Title: Symmetry Principles in Physics - Lecture 4A

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Abstract:

Currents, charges and symmetries in QFT

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May 5, 2008

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  The charge associated to a conserved current is the generator of a one-parameter continuous group of **symmetry transformations***:*

  $$U(\tau) = \exp(iQ\tau)$$

Charges, generators and the Ward identity

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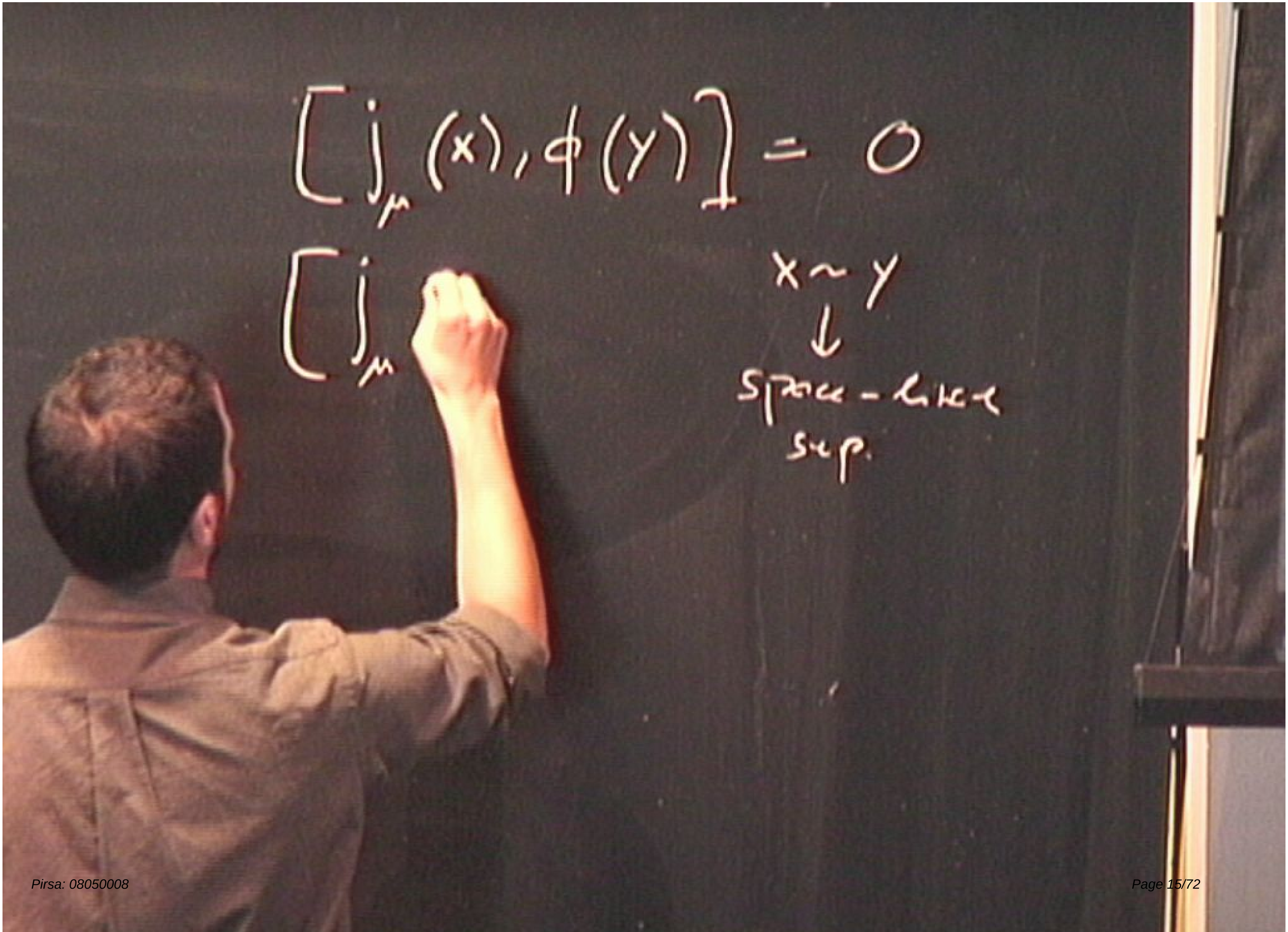
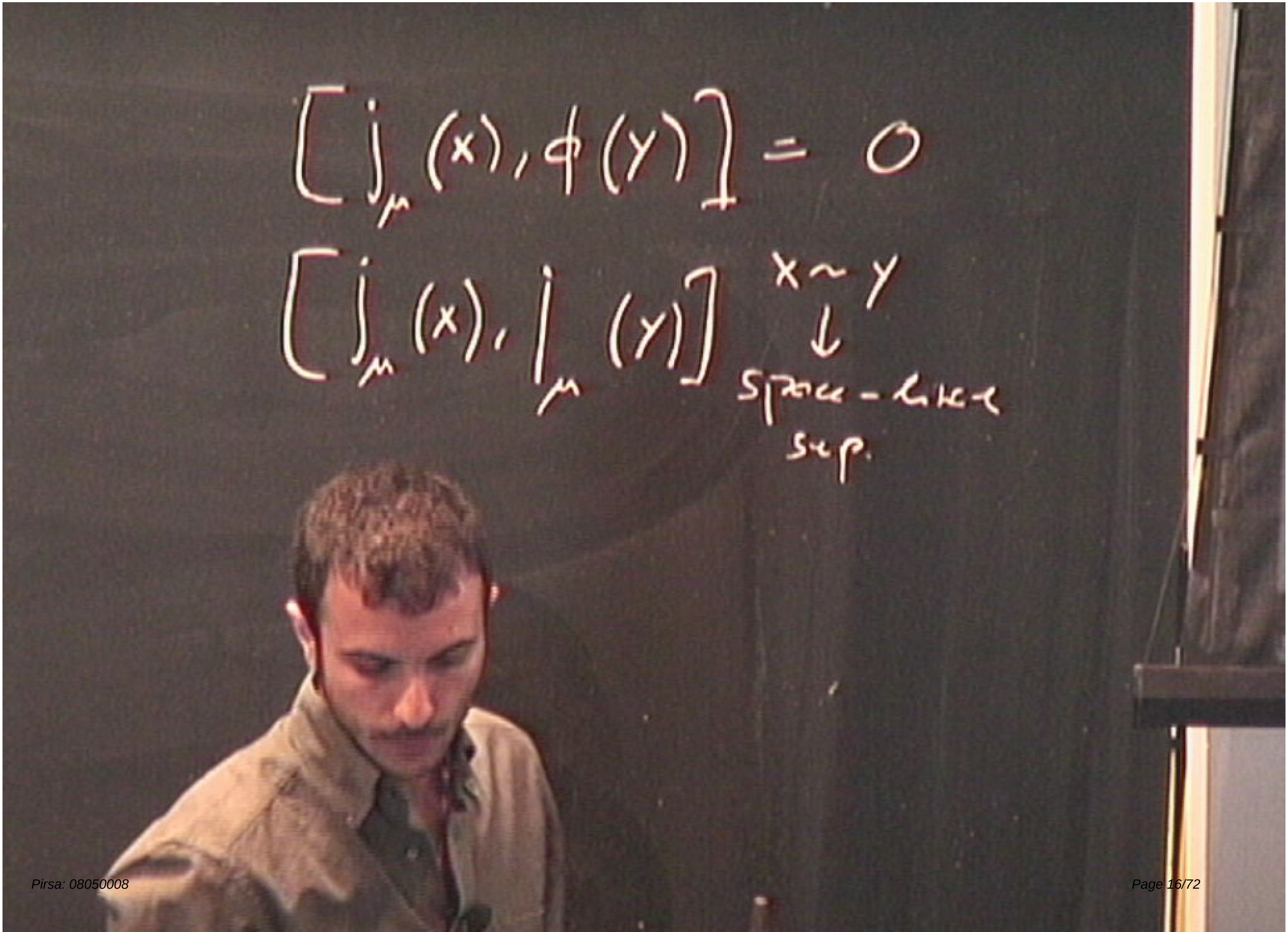
KRS Theorem (Kastler, Robinson and Swieca, 1966)

Given a local and locally conserved current $\hat{j}_\mu(x)$ the charge $\hat{Q}$ defines a symmetry generator G

$$G \rhd \phi(x)|0\rangle \equiv [\hat{Q}, \phi(x)]|0\rangle$$


[j_μ (x), φ(y)] = 0
[j_μ
x ~ y
space-like
sep.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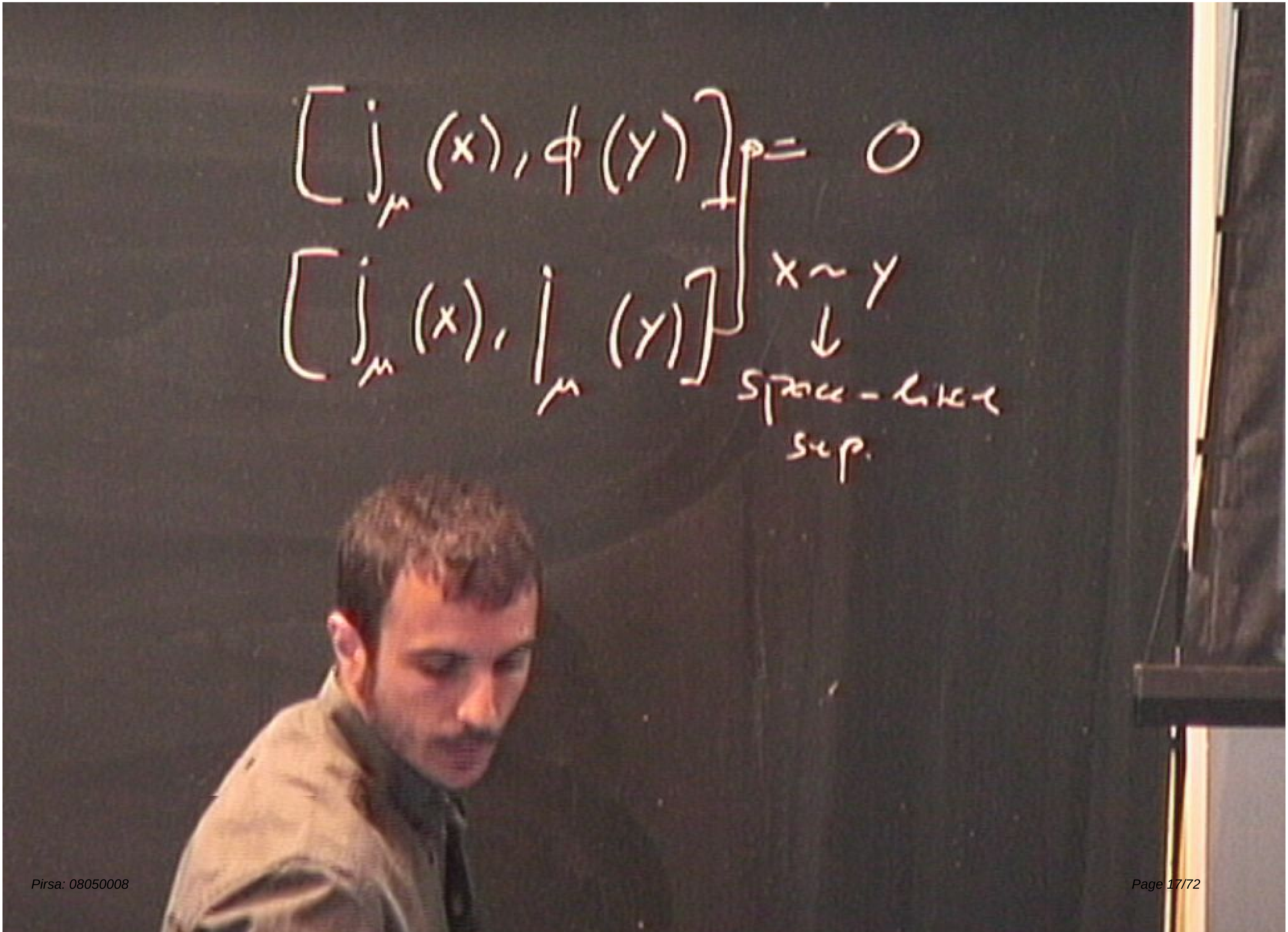
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Integrating the Ward identity

$$G \triangleright \phi(x') = \int_V dx\, \partial_\mu \langle j^\mu(x)\phi(x')\rangle = \int_{\partial V} d^3\vec{x}\, \langle j^0(x)\phi(x')\rangle$$

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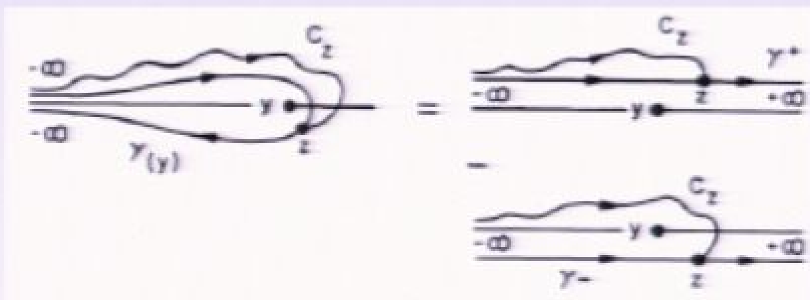

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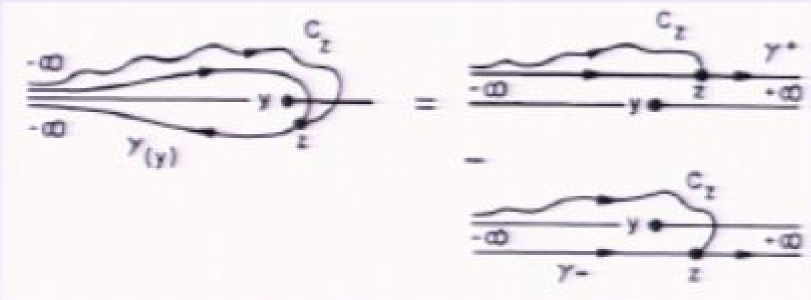

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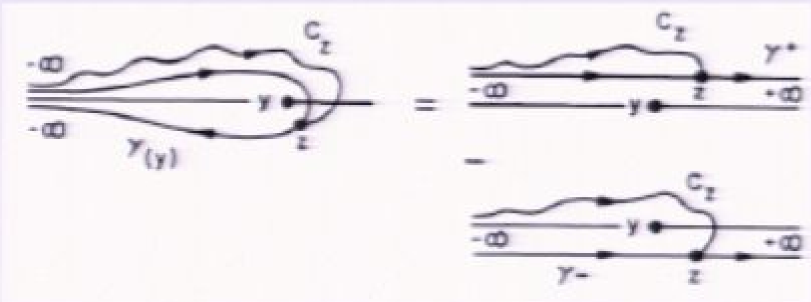

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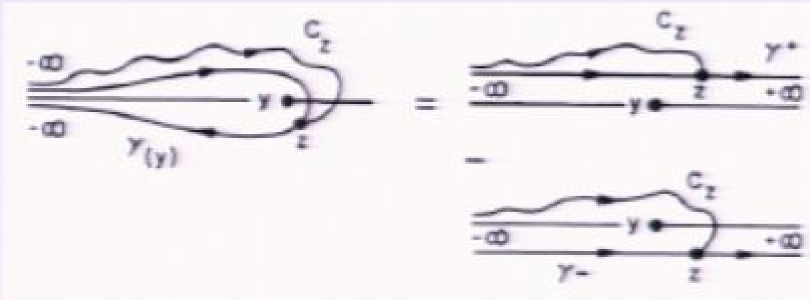

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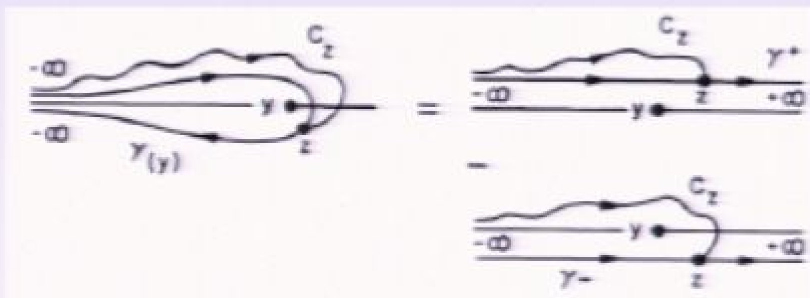

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Bibliography

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- Common belief in the QG community: **QFT observables** (i.e. operators) obtained from a full theory of QG exhibit non-locality
- treatment similar to 1+1-d case to charges associated to **space-time symmetries**

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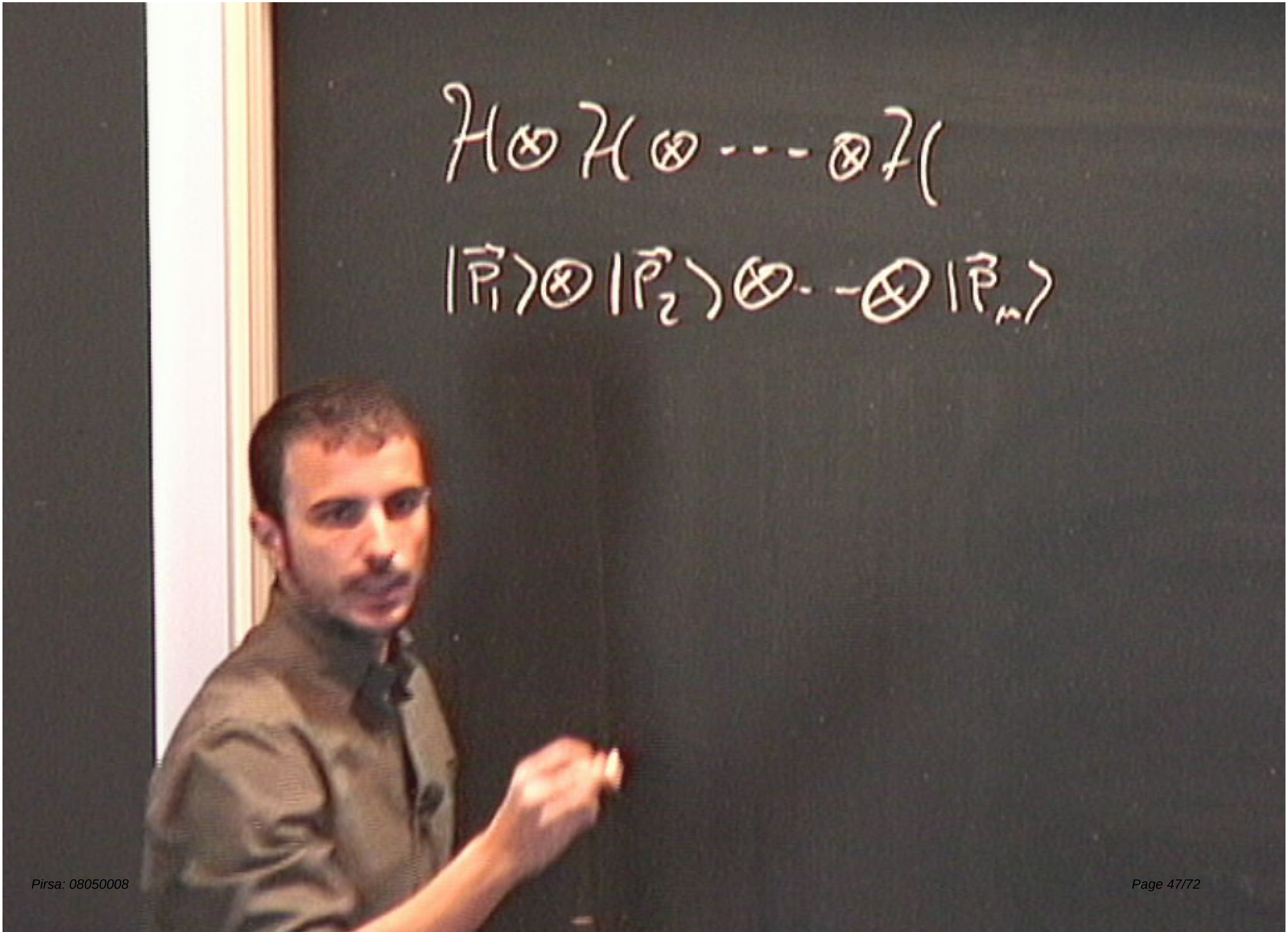
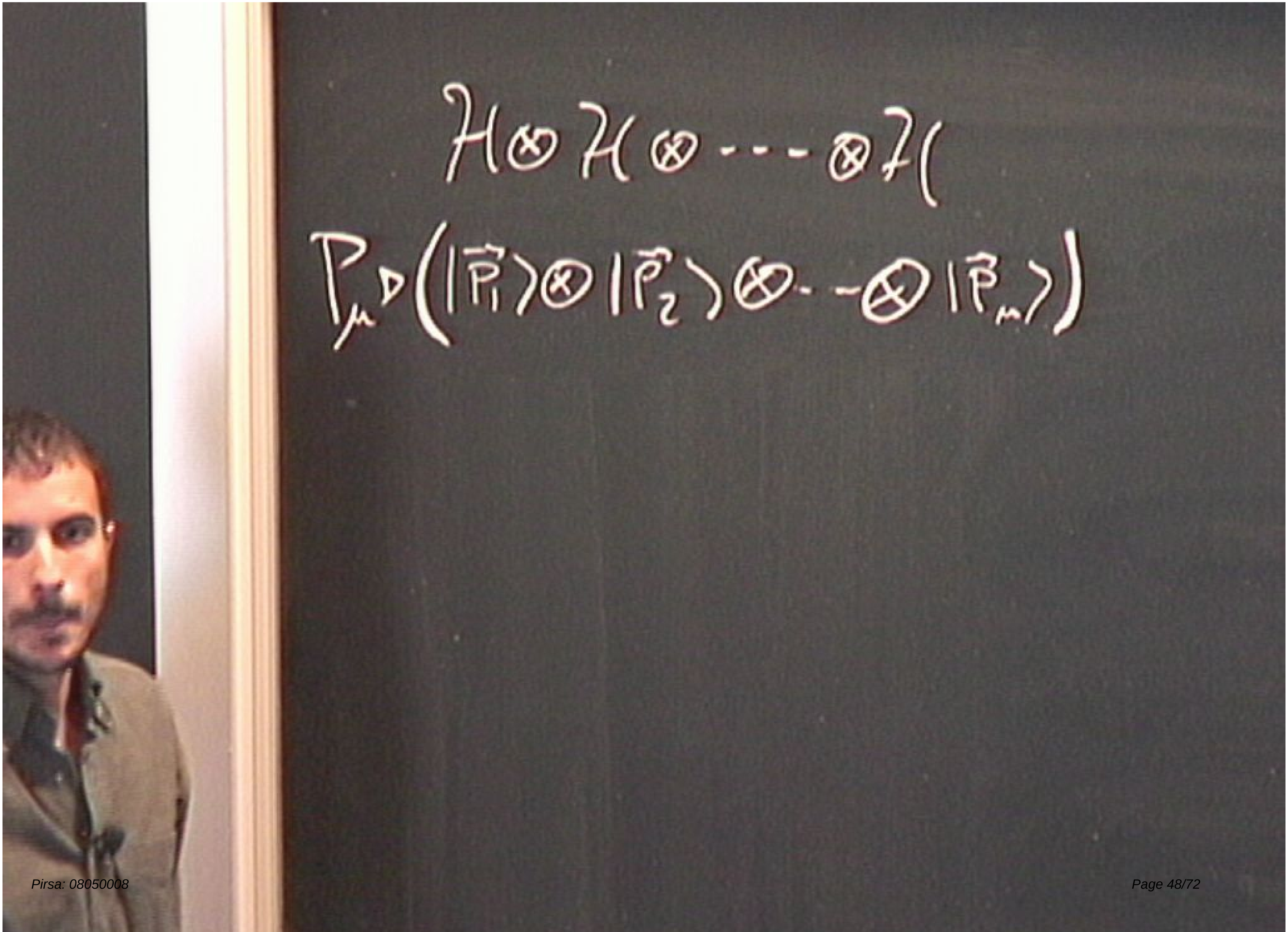
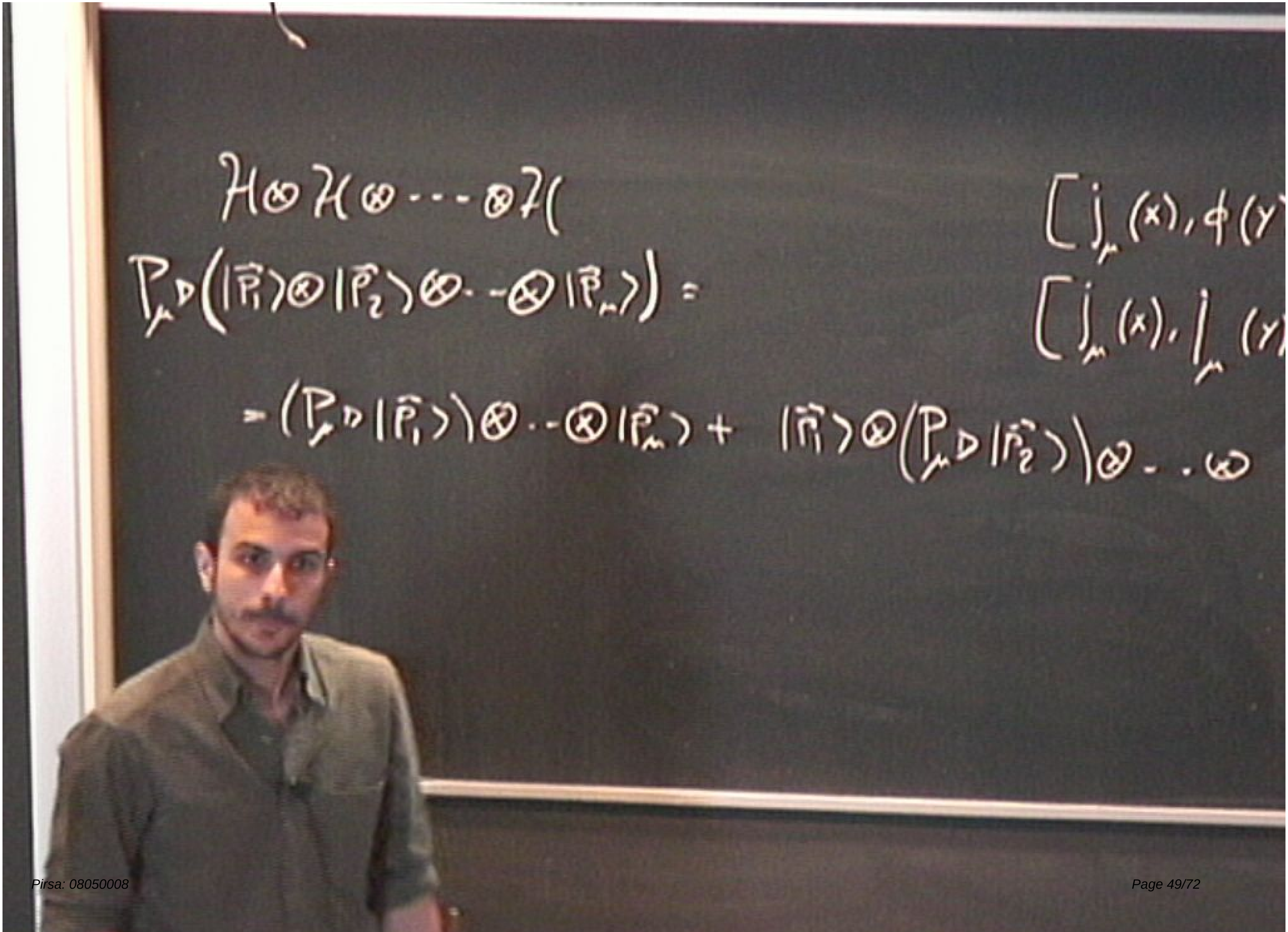

$\mathcal{H}\otimes\mathcal{H}\otimes\cdots\otimes\mathcal{H}$
$|\vec{p}_1\rangle\otimes|\vec{p}_2\rangle\otimes\cdots\otimes|\vec{p}_m\rangle$


$\mathcal{H} \otimes \mathcal{H} \otimes \cdots \otimes \mathcal{H}$
$P_{\mu}^{D}\left(|\vec{p}_1\rangle \otimes |\vec{p}_2\rangle \otimes \cdots \otimes |\vec{p}_n\rangle\right)$


$\mathcal{H} \otimes \mathcal{H} \otimes \cdots \otimes \mathcal{H}$
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$= (P_\mu \triangleright |\vec{p}_1\rangle) \otimes \cdots \otimes |\vec{p}_n\rangle + |\vec{p}_1\rangle \otimes (P_\mu \triangleright |\vec{p}_2\rangle) \otimes \cdots \otimes$
$[j_\mu(x), \phi(y) \ldots$
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$$\mathcal{H}\otimes\mathcal{H}\otimes\cdots\otimes\mathcal{H}$$

$$[j_\mu(x), \phi(y)$$

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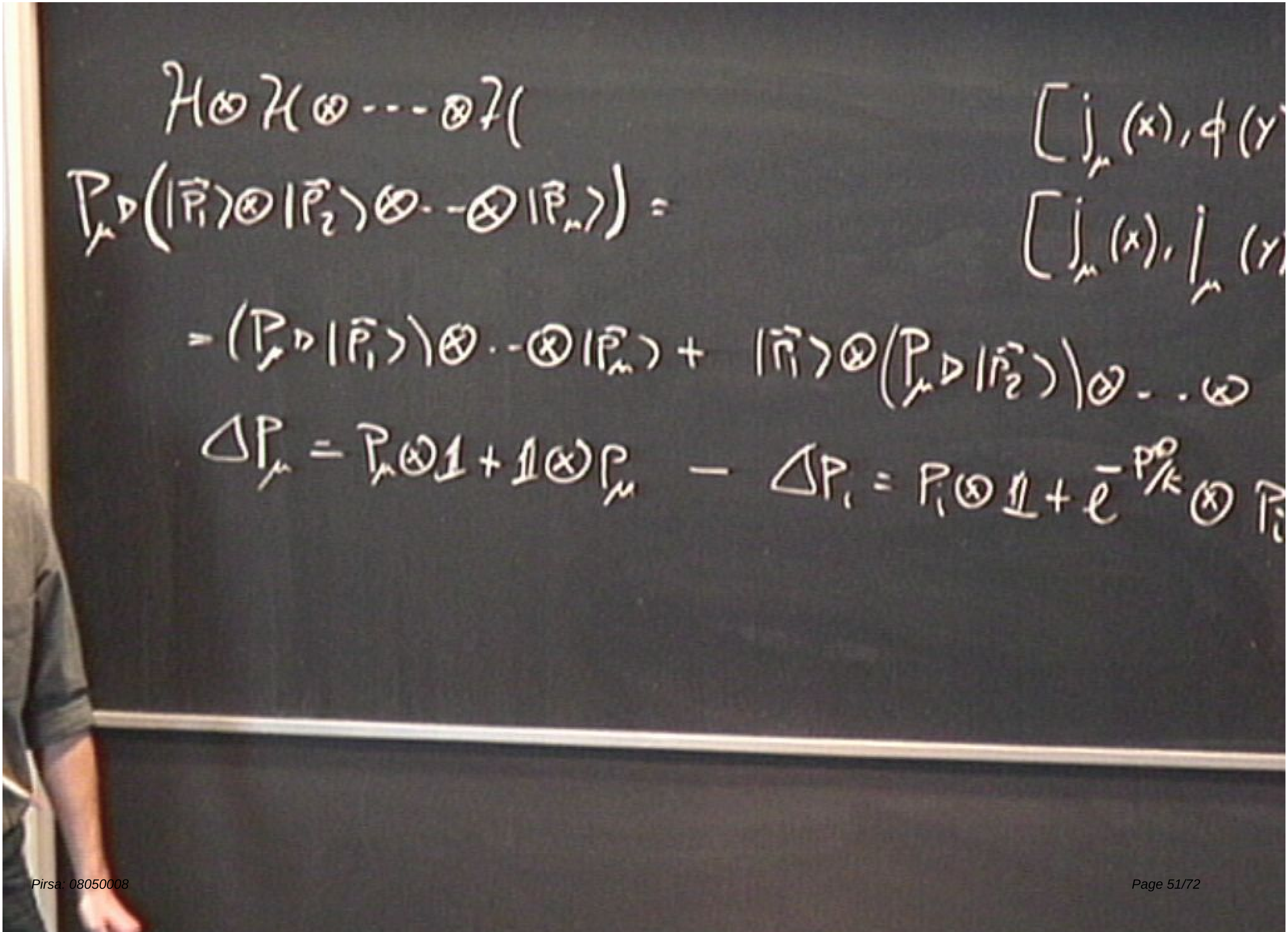
$$\Delta P_\mu = P_\mu \otimes 1 + 1 \otimes P_\mu$$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$\Delta P_\mu = P_\mu \otimes 1 + 1 \otimes P_\mu$ — $\Delta P_i = P_i \otimes 1 + e^{-P^0/k} \otimes P_i$
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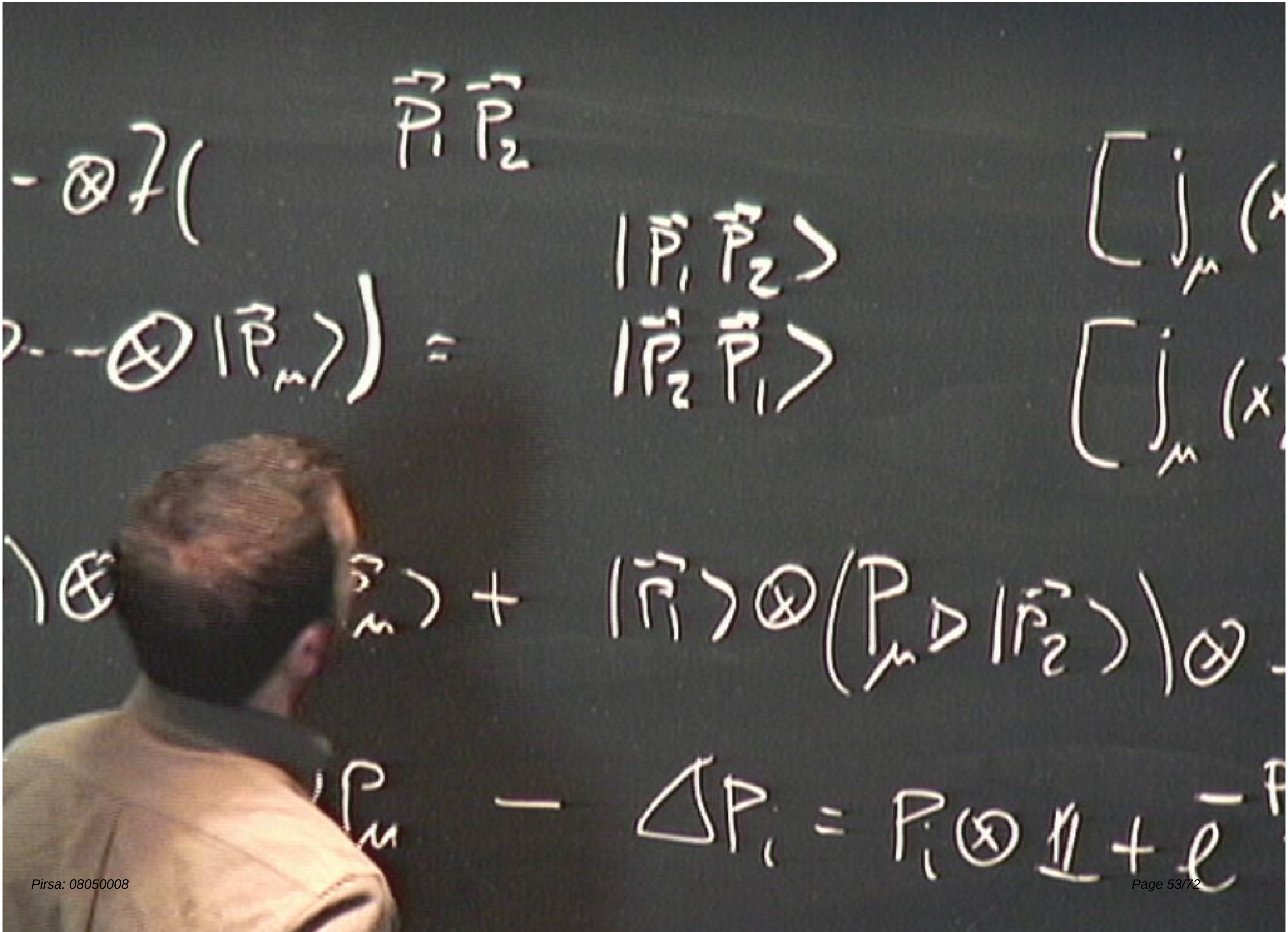
$$\Delta P_\mu = P_\mu \otimes \mathbb{1} + \mathbb{1} \otimes P_\mu \quad - \quad \Delta P_i = P_i \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes P_i$$



$\vec{p}_1 \vec{p}_2$
$|\vec{p}_1 \vec{p}_2\rangle$
$|\vec{p}_2 \vec{p}_1\rangle$
$|\vec{p}_1\rangle \otimes (P_\mu \triangleright |\vec{p}_2\rangle) \otimes$
$\triangle P_i = P_i \otimes \mathbb{1} + e$

$\vec{P}_1 \; \vec{P}_2$

$- \otimes \mathcal{H}($

$\cdots - \otimes |\vec{P}_n\rangle) = \begin{cases} |\vec{P}_1 \vec{P}_2\rangle \\ |\vec{P}_2 \vec{P}_1\rangle \end{cases}$

$[j_\mu (x$

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$) \otimes \cdots \otimes |\vec{P}_n\rangle + |\vec{P}_1\rangle \otimes (P_\mu \triangleright |\vec{P}_2\rangle) \otimes$

$\otimes 1 + 1 \otimes P_\mu \quad — \quad \Delta P_i = P_i \otimes 1 + e^{-P}$

$$\frac{1}{\sqrt{2}}\left(|\vec{p}_1\rangle \otimes |\vec{p}_2\rangle + |\vec{p}_2\rangle \otimes |\vec{p}_1\rangle\right)$$

$$\vec{p}_1 \; \vec{p}_2$$

$$\mathcal{H} \otimes \mathcal{H} \otimes \cdots \otimes \mathcal{H}$$

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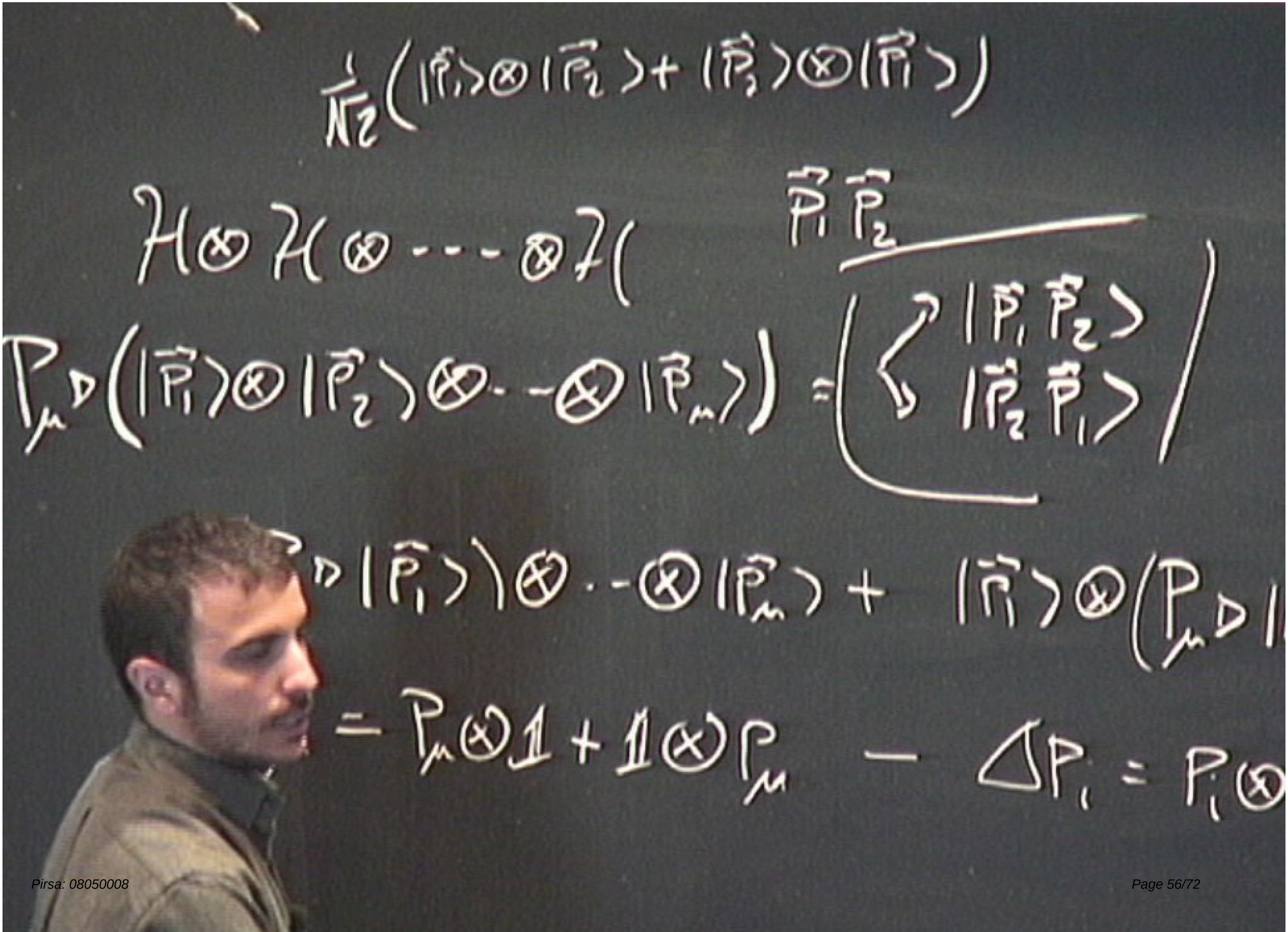
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$$[j_\mu(x), \phi(y)] \neq 0$$

$$[j_\mu(x), j_\mu(y)] \quad x \sim y \text{ space-like sep.}$$

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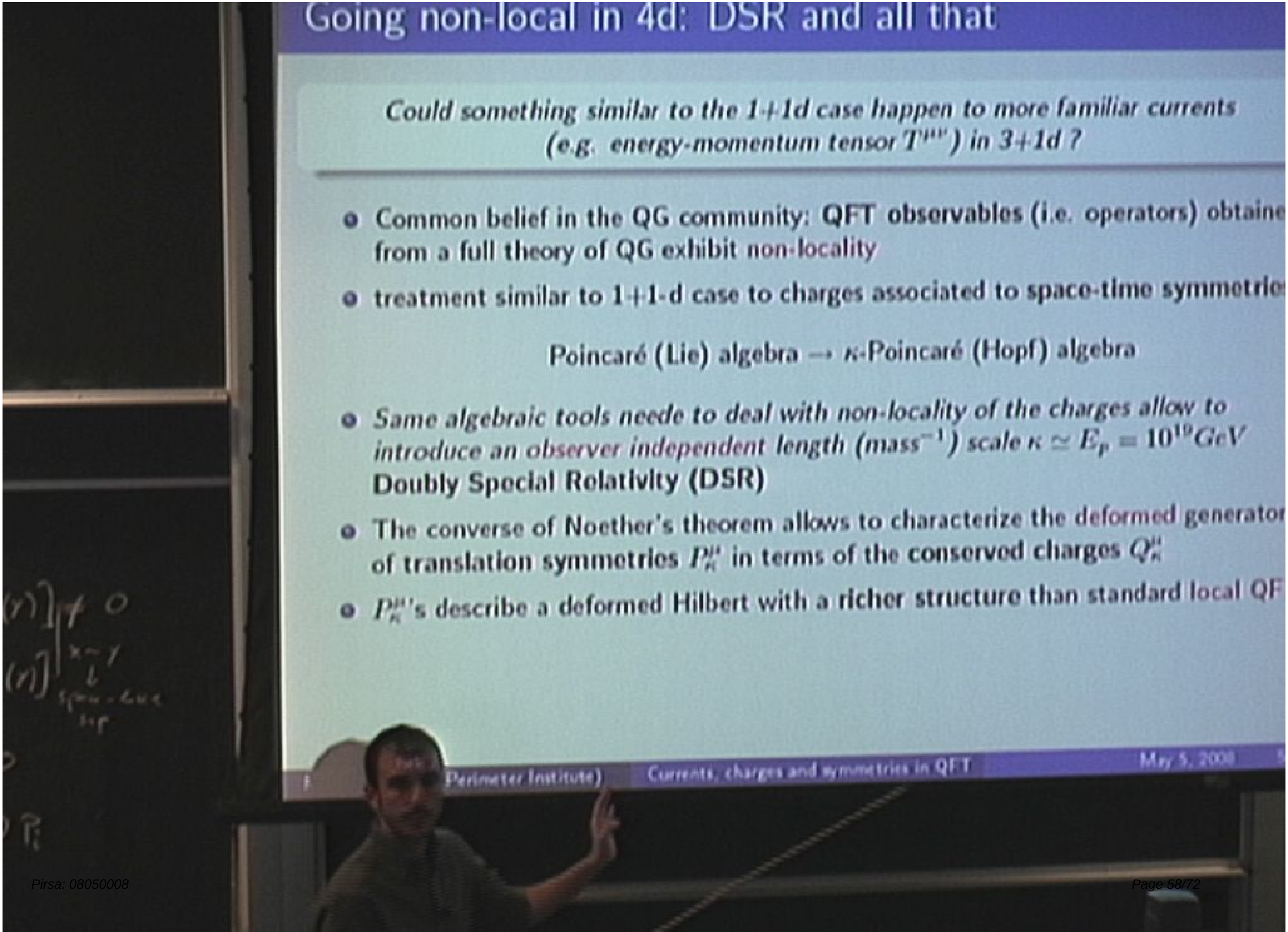
$$P^0/k$$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Perimeter Institute)
Currents, charges and symmetries in QFT
May 5, 2008

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- P^{μ}_{κ}'s describe a deformed Hilbert with a **richer structure** than standard local QFT

Emmy Noether 1882-1935

"She was not clay, pressed by the artistic hands of God into a harmonious form, but rather a chunk of human primary rock into which he had blown his creative breath of life."

Hermann Weyl 1935

Emmy Noether 1882—1935.

Mathematician

- Born in Ertlangen. Father Max Noether distinguished professor of mathematics at University of Erlangen.
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LOCAL SYMMETRIES

Recall Noether condition for quasi-invariance:

$$\underbrace{\sum_i E_i \bar{\delta}\phi_i}_{\text{interior (bulk)}} + \underbrace{d_\mu \Big[\sum_i \Big(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \bar{\delta}\phi_i \Big) + \mathcal{L}\delta x^\mu - C^\mu \Big]}_{\text{boundary}} = 0$$

Now δx^μ, $\delta\phi_i$ depend on arbitrary functions $f_k(x)$ $(k = 1, \cdots, p)$

and $\bar{\delta}\phi_i = \sum_k \big[a_{ki} f_k(x) + b^\nu{}_{ki}\, \partial_\nu f_k(x) \big]$

Arbitrariness of f_k $\Rightarrow$ interior, boundary terms vanish independently

Vanishing of interior term $\Rightarrow$ $\boxed{\sum_i E_i a_{ki} = d_\mu \sum_i \big(E_i b^\mu{}_{ki} \big)}$

NOETHER'S 2ND THEOREM.

"BIANCHI IDENTITIES"

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"BIANCHI IDENTITIES"

HILBERT 1915.

$$\mathcal{L}_{grav}^{HIL} = R\sqrt{-g}$$

- second order.

$$R = g^{\mu\nu} R_{\mu\nu}$$
$$= g^{\mu\nu} R^{\alpha}_{\mu\nu\alpha}.$$

$$\mathcal{L}_{grav}^{HIL} - \mathcal{L}_{grav}^{EIN} = \frac{d}{dx_\mu}(F_\mu)$$

Contains all second order terms in $\mathcal{L}_{grav}^{HIL}$

EINSTEIN

$\sqrt{-g} = 1$:

$$\mathcal{L}_{grav} = g^{\mu\nu} \Gamma^{\alpha}{}_{\mu\beta} \Gamma^{\beta}{}_{\nu\alpha} \sqrt{-g}$$

$$\mathcal{L}_{EM} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \sqrt{-g}$$

$$F_{\mu\nu} \equiv A_{\nu,\mu} - A_{\mu,\nu}$$

1913 : $\Gamma^{\alpha}{}_{\mu\beta} = \tfrac{1}{2} g^{\alpha\lambda} g_{\lambda\mu,\beta}$

1915 $\Gamma^{\alpha}{}_{\mu\beta} = \tfrac{1}{2} g^{\alpha\lambda} \left(g_{\lambda\mu,\beta} + g_{\lambda\beta,\mu} - g_{\mu\beta,\lambda} \right)$

Arbitrary coordinates :

$$\mathcal{L}^{Ein}_{grav} = g^{\mu\nu} \left(\Gamma^{\alpha}{}_{\mu\beta} \Gamma^{\beta}{}_{\nu\alpha} - \Gamma^{\beta}{}_{\mu\nu} \Gamma^{\alpha}{}_{\alpha\beta} \right) \sqrt{-g}$$

- first order in derivs of $g_{\mu\nu}$
- not a scalar density

Γ-Γ Lagrangian

HAMILTON'S PRINCIPLE IN MATTER-FREE GR :

$$E_{\mu\nu} = G_{\mu\nu}\sqrt{-g} \qquad G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R$$

Euler-Lagrange equations : $\boxed{G_{\mu\nu} = 0 \Leftrightarrow R_{\mu\nu} = 0.}$

VACUUM FIELD EQNS.

D. Lovelock 1969.

(Strictly) invariant action in $\leq$ 4-dim spacetime must take form :

$$S_{grav} = \int_\Omega (aR + \lambda)\sqrt{|g|}\, d^4x$$

if it is second-order & gives rise to second-order field equations

D. Grigore 1992

Quasi-invariant action, first order, must take form

$$S_{grav} = \int_\Omega (a\Gamma\text{-}\Gamma + \lambda)\sqrt{|g|}\, d^4x.$$

Common assumption :

only $g_{\mu\nu}$ & derivs count.

THE CASE OF GENERAL RELATIVITY I PURE GRAVITY

$$S_{grav} = \int \mathcal{L}_{grav}(g^{\mu\nu}, g^{\mu\nu}{}_{,\lambda}, g^{\mu\nu}{}_{,\lambda\sigma}, x)\, d^4x$$

Arbitrary transformation of coordinates $x^\mu \to x'^\mu = x^\mu + h^\mu(x)$, h small

NOETHER CONDITIONS for quasi-invariance of S

$$-g^{\mu\nu}{}_{;\alpha} E_{\mu\nu} \doteq 2\left(E_{\mu\alpha}\, g^{\sigma\mu}\right)_{;\sigma}$$

$$E_{\mu\nu} = \frac{\delta \mathcal{L}_{grav}}{\delta g^{\mu\nu}}$$

tensor density

Given metric compatibility $g^{\mu\nu}{}_{;\alpha} = 0$, we get

$$E^{\sigma}{}_{\mu;\sigma} \doteq 0.$$

Assuming $\mathcal{L}_{grav} = R\sqrt{-g}$ (Hilbert) OR $\mathcal{L}_{grav} = g^{\mu\nu}\left(\Gamma^{\alpha}{}_{\mu\beta}\Gamma^{\beta}{}_{\nu\alpha} - \Gamma^{\beta}{}_{\mu\nu}\Gamma^{\alpha}{}_{\alpha\beta}\right)\sqrt{-g}$ (Einstein)

$$E_{\mu\nu} \doteq G_{\mu\nu}\sqrt{-g}\ , \qquad G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu}$$

so: $$G^{\sigma}{}_{\mu;\sigma} \doteq 0$$ (twice) contracted BIANCHI IDENTITY

General covariance as *explanation* of Bianchi identity?

Leopold Infeld and Jerzy Plebanksy, *Motion and Relativity*, 1960.

p. 42.
"One could ask: what is the source of the [twice contracted] Bianchi identities? The answer is: they follow from the covariant character of Einstein's tensor $G^{\alpha\beta}$ and from the existence of a variational principle leading to $G^{\alpha\beta}$. We shall prove the identities in such a way as to exhibit the reason for their validity: covariance with respect to all the transformations."

p.44.
"The proof also shows the character of the Bianchi identities. They are a consequence of the invariance of the gravitational action, that is, they are a consequence of the temporal character of the theory which is covariant with respect to all transformations."

Robert Wald, *General Relativity*, 1984.

The Bianchi identity "may be viewed as a consequence of the invariance of the Hilbert action under diffeomorphisms." p. 345.

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