Title: Symmetry Principles in Physics - Lecture 4B

Date: May 05, 2008 12:00 PM

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Abstract:

GENERAL RELATIVITY II THE PALATINI PROCEDURE

$$S_{\text{grav.}} = \int R\sqrt{-g}\,d^4x = \int \left(\Gamma^{\sigma}_{\mu\nu,\sigma} - \Gamma^{\sigma}_{\mu\sigma,\nu} + \Gamma^{\sigma}_{\lambda\sigma}\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\sigma}_{\lambda\nu}\Gamma^{\lambda}_{\mu\sigma}\right) g^{\mu\nu}\sqrt{-g}\,d^4x .$$

Vary $\Gamma^{\sigma}_{\mu\nu}$ and $g^{\mu\nu}$ independently!

$$E_{\mu}^{\ \nu\lambda} = \frac{\delta S_{\text{grav}}}{\delta \Gamma^{\mu}_{\nu\lambda}} , \qquad E_{\mu\nu} = \frac{\delta S_{\text{grav}}}{\delta g^{\mu\nu}}$$

NOETHER CONDITIONS

$$\boxed{R^{\mu}_{\ \nu\lambda\rho} E_{\mu}^{\ \nu\lambda} - g^{\mu\nu}_{\ \ ;\rho} E_{\mu\nu} = E_{\rho\ \ ;\nu\lambda}^{\ \nu\lambda} + 2\left(E_{\mu\rho}\, g^{\mu\nu}\right)_{;\nu}}$$

G. Svetlichny (2001).

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Einstein's change in attitude

- Einstein 1916.
- **"important simplification of the laws of nature"** and **"considerable simplification of the formulae and calculations"** provided by choice of volume-preserving gauge
- $$\sqrt{-g} = 1 \,.$$

- Einstein 1918
- **Reply to Kretschmann:**
  General covariant formulation of a theory should be the **"simplest and most transpartent"** one available to it.

  Newtonian mechanics and gravitation ruled out **"practically if not theoretically"** by this principle.

Einstein's change of heart

In November 1916, Einstein wrote to Weyl:

"I also came belatedly to the view that the theory [GR] becomes more perspicuous when Hamilton's scheme is applied and when no restrictions are put on the choice of the frame of reference. It is true that the formulas then become somewhat more complicated but more suitable for applications; for it appears that the free choice of the reference system is advantageous in the calculations. The connection between the general covariance requirement and the conservation laws also becomes clearer."

GENERAL RELATIVITY WITH MATTER.

$$\mathcal{L} = \underbrace{\mathcal{L}_{grav}(g^{\mu\nu}, g^{\mu\nu}{}_{,\lambda}, g^{\mu\nu}{}_{,\lambda\sigma}, x) + \mathcal{L}_{matter}(g^{\mu\nu}, \phi_i, \phi_{i,\mu}, x)}_{\text{quasi-invariant}}$$

Ⓘ $\mathcal{L}_{grav}$ (Einstein 1916)

Noether condition (& metric compat.)

$$\Rightarrow \boxed{(E^{\sigma}{}_{\alpha}\ \text{[redacted]})_{;\sigma} = 0.}$$

$$E_{\mu\nu} = 0 \Rightarrow T^{\sigma}{}_{\alpha;\sigma} = 0.$$

Ⓘⓘ $\mathcal{L}_{matter}$

Noether condition & metric compat

$$\Rightarrow \boxed{\sum_i \left\{ E^i a_{\alpha i} - (E^i b^{\mu}{}_{\alpha i})_{,\mu} \right\} = 2 N^{\sigma}{}_{\alpha;\sigma}}$$

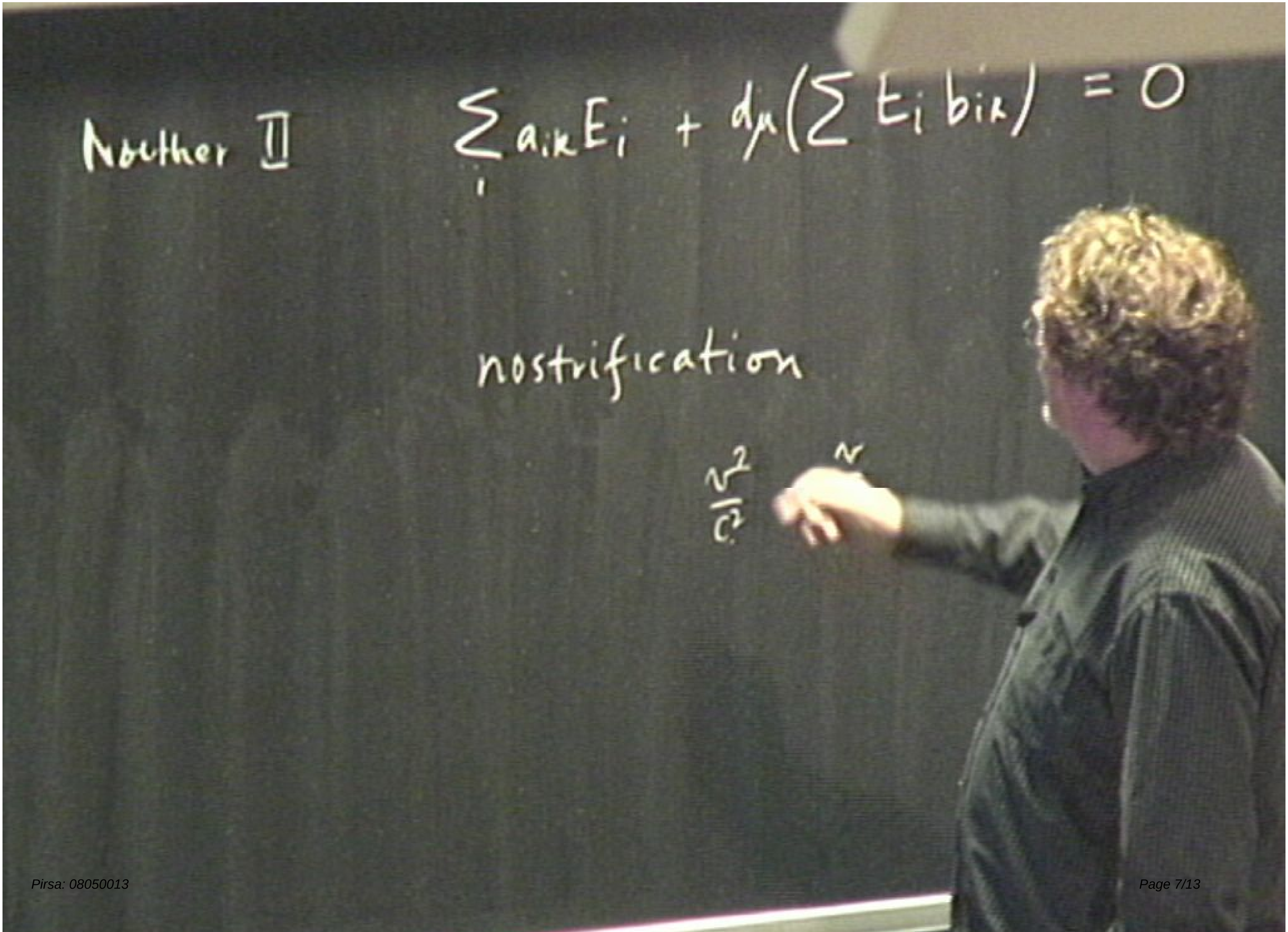
$$E^i = 0 \Rightarrow T^{\sigma}{}_{\alpha;\sigma} = 0.$$

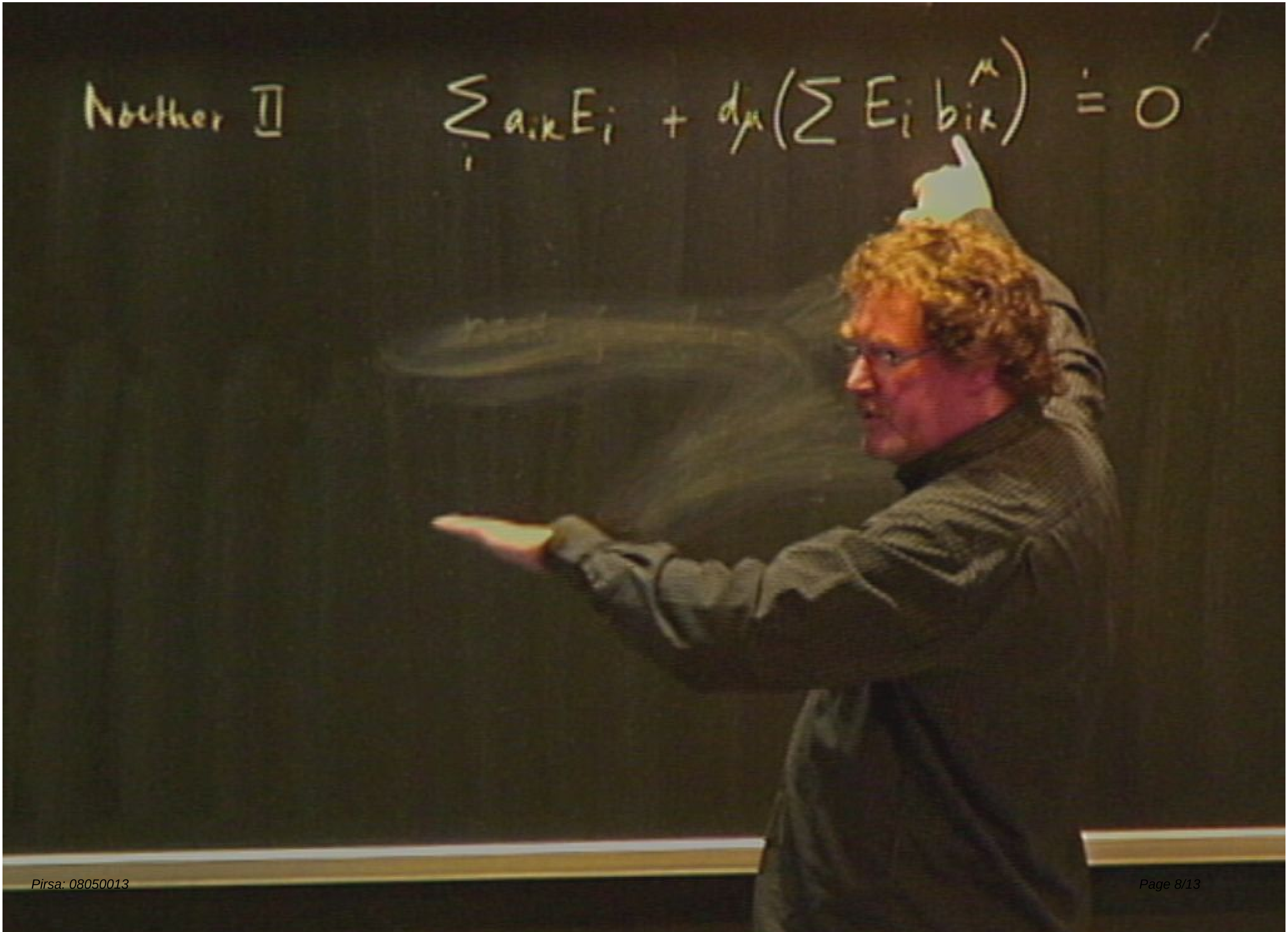
$$E_{\mu\nu} \equiv \frac{\delta \mathcal{L}_{grav}}{\delta g^{\mu\nu}}$$

$$N_{\mu\nu} \equiv \frac{\delta \mathcal{L}_{matter}}{\delta g^{\mu\nu}} = -\frac{\sqrt{-g}}{2} T_{\mu\nu}$$

$$E^i \equiv \frac{\delta \mathcal{L}_{matter}}{\delta \phi_i}$$

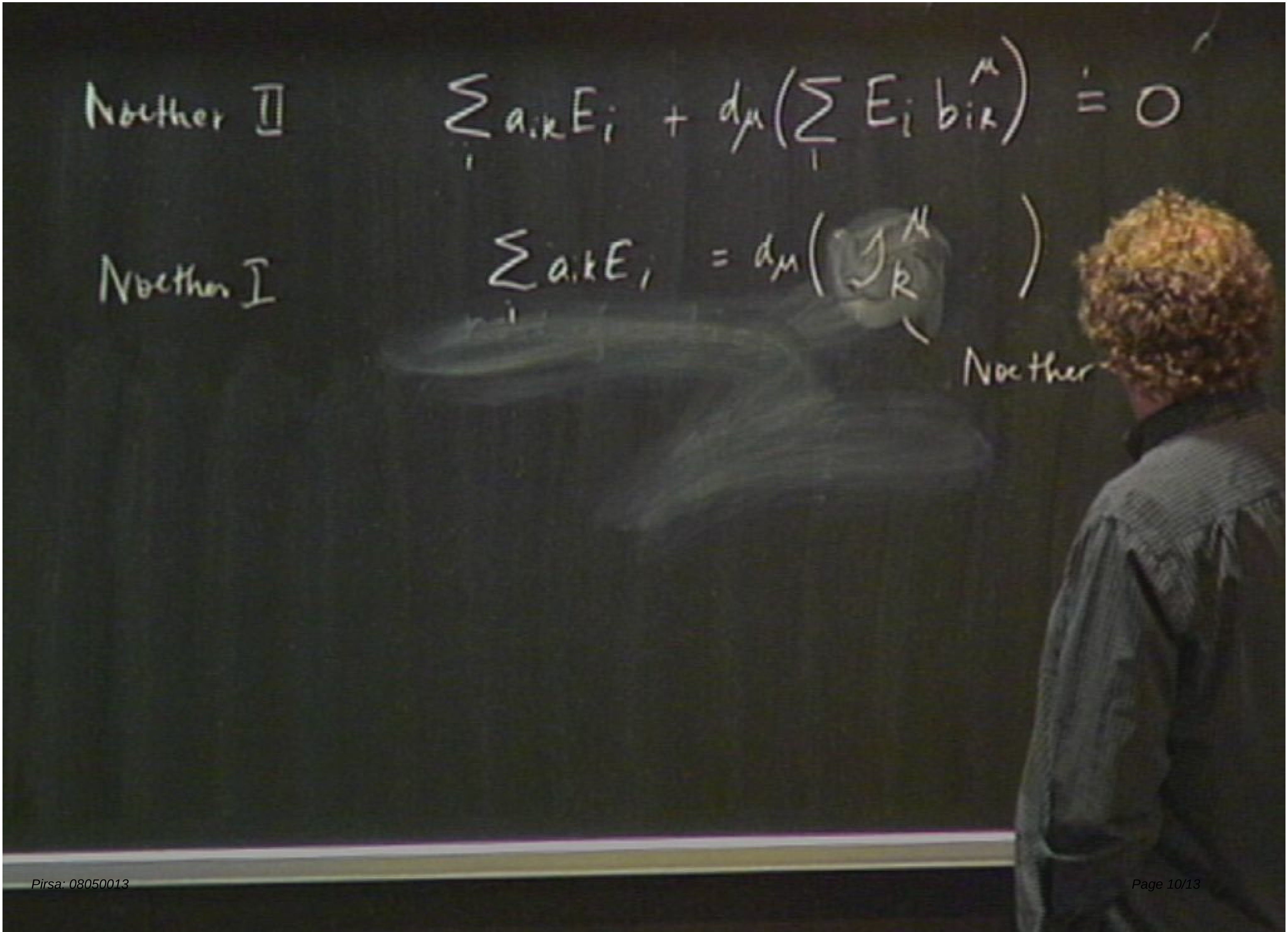
HRB & K. Brading (2002)
H.R.B. (2005)

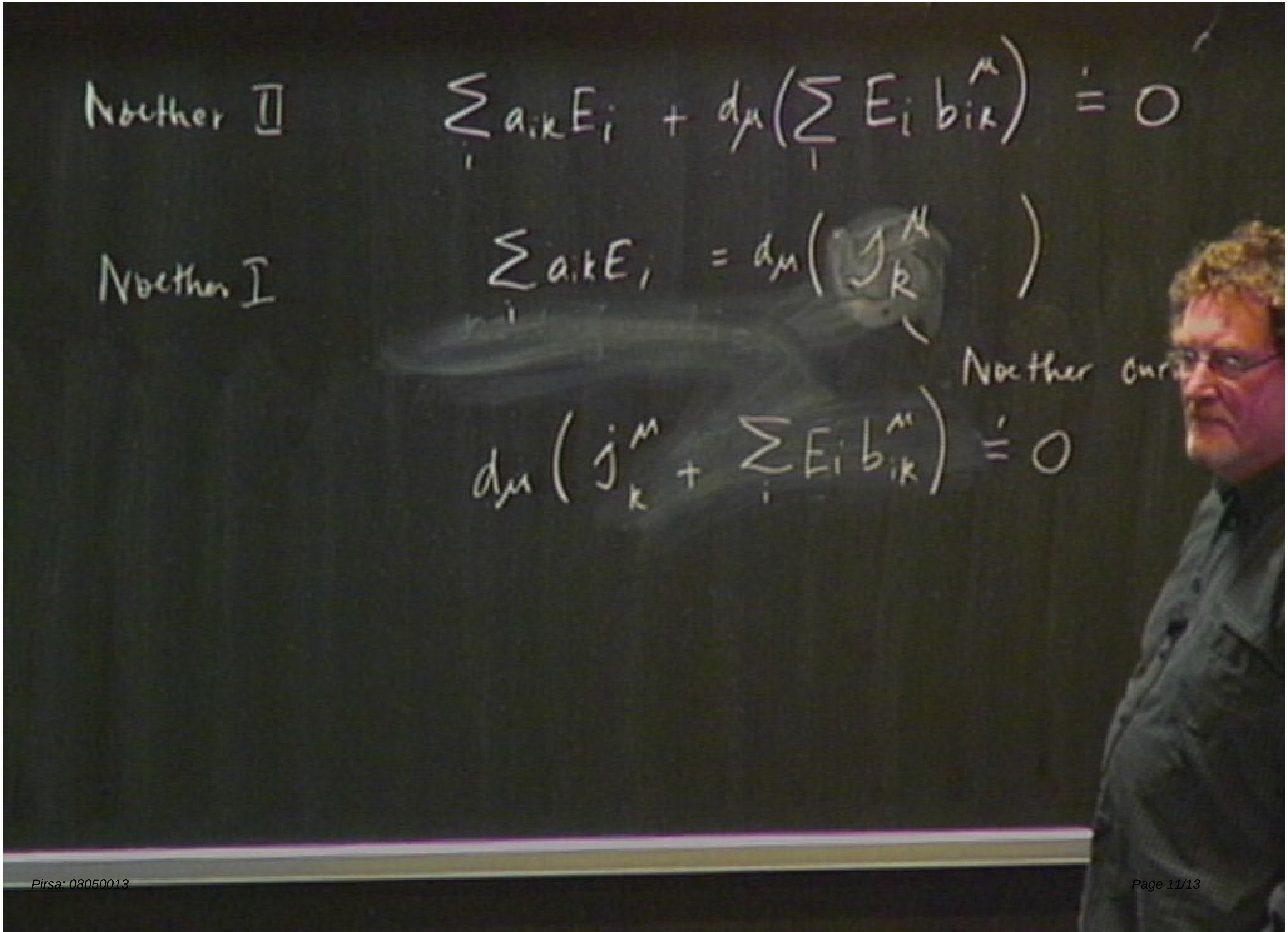

Noether II
$\sum_i a_{ik} E_i + d_\mu(\sum E_i b_{ik}) = 0$
nostrification
$\frac{v^2}{c^2}$


Noether II
$\sum_i a_{ik} E_i + d_\mu \left(\sum E_i b_{ik}^\mu\right) \doteq 0$

Noether II $\qquad \sum_i a_{ik} E_i + d_\mu \Big(\sum_i E_i \, b_{ik}^{\mu} \Big) \doteq 0$

Noether I $\qquad \sum_i a_{ik} E_i = d_\mu \Big(\sum_i E_i$


Noether II
$\sum_i a_{ik} E_i + d_\mu\left(\sum_i E_i \, b_{ik}^{\mu}\right) \doteq 0$
Noether I
$\sum_i a_{ik} E_i = d_\mu\left(J_k^{\mu} \right)$
Noether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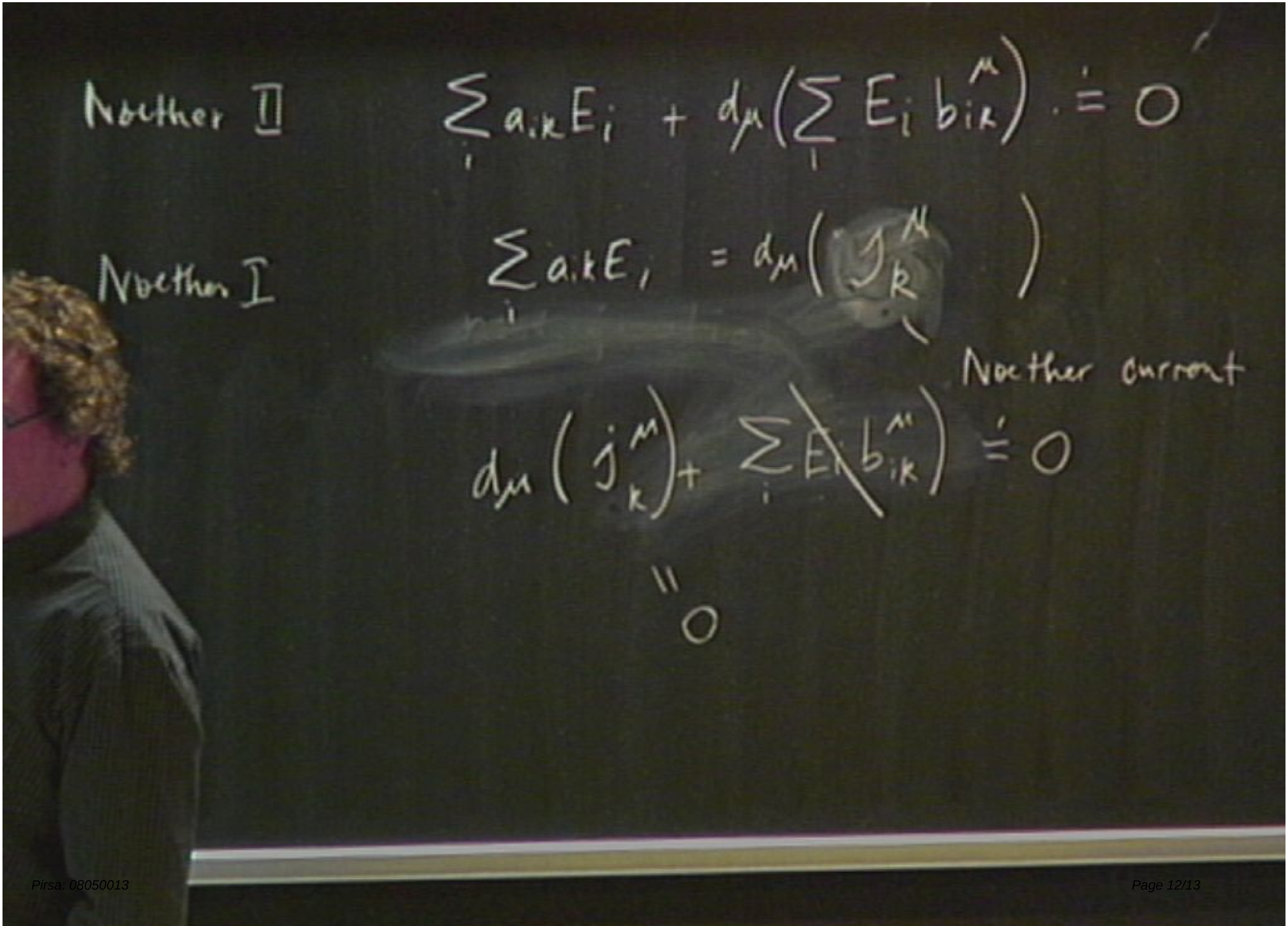
Noether I $\sum_i a_{ik} E_i = d_\mu \left(j^\mu_k \right)$
Noether cur
$d_\mu \left(j^\mu_k + \sum_i E_i b^\mu_{ik} \right) \doteq 0$



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Noether current
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