Title: Setting the boundary free in AdS/CFT

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Abstract: TBA

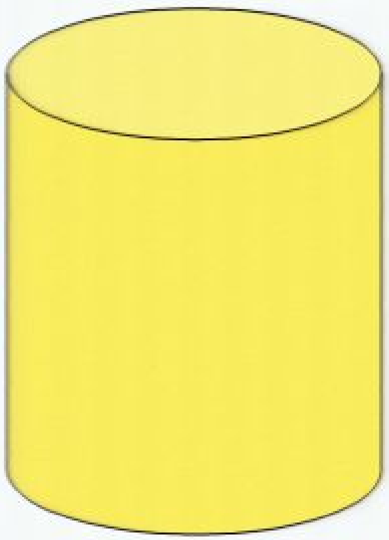
A gravity/string duality: Setting the boundary free in AdS/CFT

Don Marolf 4/30/08
UCSB

Work in progress
w/ Geoffrey Compere

Overview

M asymptotically AdS_{d+1}



- Fefferman-Graham:

$$g_{ij} = g^{(0)}_{ij}\, r^2 + g^{(1)}_{ij}\, r + g^{(2)}_{ij} + g^{(3)}_{ij} r^{-1} + \dots$$

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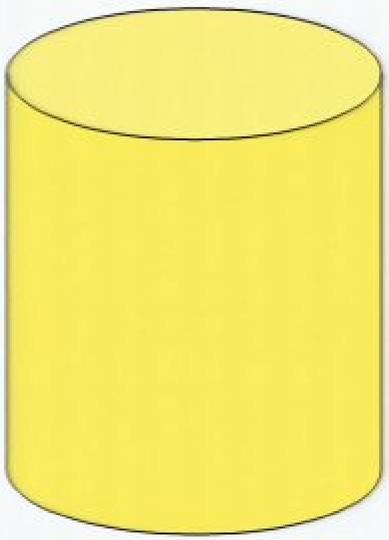

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- Usual boundary conditions fix $g^{(0)}_{ij}$, but other BCs are allowed!

- $g^{(0)}_{ij}$ can fluctuate: gravity/string duality

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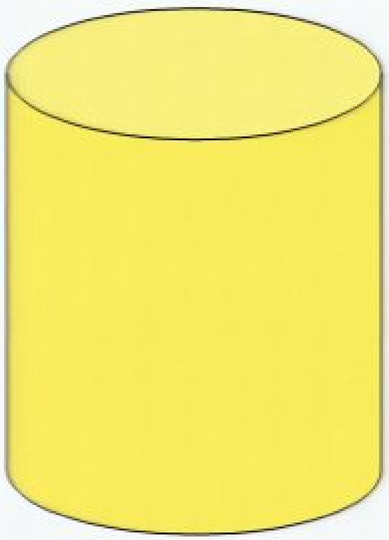

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- $g^{(0)}_{ij}$ can fluctuate: gravity/string duality
- Boundary dual:

  Usual CFT coupled to gravity.

  Neumann theory is UV complete for odd d.

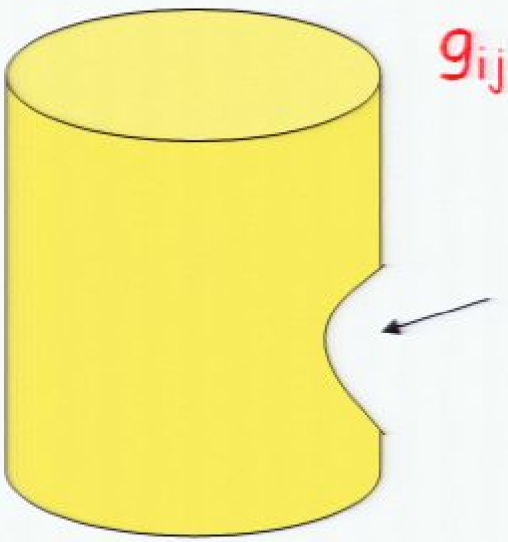
Work in progress: What can we learn?

Outline

1. AdS/CFT basics: A review
2. Setting the boundary free
3. Checks and Observations
4. Other dimensions

1. AdS/CFT basics: A review

AdS Boundary Conditions ⟺ CFT Sources...



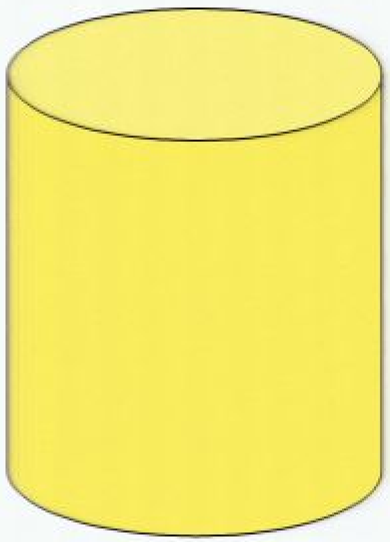
$$g_{ij} = g^{(0)}{}_{ij}\, r^2 + g^{(1)}{}_{ij}\, r + g^{(2)}{}_{ij} + g^{(3)}{}_{ij}\, r^{-1} + \ldots$$

$$\delta g^{(0)}{}_{ij} \Longleftrightarrow \delta S_{CFT} = \int T_{ij}\, \delta g^{(0)}{}_{ij} \sqrt{g^{(0)}}$$

... or other deformations of the CFT.

1. AdS/CFT basics: A review

AdS_{d+1} Boundary Conditions $\Longleftrightarrow$ CFT deformations



Scalar field toy models

$$\nabla^2\phi - m^2\phi = 0$$

$$\phi = a(x)\, r^{-\lambda_-} + b(x)\, r^{-\lambda_+} + \ldots$$

$$\lambda_{\pm} = d/2 \pm \sqrt{(d/2)^2 + m^2}$$

1. AdS/CFT basics: A review

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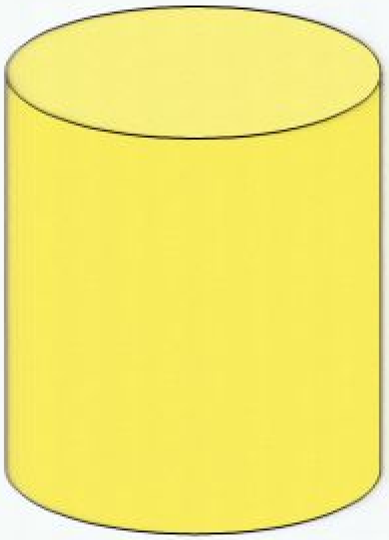

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Case 1) $m^2 \geq -(d/2)^2 + 1$

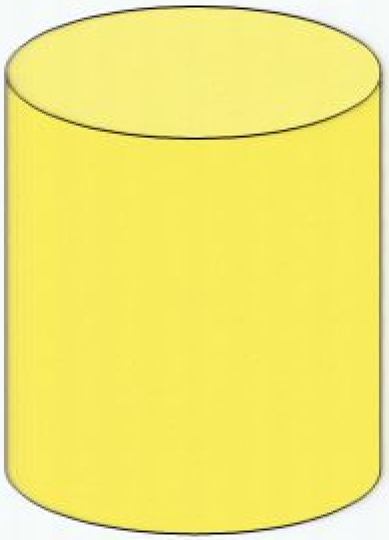
Only the λ_+ mode is normalizeable.

$a(x)$ must be fixed as a boundary conditions, with $b(x)$ free.

$a(x)$ ⟺ a CFT source

1. AdS/CFT basics: A review

AdS_{d+1} Boundary Conditions ⟺ CFT deformations



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Case 2) $-(d/2)^2 + 1 > m^2 \geq -(d/2)^2$

Both modes are normalizeable.

Any combination can be fixed as a boundary condition.

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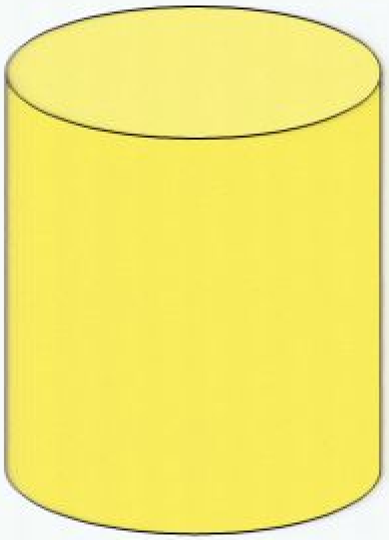

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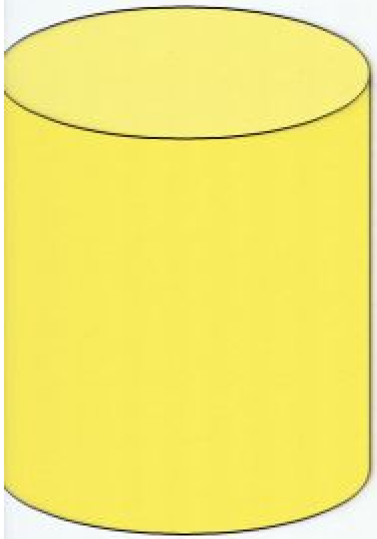
Case 3) $-(d/2)^2 > m^2$ Unstable, BF bound

1. AdS/CFT basics: A review

Case 2) BF range — Breitenlohner and Freedman, Witten, Sever, Shomer, Berkooz, ...



Both modes normalizeable,
but still need a BC to conserve probability (Unitarity)

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KG flux through boundary = $\int a^1_{prop}(x)\, b^2_{prop}(x) \sqrt{g^{(0)}}$ - $(1 \leftrightarrow 2)$

Options for BCS

1. AdS/CFT basics: A review

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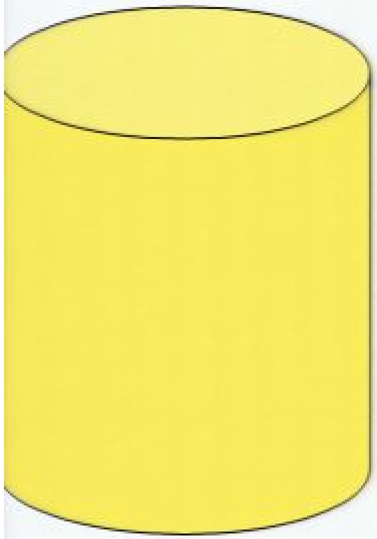

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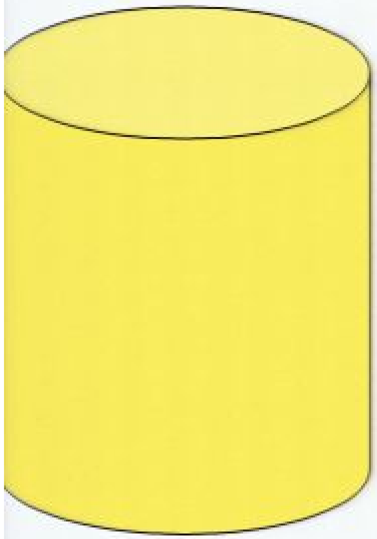

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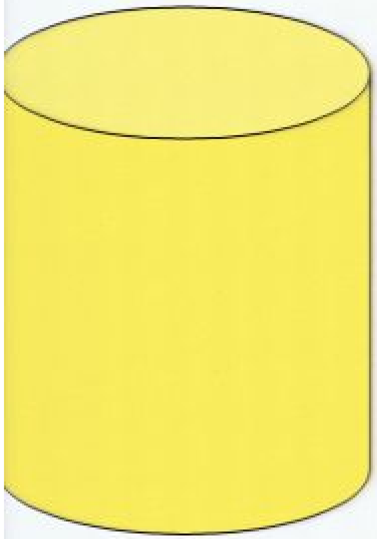

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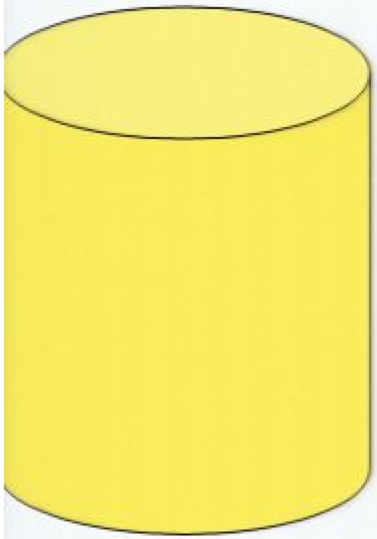
- Mixed BCs: Fix $a(x) = \delta W/\delta b$ for some W[b].
  $S_{CFT} = S_{CFT}^{Neumann} + c^{-1} W$ for $c = (\lambda_+ - \lambda_-)$.
  Typically not conformal.
  RG flow from Neumann BC in UV to Dirichlet in IR

1. AdS/CFT basics: A review

What happens for higher spin fields?
"Known" results – may need to be revisited



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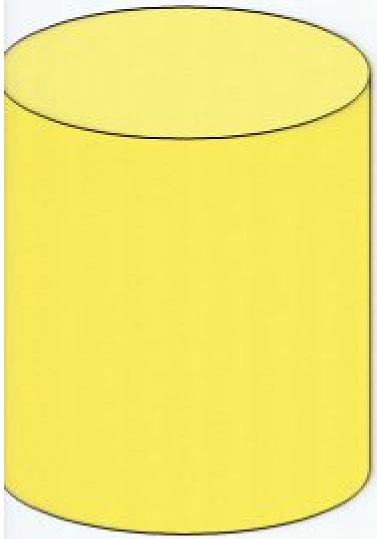

- Spin $\frac{1}{2}$: For AdS_4 w/ m=0, all modes normalizeable.
  Breitenlohner & Freedman (Maximal SUGRA)
  Similar for $m < m_{crit}$ for all d. A. Amsel in progress

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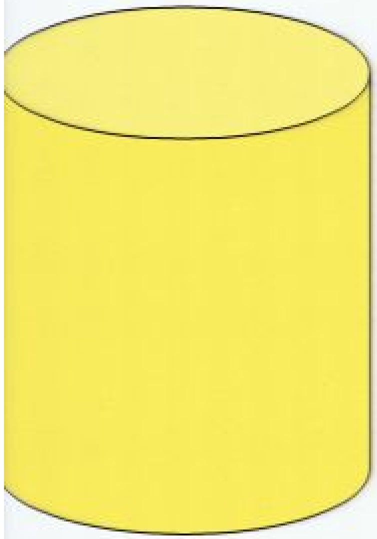

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- Spin 1: For AdS_4 w/ m=0, all modes normalizeable.
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  Ishibashi & Wald: Not true in higher dimensions!

  AdS/CFT work by Witten, Leigh & Petkou,
  Yee, DM & Ross

$$A = \underline{\underline{A}}^{(0)} r^3 + \ldots A^{(d)}$$

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Gauge Field ↓ Background

↓ Current

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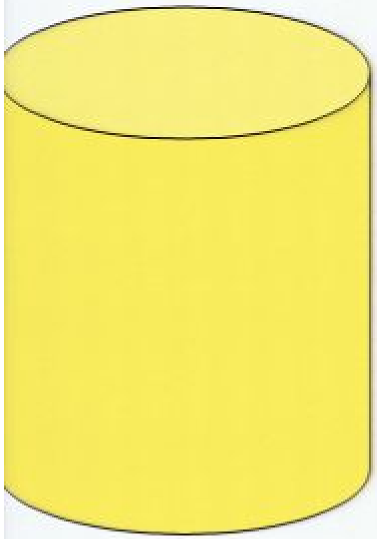

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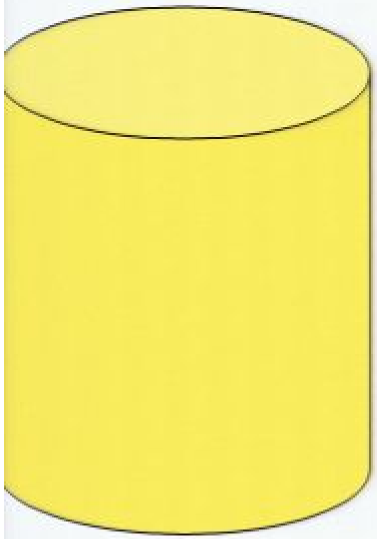
- Spin 2: BF conjectured all modes normalizeable
  for AdS_4 w/ m=0. (forgotten?)

  Ishibashi and Wald: Lin. EOM allow multiple
  self-adjoint extensions for AdS_4.

  Brief comments by Leigh & Petkou, DM & Ross

2. Setting the boundary free



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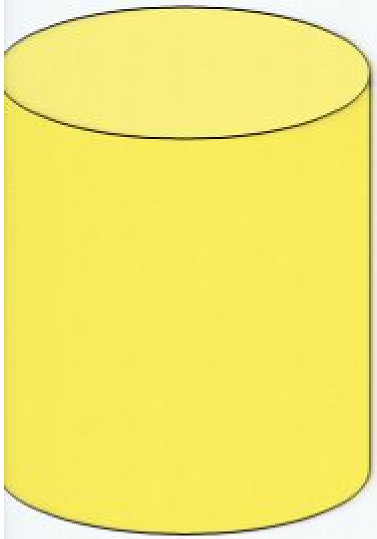
2. Setting the boundary free



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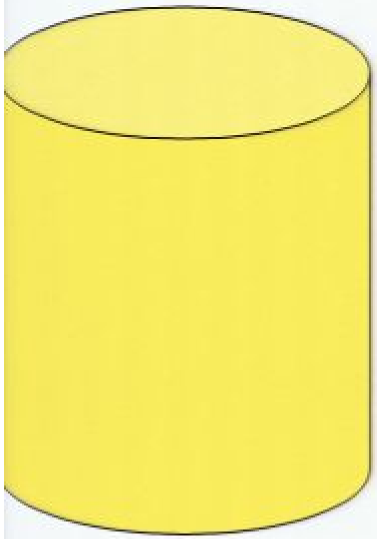
2. Setting the boundary free

Recall how to make an induced gravity theory:



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"Induced gravity;" no explicit kinetic terms for metric

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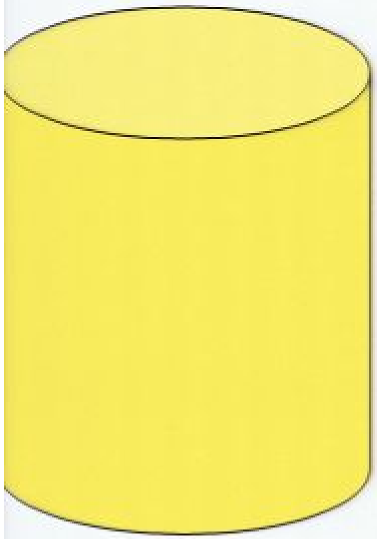

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Let $g^{(0)}{}_{ij} = \sigma\, \overline{g^{(0)}}{}_{ij}$.

Note for odd d: no trace anomaly;

integral independent of σ.

For odd d, Weyl transformations are gauge ⟹ Induced conformal gravity.

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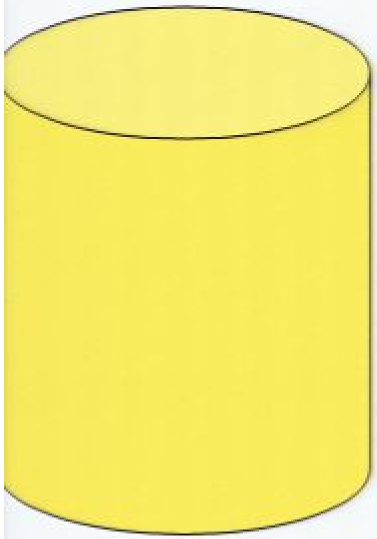

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UV complete theory for odd d?

(What could go wrong?)

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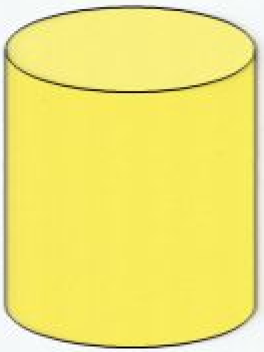
AdS_{d+1} $g_{ij} = g^{(0)}_{ij}\, r^2 + g^{(1)}_{ij}\, r + g^{(2)}_{ij} + g^{(3)}_{ij} r^{-1} + h_{ij}\, r^{2-d} \log r \ldots$



Let's try the same operation in the bulk.
(c.f. Witten for spin 1)

$$Z^{Dir}_{bulk}\,[g^{(0)}] = \int_{g \rightarrow g^{(0)}} D\phi Dg\; e^{iS_{bulk}}$$

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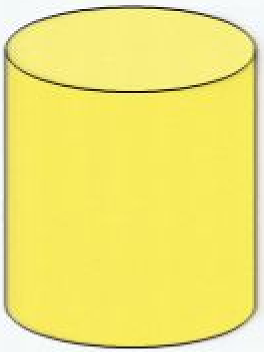

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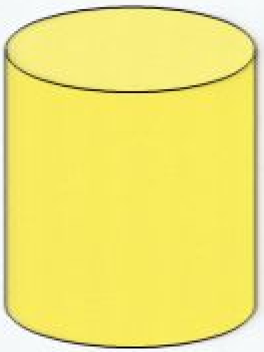

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$$16\pi G\, S_{bulk} = \int_{bulk} (R + d(d-1)/\ell^2)\sqrt{g} + \int_{boundary} (2K - 2(d-2)/\ell - {}^dR/(d-2)\ell + \ldots)\sqrt{{}^d g}$$

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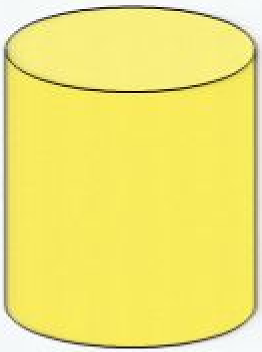

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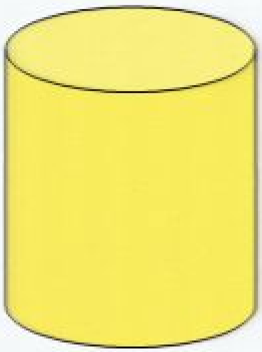

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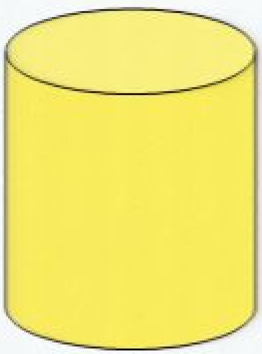

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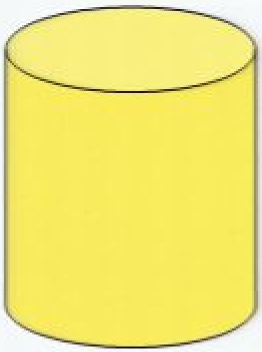

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bulk boundary

Semi-classical Limit

$$\delta S_{\text{bulk}} = \text{Eq. of Motion} + 2\int T^{ij}\, \delta g_{ij}{}^{(0)} \sqrt{g^{(0)}}$$

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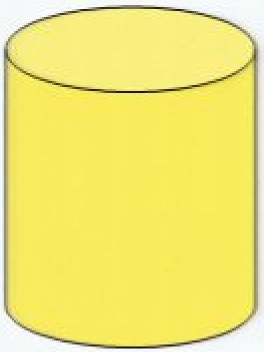

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Semi-classical Limit

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$g_{ij}{}^{(0)}$ free $\Longrightarrow$ $0 = T_{ij}$

("Neumann" theory, conformal for odd d)

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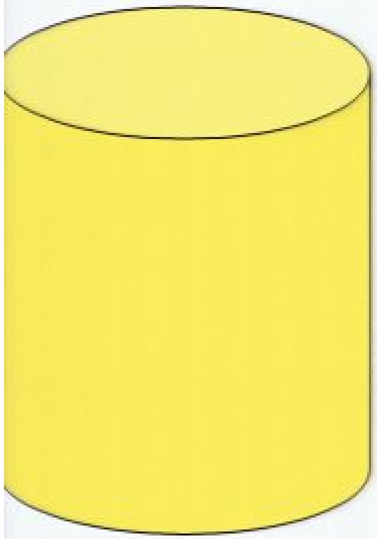

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$$Z_{ind\ grav} = \int Dg^{(0)}\, Z_{CFT}\,[g^{(0)}]$$

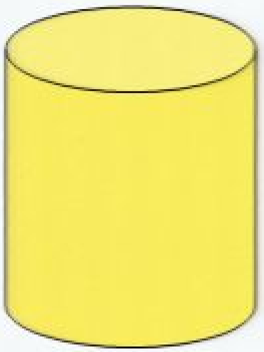
"Induced gravity;" no explicit kinetic terms for metric

Let $g^{(0)}{}_{ij} = \sigma\, \overline{g^{(0)}}{}_{ij}$.
Note for odd d: no trace anomaly;
integral independent of σ.

For odd d, Weyl transformations are gauge $\Longrightarrow$ Induced conformal gravity.

2. Setting the boundary free

AdS_{d+1} $g_{ij} = g^{(0)}{}_{ij}\, r^2 + g^{(1)}{}_{ij}\, r + g^{(2)}{}_{ij} + g^{(3)}{}_{ij} r^{-1} + h_{ij}\, r^{2-d} \log r \ldots$



Let's try the same operation in the bulk.
(c.f. Witten for spin 1)

$$Z^{Neu}_{bulk} = \int Dg^{(0)}\ Z^{Dir}_{bulk}\,[g^{(0)}] = \int D\phi Dg\ e^{iS_{bulk}}$$

$$16\pi G\ S_{bulk} = \int_{bulk} (R + d(d-1)/\ell^2)\sqrt{g} + \int_{boundary} (2K - 2(d-2)/\ell - {}^dR/(d-2)\ell + \ldots)\sqrt{{}^dg}$$

Semi-classical Limit

$$\delta S_{bulk} = \text{Eq. of Motion} + 2\int T^{ij}\,\delta g_{ij}{}^{(0)}\sqrt{g^{(0)}}$$

$$g_{ij}{}^{(0)}\ \text{free} \quad \Longrightarrow \quad 0 = T_{ij}$$

("Neumann" theory, conformal for odd d)

2. Setting the boundary free

Normalizeability?

$$g_{ij} = g^{(0)}_{ij}\, r^2 + g^{(1)}_{ij}\, r + g^{(2)}_{ij} + g^{(3)}_{ij} r^{-1} + h_{ij}\, r^{2-d} \log r \ldots$$

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Recall: $g_{ab} = \overline{g_{ab}} + h_{ab}$

Field algebra $[h_{ab}(x), \pi^{cd}(y)] = \delta(x,y)$

⇓

a, $a^\dagger$

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Fock space and norm.

$|a^\dagger|0\rangle|^2 = \langle 0|a\, a^\dagger|0\rangle = [a, a^\dagger]$

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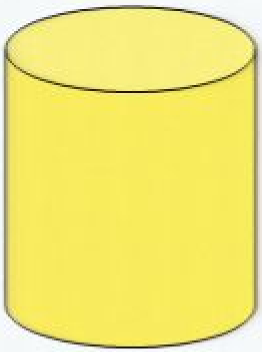
Deformations of the theory?



Compute conformal dimensions

$$g_{ij} = g^{(0)}_{ij}\, r^2 + g^{(1)}_{ij}\, r + g^{(2)}_{ij} + g^{(3)}_{ij} r^{-1} + h_{ij}\, r^{2-d} \log r \ldots$$

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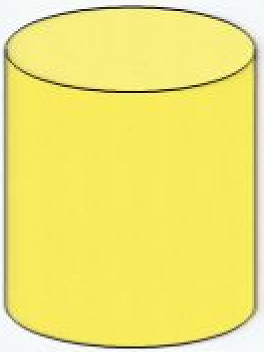

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Relevant deformations have $\leq$ d derivatives.

E.g., for d =3, we can add

$$S_{Bndy\ Grav} = \frac{\Lambda_{bndy}}{8\pi G_{bndy}} \int d^3x \sqrt{g^{(0)}} + \frac{1}{16\pi G_{bndy}} \int d^3x\, R^{(0)} \sqrt{g^{(0)}} + k \int \omega \wedge d\omega + \omega^3$$

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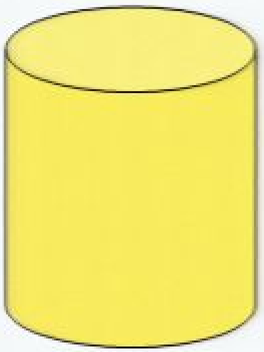

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$\delta S_{Bndy\ grav} / \delta g^{(0)ij} = T_{ij}$

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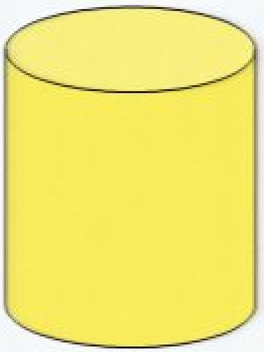

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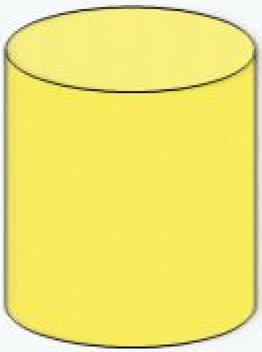
$S_{Bndy\ grav}$ also makes (finite) contributions to the symplectic structure $\Longrightarrow$ conservation.

3. Checks and ...



Consistency Checks

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Global Coordinates ⟹ $S^2 \times R$ Bndy

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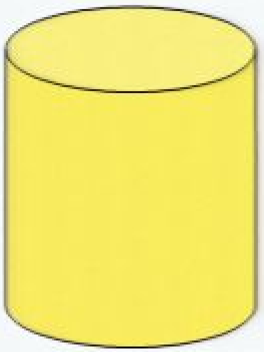

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Suppose ξ^a induces a bndy diffeo:

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Find $\omega(\delta g_1, \mathcal{L}_\xi g_2) = 0$ and $Q[\xi] = 0$.

Gauge transformations

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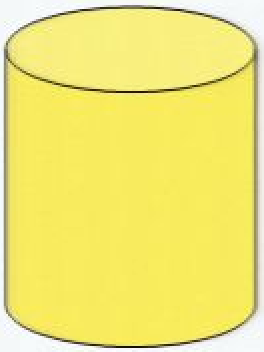

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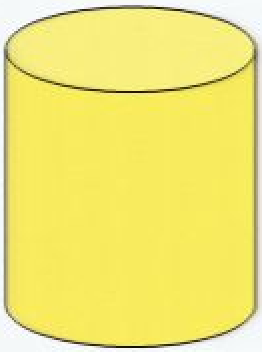
Gauge transformations

"Neumann BCs" for odd d with Weyl-inv $S_{\text{Bndy grav}}$

Also Weyl transformations: $\delta g^{(0)}_{ij} = \sigma(x)\, g^{(0)}_{ij}$

3. ... Observations



Leading 1/N (Classical bulk gravity):

At least for odd d,
pure induced-gravity is ghost- and tachyon-free.

(Even d $\geq$ 4 does have ghosts and tachyons)

Deformed theory? Consider d=3: $\Lambda_{bndy} = 0$, $k_{CS} = 0$, G_{bndy},

UV complete theory for all values (For large N.)

(Euclidean propagator)

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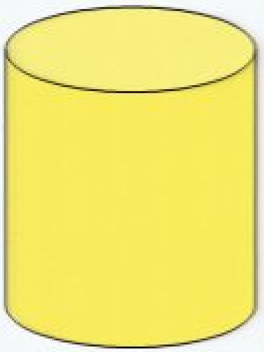

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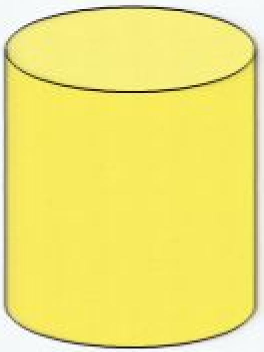

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TT tachyon for $G_4/G_B > 0$ ⟹ ghost/normal pair

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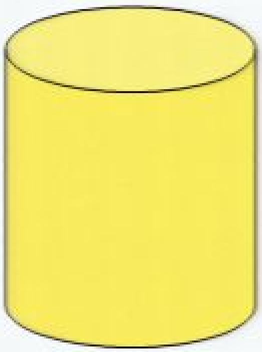

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Similar to topologically massive gravity.

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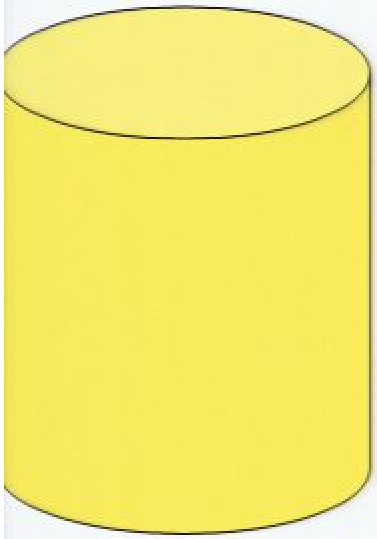
Complex instability.

Summary



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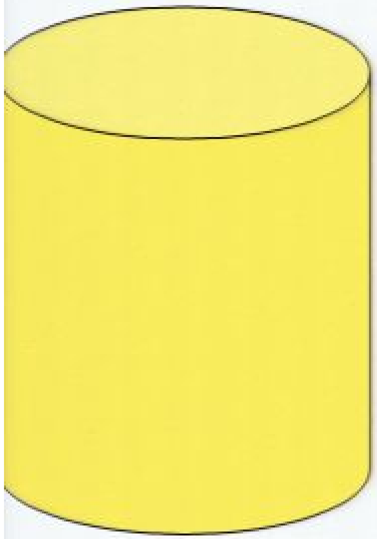

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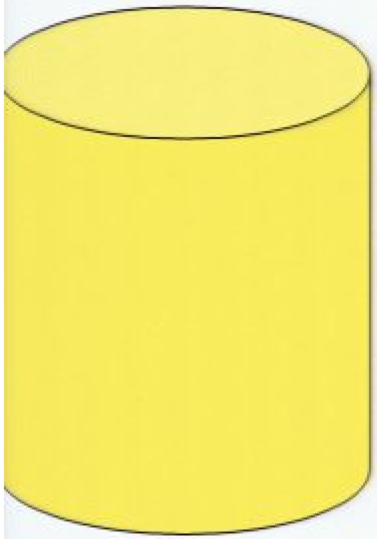

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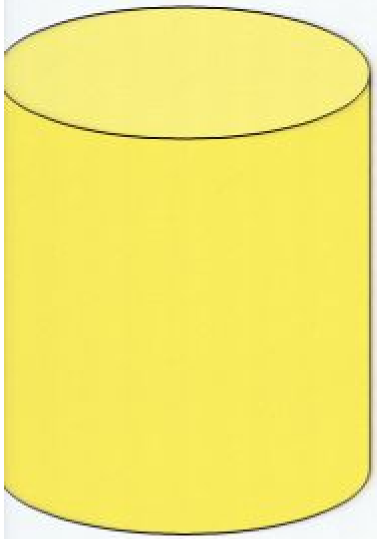

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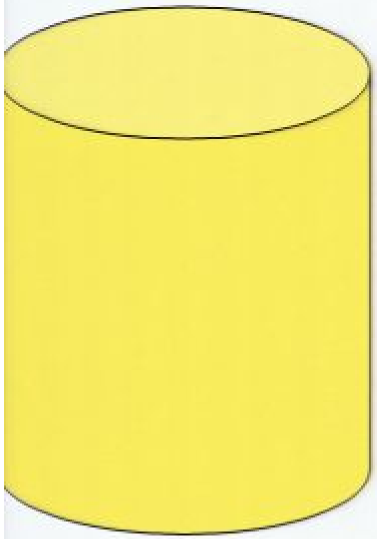

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Can ghosts/tachyons be removed?
(Condensed or tuned away?)

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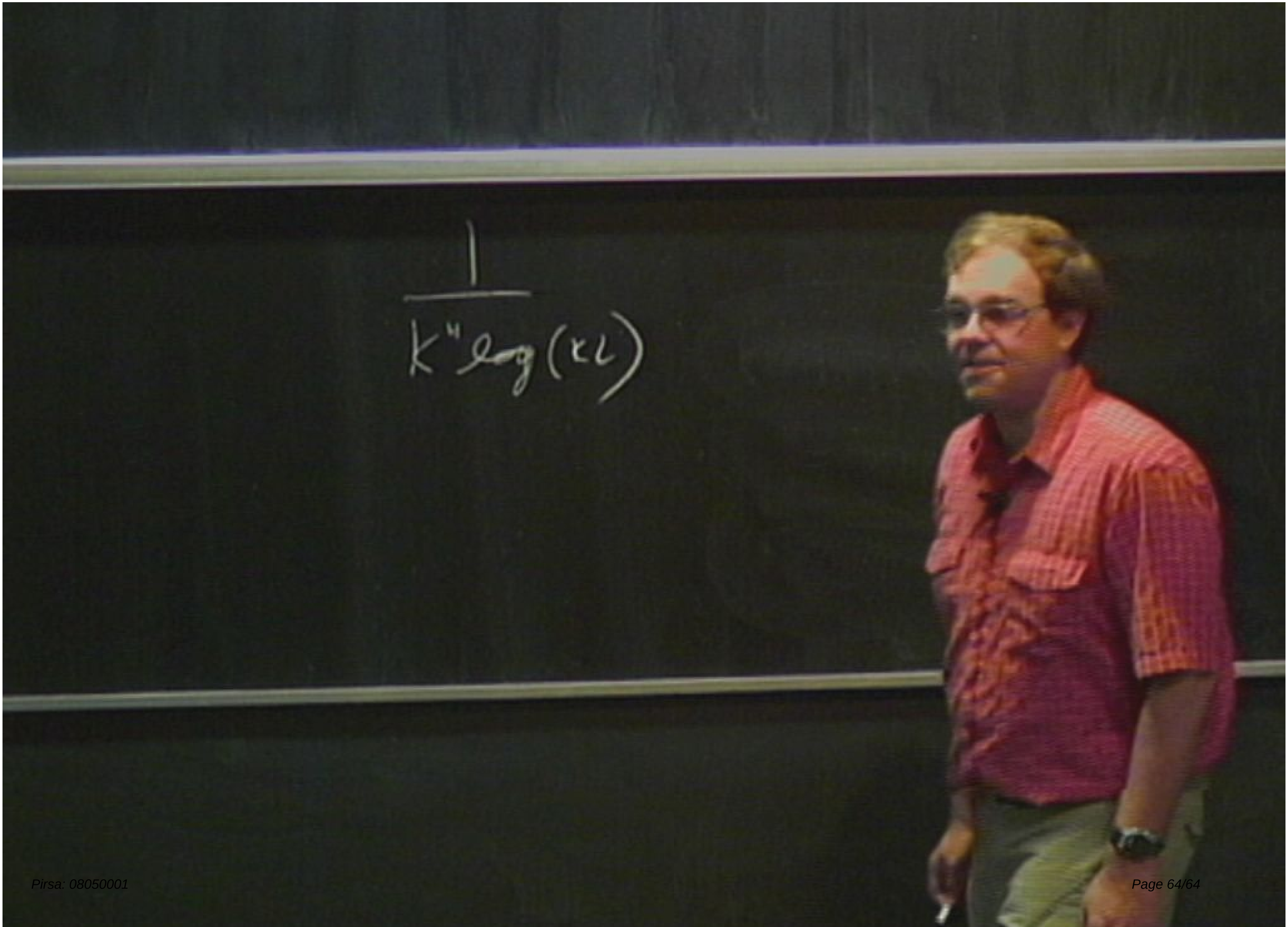
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