Title: Particle dynamics in a relativistic invariant stochastic medium

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Abstract: The dynamics of particles moving in a medium defined by its relativistically invariant stochastic properties is investigated. For this aim, the force exerted on the particles by the medium is defined by a stationary random variable as a function of the proper time of the particles. The equations of motion for a single one-dimensional particle are obtained and numerically solved. A conservation law for the drift momentum of the particle during its random motion is shown. Moreover, the conservation of the mean value of the total linear momentum for two particles repelling each other according to the Coulomb interaction also follows. Therefore, the results indicate the realization of a kind of stochastic Noether theorem in the system under study.

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The above properties suggest a possible use of the kind of construction employed in this work, to explore the realization of ensemble descriptions of QM and QFT along the D'Broglie, Einstein and Bohm lines of though.

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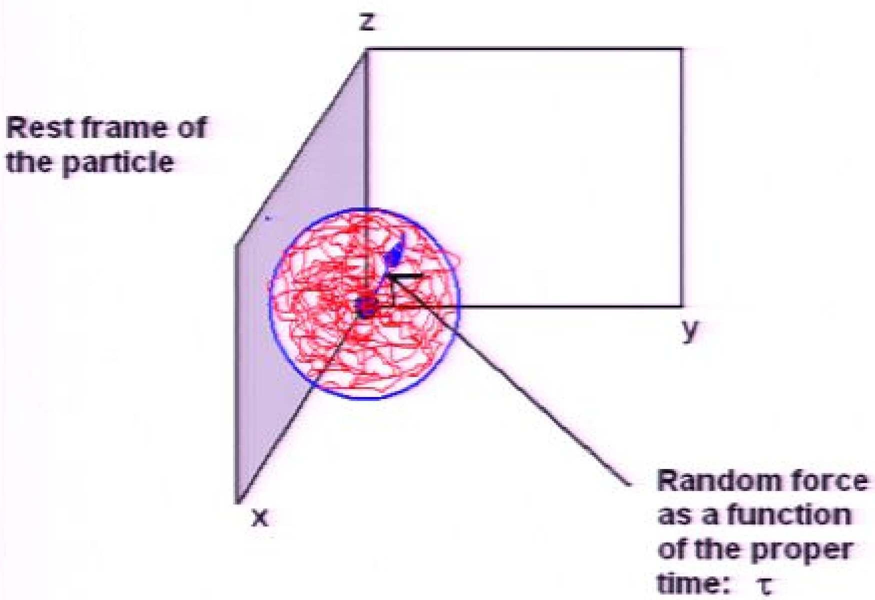
# 1. Introduction

The objetive of this work aims to call for the attention about a particular point of view to the very old problem of giving a stochastical interpretation of Quantum Mechanics (QM). A large quantity of studies have been done in the literature since the original times of QM. Two of the most relevant steps were the D'Broglie "Pilot wave" and the Bohm interpretions. The argues given here are far from to be complete and we only intend to present some issues which extension in our view are worth considering.




The idea: let us assume the Einstein's view about that a matter particle could have always a well definite position in space. Then the only apparently possible way in which the QM phenomenology could be implied, seems to necessarily corresponds to a situation in which the vacuum exerts a relativistic invariant random force on the particle. This view is the idea behind the classical works of Boyer, de la Pena, and many other authors.

The present discussion only attempts to consider a particular way of constructing the stochastic force which acts on the particle.

Basically, we study some consequences of assuming that the stochastic force is defined as as an homogeneous random variable in the rest frame of the particle as a function of its proper time. It will be argued that in this case a conservation law of a drift momentum of the particle arises and also that the total sum of the drift momenta of a pair of particles conserves under a collision. The results also rises some interesting possibilities for the extension of the work, that will be commented.

## 2. Equation of motion

The equation of motion

$$F_p(\tau)\,d\tau = m_0\,dv';$$

Increase of the velocity in the rest frame

Random force in the rest frame as a function of the proper time

$$v + dv = \frac{v + dv'}{1 + \frac{v\,dv'}{c^2}},$$
$$\cong (v + dv') \times \left(1 - \frac{v\,dv'}{c^2}\right),$$
$$\cong v + \left(1 - \frac{v^2}{c^2}\right) dv',$$

Relativistic addition of the increment in the velocity in the rest frame and the the velocity v of this rest frame with respect laboratory frame.

$$dv = \left(1 - \frac{v^2}{c^2}\right) dv'.$$

Increase of the velocity in the laboratory frame

$$F_p(\tau) = \frac{m_0}{(1-\frac{v^2}{c^2})}\frac{dv}{d\tau},$$

$$v = \frac{dx(t)}{dt}$$

Equation of motion in terms of the velocity in the laboratory frame and the proper time

$$F_p(\tau) = \sqrt{1-\frac{v^2}{c^2}}\,F_L(\tau).$$

Generel relation between the spatial component of the four-force in the rest frame and the laboratory one

$$\int F_p(\tau)\,d\tau + \hat{C} = m_0 \int \frac{1}{(1-\frac{v^2}{c^2})}\,dv,$$

$$= \frac{m_0 c}{2} \ln\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right),$$

The velocity of the laboratory frame can be analytically determined in terms of the integral over the proper time of the force in the rest frame.

$$v(\tau) = c \cdot \tanh\left[\frac{1}{m_0 c}\cdot\left(\int F_p(\tau)\cdot d\tau + \hat{C}\right)\right],$$

C is an arbitrary constant

## 3. The random force

$$f_N(\tau,\varphi_i) = \frac{f_0}{N}\sum_{i=1}^{N}\cos\big(w_i(N)\tau + \varphi_i\big),$$

$$w_i(N) = \frac{8\pi i}{N}, \quad i = 1,\ldots,N,$$

The expression employed for the random force in the rest frame as a function of the proper time:

Basically, it is a white noise with a finite frequency bandwidth, approximately generated by N uniformly spaced values of the frequency components with random phases assigned to them.

Plot of the value of force as a function of the proper time for a number of frequency components N=200.

The distribution functions (y axis) for the values of force (x axis) in the rest frame for N=250

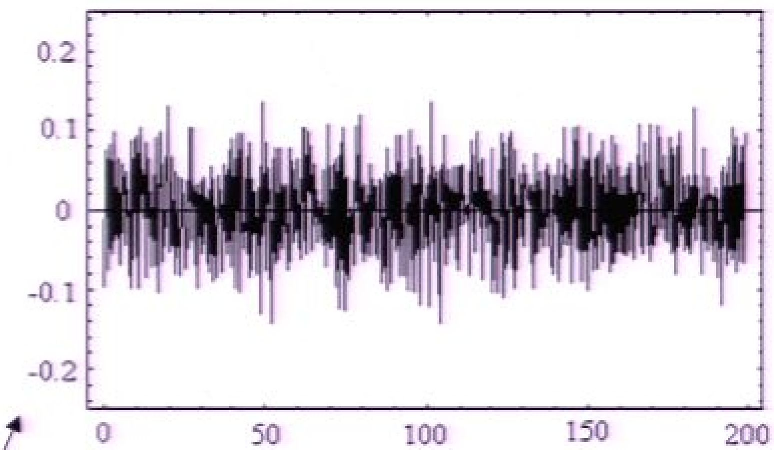



Fig. 1. The figure shows a *realization* of the force field corresponding to a spectrum of $N = 200$ frequencies. The horizontal coordinate is the time in seconds and the vertical one is the force in Newtons. The amplitude was fixed $f_0 = 1$ $(N_t)$ $(N_t = \text{Newton})$.

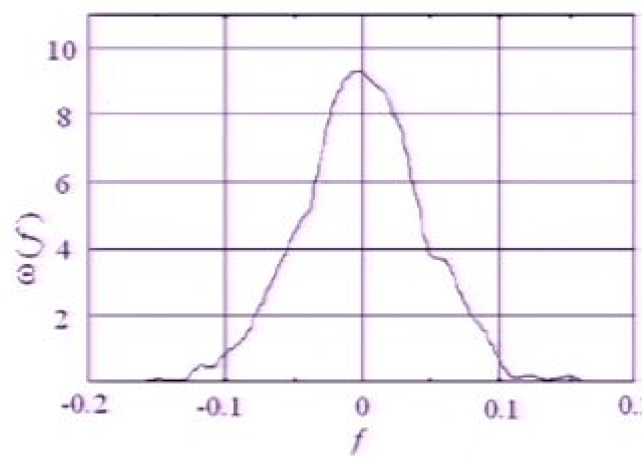



Fig. 2. The distribution function of the ensemble of forces for the set of frequencies $w_i = \frac{8\pi}{250} i$ $(s^{-1})$, $i = 1, \ldots, 250$ and $f_0 = 1$ $(N_t)$.

## 4. A particle in the medium

The expression of the random force can be directly integrated to the form:

$$I_{F_p}(\tau) = \int_0^\tau F_p(\tau)\,d\tau = \frac{f_0}{N}\int_0^\tau \left[\sum_{i=1}^{N}\cos\left(\frac{8\pi i}{N}\tau + \varphi_i\right)\right] d\tau$$
$$= \frac{f_0}{8\pi}\left[\sum_{i=1}^{N}\frac{1}{i}\sin\left(\frac{8\pi i}{N}\tau + \varphi_i\right)\right].$$

Then, after substituting in the formula previously obtained, the velocity in the laboratory frame can be explicitly expressed as a function of the proper time as follows:

$$v(\tau) = c\tanh\left[\frac{1}{m_0 c}\left(I_{F_p}(\tau) + \hat{C}\right)\right].$$
$$= c\tanh\left[\frac{1}{m_0 c}\left(\frac{f_0}{8\pi}\left[\sum_{i=1}^{N}\frac{1}{i}\sin\left(\frac{8\pi i}{N}\tau + \varphi_i\right)\right] + \hat{C}\right)\right].$$

Velocity in the laboratory system of reference as a function of the proper time and the constant C:

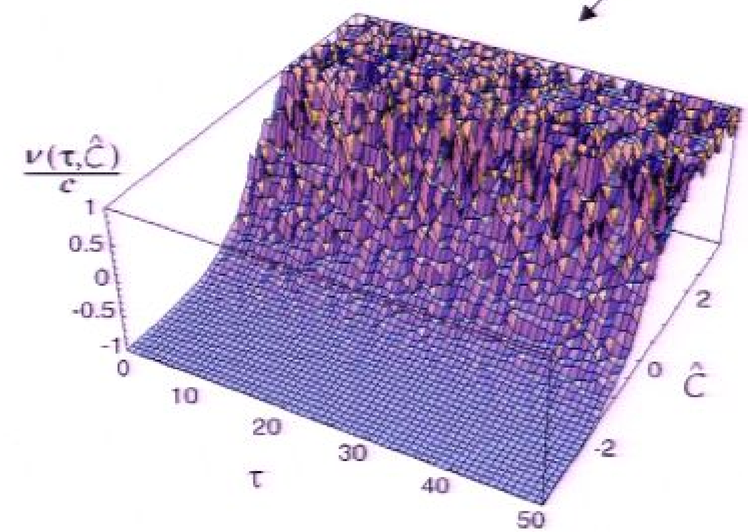



Fig. 3. The velocity of the particle in the laboratory frame against the proper time and the arbitrary constant $\hat{C}$. The parameters for the random force were $\frac{f_0}{8\pi m_0 c} = 0.1$ (s$^{-1}$), $w_i = \frac{8\pi i}{N}$ (s$^{-1}$), $i = 1, \ldots, 250$.

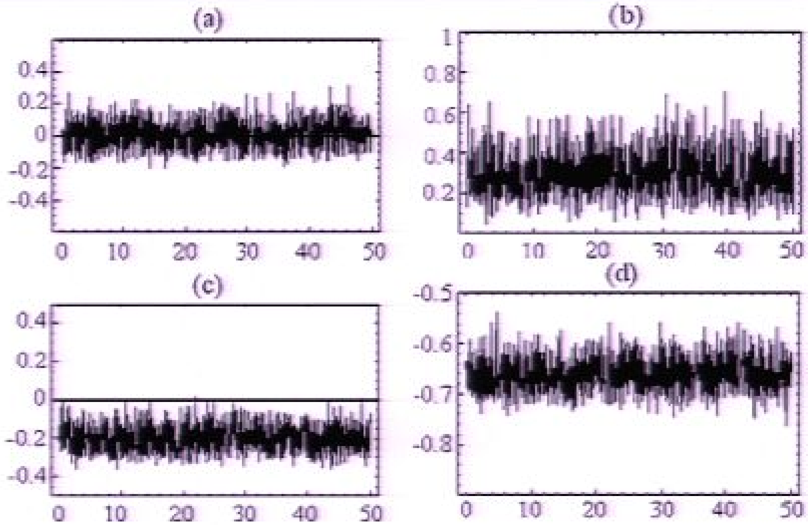



Fig. 4. The velocities of the particles (divided by $c$) in the laboratory system vs. the proper time for four specific values of $\hat{C}$: (a) $\hat{C} = 0$, (b) $\hat{C} = 0.3$, (c) $\hat{C} = -0.2$ and (d) $\hat{C} = -0.8$. Note the non-vanishing value of the mean velocity for $\hat{C}$ different from zero.

The plot of the velocity as a function of the proper time for few values of the constant C.

Note that each value of C determines a conserved non vanishing value of the velocity of the particle. Therefore, a kind of stochastic conservation law of the mean momentum follows. Thus, the Lorentz invariance of the equations of motion suggests the validity of a kind of "stochastic" Noether Theorem.

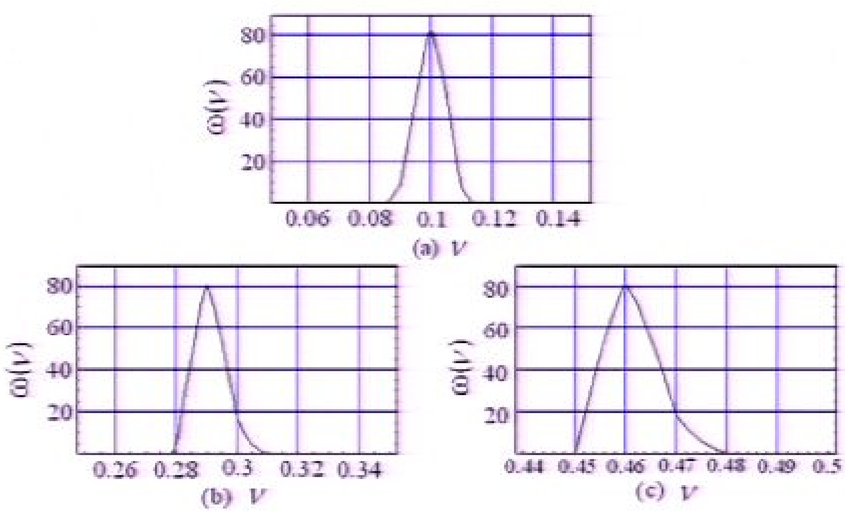



Fig. 5. The distribution functions: (a) $w(v)$ for $\hat{C} = 0.1$, (b) $w(v)$ for $\hat{C} = 0.3$ and (c) $w(v)$ for $\hat{C} = 0.5$. Note the distortion of the symmetry around the center, when $\hat{C}$ grows.

The distribution functions for the velocity for various values of the constant C, showing a non vanishing average in them. Note, that due to the Lorentz invariance of the system, it should be valid that the distributions of velocities for two different values of C, must be related by a particular Lorentz transformation between the solutions for the velocity for such values of C.

In order to determine the velocities and coordinates as functions of the time in the laboratory system, the proper time should be found in terms of the laboratory time for each solution. First, the laboratory time was numerically found as a function of the proper time through the usual formula

$$t(\tau,\hat{C}) = \int_0^{\tau} \frac{d\tau'}{\left(1 - \frac{v(\tau',\hat{C})^2}{c^2}\right)^{1/2}},$$

$$t(0) = 0,$$

Then, the inverse mapping expressing the proper time as a function of the laboratory one, was also numerically evaluated. It is illustrated in the figure:

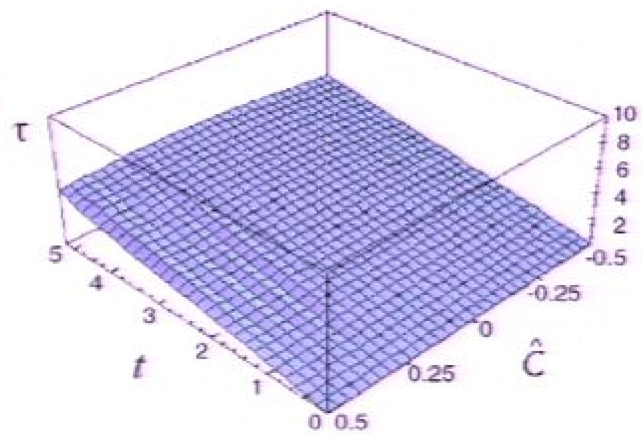
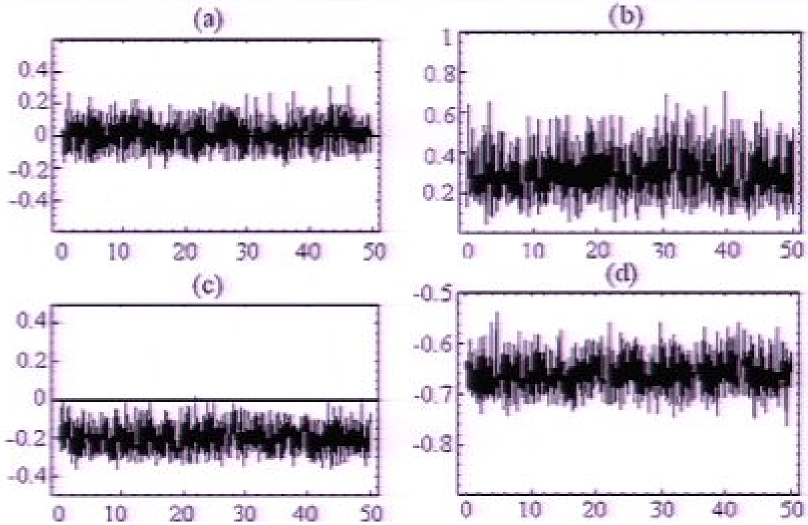



Fig. 6. Dependence on $t$ of the proper time in the laboratory $\tau = \tau(t,\hat{C})$, with $\hat{C}$ running along a third axis. Note that, since the dependence, by definition, should always be monotonous, the random oscillation are not apparent. They are however present in the local slope of the curves.

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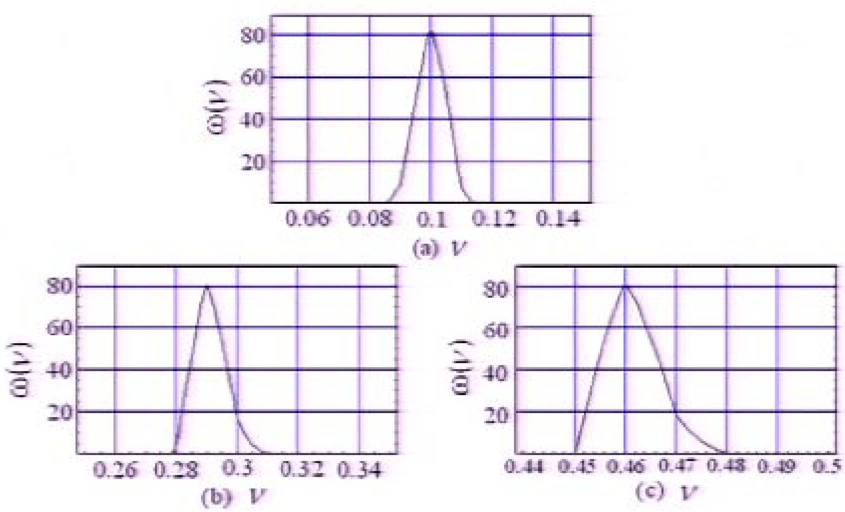


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The velocities of the particle or various values of C, now plotted as functions of the observer time in the laboratory frame.

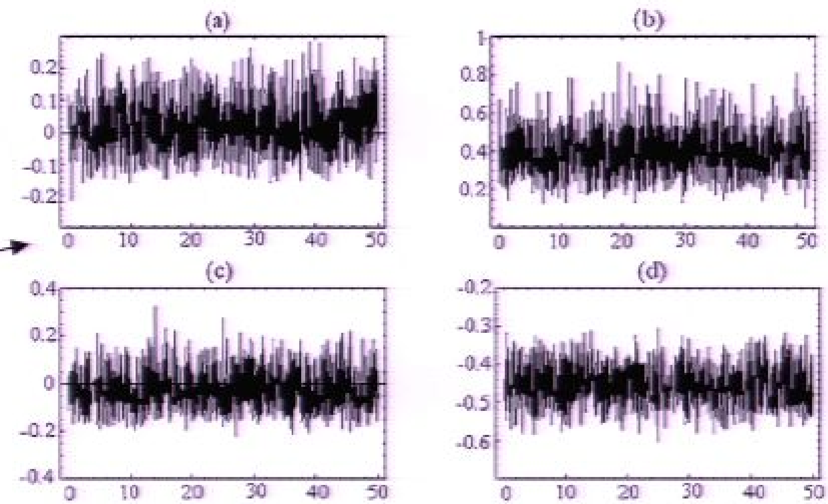


Fig. 7. The velocities in the laboratory frame (divided by $c$), but now plott as functions of the time $t$ in this same frame of reference, for the values of ( (a) $\hat{C} = 0.01$, (b) $\hat{C} = 0.4$, (c) $\hat{C} = -0.03$, (d) $\hat{C} = -0.5$.

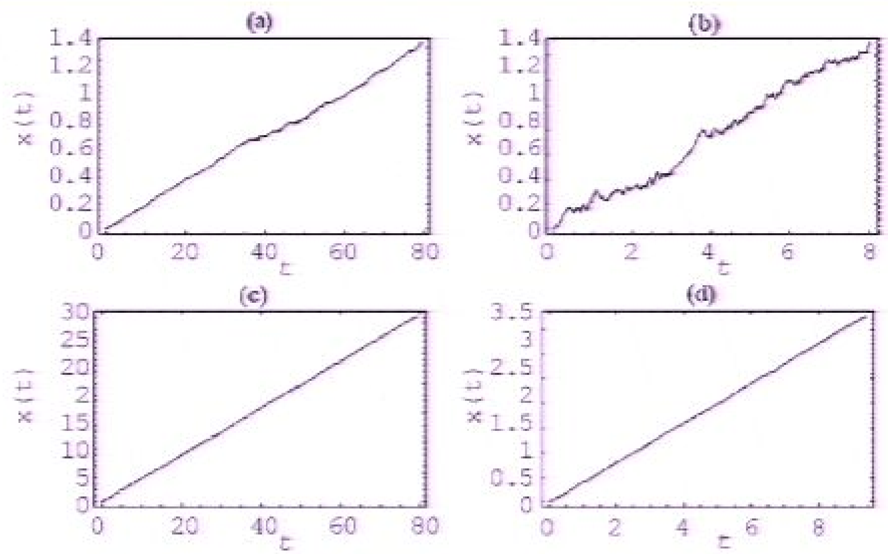


Fig. 8. The particle coordinates divided by $c$: $\frac{x}{c} = \frac{1}{c}x(t)$ as functions of the time in the laboratory frame. In (a) and (b) $\hat{C} = 0.01$ but the time scale is different. For (c) and (d) $\hat{C} = 0.4$ and again the two scales are different. Note that for $\hat{C}$ small the randomness seems to be greater than for larger $\hat{C}$.

The coordinates of the particle are also plotted in these figures as functions of the time in the laboratory frame for various values of C.

## 5. Two particles in the medium

$$\vec{F}_{\text{rep}_1}(x_1, x_2) = -\vec{F}_{\text{rep}_2}(x_1, x_2) = \frac{\alpha}{|x_1 - x_2|^3}(\vec{x}_1 - \vec{x}_2).$$

The Columb force between two particles

$$\frac{m_0}{(1 - \frac{x_1'(t)^2}{c^2})^{3/2}} \frac{d^2 x_1}{dt^2} = F_{p_1}(\tau_1) + \left(1 - \frac{x_1'(t)^2}{c^2}\right)^{1/2} F_{\text{rep}_1}(x_1, x_2),$$

$$\frac{m_0}{(1 - \frac{x_2'(t)^2}{c^2})^{3/2}} \frac{d^2 x_2}{dt^2} = F_{p_2}(\tau_2) + \left(1 - \frac{x_2'(t)^2}{c^2}\right)^{1/2} F_{\text{rep}_2}(x_1, x_2),$$

The equations of motions for the two particles interacting through a repulsive Coulomb potential. The radical factors multiply the Coulomb force in order to express the laboratory frame forces in terms of the forces acting in the rest frames of each particle. It was assumed that the velocities and the Coulomb forces are small in order to disregard retardation effects.

$$\frac{d\tau_1}{dt} = \left(1 - \frac{x_1'(t)^2}{c^2}\right)^{1/2},$$

$$\tau_1(t_0) = \tau_{1_0}, \qquad x_1(t_0) = x_{1_0}, \qquad x_1'(t_0) = v_{1_0},$$

$$\frac{d\tau_2}{dt} = \left(1 - \frac{x_2'(t)^2}{c^2}\right)^{1/2},$$

$$\tau_2(t_0) = \tau_{2_0}, \qquad x_2(t_0) = x_{2_0}, \qquad x_2'(t_0) = v_{2_0}.$$

The relations between the proper times of each of the particles and the time in the laboratory frame.

The initial conditions for the proper time and particle coordinates.

The plot of the velocities of the two particles as functions of the laboratory frame. Note that the velocities are exchanged. That is, the impact is elastic and the addition of the drift momenta conserves. Thus, again the Noether Theorem result appears: The Lorentz invariance of the system seems to determine the conservation of the total momentum associated to the conserved drift velocities.

$$\langle v_{1_0} \rangle = 0, \qquad \langle v_{2_0} \rangle = 0.152,$$
$$\langle v_{1_f} \rangle = 0.152, \qquad \langle v_{2_f} \rangle = 0,$$

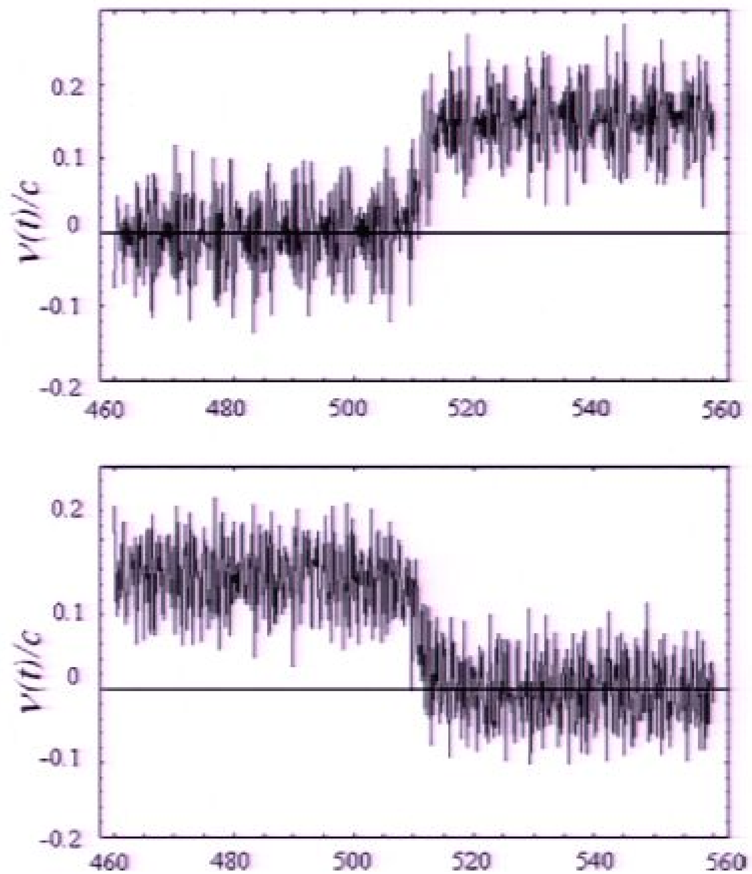


Fig. 10. The time dependence for the velocities (divided by $c$) of both particles: (a) $v_1(t)/c$ and (b) $v_2(t)/c$ in the considered shock. Note, the exchange of their mean velocity values.

## Possible extensions of the work

1) To generalize the discussion for considering more spatial dimensions.

2) To study the formulation of a "stochastic" Noether theorem in which the symmetries of the system extends to be symmetries of the random forces acting on it.

3) The ability of formulating that theorem, could open the possibitily of considering the conservation of the total angular momentum (spin plus orbital one) and general internal charges.

4) In the cases of the number of spatial dimensions being greater than one, the following interesting possibility is suggested.

- Consider the preparation of initial states for the collision of two particles with given drift velocities.
- In 1+1 dimension, the only result of the collision should be the exchange of the velocities. However, for more than one spatial dimensions, the resulting conserved drift velocities will be oriented stocastically as the they should depend on the specific realization of the stochastic force when they are near one from another.
- Then, let us consider an ensemble of a large number of those identical collisions and the associated spatial distributions of the particle density (or any other local quantity) at any arbitrarily given laboratory instant of time.
- It should be the case that if a particle is detected at a far away position of the region of collision (which can be estimated by the form of the construction of the experience), another particle should be detected in a postion which can be approximately calculated given the initial velocities of the particles.
- The same is true for the momenta, due to the conservation of the total drift momenta: measuring the impulse of one of the particles allows to know the other momentum.
- Therefore, if the described ensemble can be shown to be dynamically driven by the laws of Quantum Mechanics, then, it seems that the EPR paradox could have a solution in the picture.

5) Some observations suggests the possibility that the stochastic properties of the considered ensembles could be determined by relativistic wave equations, perhaps after properly choosing a required structure for the stochastic force. Some of these observation are:

- It naturally follows that all the ensembre densities for a given laboratory time t after the collision, vanish outside a circle of radius R= c t with c the velocity of light.
- Assumed the validiy of the Noether Theorem, it could be the case that all the symmetries of the stochastic forces will determine associated conserved quantities.
- The above two observations suggests that perhaps, the appropriate way of assuring these properties is that the particle density in the ensembles could be be associated to the known conserved particle densities following from linear wave equations, like the Dirac one by example. The search for this possibility seems to be an interesting extension of the work.

6) In the case of small mass of the particle, it follows that a small force can accelerate it to the velocity of light. Then, it seems motivating to explore the possibity that in this limit of zero mass, the stochastic description could lead to a picture in which, the massless particles (by example , the photons) move in stright lines with given values of their internal propeties (frequency, helicity… ). This view seems to be compatible with the Dirac's Delta like form of the propagator in the light cone for the D'Alembert equation. Thus, one could also ask that, up to what measure the Maxwell equations for photons, by example, could be represented as a stochastic distributions of point particles following stright line trajectories, and having given helicities,and frequencies. In this case the EM field components could turn to be no other things that some mean values of the internal properties of particles (helicities, energies…).

7) More in general, the discussion also suggests, the (remote perhaps!) possibility of a consistent picture for QM and QFT in which the particles udergo movements guided by random stationary states, defined by some stochastically conserved quantities (quantum numbers, eigenfuntions?), and eventually they are able to emit (by causal contact interactions perhaps) other kind of particles. After the emission, the stochastic vacuum force could pilot the particle, but now undergoing a different