Title: On Spectral Triples in Quantum Gravity

Date: May 29, 2008 04:00 AM

URL: http://pirsa.org/08050004

Abstract: This talk is concerned with the existence of spectral triples in quantum gravity. I will review the construction of a spectral triple over a functional space of connections. Here, the \*-algebra is generated by holonomy loops and the Dirac type operator has the form of a global functional derivation operator. The spectral triple encodes the Poisson structure of General Relativity when formulated in terms of Ashtekars variables. Finally I will argue that the Hamiltonian of General Relativity may emerge from the construction via the requirement that inner automorphisms vanish on the vacuum sector.

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## On Spectral Triples in Quantum Gravity

Jesper Møller Grimstrup

The Niels Bohr Institute. Copenhagen. Denmark

Collaboration with Johannes Aastrup and Ryszard Nest

Perimeter Institute 29.05.08

### On Spectral Triples in Quantum Gravity

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[Connes. Lott. Chamseddine, Marcolli, ...]

- ▶  $B = C^{\infty}(M) \otimes B_F$ , almost commutative algebra  $B_F = \mathbb{C} \oplus \mathbb{H} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$
- $D = D_M \otimes 1 + \gamma_5 \otimes D_F$ ,  $D_F$  is the Yukawa coupling matrix
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- this would presumably involve Quantum Gravity.

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### Our goal

to construct a model which combines Noncommutative Geometry with elements of Quantum Gravity.

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"A striking aspect of this approach to geometry of  $\bar{\mathcal{A}}/\mathcal{G}$  is that its general spirit is the same as that of non-commutative geometry and quantum groups: even though there is no underlying differential manifold, geometrical notions can be developed by exploiting the properties of the *algebra* of functions."

[Ashtekar, Lewandowski, 1996]

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## Open questions

- The construction is based on a countable system of embedded graphs (lattices, simplicial complexes). The construction is essentially combinatorial.
- The structure of the Hamiltonian of GR emerges from a condition which restricts the triple to a sector where inner automorphisms play no role (turning off interactions vacuum).
- ▶ The triple  $(\mathcal{B}_{\Delta}, \mathcal{D}_{\Delta}, \mathcal{H}_{\Delta})$  depends on a set  $\{a_i\}$  of scaling parameters. This resembles a regularization scheme.
- Connes distance formula: distances between "geometries".

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### he Hamiltonian

Then (heuristically)

$$Tr\{D_{\Delta}, W\} + \{W, W\}\Big|_{\text{one vertex}} \stackrel{\text{classical}}{\sim} \epsilon^{ij}_{\ k} F_{\mu\nu}^k E_j^\mu E_k^\nu + \mathcal{O}\left(\alpha^2\right)$$

(we set 
$$\alpha = a_n$$
 for  $n - \infty$ )

This has the form of the Hamilton constraint of GR.

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### Graphs

Shift focus from connections to holonomies and flux variables

$$h_L(A) = \operatorname{Hol}(L, A)$$

L loop on  $\Sigma$ 

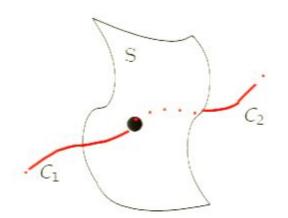
$$F_S^a(E) = \int_S \epsilon_{mnp} E^{ma} dx^n dx^p$$

S surface in  $\Sigma$ .

Poisson brackets

$$\{F_S^a(E), h_C(A)\} = \pm h_{C_1}(A)\tau^a h_{C_2}(A)$$

 $\tau^a$  generator of  $\mathfrak{su}(2)$ ,  $C = C_1 C_2$  are curves in  $\Sigma$ .



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### Main point

Formulation of the Standard Model coupled to General Relativity as a single gravitational theory. The Standard Model emerges from a very simple modification of space-time geometry:

$$C^{\infty}(M) - C^{\infty}(M) \otimes B_F$$

## Two questions

- How does the quantization procedure translate into the language of Noncommutative Geometry?
- this would presumably involve Quantum Gravity.
- How to explain the finite algebra  $B_F$ ?

## Our goal

to construct a model which combines Noncommutative Geometry with elements of Quantum Gravity.

### On Spectral Triples in Quantum Gravity

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to construct a model which combines Noncommutative Geometry with elements of Quantum Gravity.

## Inspiration

Lore antum Gravity (algebra, mathematical techniques, ideas)

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## .oop Quantum Gravity

- ▶ Foliation:  $M = \mathbb{R} \times \Sigma$
- Ashtekar variables

$$A_j^i$$
  $SU(2)$ -connection on  $\Sigma$ .  
 $E_j^i = |\det e_j^i|^{\frac{1}{2}} e_j^i$   $e_j^i$  orthonormal frame field.

Poisson brackets

$$\{A_j^i(x), E_l^k(y)\} = \delta_l^i \delta_j^k \delta(x - y)$$

+ constraints (diffeomorphism, Hamilton, Gauss)

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 Shift focus from connections to holonomies and flux variables

$$h_L(A) = \operatorname{Hol}(L, A)$$

L loop on  $\Sigma$ 

$$F_S^a(E) = \int_S \epsilon_{mnp} E^{ma} dx^n dx^p$$

S surface in  $\Sigma$ .

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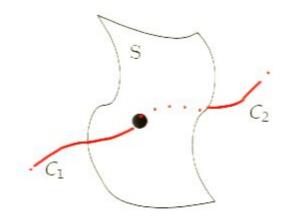
$$F_S^a(E) = \int_S \epsilon_{mnp} E^{ma} dx^n dx^p$$

S surface in  $\Sigma$ .

Poisson brackets

$$\{F_S^a(E), h_C(A)\} = \pm h_{C_1}(A)\tau^a h_{C_2}(A)$$

 $\tau^a$  generator of  $\mathfrak{su}(2)$ ,  $C = C_1 C_2$  are curves in  $\Sigma$ .



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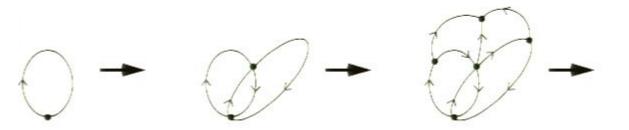
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## Graphs

The algebra of holonomy loops is described via the inductive system of all finite, piece-wise analytic graphs



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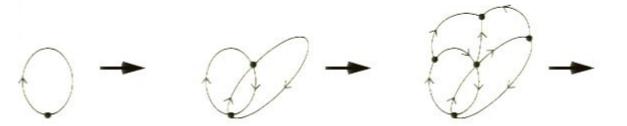
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## Graphs

The algebra of holonomy loops is described via the inductive system of all finite, piece-wise analytic graphs



Let  $\mathcal A$  be the space of smooth connections with gauge group G. Denote by  $\mathcal A_\Gamma$  the restriction of  $\mathcal A$  to a finite graph  $\Gamma$ . Seen from  $\Gamma$  a connection  $\nabla \in \mathcal A$  can be seen as a point in the space  $G^n$ 

$$\nabla = (g_1, \ldots, g_n) \in G^{n(\Gamma)} \simeq A_{\Gamma}$$

where  $n(\Gamma)$  is the number of edges  $\epsilon_i$  in  $\Gamma$  and where  $g_i = Hol(\nabla, \epsilon_i)$  is the holonomy of  $\nabla$  along  $\epsilon_i$ .

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Projective system of coarse grained spaces of connections:

with structure maps

$$P_{\Gamma\Gamma'}:G^{n(\Gamma')}-G^{n(\Gamma)}$$

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Projective system of coarse grained spaces of connections:

with structure maps

$$P_{\Gamma\Gamma'}:G^{n(\Gamma')}-G^{n(\Gamma)}$$

Example:

$$P: G^4 - G$$
  
 $(g_1, g_2, g_3, g_4) - g_1 \cdot g_3$ 





### Result:

$$\mathcal{A} \hookrightarrow \varliminf \mathcal{A}_{\Gamma} =: \overline{\mathcal{A}}^{\mathfrak{a}}$$

[Ashtekar, Lewandowski]

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### Result:

$$\mathcal{A} \hookrightarrow \varliminf \mathcal{A}_{\Gamma} =: \overline{\mathcal{A}}^{\mathfrak{a}}$$

### [Ashtekar, Lewandowski]

Point: The space of connections is densely imbedded in a pro-manifold \( \overline{A}^a - Ashtekar-Lewandowski measure, Hilbert space structure ...

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▶ Aim: To construct a spectral triple that involves an algebra of loops, i.e. functions on A:

$$L: \nabla \longrightarrow \operatorname{Hol}(\nabla, L) \in M_n(\mathbb{C})$$

where the interaction between Dirac type operator and the loop algebra reproduces the Poisson structure of GR.

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Strategy: Exploit the pro-manifold structure of A

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- **Strategy:** Exploit the pro-manifold structure of  $\mathcal A$ 
  - 1. Construct a spectral triple  $(\mathcal{B}, \mathcal{D}, \mathcal{H})_{\Gamma}$  at the level of each finite graph  $\Gamma$ . Since

$$A_{\Gamma} \simeq G^{n}$$

this is easy (Haar measure, Dirac operator etc.)

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2. Ensure compatibility with the structure maps

$$P_{\Gamma_n\Gamma_m}: \mathcal{A}_{\Gamma_n} - \mathcal{A}_{\Gamma_m}$$
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for all structures (Hilbert space, algebra, Dirac operator)

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Pirsa: 0805004 take the projective/inductive limit to obtain a spectral triple over the space of connections  $\mathcal{A}$ .

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<u>Problem:</u> Too many different embeddings between graphs to permit a Dirac type operator. — The setup is <u>overcountable</u>.

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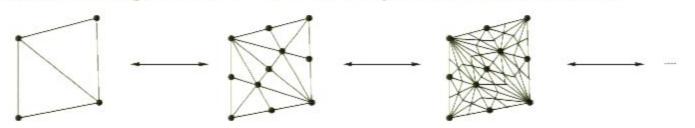
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- New Approach: Consider a restricted, countable system of graphs.
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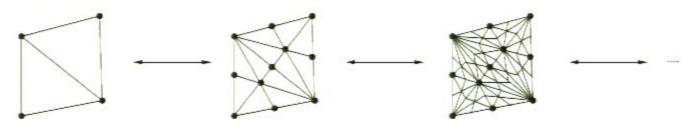
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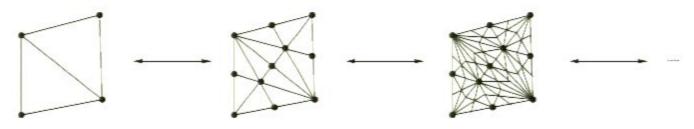
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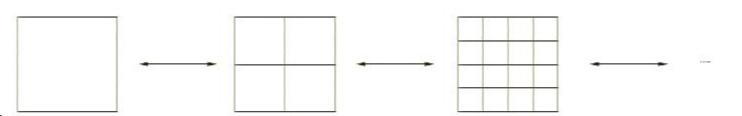
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Here: take a projective system of cubic lattices.



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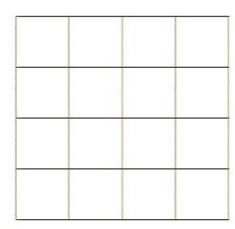
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## he construction

Let Γ be a finite d-dim lattice with edges  $\{\epsilon_i\}$  and vertices  $\{v_i\}$  with

$$\epsilon_i : \{0, 1\} - \{v_i\}$$



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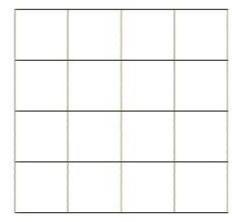
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▶ Assign to each edge  $\epsilon_i$  a group element  $g_i \in G$ .

$$\nabla : \epsilon_i - g_i$$

G is a compact Lie-group. The space of such maps is denoted  $A_{\Gamma}$ . Notice again:

$$\mathcal{A}_{\Gamma} \simeq G^n$$
 because  $\mathcal{A}_{\Gamma} \ni \nabla - (\nabla(\epsilon_1), \dots, \nabla(\epsilon_n)) \in G^n$ 

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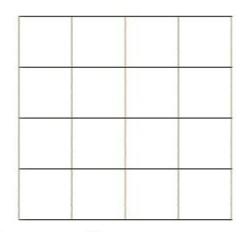
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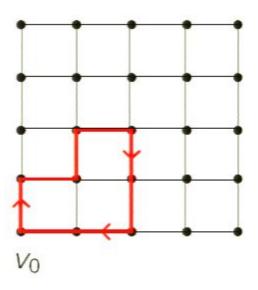
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### Algebra: A loop L is a finite sequence of edges L = {ε<sub>i1</sub>, ε<sub>i2</sub>,..., ε<sub>in</sub>} running in Γ (choose basepoint v<sub>0</sub>). Discard trivial backtracking.



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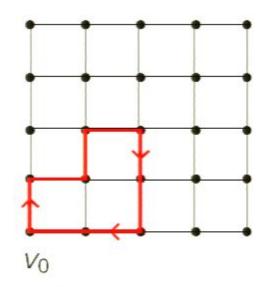
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Product by gluing

$$L_1 \circ L_2 = \{L_1, L_2\}$$

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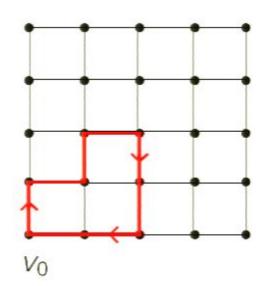
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Inversion:

$$L^* = \{\epsilon_{i_n}^*, \ldots, \epsilon_{i_j}^*, \ldots, \epsilon_{i_1}^*\}$$

with

$$\epsilon_j^*(\tau) = \epsilon_j(1-\tau), \quad \tau \in \{0,1\}$$

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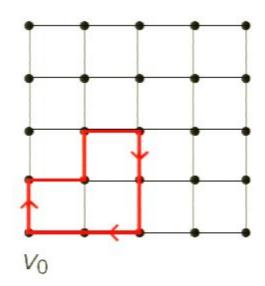
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with

$$\epsilon_j^*(\tau) = \epsilon_j(1-\tau) , \quad \tau \in \{0,1\}$$

Consider formal, finite series of loops

$$a = \sum_{i} a_i L_i$$
 .  $a_i \in \mathbb{C}$ 

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### ► The product between two elements a and b is defined

$$a \circ b = \sum_{i,j} (a_i \cdot b_j) L_i \circ L_j$$

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Conclusation

► The product between two elements a and b is defined

$$a \circ b = \sum_{i,j} (a_i \cdot b_j) L_i \circ L_j$$

The involution of a is defined

$$a^* = \sum_i \bar{a}_i L_i^*$$

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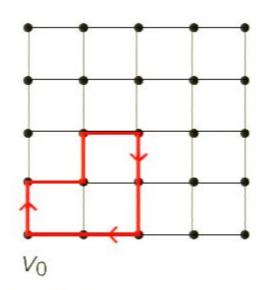
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# ▶ Algebra: A loop L is a finite sequence of edges

 $L = \{\epsilon_{i_1}, \epsilon_{i_2}, \dots, \epsilon_{i_n}\}$  running in  $\Gamma$ (choose basepoint  $v_0$ ). Discard trivial backtracking.



▶ Product by gluing  $L_1 \circ L_2 = \{L_1, L_2\}$ 

$$L_1 \circ L_2 = \{L_1, L_2\}$$

Inversion:

$$L^* = \{\epsilon_{i_n}^*, \ldots, \epsilon_{i_j}^*, \ldots, \epsilon_{i_1}^*\}$$

with

$$\epsilon_j^*(\tau) = \epsilon_j(1-\tau), \quad \tau \in \{0,1\}$$

Consider formal, finite series of loops

$$a = \sum_i a_i L_i$$
 .  $a_i \in \mathbb{C}$ 

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These elements have a natural norm

$$||a|| = \sup_{\nabla \in \mathcal{A}_{\Gamma}} ||\sum a_i \nabla (L_i)||_G$$

where the norm on the rhs is the matrix norm in G. The closure of the  $\star$ -algebra of loops with respect to this norm is a  $C^{\star}$ -algebra. We denote this loop algebra by  $\mathcal{B}$ .

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ToP Cursion

Hilbert space: There is the (somewhat) natural Hilbert space

$$\mathcal{H} = L^2(G^n, Cl(T^*G^n) \otimes M_l(\mathbb{C}))$$

involving the Clifford bundle over  $G^n$  (I size of rep. of G).  $L^2$  is with respect to the Haar measure on  $G^n$ .

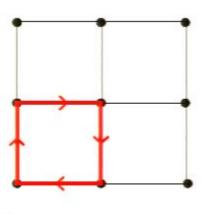
lacktriangleright The loop algebra  ${\cal B}$  has a natural representation on  ${\cal H}$ 

$$f_L \cdot \psi(\nabla) = (1 \otimes \nabla(L)) \cdot \psi(\nabla)$$
.  $\psi \in \mathcal{H}$ 

where the first factor acts on the Clifford-part of the Hilbert space and the second factor acts by matrix multiplication on the matrix part of the Hilbert space

$$L = \{\epsilon_1, \epsilon_4, \epsilon_6^*, \epsilon_3^*\}$$

$$f_L \sim g_1 \cdot g_4 \cdot (g_6)^{-1} \cdot (g_3)^{-1}$$



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▶ **Dirac operator:** We choose the Dirac operator *D* on *G*<sup>n</sup>

$$D(\xi) = \sum e_i \nabla_{e_i}(\xi) . \quad \xi \in \mathcal{H}$$

(choose a metric on G and use Levi-Civita) and obtain

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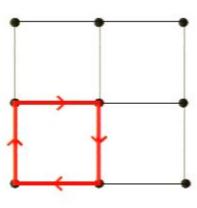
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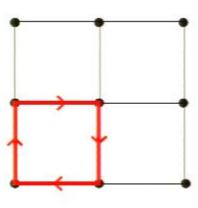
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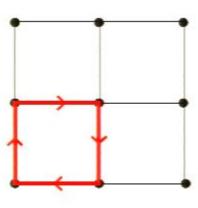
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the spectral triple

$$(\mathcal{B}, \mathcal{D}, \mathcal{H})_{\Gamma}$$
.

on the level of the simplicial complex  $\Gamma$ .

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.

on the level of the simplicial complex  $\Gamma$ .

- Second step: is to allow the complexity of the simplicial complex to grow infinitely while keeping control of the spectral triple.
  - $\Rightarrow$  More refined version of the functional space  $\mathcal{A}_{\Gamma}$ .

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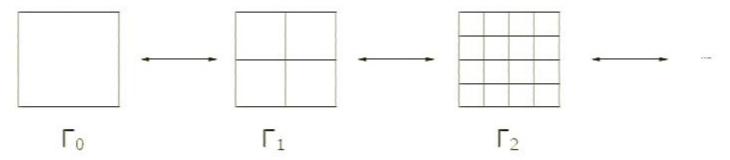
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Consider a system of nested, lattices

$$\Gamma_0 - \Gamma_1 - \Gamma_2 - \dots$$

with  $\Gamma_i$  a subdivision of  $\Gamma_{i-1}$ 



On the level of the associated manifolds  $\mathcal{A}_{\Gamma_i}$  this gives rise to projections

$$G^{n_0} \stackrel{P_{10}}{\leftarrow} G^{n_1} \stackrel{P_{21}}{\leftarrow} G^{n_2} \stackrel{P_{32}}{\leftarrow} G^{n_3} \stackrel{P_{43}}{\leftarrow} \dots$$

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Consider next a corresponding system of spectral triples

$$(\mathcal{B}, \mathcal{D}, \mathcal{H})_{\Gamma_0} - (\mathcal{B}, \mathcal{D}, \mathcal{H})_{\Gamma_1} - (\mathcal{B}, \mathcal{D}, \mathcal{H})_{\Gamma_2} - \dots$$

with the additional condition that the spectral triples are compatible with the projections/embeddings between them.

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Consider next a corresponding system of spectral triples

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with the additional condition that the spectral triples are compatible with the projections/embeddings between them.

 For the Hilbert space compatibility is easily obtained (weighted inner product) and compatibility for the algebra is clear.

To obtain compatibility for the Dirac operator we need to work a little:

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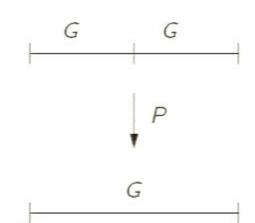
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It all boils down to study the simple case

$$P: G^2 - G$$
.  $(g_1, g_2) - g_1 \cdot g_2$ 



corresponding to the compatibility condition

$$P^*(D_1v)(g_1,g_2)=D_2(P^*v)(g_1,g_2)$$
,  $v\in L^2(G,CI(T^*G))$ 

where  $D_1$  is a Dirac operator on G using the Levi-Civita connection and  $D_2$  is a Dirac operator on  $G^2$  to be constructed.  $D_2$  has the form

$$D_2 = D_{\parallel} + aD_{\perp}$$
.  $a \in \mathbb{R}$ 

where  $D_{||}$  probes the embedded G and  $D_{\perp}$  its orthogonal complement. a is a free parameter.

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Consider next a corresponding system of spectral triples

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After repeated subdivisions this gives rise to an infinite series of free parameters {a<sub>i</sub>}.

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- After repeated subdivisions this gives rise to an infinite series of free parameters {a<sub>i</sub>}.
- By solving the G<sup>2</sup> G problem repeatedly we end up with a Dirac-like operator on the level of Γ<sub>i</sub>



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$$D = \sum_{k} a_{k} \bar{\mathcal{E}}_{k} \nabla_{\mathcal{E}_{k}}$$

where  $\mathcal{E}_k$  is an orthonormal set of covectors over  $G^n$  and  $\bar{\mathcal{E}}_k$  the corresponding element in  $Cl(T^*G^n)$ . (exact form is complicated).

# In the limit, this gives us a candidate for a spectral triple

 $(\mathcal{B}_{\Delta}, \mathcal{D}_{\Delta}, \mathcal{H}_{\Delta})$ 

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In the limit, this gives us a candidate for a spectral triple

$$(\mathcal{B}_{\Delta}, \mathcal{D}_{\Delta}, \mathcal{H}_{\Delta})$$

- ▶ Result: For a compact Lie-group G the triple  $(\mathcal{B}_{\Delta}, D_{\Delta}, \mathcal{H}_{\Delta})$  is a semi-finite\* spectral triple:
  - $D_{\Delta}$ 's resolvent  $(1+D_{\Delta}^2)^{-1}$  is compact (wrt. trace) and
  - ▶ the commutator  $[D_{\Delta}, a]$  is bounded

Provided the sequence  $\{a_i\}$  approaches  $\infty$  sufficiently fast. For G = U(1) we find

$$a_n = 2^n b_n$$
.  $\lim_{n \to \infty} b_n = \infty$ 

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<sup>\*</sup>semi-finite: everything works up to a certain symmetry group with a trace.

### paces of connections

Denote

$$\overline{\mathcal{A}}^{\Delta} := \lim_{\underline{\kappa}} \mathcal{A}_{\mathcal{K}}$$

or roughly:

$$G^{n_1}-G^{n_2}-\ldots-G^{\infty}\sim \overline{\mathcal{A}}^{\Delta}$$

 $ightharpoonup \overline{\mathcal{A}}^{\Delta}$  is a space of generalised connections. To see this map the graphs  $\{\Gamma_i\}$  into a manifold  $\mathcal{M}$ 

$$h: \Gamma_i - \Gamma_i \in \mathcal{M}$$

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Denote by A the space of smooth G-connections. There is a natural map

$$\chi: \mathcal{A} \to \overline{\mathcal{A}}^{\Delta}$$
.  $\chi(\nabla)(\epsilon_i) = Hol(\nabla, \epsilon_i)$ 

where  $Hol(\nabla, \epsilon_i)$  is the holonomy of  $\nabla$  along  $\epsilon_i$  (now in  $\mathcal{M}$ ).

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$$\mathcal{A} \subseteq \overline{\mathcal{A}}^{\Delta}$$

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Result: γ is an embedding

$$A \subseteq \overline{A}^{\Delta}$$

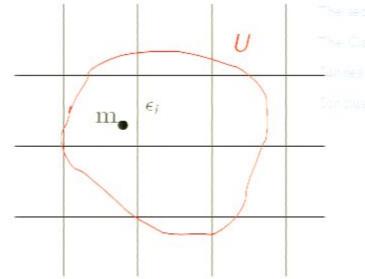
**Argument:** given  $\nabla_1, \nabla_2 \in \mathcal{A}$ they will differ in a point  $m \in \mathcal{M}$ and in a neighbourhood U of m. Choose a small edge  $\epsilon_i$  in a graphs  $\Gamma_i$  so that  $\epsilon_i \in U$ . Thus

$$Hol(\nabla_1, \epsilon_i) \neq Hol(\nabla_2, \epsilon_i)$$

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Spaces of Connections



### ▶ Thus: $\overline{\mathcal{A}}^{\triangle}$ contains all smooth connections. This implies:

 $\triangleright$  The Dirac operator is a kind of functional derivation operator over  ${\cal A}$ 

$$D_{\Delta} \sim \frac{\delta}{\delta \nabla}$$

of connections.

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▶ Thus:  $\overline{\mathcal{A}}^{\Delta}$  contains all smooth connections. This implies:

The Dirac operator is a kind of functional derivation operator over A

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of connections. In fact, it is a global operator

$$D_{\Delta} \sim \sum_{x} \overline{\nabla(x)} \cdot \frac{\delta}{\delta \nabla(x)}$$

where  $\overline{\nabla(x)}$  represents a degree of freedom in each point.

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where  $\overline{\nabla(x)}$  represents a degree of freedom in each point.

The inner product of the Hilbert space is a functional integral over A

$$\langle \Psi | ... | \Psi \rangle \sim \int_{\overline{\mathcal{A}}^{\Delta}} [d\nabla] \mathsf{Tr} ...$$

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► The inner product of the Hilbert space is a functional integral over A

$$\langle \Psi | ... | \Psi \rangle \sim \int_{\overline{\mathcal{A}}^{\Delta}} [d \nabla] \mathsf{Tr} ...$$

**Remark:** The Dirac-type operator  $D_{\Delta}$  is gauge invariant.

▶ Loop Quantum Gravity works with the space A of generalised connections based on a projective system of piecewise analytic graphs.

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- Loop Quantum Gravity works with the space \(\overline{A}^a\) of generalised connections based on a projective system of piecewise analytic graphs.
- ▶ Thus, there are *two different* completions of the space  $\mathcal{A}$  of smooth connections:  $\overline{\mathcal{A}}^a$  and  $\overline{\mathcal{A}}^\Delta$

$$A \hookrightarrow \overline{A}^{a}$$
,  $A \hookrightarrow \overline{A}^{\Delta}$ 

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- ▶ Loop Quantum Gravity works with the space A of generalised connections based on a projective system of piecewise analytic graphs.
- ▶ Thus, there are *two different* completions of the space  $\mathcal{A}$  of smooth connections:  $\overline{\mathcal{A}}^a$  and  $\overline{\mathcal{A}}^\Delta$

$$A \hookrightarrow \overline{A}^{a}$$
,  $A \hookrightarrow \overline{A}^{\Delta}$ 

- The difference between these completions is their corresponding symmetry groups:
  - In LQG: Analytic diffeomorphisms
  - Here: discrete diffeomorphisms which preserve the graph structure: Diff(△).

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We observe the following:

 $\mathcal{A}$ : - action of diff( $\mathcal{M}$ )

- no Hilbert space structure

- no Dirac-like operator

 $\overline{\mathcal{A}}^a$ : - action of (analytic) diff( $\mathcal{M}$ )

- Hilbert space structure (non-separable)

- no Dirac-like operator

 $\overline{\mathcal{A}}^{\Delta}$ : - no action of diff( $\mathcal{M}$ ) (few discrete)

- Hilbert space structure (separable)

- Dirac-like operator

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- ▶ In short: the choice of completion of the space of connections A is decisive for:
  - the amount of remaining diffeomorphisms
  - by the separability of the corresponding Hilbert space
  - by the existence of a Dirac-like operator.

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- In short: the choice of completion of the space of connections A is decisive for:
  - the amount of remaining diffeomorphisms
  - the separability of the corresponding Hilbert space
  - the existence of a Dirac-like operator.
- It appears that the use of a restricted system of graphs (simplicial complexes or cubic lattices) correspond to a kind of (partly) gauge fixing of the diffeomorphism group.

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Alternative interpretation: Notice that a cubic graph Γ is also a piecewise analytic graph. Thus:

$$L^2(\overline{\mathcal{A}}^{\Delta}) \stackrel{\iota}{\smile} L^2(\overline{\mathcal{A}}^{a}).$$

In LQG there is the Hilbert space  $\mathcal{H}_{diff}$  of (spatial) diffeomorphism invariant states. A surjection:

$$L^2(\overline{\mathcal{A}}^a) \stackrel{q}{-} \mathcal{H}_{diff}$$

We therefore get a map

$$L^2(\overline{\mathcal{A}}^{\Delta}) \stackrel{\Xi}{\longrightarrow} \mathcal{H}_{diff}$$

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### ▶ We find the diagram:

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We find the diagram:

The amount by which the map ≡ fails to be injective is exactly the symmetry group diff(△) of discrete diffeomorphisms of graphs.

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We find the diagram:

$$L^{2}(\overline{\mathcal{A}}^{a})$$

$$\downarrow q$$

$$L^{2}(\overline{\mathcal{A}}^{\Delta}) \stackrel{\Xi}{\longrightarrow} \mathcal{H}_{diff}$$

- The amount by which the map ≡ fails to be injective is exactly the symmetry group diff(△) of discrete diffeomorphisms of graphs.
- This means that  $\mathcal{H}_{\Delta}$  is directly related to the Hilbert space of (spatial) diffeomorphism invariant states known from LQG (here we set G = SU(2)).

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- The amount by which the map ≡ fails to be injective is exactly the symmetry group diff(△) of discrete diffeomorphisms of graphs.
- This means that  $\mathcal{H}_{\Delta}$  is directly related to the Hilbert space of (spatial) diffeomorphism invariant states known from LQG (here we set G = SU(2)).
  - so we should view a loop in  $\mathcal{B}_{\Delta}$  as an equivalence class of loops, up to diffeomorphisms.

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► First, for a single group element g corresponding to the i'th copy of G in G<sup>n</sup> we find

$$[D_{\Delta}, g] = \frac{1}{n} \sum_{k} (\pm g \mathfrak{E}_{k}) \cdot \bar{\mathcal{E}}_{k} \qquad (a_{i} \equiv 1)$$

where  $\bar{\mathcal{E}}_k \in Cl(T^*G^n)$  and  $\mathfrak{E}$  'twisted' generator of the Lie algebra  $\mathfrak{g}$ .

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where  $\bar{\mathcal{E}}_k \in Cl(T^*G^n)$  and  $\mathfrak{E}$  'twisted' generator of the Lie algebra  $\mathfrak{g}$ .

▶ Next, the commutator between D and the loop L is

$$[D_{\Delta}, f_{L}] = [D, g_{i_1}]g_{i_2} \dots g_{i_k} + g_{i_1}[D, g_{i_2}] \dots g_{i_k} + \dots$$

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▶ In short: the action of D is to insert generators  $\mathfrak{E}_k$  at each vertex in the loop.

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▶ In short: the action of D is to insert generators  $\mathfrak{E}_k$  at each vertex in the loop. Fx:  $f_L \sim g_1g_2g_3$ 

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▶ Quantization: assume the operators F<sup>a</sup><sub>S</sub>. C exist and satisfy

$$\Rightarrow$$
 [ $\mathbf{F}_{S}^{a}$ ,  $\mathbf{C}$ ] =  $\pm \mathbf{C}_{1} \tau^{a} \mathbf{C}_{2}$ 

where  $\mathbf{C} = \mathbf{C}_1 \cdot \mathbf{C}_2$  and where S intersects C in  $C_1 \cap C_2$ .

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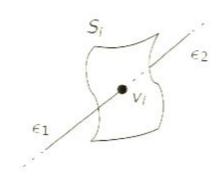
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**Quantization:** assume the operators  $F_S^a$ . C exist and satisfy

$$\Rightarrow$$
 [ $\mathbf{F}_{S}^{a}$ ,  $\mathbf{C}$ ] =  $\pm \mathbf{C}_{1} \tau^{a} \mathbf{C}_{2}$ 

where  $\mathbf{C} = \mathbf{C}_1 \cdot \mathbf{C}_2$  and where S intersects C in  $C_1 \cap C_2$ .

Consider curves restricted to a lattice Γ<sub>i</sub> and surfaces S<sub>i</sub> which intersects loops only at vertices.



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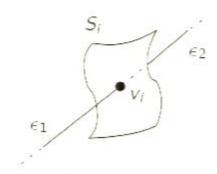
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where  $\mathbf{C} = \mathbf{C}_1 \cdot \mathbf{C}_2$  and where S intersects C in  $C_1 \cap C_2$ .

Consider curves restricted to a lattice Γ<sub>i</sub> and surfaces S<sub>i</sub> which intersects loops only at vertices.



Expand the twisted generators (ref  $\mathcal{E}_{j}^{i}$ )  $\mathfrak{E}_{j}^{i} = b_{jk}^{i} \tau^{k}$  and define the new operators

$$\mathbf{F}_{j}^{i} = \sum_{a} b_{jk}^{i} \, \mathbf{F}_{S_{i}}^{k}$$

### ▶ Then the operator

$$\Delta_{\Delta} = \frac{1}{n} \sum_{k} \bar{\mathcal{E}}_{k} \cdot (\mathbf{F}_{i}^{1} \pm \mathbf{F}_{i}^{2} \pm \mathbf{F}_{i}^{3} \pm \dots)$$

satisfy the algebra

$$[\Delta_{\Delta}, \mathbf{C}_j] = \frac{1}{n} \sum_{i} (\pm \mathbf{C}_j \mathfrak{E}_i^j) \cdot \bar{\mathcal{E}}_i$$

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# ► Then the operator

$$\Delta_{\Delta} = \frac{1}{n} \sum_{k} \bar{\mathcal{E}}_{k} \cdot (\mathbf{F}_{i}^{1} \pm \mathbf{F}_{i}^{2} \pm \mathbf{F}_{i}^{3} \pm \dots)$$

satisfy the algebra

$$[\Delta_{\Delta}, \mathbf{C}_j] = \frac{1}{n} \sum_{i} (\pm \mathbf{C}_j \mathfrak{E}_i^j) \cdot \bar{\mathcal{E}}_i$$

▶ This is exactly the commutator between the Dirac operator  $D_{\Delta}$  and a line segment  $\epsilon_i$ .

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In the limit we obtain a representation of the Poisson brackets of General Relativity. This representation is based on a more restrictive choice of graphs than is the representation used in LQG.

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- In the limit we obtain a representation of the Poisson brackets of General Relativity. This representation is based on a more restrictive choice of graphs than is the representation used in LQG.
- ▶ Recall that the Hilbert space H<sub>△</sub> corresponds to a partial solution to the (spatial) diffeomorphism constraint.

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- In the limit we obtain a representation of the Poisson brackets of General Relativity. This representation is based on a more restrictive choice of graphs than is the representation used in LQG.
- Necall that the Hilbert space H<sub>∆</sub> corresponds to a partial solution to the (spatial) diffeomorphism constraint.
  - Thus, we can think of our construction as a quantization scheme which deals first with the constraints (partially) and next with the actual quantization (not a Dirac-type quantization).

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In LQG the area operators play an important role

$$\mathbf{A}(S) = \sum_{n} \sqrt{\mathbf{F}_{S_{n}}^{i} \mathbf{F}_{S_{n}}^{j} \delta_{ij}} .$$

where 
$$S = \bigcup_n S_n$$
.

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$$\mathbf{A}(S) = \sum_{n} \sqrt{\mathbf{F}_{S_{n}}^{i} \mathbf{F}_{S_{n}}^{j} \delta_{ij}} .$$

where  $S = \bigcup_n S_n$ . We find that

$$D_{\Delta}^2 = \sum_{v} \dots \mathbf{A}^2(S_v) \sim \int_{\mathcal{M}} [d\text{Vol}] \mathbf{A}^2(x)$$

where  $\mathbf{A}(x)$  is a kind of area density operator.

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where  $\mathbf{A}(x)$  is a kind of area density operator.

The spectral action has the form of a Feynman integral

$$Tr \exp(-s\sqrt{(D_{\Delta})^2}) \sim \int_{\overline{\mathcal{A}}^{\Delta}} [d\nabla] \exp\left(-s\sqrt{(D_{\Delta})^2}\right) \delta_{\nabla}(\nabla)$$

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Pirsa: Obbishore  $D_{\Delta}^2$  plays the role of an action or an energy.

► The algebra B<sub>∆</sub> will, due to its noncommutativity, contain inner automorphisms of the form

$$\alpha_u(b) = ubu^* \cdot b \in \mathcal{B}_{\Delta}$$

where u is a unitary element of  $\mathcal{B}_{\Delta}$ .  $u^*u = uu^* = 1$ .

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Inner automorphisms generate fluctuations of the Dirac type operator  $D_{\Delta}$ 

$$\tilde{D}_{\Delta} = D_{\Delta} + W$$

where  $W=W^*$  has the general form

$$W = \sum_{ij} n_{ij} b_i [b_j, D_{\Delta}] , \quad b_i, b_j \in \mathcal{B}_{\Delta} , n_{ij} \in \mathbb{R}$$

W is, in the terminology of noncommutative geometry, a one-form.

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▶ Gravity has no inner automorhisms. Therefore, let us consider a sector of  $\mathcal{H}_{\Delta}$  which is not affected by these fluctuations. We consider the operator constraint

$$\mbox{Tr}\left( \mbox{$D_{\!\scriptscriptstyle \Delta}^2$} - \mbox{$\tilde{D}_{\!\scriptscriptstyle \Delta}^2$} \right) \Psi = 0 \; , \quad \Psi \in \mathcal{H}_{\!\scriptscriptstyle \Delta} \label{eq:power_power}$$

(Tr is the matrix trace) which implies

$$Tr(\{D_{\Delta}, W\} + \{W, W\})\Psi = 0.$$

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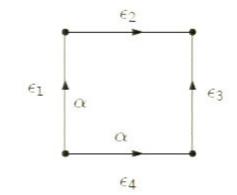
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# Let us find a *local*, *classical* interpretation of this expression. Consider a fluctuation

### $W = L_1[D_{\Delta}, L_2]$



$$L = \epsilon_1 \cdot \epsilon_2 \cdot \epsilon_3^{-1} \cdot \epsilon_4^{-1}$$

and write

$$D_{\!\scriptscriptstyle \Delta} \sim \sum \mathcal{E}_{\!\scriptscriptstyle j}^{i} E_{\!\scriptscriptstyle i}^{j} \; . \quad L \sim 1 + lpha^{2} F_{\mu 
u} {\it ds} + \mathcal{O}(lpha^{4})$$

where  $E_i^j$ ,  $F_{\mu\nu}^i$  are the classical fields.

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▶ Gravity has no inner automorhisms. Therefore, let us consider a sector of  $\mathcal{H}_{\Delta}$  which is not affected by these fluctuations. We consider the operator constraint

$$\mbox{Tr}\left(\textit{D}_{\Delta}^{2}-\tilde{\textit{D}}_{\Delta}^{2}\right)\Psi=0\;,\quad\Psi\in\mathcal{H}_{\Delta}$$

(Tr is the matrix trace) which implies

$$Tr(\{D_{\Delta}, W\} + \{W, W\})\Psi = 0.$$

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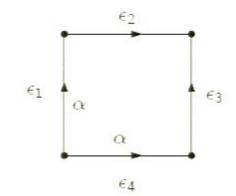
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$$W = L_1[D_{\Delta}, L_2]$$



$$L = \epsilon_1 \cdot \epsilon_2 \cdot \epsilon_3^{-1} \cdot \epsilon_4^{-1}$$

and write

$$D_{\!\scriptscriptstyle \Delta} \sim \sum \mathcal{E}_{\!\scriptscriptstyle j}^{i} E_{\!\scriptscriptstyle i}^{j} \; . \quad L \sim 1 + \alpha^{2} F_{\mu 
u} {\it ds} + \mathcal{O}(\alpha^{4})$$

where  $E_i^j$ ,  $F_{\mu\nu}^i$  are the classical fields.

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Then (heuristically)

$$Tr\{D_{\Delta}, W\} + \{W, W\}\Big|_{\text{one vertex}} \stackrel{\text{classical}}{\leadsto} \epsilon^{ij}_{\ k} F_{\mu\nu}^k E_j^\mu E_k^\nu + \mathcal{O}\left(\alpha^2\right)$$

(we set 
$$\alpha = a_n$$
 for  $n - \infty$ )

▶ This has the form of the Hamilton constraint of GR.

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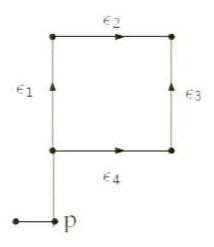
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Tonnes Distance Formula

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# ▶ Consider next the sum over all small loops of area $\alpha^2$

$$L = p \cdot \epsilon_1 \cdot \epsilon_2 \cdot \epsilon_3^{-1} \cdot \epsilon_4^{-1} \cdot p^{-1}$$



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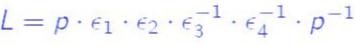
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 $\epsilon_4$ 

#### The Hamiltonian

### Consider next the sum over all small loops of area $\alpha^2$

$$L = p \cdot \epsilon_1 \cdot \epsilon_2 \cdot \epsilon_3^{-1} \cdot \epsilon_4^{-1} \cdot p^{-1}$$



Then

$$Tr\Big(\{D_{\Delta},W\}+\{W,W\}\Big)\stackrel{\text{classical}}{\leadsto}\int_{M}d^{3}xN\epsilon^{ij}_{\phantom{ij}k}F_{\mu\nu i}^{\phantom{ij}k}E_{j}^{\phantom{ji}\mu}E_{k}^{\phantom{k}\nu}+\mathcal{O}(\alpha^{2})$$

€1

where N(x) is the weighting of each small loop.

### Consider next the sum over all small loops of area \(\alpha^2\)

$$L = p \cdot \epsilon_1 \cdot \epsilon_2 \cdot \epsilon_3^{-1} \cdot \epsilon_4^{-1} \cdot p^{-1}$$

► Then

$$Tr\Big(\{D_{\Delta},W\}+\{W,W\}\Big)\stackrel{\text{classical}}{\leadsto}\int_{M}d^{3}xN\epsilon^{ij}{}_{k}F_{\mu\nu i}^{k}E_{j}^{\mu}E_{k}^{\nu}+\mathcal{O}(\alpha^{2})$$

E1

where N(x) is the weighting of each small loop.

This has the form of the Hamiltonian of GR. N plays the role of the lapse field.

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Warning: This is not rigorous.

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Warning: This is not rigorous.

A) Additional structure is needed for this to work (Real structure, extended Dirac operator, extended action of the algebra...)

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- Warning: This is not rigorous.
  - A) Additional structure is needed for this to work (Real structure, extended Dirac operator, extended action of the algebra...)
  - B) A rigorous formulation of a classical limit ...
- However, the general structure is clear.

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### Remarks:

▶ The expression  $D_{\Delta}^2 - \tilde{D}_{\Delta}^2$  for a Hamilton constraint is canonical.

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- ▶ The expression  $D_{\Delta}^2 \tilde{D}_{\Delta}^2$  for a Hamilton constraint is canonical.
- The expression is free of ordering ambiguities and requires no regularization.

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- ▶ The expression  $D_{\Delta}^2 \tilde{D}_{\Delta}^2$  for a Hamilton constraint is canonical.
- The expression is free of ordering ambiguities and requires no regularization.
- The 'full' system has inner automorphisms = additional degrees of freedom.
  - It is only when one 'turns off' the inner automorphisms that we obtain the structure of the Hamilton constraint

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- We can think of either

$$D_{\Delta}^2 - \tilde{D}_{\Delta}^2$$
 or  $\tilde{D}_{\Delta}^2$ 

as the Hamiltonian. The advantage of the latter is that it is positive.

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Time evolution operator

$$\mathcal{U}(t) = \exp\left(\mathrm{it}\tilde{\mathrm{D}}_{\scriptscriptstyle\Delta}^2\right)$$

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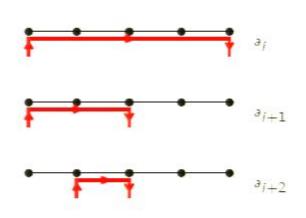
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# The sequence $\{a_i\}$

► The role of the parameters {a<sub>i</sub>} is to set a scale. A 'coarse grained' loop corresponds to small a's. A 'refined' loop corresponds to large a's.



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Natural choice:

$$a_n = 2^{-n}$$

However, exactly here  $D_{\Delta}$  fails to be spectral — infinities

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Natural choice:

$$a_n = 2^{-n}$$

However, exactly here  $D_{\Delta}$  fails to be spectral — infinities

Try instead

$$a_n = (2 + \epsilon)^{-n}$$

and take the limit  $\epsilon \to 0$ . regularization.

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## The Classical Limit

▶ The Goal is to obtain a classical limit characterized by an almost commutative algebra

$$B = C^{\infty}(\mathcal{M}) \otimes B_F$$

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The Classical Limit

## Ionnes Distance Formula

▶ Connes distance formula: Given a spectral triple  $(A, D, \mathcal{H})$  over a manifold  $\mathcal{M}$  the distance formula reads

$$d(\xi_x, \xi_y) = \sup_{a \in \mathcal{A}} \left\{ |\xi_x(a) - \xi_y(a)| \left| |[D, a]| \le 1 \right\} \right.$$

where  $\xi_x, \xi_y$  are homomorphisms  $\mathcal{A} - \mathbb{C}$ . This can be generalized to noncommutative spaces/algebras.

- Question: What about Connes distance formula for the spectral triple (B<sub>Δ</sub>, D<sub>Δ</sub>, H<sub>Δ</sub>)? A distance between field configurations? Yes.
- If two geometries differ on a large scale, then the distance  $d(\nabla_1, \nabla_2)$  between their Levi-Civita connections will be 'large' (difference weighted with small a's large distance)
- If they differ only on short scales, then the distance will be 'small' (difference weighted with large a's small distance).

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• We have constructed a semi-finite spectral triple  $(\mathcal{B}_{\Delta}, D_{\Delta}, \mathcal{H}_{\Delta})$  where:

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- We have constructed a semi-finite spectral triple  $(\mathcal{B}_{\Delta}, D_{\Delta}, \mathcal{H}_{\Delta})$  where:
  - $\blacktriangleright$   $\mathcal{B}_{\Delta}$  is an algebra of (holonomy) loops.

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- We have constructed a semi-finite spectral triple  $(\mathcal{B}_{\Delta}, D_{\Delta}, \mathcal{H}_{\Delta})$  where:
  - B<sub>△</sub> is an algebra of (holonomy) loops.
  - ▶ D<sub>△</sub> resembles a global functional derivation operator.

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  - $\mathcal{H}_{\Delta}$  corresponds (up to a descrete symmetry group) to the Hilbert space of (spatial) diffeomorphism invariant states.

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  - ▶  $\mathcal{H}_{\Delta}$  corresponds (up to a descrete symmetry group) to the Hilbert space of (spatial) diffeomorphism invariant states.
  - The interaction between B<sub>△</sub> and D<sub>△</sub> encodes the Poisson structure of GR.
  - ▶  $D_{\Delta}$  is gauge invariant (Gauss constraint).

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The construction is based on a countable system of embedded graphs (lattices, simplicial complexes). The construction is essentially combinatorial.

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- ► The structure of the Hamiltonian of GR emerges from a condition which restricts the triple to a sector where inner automorphisms play no role (turning off interactions vacuum).

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- ▶ The triple  $(\mathcal{B}_{\Delta}, D_{\Delta}, \mathcal{H}_{\Delta})$  depends on a set  $\{a_i\}$  of scaling parameters. This resembles a regularization scheme.
- Connes distance formula: distances between "geometries".

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- Exact formulation of the Hamiltonian.
- The spectral action. It resembles a Feynman integral what exactly is it?
- Computations of the inner fluctuations. What kind of degrees of freedom do they represent?
- Formulation of a classical limit.
- Noncompact structure group?
- **.**..

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