Title: Emergence of non-abelian statistics from an abelian model

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URL: http://pirsa.org/08040060

Abstract: It is well known that the toric code model supports abelian anyons. It can be realized on a square lattice of qubits, where the anyons are represented by the endpoints of strings of Pauli operators. We will demonstrate that the non-abelian Ising model can be realized in a similar way, where now the string operators are elements of the Clifford group. The Ising anyons are shown to be essentially superpositions of the abelian toric code ones, reproducing the required fusion, braiding and statistical properties. We propose a string framing and ancillary qubits to implement the non-trivial chirality of this model.

Emergence of non-abelian statistics from an abelian model

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Ville Lahtinen
Zhenghan Wang
Jiannis K. Pachos

arXiv:0804.0931

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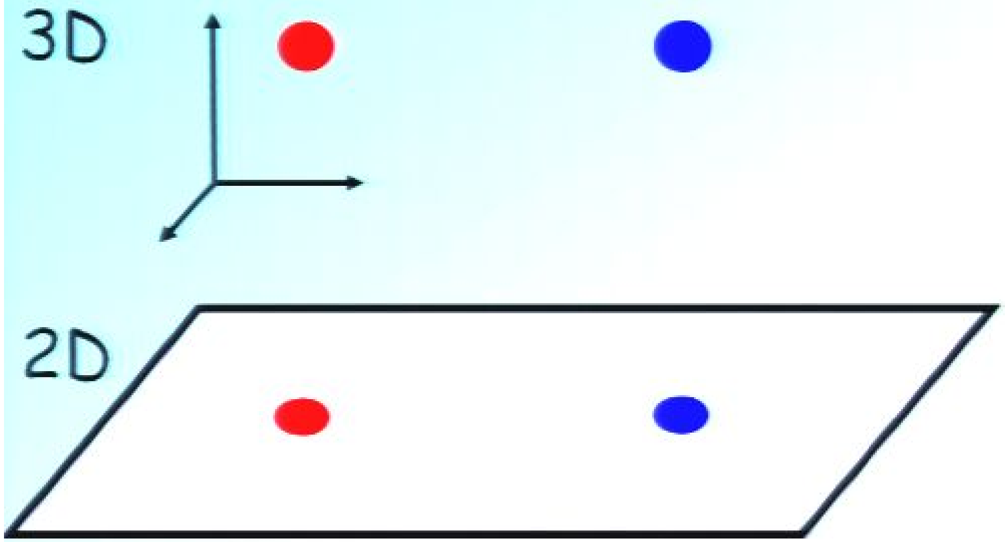
THE ROYAL SOCIETY

Anyons

Topological theories describe particles (anyons) that do not interact, but they satisfy **exotic statistics**.



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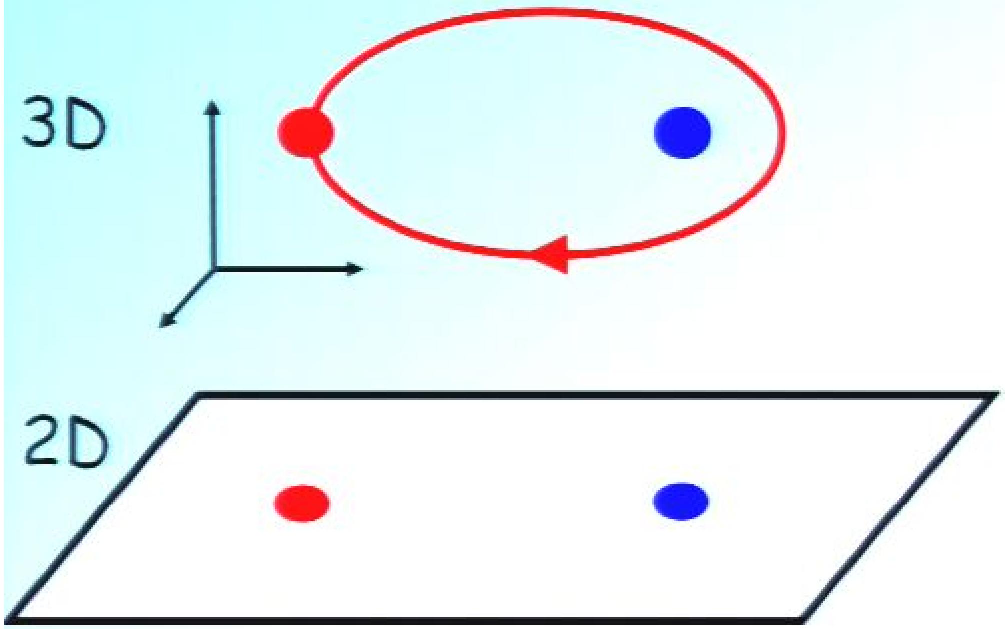

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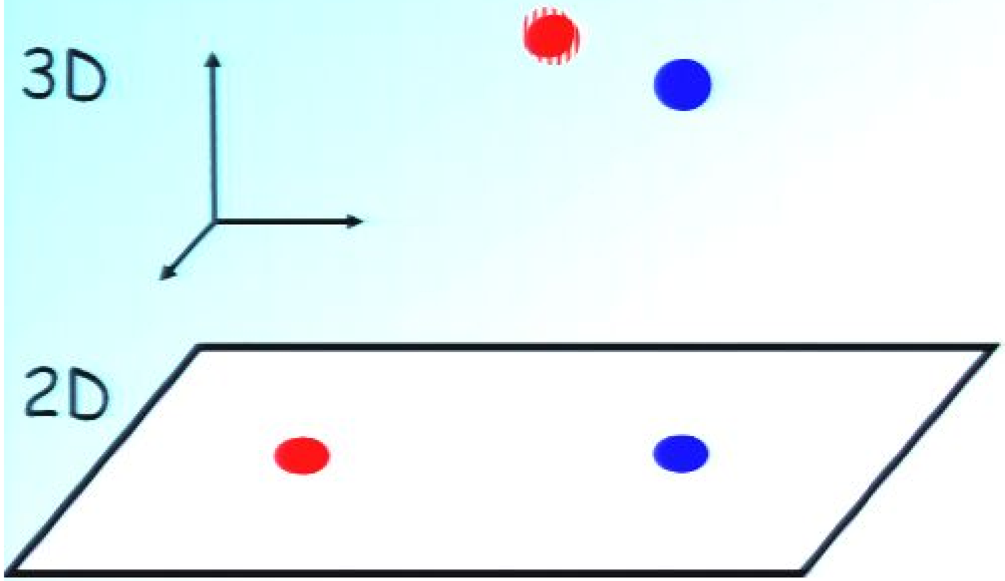

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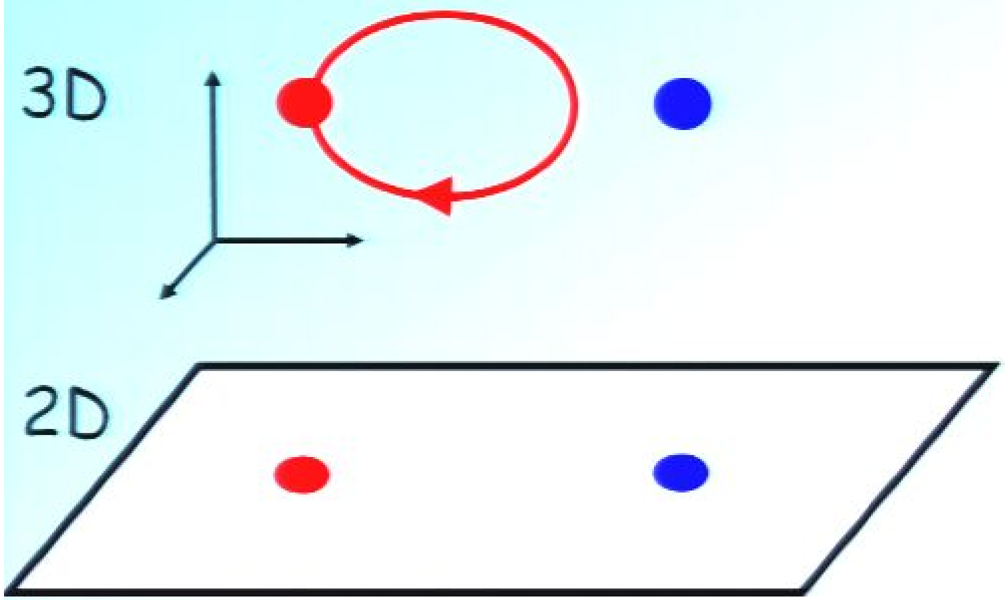

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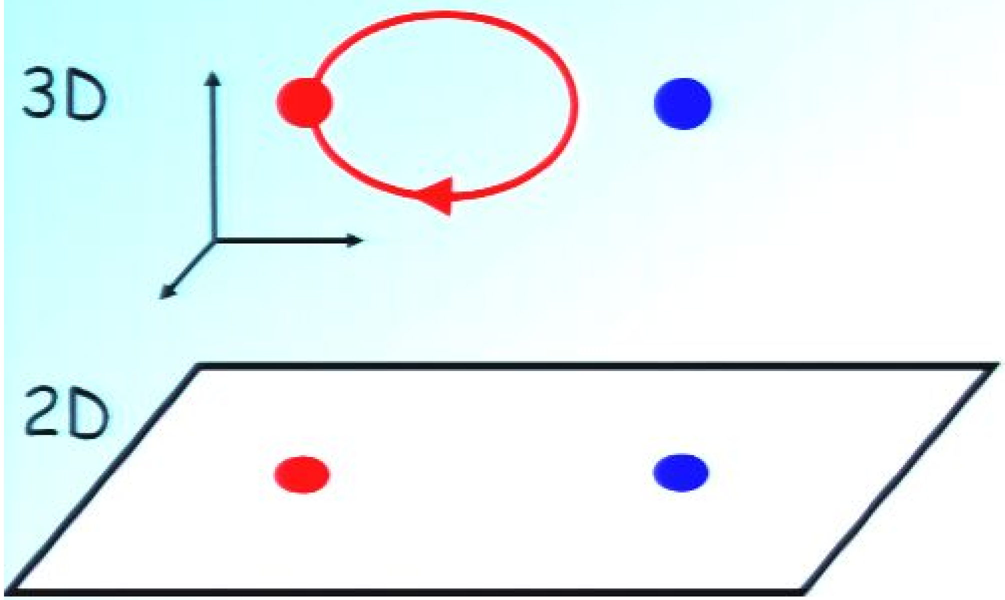

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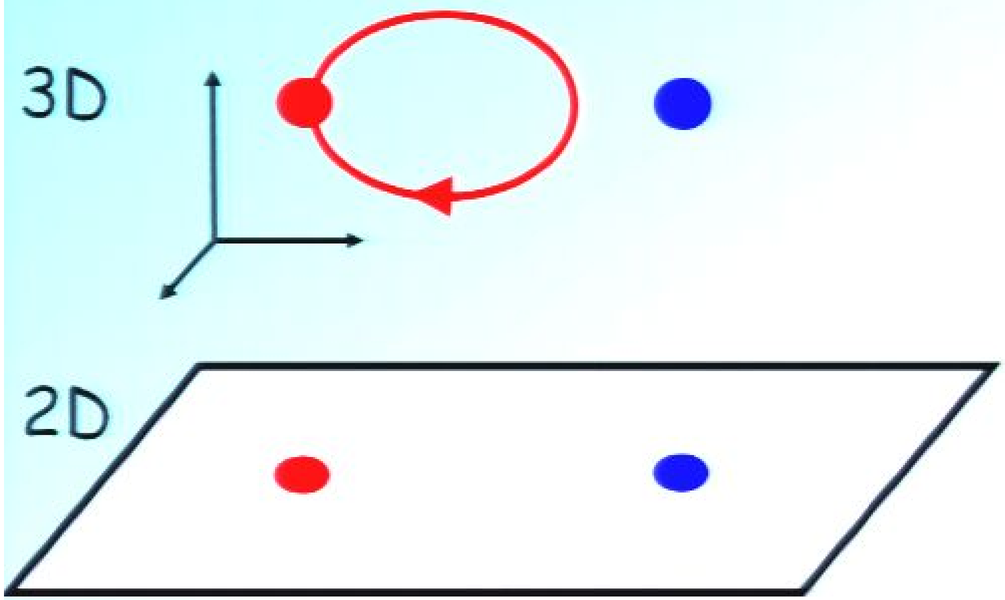

Bosons $|\Psi\rangle \rightarrow |\Psi\rangle$

Fermions $|\Psi\rangle \rightarrow e^{i2\pi}|\Psi\rangle$

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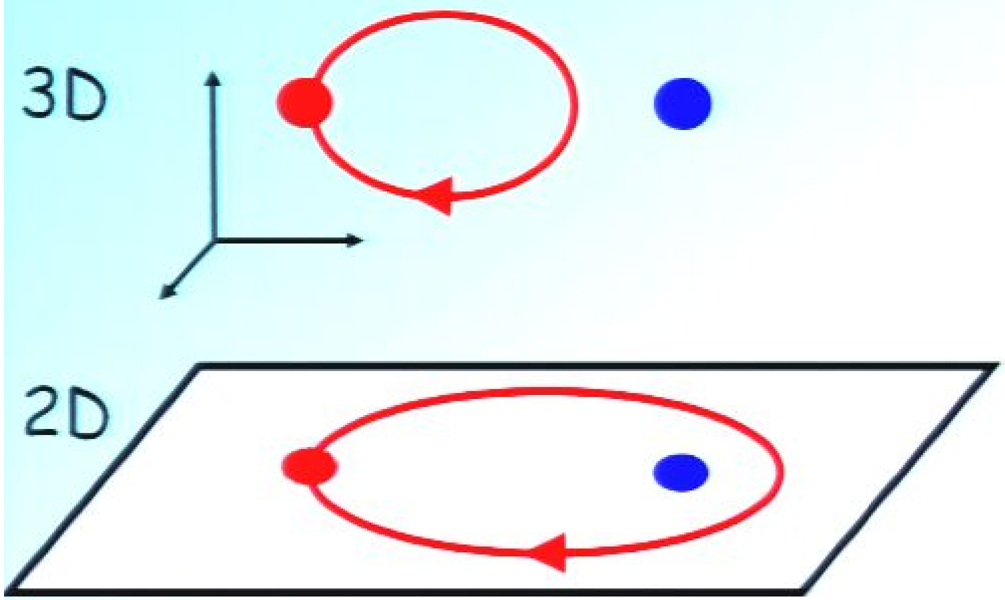

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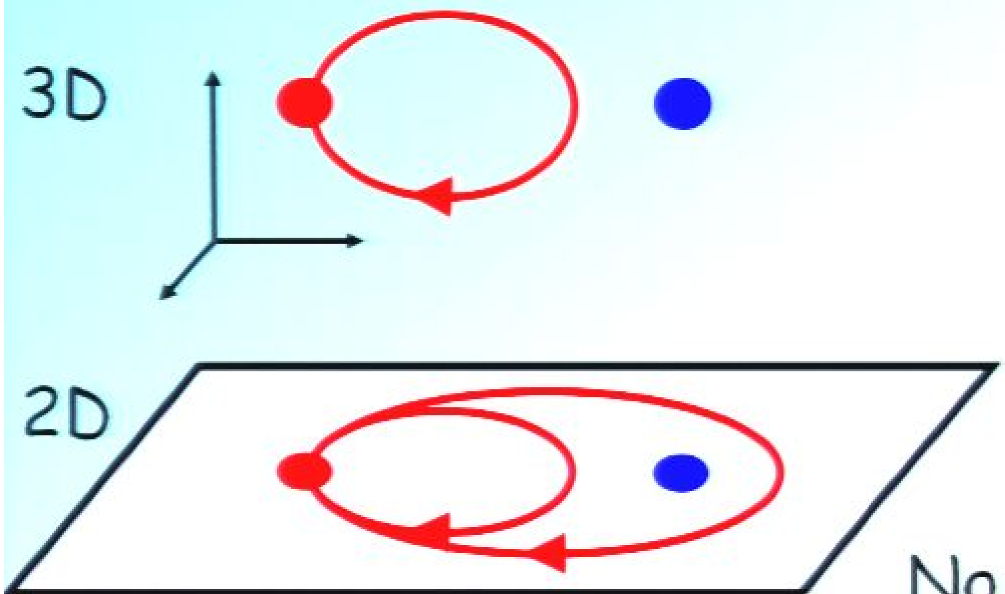

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Abelian Anyons $|\Psi\rangle \to e^{i2\varphi}|\Psi\rangle$

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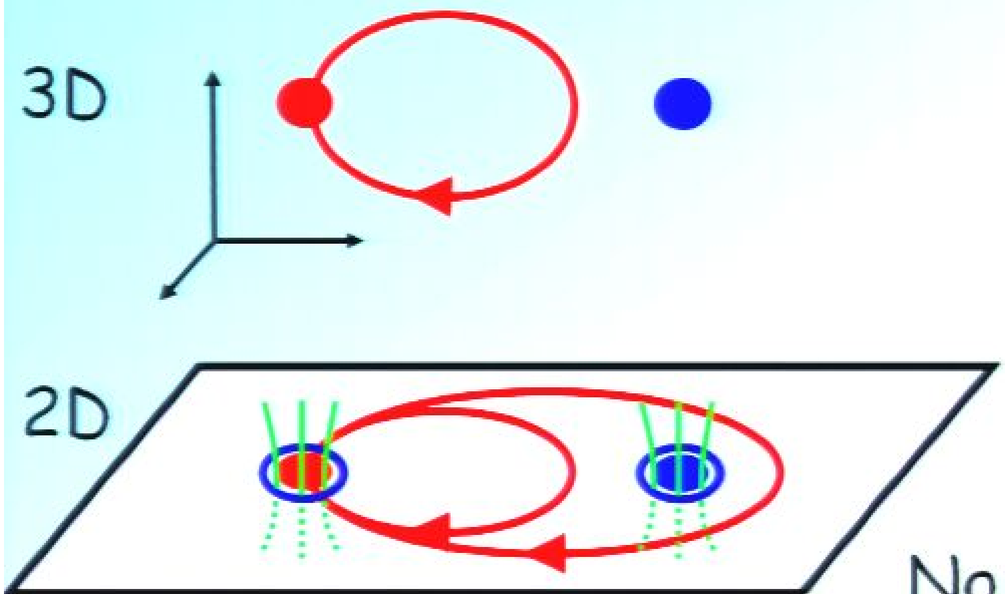

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Consider particles as composites of flux and charge.

Then phase is like the **Aharonov-Bohm effect**.

Simulate Anyons

- **Anyons** can be encoded in 2D systems:
  - **Superconducting** electrons in a strong magnetic field (Fractional Quantum Hall Effect)
  - **Lattice** systems (Kitaev's toric code/hexagonal lattice, Wen, Ioffe, Freedman-Nayak-Shtengel, Bombin-Delgado)

- Anyons are **quasiparticles** that can be identified and transported by **local operators**.
- The quantum states of the corresponding systems are **highly entangled** with long range correlations.
- **Anyonic statistics is possible due to entanglement in the underlined system.**

Simulate Anyons

- **Alexei Kitaev's** honeycomb lattice model:
  By varying the couplings of a Hamiltonian it can support abelian or non-abelian anyons.
  - Abelian: **toric code model {1, e, m, ε}**
  - Non-abelian: **Ising model {1, σ, ψ}**
- Toric code is *easy* to simulate on a qubit lattice.
- Ising model is illusive.
- **Here**: reproduce the Ising model properties by performing *complex* manipulations on toric code states.
- Answer question: *"what is non-abelian statistics?"*

[A. Kitaev, Ann. Phys. 321, 2 (2006)]

Toric Code: ECC

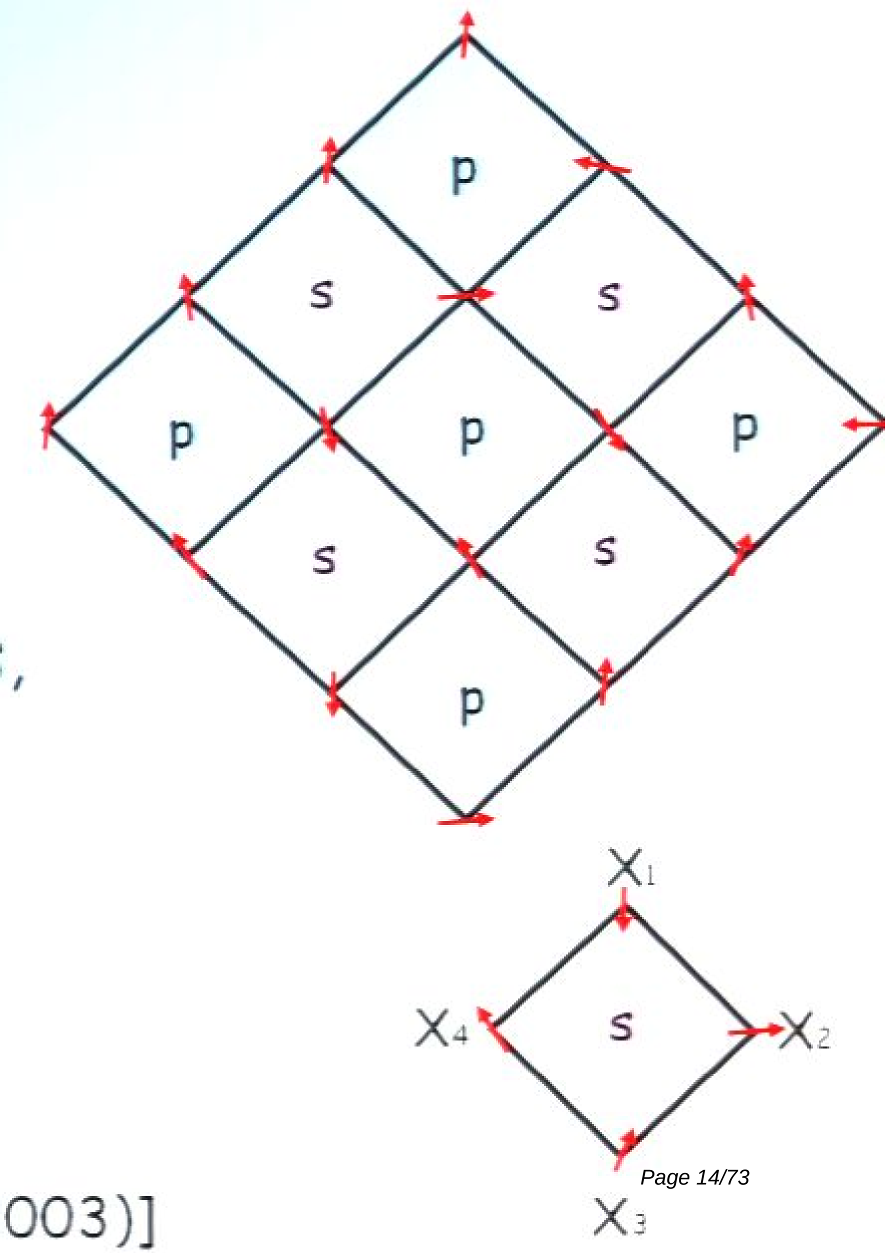
Consider the lattice Hamiltonian

$$H = -\sum_{p} Z_1 Z_2 Z_3 Z_4 - \sum_{s} X_1 X_2 X_3 X_4$$

Spins (qubits) on the vertices.

Two different types of plaquettes, p and s, with ZZZZ or XXXX interactions, respectively.

The four spin interactions involve spins at the same plaquette.



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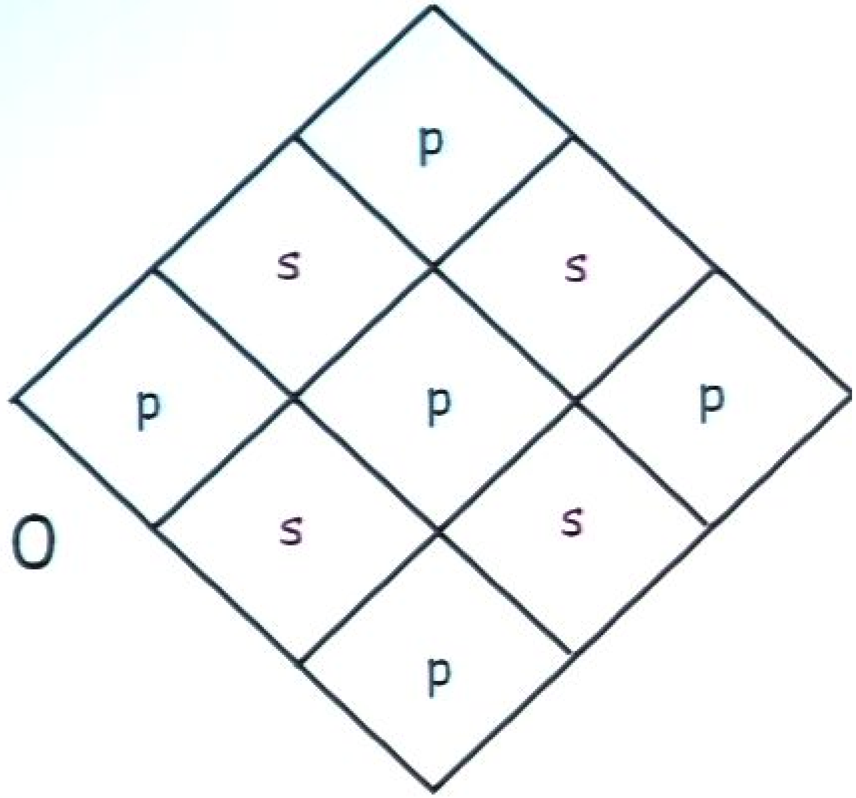
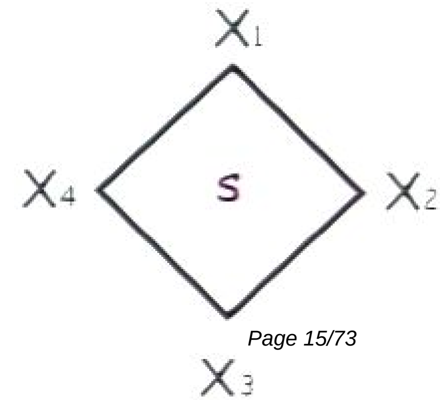


$$(X_1 X_2 X_3 X_4)^2 = (Z_1 Z_2 Z_3 Z_4)^2 = 1$$

$$[H, Z_1 Z_2 Z_3 Z_4] = 0, \ [H, X_1 X_2 X_3 X_4] = 0$$

$\Rightarrow$ Good **quantum numbers** are eigenvalues of XXXX and ZZZZ: $\pm$**1**.

Hamiltonian **exactly solvable**:



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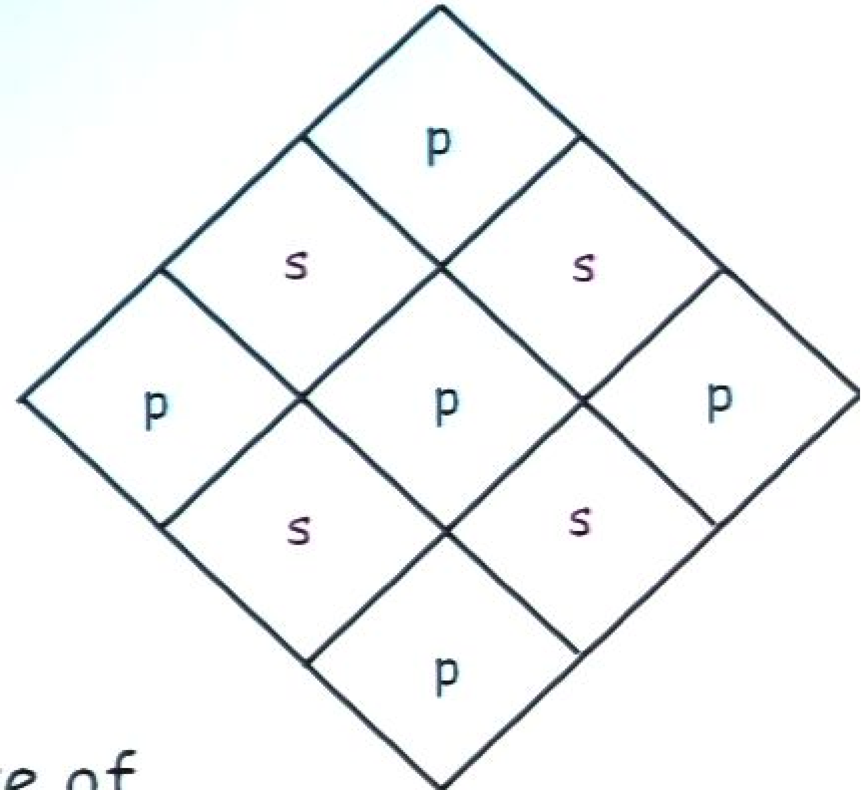
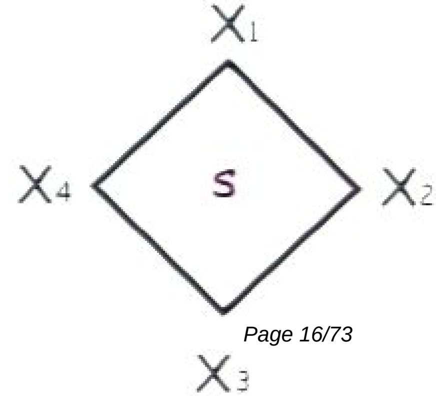


The **ground state** is:

$$\left|\xi\right\rangle = \prod_{s} \frac{1}{\sqrt{2}} (I + X_1 X_2 X_3 X_4)_s \left|00...0\right\rangle$$

The |00...0> state is a ground state of the ZZZZ term and the (I+XXXX) term projects that state to the ground state of the XXXX term.

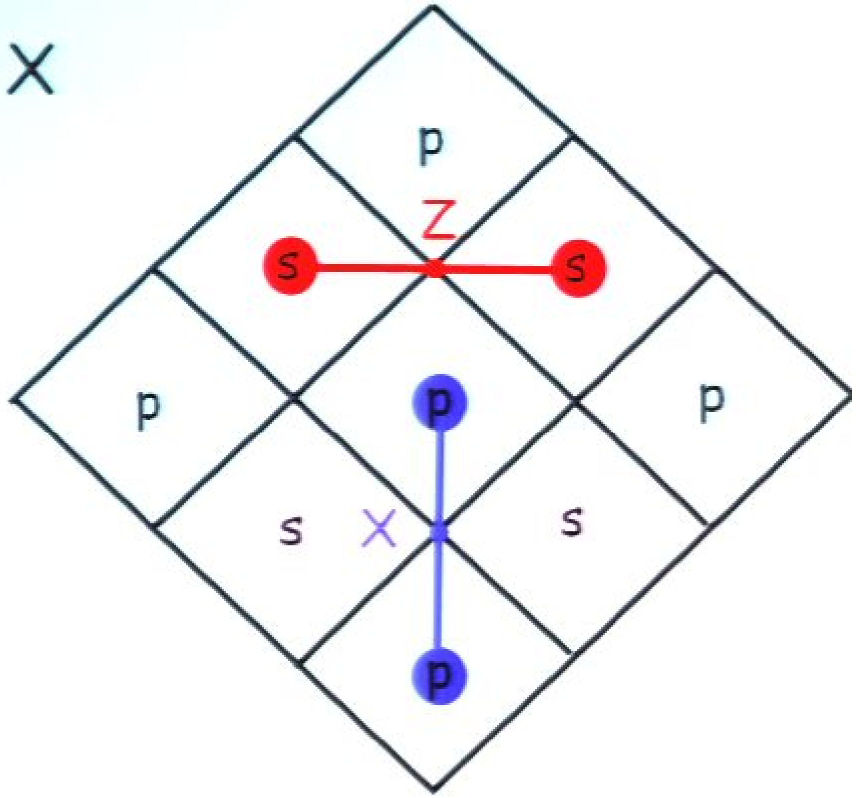


[F. Verstraete, *et al.*, PRL, 96, 220601 (2006)]

Toric Code: ECC

- **Excitations** are produced by Z or X rotations of one spin.

- These rotations anticommute with the X- or Z- part of the Hamiltonian and thus increase the energy of certain plaquettes.



- Z excitations (**m**) live on *s* plaquettes.
- X excitations (**e**) live on *p* plaquettes.

- Anyons are **transported** by a series of X or Z rotations.

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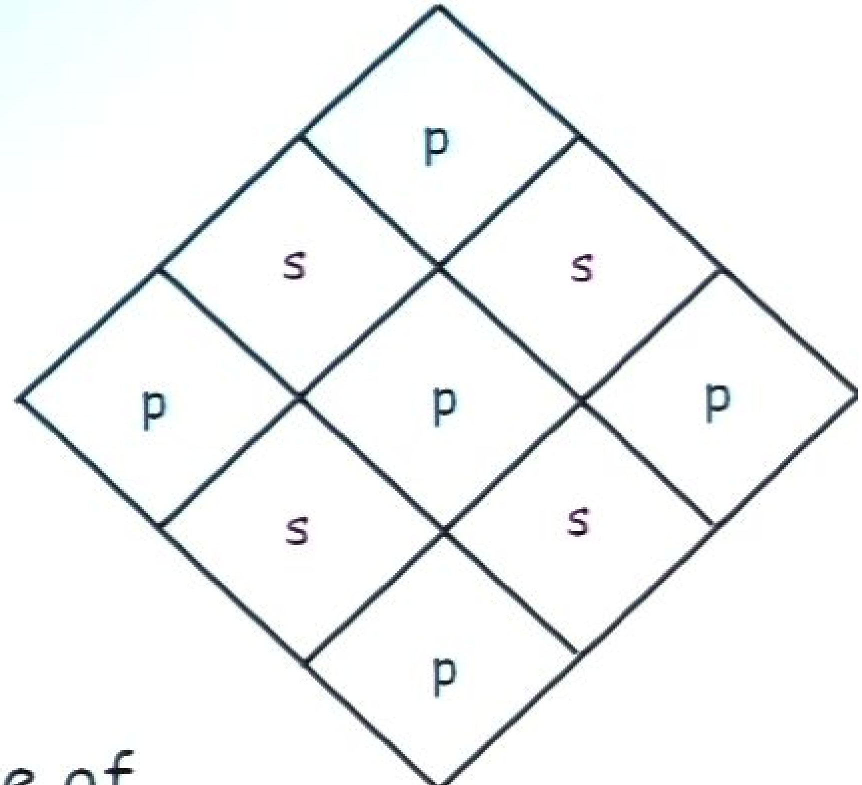

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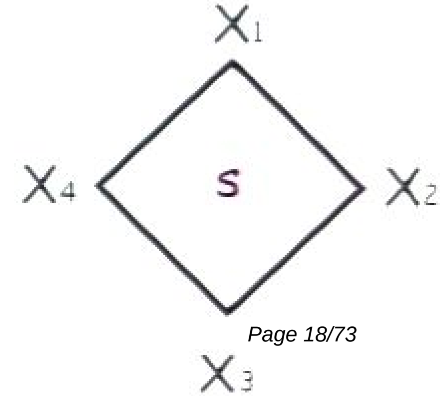

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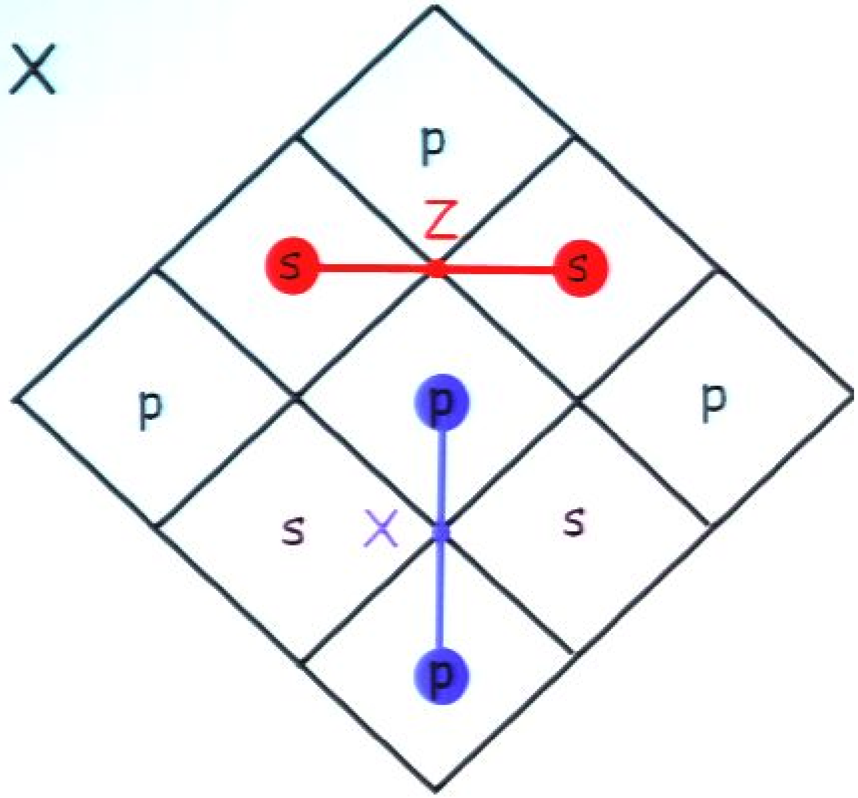

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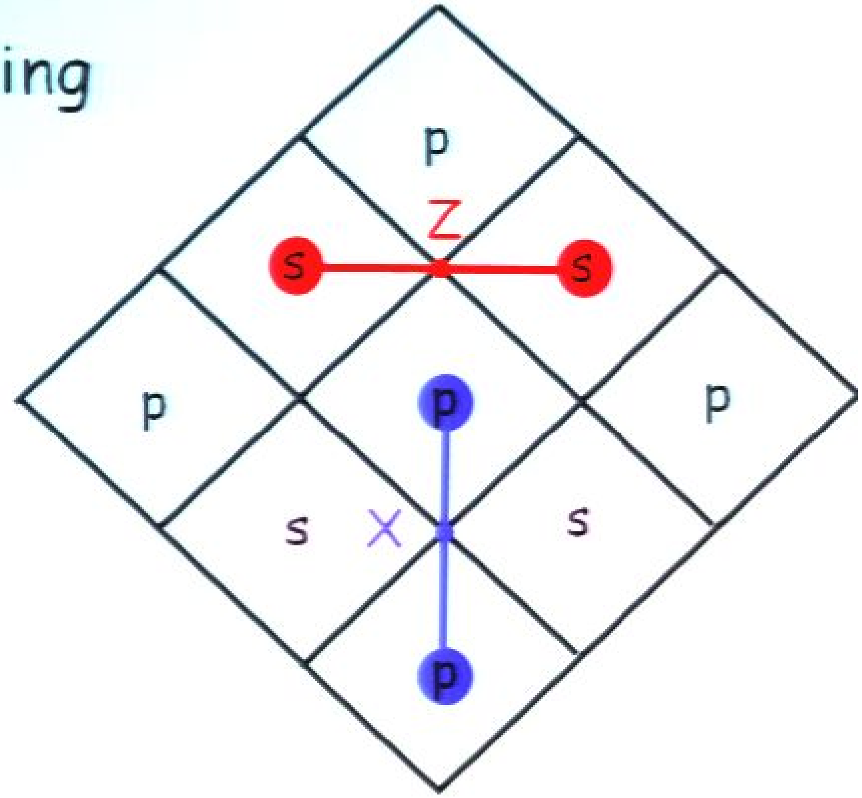
- Reveal anyonic statistics by braiding the anyons:

  move **e** anyon around **m**.



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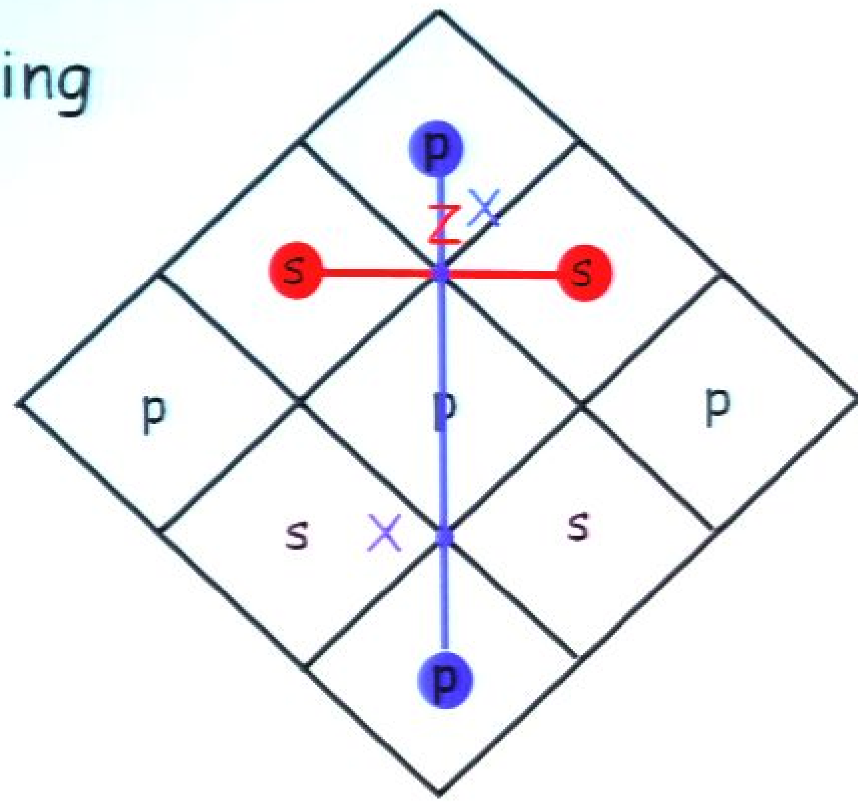

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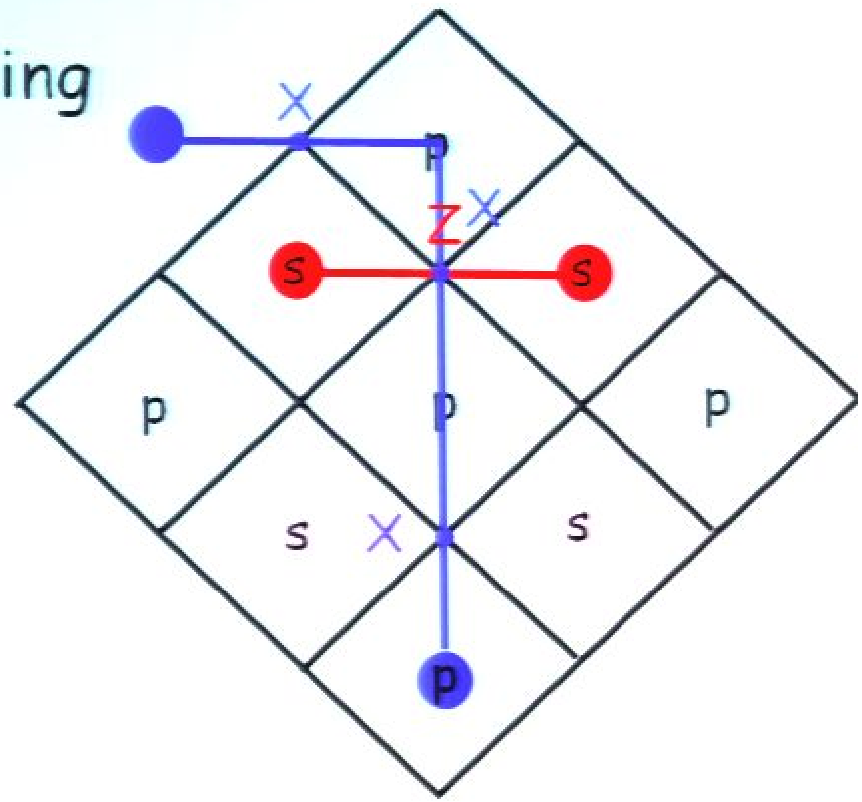

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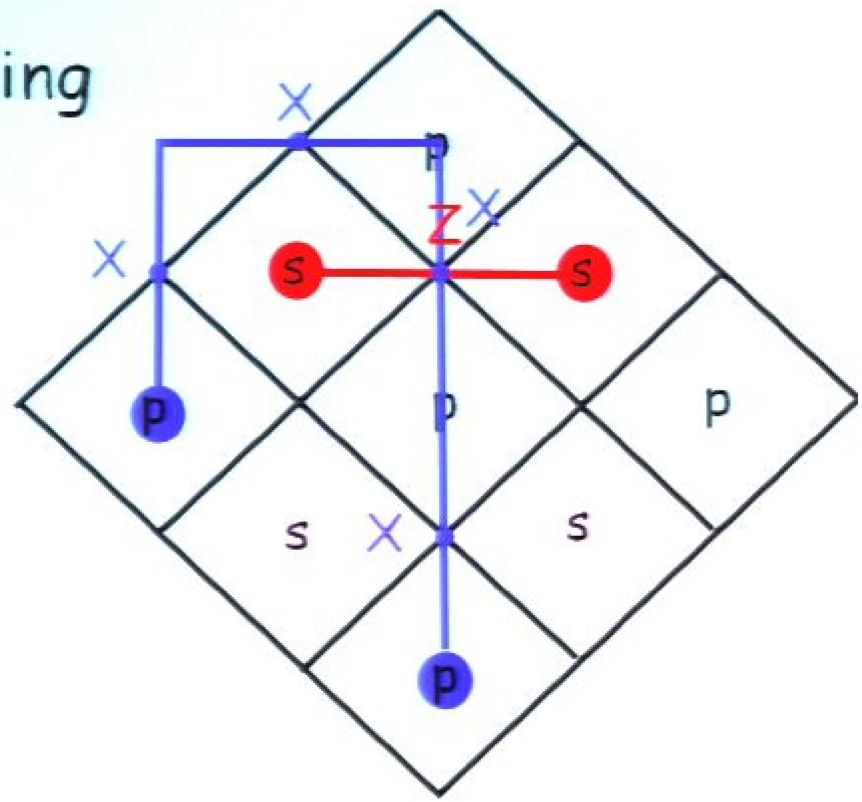

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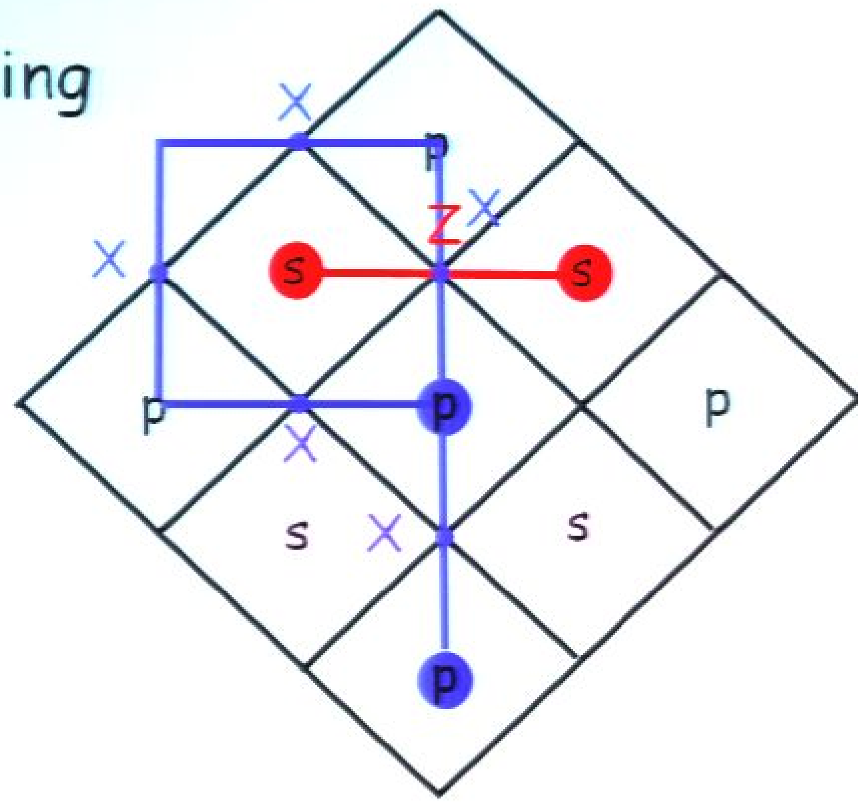

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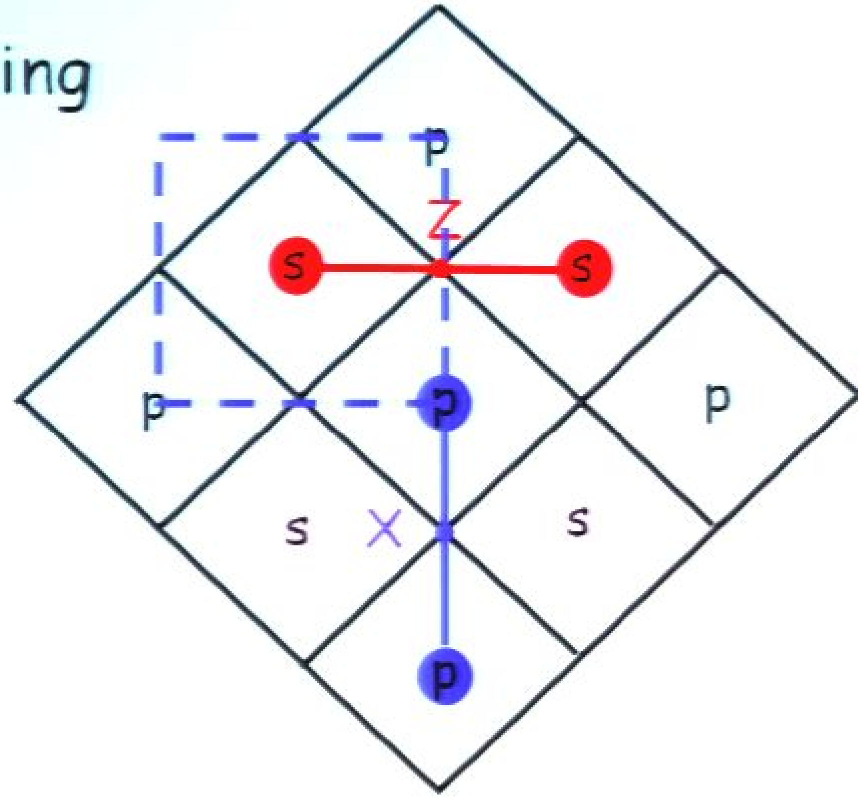

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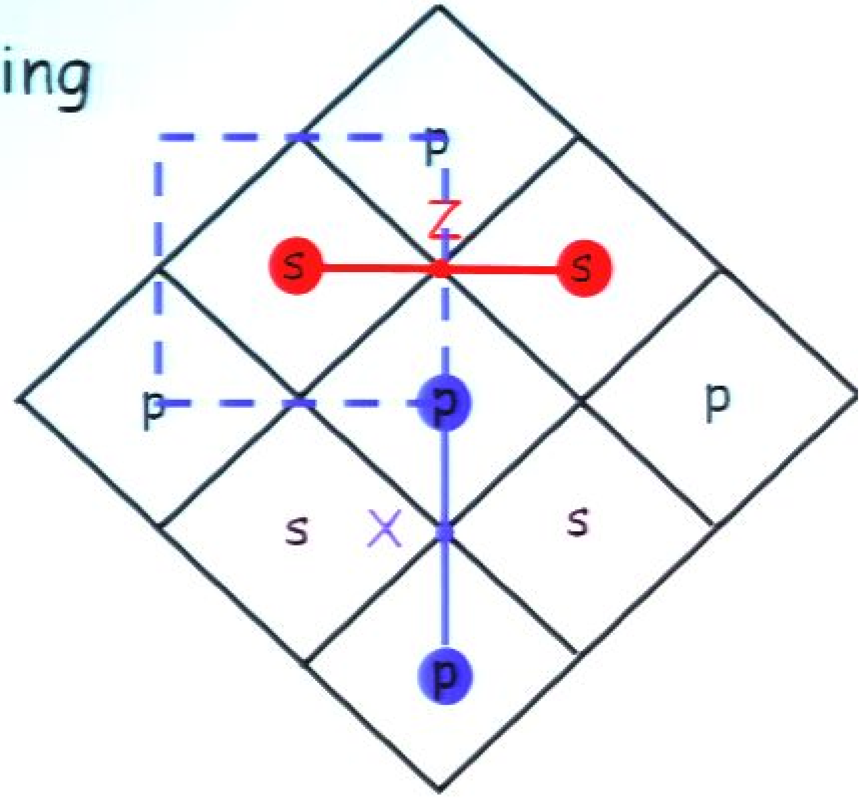

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Anyonic statistics

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- Reveal anyonic statistics by braiding the anyons:

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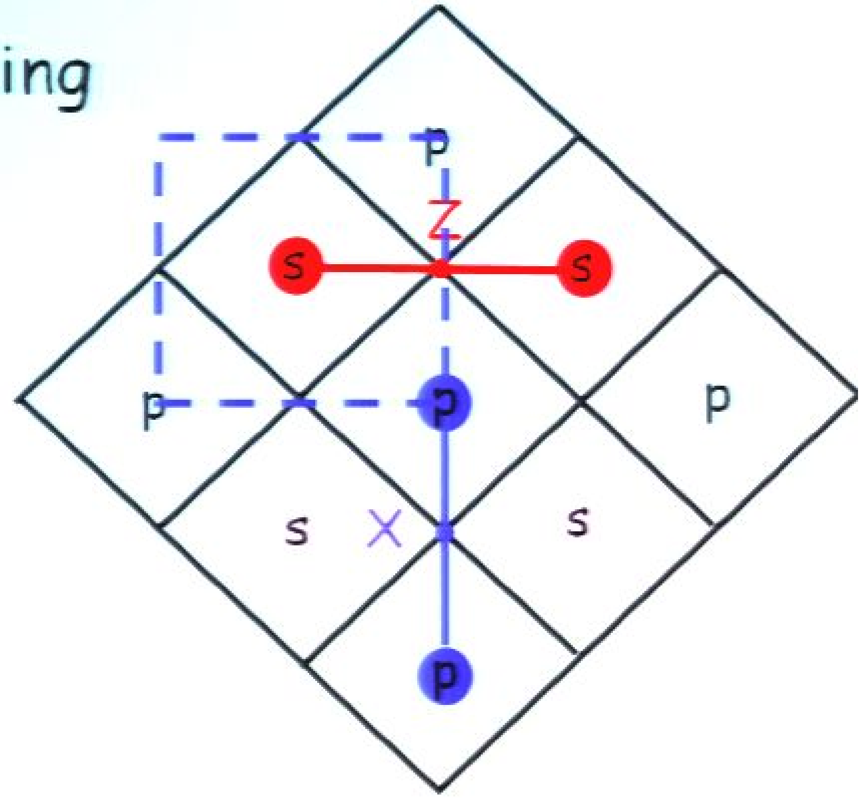
- Full circulation = two exchanges:

$$(R^{em})^2 = -1 \Rightarrow R^{em} = i$$



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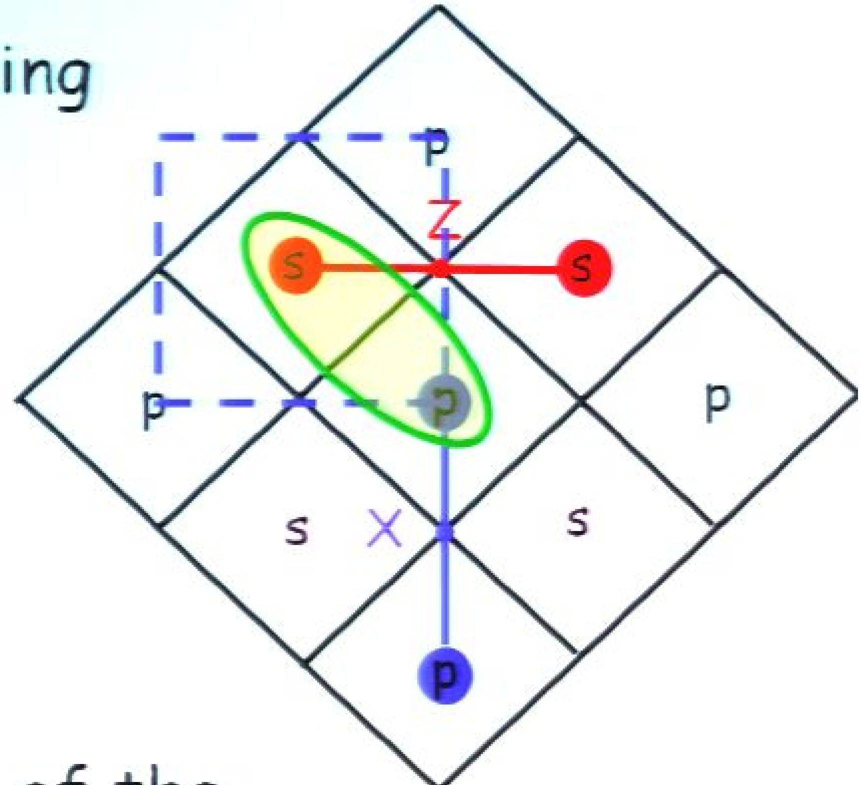


$$(R^{em})^2 = -1 \Rightarrow R^{em} = i$$

- Can be also viewed as the rotation of the composite particle, ε=e×m, by 2π

=> The "-1" factor reveals the fermionic statistics of ε.

Topological properties of Toric Code

1) Particles:

1, e (abelian), m (abelian), ε (fermion)

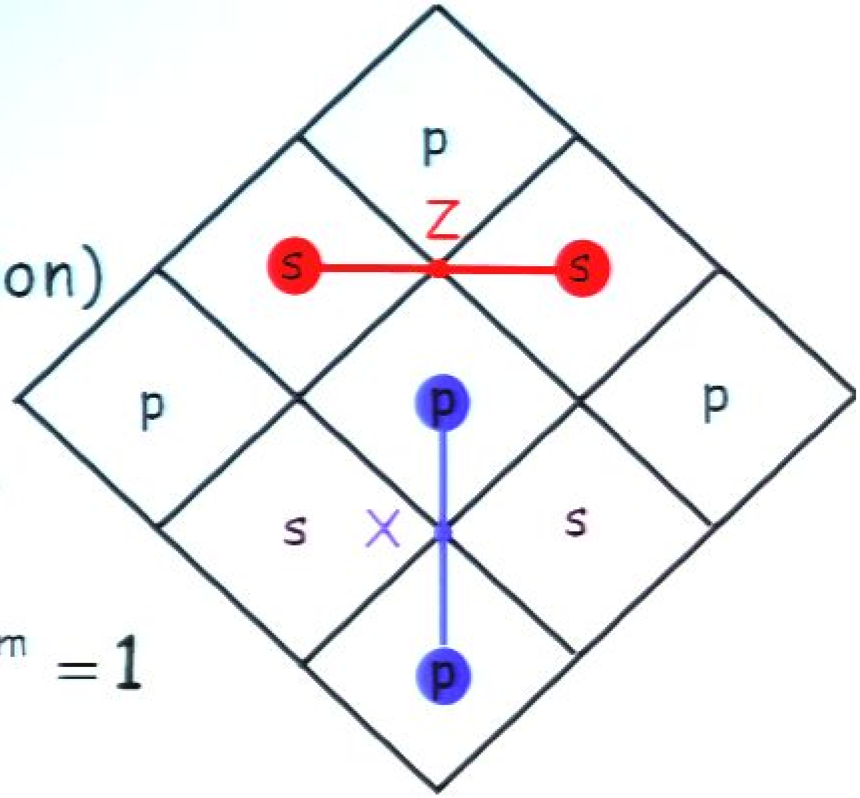
2) Fusion:

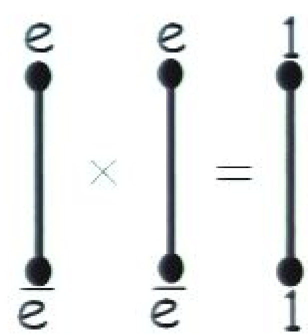
m×m=e×e=1, e×m=ε, e×ε=m, m×ε =e

3) Braiding:

$$R^{\varepsilon\varepsilon} = -1, (R^{em})^2 = -1, R^{ee} = 1, R^{mm} = 1$$

4) Fusion (non-)commutativity:





gates
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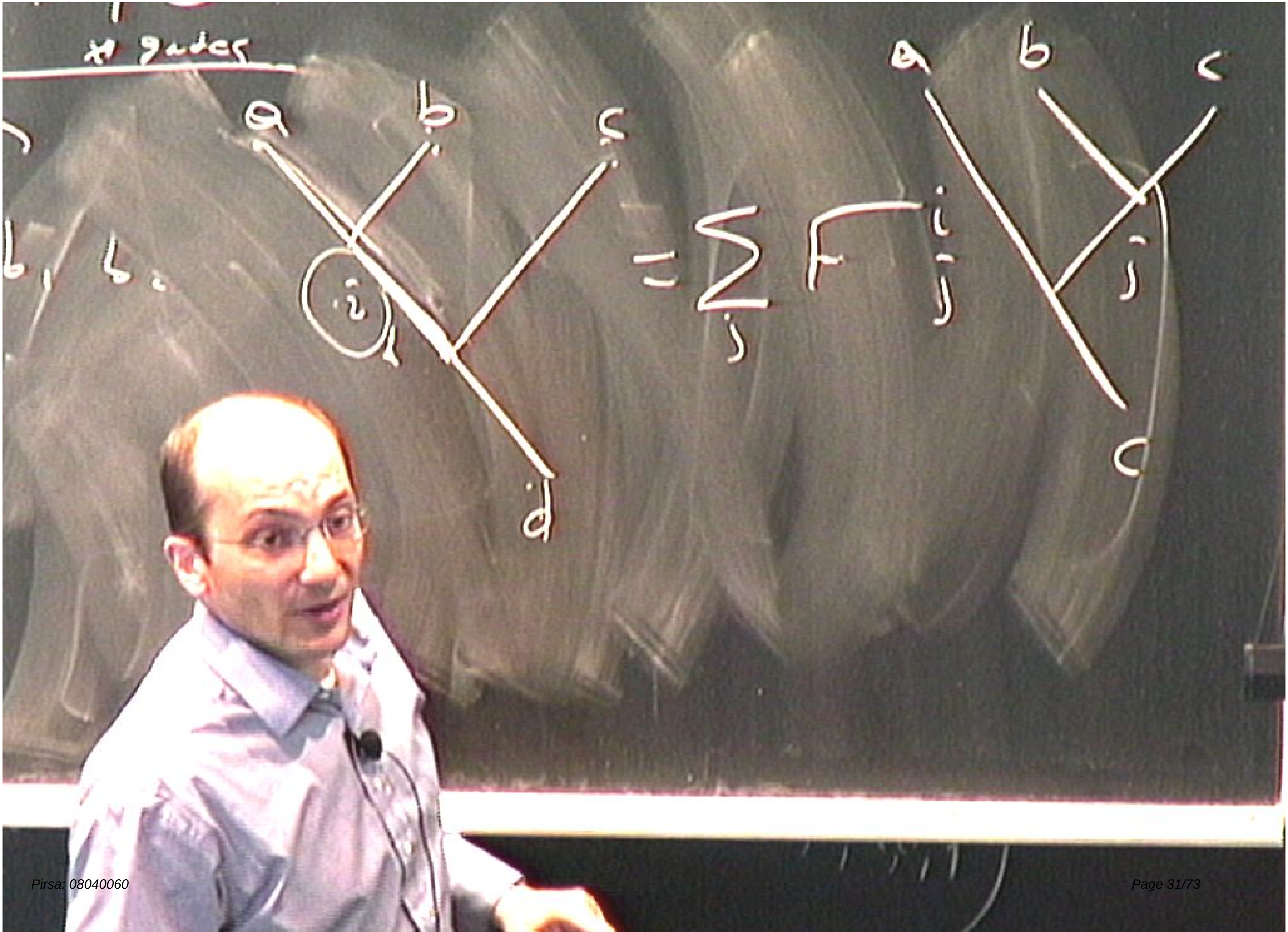

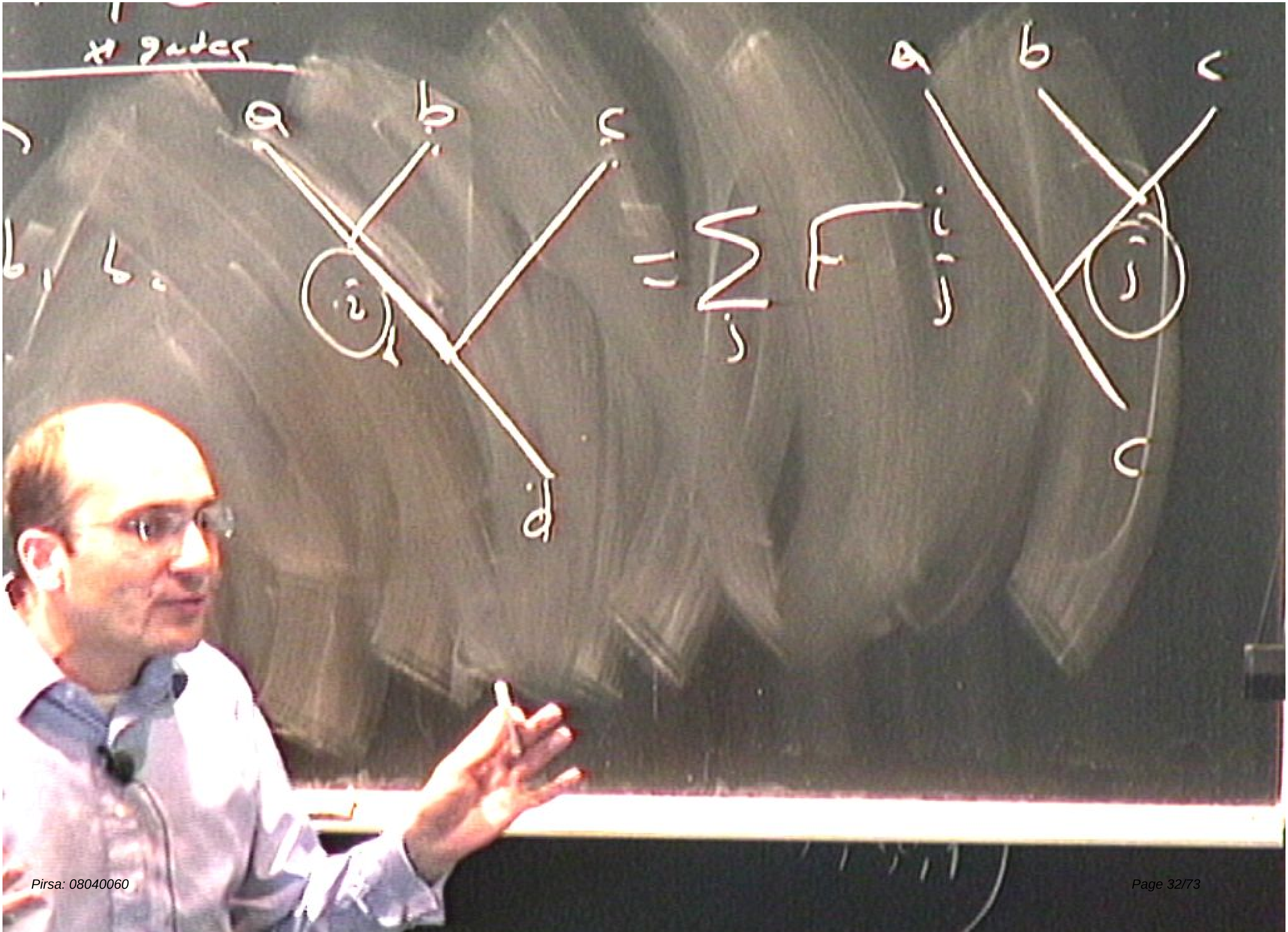
gates
a
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$= \sum_j F$
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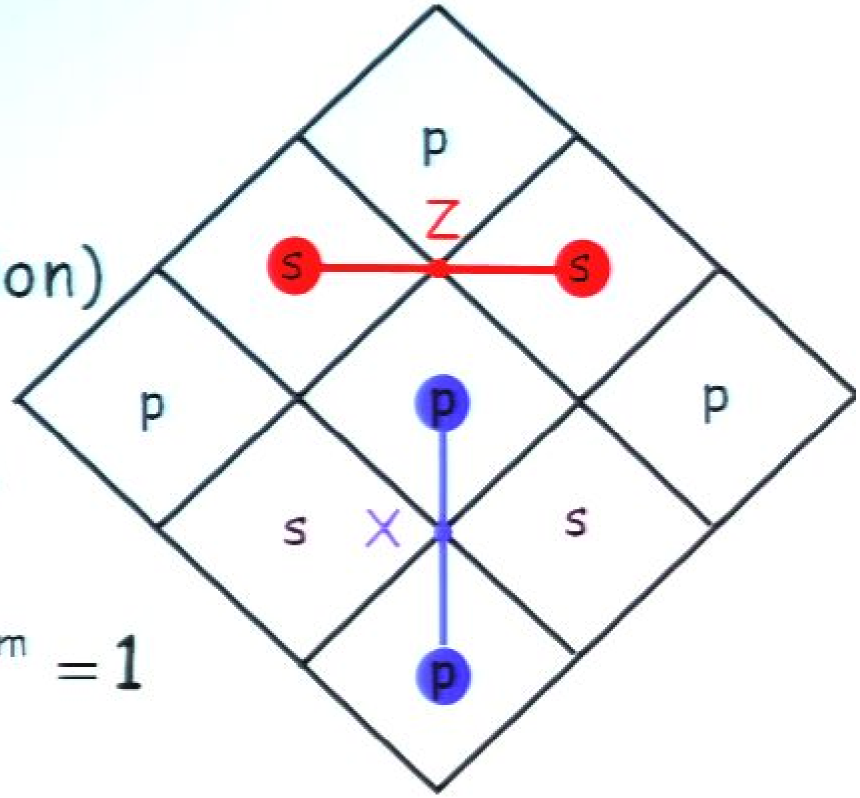

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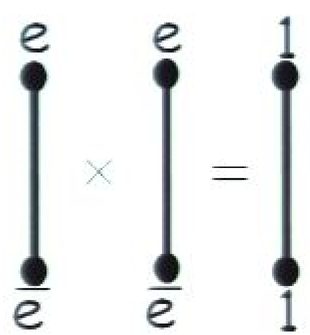
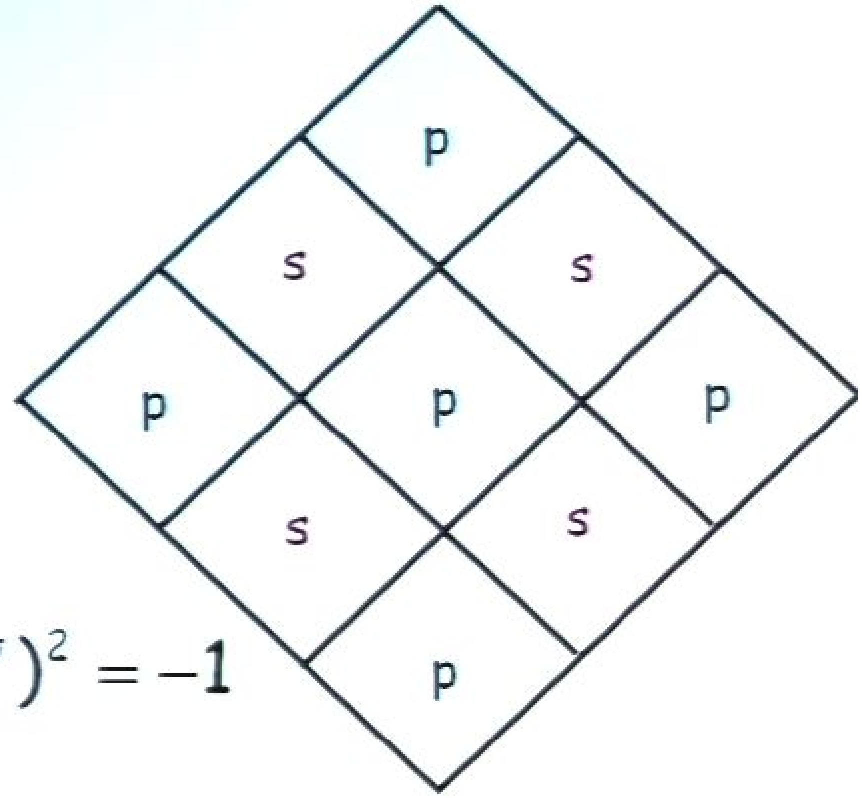


Topological properties of Ising model



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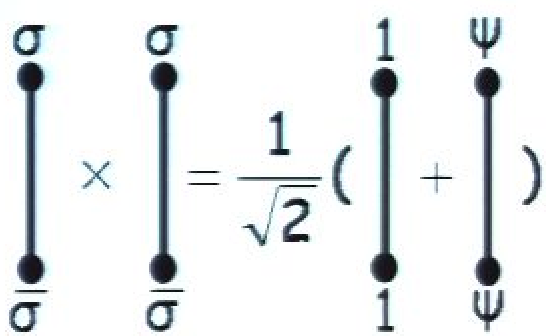
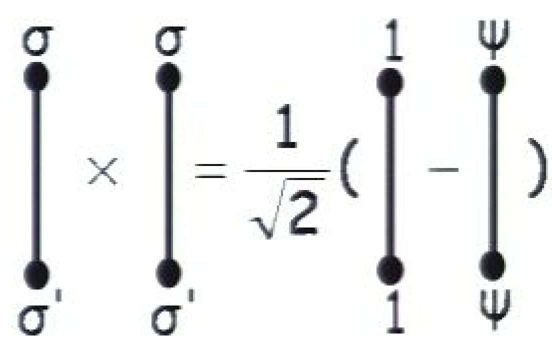
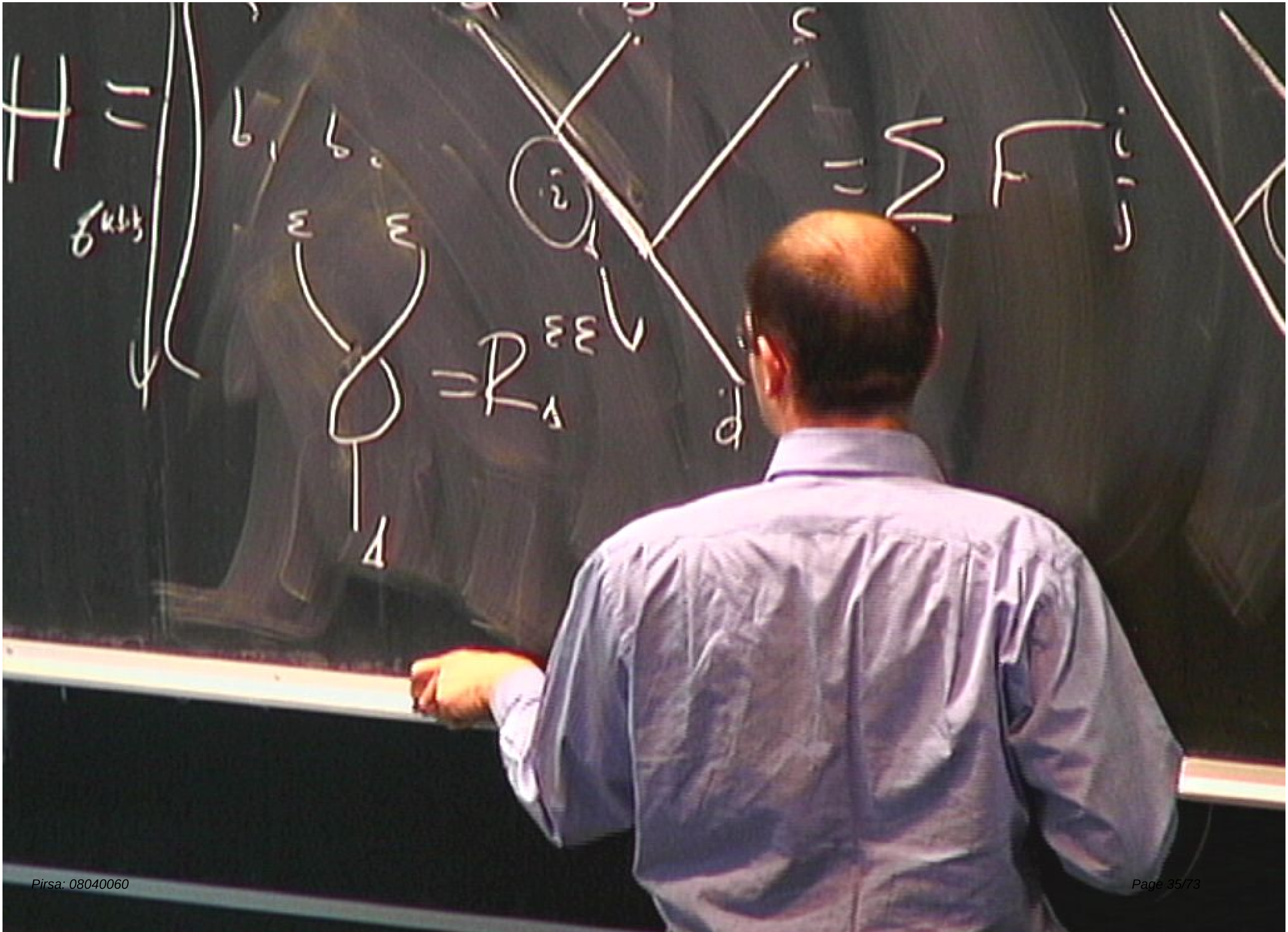
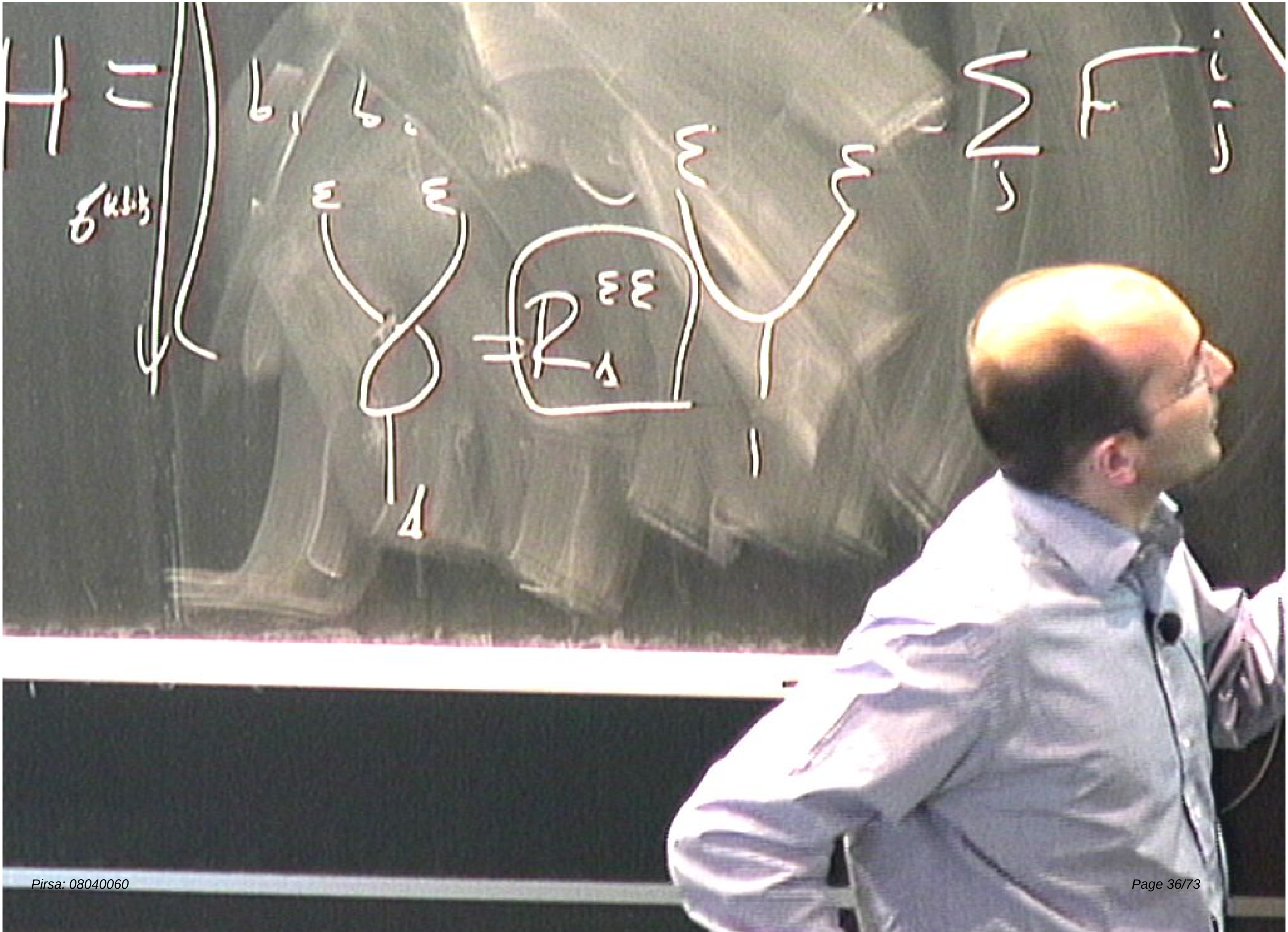






H =
ε ε
= R
d
= Σ F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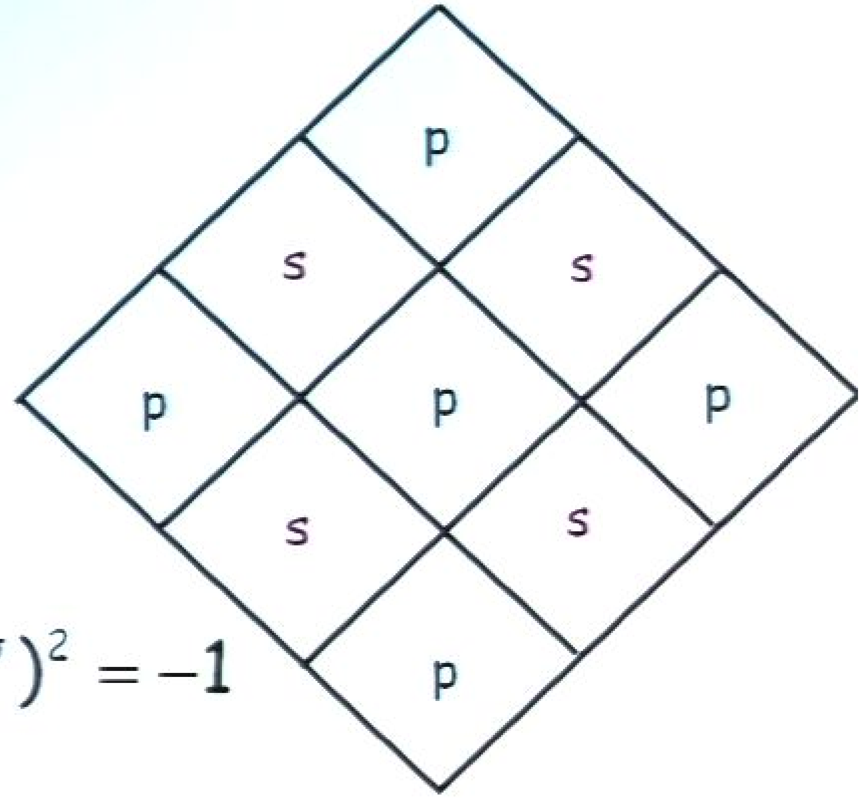
Topological properties of Ising model



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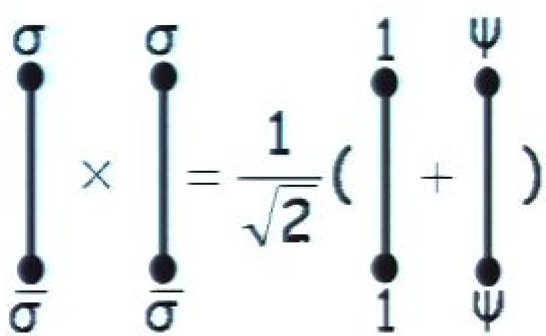

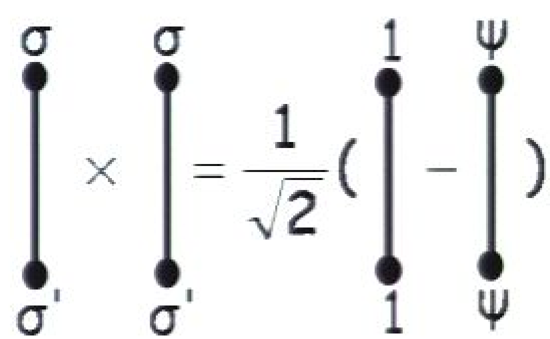

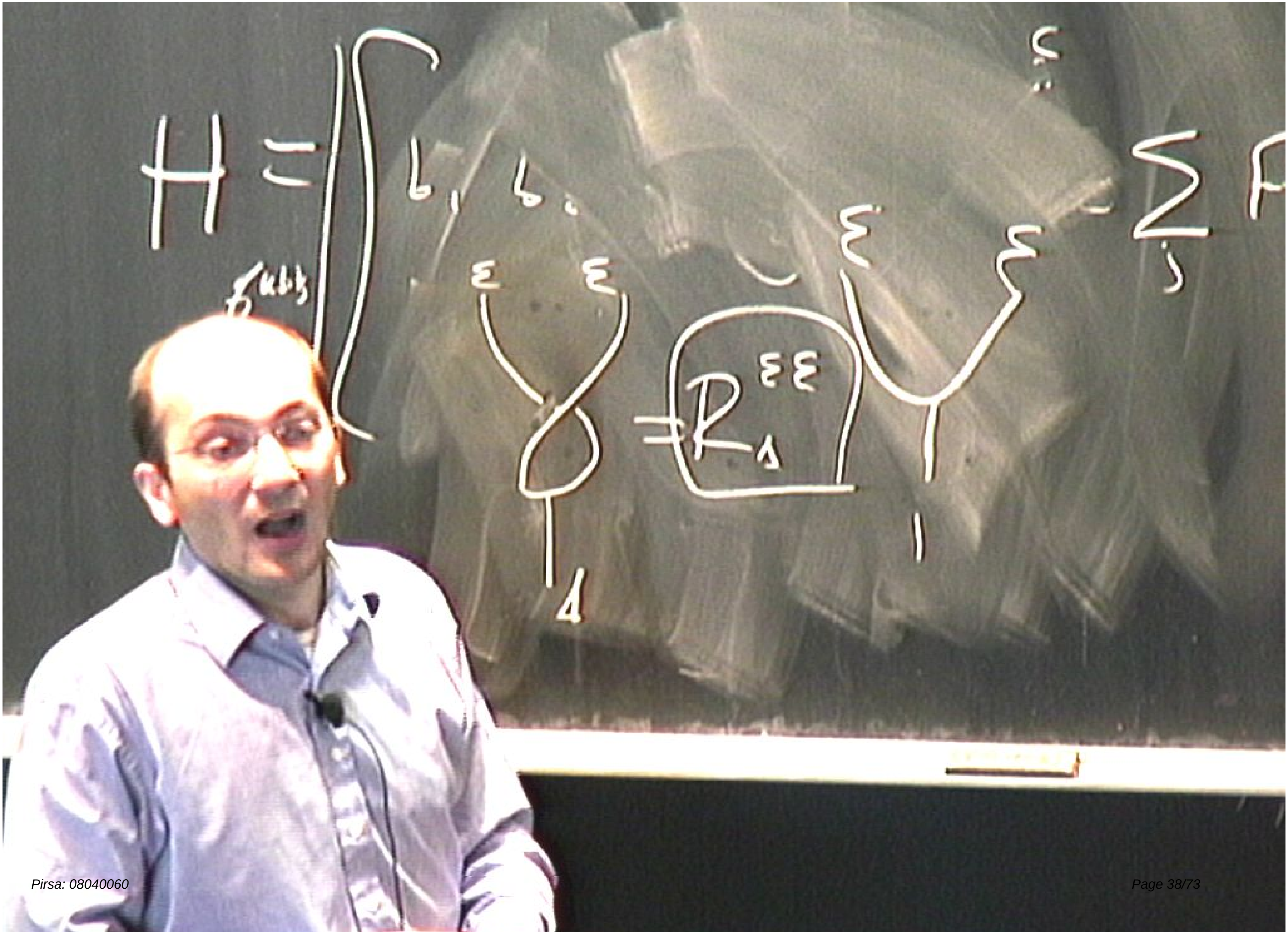
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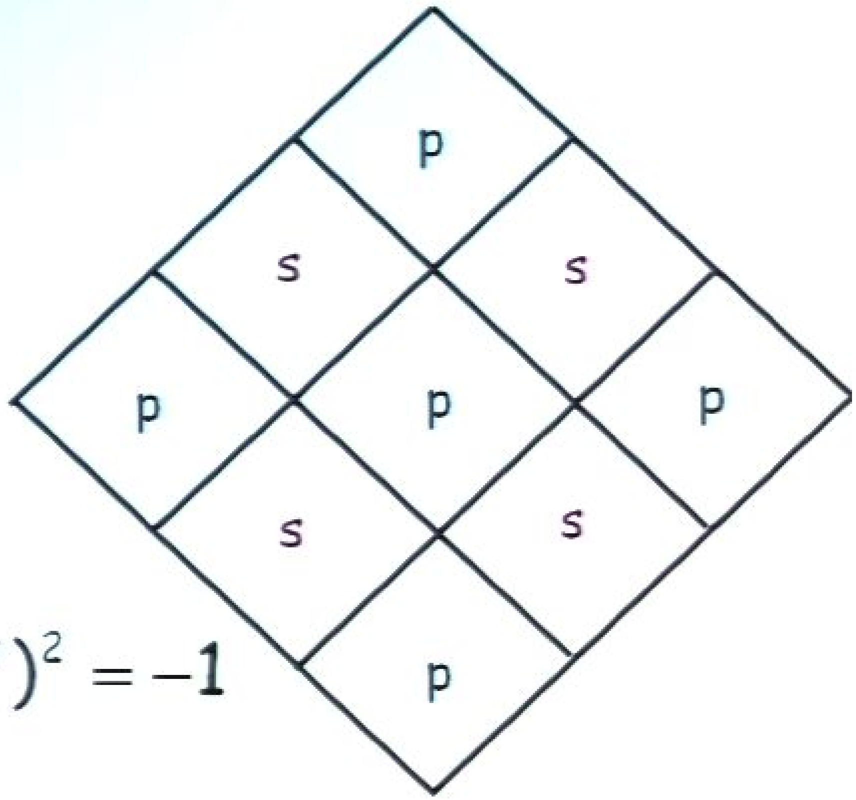

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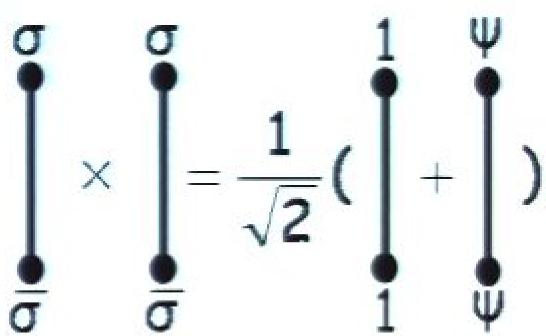
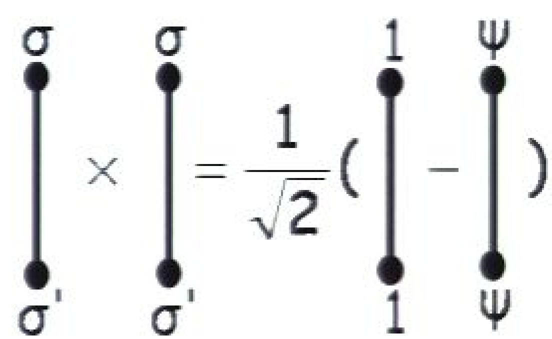
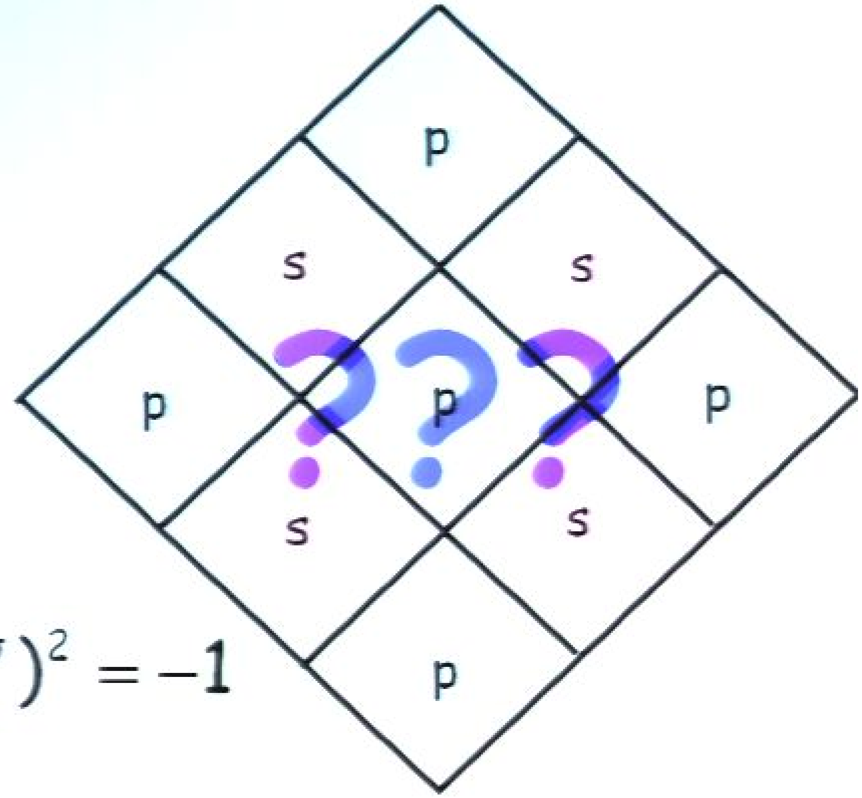




Topological properties of Ising model



1) Particles:

1, σ (non-abelian), ψ (fermion)

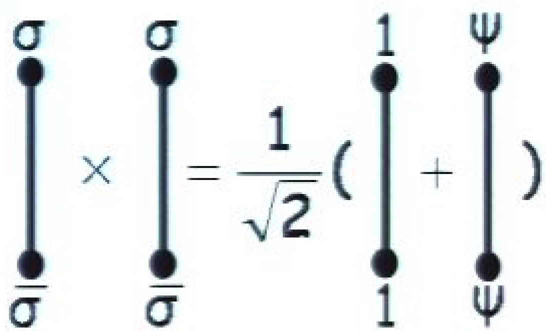
2) Fusion:

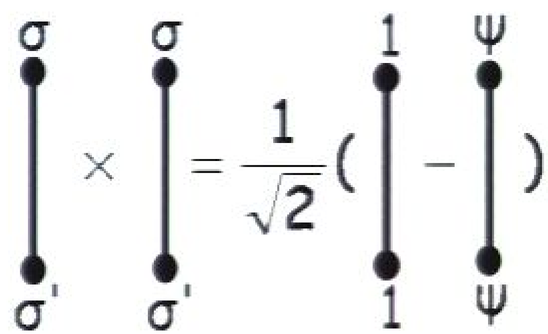
$\sigma \times \sigma = 1 + \psi$, $\sigma \times \psi = \sigma$, $\psi \times \psi = 1$

3) Braiding (up to overall phases):

$$R_1^{\psi\psi} = -1, (R_1^{\sigma\sigma})^2 = 1, (R_\psi^{\sigma\sigma})^2 = -1, (R_\sigma^{\psi\sigma})^2 = -1$$

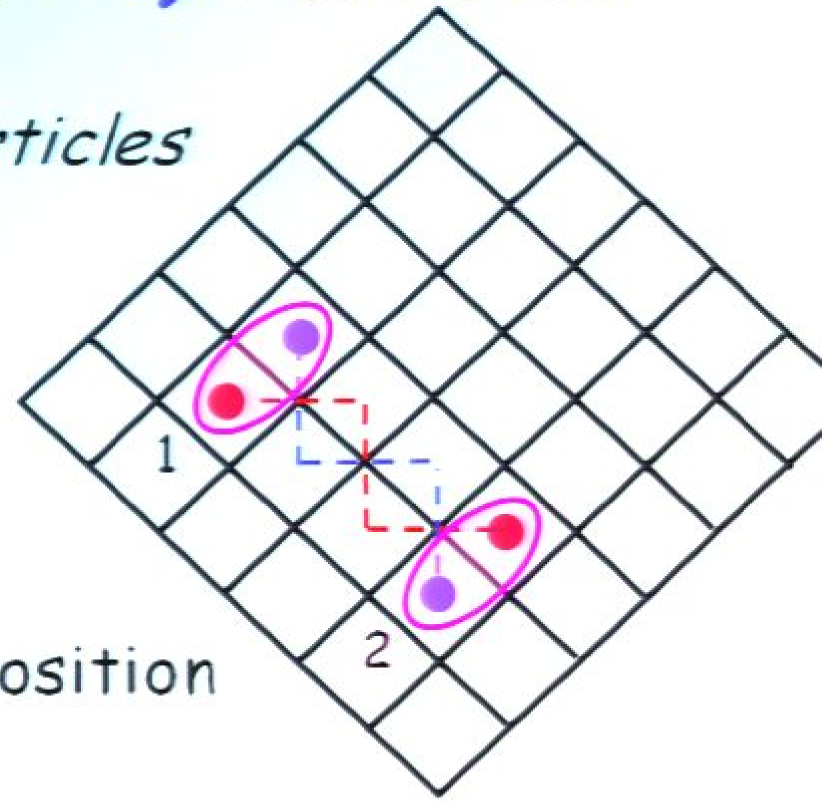
4) Fusion non-commutativity:





Superposition principle: 1) Particles

Obtain a rep. of the Ising model particles from the toric code ones.



- Particles:

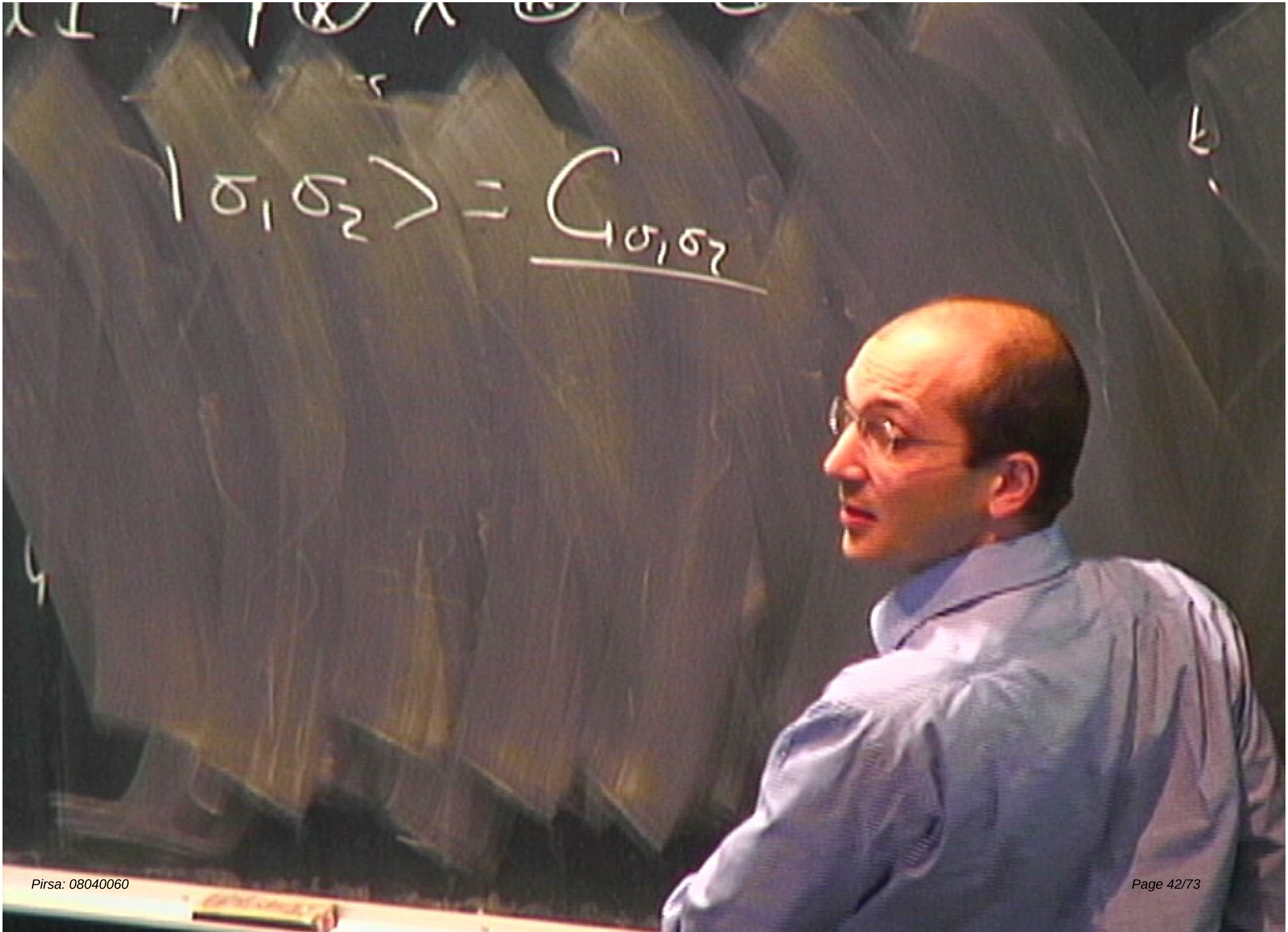
  1, σ (non-abelian), ψ (fermion)

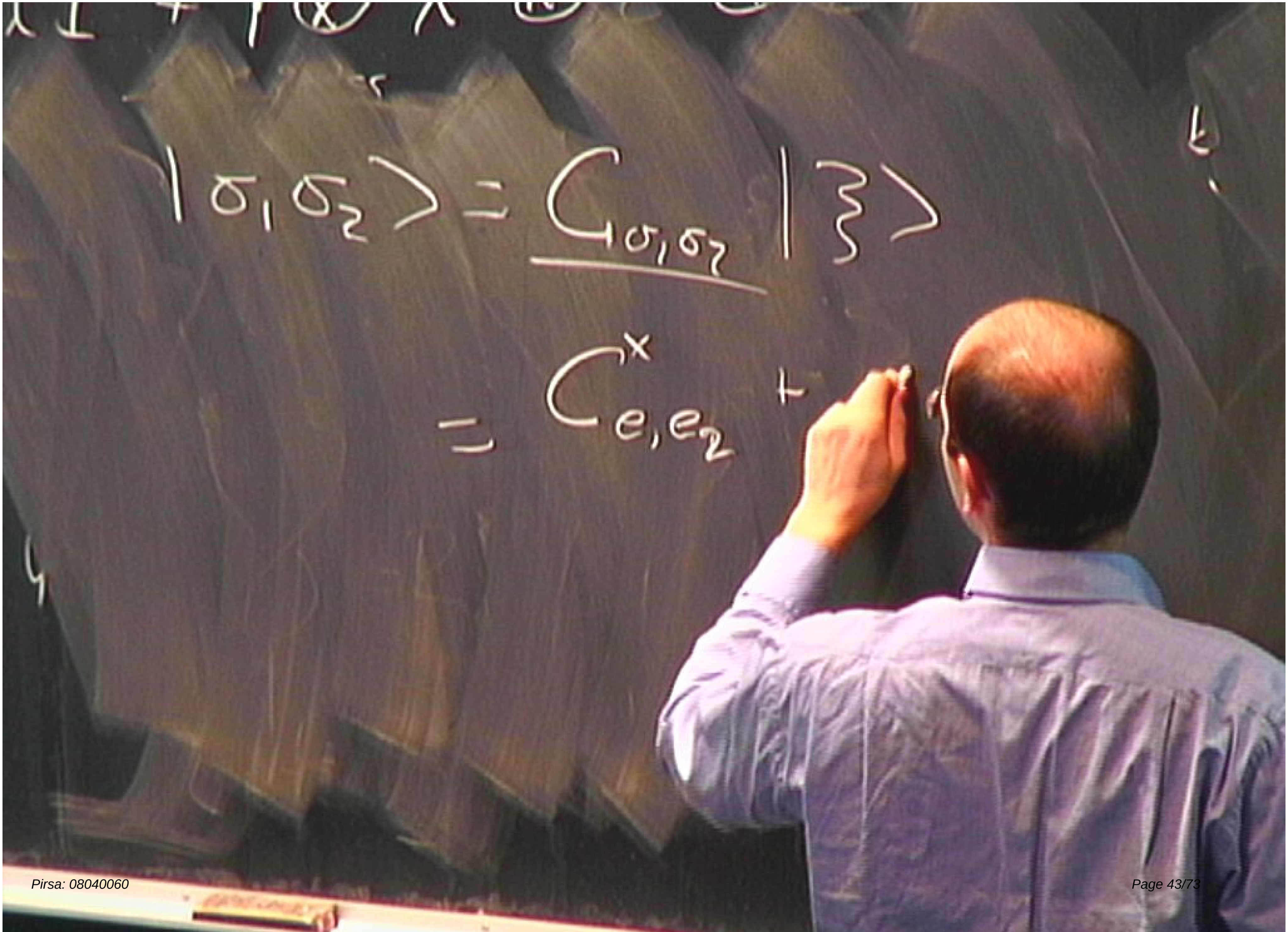
  - Identify the fermions: $\psi \equiv \varepsilon$

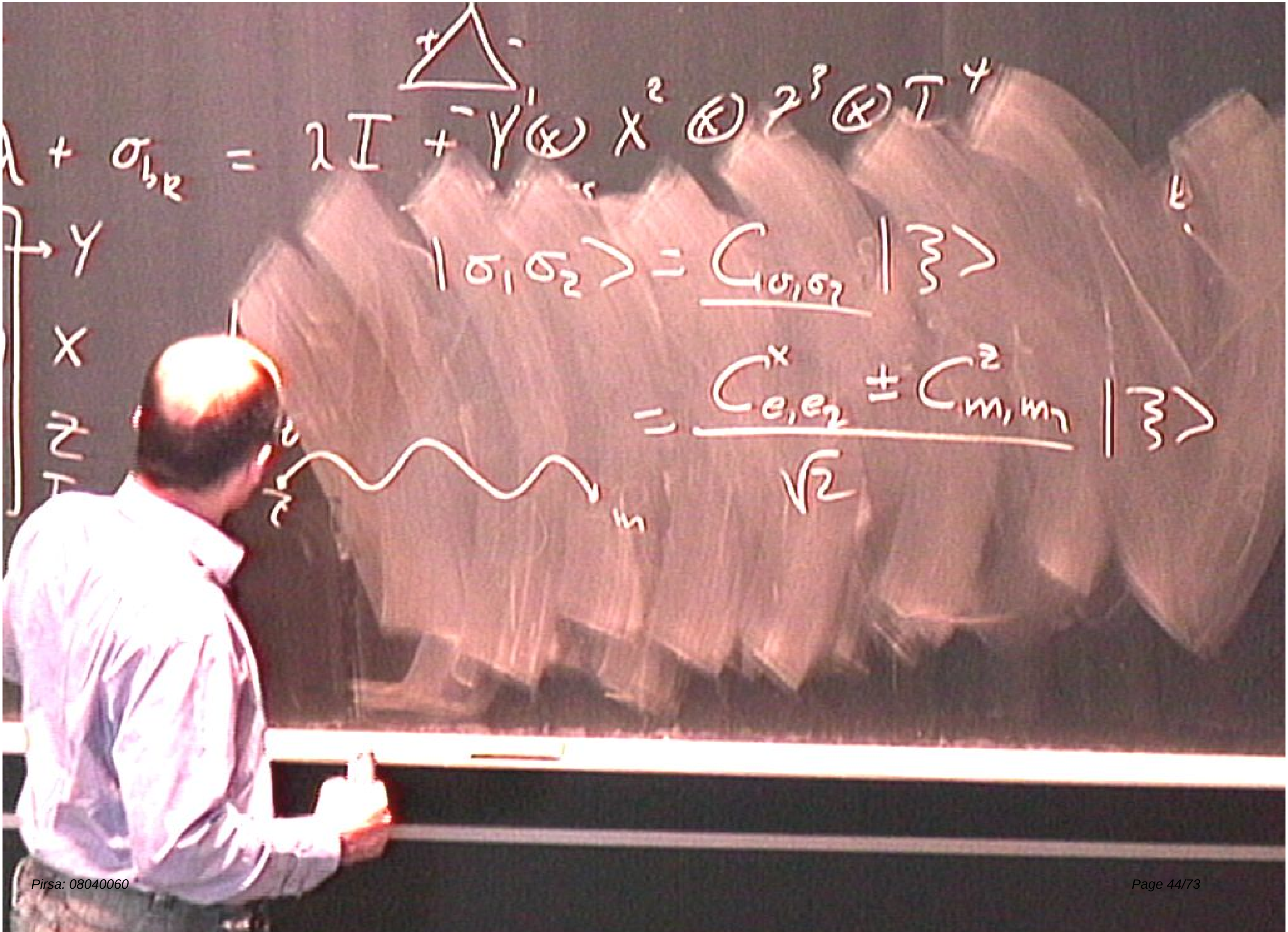
  - Take a **σ string** to be the superposition of two **e and m strings**:

$$|\sigma_1,\sigma_2;\pm\rangle = \frac{1}{\sqrt{2}}(|e_1,e_2\rangle \pm |m_1,m_2\rangle)$$

The $\pm$ sign is a non-local property. The endpoints of these new strings **can be moved with local (controlled) operations** so they can be interpreted as quasiparticles.

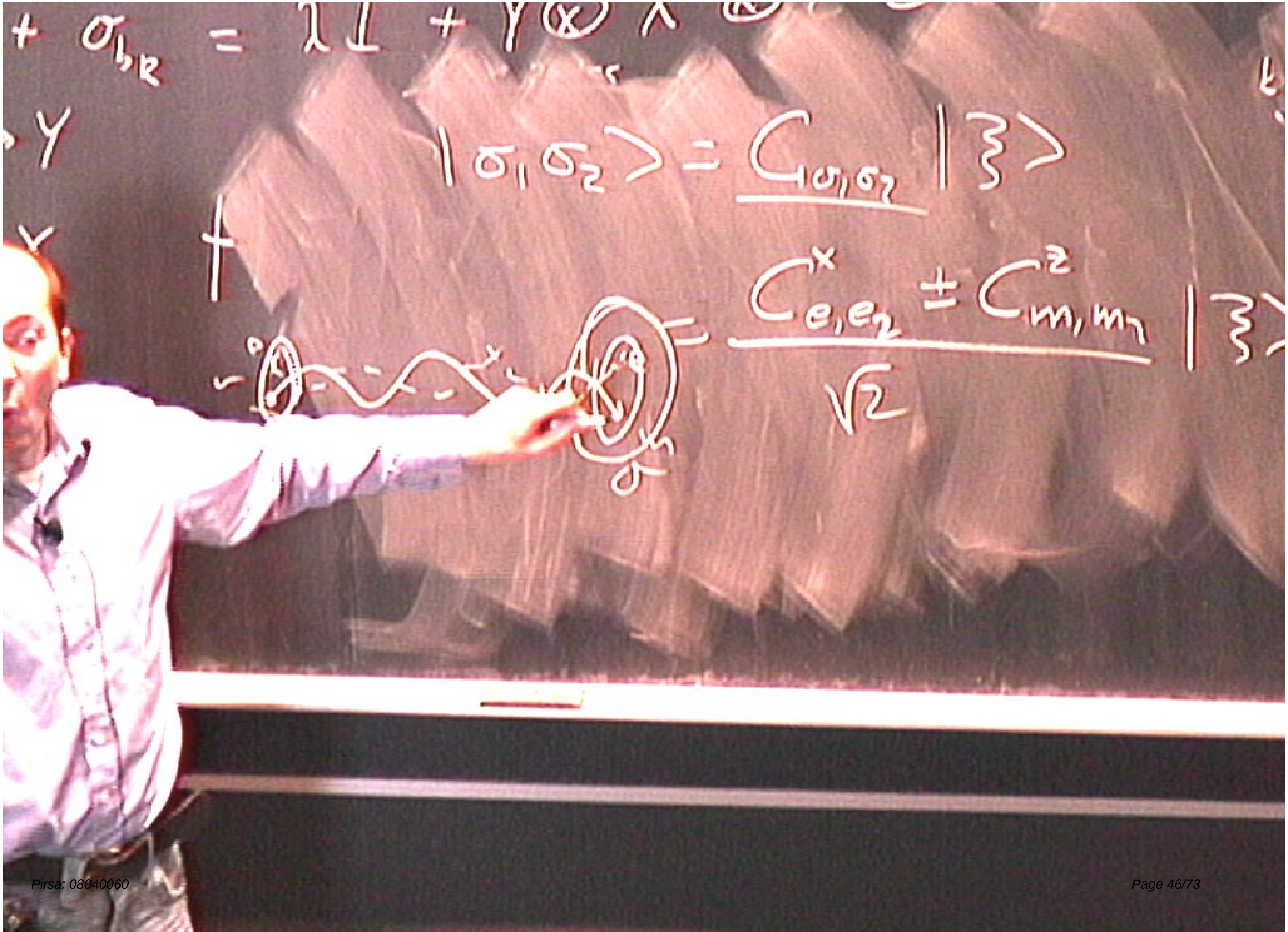

$|\sigma_1 \sigma_2\rangle = C_{\sigma_1 \sigma_2}$


$|\sigma_1, \sigma_2\rangle = C_{\sigma_1 \sigma_2} |\xi\rangle$
$= C^{x}_{e_1 e_2} +$


|σ₁σ₂> = C_{σ₁σ₂} |ξ>
= (C^x_{e₁e₂} ± C^z_{m₁m₂}) / √2 |ξ>
Y
X
Z

$$|\sigma_1 \sigma_2\rangle = \underline{C_{\sigma_1,\sigma_2}}\, |\xi\rangle$$

$$= \frac{C^x_{e_1,e_2} \pm C^z_{m_1,m_1}}{\sqrt{2}}\, |\xi\rangle$$


|σ₁σ₂> = C_{σ₁σ₂} |ξ>
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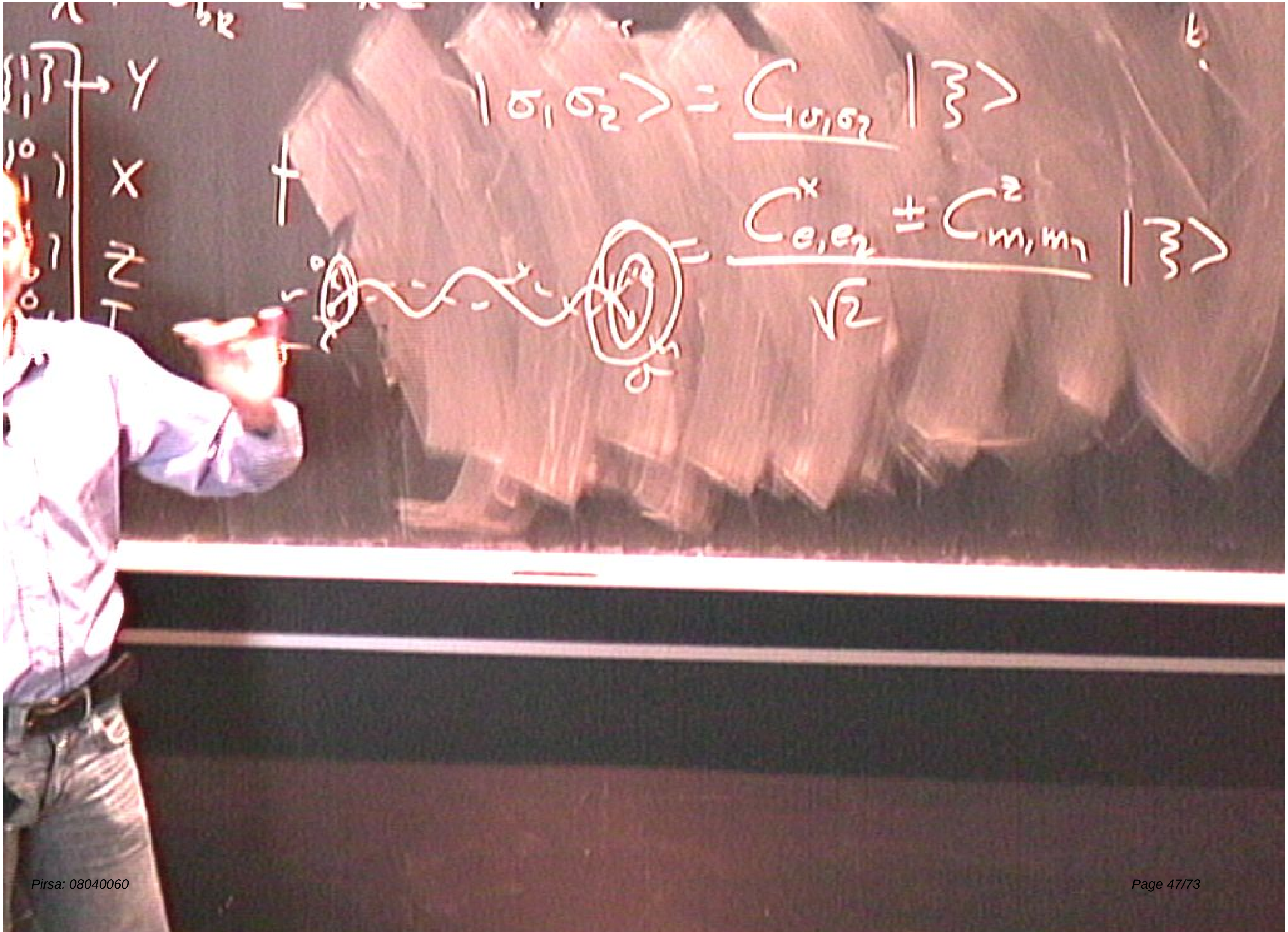
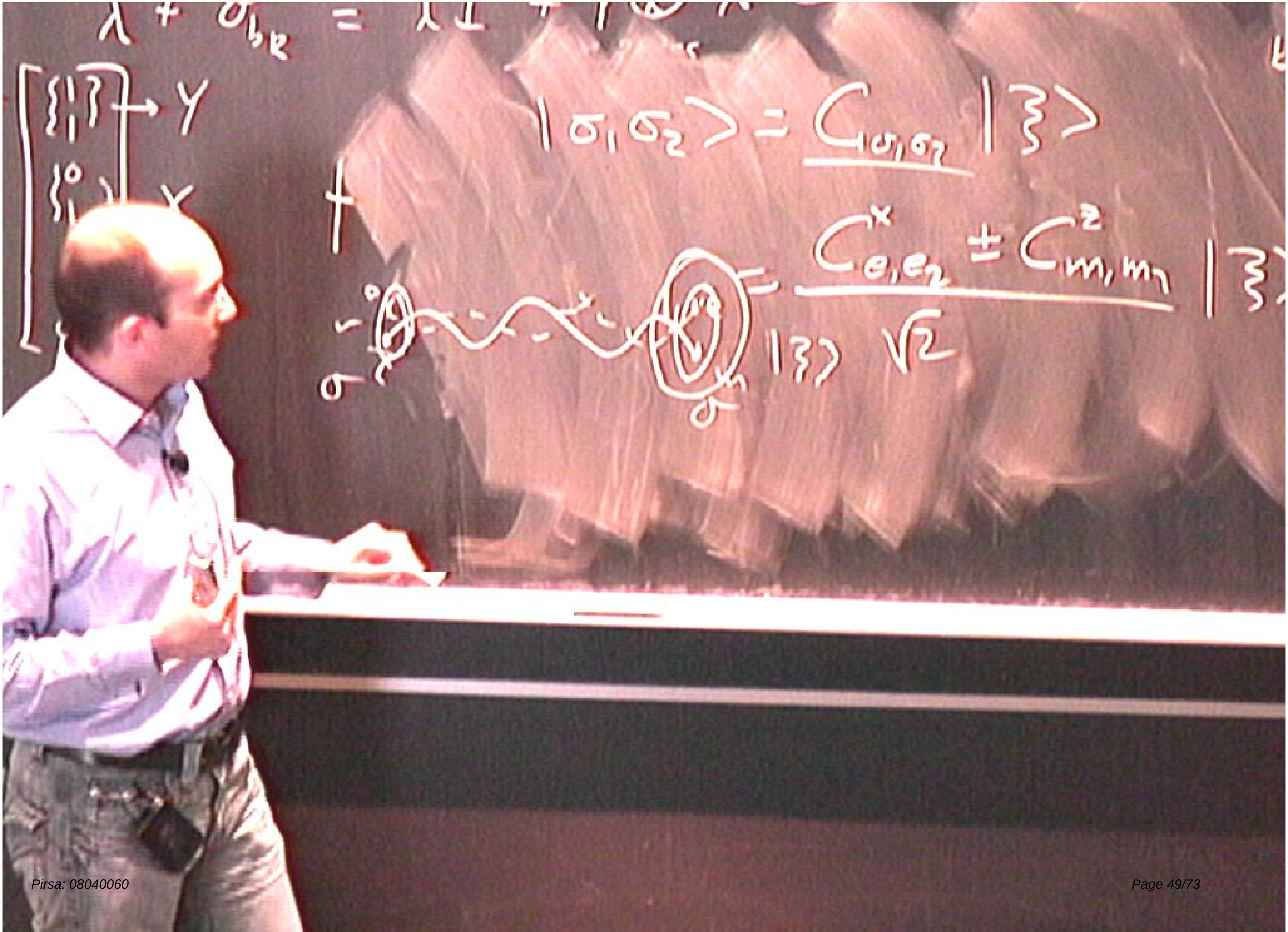
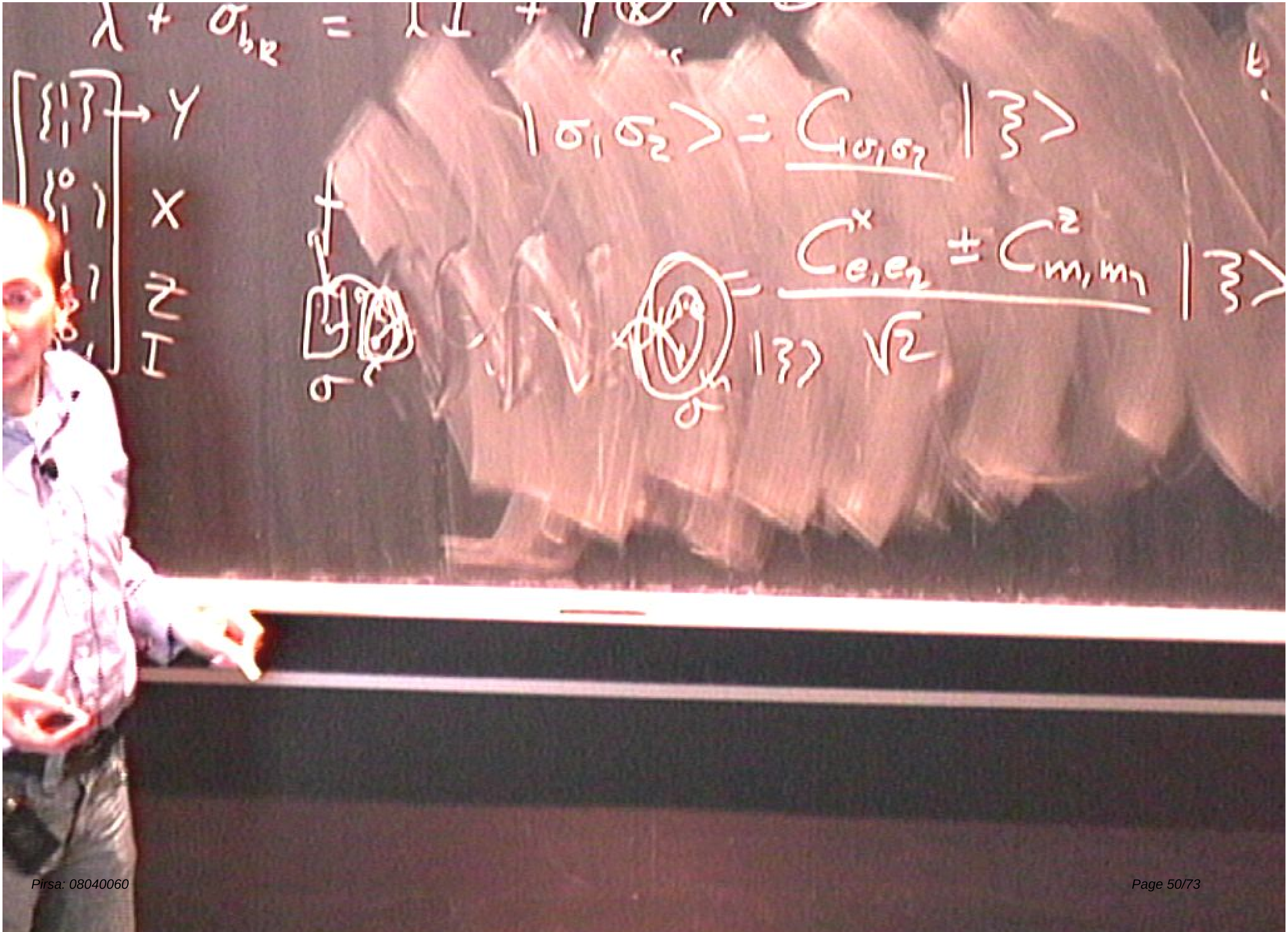

Y
X
Z
|σ1, σ2> = C_{σ1,σ2} |ξ>
= (C^x_{e1,e2} ± C^z_{m1,m2}) / √2 |ξ>

$\sigma_{b,k} = \lambda I + Y \otimes X \otimes \ldots$

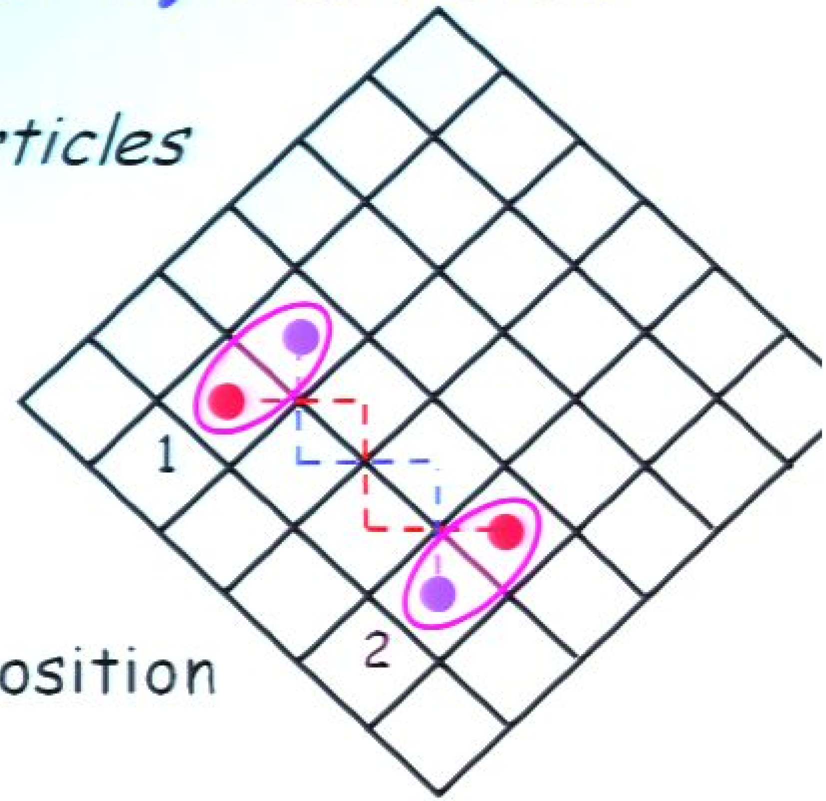
$|\sigma_1 \sigma_2\rangle = \underline{C_{\sigma_1 \sigma_2}}\, |\xi\rangle$

$= \dfrac{C^{x}_{e_1 e_2} \pm C^{z}_{m_1 m_2}}{\sqrt{2}}\, |\xi\rangle$


|σ1 σ2> = Cσ1σ2 |ξ>


Y
X
Z
I
|σ1 σ2> = Cσ1σ2 |ξ>
Cx e1e2 ± Cz m1m2 / √2 |ξ>

Superposition principle: 1) Particles



Obtain a rep. of the Ising model particles from the toric code ones.

- Particles:

  1, σ (non-abelian), ψ (fermion)

  - Identify the fermions: $\psi \equiv \varepsilon$
  - Take a **σ string** to be the superposition of two **e and m strings**:

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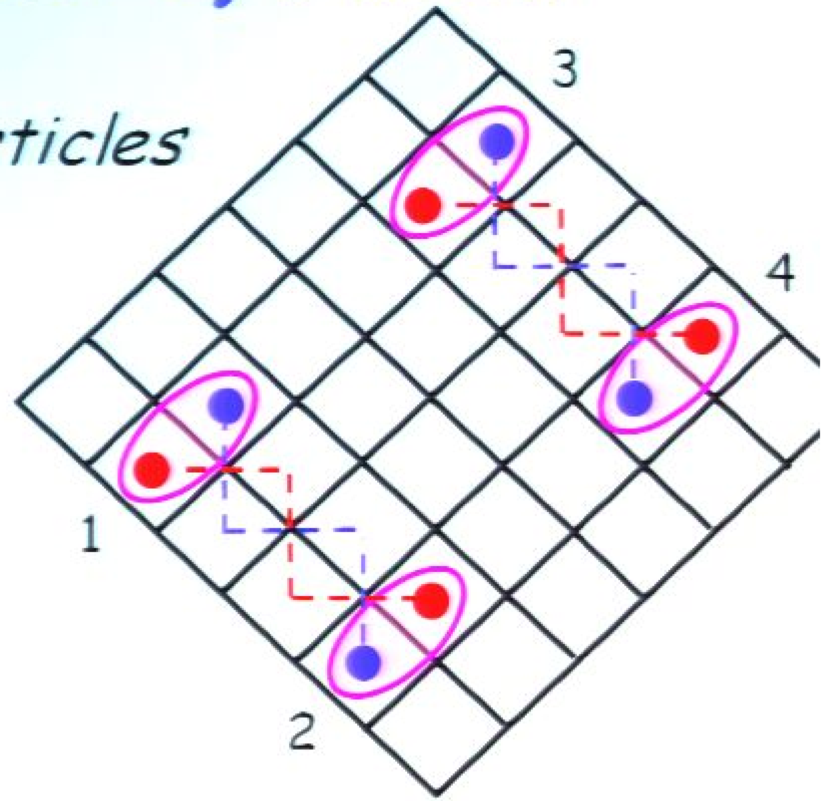
The $\pm$ sign is a non-local property. The endpoints of these new strings **can be moved with local (controlled) operations** so they can be interpreted as quasiparticles.

Superposition principle: 2) Fusion

Obtain a rep. of the Ising model particles from the toric code ones.



- String fusion and

fusion non-commutativity:

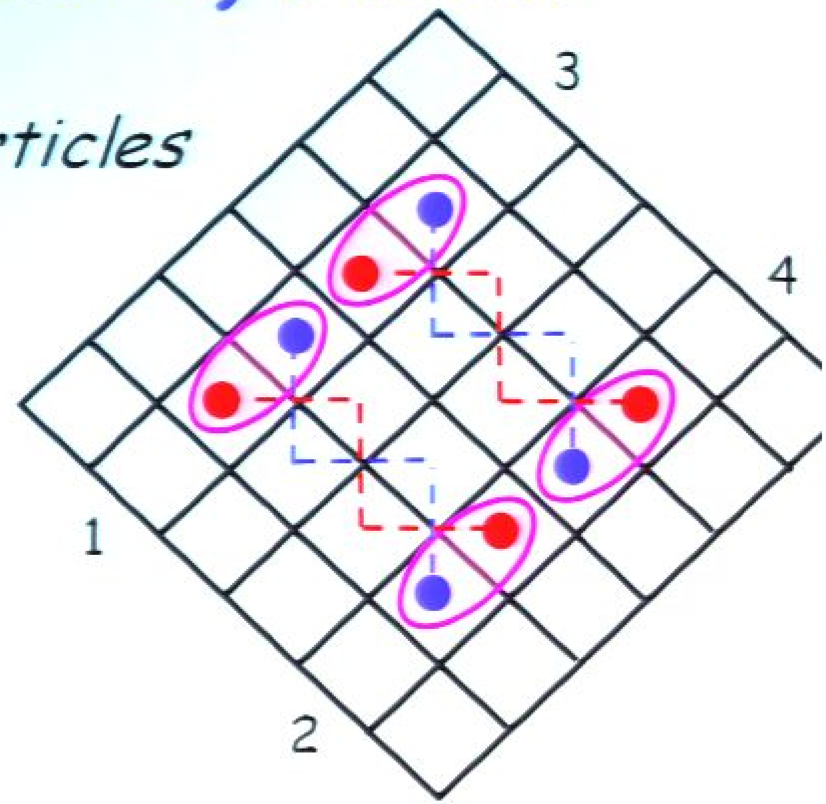
$$\left|(\sigma_1,\sigma_2;j)(\sigma_3,\sigma_4;j)\right\rangle =$$

$$=\frac{1}{\sqrt{2}}\left(\frac{\left|e_1,e_2,e_3,e_4\right\rangle+\left|m_1,m_2,m_3,m_4\right\rangle}{\sqrt{2}}+j\frac{\left|e_1,e_2,m_3,m_4\right\rangle+\left|m_1,m_2,e_3,e_4\right\rangle}{\sqrt{2}}\right)$$

$$\equiv\frac{1}{\sqrt{2}}\left(\left|1_{1,3},1_{2,4}\right\rangle+j\left|\psi_{1,3},\psi_{2,4}\right\rangle\right)$$

=> σ strings with j=+1 fuse to 1 and with j=-1 to ψ

Superposition principle: 2) Fusion



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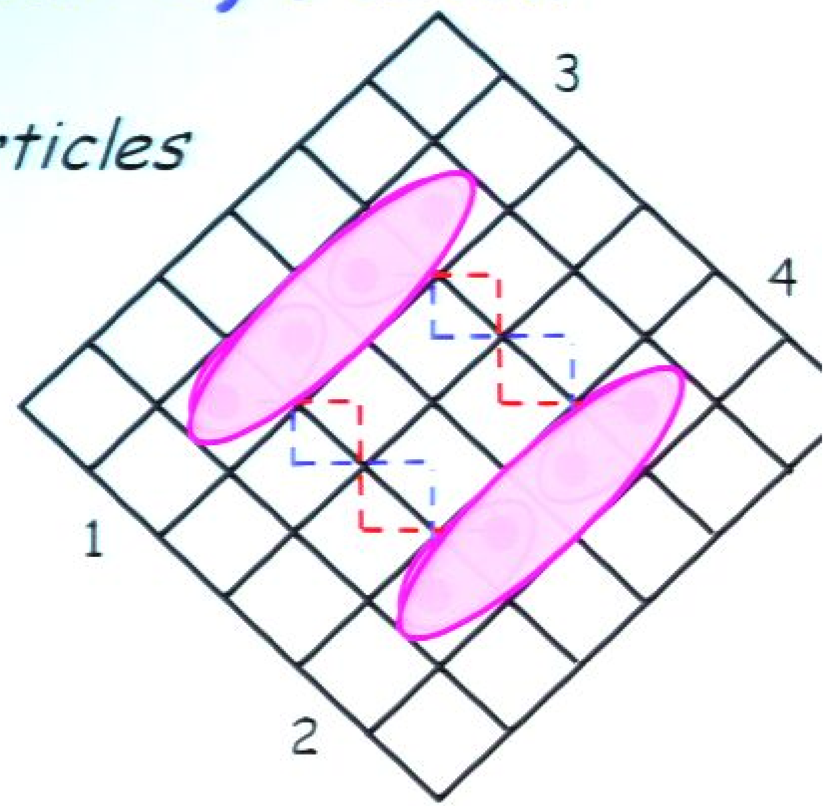

- String fusion and

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$$\left|(\sigma_1,\sigma_2;j)(\sigma_3,\sigma_4;j)\right\rangle =$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\left|e_1,e_2,e_3,e_4\right\rangle + \left|m_1,m_2,m_3,m_4\right\rangle}{\sqrt{2}} + j\frac{\left|e_1,e_2,m_3,m_4\right\rangle + \left|m_1,m_2,e_3,e_4\right\rangle}{\sqrt{2}}\right)$$

$$\equiv \frac{1}{\sqrt{2}}\left(\left|1_{1,3},1_{2,4}\right\rangle + j\left|\psi_{1,3},\psi_{2,4}\right\rangle\right)$$

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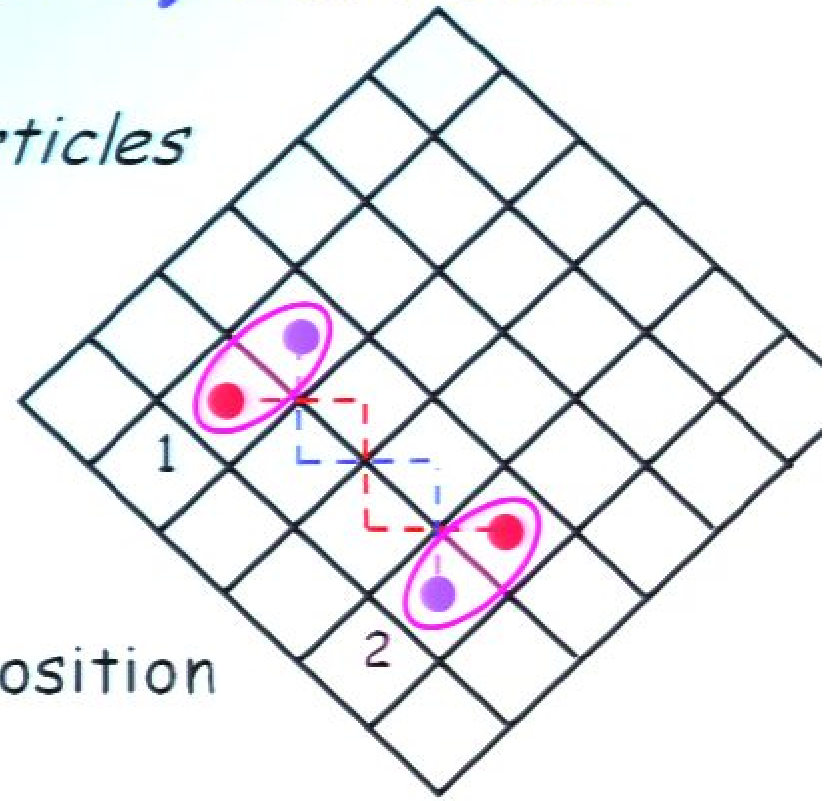
Superposition principle: 1) Particles



Obtain a rep. of the Ising model particles from the toric code ones.

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  1, σ (non-abelian), ψ (fermion)

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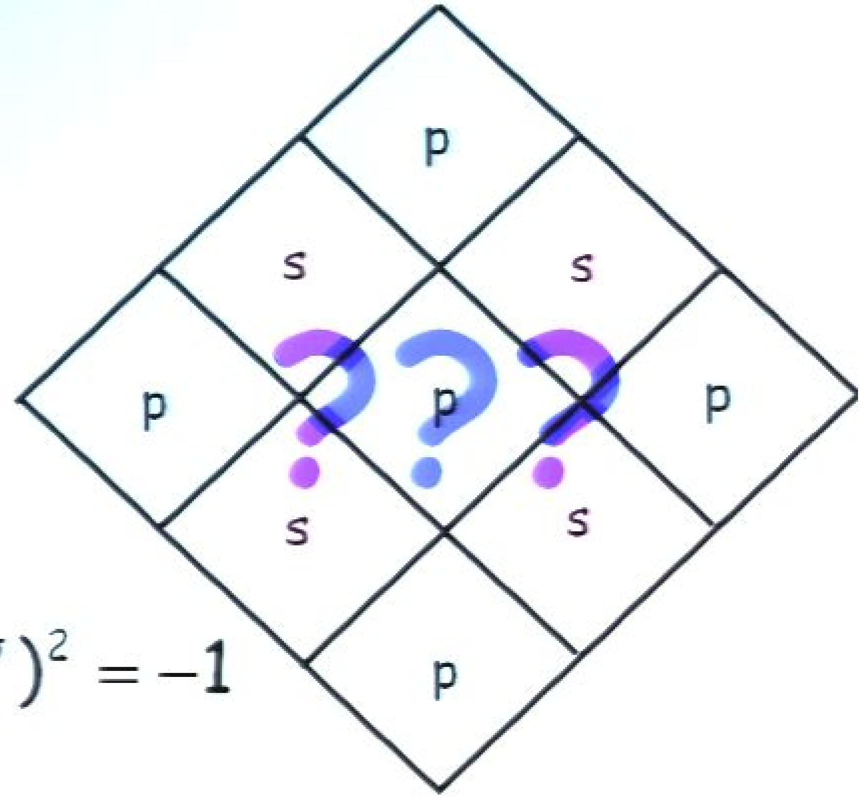
The ± sign is a non-local property. The endpoints of these new strings **can be moved with local (controlled) operations** so they can be interpreted as quasiparticles.

Topological properties of Ising model



1) Particles:

1, σ (non-abelian), ψ (fermion)

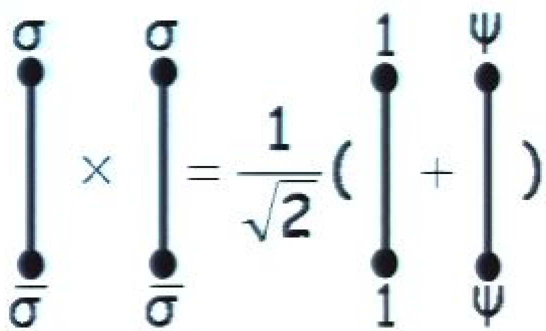
2) Fusion:

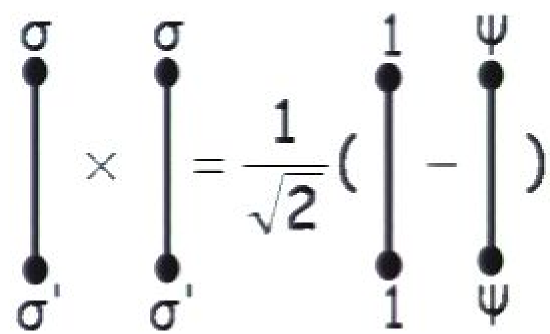
$\sigma\times\sigma=1+\psi$, $\sigma\times\psi=\sigma$, $\psi\times\psi=1$

3) Braiding (up to overall phases):

$$R_1^{\psi\psi}=-1,(R_1^{\sigma\sigma})^2=1,(R_\psi^{\sigma\sigma})^2=-1,(R_\sigma^{\psi\sigma})^2=-1$$

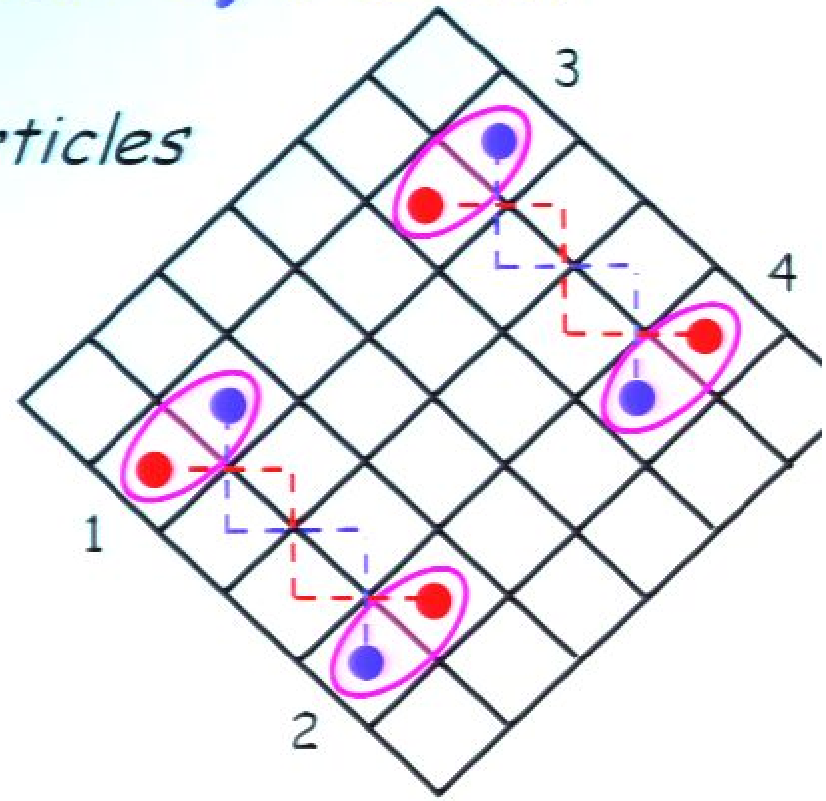
4) Fusion non-commutativity:





Superposition principle: 2) Fusion

Obtain a rep. of the Ising model particles from the toric code ones.



- String fusion and

fusion non-commutativity:

$$\left|(\sigma_1,\sigma_2;j)(\sigma_3,\sigma_4;j)\right\rangle =$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\left|e_1,e_2,e_3,e_4\right\rangle + \left|m_1,m_2,m_3,m_4\right\rangle}{\sqrt{2}} + j\,\frac{\left|e_1,e_2,m_3,m_4\right\rangle + \left|m_1,m_2,e_3,e_4\right\rangle}{\sqrt{2}}\right)$$

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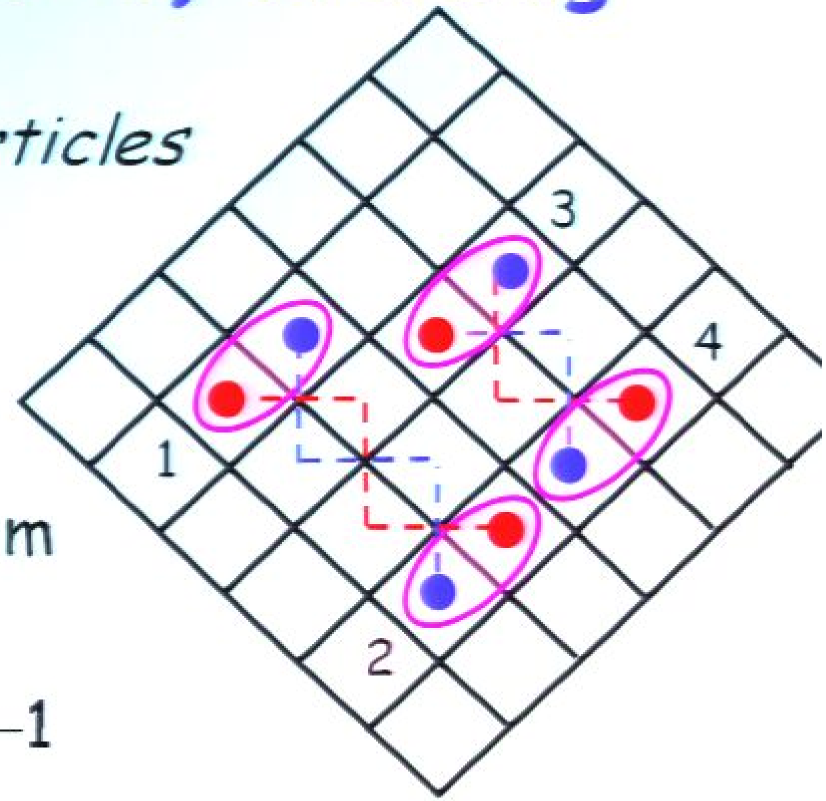
=> σ strings with j=+1 fuse to 1 and with j=-1 to ψ

Superposition principle: 3) Braiding

Obtain a rep. of the Ising model particles from the toric code ones.

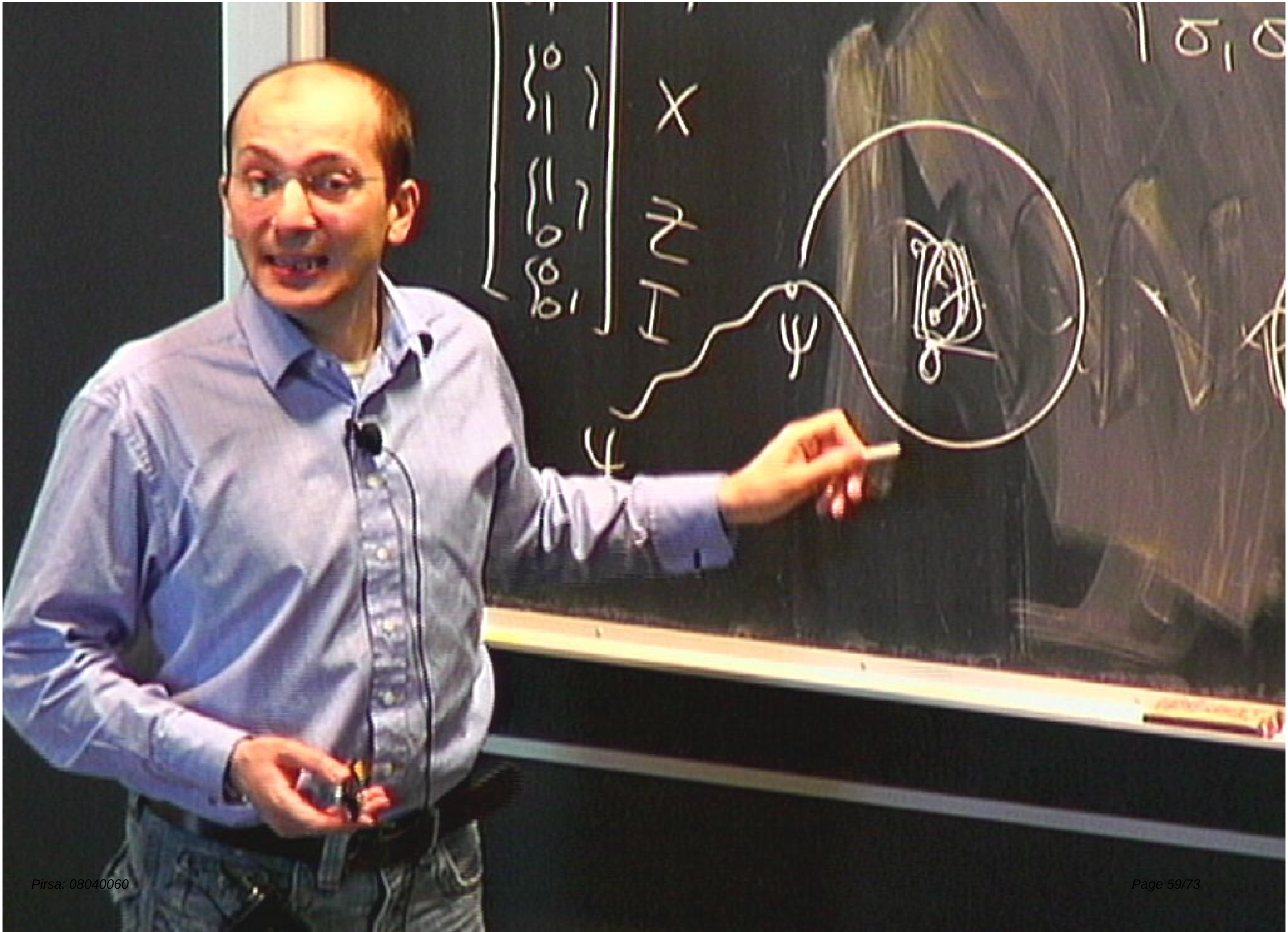
- Braiding:
  - **ψ around ψ:** $R_1^{\psi\psi} = -1$
  - **ψ around σ:** σ consists of e or m $(R_\sigma^{\psi\sigma})^2 = -1$
  - **σ around σ:** $(R_1^{\sigma\sigma})^2 = 1, (R_\psi^{\sigma\sigma})^2 = -1$

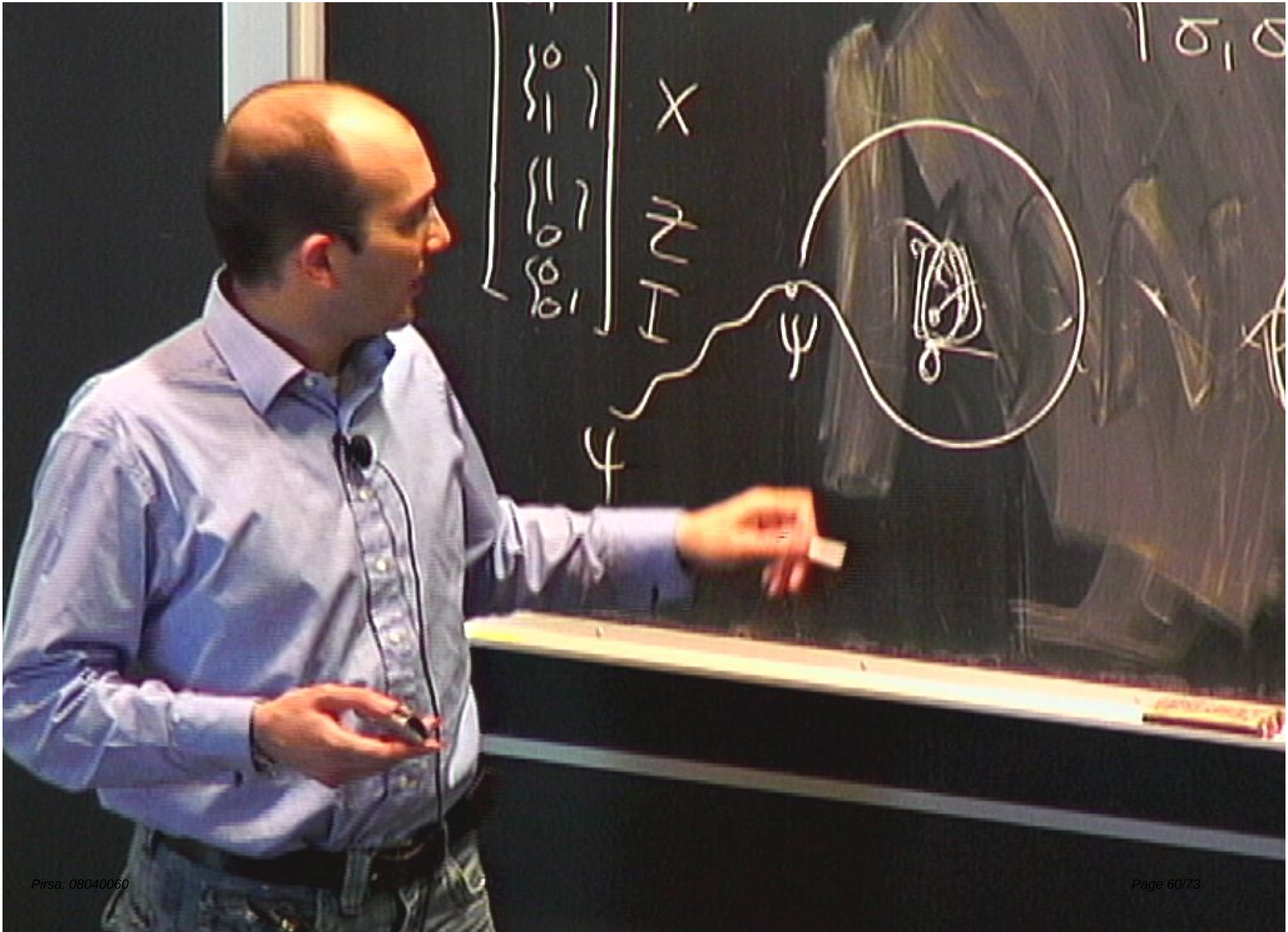


$$\left|(\sigma_1,\sigma_2;j)(\sigma_3,\sigma_4;j)\right\rangle =$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\left|e_1,e_2,e_3,e_4\right\rangle + \left|m_1,m_2,m_3,m_4\right\rangle}{\sqrt{2}} + j\frac{\left|e_1,e_2,m_3,m_4\right\rangle + \left|m_1,m_2,e_3,e_4\right\rangle}{\sqrt{2}}\right.$$

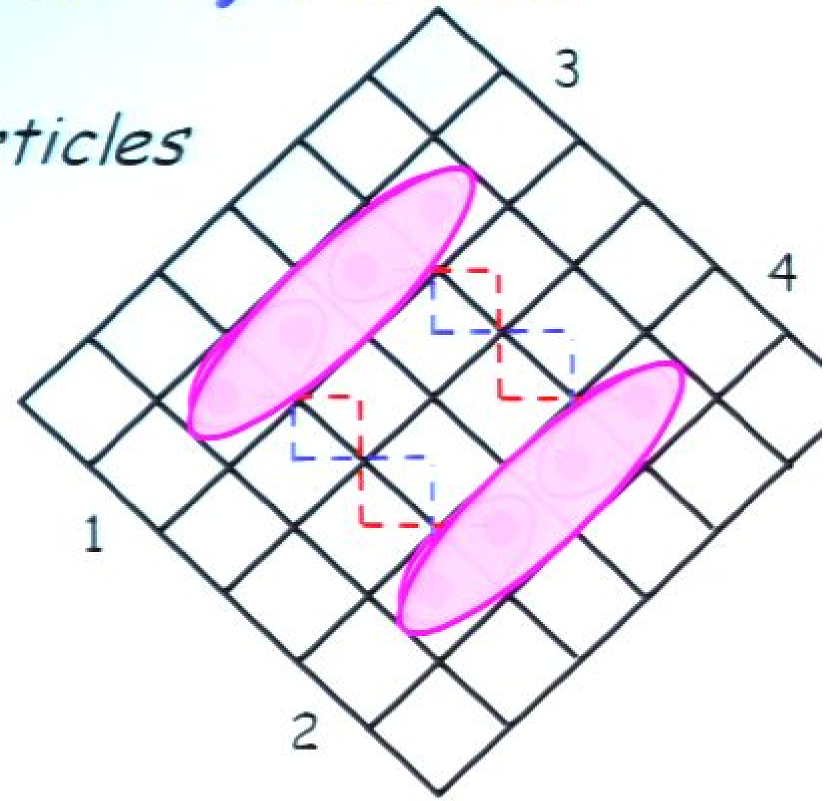
$$R^{\sigma_1\sigma_3}\ \frac{1}{\sqrt{2}}\left(\frac{\left|e_1,e_2,e_3,e_4\right\rangle + \left|m_1,m_2,m_3,m_4\right\rangle}{\sqrt{2}} - j\frac{\left|e_1,e_2,m_3,m_4\right\rangle + \left|m_1,m_2,e_3,e_4\right\rangle}{\sqrt{2}}\right.$$





Superposition principle: 2) Fusion

Obtain a rep. of the Ising model particles from the toric code ones.



- String fusion and

fusion non-commutativity:

$$\left|(\sigma_1,\sigma_2;j)(\sigma_3,\sigma_4;j)\right\rangle =$$

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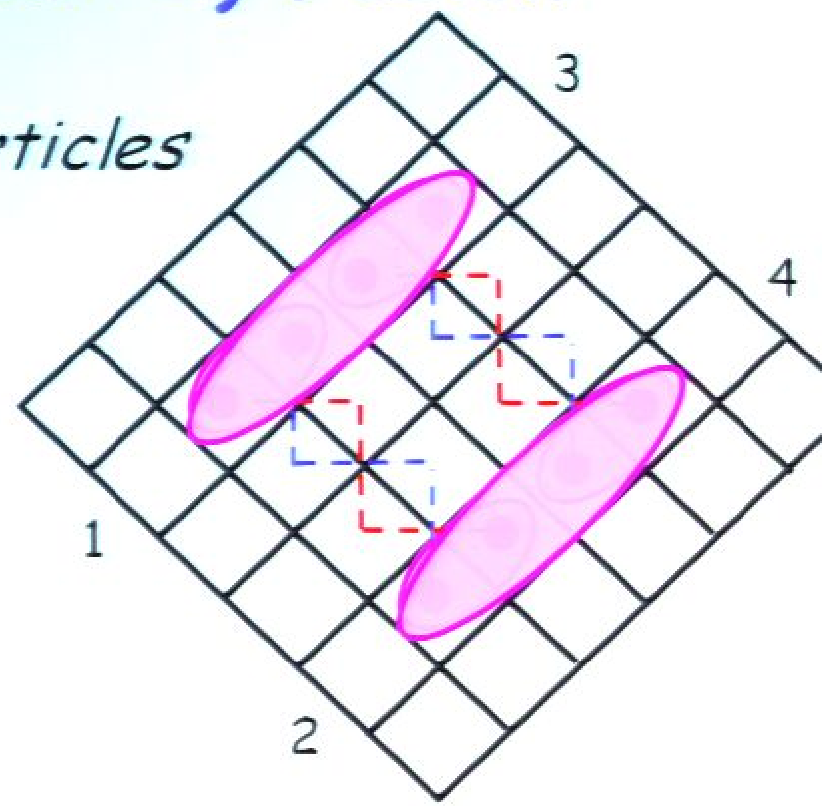
=> σ strings with j=+1 fuse to 1 and with j=-1 to ψ

Superposition principle: 2) Fusion

Obtain a rep. of the Ising model particles from the toric code ones.



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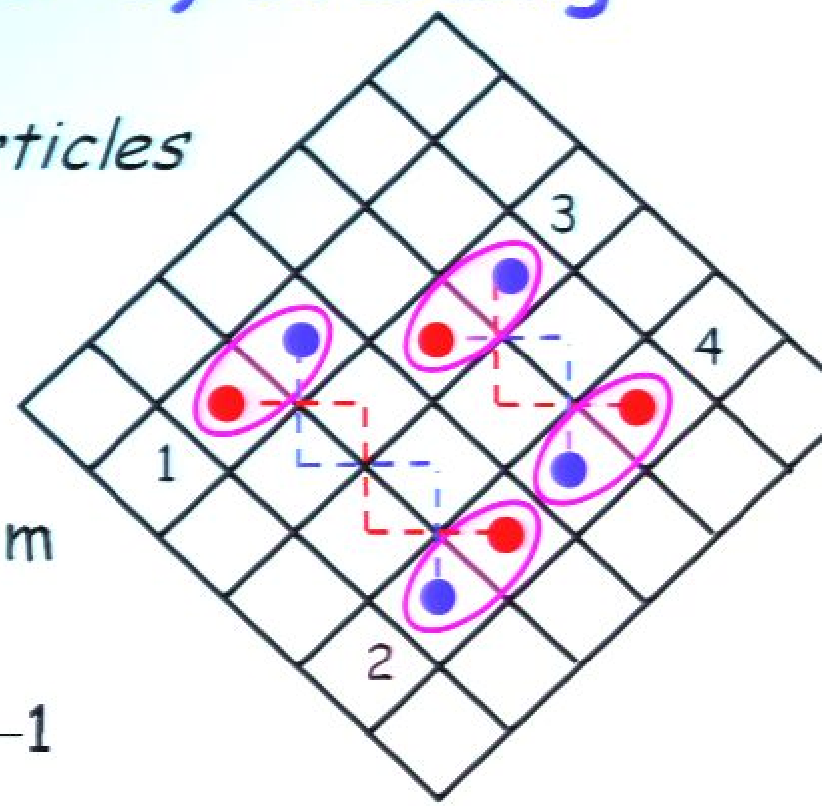
Superposition principle: 3) Braiding

Obtain a rep. of the Ising model particles from the toric code ones.

- Braiding:
  - **ψ around ψ:** $R_1^{\psi\psi} = -1$
  - **ψ around σ:** σ consists of e or m
    $(R_\sigma^{\psi\sigma})^2 = -1$
  - **σ around σ:** $(R_1^{\sigma\sigma})^2 = 1, (R_\psi^{\sigma\sigma})^2 = -1$



$$\left|(\sigma_1,\sigma_2;j)(\sigma_3,\sigma_4;j)\right\rangle =$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\left|e_1,e_2,e_3,e_4\right\rangle + \left|m_1,m_2,m_3,m_4\right\rangle}{\sqrt{2}} + j\frac{\left|e_1,e_2,m_3,m_4\right\rangle + \left|m_1,m_2,e_3,e_4\right\rangle}{\sqrt{2}}\right.$$

$$R^{\sigma_1\sigma_3}\ \frac{1}{\sqrt{2}}\left(\frac{\left|e_1,e_2,e_3,e_4\right\rangle + \left|m_1,m_2,m_3,m_4\right\rangle}{\sqrt{2}} - j\frac{\left|e_1,e_2,m_3,m_4\right\rangle + \left|m_1,m_2,e_3,e_4\right\rangle}{\sqrt{2}}\right.$$

Superposition principle: 3) Braiding

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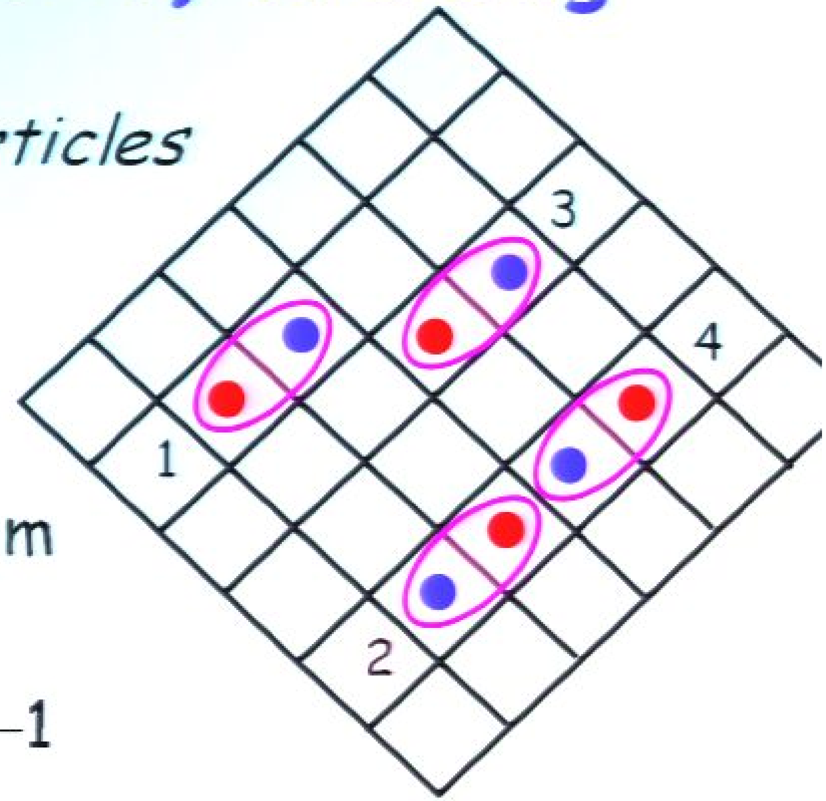

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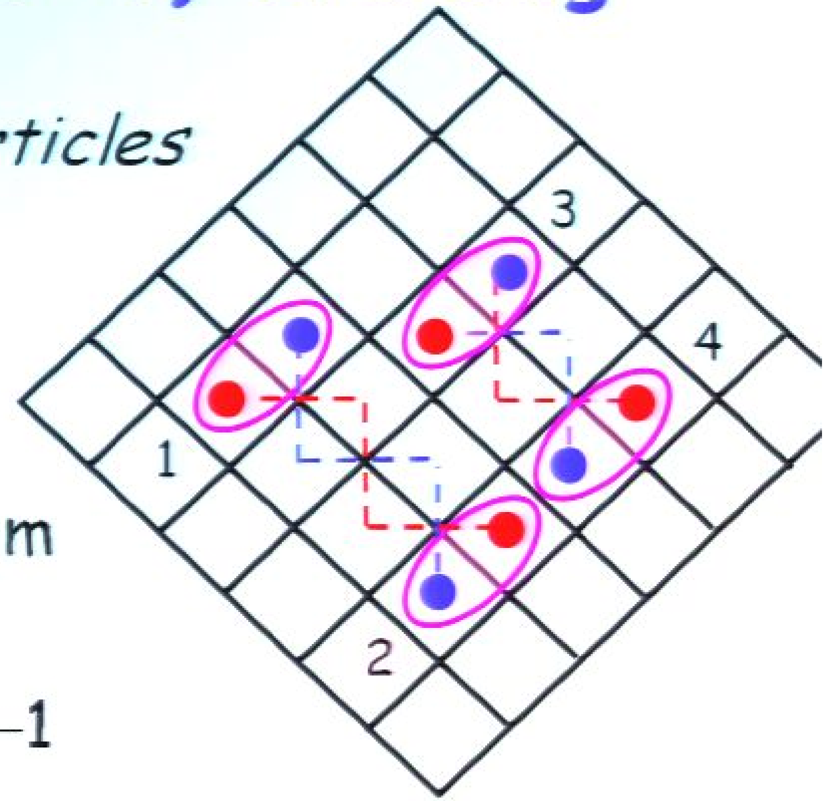

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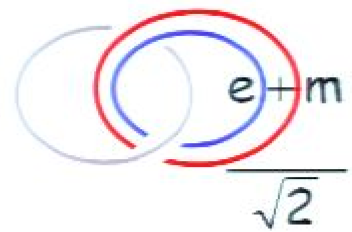
Conclusions

- It is possible to relate their corresponding S-matrices by the superposition principle:

1  $\sigma \;\; \equiv S_{1\sigma}^{\mathrm{Ising}} = \frac{S_{1e}^{t.c.} + S_{1m}^{t.c.}}{\sqrt{2}} \equiv \;\; 1$ 

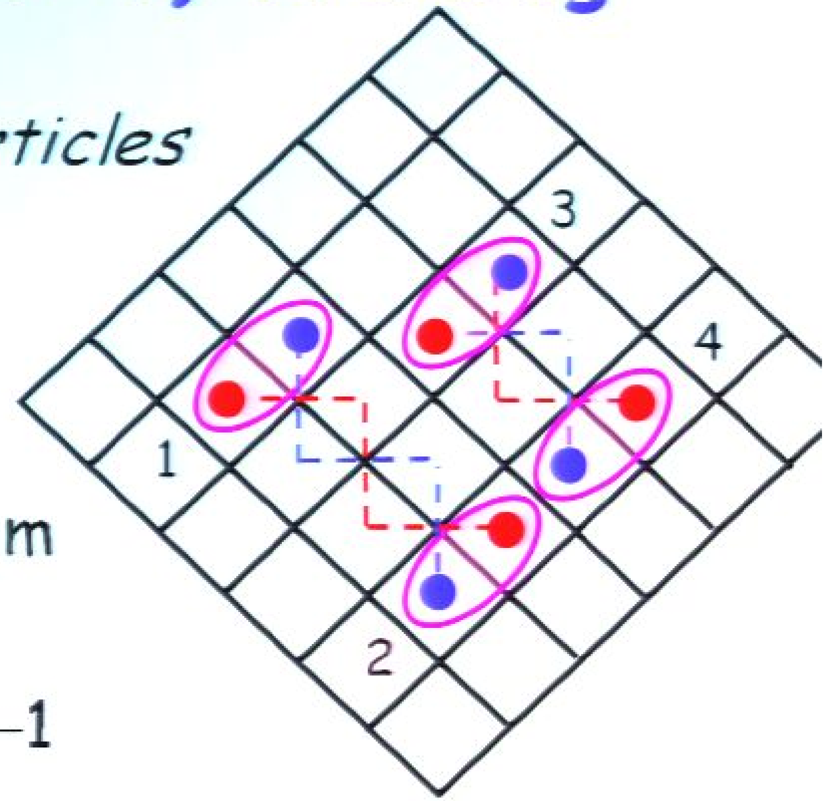
$= \sqrt{2}$

- **Clifford group** representation of non-abelian Anyons.
- Implement **chirality** with additional ancilla and controlled operations to implement overall phase factors.

- These models have the **same Quantum Dimension** $D^2 = 4$
- Relate other topological models with equal QD.

Superposition principle: 3) Braiding

Obtain a rep. of the Ising model particles from the toric code ones.



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    $$(R_\sigma^{\psi\sigma})^2 = -1$$
  - **σ around σ:** $(R_1^{\sigma\sigma})^2 = 1, (R_\psi^{\sigma\sigma})^2 = -1$

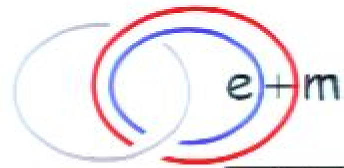
$$\left|(\sigma_1,\sigma_2;j)(\sigma_3,\sigma_4;j)\right\rangle =$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\left|e_1,e_2,e_3,e_4\right\rangle + \left|m_1,m_2,m_3,m_4\right\rangle}{\sqrt{2}} + j\,\frac{\left|e_1,e_2,m_3,m_4\right\rangle + \left|m_1,m_2,e_3,e_4\right\rangle}{\sqrt{2}}\right)$$

$$R^{\sigma_1\sigma_3}\ \frac{1}{\sqrt{2}}\left(\frac{\left|e_1,e_2,e_3,e_4\right\rangle + \left|m_1,m_2,m_3,m_4\right\rangle}{\sqrt{2}} - j\,\frac{\left|e_1,e_2,m_3,m_4\right\rangle + \left|m_1,m_2,e_3,e_4\right\rangle}{\sqrt{2}}\right)$$

Conclusions

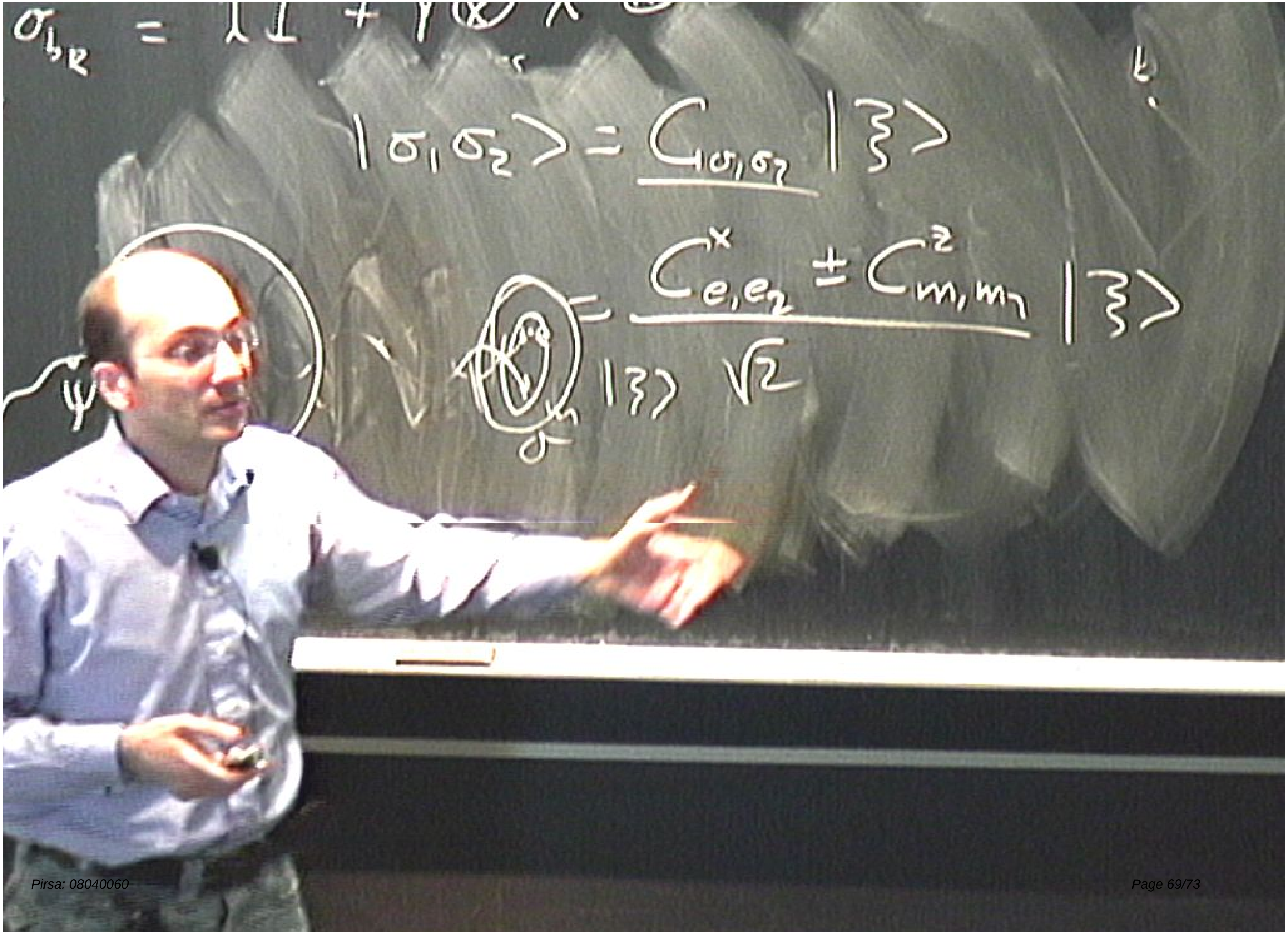
- It is possible to relate their corresponding S-matrices by the superposition principle:

1  σ $= S^{Ising}_{1\sigma} = \dfrac{S^{t.c.}_{1e} + S^{t.c.}_{1m}}{\sqrt{2}} \equiv$ 1 

$\overline{\sqrt{2}}$ $= \sqrt{2}$

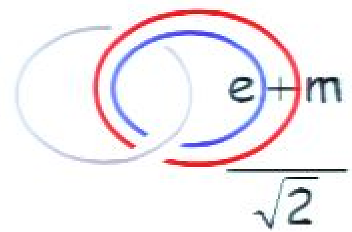
- **Clifford group** representation of non-abelian Anyons.
- Implement **chirality** with additional ancilla and controlled operations to implement overall phase factors.

- These models have the **same Quantum Dimension** $D^2 = 4$
- Relate other topological models with equal QD.


$|\sigma_1 \sigma_2\rangle = C_{\sigma_1 \sigma_2} |\xi\rangle$
$= \frac{C^x_{e_1 e_2} \pm C^z_{m_1 m_2}}{\sqrt{2}} |\xi\rangle$
$|\xi\rangle$

Conclusions

- It is possible to relate their corresponding S-matrices by the superposition principle:

1  σ $\equiv S_{1\sigma}^{Ising} = \frac{S_{1e}^{t.c.} + S_{1m}^{t.c.}}{\sqrt{2}} \equiv$ 1  $= \sqrt{2}$

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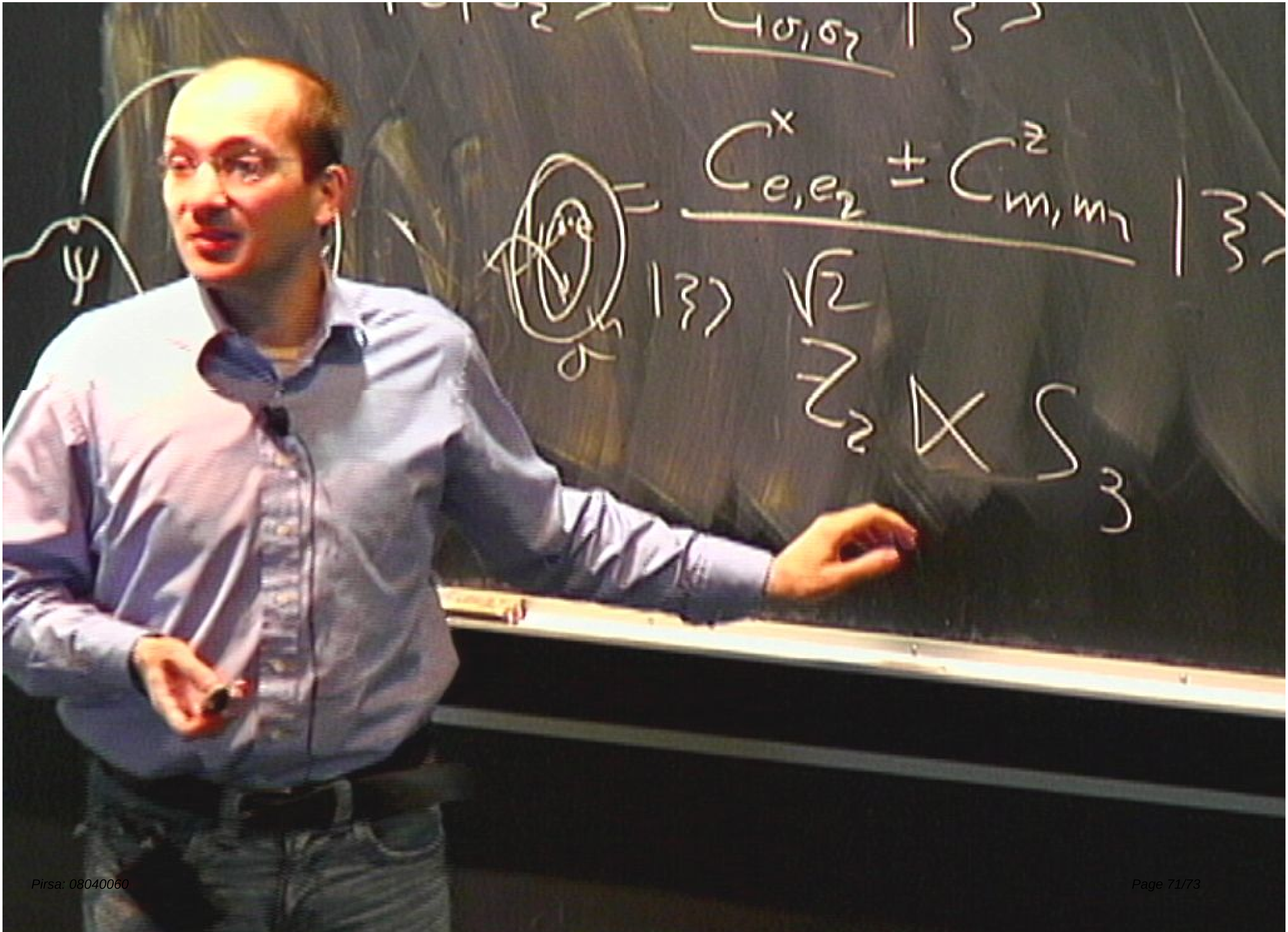
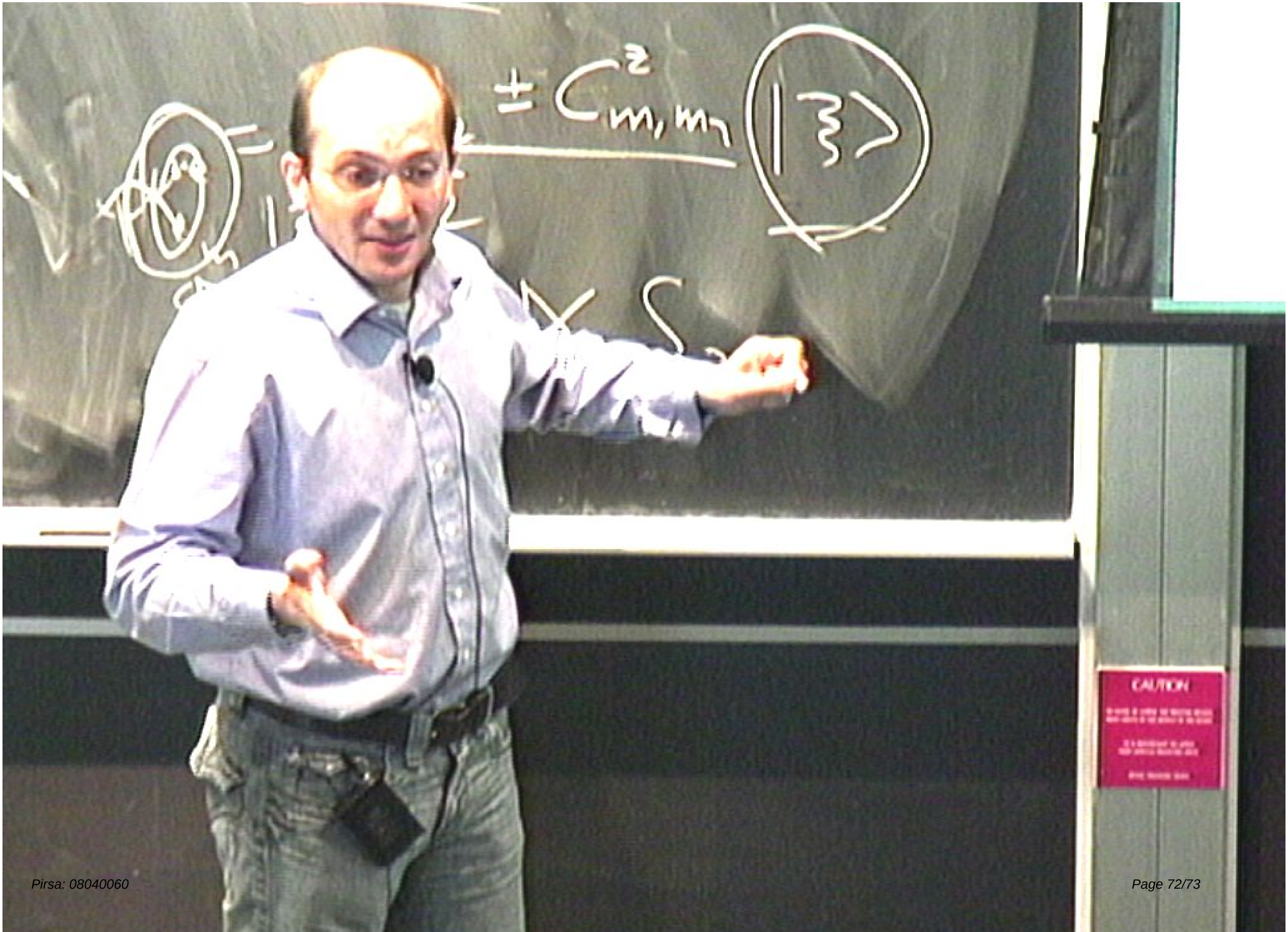
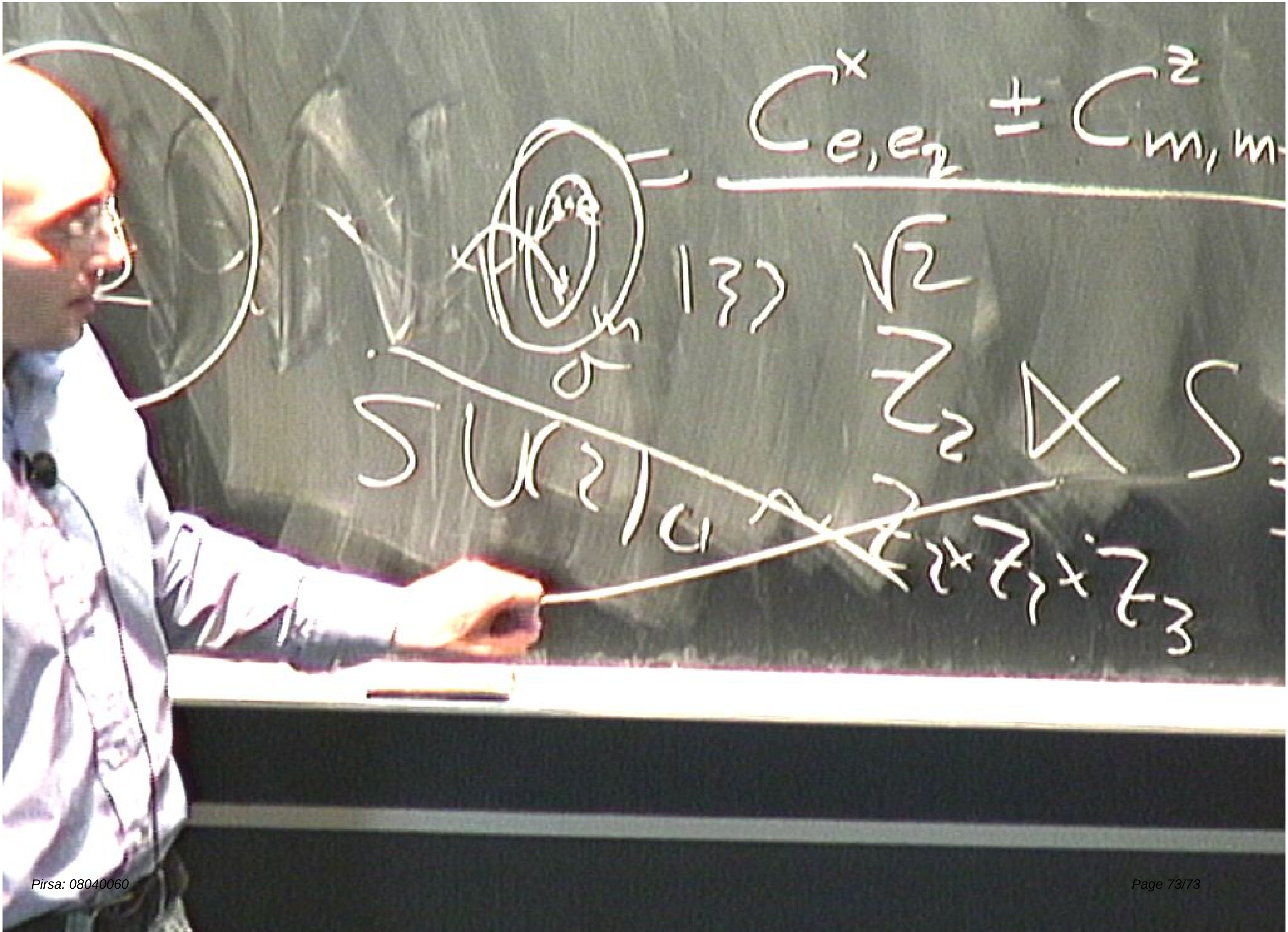
Thank you for your attention!



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