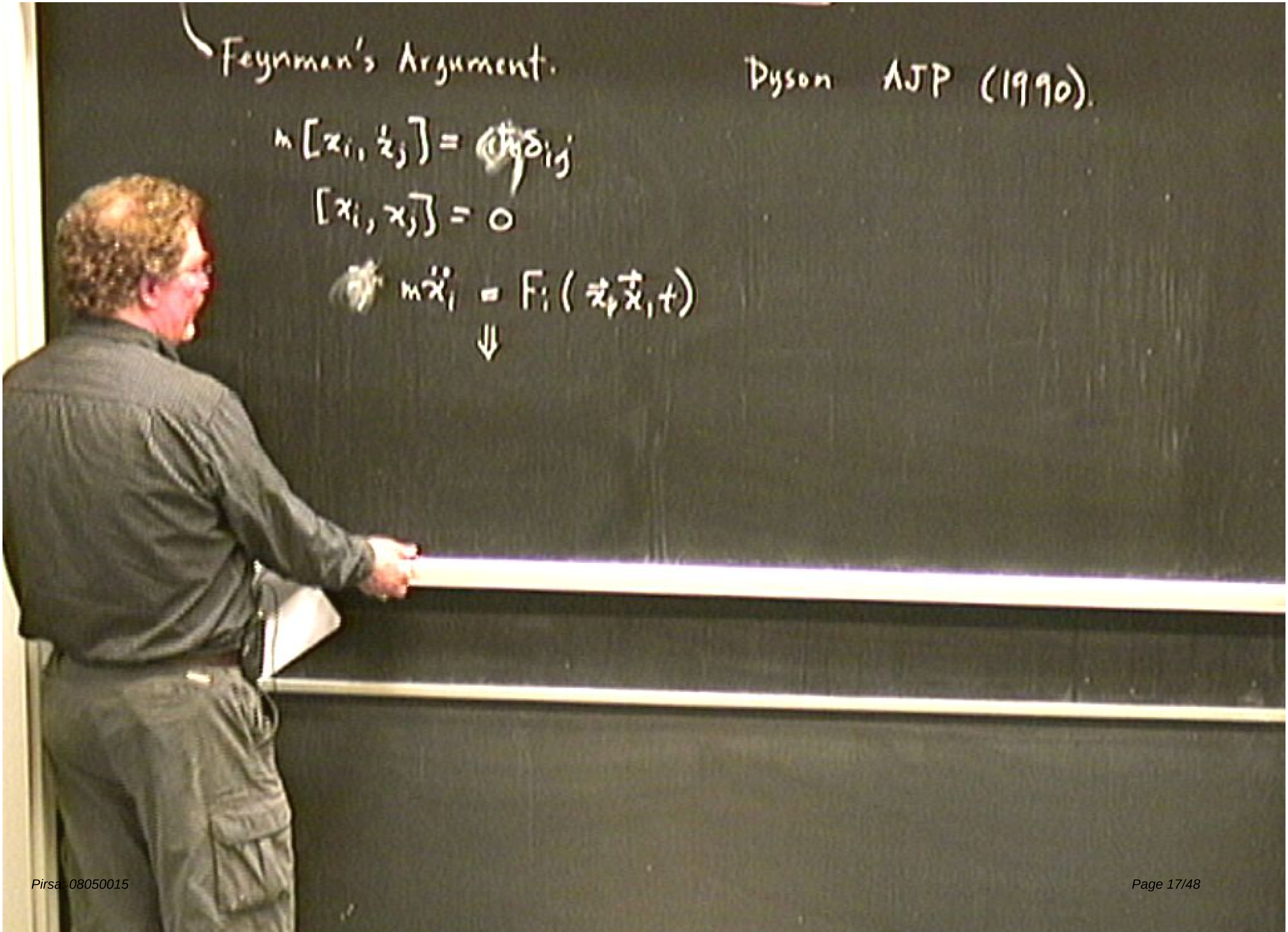
Feynman's Argument.

Dyson AJP (1990).

$$m[x_i, \dot{x}_j] = i\hbar\,\delta_{ij}$$

$$[x_i, x_j] = 0$$


Feynman's Argument.
Dyson AJP (1990).
$m[x_i, \dot{x}_j] = i\hbar\delta_{ij}$
$[x_i, x_j] = 0$
$m\ddot{x}_i = F_i(\vec{x}, \dot{\vec{x}}, t)$


Feynman's Argument.
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$m[x_i, \dot{x}_j] = i\hbar\delta_{ij}$
$[x_i, x_j] = 0$
$m\ddot{x}_i = F_i(\vec{x}, \dot{\vec{x}}, t)$
$\Downarrow$

Galilean covariant E.M.

$$m[x_i, \dot{x}_j] = i\hbar\delta_{ij}$$

$$[x_i, x_j] = 0$$

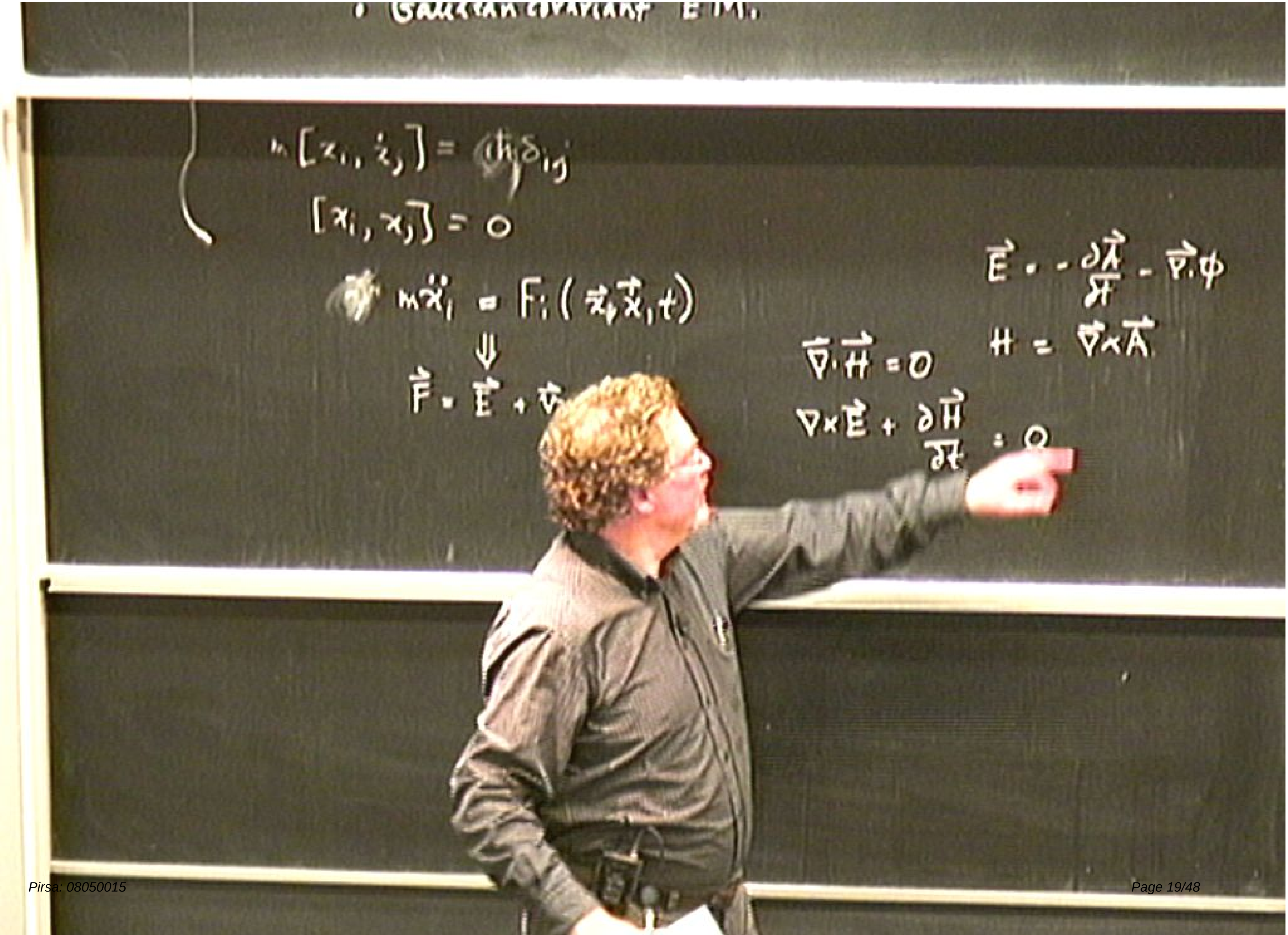
$$m\ddot{x}_i = F_i(\vec{x}, \dot{\vec{x}}, t)$$

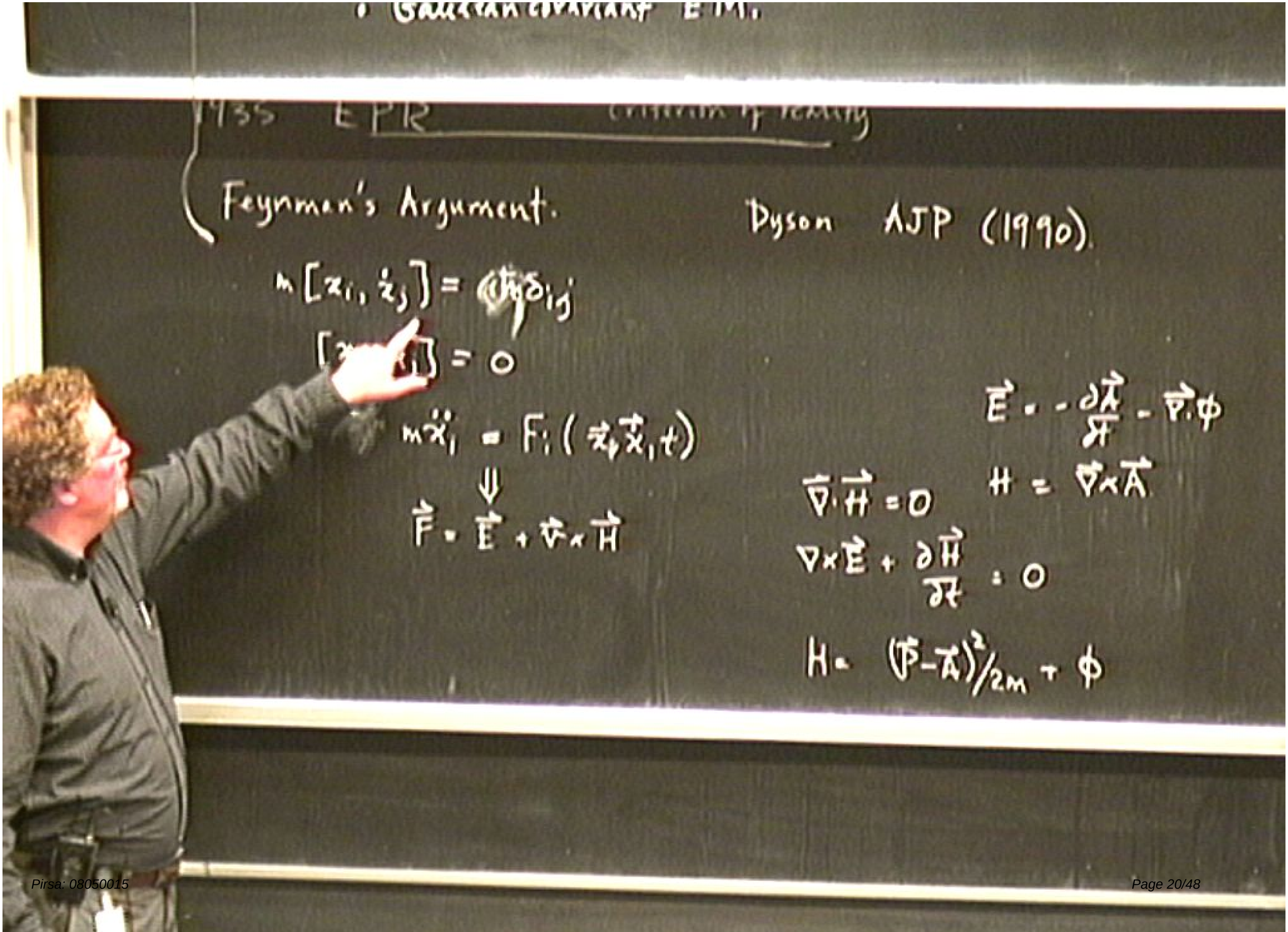
$$\Downarrow$$

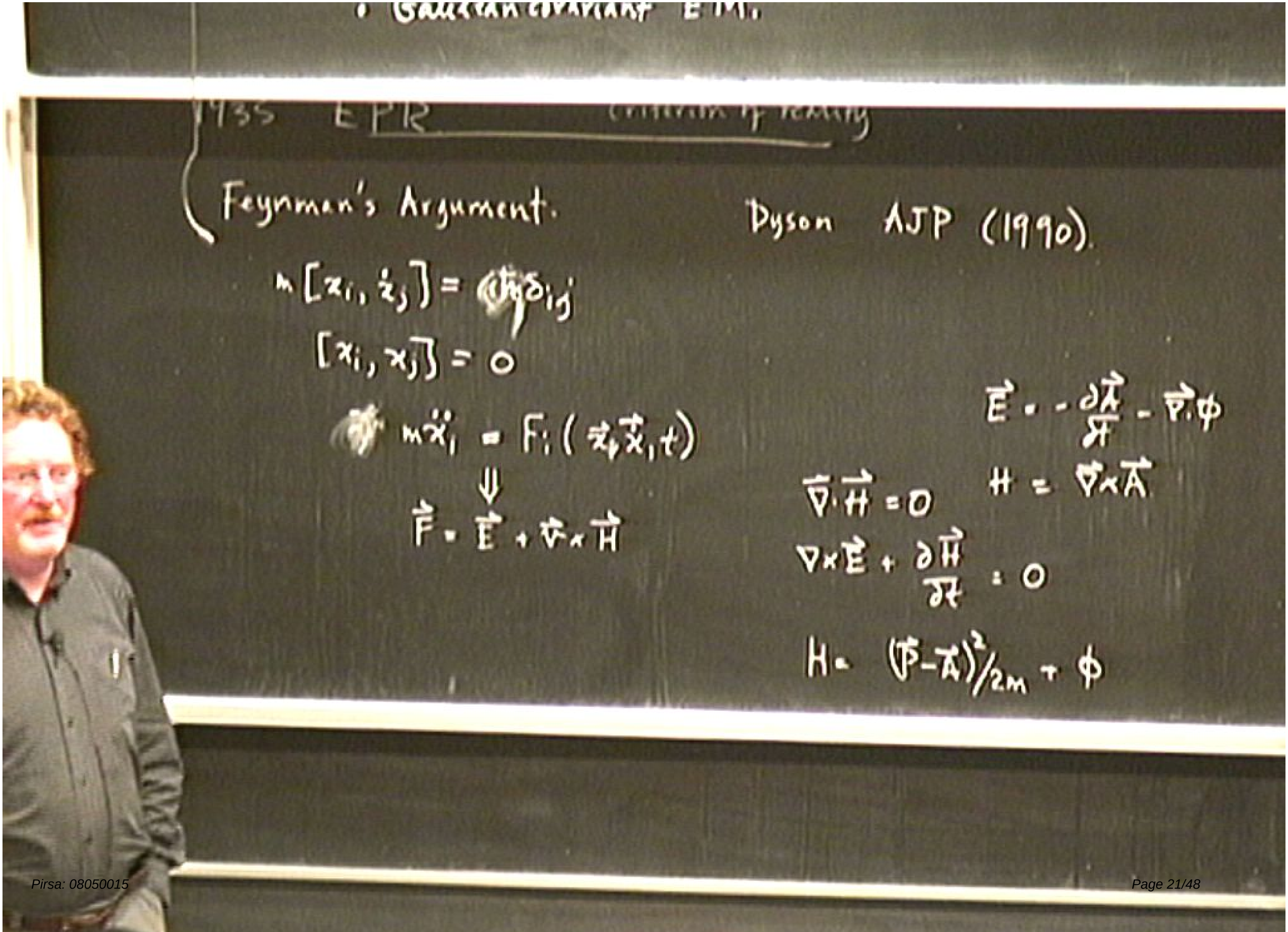
$$\vec{F} = \vec{E} + \vec{v} \times \vec{H}$$

$$\vec{\nabla} \cdot \vec{H} = 0$$

$$\nabla \times \vec{E} + \frac{\partial \vec{H}}{\partial t} = 0$$


$m[x_i, \dot{x}_j] = i\hbar\delta_{ij}$
$[x_i, x_j] = 0$
$m\ddot{x}_i = F_i(\vec{x}, \dot{\vec{x}}, t)$
$\Downarrow$
$\vec{F} = \vec{E} +$
$\vec{E} = -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\phi$
$H = \vec{\nabla}\times\vec{A}$
$\vec{\nabla}\cdot\vec{H} = 0$
$\nabla\times\vec{E} + \frac{\partial\vec{H}}{\partial t} = 0$


EPR
Feynman's Argument.
Dyson AJP (1990).
$m[x_i, \dot{x}_j] = i\hbar\delta_{ij}$
$= 0$
$m\ddot{x}_i = F_i(\vec{x}, \dot{\vec{x}}, t)$
$\Downarrow$
$\vec{F} = \vec{E} + \vec{v} \times \vec{H}$
$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi$
$H = \vec{\nabla} \times \vec{A}$
$\vec{\nabla} \cdot \vec{H} = 0$
$\nabla \times \vec{E} + \frac{\partial \vec{H}}{\partial t} = 0$
$H = (\vec{p} - \vec{A})^2/2m + \phi$


1935 EPR
Feynman's Argument.
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$m[x_i, \dot{x}_j] = i\hbar\delta_{ij}$
$[x_i, x_j] = 0$
$m\ddot{x}_i = F_i(\vec{x}, \dot{\vec{x}}, t)$
$\Downarrow$
$\vec{F} = \vec{E} + \vec{v} \times \vec{H}$
$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi$
$H = \nabla \times \vec{A}$
$\vec{\nabla} \cdot \vec{H} = 0$
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$H = (\vec{p} - \vec{A})^2/2m + \phi$

Galilean covariant E.M.

1935 EPR criterion of reality

Feynman's Argument.

Dyson AJP (1990).

$$m[x_i, \dot{x}_j] = i\hbar\,\delta_{ij}$$

$$[x_i, x_j] = 0$$

$$m\ddot{x}_i = F_i(\vec{x}, \dot{\vec{x}}, t)$$

$$\Downarrow$$

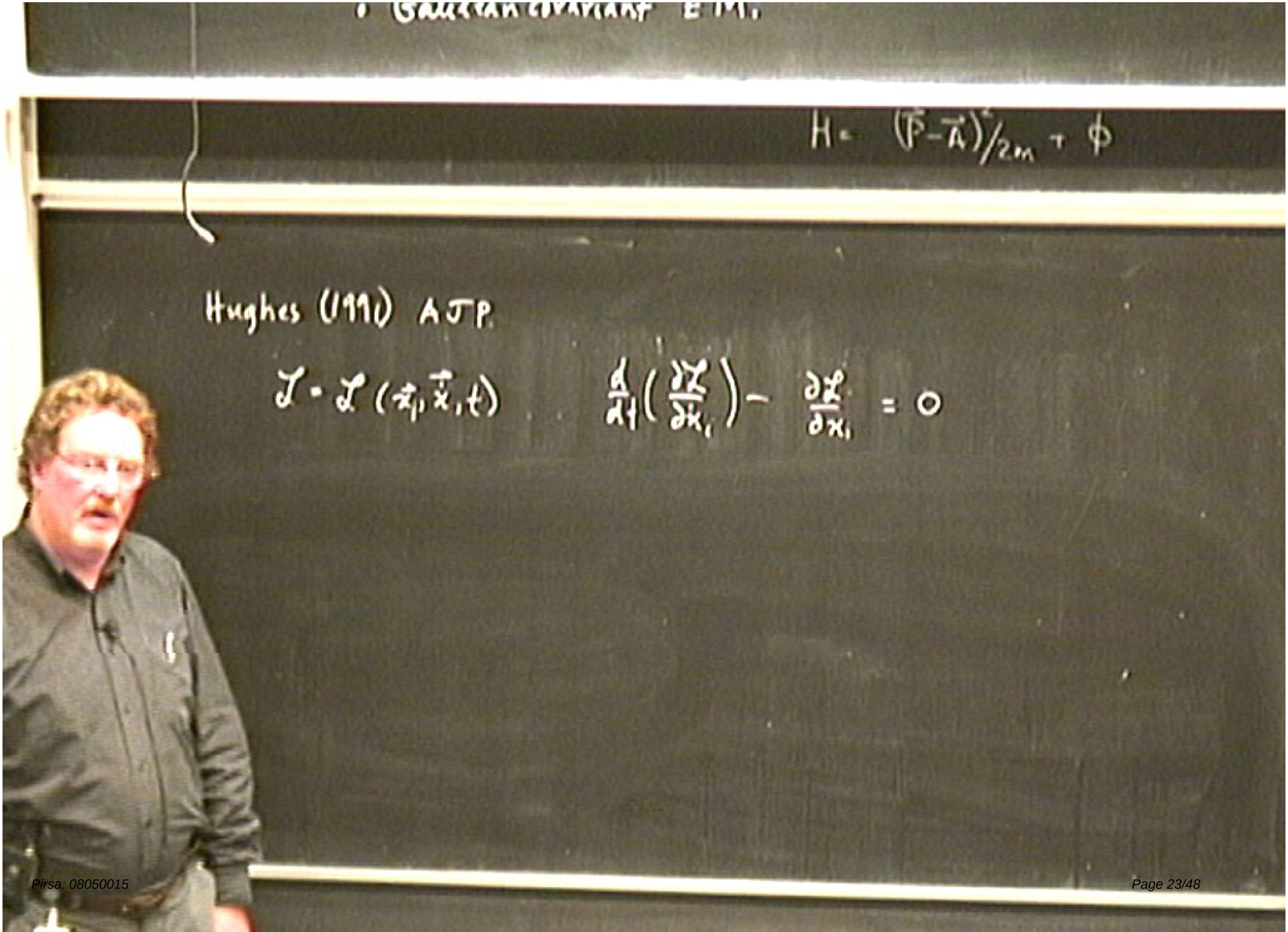
$$\vec{F} = \vec{E} + \vec{v}\times\vec{H}$$

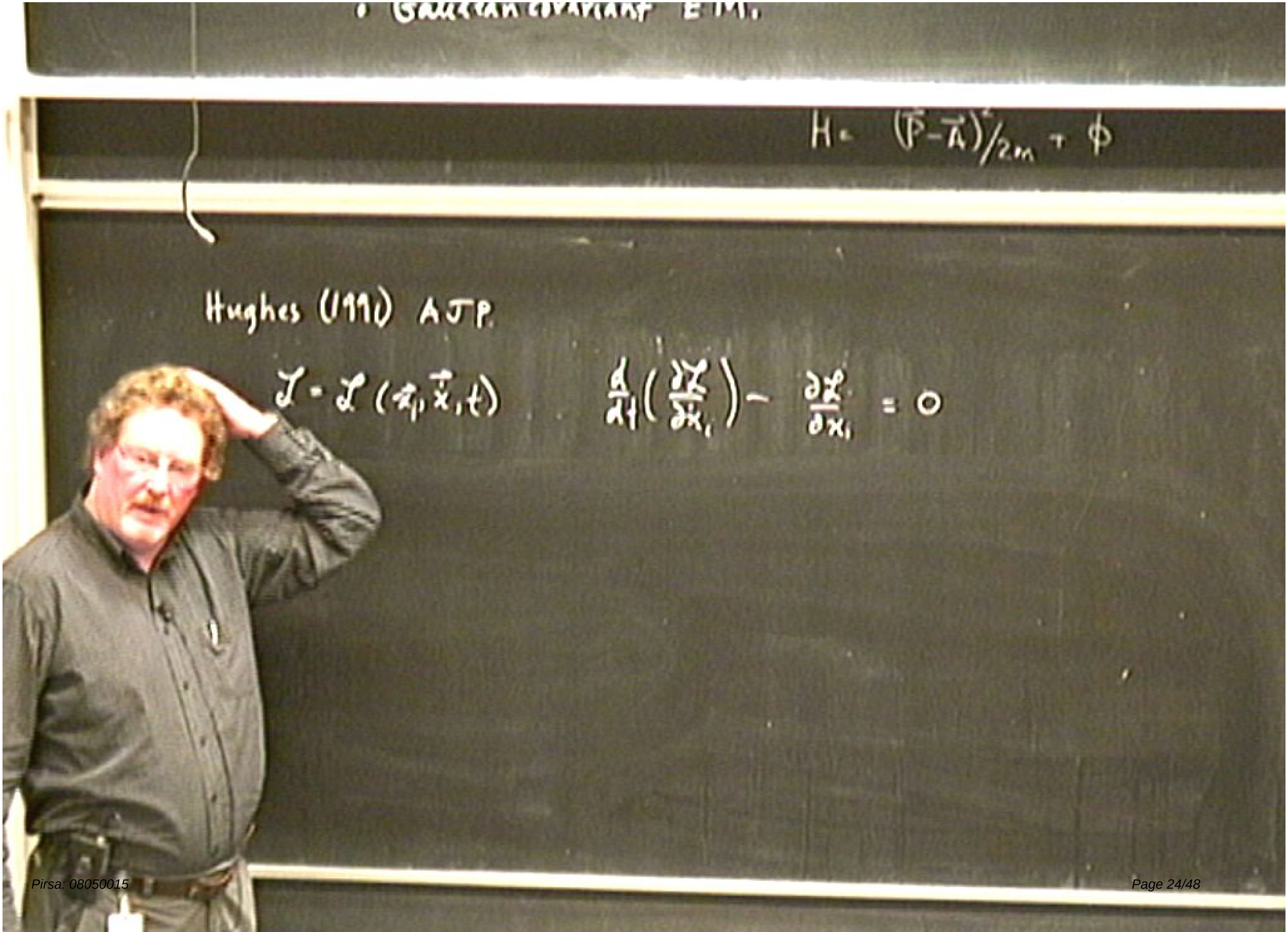
$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\phi$$

$$H = \vec{\nabla}\times\vec{A}$$

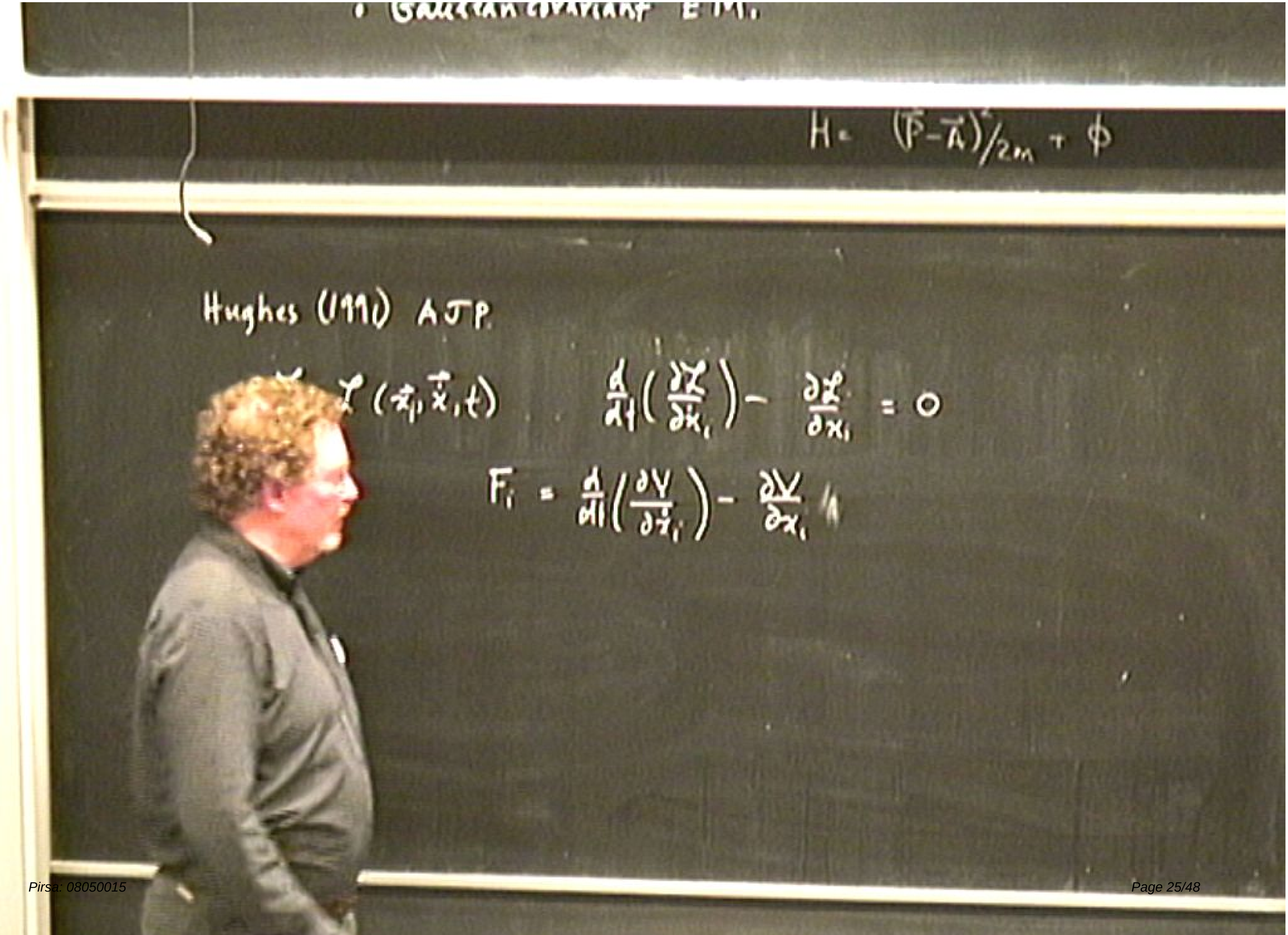
$$\boxed{\vec{\nabla}\cdot\vec{H} = 0 \qquad \nabla\times\vec{E} + \frac{\partial\vec{H}}{\partial t} = 0}$$

$$H = (\vec{p}-\vec{A})^2/2m + \phi$$


• Galilean covariant E.M.
$H = (\vec{p}-\vec{A})^2/2m + \phi$
Hughes (1991) AJP
$\mathcal{L} = \mathcal{L}(x_i, \dot{x}_i, t)$
$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right) - \frac{\partial \mathcal{L}}{\partial x_i} = 0$



Galilean covariant E.M.
H = (p − A)²/2m + φ
Hughes (1991) AJP
L = L(ẋ, x, t)
d/dt(∂L/∂ẋᵢ) − ∂L/∂xᵢ = 0


$H = (\vec{p} - \vec{A})^2/2m + \phi$
Hughes (1991) AJP
$L(\dot{x}_i, \vec{x}, t)$
$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = 0$
$F_i = \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{x}_i}\right) - \frac{\partial V}{\partial x_i}$

• Galilean covariant E.M.

$H = (\vec{p}-\vec{A})^2/2m + \phi$

$\mathcal{L} = \frac{1}{2}m\dot{\vec{x}}^2 + V$

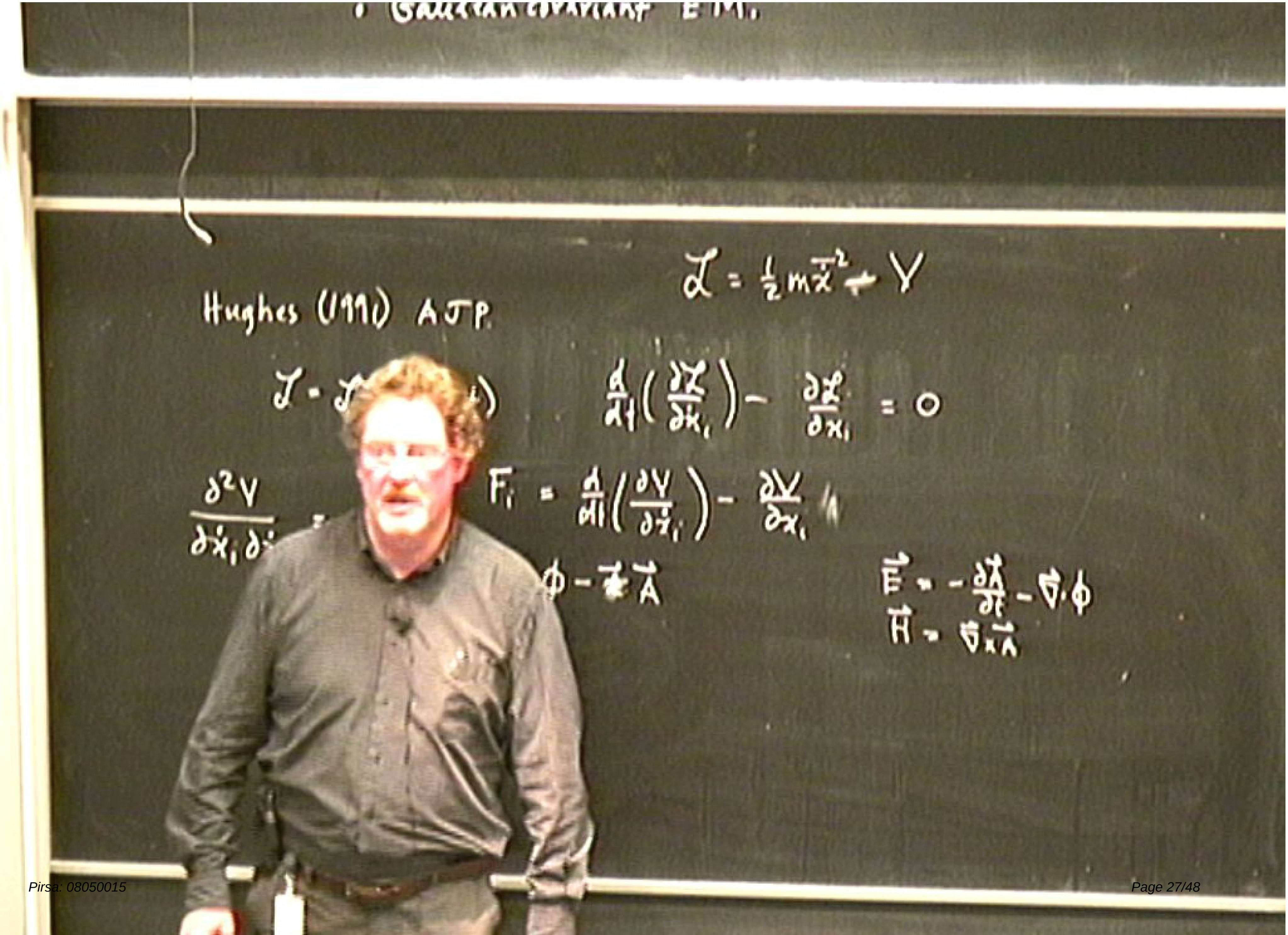
Hughes (1991) AJP

$\mathcal{L} = \mathcal{L}(\dot{x}_i, \vec{x}, t)$

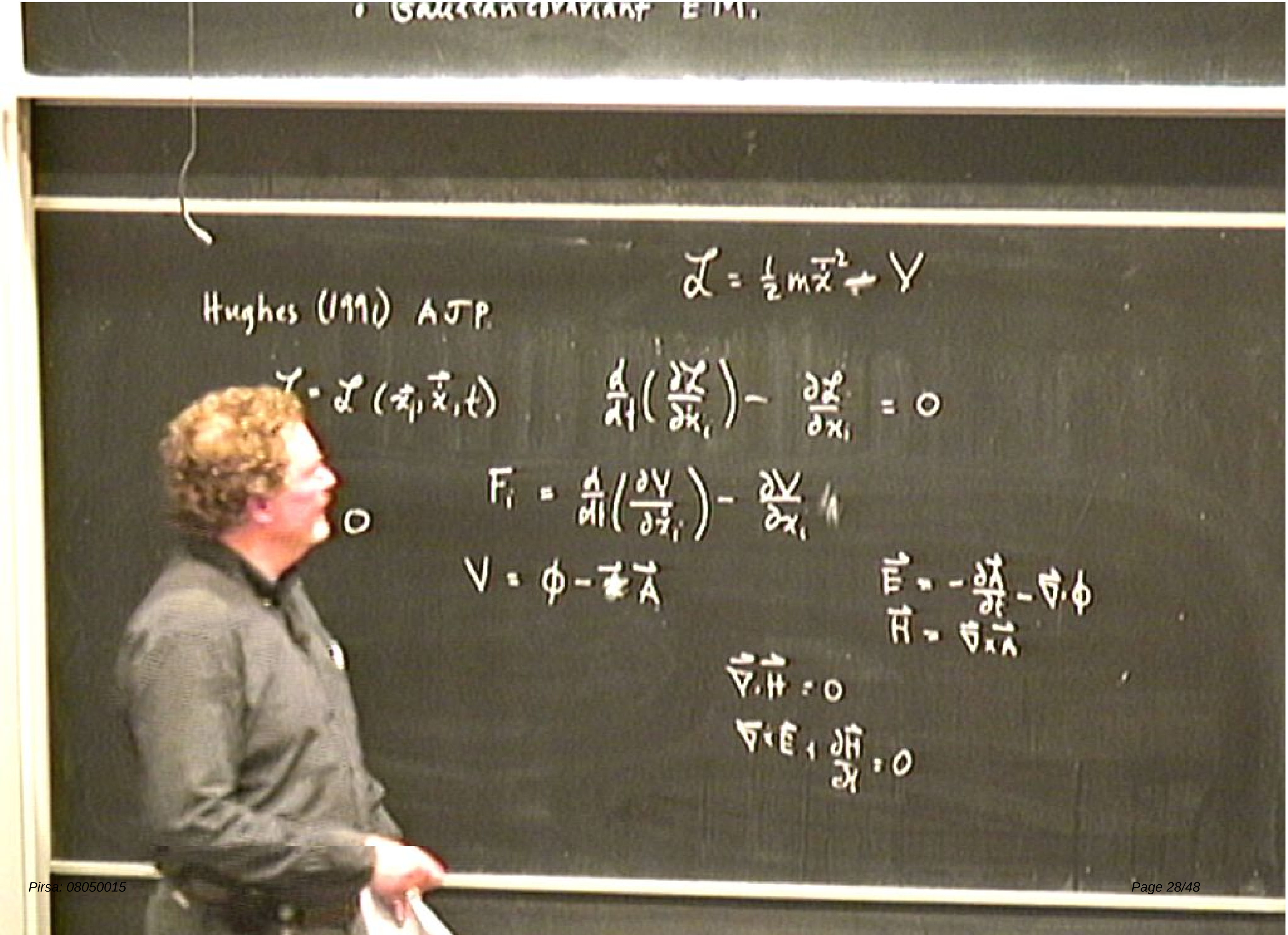
$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right) - \frac{\partial \mathcal{L}}{\partial x_i} = 0$

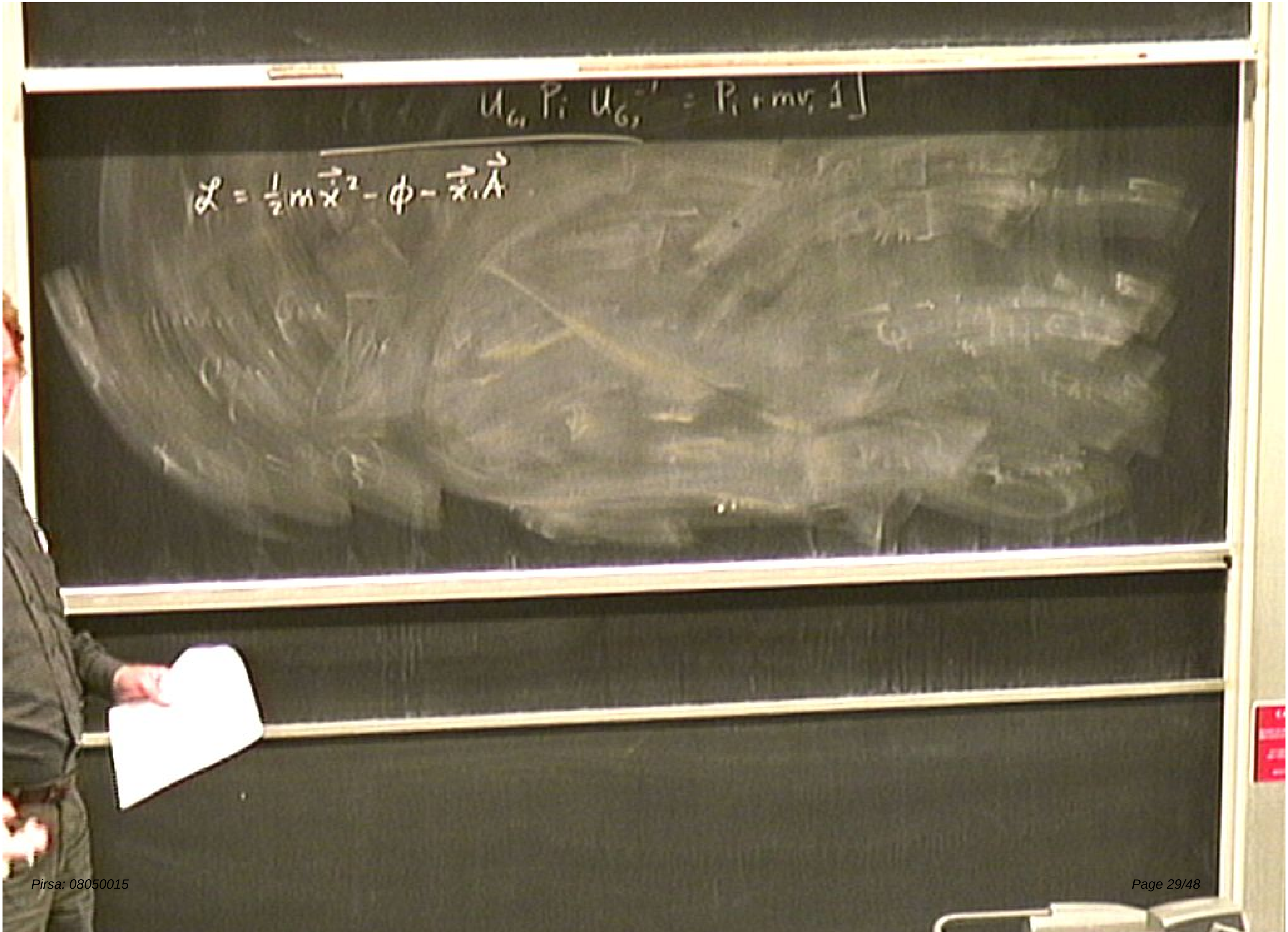
$\frac{\partial^2 V}{\partial \dot{x}_i \partial \dot{x}_j} = 0$

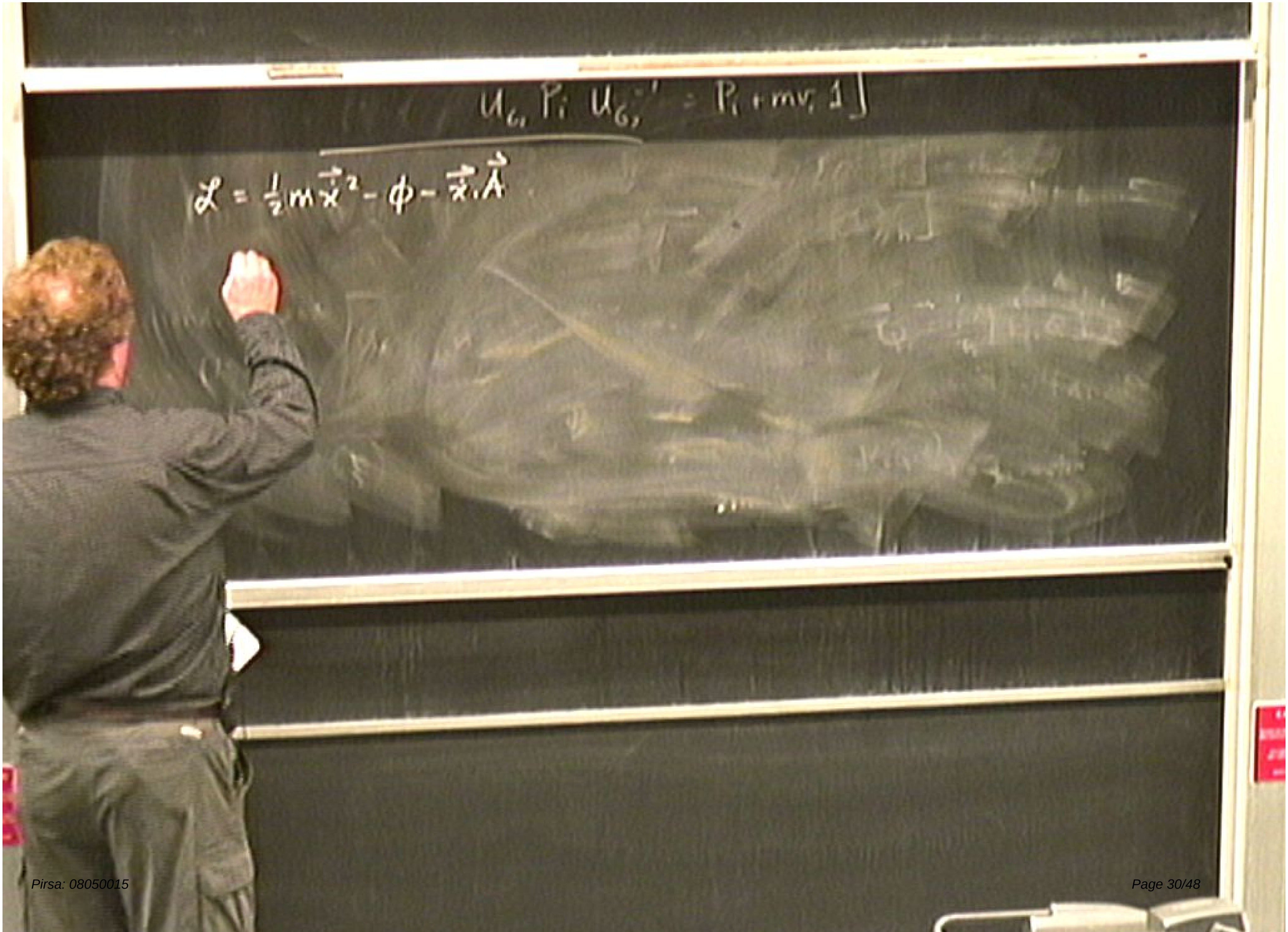
$F_i = \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{x}_i}\right) - \frac{\partial V}{\partial x_i}$



Hughes (1991) AJP
$\mathcal{L} = \frac{1}{2} m \dot{\vec{x}}^2 - V$
$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right) - \frac{\partial \mathcal{L}}{\partial x_i} = 0$
$F_i = \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{x}_i}\right) - \frac{\partial V}{\partial x_i}$
$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \phi$
$\vec{H} = \nabla \times \vec{A}$


Hughes (1991) AJP
$\mathcal{L} = \frac{1}{2} m \dot{\vec{x}}^2 + V$
$\mathcal{L} = \mathcal{L}(x_i, \dot{\vec{x}}, t)$
$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right) - \frac{\partial \mathcal{L}}{\partial x_i} = 0$
$F_i = \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{x}_i}\right) - \frac{\partial V}{\partial x_i}$
$V = \phi - \dot{\vec{x}} \cdot \vec{A}$
$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi$
$\vec{H} = \vec{\nabla} \times \vec{A}$
$\vec{\nabla} \cdot \vec{H} = 0$
$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{H}}{\partial t} = 0$


$U_{G_1} P_i U_{G_1}^{-1} = P_i + m v_i 1]$
$\mathcal{L} = \frac{1}{2} m \dot{\vec{x}}^2 - \phi - \dot{\vec{x}} \cdot \vec{A}$


$U_{G_i} P_i U_{G_i}^{-1} = P_i + m v_i 1]$
$\mathcal{L} = \frac{1}{2} m \dot{\vec{x}}^2 - \phi - \dot{\vec{x}} \cdot \vec{A}$

$$U_G, P_i \, U_G^{-1} = P_i + m v_i \, \mathbb{1}]$$

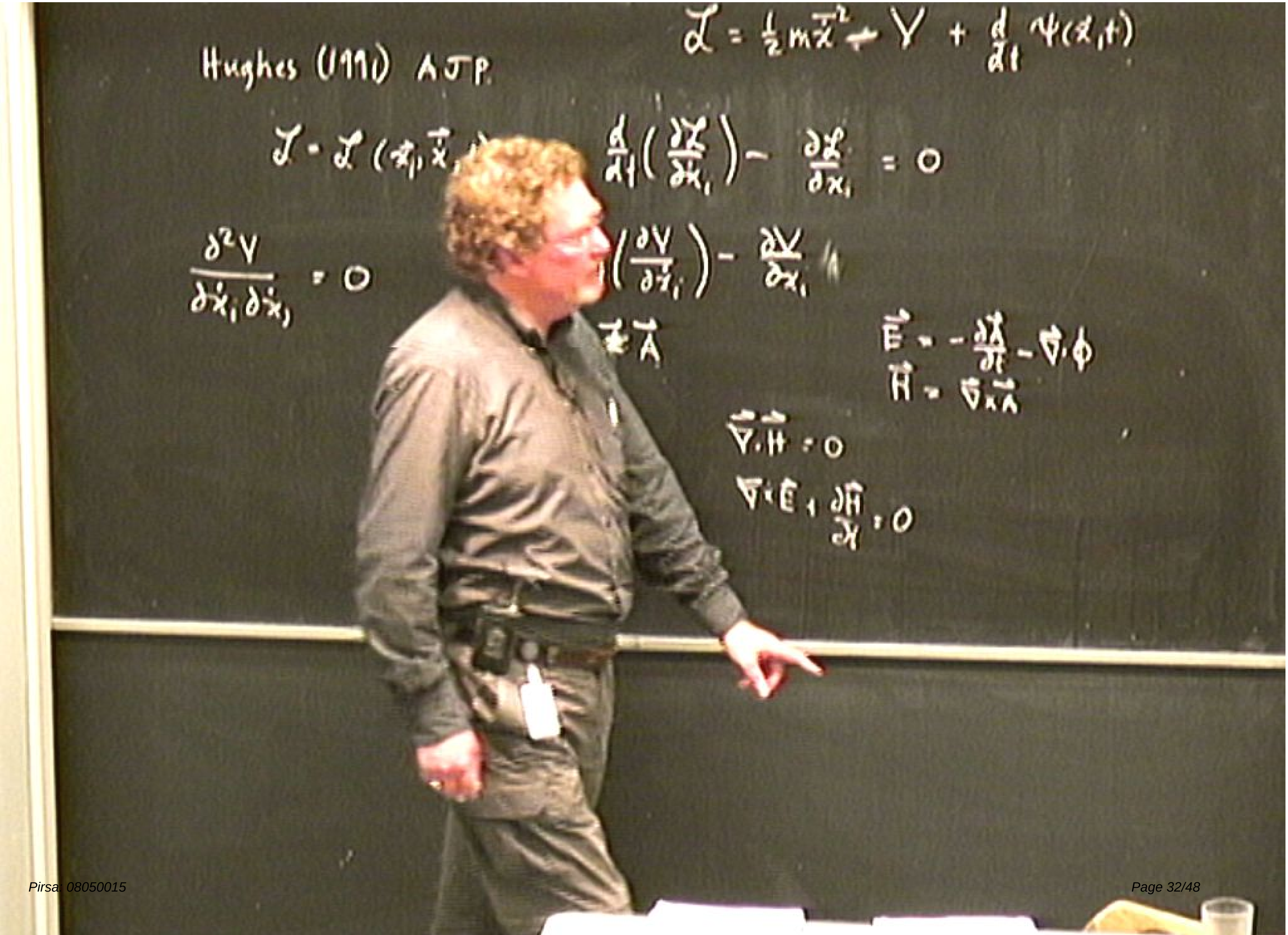
$$\mathcal{L} = \tfrac{1}{2} m \dot{\vec{x}}^2 - \phi - \dot{\vec{x}} \cdot \vec{A}$$

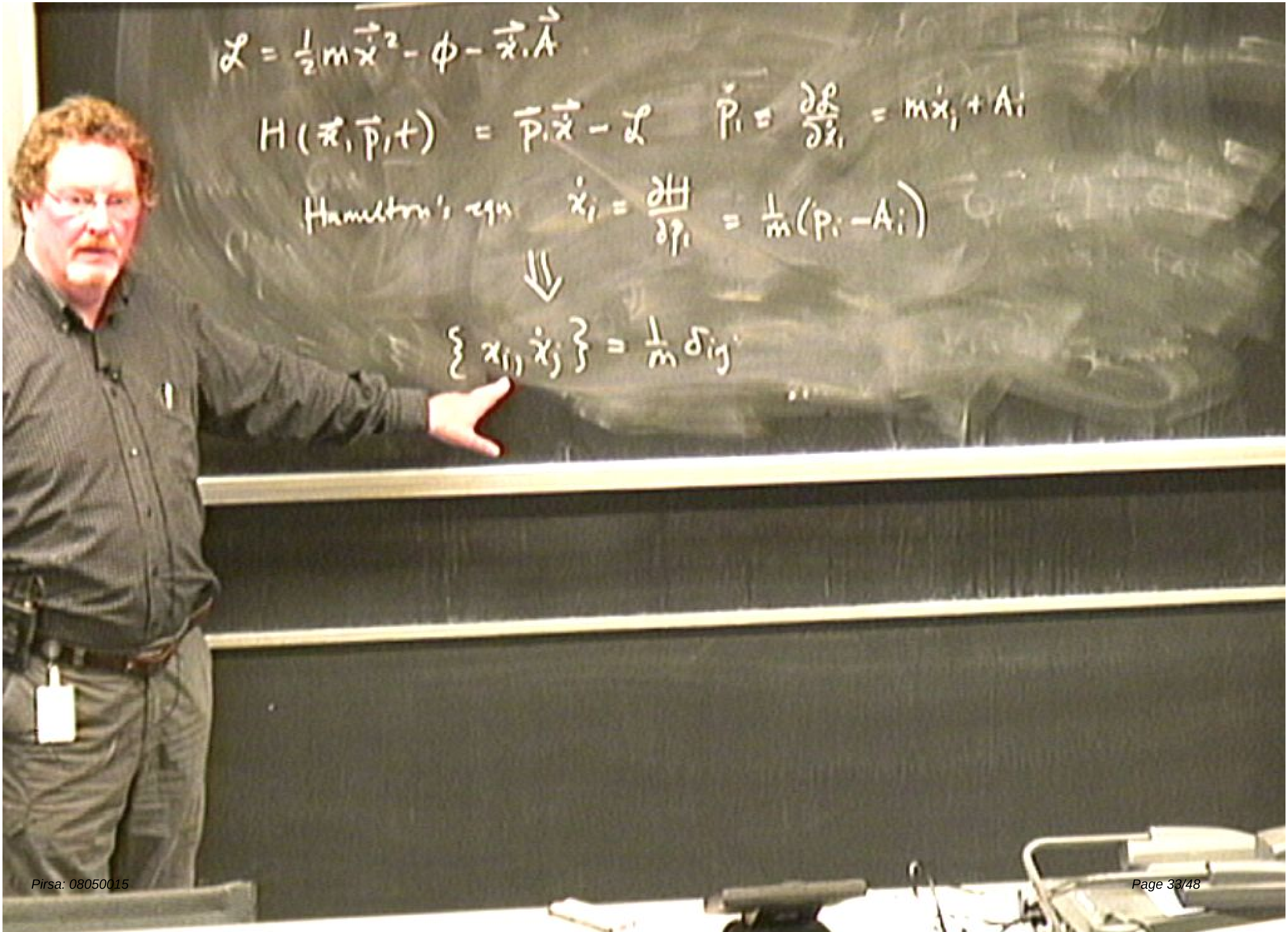
$$H(\vec{x}, \vec{p}, t) = \vec{p} \cdot \dot{\vec{x}} - \mathcal{L} \qquad p_i = \frac{\partial \mathcal{L}}{\partial \dot{x}_i} = m \dot{x}_i + A_i$$

Hamilton's eqn $\quad \dot{x}_i = \dfrac{\partial H}{\partial p_i} = \dfrac{1}{m}(p_i - A_i)$

$$\Downarrow$$

$$\{ x_i, \dot{x}_j \} = \frac{1}{m} \delta_{ij}$$


Hughes (1991) AJP
$\mathcal{L} = \frac{1}{2} m \dot{\vec{x}}^2 - V + \frac{d}{dt} \psi(x,t)$
$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right) - \frac{\partial \mathcal{L}}{\partial x_i} = 0$
$\frac{\partial^2 V}{\partial \dot{x}_i \partial \dot{x}_j} = 0$
$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi$
$\vec{H} = \vec{\nabla} \times \vec{A}$
$\vec{\nabla} \cdot \vec{H} = 0$
$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{H}}{\partial t} = 0$



$\mathcal{L} = \frac{1}{2} m \dot{\vec{x}}^2 - \phi - \dot{\vec{x}} \cdot \vec{A}$
$H(\vec{x}, \vec{p}, t) = \vec{p} \cdot \dot{\vec{x}} - \mathcal{L}$
$p_i = \frac{\partial \mathcal{L}}{\partial \dot{x}_i} = m\dot{x}_i + A_i$
Hamilton's eqn $\dot{x}_i = \frac{\partial H}{\partial p_i} = \frac{1}{m}(p_i - A_i)$
$\Downarrow$
$\{ x_i, \dot{x}_j \} = \frac{1}{m} \delta_{ij}$

$$U_{G_i} P_i U_{G_i}^{-1} = P_i + m v_i \mathbb{1} \Big]$$

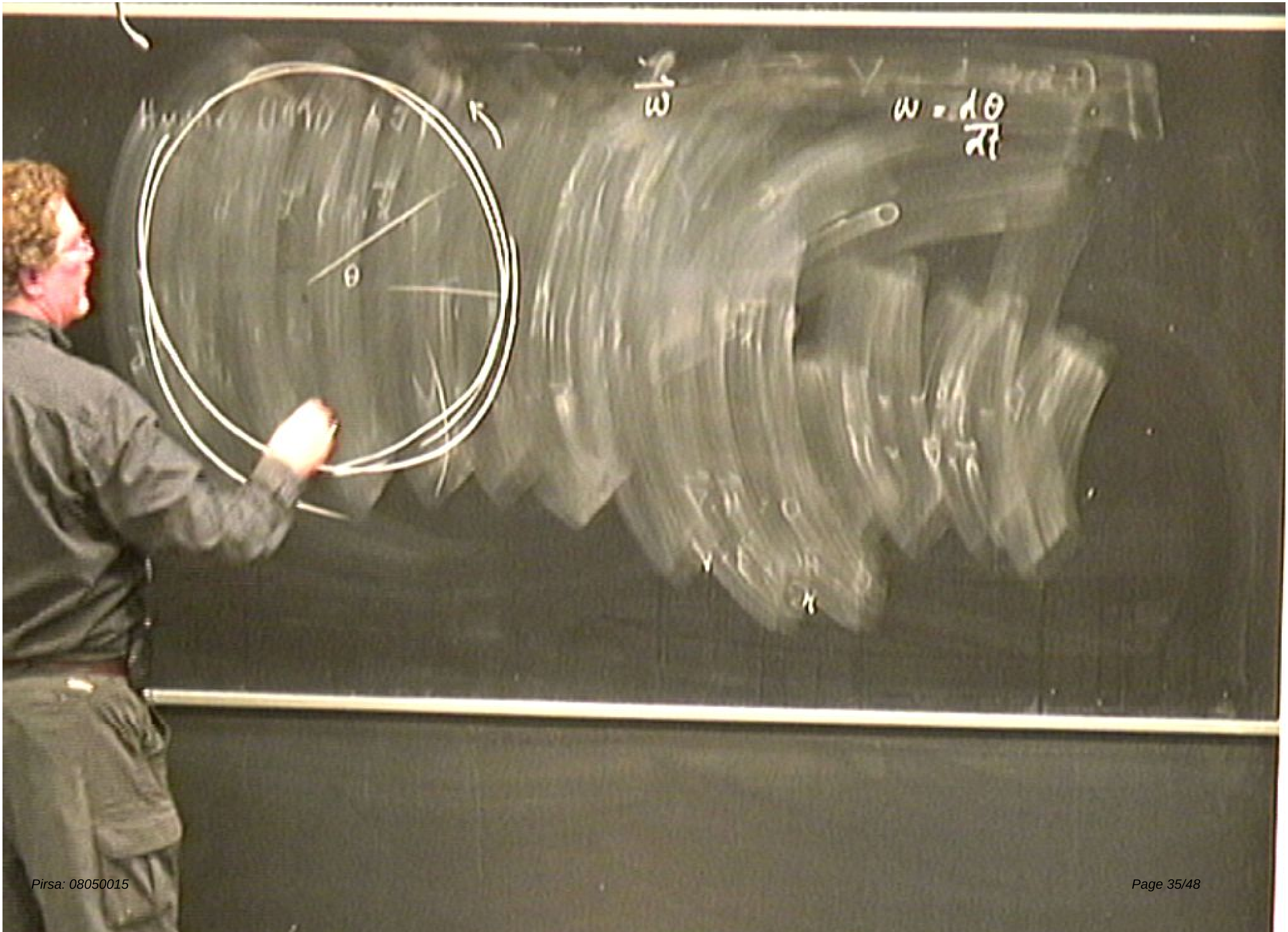
$$\mathcal{L} = \frac{1}{2} m \dot{\vec{x}}^2 - \phi - \dot{\vec{x}} \cdot \vec{A}$$

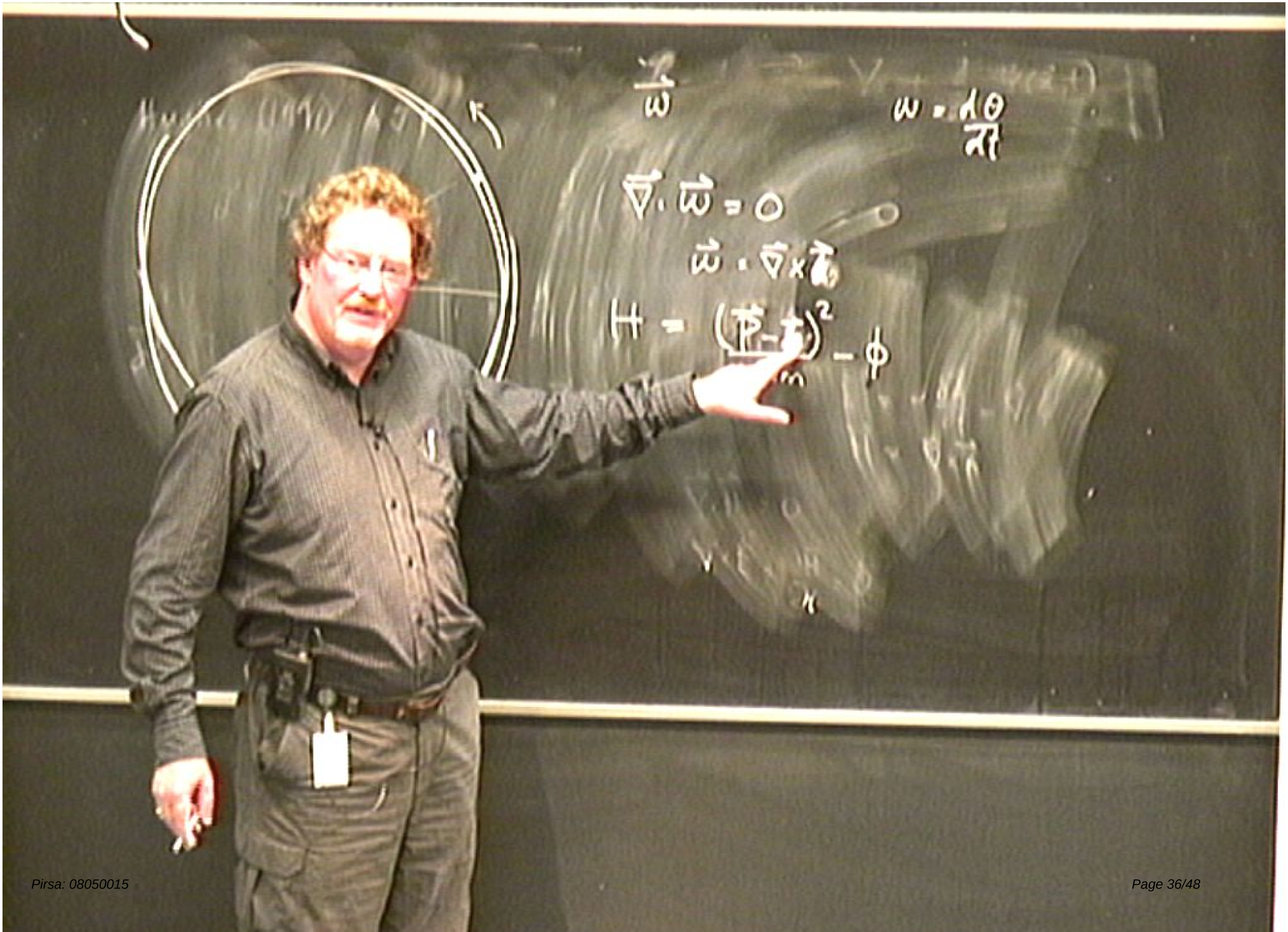
$$H(\vec{x}, \vec{p}, t) = \vec{p} \cdot \dot{\vec{x}} - \mathcal{L} \qquad p_i = \frac{\partial \mathcal{L}}{\partial \dot{x}_i} = m \dot{x}_i + A_i$$

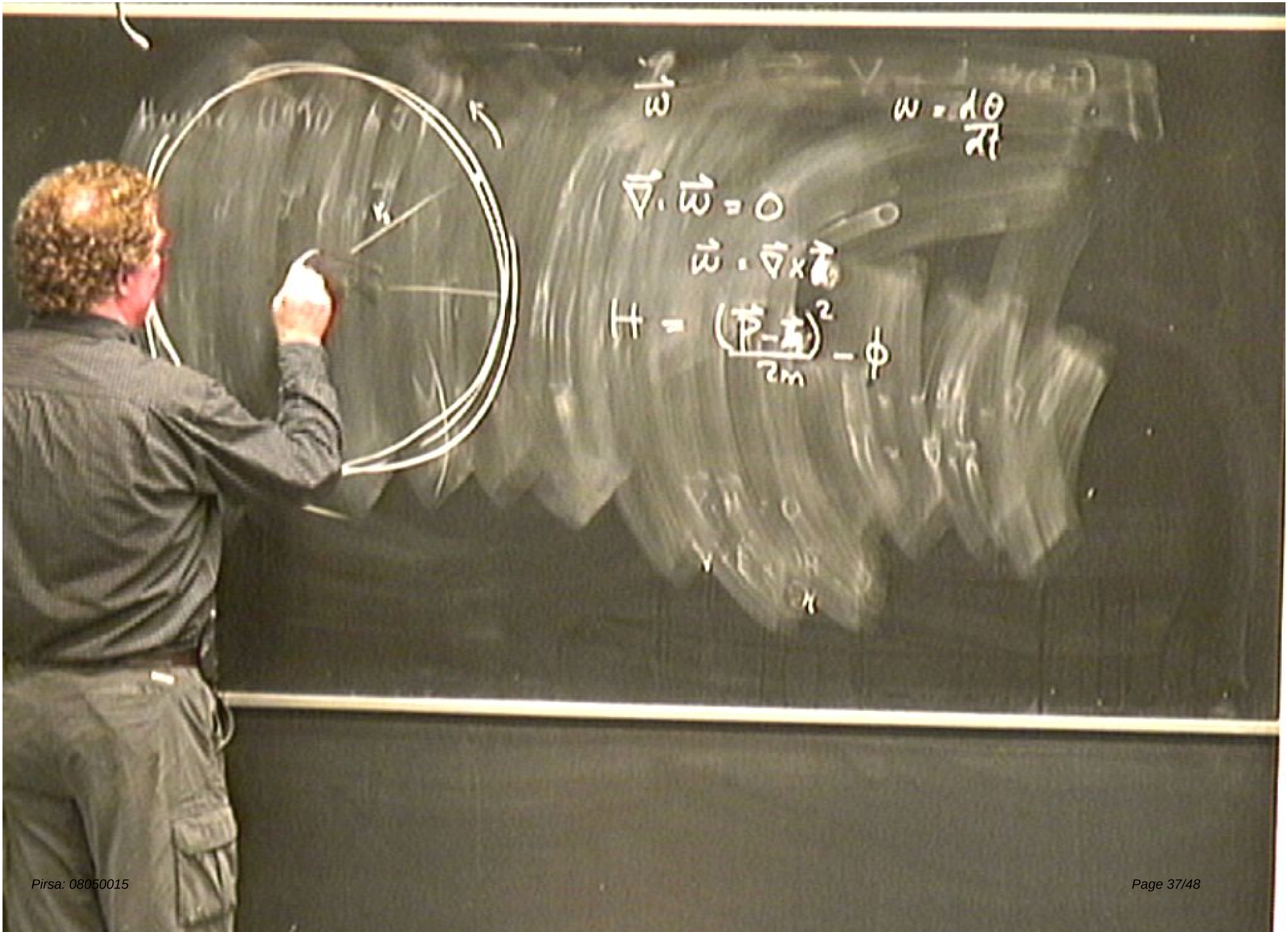
Hamilton's eqn $\dot{x}_i = \dfrac{\partial H}{\partial p_i} = \dfrac{1}{m}(p_i - A_i)$

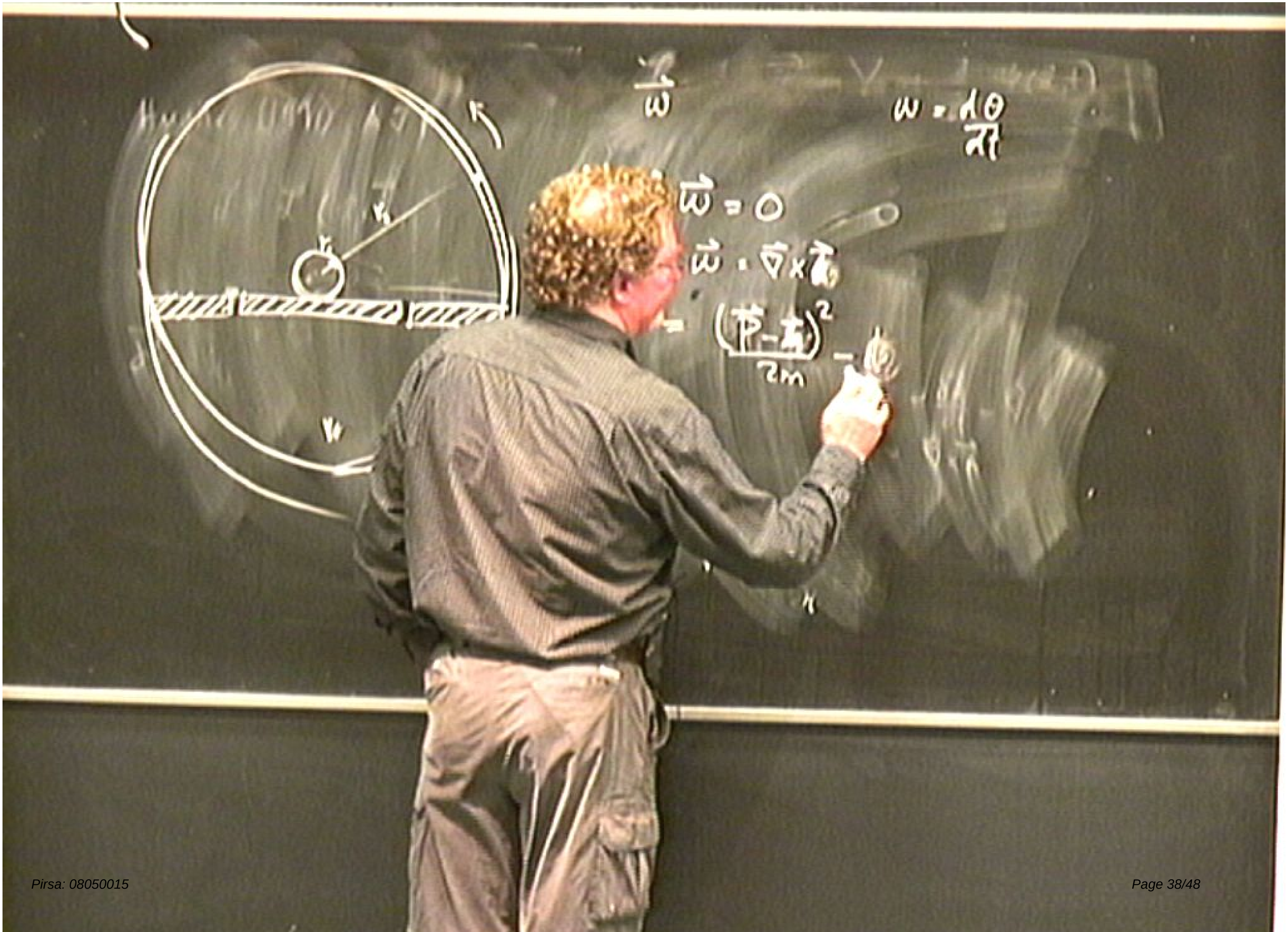
$$\Downarrow$$

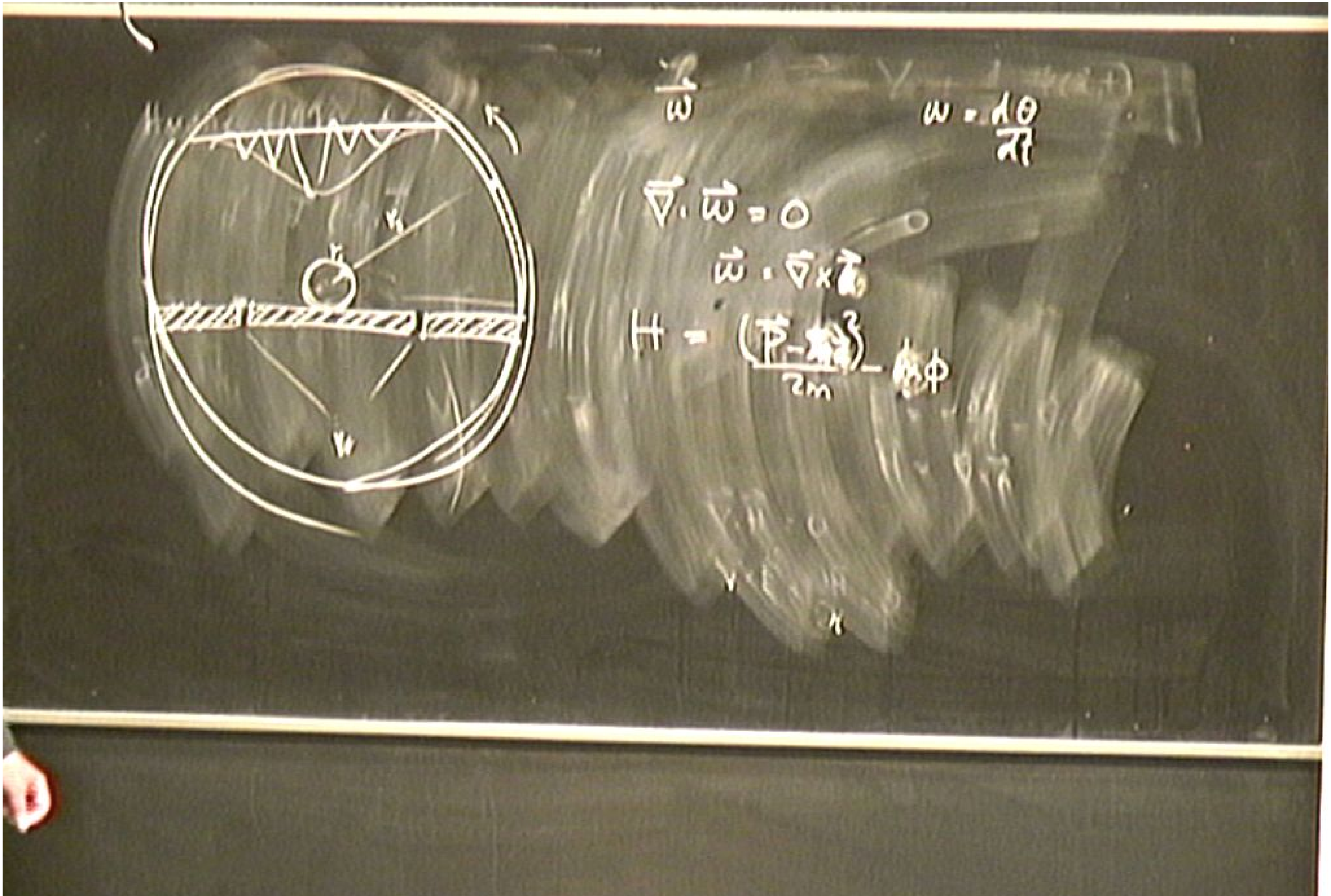
$$\{ x_i, \dot{x}_j \} = \frac{1}{m} \delta_{ij}$$


θ
ω = dθ/dt

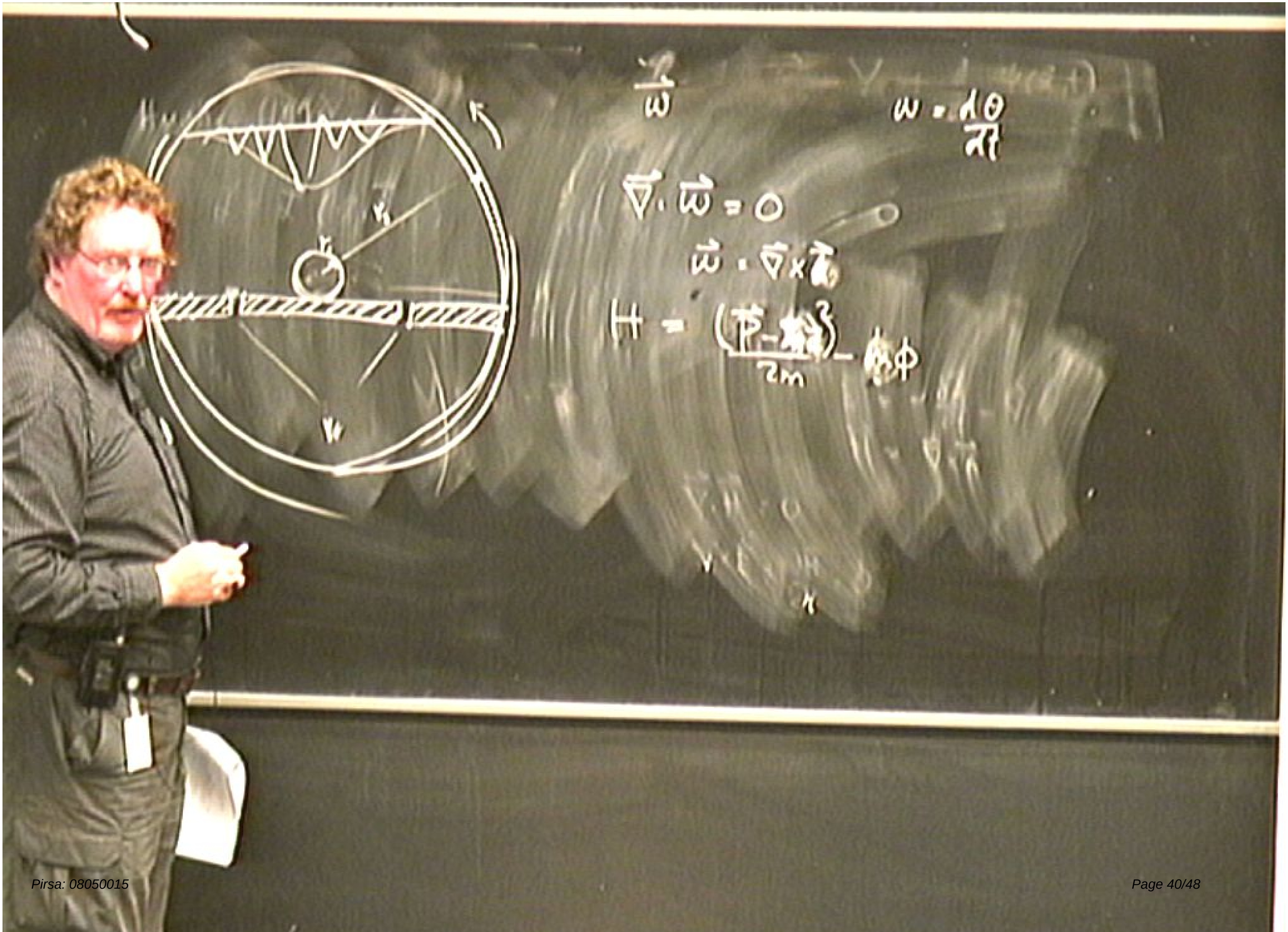

ω = dθ/dt
∇·ω = 0
ω = ∇×a
H = (p − a)²/2m − φ

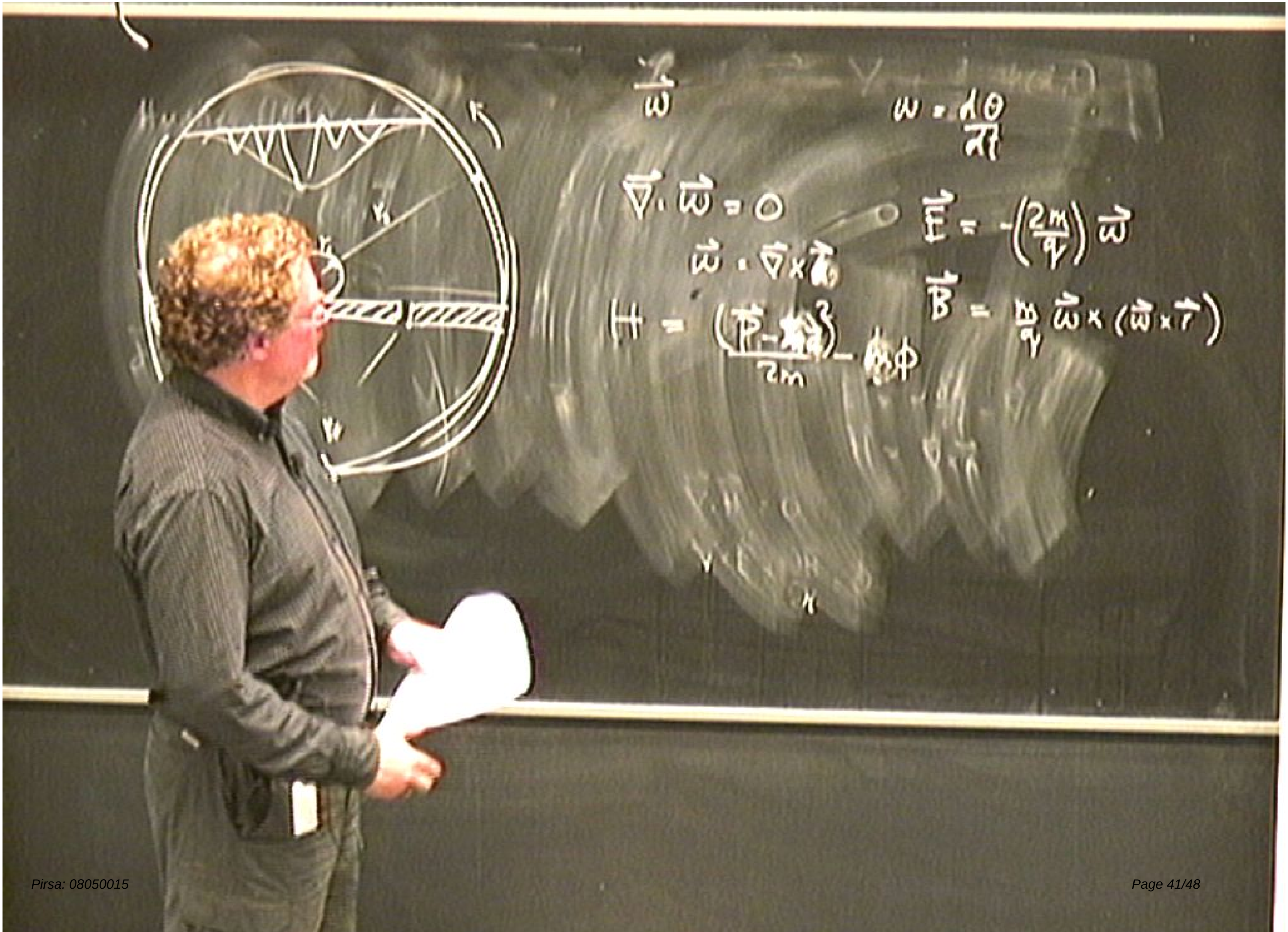

ω
ω = dθ/dt
∇ · ω = 0
ω = ∇ × a
H = (p − a)²/2m − φ

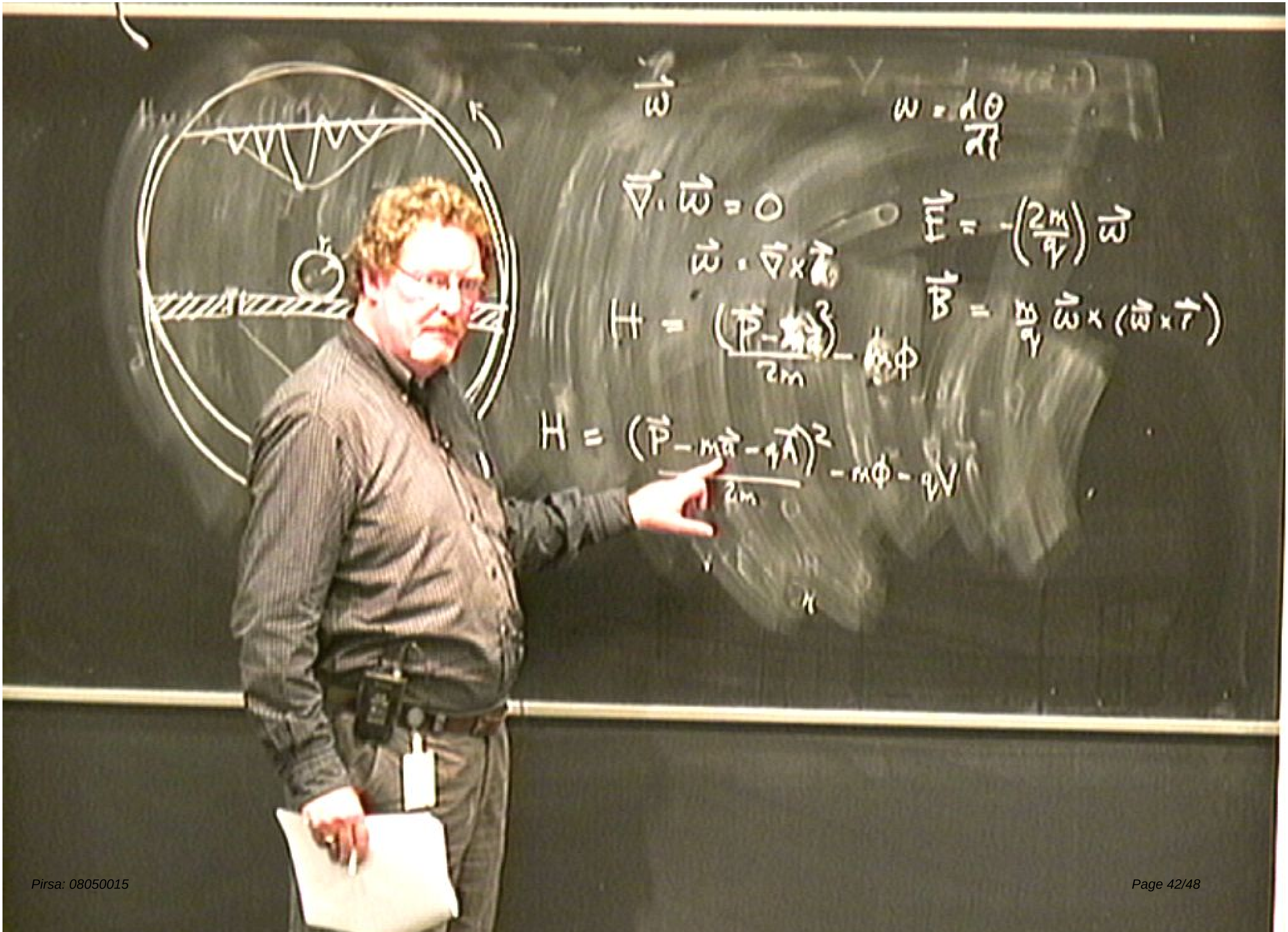

ω = dθ/dt
ω = 0
ω = ∇ × A
(p − A)²/2m

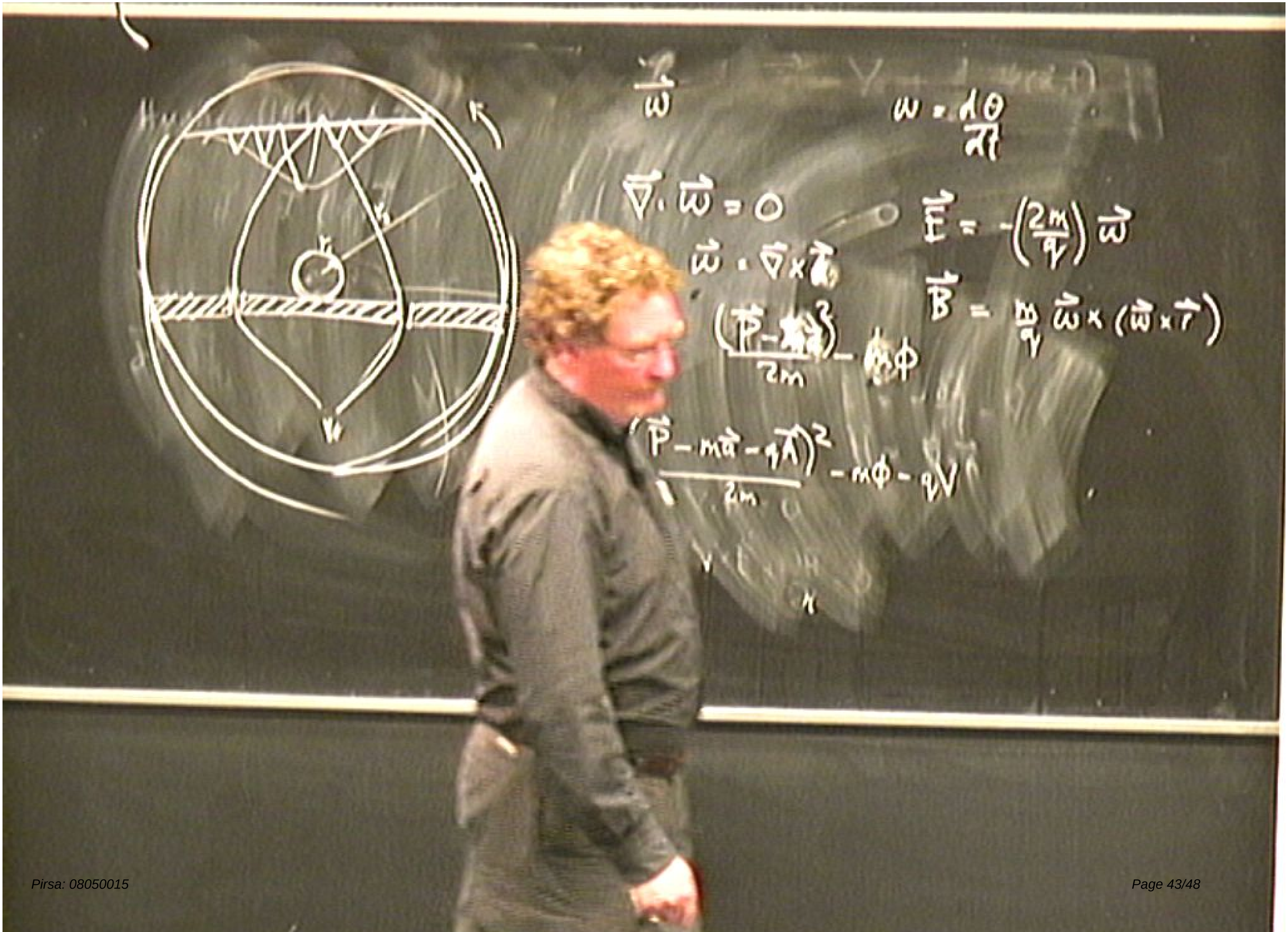


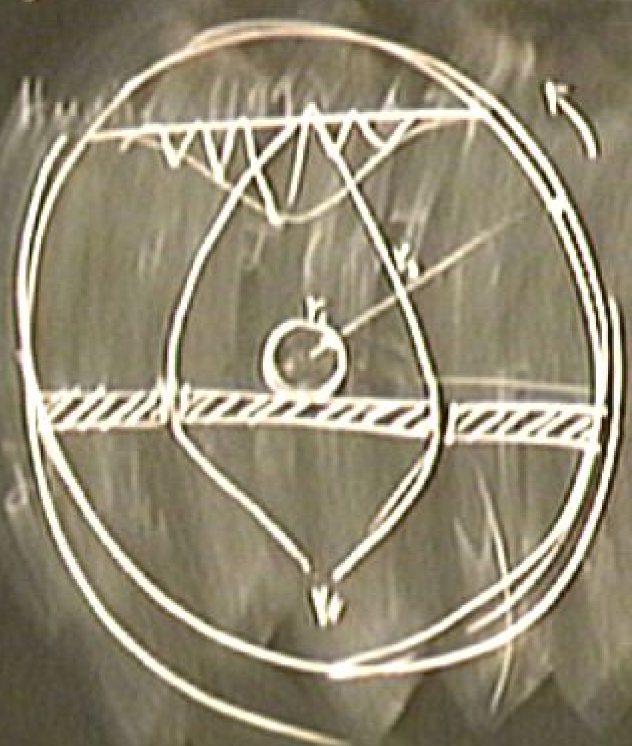
ω = dθ/dt
∇ · ω = 0
ω = ∇ × a


ω = dθ/dt
∇·ω = 0
ω = ∇×A
H = (p − eA)²/2m − eφ


ω = dθ/dt
∇·ω = 0
E = -(2m/q) ω
B = m/q ω × (ω × r)


ω = dθ/dt
∇ · ω = 0
E = −(2m/q) ω
B = (m/q) ω × (ω × r)
H = (P − mr − qA)²/2m − mΦ − qV


ω = dθ/dt
∇ · ω = 0
E = −(2m/q) ω
B = m/q ω × (ω × r)
2m
−mΦ − qV



$\vec{\omega}$

$\omega = \frac{d\theta}{dt}$

$\vec{\nabla}\cdot\vec{\omega} = 0$

$\vec{\omega} = \vec{\nabla}\times\vec{a}$

$\vec{E} = -\left(\frac{2m}{q}\right)\vec{\omega}$

$\vec{B} = \frac{m}{q}\vec{\omega}\times(\vec{\omega}\times\vec{r})$

$$H = \frac{(\vec{P} - m\vec{a})^2}{2m} - m\phi$$

$$H = \frac{(\vec{P} - m\vec{a} - q\vec{A})^2}{2m} - m\phi - qV$$

$$q\vec{\nabla}\times\vec{A} = -m\vec{\nabla}\times\vec{R}$$

$$\Downarrow$$

$$q\vec{A} = -m\vec{R}$$

$$q\vec{\nabla}\times\vec{A} = -m\vec{\nabla}\times\vec{a}$$

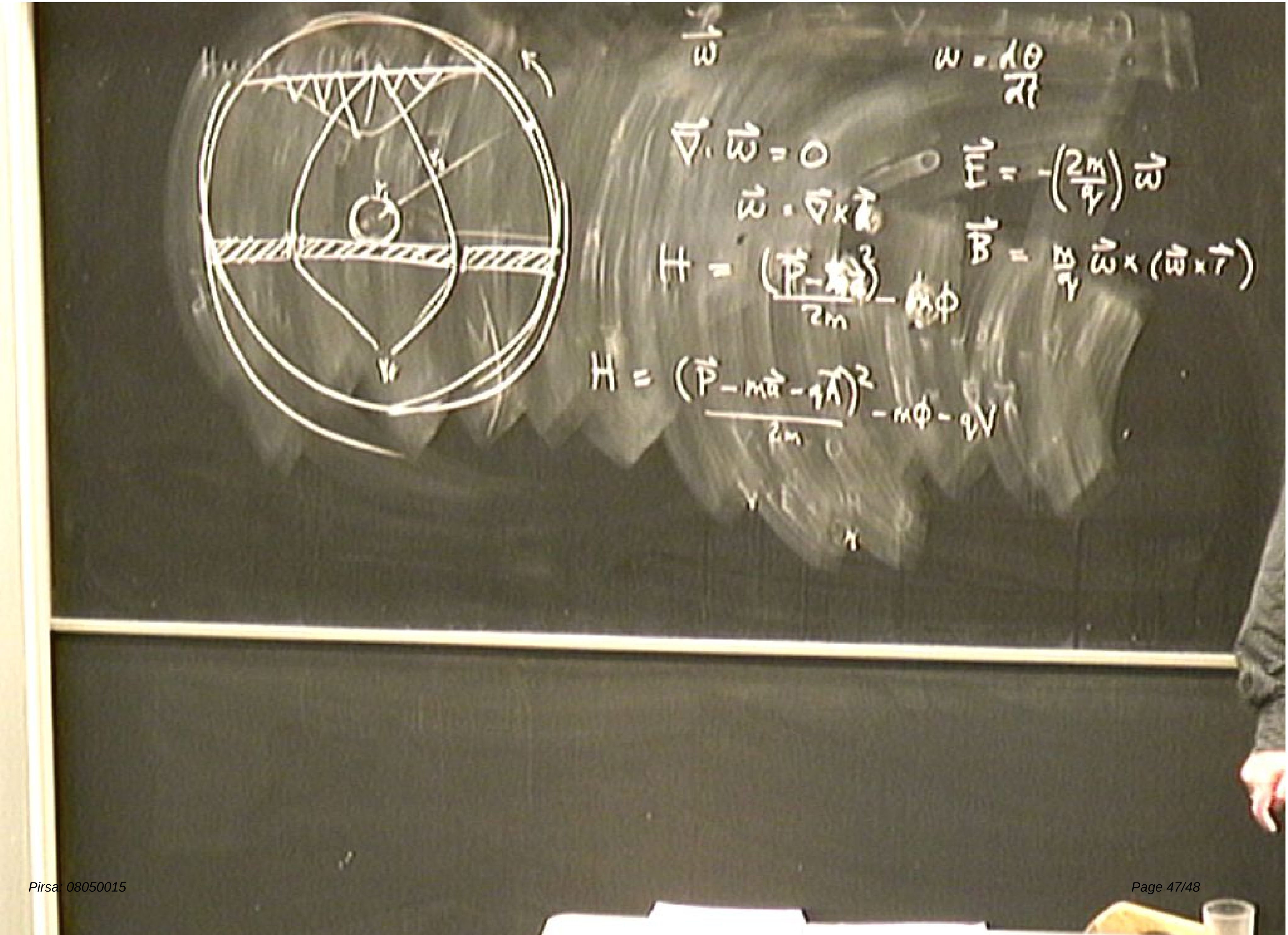
$$\Downarrow$$

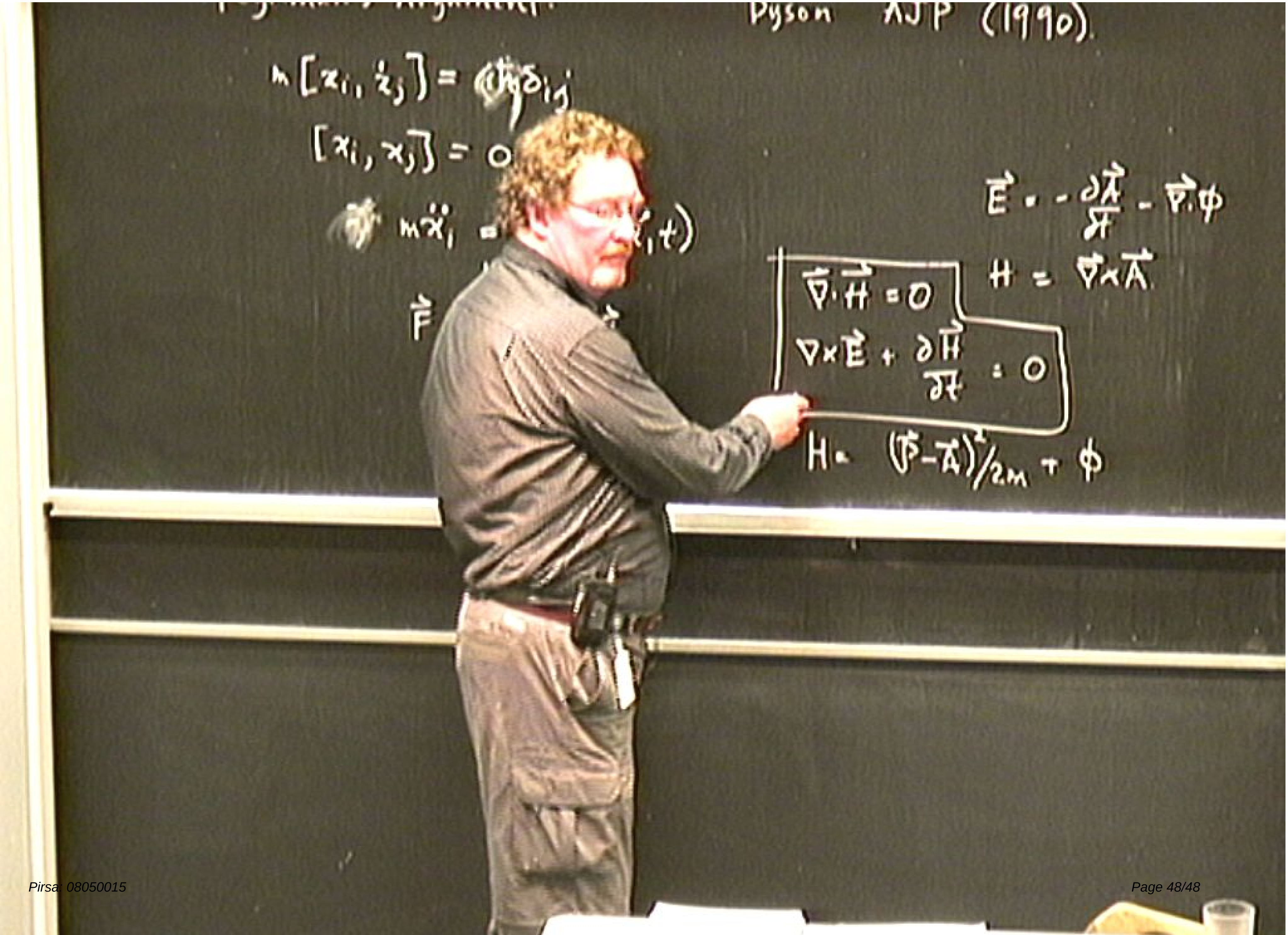
$$q\vec{A} = -m\vec{a}$$

$$e^{i\Phi}$$

$$\Phi = \frac{1}{\hbar}\oint\left(q\vec{A} + m\vec{a}\right)\cdot d\ell$$

$$= 2m\omega\pi r_1^2$$


$\vec{\omega}$
$\omega = \frac{d\theta}{dt}$
$\vec{\nabla}\cdot\vec{\omega} = 0$
$\vec{\omega} = \vec{\nabla}\times\vec{a}$
$\vec{E} = -\left(\frac{2m}{q}\right)\vec{\omega}$
$\vec{B} = \frac{m}{q}\vec{\omega}\times(\vec{\omega}\times\vec{r})$
$H = \frac{(\vec{P}-m\vec{a})^2}{2m} - m\phi$
$H = \frac{(\vec{P}-m\vec{a}-q\vec{A})^2}{2m} - m\phi - qV$


Dyson AJP (1990).
[x_i, x_j] = 0
E = - ∂A/∂t - ∇φ
H = ∇×A
∇·H = 0
∇×E + ∂H/∂t = 0
H = (p - A)²/2m + φ