Title: Quantum Information and Graph Theory - Introduction

Date: Apr 28, 2008 09:20 AM

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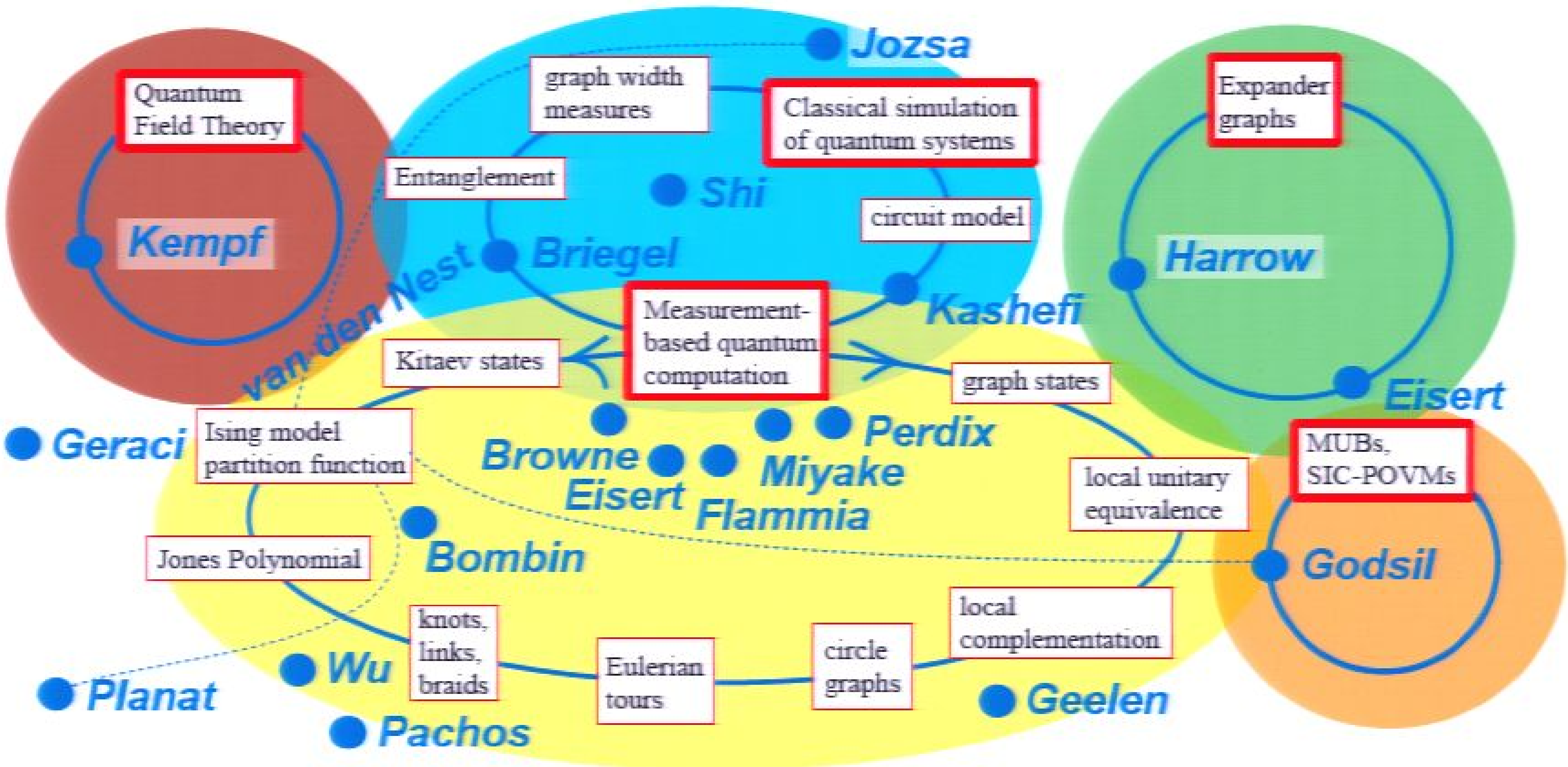
Abstract:

Overview



Quantum Information and Graph Theory: Emerging connections.

Perimeter Institute Waterloo, April 28 - May 2, 2008.

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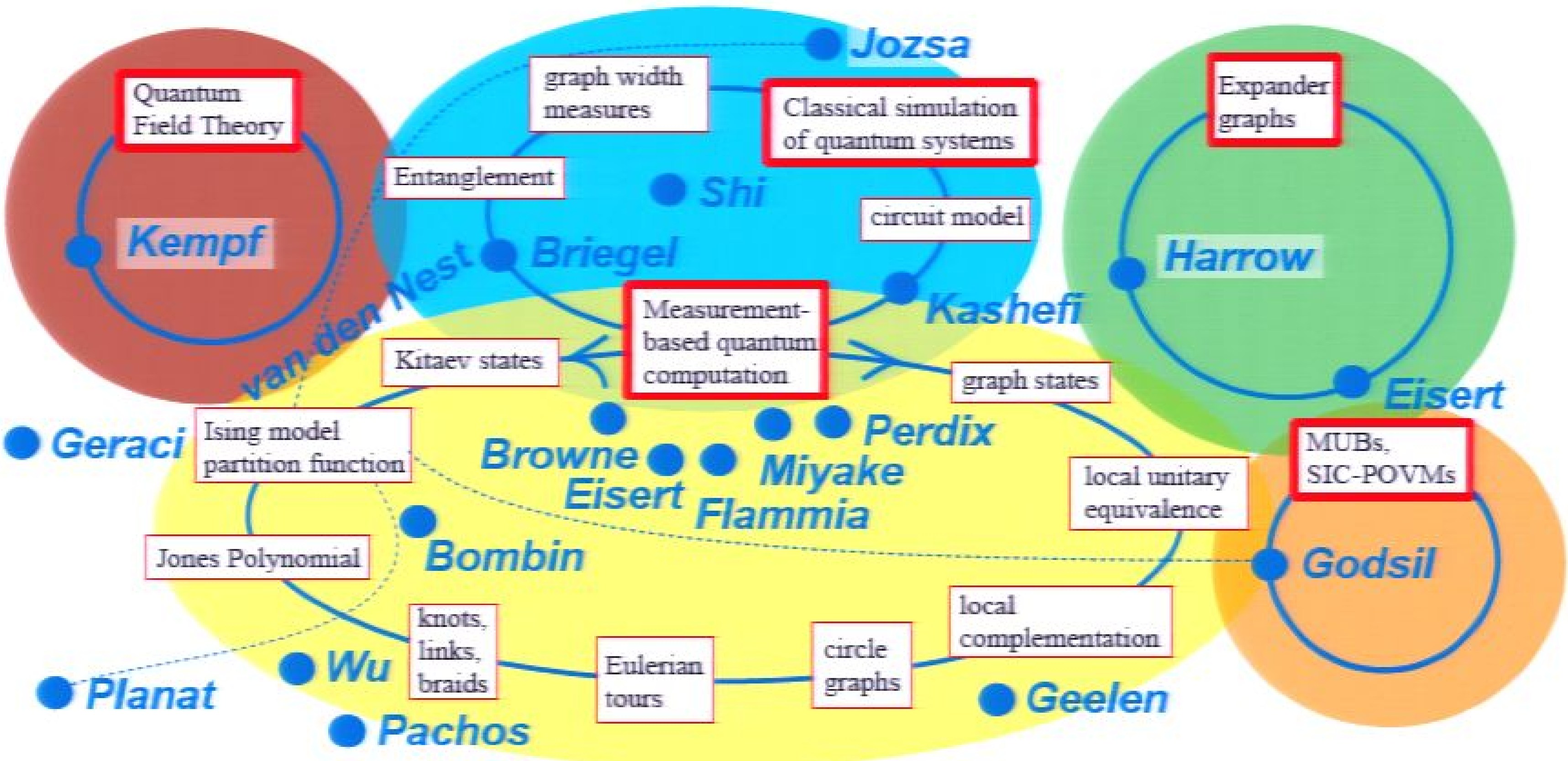

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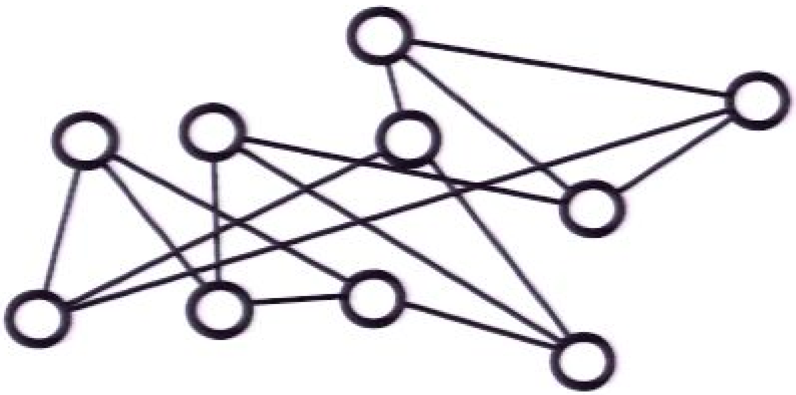
Expander graphs



Random graph of degree 3

Rumors spread exponentially fast in expander graphs.

- Many applications in the theory of classical computation, e.g.:
  - less randomness in probabilistic algorithms.
  - error-correcting codes.

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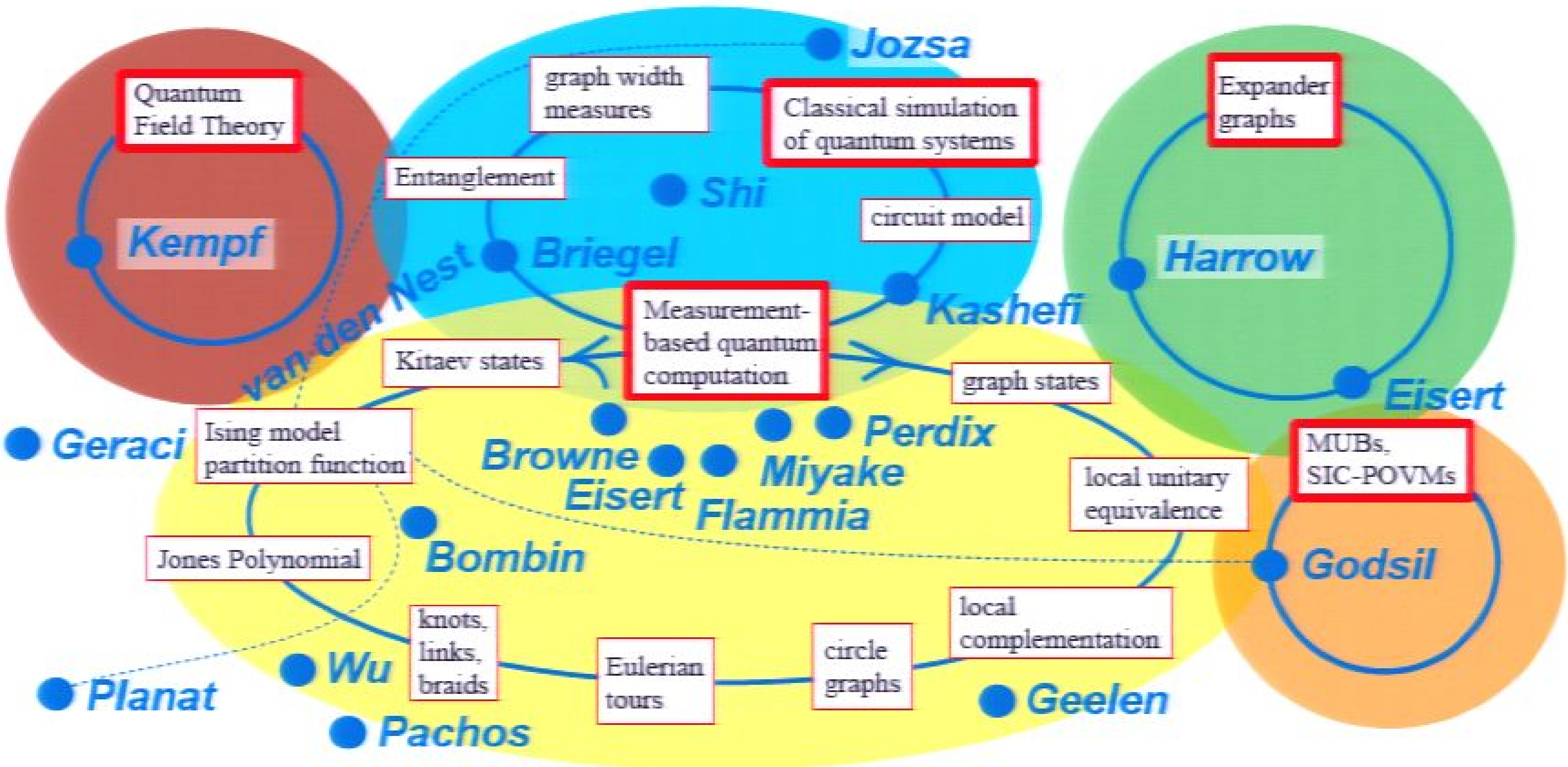

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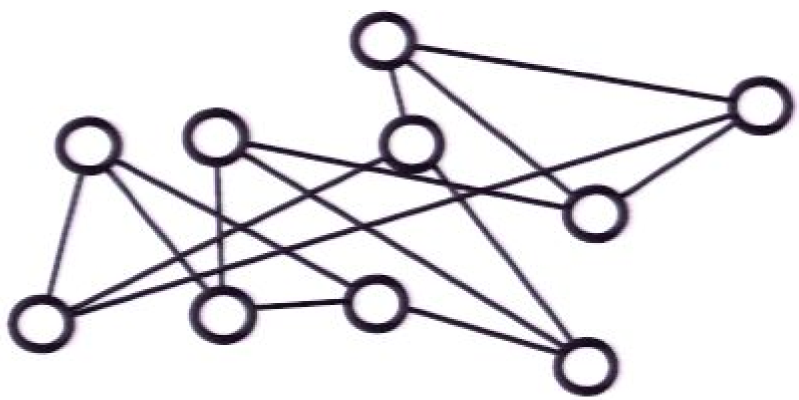

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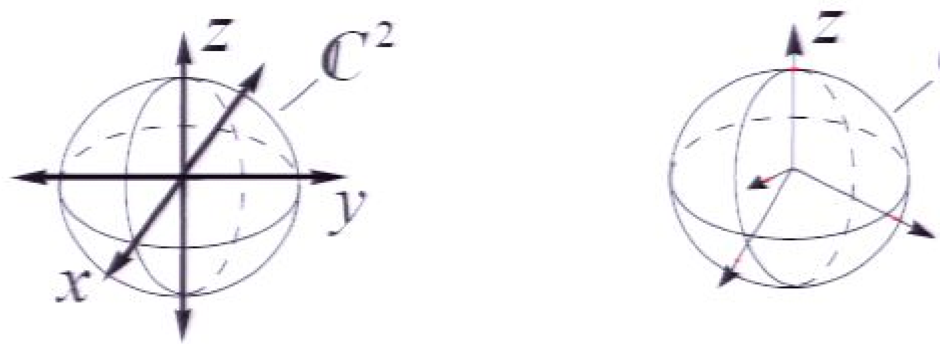
Sets of equiangular lines - MUBs and SIC-POVMs

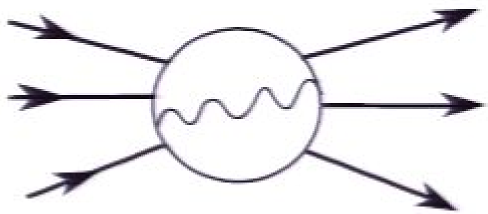


Appear in the contexts of quantum measurement and quantum cryptography.

- Equiangular lines in real space $\Leftrightarrow$ combinatorial problems
- Equiangular lines in complex space $\Leftrightarrow$?!

Godsil

Generating functionals in QFT



In quantum field theory, Feynman graphs encode amplitudes of scattering events of physical particles.

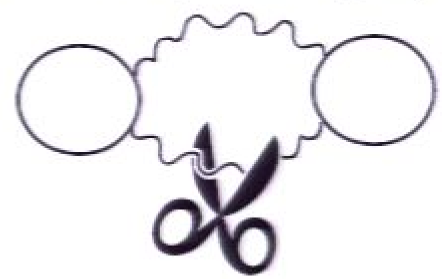
Energy $E[J]$ $\overset{\mathcal{L}}{\longleftrightarrow}$ Effective action $\Gamma[\Phi]$

Energy $E[J]$

- Yields correlation functions.
- Contains all connected Feynman graphs.

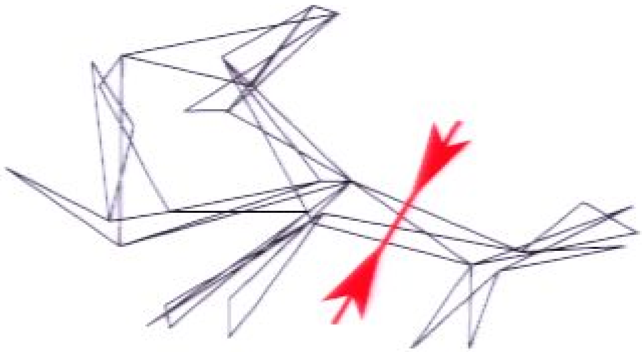
Effective action $\Gamma[\Phi]$

- Yields EoM for EV Φ.
- Contains all 1-particle irreducible Feynman graphs.



Graph width measures

... appear in the classical simulation of quantum systems.



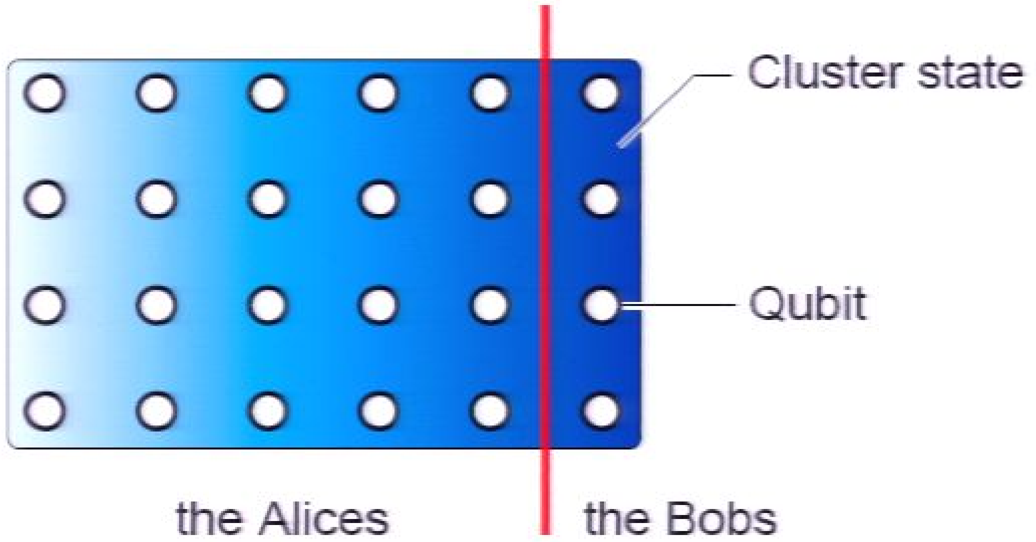
Treewidth measures the deviation of a graph from tree-ness.

- Some NP-hard problems in graph theory become efficiently solvable on graphs of bounded width (treewidth, rankwidth).
- Classical simulation of certain quantum systems is a case.

Shi • Van den Nest; Q-sim: Geraci • Bombin • Jozsa

Measurement-based quantum computation

The one-way quantum computer ($QC_{\mathcal{C}}$):



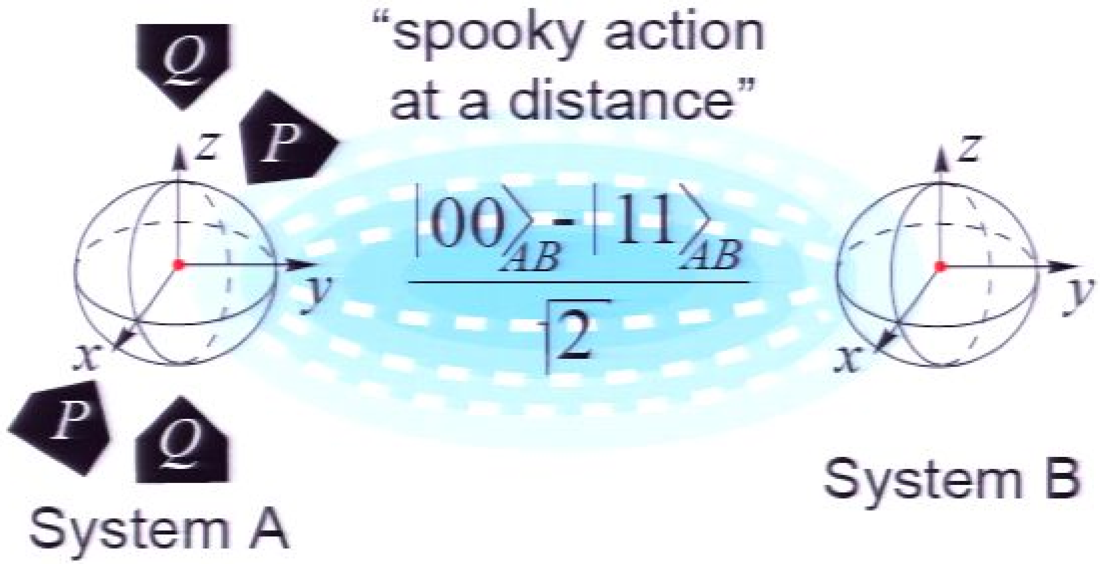
- Quantum computation is driven by *measurement* instead of unitary evolution.

Kashefi • Browne • Eisert • Flammia • Shi • Miyake

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)



- EPR state $\Psi(x_A, x_B) = \int_{-\infty}^{\infty} e^{\frac{i}{\hbar}(x_A - x_B)p} dp \longrightarrow |\Psi\rangle = \frac{|00\rangle_{AB} - |11\rangle_{AB}}{\sqrt{2}}$ (Bohm).

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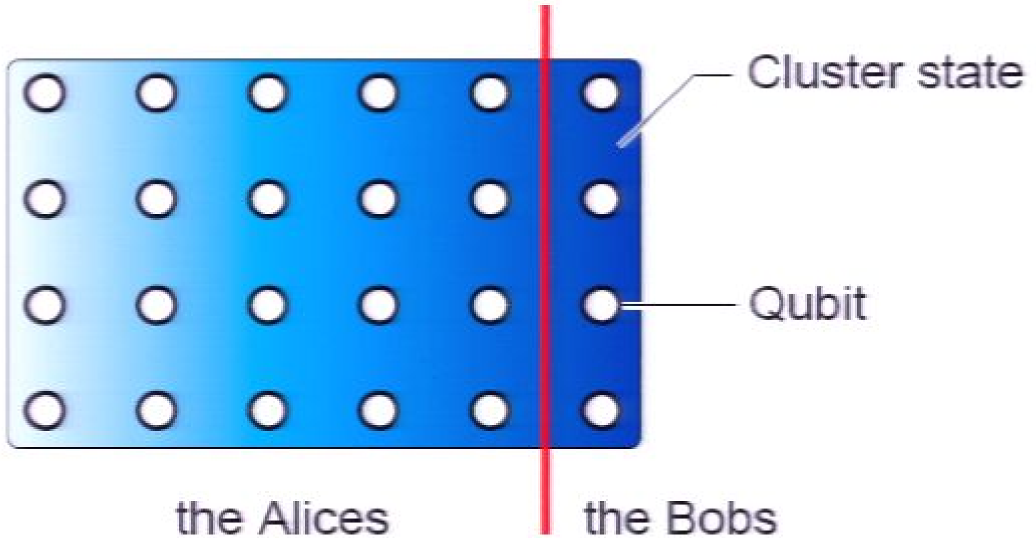

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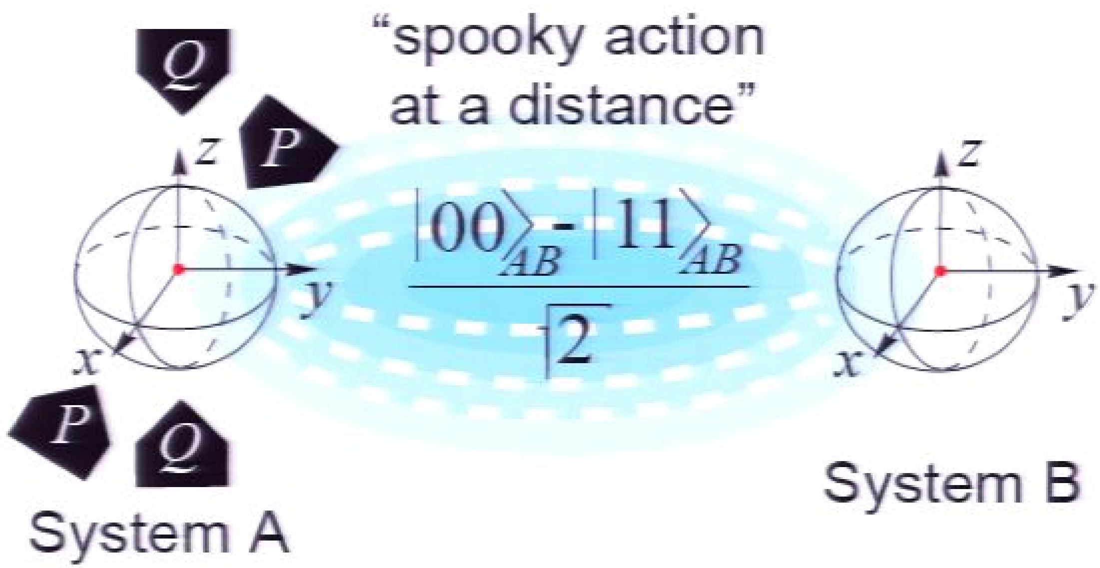

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The Einstein-Podolski-Rosen paradox

Requirement for completeness of a physical theory:

> Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory.*

'Element of reality':

> We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*

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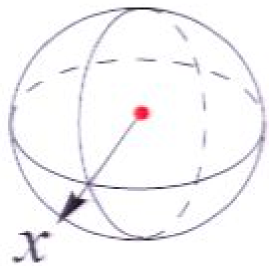

$$P \equiv \sigma_x,\ Q \equiv \sigma_z.\ [\sigma_z, \sigma_x] = 2i\sigma_y$$

- $\sigma_x|+\rangle = |+\rangle$: σ_x has value of $+1$ with certainty.
- σ_z has no definite value.

From this follows that either (1) *the quantum-mechanical description of reality given by the wave function is not complete* or (2) *when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality.*

$$|\Psi\rangle = \frac{|00\rangle_{AB} + |11\rangle_{AB}}{\sqrt{2}} = \frac{|+-\rangle_{AB} + |-+\rangle_{AB}}{\sqrt{2}}$$

- Assume (1) wrong: $|\Psi\rangle$ complete.

(a) A measures $\sigma_x^A \to \sigma_x^B$ EoR.

(b) A measures $\sigma_z^A \to \sigma_z^B$ EoR.

- (a) and (b) belong to same reality.

$\to$ (2) is wrong.

Thus the negation of (1) leads to the negation of the only other alternative (2). We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete.

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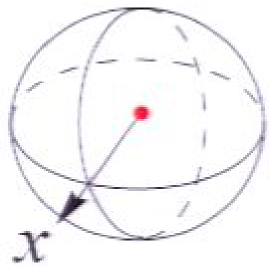

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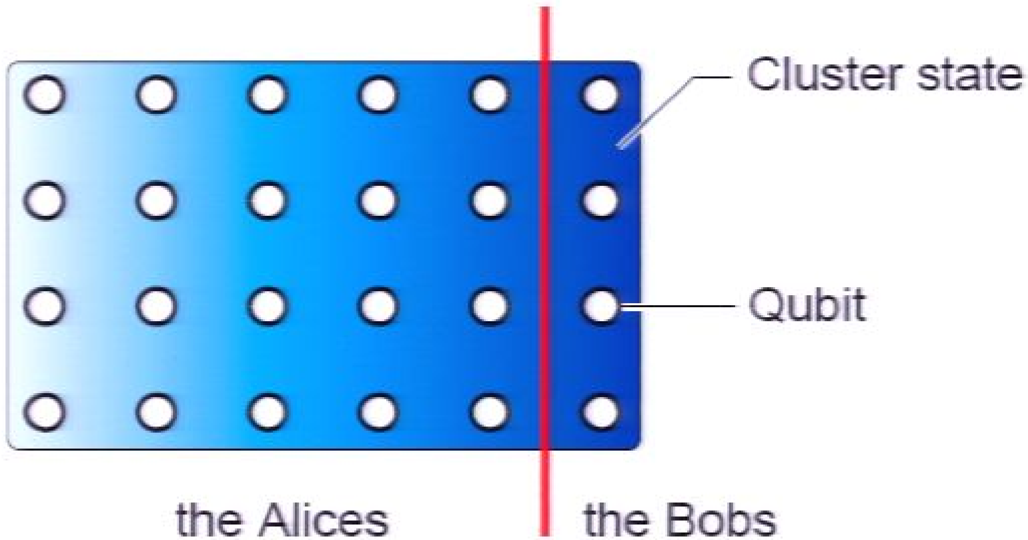
One could object to this conclusion on the grounds that our criterion of reality is not sufficiently restrictive. Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted*. On this point of view, since either one or the other, but not both simultaneously, of the quantities P and Q can be predicted, they are not simultaneously real. This makes the reality of P and Q depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this.

The one-way quantum computer ($QC_{\mathcal{C}}$)

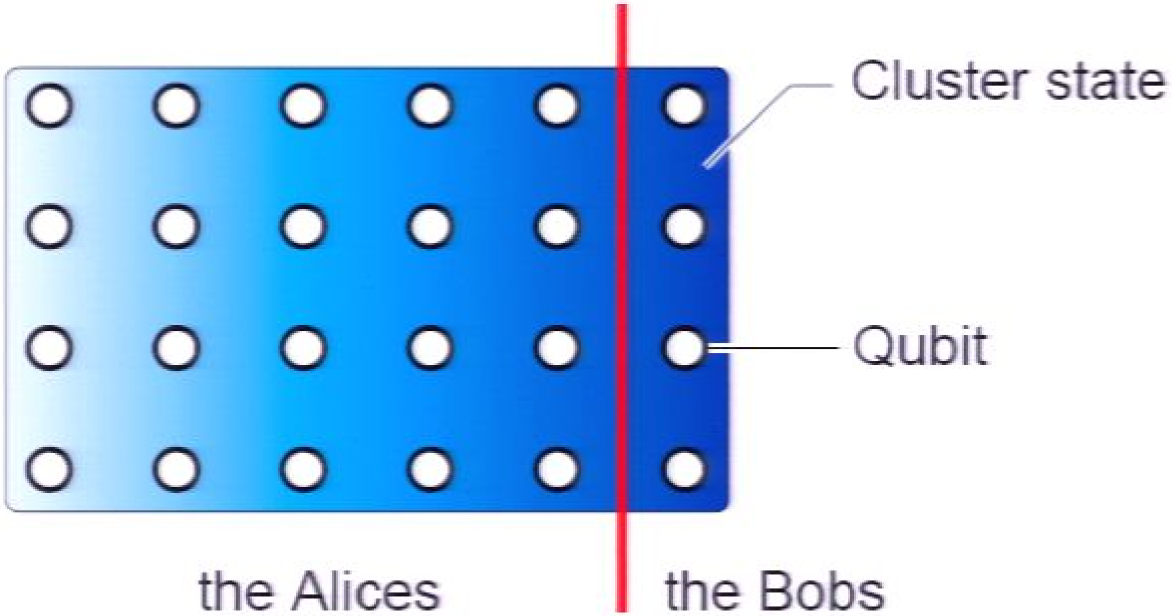
... is a generalization of the EPR setting to many A's and B's



The A's and B's can perform *universal quantum computation*, where

- The A's, by local measurements and broadcast of their outcomes to the B's, *set up a quantum algorithm*; and
- The B's, by measuring their respective qubits, *read off the computational results.*

The one-way quantum computer is universal



- *Every* quantum compuation can be implemented on a cluster state by local measurements and classical communication.
- The cluster state is thus a universal resource for the $QC_{\mathcal{C}}$.

The cluster state as entanglement resource

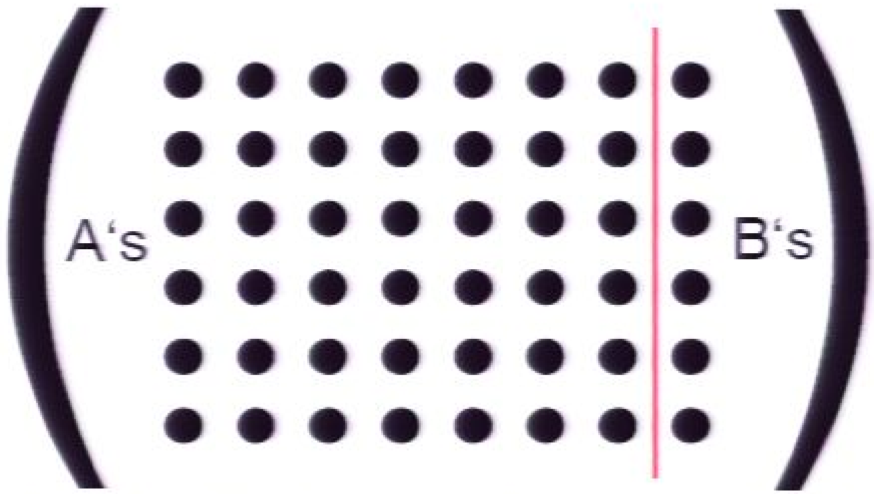
Why see the cluster state as a resource?

- *It is special:* Only a few resource states for the universal $QC_{\mathcal{C}}$ are known.
- *It is consumable:* After compuation is done, the remaining quantum state is useless for a subsequent $QC_{\mathcal{C}}$ computation.

What is the essential quantum mechanical property that makes these states universal resources? Can it be quantified?

- Whatever this essence is, it must not increase under LOCC and must be zero for product states.
- Suggestive candidate: Entanglement.

It's all in the correlations



$$\{A\},\{B\} \longrightarrow \{A\} \cup \{B\}$$

- For classical to classical computation, the distinction between the A's (setting up) and the B's (read-out) is *artifical*.

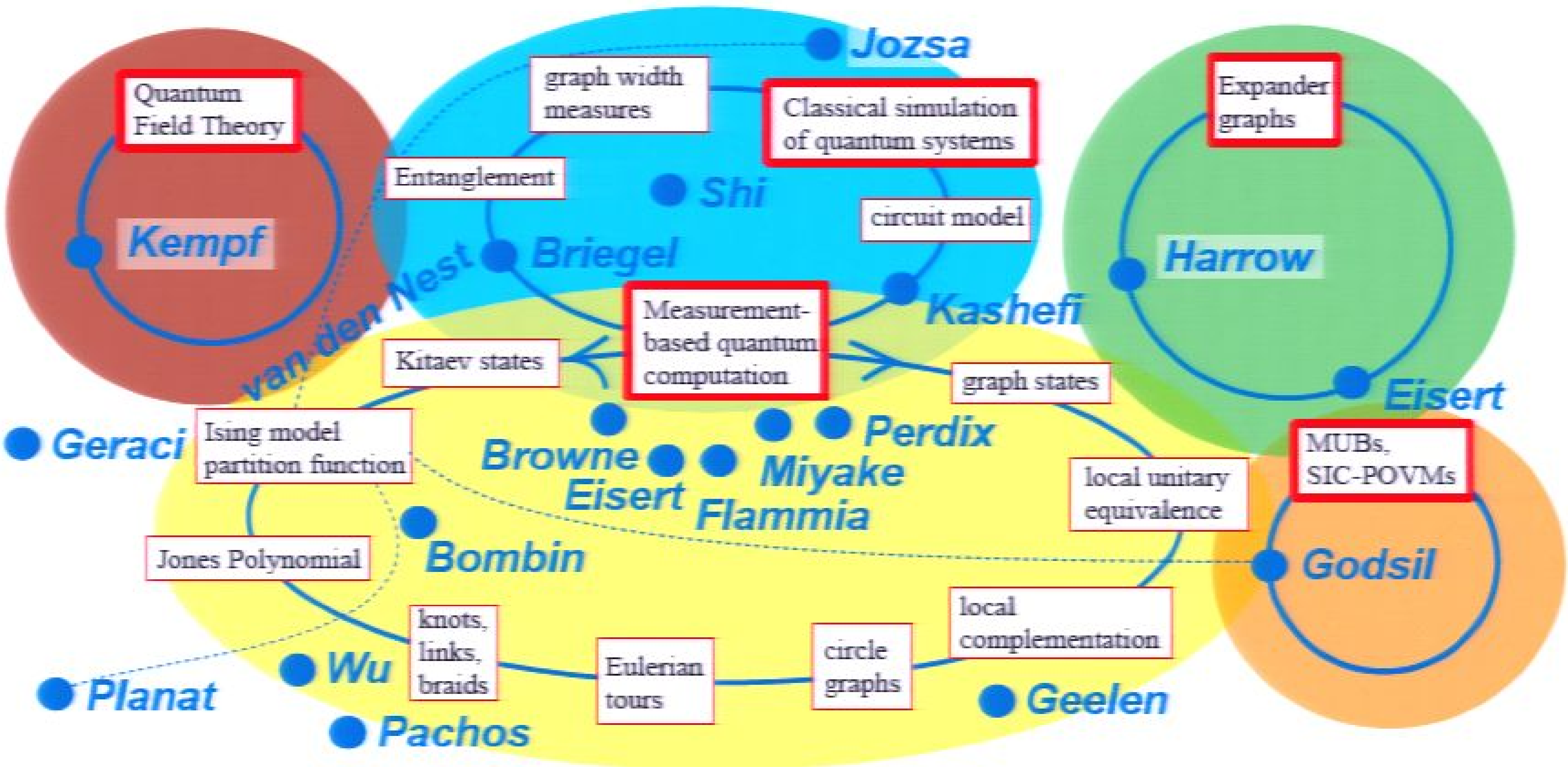
- The result of the computation is derived from the *correlations* of measurement outcomes on $\{A\} \cup \{B\}$.

What are the open questions for the $QC_{\mathcal{C}}$?

- Can we construct novel algorithms in the $QC_{\mathcal{C}}$ directly?
- What are the logical building blocks of the $QC_{\mathcal{C}}$ and what is their composition?
- Where does the quantum speed-up come from?

(Solved: Universality and Fault-tolerance.)

Overview and workshop program



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