

Title: The Exact Renormalization Group - Lecture 4: Gauge Theories

Date: May 07, 2008 10:30 AM

URL: <http://pirsa.org/08050007>

Abstract: At first sight, the ERG does not sit well with gauge theories: a naive implementation of the momentum cutoff central to the ERG breaks gauge invariance. However, things are not as they seem. Not only is it possible to construct a gauge invariant cutoff, but it is possible to construct manifestly gauge invariant ERGs. I will discuss the formulation, what has been achieved to date, and what can reasonably be hoped for in the future.

Useful Literature

- T. R. Morris, "A gauge invariant exact renormalization group. I," Nucl. Phys. **B 573** (2000) 97, [hep-th/9910058](#).
- S. Arnone, A. Gatti, and T. R. Morris, "A proposal for a manifestly gauge invariant and universal calculus in Yang-Mills theory," Phys. Rev. **D 67** (2003) 085004, [hep-th/0209162](#).
- S. Arnone, T. R. Morris, and OJR, "A Generalised manifestly gauge invariant exact renormalisation group for SU(N) Yang-Mills," Eur. Phys. J. **C 50** (2007) 467, [hep-th/0507154](#).
- T. R. Morris and OJR, "Manifestly gauge invariant QCD," J. Phys. **A 39** (2006) 11657, [hep-th/0606189](#).

Continued...

- N. Evans, T. R. Morris and OJR, "Gauge invariant regularization in the AdS/CFT correspondence and ghost D-branes," Phys. Lett. B **635** (2006) 148, hep-th/0601114.
- OJR, "A manifestly gauge invariant and universal calculus for SU(N) Yang-Mills," Int. J. Mod. Phys. **A 21** (2006) 4627, hep-th/0602229.
- OJR, "General computations without fixing the gauge," Phys. Rev. **D 74** (2006) 125006, hep-th/0604183.
- OJR, "Universality from very general nonperturbative flow equations in QCD," Phys. Lett. **B 645** (466) 2007, hep-th/0611323.

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- J. M. Pawłowski, “Aspects of the functional renormalisation group,” *Annals Phys.* **332** (2007) 2831, [hep-th/0512261](#).
- H. Gies, “Introduction to the functional RG and applications to gauge theories,” [hep-ph/0611146](#).

Useful Concepts from Earlier Lectures

Useful Concepts from Earlier Lectures

Very General ERGs

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta\varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
- defines our ERG

- For scalar field theory we chose $\Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$

- \hat{S} is the seed action

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- **defines our ERG**

- parametrizes blocking procedure
- huge freedom in precise form—adapt to suit our needs

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• The ERG reveals important properties of the flow

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• ERG kernel incorporates cutoff function

• Σ is the action

- \hat{S} is the seed action

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• Some authors use \hat{S} for the flow

• Only a small number of ERGs are known

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 - ERG kernels: incorporate cutoff functions
 - $\Sigma = S - 2\hat{S}$
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 - Same structure and symmetries as S
 - Only a kinetic term is required for scalar field theory

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- 1 Motivation
- 2 Technicalities
- 3 Manifestly Gauge Invariant ERGs
 - Formulation
 - Diagrammatics

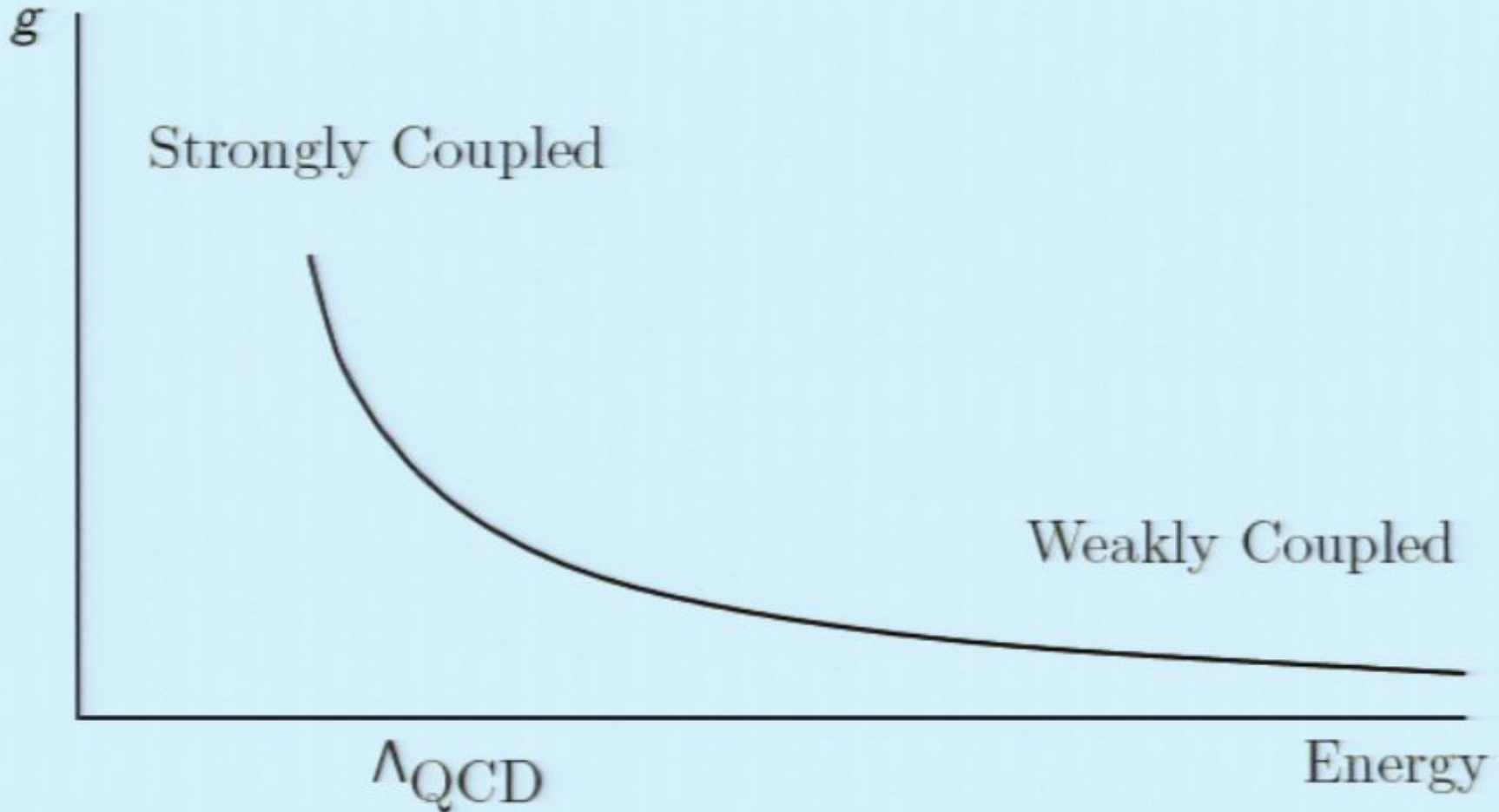
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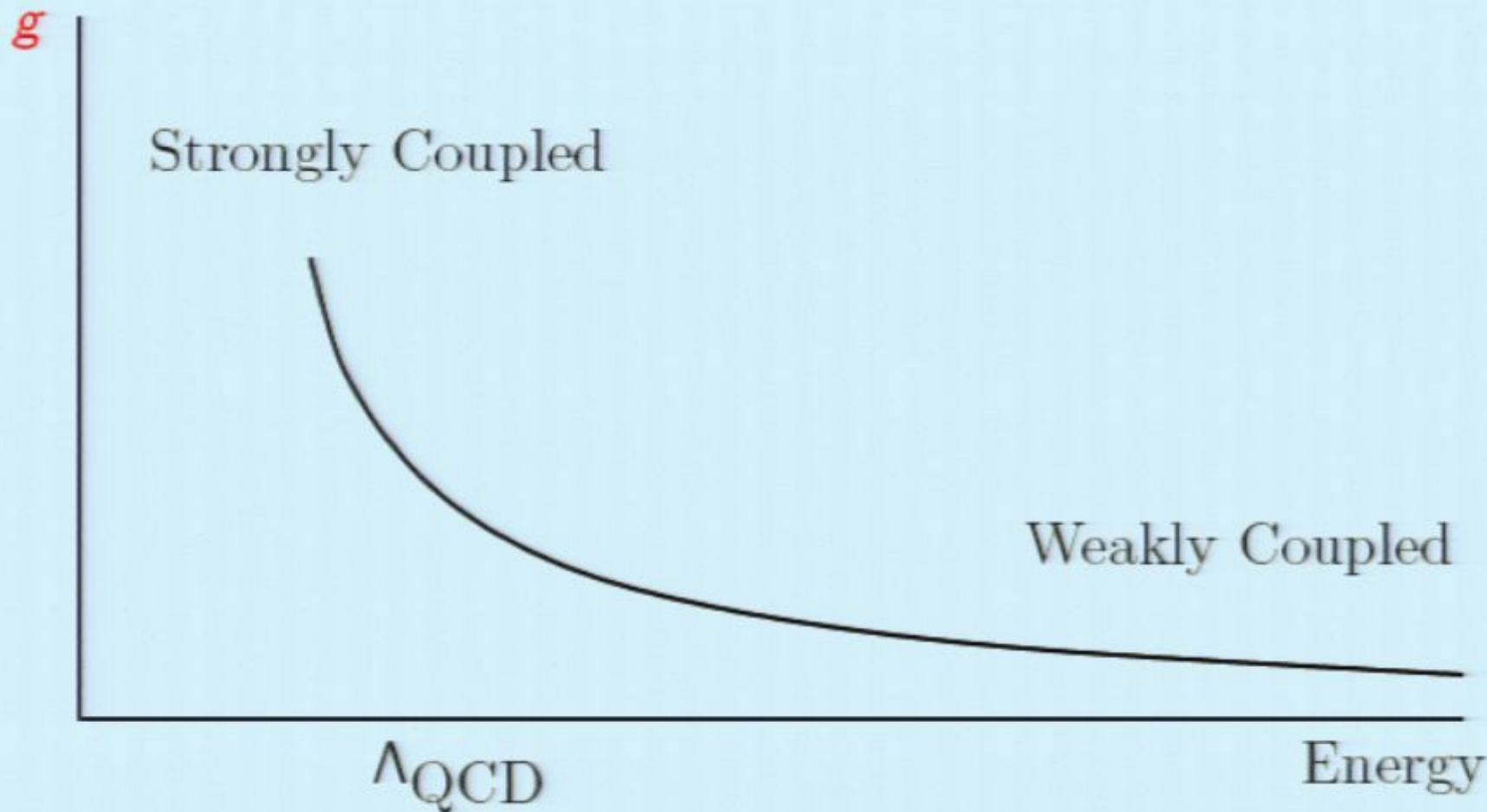
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- QCD coupling constant
- quarks & gluons
- hadrons

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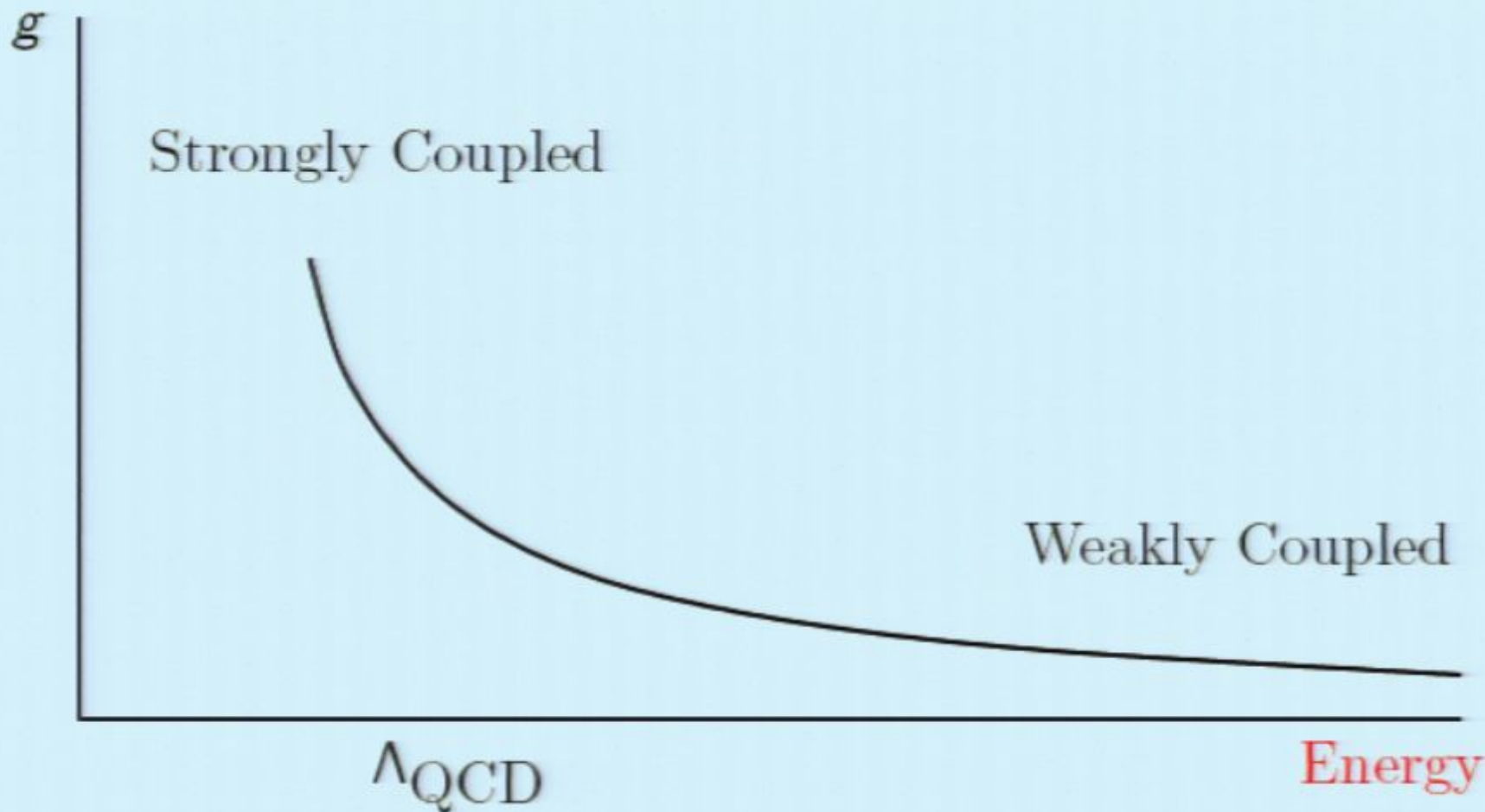


- QCD coupling constant versus energy

- quarks & gluons

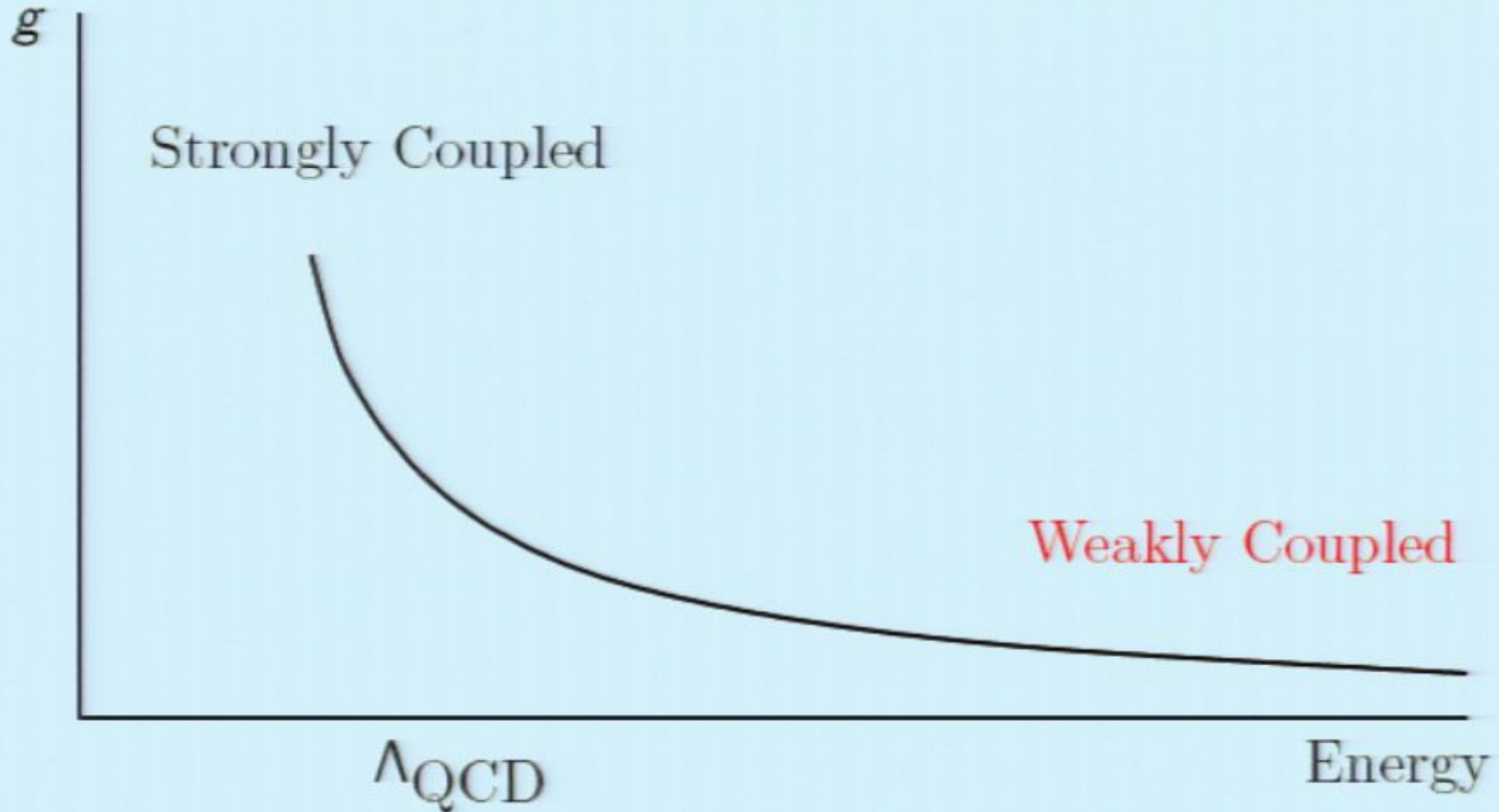
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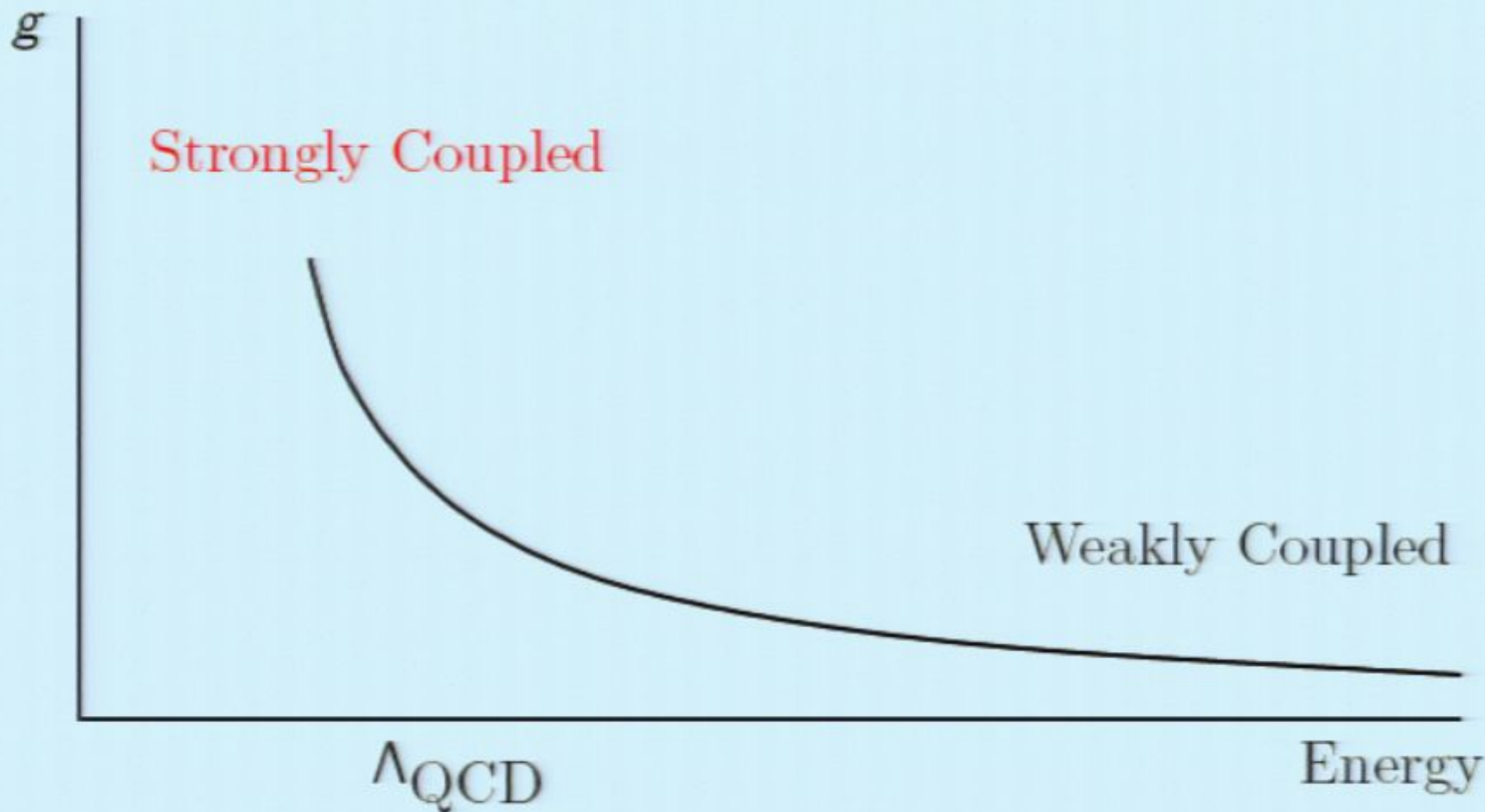
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Why the ERG?

Why the ERG?

General Characteristics of the ERG

- The ERG is nonperturbative in essence
- The ERG allows us to work directly with renormalized variables

- The flow starts from the Gaussian fixed point and flows to the interacting fixed point
- One can exploit self-similarity to write
 - $\Gamma_k = \Gamma_{k_0} + \dots$

An Amazing Added Benefit

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 - The flow emanates from the Gaussian fixed point, along the relevant direction(s)
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$$S_\Lambda[\varphi] = S[\varphi](g_1(\Lambda), \dots, g_n(\Lambda), \gamma(\Lambda))$$

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Gauge Fixing

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- The partition function contains an integral over field configurations
- For each field configuration, there are an infinite number which are gauge equivalent
- Gauge fixing \Rightarrow (try to) pick a unique representative for each orbit

Gauge Fixing

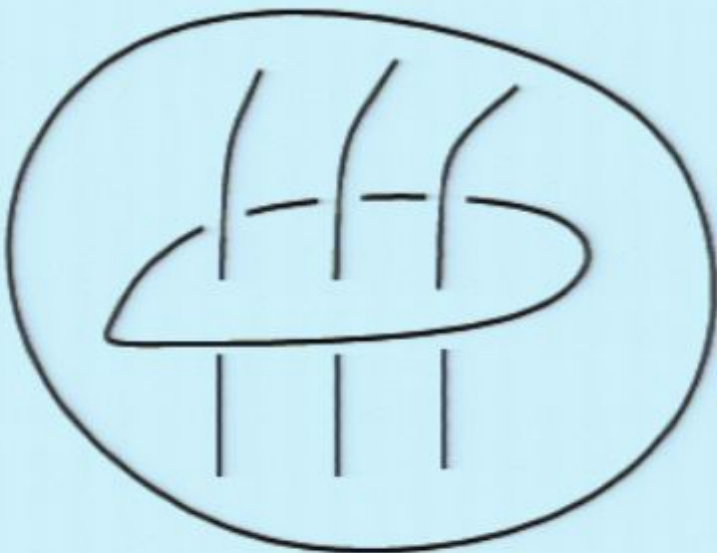
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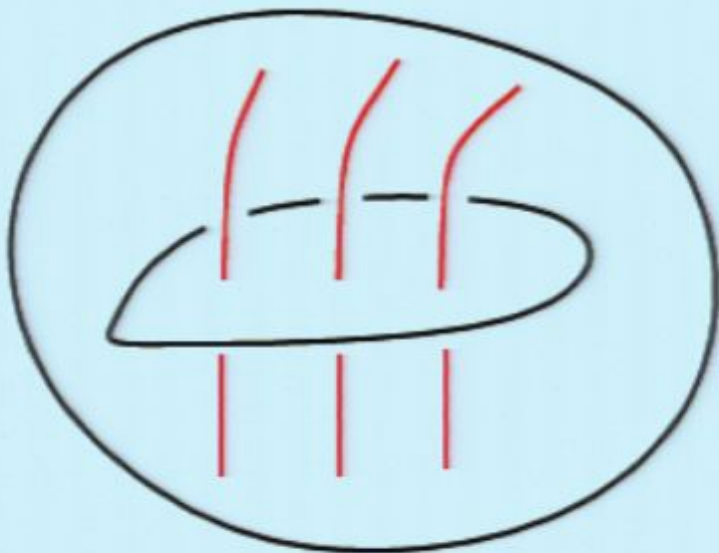
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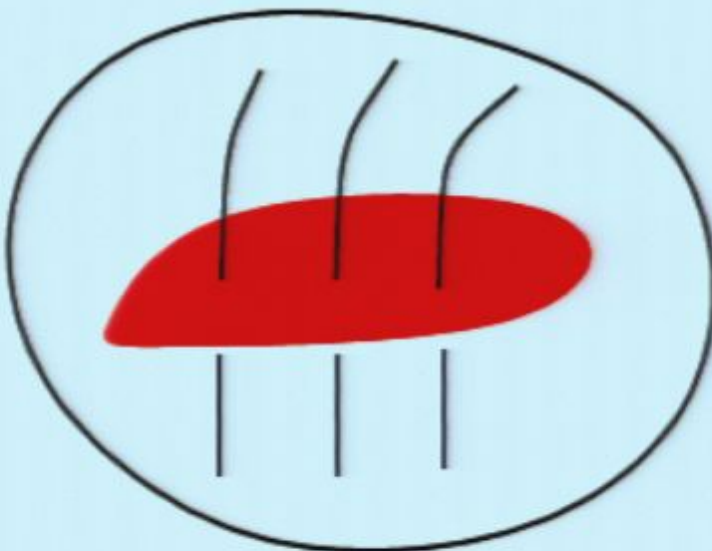
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- Space of gauge configurations
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- Space of representative configurations

Why Fix the Gauge?

Gauge fixing allows us to

compute the Green's functions of the theory. This is necessary for the calculation of the effective action and the renormalization group equations. The gauge fixing is a choice of a representative from each gauge orbit, which is necessary to define the path integral.

Is there anything we can compute without fixing gauge?

Yes, we can compute the gauge invariant observables. These are the observables that are invariant under the gauge transformations. They are the observables that are physically measurable. The gauge fixing is not necessary for the calculation of these observables.

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Gauge fixing allows us to

- define the gluon propagator
- compute Green's functions (in perturbation theory)

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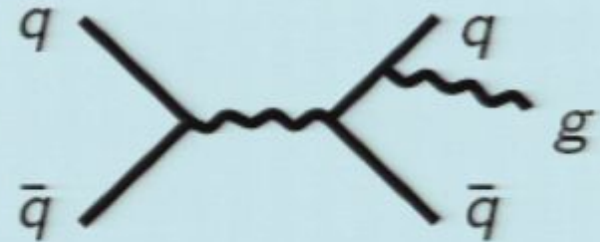
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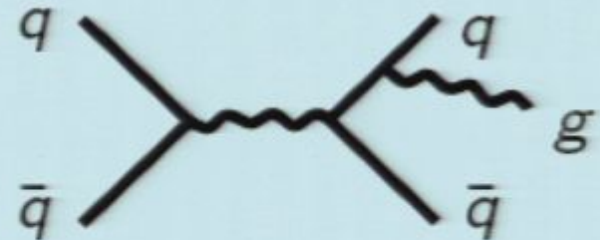
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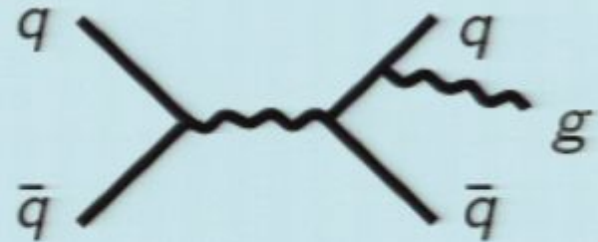
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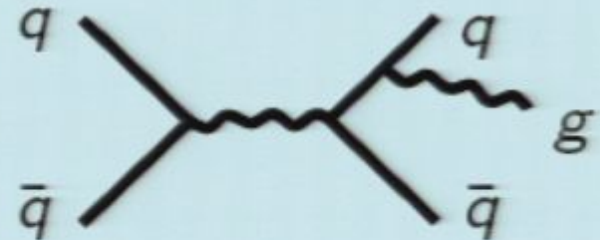
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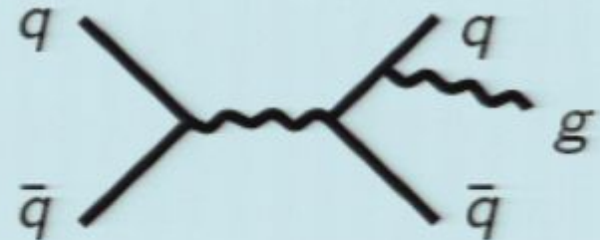
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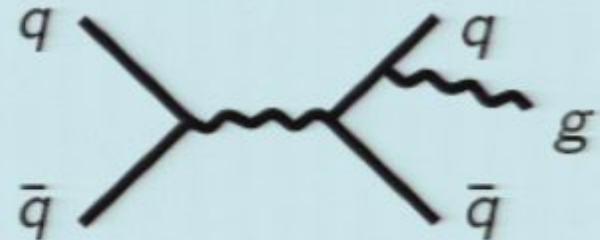
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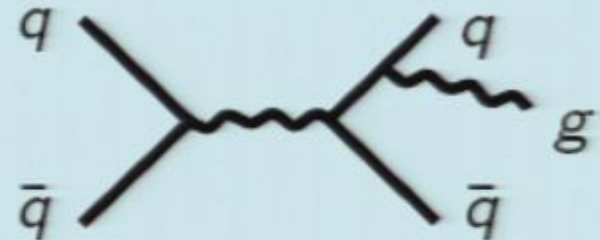
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Technical Advantages

- The gauge field does not renormalize
- The Ward identities take a very simple form
- Gribov copies are entirely avoided

Natural Language for Non-Perturbative Phenomena

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$$SA_{\mu} = [\nabla_{\mu}, \omega] = \partial_{\mu} \omega - i[A_{\mu}, \omega]$$

$$A_{\mu} \mapsto \left(\frac{A_{\mu}}{Z} \right)$$

$$\frac{\delta A_{\mu}}{\delta Z} \mapsto \left((Z^{-1}) \partial_{\mu} \omega + \partial_{\mu} \omega - i[A_{\mu}, \omega] \right)$$

$$[\nabla_{\mu}, \omega]$$

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The Practical Solution

- Accept this breaking of gauge invariance
- Keep track of the breaking using modified Ward identities
- Recover gauge invariance in the limit that all fluctuations are integrated out (if no truncations!)
- This is the approach pioneered in M. Reuter and C. Wetterich, "Effective average action for gauge theories and exact evolution equations," Nucl. Phys. B 417 (1994) 181.
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The Elegant Solution

- Covariantize the cutoff
- Utilize the immense freedom of the ERG to construct a covariantized blocking procedure which does not require gauge fixing
- This approach was pioneered by Tim Morris
- It is not so simple to implement
- No nonperturbative physics has been done
- But nonperturbative aspects of the ERGE have been explored
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Covariantizing the Cutoff for $SU(N)$ Gauge Theory

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The Basic Idea

- Step 1: Add in Covariant Higher Derivatives
 $\text{tr} F_{\mu\nu} F_{\mu\nu} \rightarrow \text{tr} F_{\mu\nu} c^{-1} (\nabla^2 / \Lambda^2) F_{\mu\nu}$
- Problem: a set of one-loop divergences slip through
- Step 2: Add Pauli-Villars (PV) fields to remove these divergences

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SU(N|M) Field Content

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- In the defining representation:

$$\mathcal{A}_\mu = \begin{pmatrix} A^1_\mu & B_\mu \\ \bar{B}_\mu & A^2_\mu \end{pmatrix} + \mathcal{A}_0 \mathbb{1}$$

- Supergauge field valued in $SU(N|N)$
- The physical gauge field
- An unphysical copy, with wrong sign action
- Wrong statistics fermion field
- A central term

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Symmetries of $SU(N|N)$ Gauge Theory

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'no- \mathcal{A}^0 ' symmetry

- $S \sim \text{str} F_{\mu\nu} F_{\mu\nu}$
- Combination of

- But \mathcal{A}_μ^0 cannot simply be dropped from the action, since it is generated by gauge transformations!
- Leads to non-trivial constraints on the action
- Can essentially be forgotten about if we adopt a convenient and natural diagrammatic prescription

Local $SU(N|N)$ Invariance

$$A_{\mu} = \begin{pmatrix} A_{\mu}^1 & B \\ \bar{B} & A_{\mu}^2 \end{pmatrix} + A_{\mu}^0 \mathbb{I}$$

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means that \mathcal{A}^0 lacks a kinetic term

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- $\delta \mathcal{A}_\mu = [\nabla_\mu, \Omega(x)]$

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(Super) Higgs Mechanism

- Introduce a superscalar, \mathcal{C} , valued in $U(N|N)$

$$\mathcal{C} = \begin{pmatrix} C^1 & D \\ \bar{D} & C^2 \end{pmatrix}$$

- Arrange for \mathcal{C} to acquire a vev in the direction

$$\langle \mathcal{C} \rangle = \sigma \equiv \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$$

- $SU(N|N) \rightarrow SU(N) \times SU(N) \times U(1)$

• Physical fermions carried by \mathbf{N}

• Unphysical fermions carried by $\bar{\mathbf{N}}$

• Physical bosons carried by \mathbf{N}^2

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Outline of this Lecture

- 1 Motivation
- 2 Technicalities
- 3 Manifestly Gauge Invariant ERGs**
 - Formulation
 - Diagrammatics

Strategy

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- Pure U1 gauge theory can be regularized by a naïve cutoff
- We can use this theory to understand the basics of constructing manifestly gauge invariant ERGs
- We can then combine this knowledge with the $SU(N|N)$ regularization to write down a flow equation for QCD

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A Flow Equation for pure U(1) Gauge Theory

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$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta \varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- For scalar field theory we chose

$$\Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$

- The flow equation is

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- $\delta/\delta A_\mu$ is gauge invariant
- No gauge fixing has been performed
- Therefore we have manifest gauge invariance!

Manifest Gauge Invariance

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$$-\Lambda \partial_\Lambda S_\Lambda = \frac{1}{2} \frac{\delta S}{\delta A_\mu} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta A_\mu} - \frac{1}{2} \frac{\delta}{\delta A_\mu} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta A_\mu}$$

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- It is convenient to rescale $A_\mu \rightarrow A_\mu/g$
 - In the non-Abelian case, $D_\mu = \partial_\mu - iA_\mu$ cannot now renormalize
- This adds a term to the left-hand side of the ERGE

$$\frac{\beta}{g} \int d^D x A_\mu(x) \frac{\delta S}{\delta A_\mu(x)}$$

- The solution is to drop this term!
- The resulting flow equation is perfectly valid

- Rescaling $S \rightarrow S/g^2$, $\hat{S} \rightarrow \hat{S}/g^2$:

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The Effective Propagator

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- In scalar field theory, the ERG kernel is usually **chosen** to be the derivative of the regularized propagator

$$\dot{\Delta} = -\Lambda \partial_{\Lambda} \frac{C_{UV}}{p^2}$$

- How can this make sense in gauge theory?
- We have not fixed the gauge and so do not have a propagator!
- First and foremost, $\dot{\Delta}$ is an ERG kernel
- It can be integrated to give Δ , which may or may not be directly identifiable with a propagator
- For this reason, Δ is often called an effective propagator

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The Effective Propagator Relationship

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- In scalar field theory, we found it convenient to write the action as

$$S[\varphi] = \frac{1}{2} \varphi \cdot \Delta^{-1} \cdot \varphi + S^R[\varphi]$$

- With the trivial relationship $\Delta^{-1} \Delta = 1$
- In pure $U(1)$ gauge theory, this becomes

$$S[A] = \frac{1}{2} A_\mu \cdot (\Delta^{-1})_{\mu\nu} \cdot A_\nu + S^R[A]$$

- With the non-trivial effective propagator relationship

$$(\Delta^{-1})_{\mu\nu} \Delta = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \equiv \frac{1}{p^2} \square_{\mu\nu}(p)$$

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- This will not work in the non-Abelian case because now $\delta/\delta A_\mu$ does gauge transform
- We know the solution!

→ Embed $U(1)$ in $U(N)$ gauge field

→ Introduce a dimensionless breaking field

→ Covariantize

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1. Start with S_Λ for a $U(N)$ gauge field

2. Incorporate the ghost fields

3. Covariantize

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 - Incorporate the symmetry breaking field, \mathcal{C}
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 - Incorporate the symmetry breaking field, \mathcal{C}
 - **Covariantize**

$$\frac{\delta}{\delta \mathcal{A}_\mu} \cdot \dot{\Delta} \cdot \frac{\delta}{\delta \mathcal{A}_\mu} \equiv \int_{x,y} \frac{\delta}{\delta \mathcal{A}_\mu(x)} \dot{\Delta}(x,y) \frac{\delta}{\delta \mathcal{A}_\mu(y)}$$

The Non-Abelian Case

Manifestly Gauge Invariant Flow Equation

$$\begin{aligned}
 -\Lambda \partial_\Lambda S_\Lambda &= \frac{1}{2} \frac{\delta S}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} - \frac{1}{2} \frac{\delta}{\delta \mathcal{A}_\mu} \{ \dot{\Delta}^{\mathcal{A}\mathcal{A}} \} \frac{\delta \Sigma_g}{\delta \mathcal{A}_\mu} \\
 &+ \frac{1}{2} \frac{\delta S}{\delta \mathcal{C}} \{ \dot{\Delta}^{\mathcal{C}\mathcal{C}} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}} - \frac{1}{2} \frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{\mathcal{C}\mathcal{C}} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}}
 \end{aligned}$$

- The fields come with differing kernels
- Denotes covariantization of the kernel
- The seed action is required to have interaction terms

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 &+ \frac{1}{2} \frac{\delta S}{\delta \mathcal{C}} \{ \dot{\Delta}^{c\mathcal{C}} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}} - \frac{1}{2} \frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{c\mathcal{C}} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}}
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 &+ \frac{1}{2} \frac{\delta S}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}} - \frac{1}{2} \frac{\delta}{\delta \mathcal{C}} \{ \dot{\Delta}^{cc} \} \frac{\delta \Sigma_g}{\delta \mathcal{C}}
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MDW



$$\begin{aligned}
 \text{MDW} &= \int d^0x d^0y \int d^1x_1 \dots d^1x_n \int d^1y_1 \dots d^1y_n \underbrace{\Delta_{ij}(x_i, y_j)}_{\text{propagator}} \\
 &= \text{str} \left(u(x) \underbrace{A_{\mu\nu}(x)}_{\uparrow} \underbrace{A_{\mu\nu}(y)}_{\leftarrow} v(y) \underbrace{A_{\mu\nu}(y)}_{\uparrow} \dots \right)
 \end{aligned}$$

$$\Delta_{ij}(x) = -N_x \frac{c(p, \mu)}{p^2}$$

$$S_{\mu\nu} = i[\nabla_{\mu}, \nabla_{\nu}]$$

$$\square_{\mu\nu}(p) = p^2 S_{\mu\nu} - p_{\mu} p_{\nu}$$

$$\left. \begin{aligned}
 \Delta_{ij} &= \sigma_{ij}(p, \mu) \square_{\mu\nu} \\
 \Delta &= \frac{c(p, \mu)}{p^2}
 \end{aligned} \right\}$$

MDW



$$\begin{aligned}
 \text{MDW} & \rightarrow \int d^0x d^0y \int d^1x_1 \dots x_n d^1y_1 \dots y_n \underbrace{\Delta_{xy}^i}_{\text{agrees}} \\
 & \text{str} \left(u(x) \underbrace{A_{x_1}(x_1)}_{\uparrow} \dots A_{x_n}(x_n) v(y) \underbrace{A_{y_1}(y_1) \dots A_{y_n}(y_n)}_{\uparrow} \right)
 \end{aligned}$$

$$\Delta_{xy}^i = -A_x^i \frac{c(p^i u)}{p^i}$$

$$S_{p^i} = i[\square_{p^i}, \mathcal{L}]$$

$$\square_{p^i}(p) = p^2 S_{p^i} - p_i p_i$$

$$\left. \begin{aligned}
 \Delta_{p^i}^i = \sigma^i(p^i u) \square_{p^i} \\
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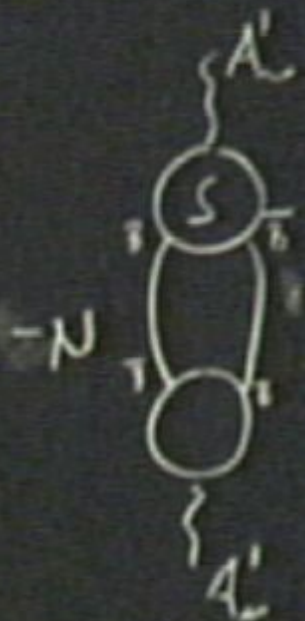
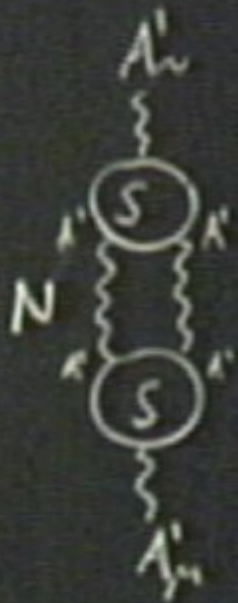
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- The seed action is required to have interaction terms

$$\text{str} \begin{pmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{pmatrix} = \mathbb{N}$$



$$\text{str} \begin{pmatrix} I_1 & 0 \\ 0 & 0 \end{pmatrix} = +N$$

$$\text{str} \begin{matrix} A'_1 & A'_2 & A'_\alpha \\ \times & \times & \times \end{matrix}$$

$$\begin{pmatrix} A' & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & A' \end{pmatrix}$$

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Non-Universal Details

Non-Universal Details

Structural Elements

- The seed action
- The cutoff functions
- The details of the covariantization of the cutoff

Unphysical Fields

- Fields with mass A, B, C, D
- Masses m_A, m_B, m_C, m_D
- λ does not appear in definition

The Crucial Question

How do we ensure that we are dealing only with the physical fields?

→ How to fix m_A, m_B, m_C, m_D ?

Non-Universal Details

Structural Elements

- The seed action
- The cutoff functions
- The details of the covariantization of the cutoff

Unphysical Fields

- Fields with a mass $\sim 5 \text{ GeV}$
- Masses $\sim 10^3 \text{ GeV}$
- λ does not appear in the action

The Crucial Question

How do we ensure that we are dealing only with the physics of the SM Yang-Mills?

Non-Universal Details

Structural Elements

- The seed action
- The cutoff functions
- The details of the covariantization of the cutoff

Unphysical Fields

- ϕ fields with mass m , $\Delta = \square + m^2$
- Massless fermions ψ , $\Delta = \not{\partial}$
- λ does not appear in the action

The Crucial Question

How do we ensure that we are dealing only with the physical fields?

Non-Universal Details

Structural Elements

- The seed action
- The cutoff functions
- The details of the covariantization of the cutoff

Unphysical Fields

- Fields $\omega, \eta, \bar{\eta}, \bar{A}, \bar{B}, \bar{C}, \bar{D}$ are not physical
- Massless field \bar{A} is not physical
- \bar{A} does not appear in the full on-shell action

The Crucial Question

How do we argue that we're dealing only with the physics of $U(1)$ Yang-Mills?

Non-Universal Details

Structural Elements

- The seed action
- The cutoff functions
- The details of the covariantization of the cutoff

Unphysical Fields

- Fields with a mass, Λ : $B, C^{1,2}, D$
- Massless field: A_μ^2
- A_μ^0 does not appear in the action

The Crucial Question

How do we ensure that we're dealing properly with the ways of...

My Name is A...

Non-Universal Details

Structural Elements

- The seed action
- The cutoff functions
- The details of the covariantization of the cutoff

Unphysical Fields

- Fields with a mass, Λ : $B, C^{1,2}, D$
- Massless field: A_μ^2
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The Crucial Question

How do we ensure that we are dealing and with the physics of

the full Yang-Mills theory?

Non-Universal Details

Structural Elements

- The seed action
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Unphysical Fields

- Fields with a mass, Λ : $B, C^{1,2}, D$
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The Crucial Question

How do we ensure that we are dealing only with the physical fields of $N=1$ Yang-Mills?

Non-Universal Details

Structural Elements

- The seed action
- The cutoff functions
- The details of the covariantization of the cutoff

Unphysical Fields

- Fields with a mass, Λ : $B, C^{1,2}, D$
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The Crucial Question

How do we find a suitable regulator that is only with the one-loop

divergences?

Non-Universal Details

Structural Elements

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- The cutoff functions
- The details of the covariantization of the cutoff

Unphysical Fields

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The Crucial Question

How do we ensure that we are dealing only with the physics of $SU(N)$ Yang-Mills?

Non-Universal Details

Structural Elements

- The seed action
- The cutoff functions
- The details of the covariantization of the cutoff

Unphysical Fields

- Fields with a mass, Λ : $B, C^{1,2}, D$
- Massless field: A_{μ}^2
- A_{μ}^0 does not appear in the action

The Crucial Question

How do we ensure that we are dealing only with the physics of $STI(N)$ Yang-Mills?

Guaranteeing Physicality

Guaranteeing Physicality

The Key Concepts

- The partition function is invariant under the flow
- If we write the action in self-similar form
 - We are guaranteed that we are on a renormalizable Trajectory
 - The theory in the UV is automatically the same as the infrared limit of the theory at that point
- We are guaranteed to be dealing with the physical $SU(N)$ Yang-Mills everywhere along the flow if
 - We are dealing with the physical theory in the UV
 - The flow is well defined

Guaranteeing Physicality

The Key Concepts

- **The partition function is invariant under the flow**
- If we write the action in self-similar form
 - We are guaranteed that we are on a renormalizable trajectory
 - The theory in the UV is automatically the theory in the vicinity of the Gaussian fixed point
- We are guaranteed to be dealing with the physical $SU(N)$ Yang-Mills everywhere along the flow if
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 - The flows will remain

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Guaranteeing Physicality

Decoupling

- In the UV, $\Lambda \rightarrow \infty$ the fields $B, C^{1,2}, D$ become infinitely massive
- A_μ^2 remains massless but decouples
 - A_μ and A_ν communicates only via B and D
 - The lowest dimension gauge invariant effective interaction irrelevant

$$\frac{1}{\Lambda^2} \text{tr} F^2 \text{tr} D F^2$$
- If we pose a question about the physical theory at the effective scale,
 - All other fields provide regularization and nothing more
 - Particularly clear in actual calculations

Guaranteeing Physicality

Decoupling

- In the UV, $\Lambda \rightarrow \infty$ the fields $B, C^{1,2}, D$ become infinitely massive

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- A_μ^2 and A_μ^1 communicate only to B and D
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$$\frac{1}{\Lambda^2} \text{tr} F_{\mu\nu}^2 + \dots$$

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Guaranteeing Physicality

Decoupling

- In the UV, $\Lambda \rightarrow \infty$ the fields $B, C^{1,2}, D$ become infinitely massive
- A^2_μ remains massless but decouples
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 - The lowest dimension gauge invariant effective interaction is irrelevant

$$\frac{1}{\Lambda^2} \text{tr}(F_{\mu\nu}^1)^2 \text{tr}(F_{\mu\nu}^2)^2$$

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2. Particularly clear in actual calculations

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Guaranteeing Physicality

Ensuring a Well Defined Flow

- Blocking must take place only over a localized patch
- Long range interactions only appear as $\Lambda \rightarrow 0$
- Demand that each ERG step is free of IR divergences
- Require all ingredients of the flow equation have an all orders derivative expansion (quasi-locality)

Locality

- The action's dependence on the physics in the UV is the physics in the vicinity of the Gaussian fixed point
- The partition function is invariant under the flow
- A regulator in the UV guarantees a local action for all $\Lambda > 0$

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Locality

- The action is selected for the objects in the UV as the physics in the vicinity of the Wilson-Fisher point
- The partition function is invariant under IR flow
- A local action in the UV guarantees a local action for all $\Lambda > 0$

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Ensuring a Well Defined Flow

- Blocking must take place only over a localized patch
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Locality

- The action is defined by Γ_k , so the physics in the UV is the physics in the vicinity of the Gaussian fixed point
- The part of the function is invariant under renflow
- A counterterm in the UV generates a local action for all $\Lambda \rightarrow 0$



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Locality

- The action is well defined, so the physics in the UV is the physics in the IR, i.e. of the Gaussian fixed point
- The path integral is invariant under the flow
- A fixed point in the UV guarantees a flow action for all $\Lambda > 0$

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Locality

- The Wilson $Z[\phi, \Lambda]$ is power law divergent in the UV as the cutoff Λ increases
- Check in the vicinity of the Gaussian fixed point
- The partition function is invariant under the flow
- A local action in the UV guarantees a local action for all Λ

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Locality

- The action is self-similar, so the physics in the UV is the physics in the vicinity of the Gaussian fixed point
- The partition function is invariant under the flow
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The Bare Action



The Bare Action

- Recall (Lecture 1) that, for an RT, the bare action is something which we **compute**
- The bare action along an RT is the 'perfect action in the vicinity of the UV fixed point'
- When computing universal quantities, we can leave (in principle) many of the non-universal details unspecified
- By not explicitly choosing



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 - The seed action
 - The cutoff functions
 - The details of the covariantization of the cutoff

We work implicitly with an infinite number of bare actions



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- Recall (Lecture 1) that, for an RT, the bare action is something which we compute
- The bare action along an RT is the 'perfect action in the vicinity of the UV fixed point'
- When computing universal quantities, we can leave (in principle) many of the non-universal details unspecified
- By not explicitly choosing
 - The seed action
 - The cutoff functions
 - The details of the covariantization of the cutoff

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The Renormalization Conditions



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- We exploit self-similarity to formulate the renormalization condition for g

$$S[\mathcal{A} = A^1, \mathcal{C} = \sigma] = \frac{1}{2g^2} \text{tr} \int d^D x (F_{\mu\nu}^1)^2 + \dots$$

- But A^2 carries its own coupling, which renormalizes separately

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- Usually work with $\alpha \equiv g_2^2/g^2$

$$\frac{1}{g_2^2} = \frac{1}{\alpha g^2}$$

- g^2 still counts loops

\mathbb{P}^2/\mathbb{R}

QUASILOCALITY

$\Lambda \rightarrow 0$

\hat{S}

$$\int d^0x d^0y \int d^4x_1 \dots d^4x_n d^4y_1 \dots d^4y_m \Delta_{ij}$$

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- We have two dimensionless couplings, g and g_2 , which renormalize separately
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- This is not catastrophic, but it is not convenient
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Scalar Diagrammatics



Scalar Diagrammatics

$$\left(-\Lambda\partial_\Lambda + n\frac{\gamma}{2}\right) \left[\text{S} \right]^{(n)} = \frac{1}{2} \left[\begin{array}{c} \text{\Sigma}_\lambda \\ | \\ \text{S} \end{array} - \text{\Sigma}_\lambda \right]^{(n)}$$

The diagrammatic equation shows the action of the operator $(-\Lambda\partial_\Lambda + n\frac{\gamma}{2})$ on the n -loop scalar diagram $[S]^{(n)}$. The result is $\frac{1}{2}$ times the difference between two diagrams: a diagram with a Σ_λ loop connected to an S loop by a vertical line with a central dot, and a diagram with two overlapping Σ_λ loops, one above the other, with a dot on the top loop.

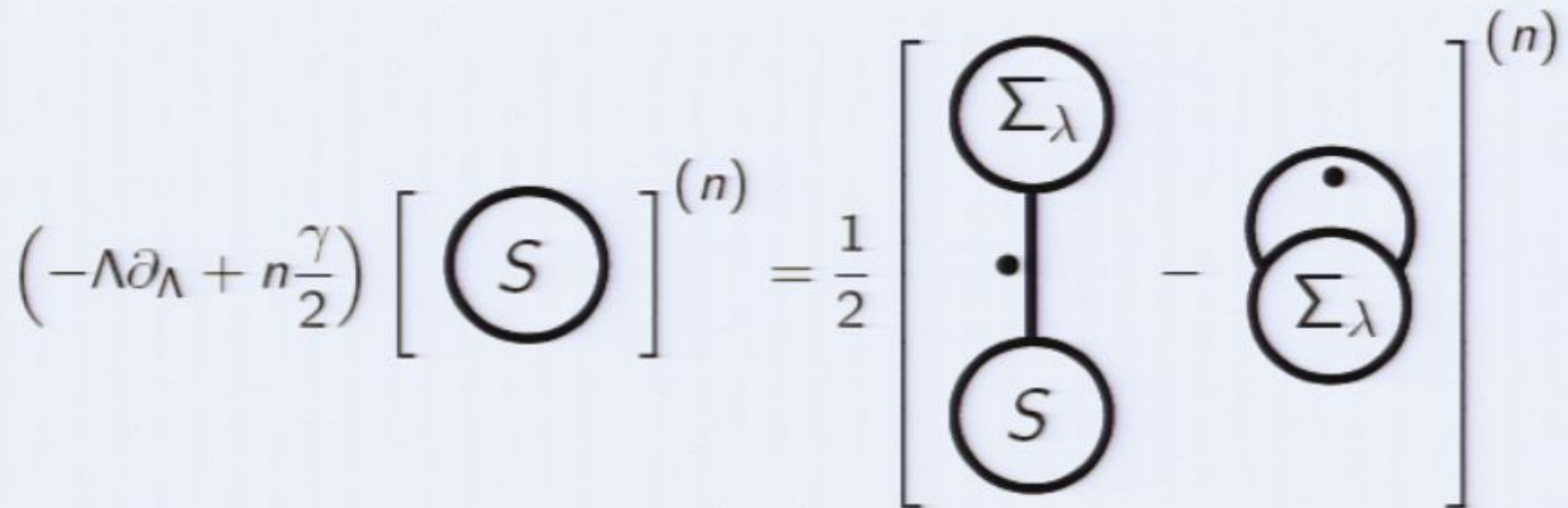


Scalar Diagrammatics

$$\left(-\Lambda\partial_\Lambda + n\frac{\gamma}{2}\right) \left[\text{circle with } S \right]^{(n)} = \frac{1}{2} \left[\begin{array}{c} \text{circle with } \Sigma_\lambda \\ | \\ \text{circle with } S \end{array} - \text{circle with } \Sigma_\lambda \text{ and a dot} \right]^{(n)}$$

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- **Effective propagator, Δ**



Non-Abelian Diagrammatics



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$$-\Lambda \partial_\Lambda \left[\text{S} \right]^{\{f\}} = \frac{1}{2} \left[\begin{array}{c} \text{\Sigma}_g \\ | \\ \text{S} \end{array} - \text{\Sigma}_g \right]^{\{f\}}$$

The diagrammatic equation shows the derivative of a circle labeled 'S' with a brace {f} next to it. This is equal to 1/2 times a large bracketed expression. Inside the large bracket, there are two terms separated by a minus sign. The first term is a vertical line with a dot in the middle, connecting a circle labeled 'Σ_g' at the top to a circle labeled 'S' at the bottom. The second term is a circle labeled 'Σ_g' with a dot on its top boundary. The entire large bracketed expression has a brace {f} to its right.



Non-Abelian Diagrammatics

$$-\Lambda\partial_\Lambda \left[\text{Diagram } S \right]_{\{f\}} = \frac{1}{2} \left[\text{Diagram } \Sigma_g \text{ --- } S - \text{Diagram } \Sigma_g \text{ with bubble} \right]_{\{f\}}$$

The diagram on the left is a circle labeled 'S'. The diagram in the first term of the bracket is a vertical line with a dot in the middle, connecting a circle labeled 'Σ_g' at the top to a circle labeled 'S' at the bottom. The diagram in the second term of the bracket is a circle labeled 'Σ_g' with a smaller circle (bubble) attached to its top edge, containing a dot.

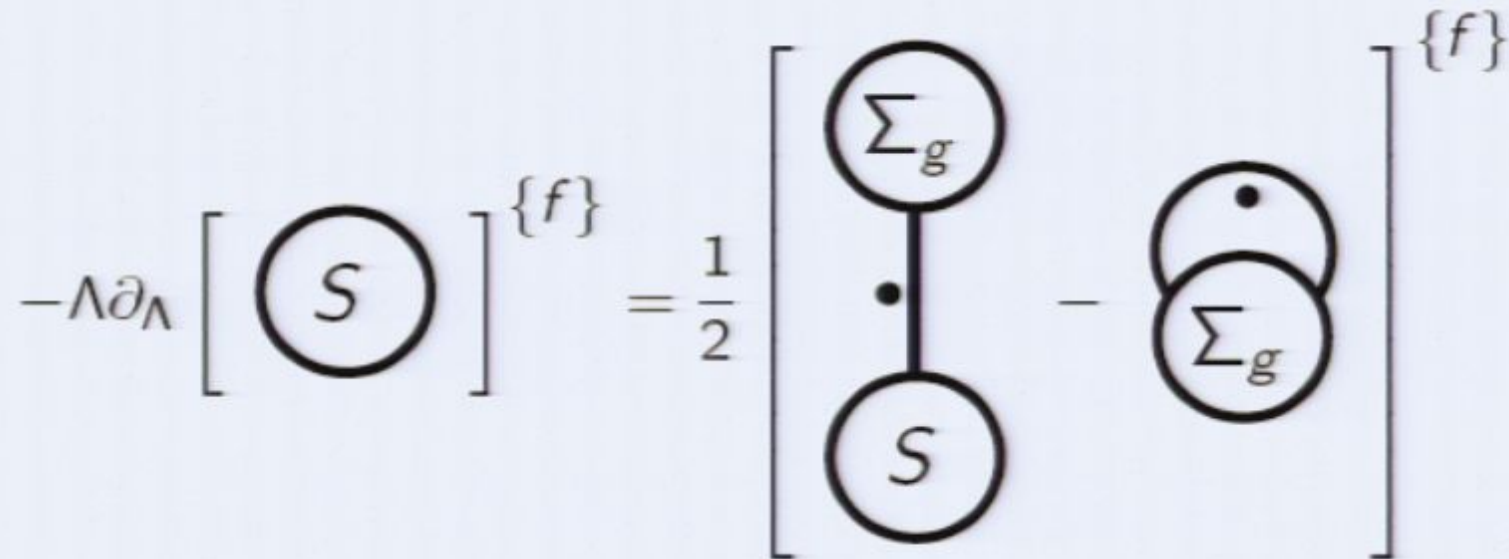
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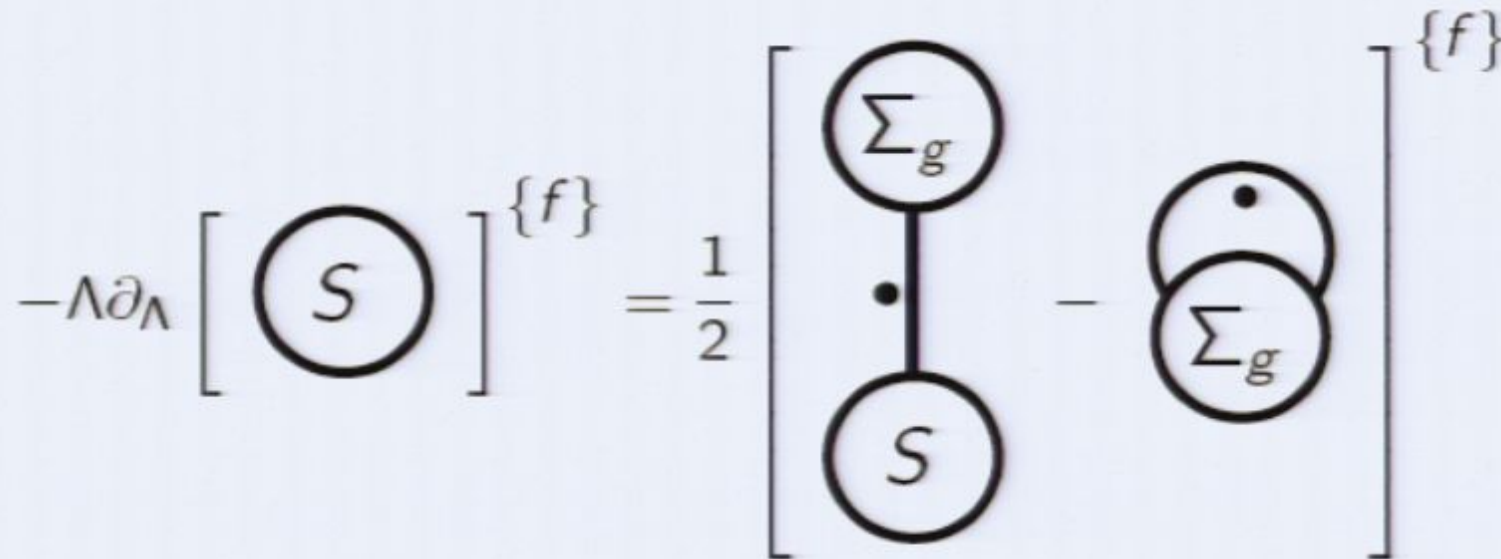
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- In pure $U(1)$ gauge theory we had:

$$(\Delta^{-1})_{\mu\nu}(p)\Delta(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \equiv \frac{1}{p^2} \square_{\mu\nu}(p)$$

- Diagrammatically

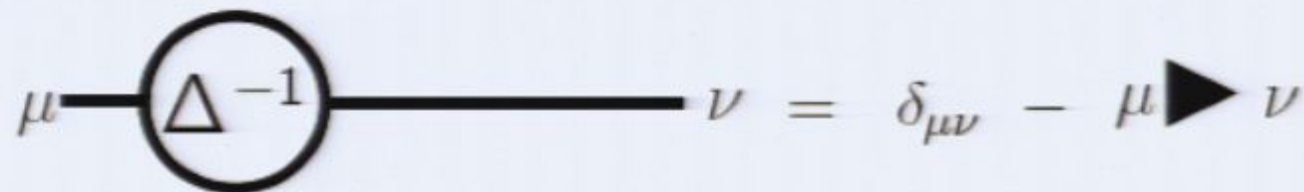
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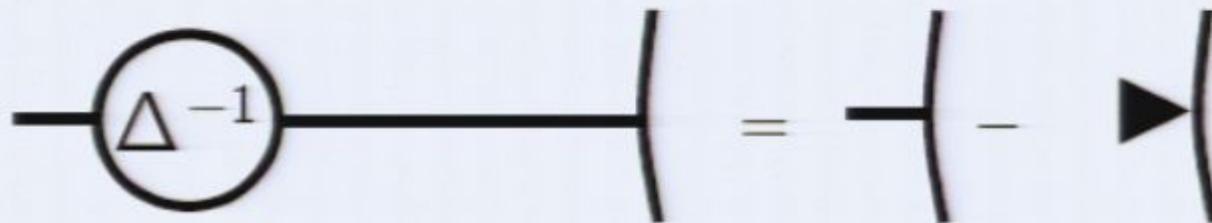
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First Attempt

- The most obvious approach is to embed each quark field in a superquark field

$$\Psi = \begin{pmatrix} \psi \\ \varphi \end{pmatrix}$$

→ Physical field

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A'

$$\overline{\Psi} A' \Psi$$

\overline{B}

H

B

$$st_1 \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

Adding Quarks

The Solution

- Embed up-like (down-like) quarks into an $N \times N$ matrix

$$\psi_u = \begin{pmatrix} u_r & c_r & t_r \\ u_g & c_g & t_g \\ u_b & c_b & t_b \end{pmatrix}, \quad \psi_d = \begin{pmatrix} d_r & s_r & b_r \\ d_g & s_g & b_g \\ d_b & s_b & b_b \end{pmatrix}$$

- There is now an unphysical flavour symmetry carried by A_μ^2
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$$\# \text{ of quarks} = \text{multiple of } N$$
- Too bad the flavour symmetry is unphysical, but this is entertaining
- Modifying the diagrammatics is very easy
 - Ensure there is an independent ERG kernel for each independent field
 - Modify the group theory prescription in the unphysical sector



Achievements to Date



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Outlook



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- **The framework is very appealing**
- It is understood how to perform efficient calculations in perturbation theory
- Understanding how best to apply the formalism beyond perturbation theory is a real challenge
- Generic truncations will spoil universality
- All the non-universal details buried in the diagrams would then need to be explicitly dealt with
- SUSY theories offer an excellent framework for trying to understand how best to proceed
- I'm very hopeful that this will not be a passive process!



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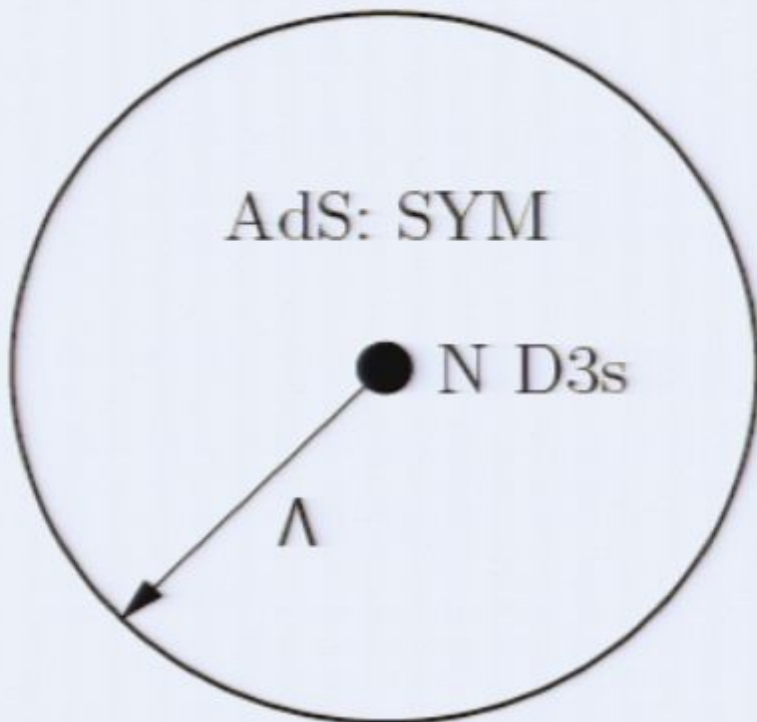
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D-Brane Interpretation

$\mathcal{N} = 4$ SYM

N ghost D3s on S^5



flat space: regularized

[Return](#)

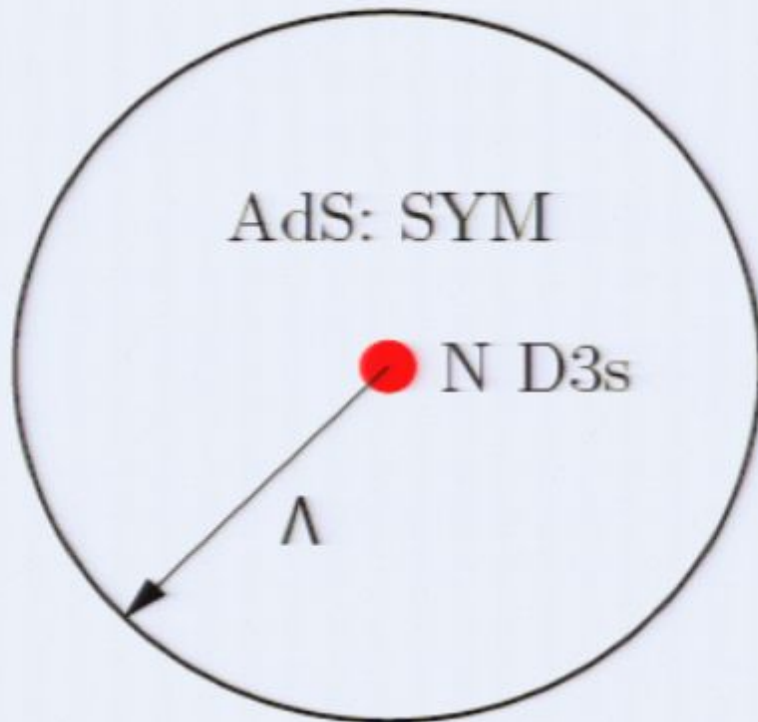
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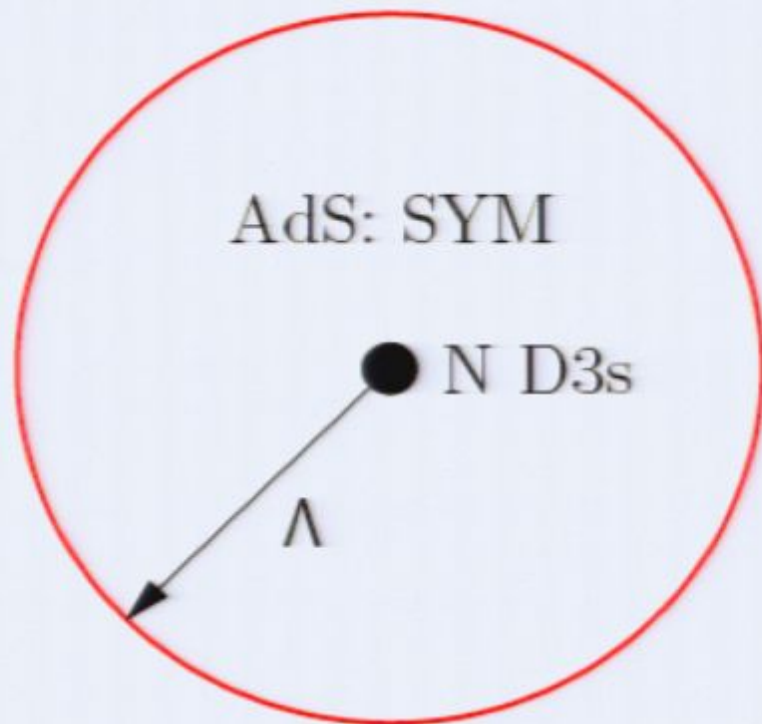
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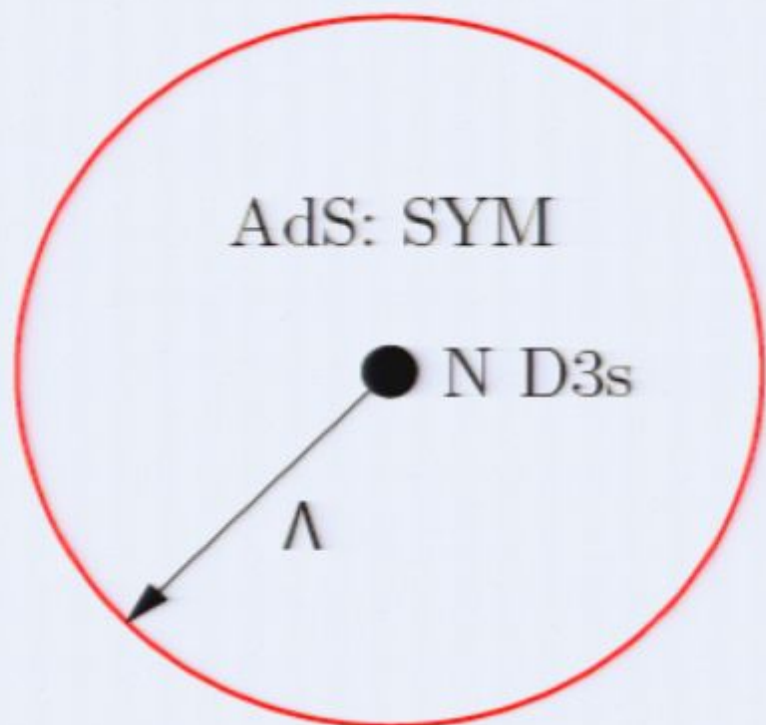
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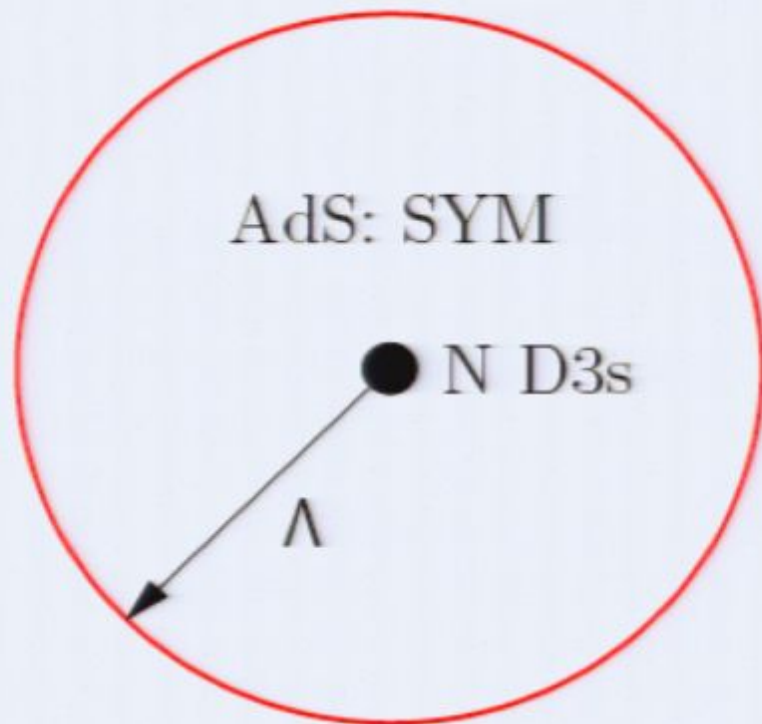
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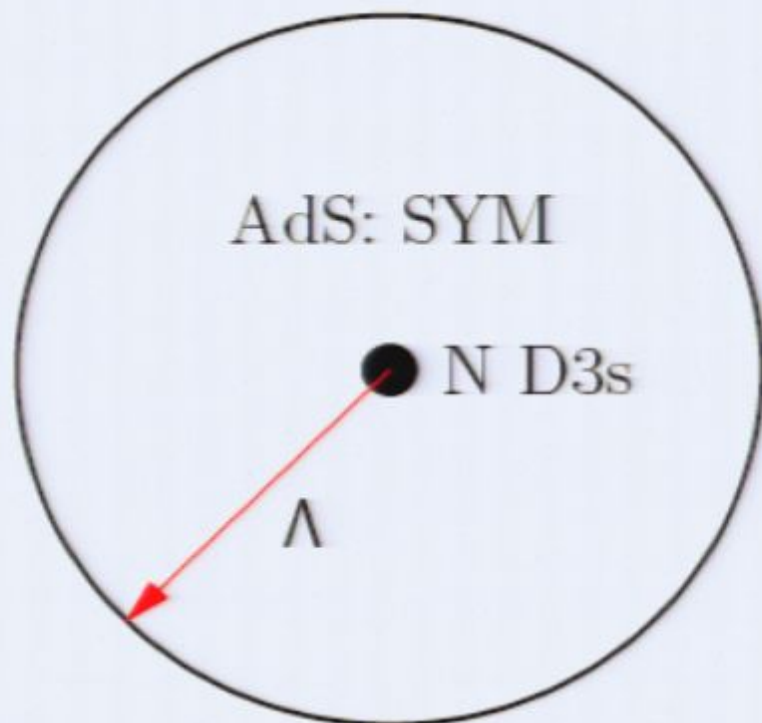
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Bookmarks

- Motivation
- Technicalities
- Manifestly Gauge Invariant ERGs
 - Formulation
 - Diagrammatics
- Appendix

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