Title: The Exact Renormalization Group - Lecture 4: Gauge Theories

Date: May 07, 2008 10:30 AM

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Abstract: At first sight, the ERG does not sit well with gauge theories: a naive implementation of the momentum cutoff central to the ERG breaks gauge invariance. However, things are not as they seem. Not only is it possible to construct a gauge invariant cutoff, but it is possible to construct manifestly gauge invariant ERGs. I will discuss the formulation, what has been achieved to date, and what can reasonably be hoped for in the future.

Useful Literature

- T. R. Morris, "A gauge invariant exact renormalization group. I," Nucl. Phys. **B 573** (2000) 97, `hep-th/9910058`.
- S. Arnone, A. Gatti, and T. R. Morris, "A proposal for a manifestly gauge invariant and universal calculus in Yang-Mills theory," Phys. Rev. **D 67** (2003) 085004, `hep-th/0209162`.
- S. Arnone, T. R. Morris, and OJR, "A Generalised manifestly gauge invariant exact renormalisation group for SU(N) Yang-Mills," Eur. Phys. J. **C 50** (2007) 467, `hep-th/0507154`.
- T. R. Morris and OJR, "Manifestly gauge invariant QCD," J. Phys. **A 39** (2006) 11657, `hep-th/0606189`.

Continued...

- N. Evans, T. R. Morris and OJR, "Gauge invariant regularization in the AdS/CFT correspondence and ghost D-branes," Phys. Lett. B **635** (2006) 148, hep-th/0601114.
- OJR, "A manifestly gauge invariant and universal calculus for SU(N) Yang-Mills," Int. J. Mod. Phys. **A 21** (2006) 4627, hep-th/0602229.
- OJR, "General computations without fixing the gauge," Phys. Rev. **D 74** (2006) 125006, hep-th/0604183.
- OJR, "Universality from very general nonperturbative flow equations in QCD," Phys. Lett. **B 645** (466) 2007, hep-th/0611323.

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- J. M. Pawlowski, "Aspects of the functional renormalisation group," Annals Phys. **332** (2007) 2831, hep-th/0512261.
- H. Gies, "Introduction to the functional RG and applications to gauge theories," hep-ph/0611146.

Useful Concepts from Earlier Lectures

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Very General ERGs

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta \varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
- defines our ERG
- For scalar field theory we chose $\Psi_x = \frac{1}{2} \dot{\Delta}(x,y) \frac{\delta \Sigma}{\delta \varphi(y)}$
- $\hat{S}$ is the seed action

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2. Technicalities
3. Manifestly Gauge Invariant ERGs
   - Formulation
   - Diagrammatics

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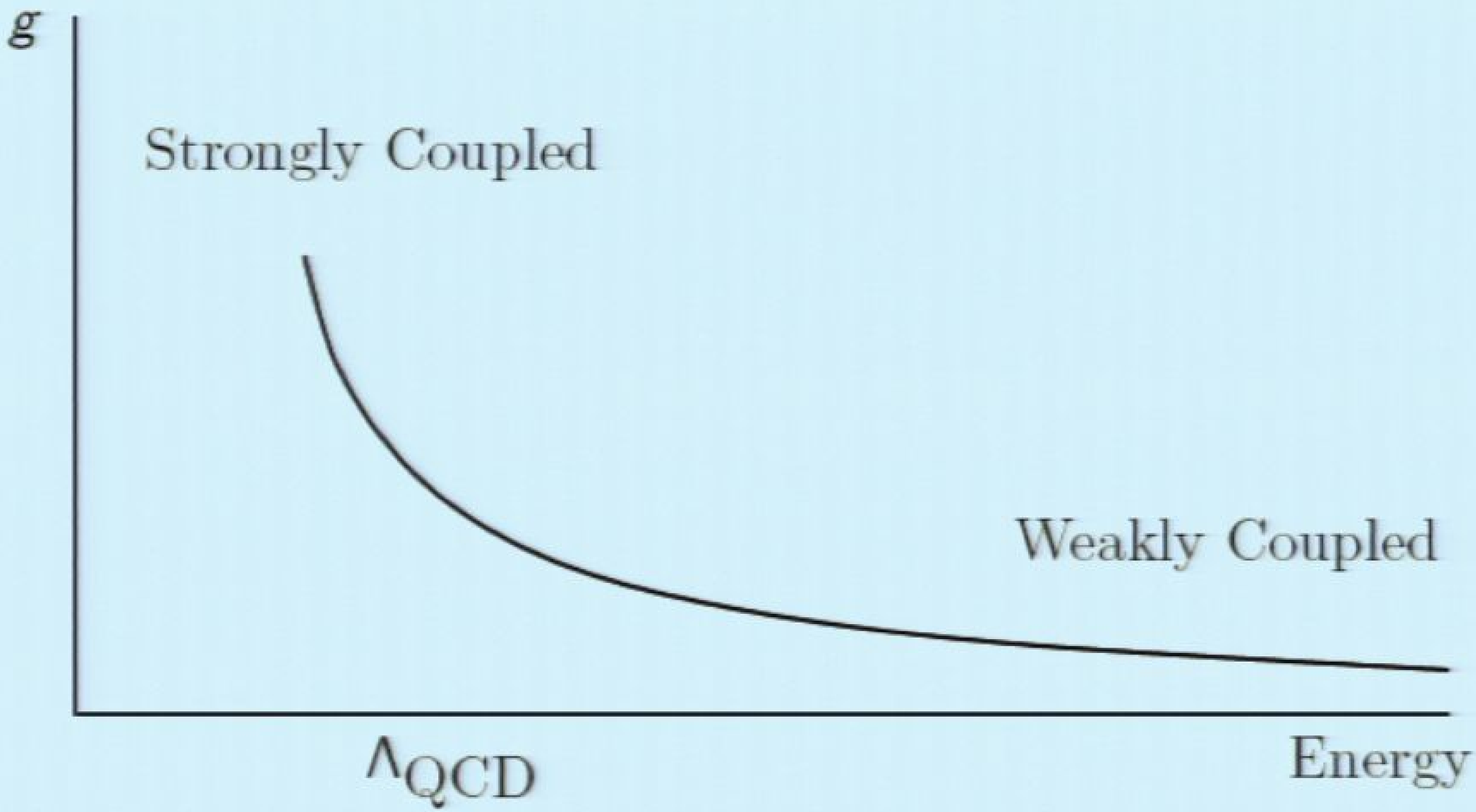

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- quarks & gluons
- hadrons

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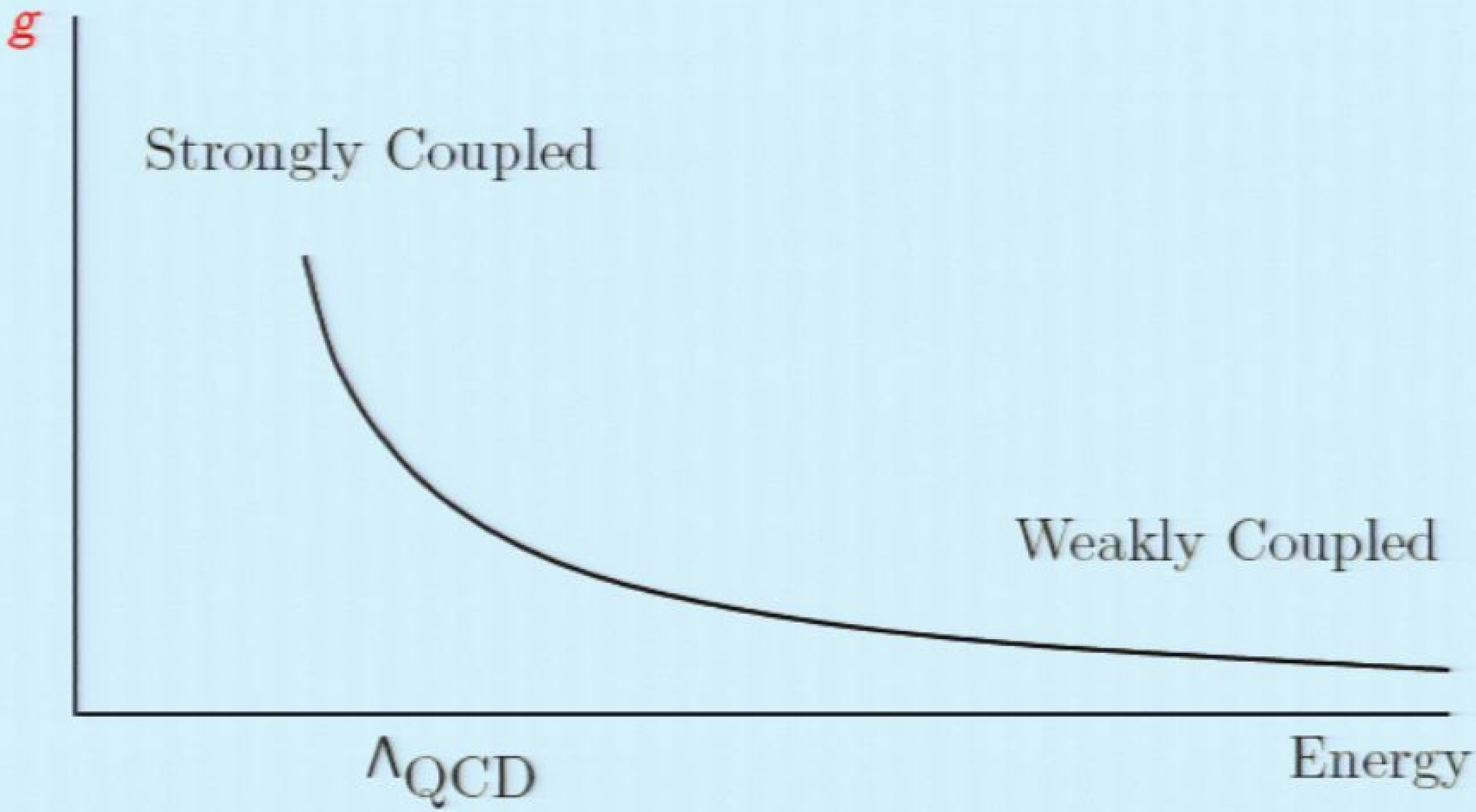

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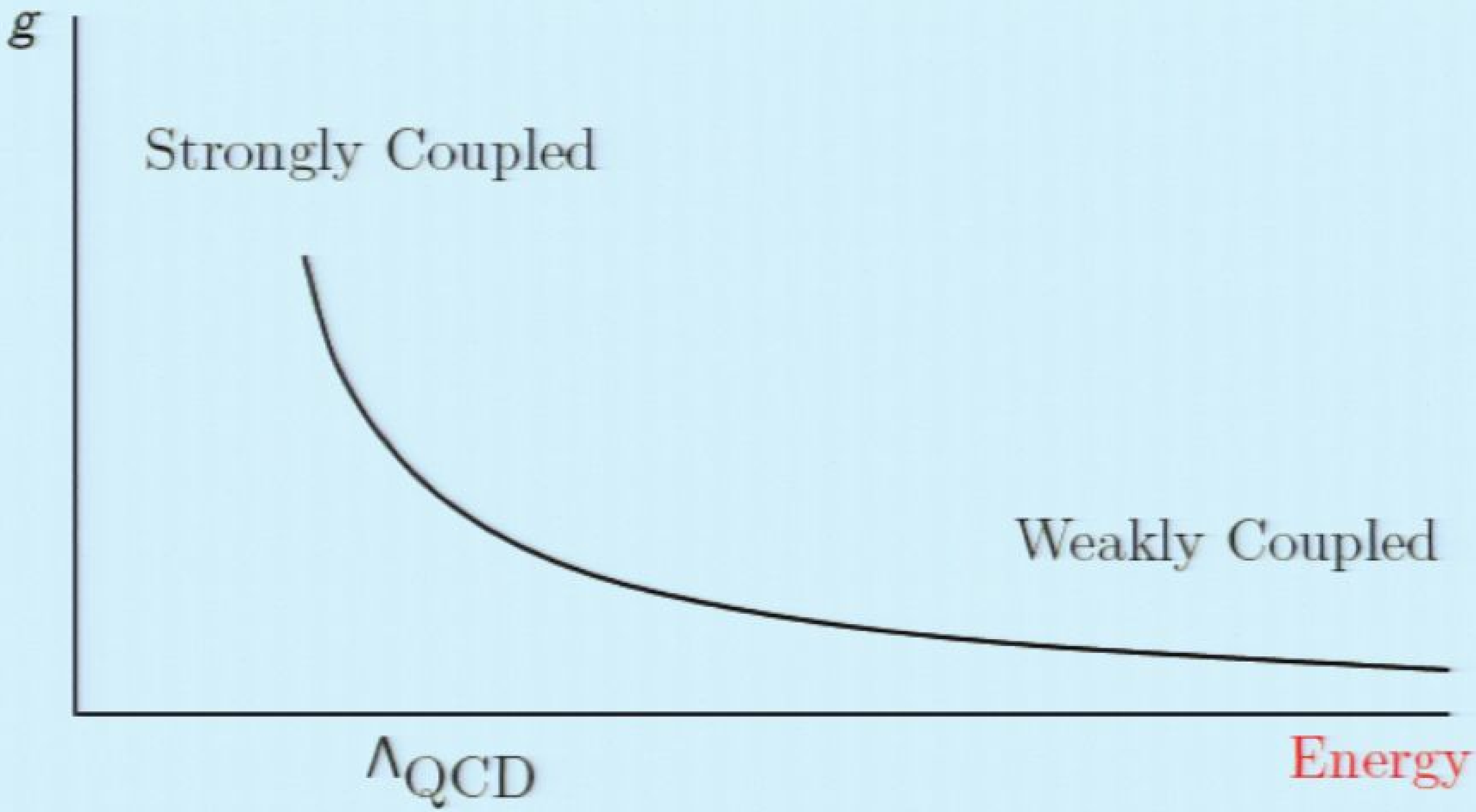

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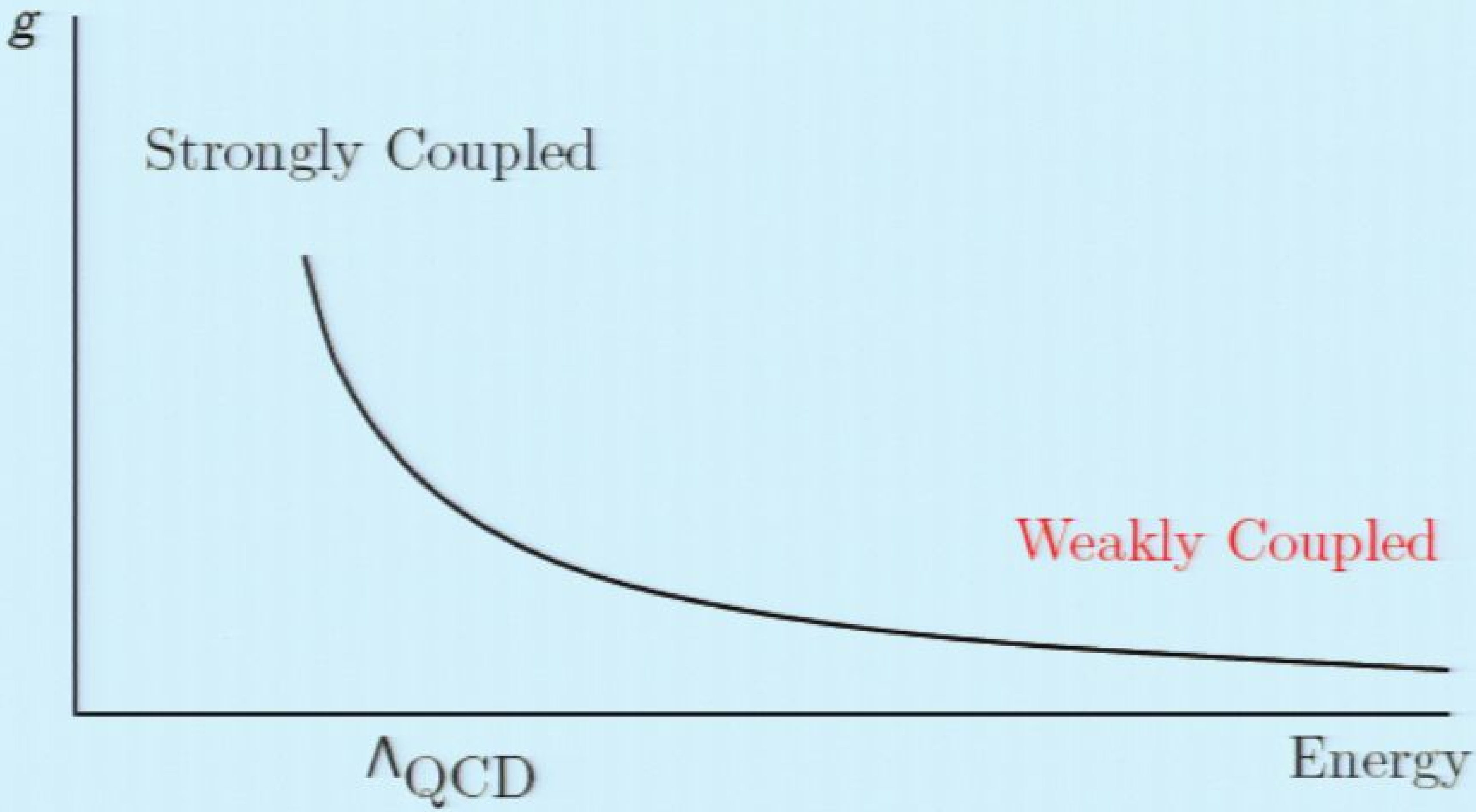

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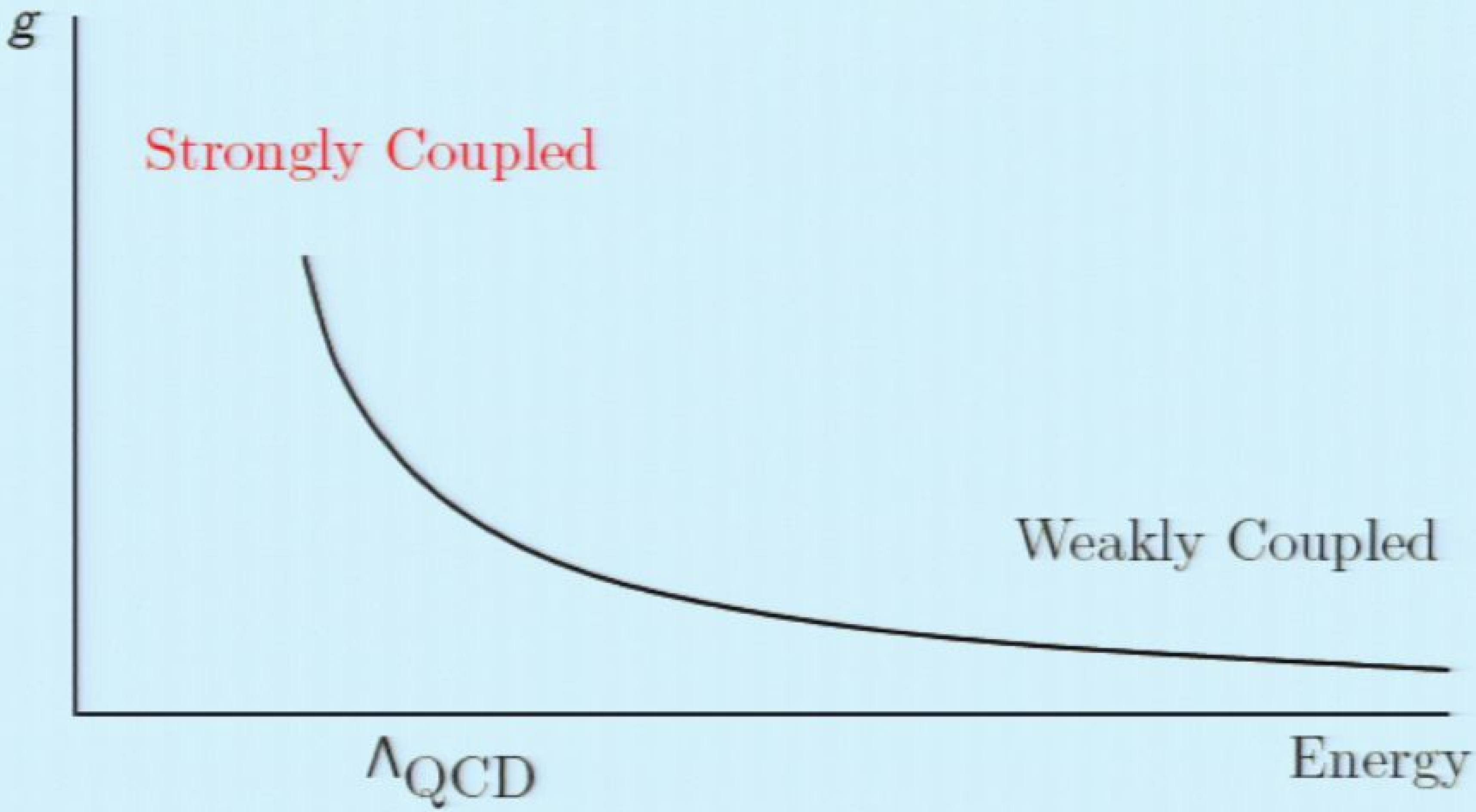

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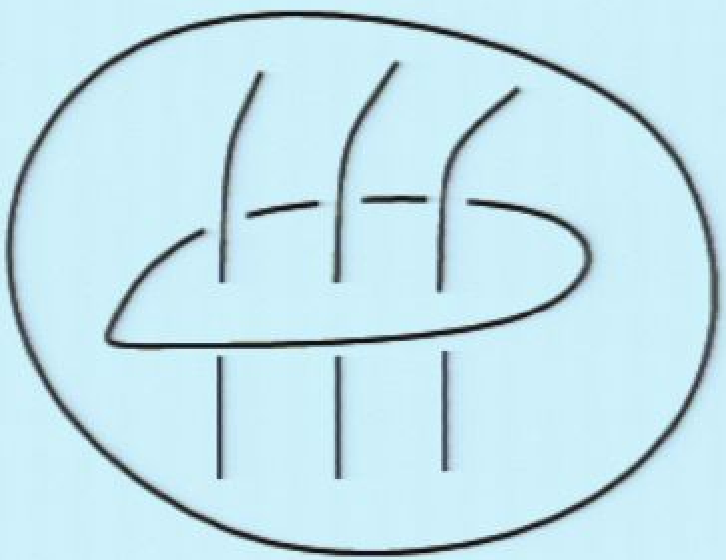

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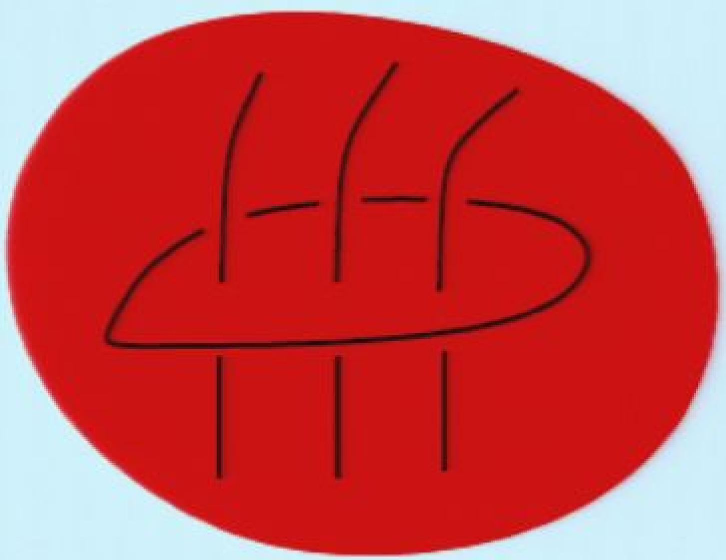

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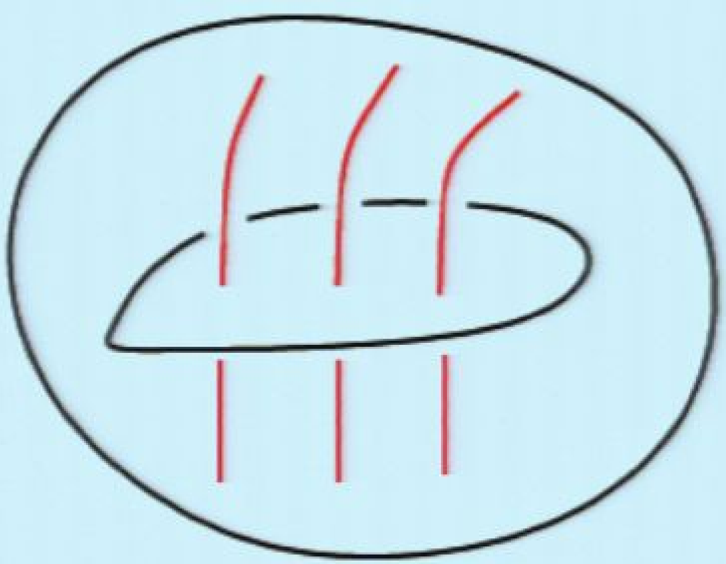

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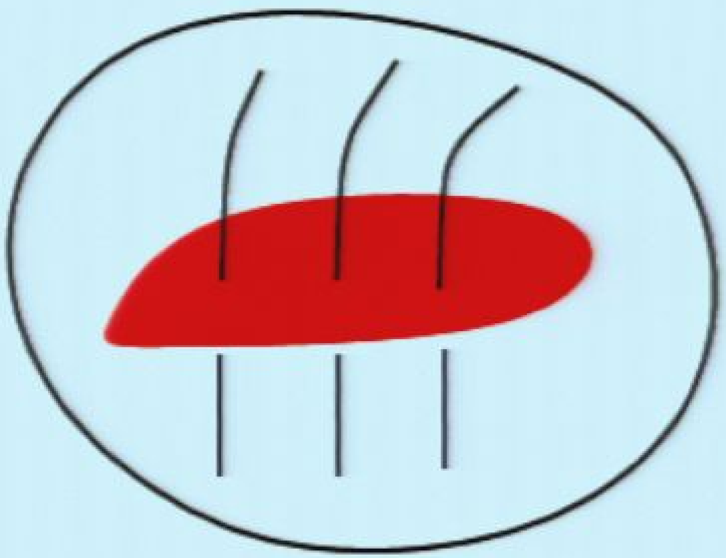

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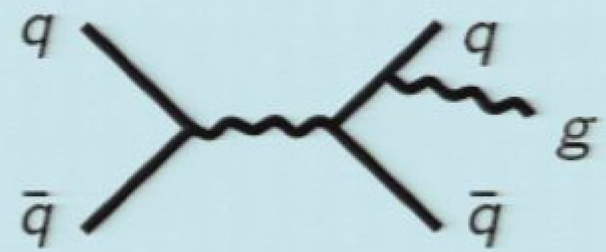

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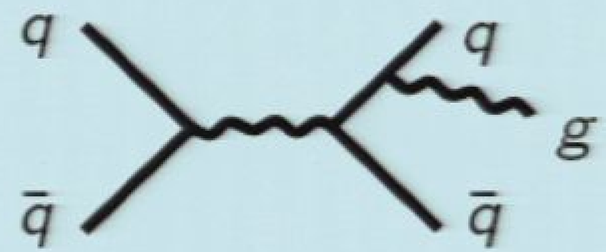

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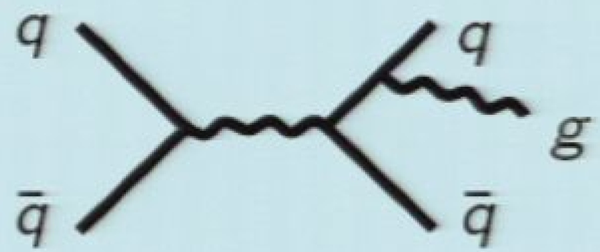

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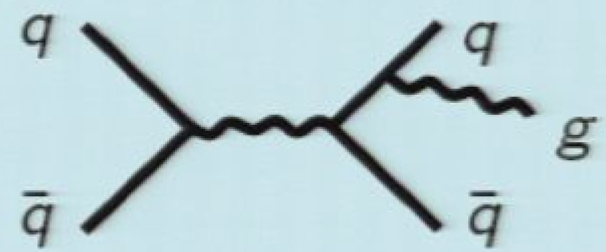

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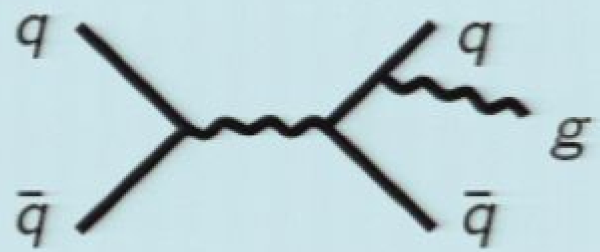

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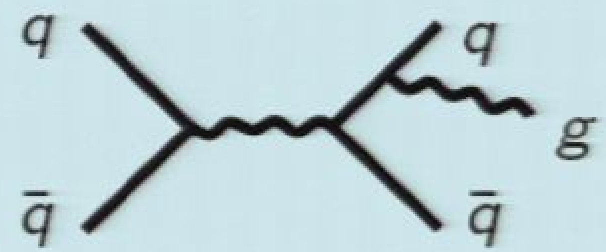

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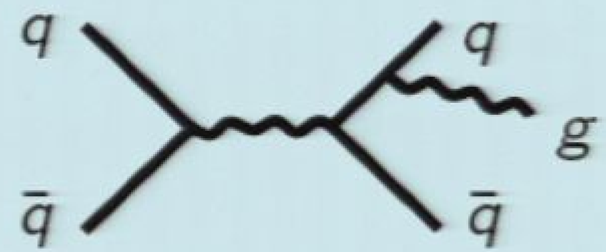

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$$\delta A_\mu = [\nabla_\mu, \omega] = \partial_\mu \omega - i[A_\mu, \omega]$$

$$A_\mu \mapsto \frac{A_\mu}{Z} \qquad \frac{\delta A_\mu}{Z} = (Z-1)\partial_\mu \omega - i[A_\mu, \omega]$$

$$+\partial_\mu \omega$$

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- Keep track of the breaking using modified Ward identities
- Recover gauge invariance in the limit that all fluctuations are integrated out (if no truncations!)
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The Basic Idea

- Step 1: Add in Covariant Higher Derivatives
  $\mathrm{tr} F_{\mu\nu} F_{\mu\nu} \to \mathrm{tr} F_{\mu\nu} c^{-1}(\nabla^2/\Lambda^2) F_{\mu\nu}$
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- In the defining representation:

$$\mathcal{A}_\mu = \begin{pmatrix} {A^1}_\mu & B_\mu \\ \bar{B}_\mu & A^2_\mu \end{pmatrix} + \mathcal{A}_0 \mathbb{1}$$

- Supergauge field valued in $SU(N|N)$
- The physical gauge field
- An unphysical copy, with wrong sign action
- Wrong statistics fermion field
- A central term

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Symmetries of SU($N|N$) Gauge Theory

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'no-$\mathcal{A}^0$' symmetry

- $S \sim \mathrm{str} F_{\mu\nu} F_{\mu\nu}$
- Combination of
- But $\mathcal{A}^0_\mu$ cannot simply be dropped from the action, since it is generated by gauge transformations!
- Leads to non-trivial constraints on the action
- Can essentially be forgotten about if we adopt a convenient and natural diagrammatic prescription

Local SU($N|N$) Invariance

$$A_\mu = \begin{pmatrix} A^1_\mu & B \\ \bar{B} & A^2_\mu \end{pmatrix} + A^0_\mu \mathbb{1}$$

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- $\delta \mathcal{A}_\mu = [\nabla_\mu, \Omega(x)]$

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(Super) Higgs Mechanism

- Introduce a superscalar, $\mathcal{C}$, valued in $\mathrm{U}(N|N)$

$$\mathcal{C} = \begin{pmatrix} C^1 & D \\ \bar{D} & C^2 \end{pmatrix}$$

- Arrange for $\mathcal{C}$ to acquire a vev in the direction

$$< \mathcal{C} > = \sigma \equiv \begin{pmatrix} 1\!\!1 & 0 \\ 0 & -1\!\!1 \end{pmatrix}$$

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Outline of this Lecture

1. Motivation
2. Technicalities
3. Manifestly Gauge Invariant ERGs
   - Formulation
   - Diagrammatics

Strategy

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- Pure U1 gauge theory can be regularized by a naïve cutoff
- We can use this theory to understand the basics of constructing manifestly gauge invariant ERGs
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A Flow Equation for pure U(1) Gauge Theory

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$$-\Lambda\partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta\varphi(x)}\left(\Psi_x[\varphi]e^{-S[\varphi]}\right)$$

- For scalar field theory we chose

$$\Psi_x = \frac{1}{2}\dot{\Delta}(x,y)\frac{\delta\Sigma}{\delta\varphi(y)}$$

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- $\delta/\delta A_\mu$ is gauge invariant
- No gauge fixing has been performed
- Therefore we have manifest gauge invariance!

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- In scalar field theory, the ERG kernel is usually chosen to be the derivative of the regularized propagator

$$\dot{\Delta} = -\Lambda \partial_\Lambda \frac{C_{\mathrm{UV}}}{p^2}$$

- How can this make sense in gauge theory?
- We have not fixed the gauge and so do not have a propagator!
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- In scalar field theory, we found it convenient to write the action as

$$S[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^{\mathrm{R}}[\varphi]$$

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Manifestly Gauge Invariant Flow Equation

$$-\Lambda\partial_\Lambda S_\Lambda = \frac{1}{2}\frac{\delta S}{\delta \mathcal{A}_\mu}\{\dot{\Delta}^{\mathcal{A}\mathcal{A}}\}\frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} - \frac{1}{2}\frac{\delta}{\delta \mathcal{A}_\mu}\{\dot{\Delta}^{\mathcal{A}\mathcal{A}}\}\frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} + \frac{1}{2}\frac{\delta S}{\delta \mathcal{C}}\{\dot{\Delta}^{\mathcal{C}\mathcal{C}}\}\frac{\delta \Sigma_g}{\delta \mathcal{C}} - \frac{1}{2}\frac{\delta}{\delta \mathcal{C}}\{\dot{\Delta}^{\mathcal{C}\mathcal{C}}\}\frac{\delta \Sigma_g}{\delta \mathcal{C}}$$

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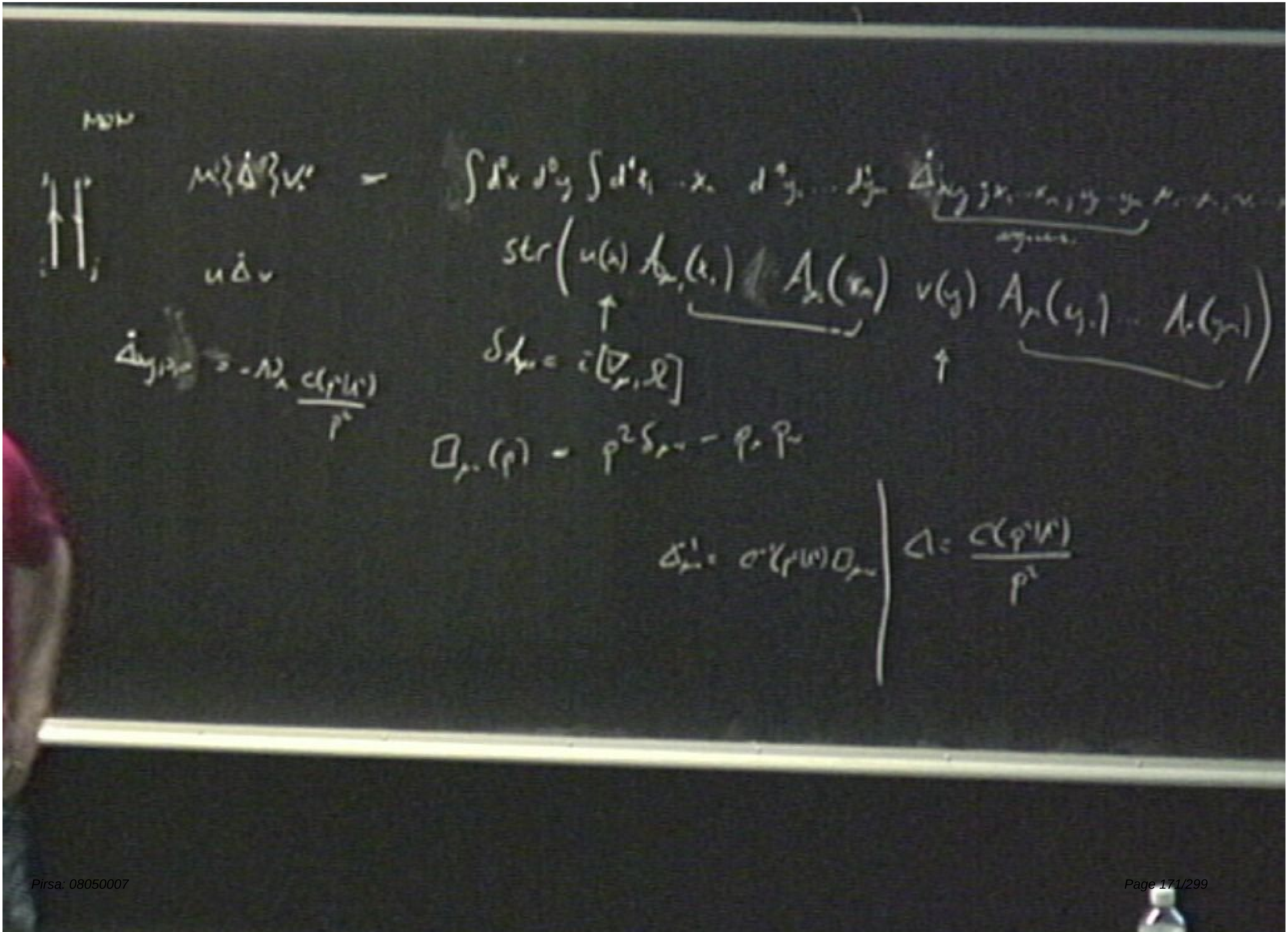
str
□μν(p) = p² δμν − pμ pν

$$u\{\dot{\Delta}\}v = \int d^D x\, d^D y \int d^D x_1 \ldots x_n\, d^D y_1 \ldots d^D y_m\, \dot{\Delta}_{xy;x_1\ldots x_n;y_1\ldots y_m}$$

$$\mathrm{str}\Big(u(x)\, A_{\mu_1}(x_1) \cdots A_{\mu_n}(x_n)\, v(y)\, A_{\nu_1}(y_1) \cdots A_{\nu_m}(y_m) \Big)$$

$u\dot{\Delta}v$

$$\dot{\Delta}_{xy;\ldots} = -\Lambda\partial_\Lambda \frac{c(p^2/\Lambda^2)}{p^2}$$

$$\delta A_\mu = i[\nabla_\mu, \Omega]$$

$$\Box_{\mu\nu}(p) = p^2\delta_{\mu\nu} - p_\mu p_\nu$$

$$\Delta^{-1}_{\mu\nu} = c^{-1}(p^2/\Lambda^2)\,\Box_{\mu\nu} \qquad \Big| \qquad \Delta = \frac{c(p^2/\Lambda^2)}{p^2}$$

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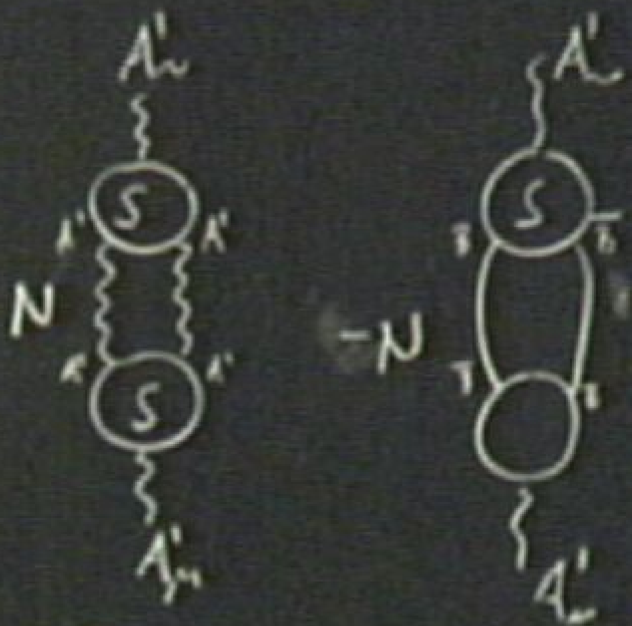

$$\mathrm{str}\begin{pmatrix} I_1 & 0 \\ 0 & 0 \end{pmatrix} = +N$$

$$\mathrm{str}\, A^1_\mu \, A^2_\nu \, A^2_\alpha$$

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Non-Universal Details

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- The seed action
- The cutoff functions
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Unphysical Fields

The Crucial Question

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- Blocking must take place only over a localized patch
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- We exploit self-similarity to formulate the renormalization condition for g

$$S[\mathcal{A} = A^1, \mathcal{C} = \sigma] = \frac{1}{2g^2} \operatorname{tr} \int d^D x \, (F^1_{\mu\nu})^2 + \dots$$

- But A^2 carries its own coupling, which renormalizes separately

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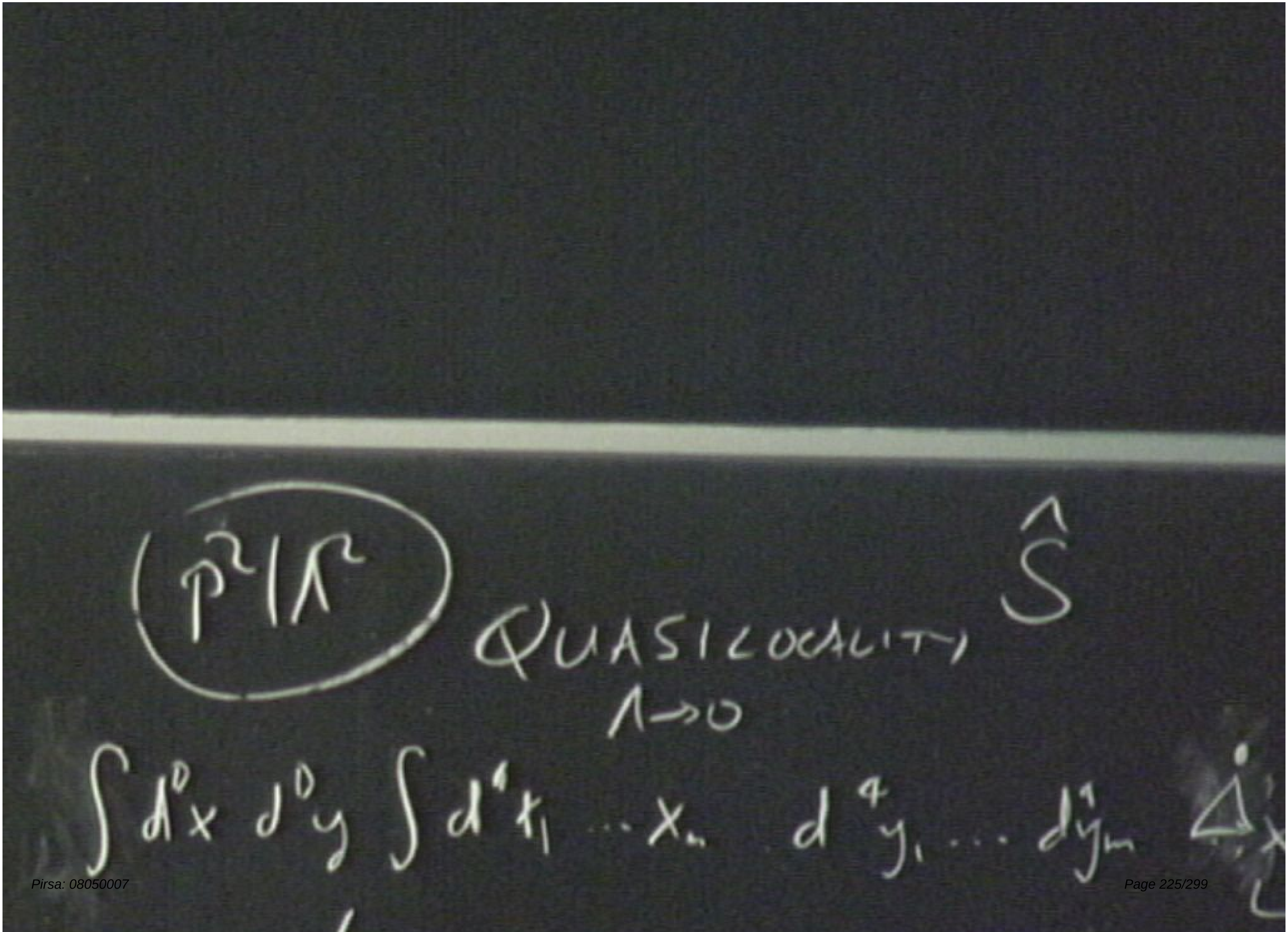
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  - This comes from the supertrace
  - Usually work with $\alpha \equiv g_2^2/g^2$

$$\frac{1}{g_2^2} = \frac{1}{\alpha g^2}$$

  - g^2 still counts loops

A More Complicated Flow Equation?

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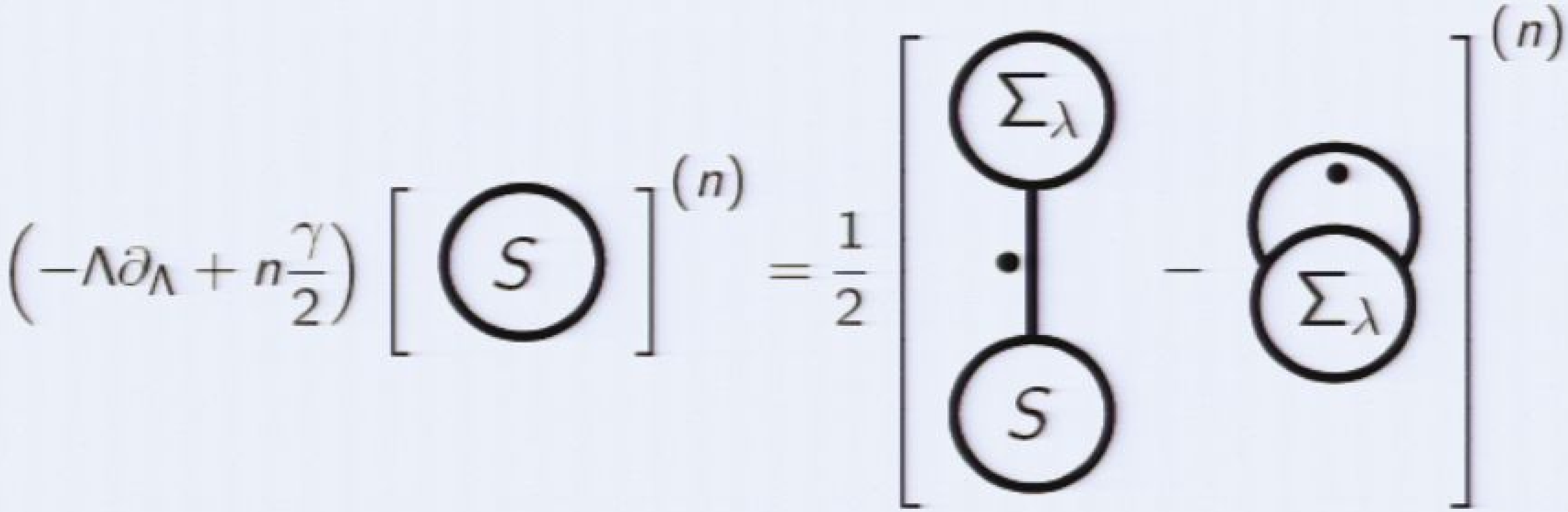

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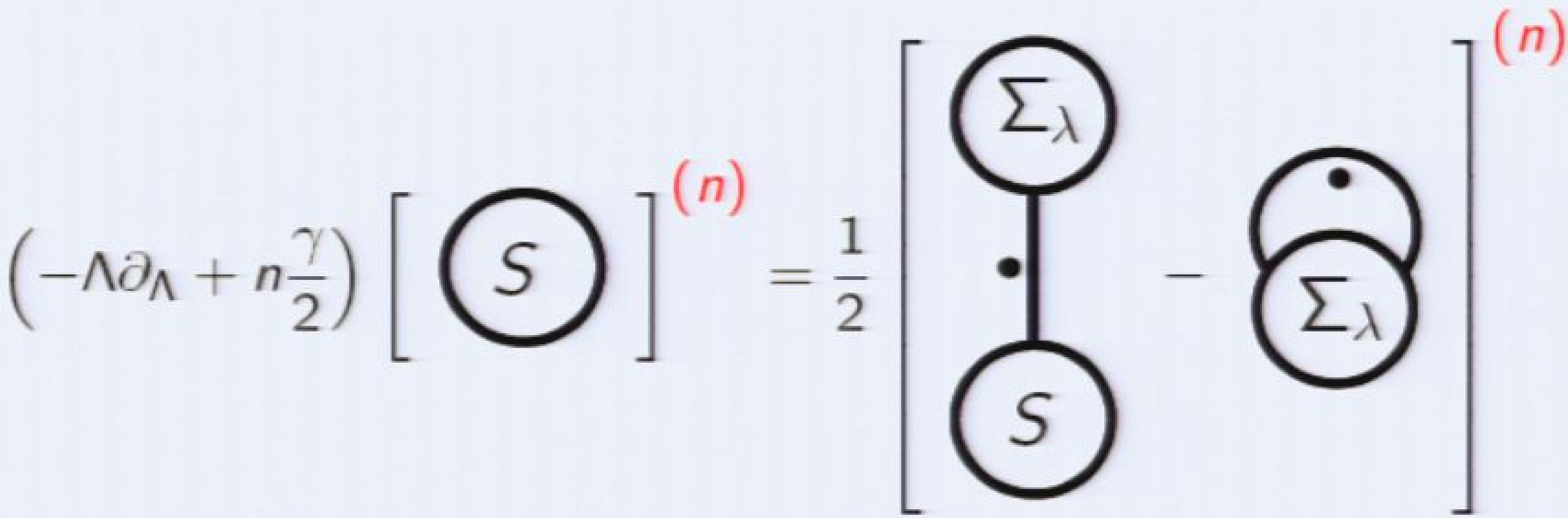

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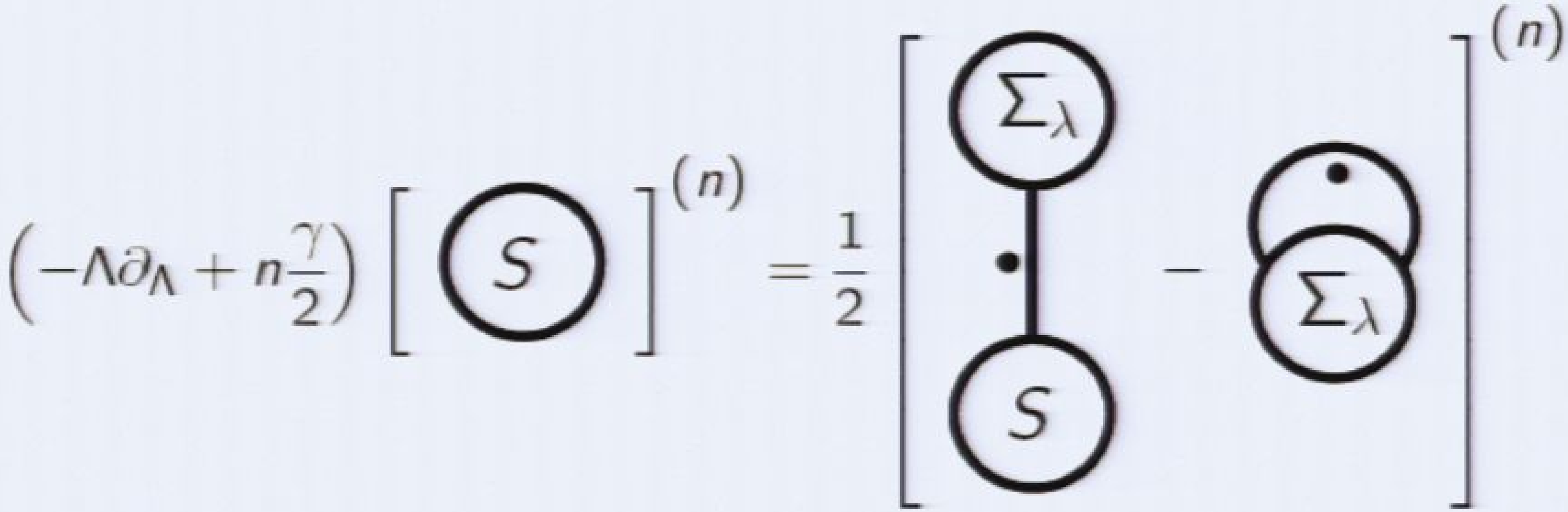

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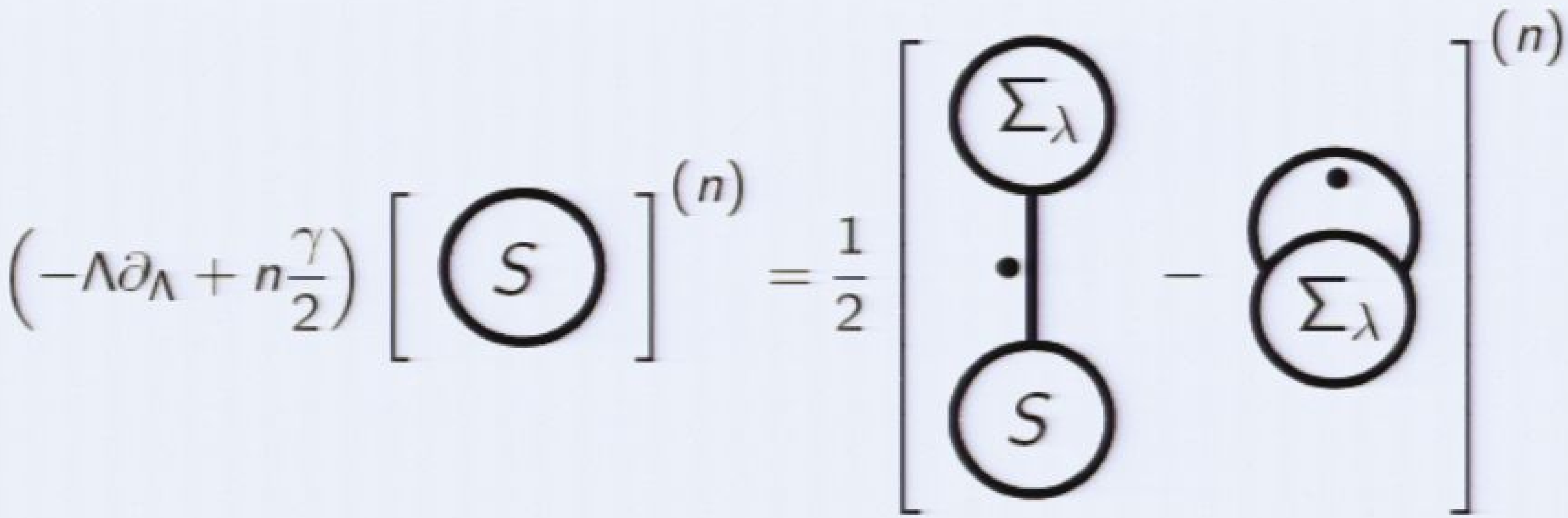

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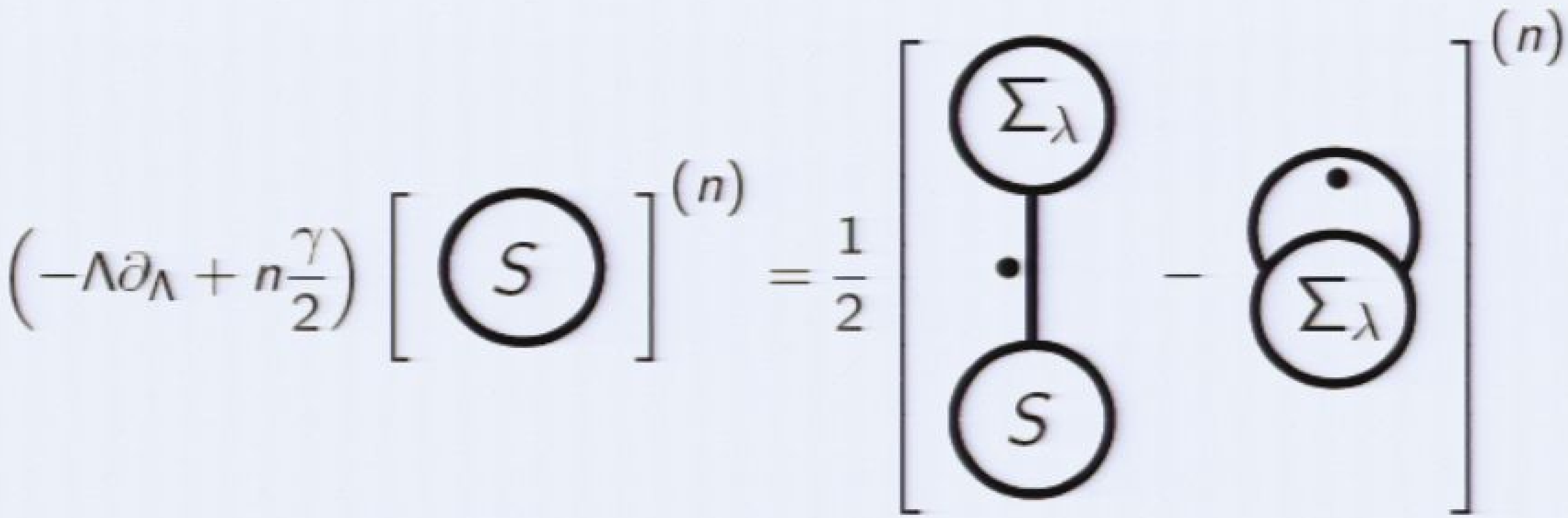

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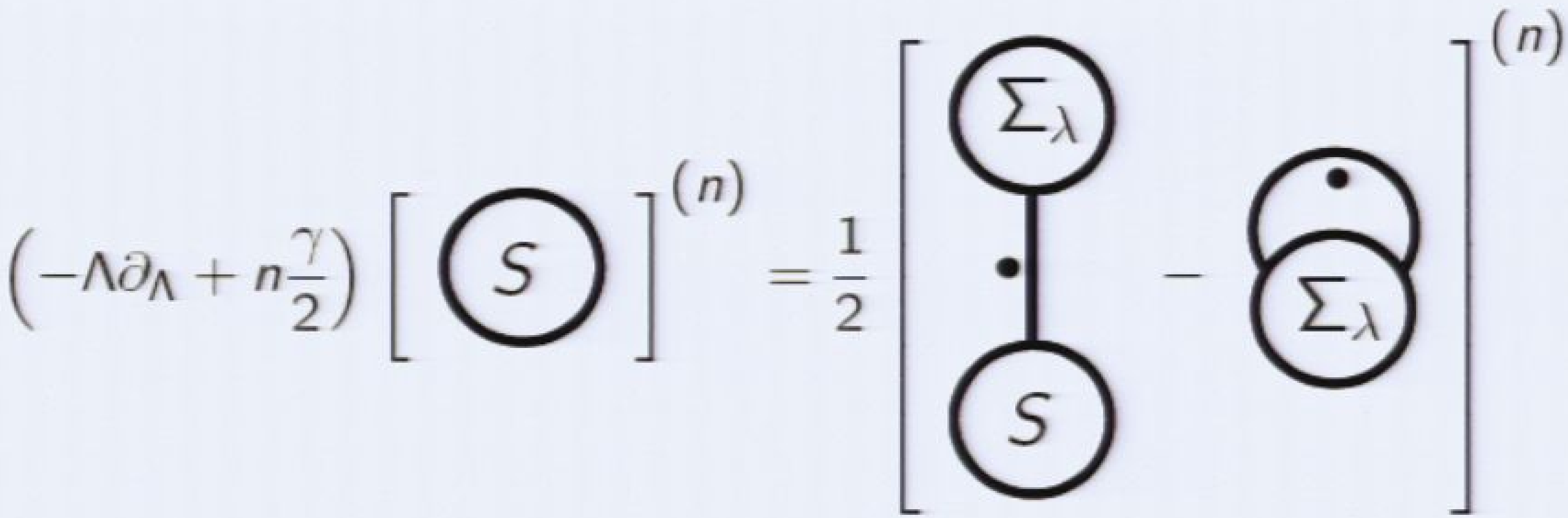

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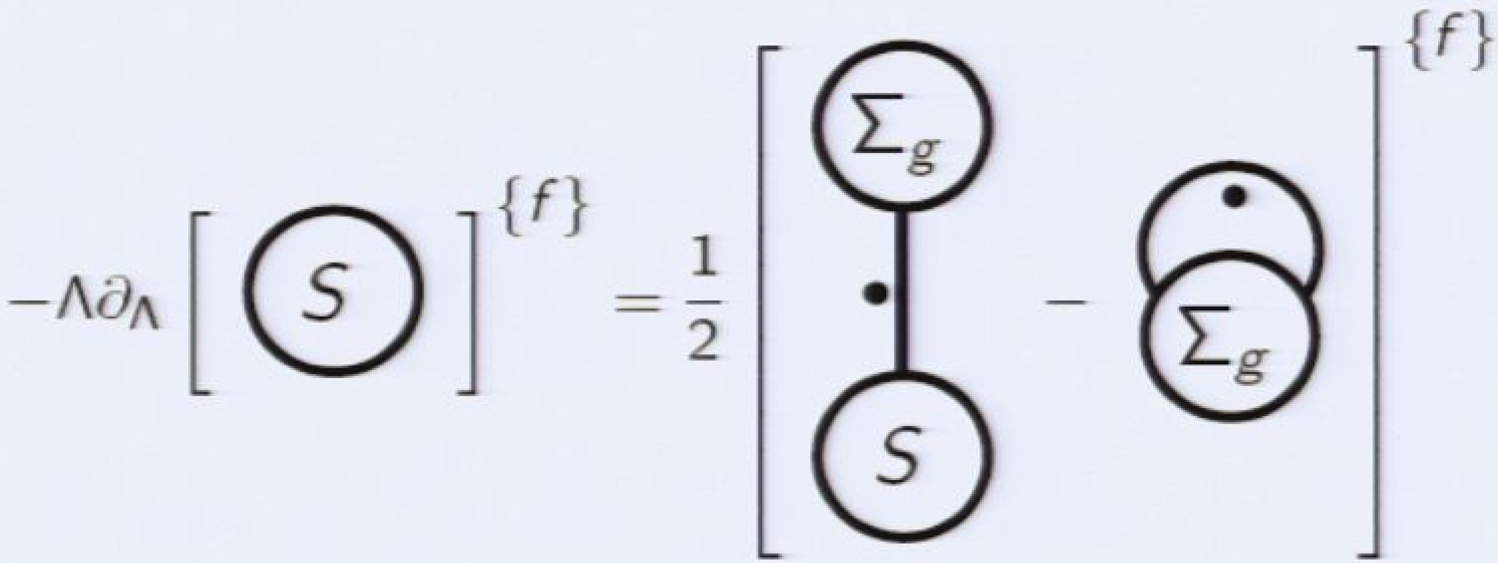

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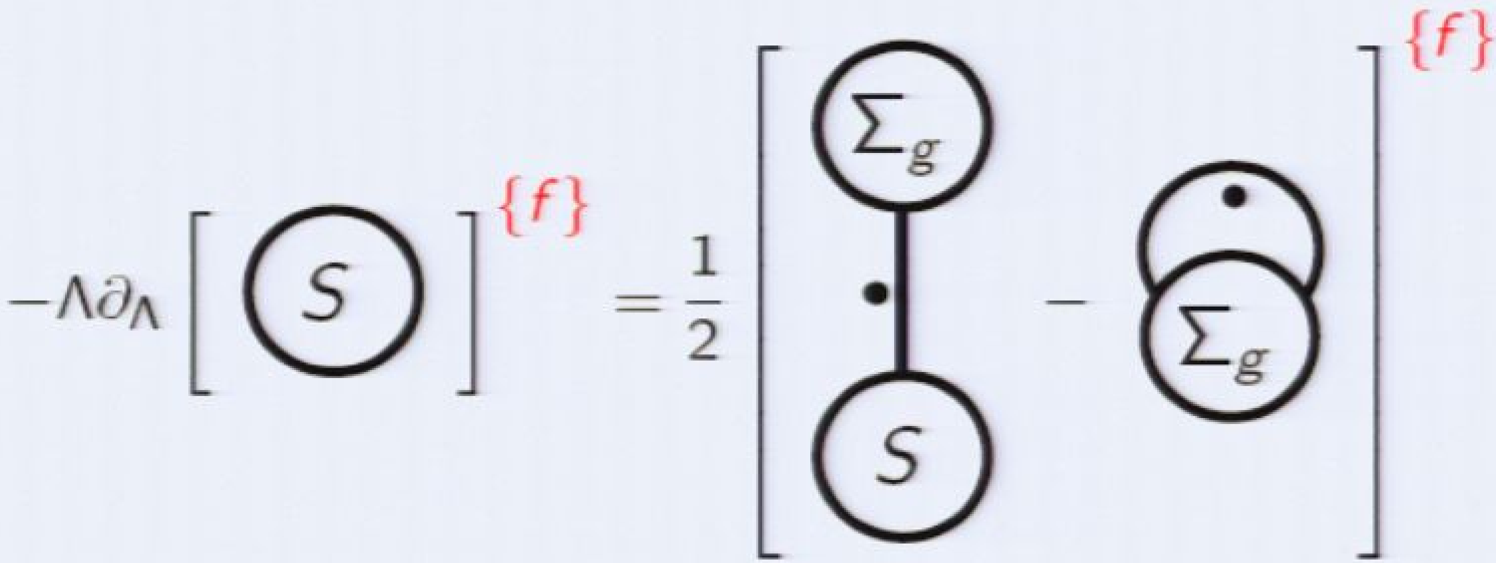

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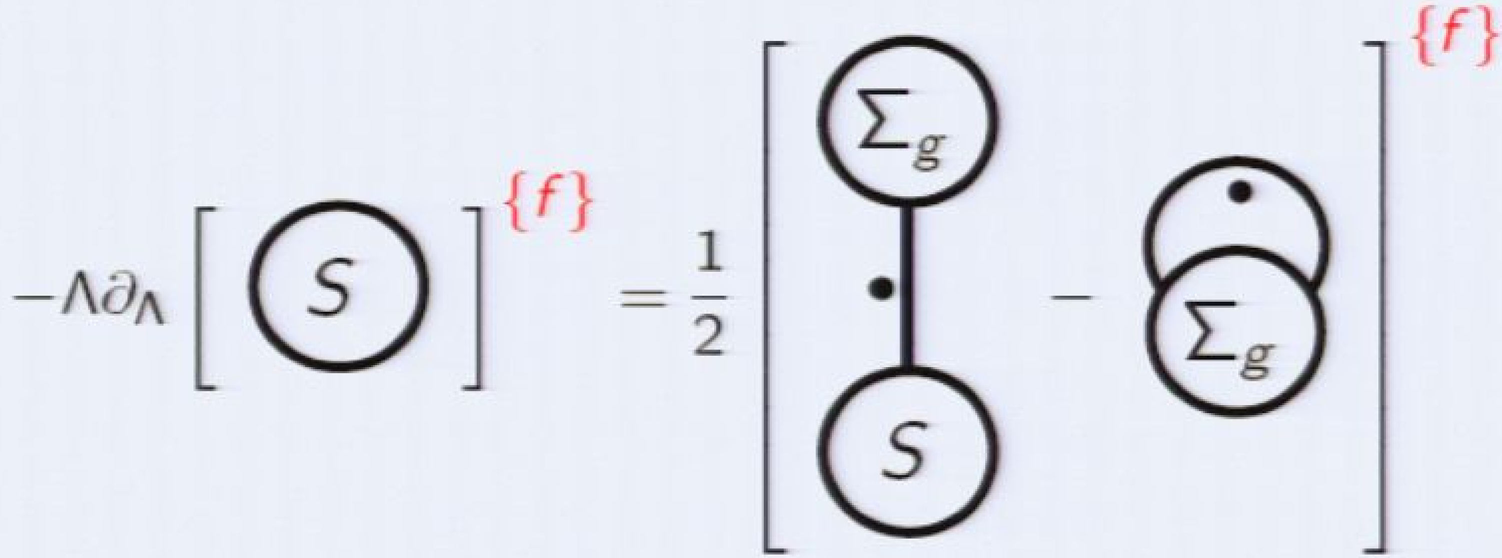

- The fields $\{f\}$
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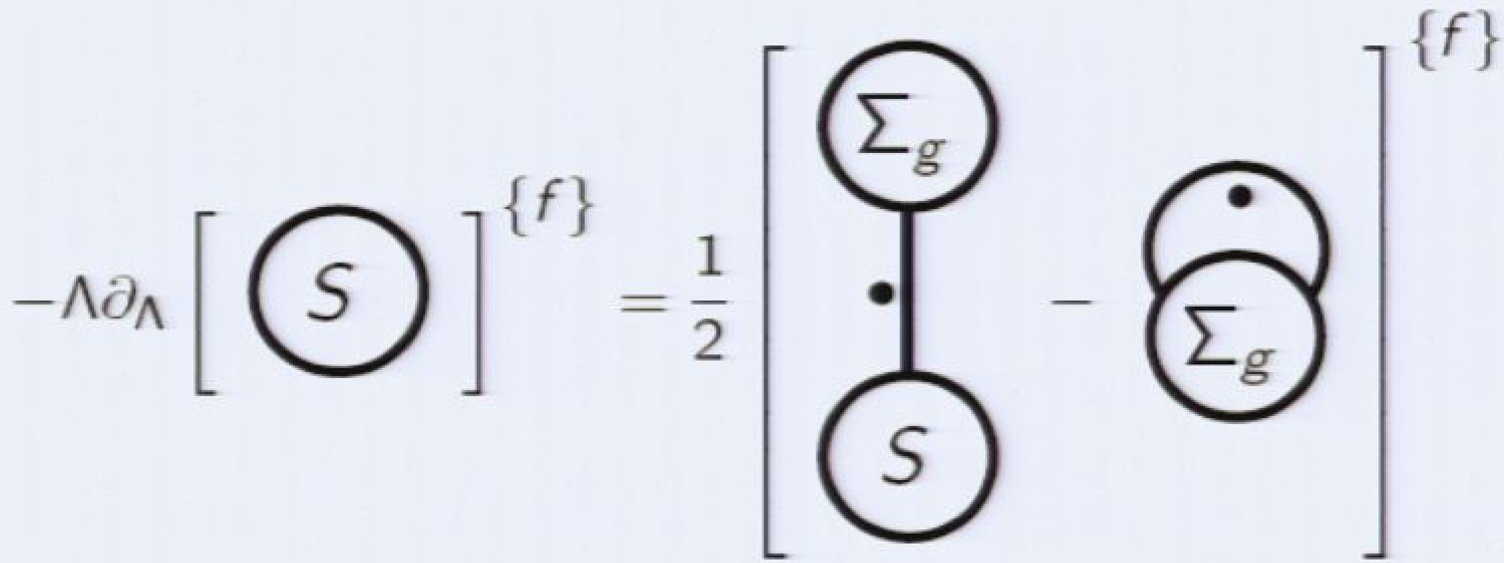

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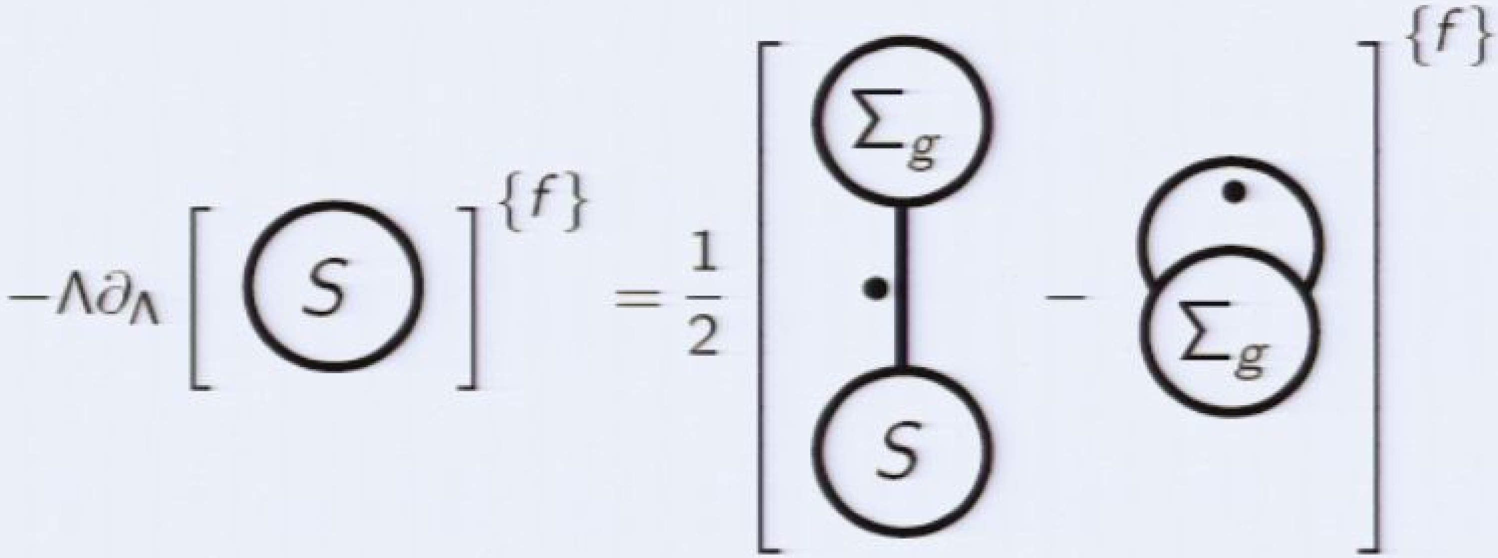

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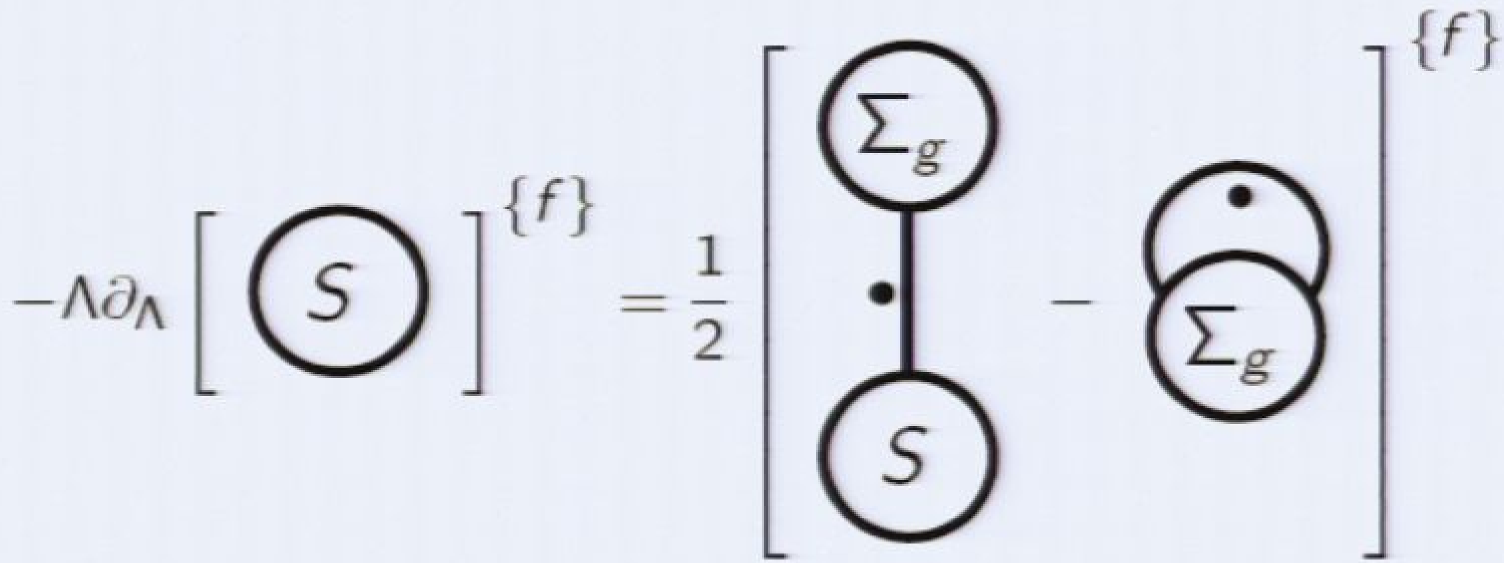

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- ERG sufficiently general to allow $\dot{\Delta}^{A^1A^1} \neq \dot{\Delta}^{A^2A^2}$

The Effective Propagator Relationship Revisited

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- In pure $U(1)$ gauge theory we had:

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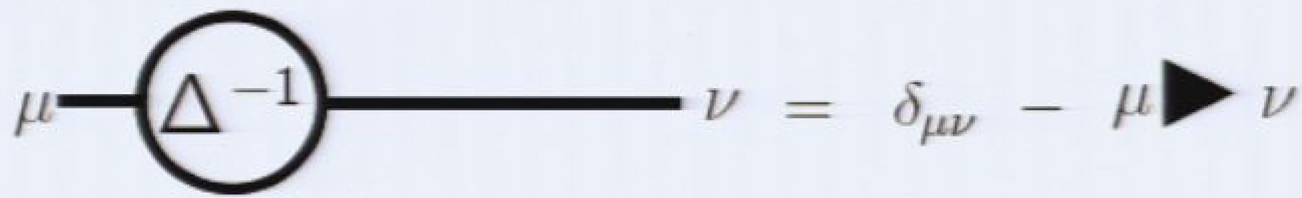

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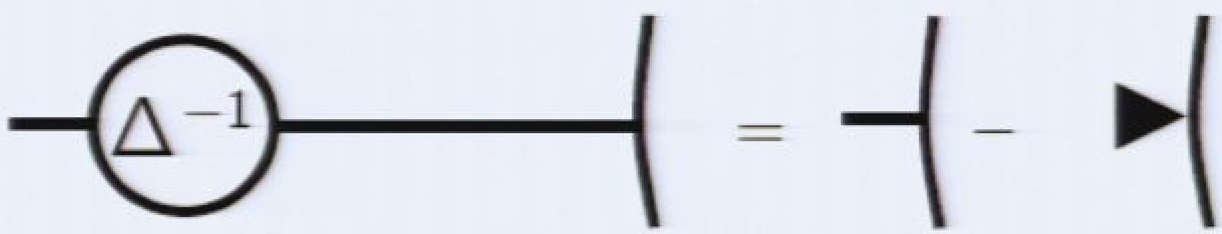

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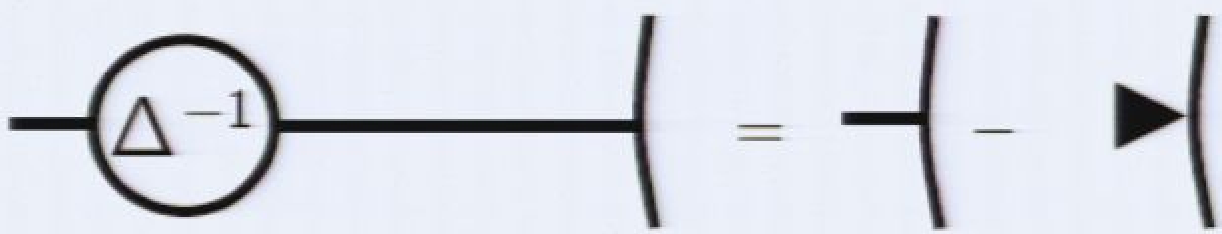

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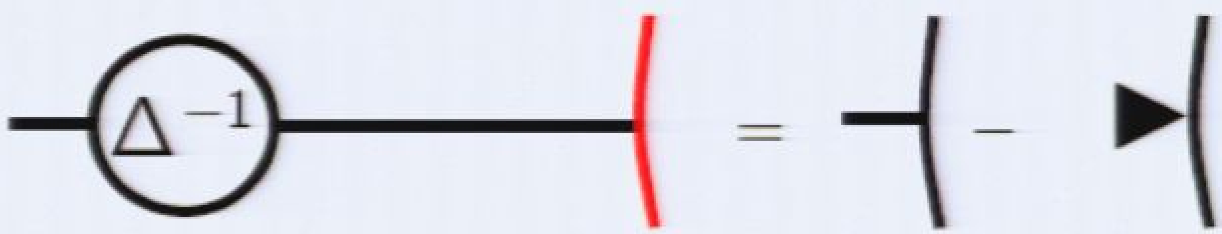

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Adding Quarks

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First Attempt

- The most obvious approach is to embed each quark field in a superquark field

$$\Psi = \begin{pmatrix} \psi \\ \varphi \end{pmatrix}$$

  - Physical fermion
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$$\bar{\Psi} A_\mu^0 \Psi$$

$$\text{st}, \begin{pmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{pmatrix} =$$

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- Embed up-like (down-like) quarks into an $N \times N$ matrix

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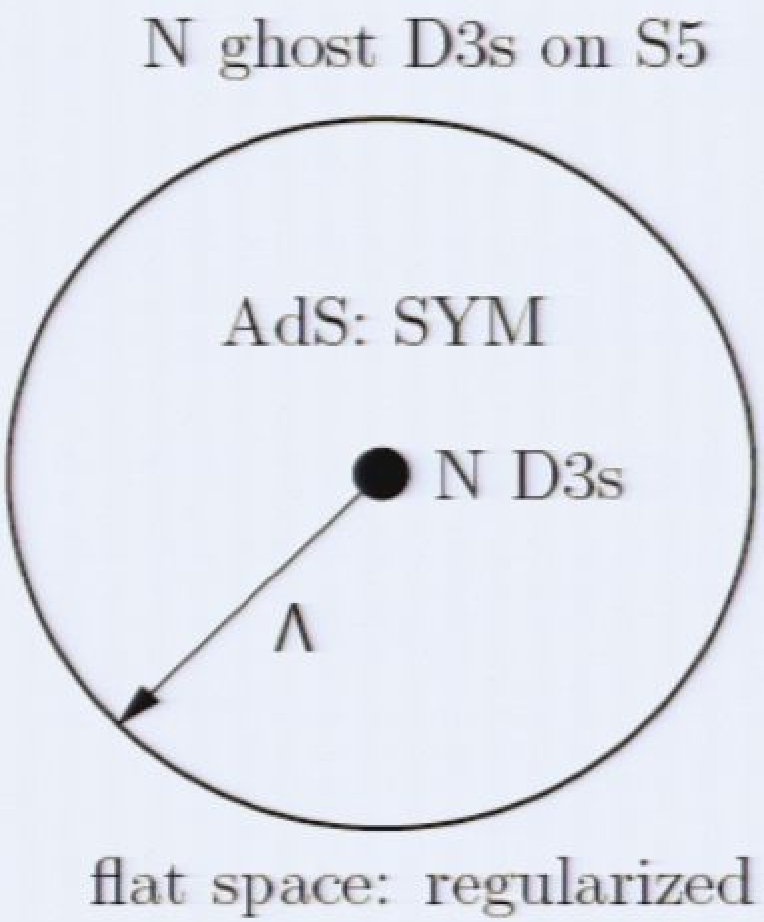
- The framework is very appealing
- It is understood how to perform efficient calculations in perturbation theory
- Understanding how best to apply the formalism beyond perturbation theory is a real challenge
- Generic truncations will spoil universality
- All the non-universal details buried in the diagrams would then need to be explicitly dealt with
- SUSY theories offer an excellent framework for trying to understand how best to proceed
- I'm very hopeful that this will not be a passive process!

D-Brane Interpretation

$\mathcal{N} = 4$ SYM

Return



- N D3 Branes generate $AdS_5 \times S^5$ geometry
- N Ghost D3 Branes
- Separation of D3s and ghost D3s $\Rightarrow$ spontaneously broken $SU(N|N)$ gauge theory

D-Brane Interpretation

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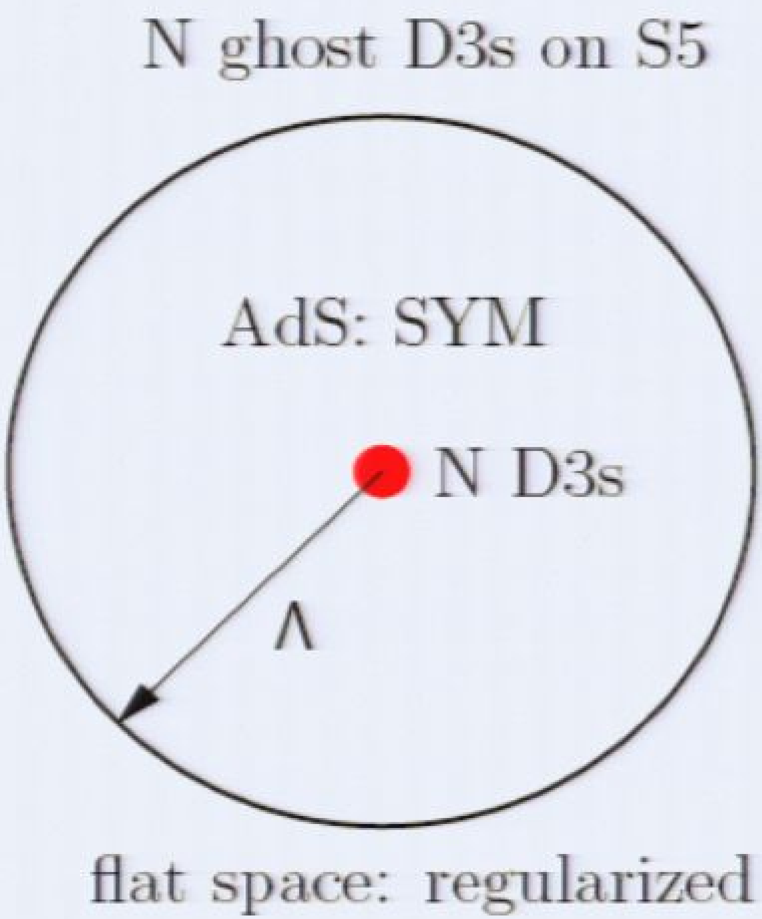

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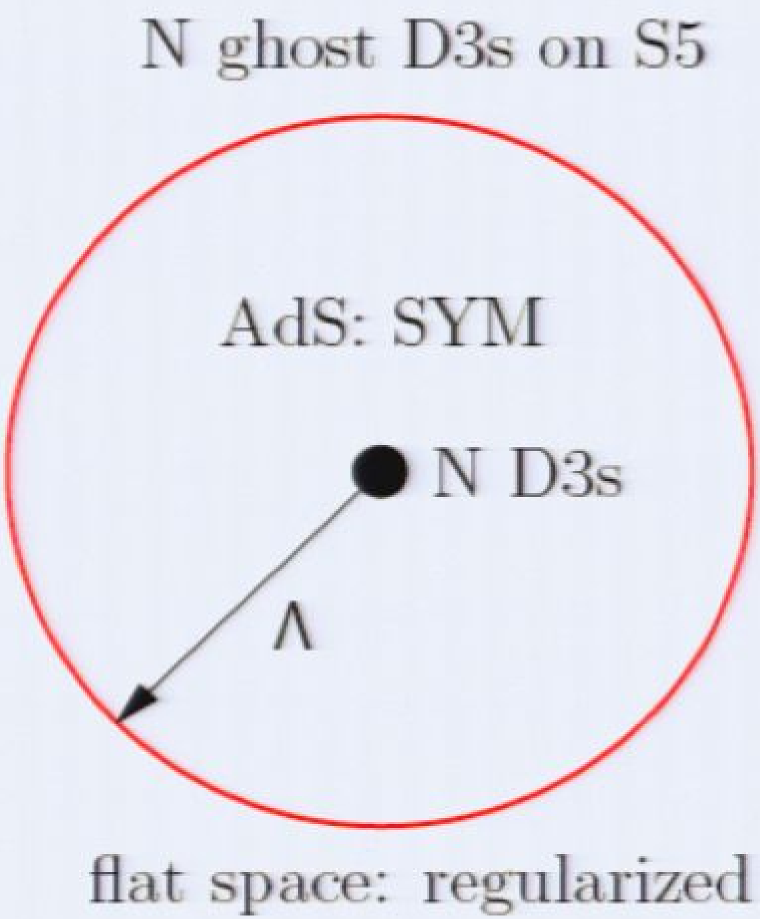

- N D3 Branes generate $AdS_5 \times S^5$ geometry
- N Ghost D3 Branes
  - Negative tension
  - No effect inside shell (Gauss' Law)
  - Cancel D3 branes outside shell
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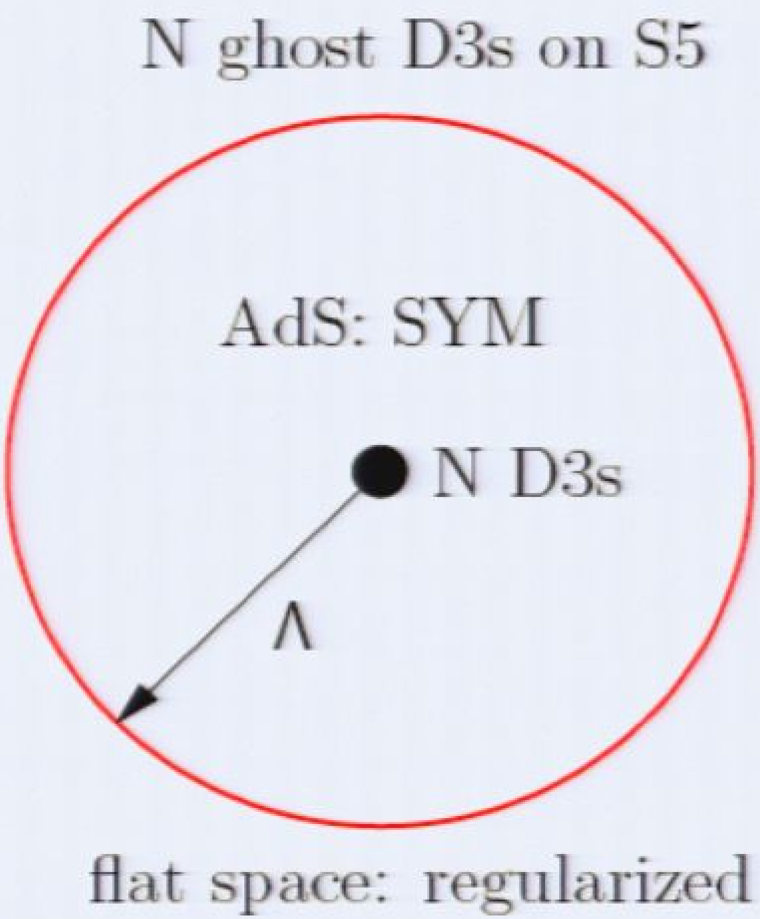

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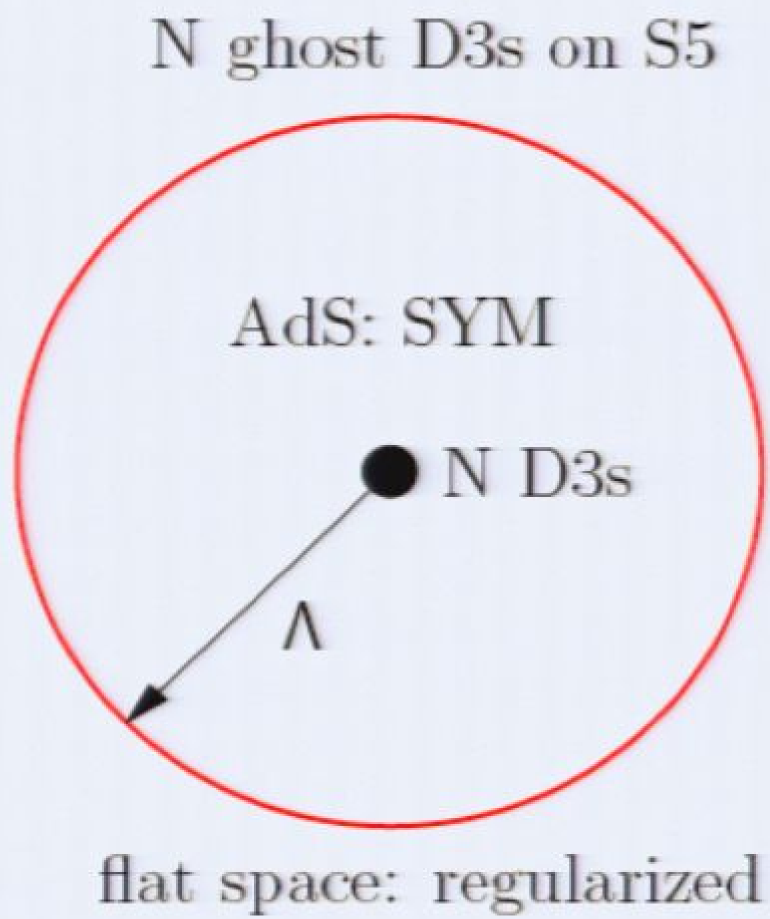

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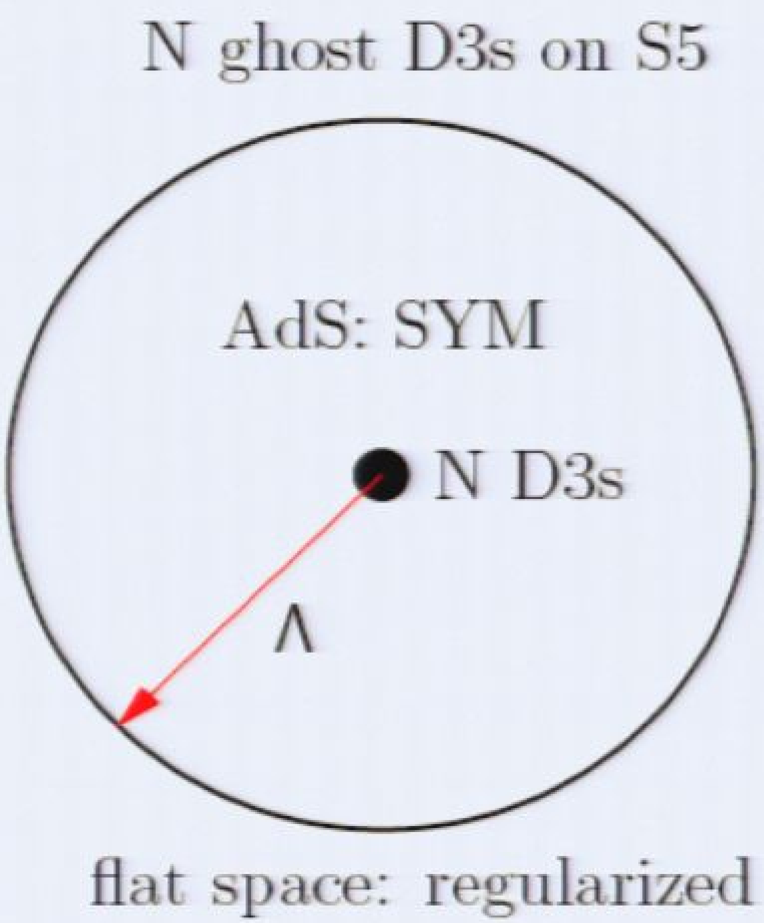

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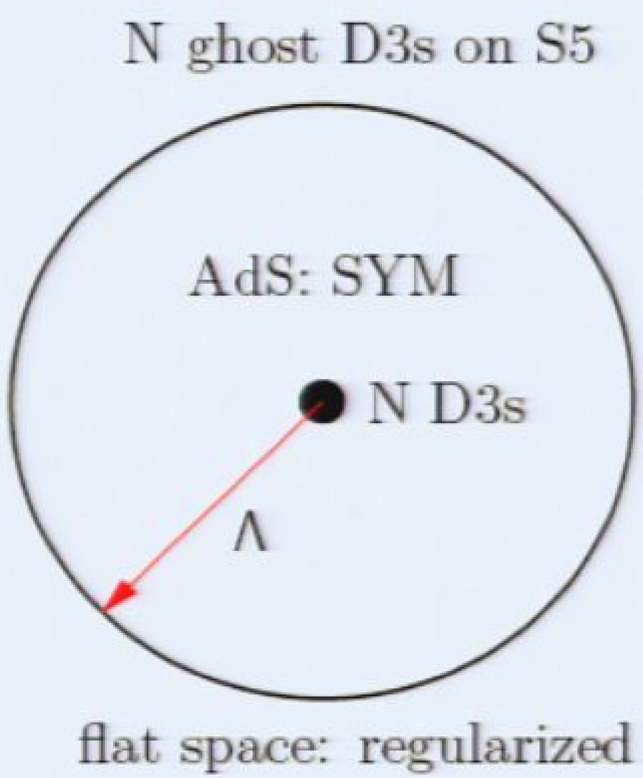

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