Title: The Effective Field Theory of Inflation.

Date: May 06, 2008 02:00 PM

URL: http://pirsa.org/08050006

Abstract: We study the effective field theory of inflation, i.e. the most general theory describing the fluctuations around a quasi de Sitter background, in the case of single field models. The scalar mode can be eaten by the metric by going to unitary gauge. In this gauge, the most general theory is built with the lowest dimension operators invariant under spatial diffeomorphisms, like g^{00} and K_{mu nu}, the extrinsic curvature of constant time surfaces. This approach allows us to characterize all the possible high energy corrections to simple slow-roll inflation, whose sizes are constrained by experiments. Also, it describes in a common language all single field models, including those with a small speed of sound and Ghost Inflation, and it makes explicit the implications of having a quasi de Sitter background. The non-linear realization of time diffeomorphisms forces correlation among different observables, like a reduced speed of sound and an enhanced level of non-Gaussianity.

The general idea

Usual approach to inflation:

1. Take a Lagrangian for a scalar $\mathcal{L}(\phi, \partial_\mu \phi, \Box\phi \ldots)$
2. Solve EOM of the scalar + FRW. Find an inflating solution $\ddot{a} > 0$
3. Study perturbations around this solution to work out predictions

We want to focus directly on the theory of perturbations around the inflating solution

- Time diffeomorphisms are broken: $t \to t + \xi^0(t, \vec{x})$ $\delta\phi \to \delta\phi + \dot{\phi}_0(t)\xi^0$
- In unitary gauge $\phi(t, \vec{x}) = \phi_0(t)$ the scalar mode is eaten by the graviton:
  3 degrees of freedom. Like in a broken gauge theory.
- Using effective field theory approach, the most generic action in unitary gauge

$$
\begin{aligned}
S \;=\; & \int d^4x \sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot{H} g^{00} - M_P^2\left(3H^2 + \dot{H}\right) + \frac{M_2(t)^4}{2!}(g^{00}+1)^2 \right.\\
& \left. + \frac{M_3(t)^4}{3!}(g^{00}+1)^3 + \ldots - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 + \ldots\right].
\end{aligned}
$$

- At short distance the scalar mode decouples: mixing with gravity can be neglected.

~ Equivalence theorem for gauge theories.

$$
S_\pi = \int d^4x \sqrt{-g}\left[M_{\rm Pl}^2 \dot{H}\,(\partial_\mu\pi)^2 + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{1}{a^2}(\partial_i\pi)^2\right) - \frac{4}{3}M_3^4\dot{\pi}^3 - \frac{\bar{M}^2}{2}\frac{1}{a^4}(\partial_i^2\pi)^2 + \ldots\right.
$$

Advantages of EFT approach

$$S_\pi = \int d^4x \sqrt{-g}\left[M_{\rm Pl}^2 \dot{H}\,(\partial_\mu\pi)^2 + 2M_2^4\left(\dot\pi^2 + \dot\pi^3 - \dot\pi\frac{1}{a^2}(\partial_i\pi)^2\right) - \frac{4}{3}M_3^4\dot\pi^3 - \frac{\bar{M}^2}{2}\frac{1}{a^4}(\partial_i^2\pi)^2 + \ldots\right.$$

- Description of a system through lower dimensional operators
- General and Simple
- Explicit what comes from symmetry
- Operators as parametrization of corrections to standard slow-roll inflation
- Knowing all possible operators, and their importance
- Unification of all models
- No field redefinitions
- Loop corrections and counter terms

Outline

$$S_\pi = \int d^4x \, \sqrt{-g} \left[M_{\rm Pl}^2 \dot{H} \, (\partial_\mu \pi)^2 + 2 M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{1}{a^2} (\partial_i \pi)^2 \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 - \frac{\bar{M}^2}{2} \frac{1}{a^4} (\partial_i^2 \pi)^2 + \ldots \right.$$

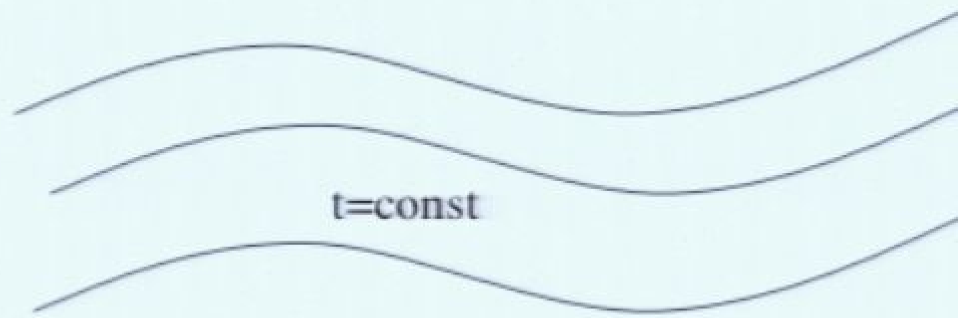
- Construction in unitary gauge and Goldstone boson
- The various limits of single field inflation, and their signatures
- Theorems: The consistency condition of the 3-point function
- Stable Violation of Null Energy Condition
- Conclusions

Construction of the action in unitary gauge

Inflation. Quasi dS phase with a privileged spatial slicing

Unitary gauge. This slicing coincides with time.

$$\delta\phi(\vec{x},t) = 0$$



Most generic Lagrangian built by metric operators invariant only under $x^i \to x^i + \xi^i(t,\vec{x})$

- Generic functions of time
- Upper 0 indices are ok. E.g. g^{00} R^{00}
- Geometric objects of the 3d spatial slices: e.g. extrinsic curvature $K_{\mu\nu}$ and covariant derivatives

$$S = \int d^4x \sqrt{-g}\, F(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t)$$

- One can isolate linear terms from the others

$$= \int d^4x\, \sqrt{-g}\Big[\boxed{\frac{1}{2}M_{\rm Pl}^2 R + c(t)g^{00} - \Lambda(t)} + \frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 +$$

$$-\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + \ldots\Big],$$

with $\delta K_{\mu\nu} = K_{\mu\nu} - a^2 H h_{\mu\nu}$

Fixing the tadpoles

Background evolution fixes c(t) and Λ(t). Higher order terms only affect perturbations

Friedman equations give:

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left[c(t)+\Lambda(t)\right]$$

$$\frac{\ddot a}{a} = \dot H + H^2 = -\frac{1}{3M_{\rm Pl}^2}\left[2c(t)-\Lambda(t)\right]$$

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Simplest case: $\int d^4x\,\sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2 - V(\phi)\right] \to \int d^4x\,\sqrt{-g}\left[-\frac{\dot\phi_0(t)^2}{2}g^{00} - V(\phi_0(t))\right]$

$$S = \int d^4x\,\sqrt{-g}\;P(\phi_0(t)^2 g^{00}, \phi(t))$$

$L = P(X,\phi)$, with $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.

$$M_n^4(t) = \phi_0(t)^{2n}\partial^n P/\partial X^n$$

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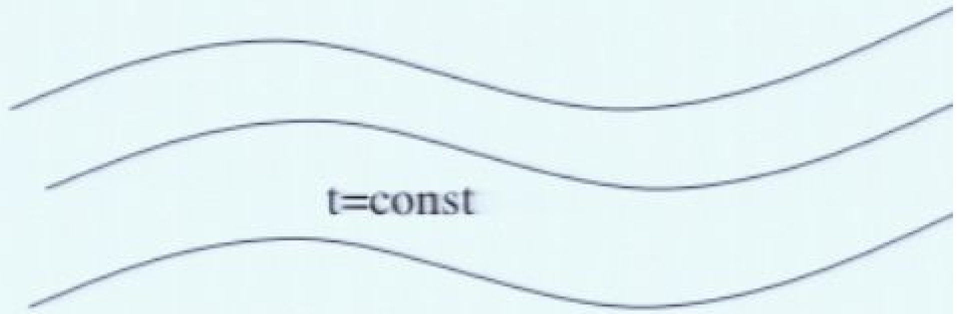

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All single field models are unified

$$= \int d^4x \sqrt{-g} \Big[M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot{H} g^{00} - M_{\rm Pl}^2 (H^2 + \dot{H}) + M_2(t)^4 (g^{00}+1)^2 + M_3(t)^4 (g^{00}+1$$

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- Slow Roll Inflation: $\int d^4x \, \sqrt{-g} \left[-\frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \to \int d^4x \, \sqrt{-g} \left[-\frac{\dot{\phi}_0(t)^2}{2} g^{00} - V(\phi_0(t)) \right.$

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- k-inflation, DBI inflation $\mathcal{L} = \phi^4 \sqrt{1 - \lambda \frac{\dot{\phi}^2}{\phi^4}}$

Alishahiha, Silverstein and Tong,
Phys.Rev.D70:123505,2004

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- Ghost Inflation $-(\partial\phi)^2 + \frac{1}{M^4}(\partial\phi)^4 + \ldots$

WRONG SIGN

Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga,
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- Something else

A simplifying limit

$$= \int d^4x \sqrt{-g} \Big[M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot{H} g^{00} - M_{\rm Pl}^2 (H^2 + \dot{H}) + M_2(t)^4 (g^{00}+1)^2 + M_3(t)^4 (g^{00}+1$$

$$-\bar{M}_1^3 (g^{00}+1)\delta K_\mu^\mu - \bar{M}_2^2 \delta K_\mu^{\mu\,2} + \ldots \Big]$$

Spontaneously Broken Gauge Symmetry

Reintroduce the Goldstone boson

Reintroducing the Goldstone

At sufficiently high energy the Goldstone mode decouples.

$$S = \int d^4x \, -\frac{1}{4}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 \mathrm{Tr}\, A_\mu A^\mu \qquad \text{where } A_\mu = A_\mu^a T^a.$$

Gauge transformation:

$$A_\mu \to U A_\mu U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger \equiv \frac{i}{g} U D_\mu U^\dagger \,. \qquad S = \int d^4x \, -\frac{1}{4}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\frac{m^2}{g^2}\mathrm{Tr} D_\mu U^\dagger D_\mu U \,.$$

Gauge invariance is "restored" introducing the Goldstones: $U = \exp\left[iT^a \pi^a(t,\vec{x})\right]$

Under a gauge trans. Λ we impose: $e^{iT^a \tilde{\pi}^a(t,\vec{x})} = \Lambda(t,\vec{x})\, e^{iT^a \pi^a(t,\vec{x})}$

Going to canonical normalization: $\pi_c \equiv m/g \cdot \pi$ Cutoff: $4\pi m/g$

Mixing with transverse component: $\frac{m^2}{g} A_\mu^a \partial^\mu \pi^a = m A_\mu^a \partial^\mu \pi_c^a$ Irrelevant for $E \gg m$

In the window: $m \ll E \ll 4\pi m/g$ The physics of the Goldstones is perturbative and decoupled from transverse modes

Doing the same for inflation

Consider for example: $\int d^4x \sqrt{-g}\,[A(t) + B(t) g^{00}(x)]$

Time diff: $t \to \tilde{t} = t + \xi^0(x),\ \vec{x} \to \tilde{\vec{x}} = \vec{x}$ $\qquad g^{00}(x) \to \tilde{g}^{00}(\tilde{x}(x)) = \frac{\partial \tilde{x}^0(x)}{\partial x^\mu}\frac{\partial \tilde{x}^0(x)}{\partial x^\nu} g^{\mu\nu}(x)$

We get: $$\int d^4\tilde{x}\,\sqrt{-\tilde{g}(\tilde{x})}\left[A(\tilde{t}-\xi^0(x(\tilde{x}))) + B(\tilde{t}-\xi^0(x(\tilde{x})))\frac{\partial(\tilde{t}-\xi^0(x(\tilde{x})))}{\partial \tilde{x}^\mu}\frac{\partial(\tilde{t}-\xi^0(x(\tilde{x})))}{\partial \tilde{x}^\nu}\tilde{g}^{\mu\nu}(\tilde{x})\right]$$

To restore diff invariance we promote ξ to a field: $\xi^0(x(\tilde{x})) \to -\tilde{\pi}(\tilde{x})$

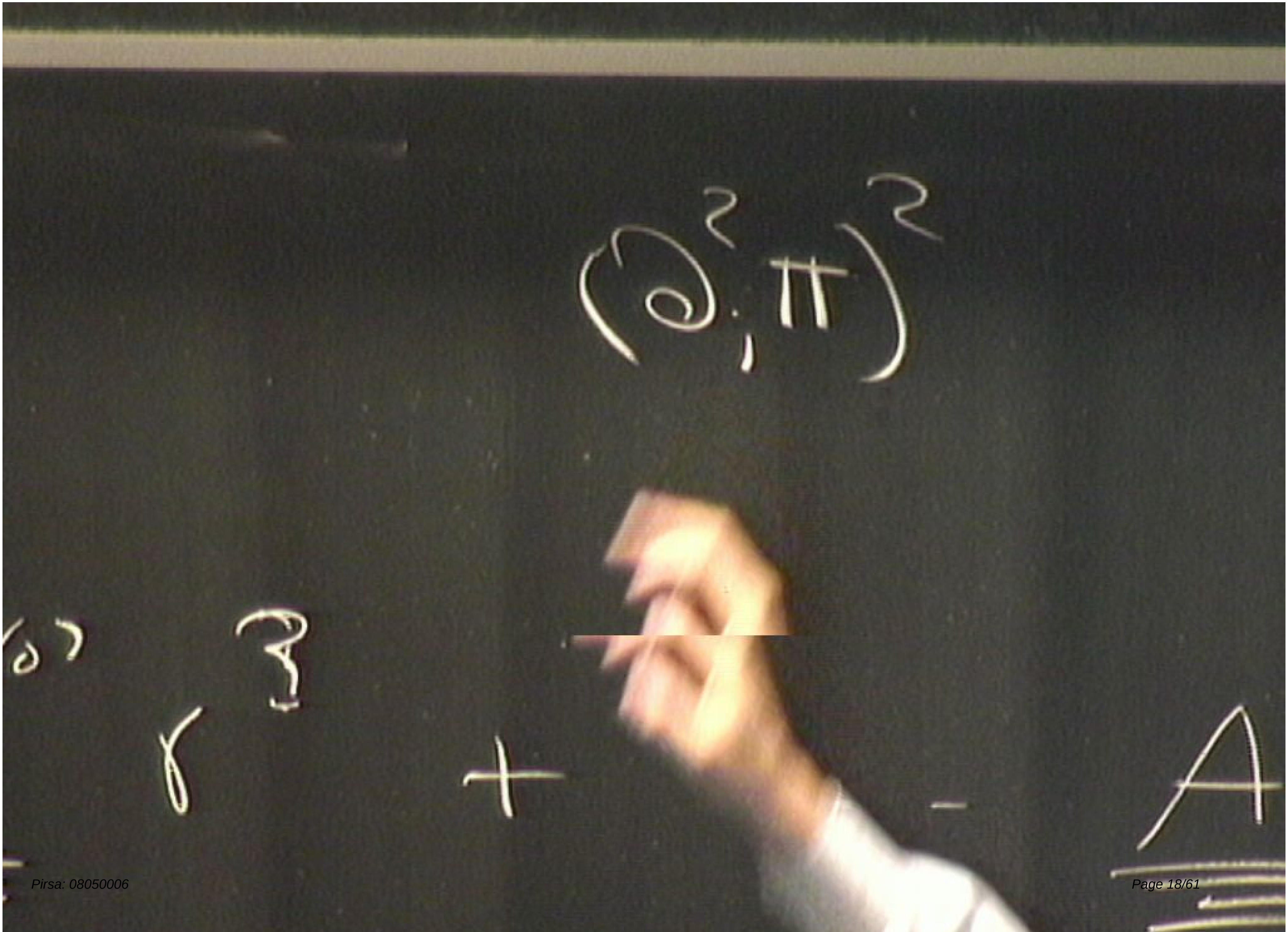
The action $$\int d^4x\,\sqrt{-g(x)}\left[A(t+\pi(x)) + B(t+\pi(x))\frac{\partial(t+\pi(x))}{\partial x^\mu}\frac{\partial(t+\pi(x))}{\partial x^\nu} g^{\mu\nu}(x)\right]$$

is invariant if π transforms non-linearly: $\pi(x) \to \tilde{\pi}(\tilde{x}(x)) = \pi(x) - \xi^0(x)$

Decoupling limit.
At high energy, no mixing with gravity.

Cosmological perturbations probe the theory at E ~ H

$$S_\pi = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - M_{\rm Pl}^2 \dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3 + \ldots\right]$$


$(\partial_i^2 \pi)^2$

The various limits of single field inflation: Slow-roll inflation…

Set to zero all additional operators: $M_2 = M_3 = \bar{M}_1 = \bar{M}_2 \ldots = 0$

$$S = \int d^4x \, \sqrt{-g} \quad \left[\frac{1}{2} M_{\rm Pl}^2 R - M_{\rm Pl}^2 \left(3H^2(t+\pi) + \dot{H}(t+\pi)\right) + \right.$$
$$\left. + M_{\rm Pl}^2 \dot{H}(t+\pi) \left((1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi})\partial_i \pi g^{0i} + g^{ij}\partial_i \pi \partial_j \pi\right)\right]$$

From terms of the form: $\sim M_{\rm Pl}^2 \dot{H} \dot{\pi} \delta g^{00}$ mixing is relevant at $E_{mix} \sim \varepsilon^{1/2} H$

At E~H + leading order in slow-roll: $S_\pi = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\rm Pl}^2 R - M_{\rm Pl}^2 \dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2}\right)\right]$

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After horizon crossing one switch to ζ which is (non-linearly) conserved

$\pi = 0 \qquad g_{ij} = a^2(t)\left[(1 + 2\zeta(t,\vec{x}))\delta_{ij} + \gamma_{ij}\right]$

$t \to t - \pi(t,\vec{x}) \qquad \zeta(t,\vec{x}) = -H\pi(t,\vec{x})$

Standard results:

$$\langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{H_*^2}{4\epsilon_* M_{\rm Pl}^2} \frac{1}{k_1^3}$$

$$n_s - 1 = \frac{d}{d \log k} \log \frac{H_*^4}{|\dot{H}_*|} = 4\frac{\dot{H}_*}{H_*^2} - \frac{\ddot{H}_*}{H_* \dot{H}_*}$$

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Doing the same for inflation

Consider for example: $\int d^4x\,\sqrt{-g}\,[A(t)+B(t)g^{00}(x)]$

Time diff: $t \to \tilde{t} = t + \xi^0(x),\ \vec{x} \to \tilde{\vec{x}} = \vec{x}$ $\qquad g^{00}(x) \to \tilde{g}^{00}(\tilde{x}(x)) = \frac{\partial \tilde{x}^0(x)}{\partial x^\mu}\frac{\partial \tilde{x}^0(x)}{\partial x^\nu} g^{\mu\nu}(x)$

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At high energy, no mixing with gravity.

Cosmological perturbations probe the theory at E ~ H

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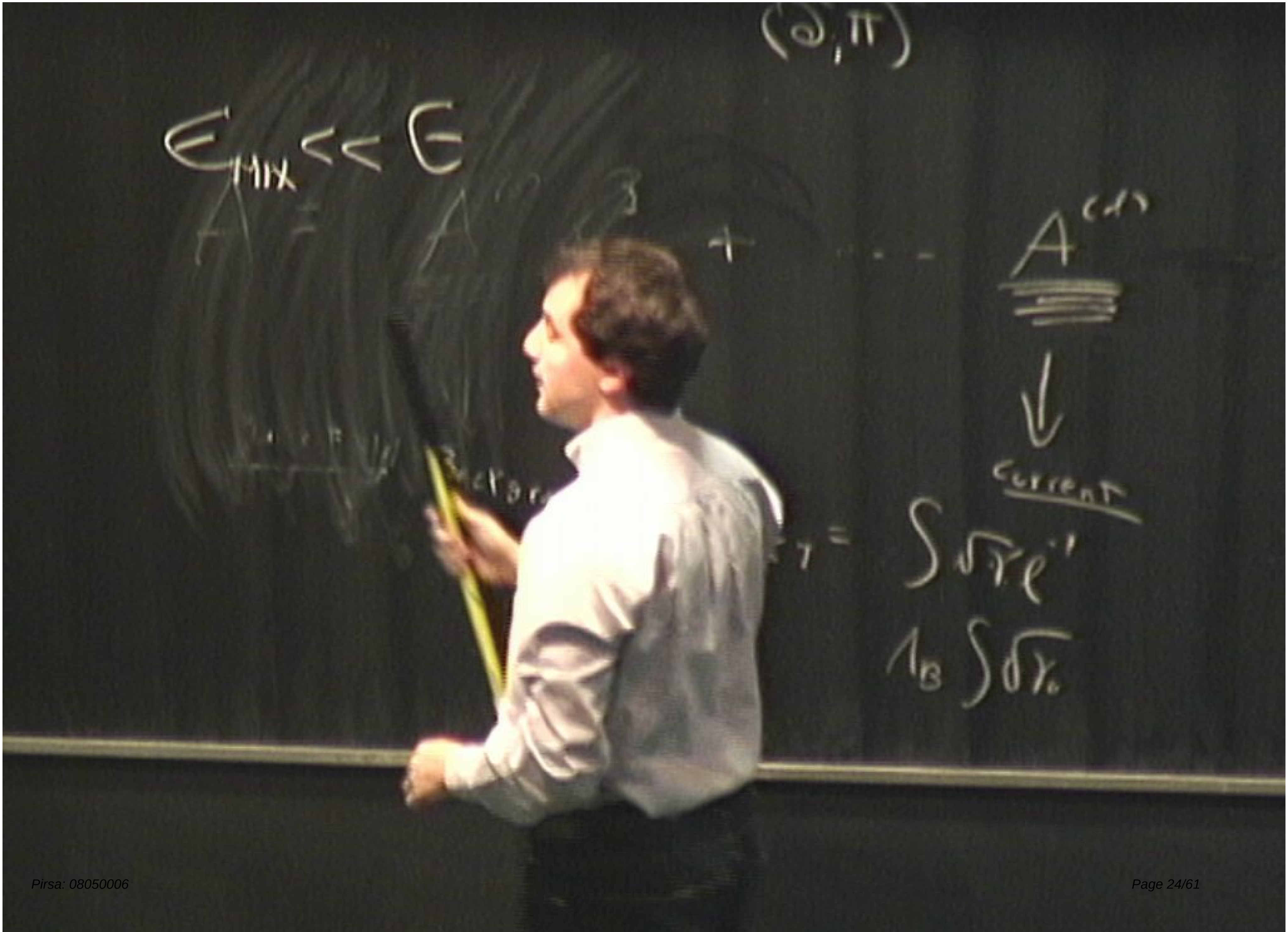
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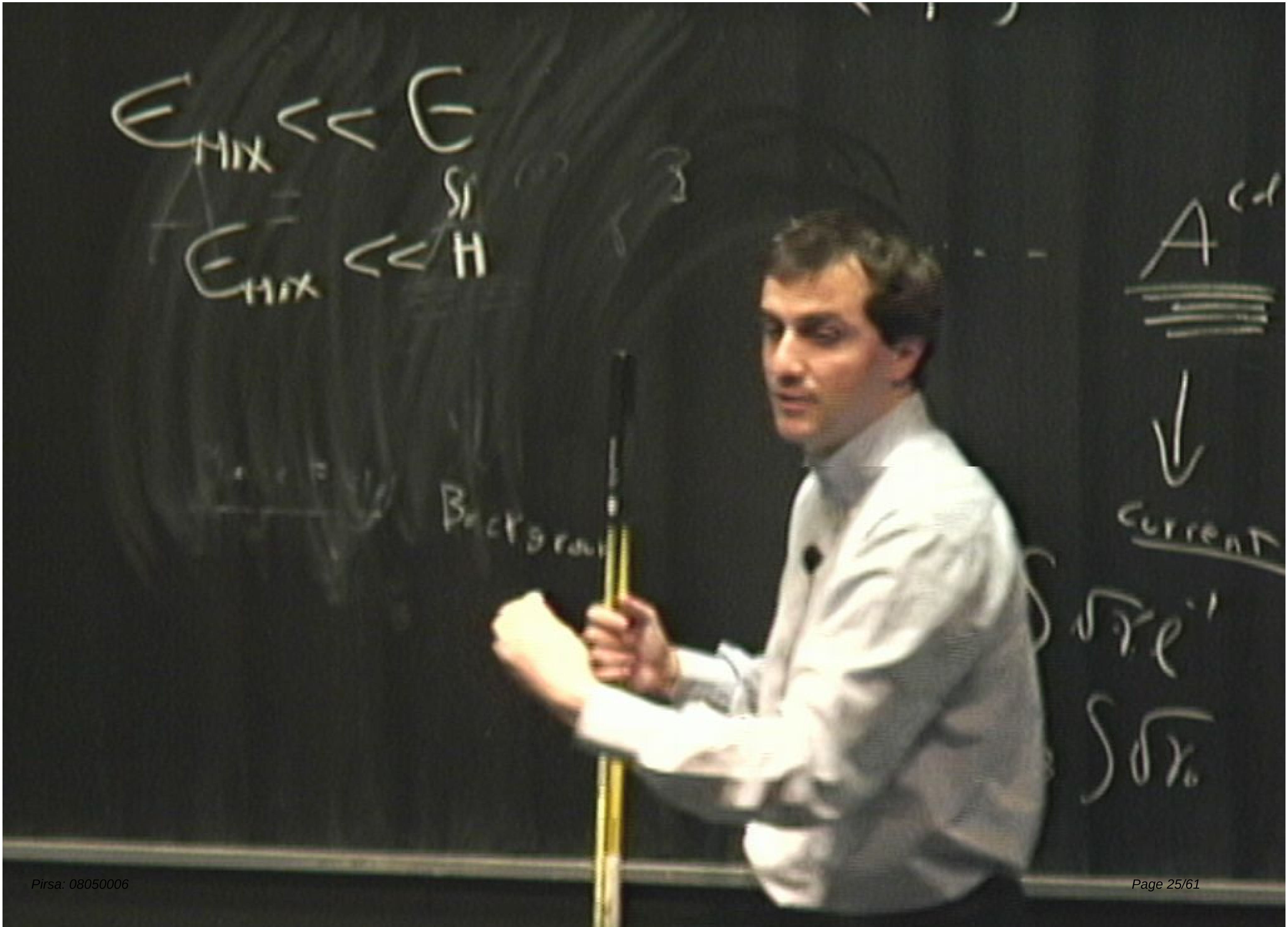

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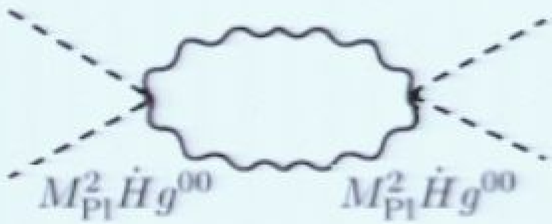

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't Hooft, Veltman
Annales Poincare Phys.Theor.A20:69-94,19

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Additional operators may be much bigger with new physics below M_P

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~ Physics beyond SM

Weinberg **0804.4291 [hep-th]**: R^2 again $1 - c_s \lesssim \epsilon\, 10^{-10}$

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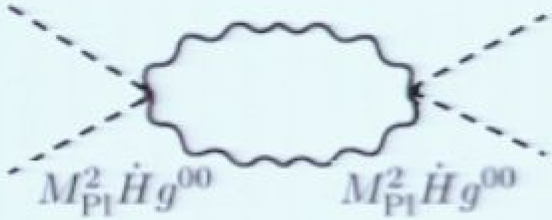

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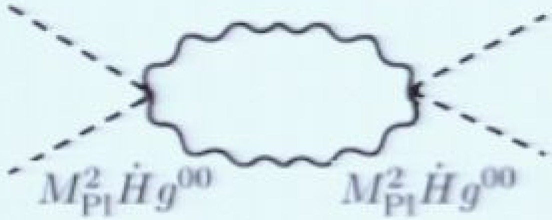

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…and large Non-Gaussianities

Cubic terms for the Goldstone:

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Experiments set limits on M_2 or equivalently on c_s

Explicit calculation gives: $$f_{\rm NL}^{\rm equil.} = \frac{85}{324}\cdot\frac{1}{c_s^2}$$
(see Chen, Huang, Kachru and Shiu hep-th/0605045)

WMAP5 limits: $-151 < f_{\rm NL}^{\rm equil.} < 253$ at 95% C.L. ⟶ $c_s > 0.028$

(technique based on P. Creminelli, A. Nicolis, L.Senatore, M.Zaldarriaga and M.Tegmark **JCAP 0604:004,2006**)

(barring cancellations with M_3 operator)

Similarly for 4-point function. At leading order in slow-roll: $(g^{00}+1)^2$, $(g^{00}+1)^3$, $(g^{00}+1)^4$

Contribution linked to c_s: $$\frac{\mathcal{L}_{(\nabla\pi)^4}}{\mathcal{L}_2} \sim \frac{\left(\frac{H}{c_s}\pi\right)^4}{H^2\pi^2} \sim \frac{H^2}{c_s^4}\pi^2 \sim \frac{1}{c_s^4}\zeta^2$$

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(see Chen, Huang, Kachru and Shiu hep-th/0605045)

WMAP5 limits: $-151 < f_{\rm NL}^{\rm equil.} < 253$ at 95% C.L. $\longrightarrow$ $c_s > 0.028$

(technique based on P. Creminelli, A. Nicolis, L.Senatore, M.Zaldarriaga and M.Tegmark **JCAP 0604:004,2006**)

(barring cancellations with M_3 operator)

Similarly for 4-point function. At leading order in slow-roll: $(g^{00}+1)^2$, $(g^{00}+1)^3$, $(g^{00}+1)^4$

Contribution linked to c_s: $\frac{\mathcal{L}_{(\nabla\pi)^4}}{\mathcal{L}_2} \sim \frac{\left(\frac{H}{c_s}\pi\right)^4}{H^2\pi^2} \sim \frac{H^2}{c_s^4}\pi^2 \sim \frac{1}{c_s^4}\zeta^2$

(Huang, Shiu hep-th/0610235)

Cutoff, Naturalness, and the dS limit

For $c_S<1$ the π theory is not renormalizable. It becomes strongly coupled at Λ

$$\Lambda^4 \simeq 16\pi^2 M_2^4 \frac{c_s^7}{(1-c_s^2)^2} \simeq 16\pi^2 M_{\rm Pl}^2 |\dot H| \frac{c_s^5}{1-c_s^2}$$

For cosmology we need: $H^4 \ll M_{\rm Pl}^2 |\dot H| c_s^5 \qquad c_s \gg P_\zeta^{1/4} \simeq 0.003$

Not surprisingly
$NG_3, NG_4 \sim 1$

No large radiative corrections $\delta c_s^2 \sim \; c_s^{-5}\Lambda^4/(16\pi^2 M_2^4) \;\lesssim c_s^2$

Goldstone theory is natural: - No large radiative corrections
- Irrelevant operators
- Approximate shift symmetry

$$M_{\rm Pl}^2 \dot H \left(1-\frac{1}{c_s^2}\right)\left(\dot\pi^3 - \dot\pi\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot\pi$$

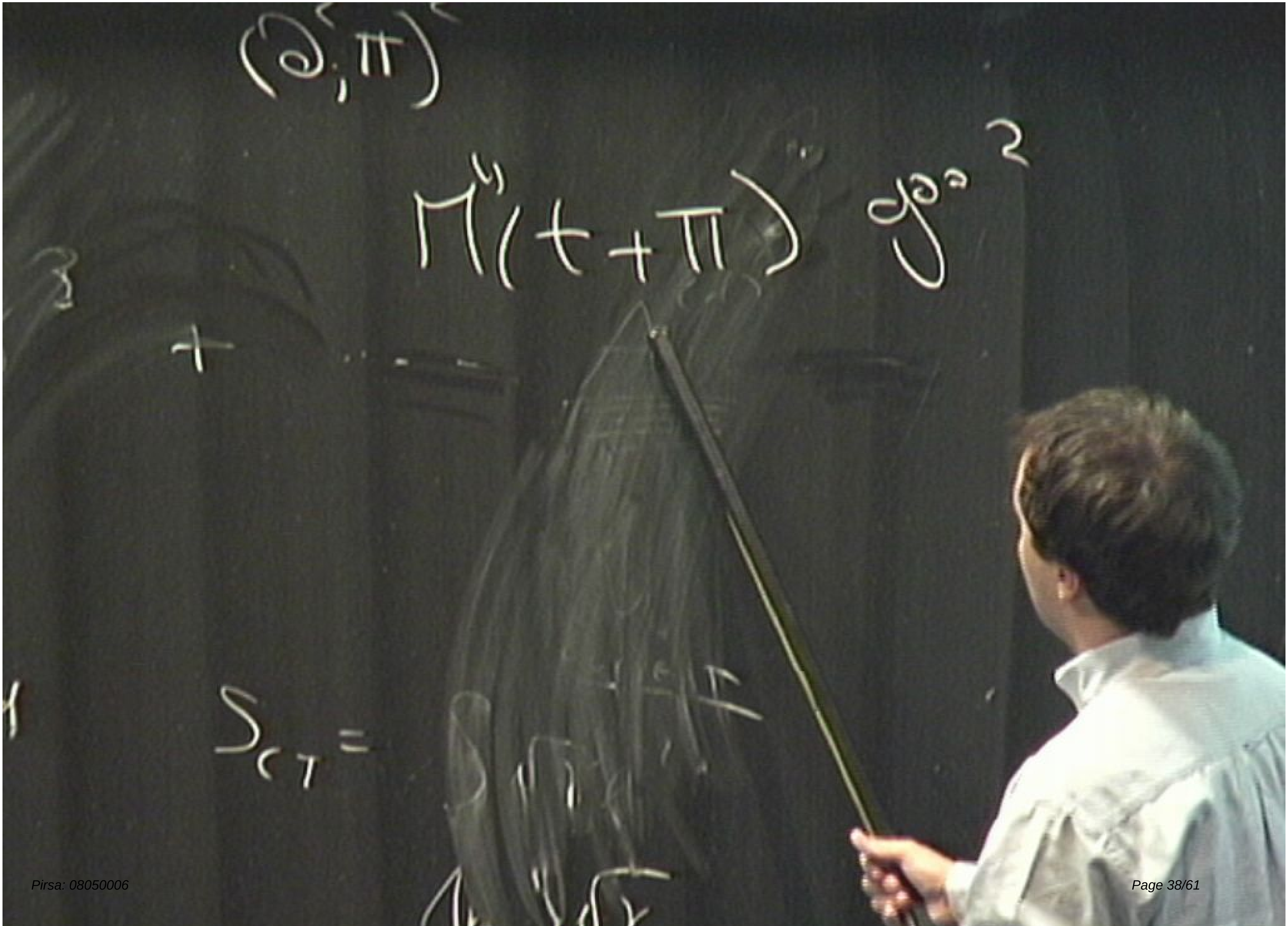
$$c_s^{-2} = 1 - \frac{2M_2^4}{M_{\rm Pl}^2 \dot H} \longrightarrow$$ dS limit pathological? We have to consider higher derivative terms:

$$\int d^4x \sqrt{-g}\left(-\frac{\bar M_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar M_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu\right) \longrightarrow \int d^4x\sqrt{-g}\left[-\frac{\bar M^2}{2}\frac{1}{a^4}(\partial_i^2\pi)^2\right]$$

Non-relativistic dispersion relation: $\omega \propto k^2$

We can impose a new symmetry: $\longrightarrow \quad \pi(\vec x, t) \to \pi(\vec x, t) + \text{const.}$
time invariance of unitary gauge coefficients

Ghost condensation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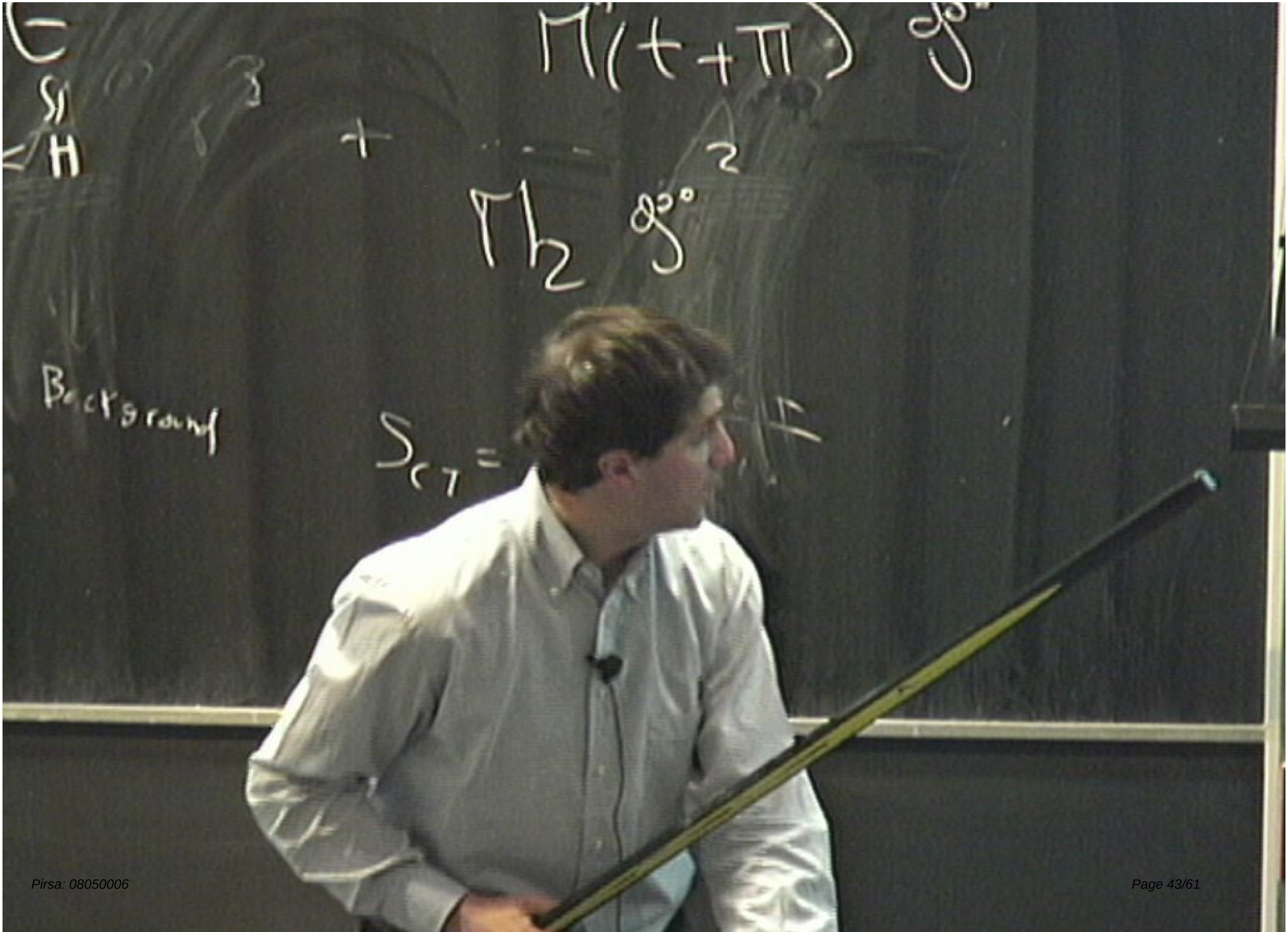
Background

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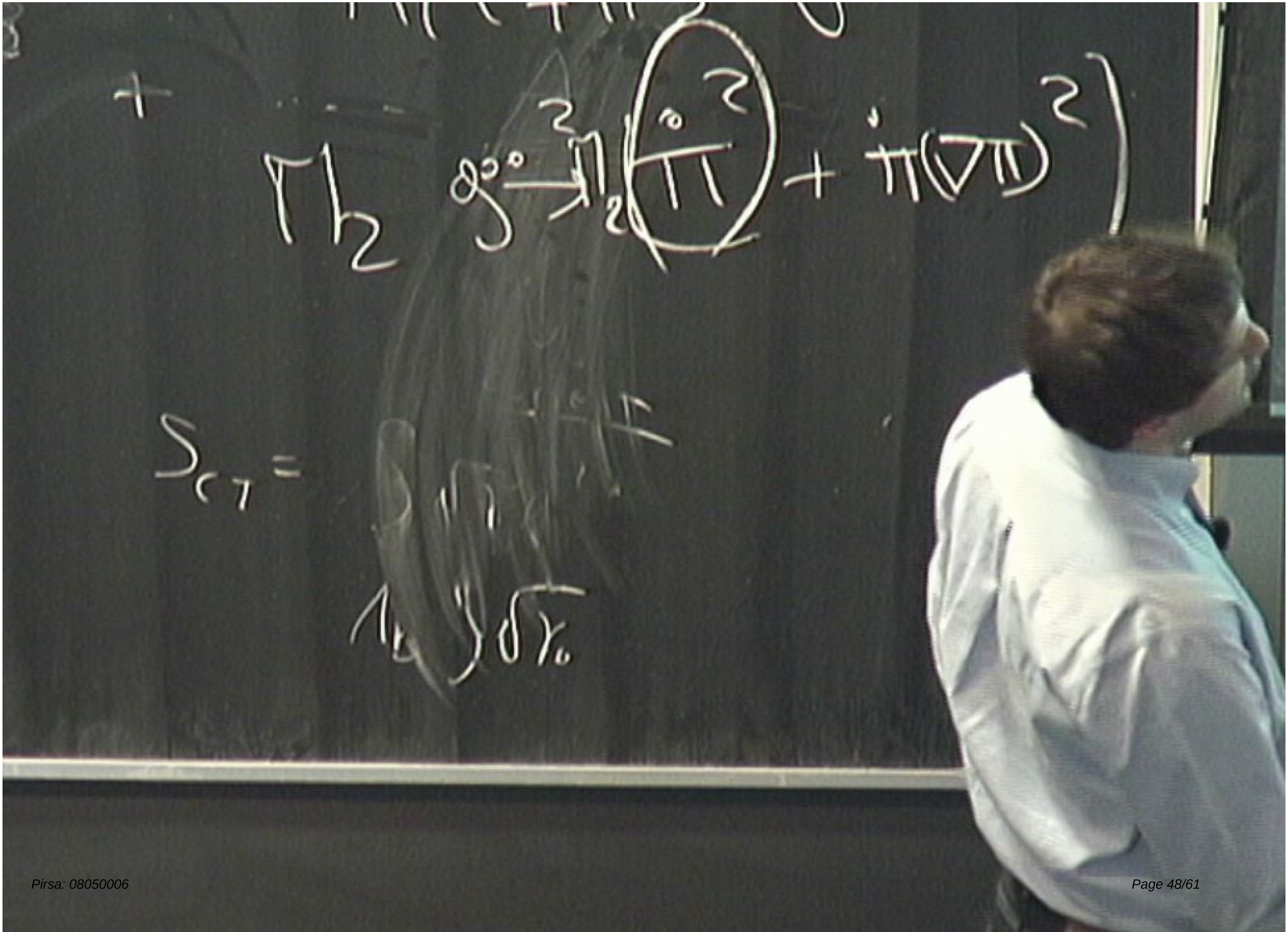
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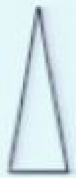
Theorems on signatures: The consistency condition of the 3-point function of single field inflation



with C. Cheung, A.L. Fitzpatrick, J. Kapla
JCAP 0802:021,2008.

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Maldacena **JHEP 0305:013,2003**
Creminelli, Zaldarriaga **JCAP 0410:006,20**

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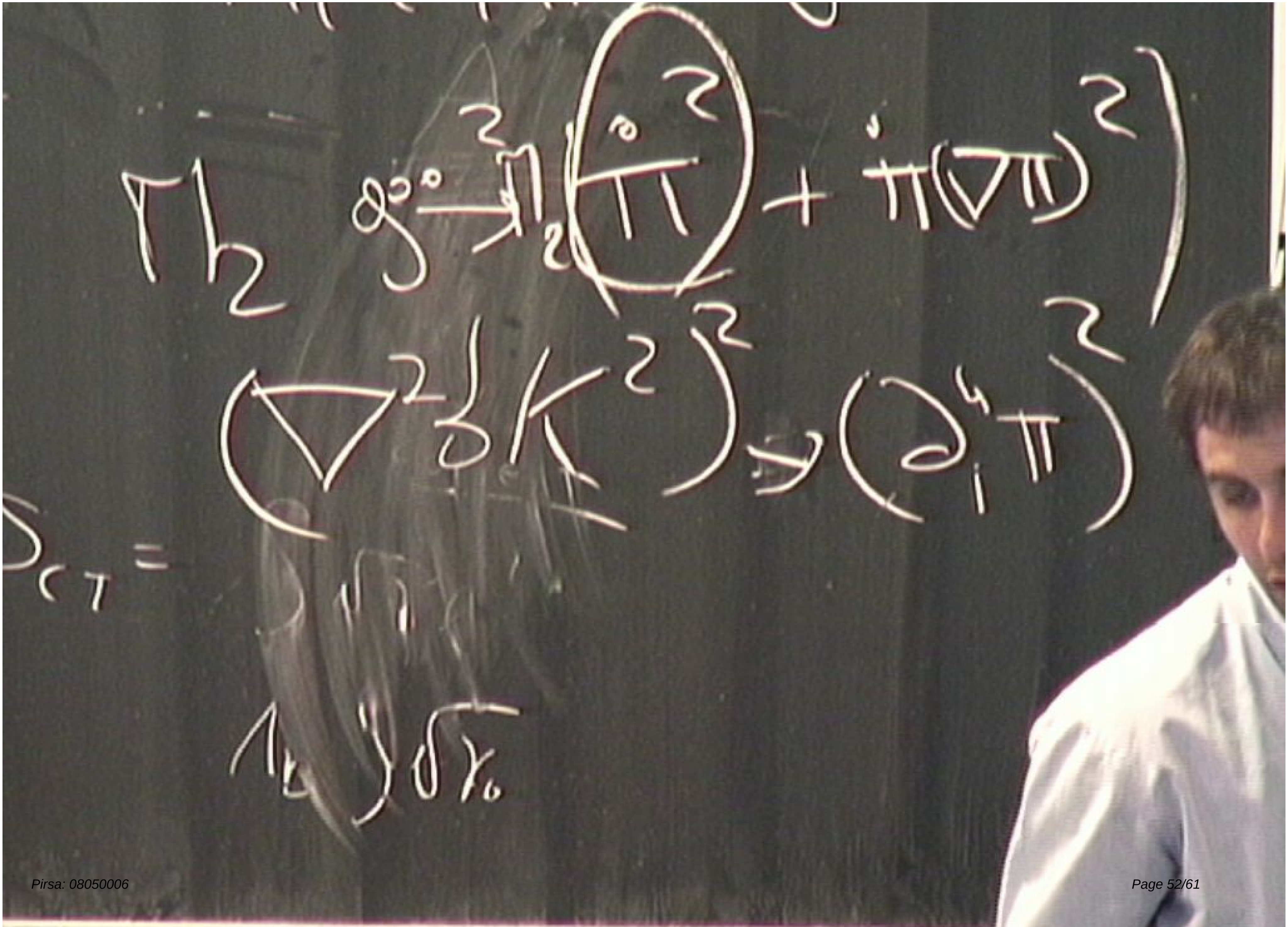

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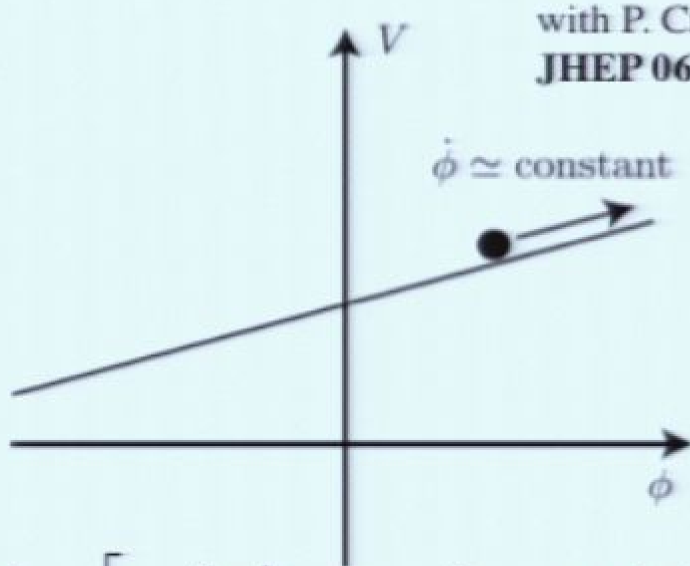
Violation of Null Energy Condition

with P. Creminelli, M. Luty, A.Nicolis,
JHEP 0612:080,2006

Is $\dot{H} > 0$ possible ?

Imagine:



$\dot{H} > 0 \quad \Longrightarrow \quad w < -1$

Low Energy Action: $$S_\pi = \int d^4x \left[M_{\rm Pl}^2 \dot{H} (\partial_\mu \pi)^2 + M^4 \dot{\pi}^2 - \bar{M} (\partial_i^2 \pi)^2 \right]$$

Dispersion relation: $$\omega^2 = -\frac{M_{\rm Pl}^2 \dot{H}}{M^4} k^2 + \frac{\bar{M}^2}{M^4} k^4$$

- Stable for $\frac{\dot{H}}{H} \lesssim \frac{\overline{M}(t) M(t)^2}{M_{\rm Pl}^2} \lesssim H$.

- Violation of NEC ⟶ New Ekpirotic Models

with Creminelli **JCAP 0711:010,2007**
Buchbinder, Khoury, Ovrut **PRD76:123503,2007**
Lehners, McFadden, Turok, Steinhardt **PRD76:103501,20**

- Unique low energy prediction for today's $w < -1$

- Study effects of perturbations: in progress

Summary. Advantages of this approach

$$S_\pi = \int d^4x \sqrt{-g} \left[M_{\rm Pl}^2 \dot{H} (\partial_\mu \pi)^2 + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{1}{a^2} (\partial_i \pi)^2 \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 - \frac{\bar{M}^2}{2} \frac{1}{a^4} (\partial_i^2 \pi)^2 + \ldots \right.$$

1. The systematic way of parametrizing high energy effects on simple slow-roll inflation. Experiment sets limits on the additional operators (NG, GWs…).

2. What is forced by symmetries is made explicit. E.g.
   - The spatial kinetic term is fixed by the background: $-M_{\rm Pl} \dot{H} (\partial_i \pi)^2$
     (relation NEC/instabilities, discovery on how to violate NEC)
   - $(g^{00}+1)^2$ reduces c_S and gives 3 and 4 point functions

3. The number of relevant operators is explicit. E.g. the 3-point function can be changed by $(g^{00}+1)^2$ and $(g^{00}+1)^3$ (at leading order in derivatives)

4. All single field models are unified: DBI, ghost inflation. Prove Theorems on signal.

5. In ϕ language one can perform fieldredefinitions: $\phi \to \tilde{\phi}(\phi)$
   (E.g. $f(\phi)^2 (\partial\phi)^2 - V(\phi)$ is equivalent to a minimal model)
   While π gives a standard non-linear representation of time diff

Summary. Advantages of this approach

6. In ϕ language it is not obvious how to assess importance of operators
   E.g. All $(\partial\phi)^{2n}$ may be comparable as some legs can be put on background
   (in DBI for example). In ghost inflation the scaling is only clear in π.

7. Loop corrections of cosmological perturbations. UV divergence are easy to reabsorb.
   For ϕ one would have to consider infinite counterterms.

$$S_\pi = \int d^4x \sqrt{-g}\left[M_{\rm Pl}^2 \dot{H}\,(\partial_\mu\pi)^2 + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{1}{a^2}(\partial_i\pi)^2\right) - \frac{4}{3}M_3^4\dot{\pi}^3 - \frac{\bar{M}^2}{2}\frac{1}{a^4}(\partial_i^2\pi)^2 + \ldots\right.$$

Summary. Advantages of this approach

$$S_\pi = \int d^4x \sqrt{-g} \left[M_{\rm Pl}^2 \dot{H} (\partial_\mu \pi)^2 + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{1}{a^2} (\partial_i \pi)^2 \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 - \frac{\bar{M}^2}{2} \frac{1}{a^4} (\partial_i^2 \pi)^2 + \ldots$$

1. The systematic way of parametrizing high energy effects on simple slow-roll inflation. Experiment sets limits on the additional operators (NG, GWs…).

2. What is forced by symmetries is made explicit. E.g.
   - The spatial kinetic term is fixed by the background: $-M_{\rm Pl} \dot{H} (\partial_i \pi)^2$

     (relation NEC/instabilities, discovery on how to violate NEC)
   - $(g^{00}+1)^2$ reduces c_s and gives 3 and 4 point functions

3. The number of relevant operators is explicit. E.g. the 3-point function can be changed by $(g^{00}+1)^2$ and $(g^{00}+1)^3$ (at leading order in derivatives)

4. All single field models are unified: DBI, ghost inflation. Prove Theorems on signal.

5. In ϕ language one can perform fieldredefinitions: $\phi \to \tilde{\phi}(\phi)$

   (E.g. $f(\phi)^2 (\partial\phi)^2 - V(\phi)$ is equivalent to a minimal model)

   While π gives a standard non-linear representation of time diff

The various limits of single field inflation: Slow-roll inflation…

Set to zero all additional operators: $M_2 = M_3 = \bar{M}_1 = \bar{M}_2 \ldots = 0$

$$S = \int d^4x \sqrt{-g} \quad \left[\frac{1}{2}M_{\rm Pl}^2 R - M_{\rm Pl}^2 \left(3H^2(t+\pi) + \dot{H}(t+\pi)\right) + \right.$$
$$\left. + M_{\rm Pl}^2 \dot{H}(t+\pi)\left((1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi})\partial_i \pi g^{0i} + g^{ij}\partial_i\pi\partial_j\pi\right)\right]$$

From terms of the form: $\sim M_{\rm Pl}^2 \dot{H}\dot{\pi}\delta g^{00}$ mixing is relevant at $E_{\rm mix} \sim \varepsilon^{1/2} H$

At E~H + leading order in slow-roll: $$S_\pi = \int d^4x \sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - M_{\rm Pl}^2 \dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2}\right)\right]$$

$$\langle \pi_c(\vec{k}_1)\pi_c(\vec{k}_2)\rangle = (2\pi)^3\delta(\vec{k}_1 + \vec{k}_2)\frac{H_*^2}{2k_1^3}$$ ⟵ A free scalar in dS!

After horizon crossing one switch to ζ which is (non-linearly) conserved

$\pi = 0 \qquad g_{ij} = a^2(t)\left[(1+2\zeta(t,\vec{x}))\delta_{ij} + \gamma_{ij}\right]$

$t \to t - \pi(t,\vec{x}) \qquad \zeta(t,\vec{x}) = -H\pi(t,\vec{x})$

Standard results:

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\rangle = (2\pi)^3\delta(\vec{k}_1 + \vec{k}_2)\frac{H_*^2}{4\epsilon_* M_{\rm Pl}^2}\frac{1}{k_1^3}$$

$$n_s - 1 = \frac{d}{d\log k}\log\frac{H_*^4}{|\dot{H}_*|} = 4\frac{\dot{H}_*}{H_*^2} - \frac{\ddot{H}_*}{H_*\dot{H}_*}$$

Not everything in decoupling limit: e.g. 3-point function comes from mixing with gravity

Fixing the tadpoles

Background evolution fixes c(t) and Λ(t). Higher order terms only affect perturbations

Friedman equations give:

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left[c(t)+\Lambda(t)\right]$$

$$\frac{\ddot a}{a} = \dot H + H^2 = -\frac{1}{3M_{\rm Pl}^2}\left[2c(t)-\Lambda(t)\right]$$

$$S = \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot H g^{00} - M_{\rm Pl}^2(3H^2+\dot H) + \frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 + \\ -\frac{\bar M_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar M_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar M_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + \ldots\Big].$$

Simplest case: $\int d^4x\,\sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2 - V(\phi)\right] \to \int d^4x\,\sqrt{-g}\left[-\frac{\dot\phi_0(t)^2}{2}g^{00} - V(\phi_0(t))\right]$

$$S = \int d^4x\,\sqrt{-g}\,P(\phi_0(t)^2 g^{00}, \phi(t))$$

$L = P(X,\phi)$, with $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi.$

$$M_n^4(t) = \phi_0(t)^{2n}\partial^n P/\partial X^n$$