Title: Exotic Phases and Entanglement Properties of Condensed Matter Systems Living on Graphs

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Abstract: New and exotic phases as well as remarkable entanglement behaviors emerge in condensed matter systems (and quantum devices) living (fabricated) on graphs. To illustrate this, I will discuss the properties of Josephson junction networks fabricated on comb and star graphs and of spin models living on pertinent fiber-graphs.

Exotic Phases and Entanglement Properties of C.M. Systems on Graphs

(i.e. Inhomogeneneity Effects on Networks of Josephson Junctions and Spin models)

Pasquale Sodano
(Perugia)

PI, 29 April 2008

Inhomogeneous network = non-translationally invariant network

Inhomogeneity due to topology (= how the lattice sites are connected) and/or to external fields

- **Long term goal**

To induce desired macroscopic coherent behaviors by fabricating devices on pertinent networks.

For this purpose evidence:

Effects induced by the topology (i.e.: not observable on a regular lattice) on bosonic systems: ultracold bosonic gases and Josephson networks

Rather new area:

A. Kitaev, quant-ph/9707021

P. Sodano *et al.*, *EPL* **52**, 251 (2000)

L. B. Ioffe *et al.*, *Nature* **415**, 503 (2002)

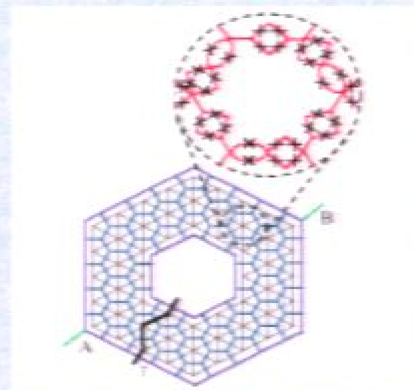
B. Doucout *et al.*, *Phys. Rev. Lett.* **90**, 107003 (2003)

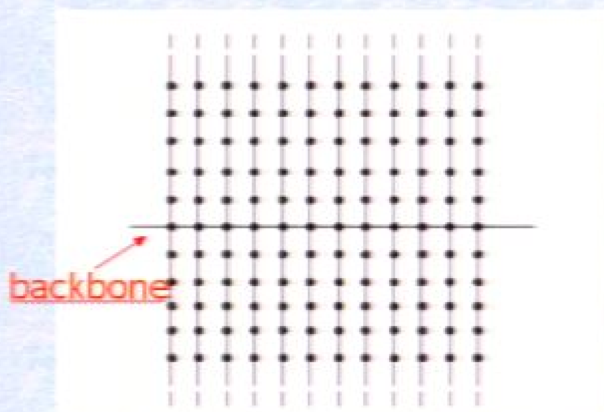
P. Sodano *et al.*, *New J. Phys.* **8**, 327 (2006)

Examples of inhomogeneity effects (I)

Topological Order in JJNs

[P.Sodano et al, Nucl. Phys. B474, 641 (1996);L.B. Ioffe and M.V. Feigel'man, *PRB* **66**, 224503 (2002); B. Doucot, M.V. Feigel'man, and L.B. Ioffe, *PRL* **90**, 107003 (2003); P. Sodano et al. Eur. Phys. JB53,19 (2006)]





Free bosons undergo Bose-Einstein condensation: they condense on the comb's backbone

[P. Sodano et al., *EPL* **52**, 251 (2000)]

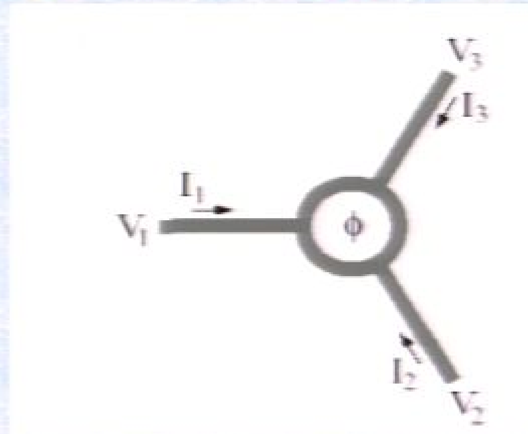
Comb-Shaped JJNs: increase of the Josephson critical current along the backbone

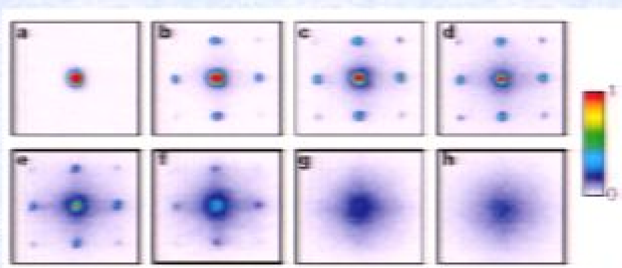
[G. Giusiano, F.P. Mancini, P. Sodano and A. Trombettoni, *Int. J. Mod. Phys. B* **18**, 691 (2004); P. Sodano et al., *New J. Phys.* **8**, 327 (2006)]

Examples of inhomogeneity effects (II)

Junction of three wires: new fixed points

[M. Oshikawa, C. Chamon, and I. Affleck, *PRL* **91**, 206403 (2003); *J. Stat. Mech.* **0602**, P008 (2006)]; D. Giuliano, P. Sodano, 0710.5554

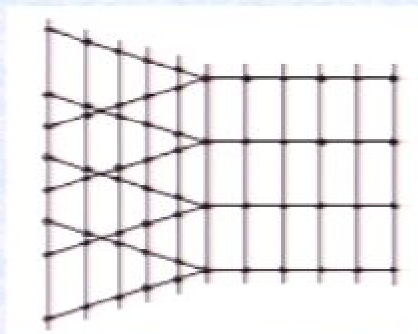




"Wedding cake" of Mott domains surrounded by superfluid regions for bosons in a lattice + a magnetic trap

[M. Greiner et al., *Nature* **415**, 39 (2002) – Batrouni et al., *PRL* **89**, 117203 (2003)]

Examples of inhomogeneity effects (III)



Critical behaviour at the junction of spin networks: local magnetization on the "backbone"

[R. Marchetti, M. Rasetti, P. Sodano and A. Trombettoni, in progress]

Plan of talk:

1) An experimentally accessible system:
- Superconducting Josephson junctions on inhomogeneous insulating supports

2) Spectral and Thermodynamical properties:
- Spectrum of quantum particles on simple inhomogeneous networks
- Superconducting JJNs on inhomogeneous insulating supports: microscopic theory.
- Comparison with available experimental results

3) Inhomogeneity effects in statistical mechanics models:
- Local order parameter induced by the inhomogeneity
- A case study: the spherical and the Ising model on the star / book graph

4) Entanglement distribution in inhomogeneous networks:
- Free fermions on inhomogeneous networks
- Bundled graphs: the example of a comb

Superconducting weak links: a Josephson junction

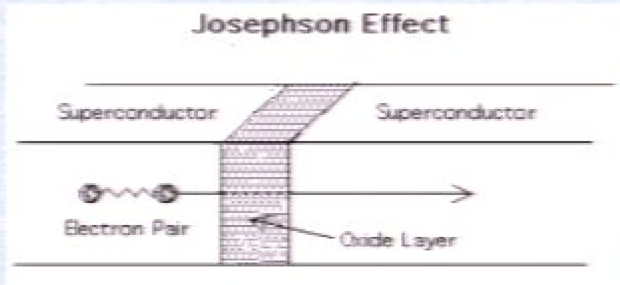


Fig. 13

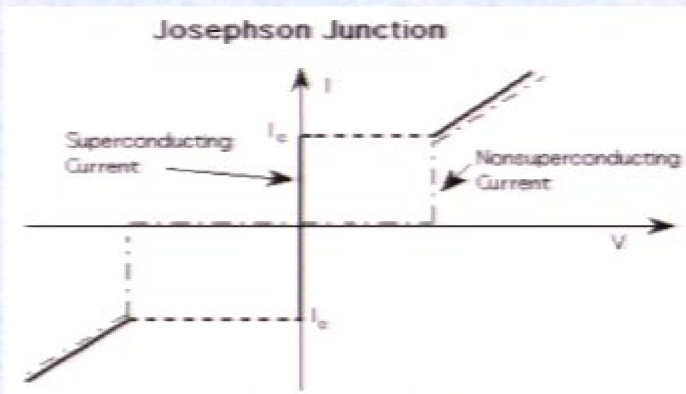


Fig. 14

Josephson current at $T<T_{BCS}$

A superconducting Josephson junction

-) In absence of fields: $I = I_C \sin(\varphi_1 - \varphi_2)$

-) The critical current is proportional to the gap Δ

-) In a SQUID the critical current can be tuned using a magnetic field:

$$I_C = I_C(\Phi = 0)\left|\cos\left(\frac{\pi\Phi}{\Phi_0}\right)\right|$$

-) At finite temperature

$$\frac{I_C(T)}{I_C(0)} = \frac{\Delta(T)}{\Delta(0)}\tanh\left(\frac{\Delta(T)}{2k_B T}\right)$$

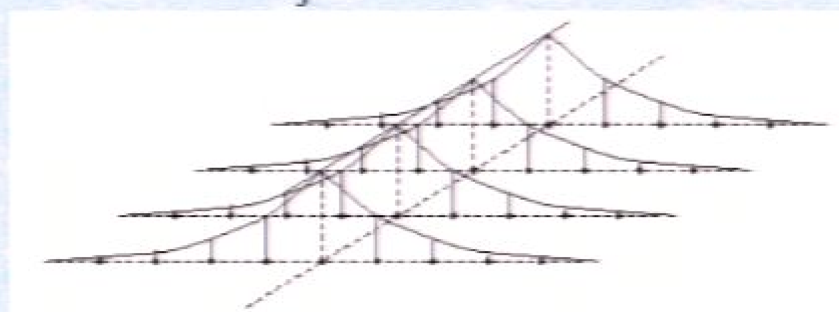
Ambegaokar-Baratoff

Free particles on a comb lattice

$$\hat{H} = -t\sum_{i,j} A_{ij}\hat{a}_i^{+}\hat{a}_j$$

$$-t\sum_{j} A_{ij}\psi_\nu(j) = E_\nu\psi_\nu(i)$$

Ground-state eigenfunction



$$T_C \approx \frac{E_J}{k_B}$$

Bose-Einstein critical temperature for free bosons

$$\frac{I_c^B(x >> 1; T/T_C)}{I_c^A} \approx \frac{T}{T_C}$$

far from (close to) the center: critical current decrease (enhancement)

P. Sodano et al, EPL 52, 251 (2000); G. Giusiano, F.P. Mancini, P. Sodano and A. Trombettoni, *Int. J. Mod. Phys. B* **18**, 691 (2004);on star lattices: I. Brunelli, G. Giusiano, F. P. Mancini, P.Sodano, and A. Trombettoni, *J. Phys. B* **37**, S275 (2004)

Fabricate a comb-shaped network with superconducting Josephson junctions

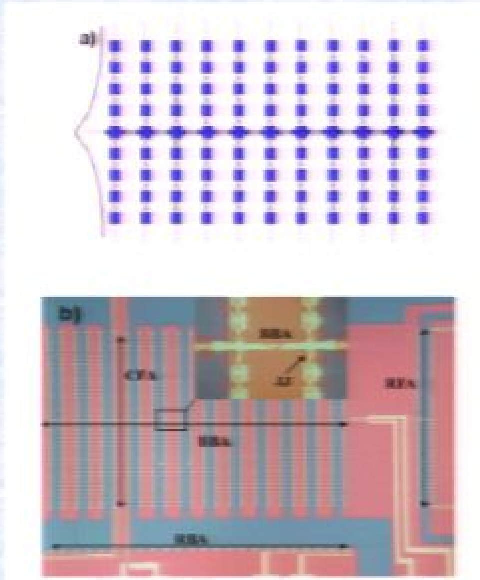


FIGURE 1

- -Nb trilayer technology
- -Josephson critical currents $I_c \sim 10\,\mu A$
- - capacitance $C \sim 2\,pF$
- - classical regime

$$E_J = \frac{\hbar}{2e} I_c$$

$$E_c = \frac{e^2}{2C}$$

$$E_c / E_J < 0.001$$

P. Silvestrini , P. Sodano *et al.*, Phys. Lett. A 370, 499 (2007)
P. Sodano *et al.*, New J. Phys. 8, 327 (2006)

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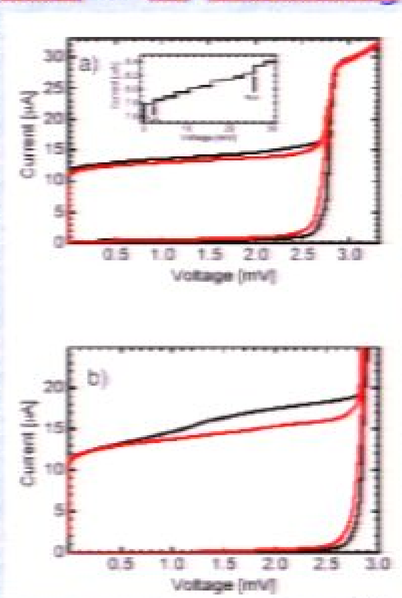
Superconducting Josephson junctions on a comb lattice: experimental results

On a comb of superconducting Josephson networks, one expects that critical currents along the backbone increase and along the fingers decrease:

need of an interacting fermionic theory to quantitatively explain the data!

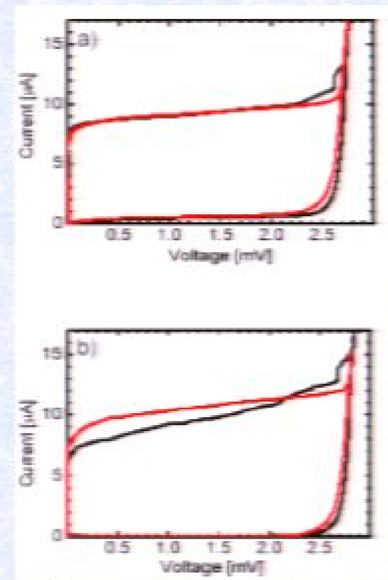


a) 4.2 K
b) 1.2 K

backbone vs. chain

finger vs. chain

(chain=red line)



P. Silvestrini, P. Sodano *et al.*, Phys. Lett. A 370, 499 (2007); P. Sodano *et al.*, New J. Phys. 8, 327 (2006)

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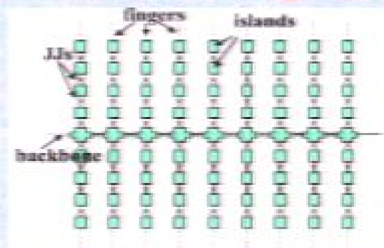
Bogoliubov-de Gennes theory for the critical current enhancement in comb shaped Josephson networks



Inhomogeneous Comb

$i = (x, y)$

x = position on the backbone

y = position on the finger

$$A_{ij} = \delta_{xx'}(\delta_{y,y'+1} + \delta_{y,y'-1}) + \delta_{y0}\delta_{y'0}(\delta_{x,x'+1} + \delta_{x,x'-1})$$

Homogeneous Chain

i = position on the chain

$$A_{ij} = \delta_{i,j+1} + \delta_{i,j-1}$$

Spectrum

$$-t\sum_{j} A_{ij}\psi_{\alpha}(j) = e_{\alpha}\psi_{\alpha}(i)$$

Ground state localized around the backbone – "Hidden" spectrum of localized states

Planewave solutions

Bogoliubov-de Gennes equations: continuous case

For an inhomogeneous fermionic systems with attractive interactions

$$H = H_0 + H_1$$

$$H_0 = \int d\vec{r} \sum_{\sigma} \psi^{+}(\vec{r}\sigma) h_0 \psi(\vec{r}\sigma) \qquad H_1 = -\frac{V}{2}\int d\vec{r} \sum_{\sigma\sigma'} \psi^{+}(\vec{r}\sigma)\psi^{+}(\vec{r}\sigma')\,\psi(\vec{r}\sigma')\psi(\vec{r}\sigma)$$

$$h_0 = -\hbar^2 \nabla^2 / 2m + U_0(\vec{r}) - \mu$$

$$\varepsilon_\alpha u_\alpha(\vec{r}) = \left[h_0 + U(\vec{r})\right] u_\alpha(\vec{r}) + \Delta(\vec{r})\, v_\alpha(\vec{r})$$

$$\varepsilon_\alpha v_\alpha(\vec{r}) = -\left[h_0 + U(\vec{r})\right] v_\alpha(\vec{r}) + \Delta^*(\vec{r})\, u_\alpha(\vec{r})$$

BdG Equations

$$\Delta(\vec{r}) = V \sum_\alpha u_\alpha(\vec{r}) v_\alpha^*(\vec{r}) \tanh\left(\frac{\beta}{2}\varepsilon_\alpha\right)$$

$$U(\vec{r}) = -V \sum_\alpha \left[|u_\alpha(\vec{r})|^2 f_\alpha + |v_\alpha(\vec{r})|^2 (1 - f_\alpha) \right]$$

$$f_\alpha = (e^{\beta\varepsilon_\alpha} + 1)^{-1}$$

Self-consistency conditions

Retrieving the standard BCS theory

For an homogeneous system, α is simply the momentum k:

$$\varepsilon_{\vec{k}} = \sqrt{\Delta^2 + E_{\vec{k}}^2} \qquad E_{\vec{k}} = \hbar^2 k^2 / 2m - \mu + U$$

$$u_{\vec{k}}(\vec{r}) = L^{-3/2} U_{\vec{k}} e^{i\vec{k}\cdot\vec{r}} \qquad v_{\vec{k}}(\vec{r}) = L^{-3/2} V_{\vec{k}} e^{i\vec{k}\cdot\vec{r}}$$

$$U_{\vec{k}}^2 = \frac{1}{2}\left(1 + \frac{E_{\vec{k}}}{\varepsilon_{\vec{k}}}\right) \qquad V_{\vec{k}}^2 = \frac{1}{2}\left(1 - \frac{E_{\vec{k}}}{\varepsilon_{\vec{k}}}\right)$$

Putting $U=0$ and $\mu=E_F$ and assuming a BCS point-like interaction, one gets the BCS equation for the gap:

$$1 = \frac{n(0)V_{BCS}}{2} \int_{-\hbar\omega_D}^{\hbar\omega_D} \frac{dE}{\sqrt{E^2+\Delta^2}} \tanh\left(\frac{\beta}{2}\sqrt{E^2+\Delta^2}\right)$$

Relation between the chemical potential and the Fermi energy

Using the equation for the number of particles

$$N = 2\sum_{\alpha}\int d\vec{r}\,|\,v_{\alpha}(\vec{r})\,|^{2} + 2\sum_{\alpha}\int d\vec{r}\,f_{\alpha}\left(|\,u_{\alpha}(\vec{r})\,|^{2} - |\,v_{\alpha}(\vec{r})\,|^{2}\right)$$

it follows at T=0 when $\Delta << E_F$

$$\mu - U \approx E_F$$

In general: since $U<0$, then $\mu<E_F$, increasing the attraction, the Hartree-Fock term U increases and the chemical potential μ decreases.

Bogoliubov-de Gennes Equations: lattice case

Discretization: $u_\alpha(\vec r) = \sum_i u_\alpha(i)\phi_i(\vec r); \quad v_\alpha(\vec r) = \sum_i v_\alpha(i)\phi_i(\vec r)$

$$\varepsilon_\alpha\, u_\alpha(i) = \sum_j \Gamma_{ij} u_\alpha(j) + \Delta(i)\, v_\alpha(i)$$

$$\varepsilon_\alpha\, v_\alpha(i) = -\sum_j \Gamma_{ij} v_\alpha(j) + \Delta^*(i)\, u_\alpha(i)$$

Lattice BdG Equations

$$\Gamma_{ij} = -t\, A_{ij} + U(i)\,\delta_{ij} - \tilde\mu\,\delta_{ij} \qquad \Delta(i) = \tilde V \sum_\alpha u_\alpha(i) v_\alpha^*(i) \tanh\left(\frac{\beta}{2}\varepsilon_\alpha\right)$$

Encoding the network´s connectivity (=topology) — **Self-consistency condition**

$t \approx -\int d\vec r\, \phi_i(\vec r)\left(-\hbar^2\nabla^2/2m + U_0(\vec r)\right)\phi_j(\vec r)$ ← **Hopping parameter**

$\tilde\mu \approx \mu - \int d\vec r\, \phi_i(\vec r)\left(-\hbar^2\nabla^2/2m\right)\phi_i(\vec r)$ ← **Lattice chemical potential**

$\tilde V \approx V\, \phi_i^2(\vec r = \vec r_i)$

Lattice Bogoliubov-de Gennes equations for the chain

$i = 1, \ldots, N_S$

$\alpha \to k$

$E_k = -2t\cos k - \tilde{\mu} + U_c; \qquad \varepsilon_k = \sqrt{\Delta^2 + E_k^2}$

$$1 = \frac{\tilde{V}}{4\pi t}\int_{-2t}^{2t} \frac{dE}{\sqrt{1-\frac{E^2}{4t^2}}\sqrt{\Delta_c^2 + (E-\tilde{\mu}+U_c)^2}} \tanh\left(\frac{\beta}{2}\sqrt{\Delta_c^2 + (E-\tilde{\mu}+U_c)^2}\right)$$

We have to set

$\varepsilon_k \approx E_k$

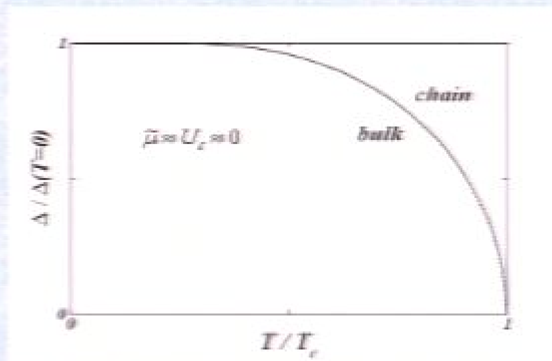
$i.e., \ \tilde{\mu} \approx U_c \approx 0$

One gets the „bulk" BCS results with $\Delta << t$

$\Delta_c(T=0) = 8t\, e^{-2\pi t/\tilde{V}}$

$k_B T_c = C t\, e^{-2\pi t/\tilde{V}} \qquad (C \approx 4.54)$

$\frac{\Delta_c(T=0)}{k_B T_c} \approx 1.76$



Lattice Bogoliubov-de Gennes equations for the comb

Away from the backbone, the fingers may be regarded as a linear chain ($U(i)=U_c$ and $\Delta(i)=\Delta_c$). Setting on the backbone $U(i)=U_b$ and $\Delta(i)=\Delta_b$, one gets with $\tilde{\mu} \approx U_b$

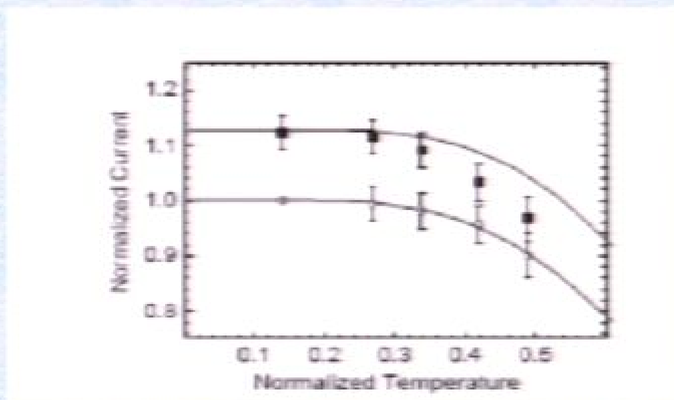
$$\Delta_b = \Delta_c + \frac{\Delta_b \tilde{V}}{\pi} \int_0^{\pi/2} dk \frac{\cos k}{\varepsilon_k \sqrt{1+\cos^2 k}} \tanh\left(\frac{\beta}{2}\varepsilon_k\right)$$

Contribution of the localized eigenstates of the adjacency matrix

At low temperature:

$$\frac{\Delta_b(T=0)}{\Delta_c(T=0)} = \frac{1}{1-\frac{\eta \tilde{V}}{2\pi t}} \qquad \left(\eta = \frac{1}{\sqrt{2}}\log\left(1+\sqrt{2}\right) = 0.62\right)$$

Comparison for the critical currents with the experimental results



P. Sodano et al., New J. Phys. 8, 327; P. Silvestrini, P. Sodano et al., Phys. Lett. A 370, 499 (2007)

Outlook

1) Experimental system:
- Superconducting Josephson junctions on inhomogeneous insulating supports

2) Thermodynamical properties:
- Spectral properties of free particles on simple inhomogeneous networks
- Superconducting grains on inhomogeneous insulating supports: microscopic theory of interacting fermions on the comb graph
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3) Inhomogeneity effects in statistical mechanics models:
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Local order parameter induced by inhomogeneities?

Inhomogeneity in a network can be induced e.g.

(a) by an external controllable potential (like the external potential for ultracold atoms) or

(b) by changing the topology of the network (e.g. the coordination number in a part of the network)

As an example of (a) bosons/fermions on a lattice + a magnetic trap

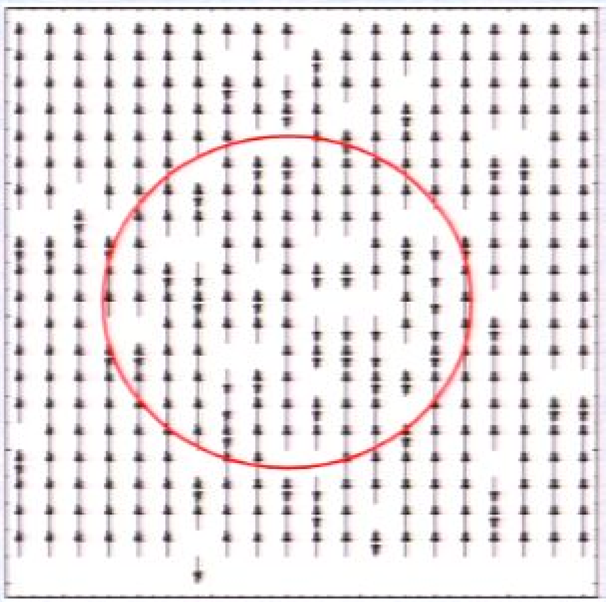
$$\hat{\mathrm{H}} = \frac{U}{2}\sum_i \hat{n}_i^2 - t\sum_{<i,j>} \hat{\mathbf{a}}_i^+ \hat{\mathbf{a}}_j + \varepsilon_i \sum_i \hat{n}_i \qquad (\varepsilon_i \propto \omega i^2)$$

$\varepsilon_i = 0$ Mott-superfluid transition

$\omega \neq 0$ Occurrence of Mott domains $\Rightarrow$ Smoothening of the phase transition

Coexistence of ordered and disordered domains in the Ising model

$$H = -J \sum_{\langle i,j \rangle} S_i S_j - \sum_i h_i S_i \qquad h_i = \Omega i^2$$



A disordered domain near the center & Ordered domains far from the center

Mean-field analysis

Local order parameter:

$$\langle S_i \rangle = \tanh\left(\beta q J \langle S_i \rangle + \beta \Omega i^2\right)$$

$$\Omega i^2 << qJ \Rightarrow \langle S_i \rangle \approx \tanh\left(\beta q J \langle S_i \rangle\right)$$

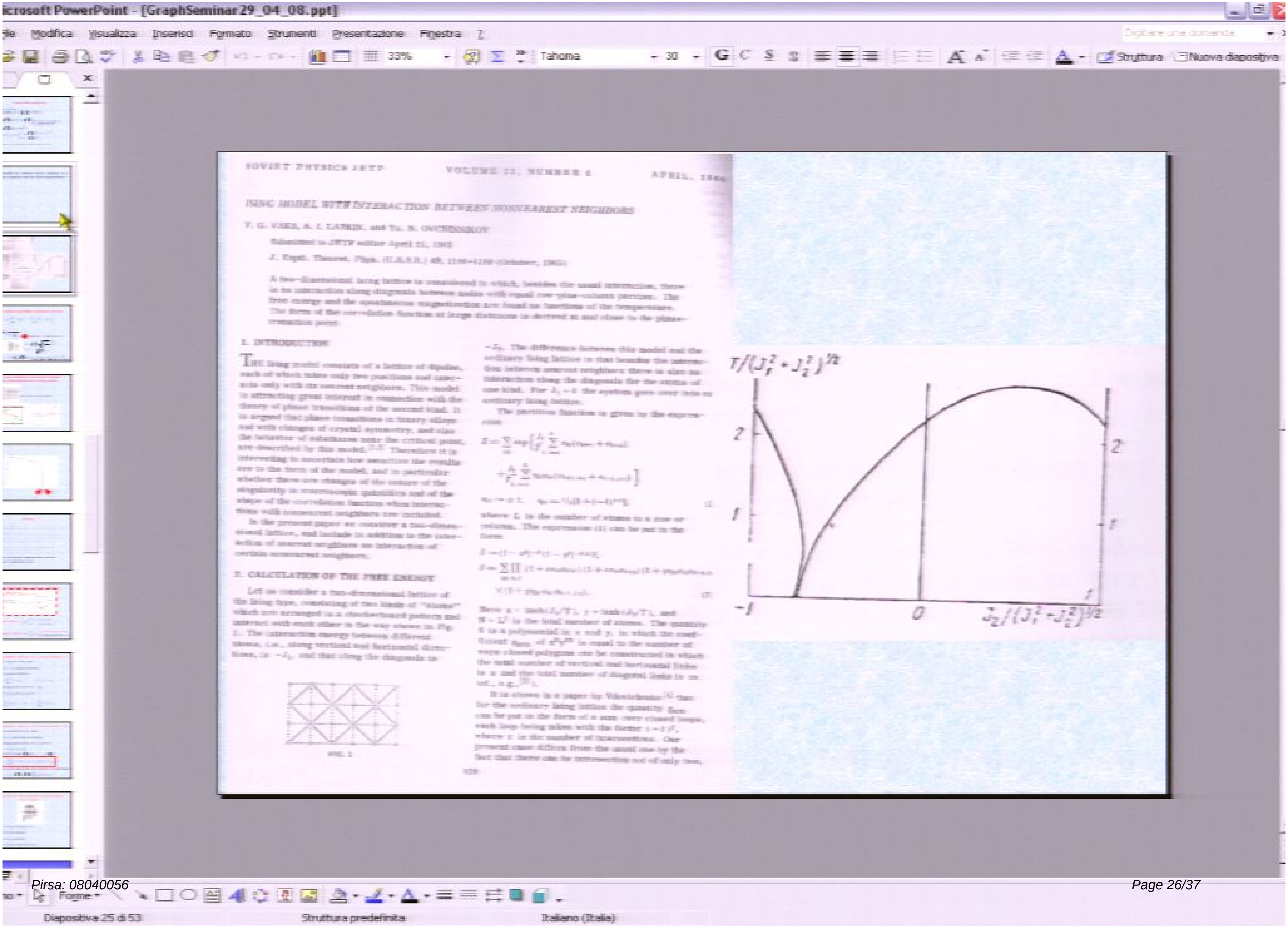
usual mean-field equations, with magnetization at low temperatures

$$\Omega i^2 >> qJ \Rightarrow \langle S_i \rangle \approx \tanh\left(\beta \Omega i^2\right)$$

$$\Omega i_c^2 = qJ \Rightarrow \begin{cases} i < i_c & \langle S_i \rangle \sim (1 - T/T_C)^{1/2} \\ i >> i_c & \langle S_i \rangle \sim 1 \end{cases}$$

Good agreement with Monte Carlo results, even in d=2

Is it possible to induce local phases in a network by engineering its inhomogeneities?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SOVIET PHYSICS JETP
VOLUME 22, NUMBER 4
APRIL, 1966
ISING MODEL WITH INTERACTION BETWEEN NONNEAREST NEIGHBORS
V. G. VAKS, A. I. LARKIN, and Yu. N. OVCHINNIKOV
1. INTRODUCTION
2. CALCULATION OF THE FREE ENERGY
FIG. 1
$T/(J_1^2+J_2^2)^{1/2}$
$J_2/(J_1^2+J_2^2)^{1/2}$
-1
0
1
2
Diapositiva 25 di 53
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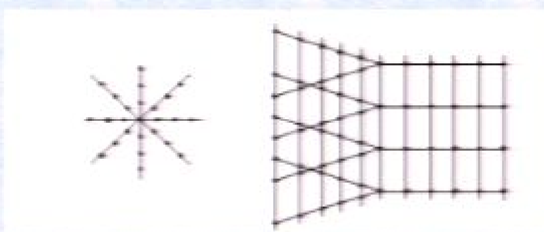
Local phase transition: spherical model

$$H = -J \sum_{<i,j>} S_i S_j \qquad \left(\sum_i S_i^2 = N \right)$$

$$1 = \frac{k_B T}{2N} \sum_\alpha \frac{1}{\mu - \frac{J}{2} e_\alpha}$$ ← eigenvalues of the adjacency matrix



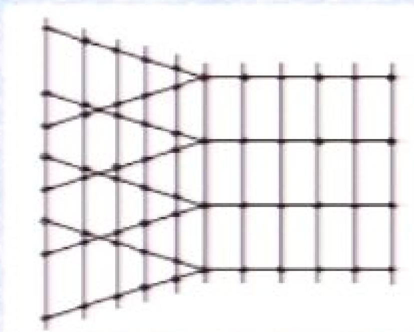
$$M_i = \langle S_i \rangle \propto \sqrt{1 - \frac{T}{T^*} e^{-i/\eta}}$$

$$k_B T^* = J \frac{p-2}{\sqrt{p-1}}$$

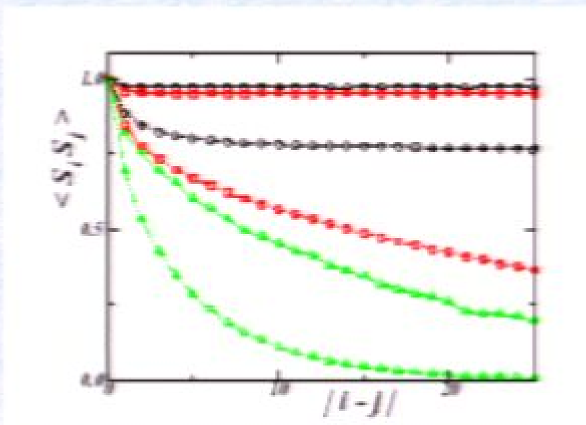
Of course $M = \frac{1}{N} \sum_i \langle S_i \rangle = 0$ for finite temperature $(T_c = 0)$

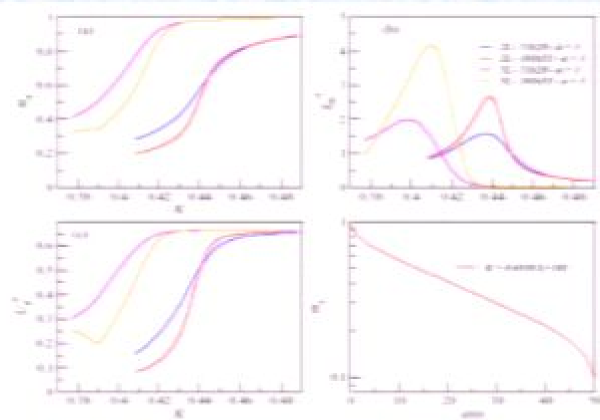
R. Marchetti, M. Rasetti, A. Trombettoni, and P. Sodano, submitted

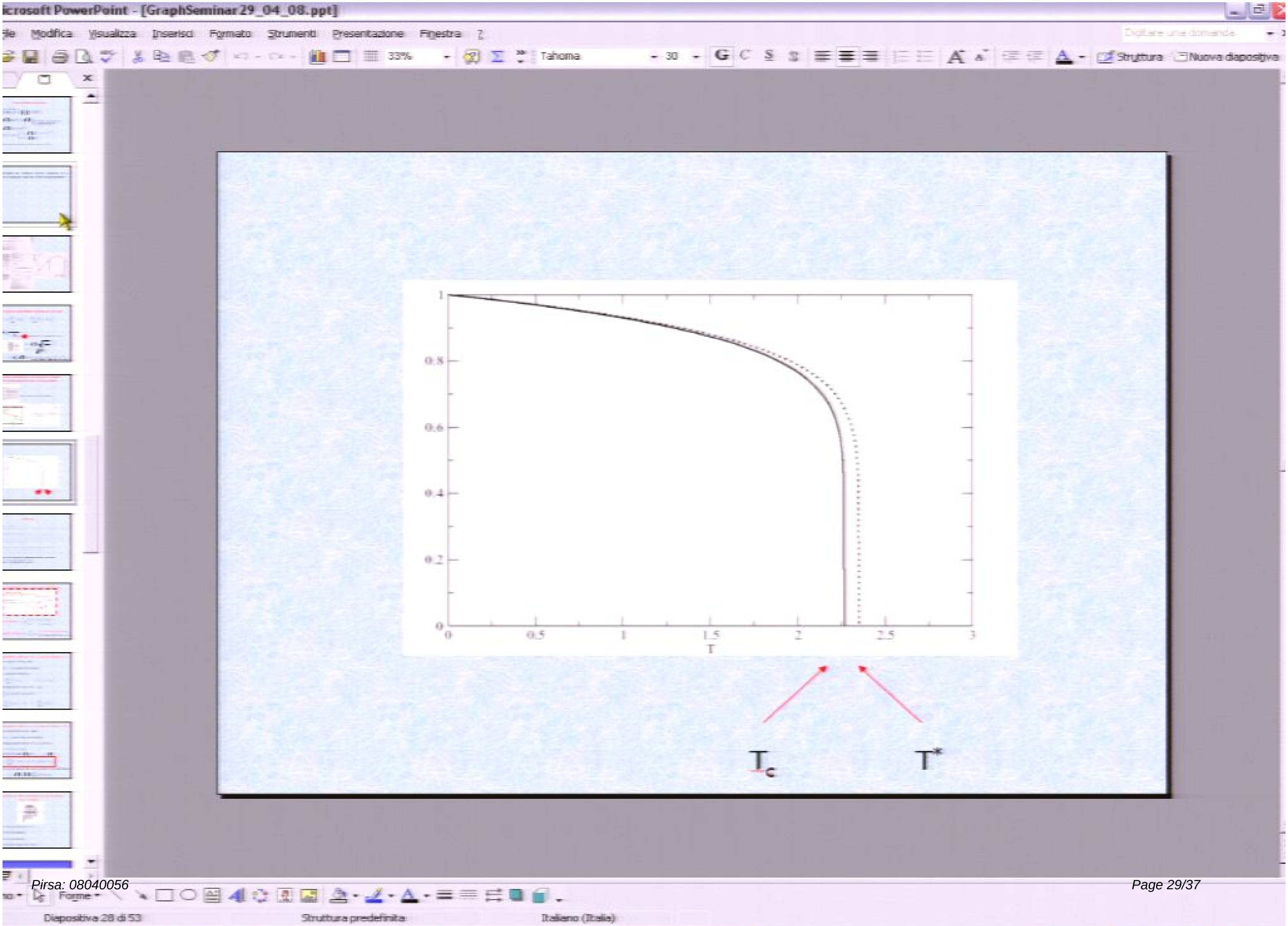
Classical statistical mechanics models with inhomogeneities: Ising model



Ising model on a book graph







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T
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T^*
Diapositiva 28 di 53
Struttura predefinita
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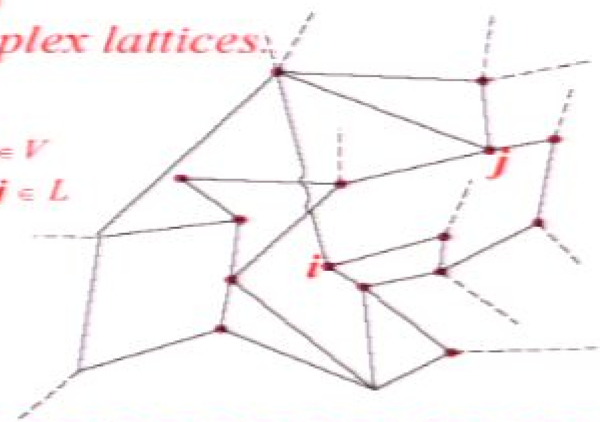
Free fermions on a general network

Some tools to describe complex lattices:

Set of sites $i \in V$

Set of links $i\text{-}j \in L$



Adjacency matrix $A_{ij} = \begin{cases} 1 \text{ if } i - j \text{ is a link} \\ 0 \quad \text{otherwise} \end{cases}$

Coordination number $z_i = \sum_j A_{ij}$

Chemical distance d_{ij} (shortest path from i to j)

Entanglement entropy on a sub-network (I)

i) G: a network with N_S sites

ii) $I,J=1,\dots,N_S$ sites of the network

iii) N_T number of fermions

$$H = -t \sum_{I,J} c_I^\dagger A_{IJ} c_J \qquad\qquad \{c_I^\dagger, c_J\} = \delta_{IJ}$$

Eigenvectors given by (with $\Gamma=1,\dots,N_S$)

$$-t \sum_J A_{IJ} \psi_\Gamma(J) = \varepsilon_\Gamma \psi_\Gamma(I)$$

$$d_\Gamma = \sum_I c_I \psi_\Gamma(I) \quad \Rightarrow \quad H = \sum_{\Gamma=1}^{N_S} \varepsilon_\Gamma d_\Gamma^\dagger d_\Gamma$$

Entanglement entropy on a sub-network (II)

i) G′: a sub-network with L sites

ii) i,j=1,…,L sites of the sub-network

The entanglement entropy of G′ is given by

$$S_{G'} = -Tr\left(\rho_{G'} \ln \rho_{G'}\right)$$

In the ground-state $\left|\Psi\right\rangle = \Pi_{\Gamma=1,\ldots,N_T} d_\Gamma^\dagger \left|0\right\rangle$

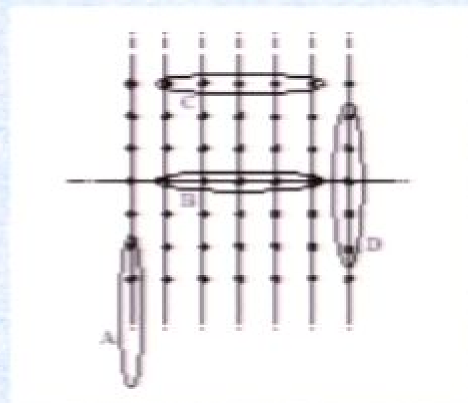
$$S_{G'} = -\sum_{\gamma=1}^{L}\left[C_\gamma \ln C_\gamma + \left(1-C_\gamma\right)\ln\left(1-C_\gamma\right)\right]$$

$C_\gamma \rightarrow$ the L eigenvalues of the (sub-network) correlation matrix

$$C_{ij} = \left\langle \Psi \left| c_i^\dagger c_j \right| \Psi \right\rangle = \sum_{\Gamma=1}^{N_T} \psi_\Gamma^*(i)\psi_\Gamma(j)$$

icrosoft PowerPoint - [GraphSeminar29_04_08.ppt]

Distribution of the entanglement entropy on a comb



Block A: S grows like (1/3) ln L

Block B: S grows like L

Block C: S grows like L

Block D: S grows like (c*/3) ln L

Conclusions

- Role of the inhomogeneity in controlling emergence of new coeherent behaviors

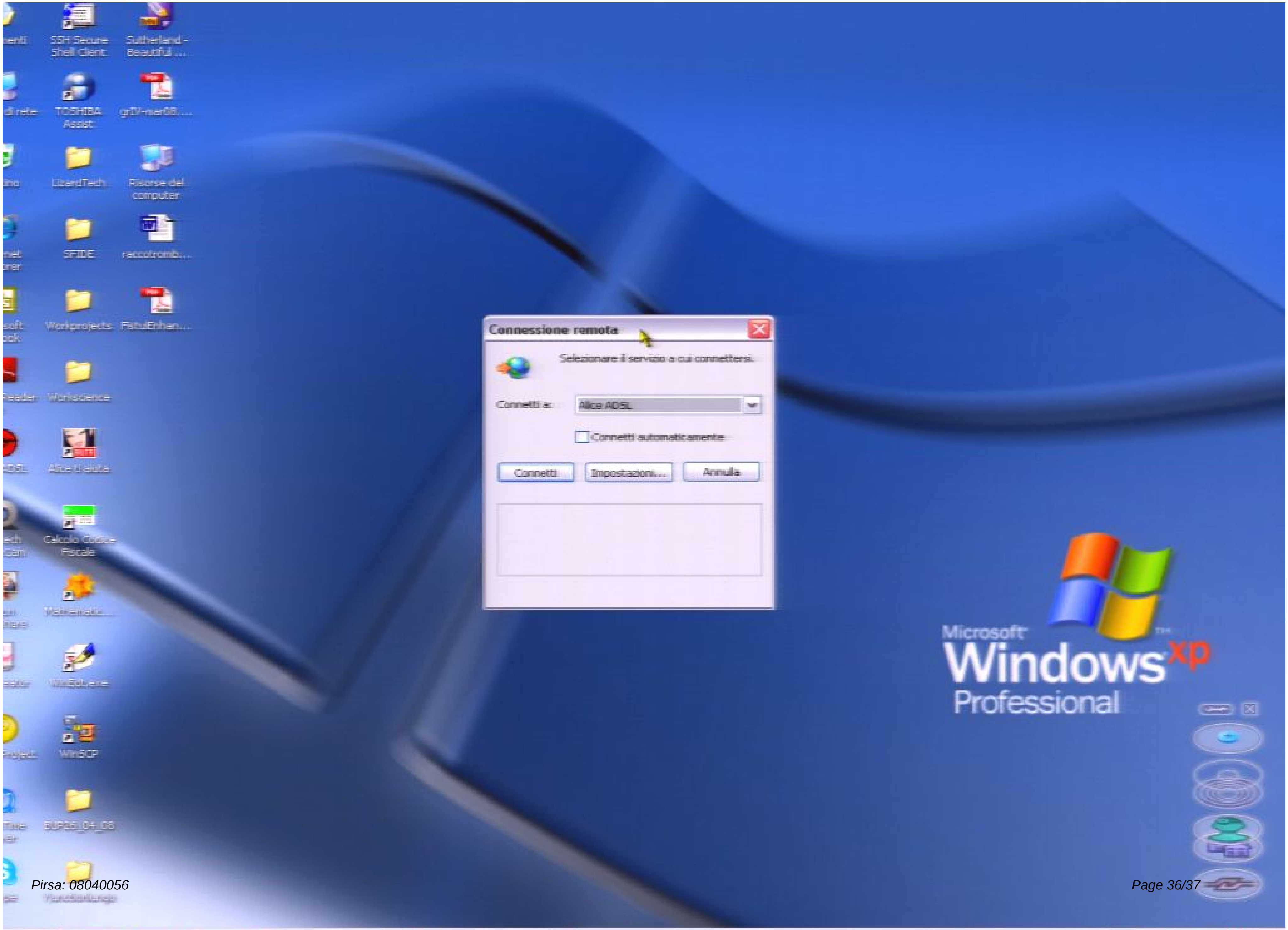
- JJNs: Observed enhancement of critical currents in inhomogeneous networks of superconducting Josephson junctions: good agreement with Bogoliubov-de Gennes results

- Statistical mechanics models on inhomogenous networks: inducing new phases even when there is zero global magnetization

- Control the entanglement distribution through the network topology

Perspectives

- Fabricate and analyze larger comb-shaped JJNs
- Analyze inhomogeneity effects in ultracold fermionic gases
- Renormalization group approach for inhomogeneous spin systems
- Relevance for QI devices.


SSH Secure Shell Client
Sutherland - Beautiful ...
TOSHIBA Assist
LizardTech
Risorse del computer
SFIDE
raccotromb...
Workprojects
FistulEnhan...
Calcolo Codice Fiscale
WinSCP
Connessione remota
Selezionare il servizio a cui connettersi.
Connetti a:
Alice ADSL
Connetti automaticamente
Connetti
Impostazioni...
Annulla
Microsoft
Windows xp
Professional


SSH Secure Shell Client
Sutherland - Beautiful ...
TOSHIBA Assist
LizardTech
Risorse del computer
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Workprojects
Workscience
WinSCP
Microsoft
Windows xp
Professional