Title: Quantum network coding, entanglement, and graphs.

Date: Apr 30, 2008 04:00 PM

URL: http://pirsa.org/08040061

Abstract: In arXiv:quant-ph/0608223, quantum network coding was proved to be no more useful than simply routing the quantum transmissions in some directed acyclic networks. This talk will connect this result, monogamity of entanglement, and graph theoretic properties of the networks involved.

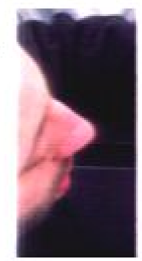
# Quantum network coding, entanglement, and graphs

Debbie Leung[1]
Jonathan Oppenheim[2]
Andreas Winter[3]

quant-ph/0608223

PI April 30, 08

1 IQC, UW

2

Cambridge

3

U. Bristol

# Plan:

- "Canonical" network coding example
  - Def of k-pair comm problem
  - Classical & quantum results
- Generalization to classes of networks

# Motivating example : the butterfly network

The 2-pair comm problem (classical):

# Motivating example : the butterfly network

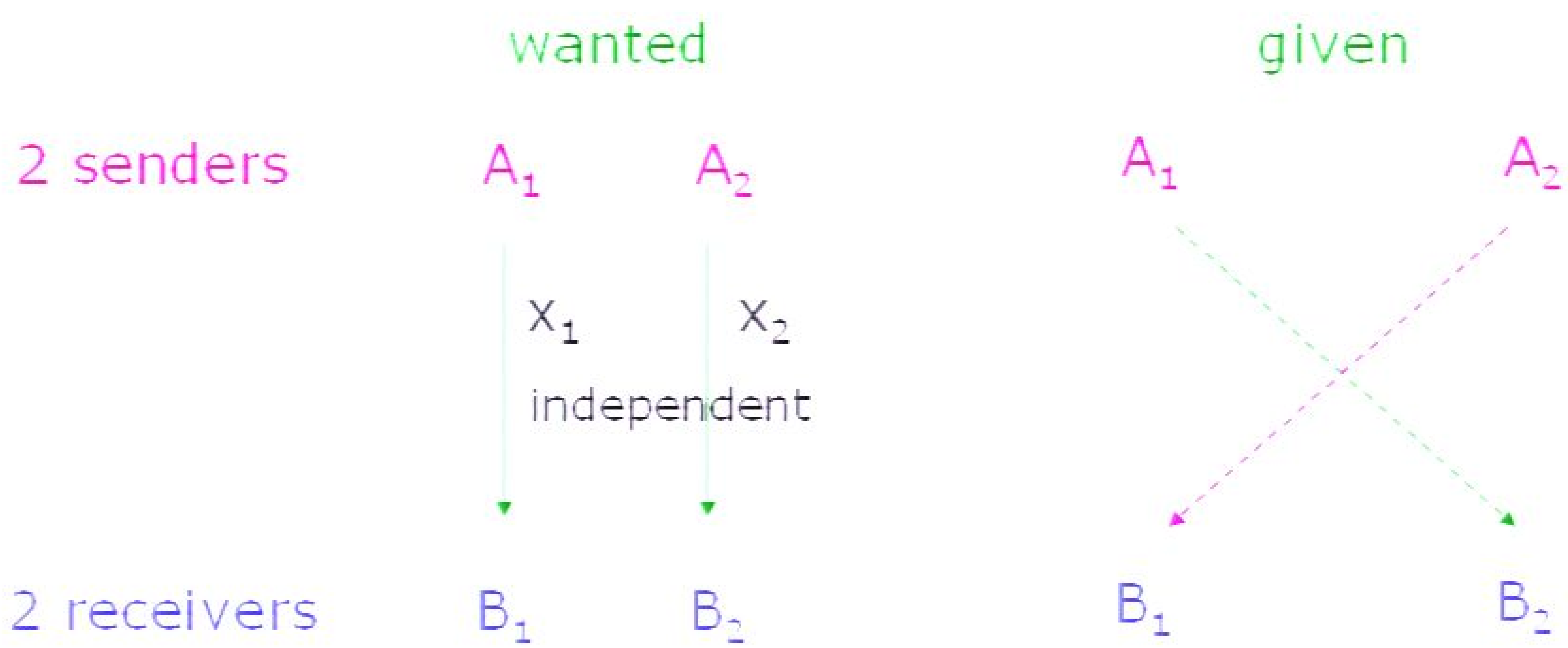
The 2-pair comm problem (classical):

2 senders $A_1$ $A_2$

2 receivers $B_1$ $B_2$

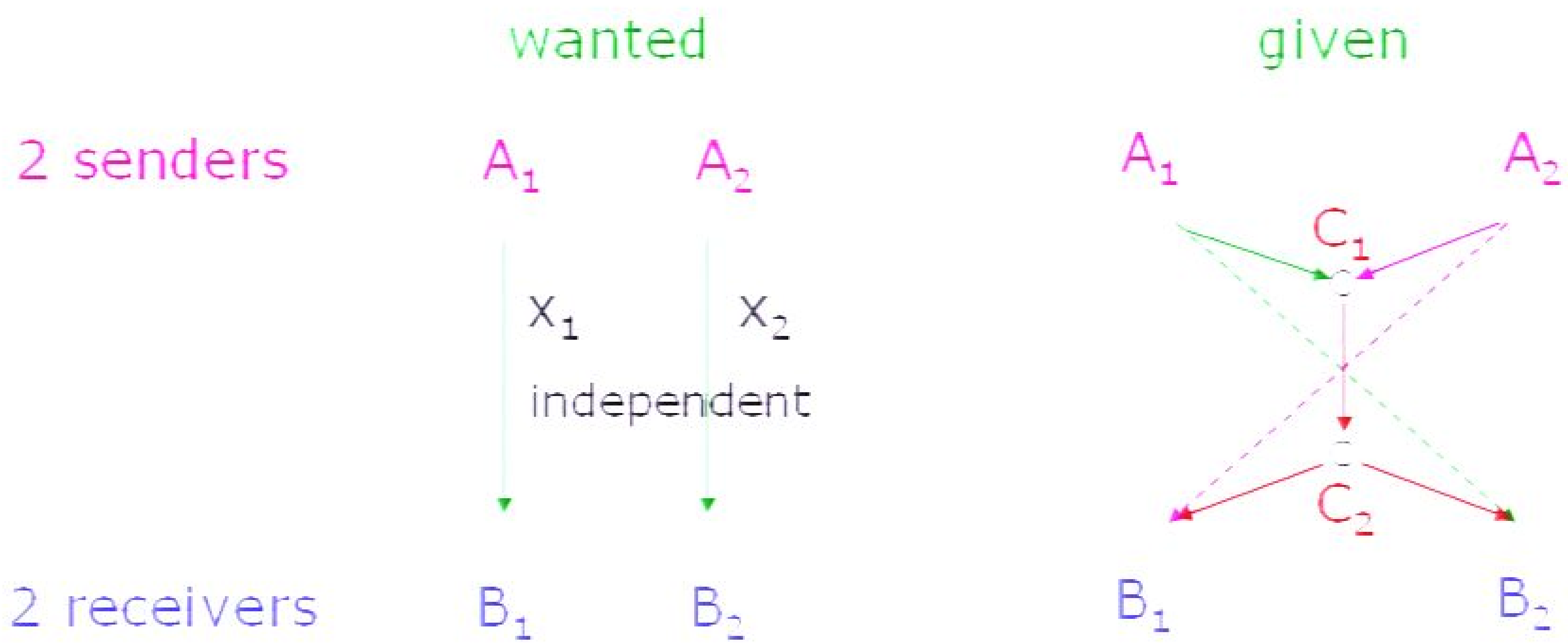
# Motivating example : the butterfly network

The 2-pair comm problem (classical):



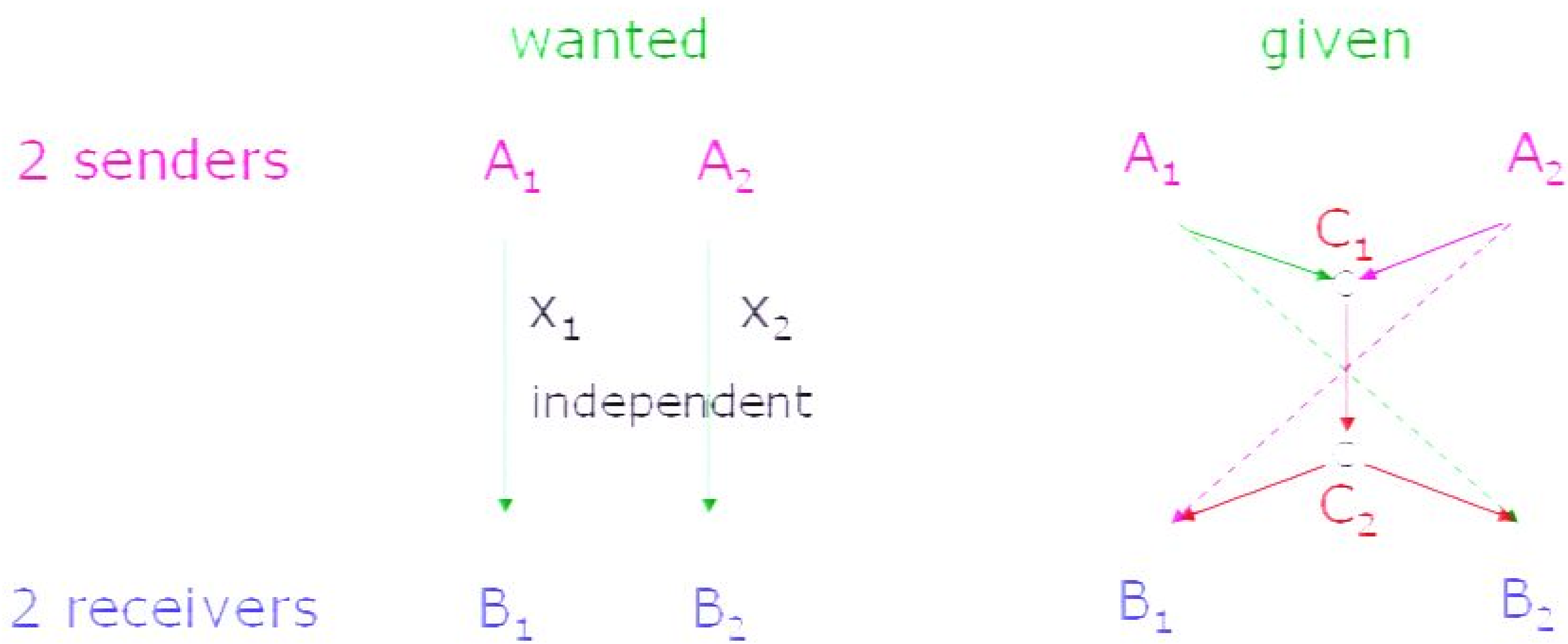
# Motivating example : the butterfly network

The 2-pair comm problem (classical):



# Motivating example : the butterfly network

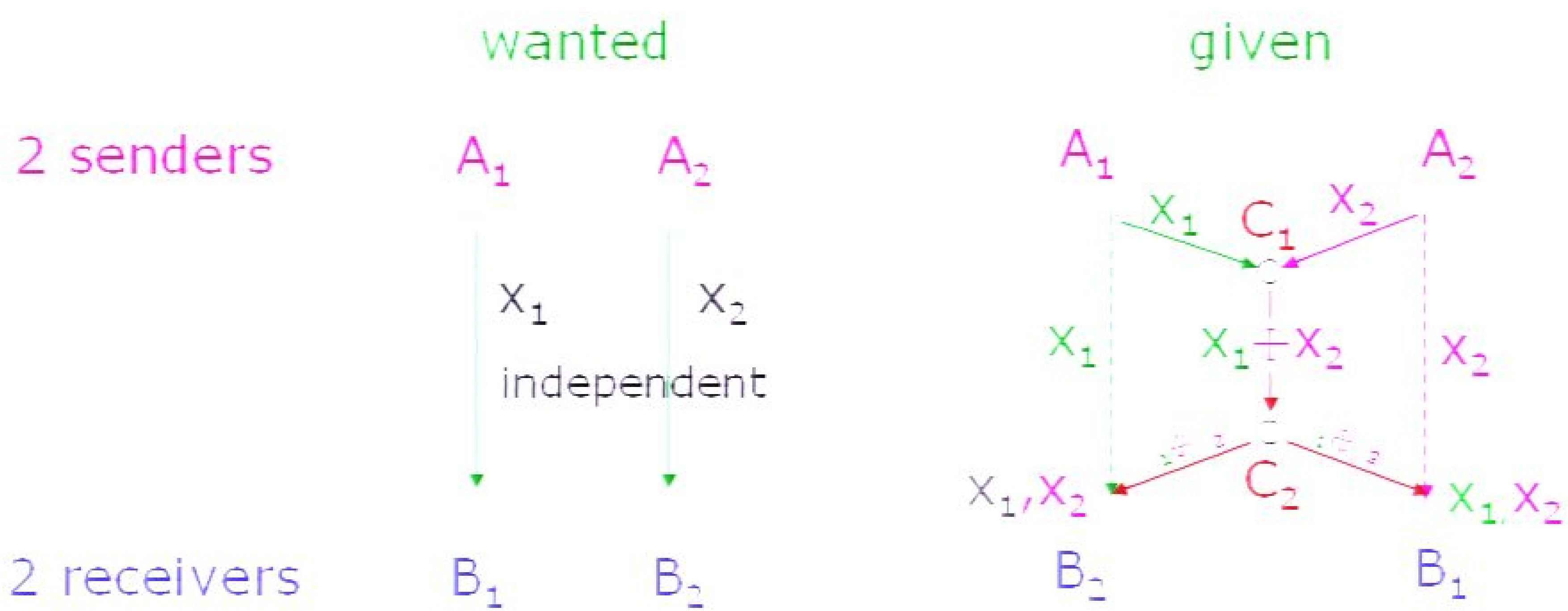
The 2-pair comm problem (classical):




Assumption: "network" (all 7 channels) called as a package
Qn: how to "do our best" ?

# Motivating example : the butterfly network

Optimal in all respects

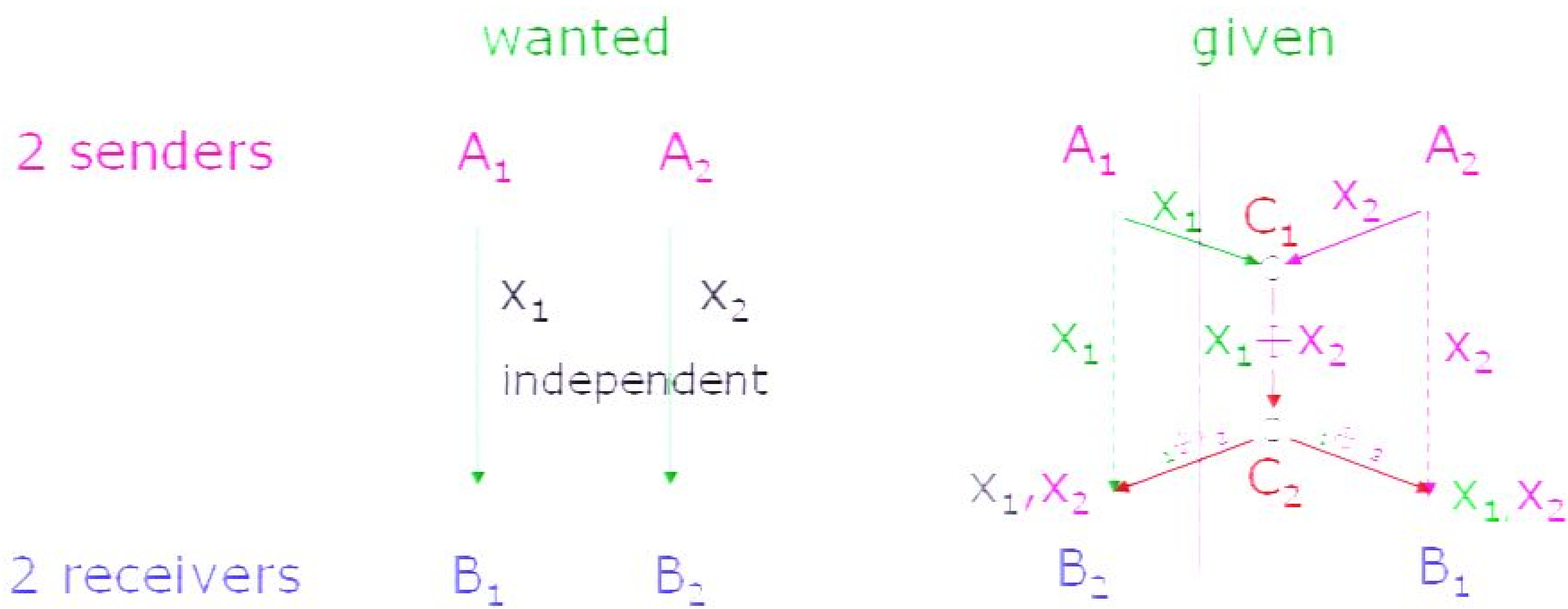



Assumption: "network" (all 7 channels) called as a package
Qn: how to "do our best" ?

# Motivating example : the butterfly network

Optimal in all respects

saturates individual rate upper bounds




Assumption: "network" (all 7 channels) called as a package

Qn: how to "do our best" ?

# Motivating example : the butterfly network

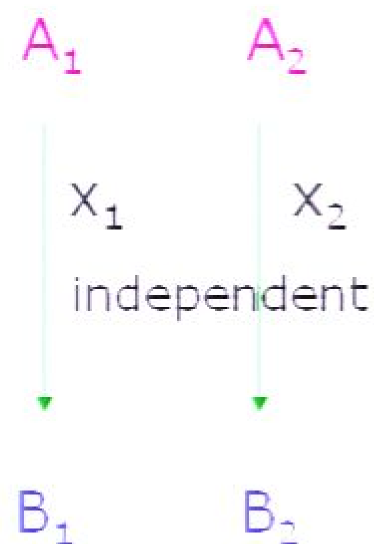
Optimal in all respects

saturates individual rate upper bounds

Network coding: $C_1$ transmits $x_1 \oplus x_2$ to $C_2$

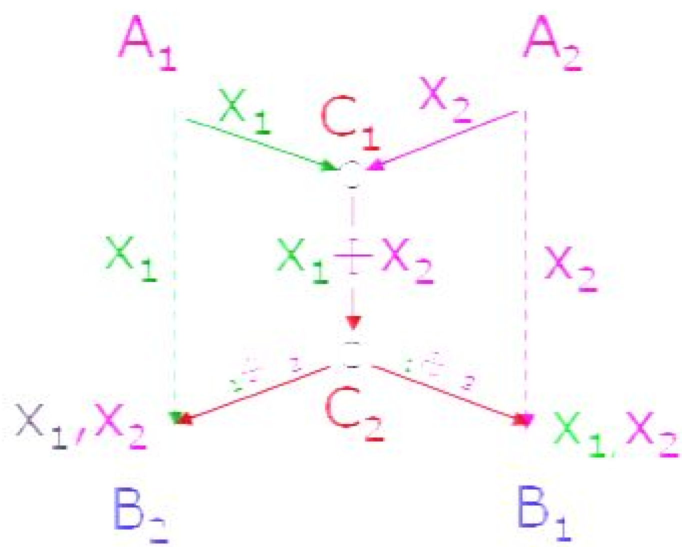
wanted

2 senders




2 receivers

given




Assumption: "network" (all 7 channels) called as a package

Qn: how to "do our best" ?

# Motivating example : the butterfly network

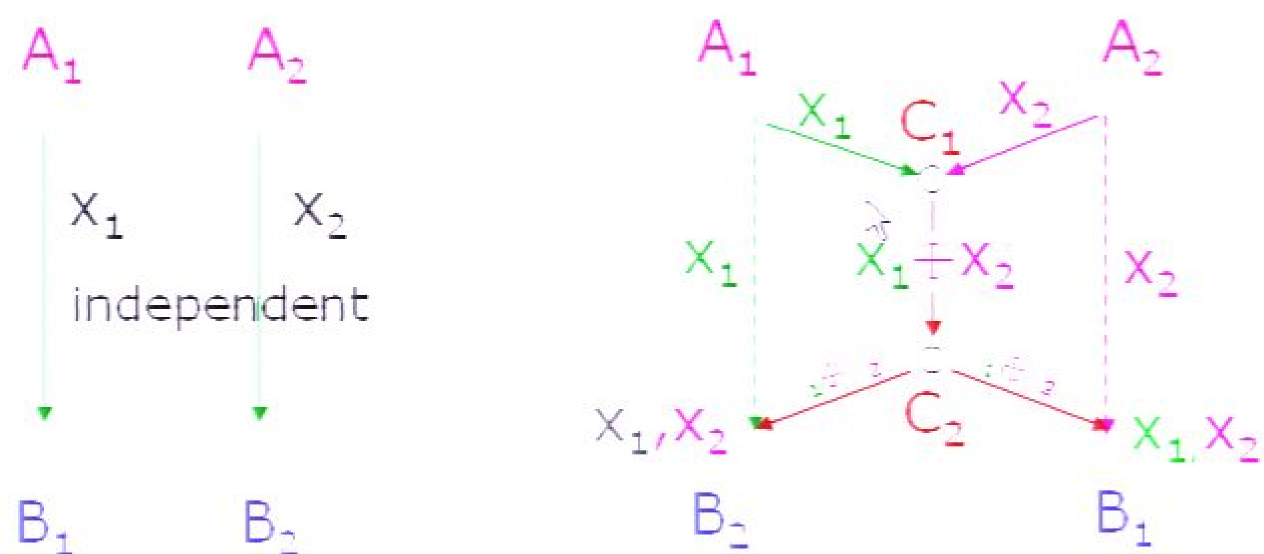
Optimal in all respects
saturates individual rate upper bounds

Network coding: $C_1$ transmits $x_1 + x_2$ to $C_2$

wanted | given

2 senders

2 receivers




Assumption: "network" (all 7 channels) called as a package
Qn: how to "do our best" ?

The butterfly network is the first (classical) example showing network coding allows comm rates strictly better than routing. (Ahlswede, Cai, Li, Yeung, 2000)

The butterfly network is the first (classical) example showing network coding allows comm rates strictly better than routing. (Ahlswede, Cai, Li, Yeung, 2000)

Will see quantum information cannot be coded as such, and routing remains optimal for classes of networks.

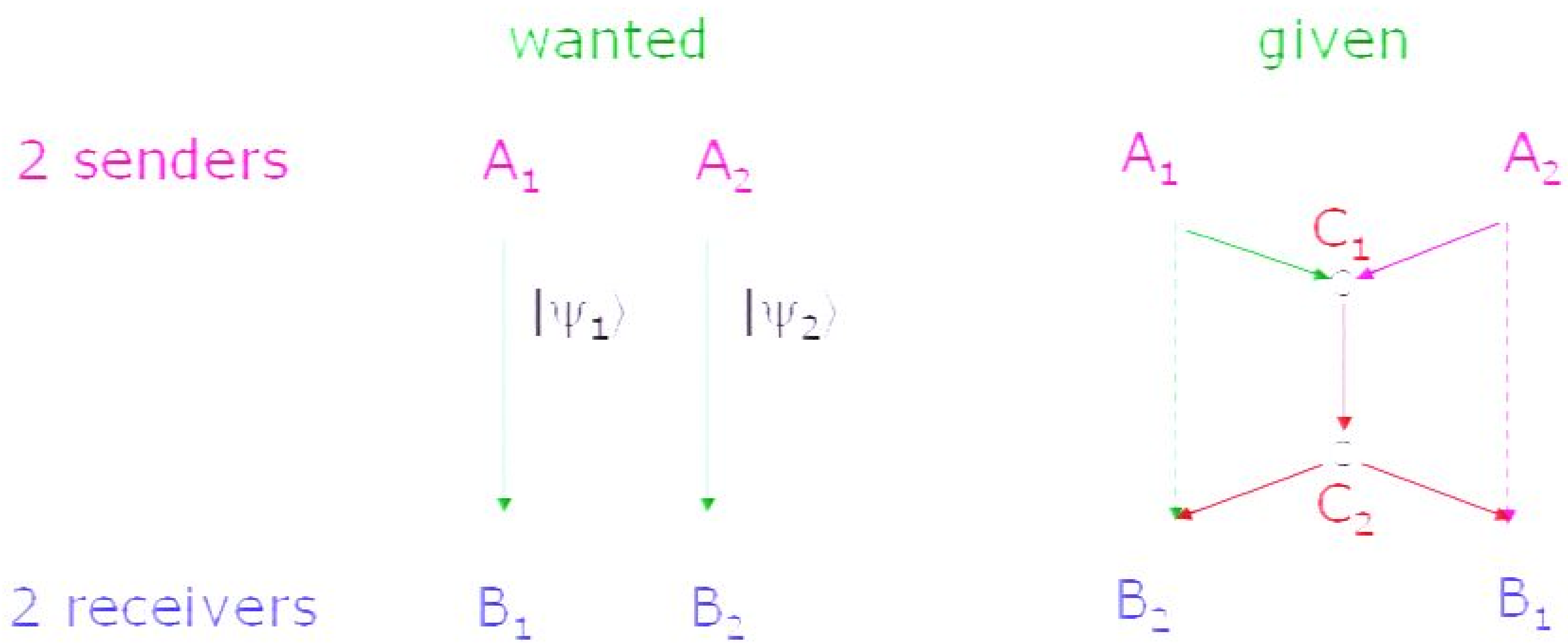
The butterfly network is the first (classical) example showing network coding allows comm rates strictly better than routing. (Ahlswede, Cai, Li, Yeung, 2000)

Will see quantum information cannot be coded as such, and routing remains optimal for classes of networks.

Quantum information, due to possible entanglement, behaves like classical commodities rather than classical information.

# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$

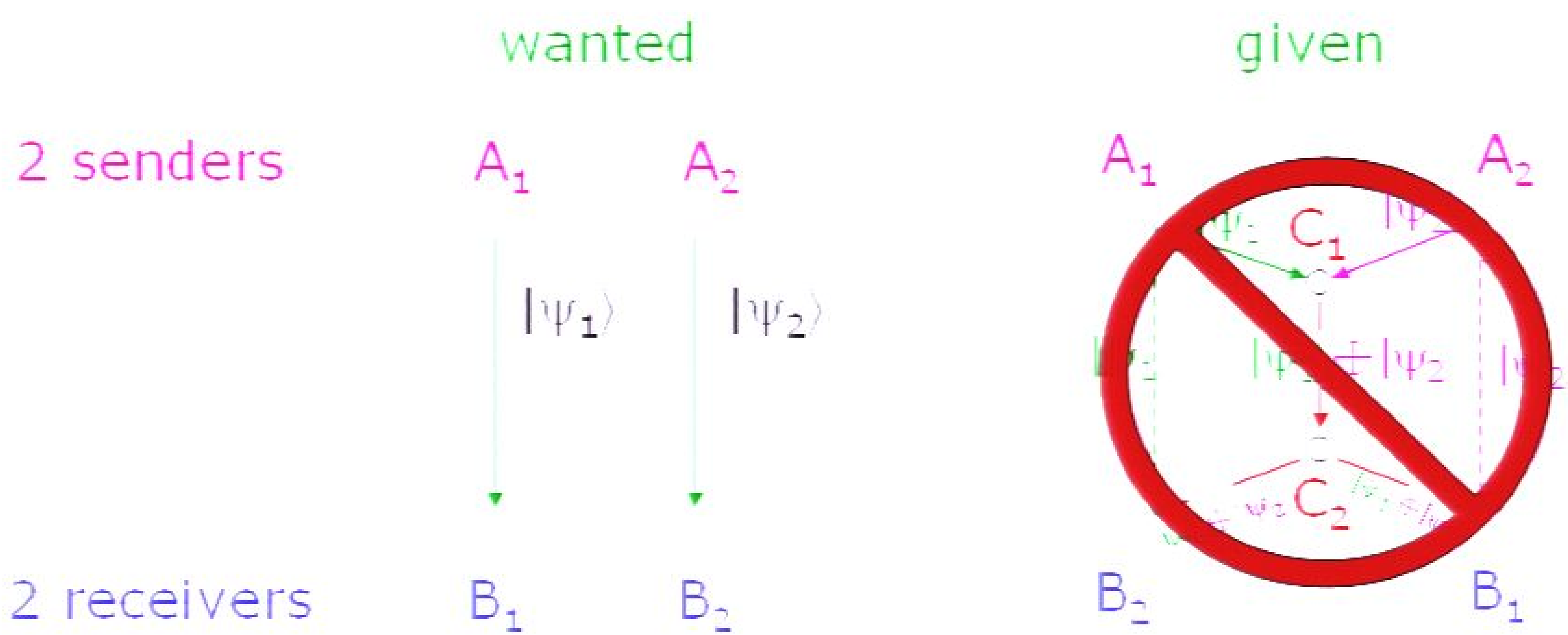



Assumption: "network" (all 7 channels) called as a package

Qn: how to "do our best" ?

# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$

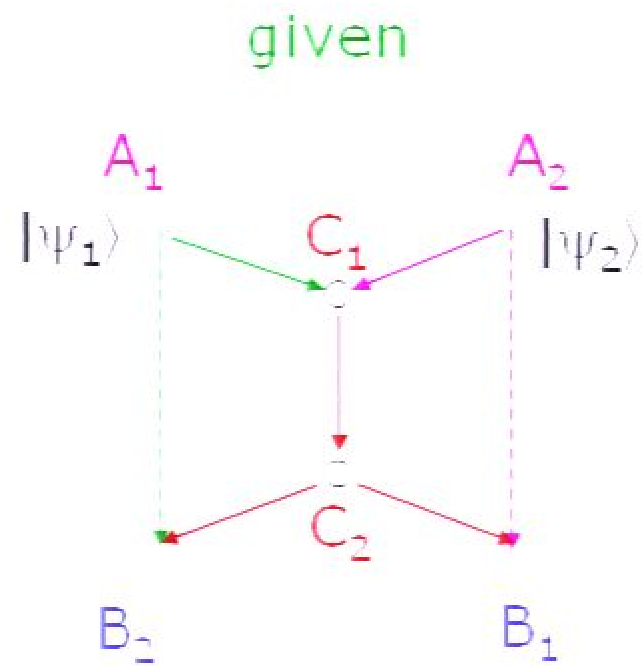



Assumption: "network" (all 7 channels) called as a package
Qn: how to "do our best" ?

# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$

QIP 2006: [Hayashi, Iwama, Nishimura, Raymond, Yamashita 0601088] 1-use of network, fixed rate, min distortion ($0.52 < \text{fidelity} < 0.983$).



# Motivating example : the butterfly network

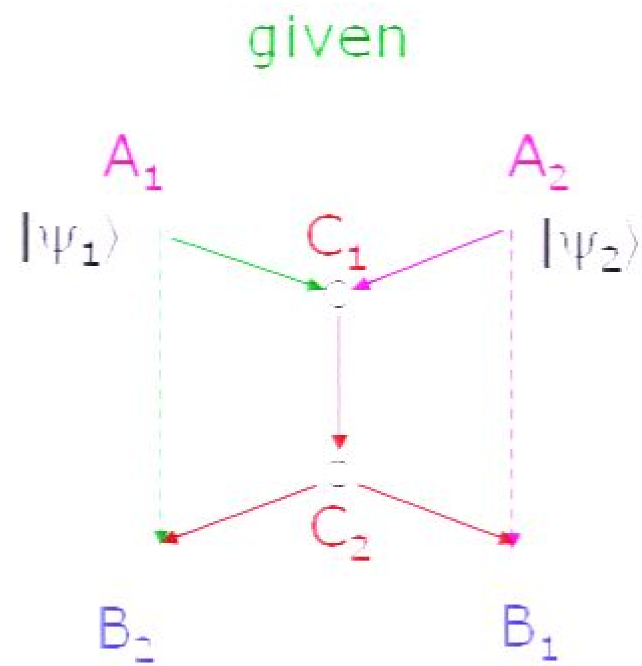
## Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$

QIP 2006: [Hayashi, Iwama, Nishimura, Raymond, Yamashita 0601088] 1-use of network, fixed rate, min distortion ($0.52 < \text{fidelity} < 0.983$).

Here:
Asymptotic (many uses, consider amount of info sent/network-use)

Demand near-perfect transmission

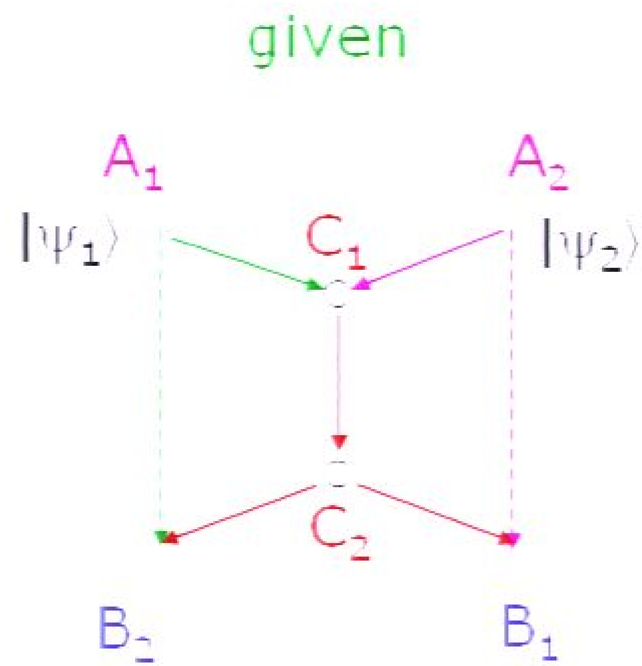
Study rate trade-off between $A_1 - B_1$ & $A_2 - B_2$ communication



# Motivating example : the butterfly network

## Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$

QIP 2006: [Hayashi, Iwama, Nishimura, Raymond, Yamashita 0601088] 1-use of network, fixed rate, min distortion ($0.52 <$ fidelity $< 0.983$).




Here:

Asymptotic (many uses, consider amount of info sent/network-use)

n

Demand near-perfect transmission

Study rate trade-off between $A_1 - B_1$ & $A_2 - B_2$ communication

$n_1$ , $n_2$

$r_1$ , $r_2$ "achievable rate pair" where $r_i = n_i/n$

# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$

QIP 2006: [Hayashi, Iwama, Nishimura, Raymond, Yamashita 0601088] 1-use of network, fixed rate, min distortion ($0.52 < \text{fidelity} < 0.983$).

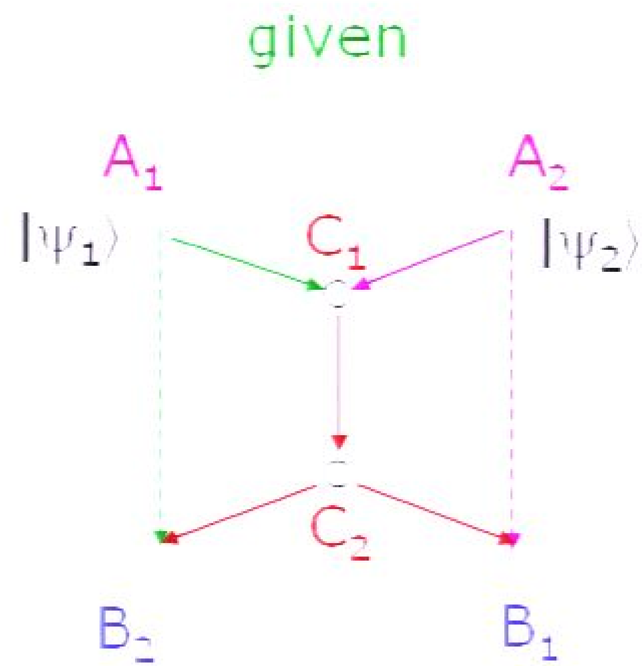
Here:

Asymptotic (many uses, consider amount of info sent/network-use) ← $n$

Demand near-perfect transmission

Study rate trade-off between $A_1 - B_1$ & $A_2 - B_2$ communication

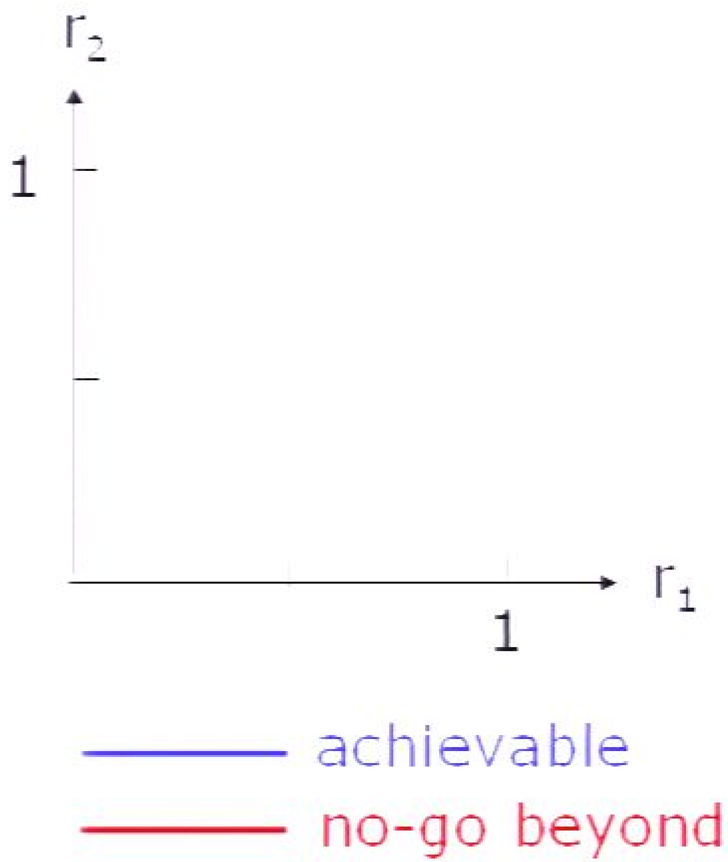
$n_1$ , $n_2$ $r_1$ , $r_2$ "achievable rate pair" where $r_i = n_i / n$

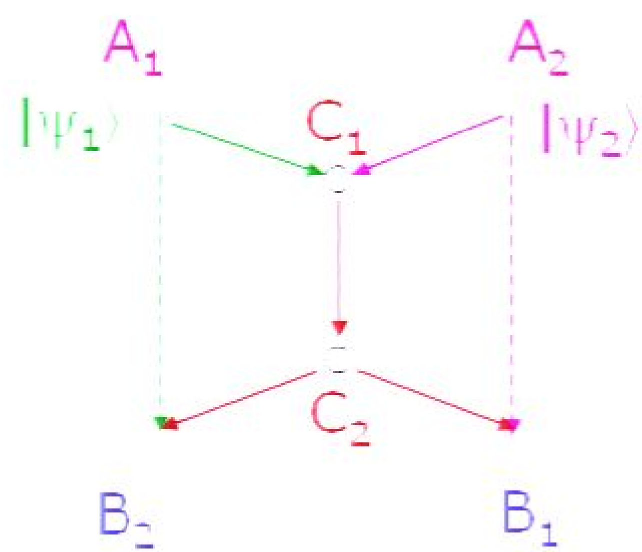



Goal: find all achievable rate pairs

# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance

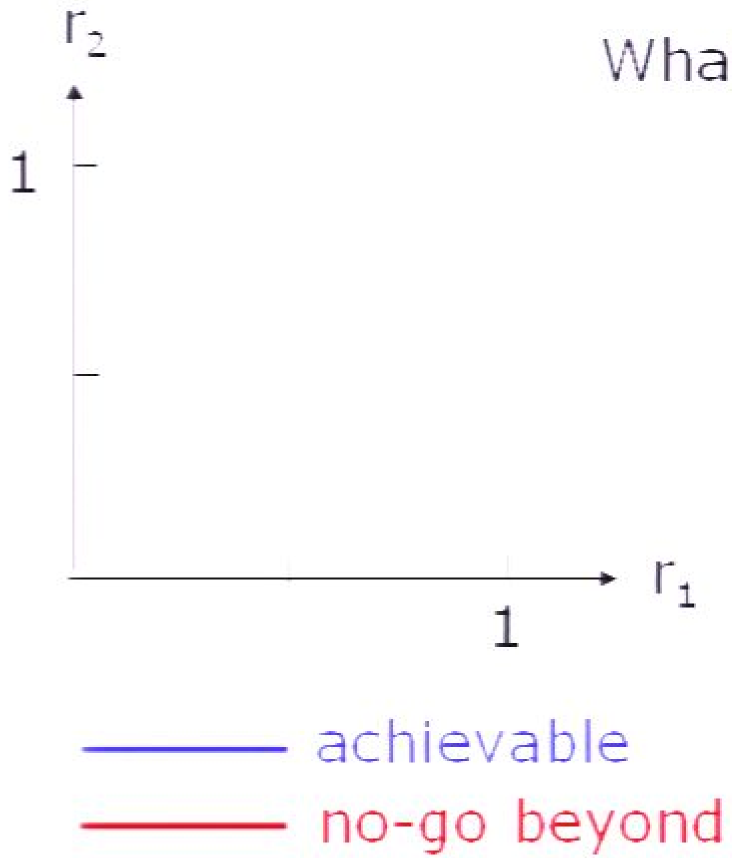


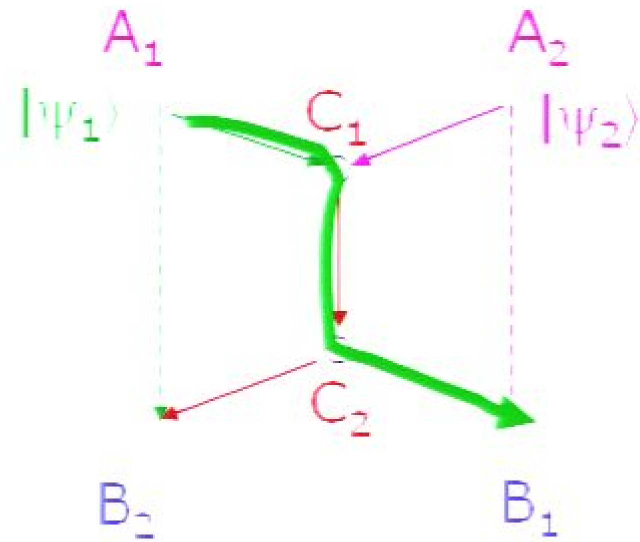



# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance

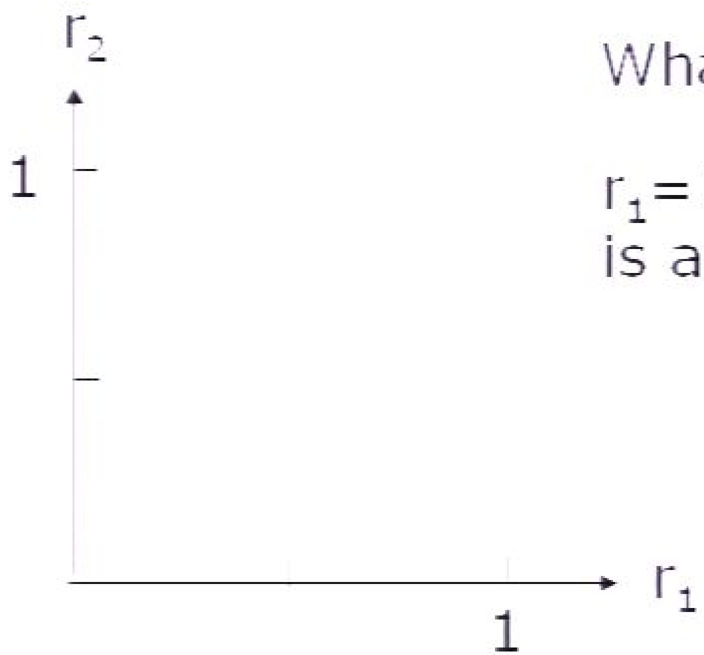
What can be done



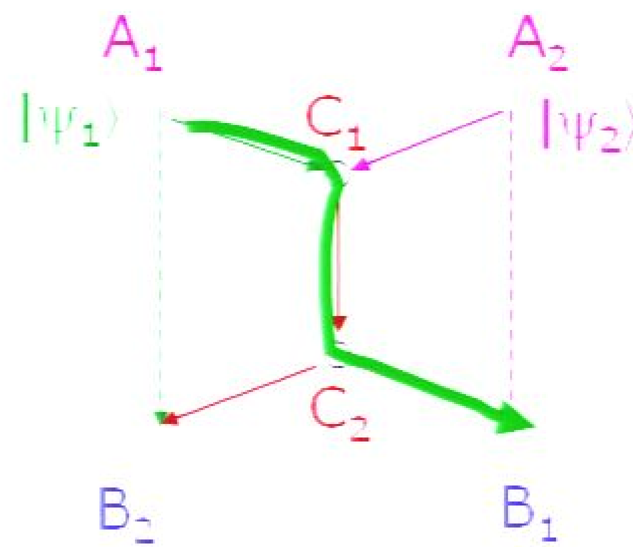
# Motivating example : the butterfly network

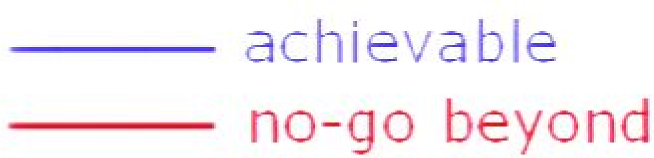
Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance




What can be done

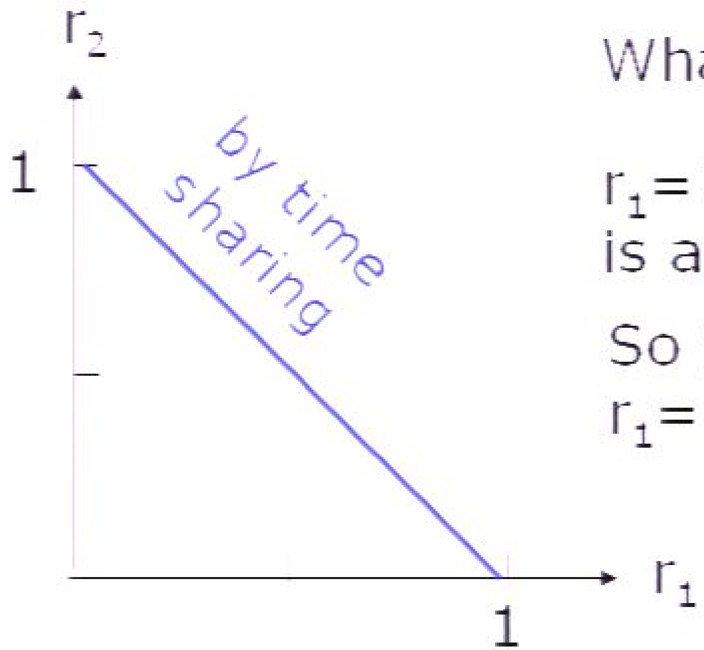
$r_1=1$, $r_2=0$
is achievable



# Motivating example : the butterfly network

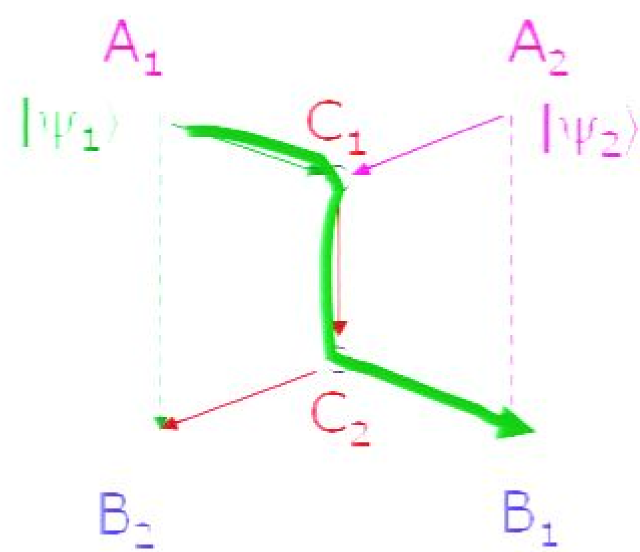
Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance




What can be done

$r_1=1$, $r_2=0$
is achievable

So is
$r_1=0$, $r_2=1$

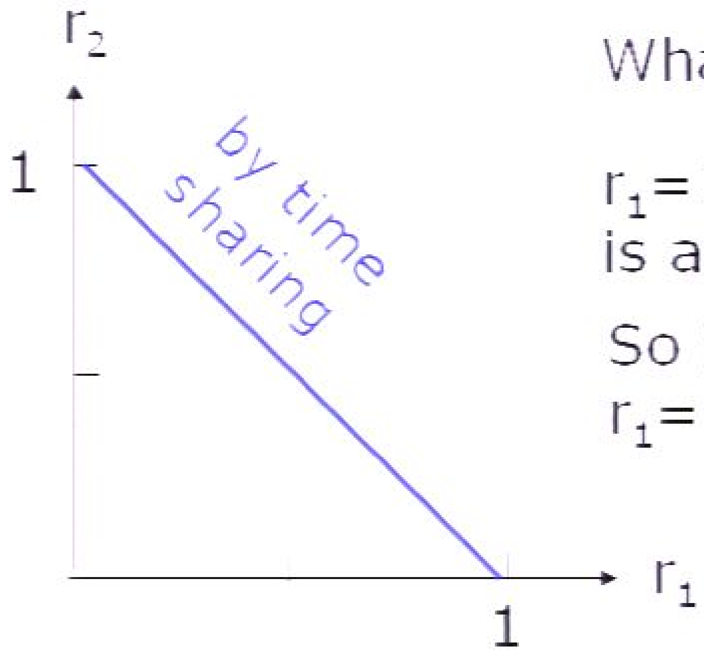



achievable

no-go beyond

# Motivating example : the butterfly network

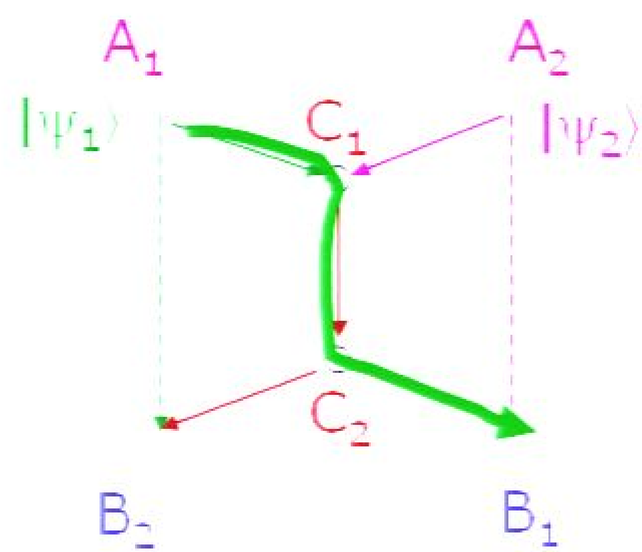
Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance




What can be done

$r_1=1$, $r_2=0$ is achievable

So is $r_1=0$, $r_2=1$




achievable

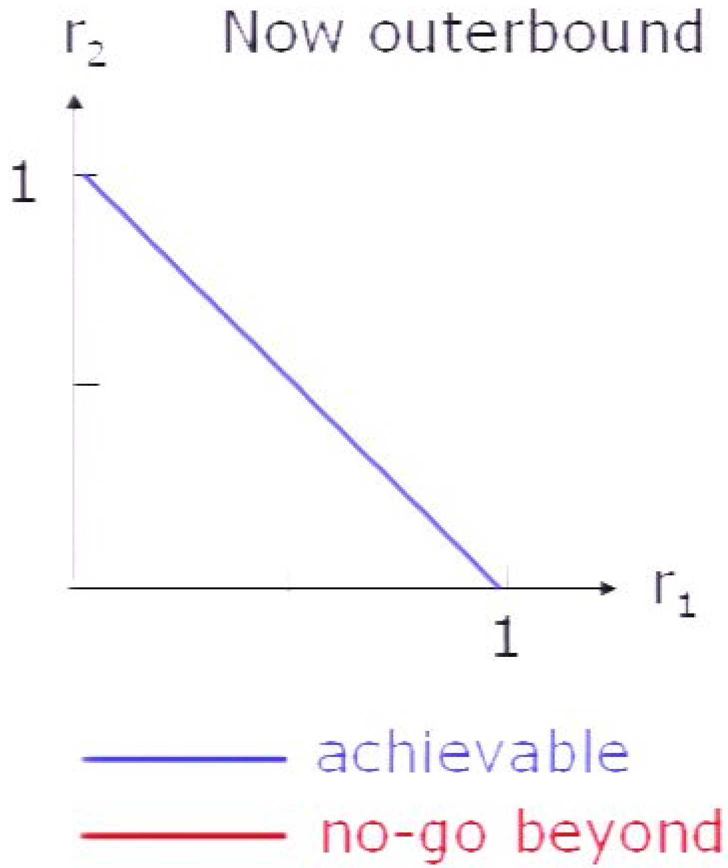
no-go beyond

PS (1) convexity (2) monotonicity of rate region hold generally

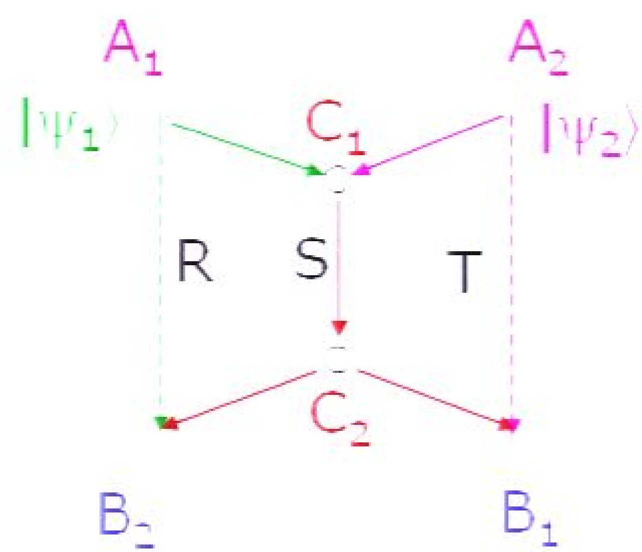
# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance

Now outerbound

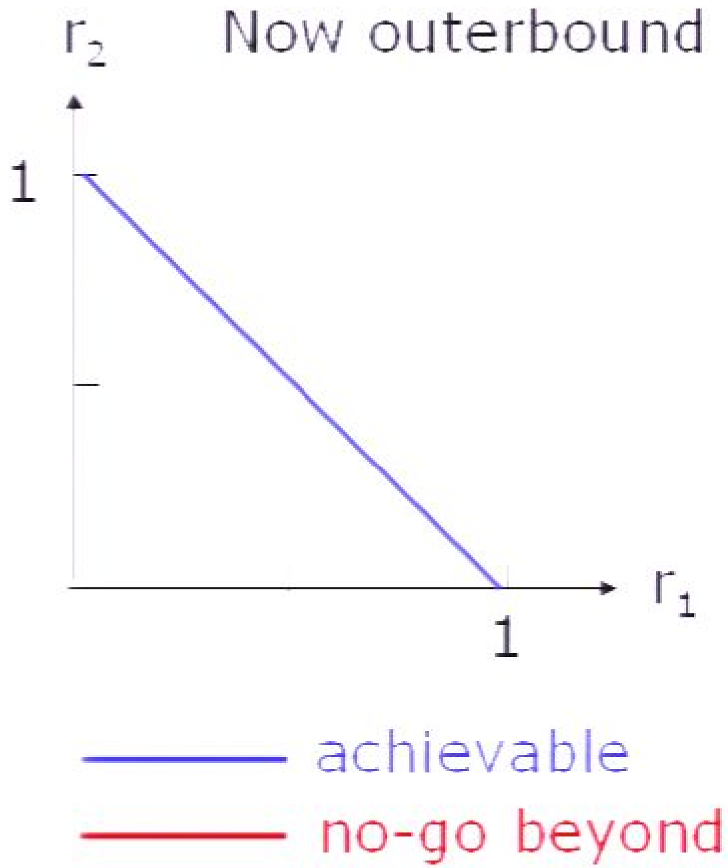



Asymptotic: n-qubit sent per edge (acyclic graph), $n_1$ qubits from $A_1$ to $B_1$

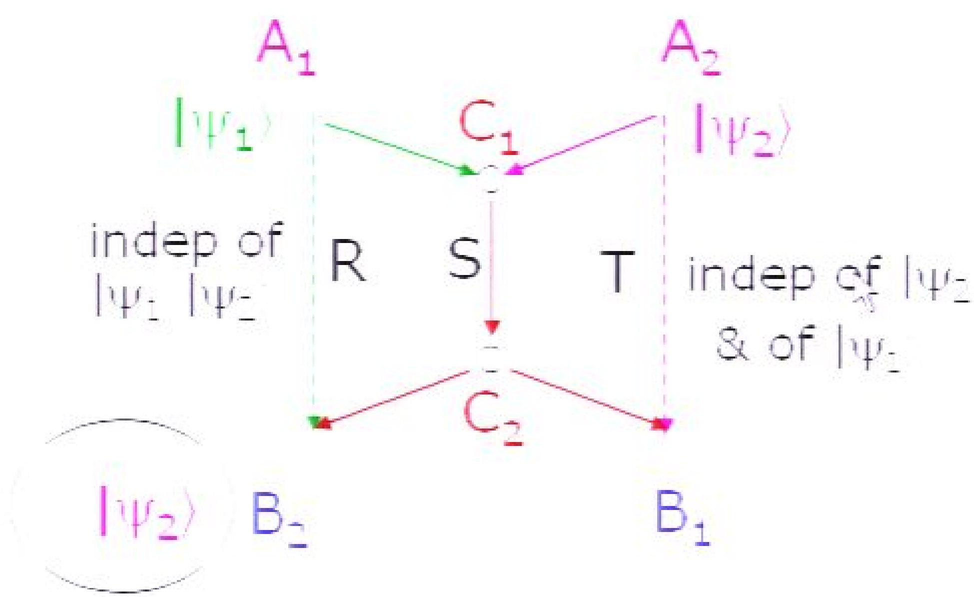


# Motivating example : the butterfly network

## Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance




Asymptotic: n-qubit sent per edge (acyclic graph), $n_1$ qubits from $A_1$ to $B_1$



# Motivating example : the butterfly network

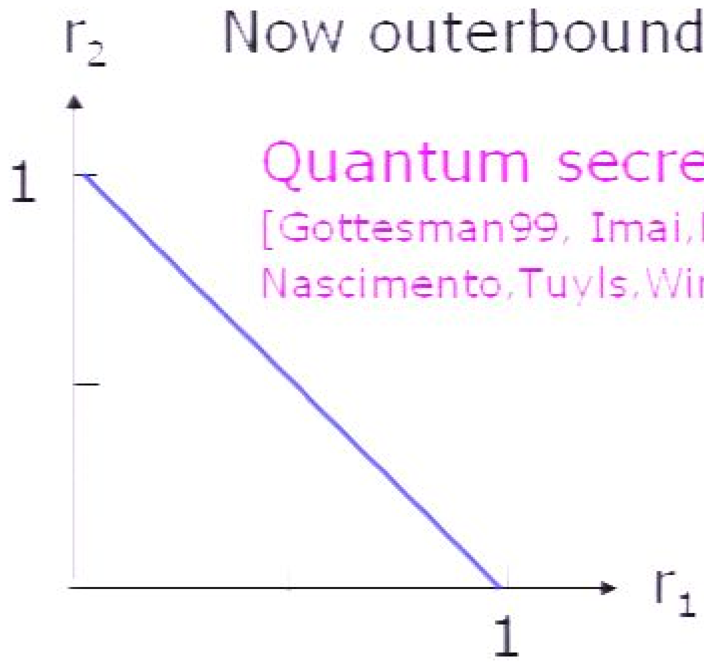
Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance

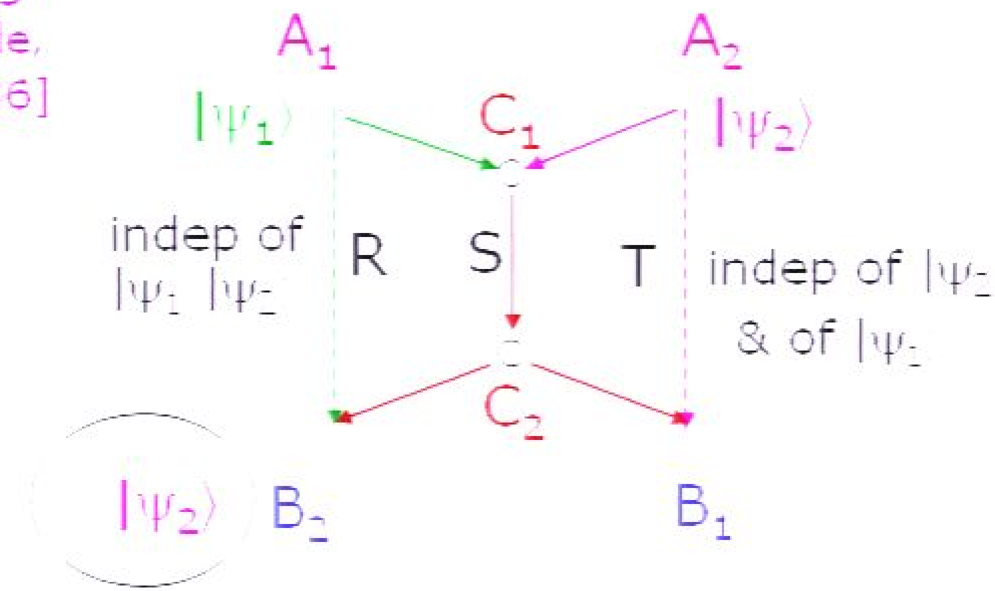
Now outerbound

Asymptotic: n-qubit sent per edge (acyclic graph), $n_1$ qubits from $A_1$ to $B_1$

Quantum secret sharing

[Gottesman99, Imai,Müller-Quade, Nascimento,Tuyls,Winter 0311136]




achievable

no-go beyond

RST authorized

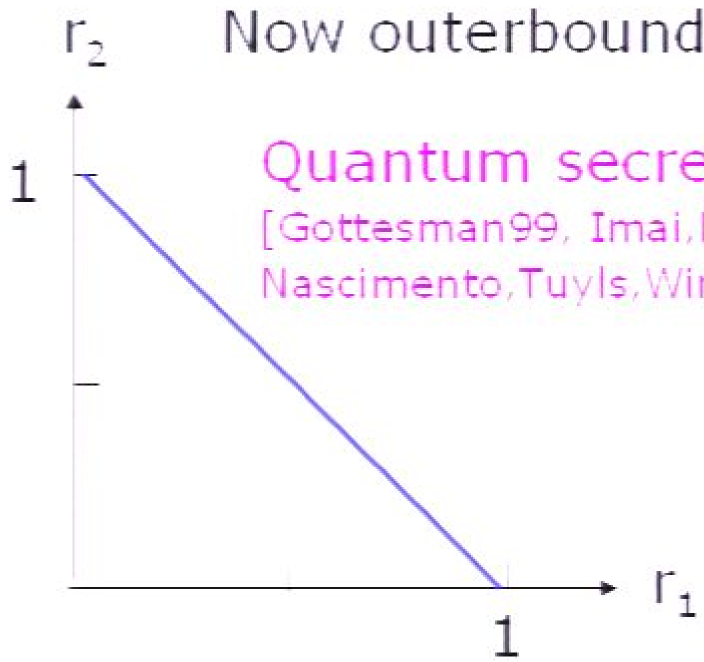
RT unauth

S significant $\geq n_1+n_2$ qubits

# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance

Now outerbound

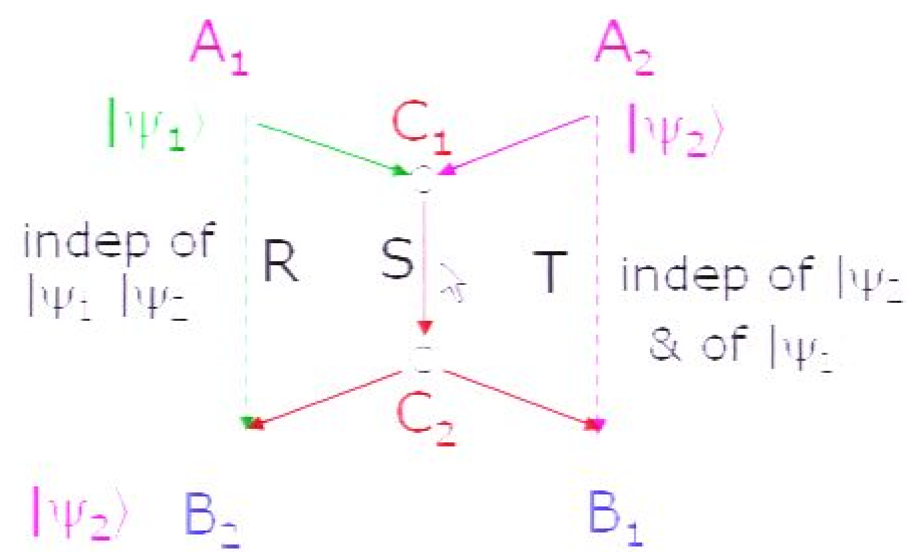



Quantum secret sharing
[Gottesman99, Imai, Müller-Quade, Nascimento, Tuyls, Winter 0311136]

achievable

no-go beyond

Asymptotic: n-qubit sent per edge (acyclic graph), $n_1$ qubits from $A_1$ to $B_1$

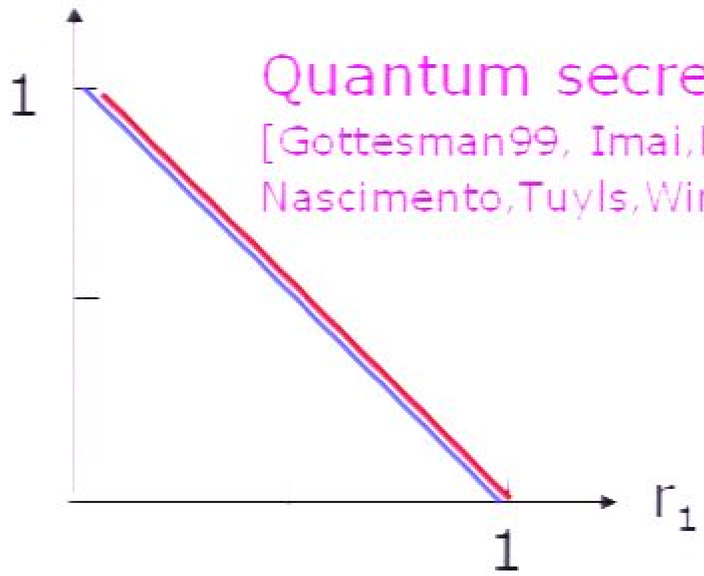



n qubits $\geq$ S significant $\geq n_1+n_2$ qubits

# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance

Now outerbound

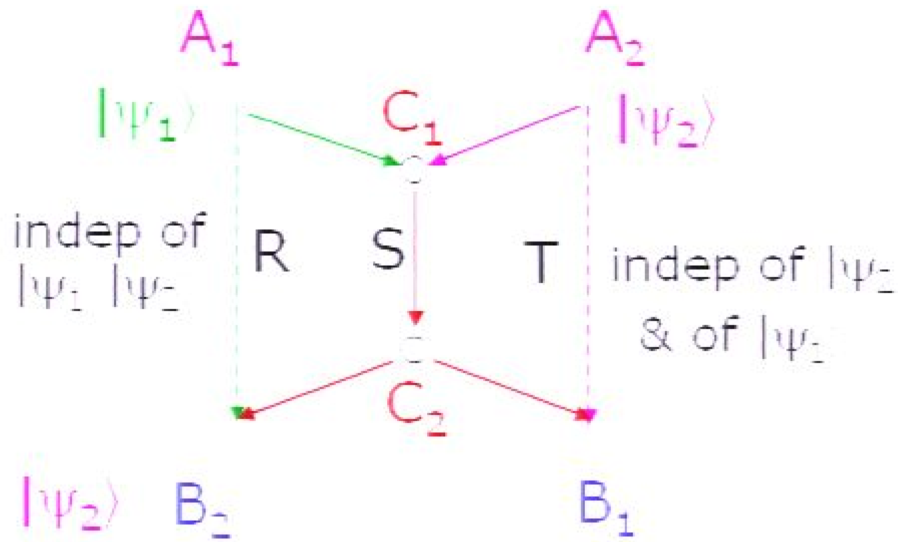



Quantum secret sharing
[Gottesman99, Imai,Müller-Quade, Nascimento,Tuyls,Winter 0311136]

achievable

no-go beyond

Asymptotic: n-qubit sent per edge (acyclic graph), $n_1$ qubits from $A_1$ to $B_1$



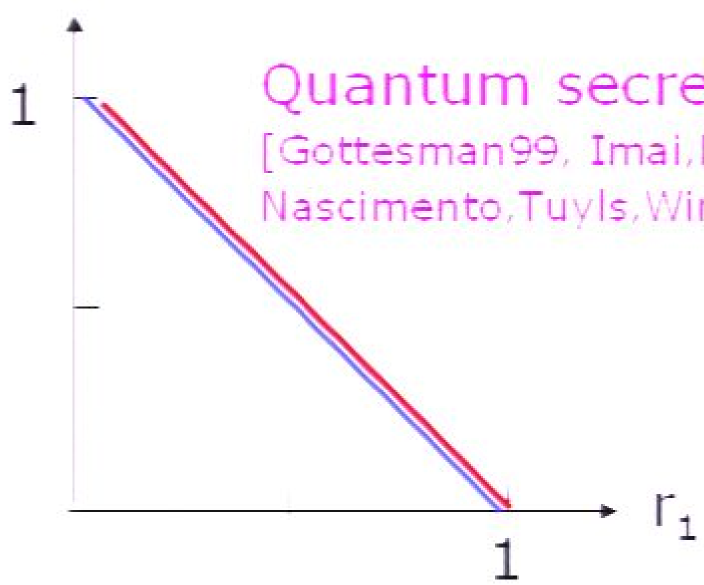

n qubits $\geq$ S significant $\geq n_1+n_2$ qubits

$1 \geq r_1 + r_2$

# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance

Now outerbound




Quantum secret sharing

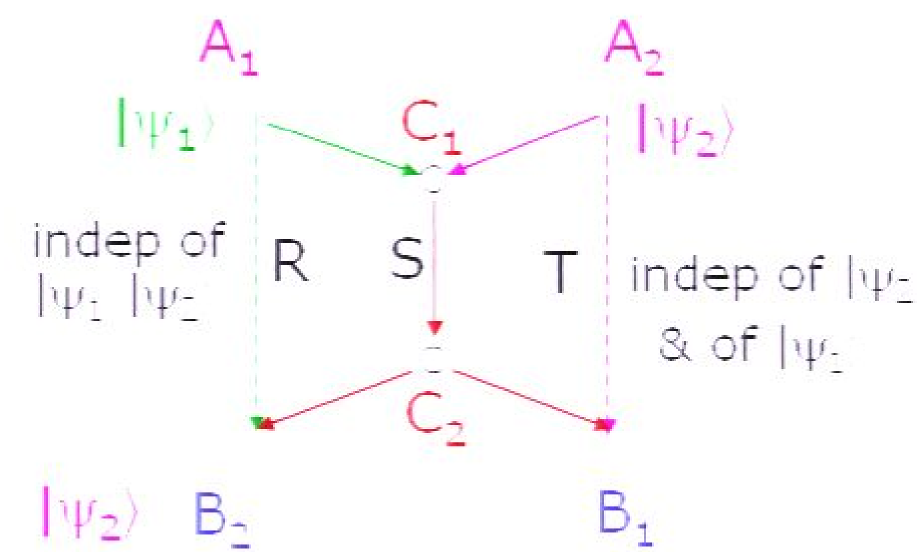
[Gottesman99, Imai,Müller-Quade, Nascimento,Tuyls,Winter 0311136]

achievable

no-go beyond

Optimal protocol: time sharing between 2 trivial 1-shot solutions

Asymptotic: n-qubit sent per edge (acyclic graph), $n_1$ qubits from $A_1$ to $B_1$




n qubits $\geq$ S significant $\geq n_1+n_2$ qubits

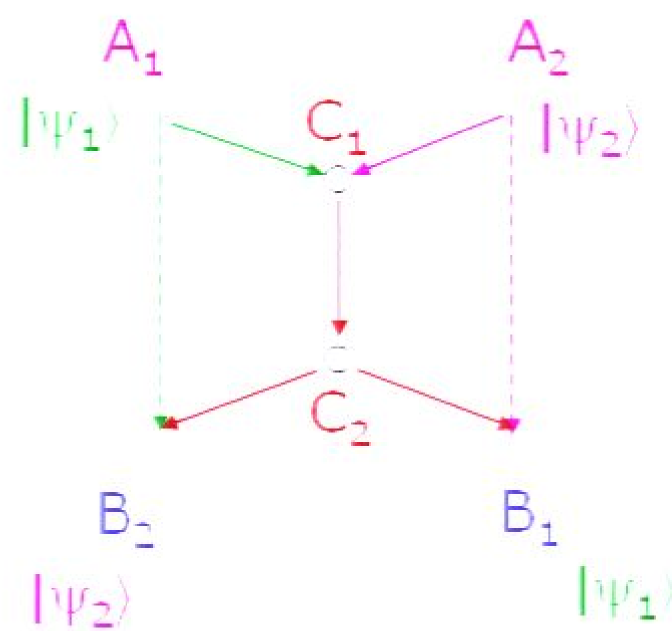
$1 \geq r_1 + r_2$

Finished unassisted case

Now consider assisted case
(i.e. having free auxiliary resources)

# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

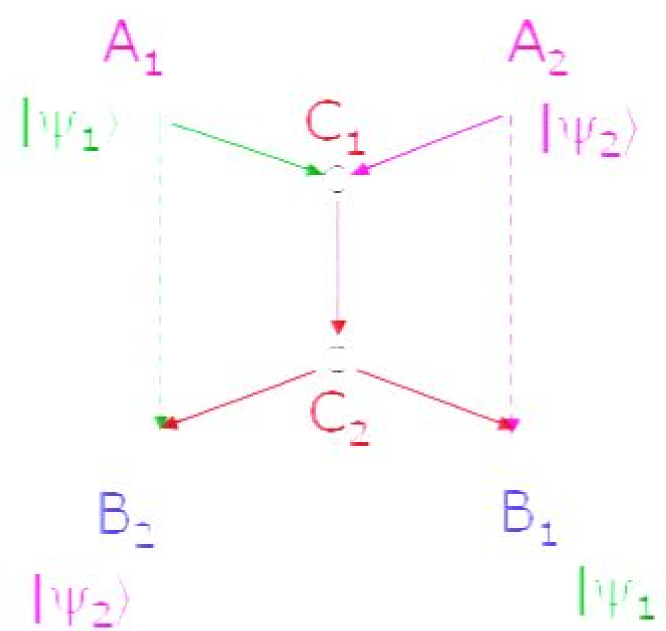


# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

What can be done

2 bits of back comm reverts a quantum channel



# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

What can be done

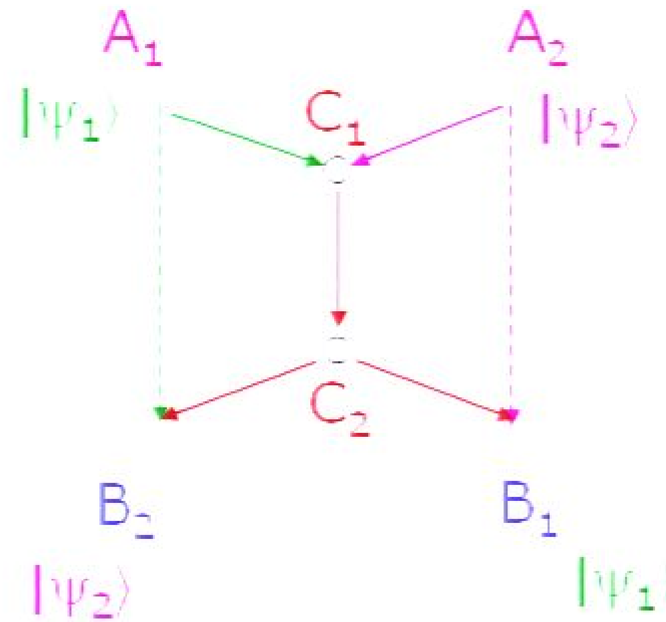
2 bits of back comm reverts a quantum channel

1 qubit forward quantum comm
+ 2 backward classical bits

gives

1 ebit of entanglement
+ 2 backward classical bits

can teleport 1 qubit backwards

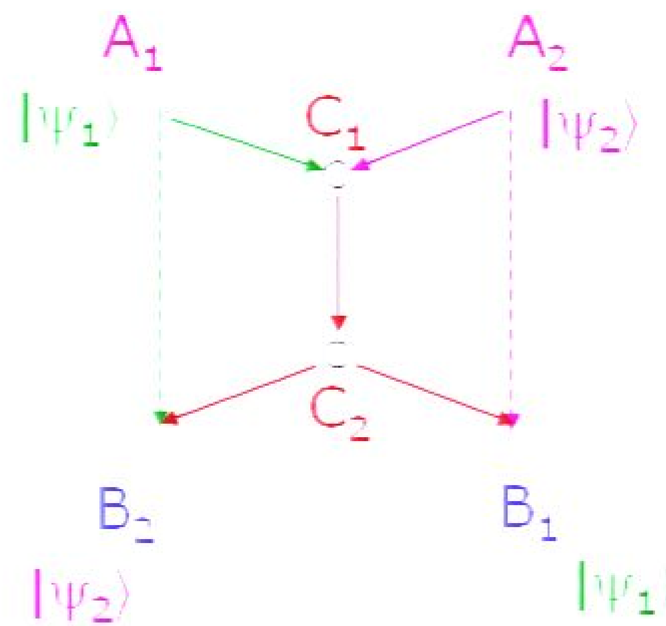


# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

What can be done

Using teleportation, 2 bits of back comm reverts a quantum channel

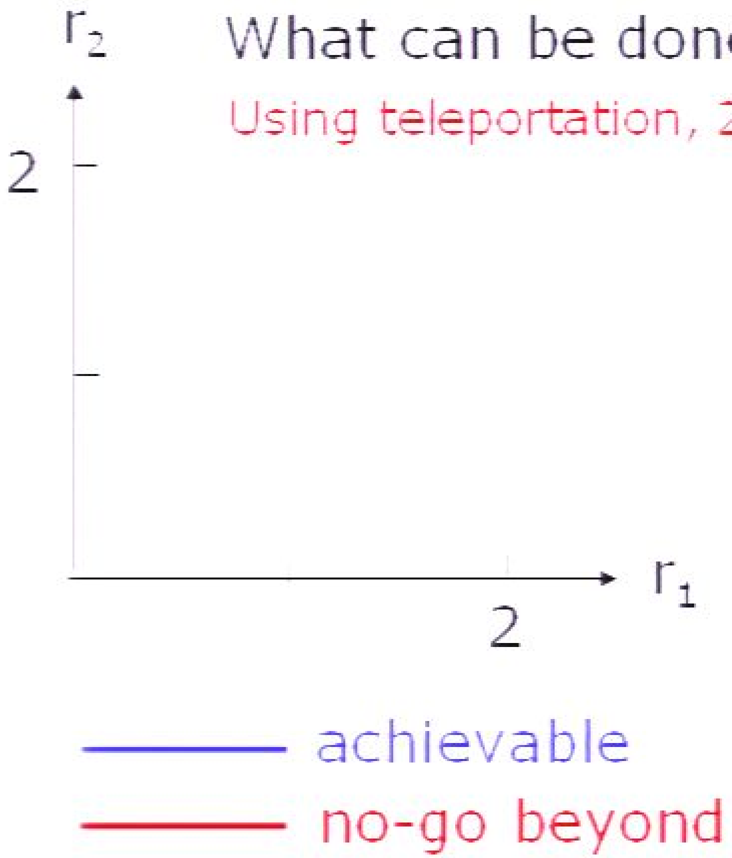


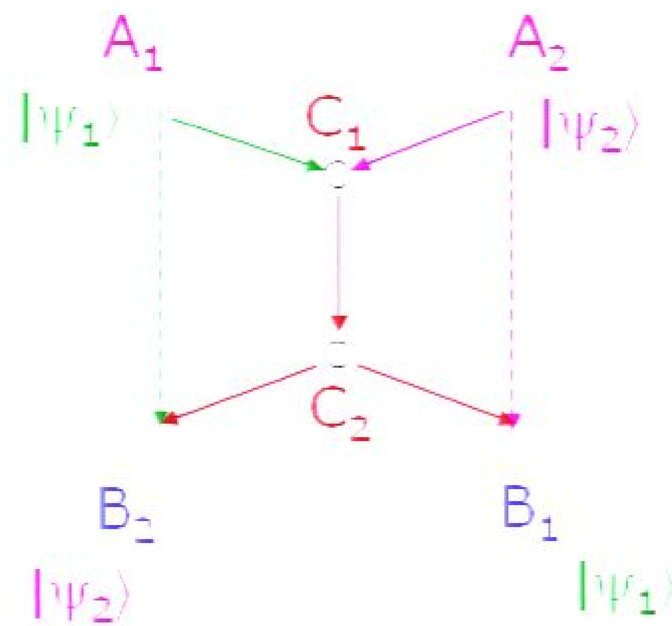
# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

What can be done

Using teleportation, 2 bits of back comm reverts a quantum channel



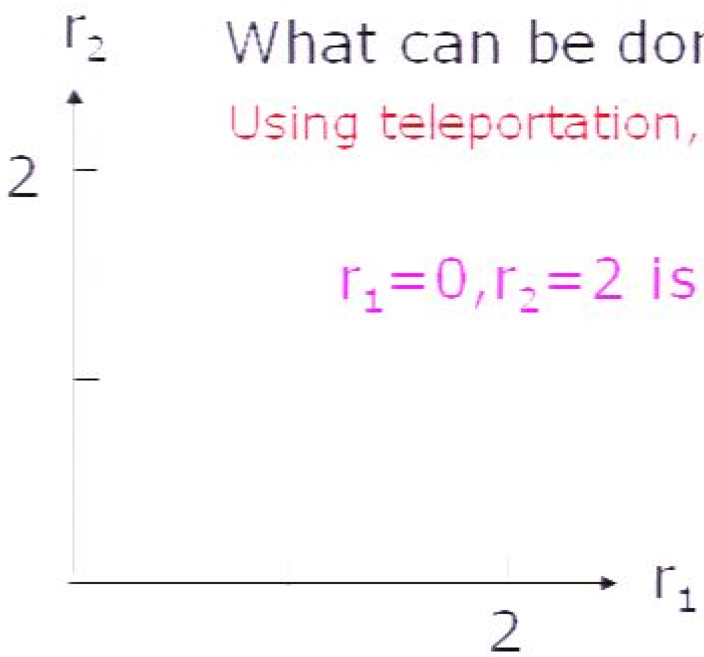
# Motivating example : the butterfly network

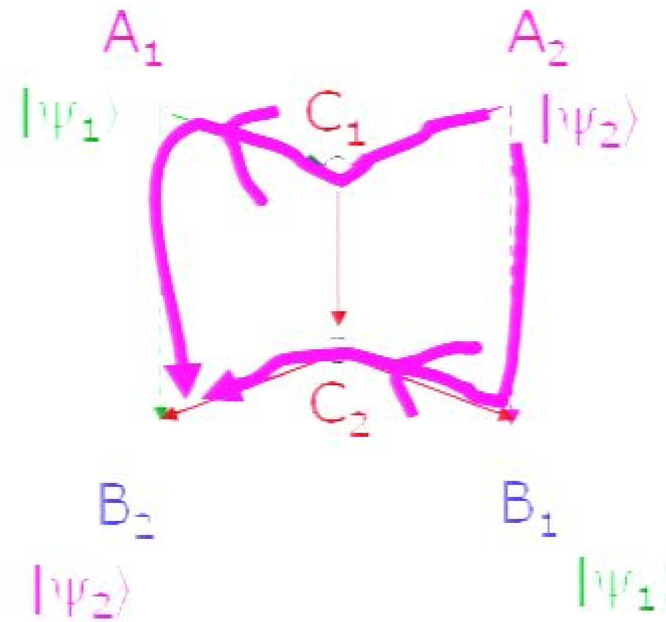
## Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

What can be done

Using teleportation, 2 bits of back comm reverts a quantum channel

$r_1=0, r_2=2$ is achievable




achievable

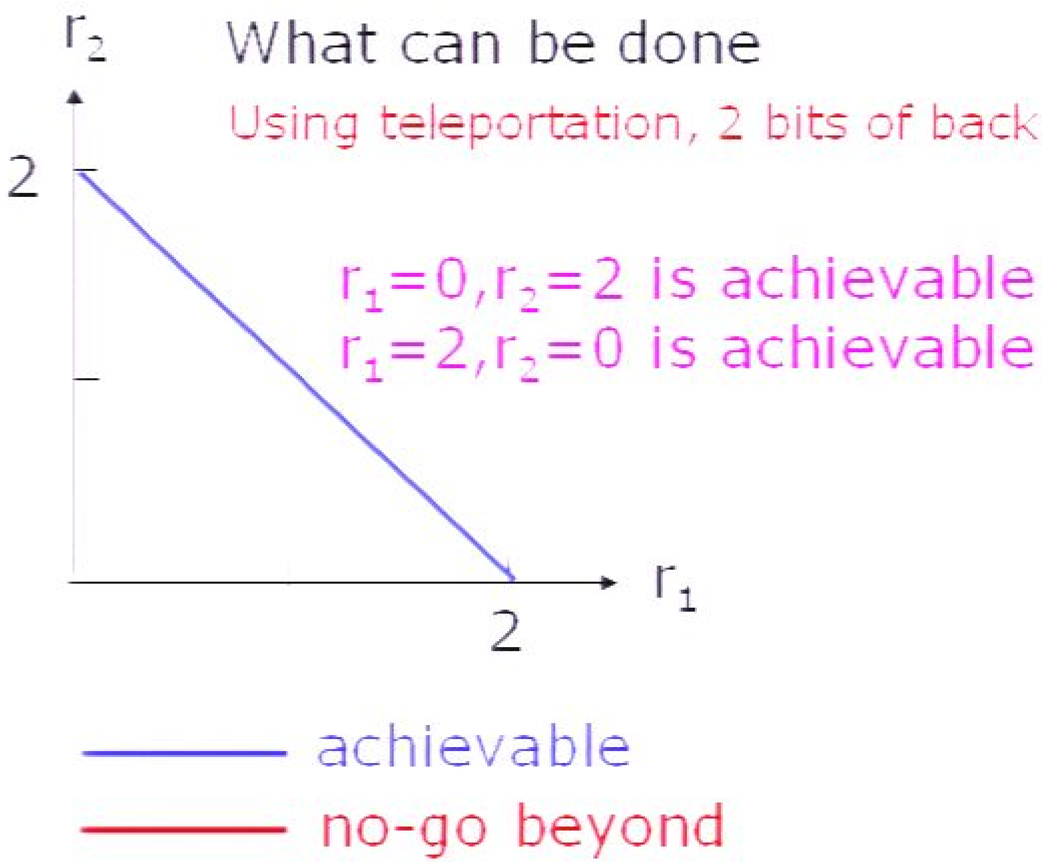
no-go beyond

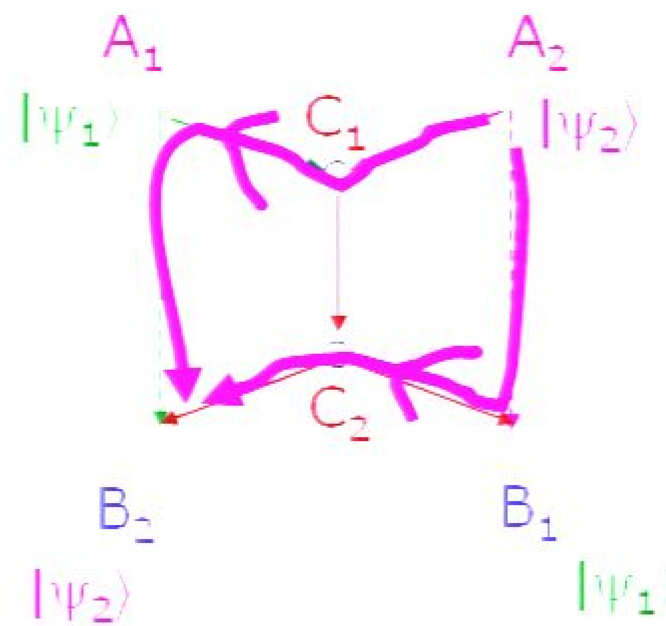
# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

What can be done

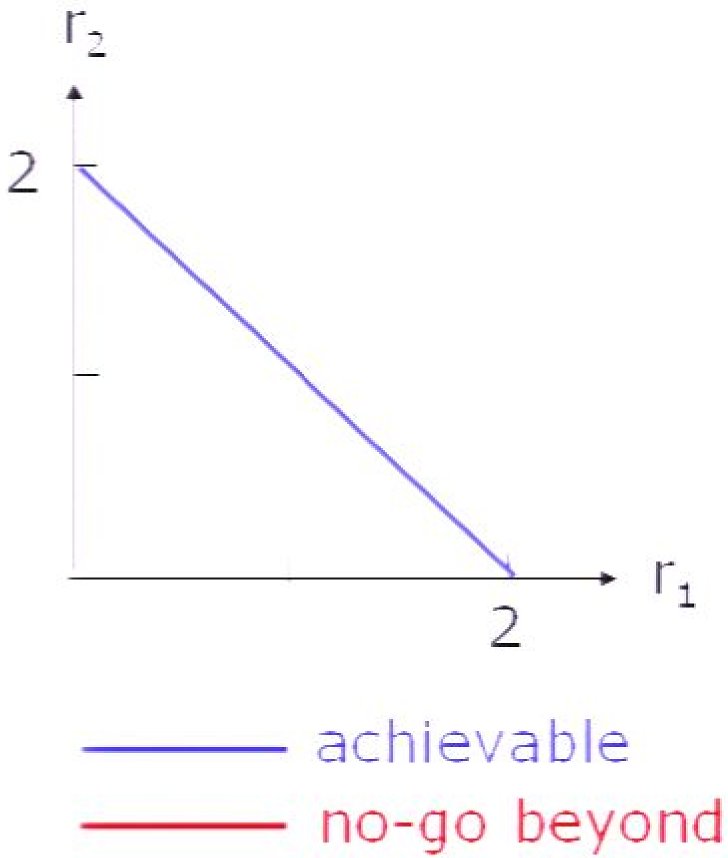
Using teleportation, 2 bits of back comm reverts a quantum channel

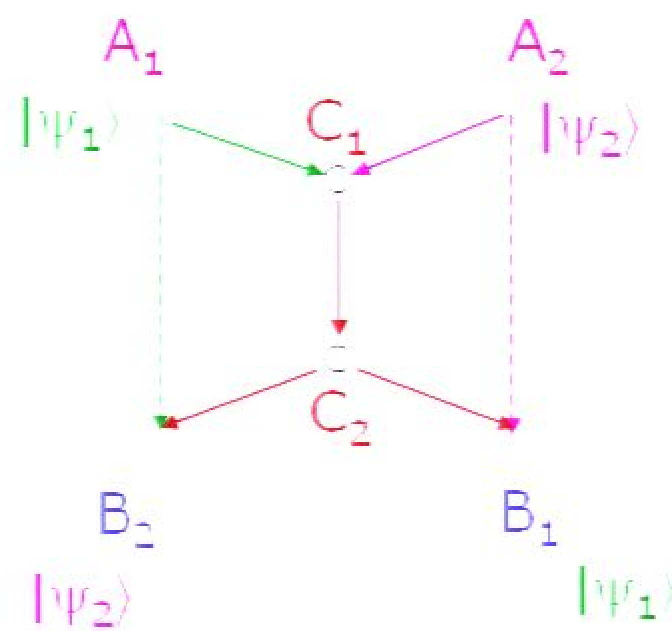


# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

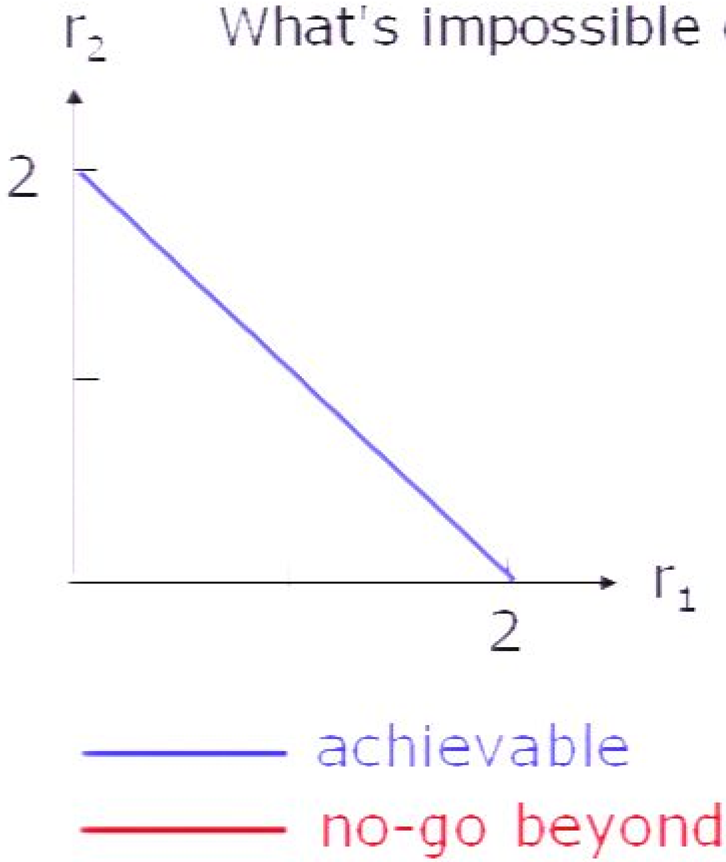


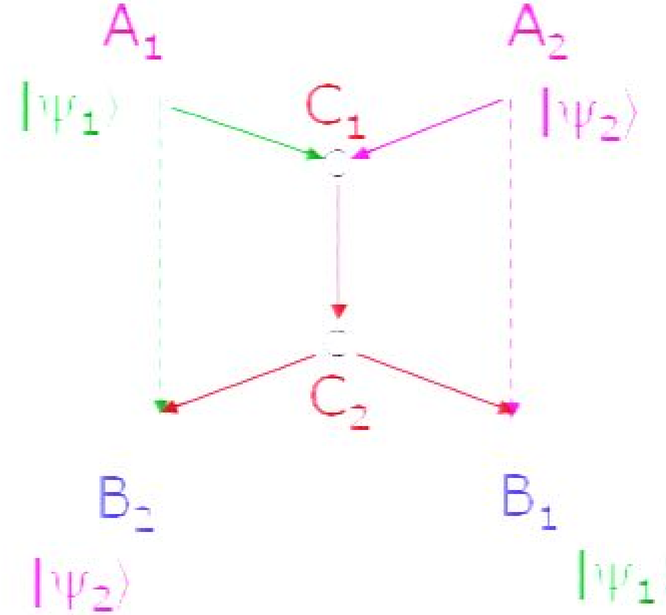



# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

What's impossible even asymptotically



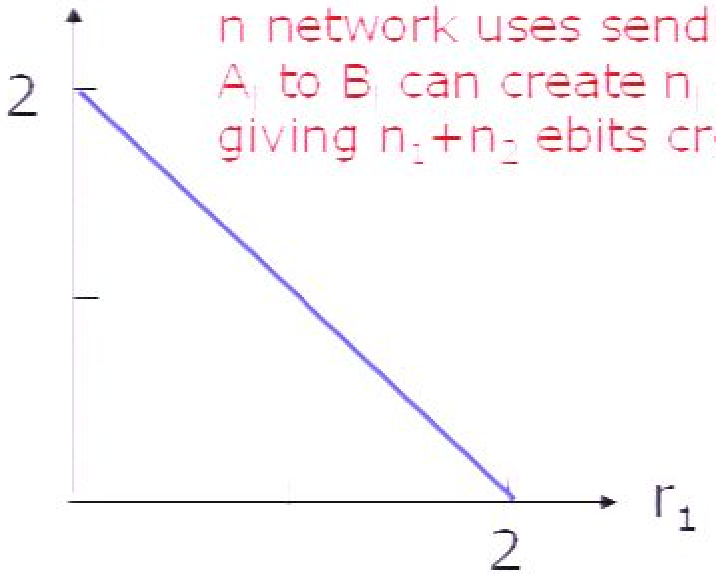



# Motivating example : the butterfly network

## Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

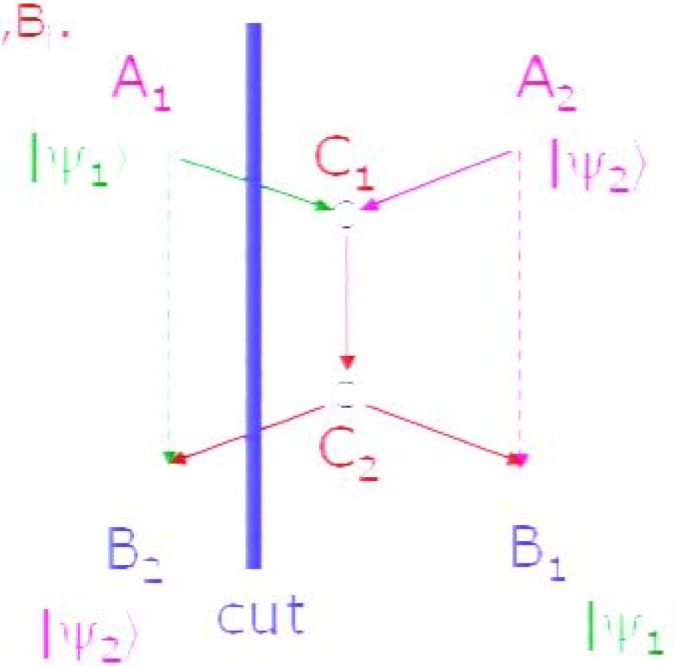
What's impossible even asymptotically

n network uses sending $n_i$ qubits from $A_i$ to $B_i$ can create $n_i$ ebits between $A_i$,$B_i$. giving $n_1+n_2$ ebits cross the cut




achievable

no-go beyond

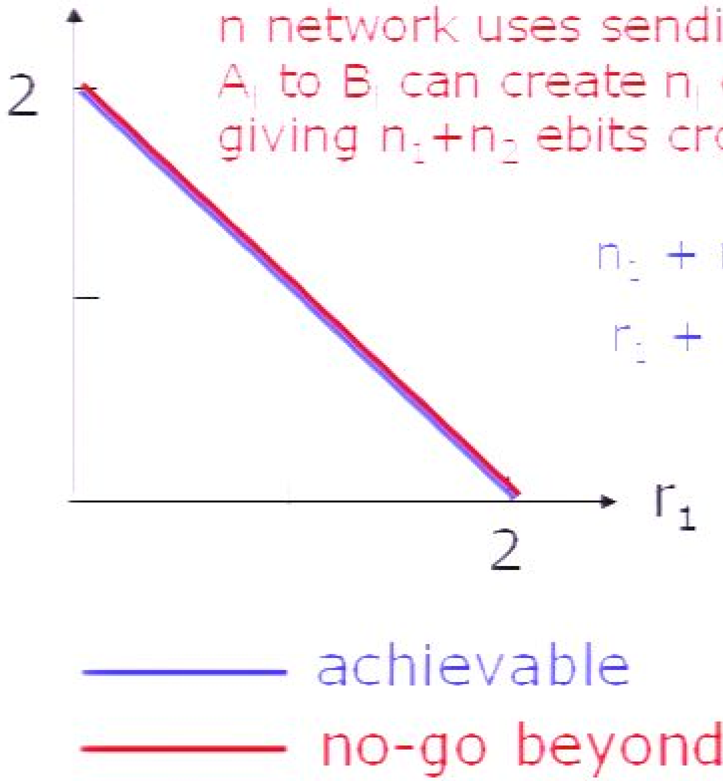


# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

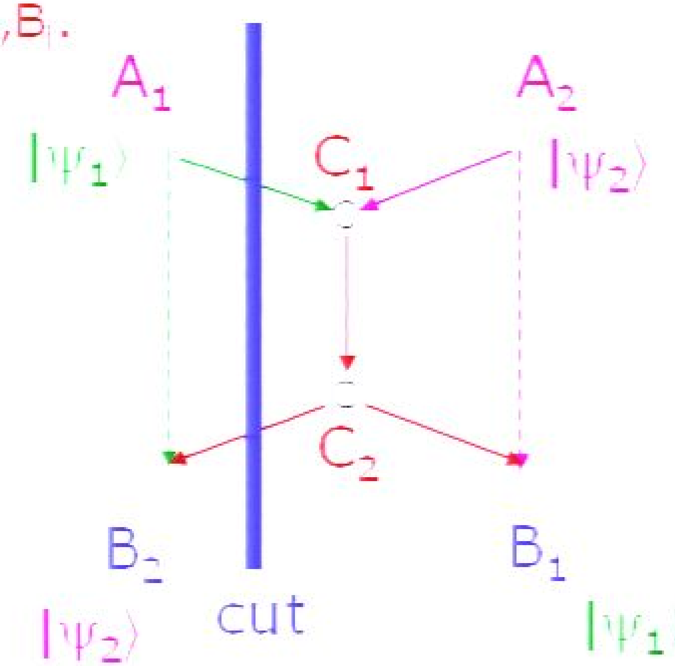
What's impossible even asymptotically

n network uses sending $n_i$ qubits from $A_i$ to $B_i$ can create $n_i$ ebits between $A_i$,$B_i$. giving $n_1+n_2$ ebits cross the cut




$n_1 + n_2 \le 2n$

$r_1 + r_2 \le 2$




Optimal solution : time sharing of two 1-shot protocols

Note: free two-way CC no better than 4 bits of back comm

# Motivating example : the butterfly network

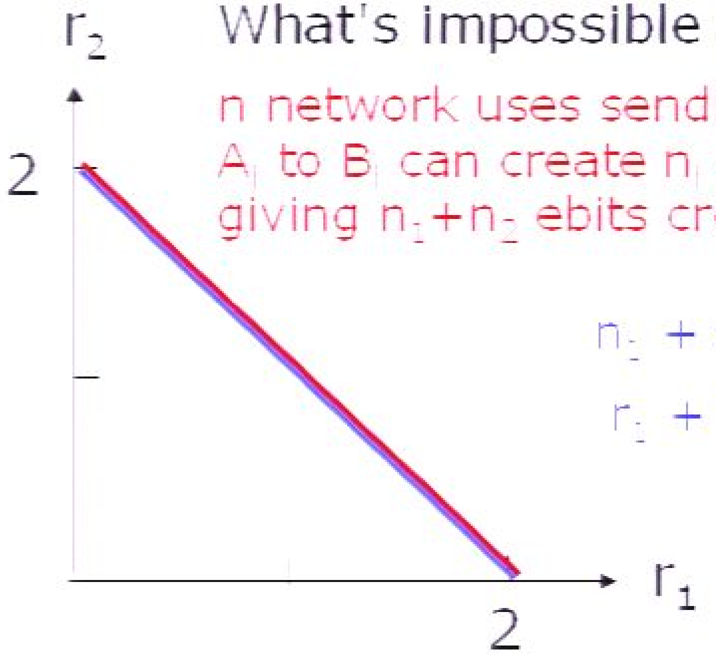
Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

What's impossible even asymptotically

n network uses sending $n_i$ qubits from $A_i$ to $B_i$ can create $n_i$ ebits between $A_i$,$B_i$. giving $n_1+n_2$ ebits cross the cut

$n_1 + n_2 \leq 2n$

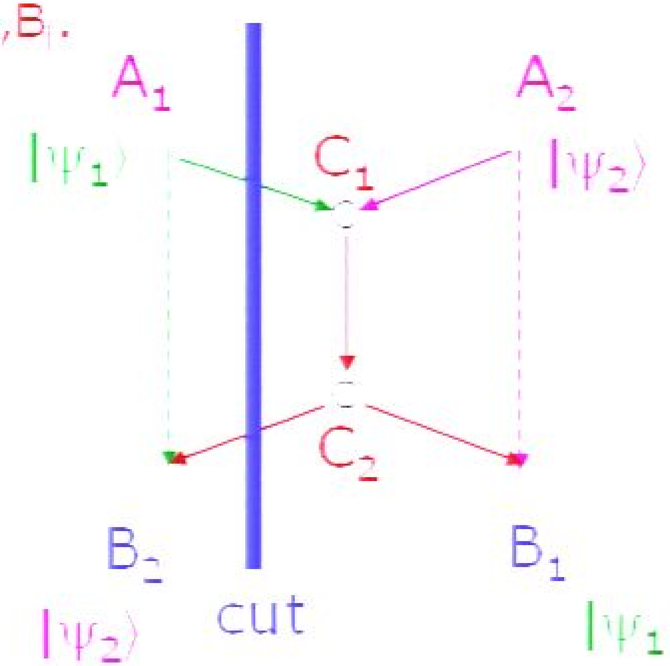
$r_1 + r_2 \leq 2$




—— achievable

—— no-go beyond

SAME ANALYSIS for free BACK CC




Optimal solution : time sharing of two 1-shot protocols

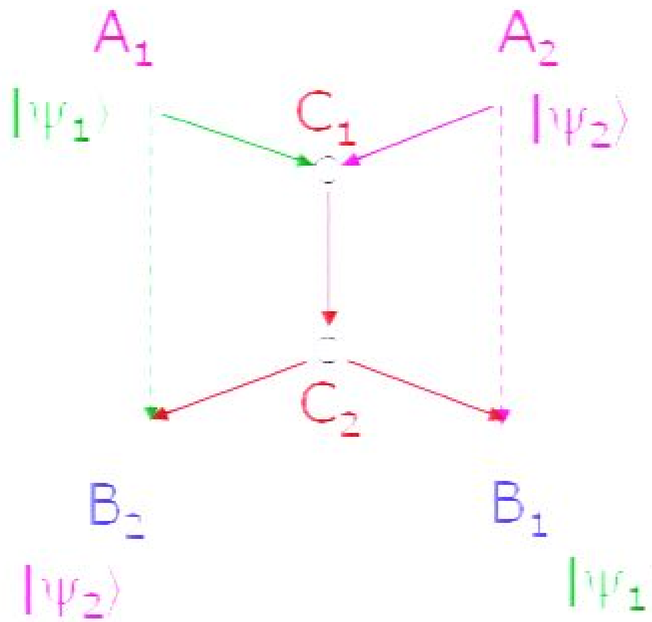
Note: free two-way CC no better than 4 bits of back comm

Finished scenario with free
2-way or backward classical comm assistance

Now consider forward assistance

# Motivating example : the butterfly network

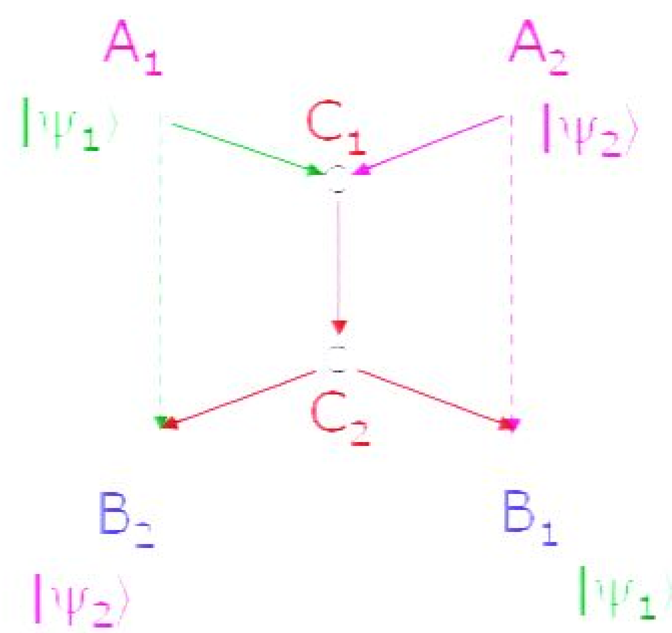
Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC



# Motivating example : the butterfly network

## Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC

In this scenario, some but not all channels can be reversed.

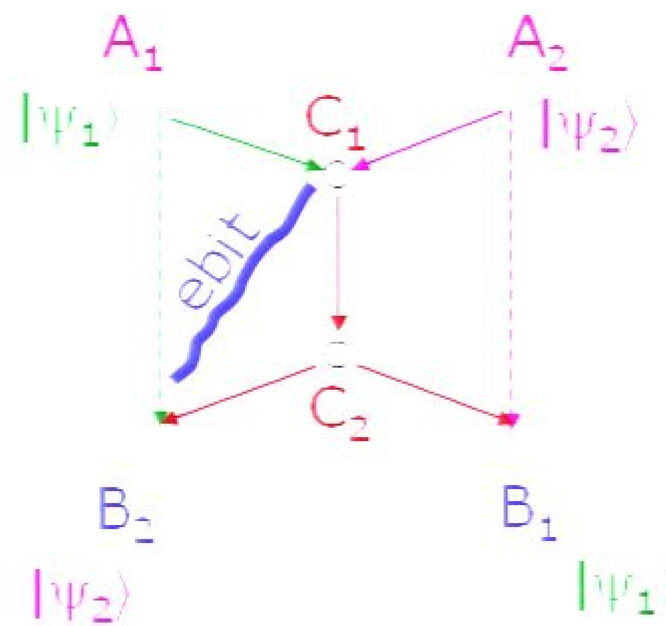


# Motivating example : the butterfly network

## Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC

In this scenario, some but not all channels can be reversed.

$A_1$ creates an ebit between $C_1$ and $B_2$

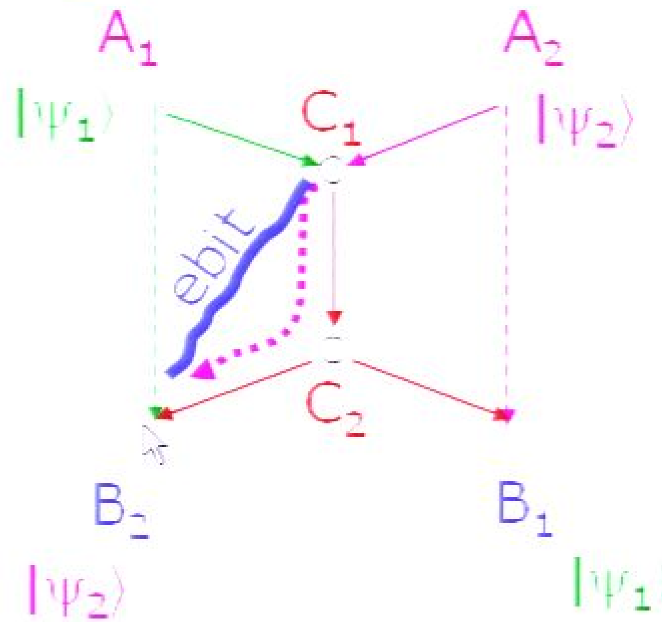


# Motivating example : the butterfly network

## Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC

In this scenario, some but not all channels can be reversed.

$A_2$ creates an ebit between $C_2$ and $B_2$
$C_1$ can teleport 1 qubit to $B_2$ by free CC via $C_2$ & $B_2$

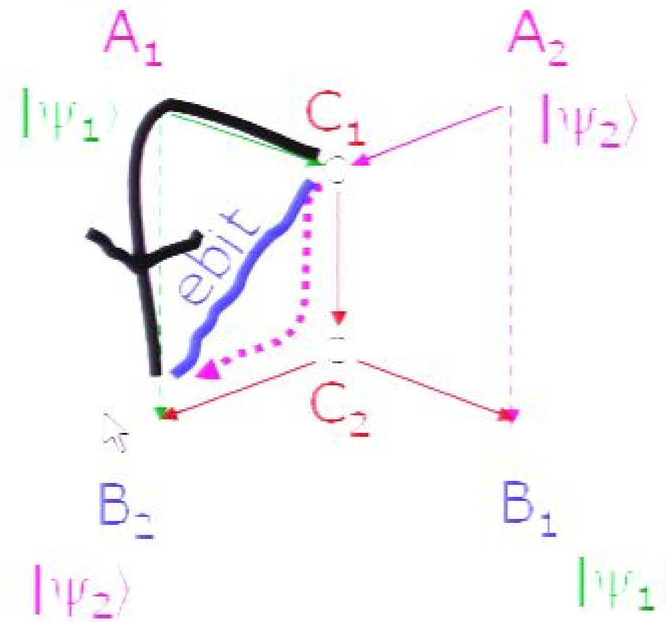


# Motivating example : the butterfly network

## Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC

In this scenario, some but not all channels can be reversed.

$A_1$ creates an ebit between $C_1$ and $B_2$
$C_1$ can teleport 1 qubit to $B_2$ by free CC via $C_2$ & $B_2$

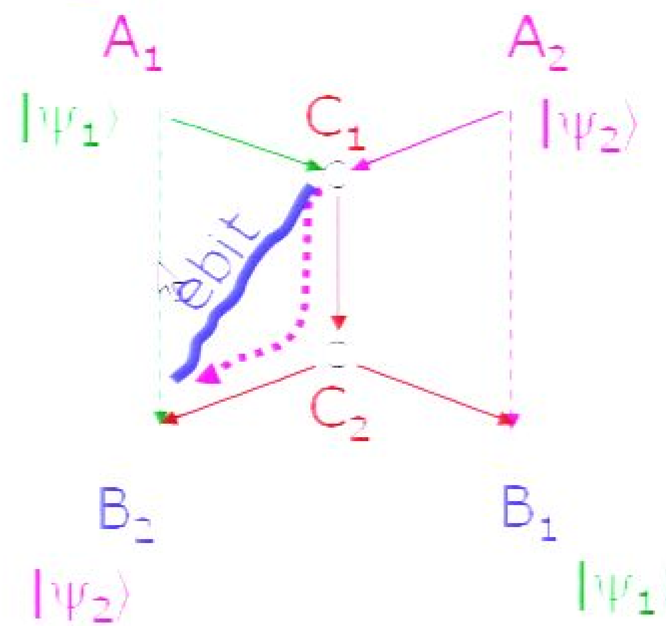


# Motivating example : the butterfly network

## Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC

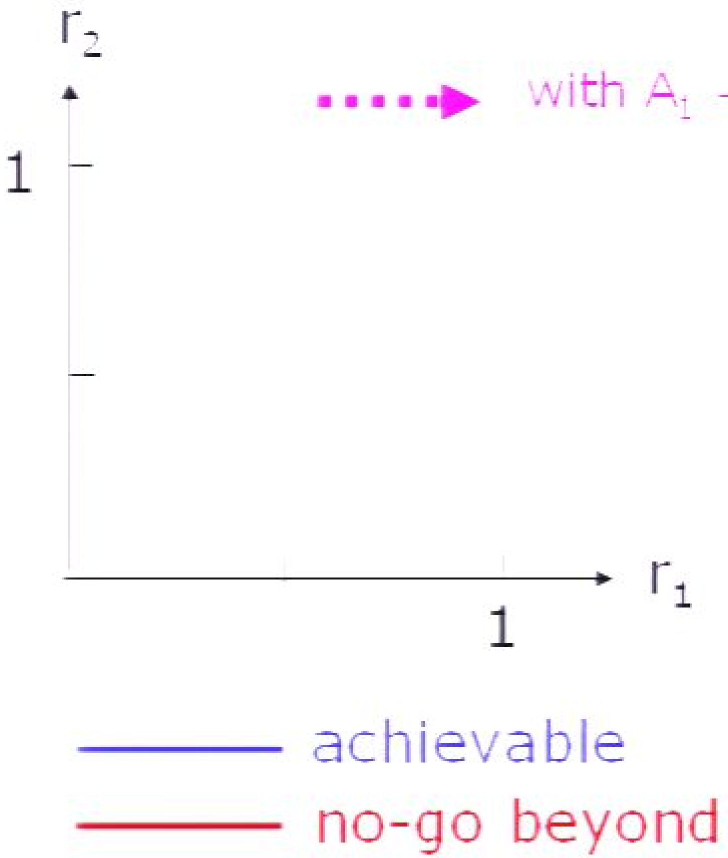
In this scenario, some but not all channels can be reversed.

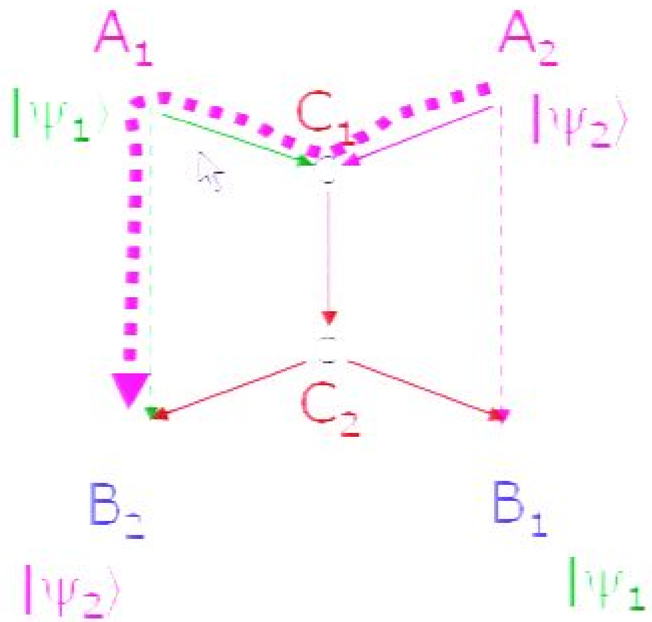
$A_1$ creates an ebit between $C_1$ and $B_2$
$C_1$ can teleport 1 qubit to $B_2$ by free CC via $C_2$ & $B_2$



# Motivating example : the butterfly network

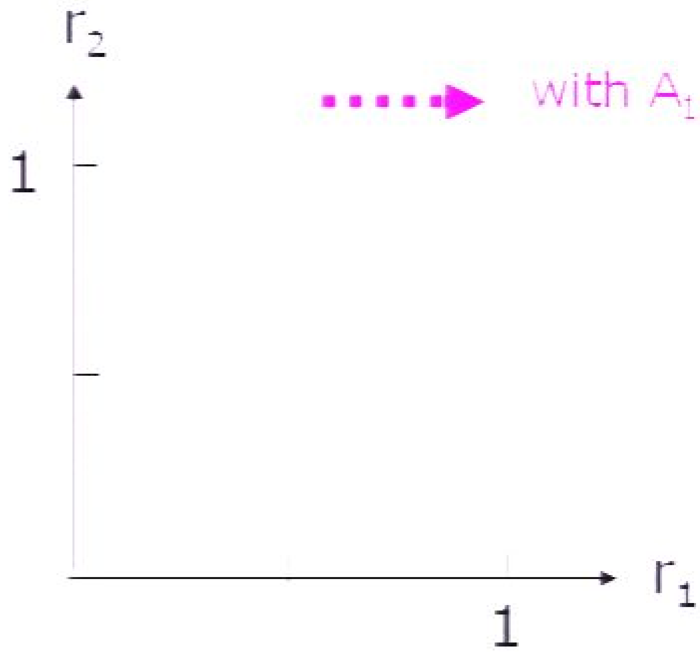
Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC



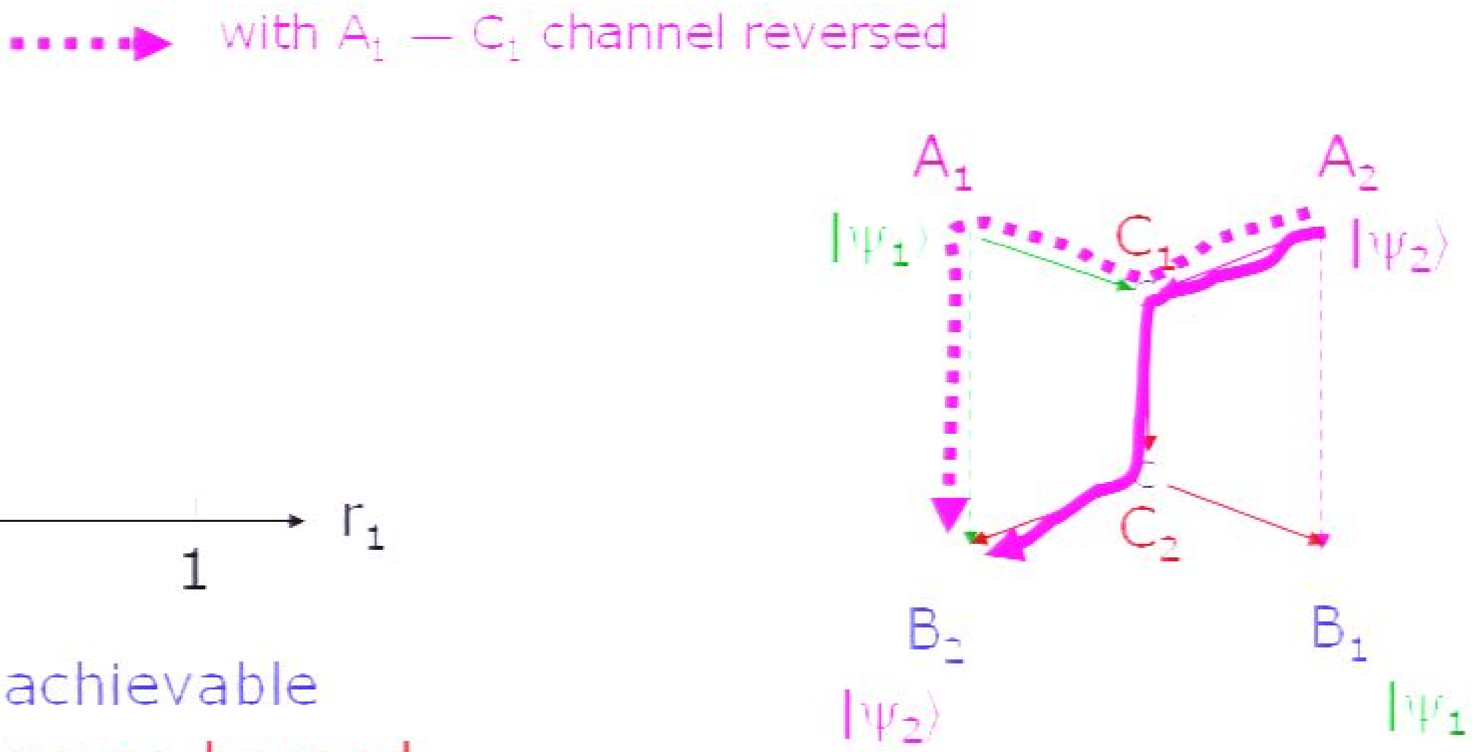
# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC



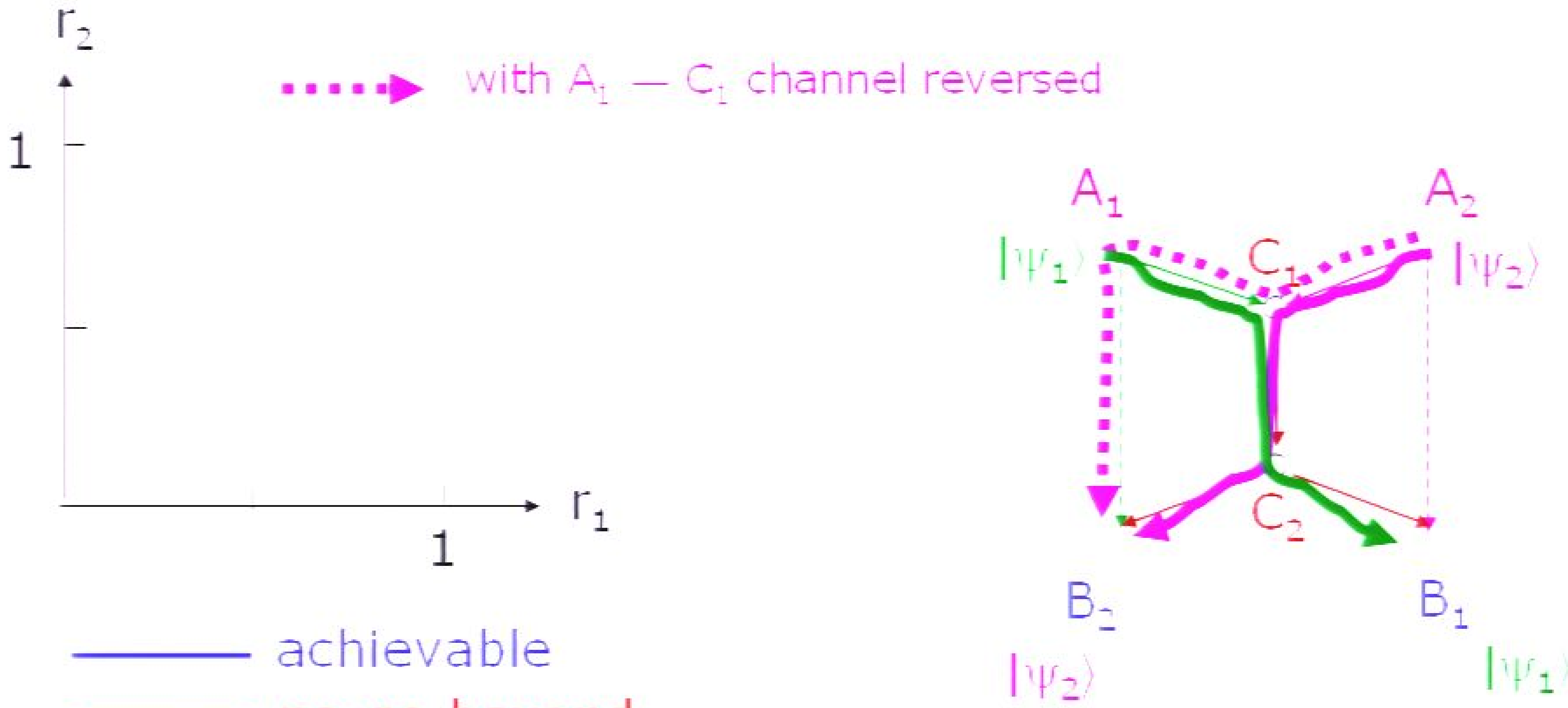

achievable

no-go beyond



# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC



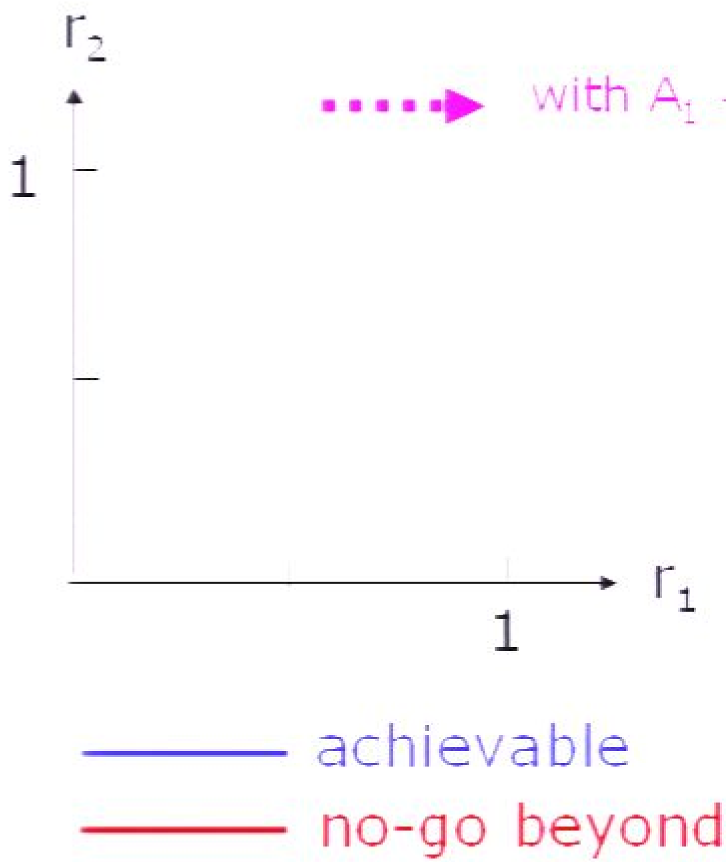

achievable

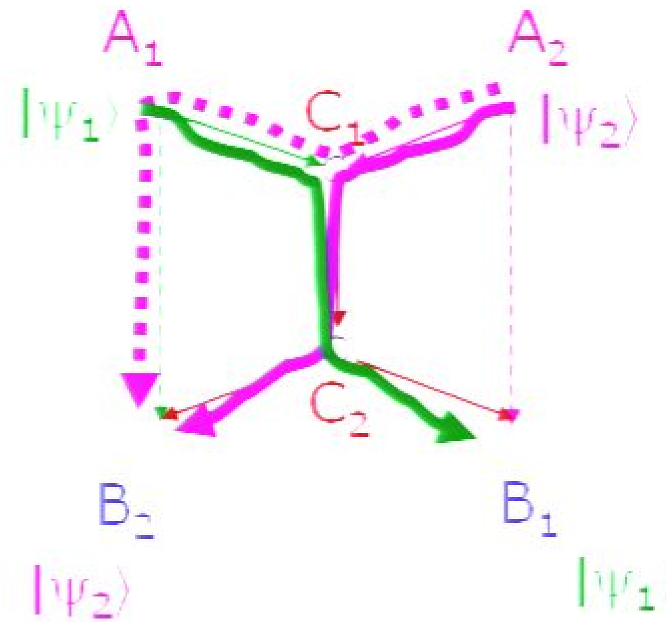
no-go beyond

$n_1=1$, $n_2=2$ for n=2, hence $(r_1, r_2) = (0.5,1)$ achievable

# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC

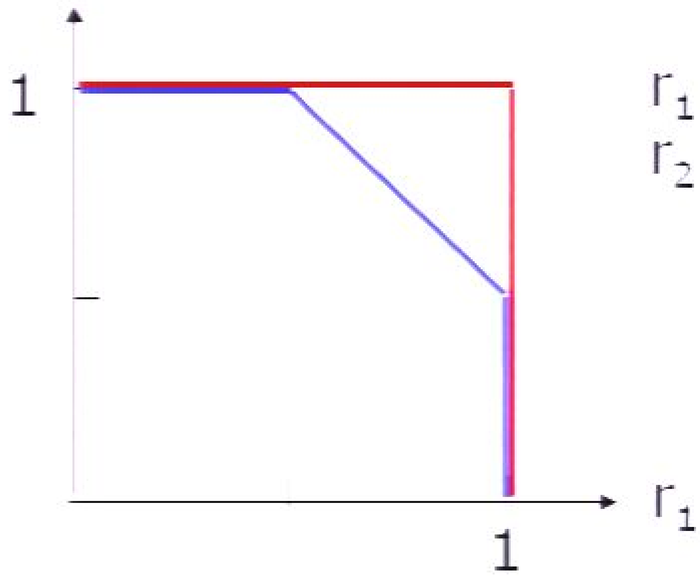



$n_1=1$, $n_2=2$ for n=2, hence $(r_1, r_2) = (0.5,1)$ achievable
So is (1,0.5) by symmetry. Time sharing the two.

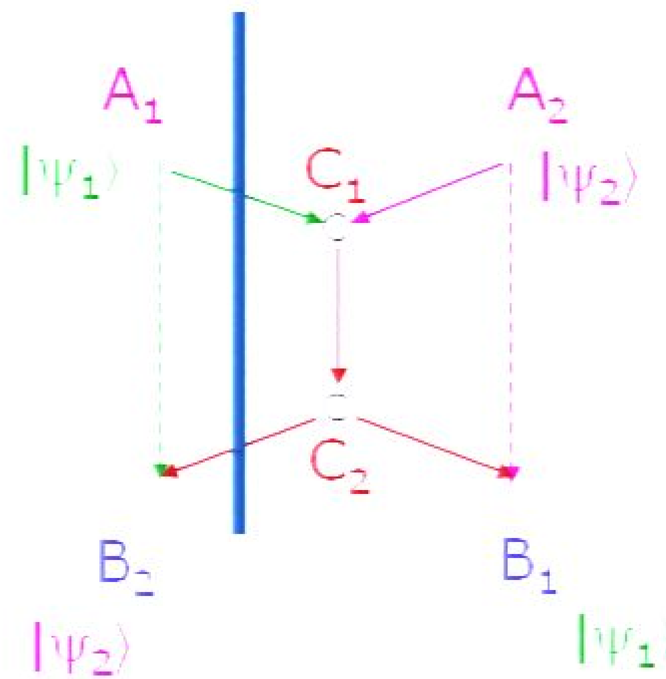
# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC




Outerbounds:

$r_1 \leq 1$
$r_2 \leq 1$




achievable

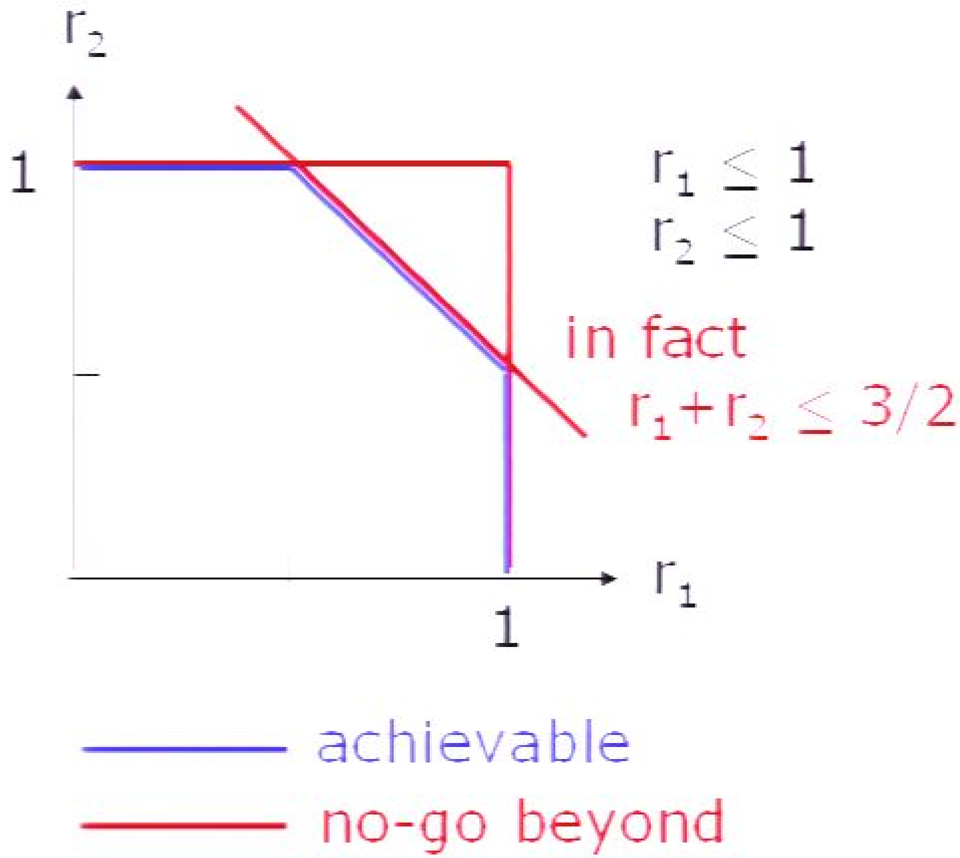
no-go beyond

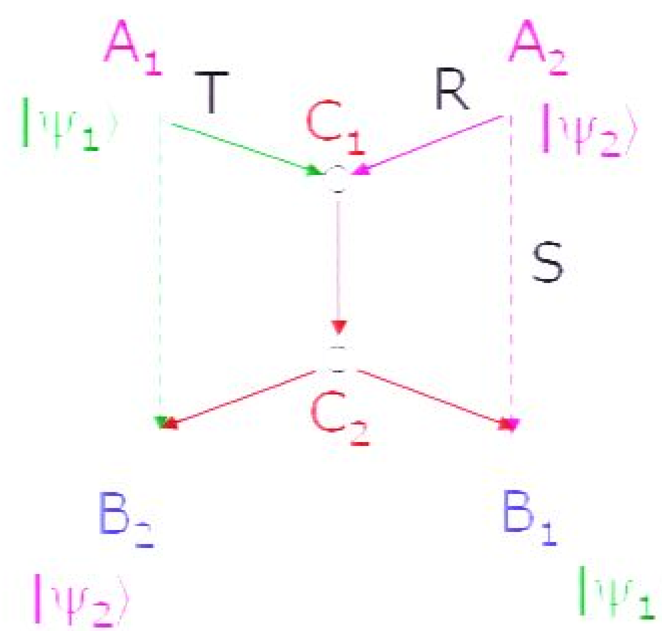
Barnum
Jozsa
Hayden
Winter 99

$A_1$ — $C_1$ communication (q+c) has to be authorized for $|\psi_1\rangle$ which has $n_1$ qubits. Also, q/c encoding of q state does not reduce the quantum part. Hence, $n_1 \leq n$.

# Motivating example : the butterfly network

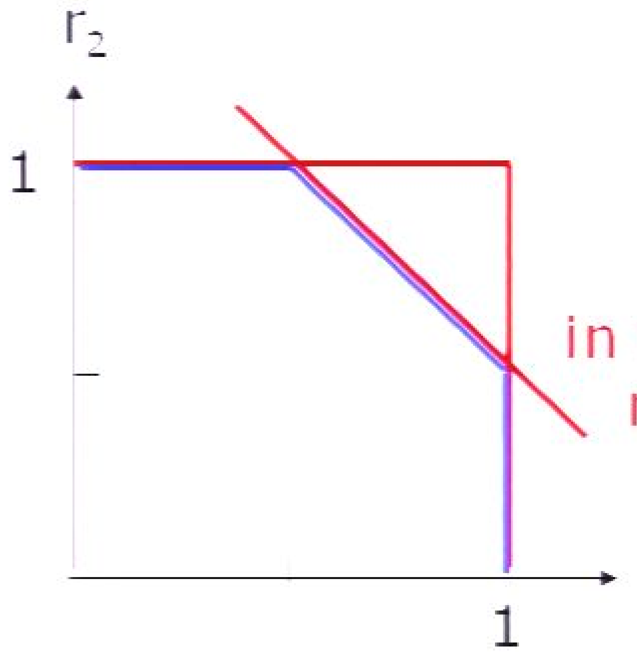
Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC



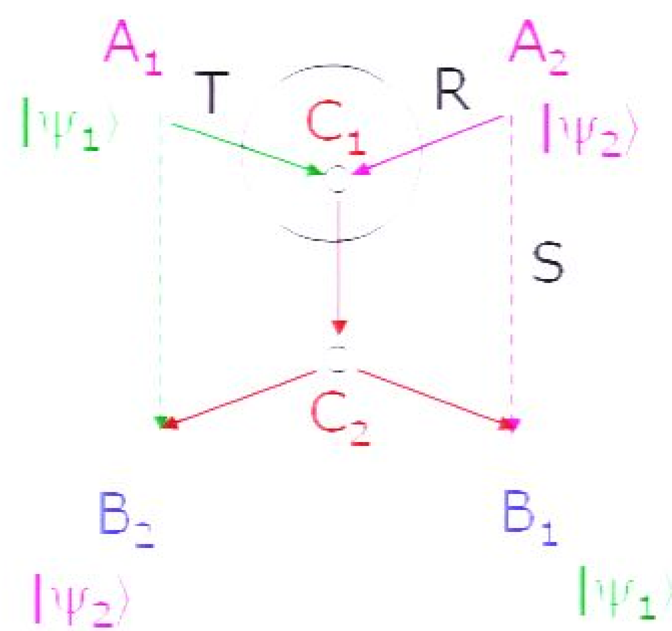
# Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC




$r_1 \leq 1$
$r_2 \leq 1$

in fact
$r_1 + r_2 \leq 3/2$



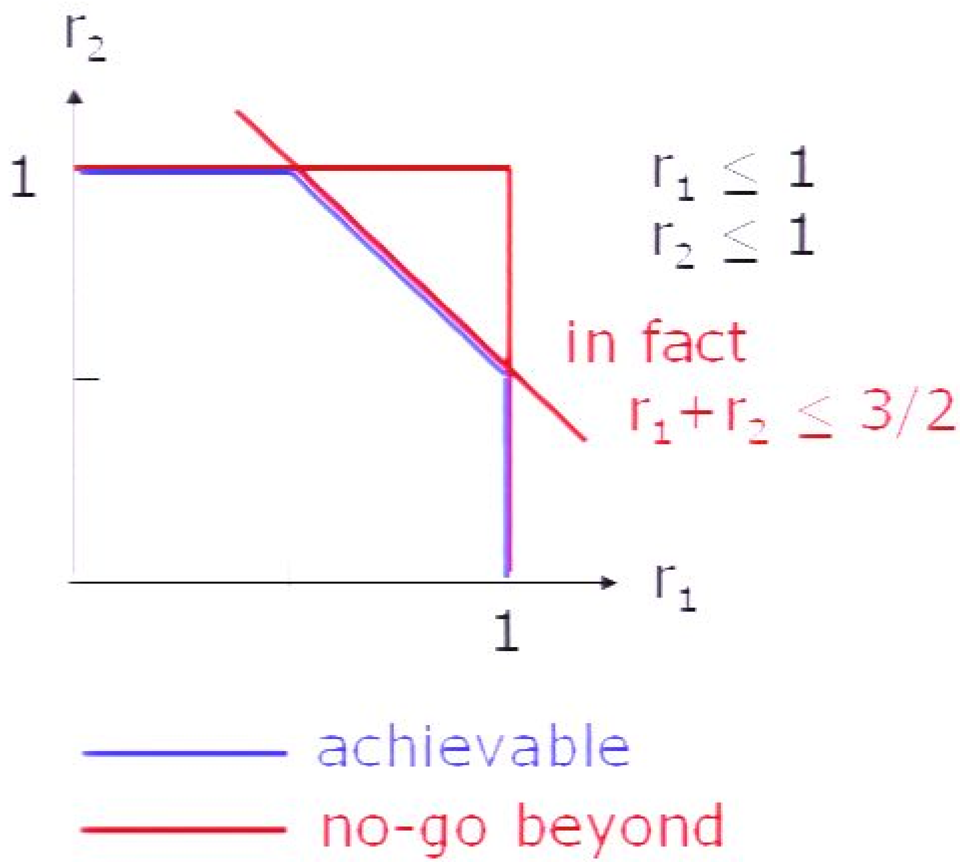

achievable

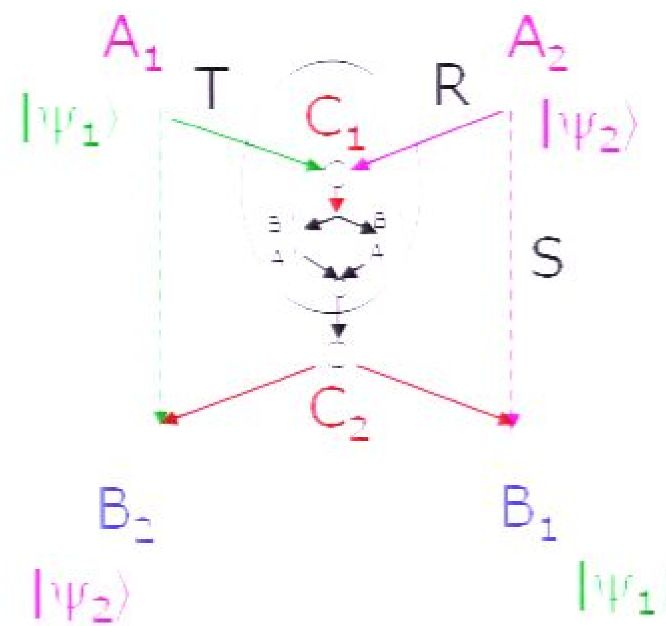
no-go beyond

S unauthorized of $|\psi_2\rangle$, thus, R authorized. Similarly T authorized of $|\psi_1\rangle$.

# Motivating example : the butterfly network

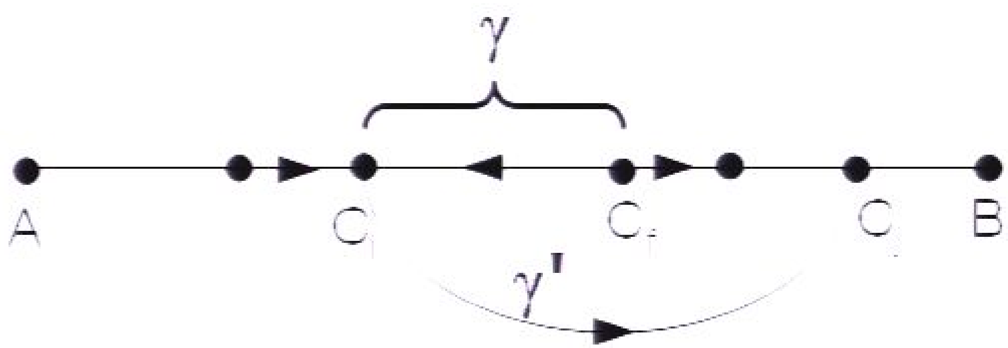
Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC



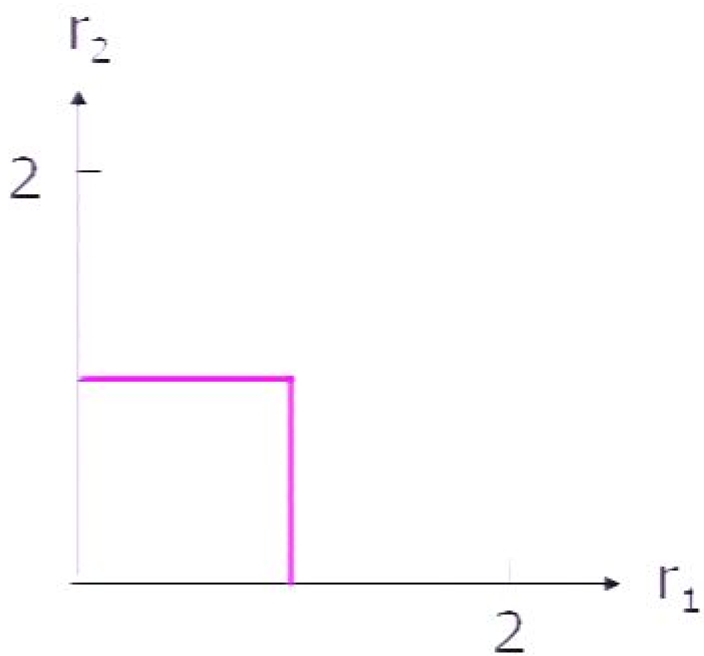

$|\psi_2\rangle$ can be reconstructed using R alone (S is not used). (Similarly for $|\psi_1\rangle$ & T). If n uses of the network sends $n_i$ qubits from $A_i$ to $B_i$, $C_1$ can instead generate $2(n_1+n_2)$ ebits with others, thus $n_1+n_2 \le 3/2$

# General reversal rule with forward assistance

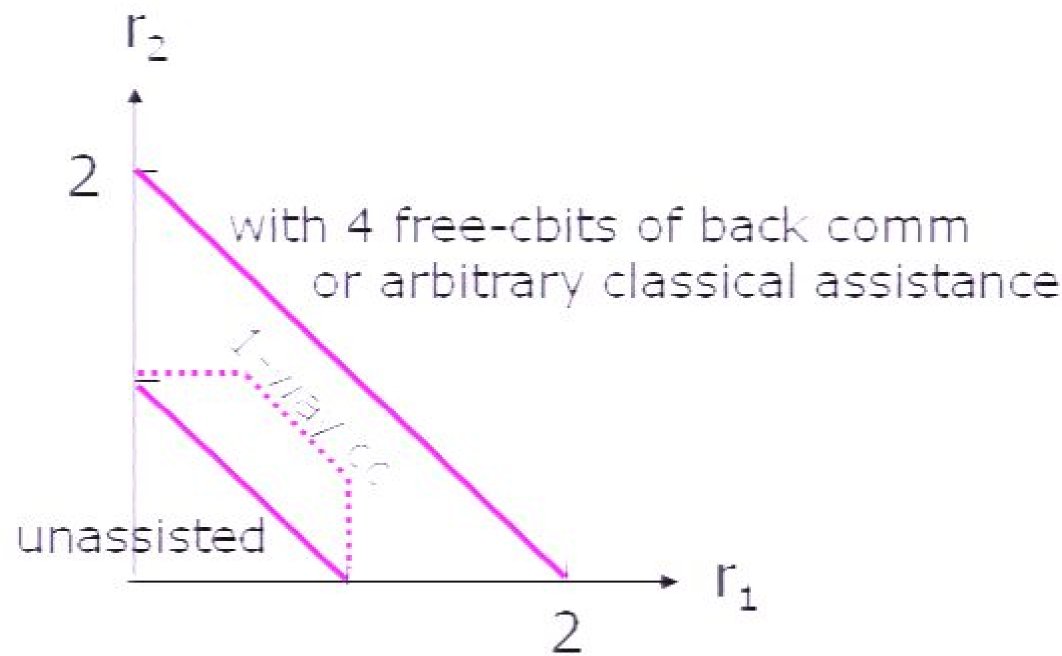


# Summary for the butterfly network:

Uniquely optimal classical rate region




Rate optimal - high fidelity quantum rate regions



# Summary for the butterfly network:

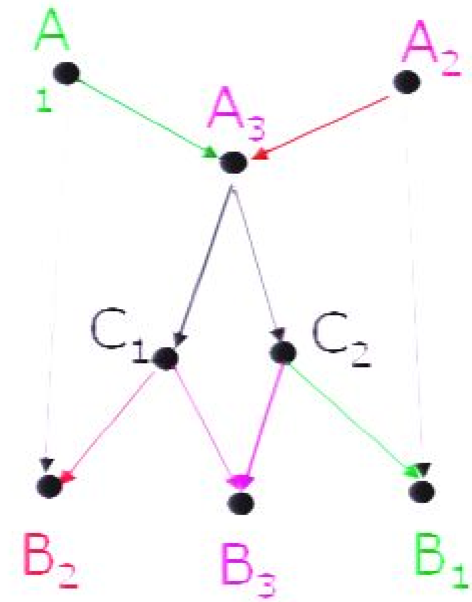
Quantum case:

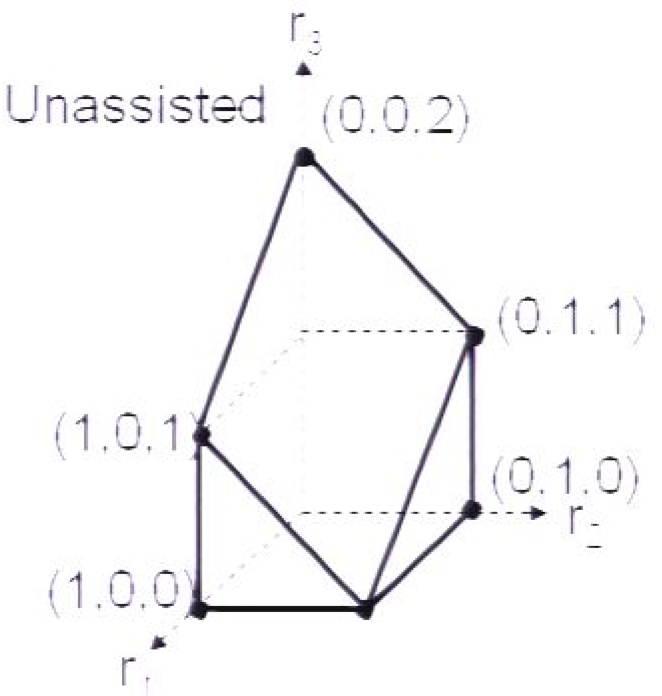
Routing is optimal : each "leg" is either used for $A_1 \to B_1$ comm or $A_2 \to B_2$ comm

Contrast to the classical case, quantum info runs down a network like commodities.

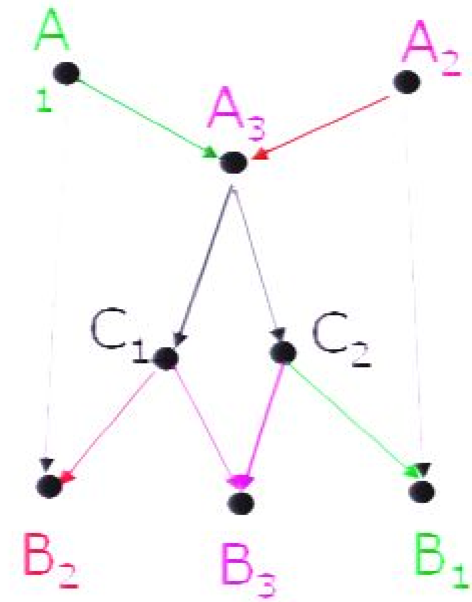
Does this simplifying feature hold in more general networks?

Inverted crown network

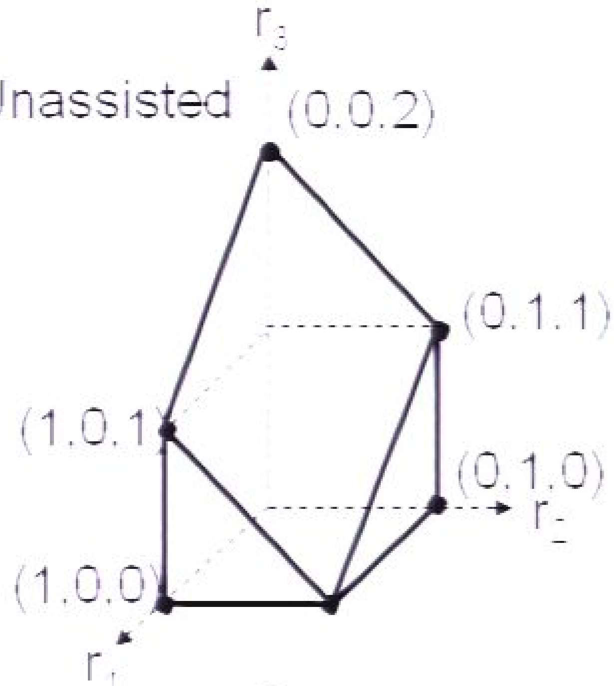




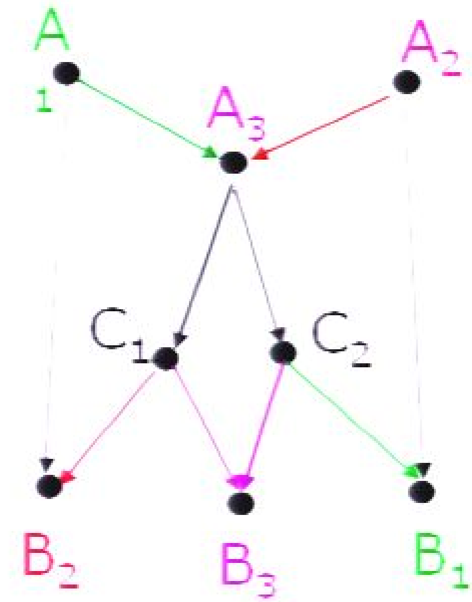

Inverted crown network



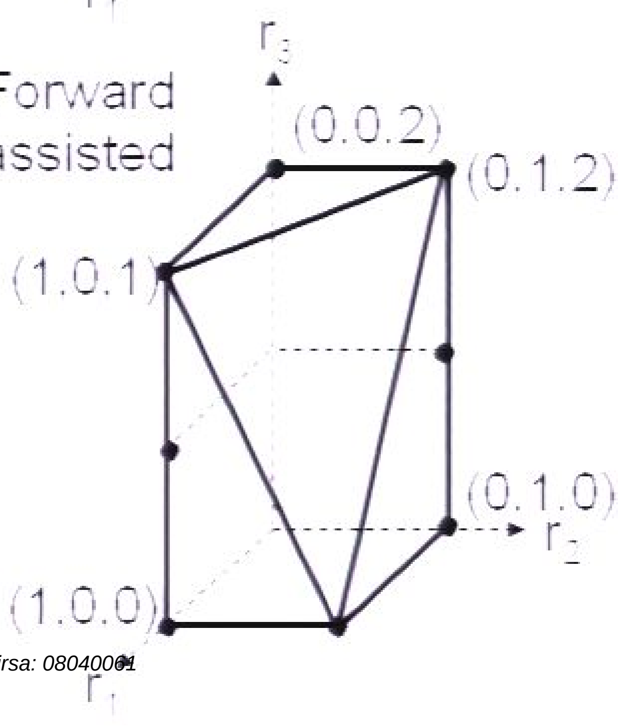
Unassisted



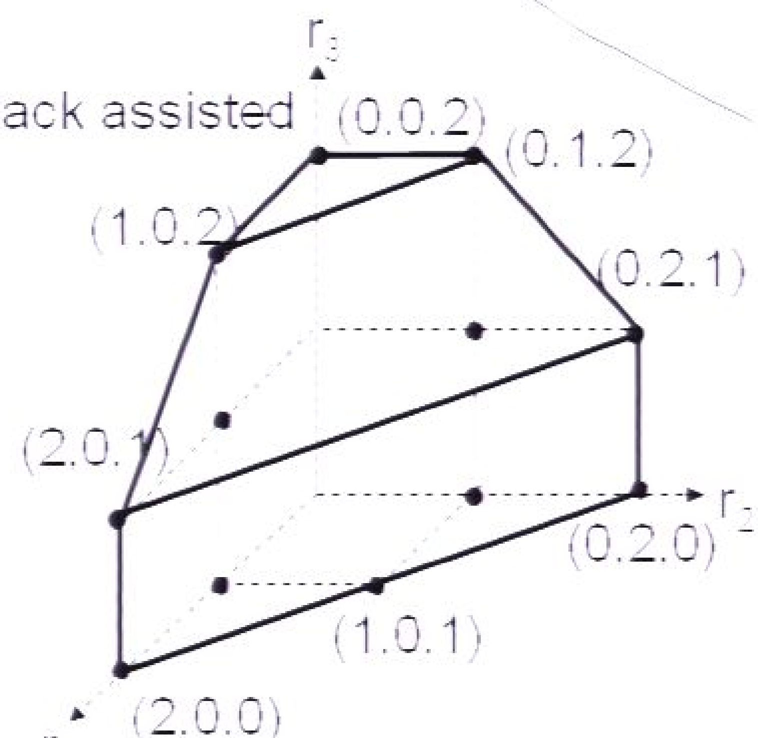

Inverted crown network




Forward assisted

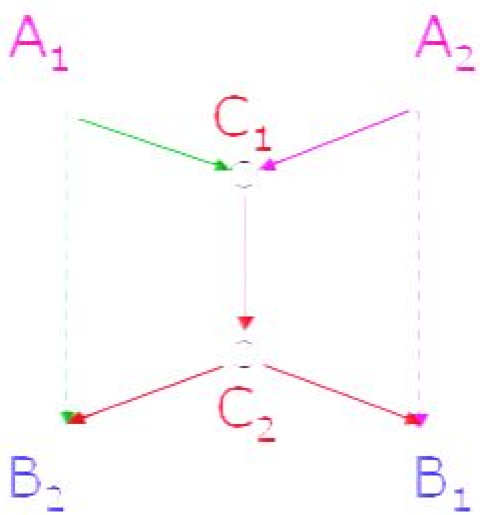



Back assisted



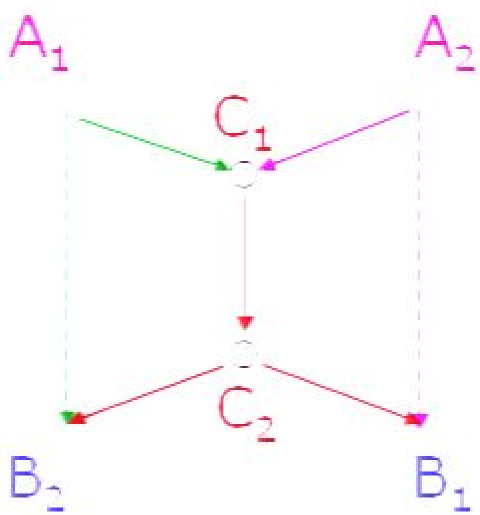
## Large gap between quantum & classical network coding (borrowing idea from Harvey, Kleinberg, Lehman)

n=1, 2-pair comm problem



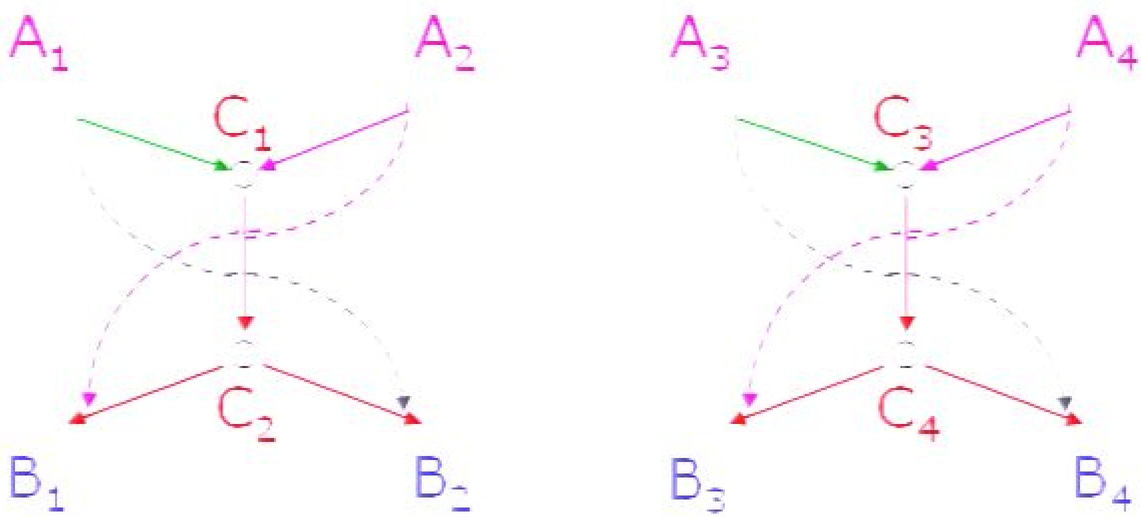
## Large gap between quantum & classical network coding (borrowing idea from Harvey, Kleinberg, Lehman)

n=1, 2-pair comm problem



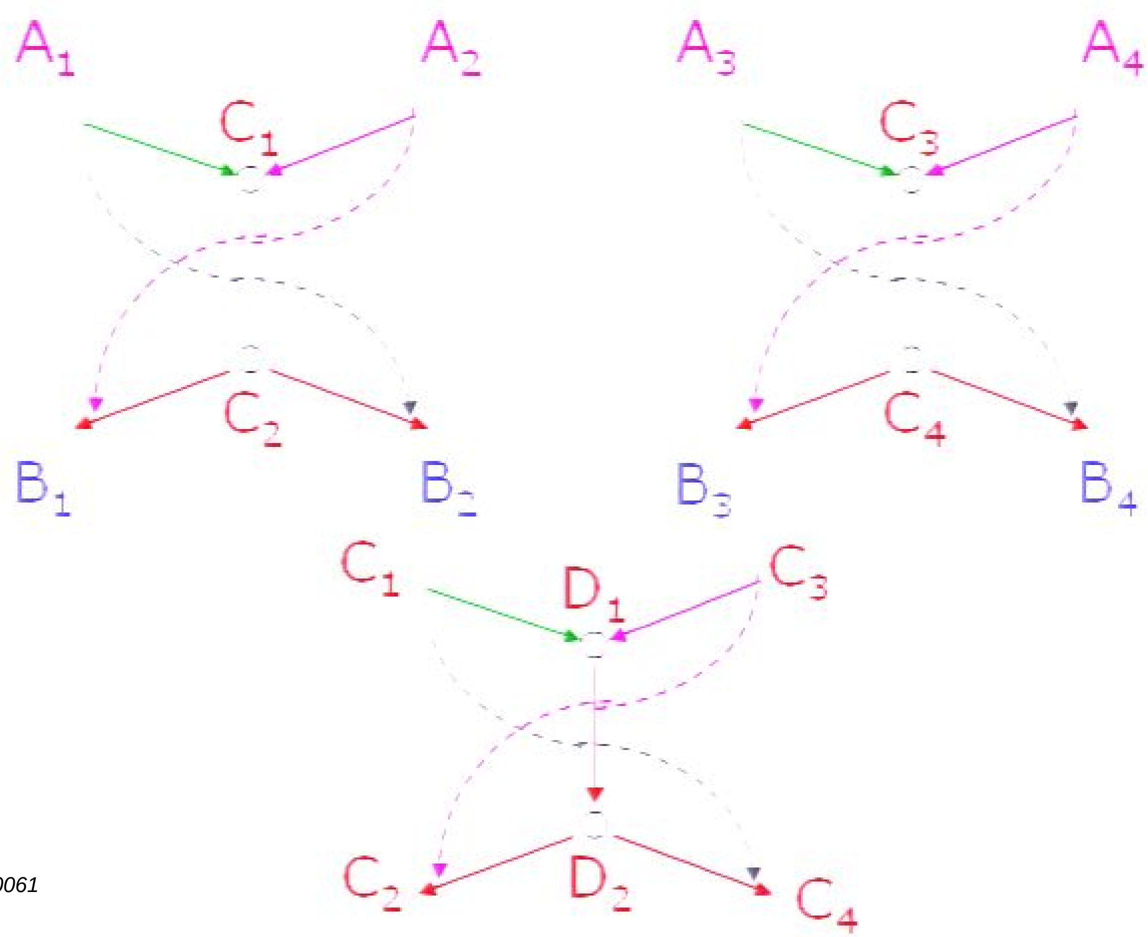
Large gap between quantum & classical network coding (borrowing idea from Harvey, Kleinberg, Lehman)

n=2, 4-pair comm problem (I)



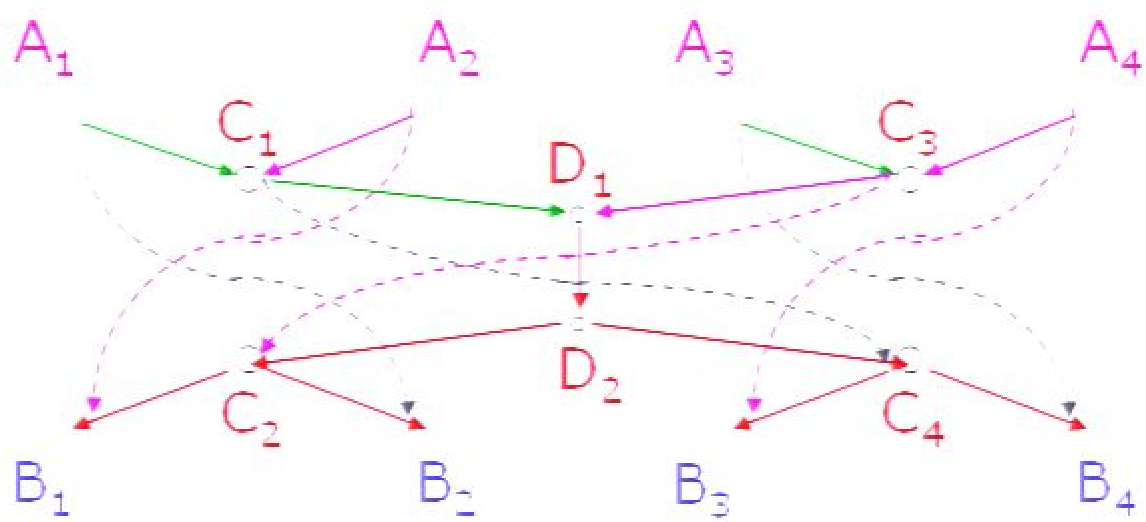
## Large gap between quantum & classical network coding (borrowing idea from Harvey, Kleinberg, Lehman)

n=2, 4-pair comm problem (III)



## Large gap between quantum & classical network coding (borrowing idea from Harvey, Kleinberg, Lehman)

n=2, 4-pair comm problem (IV)

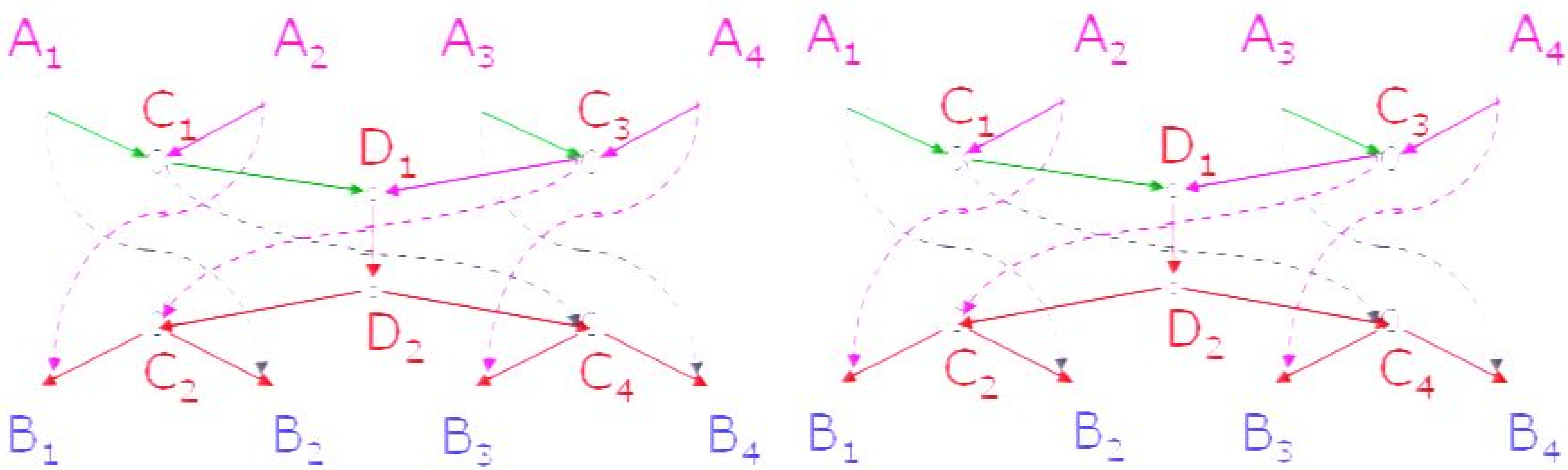



classical network -- each $A_i$ can communicate 1 bit to $B_i$

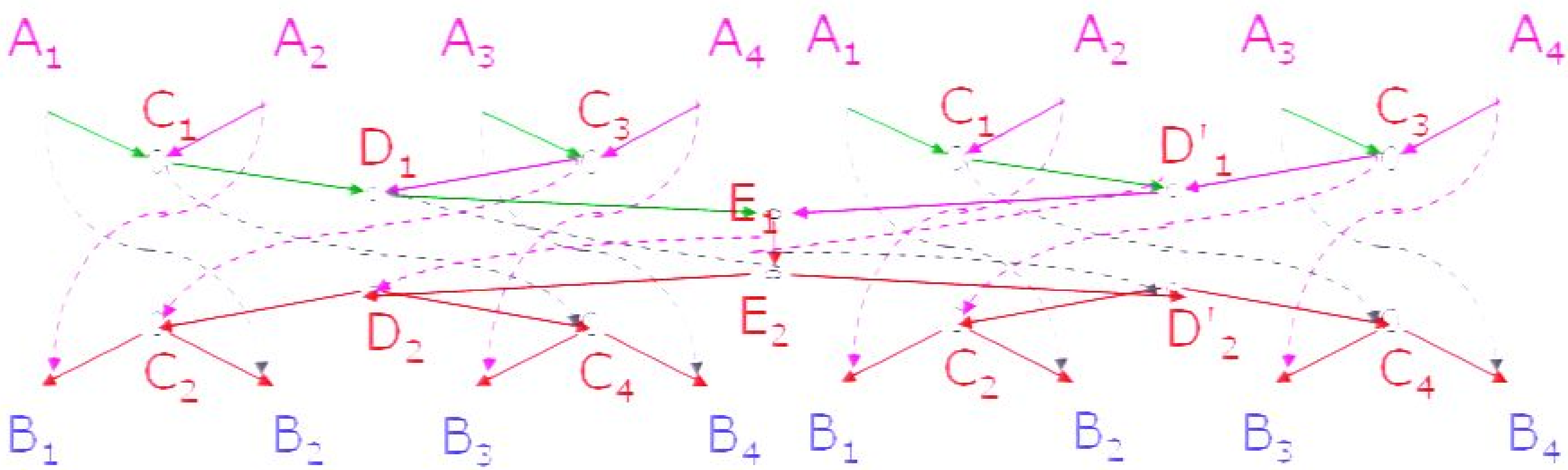
quantum -- the total rate is 1 (inductive argument on the size of the significant share)

# Large gap between quantum & classical network coding (borrowing idea from Harvey, Kleinberg, Lehman)

n=3, 8-pair comm problem (IV)



# Large gap between quantum & classical network coding (borrowing idea from Harvey, Kleinberg, Lehman)



# Possible generalizations:

- k-pair comm problem for any k
- arbitrary directed acyclic graphs
  - arbitrary number of players
  - noiseless channels with arbitrary capacities
- cyclic graphs
- undirected graphs
- noisy channels

Dream: find asymptotic achievable rate region

# Actual results (life):

# Actual results (life):

- Routing optimality for

# Actual results (life):

- Routing optimality for

  (a) "shallow" networks ($d \leq 3$) without 4-cycles

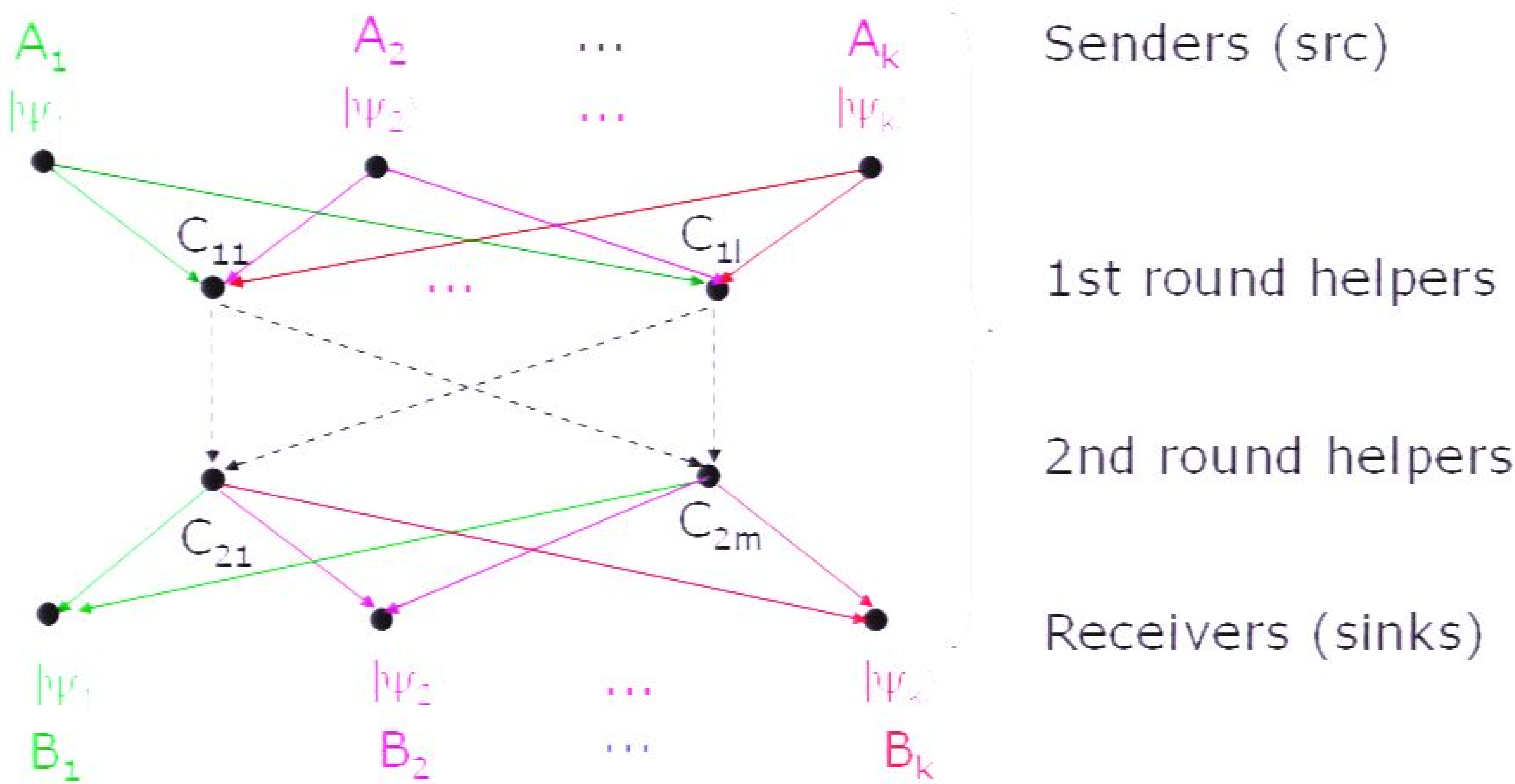
# Actual results (life):

- Routing optimality for

  (a) "shallow" networks ($d \leq 3$) without 4-cycles

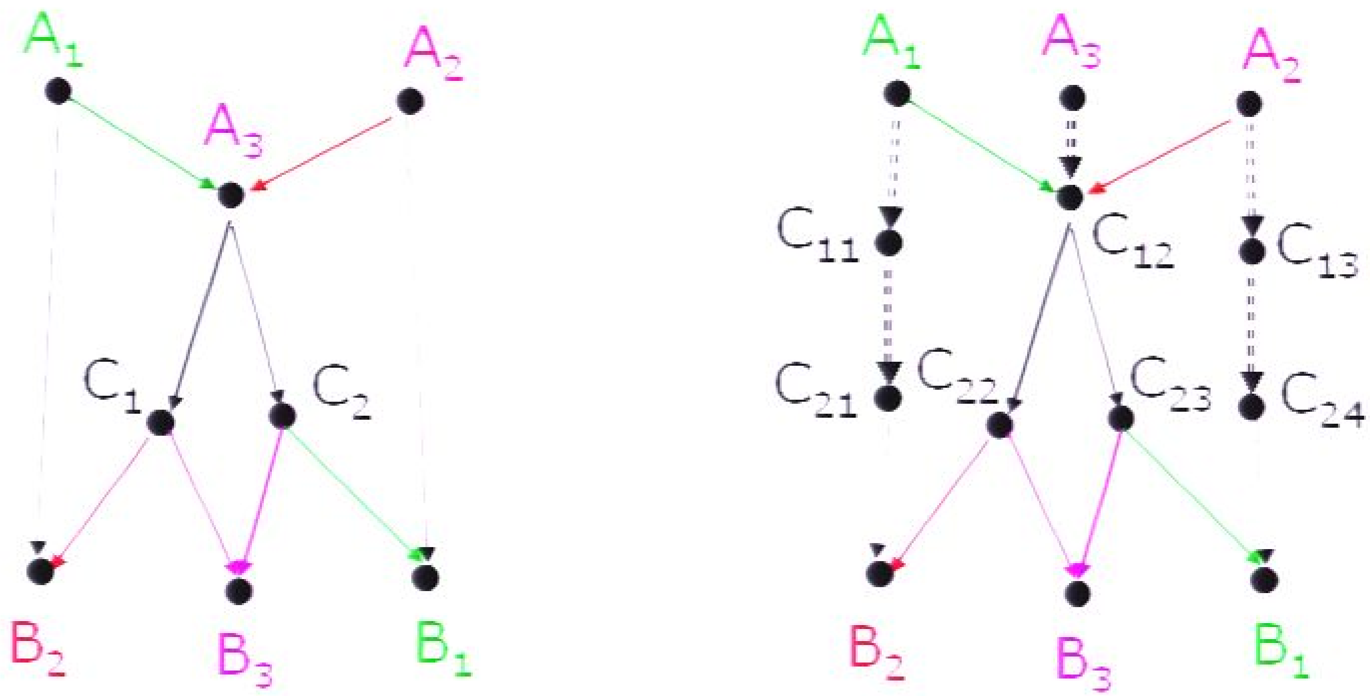
  (b) for back comm assisted, 2-pair comm problem

# Shallow networks:




These sets of players: senders, 1st & 2nd round helpers and receivers can overlap. We assign multiple vertices to each common party, linked by channels with unlimited capacities.

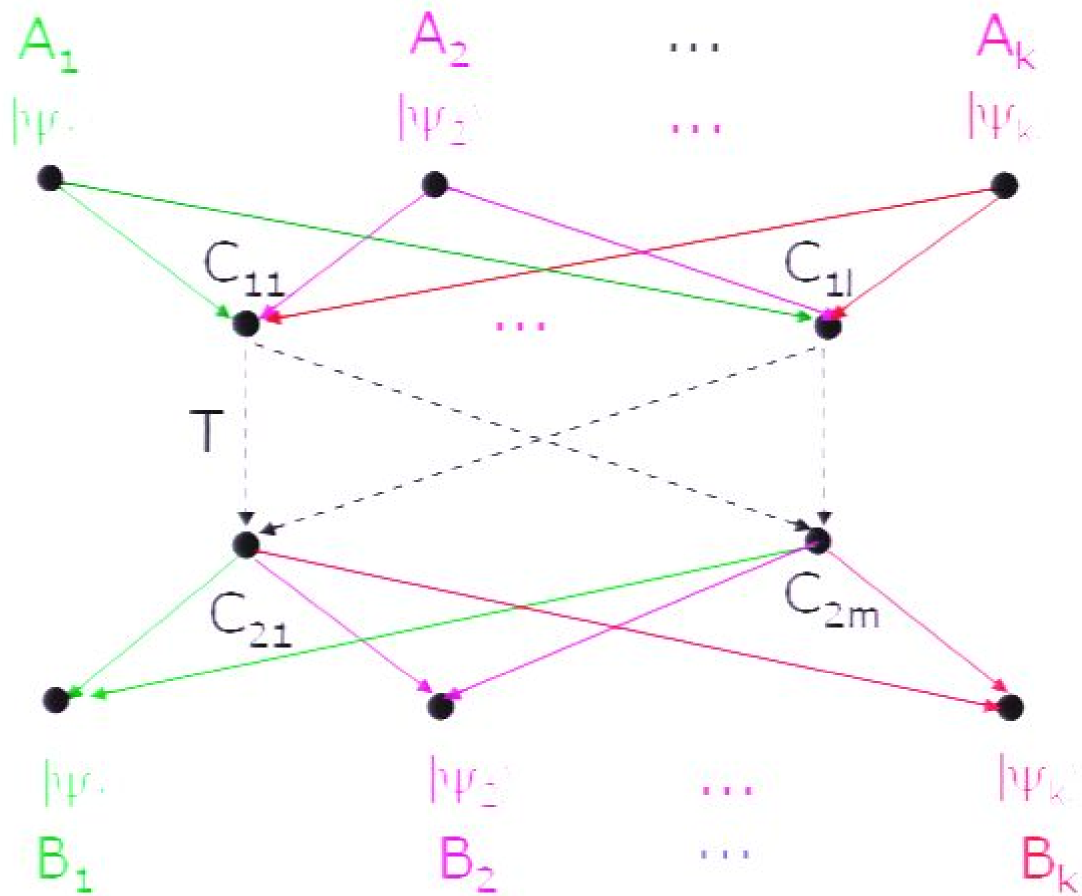
# Shallow networks (examples):



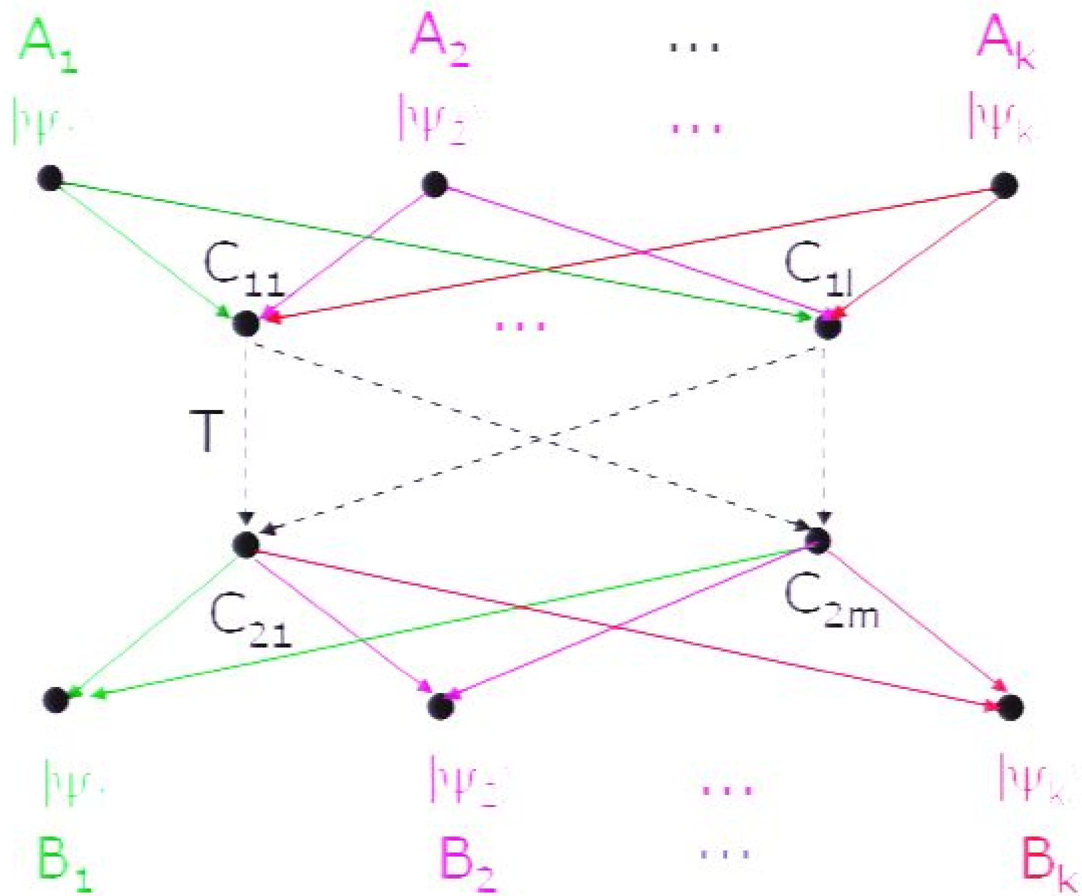


links with unlimited capacities

# Shallow networks:

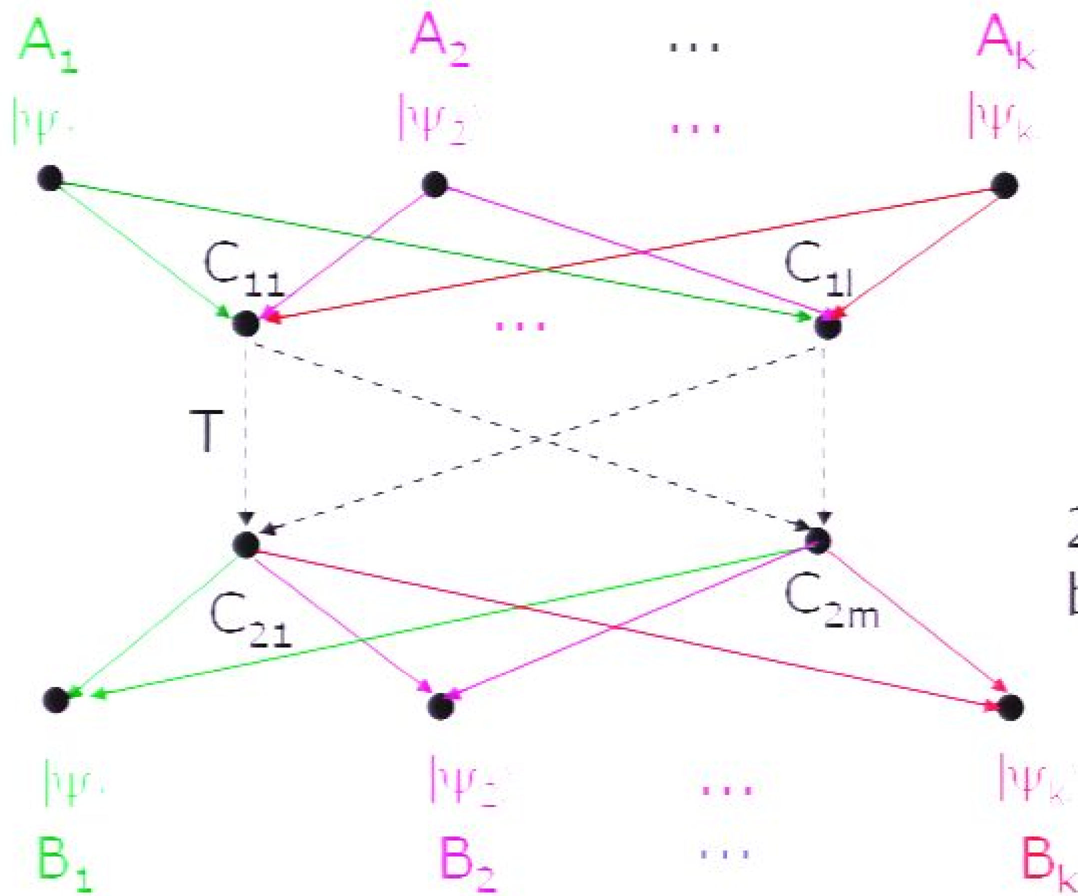


# Shallow networks:




transmissions to $B_i$
indep of $|\psi_j\rangle$ $\forall$ j = i

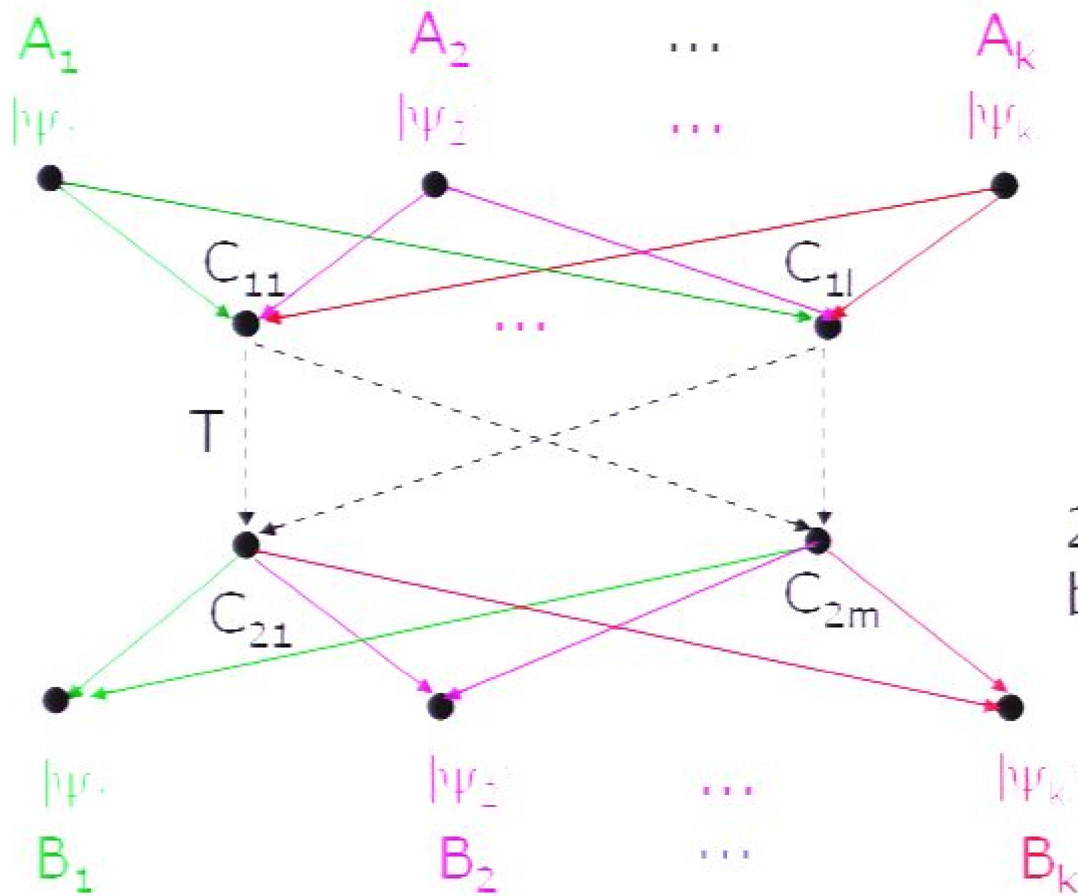
# Shallow networks:




2nd round helpers must be disentangling them

transmissions to $B_i$ indep of $|\psi_j\rangle \; \forall \; j \neq i$

# Shallow networks:



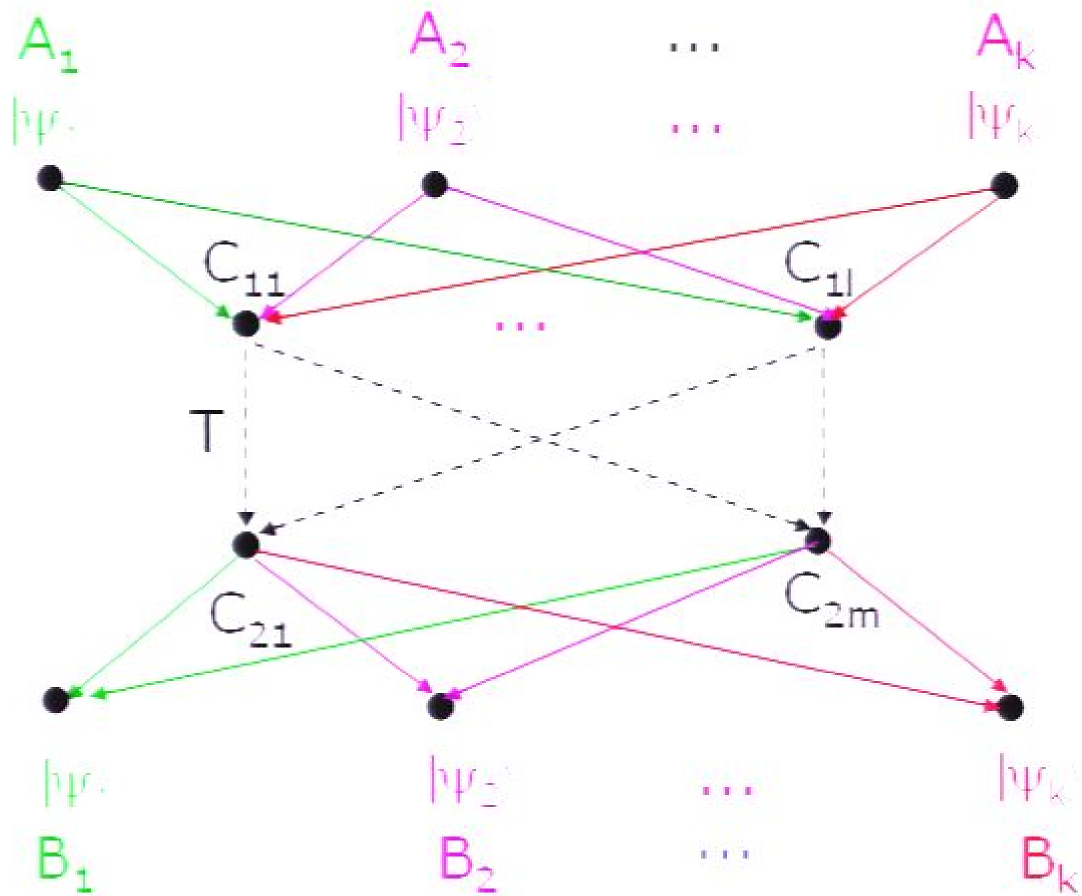

Claim: $C_{21}$ can transform T (alone) to parts each depends on only one $|\psi_i\rangle$

Corollary: coding doesn't help

2nd round helpers must be disentangling them

transmissions to $B_i$ indep of $|\psi_j\rangle \; \forall \; j \neq i$

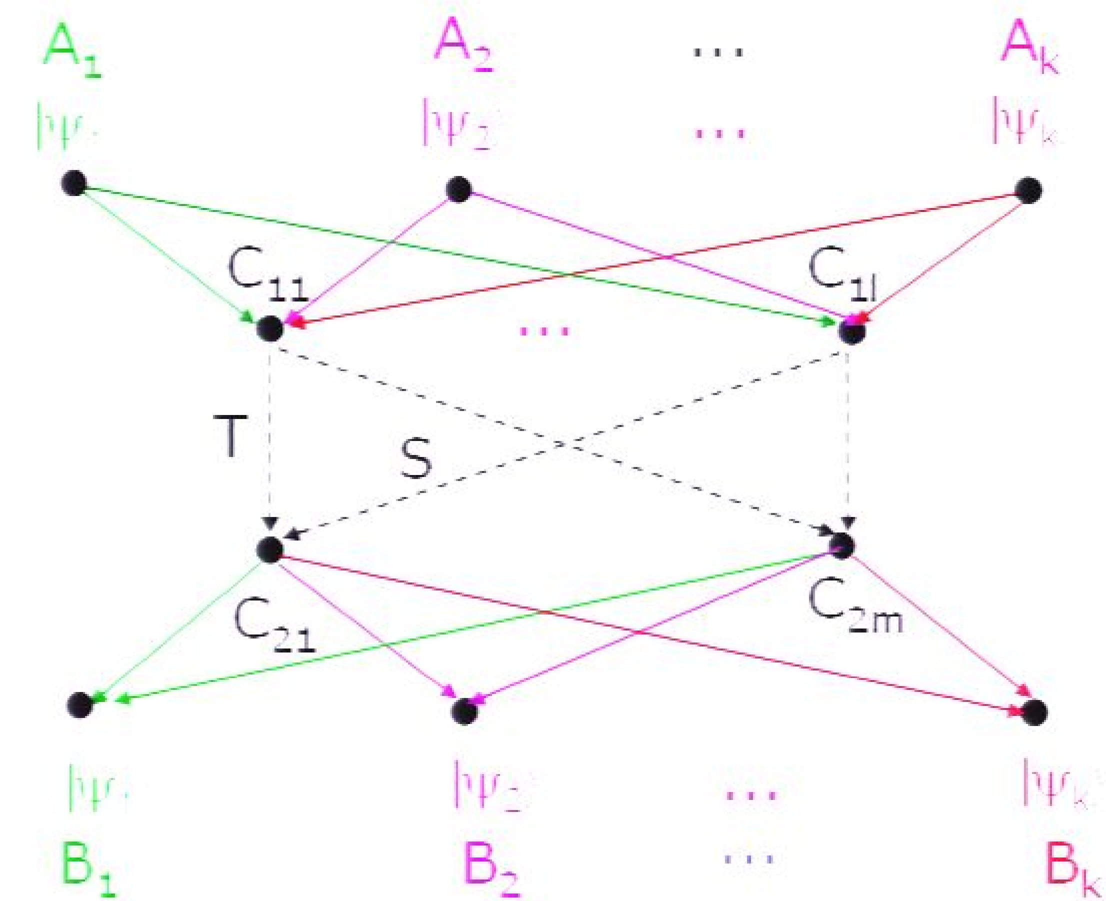
# Shallow networks:




Claim: $C_{21}$ can transform T (alone) to parts each depends on only one $|\psi_i\rangle$

Proof: if not ... say, let S also be needed for disentanglement

# Shallow networks:

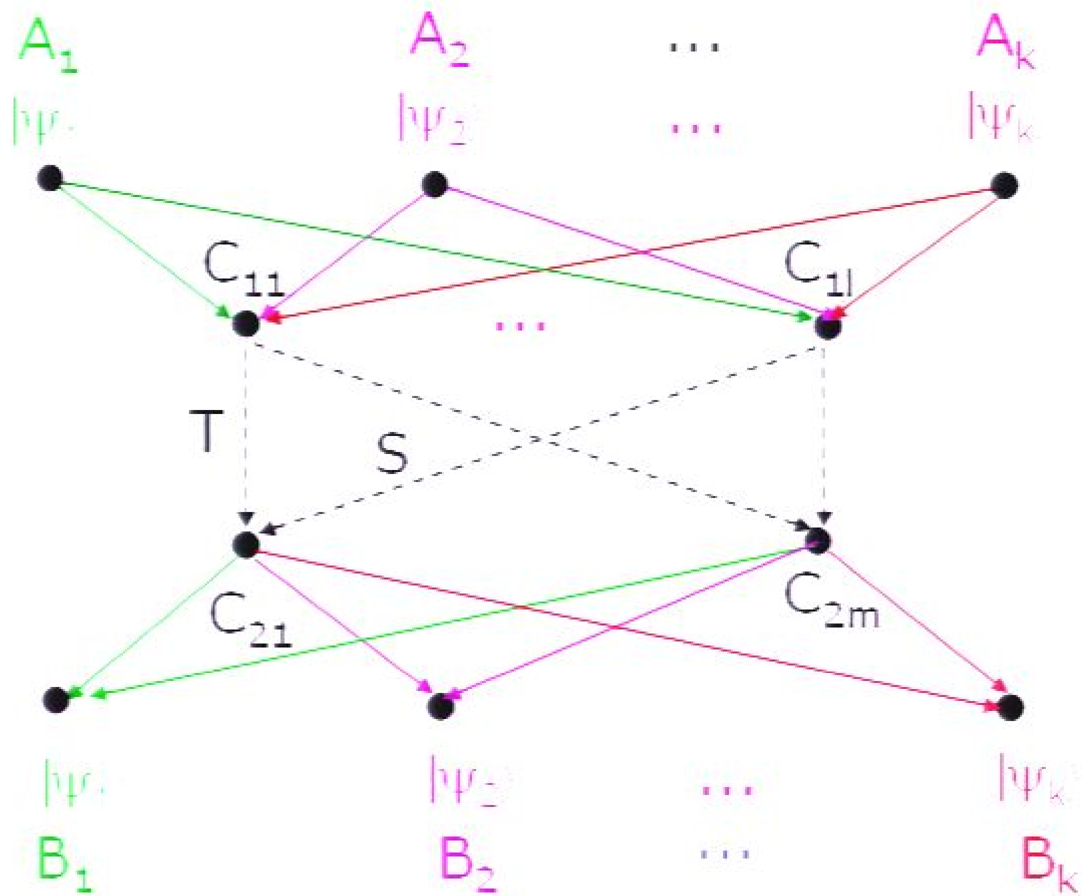



Claim: $C_{21}$ can transform T (alone) to parts each depends on only one $|\psi_i\rangle$

Proof: if not ... say, let S also be needed for disentanglement

Then, no source $A_j$ could have send to both $C_{11}$ and $C_{1l}$ else there's a 4-cycle.

# Shallow networks:



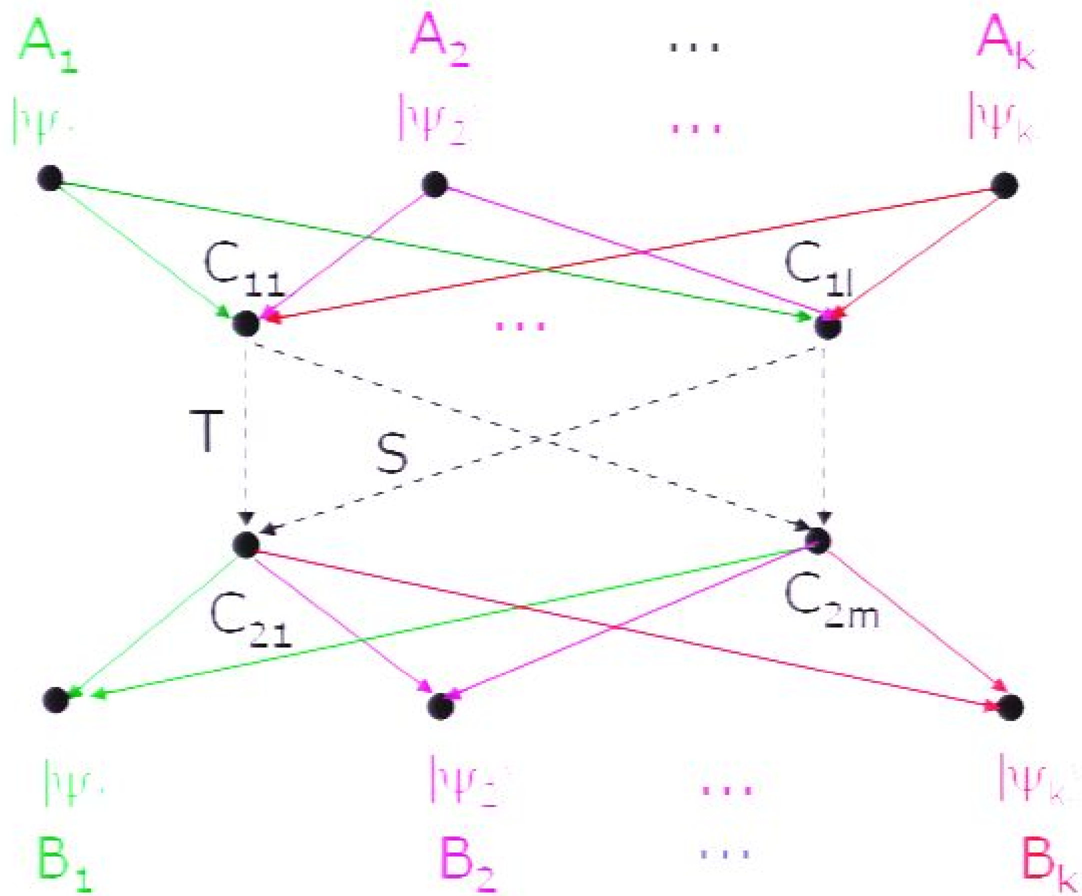

Claim: $C_{21}$ can transform T (alone) to parts each depends on only one $|\psi_i\rangle$

Proof: if not ... say, let S also be needed for disentanglement

Then, no source $A_j$ could have send to both $C_{11}$ and $C_{1l}$ else there's a 4-cycle.

So, S, T uncorrelated ... S couldn't have helped disentangle T

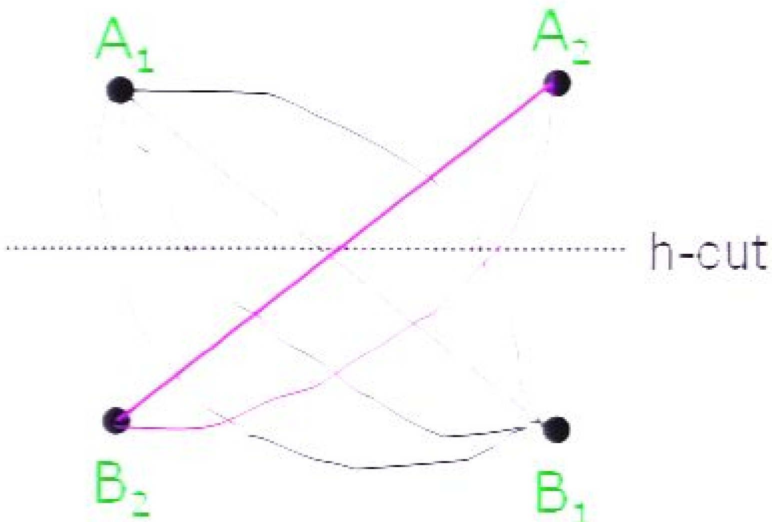
# Shallow networks:



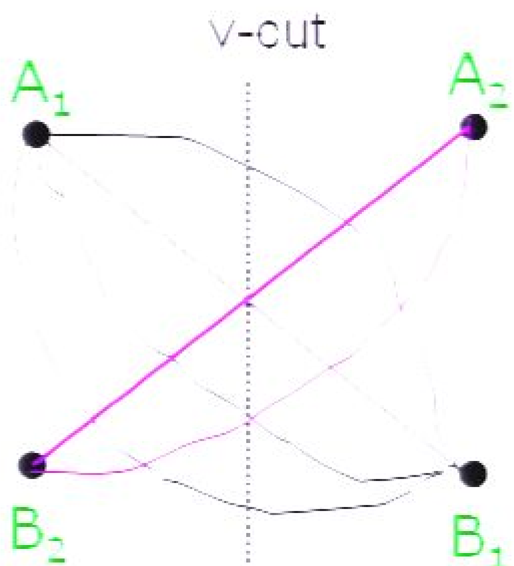

Claim: $C_{21}$ can transform T (alone) to parts each depends on only one $|\psi_i\rangle$

Proof: if not ... say, let S also be needed for disentanglement

# 2-pair back-assisted (undirected) networks:

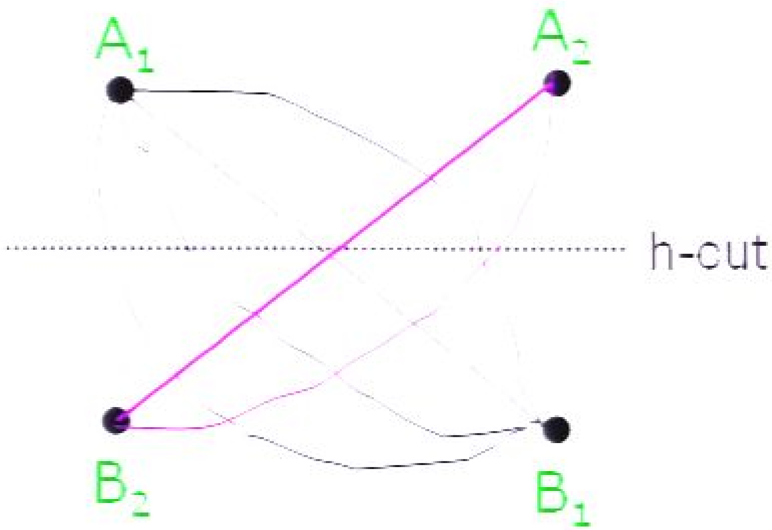



Define the h-cut as the min cut separating $A_{1,2}$ from $B_{1,2}$

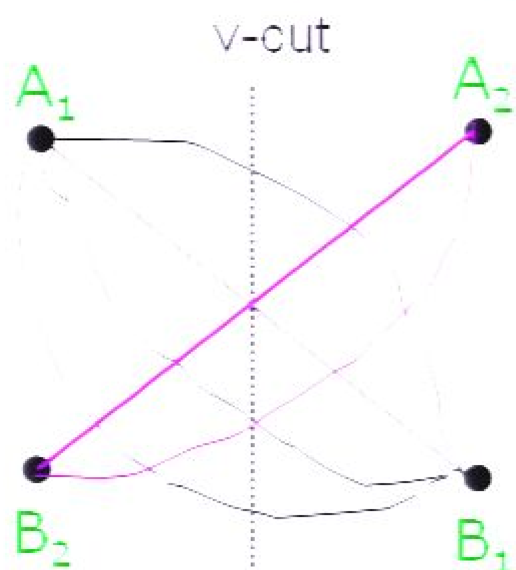



Define the v-cut as the min cut separating $A_1$,$B_2$ from $A_2$,$B_1$

# 2-pair back-assisted (undirected) networks:



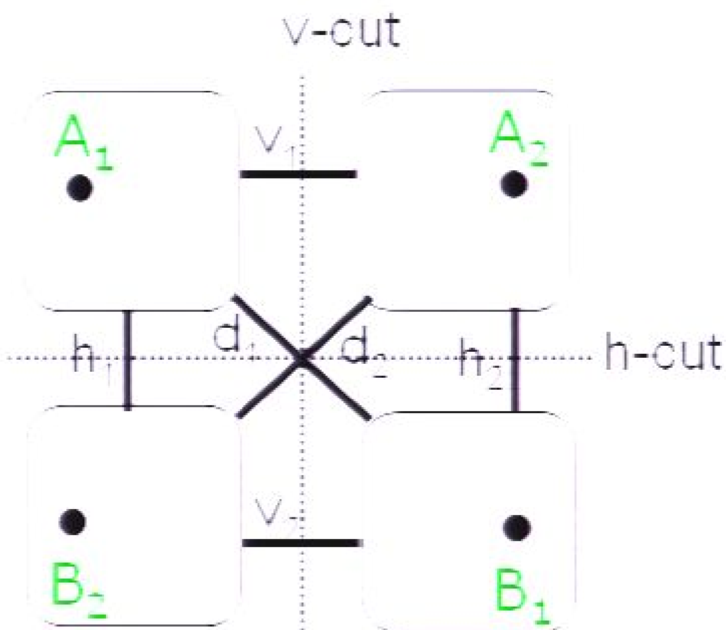

Define the h-cut as the min cut separating $A_{1,2}$ from $B_{1,2}$




Define the v-cut as the min cut separating $A_1$,$B_2$ from $A_2$,$B_1$

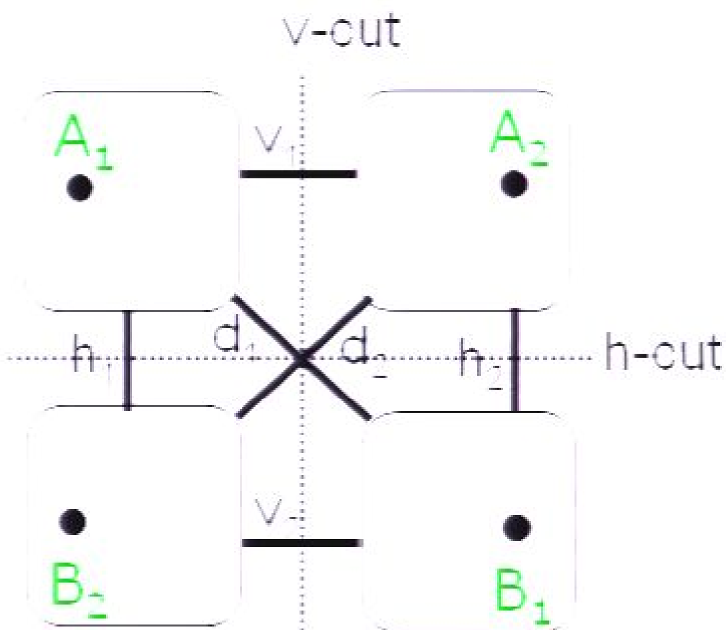
Taking the min of the 2 cuts, we won't count channels not contributing to rate sum.

# 2-pair back-assisted (undirected) networks:




2 cuts, 4 regions,
6 types of links.

# 2-pair back-assisted (undirected) networks:




2 cuts, 4 regions,
6 types of links.

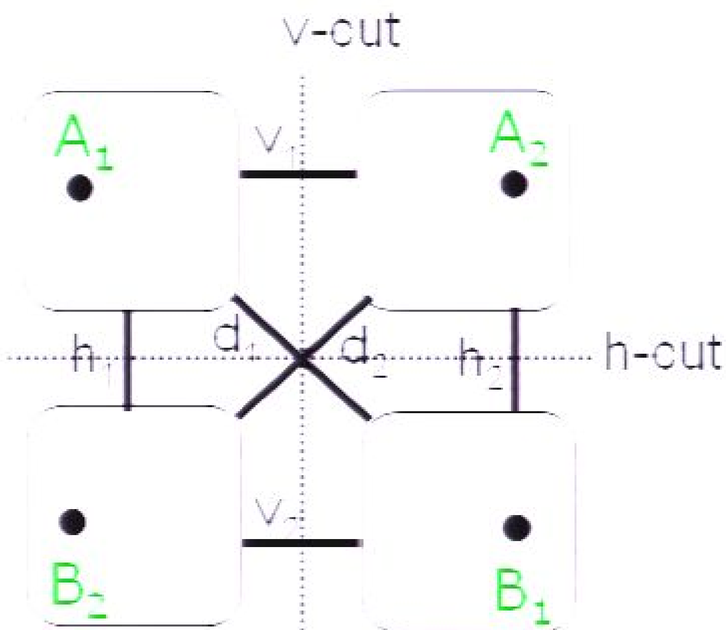
Outerbound:

$r_1 \leq$ mincut separating $A_1$, $B_1$

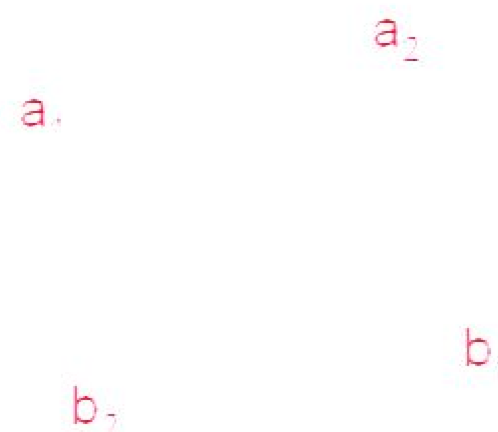
$r_2 \leq$ mincut separating $A_2$, $B_2$

$r_1 + r_2 \leq$ min (h-cut,v-cut)

# 2-pair back-assisted (undirected) networks:



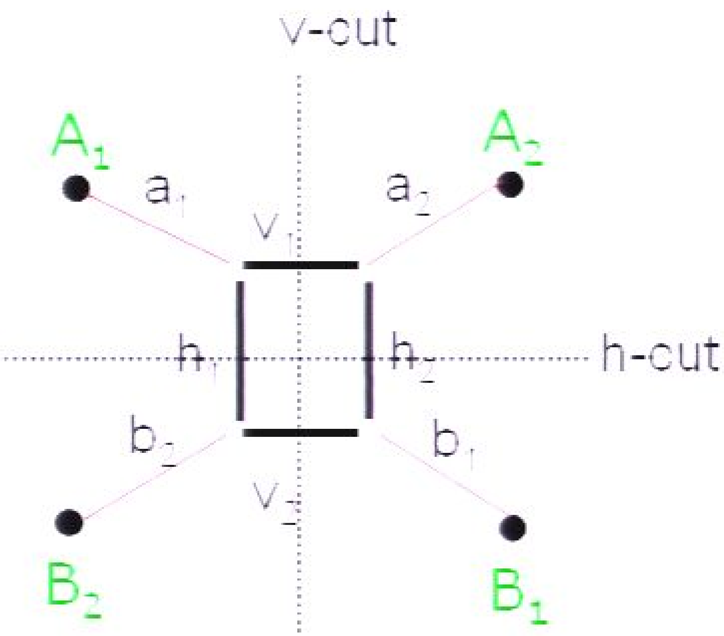

Proof:




Outerbound:
$r_1 \leq$ mincut separating $A_1$, $B_1$
$r_1 \leq$ mincut separating $A_1$, $B_1$
$r_1 + r_2 \leq$ min (h-cut,v-cut)

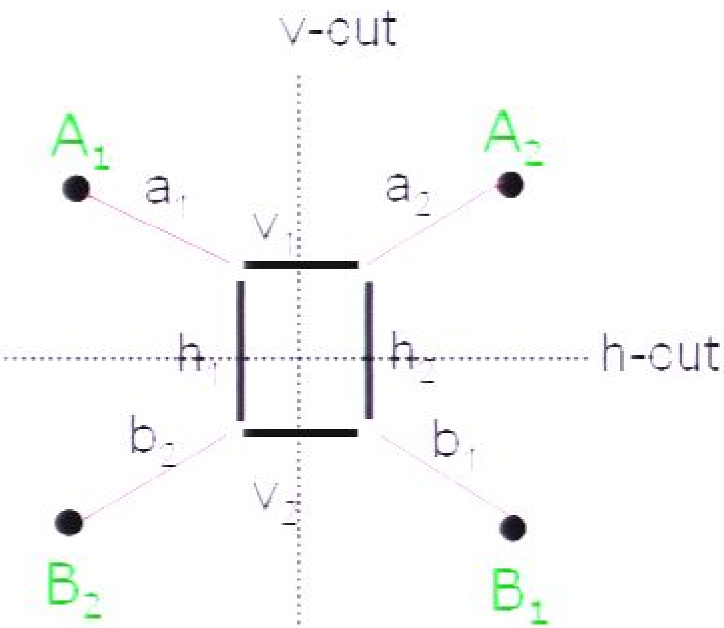
Claim: achievable by routing.

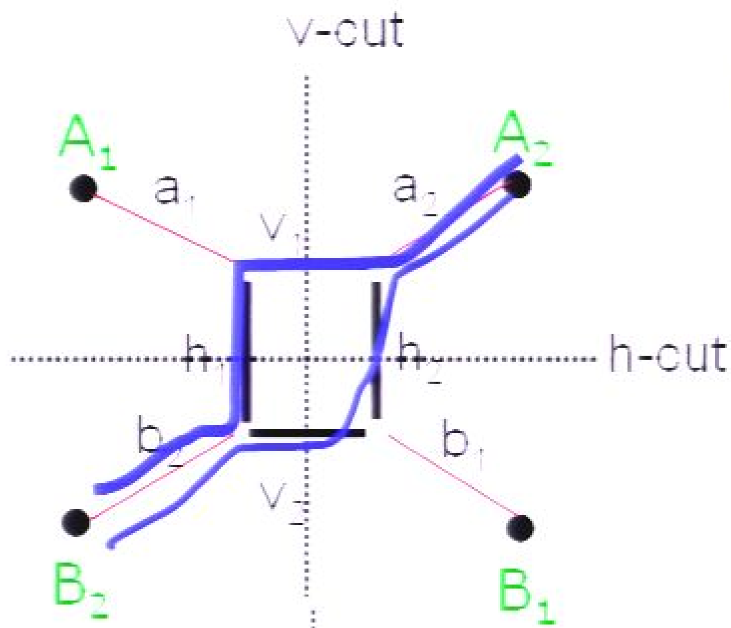
# 2-pair back-assisted (undirected) networks:



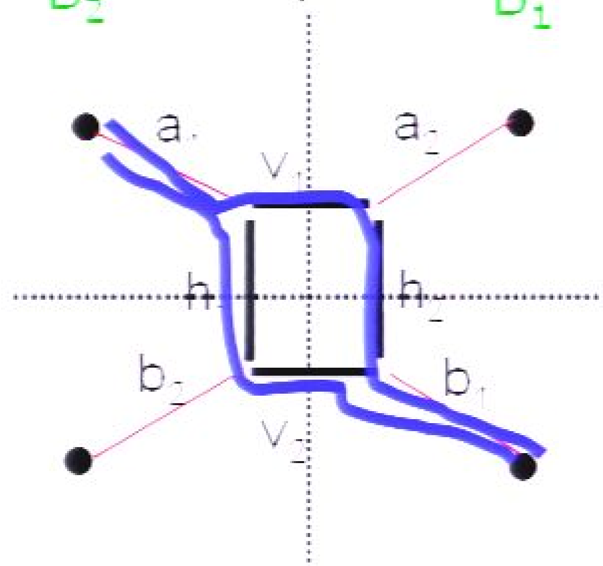

Remaining channels

# 2-pair back-assisted (undirected) networks:




for $r_2$

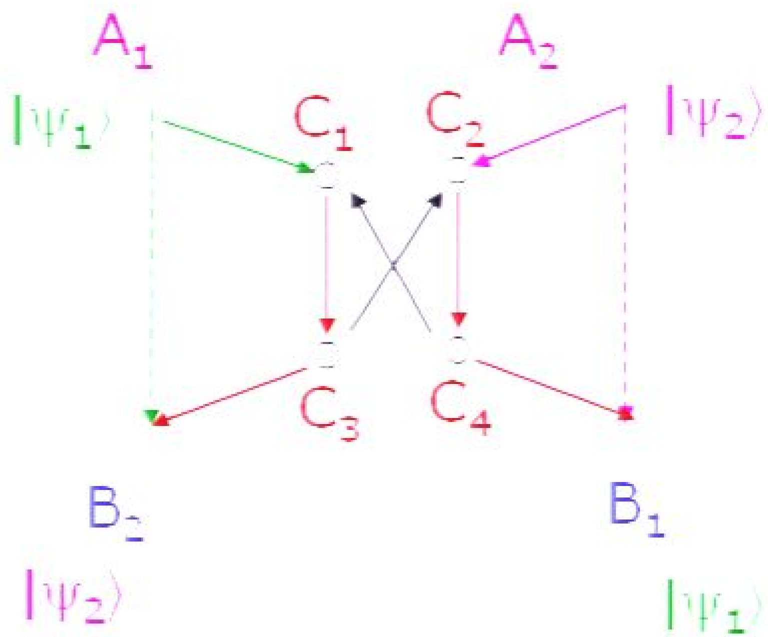



for $r_1$

Remaining channels

Can achieve any claimed rate pair by using a combination of these 4 paths

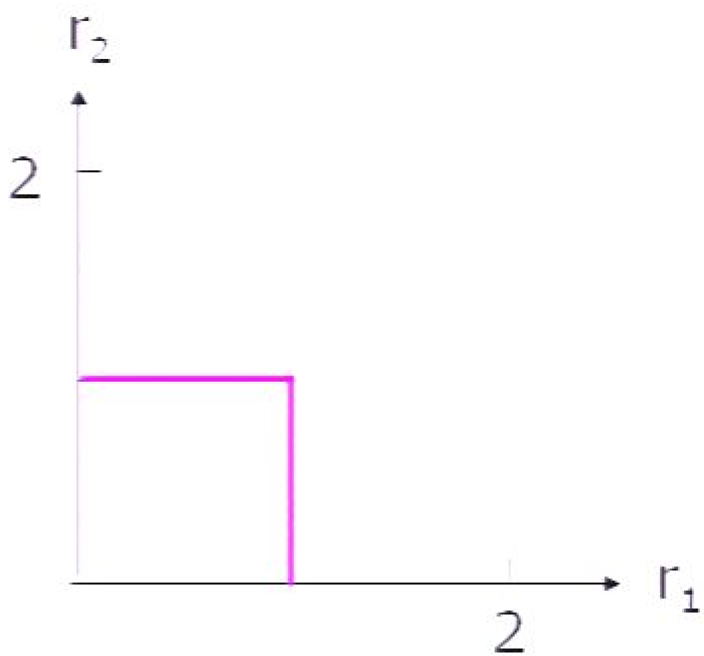
# Butterfly with cyclic body



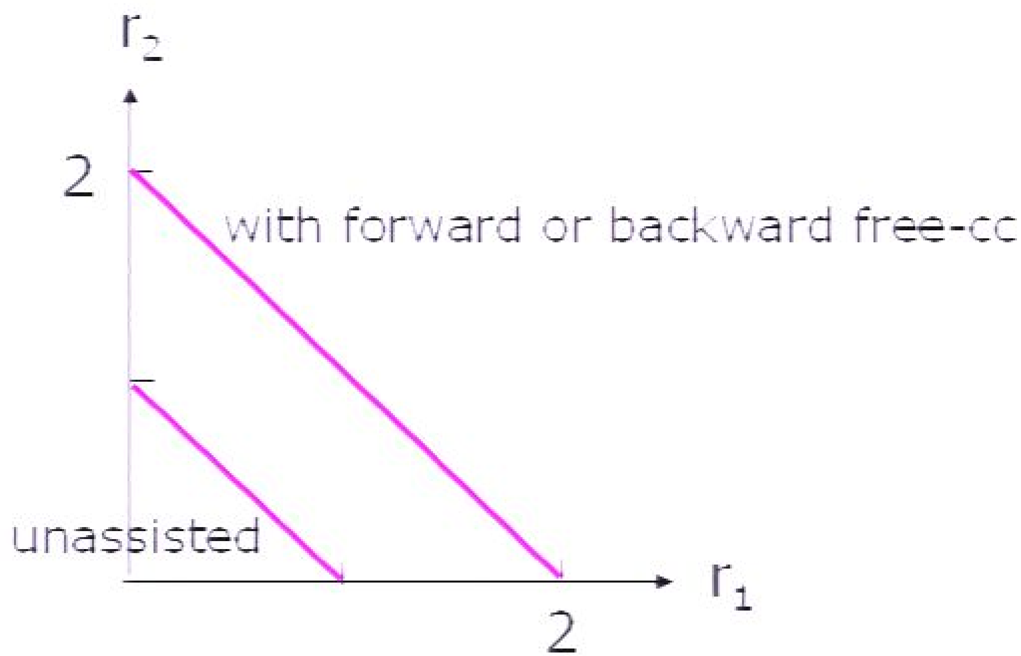

Rate sum $\leq 1$, since $C_1 \rightarrow C_3$ communication is significant for $|\psi_2\rangle$ and authorized for $|\psi_1\rangle$

# Summary for the cyclic butterfly network:

Uniquely optimal classical rate region




Rate optimal - high fidelity quantum rate regions



# To do ...

Should have done:

Resubmit & write ...
Find better graph theoretic connections.

Have tried without luck:

General shallow acyclic graphs, general acyclic graphs, general graphs.

# To do ...

Should have done:

Resubmit & write ...
Find better graph theoretic connections.

Have tried without luck:

General shallow acyclic graphs, general acyclic graphs, general graphs.