Title: A new no-go theorem for hidden variables theories

Date: Apr 29, 2008 04:00 PM

URL: http://pirsa.org/08040073

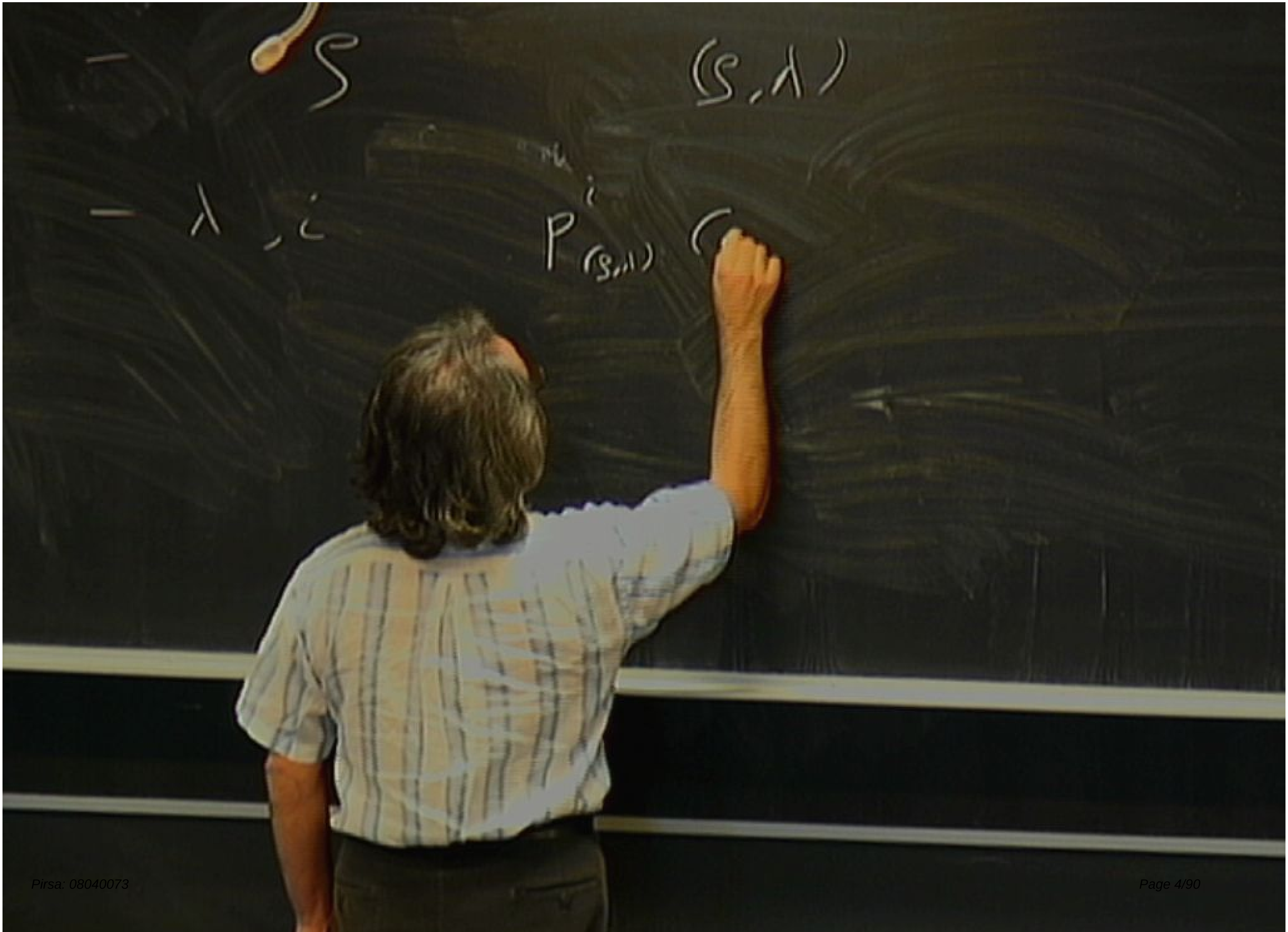
Abstract: Consider the quantum predictions for EPR-type measurements on two systems with Hilbert space of dimension at least 3 in any maximally entangled state. I show that the only possible hidden variables model of these probabilities that satisfies both Shimony\'s and Jarrett\'s condition of parameter independence (or `locality\') and Jones and Clifton\'s condition of conditional parameter independence (or `constrained locality\') is trivial, i.e. given by the quantum probabilities themselves. I shall attempt to discuss also the meaning of the conditions and of this result.

S (S,λ)

S

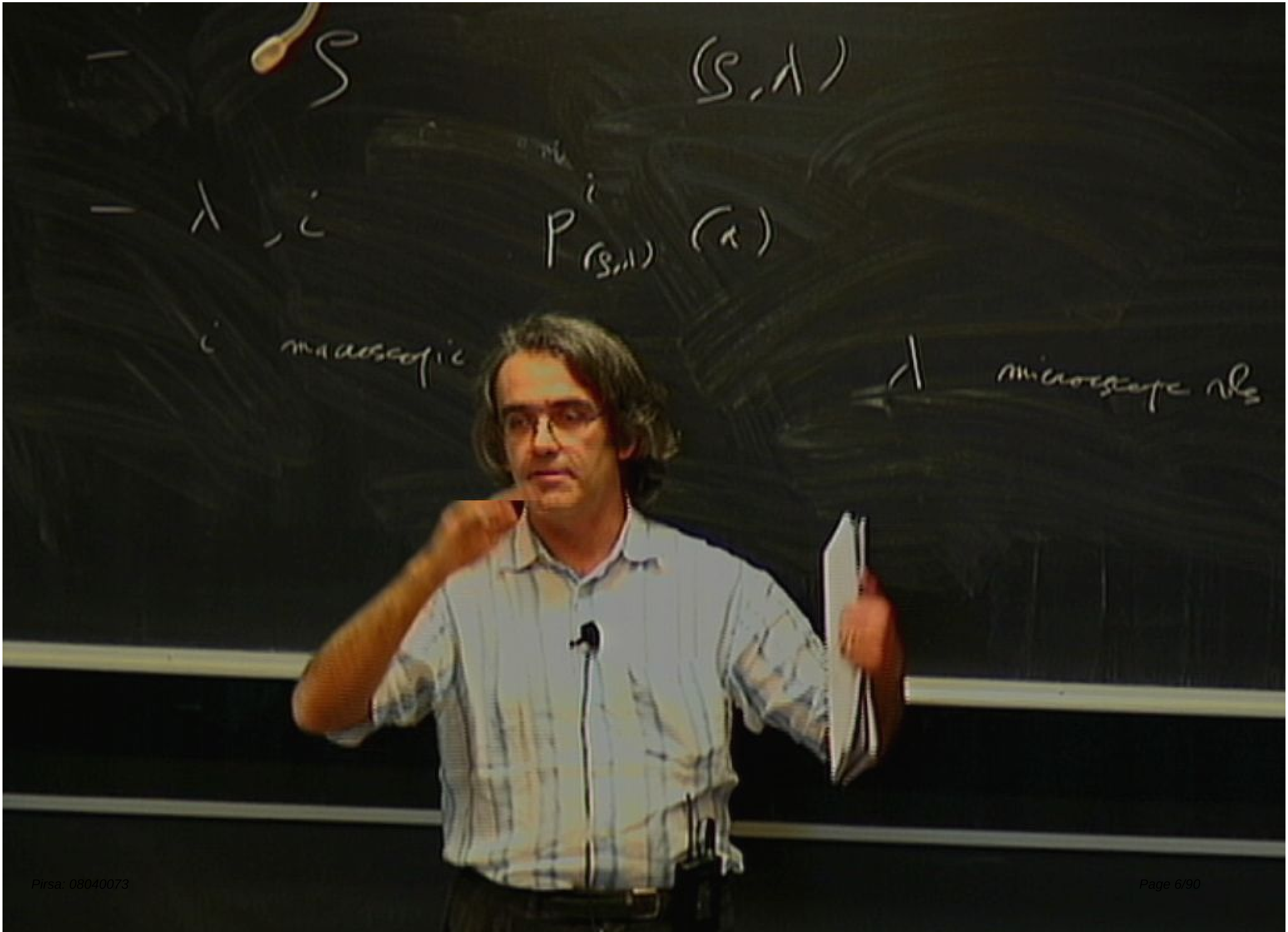
(S, λ)

λ, i

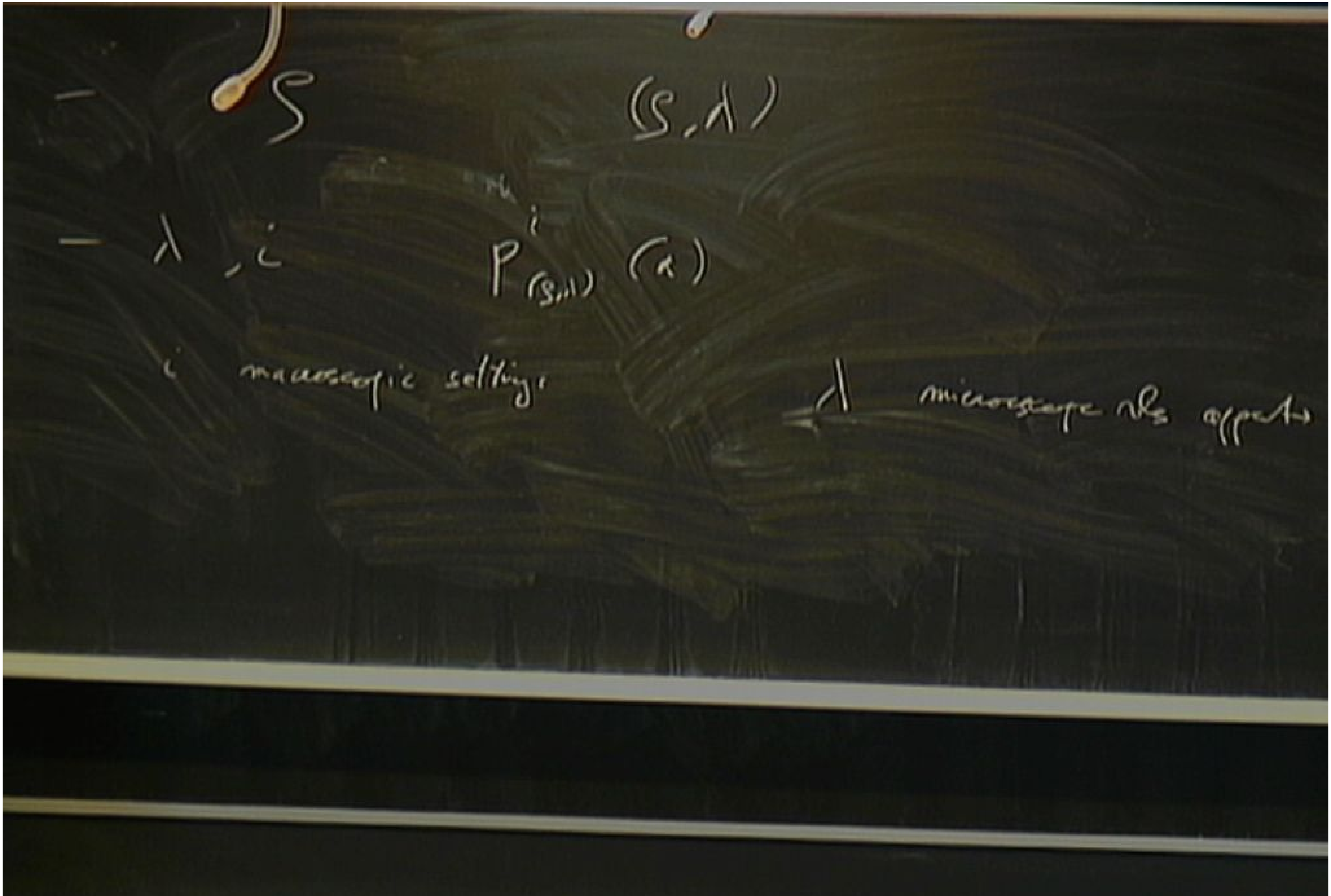

— ρ
(ρ, λ)
— λ , i
P(ρ,λ)

$-\ \rho \qquad (\rho,\lambda)$

$-\ \lambda,i \qquad P^{i}_{(\rho,\lambda)}(x)$



ρ
(ρ, λ)
λ, i
$P^{i}_{(\rho,\lambda)}(x)$
i macroscopic
λ microscopic


ρ
(ρ, λ)
λ, i
$P^{i}_{(\rho,\lambda)}(x)$
i macroscopic settings
λ microscopic

— ρ $\qquad (\rho, \lambda)$

— λ, i $\qquad P^{i}_{(\rho,\lambda)}(x)$

i macroscopic settings

λ microscopic d.o.f. aspects

i macro + micro

$\lambda, i \qquad P^{i}_{(S,\lambda)}(\alpha)$

i macroscopic settings

i macro + micro

λ

$\lambda, i \qquad P^{i}_{(S,\lambda)}(a)$

i macroscopic settings

λ macro + micro

λ

$- \; \rho \qquad\qquad (\rho, \lambda)$

$- \; \lambda, i \qquad\qquad P^{i}_{(\rho,\lambda)}(a)$

i macroscopic settings.

i macro + micro

A

— ρ (ρ, λ)

— λ, i $P^{i}_{(\rho,\lambda)}(\alpha)$

i macroscopic settings.

i macro + micro

— stat. dist.

- ρ (ρ, λ)

- λ, i $P^{i}_{(\rho,\lambda)}(\alpha)$

i macroscopic settings.

i macro + micro

- stat. distr.

— ρ (ρ, λ)

— λ, i $P^{i}_{(\rho,\lambda)}(\alpha)$

i macroscopic settings.

i macro + micro

— stat. dist. $\rho(\lambda)$

↳ macro + micro

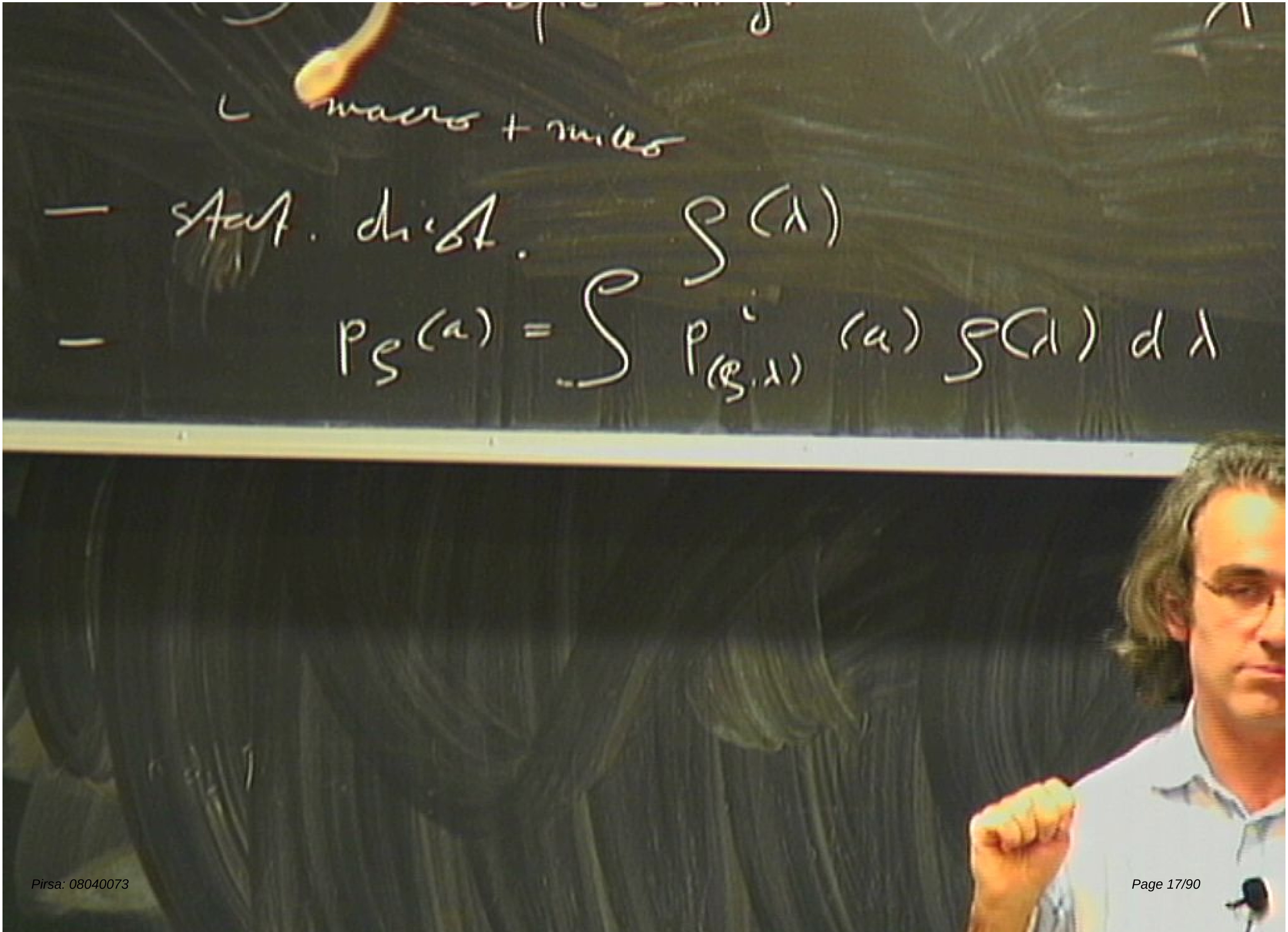
— stat. dist. $\rho(\lambda)$

— $p_\rho(a) = \int p^i_{(\rho,\lambda)}(a)\,\rho(\lambda)\,d\lambda$

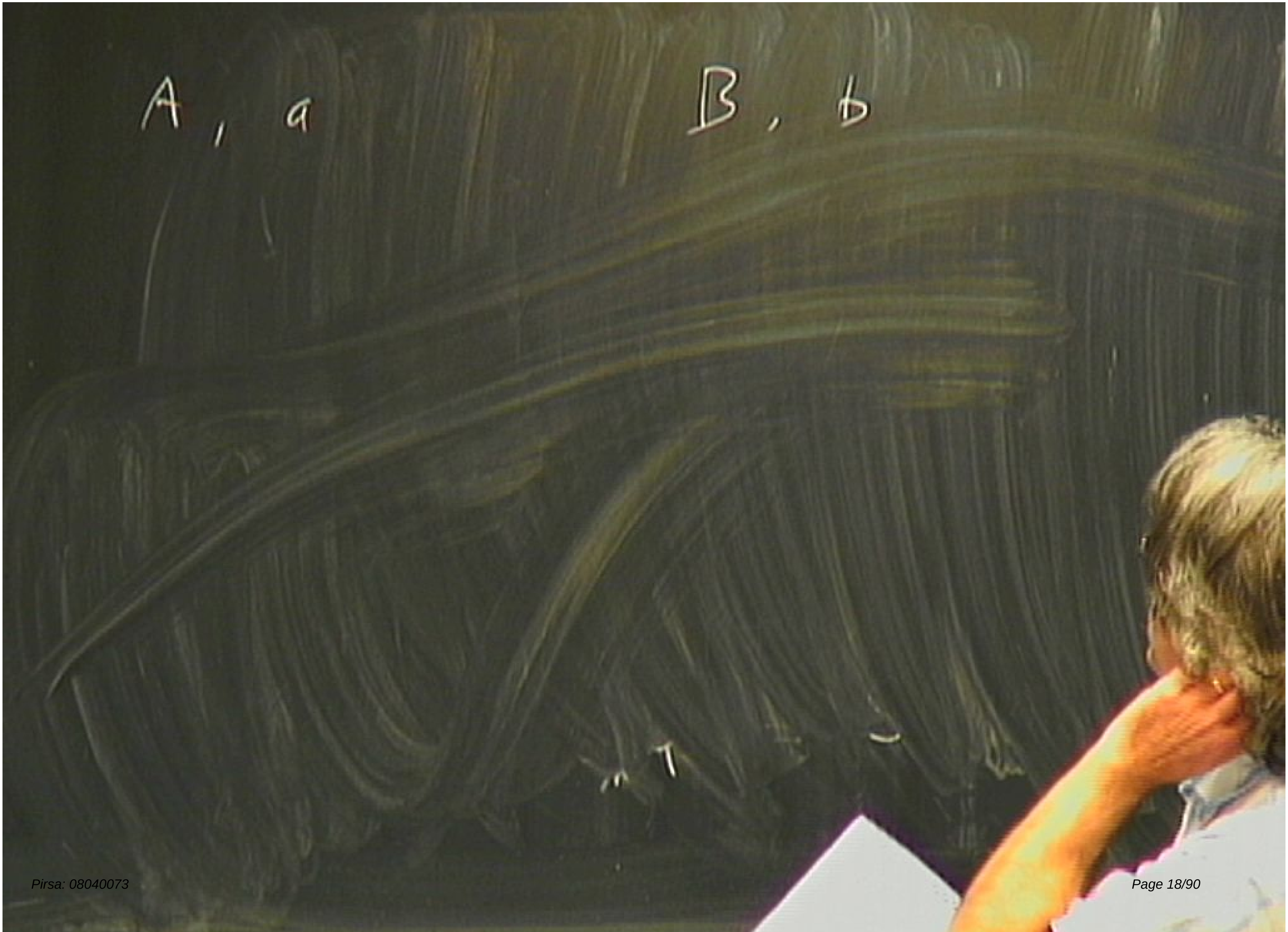
↳ macro + micro

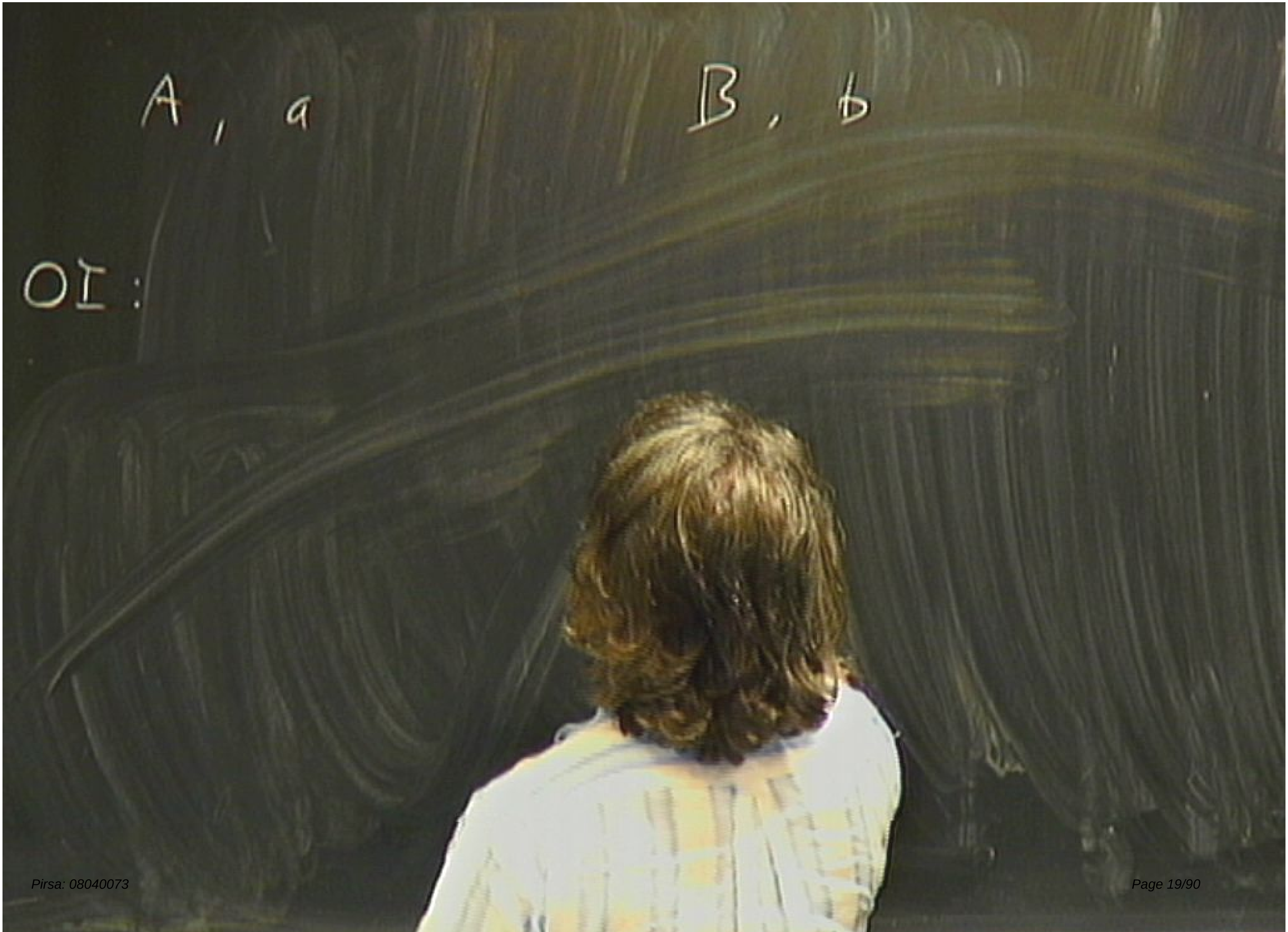
— stat. dist. $\rho(\lambda)$

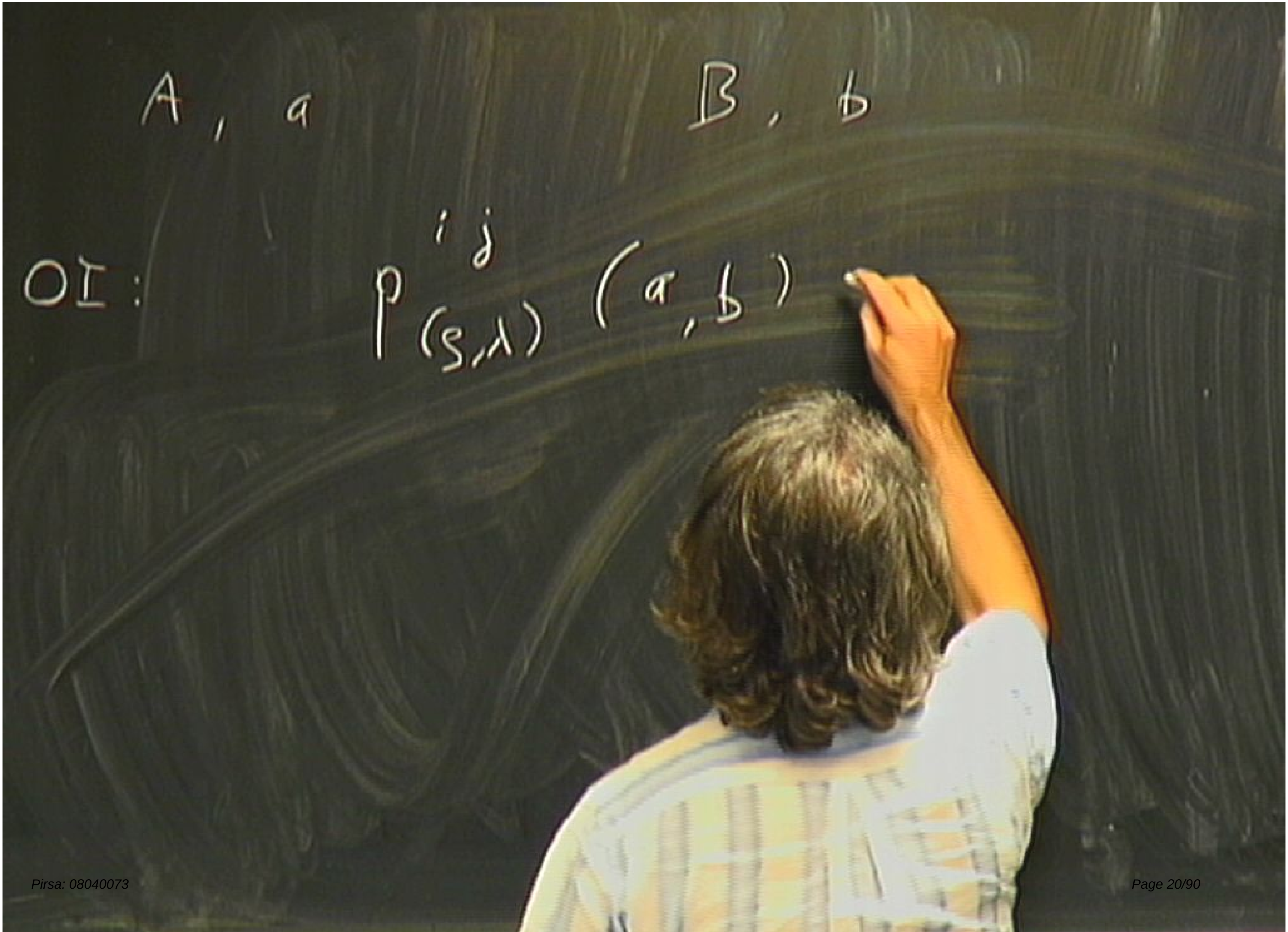
— $p_\rho(a) = \int p^{i}_{(\rho,\lambda)}(a)\,\rho(\lambda)\,d\lambda$

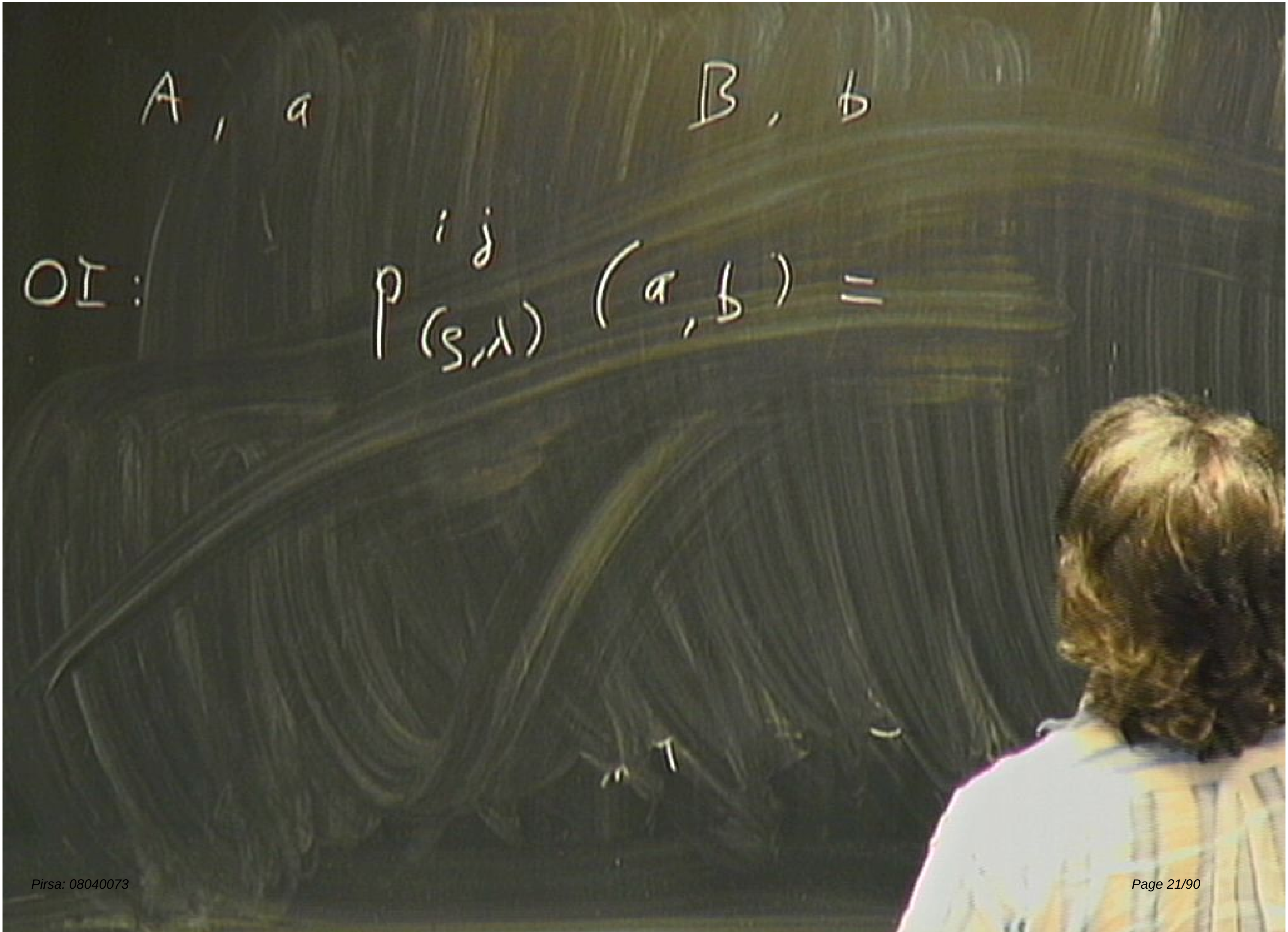


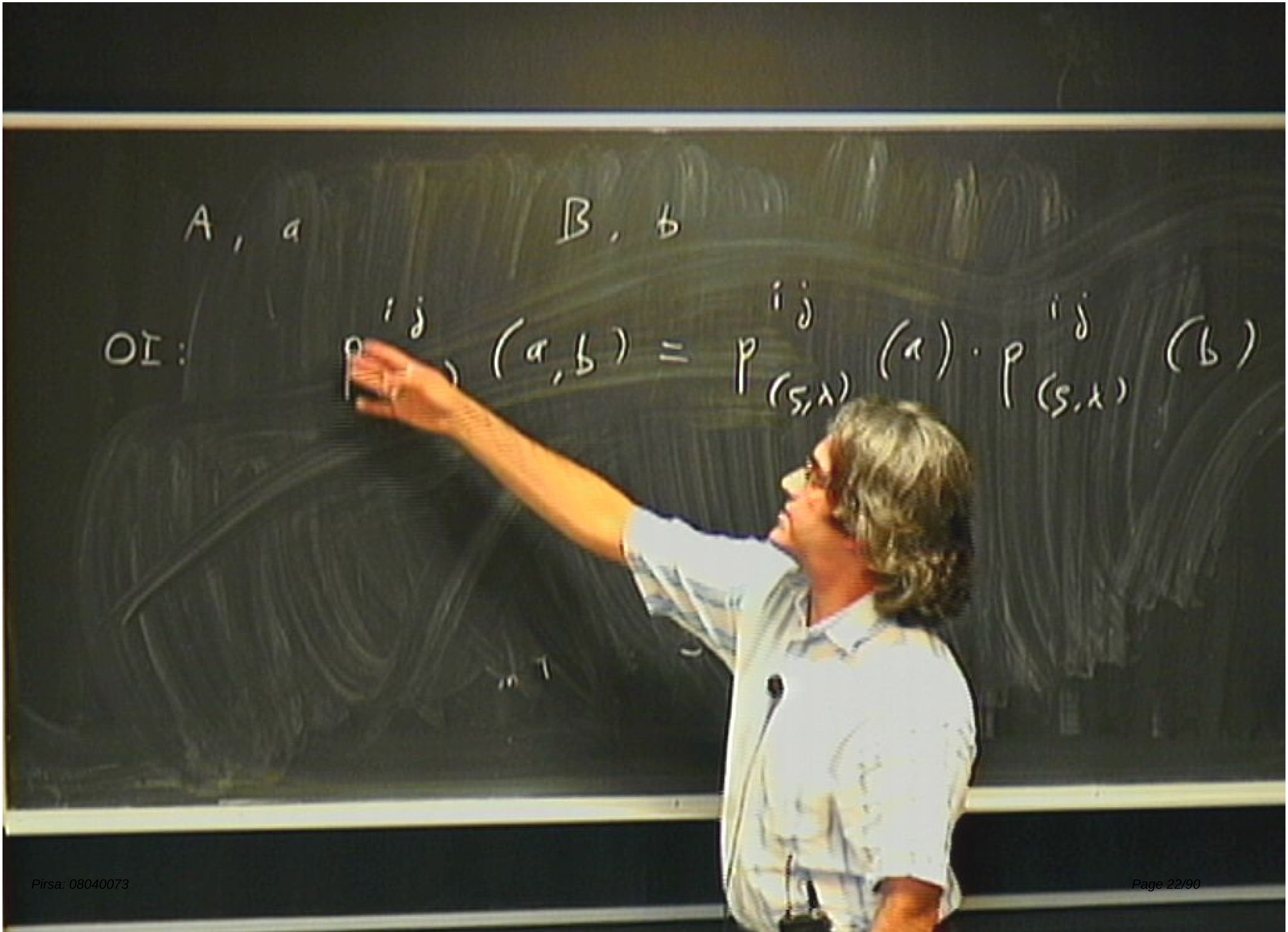
macro + micro
— stat. dist. $\rho(\lambda)$
— $p_\rho(a) = \int p^{i}_{(\rho,\lambda)}(a)\, \rho(\lambda)\, d\lambda$

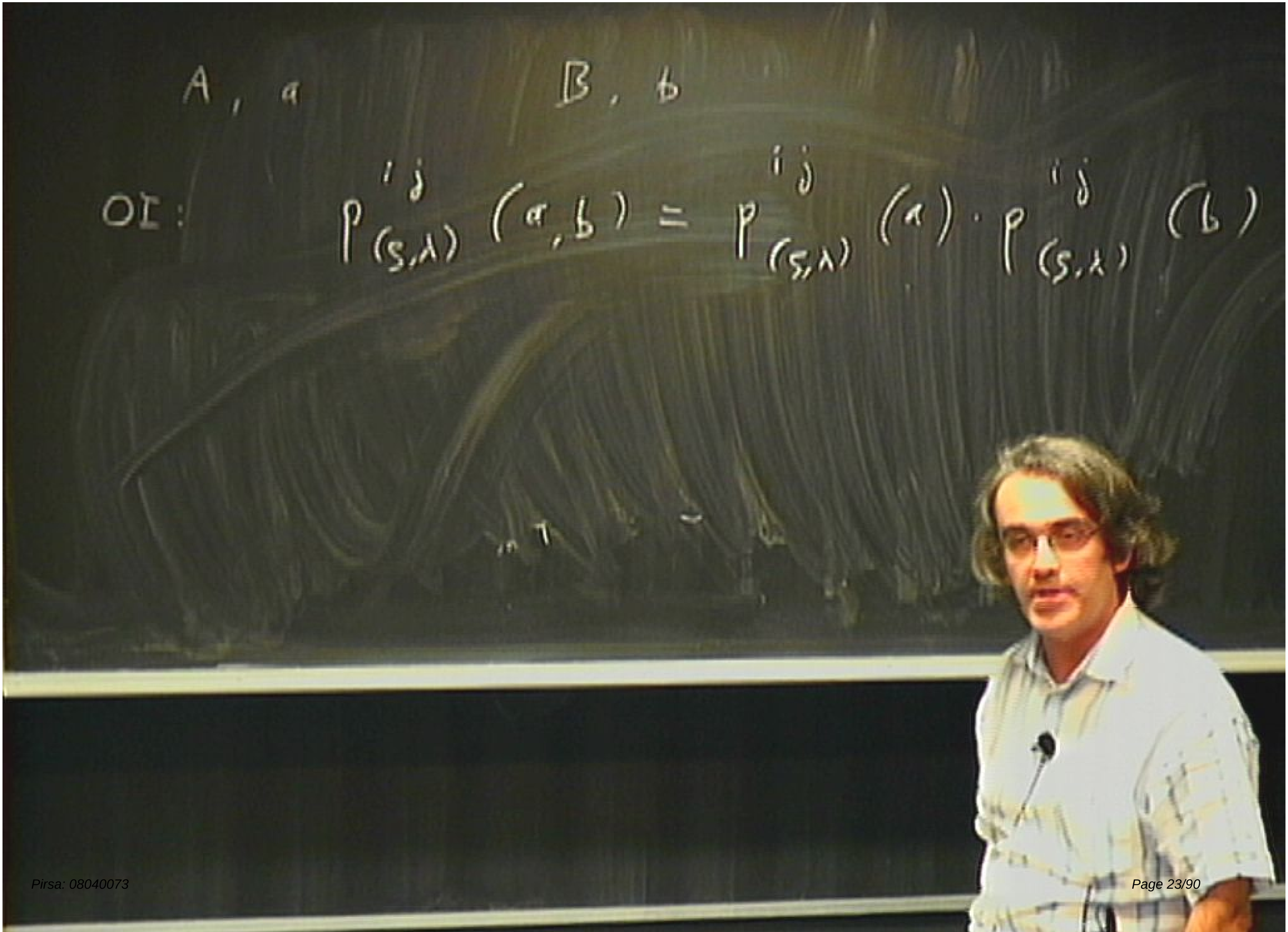

A, a
B, b

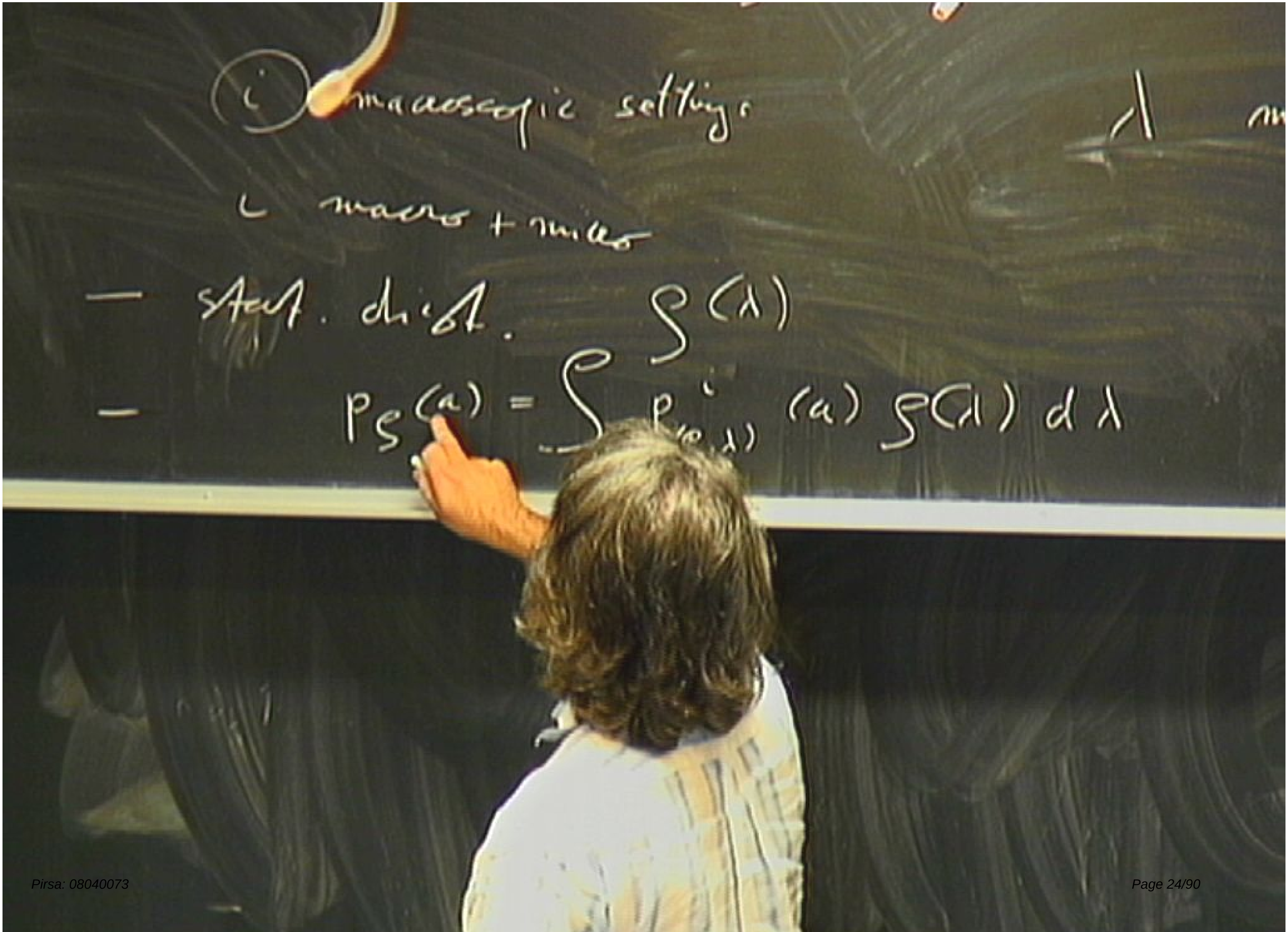

A, a
B, b
OI :

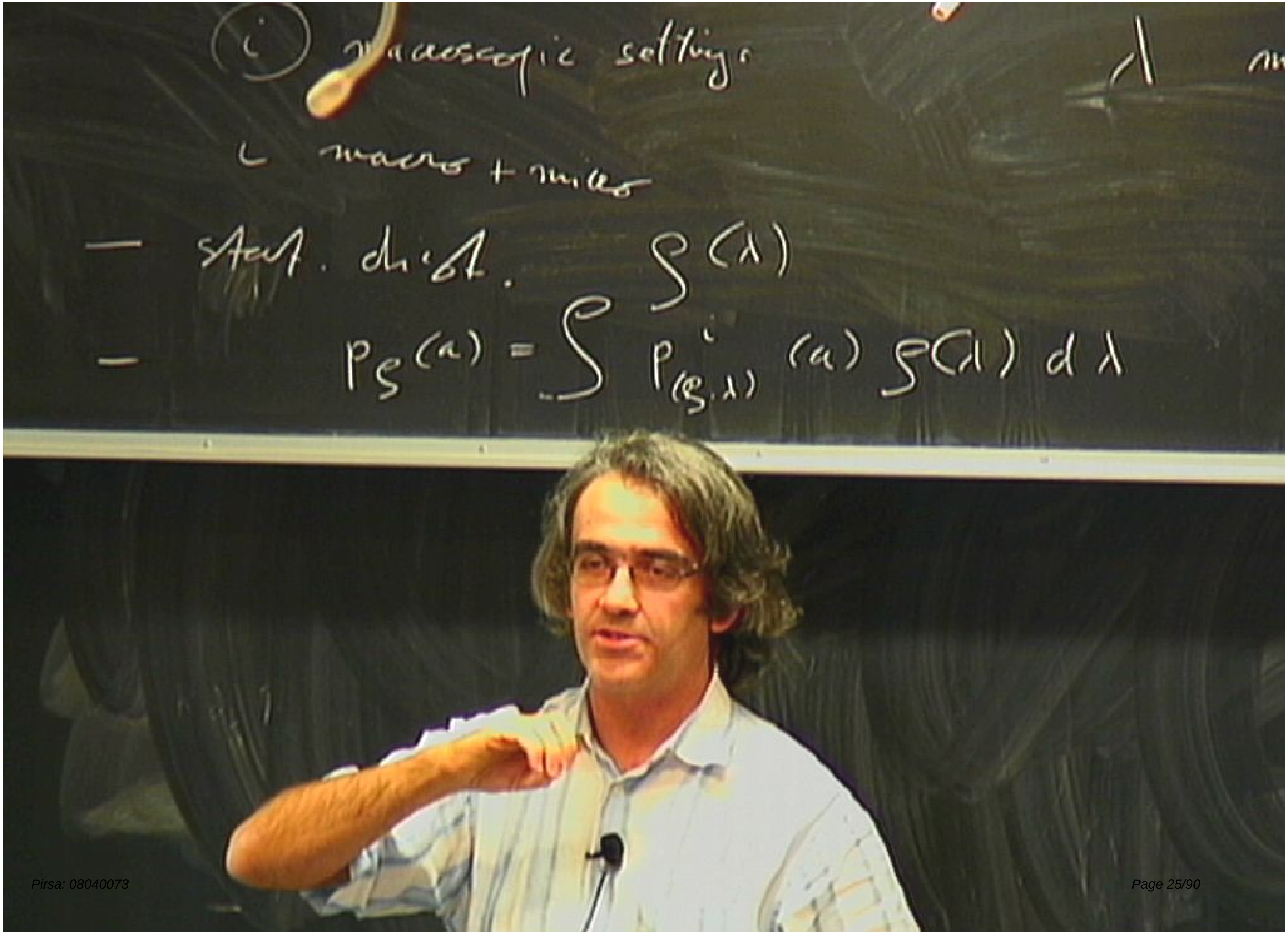

A, a
B, b
OI: $p^{ij}_{(\varsigma,\lambda)}(a,b)$

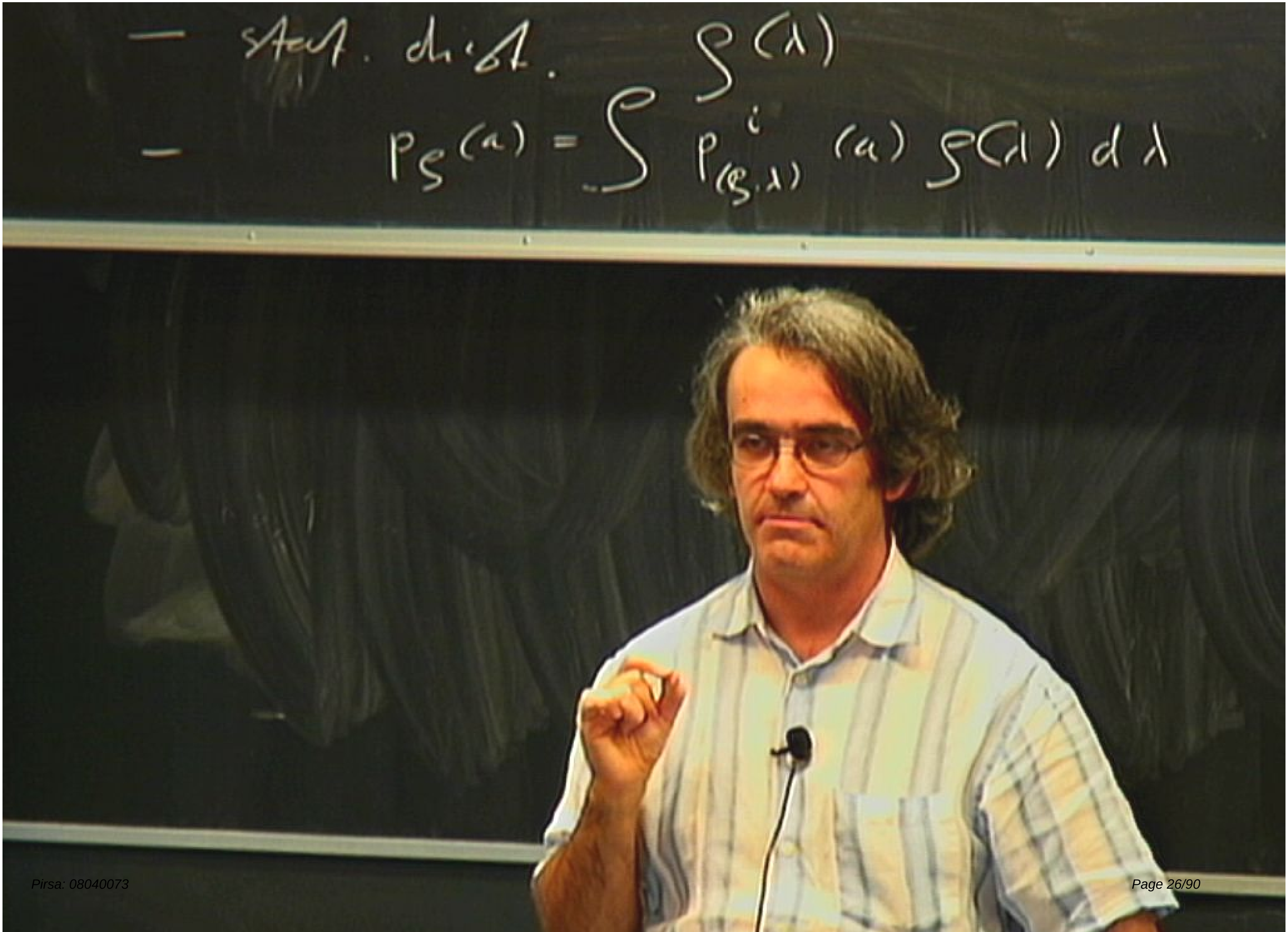

A , a
B , b
OI : $p^{ij}_{(S,\lambda)}(a,b) =$

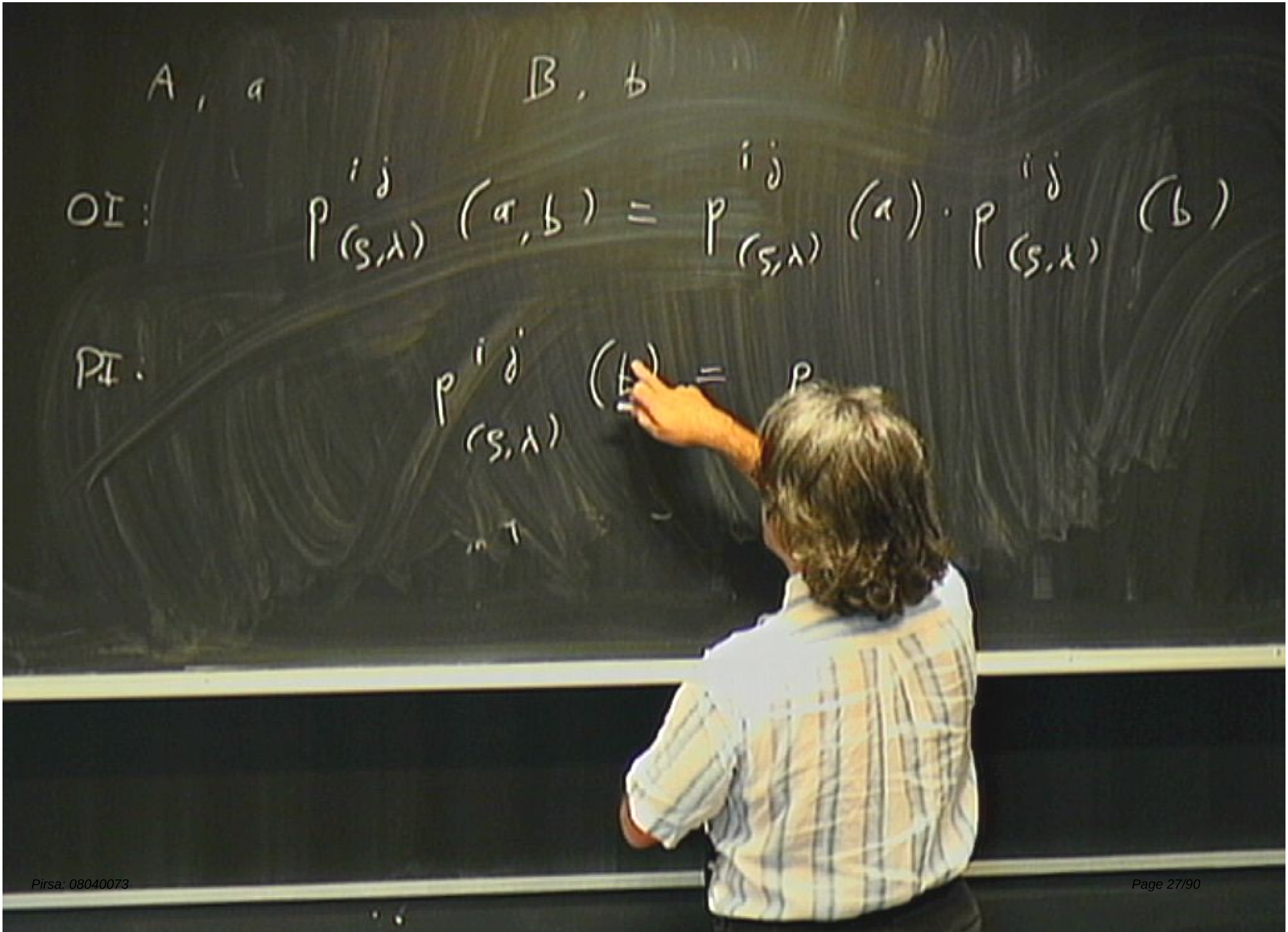

A , a
B , b
OI: $p^{ij}(a,b) = p^{ij}_{(s,\lambda)}(a) \cdot p^{ij}_{(s,\lambda)}(b)$

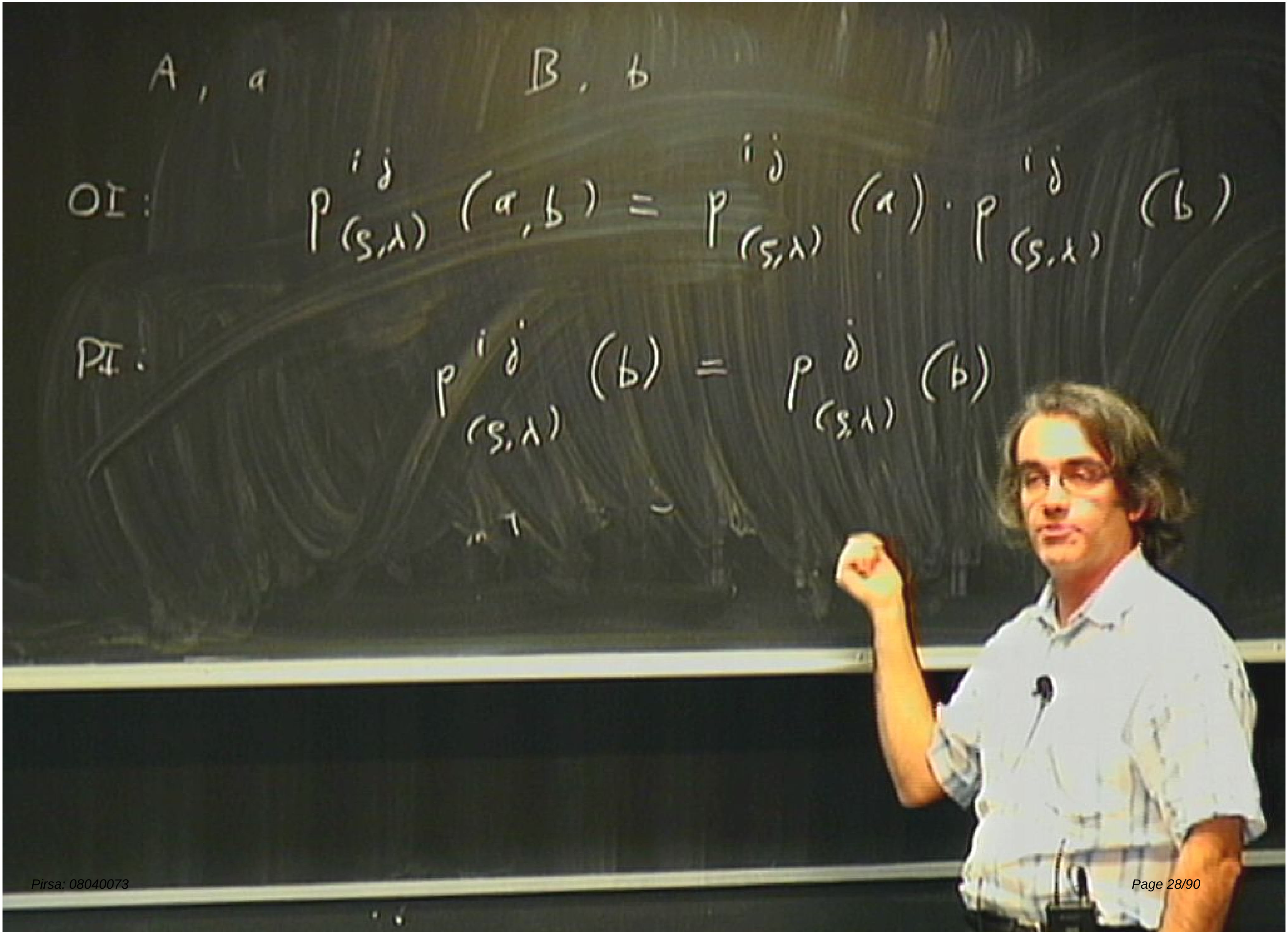

A , a
B , b
OI : $p^{ij}_{(s,\lambda)}(a,b) = p^{ij}_{(s,\lambda)}(a) \cdot p^{ij}_{(s,\lambda)}(b)$

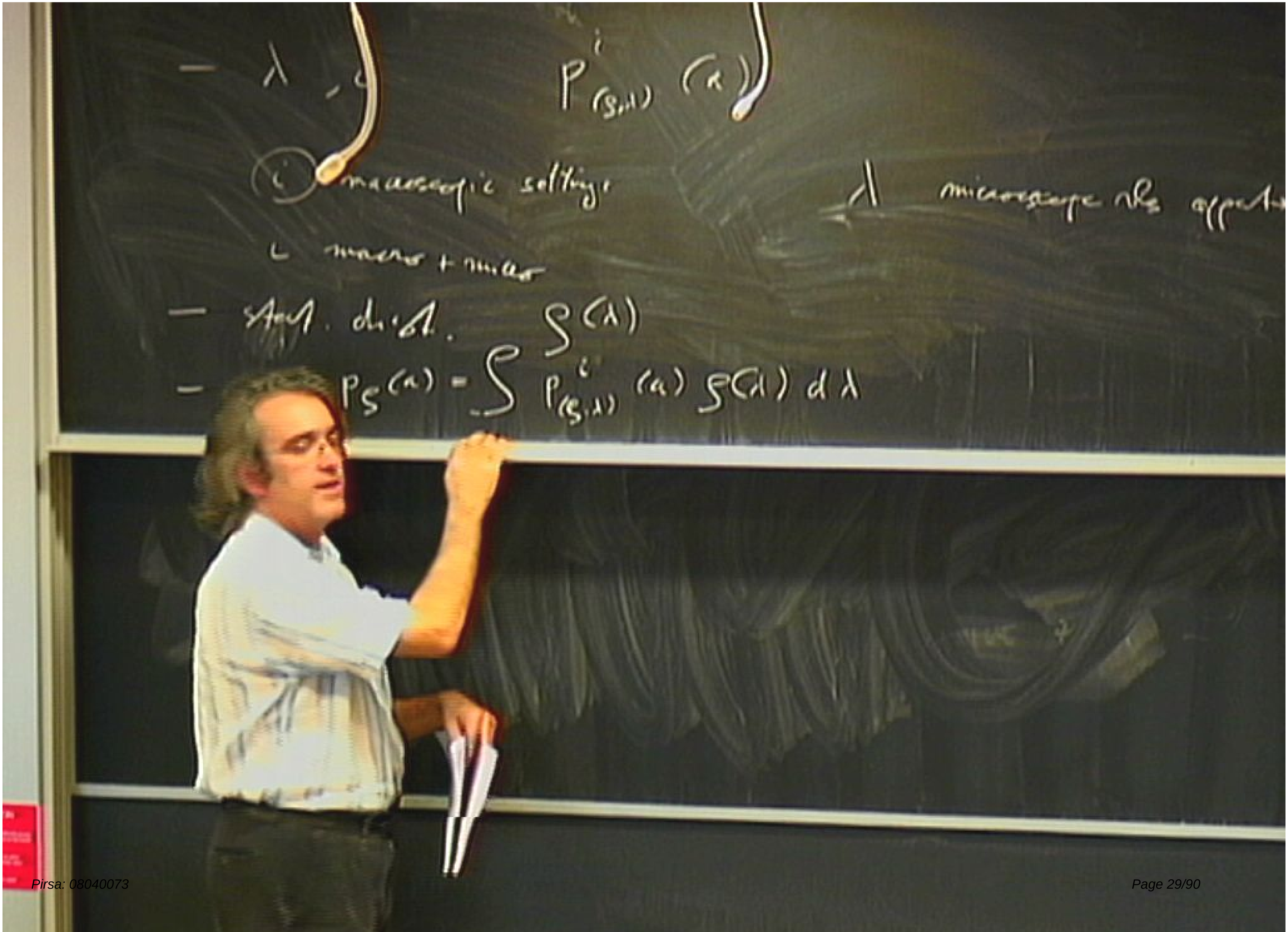

i macroscopic settings
λ
i macro + micro
— stat. distr. ρ(λ)
— P_ρ(a) = ∫ P^i_(a,λ) (a) ρ(λ) dλ

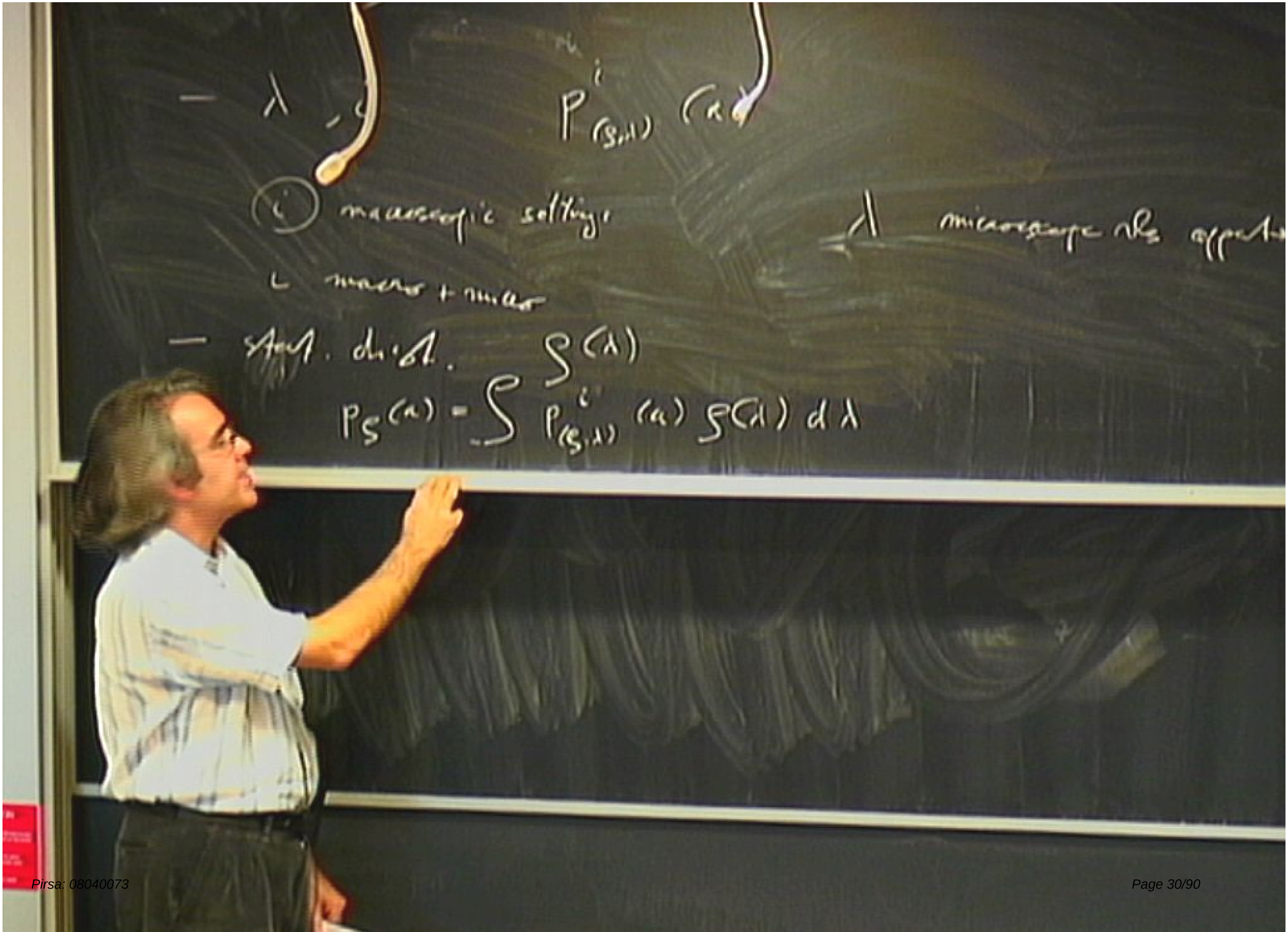

(i) macroscopic settings
λ
i macro + micro
— stat. distr. ρ(λ)
— $P_\rho(a) = \int P^{i}_{(\rho,\lambda)}(a)\,\rho(\lambda)\,d\lambda$

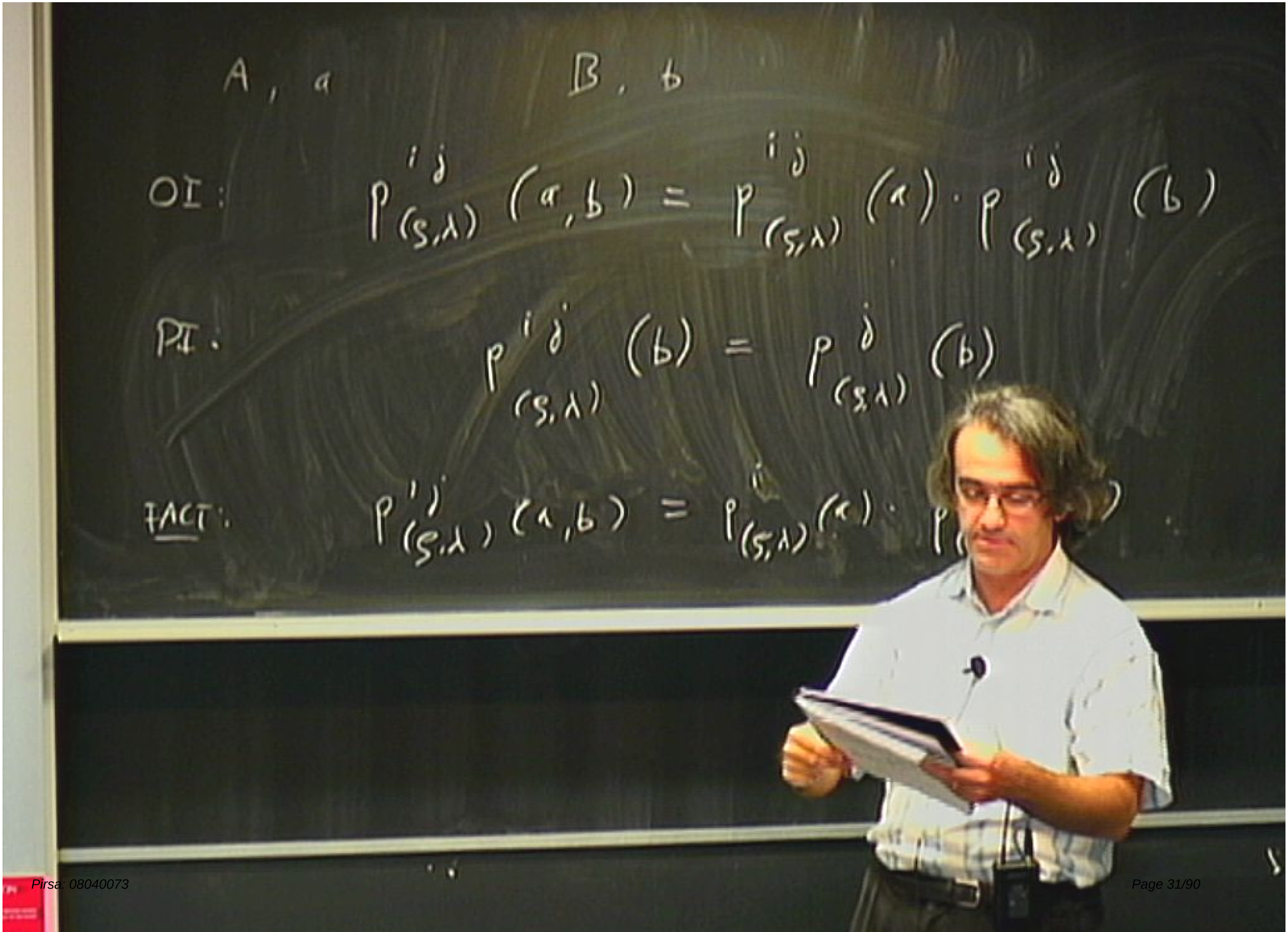

— stat. distr. $\rho(\lambda)$
— $P_\rho(a) = \int P^i_{(\rho,\lambda)}(a)\,\rho(\lambda)\,d\lambda$

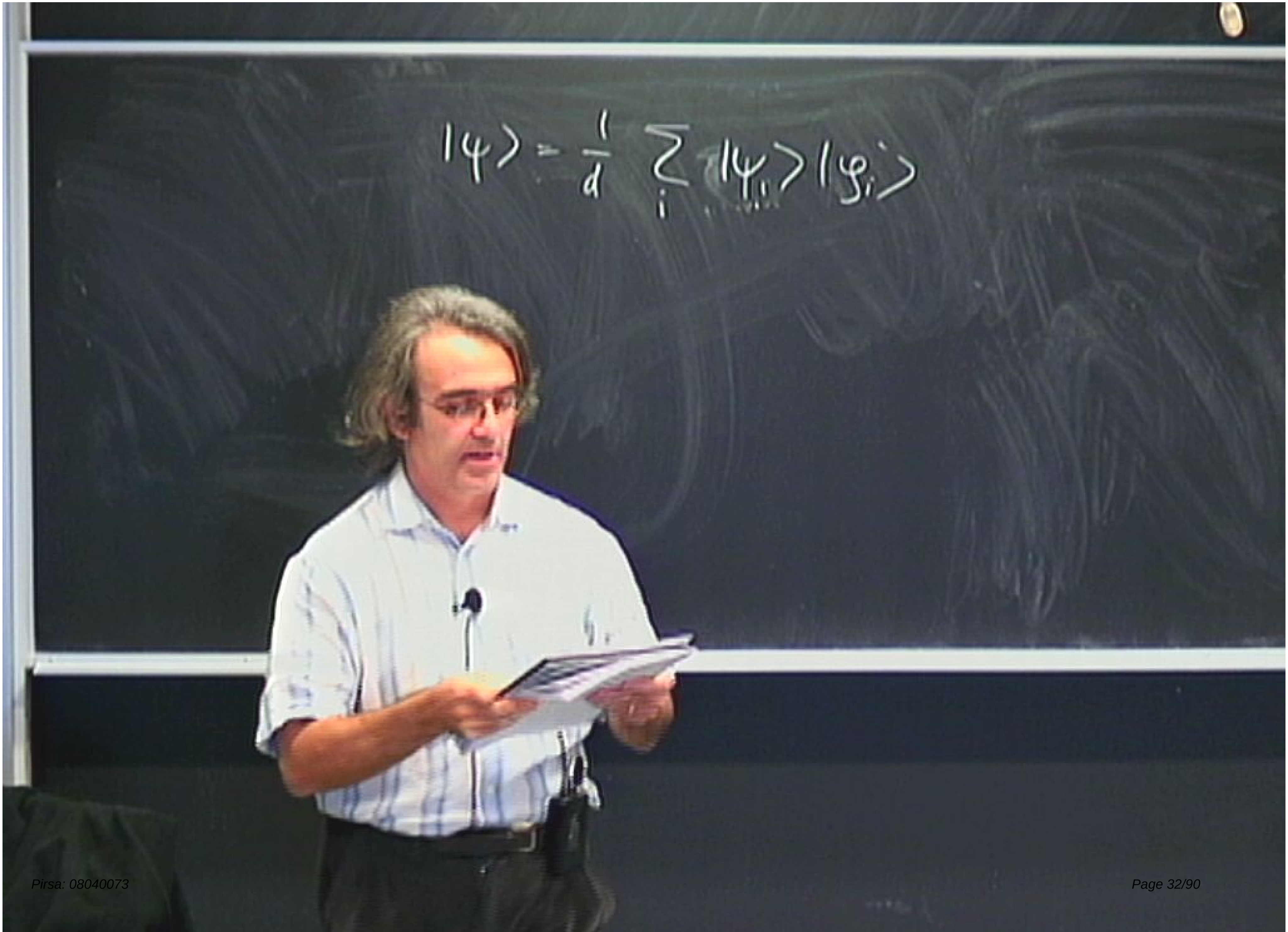

A , a
B , b
OI : $p^{ij}_{(s,\lambda)}(a,b) = p^{ij}_{(s,\lambda)}(a) \cdot p^{ij}_{(s,\lambda)}(b)$
PI : $p^{ij}_{(s,\lambda)}(b) = p$

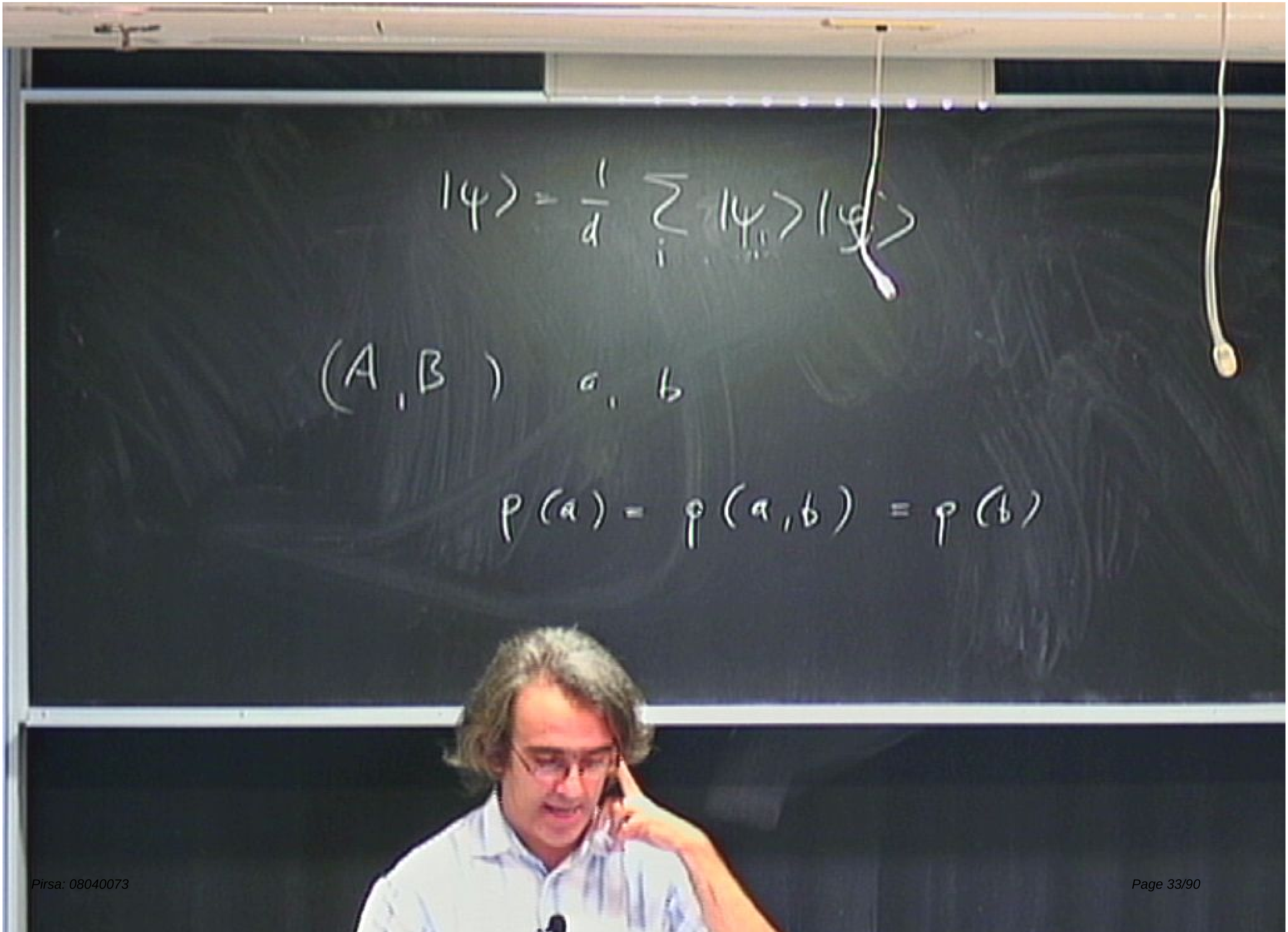

A , a
B , b
OI: $P^{ij}_{(\varsigma,\lambda)}(a,b) = P^{ij}_{(\varsigma,\lambda)}(a) \cdot P^{ij}_{(\varsigma,\lambda)}(b)$
PI: $P^{ij}_{(\varsigma,\lambda)}(b) = P^{j}_{(\varsigma,\lambda)}(b)$

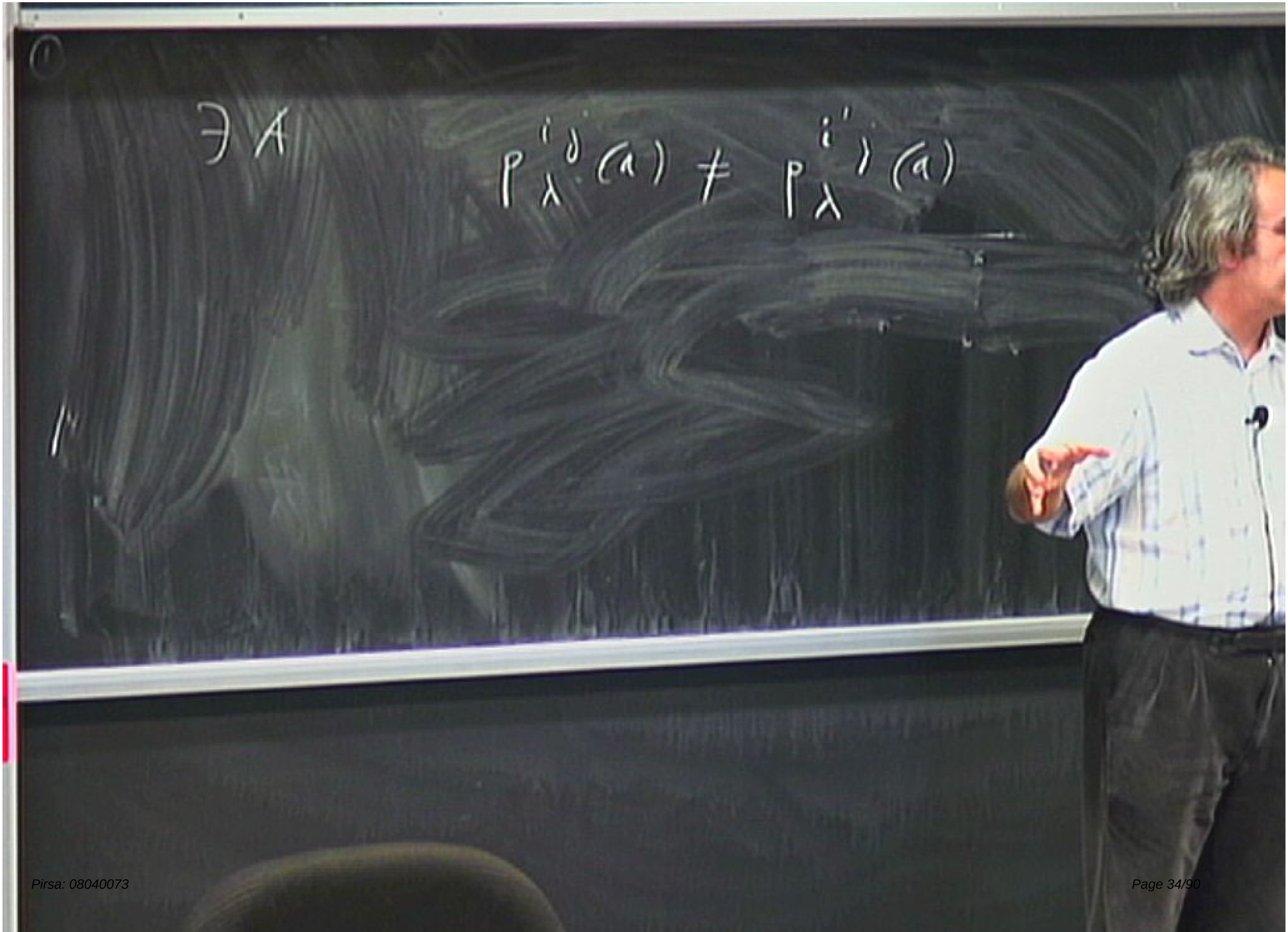

— λ , i   P^i_(S,λ) (k)
i macroscopic settings
λ microscopic vbs
i macro + micro
— stat. dist. ρ(λ)
— P_S(k) = ∫ P^i_(S,λ) (k) ρ(λ) dλ

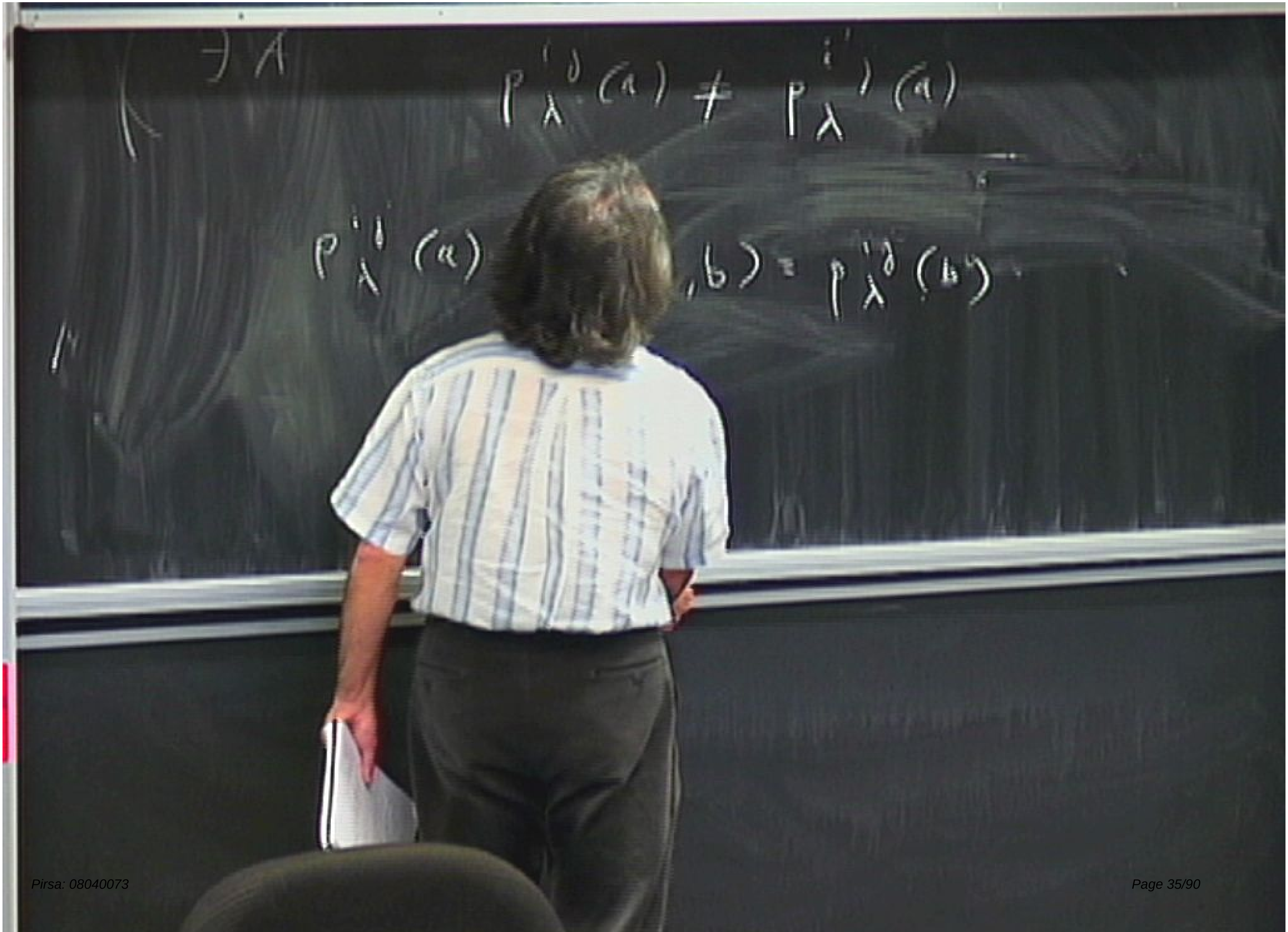

macroscopic settings
macro + micro
$P_\rho(x) = \int P^i_{(\rho,\lambda)}(x)\,\rho(\lambda)\,d\lambda$

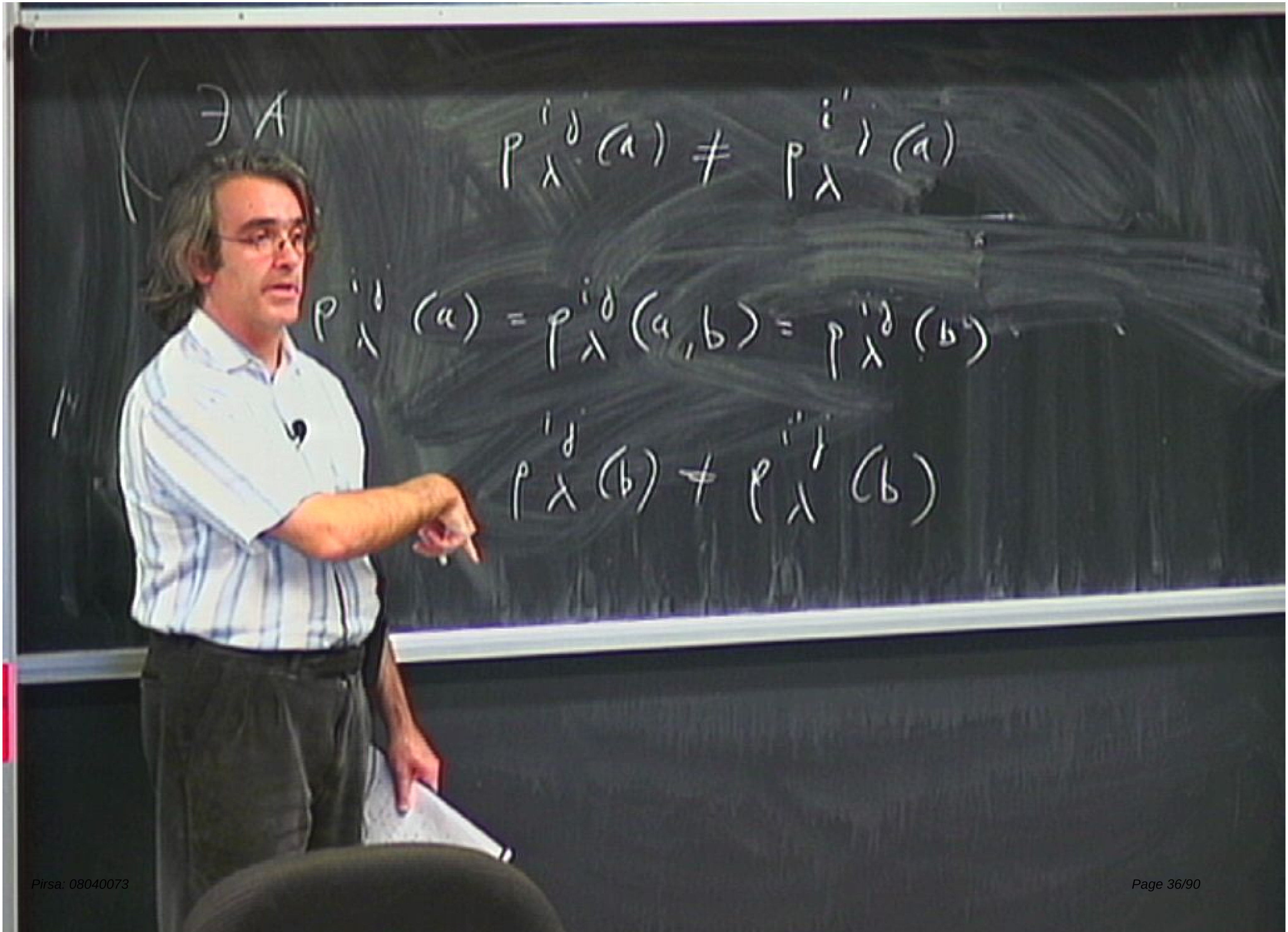

A, a   B, b
OI: $p^{ij}_{(\varsigma,\lambda)}(a,b) = p^{ij}_{(\varsigma,\lambda)}(a) \cdot p^{ij}_{(\varsigma,\lambda)}(b)$
PI: $p^{ij}_{(\varsigma,\lambda)}(b) = p^{j}_{(\varsigma,\lambda)}(b)$
FACT: $p^{ij}_{(\varsigma,\lambda)}(a,b) = p^{i}_{(\varsigma,\lambda)}(a) \cdot p$

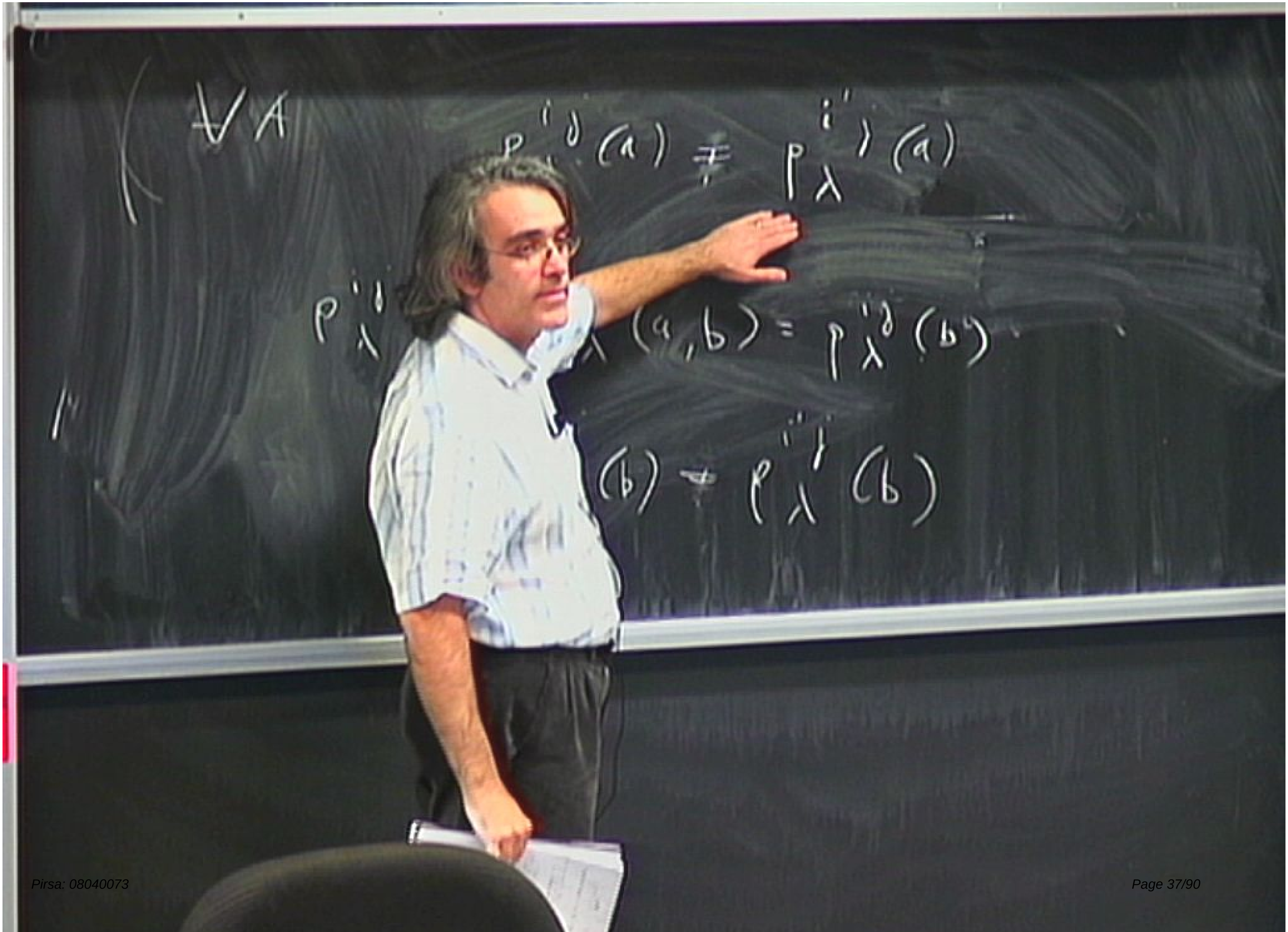

$|\psi\rangle = \frac{1}{d}\sum_i |\psi_i\rangle|\varphi_i\rangle$

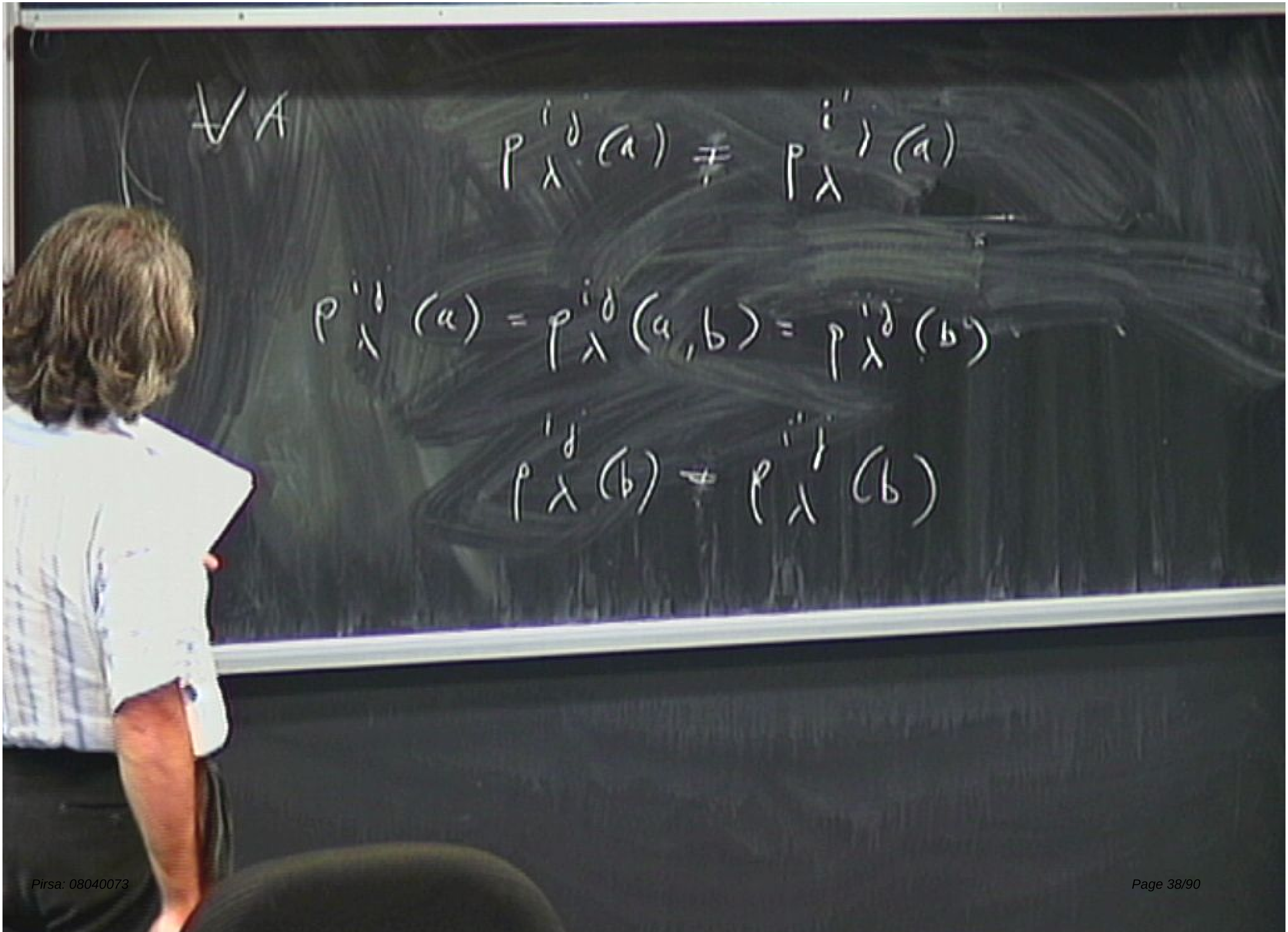

$|\psi\rangle = \frac{1}{d} \sum_i |\psi_i\rangle |\psi_i^*\rangle$
$(A, B) \quad a, b$
$p(a) = p(a, b) = p(b)$


∃ A
$p_\lambda^{ij}(a) \neq p_\lambda^{i'j}(a)$


∃ λ
$p_\lambda^{ij}(a) \neq p_\lambda^{i'j}(a)$
$p_\lambda^{ij}(a)$
$,b) = p_\lambda^{ij}(b)$


$\exists A$
$p_\lambda^{ij}(a) \neq p_\lambda^{i'j}(a)$
$p_\lambda^{ij}(a) = p_\lambda^{ij}(a,b) = p_\lambda^{ij}(b)$
$p_\lambda^{ij}(b) \neq p_\lambda^{i'j}(b)$


∀A
(a) ≠ p_λ(a)
(a,b) = p_λ(b)
(b) ≠ p_λ(b)

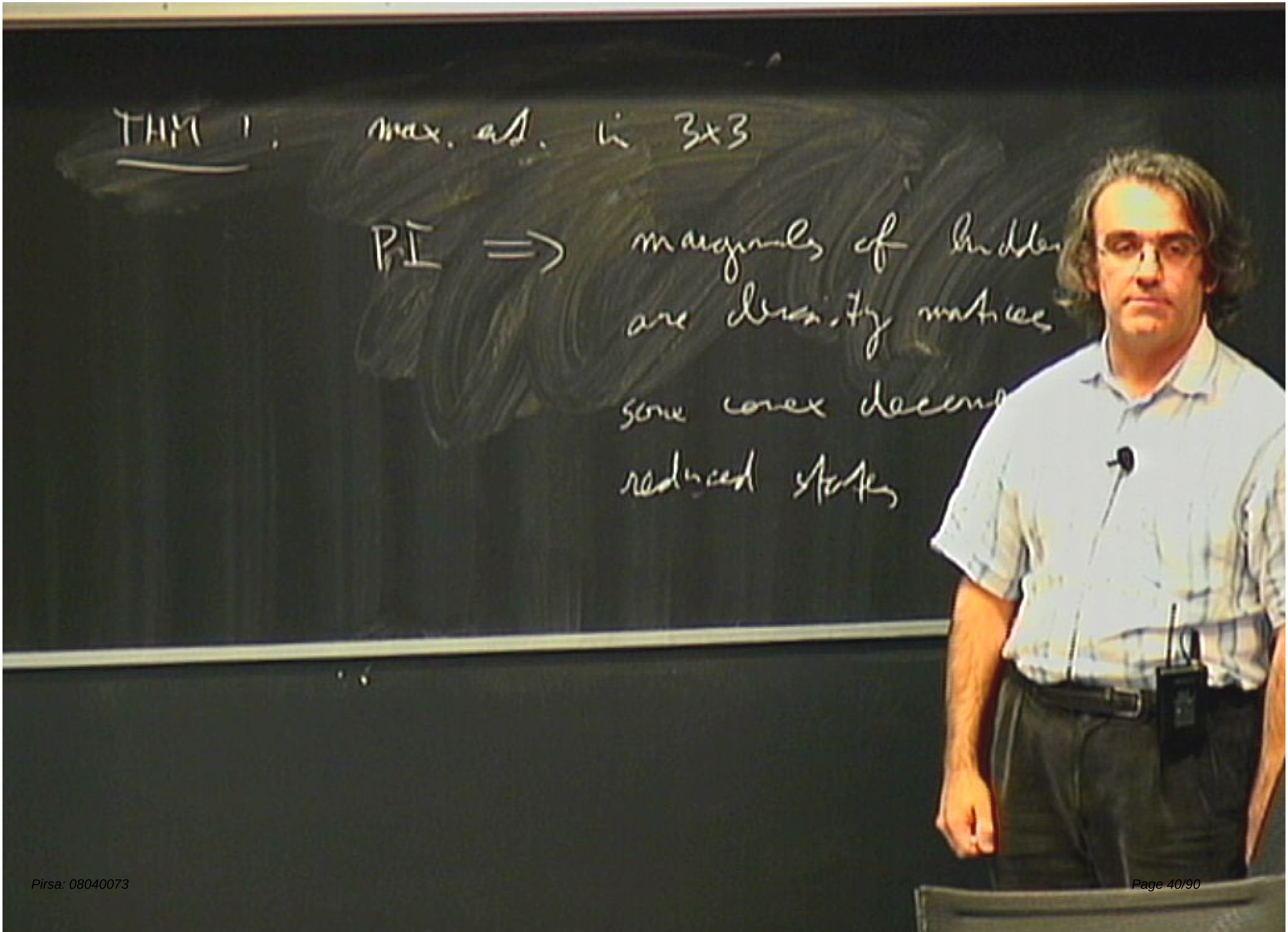

$\forall A$
$p_\lambda^{ij}(a) \neq p_\lambda^{i'j}(a)$
$p_\lambda^{ij}(a) = p_\lambda^{ij}(a,b) = p_\lambda^{ij}(b)$
$p_\lambda^{ij}(b) \neq p_\lambda^{i'j}(b)$

$\forall A$

$$\rho_\lambda^{ij}(a) \neq \rho_\lambda^{i'j}(a)$$

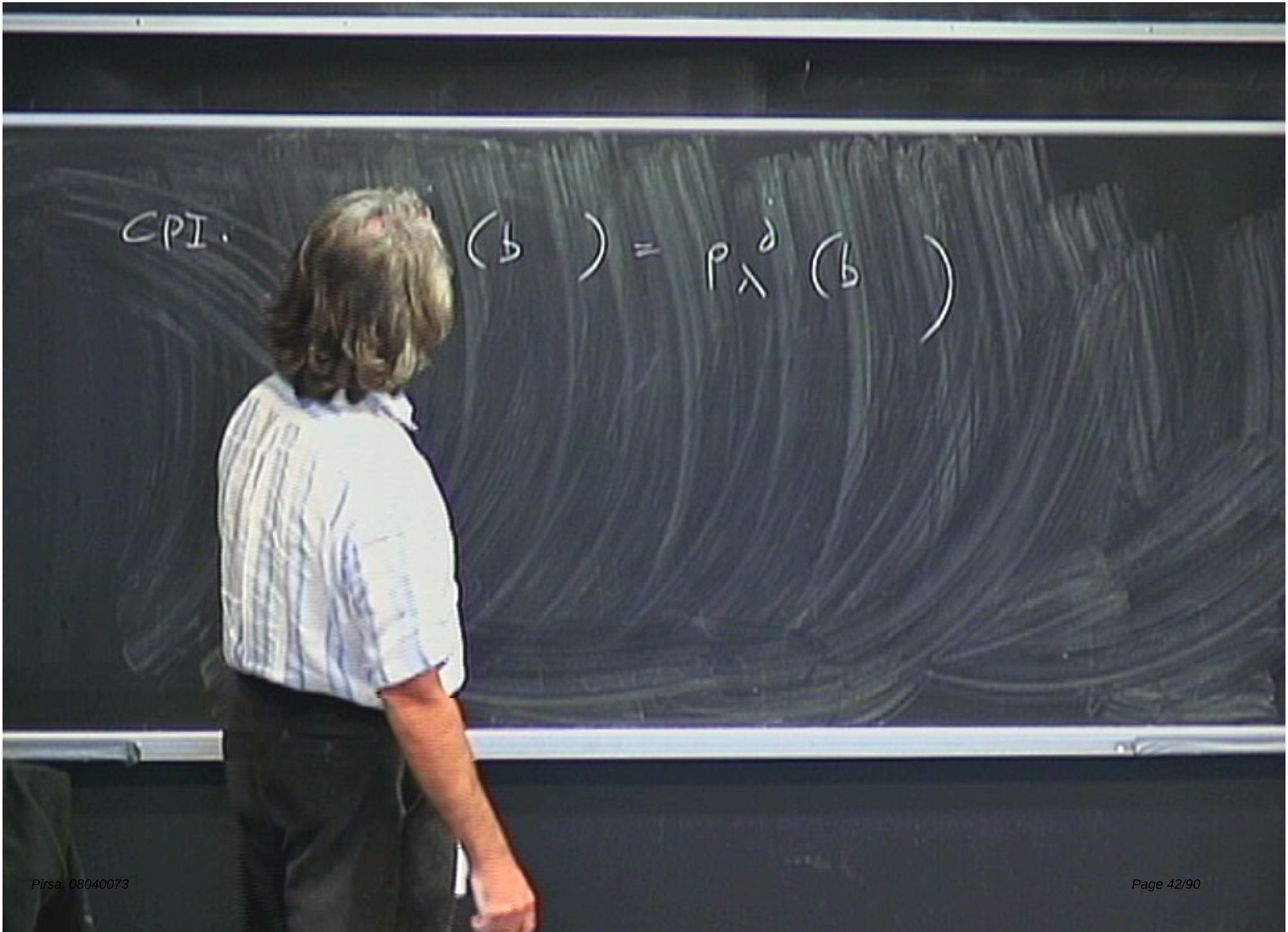
$$\rho_\lambda^{ij}(a) = \rho_\lambda^{ij}(a,b) = \rho_\lambda^{ij}(b)$$

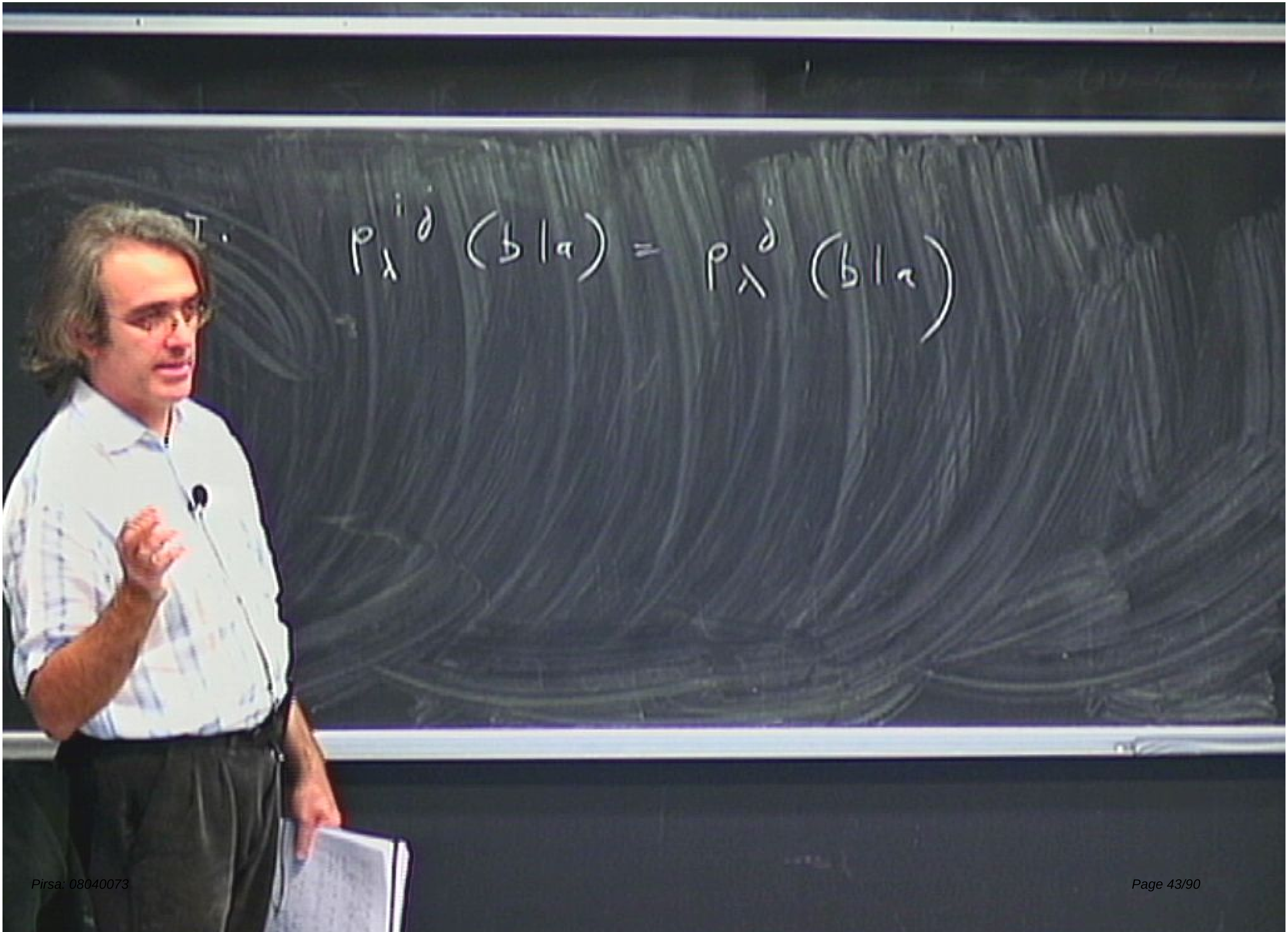
$$\rho_\lambda^{ij}(b) \neq \rho_\lambda^{i'j}(b)$$

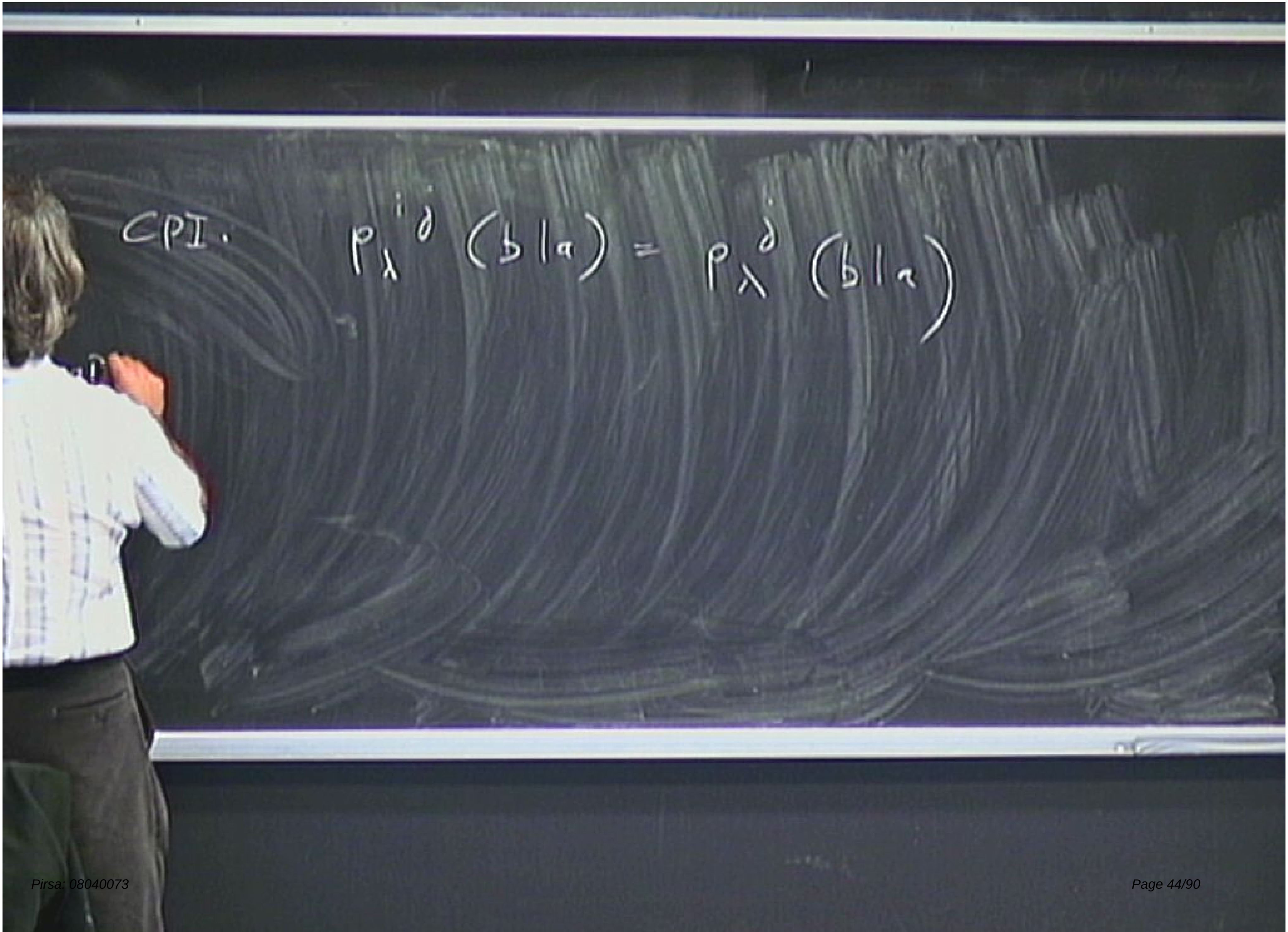

THM 1: max. ent. in 3x3
PI ⇒ marginals of
are density matrices
some convex
reduced states

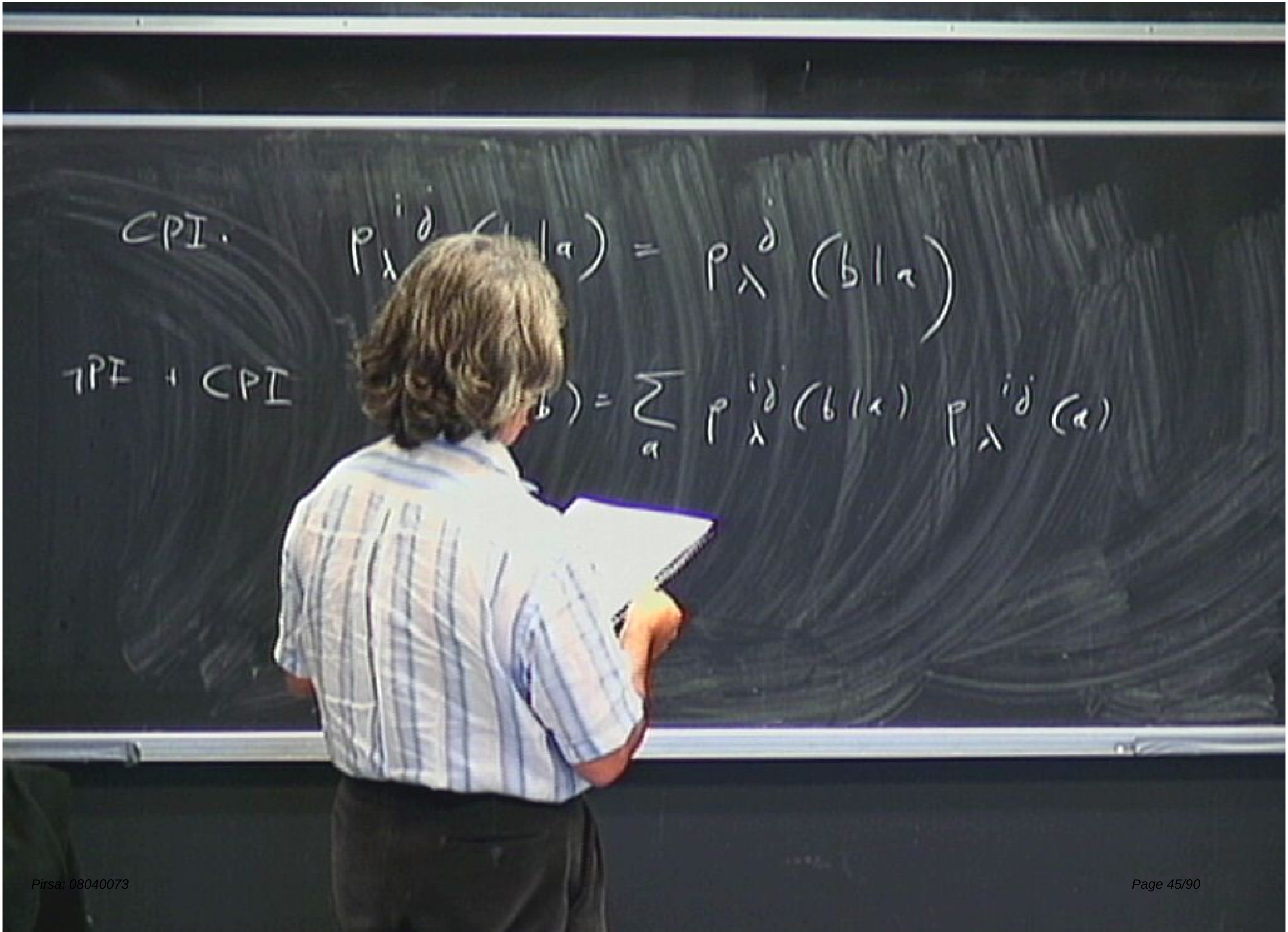
THM 1: max. ent. in 3x3

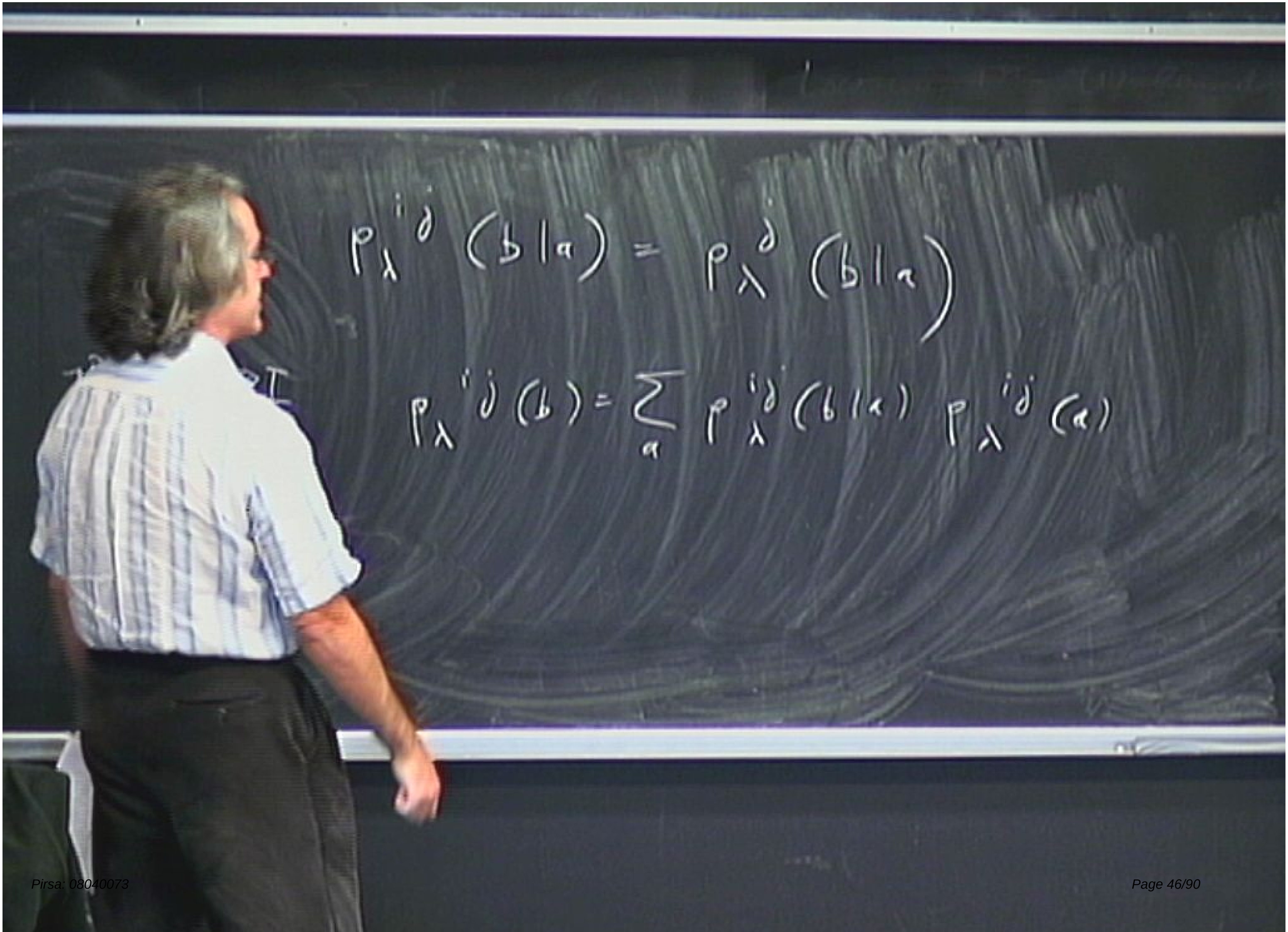
PI $\Rightarrow$ marginals of hidden states are density matrices in some convex decomp. of the reduced states

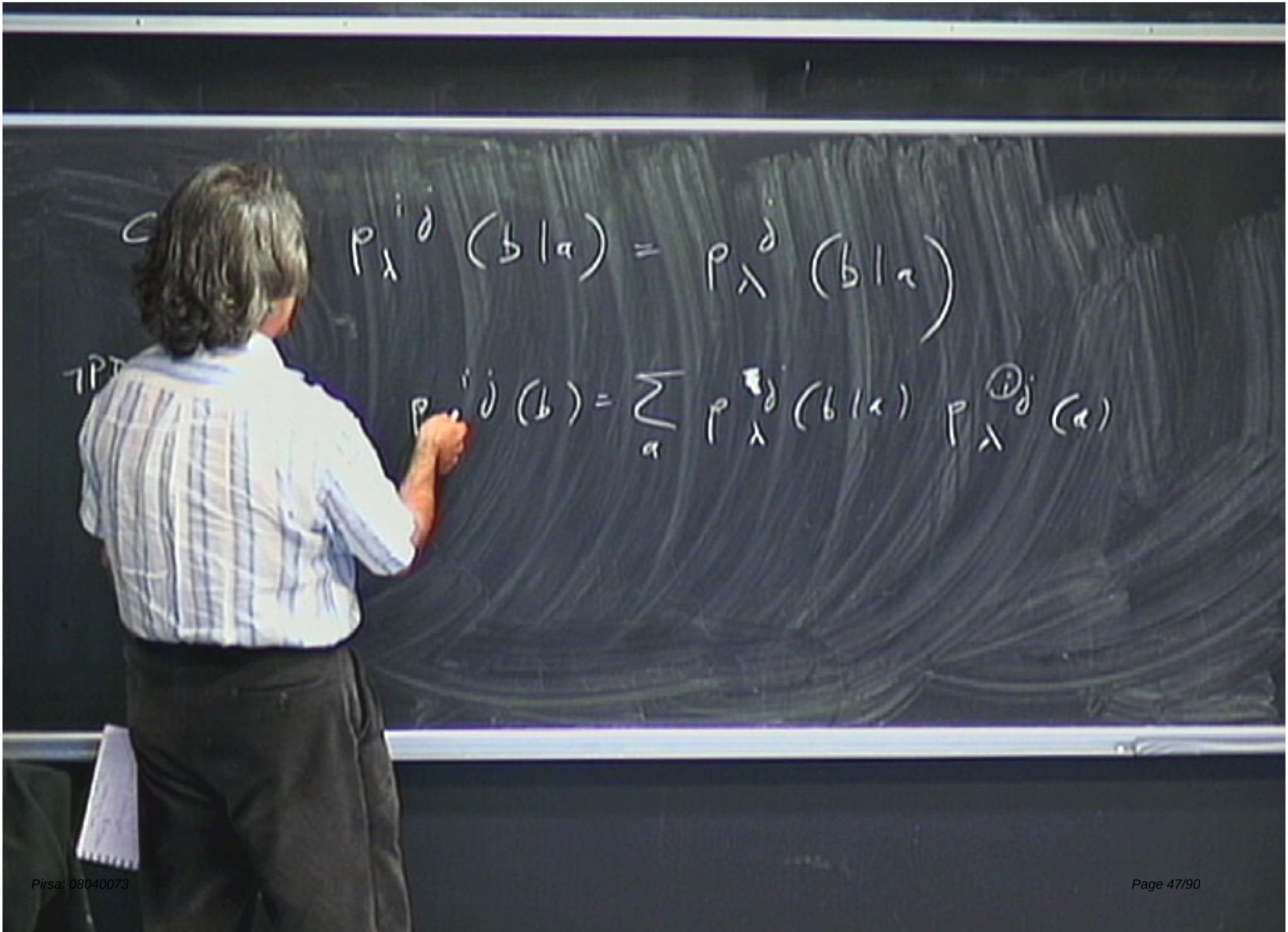

CPI.
(b) = p_λ^d (b)

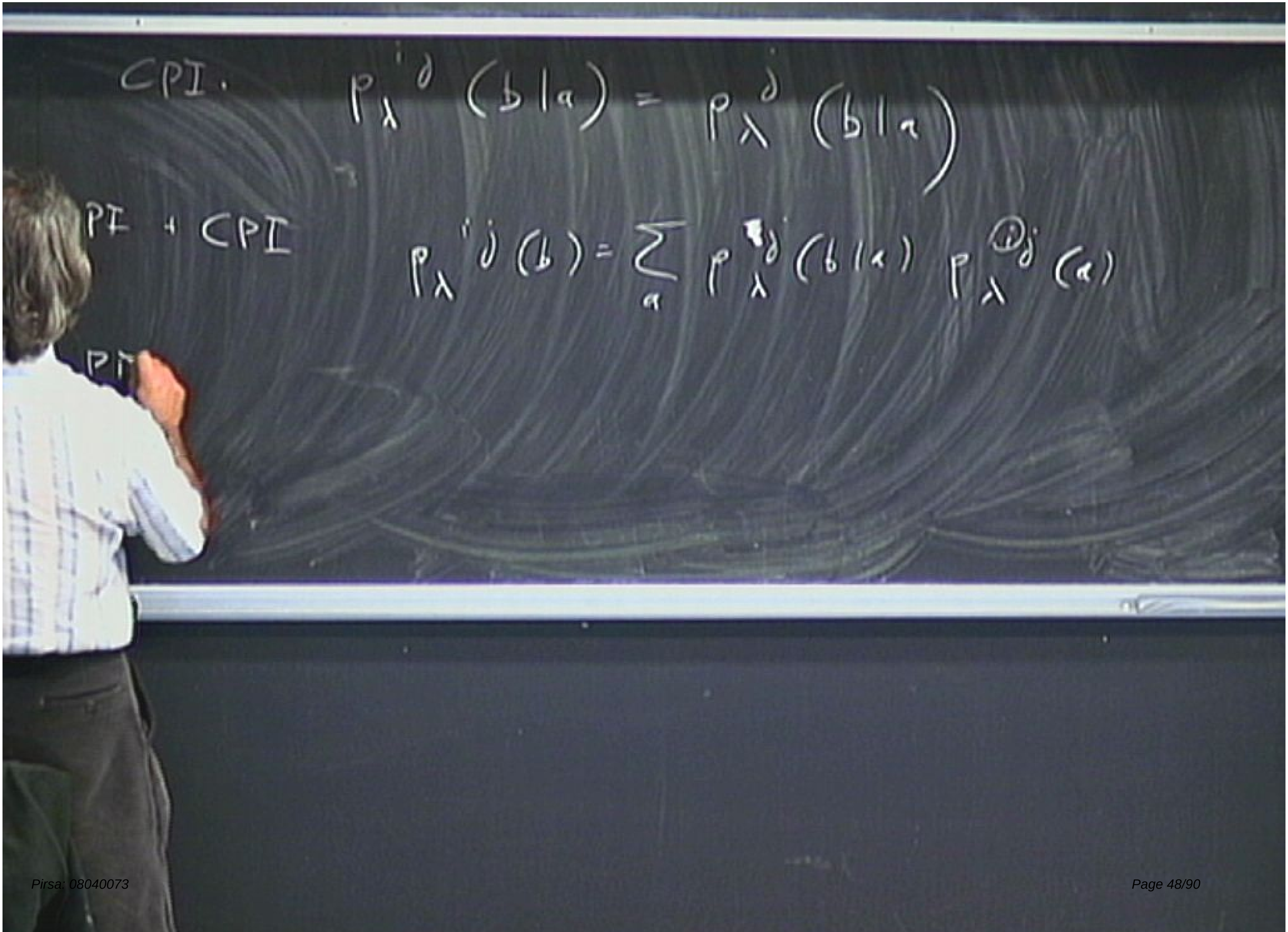

$p_\lambda^{ij}(b|a) = p_\lambda^{j}(b|a)$


CPI.
$p_\lambda^{i,d}(b|a) = p_\lambda^{d}(b|a)$


CPI.
$p_\lambda^{\,i\partial}(\ |a) = p_\lambda^{\,\partial}(b|a)$
TPI + CPI
$) = \sum_a p_\lambda^{\,i\partial}(b|a)\; p_\lambda^{\,i\partial}(a)$


$p_\lambda^{i,j}(b|a) = p_\lambda^{j}(b|a)$
$p_\lambda^{i,j}(b) = \sum_a p_\lambda^{i,j}(b|a)\, p_\lambda^{i,j}(a)$


$p_\lambda^{ij}(b|a) = p_\lambda^{j}(b|a)$
$p_\lambda^{ij}(b) = \sum_a p_\lambda^{ij}(b|a)\, p_\lambda^{ij}(a)$

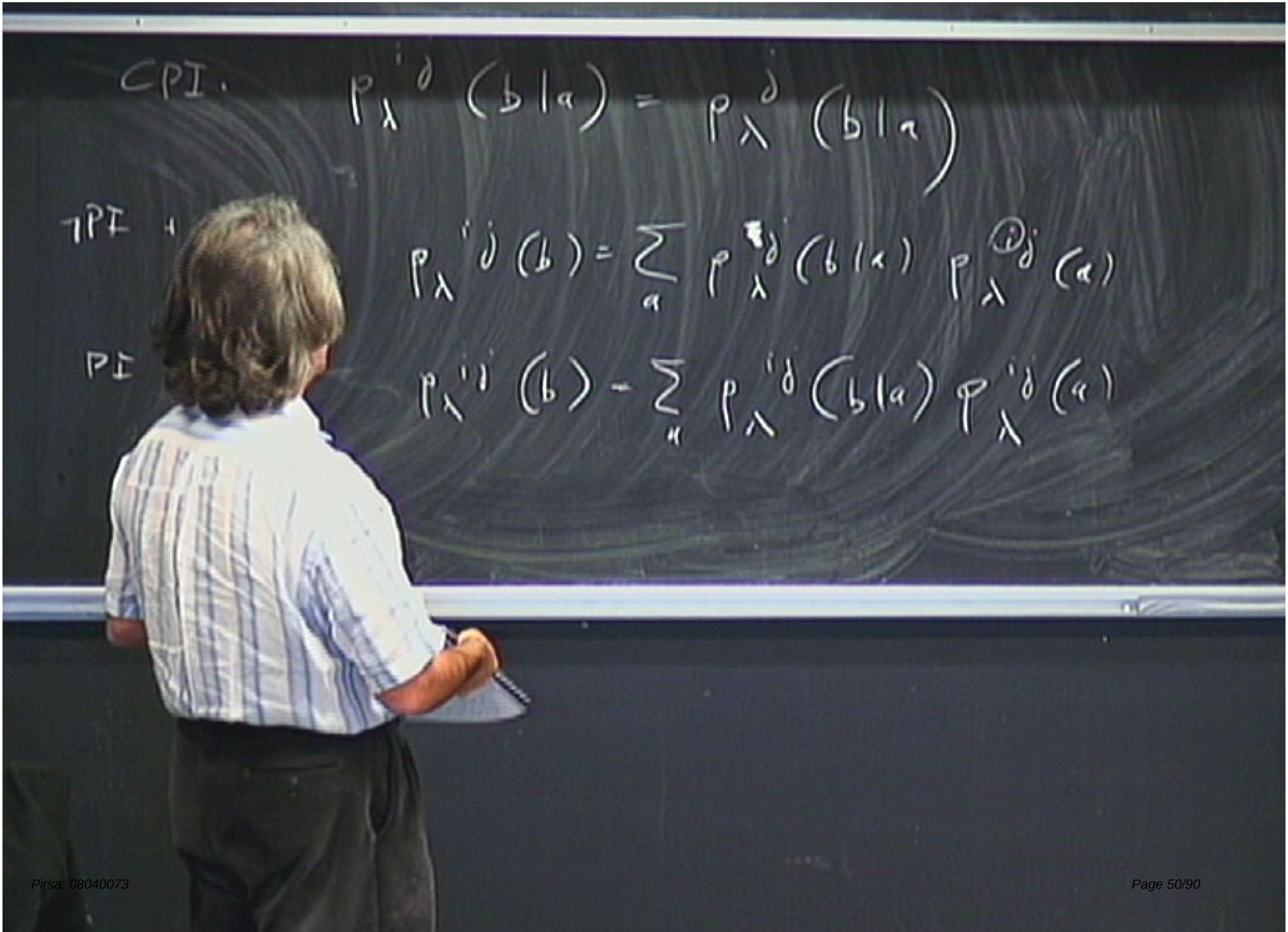


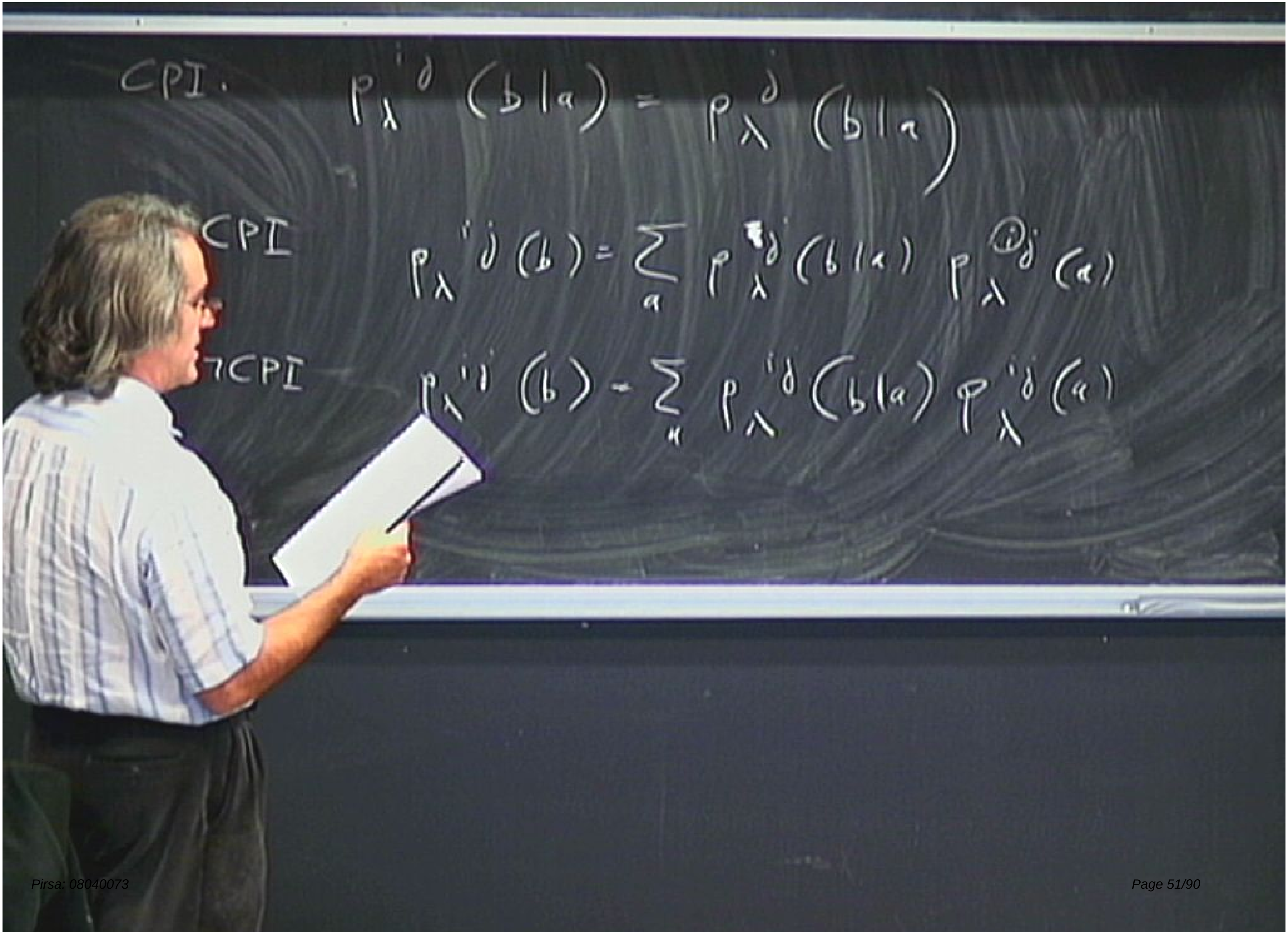
CPI.
$p_\lambda^{ij}(b|a) = p_\lambda^{j}(b|a)$
PI + CPI
$p_\lambda^{ij}(b) = \sum_a p_\lambda^{ij}(b|a)\, p_\lambda^{ij}(a)$
PI

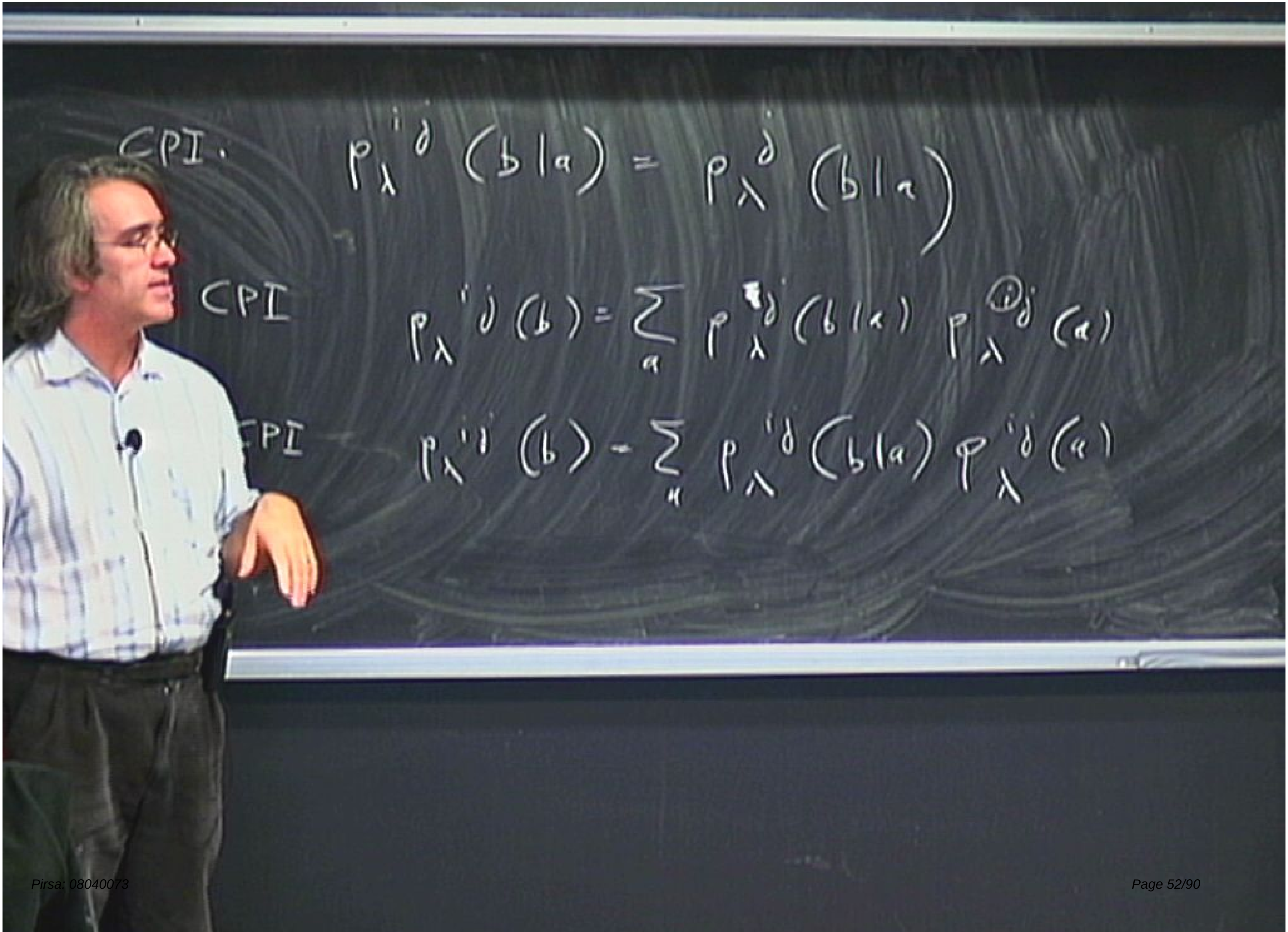
CPI. $p_\lambda^{ij}(b|a) = p_\lambda^{j}(b|a)$

¬PI + CPI $\quad p_\lambda^{ij}(b) = \sum_a p_\lambda^{ij}(b|a)\, p_\lambda^{ij}(a)$

PI + ¬CPI


CPI. $p_\lambda^{ij}(b|a) = p_\lambda^{j}(b|a)$
TPI $p_\lambda^{ij}(b) = \sum_a p_\lambda^{ij}(b|a)\, p_\lambda^{ij}(a)$
PI $p_\lambda^{ij}(b) = \sum_a p_\lambda^{ij}(b|a)\, p_\lambda^{ij}(a)$

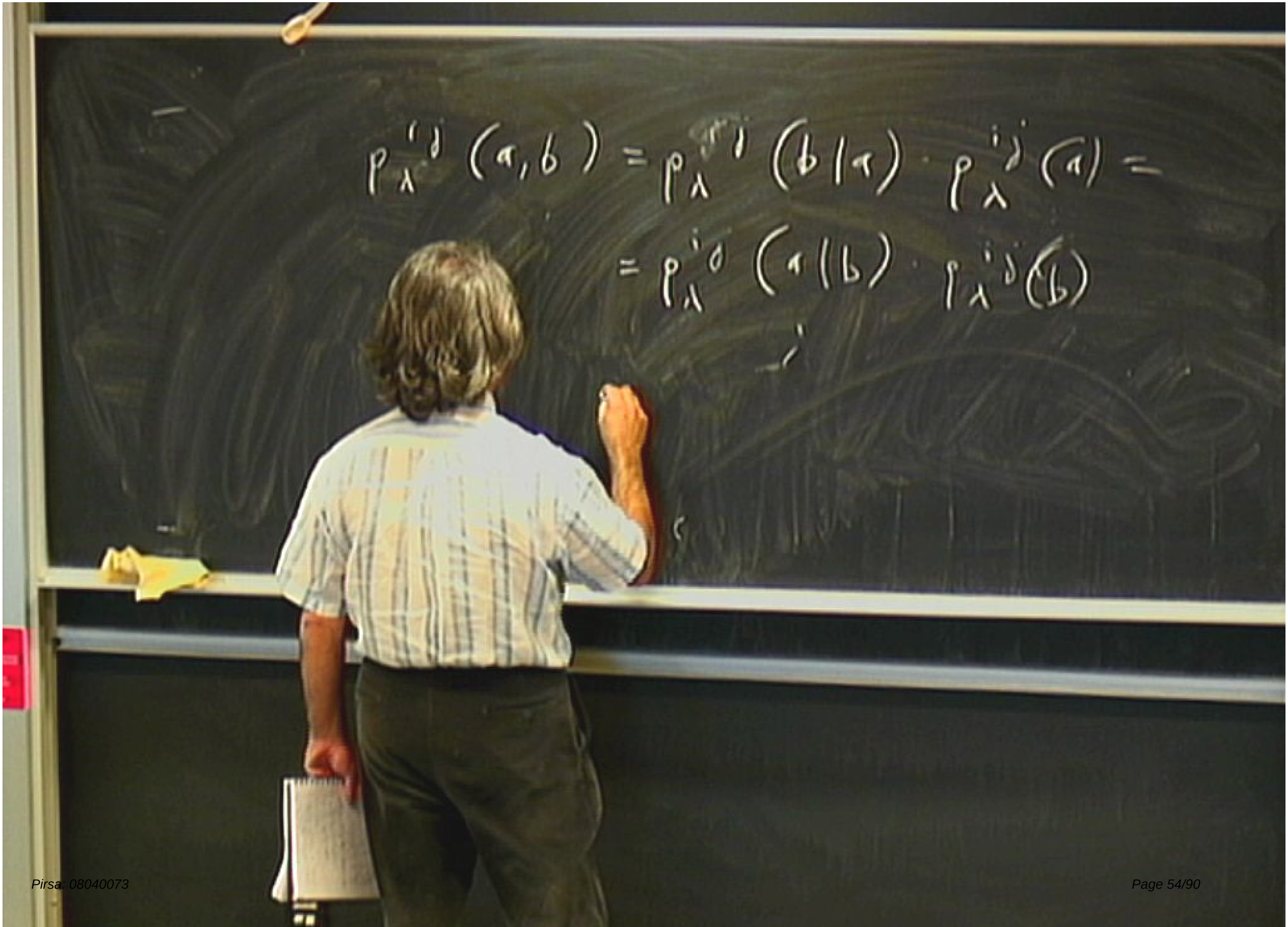

CPI: $p_\lambda^{i,j}(b|a) = p_\lambda^{j}(b|a)$
CPI $p_\lambda^{i,j}(b) = \sum_a p_\lambda^{i,j}(b|a)\, p_\lambda^{i,j}(a)$
¬CPI $p_\lambda^{i,j}(b) = \sum_a p_\lambda^{i,j}(b|a)\, p_\lambda^{i,j}(a)$

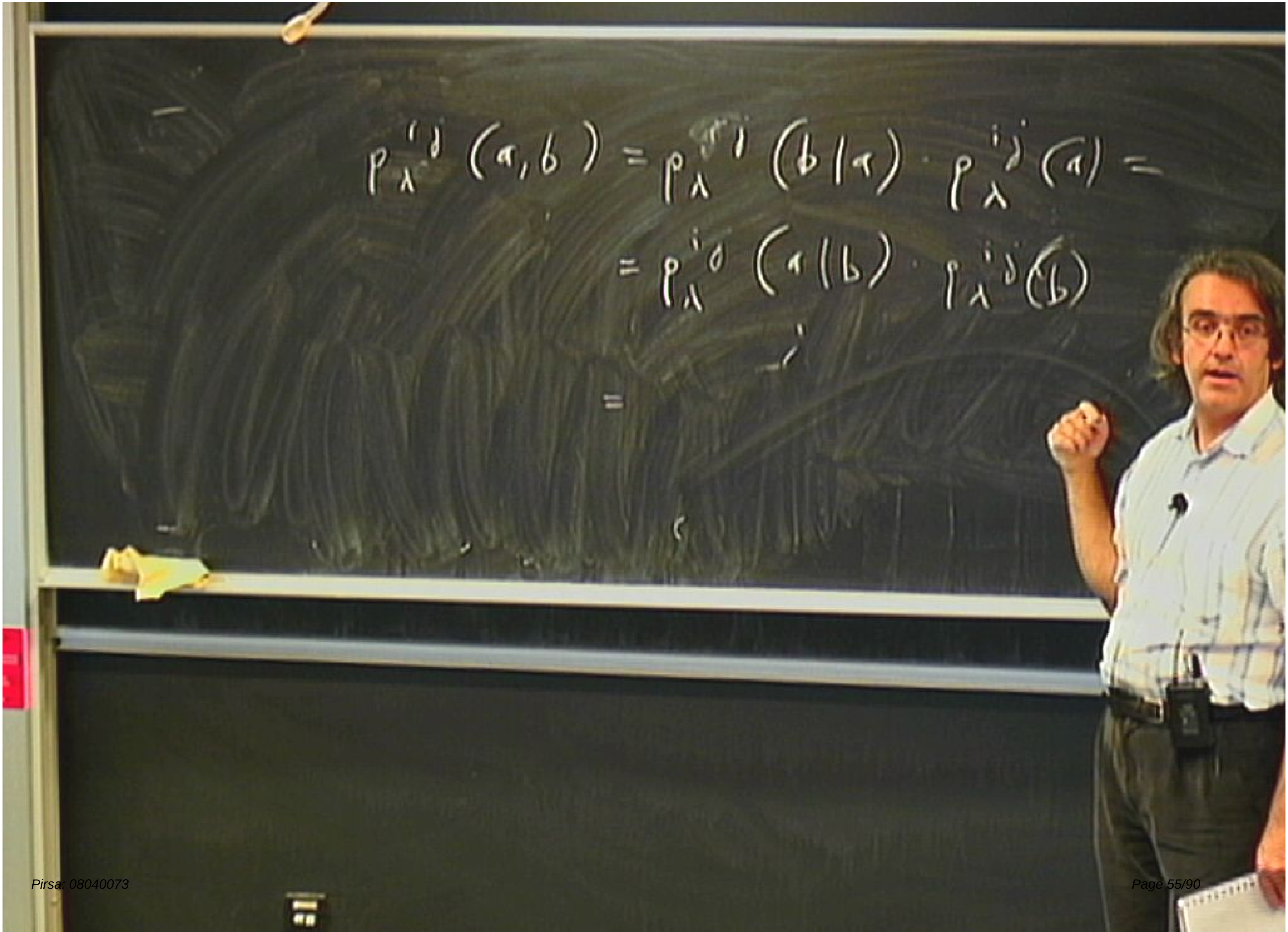

CPI: $p_\lambda^{i,j}(b|a) = p_\lambda^{j}(b|a)$
CPI $p_\lambda^{i,j}(b) = \sum_a p_\lambda^{i,j}(b|a)\, p_\lambda^{i,j}(a)$
CPI $p_\lambda^{i,j}(b) = \sum_a p_\lambda^{i,j}(b|a)\, p_\lambda^{i,j}(a)$

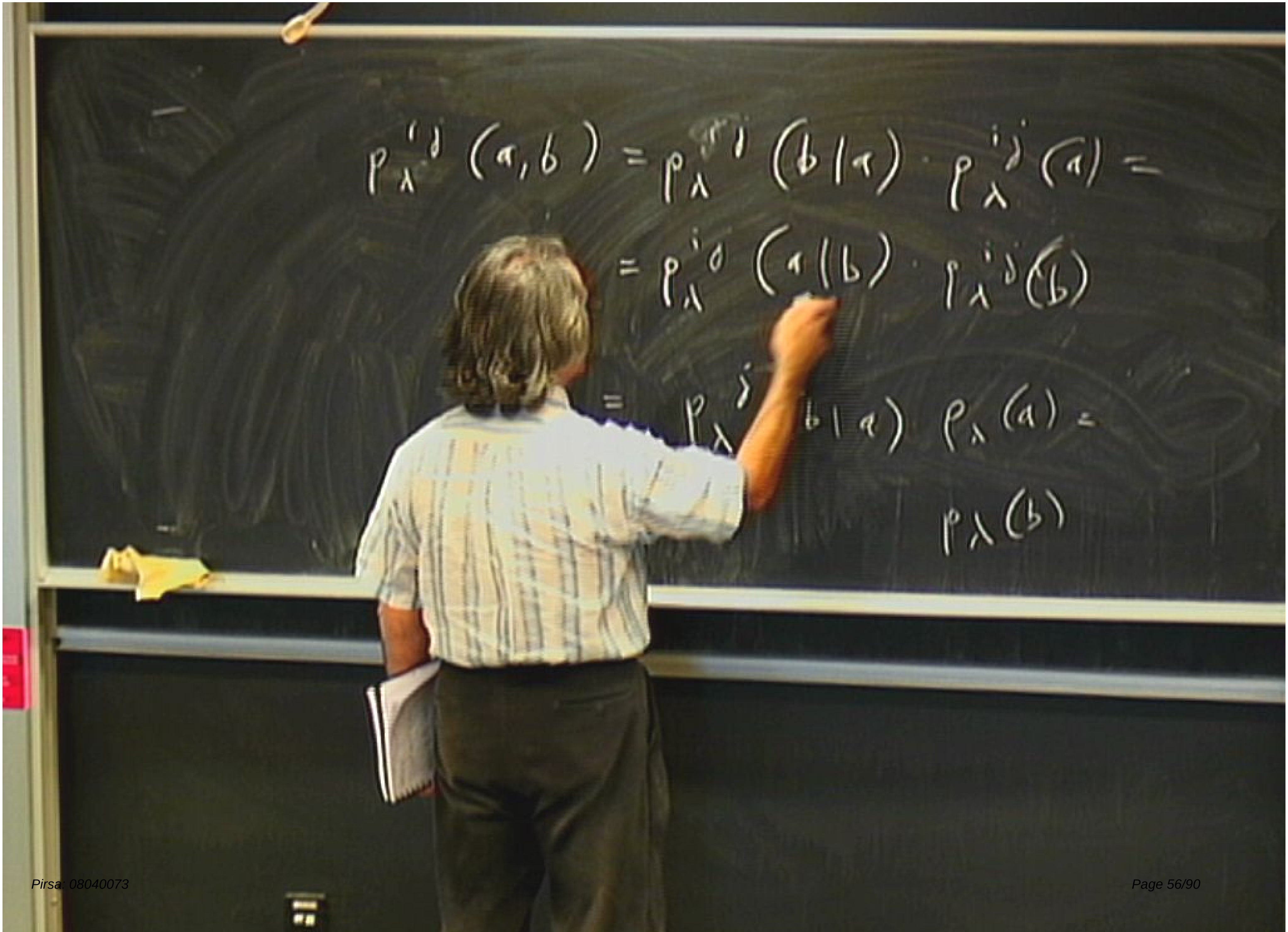
CPI. $p_\lambda^{i,j}(b|a) = p_\lambda^{j}(b|a)$

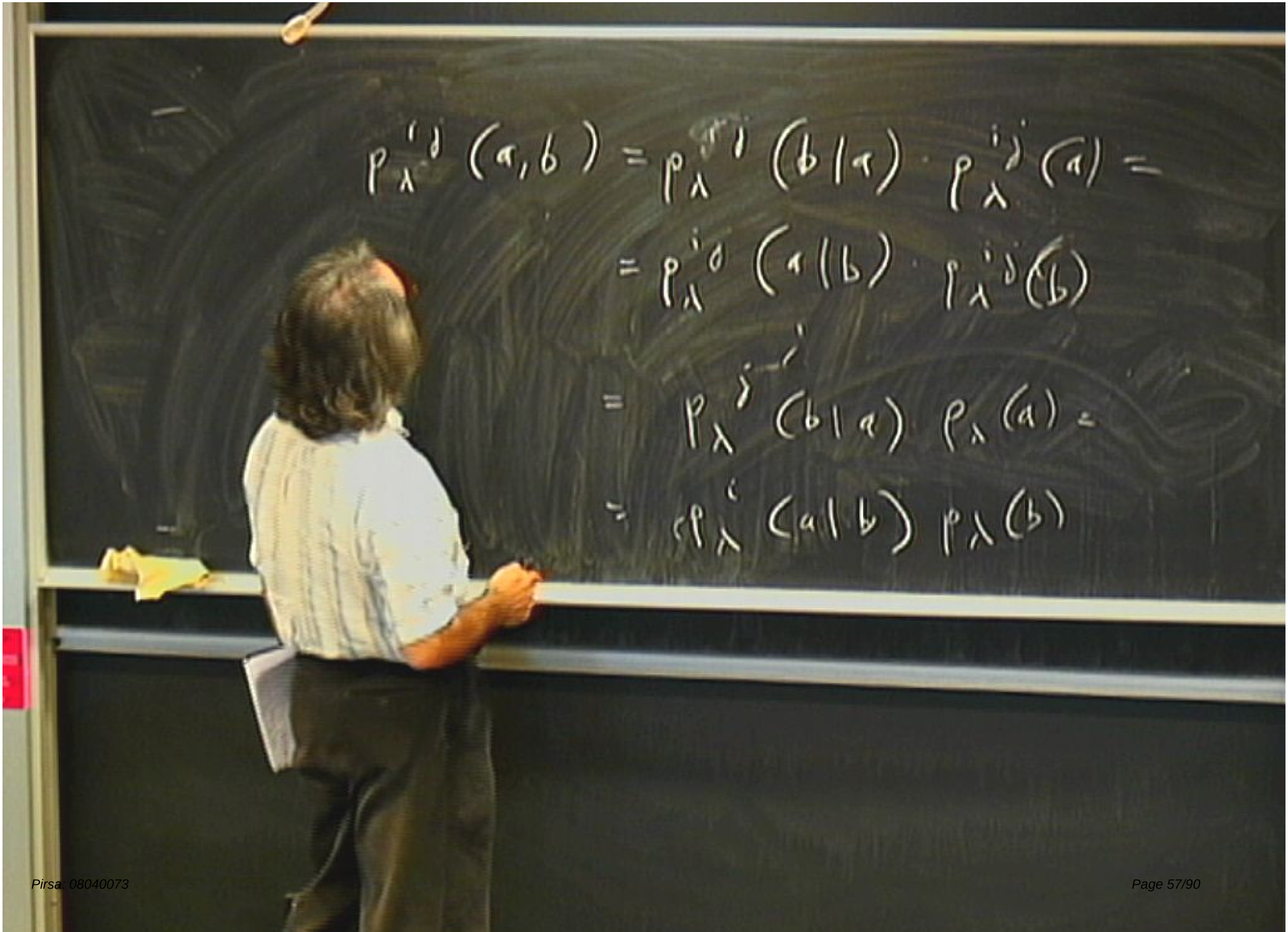
¬PI + CPI $p_\lambda^{i,j}(b) = \sum_a p_\lambda^{j}(b|a)\, p_\lambda^{i,j}(a)$

PI + ¬CPI $p_\lambda^{i,j}(b) = \sum_a p_\lambda^{i,j}(b|a)\, p_\lambda^{i,j}(a)$


$\rho_\lambda^{ij}(a,b) = \rho_\lambda^{ij}(b|a) \cdot \rho_\lambda^{ij}(a) =$
$= \rho_\lambda^{ij}(a|b) \cdot \rho_\lambda^{ij}(b)$


$p_\lambda^{ij}(a,b) = p_\lambda^{ij}(b|a) \cdot p_\lambda^{ij}(a) =$
$= p_\lambda^{ij}(a|b) \cdot p_\lambda^{ij}(b)$


$\rho_\lambda^{ij}(a,b) = \rho_\lambda^{ij}(b|a) \cdot \rho_\lambda^{ij}(a) =$
$= \rho_\lambda^{ij}(a|b) \cdot \rho_\lambda^{ij}(b)$
$= \rho_\lambda^{j}(b|a)\ \rho_\lambda(a) =$
$\rho_\lambda(b)$


$p_\lambda^{ij}(a,b) = p_\lambda^{ij}(b|a) \cdot p_\lambda^{ij}(a) =$
$= p_\lambda^{ij}(a|b) \cdot p_\lambda^{ij}(b)$
$= p_\lambda^{j}(b|a)\, p_\lambda(a) =$
$= p_\lambda^{i}(a|b)\, p_\lambda(b)$

$$p_\lambda^{ij}(a,b) = p_\lambda^{ij}(b|a) \cdot p_\lambda^{ij}(a) =$$

$$= p_\lambda^{ij}(a|b) \cdot p_\lambda^{ij}(b)$$

$$= p_\lambda^{j}(b|a)\, p_\lambda(a) =$$

$$= p_\lambda^{i}(a|b)\, p_\lambda(b)$$

$$p_\lambda^{ij}(a,b) = p_\lambda^{ij}(b|a)\cdot p_\lambda^{ij}(a) =$$

$$= p_\lambda^{ij}(a|b)\cdot p_\lambda^{ij}(b)$$

THM 2

PI + CPI

$\Rightarrow$ the joint probs given by the h. states are non-contextual

$$= p_\lambda^{j}(b|a)\; p_\lambda(a) =$$

$$= p_\lambda^{i}(a|b)\; p_\lambda(b)$$

THM 2

PI + CPI

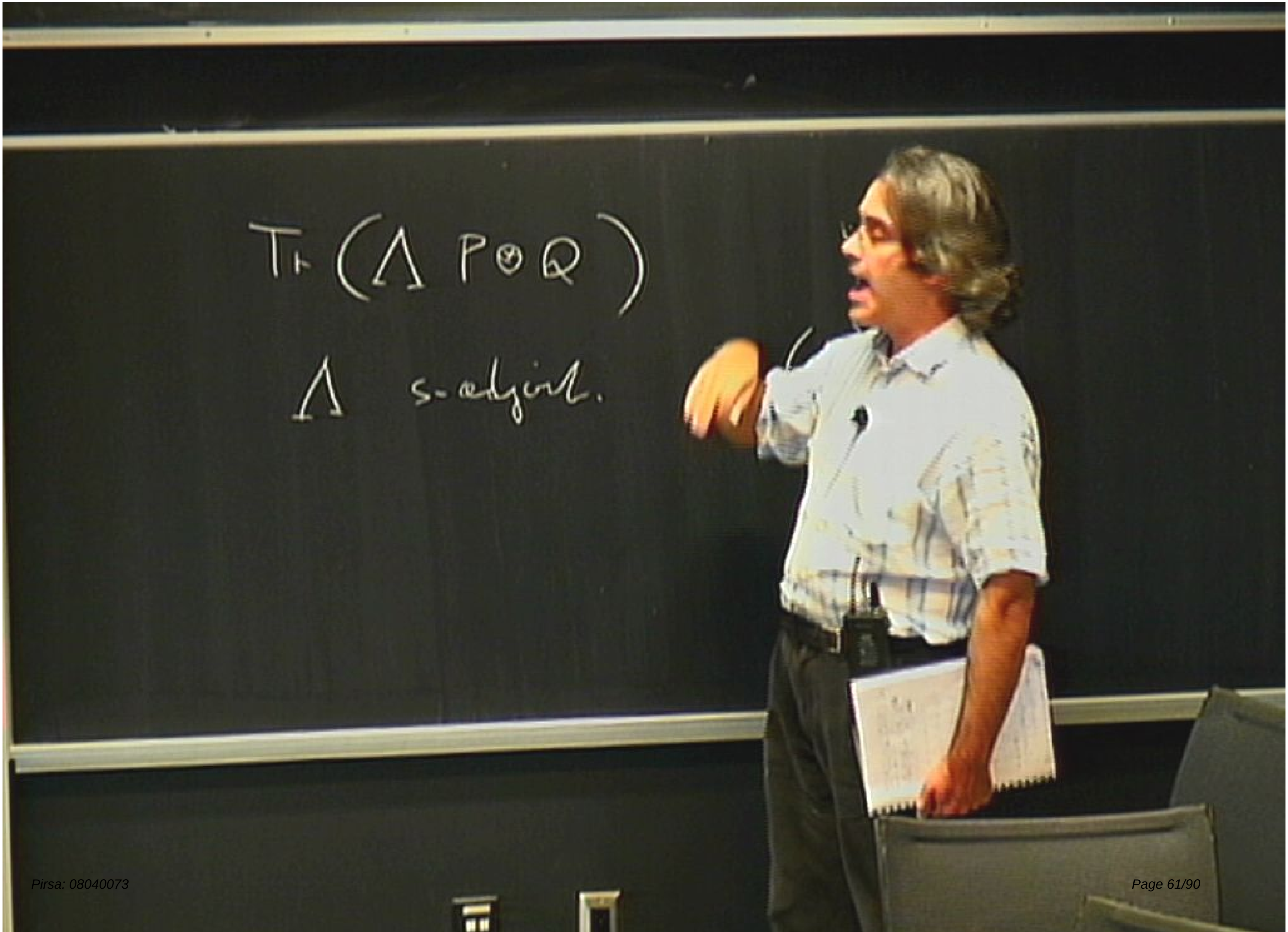
$\Rightarrow$ the joint probs given by the h. states are non-contextual

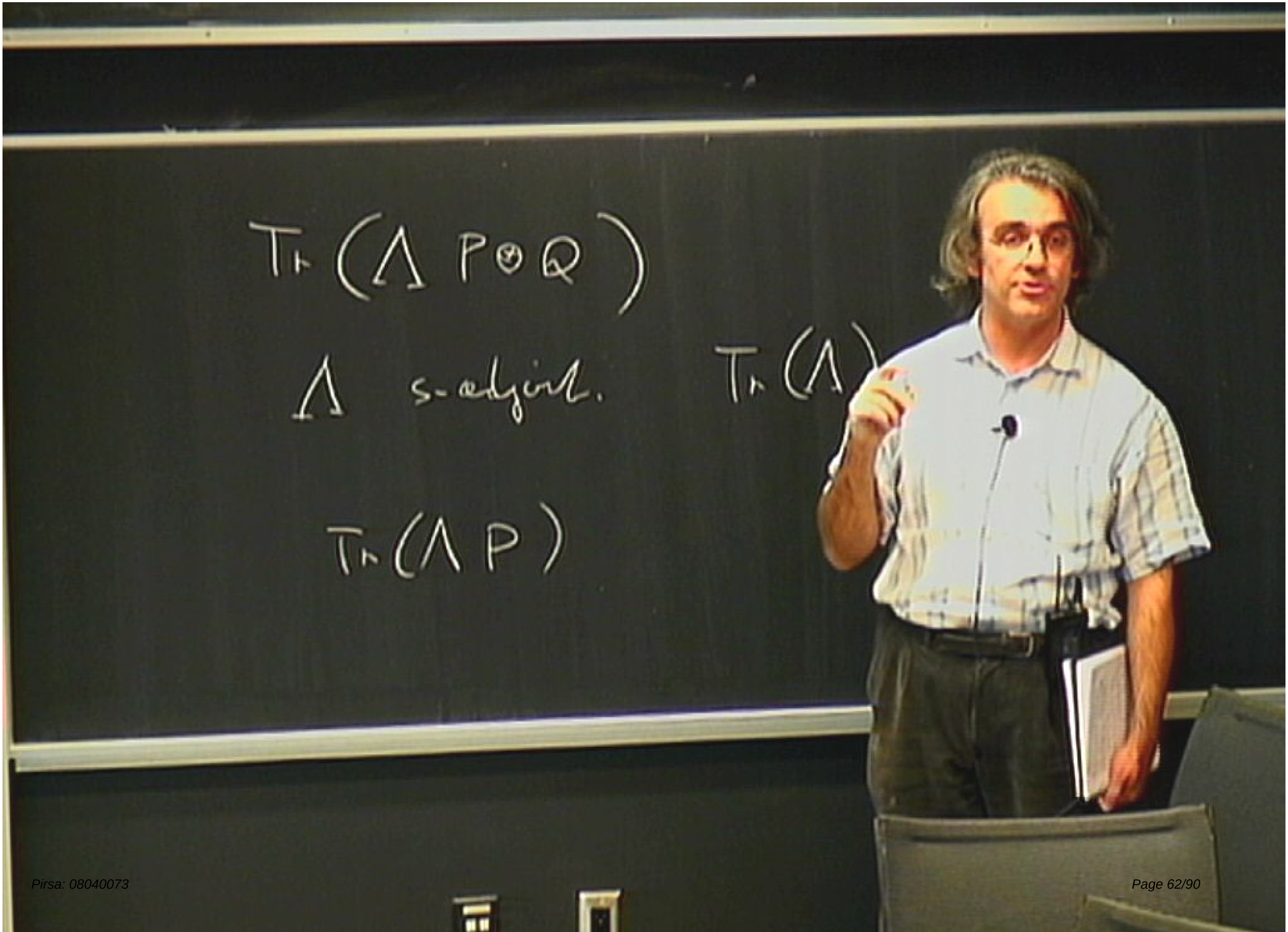
$$p_\lambda^{ij}(a,b) = p_\lambda^{ij}(b|a)\cdot p_\lambda^{ij}(a) =$$

$$= p_\lambda^{ij}(a|b)\cdot p_\lambda^{ij}(b)$$

$$= p_\lambda^{j}(b|a)\, p_\lambda(a) =$$

$$= p_\lambda^{i}(a|b)\, p_\lambda(b)$$


Tr(Λ P⊗Q)
Λ s-adjoint.


$\mathrm{Tr}(\Lambda\, P \otimes Q)$
Λ s-adjoint.
$\mathrm{Tr}(\Lambda)$
$\mathrm{Tr}(\Lambda P)$

$$\mathrm{Tr}(\Lambda\, P \otimes Q)$$

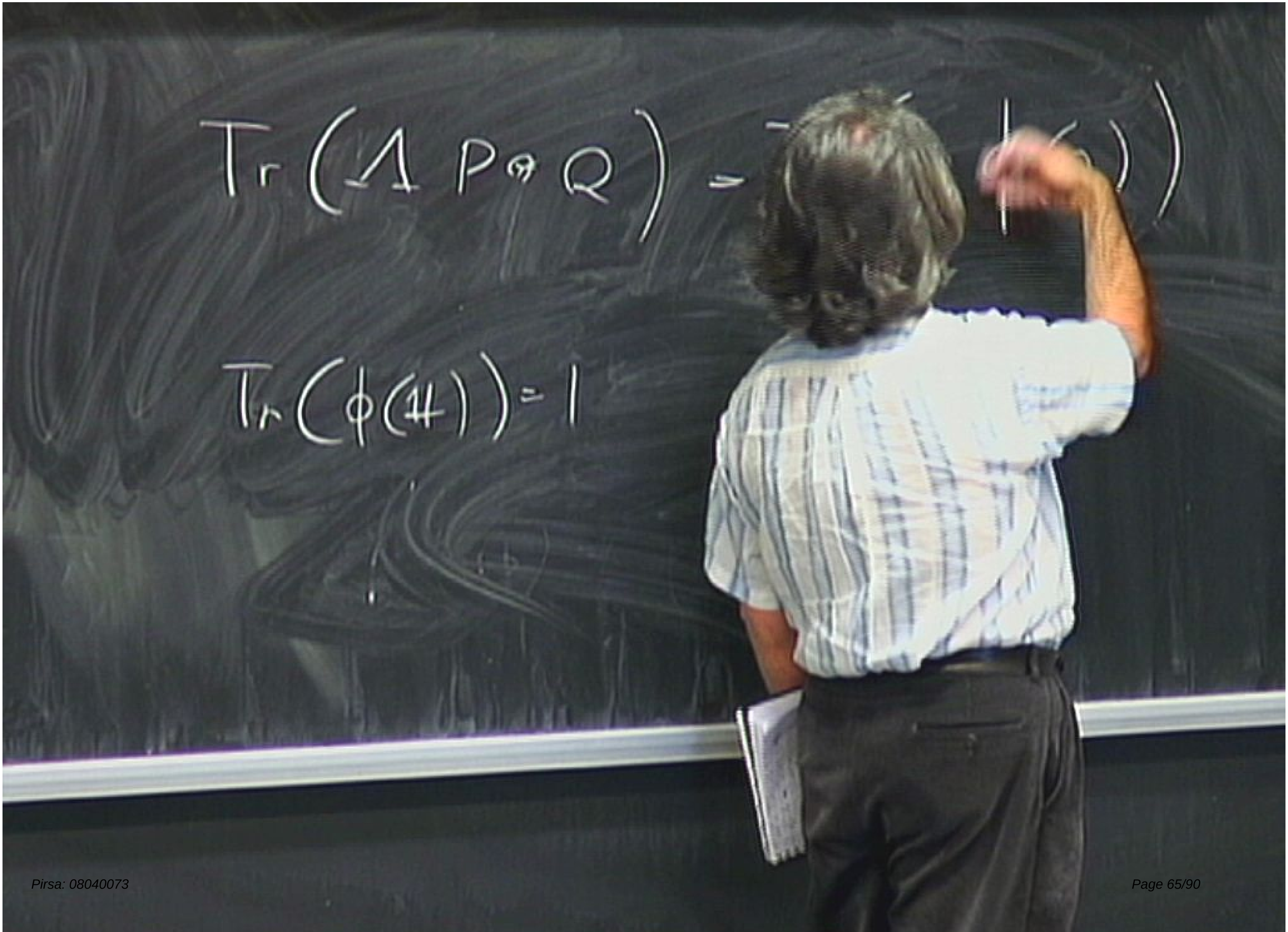
$$\Lambda = |\psi\rangle\langle\psi|$$

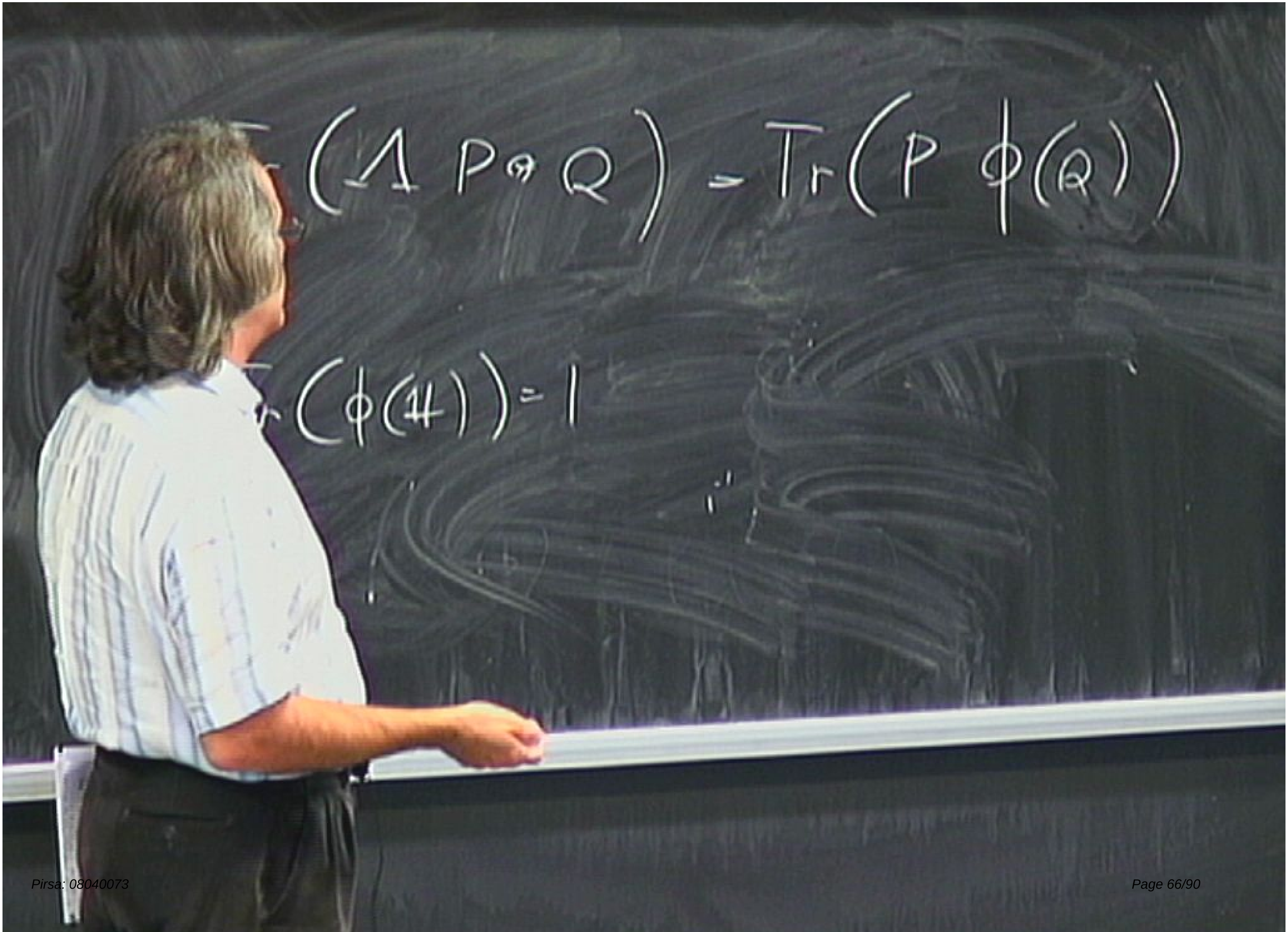
Λ s-adjoint.

$$\mathrm{Tr}(\Lambda) = 1$$

$$\mathrm{Tr}(\Lambda P)$$

$$\mathrm{Tr}(\Lambda P \phi Q) = \mathrm{Tr}(P\, \phi(Q))$$


Tr(Λ P⊗Q) =
Tr(φ(1)) = 1


(Λ P⊗Q) = Tr(P φ(Q))
(φ(1)) = 1

$$\mathrm{Tr}\left(\Lambda\, P \otimes Q\right) = \mathrm{Tr}\left(P\, \phi(Q)\right)$$

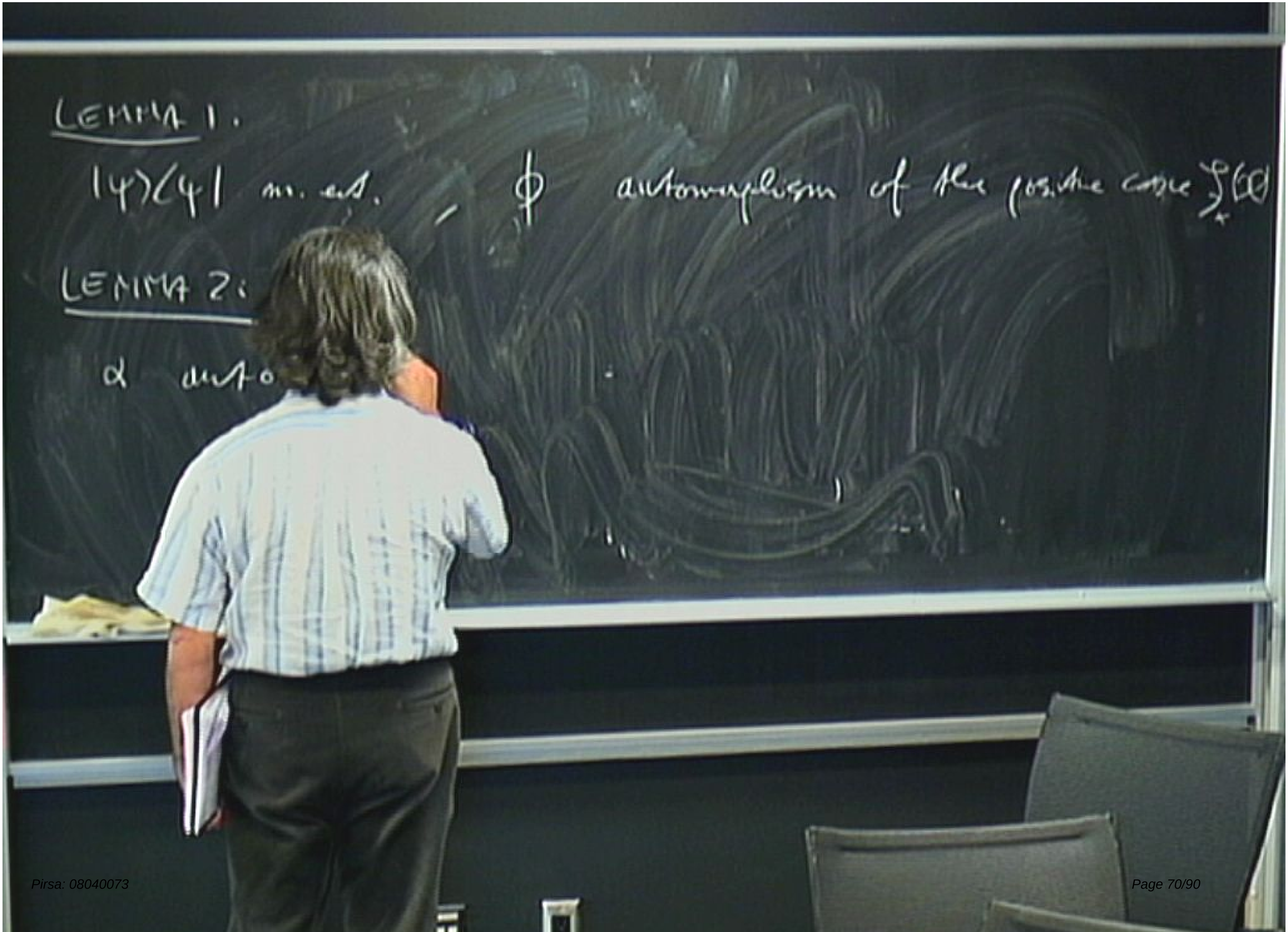
$$\mathrm{Tr}\left(\phi(\mathbb{1})\right) = 1$$

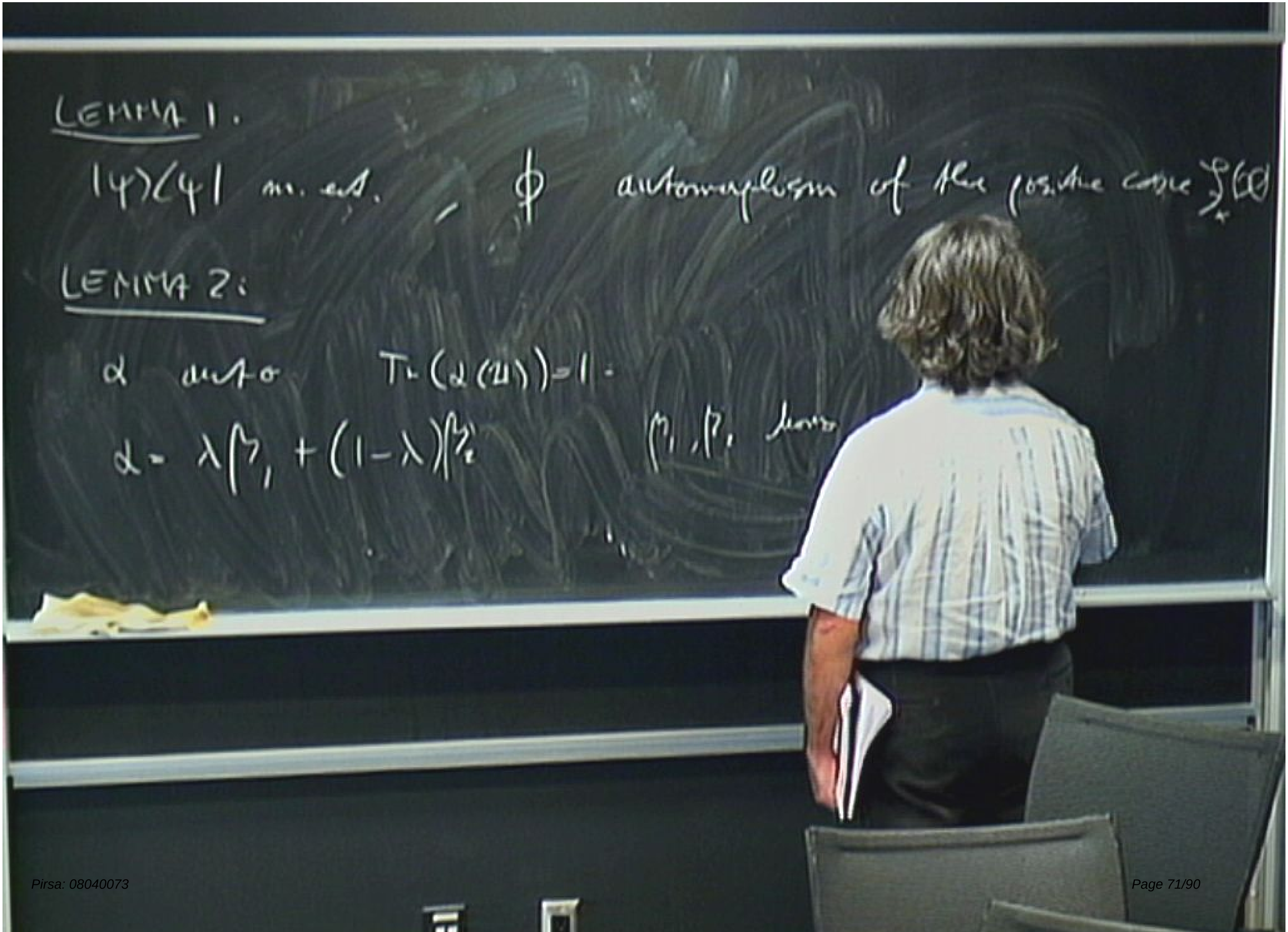
LEMMA 1.

$|\psi\rangle\langle\psi|$ m. e.s.

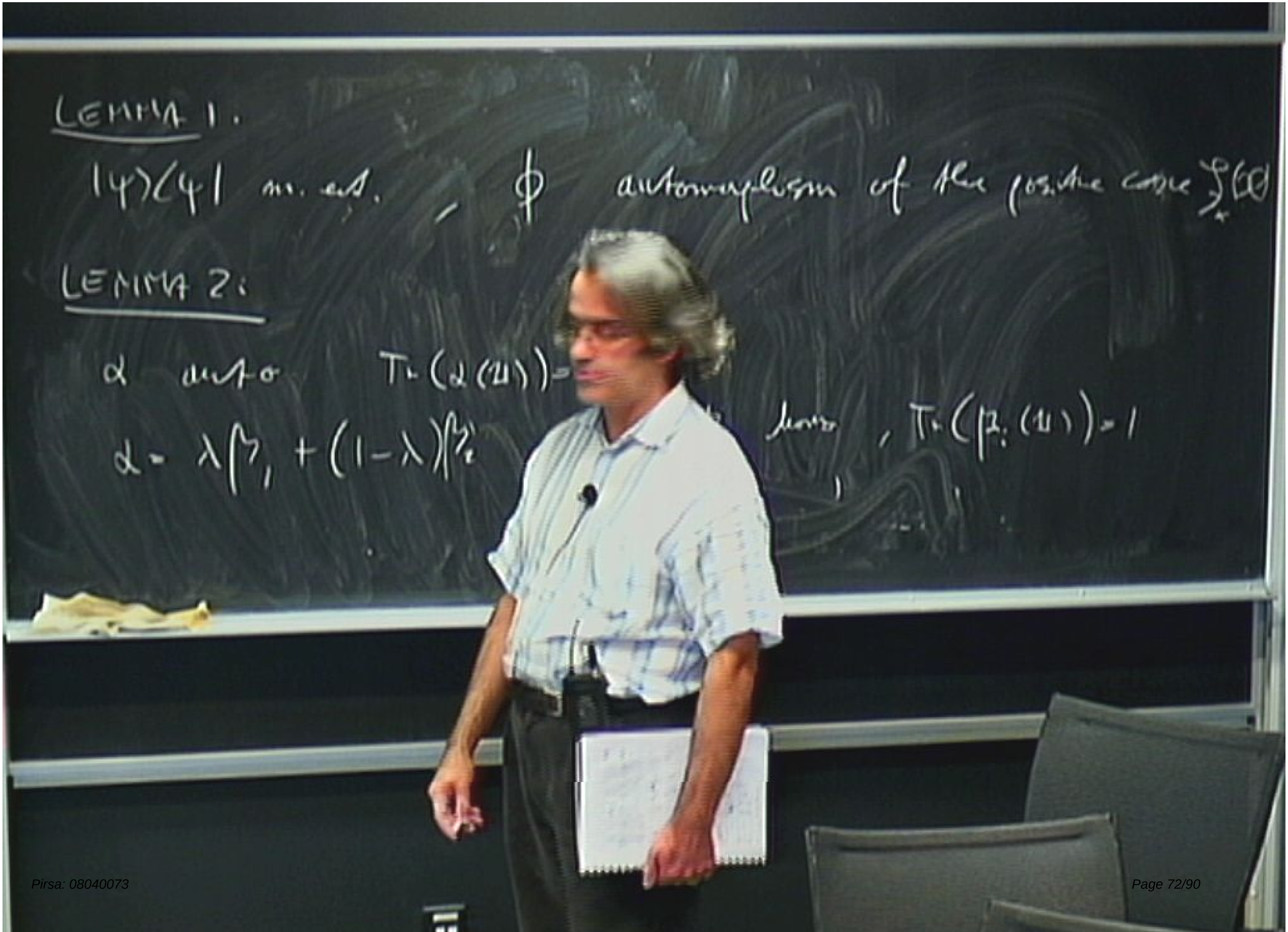
LEMMA 1.

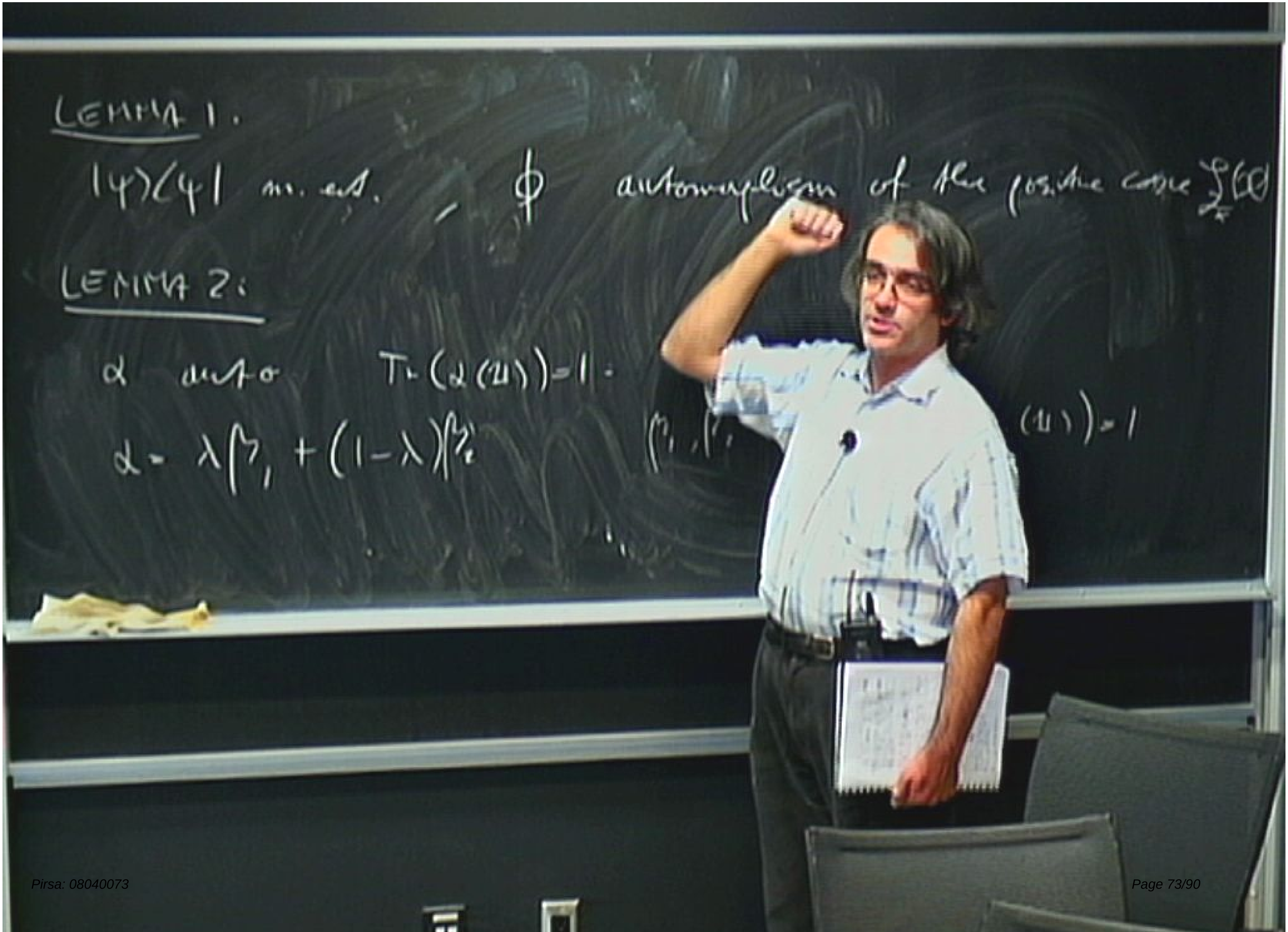
$|\psi\rangle\langle\psi|$ m.e.s. , ϕ automorphism of the positive cone $\mathfrak{S}_+(\mathcal{H})$

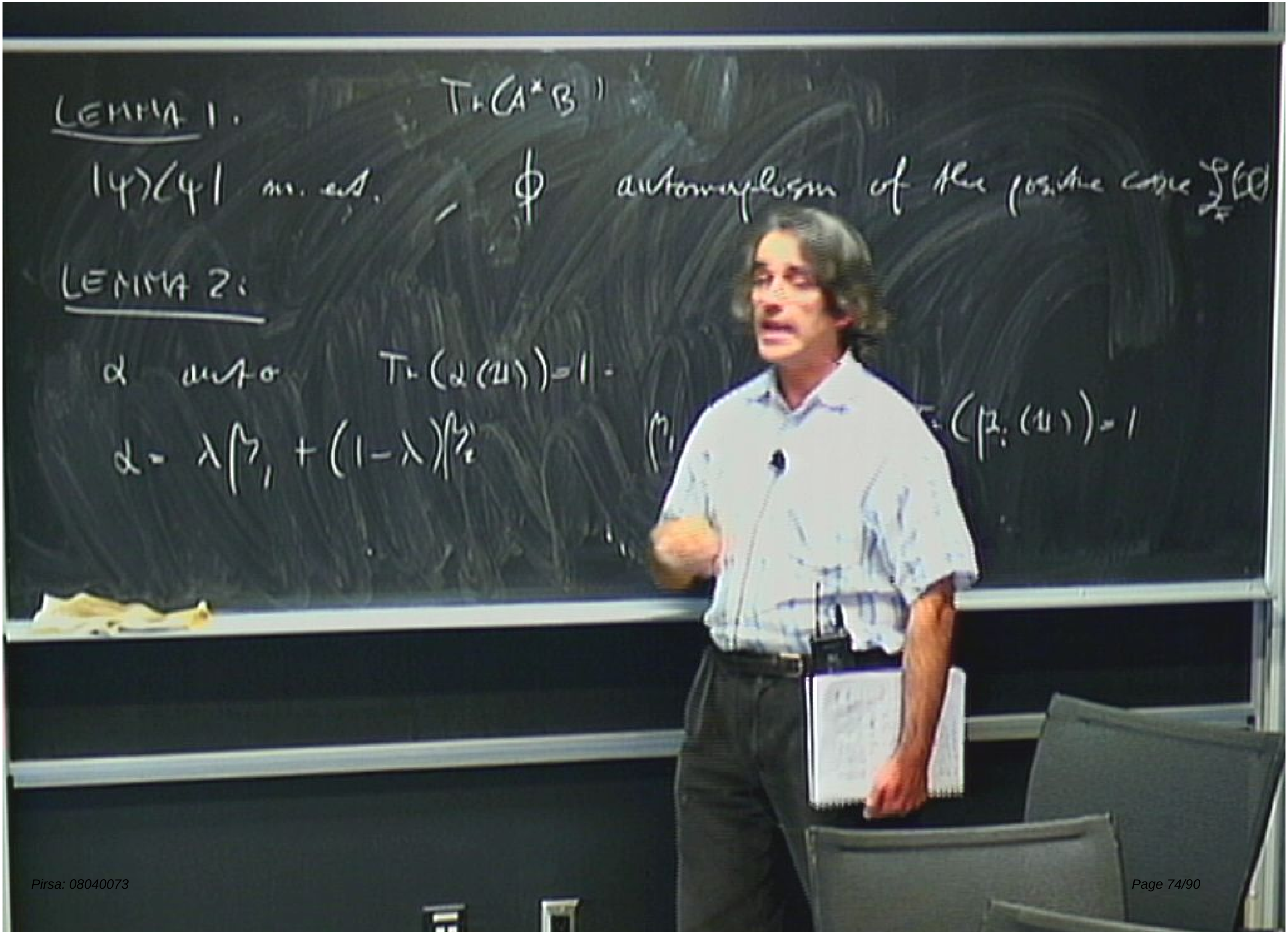

LEMMA 1.
|ψ⟩⟨ψ| m. e.s.
ϕ automorphism of the positive cone
LEMMA 2:
α auto

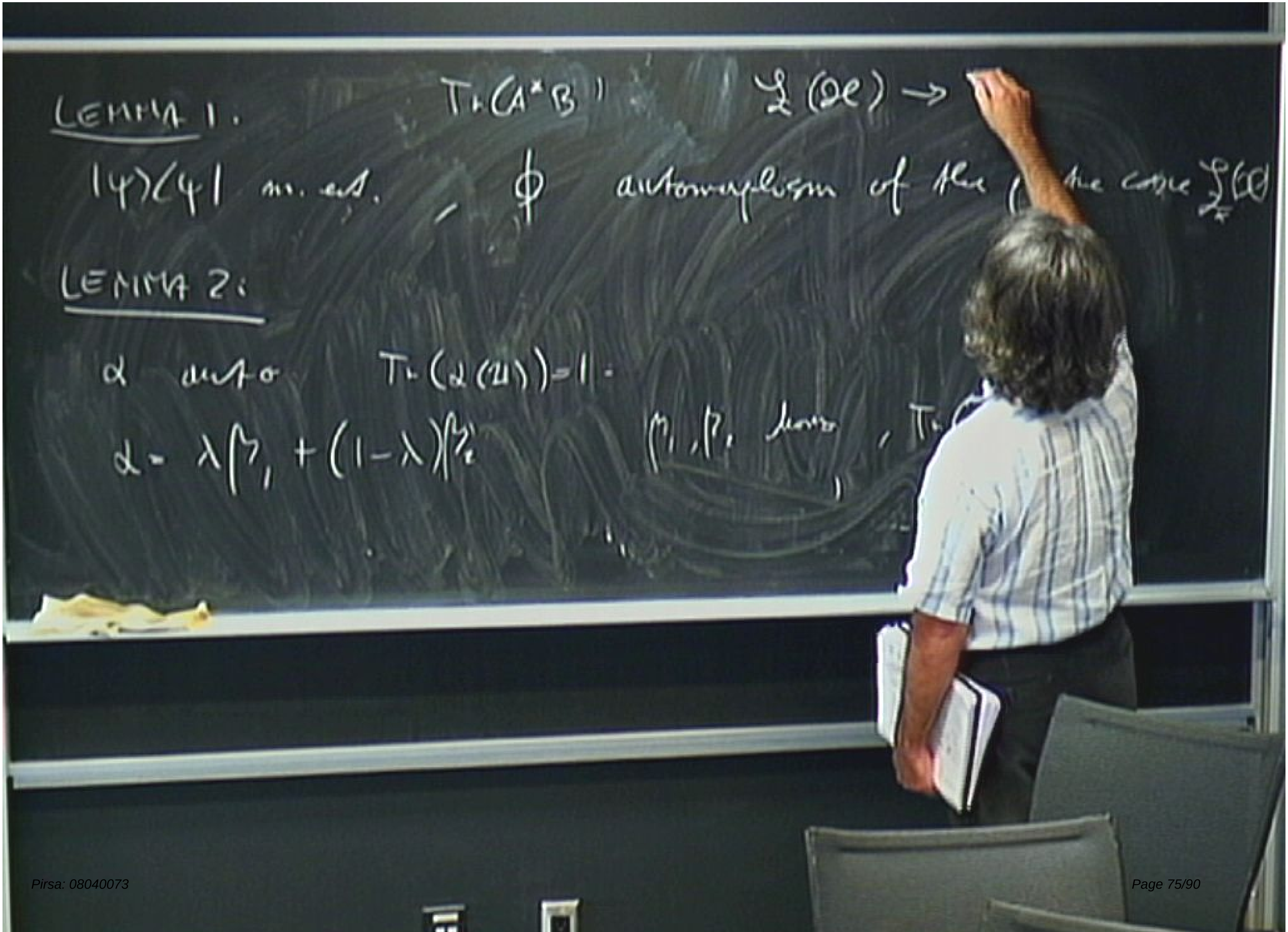


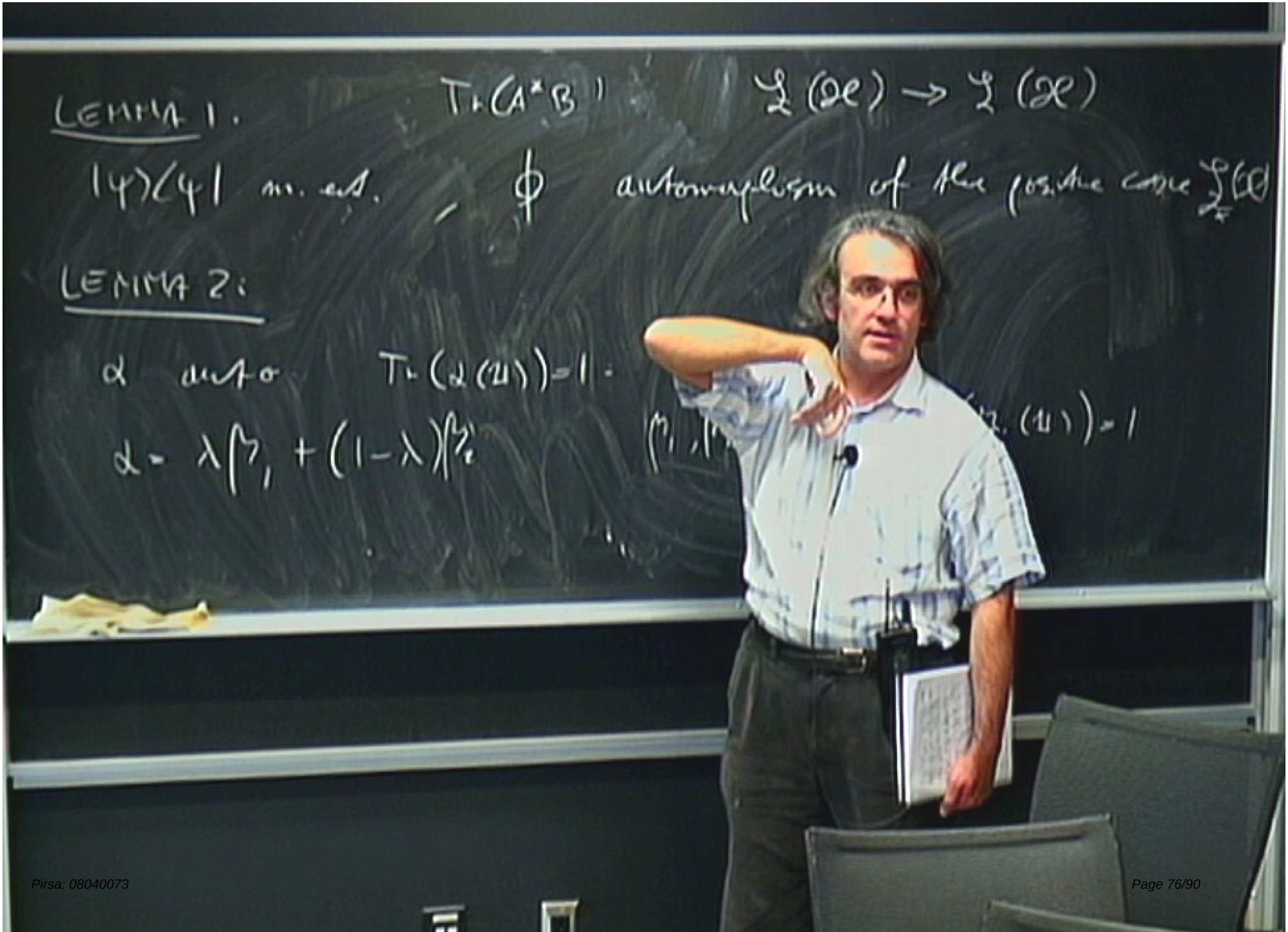
LEMMA 1.
|ψ⟩⟨ψ| m. ent.
φ automorphism of the positive cone
LEMMA 2:
α auto Tr(α(1)) = 1.
α = λβ₁ + (1 − λ)β₂
β₁, β₂

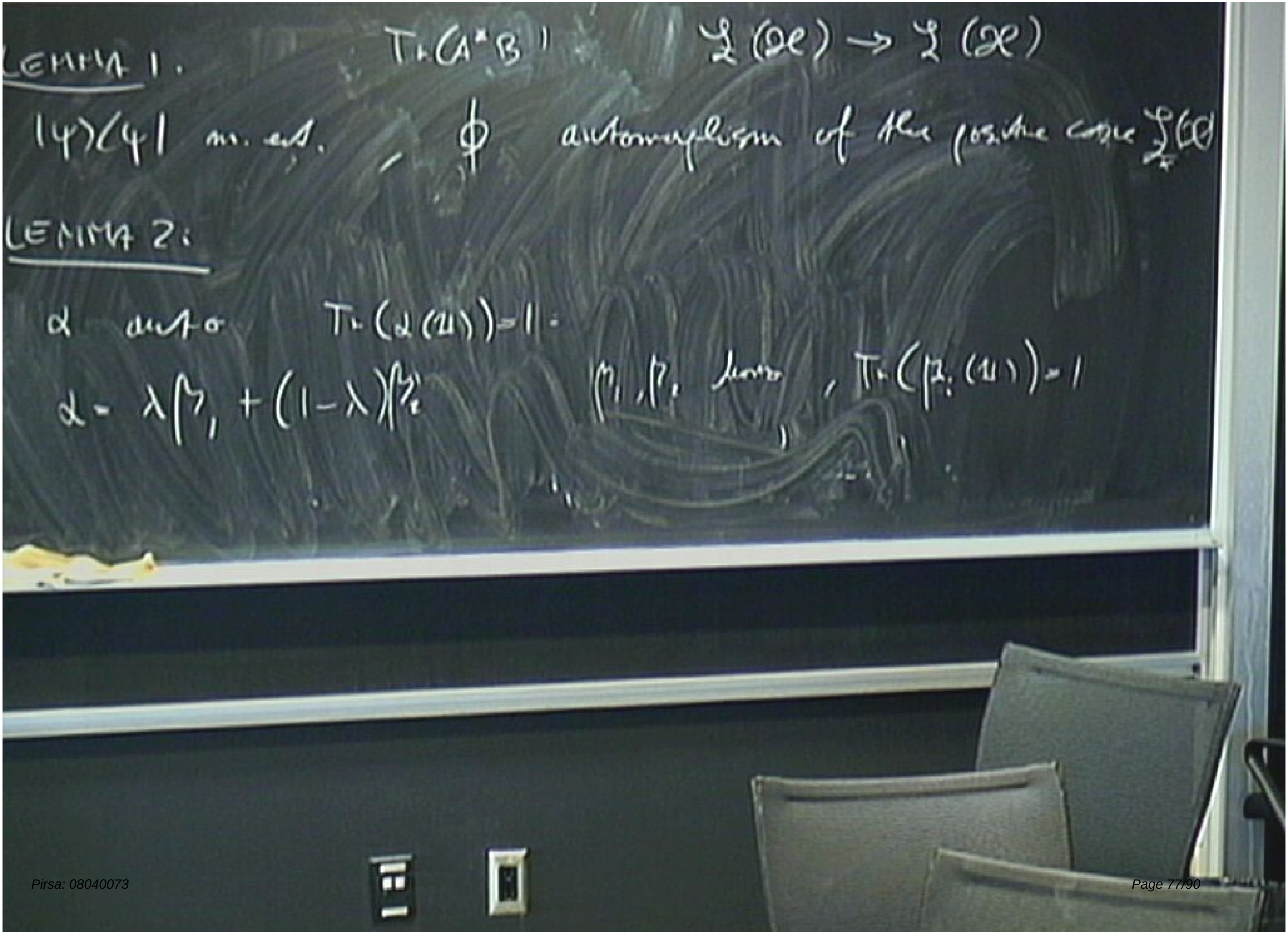

LEMMA 1:
automorphism of the positive cone
LEMMA 2:

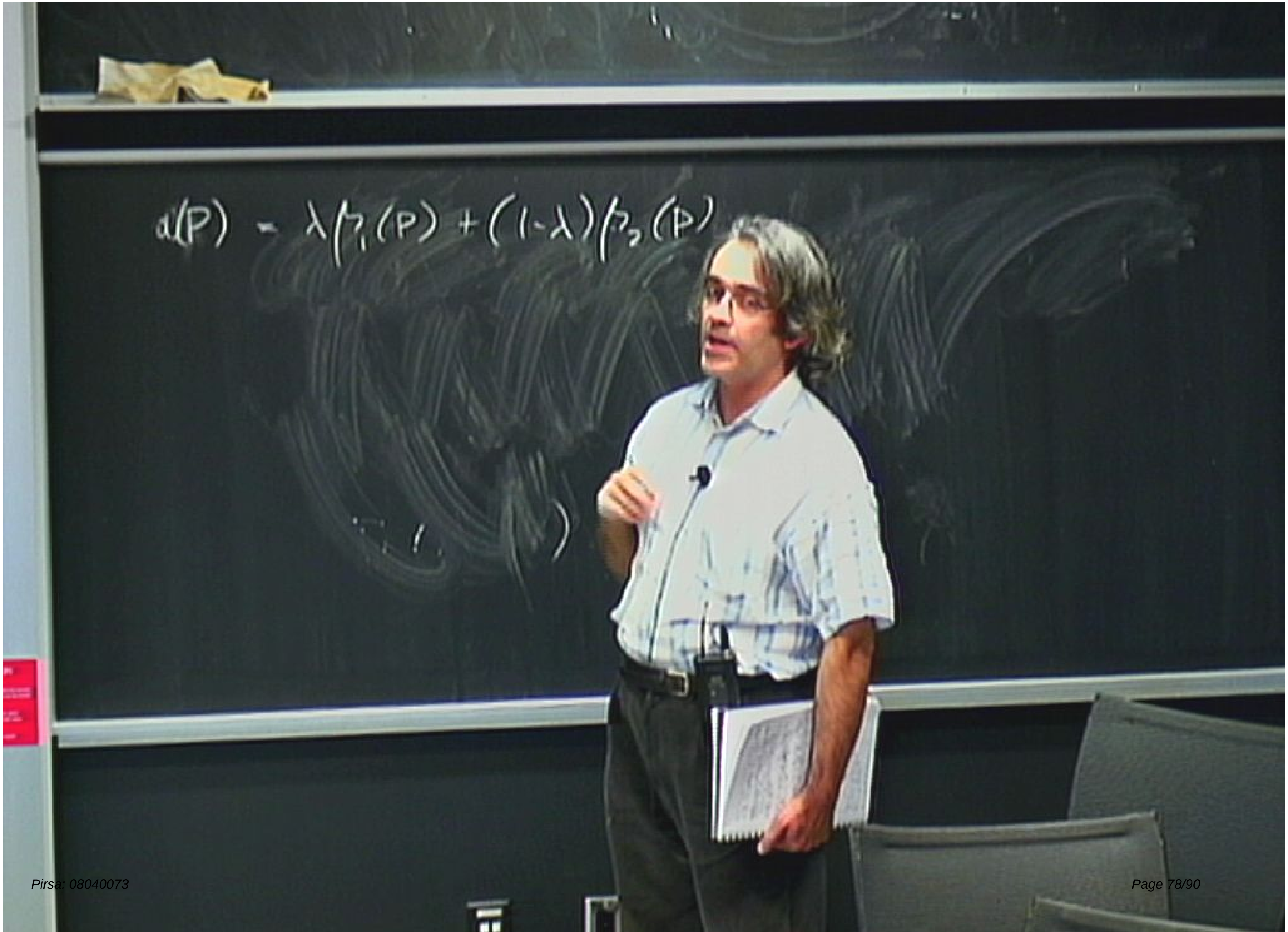

LEMMA 1.
|ψ⟩⟨ψ| m. ent.   φ automorphism of the positive cone
LEMMA 2:
α auto   Tr(α(1)) = 1.
α = λβ₁ + (1−λ)β₂
(1)) = 1

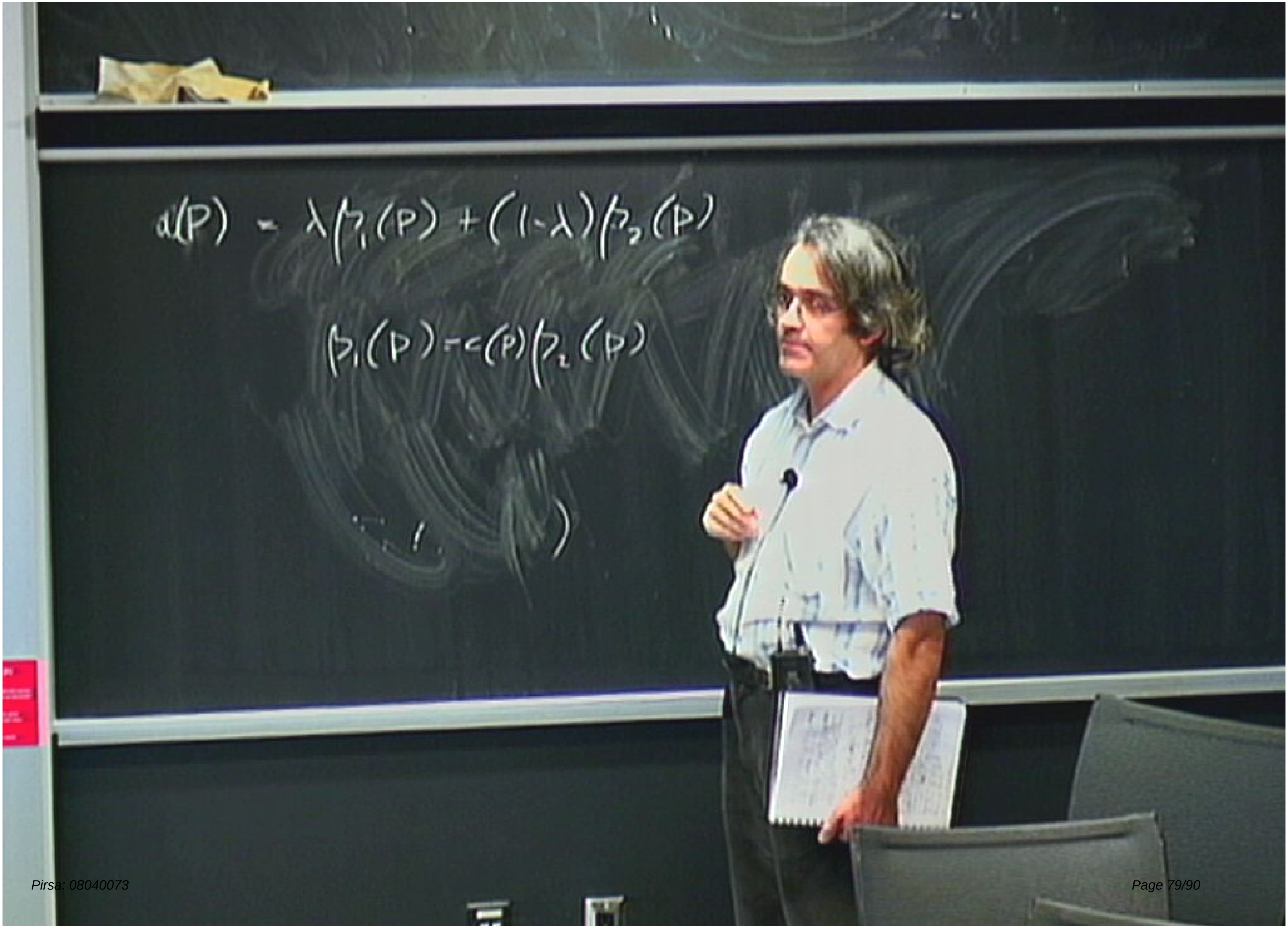

LEMMA 1.
Tr(A*B)
|ψ⟩⟨ψ| m. ent.
φ automorphism of the positive cone
LEMMA 2:
α auto Tr(α(1)) = 1.
α = λβ1 + (1−λ)β2
Tr(βi(1)) = 1

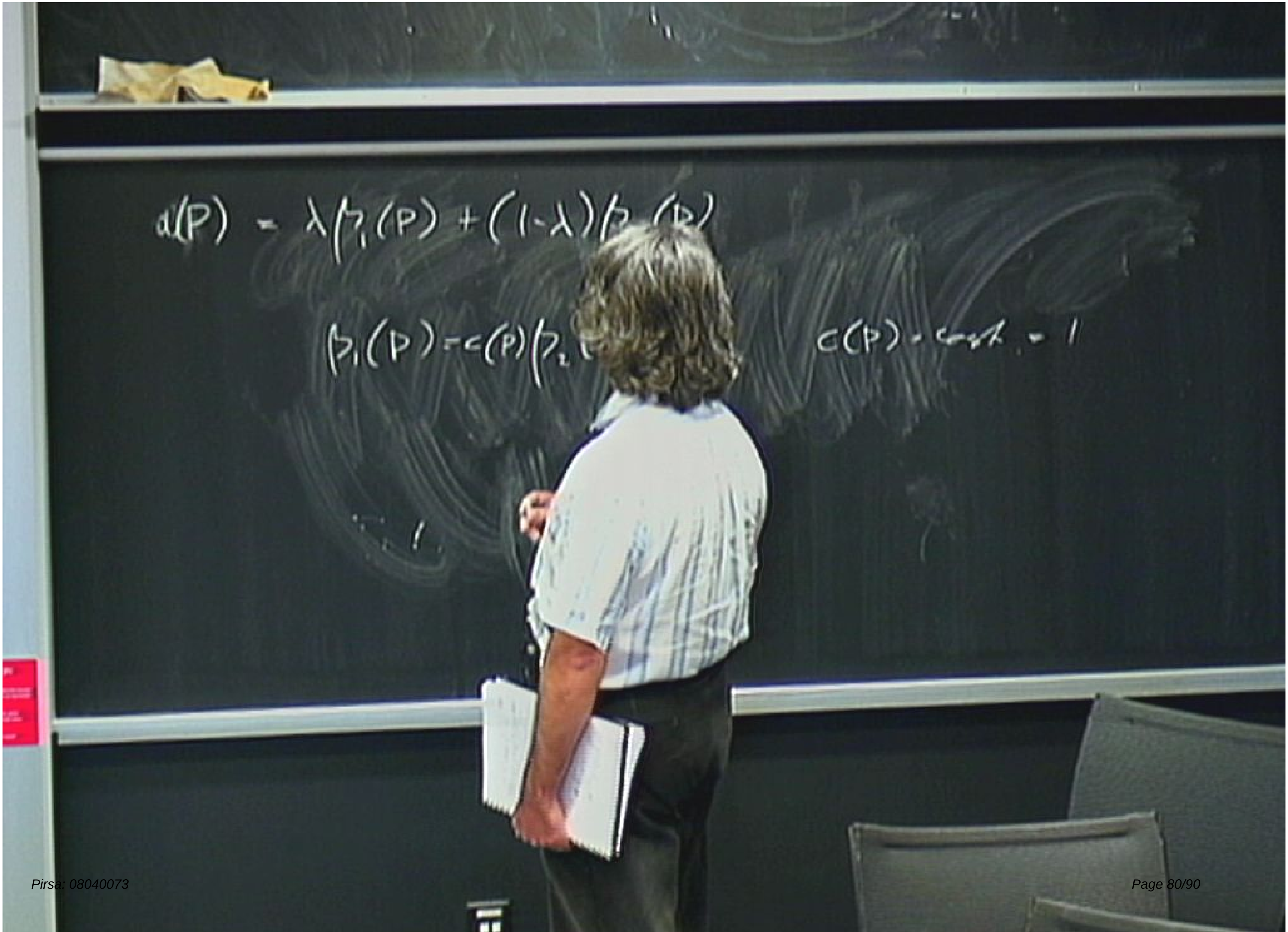

LEMMA 1.
Tr(A*B)
|ψ⟩⟨ψ| m. ext.
φ automorphism of the
LEMMA 2:
α auto
Tr(α(1))=1.
α = λβ1 + (1−λ)β2
β1, β2 homo


LEMMA 1.
Tr(A*B)
L(H) → L(H)
|ψ⟩⟨ψ| m. ent.
φ automorphism of the positive cone
LEMMA 2:
α auto Tr(α(1)) = 1.
α = λβ1 + (1 − λ)β2
(1)) = 1


LEMMA 1.
Tr(A*B)
L(H) → L(H)
|ψ⟩⟨ψ| m. ent.
φ automorphism of the positive cone L+(H)
LEMMA 2:
α auto Tr(α(1)) = 1.
α = λβ1 + (1−λ)β2
β1, β2 auto, Tr(βi(1)) = 1

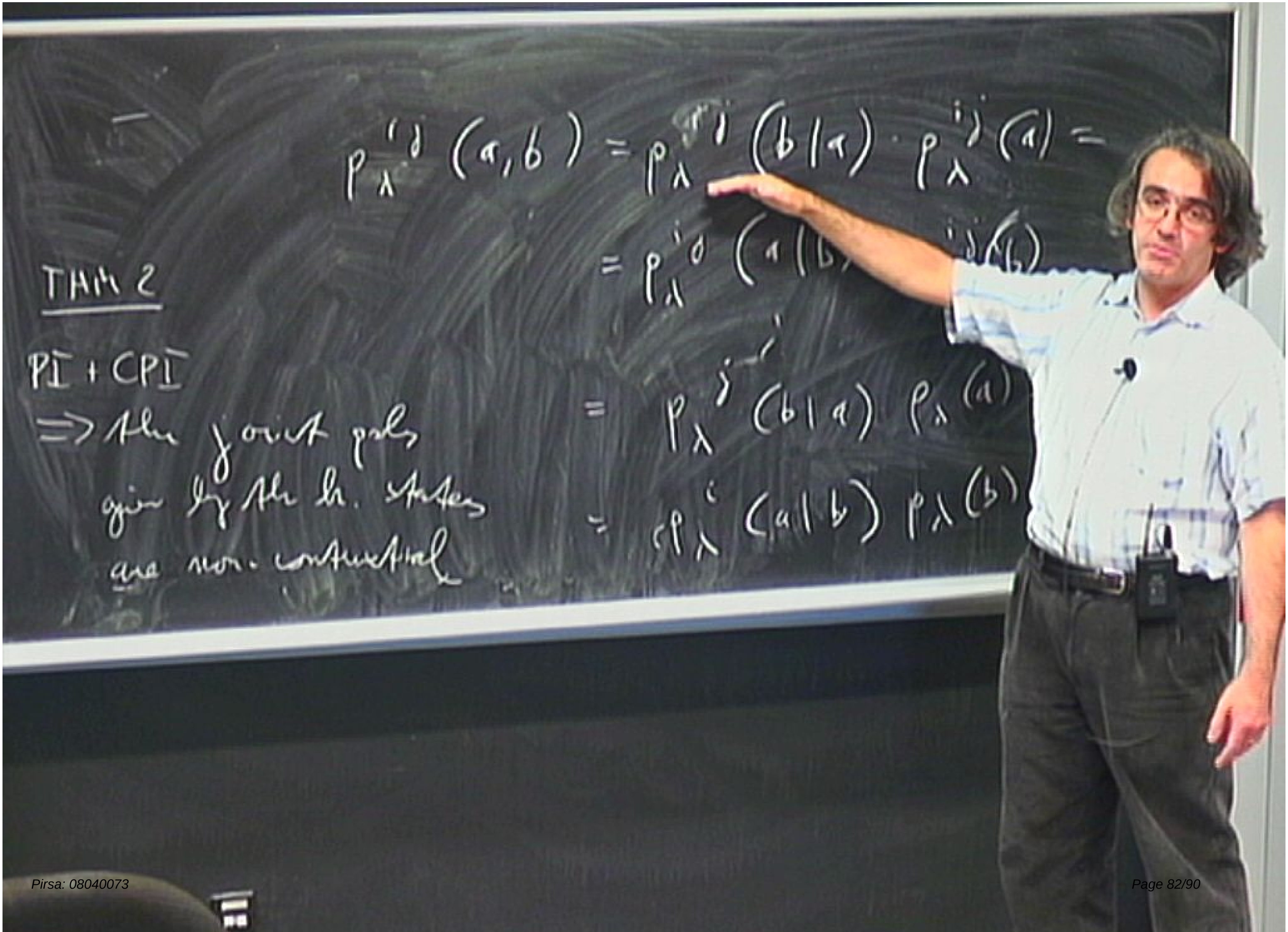

$\rho(P) = \lambda \rho_1(P) + (1-\lambda)\rho_2(P)$


$\alpha(P) = \lambda \mathbb{P}_1(P) + (1-\lambda)\mathbb{P}_2(P)$
$\mathbb{P}_1(P) = c(P)\mathbb{P}_2(P)$

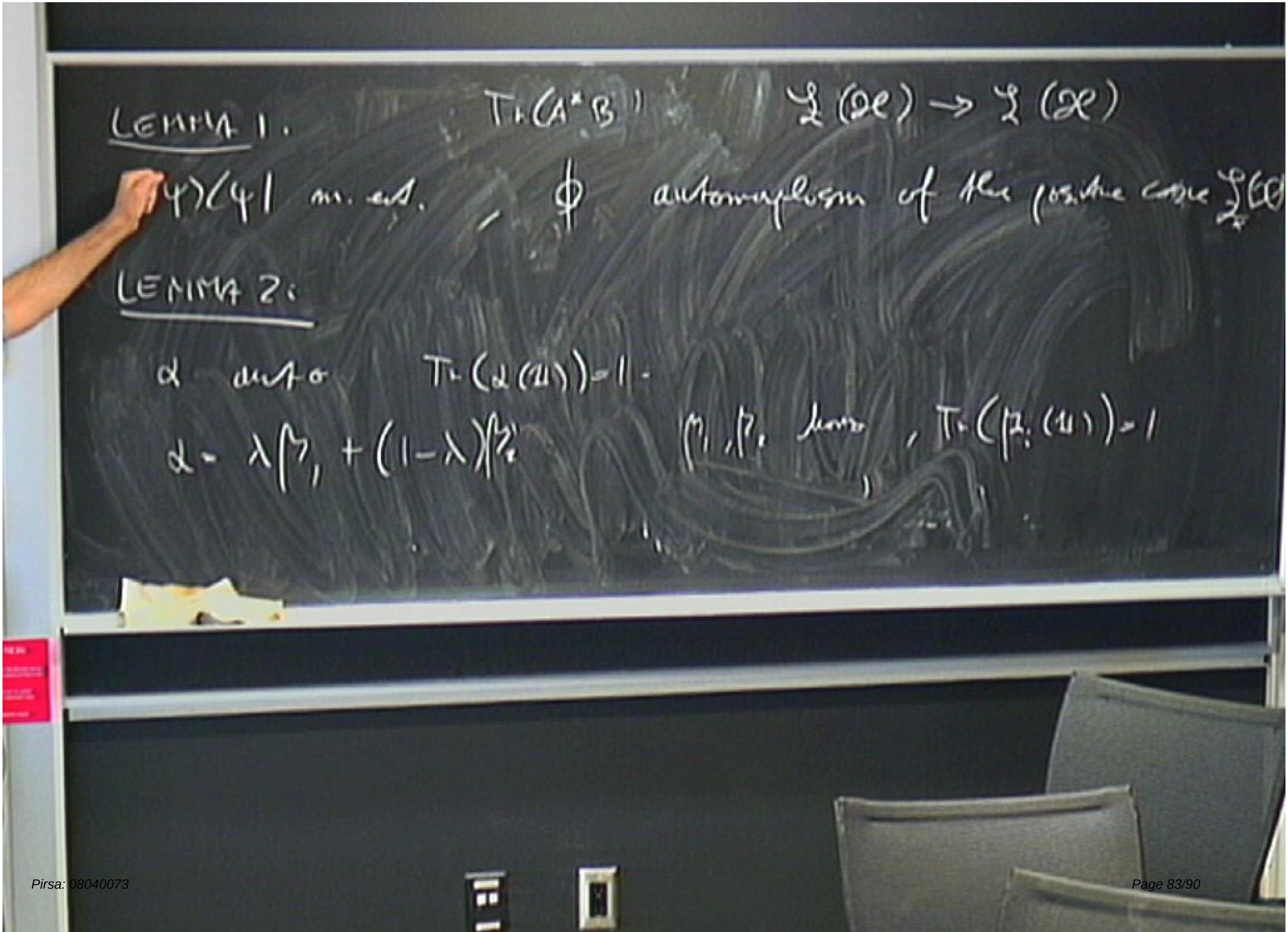

α(P) = λβ1(P) + (1-λ)β2(P)
β1(P) = c(P)β2(
c(P) = const = 1

$$\alpha(P) = \lambda \beta_1(P) + (1-\lambda)\beta_2(P)$$

$$\beta_1(P) = c(P)\beta_2(P) \qquad c(P) = \text{const} = 1$$



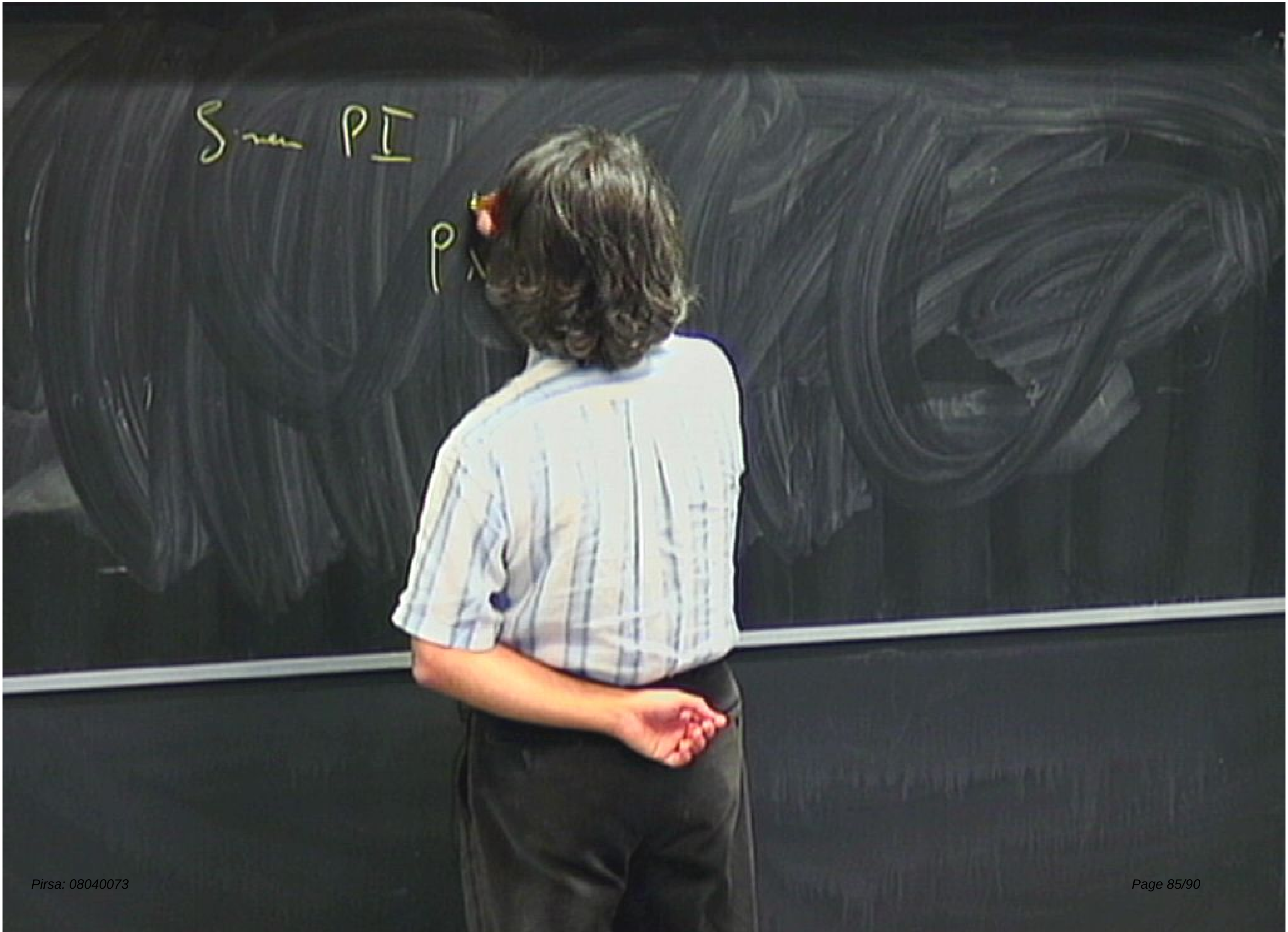
$p_\lambda^{ij}(a,b) = p_\lambda^{ij}(b|a) \cdot p_\lambda^{ij}(a) =$
THM 2
PI + CPI
⇒ the joint probs given by the h. states are non-contextual
$= p_\lambda^{j}(b|a)\; p_\lambda(a)$
$= p_\lambda^{i}(a|b)\; p_\lambda(b)$

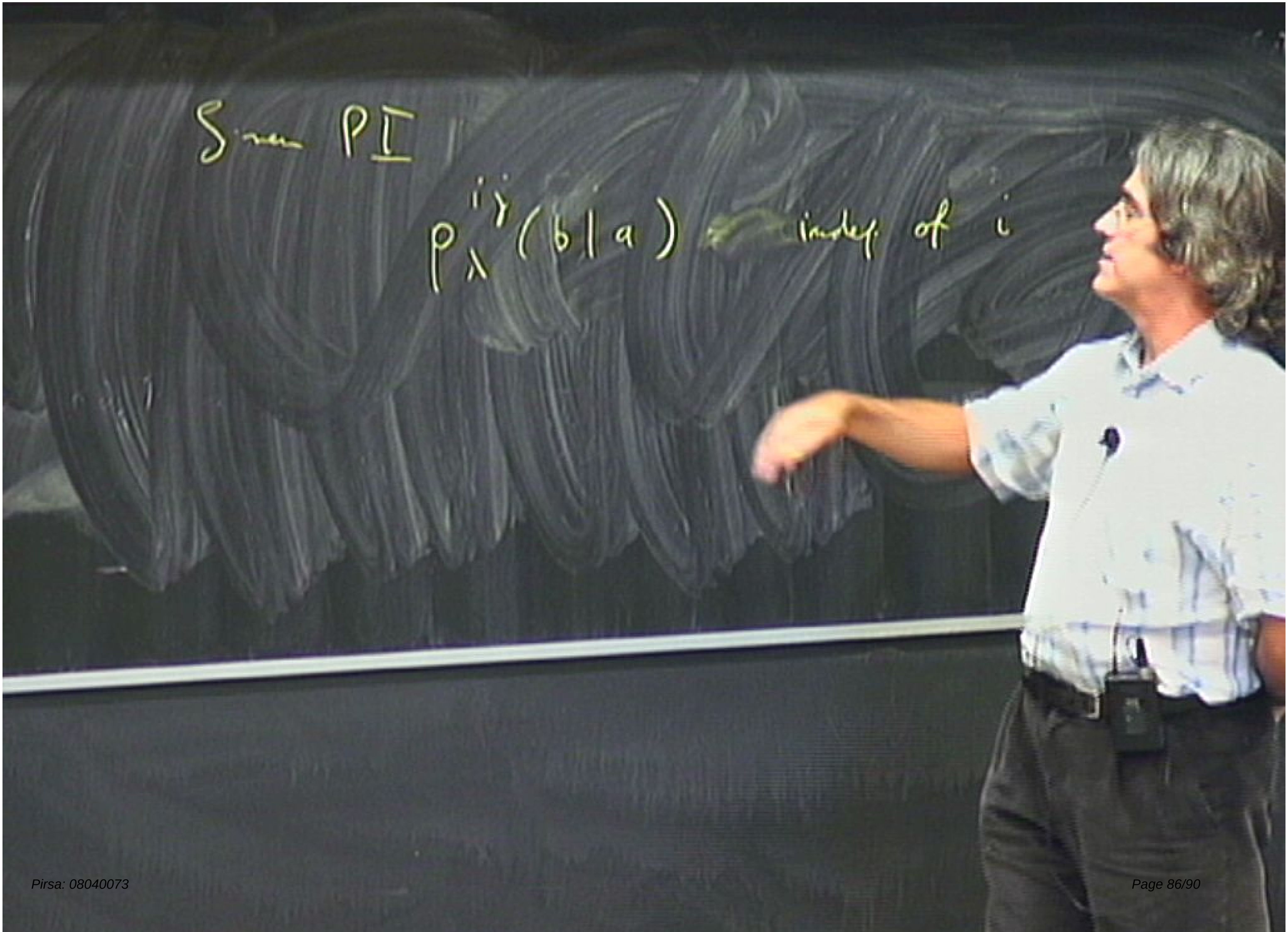

LEMMA 1.
Tr(A*B)
L(H) → L(H)
|ψ⟩⟨ψ| m. est.
φ automorphism of the positive cone
LEMMA 2:
α auto
Tr(α(1)) = 1.
α = λβ1 + (1−λ)β2
β1, β2 homo
Tr(βi(1)) = 1

CPI: $p_\lambda^{i,j}(b|a) = p_\lambda^{j}(b|a)$

$\neg$PI + CPI: $p_\lambda^{i,j}(b) = \sum_a p_\lambda^{i,j}(b|a)\, p_\lambda^{i,j}(\ldots$

PI + $\neg$CPI: $p_\lambda^{i,j}(b) = \sum_a p_\lambda^{i,j}(b|a)\, p_\lambda^{i,j}(\ldots$


PI


Since PI
$\rho_\lambda^{ij}(b|a)$ indep. of i

Since PI

$p_\lambda^{ij}(b|a)$ indep. of i

$\Leftrightarrow$ $p_\lambda^{ij}(a\|b)$ indep. of $\underline{i}$

Given PI

$$\rho_\lambda^{ij}(b|a) \quad \text{indep. of } i$$

$$\Leftrightarrow \rho_\lambda^{ij}(a||b) \quad \text{indep of } \underline{i}$$

Since PI

$$p_\lambda^{ij}(b|a) \quad \text{indep. of } i$$

$$\Longleftrightarrow \quad p_\lambda^{ij}(a|b) \quad \text{indep. of } i$$

$$\Longleftrightarrow \quad p_\lambda^{ij}(a,b) \quad \text{indep. of } i$$

Since PI

$p_\lambda^{ij}(b|a)$ indep. of i, j

$\Leftrightarrow$ $p_\lambda^{ij}(a|b)$ indep. of i, j

$\Leftrightarrow$ $p_\lambda^{ij}(a, b)$ indep. of i, j