Title: Symmetry Principles in Physics - Lecture 6A

Date: May 26, 2008 11:00 AM

URL: http://pirsa.org/08050010

Abstract:

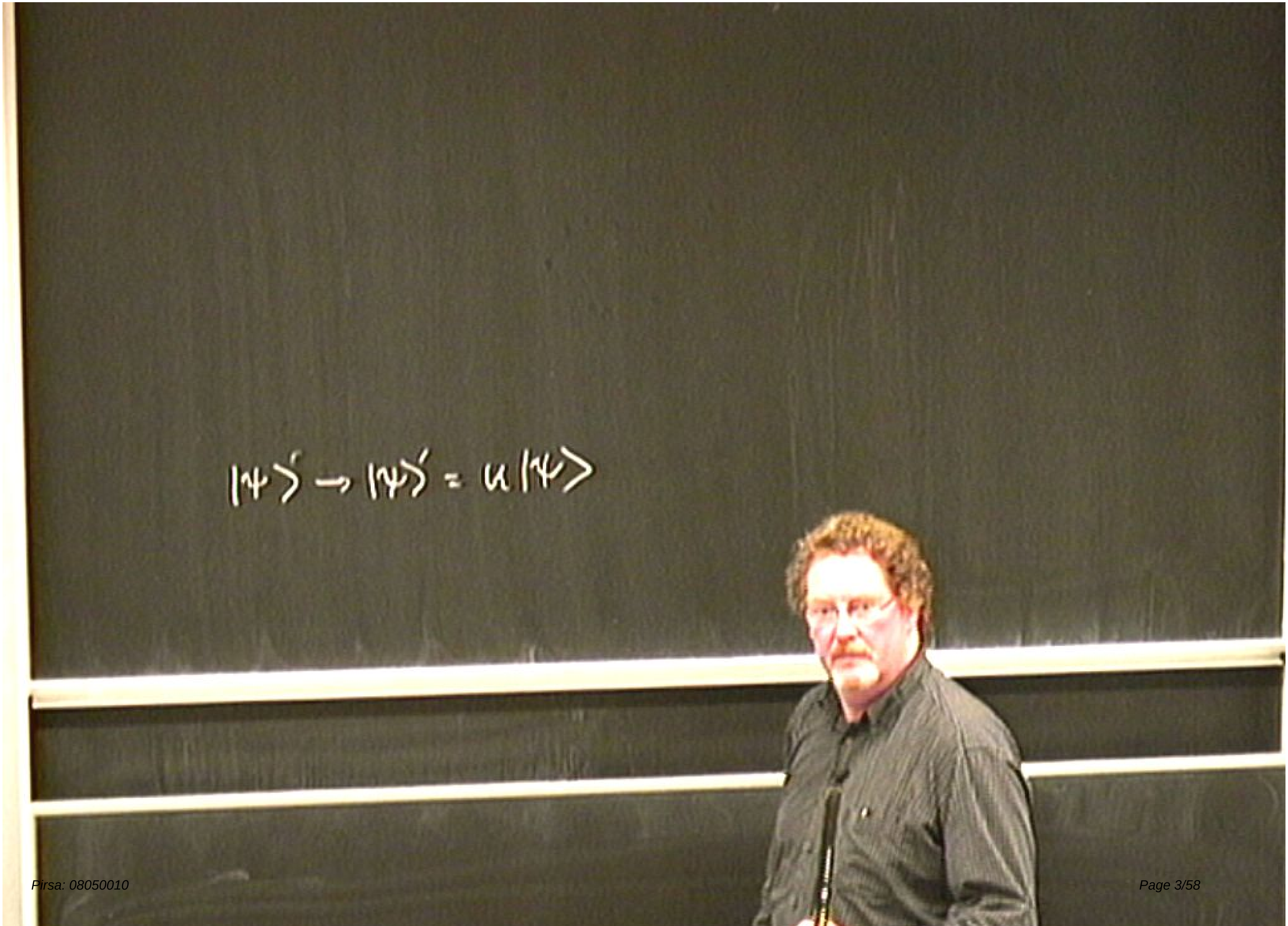
Galilean Covariance of QM : A Romp

① Covariance of Schröd Eqn

- Some implications
- Minkowski lurking in (extended) Galilean space-time
- How do operators transform?

② Jauch's (1964, 1968) (nm.) theorem (Galilean covariance ⇒ minimal EM coupling)

- the theorem
- Feynman's argument & Hughes' 1999 variant
- The inertial analogue of the AB effect.
- Galilean covariant EM.


$|\psi\rangle \to |\psi\rangle' = U|\psi\rangle$

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Galilean Covariance of QM : A Romp

① Covariance of Schröd Eqn.

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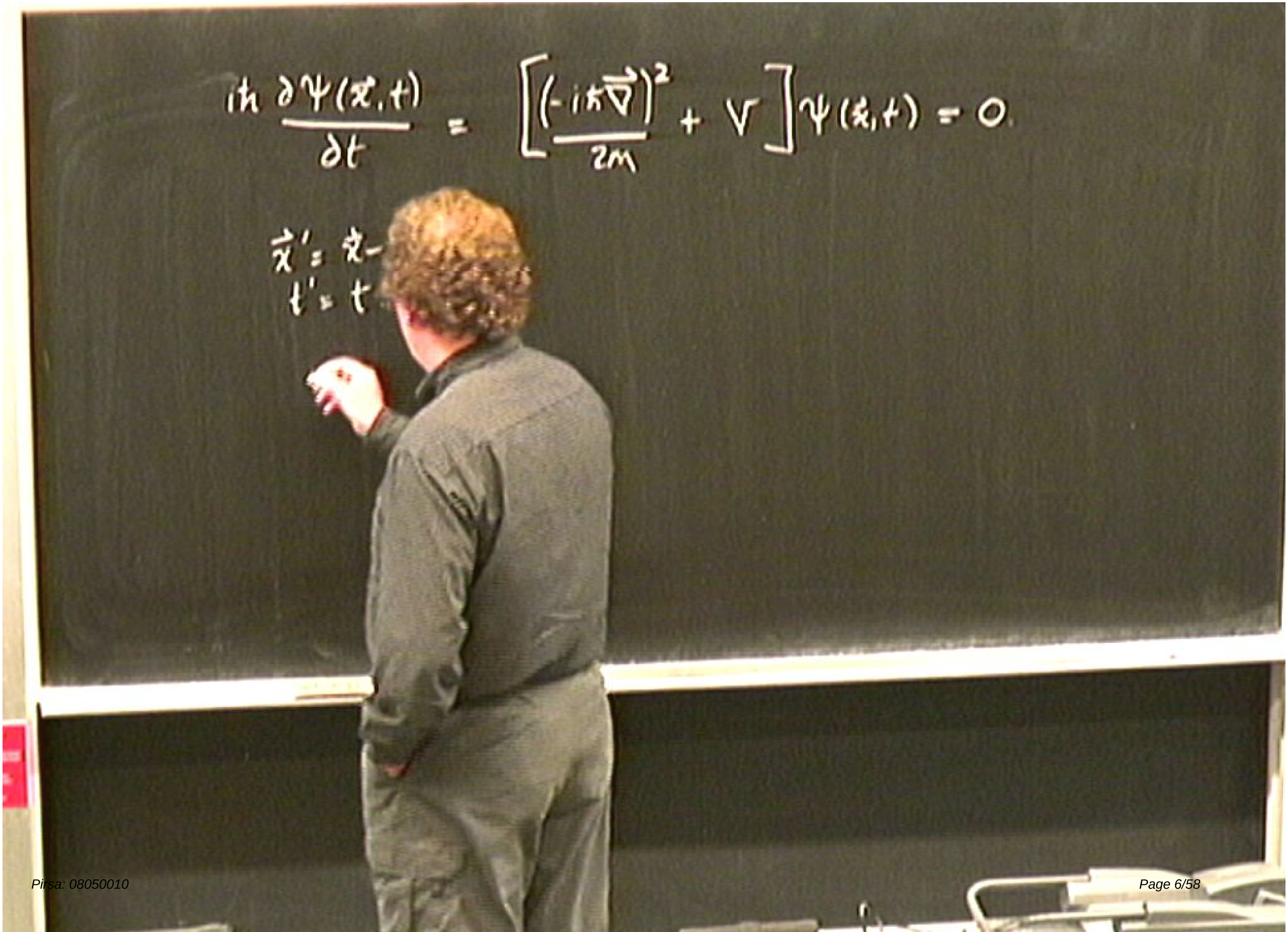
$i\hbar \frac{\partial \psi(\vec{x},t)}{\partial t} = \left[\frac{(-i\hbar\vec{\nabla})^2}{2m} + V \right] \psi(\vec{x},t) = 0$
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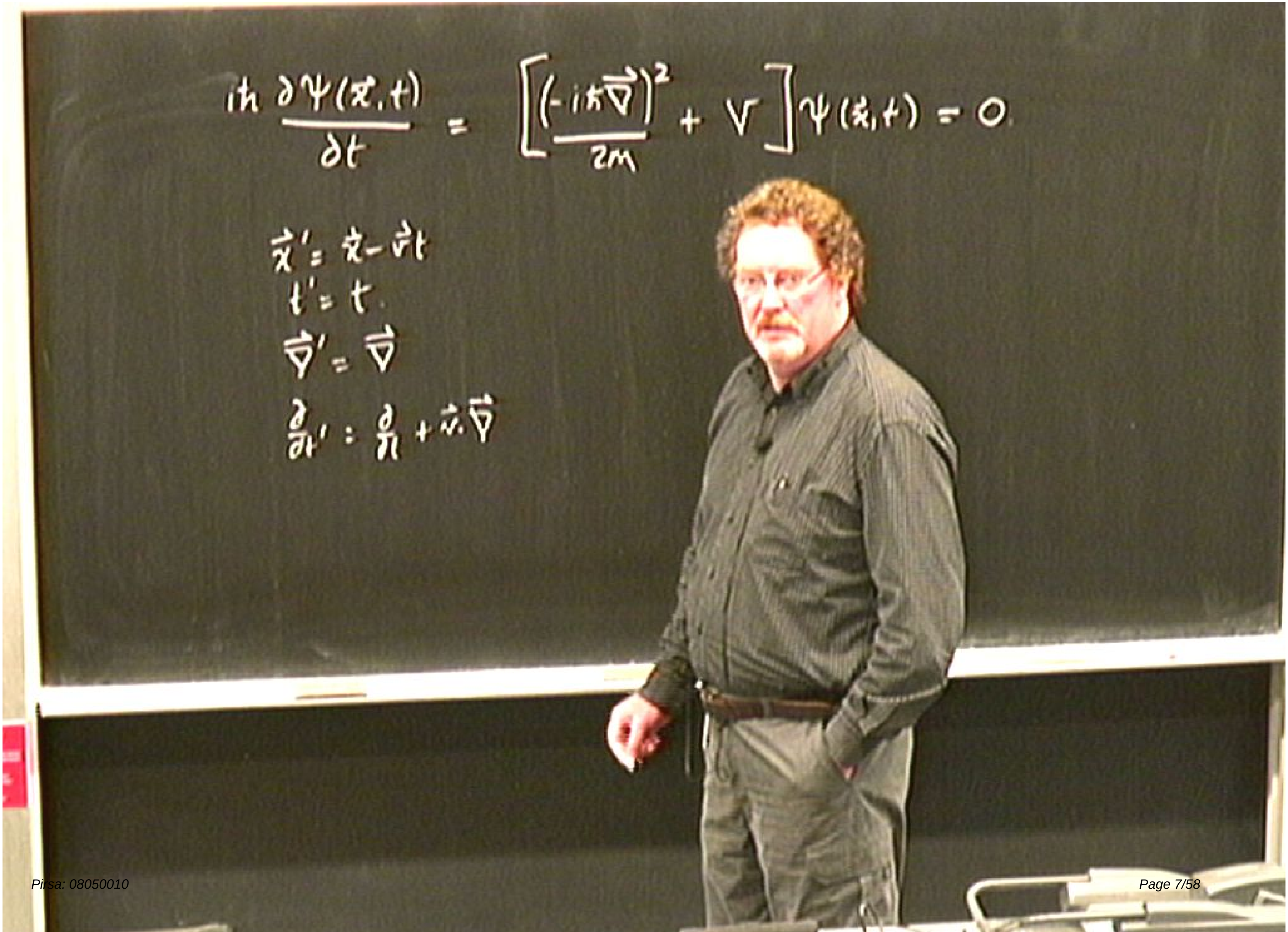
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$$\partial t \qquad \qquad 2m$$

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$$\psi'(\vec{x}', t') = e^{i\phi}\,\psi(\vec{x}, t)$$

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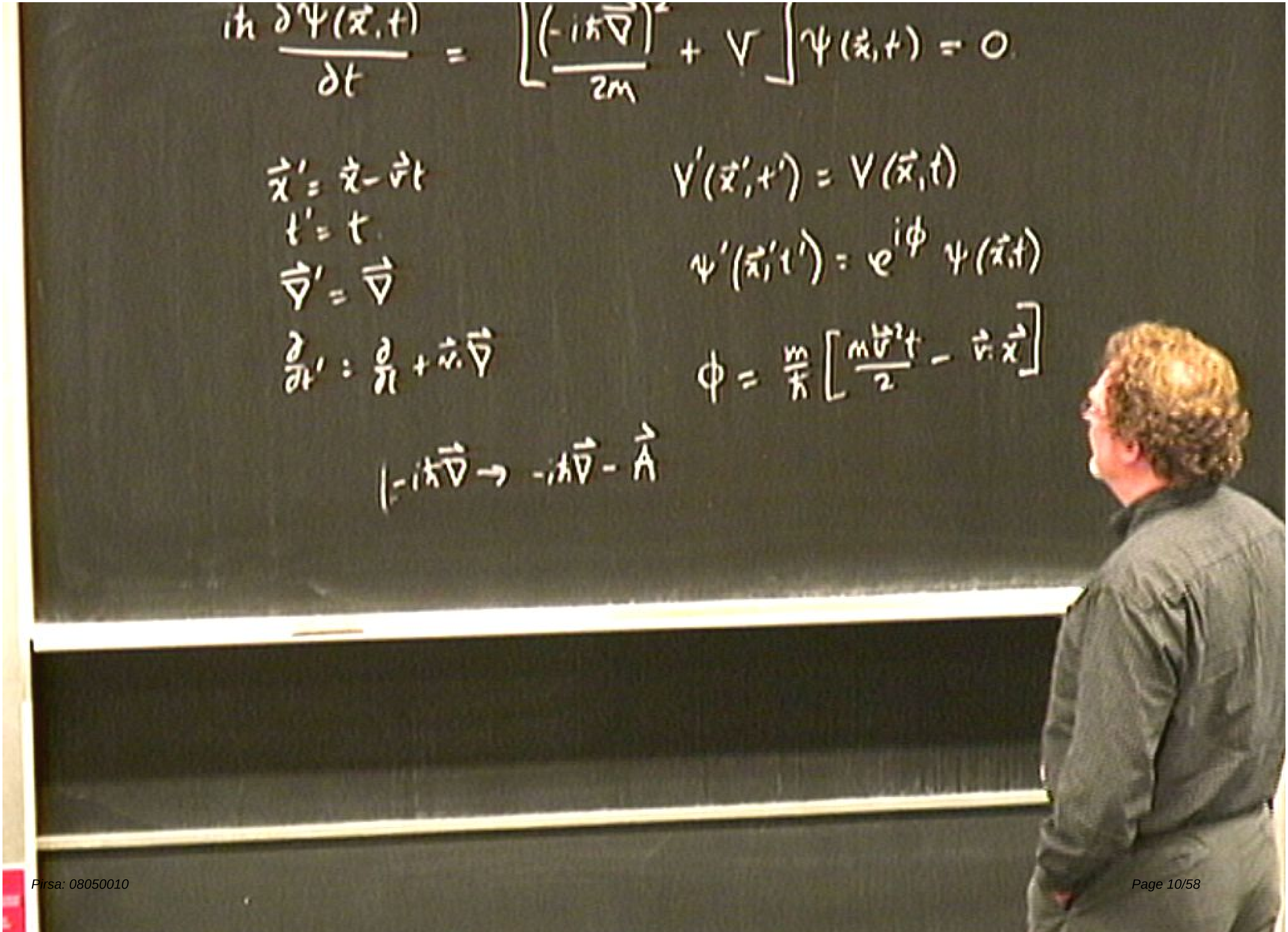
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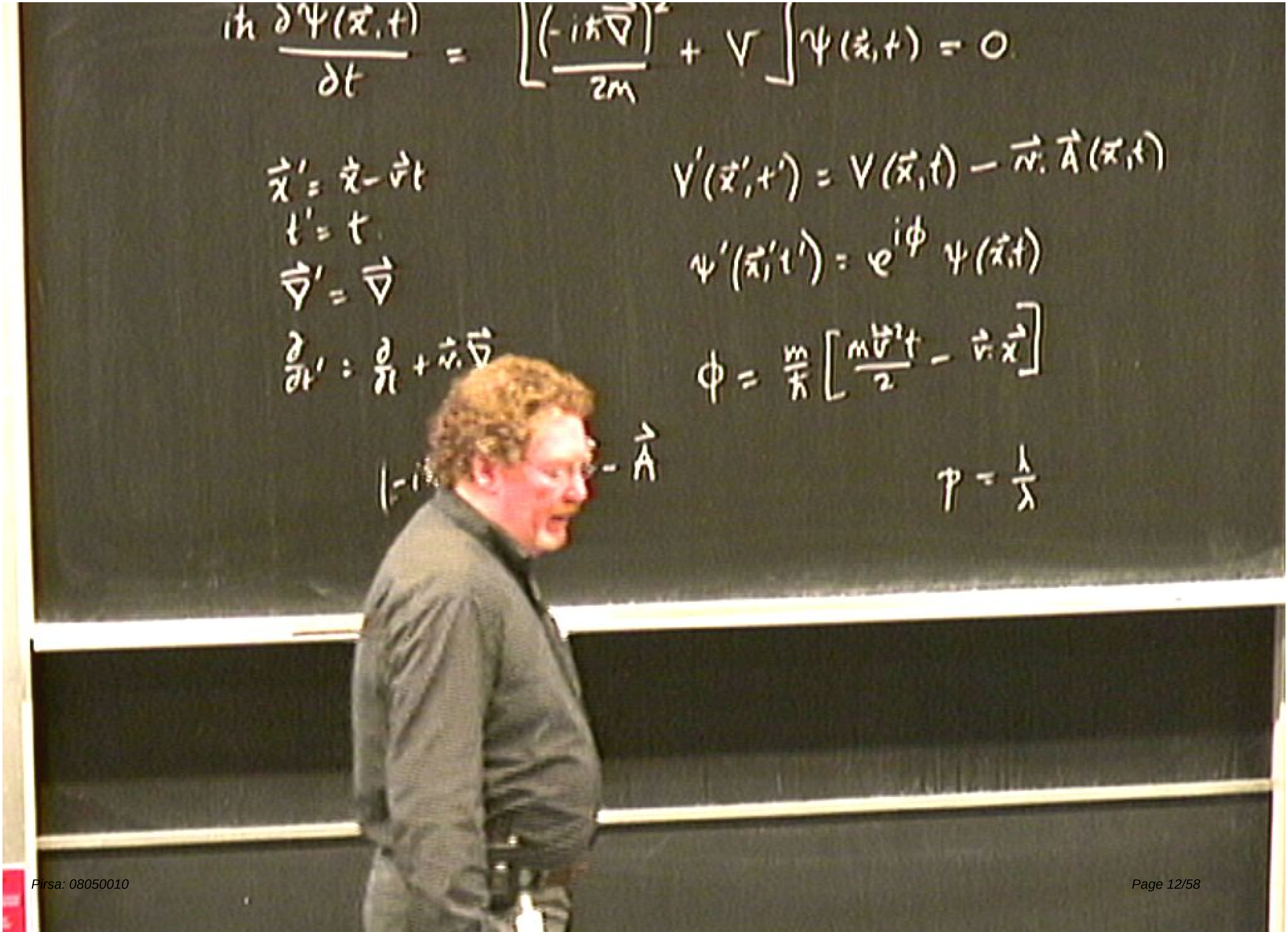
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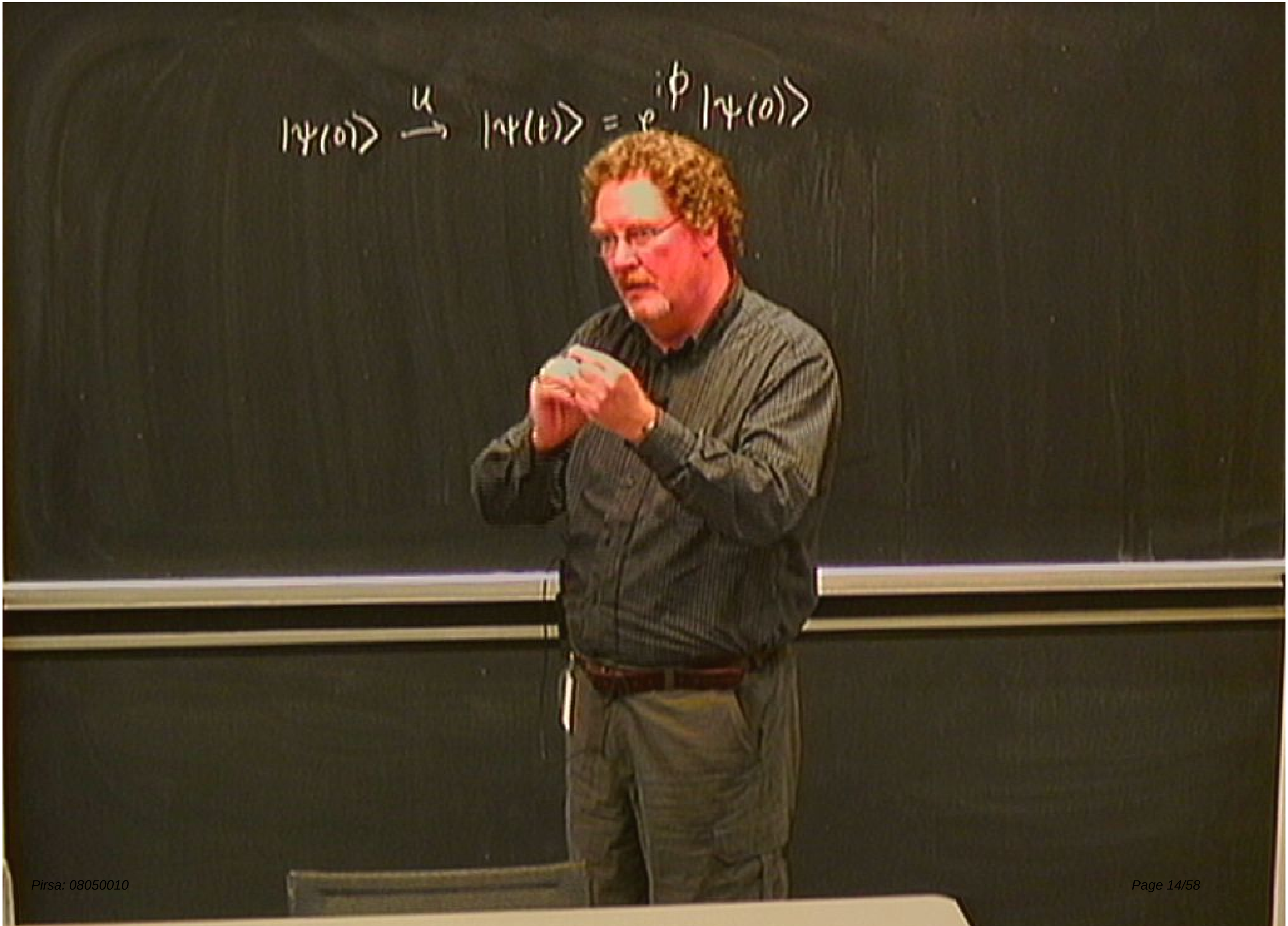
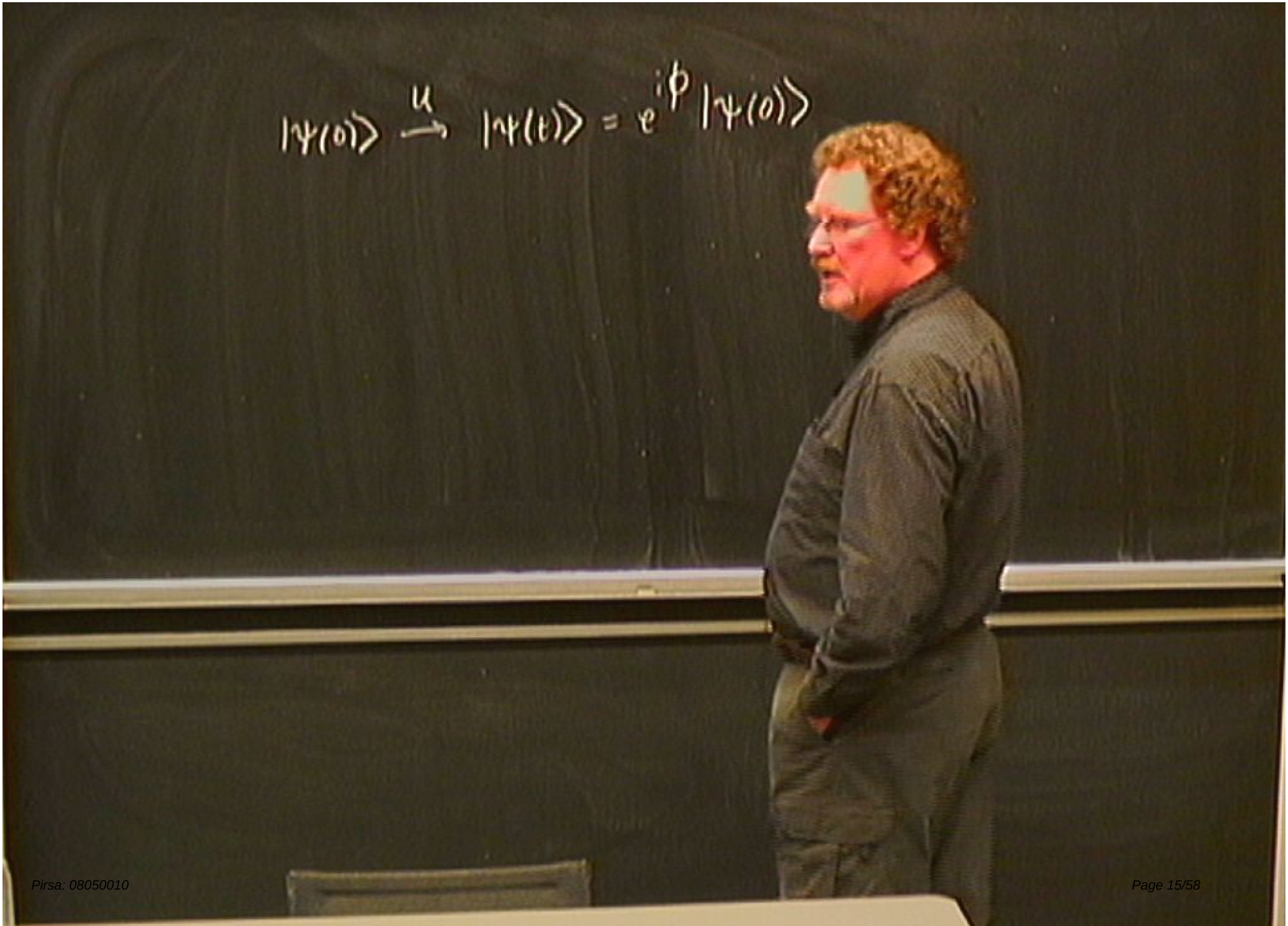

$|\psi(0)\rangle \xrightarrow{U} |\psi(t)\rangle = e^{i\phi}|\psi(0)\rangle$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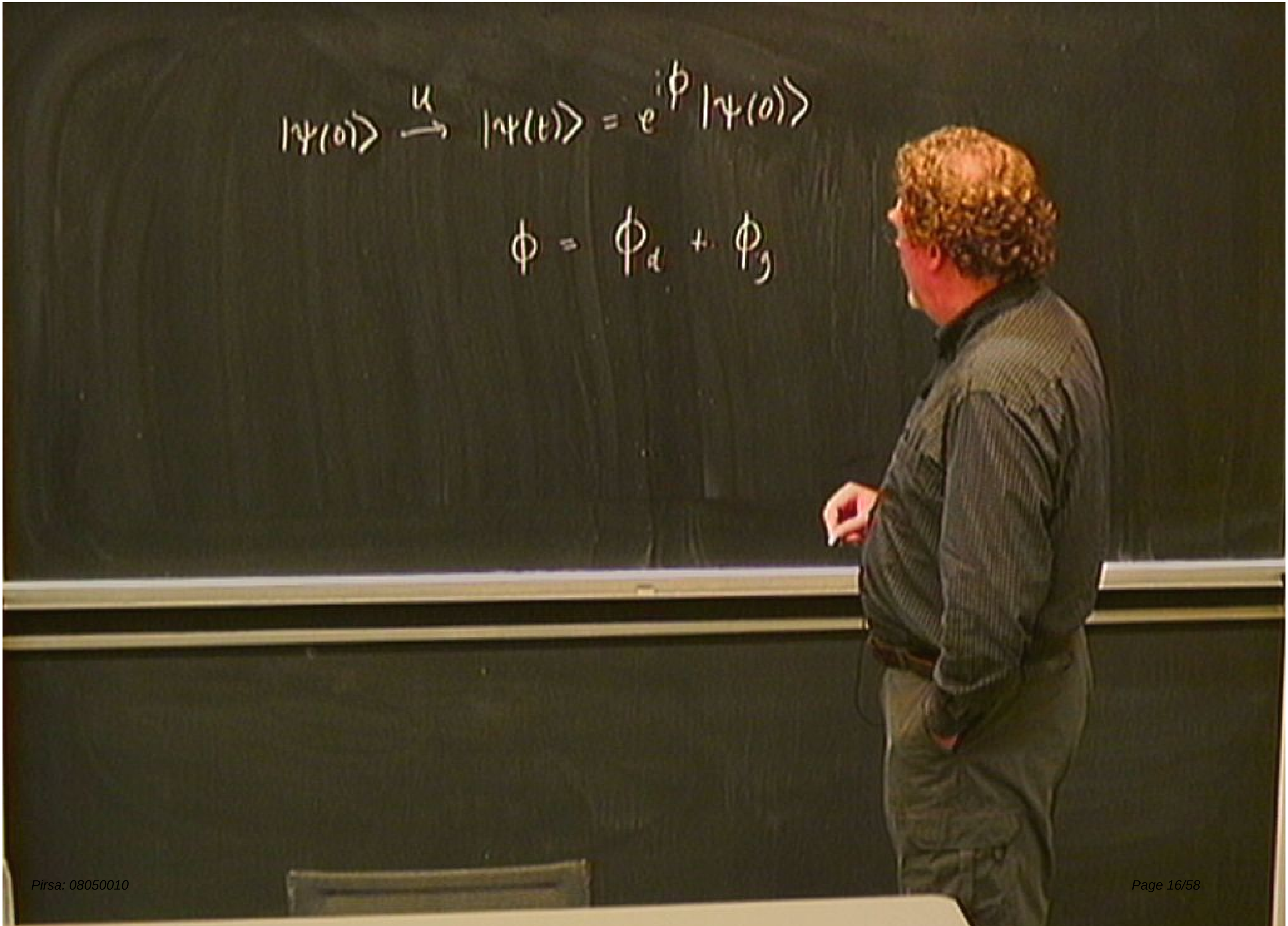
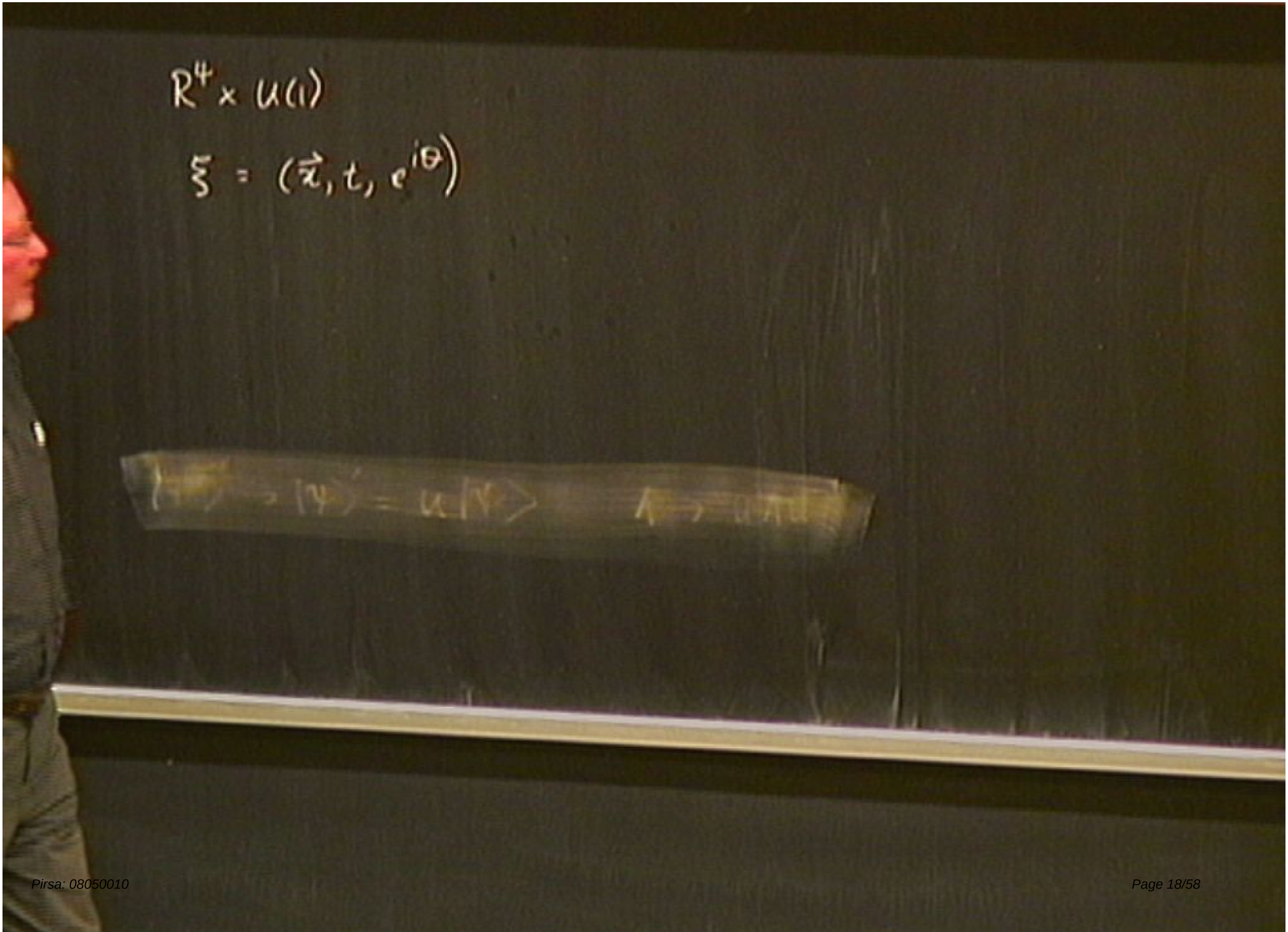


$|\psi(0)\rangle \xrightarrow{U} |\psi(t)\rangle = e^{i\phi}|\psi(0)\rangle$
$\phi = \phi_d + \phi_g$

$R^4 \times U(1)$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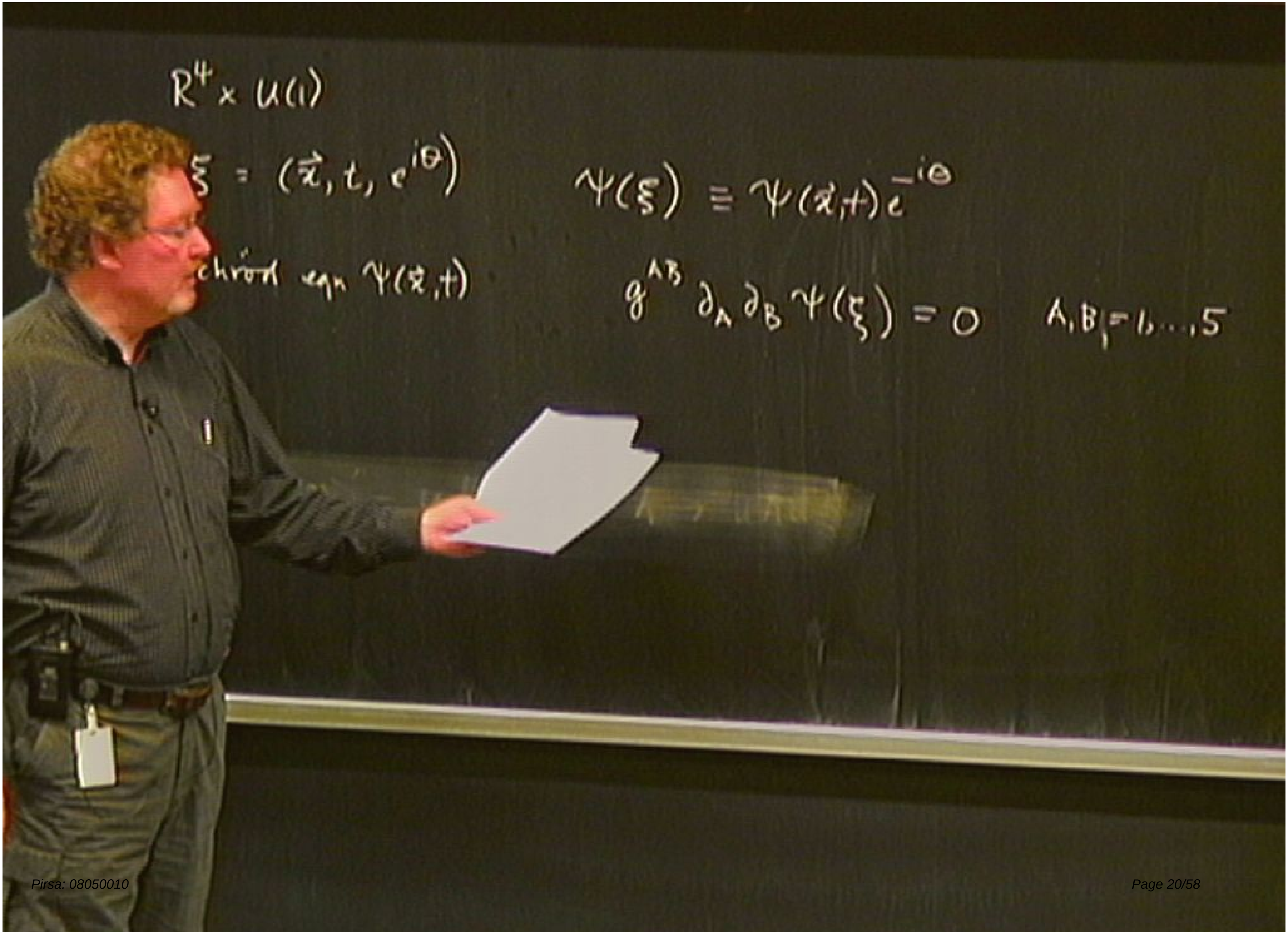
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$\xi = (\vec{x}, t, e^{i\theta})$
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chröd eqn $\psi(\vec{x},t)$
$g^{AB}\partial_A \partial_B \psi(\xi) = 0 \quad A,B = 1\ldots 5$

Free Schröd eqn $\psi(\vec{x},t)$

$$g^{AB}\,\partial_A \partial_B \psi(\xi) = 0 \qquad A,B = 1,\dots,5$$

$$g_{AB} = \begin{bmatrix} \frac{1}{2m} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2m} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2m} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & -\frac{1}{2} & 0 \end{bmatrix}$$

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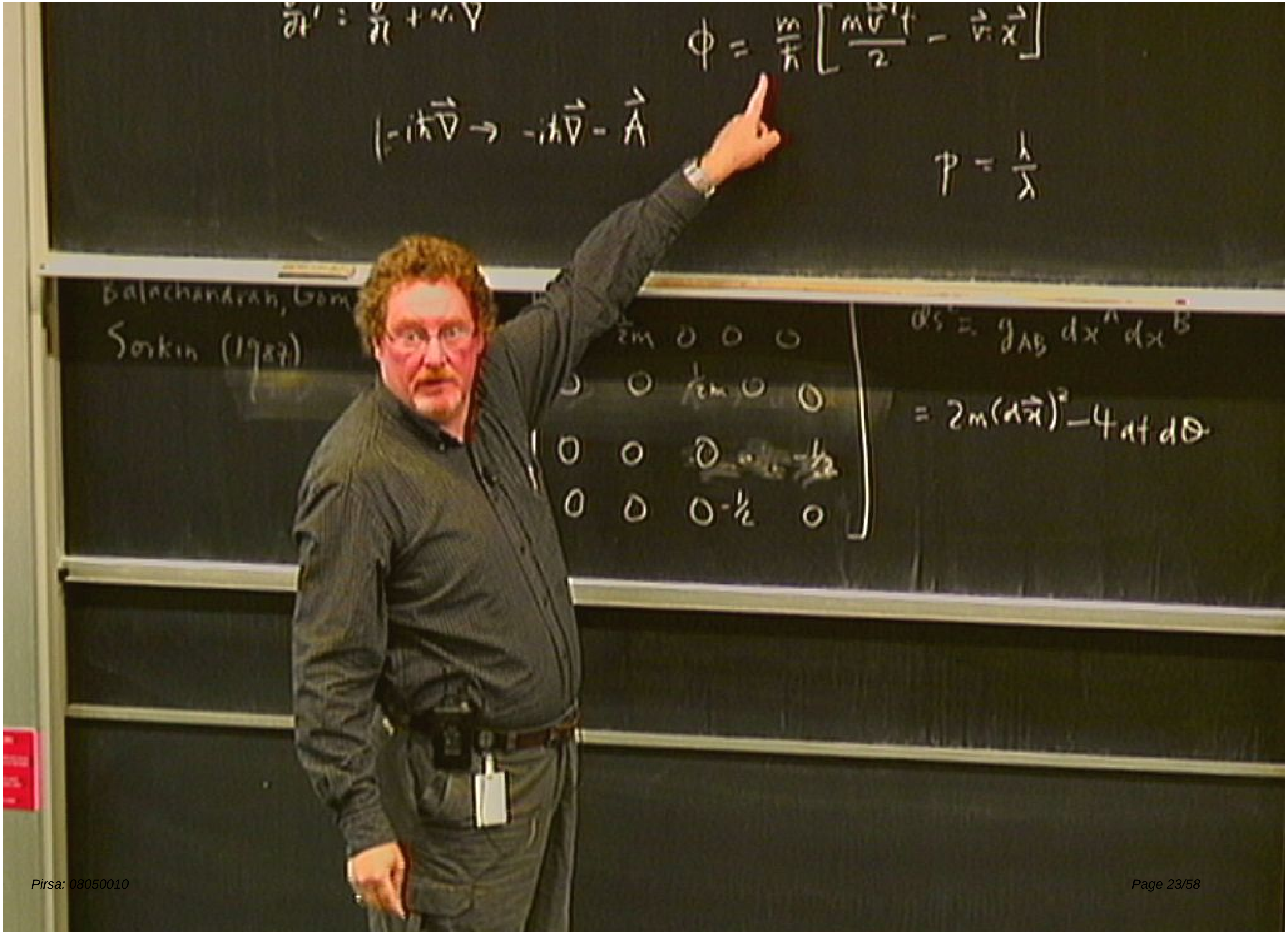
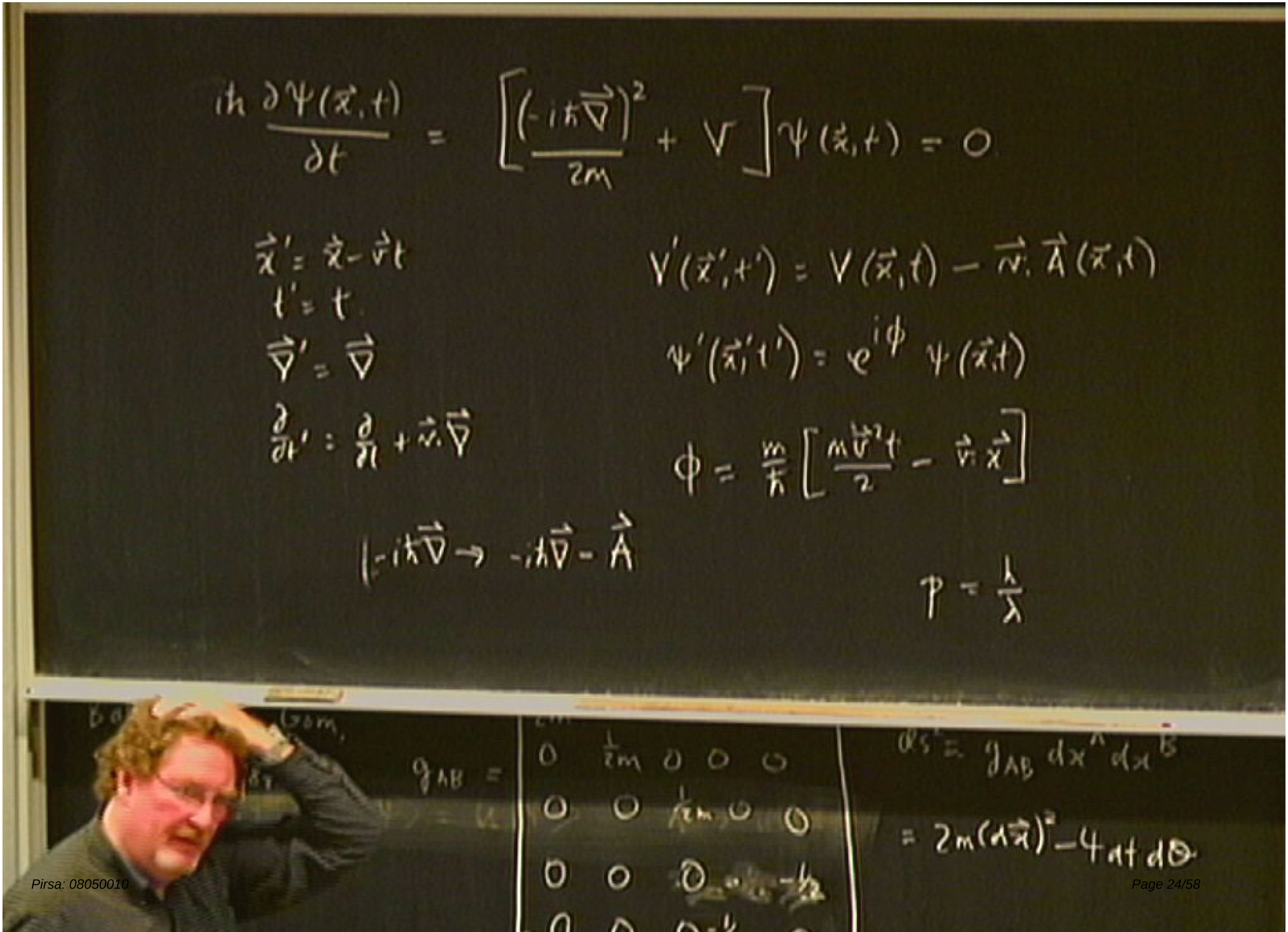
$$ds^2 = g_{AB}\, dx^A dx^B$$

$$= 2m(d\vec{x})^2 - 4\, dt\, d\theta$$


Balachandran, Gom
Sorkin (1987)



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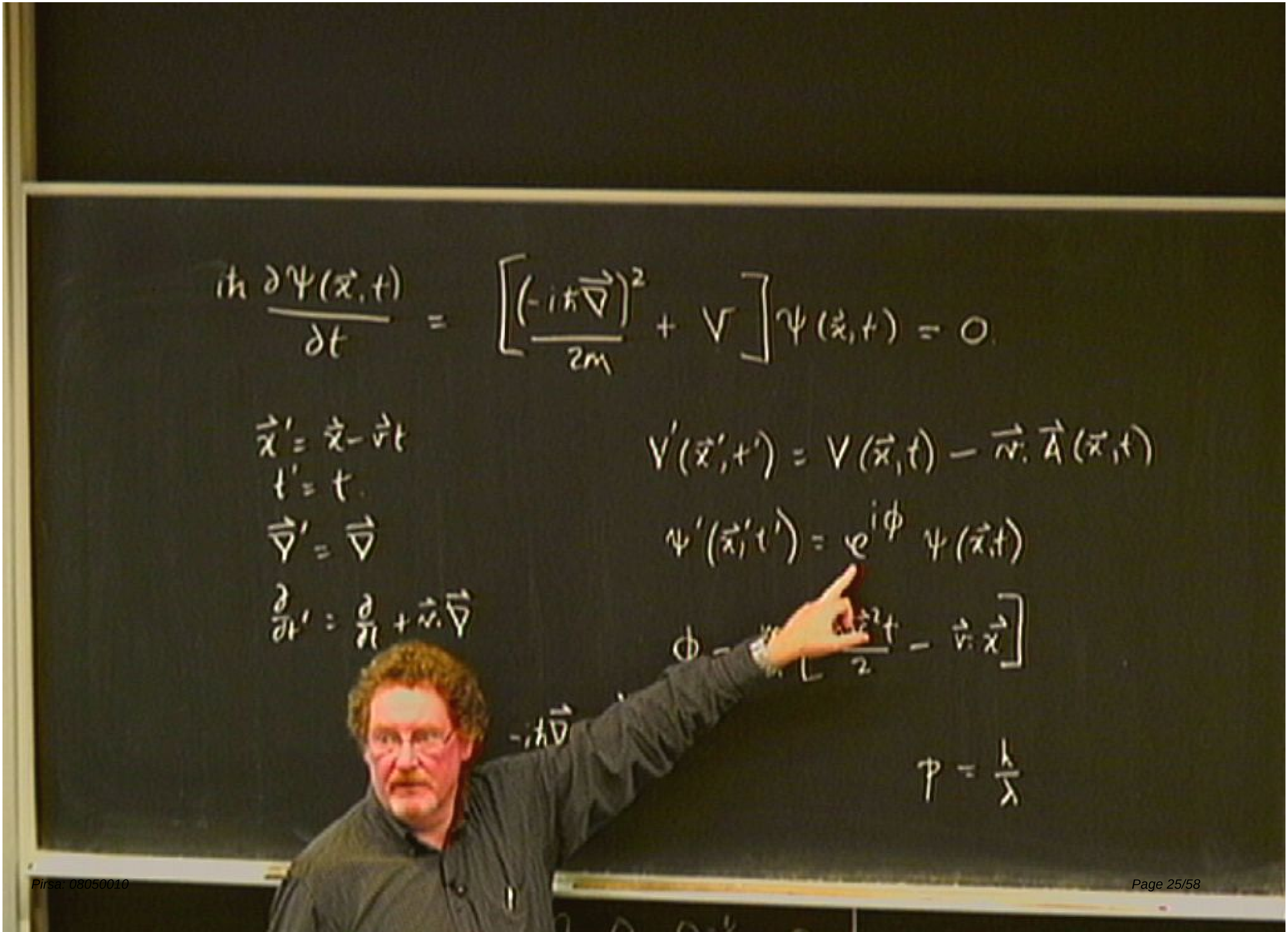
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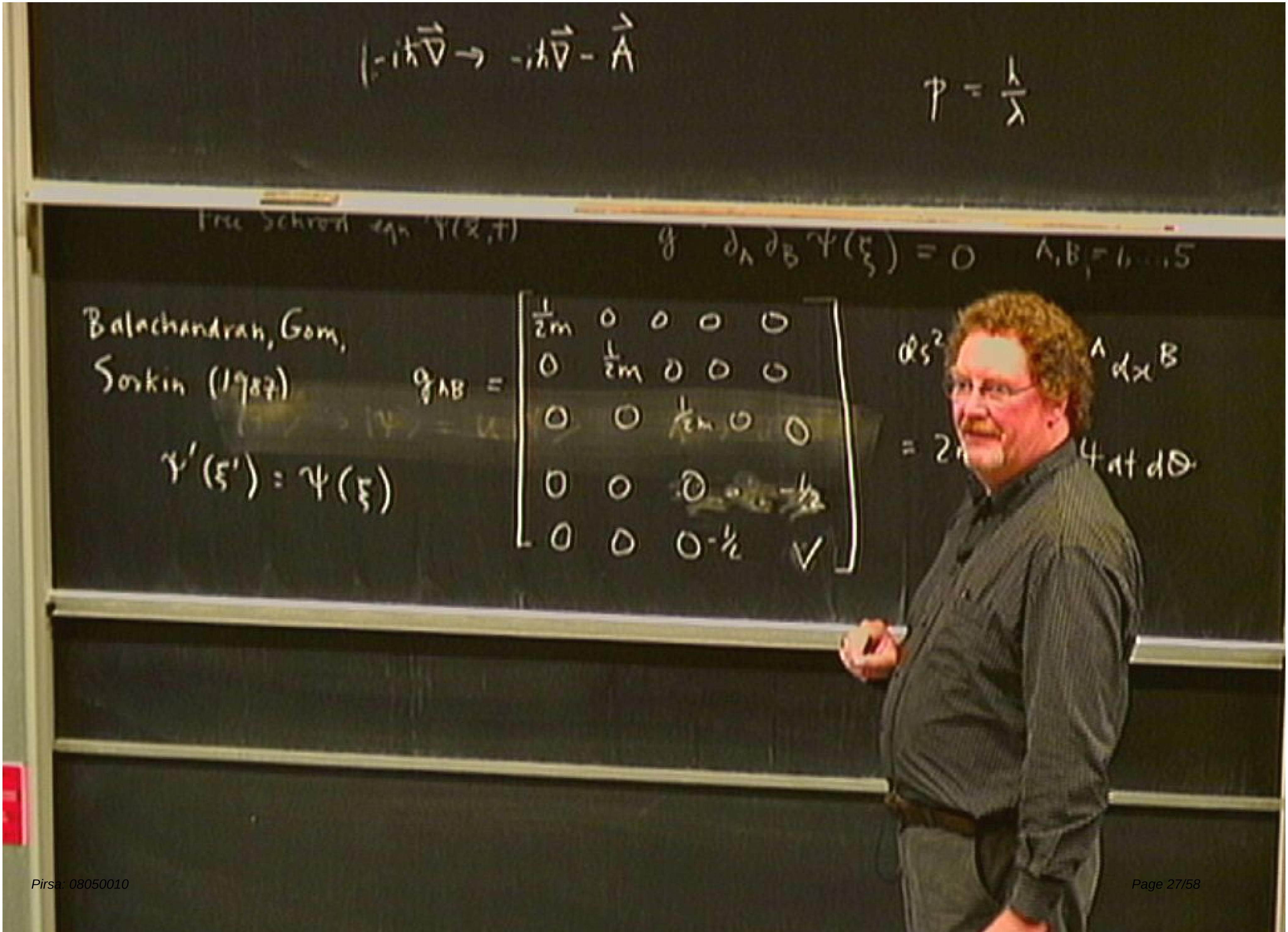
-iħ∇ → -iħ∇ - A
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Free Schrod eqn ψ(x,t)
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gAB =
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ds²
dx^B
= 2m
dt dΘ

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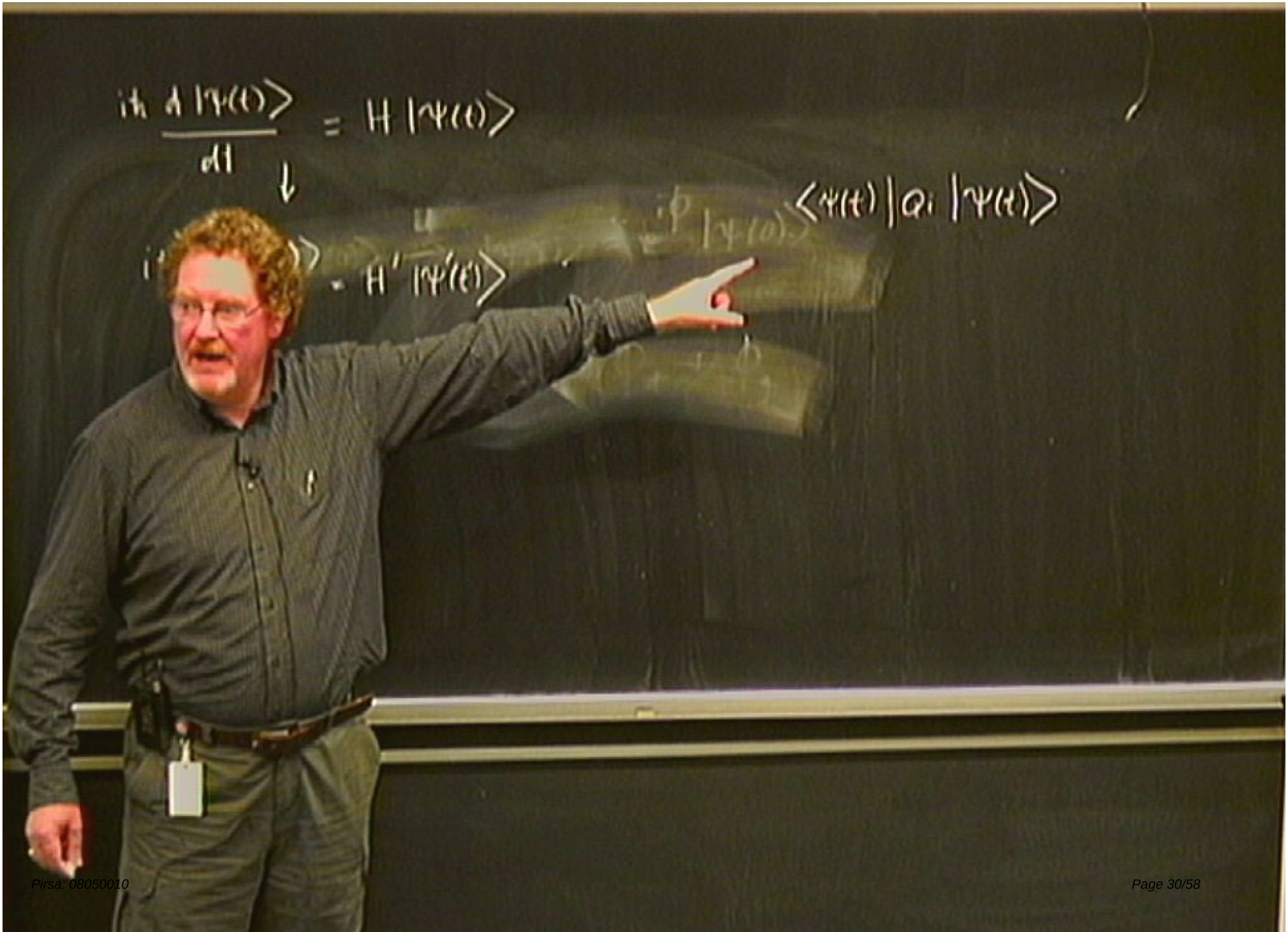
$$i\hbar \frac{d|\psi(t)\rangle}{dt} = H|\psi(t)\rangle$$

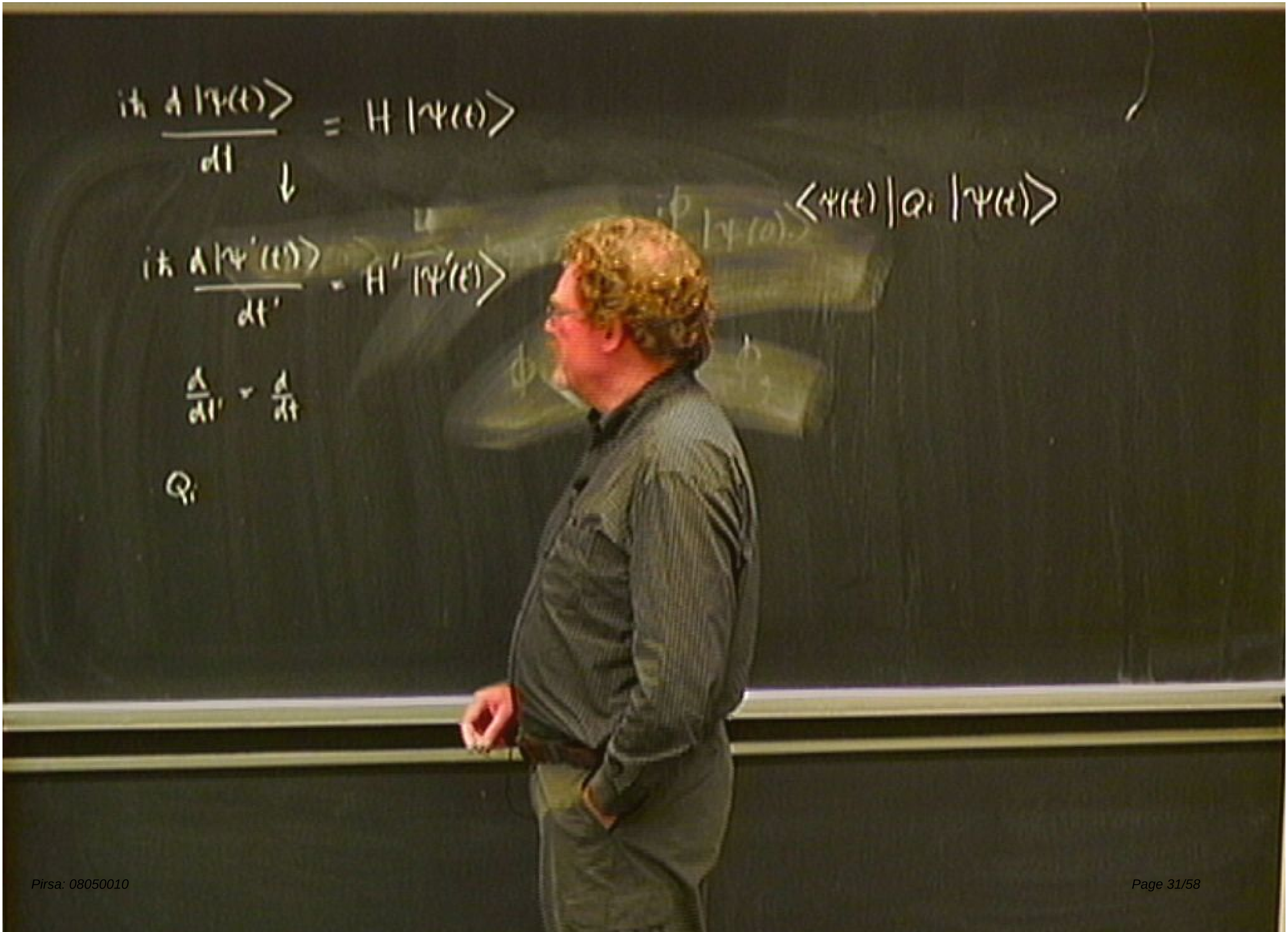
$$\downarrow$$

$$i\hbar \frac{d|\psi'(t')\rangle}{dt'} = H'|\psi'(t')\rangle$$

$$\frac{d}{dt'} = \frac{d}{dt}$$

Q_i


iħ d|Ψ(t)⟩ / dt = H |Ψ(t)⟩
⟨Ψ(t)| Q_i |Ψ(t)⟩
= H' |Ψ'(t)⟩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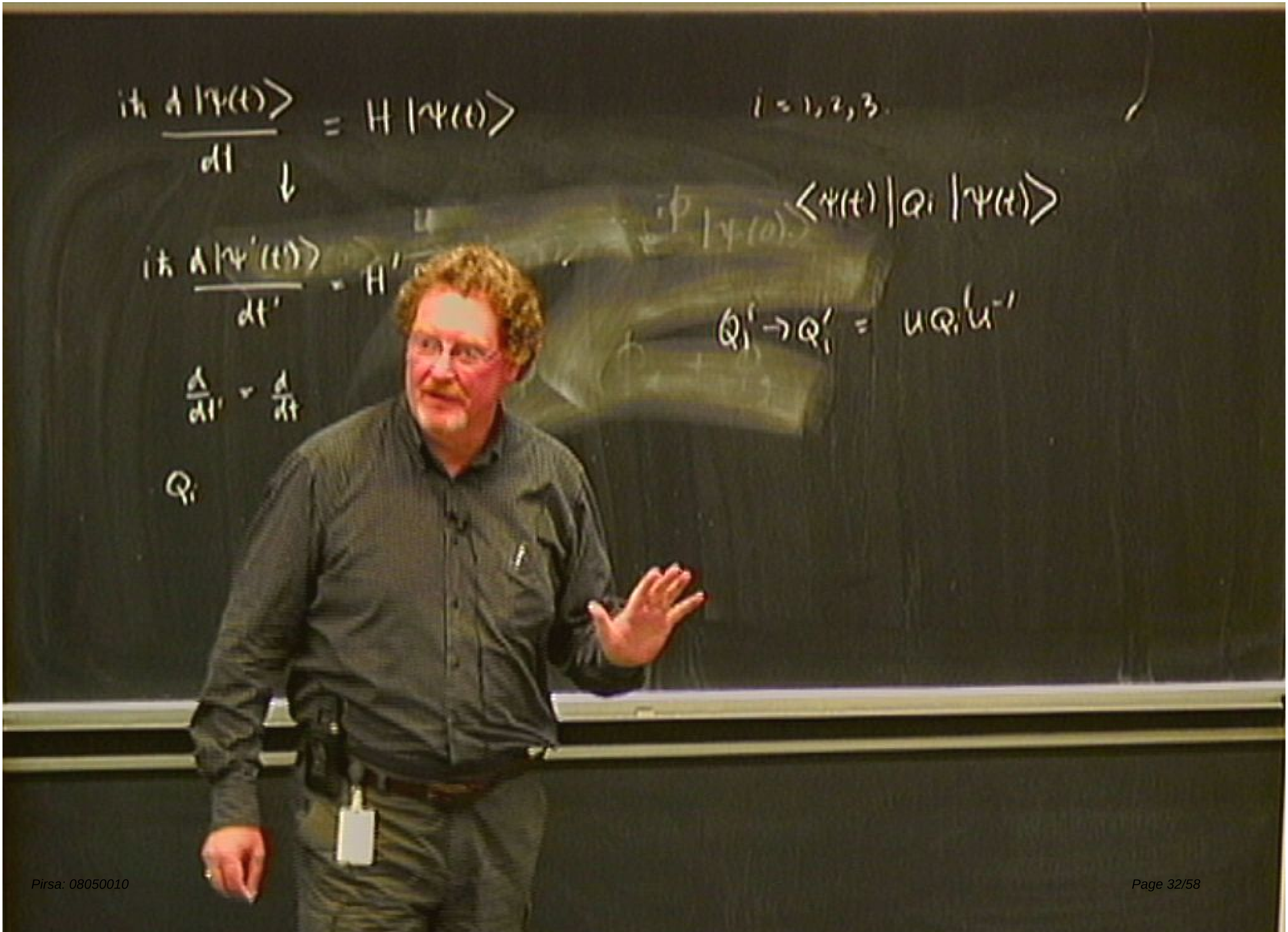
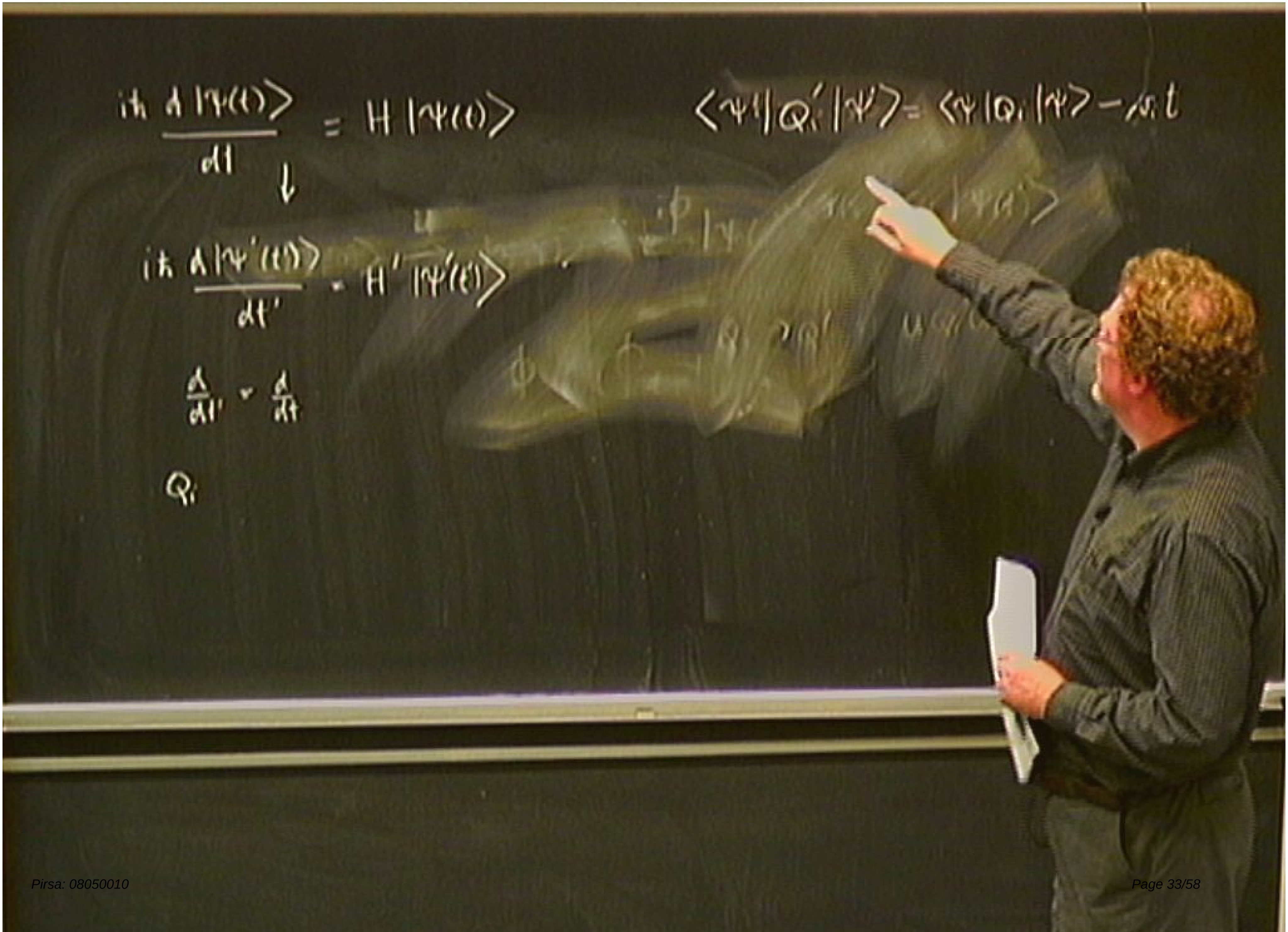
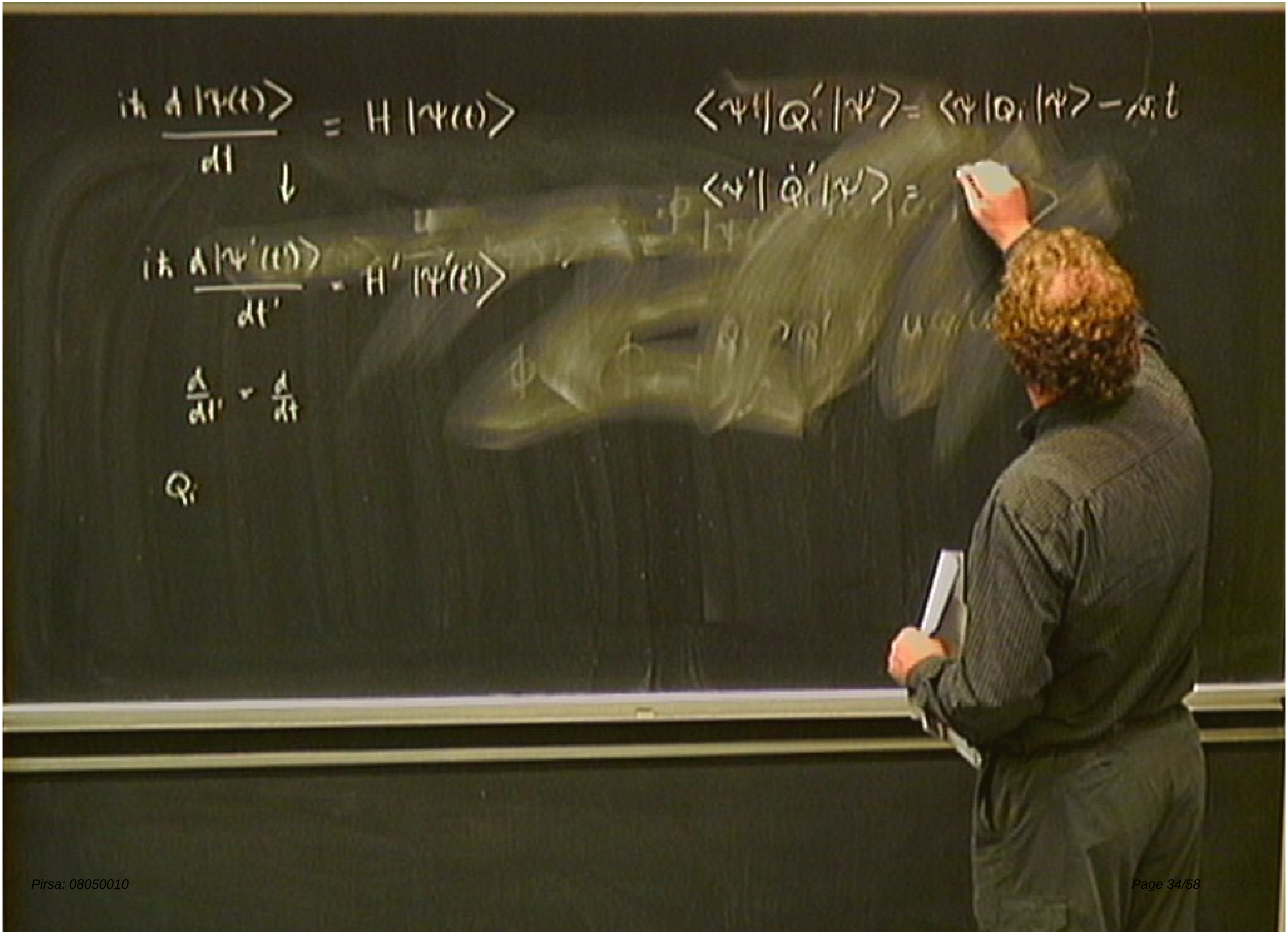
$\langle\psi(t)|Q_i|\psi(t)\rangle$
$i\hbar \frac{d|\psi'(t')\rangle}{dt'} = H'|\psi'(t')\rangle$
$\frac{d}{dt'} = \frac{d}{dt}$
Q_i



$i\hbar \frac{d|\psi(t)\rangle}{dt} = H|\psi(t)\rangle$
$i = 1, 2, 3.$
$\langle\psi(t)|Q_i|\psi(t)\rangle$
$i\hbar \frac{d|\psi'(t')\rangle}{dt'} = H'$
$Q_i \to Q_i' = U Q_i U^{-1}$
$\frac{d}{dt'} = \frac{d}{dt}$
Q_i


iħ d|ψ(t)⟩/dt = H |ψ(t)⟩
⟨ψ'|Q'ᵢ|ψ'⟩ = ⟨ψ|Qᵢ|ψ⟩ − t
iħ d|ψ'(t')⟩/dt' = H' |ψ'(t')⟩
d/dt' = d/dt
Qᵢ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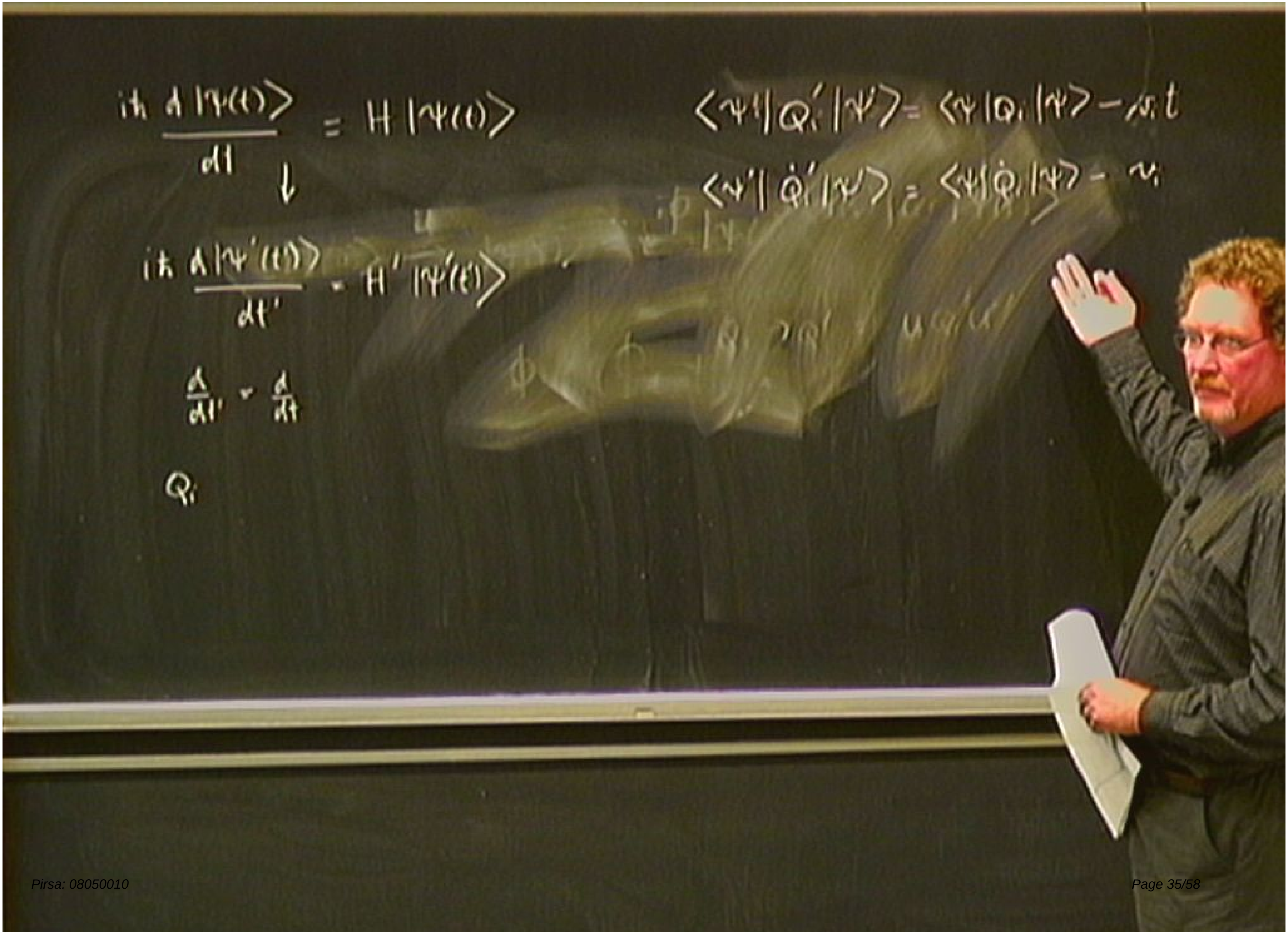
Q_i
⟨ψ'| Q'_i |ψ'⟩ = ⟨ψ| Q_i |ψ⟩ − v_i t
⟨ψ'| Q̇'_i |ψ'⟩ =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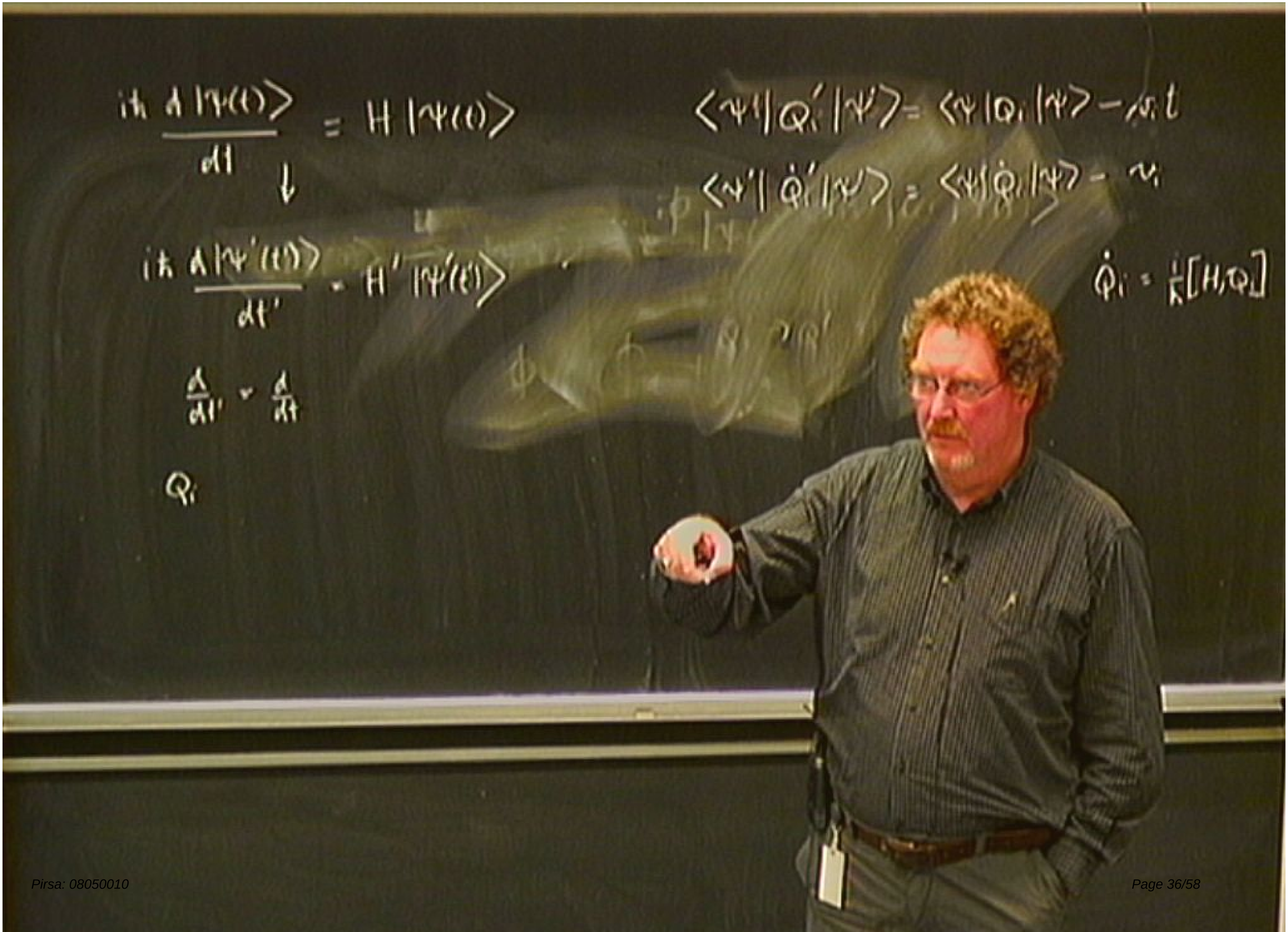
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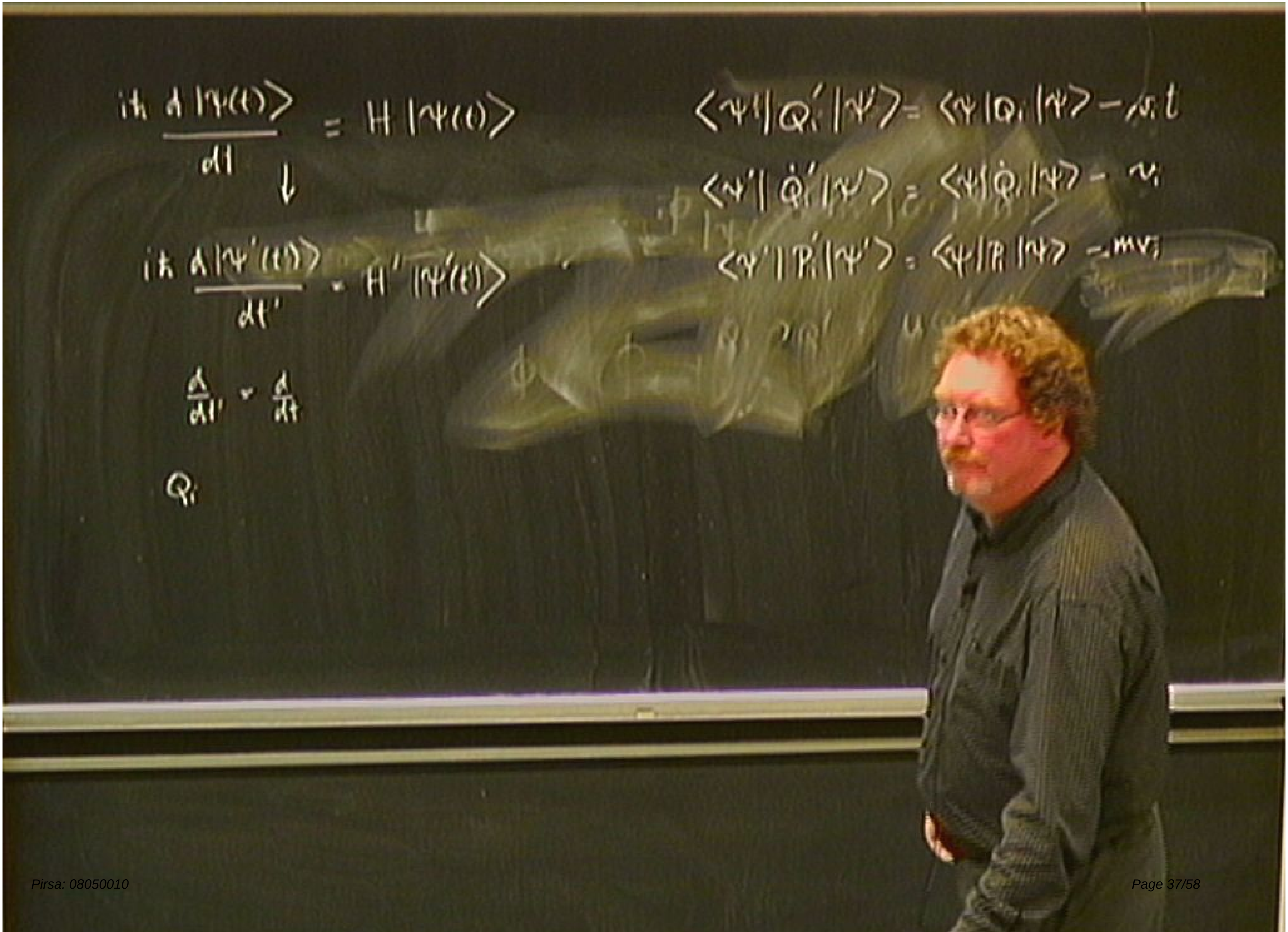
$\dot{Q}_i = \frac{i}{\hbar}[H,Q]$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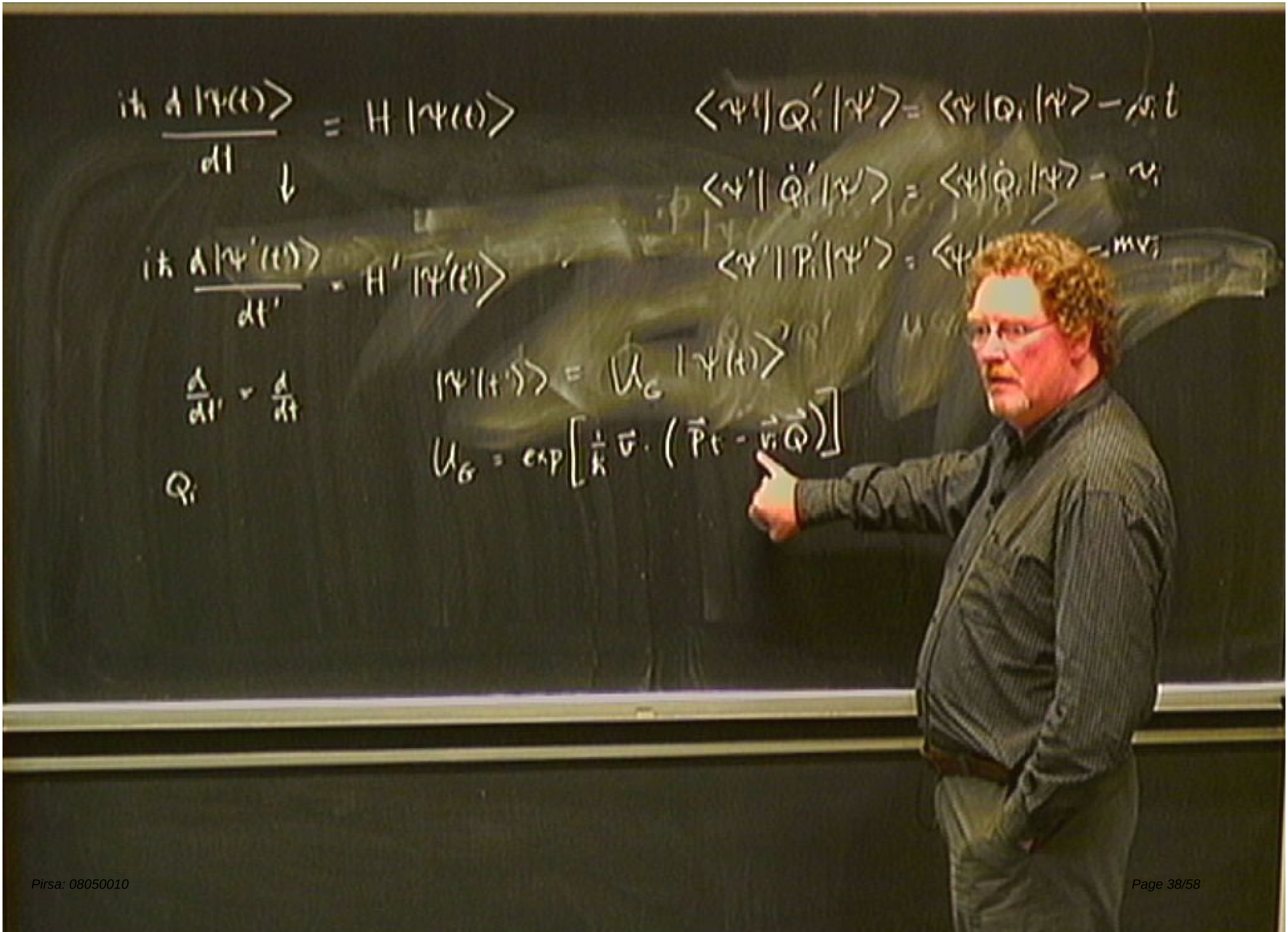
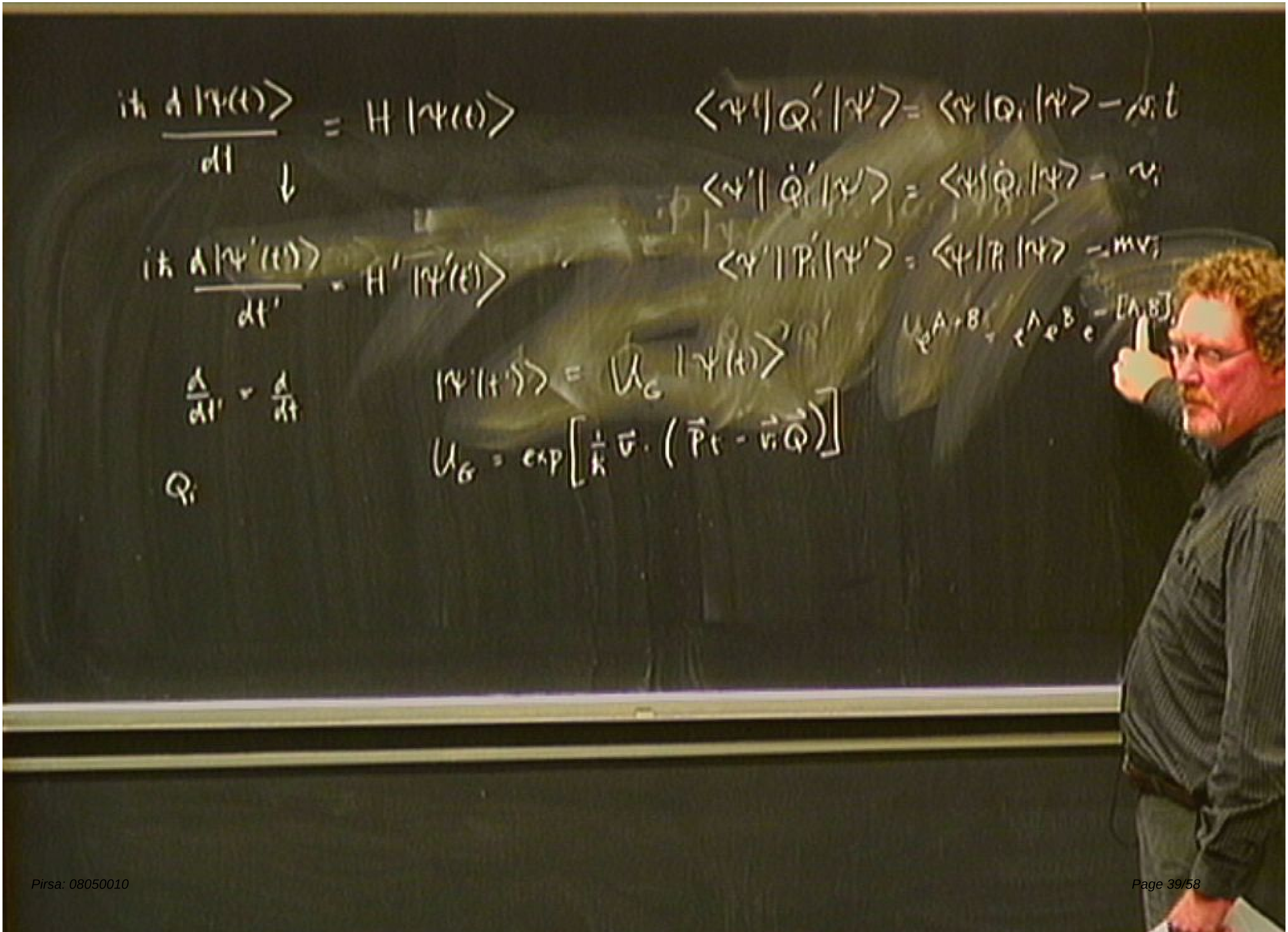
$\langle\psi'|P_i'|\psi'\rangle = \langle\psi|P_i|\psi\rangle - mv_i$


iħ d|ψ(t)>/dt = H|ψ(t)>
<ψ'|Q'_i|ψ'> = <ψ|Q_i|ψ> − v_i t
<ψ'|Q̇'_i|ψ'> = <ψ|Q̇_i|ψ> − v_i
<ψ'|P'_i|ψ'> =
− mv_i
iħ d|ψ'(t')>/dt' = H'|ψ'(t')>
d/dt' = d/dt
|ψ'(t')> = U_G |ψ(t)>
U_G = exp[(i/ħ) v · (P t − Q)]
Q_i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$\langle\psi'|P_i'|\psi'\rangle = \langle\psi|P_i|\psi\rangle - m v_i$
$e^{A+B} = e^A e^B e^{-[A,B]}$
$|\psi'(t')\rangle = U_G|\psi(t)\rangle$
$U_G = \exp\left[\frac{i}{\hbar}\vec{v}\cdot(\vec{P}t - m\vec{Q})\right]$

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$$e^{A+B} = e^A e^B e^{-[A,B]/2}$$

$$|\psi'(t')\rangle = U_G|\psi(t)\rangle$$

$$U_G = \exp\left[\frac{i}{\hbar}\vec{v}\cdot\left(\vec{P}t - m\vec{Q}\right)\right]$$

$$= \exp\left(\frac{imv^2t}{2\hbar}\right)\exp\left(\frac{i\vec{v}\cdot\vec{P}t}{\hbar}\right)\exp\left(\frac{-im\vec{v}\cdot\vec{Q}}{\hbar}\right)$$

$$\frac{\qquad}{\partial t} = \left[\frac{(\;\;)}{2m} + V \right] \psi(\vec{x},t) = 0$$

$$\vec{x}' = \vec{x} - \vec{v}t$$

$$t' = t$$

$$\vec{\nabla}' = \vec{\nabla}$$

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}$$

$$V'(\vec{x}',t') = V(\vec{x},t) - \vec{v}\cdot\vec{A}(\vec{x},t)$$

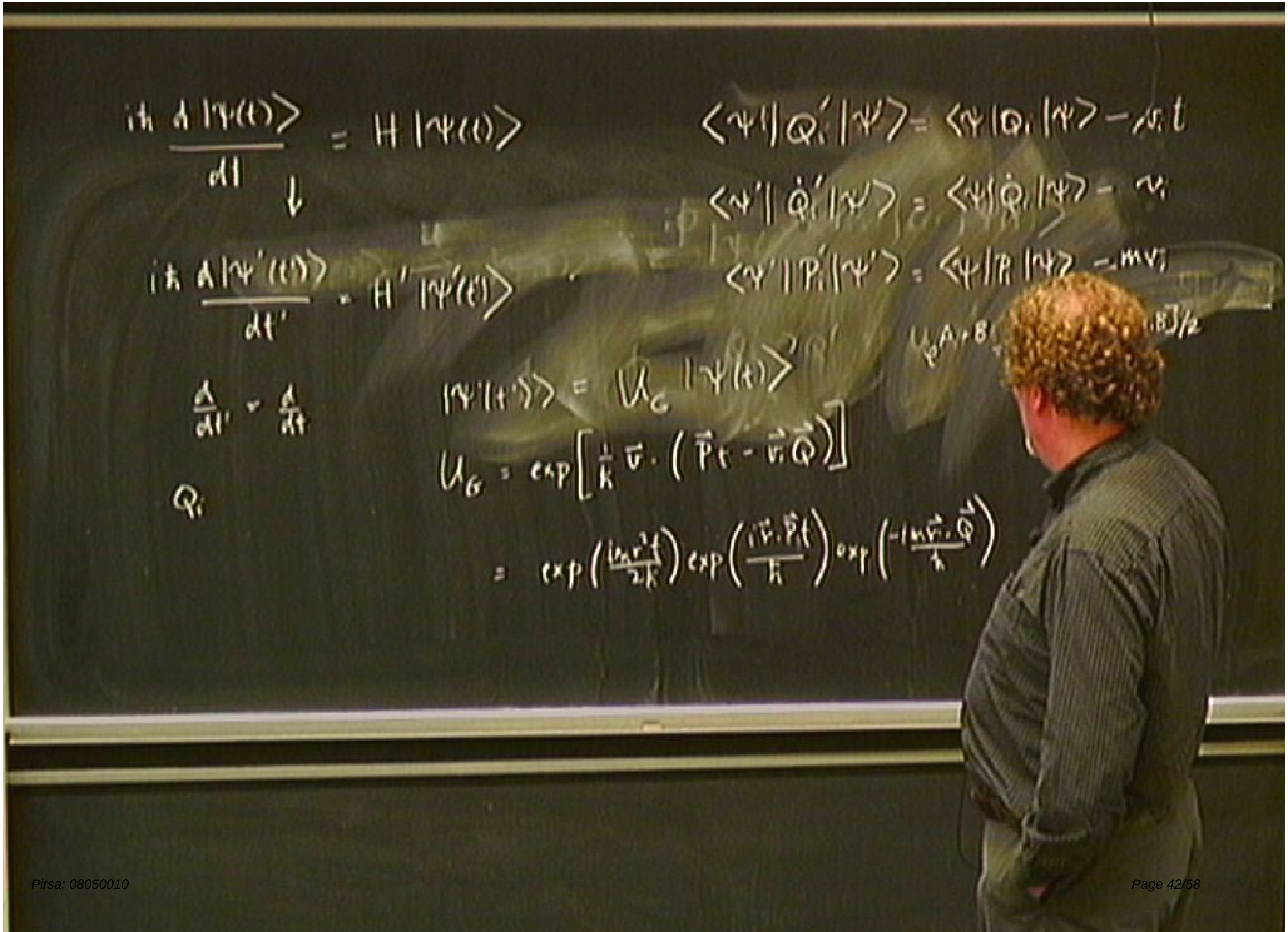
$$\psi'(\vec{x}',t') = e^{i\phi}\,\psi(\vec{x},t)$$

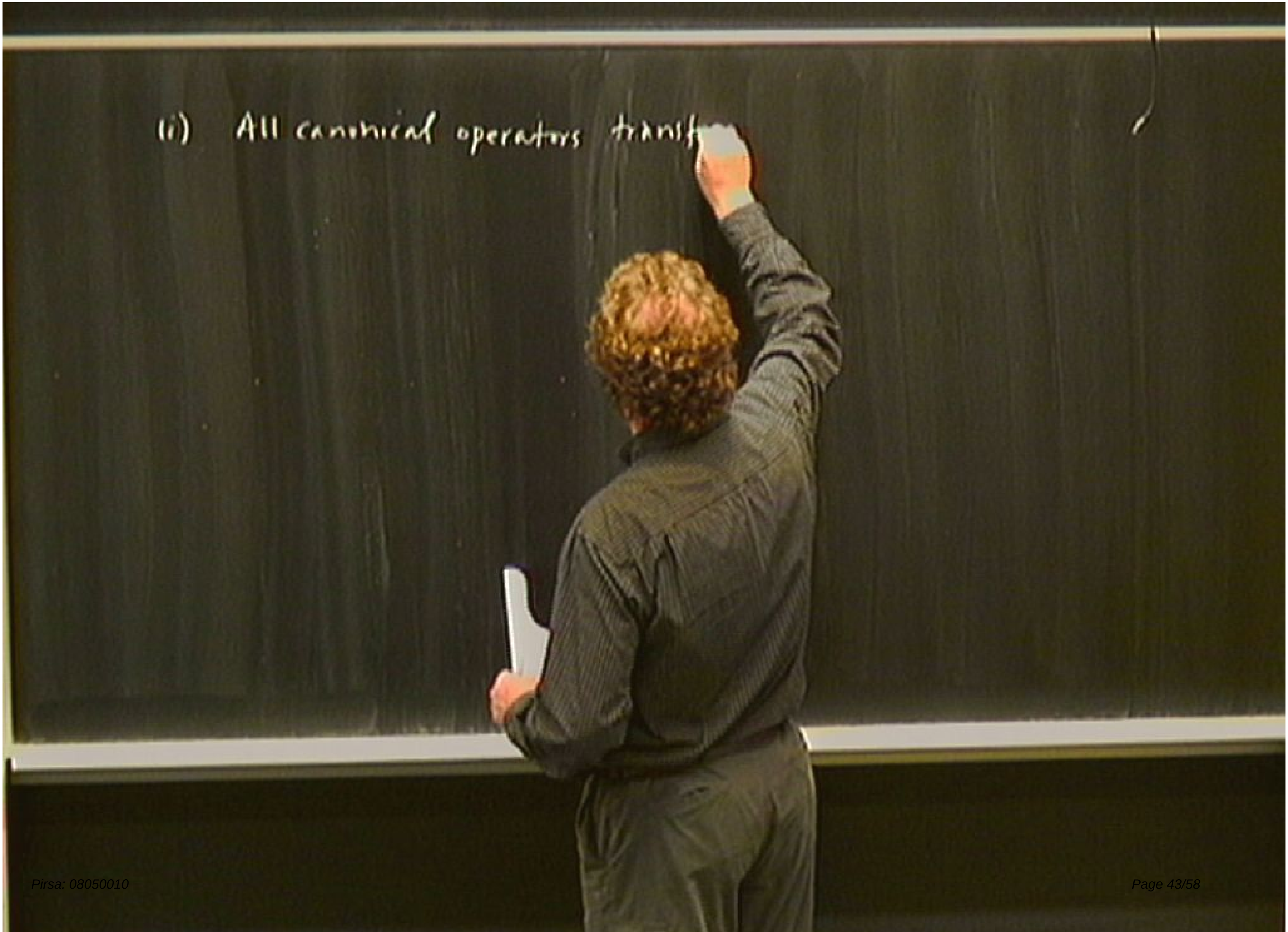
$$\phi = \frac{m}{\hbar}\left[\frac{m\vec{v}^2 t}{2} - \vec{v}\cdot\vec{x}\right]$$

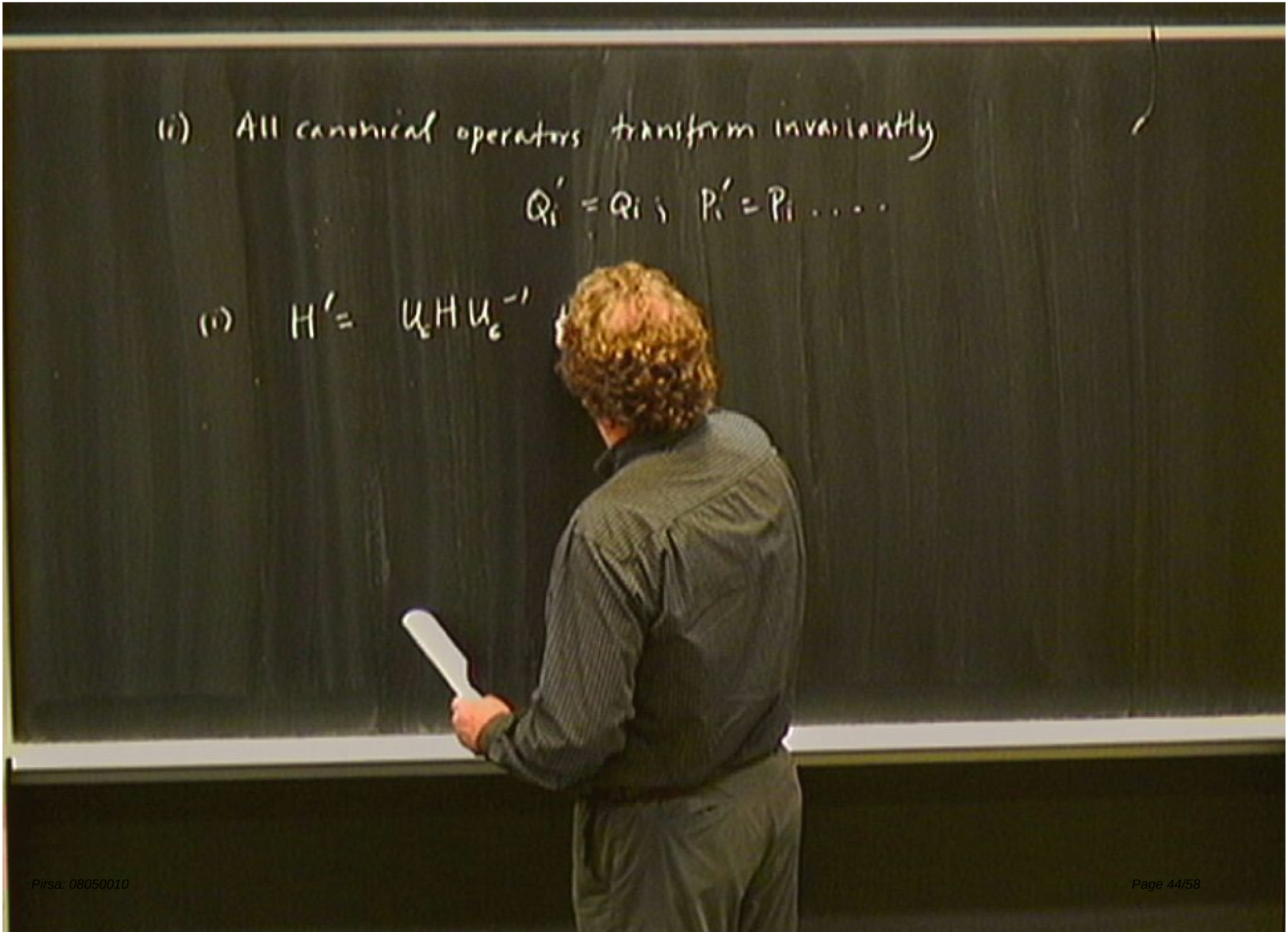
$$-i\hbar\vec{\nabla} \rightarrow -i\hbar\vec{\nabla} - \vec{A}$$

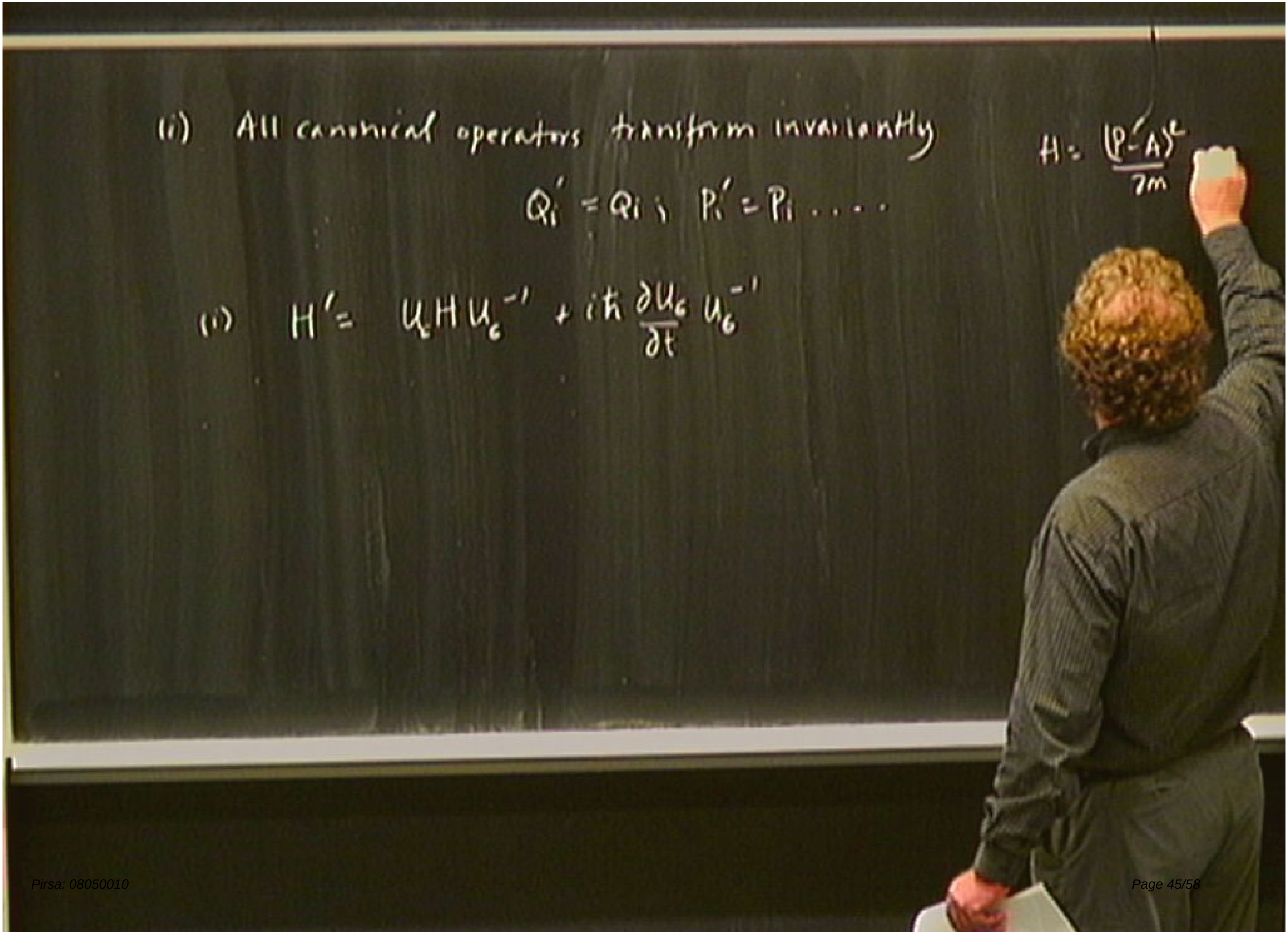
$$p = \frac{h}{\lambda}$$

$$[\,0 \quad 0 \quad 0 \quad 0\text{-}\tfrac{1}{2} \quad V\,]$$


iħ d|ψ(t)⟩/dt = H|ψ(t)⟩
iħ d|ψ'(t')⟩/dt' = H'|ψ'(t')⟩
d/dt' = d/dt
Qᵢ
⟨ψ'|Q'ᵢ|ψ'⟩ = ⟨ψ|Qᵢ|ψ⟩ − vᵢt
⟨ψ'|Q̇'ᵢ|ψ'⟩ = ⟨ψ|Q̇ᵢ|ψ⟩ − vᵢ
⟨ψ'|Pᵢ|ψ'⟩ = ⟨ψ|Pᵢ|ψ⟩ − mvᵢ
|ψ'(t')⟩ = U_G |ψ(t)⟩
U_G = exp[i/ħ v⃗ · (P⃗t − m Q⃗)]
= exp(imv²t/2ħ) exp(iv⃗·P⃗t/ħ) exp(−imv⃗·Q⃗/ħ)


(i) All canonical operators transf


(i) All canonical operators transform invariantly
$Q_i' = Q_i \; ; \; P_i' = P_i \; \ldots .$
(ii) $H' = U_G H U_G^{-1}$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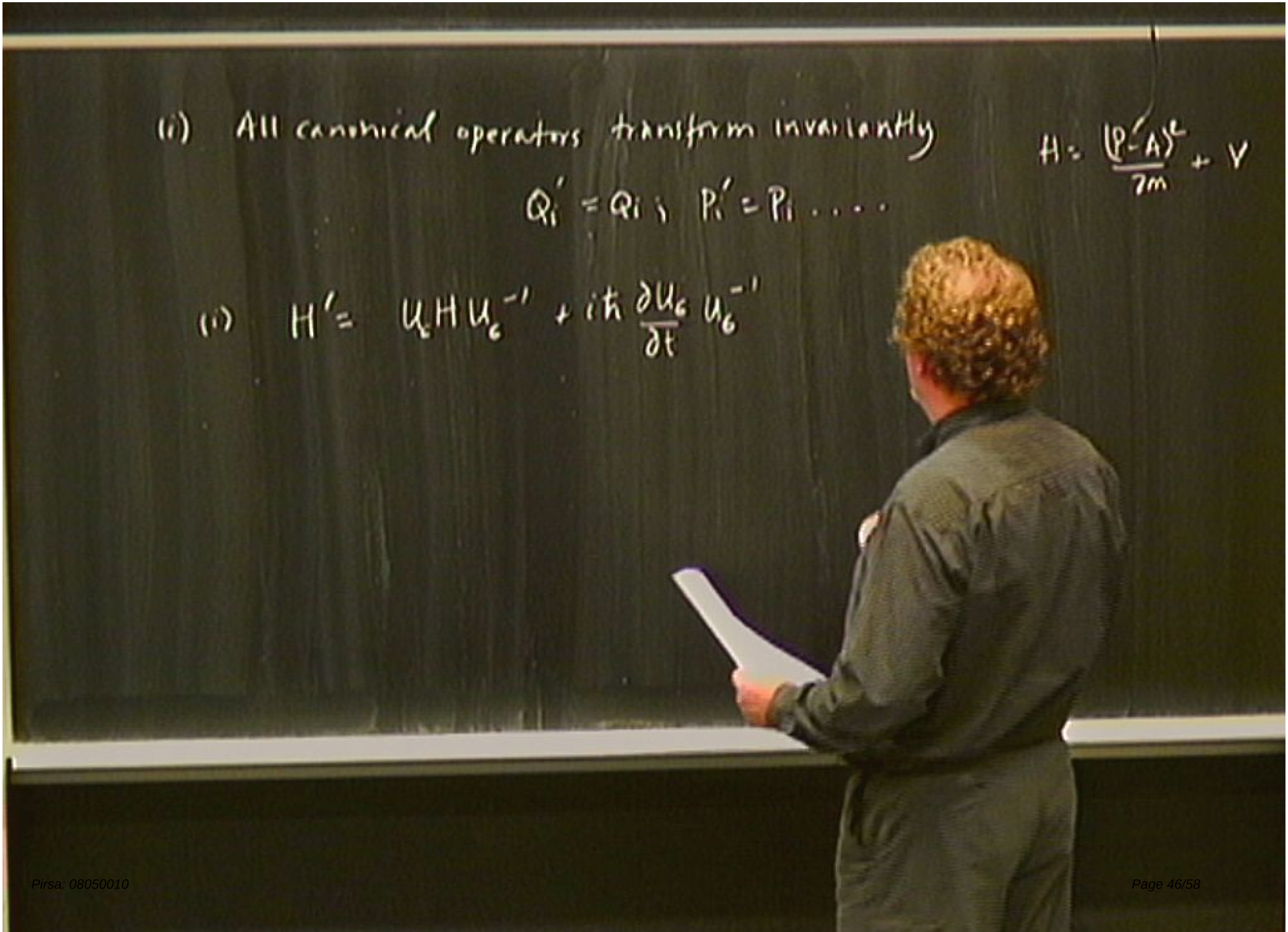
$H = \frac{(P - A)^2}{2m}$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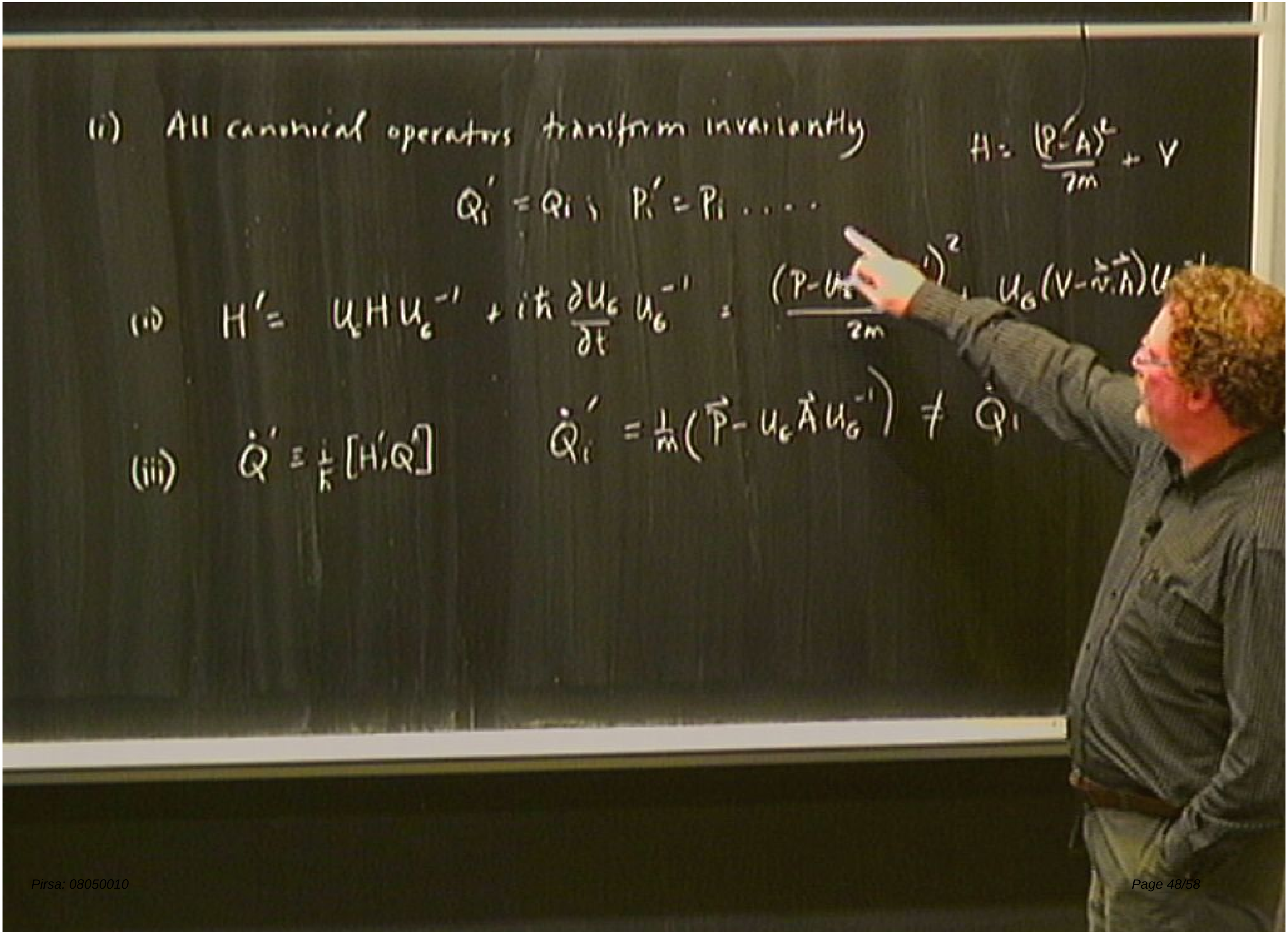
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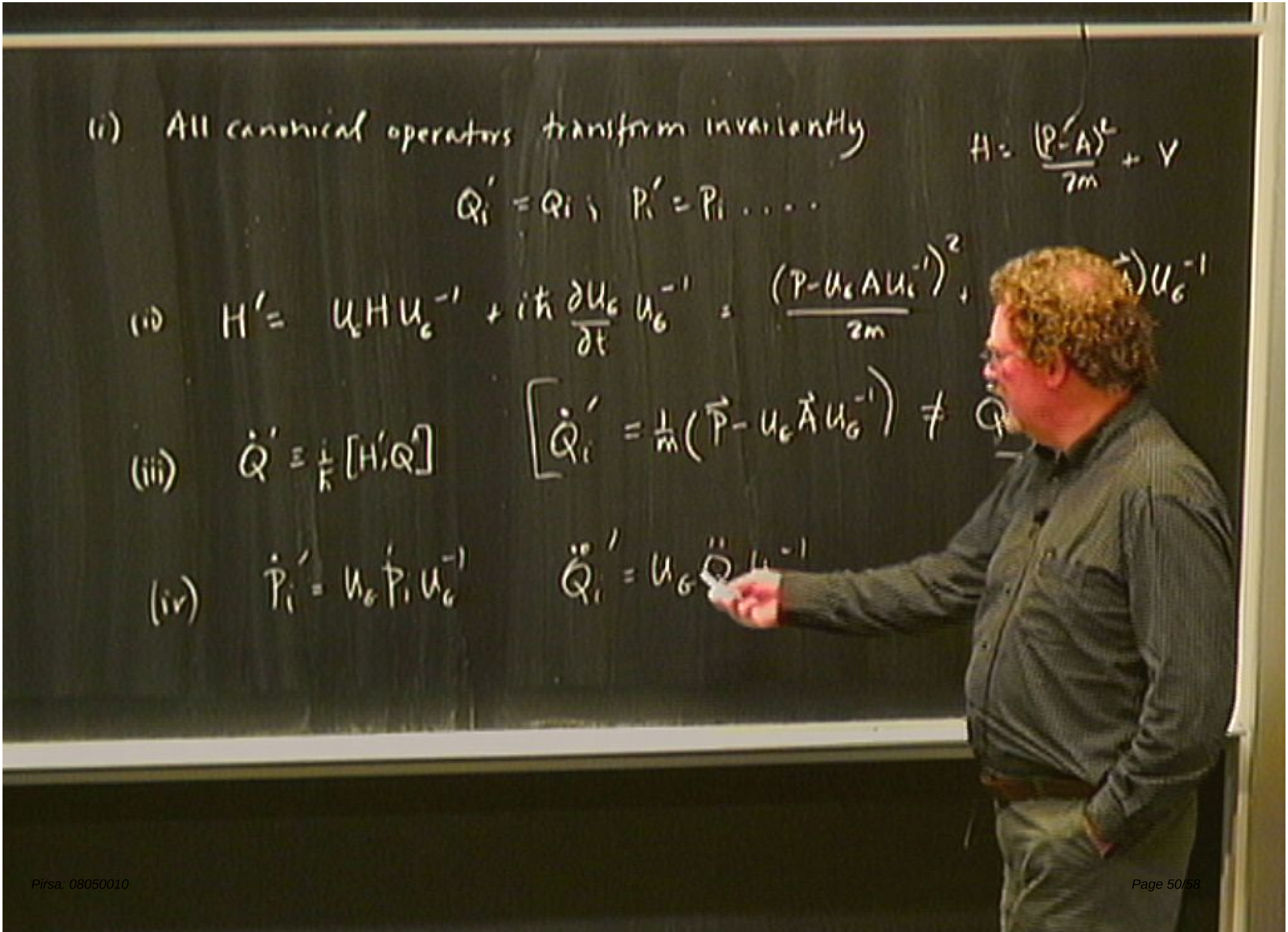
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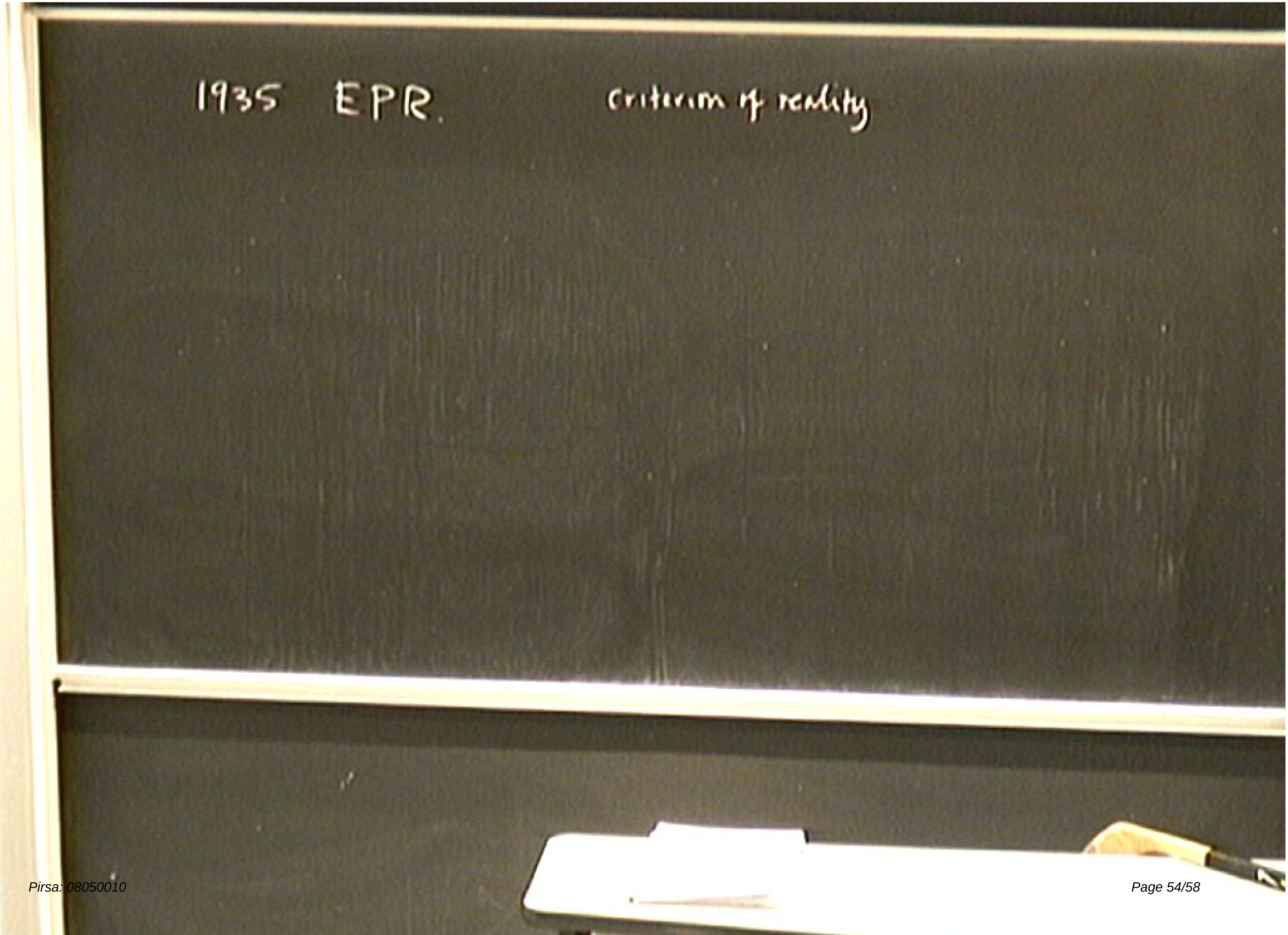
1935 EPR.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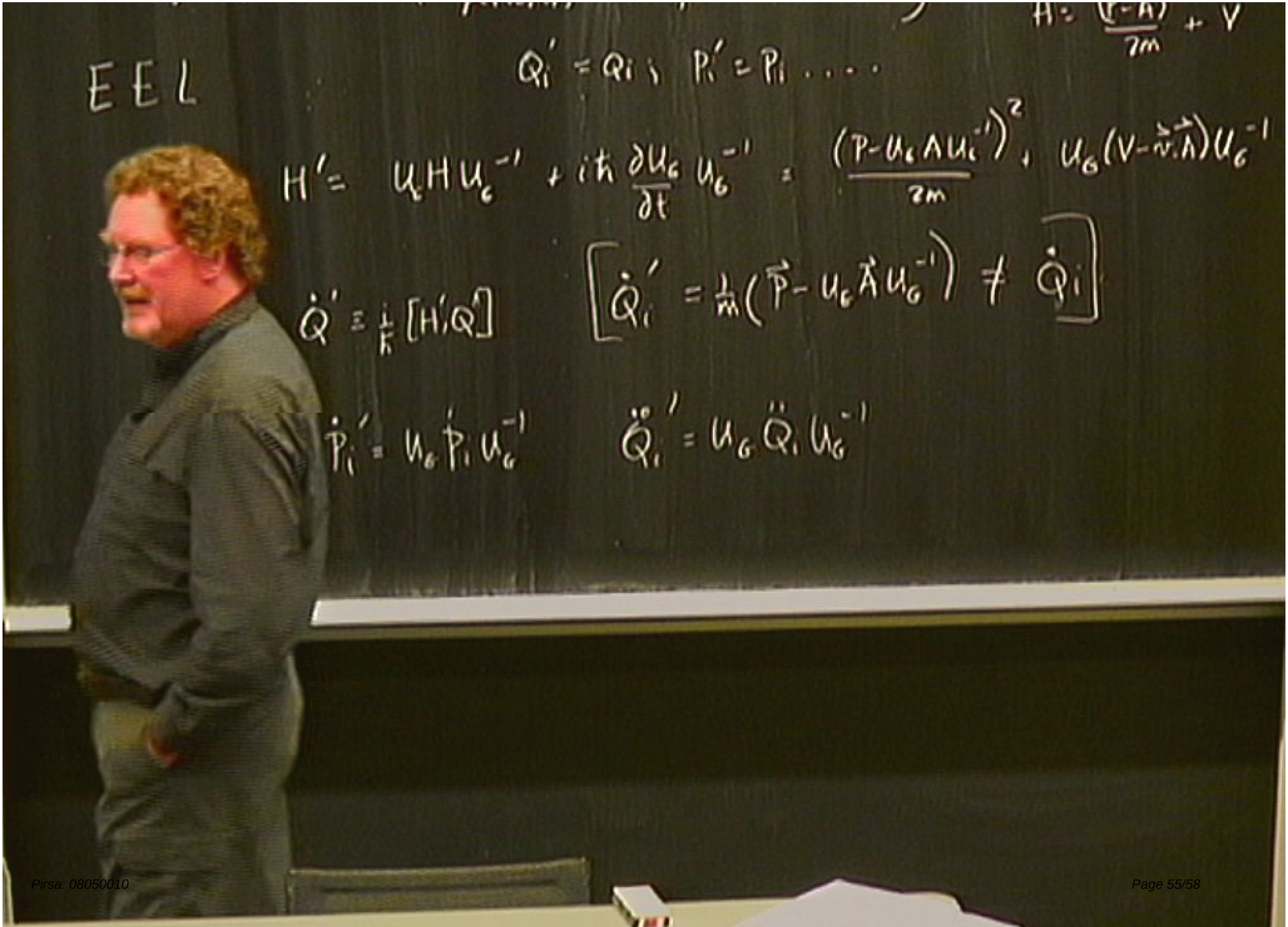
Criterium of reality



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EEL
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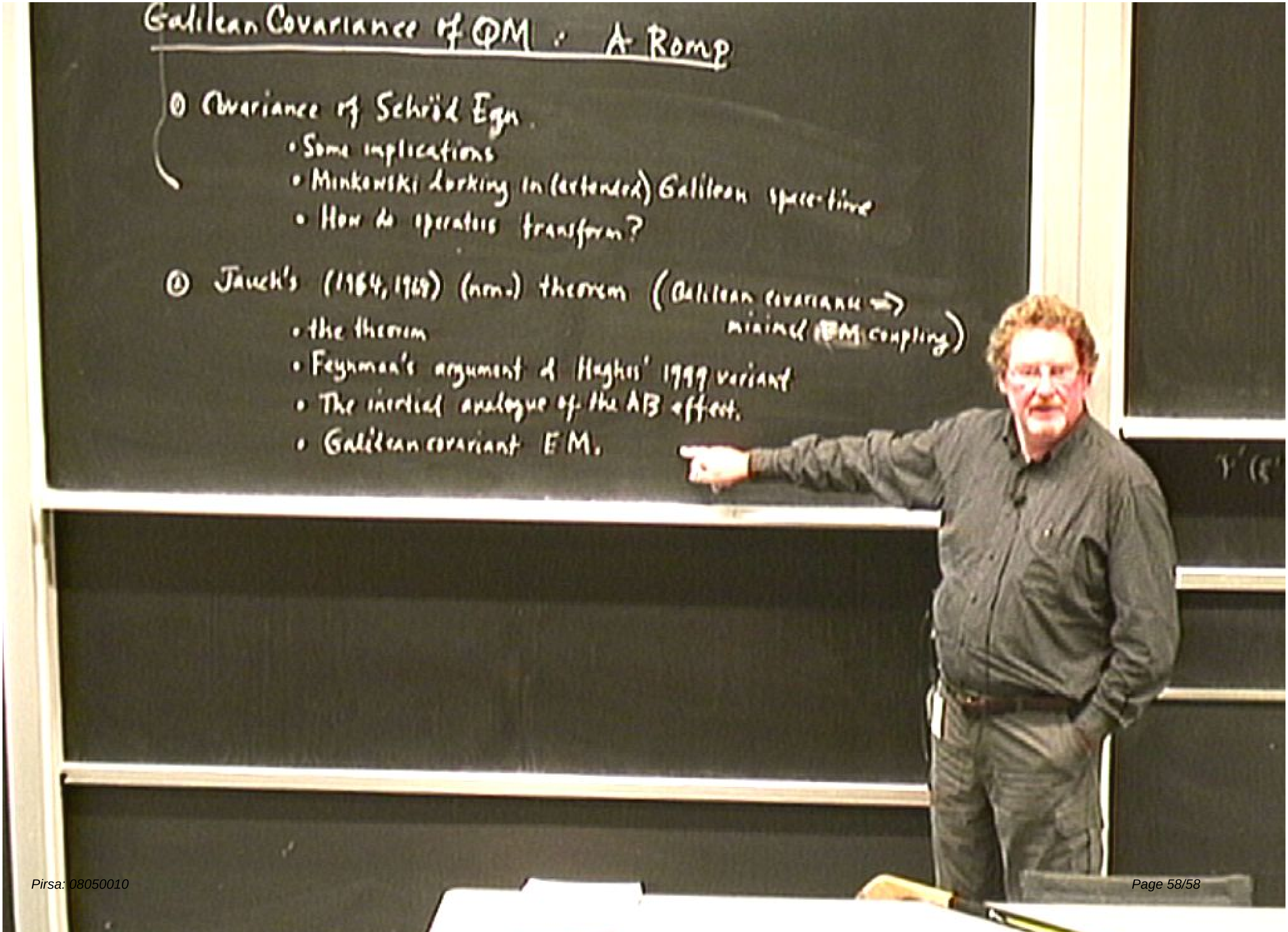
$L_z = Q_x P_y -$

$$L_z = Q_x P_y - Q_y P_x$$


Galilean Covariance of QM : A Romp
① Covariance of Schröd Eqn
• Some implications
• Minkowski lurking in (extended) Galilean space-time
• How do operators transform?
② Jauch's (1964, 1968) (nmd) theorem (Galilean covariance ⇒ minimal EM coupling)
• the theorem
• Feynman's argument & Hughes' 1999 variant
• The inertial analogue of the AB effect.
• Galilean covariant EM.