Title: Graph expansions and simulating one-way quantum computation

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Abstract: It is well-known that if a graph G_1 can be obtained from another graph G_2 by removing a degree-2 vertex and combing its two neighbors, the graph state |G_1> can be obtained from |G_2> through LOCC. In this talk, I will describe how to construct a graph G\' from a given graph G so that (a) The maximum degree of G\', Delta(G\'), is no more than 3. (b) G can be obtained from G\' by a sequence of contraction operations described above. (c) The treewidth of G\', tw(G\'), is no more than tw(G)+1. (d) The construction takes exp(O(tw(G))) time. Those properties imply that a one-way quantum computation on the graph state |G> can be simulated by a randomized algorithm in time exp(O(tw(G))). Previously, it was known that such a computation can be simulated in time exp(O(tw(G) Delta(G))) [Markov and Shi, to appear in SICOMP], which is substantially more expensive than our bound when Delta(G) is large. Joint work with Igor Markov, based on arXiv:0707.3622.

Graph expansions and simulating one-way quantum computation

Yaoyun Shi
University of Michigan

Joint work with Igor Markov
based on arXiv:0707.3622

Motivation

Is quantum computation indeed much more powerful than classical computation?

An incrementalist's program: identify largest sets of efficiently simulatable quantum computation.

- If the answer is "No", we are on the right track:
- Otherwise, we know better any non-simulatable quantum computation must be like.

Simulatable classes of quantum computation

- Quantum circuits using a restricted gate set [Gottesman and Knill '98;...]
- Reduction to the Fisher-Kasteleyn-Temperley algorithm [Valiant '02, Terhal and DiVincenzo '02;

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A quantum circuit describes a quantum algorithm

- Start with a fixed initial state in the computational basis.
- Sequentially apply quantum gates, some on multiple qubits.
- Measure on the computational basis.

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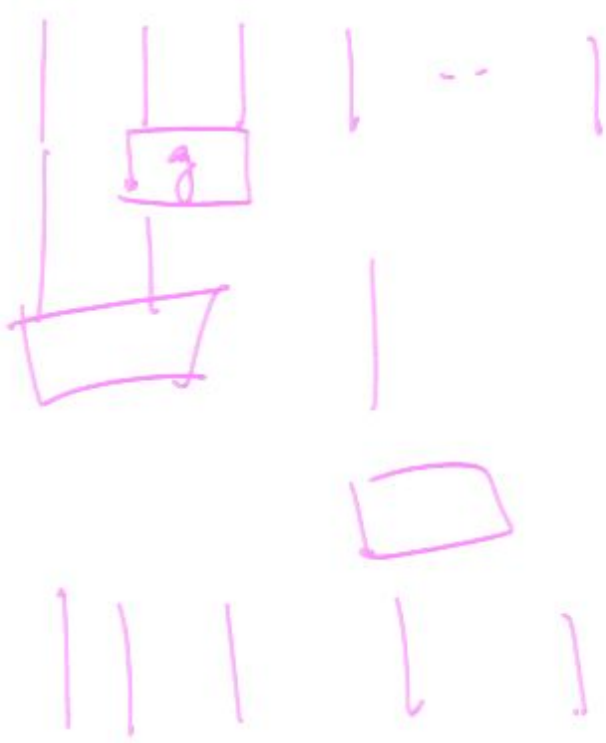

One-way quantum computation [Raussendorf and Briegel '01]

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Graph states [Briegel and Raussendorf '01]

Given an undirected simple graph $G = \langle V, E \rangle$, the graph state $|G\rangle$ is

$$|G\rangle := \frac{1}{2^{|V|\ 2}} \sum_{V' \subseteq V} (-1)^{|E(V')|} |V'\rangle,$$

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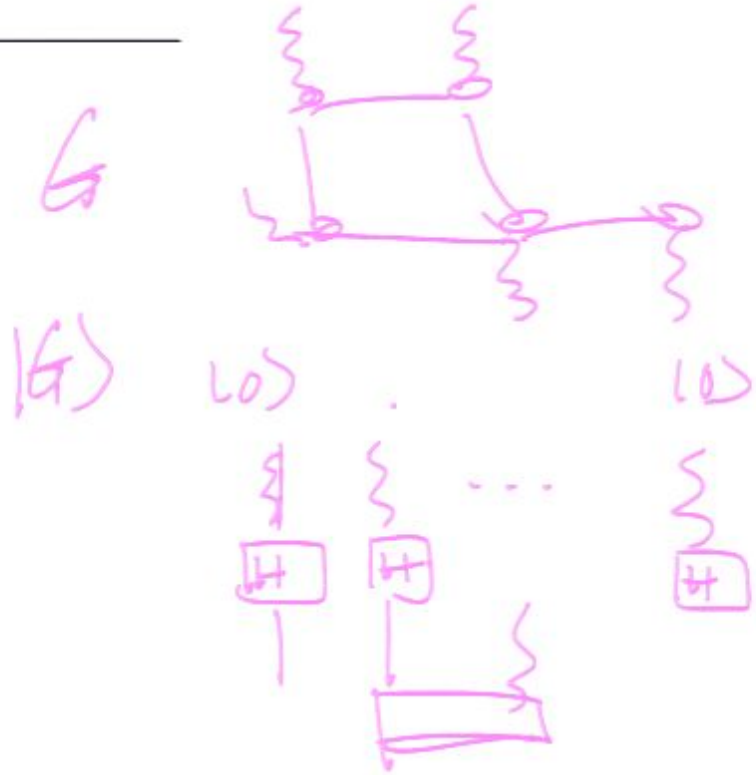

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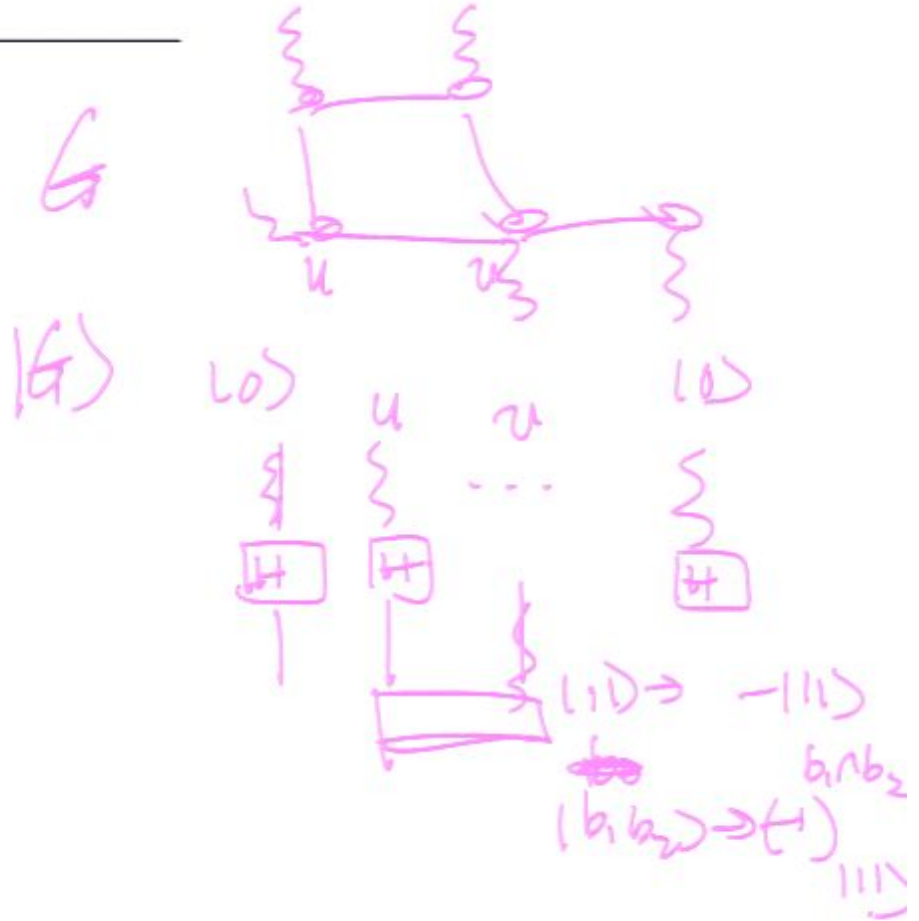

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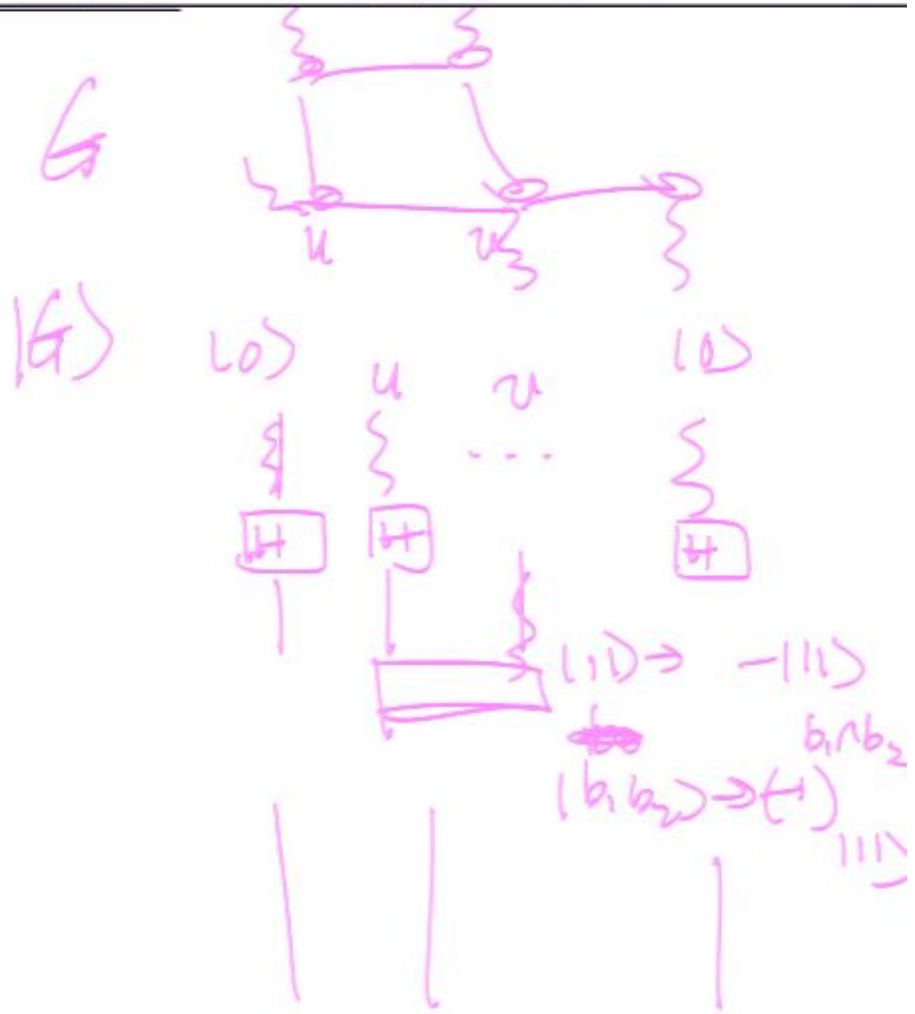

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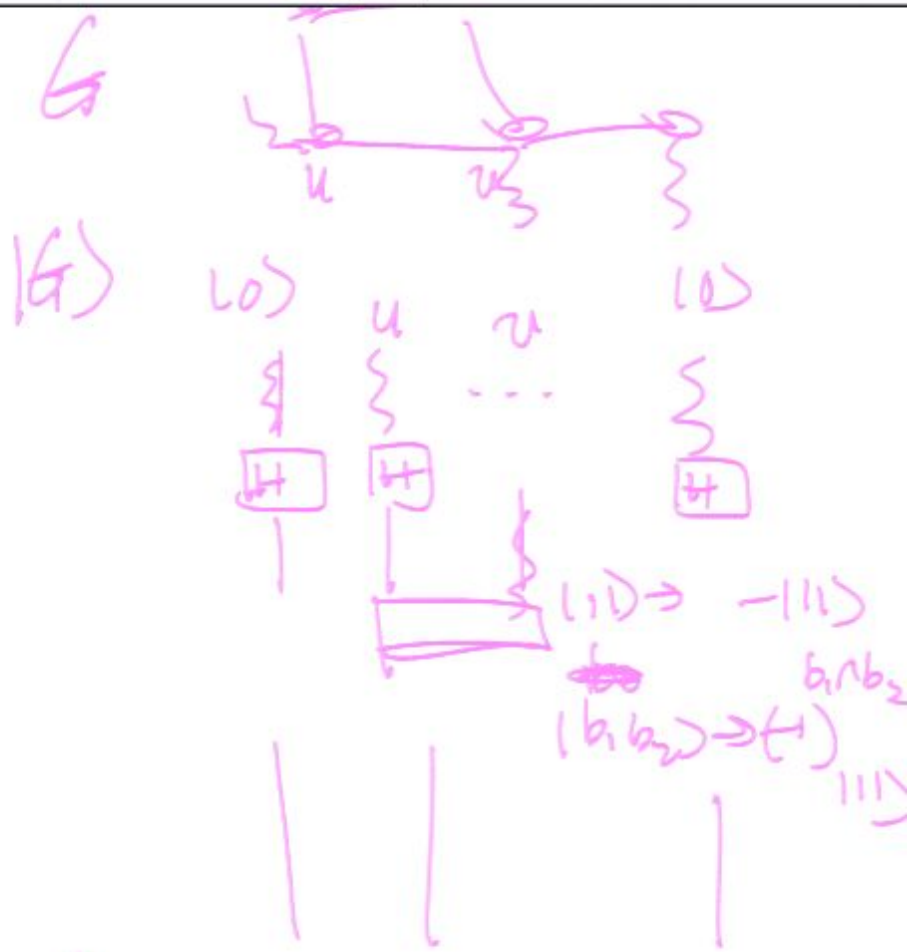

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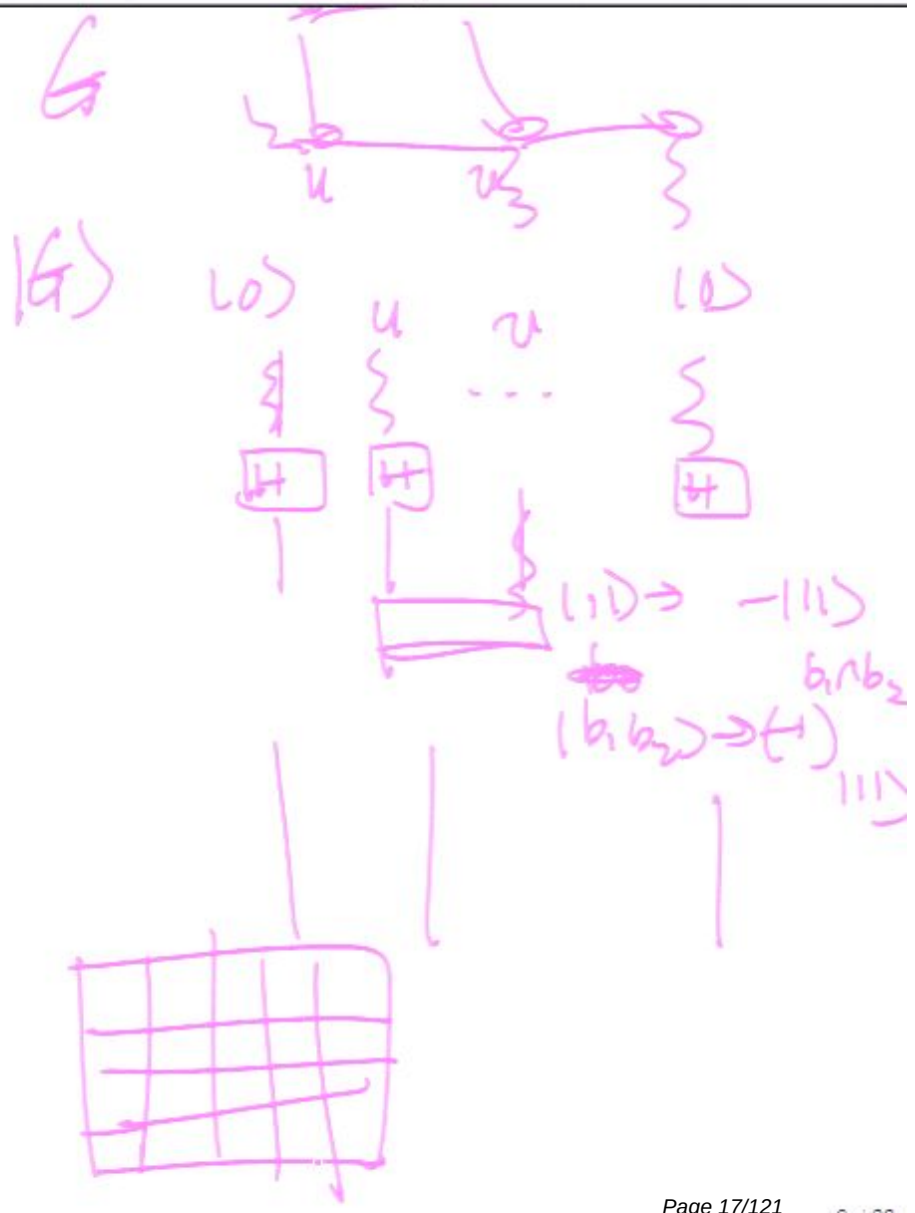

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Which states are universal?

Definition. Let $|S_n\rangle$, $n = 0, 1, ...$ be a (family of) n-qubit quantum state. We say that $|S_n\rangle$ is *simulatable* if if for any one-way quantum computation on $|S_n\rangle$, there exists a $poly(n)$ time randomized algorithm that outputs an identical distribution of the measurement outcomes.

Main Theorem (Informally): If a graph G is close to a tree then $|G\rangle$ is simulatable.

Tensor and tensor network

A rank-k *tensor* in an m-dimension space

$$g = [g_{i_1.....i_k}]_{1 \le i_1,.....i_k \le m}.$$

is a m^k-dimensional array of complex numbers.

Variants include *Matrix Product States* and Projected Entangled Paired States.

Tensor contraction: generalizes matrix multiplication.

Contracting $g = [g_{i_1....i_s.u_1.....u_k}]$ and $h = [h_{i_1.....i_s.v_1.....v_\ell}]$ gives a rank-$k-\ell$ tensor $f = [f_{u_1.....u_k.v_1.....v_\ell}]$, where

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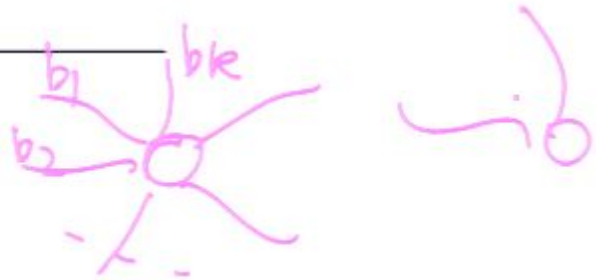

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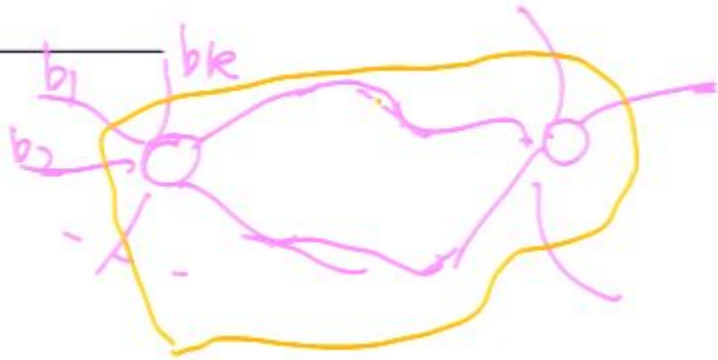

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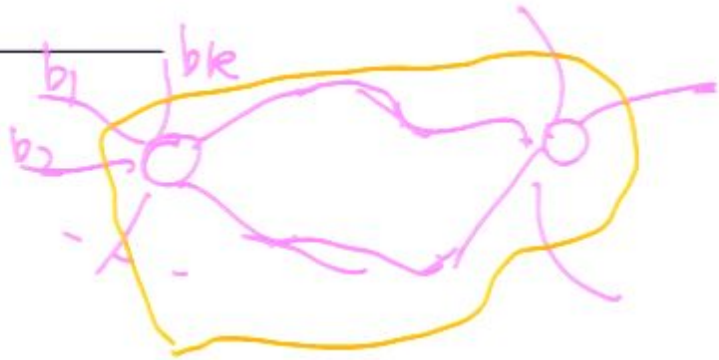

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Simulation through tensor contraction [Vidal '03; Verstraete et al. '04, Markov and Shi '05]

Given a quantum circuit, an initial (product) state, and a (product) measurement operator

- Attach the tensors for the input state to the input wires.
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- Contract the whole tensor according to a sequence of wires $\pi : e_1, e_2, \ldots, e_T$.
  - For each contraction, write down the entries of the new tensor.
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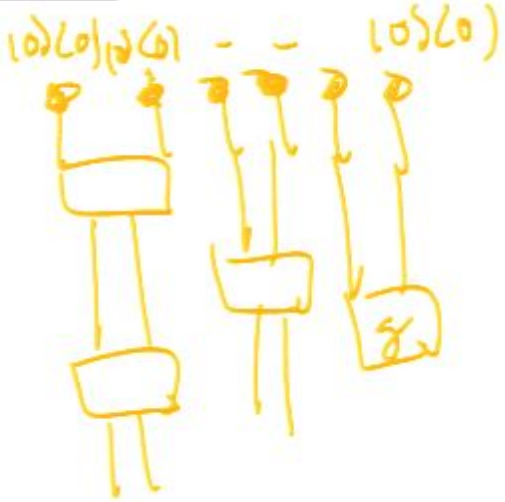

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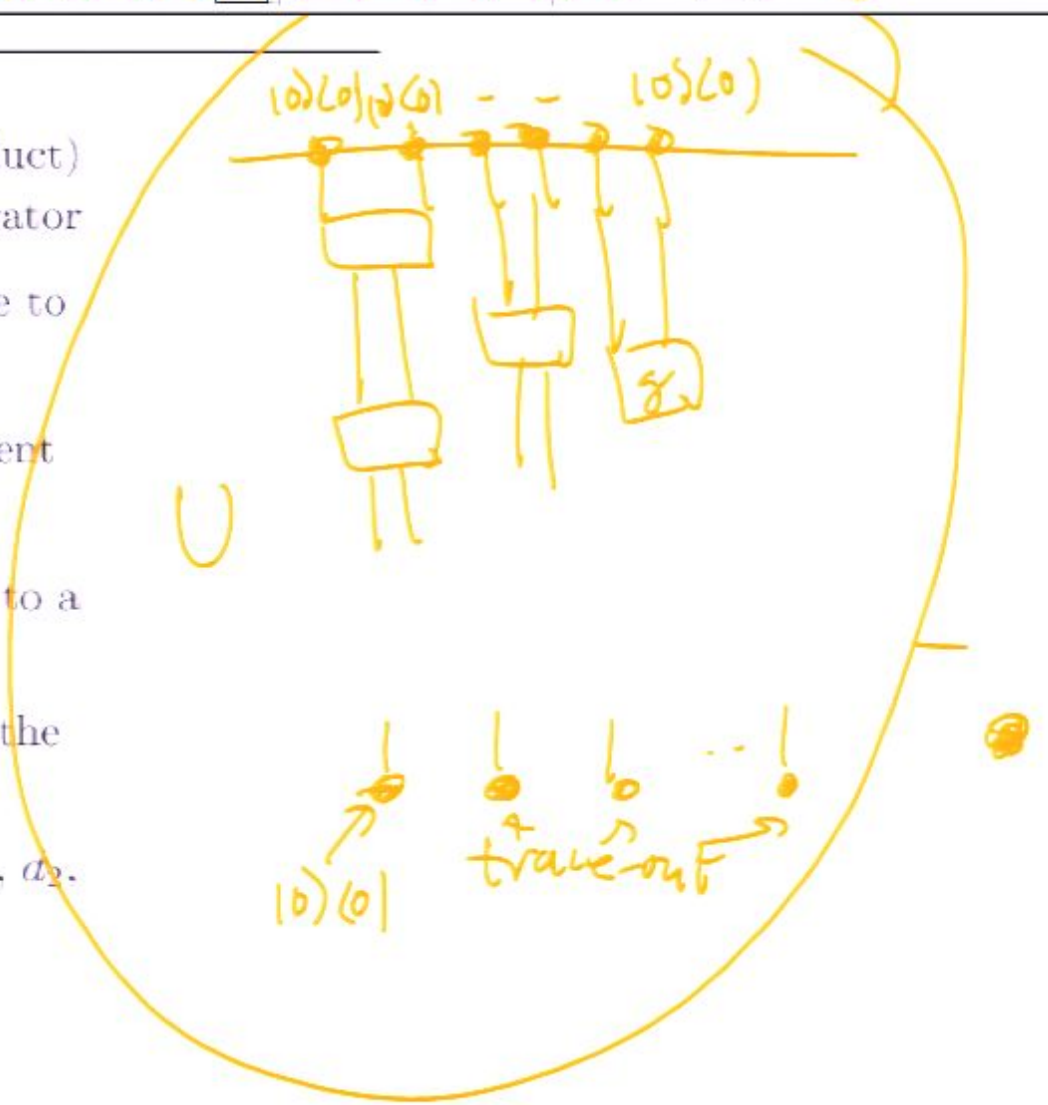

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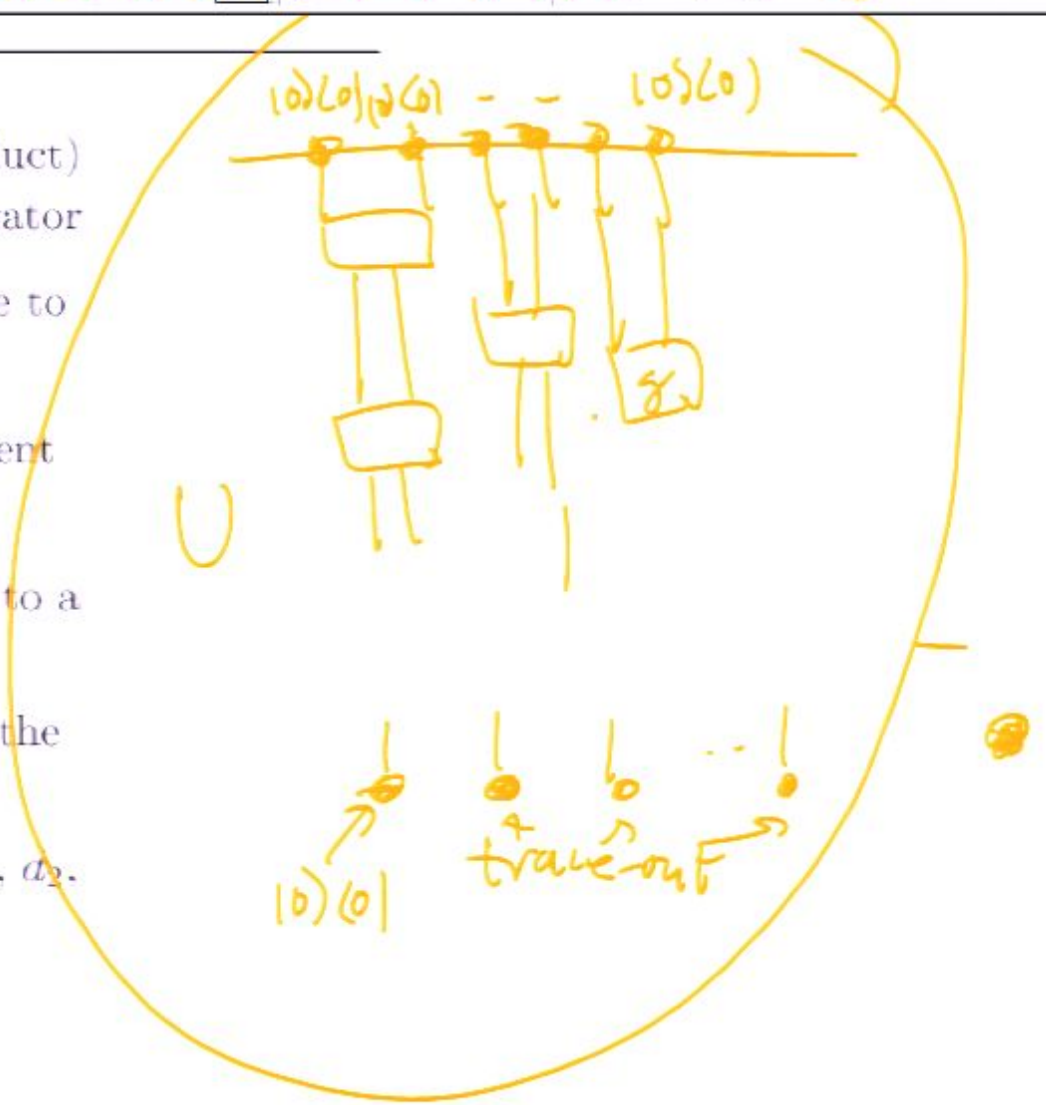

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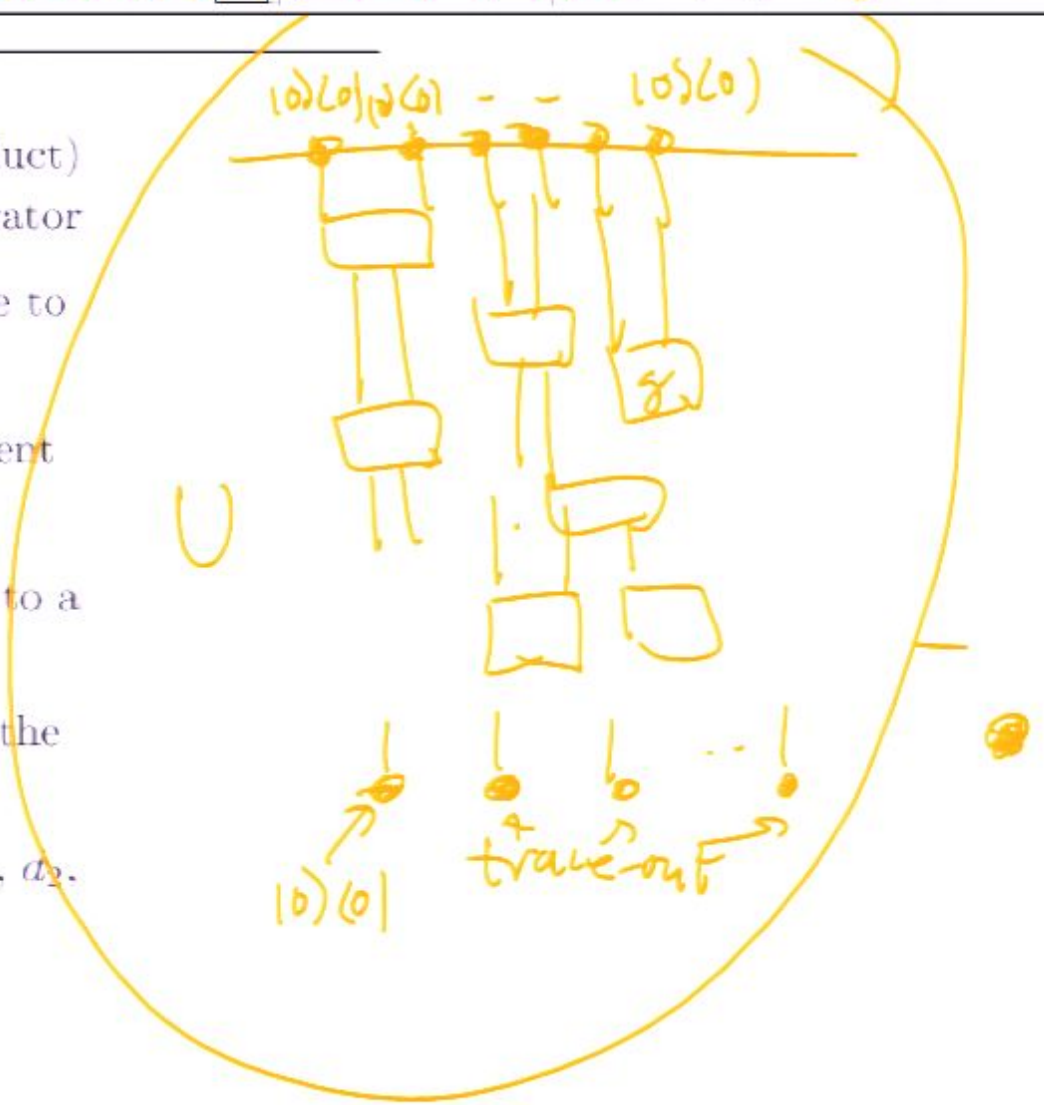

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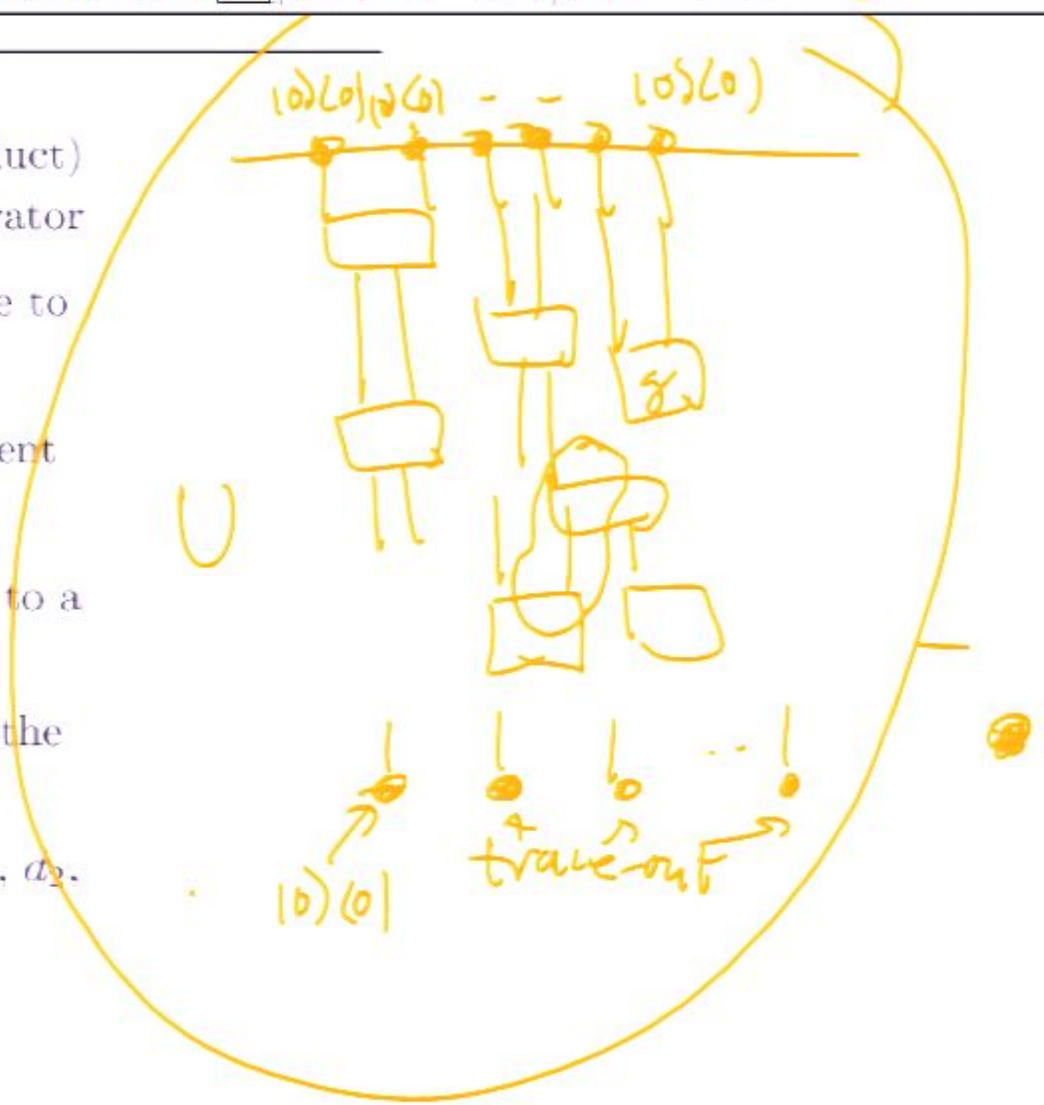

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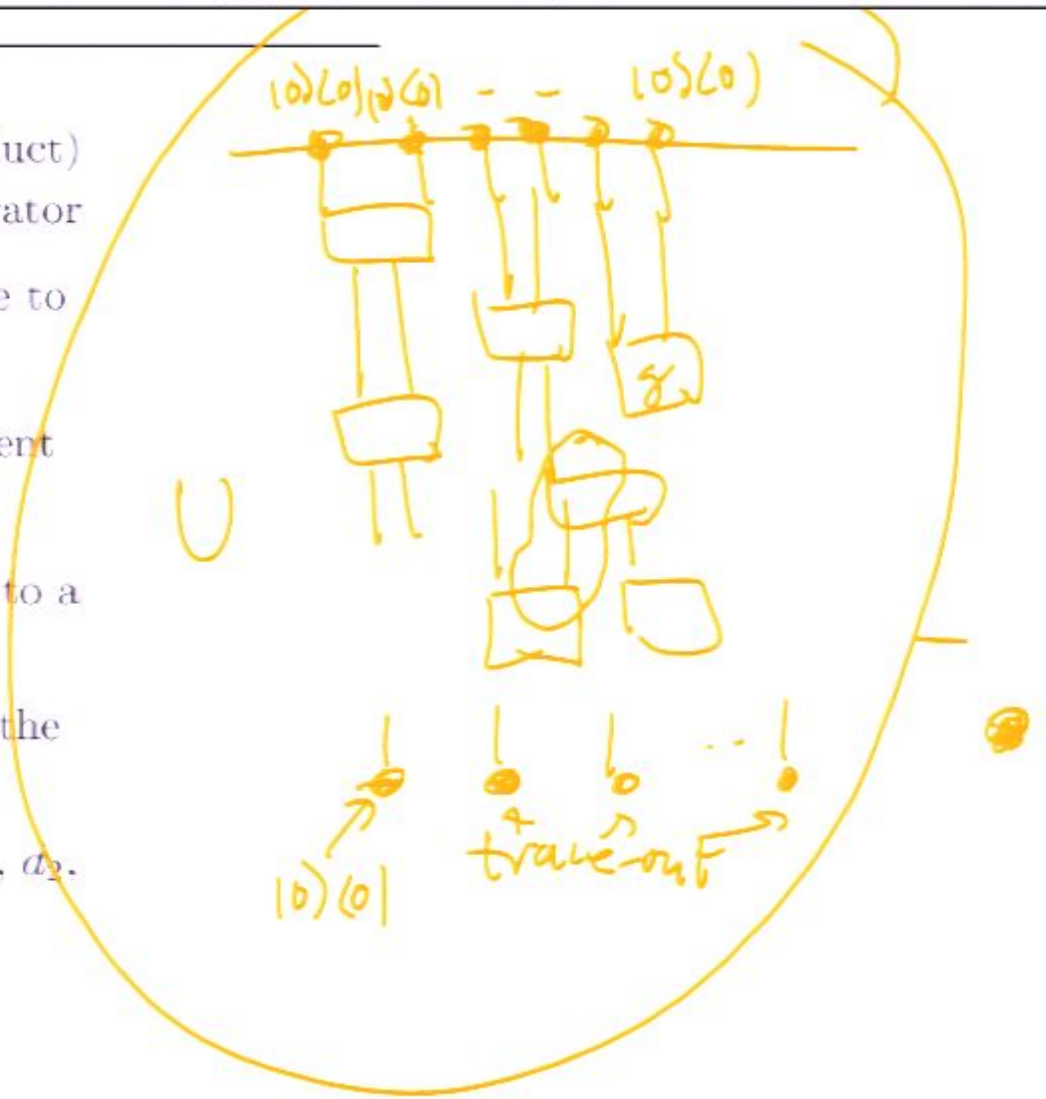

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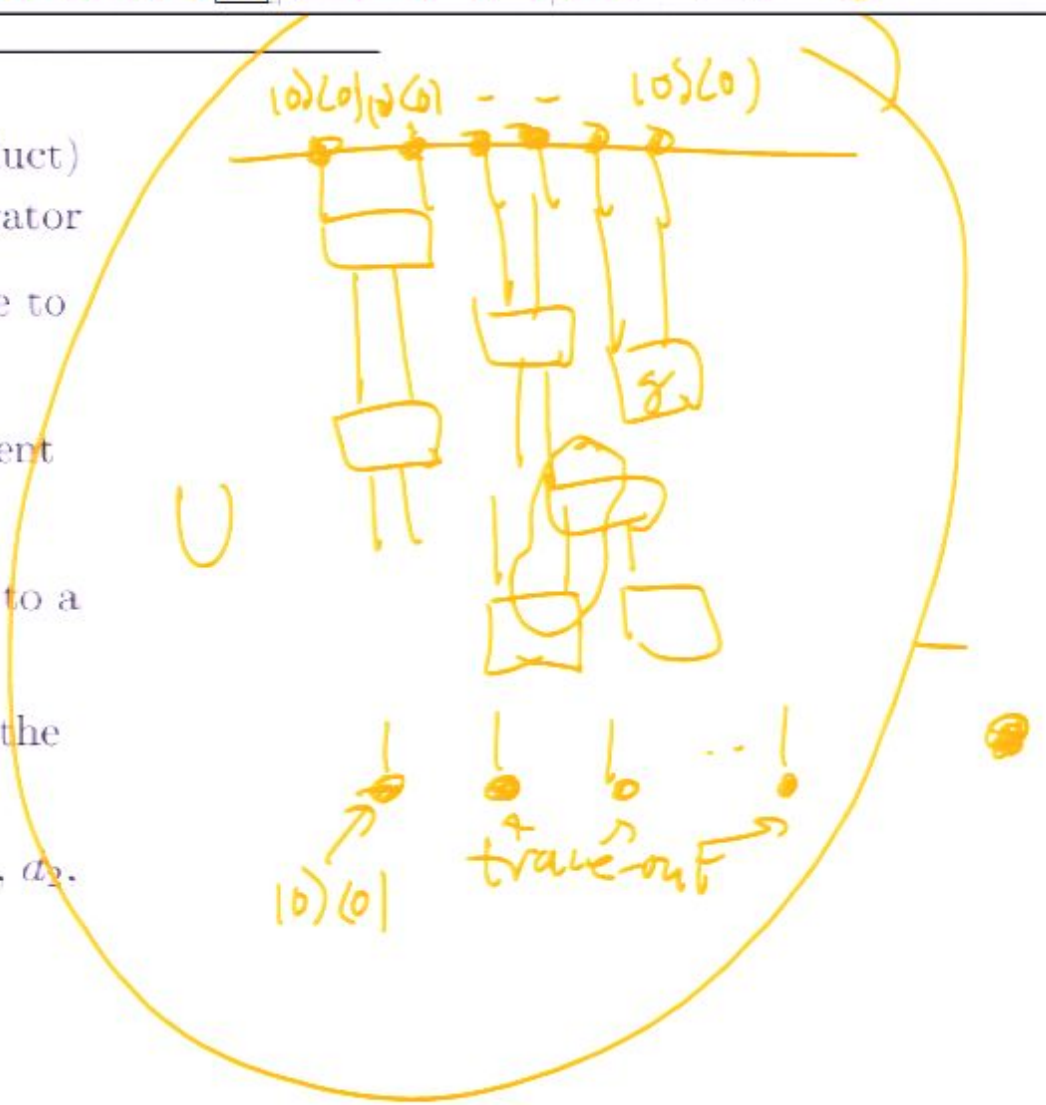

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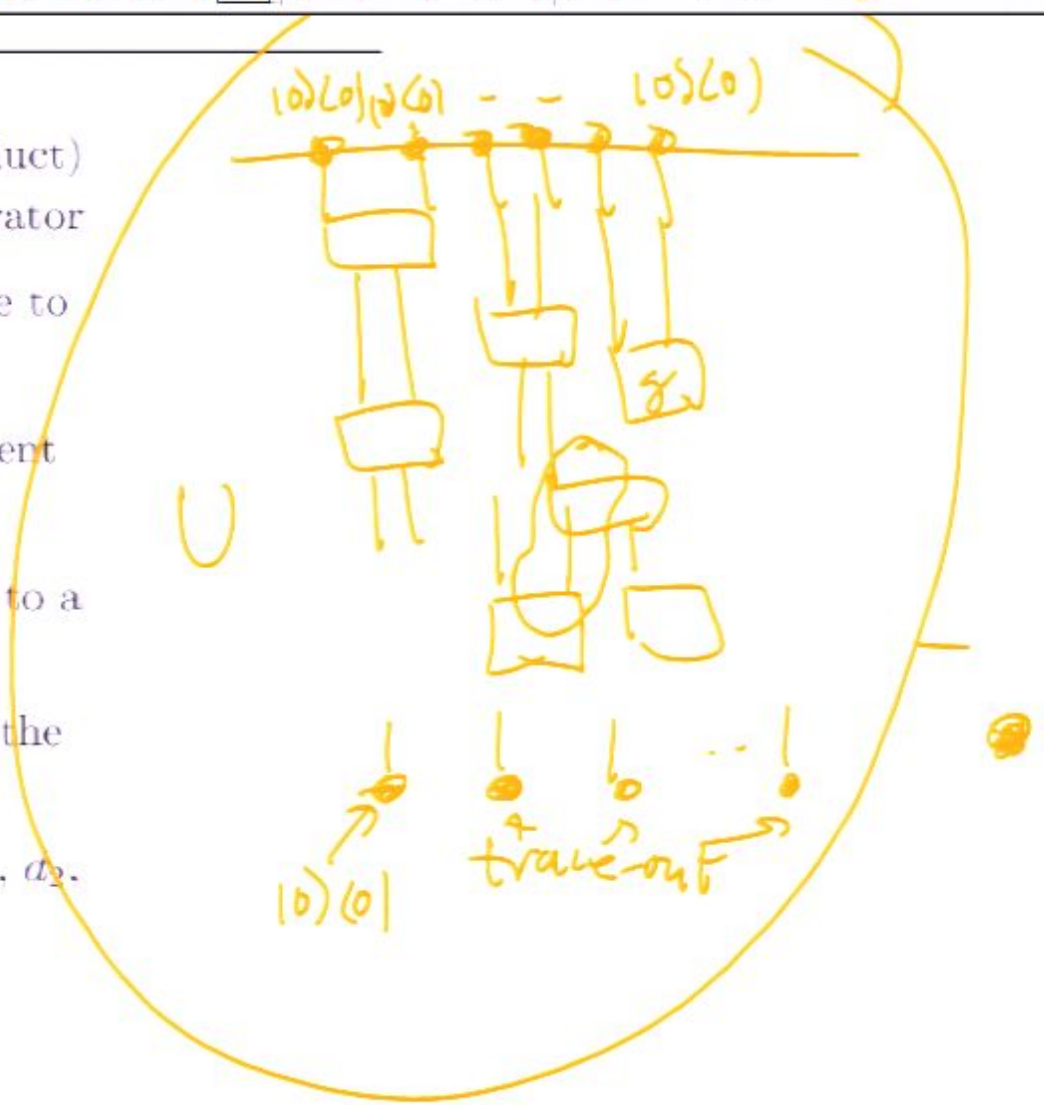

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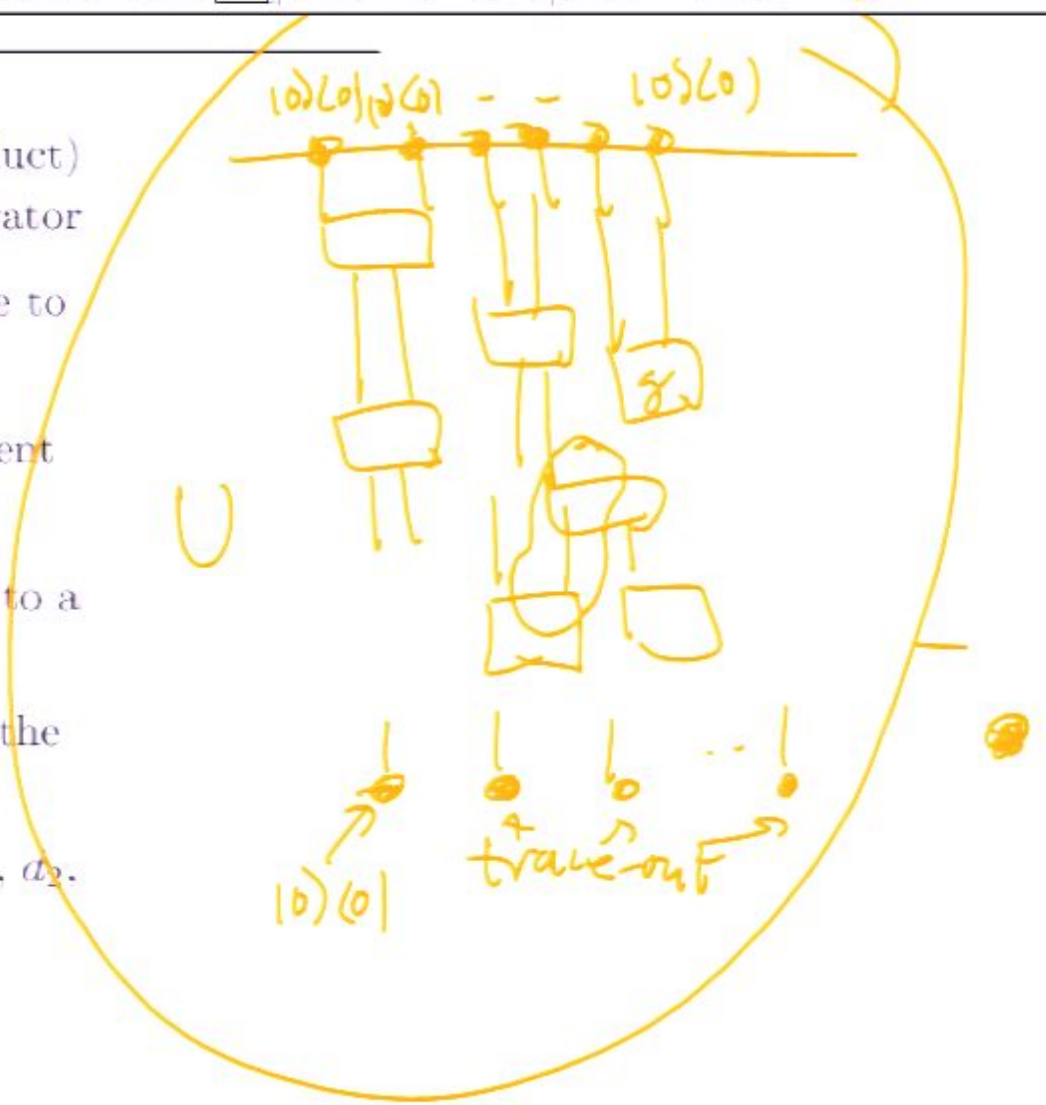

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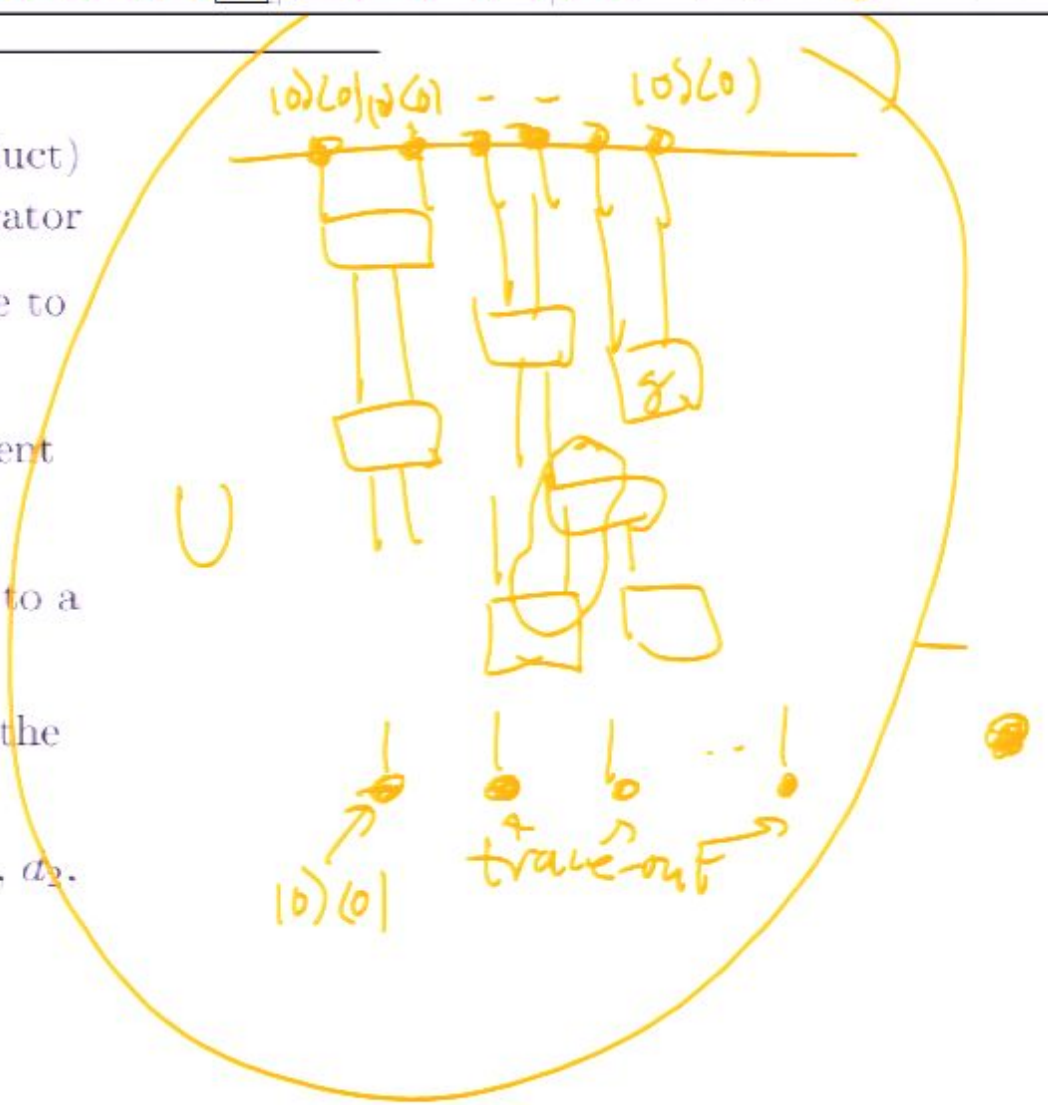

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Definition [Markov and Shi '05]. The *contraction complexity* of a graph G is

$$\mathrm{CC}(G) := \min_{\pi} \Delta(\pi).$$

Definition. The line graph of $G = \langle V, E \rangle$ is the graph G^* whose vertex set is E and $e_1 e_2$ is an edge if they are incident to a common vertex in G.

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Treewidth

The treewidth of a graph G, $\mathrm{tw}(G)$, describes how close G is to a tree.

- If G is a (nonempty) tree, $\mathrm{tw}(G) = 1$.
- If G is a series parallel graph, $\mathrm{tw}(G) \leq 2$.
- If G is fully connected, $\mathrm{tw}(G) = |V(G)| - 1$.
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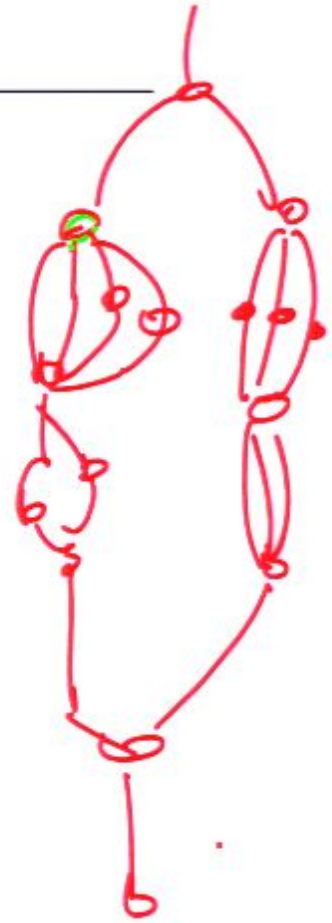

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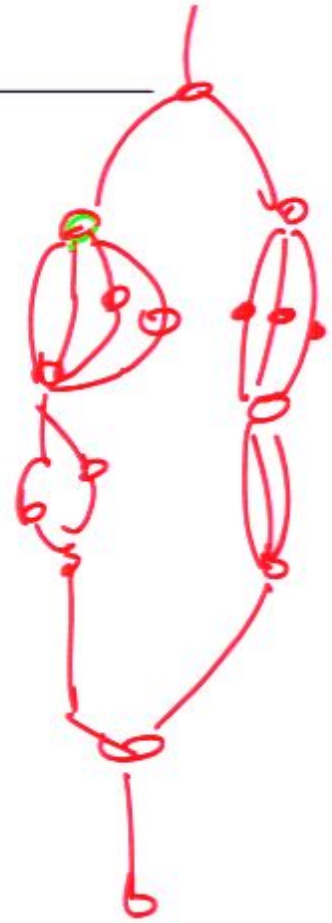

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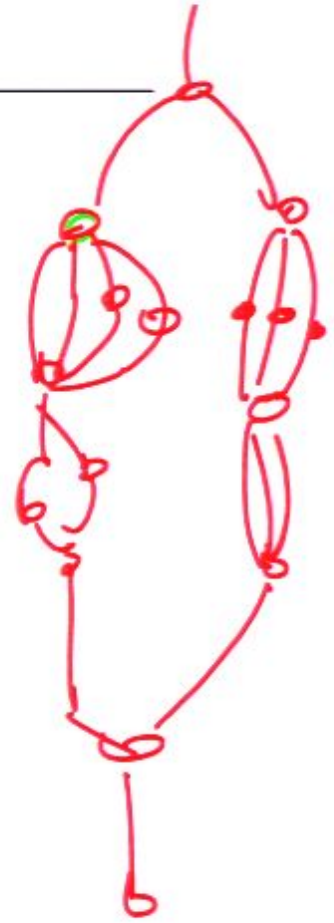
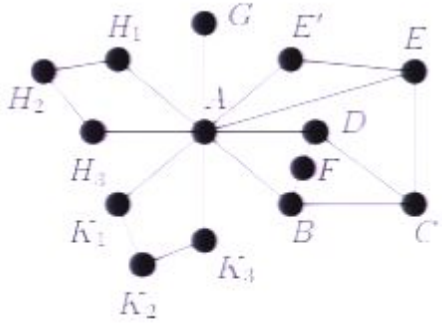
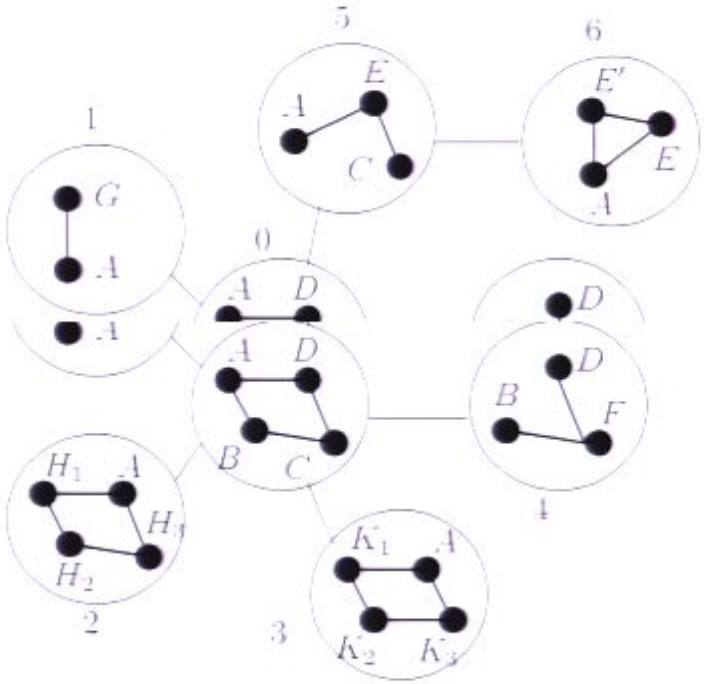


Definition [Robertson and Seymour '84]. A *tree decomposition* of G is a tree $\mathcal{T}$, together with a function that maps each vertex $w \in V(\mathcal{T})$ to a subset $B_w \subseteq V(G)$, called *bags*. In addition, the following conditions must hold.



(T1) $\bigcup_{v \in V(\mathcal{T})} B_v = V(G)$, i.e., each vertex must appear in at least one bag.

(T2) $\forall \{u, v\} \in E(G)$, $\exists w \in V(\mathcal{T})$, $\{u, v\} \subseteq B_w$, i.e., for each edge, at least one bag must contain both of its end vertices.



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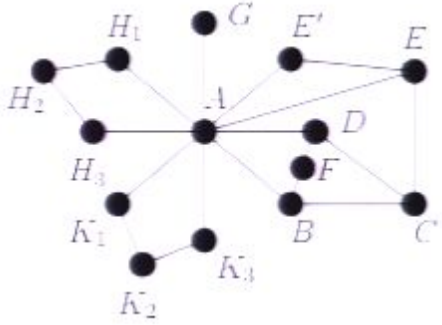

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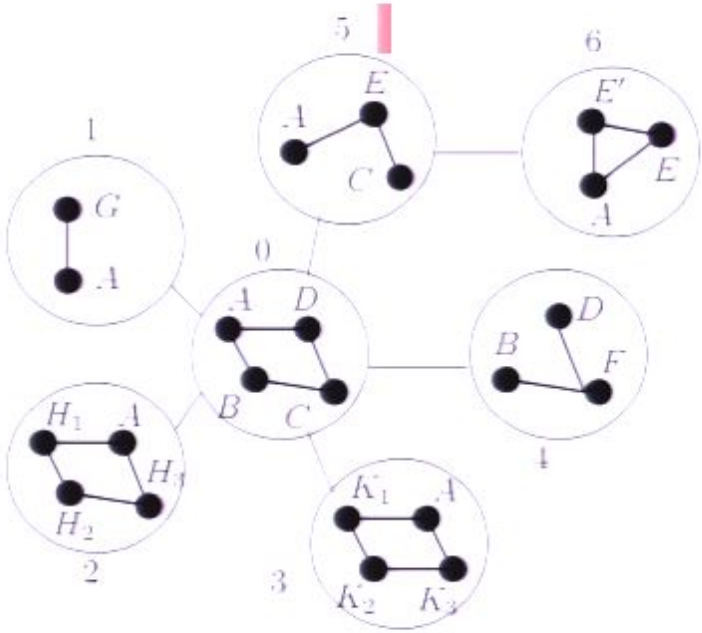

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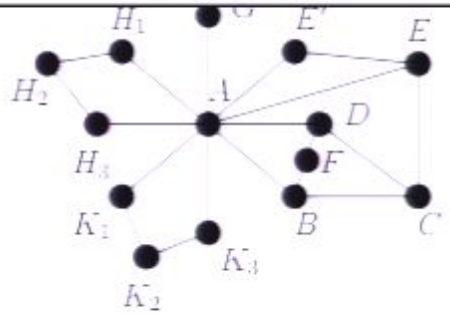

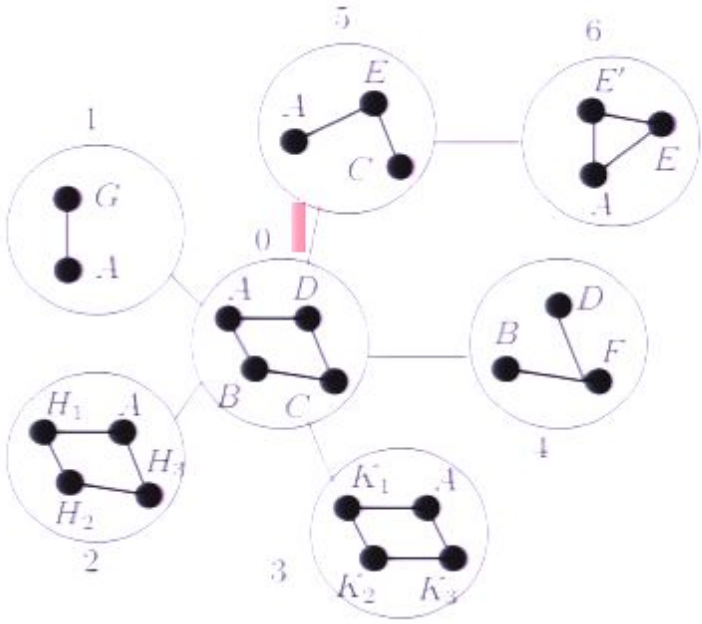

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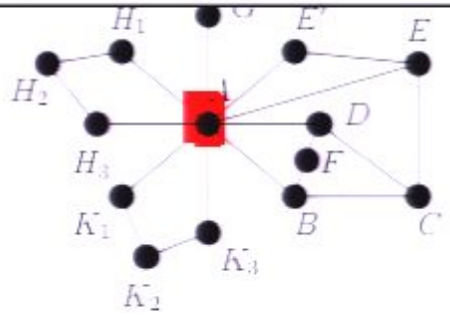

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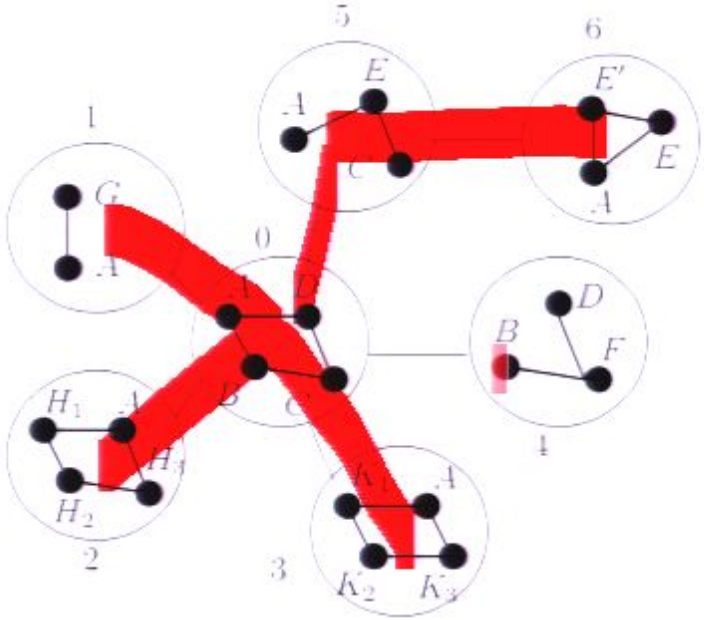

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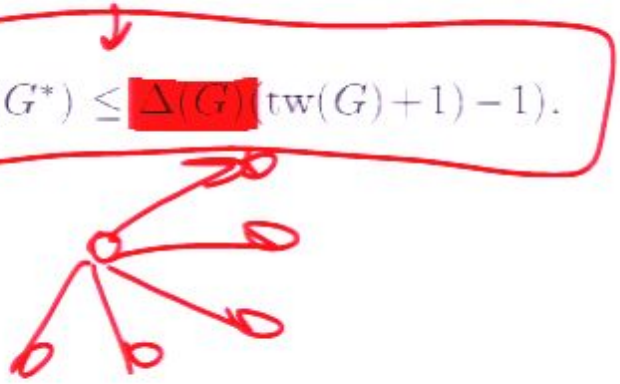

Back to the simulation of one-way computation

Fix a circuit C_G creating $|G\rangle$. A simulation algorithm:

- Select the next qubit to be measured according to the one-way algorithm.
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- Repeat till the end of the one-way algorithm.
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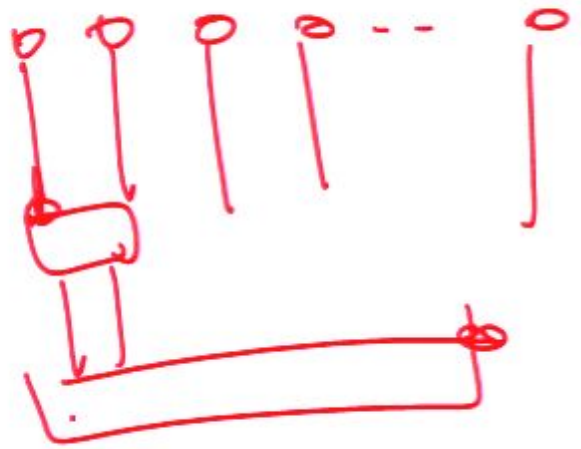

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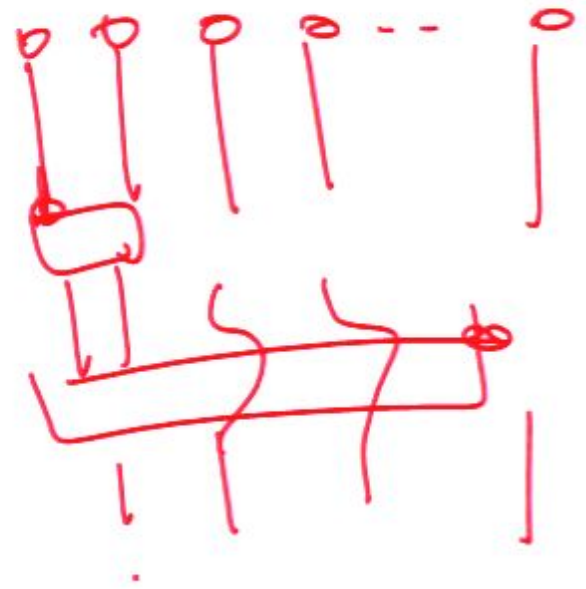

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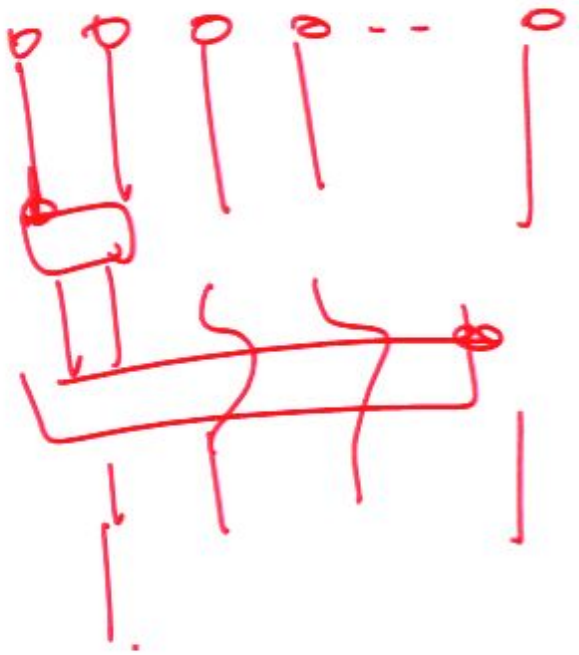

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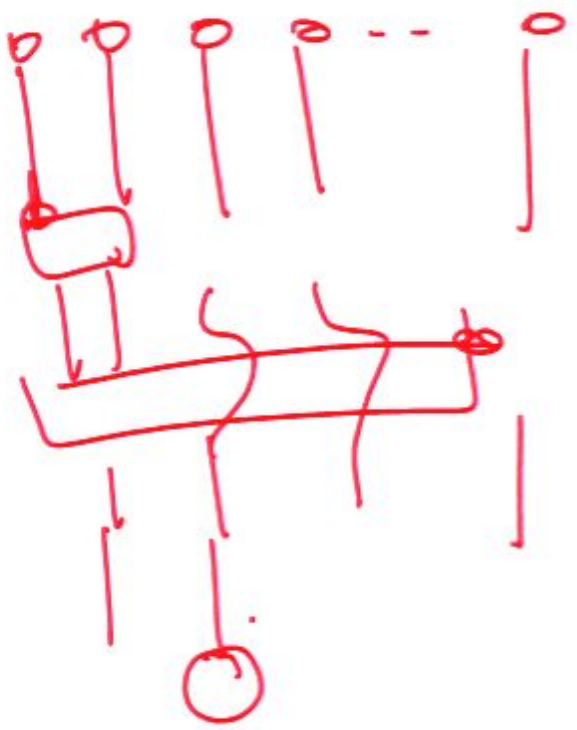

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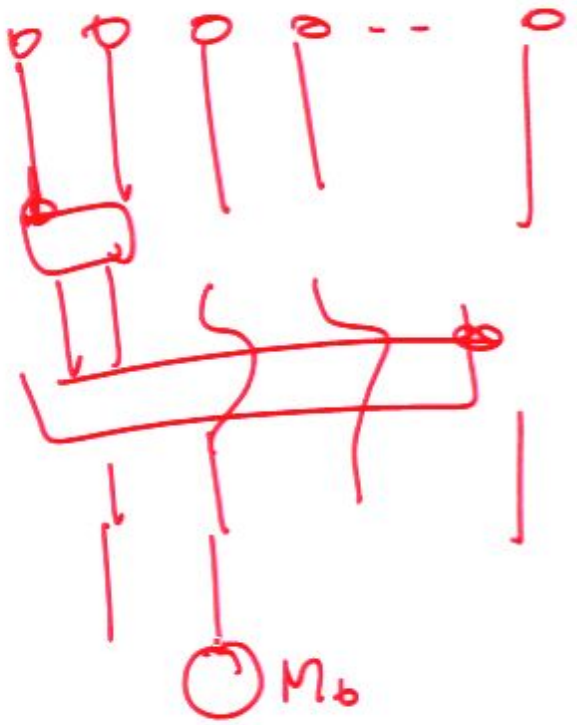

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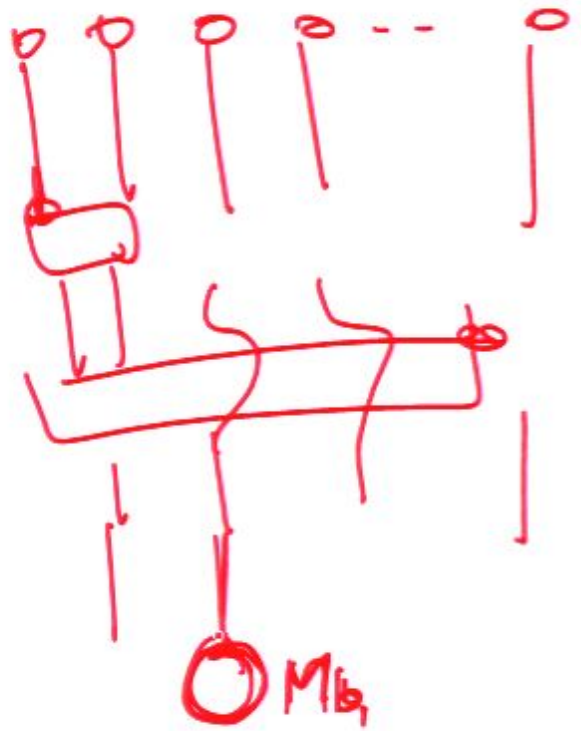

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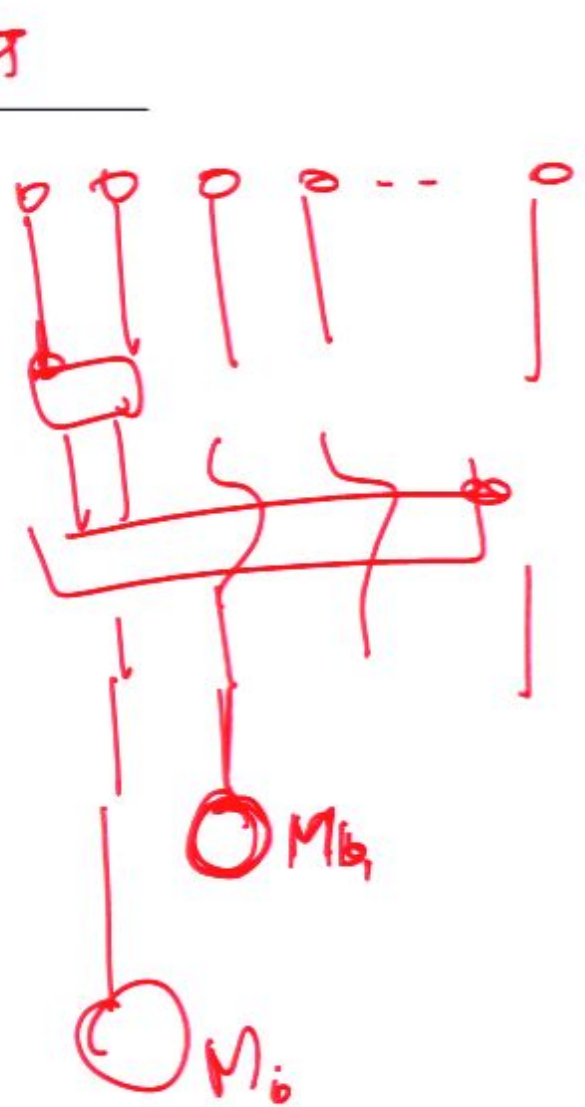

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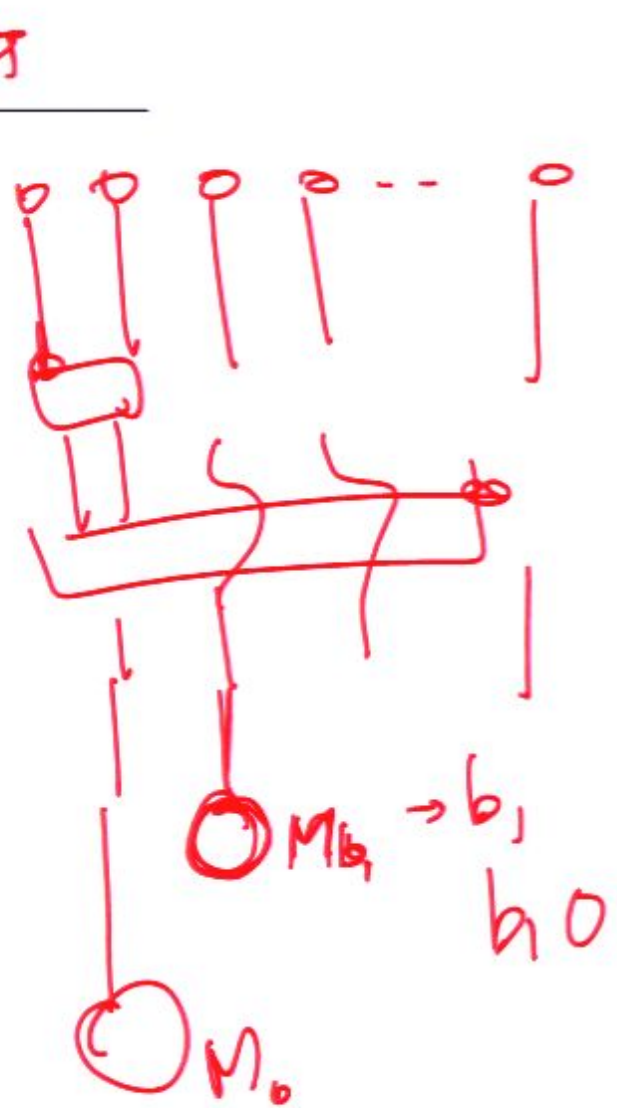

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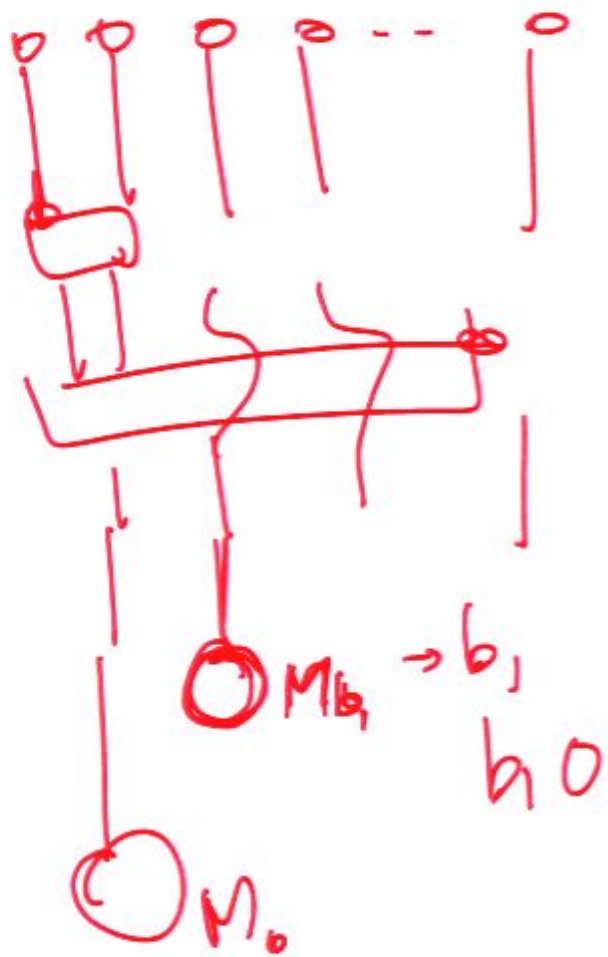

Fix a circuit C_G creating $|G\rangle$. A simulation algorithm:

- Select the next qubit to be measured according to the one-way algorithm.
- Attach the corresponding measurement element to the circuit.
- Compute the probability p_0 of outputting 0 through tensor contraction.
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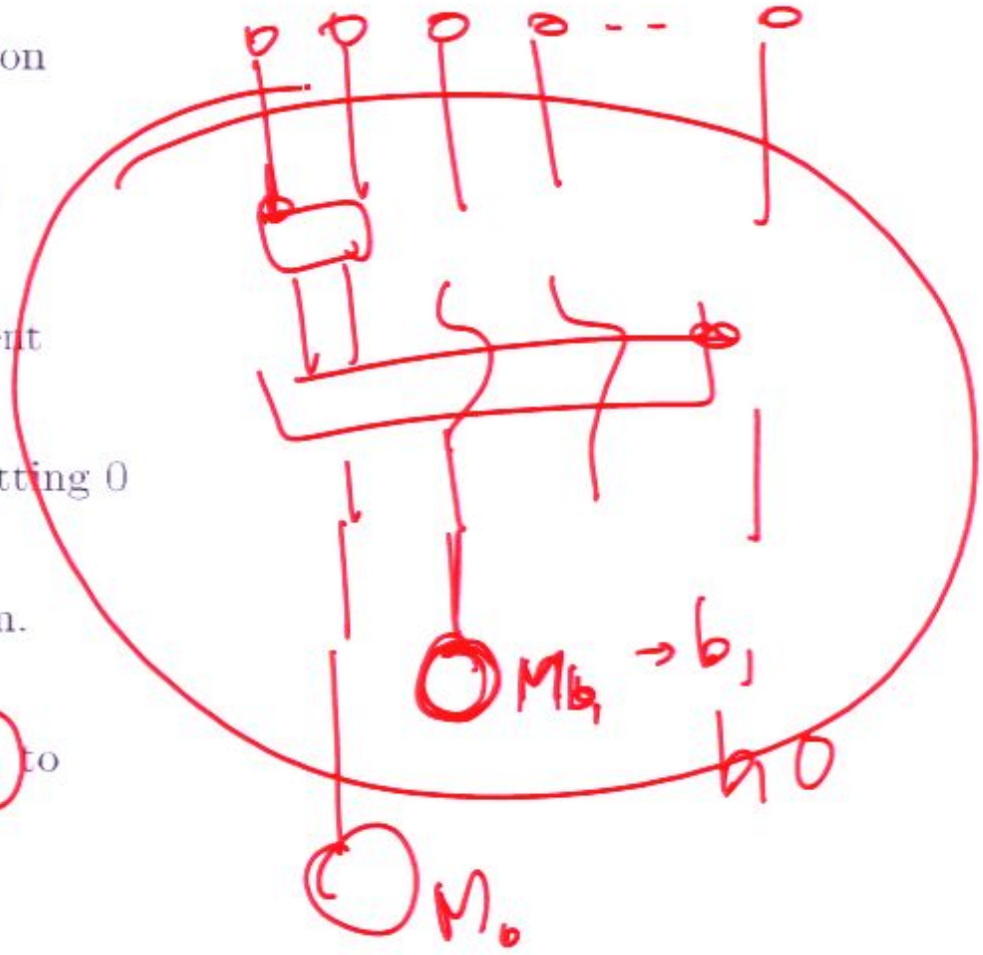

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Complexity: $O(T)\exp(\mathrm{CC}(C_G))$.

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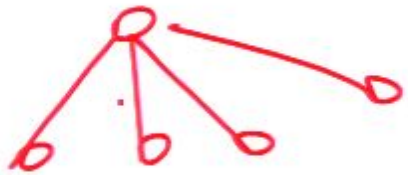

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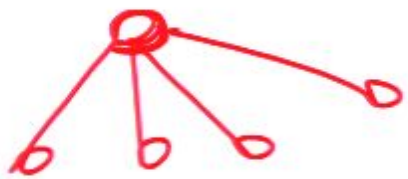

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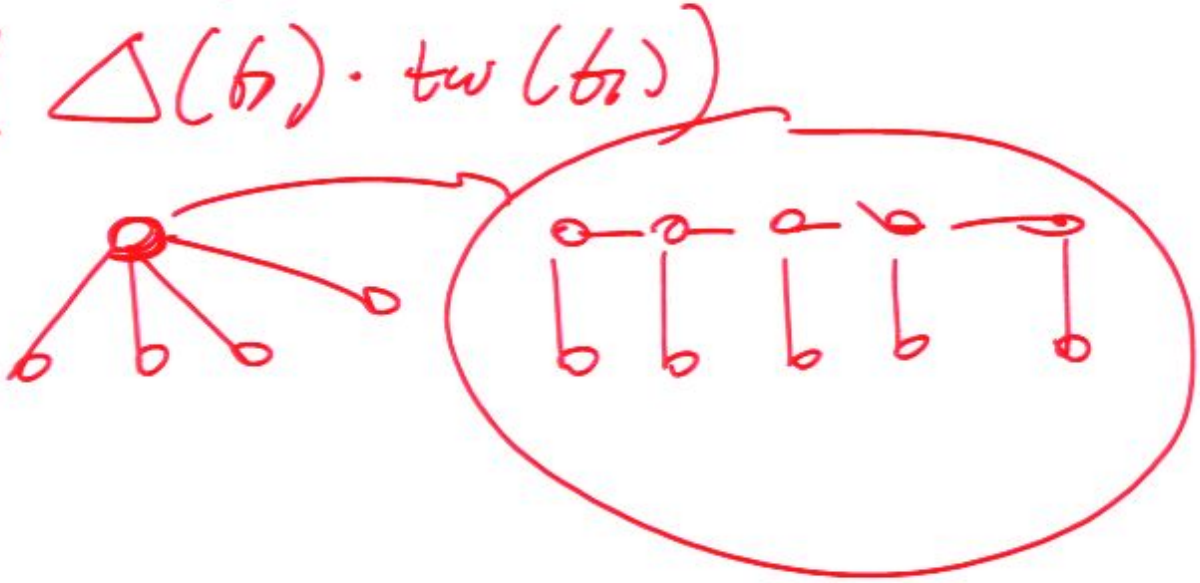

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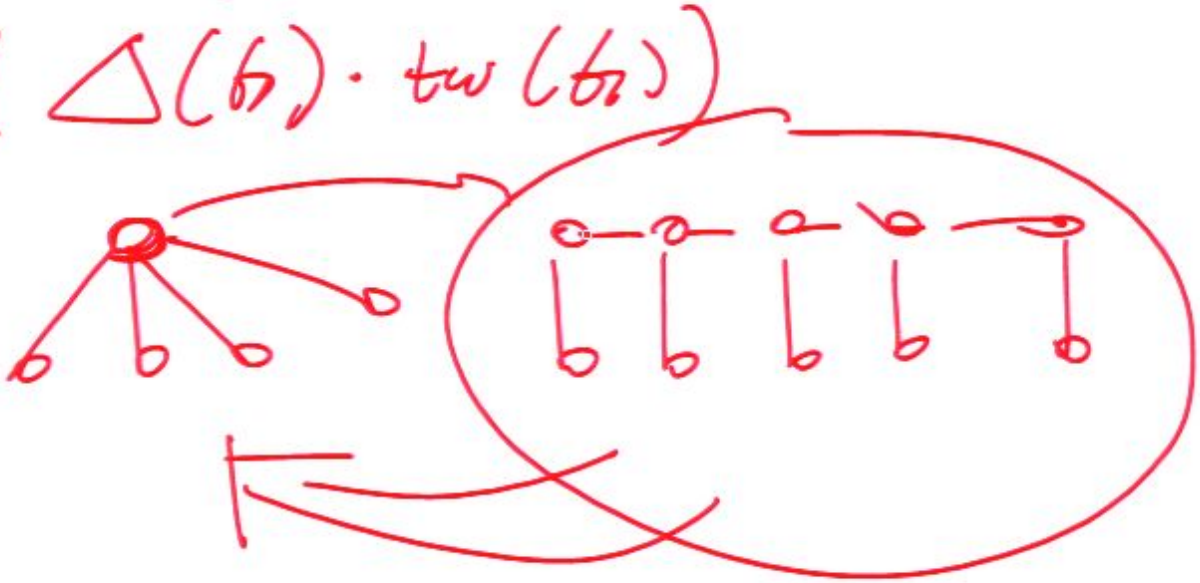

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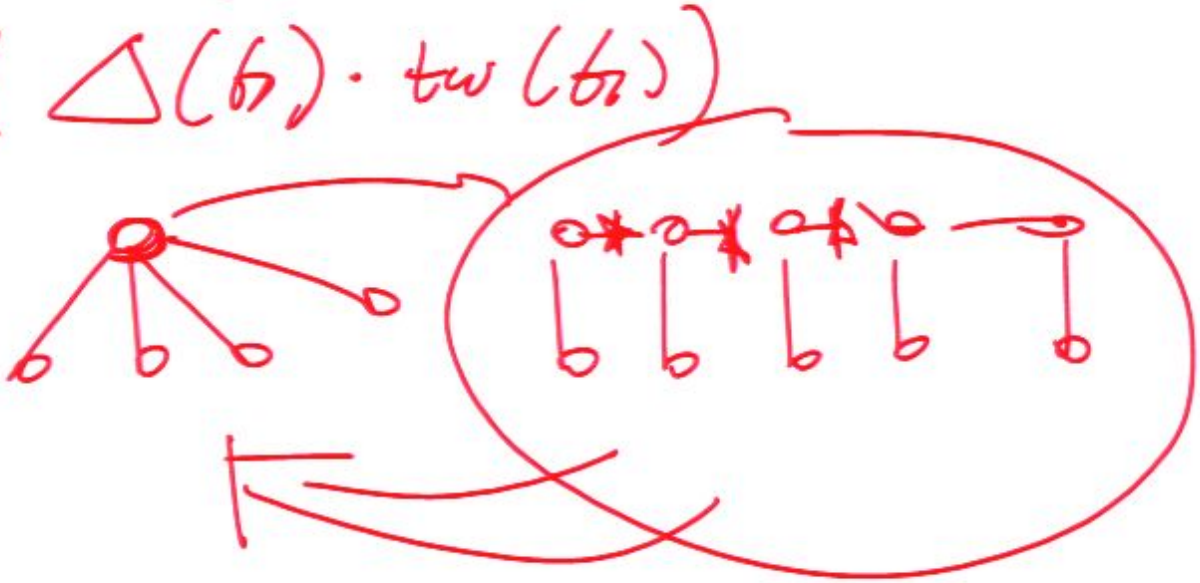

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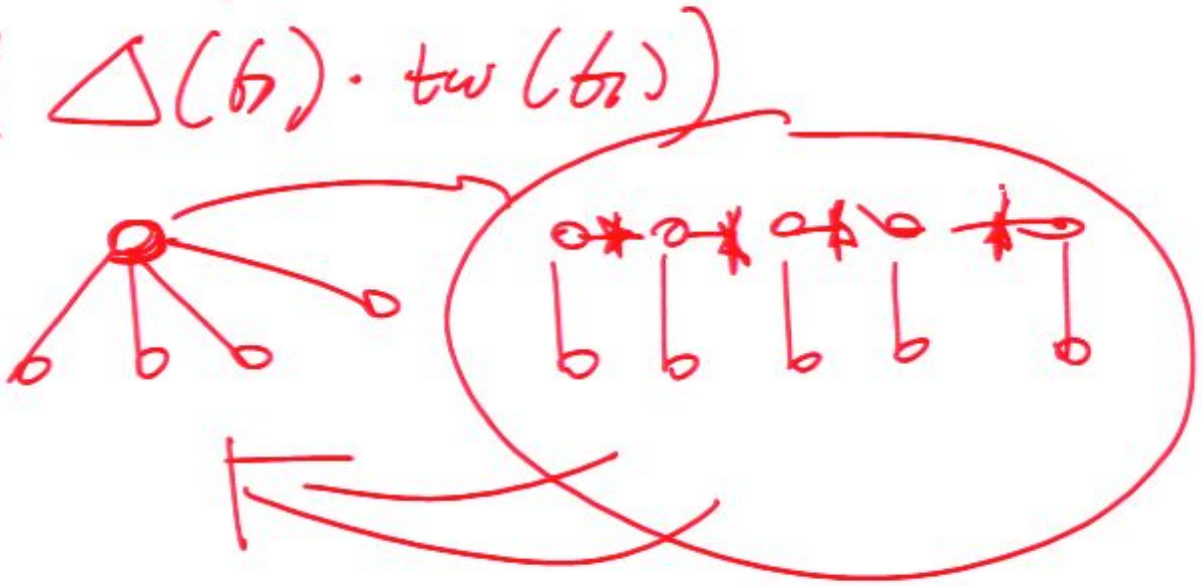

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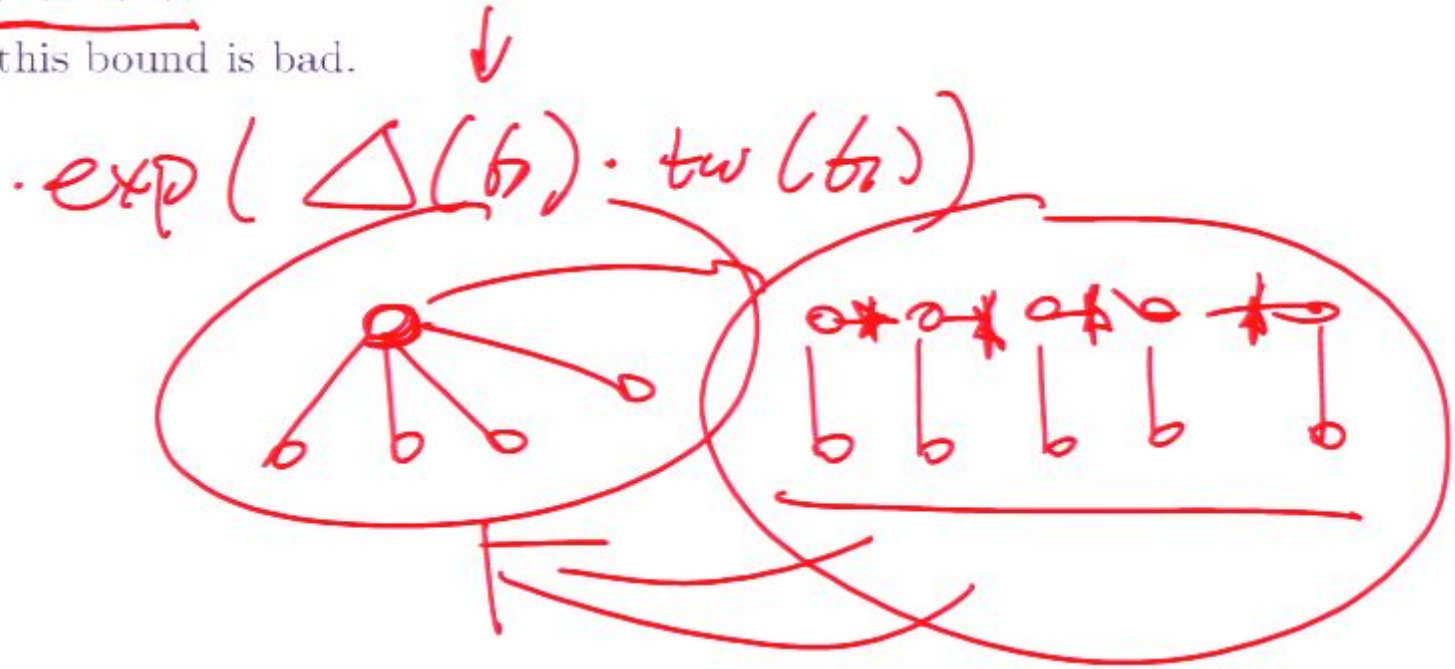
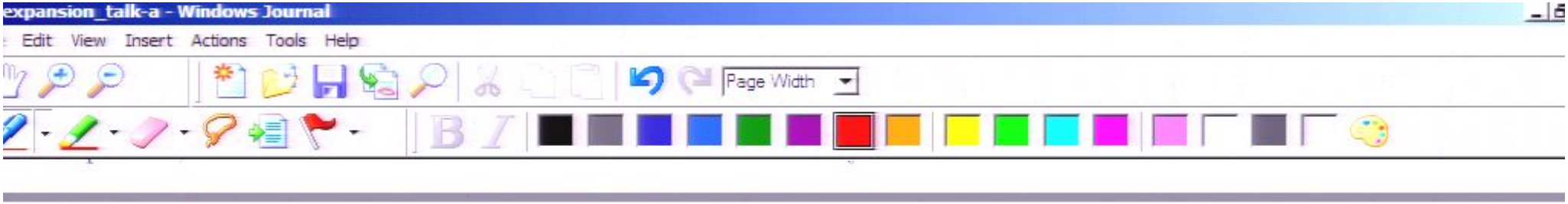


Graph expansion



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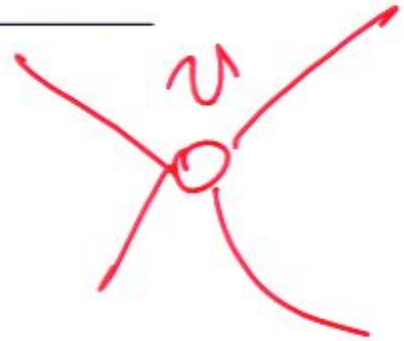

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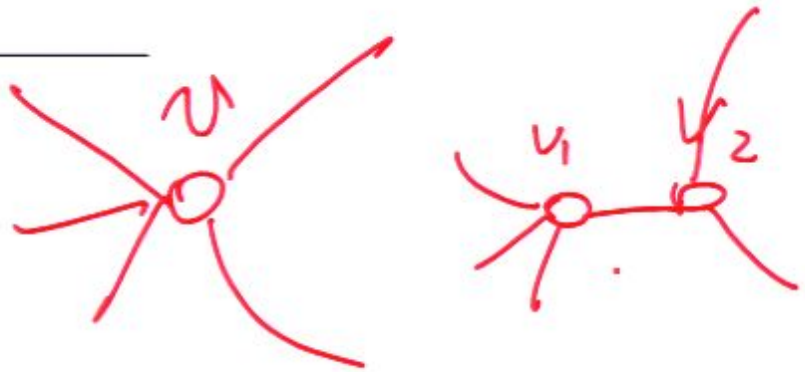

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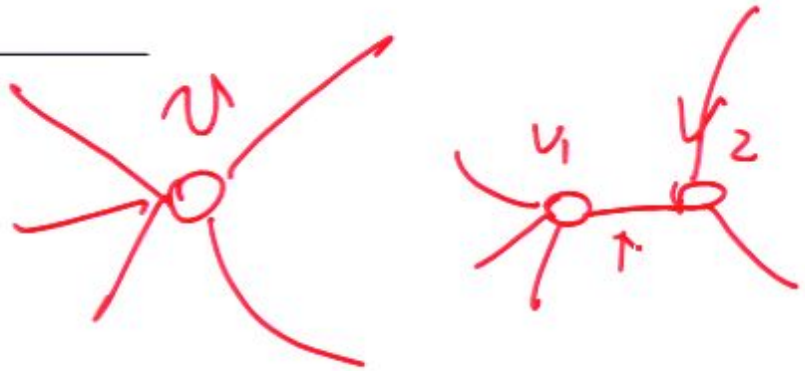

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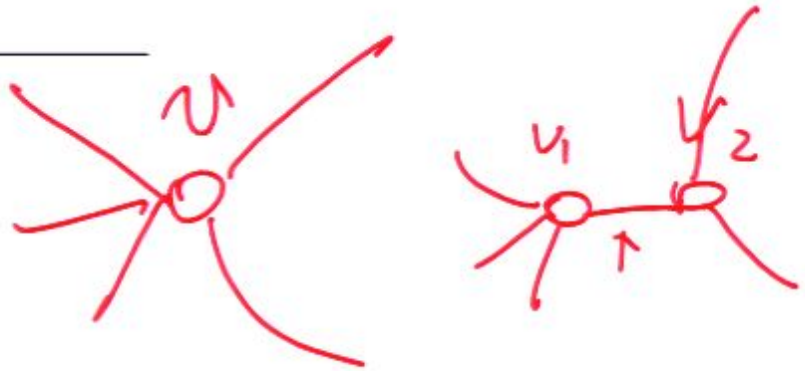

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Use a graph expansion for one-way computation

Fact [RB'01]. Suppose that G' is an expansion of G. and G'' is obtained from G' by inserting a new vertex on each edge. Then there exists a one-way computation on $|G''\rangle$ that produces $|G\rangle$.

Thus a T-step one-way computation on $|G\rangle$ can be simulated in time (and space) $O((T - |V(G'')|)\exp(\Delta(G'')\mathrm{tw}(G'')))$.

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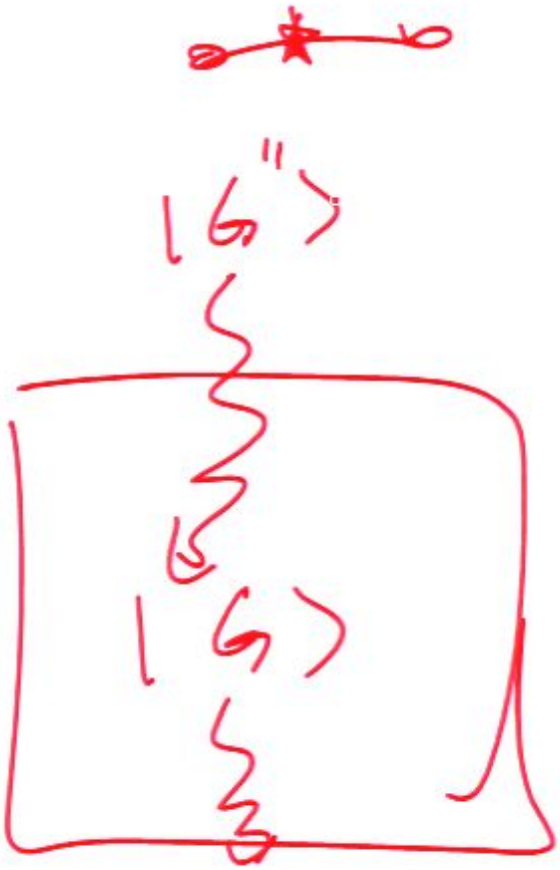

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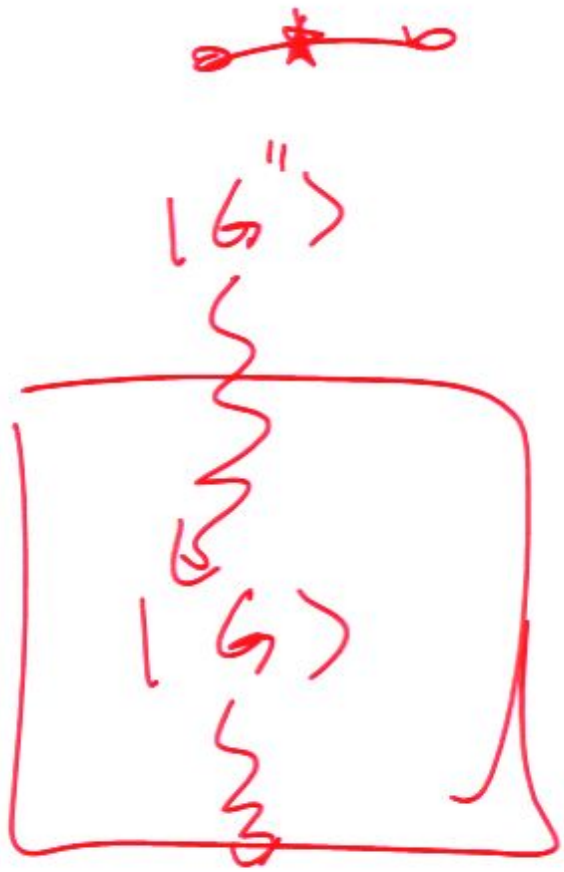

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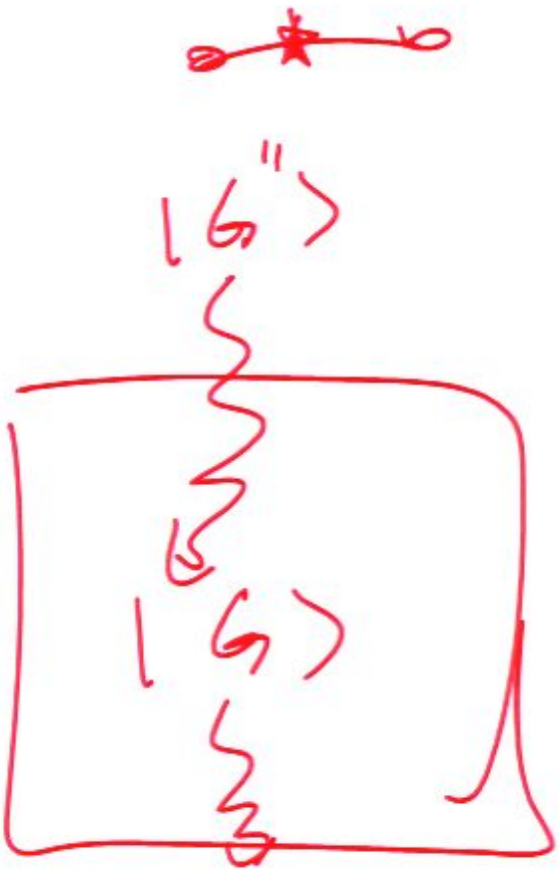

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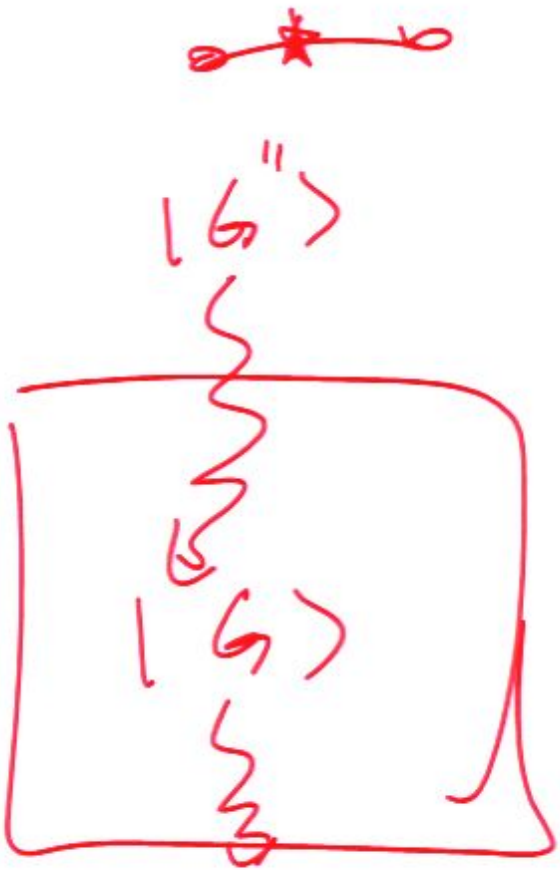

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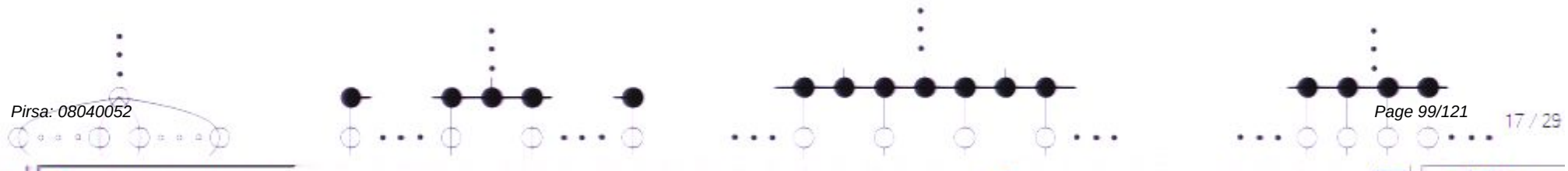

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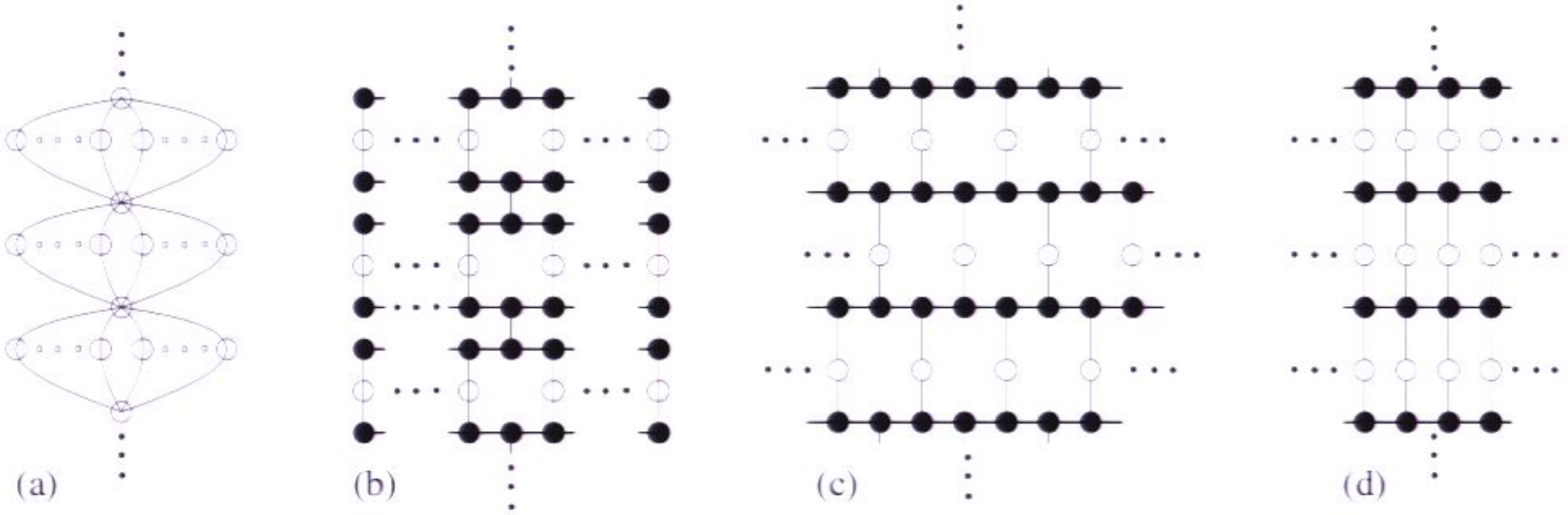

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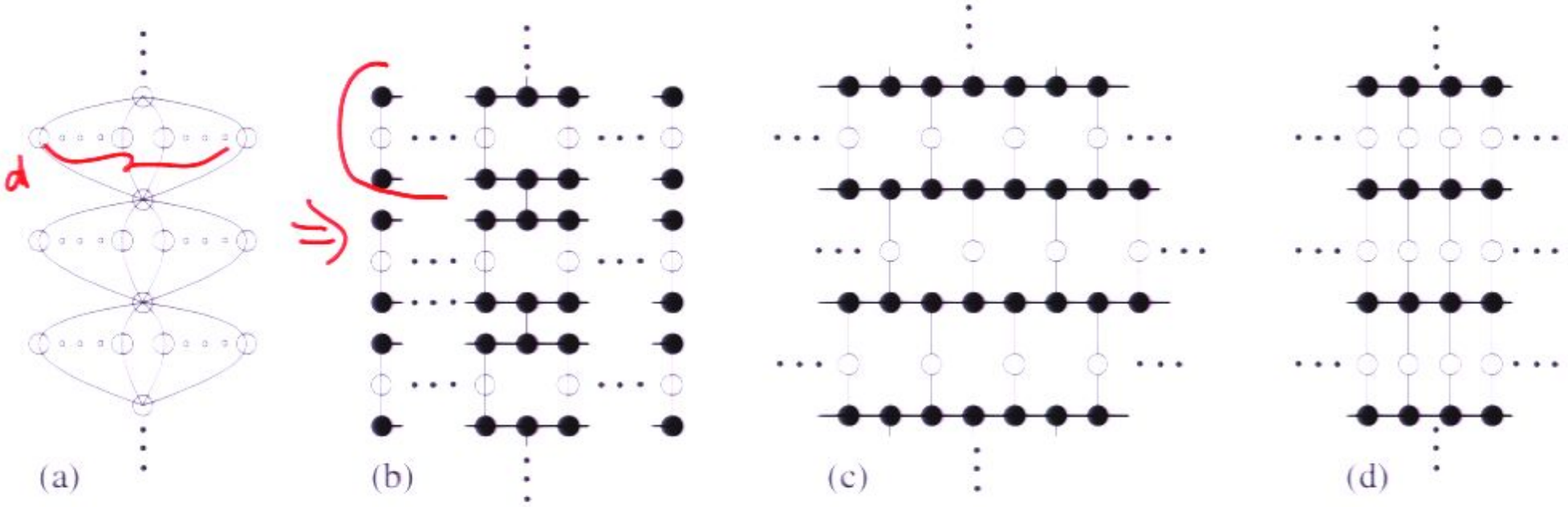

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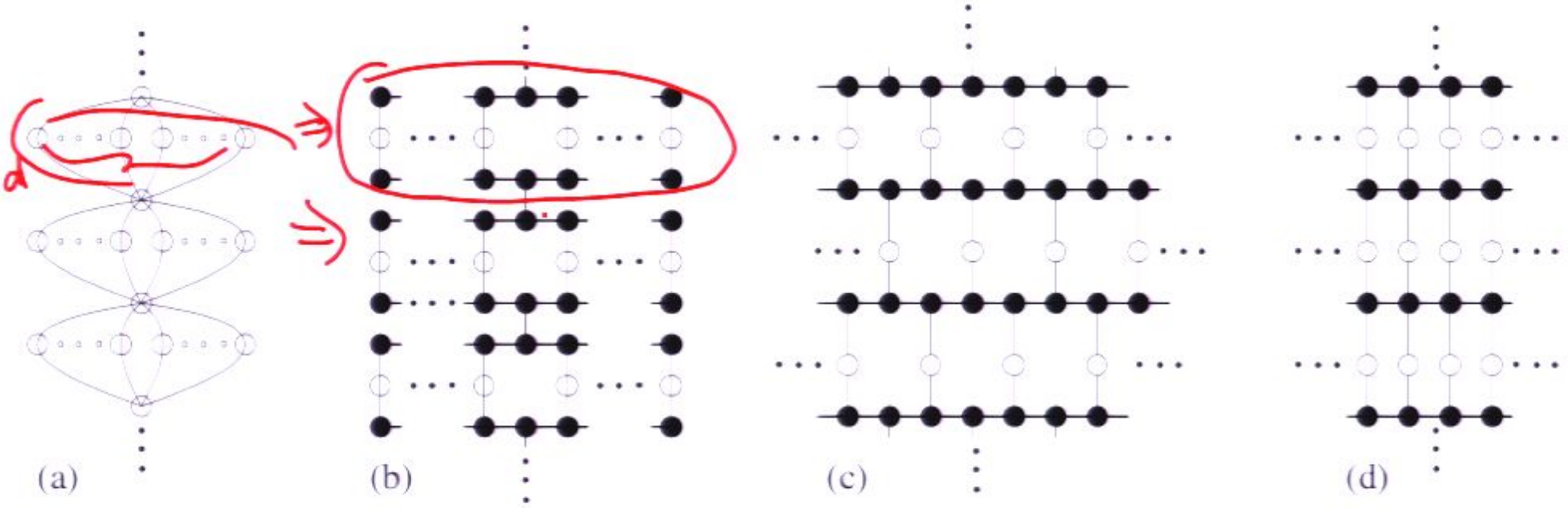

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Main Result

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Reducing in-bag max degree

Fix a tree decomposition $\mathcal{T}$ of G.

For each bag B and each $v \in B$, if v has ≥ 3 neighbors in B.

- Split $v \to v_1 v_2$.
- Attach a neighbor $u \in B$ to v_2, and all other neighbors to v_1.
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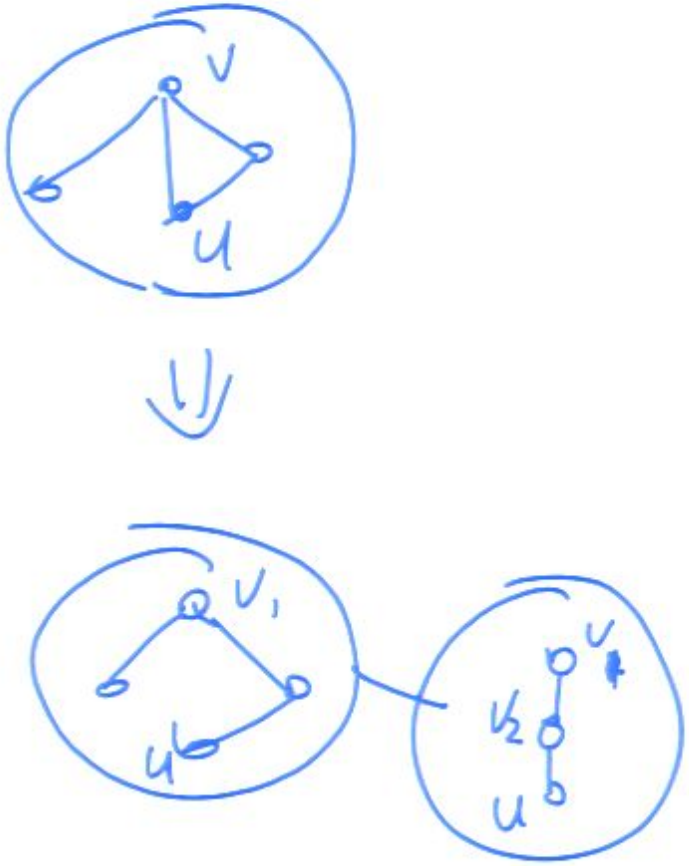
We can now assume that each vertex has ≤ 2 neighbors in each bag.



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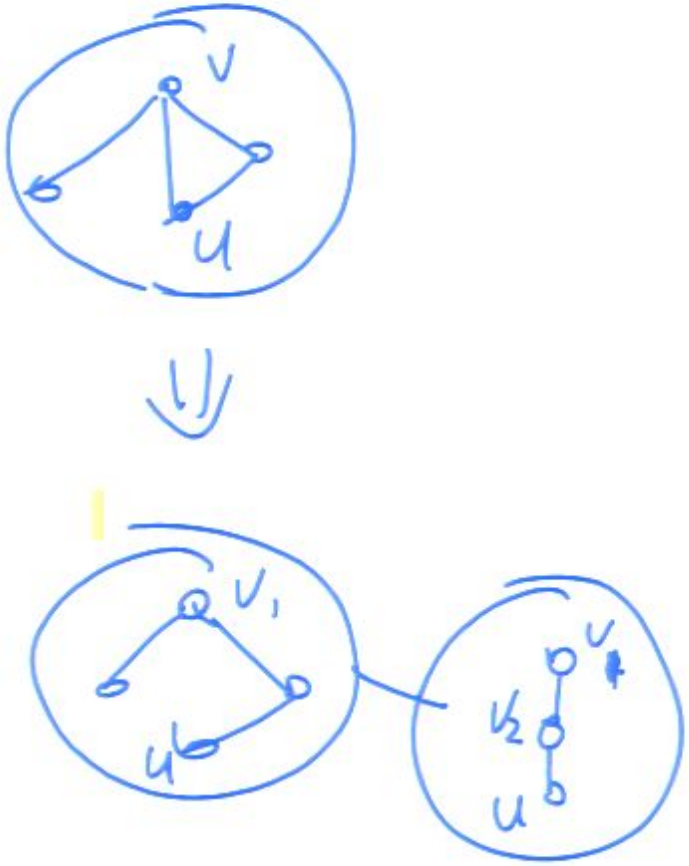

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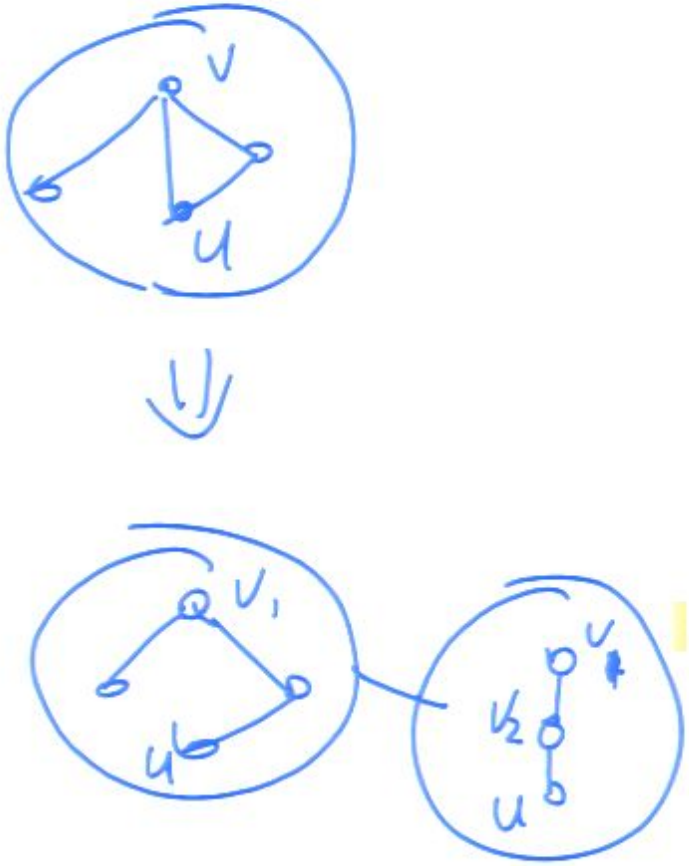


Constructing the expansion

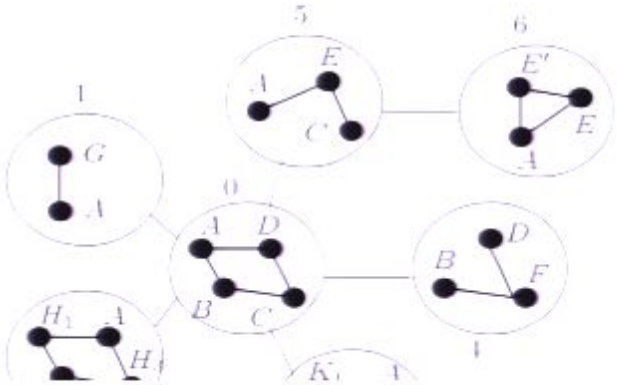
For each bag B_0 with adjacent bags B_1, B_2, ... B_t, and each $v \in B_0$, create a path graph

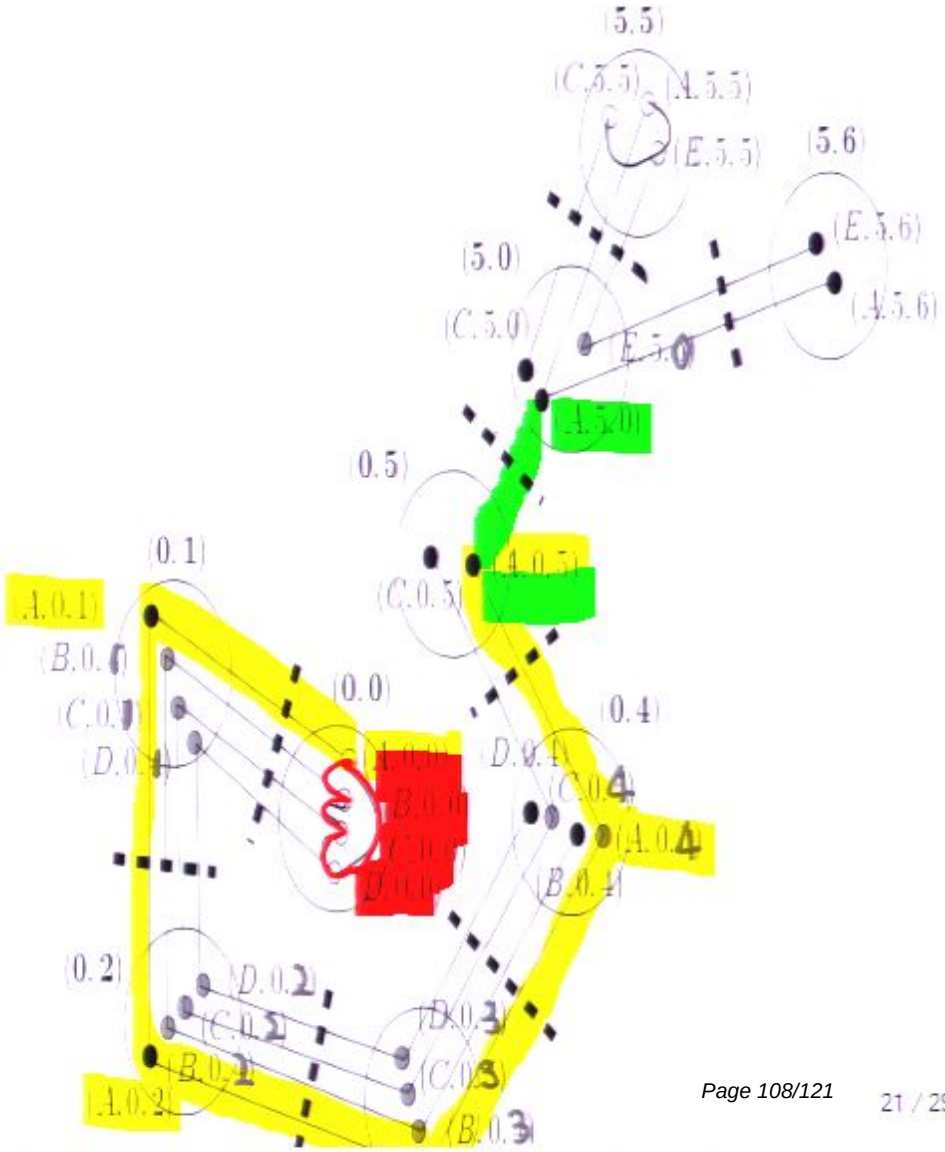
$$(v, B_0, B_0) - (v, B_0, B_1) - \cdots - (v, B_0, B_t).$$

Add the following edges

- If $v \in B \cap B'$, connect $(v, B, B') - (v, B', B)$.
- For each edge $uv \in E(G)$, fix a bag B_0 with $u, v \in B_0$, connect $(u, B_0, B_0) - (v, B_0, B_0)$.

Call the resulting graph G'.





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B_t, and each $v \in B_0$, create a path graph

$$(v, B_0, B_0) - (v, B_0, B_1) - \cdots - (v, B_0, B_t).$$

Add the following edges

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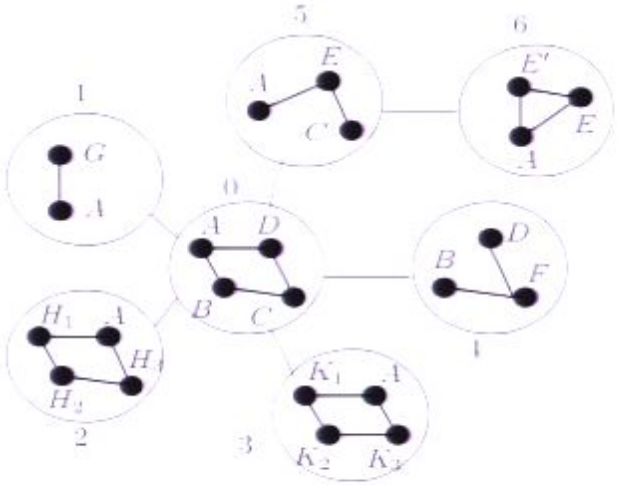

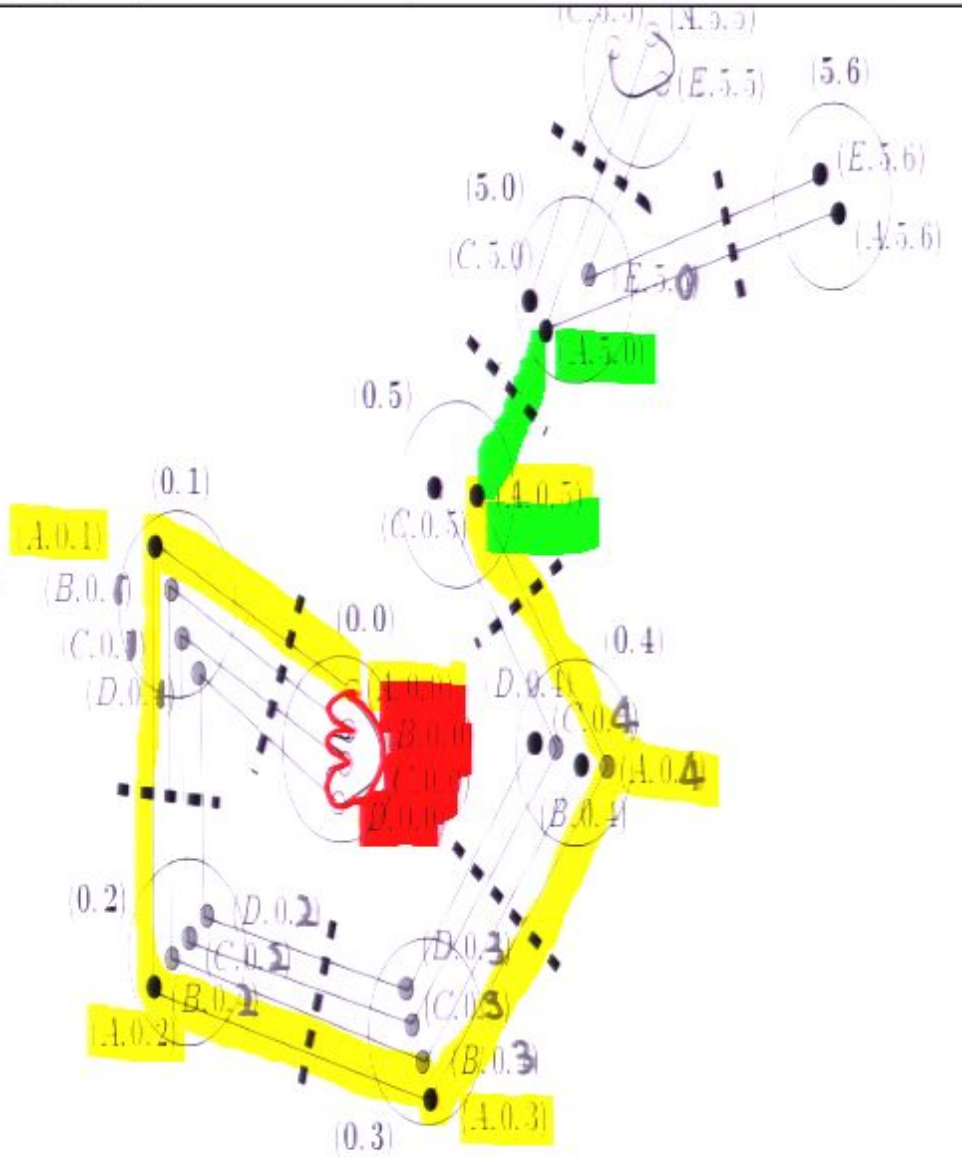

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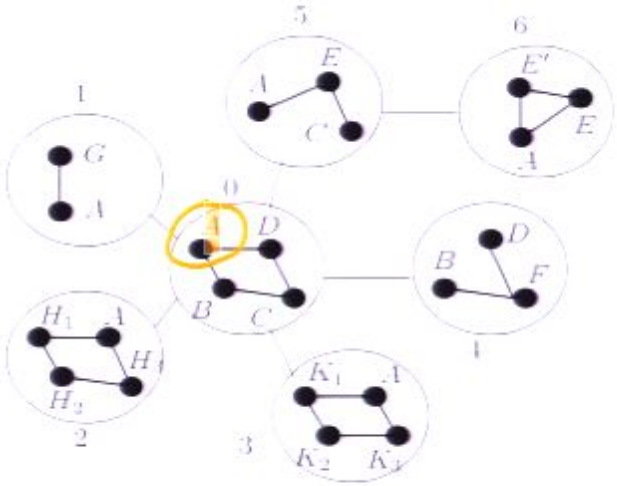

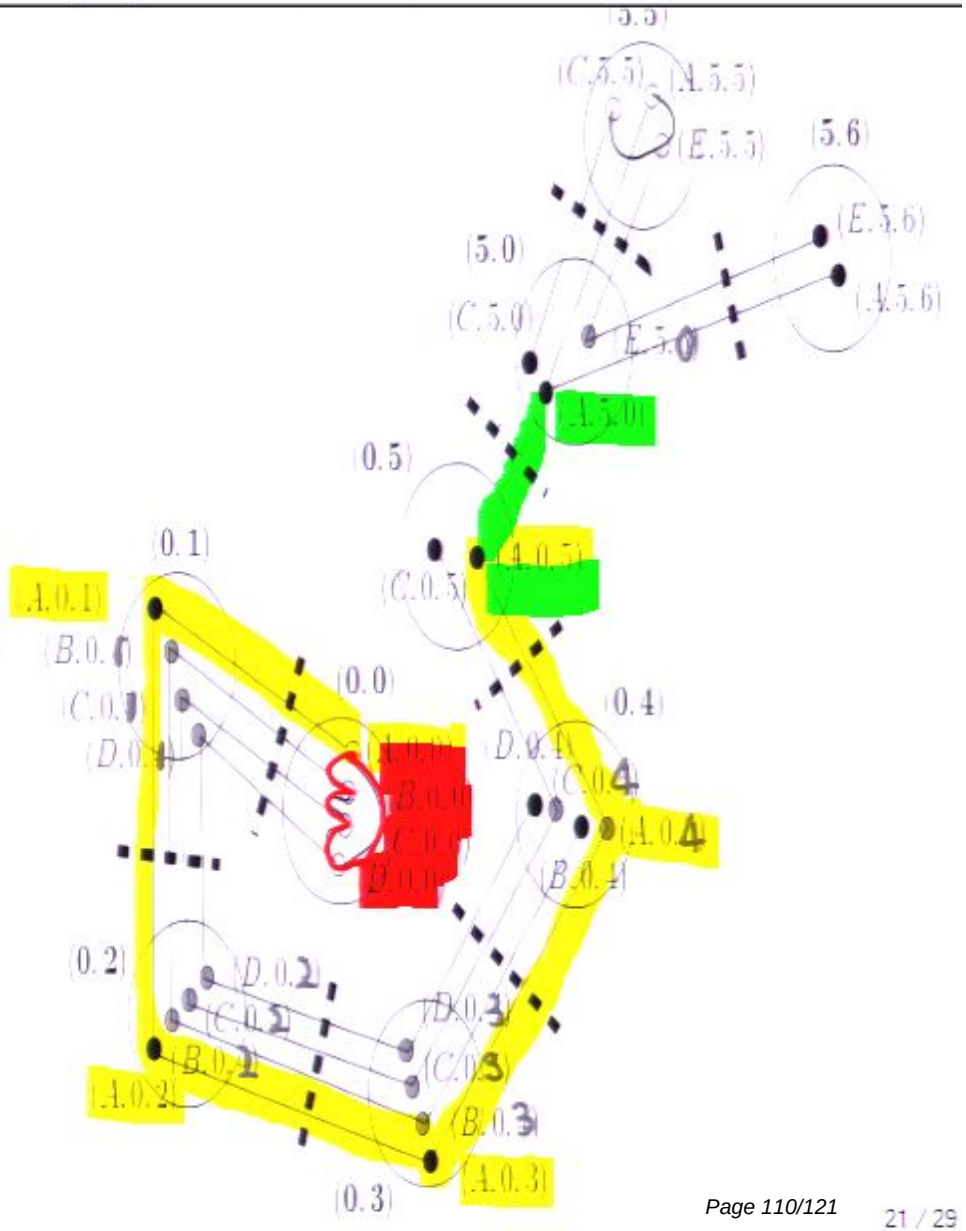

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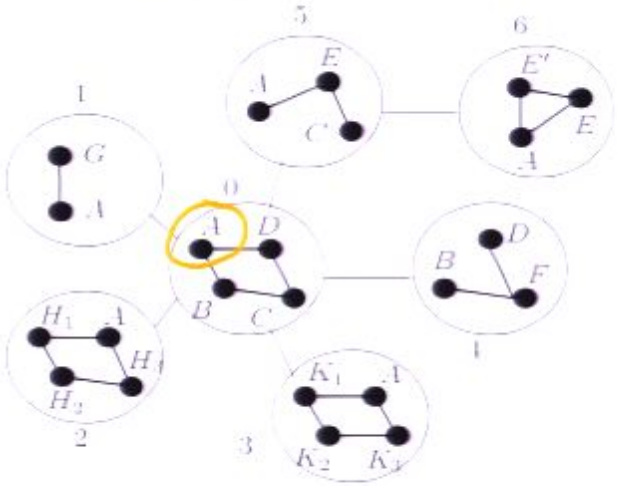

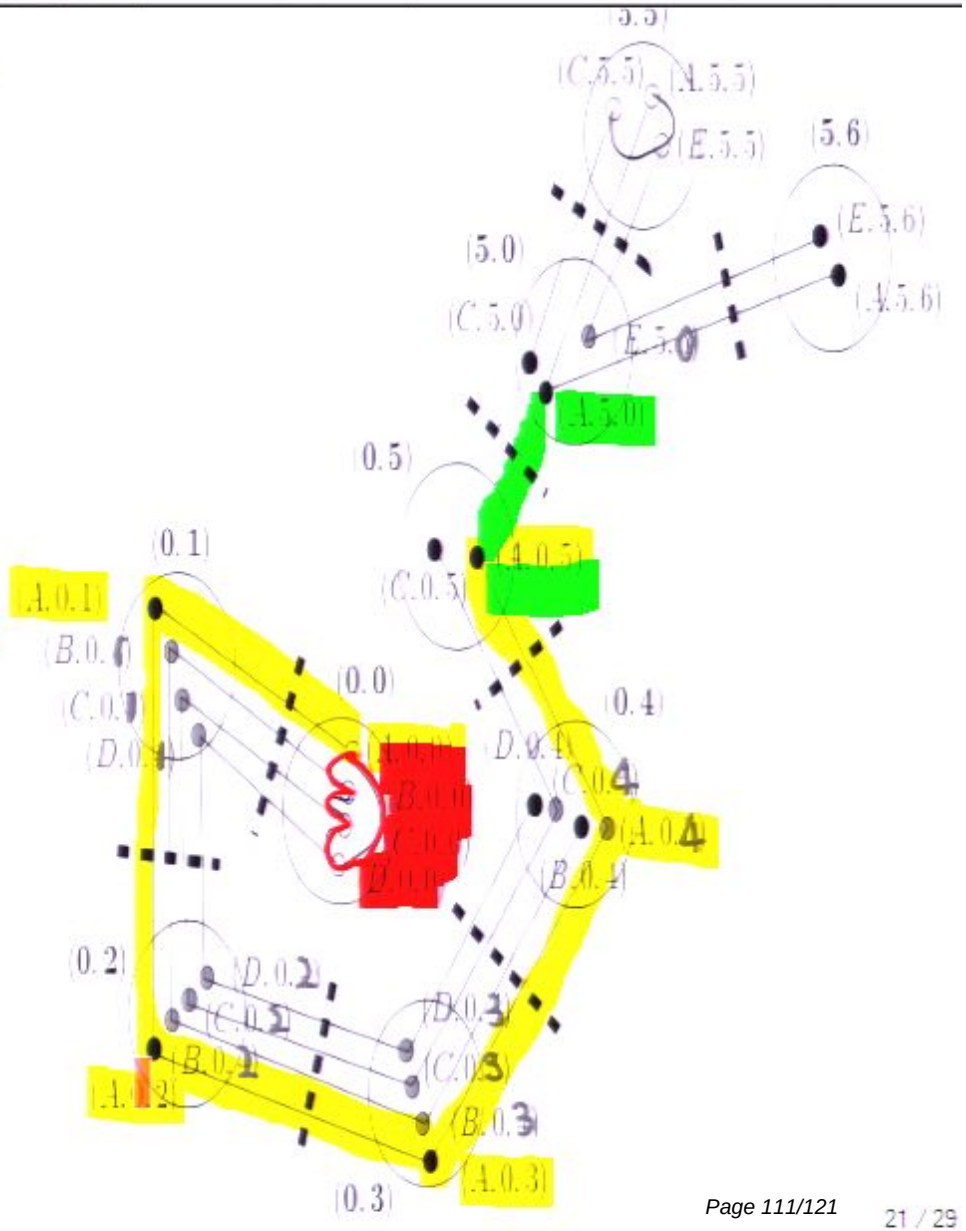

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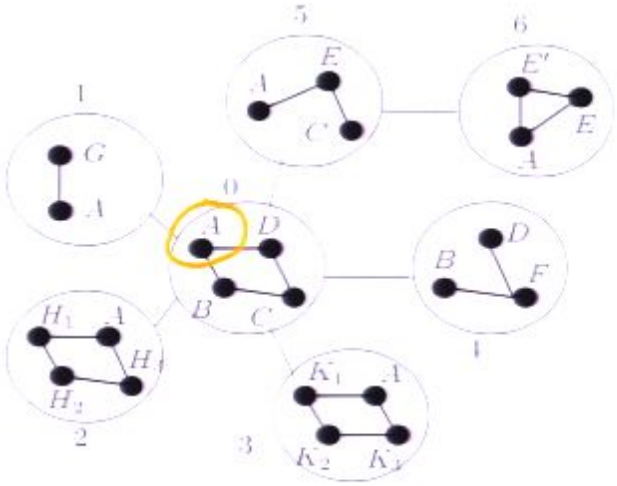
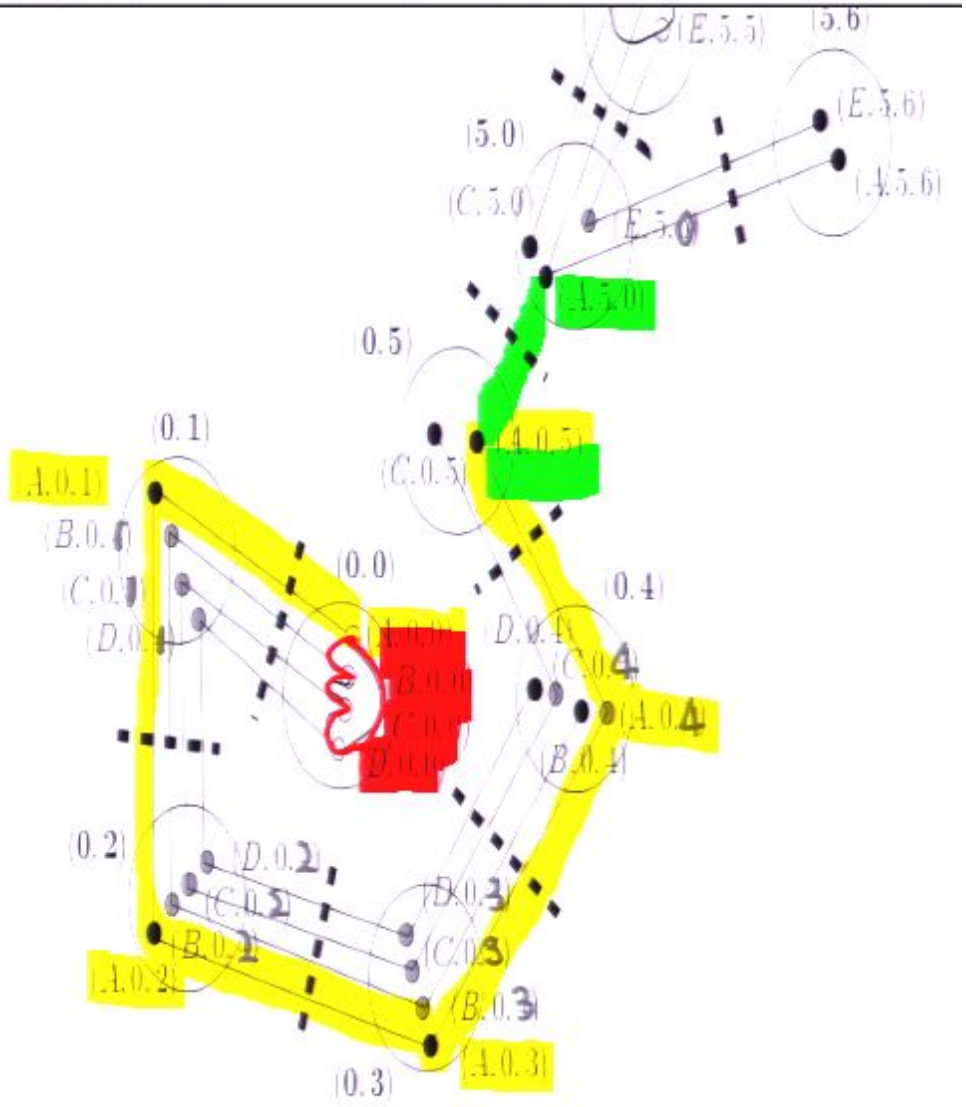




Three types of bags:

- For each bag B of $\mathcal{T}$, create a bag

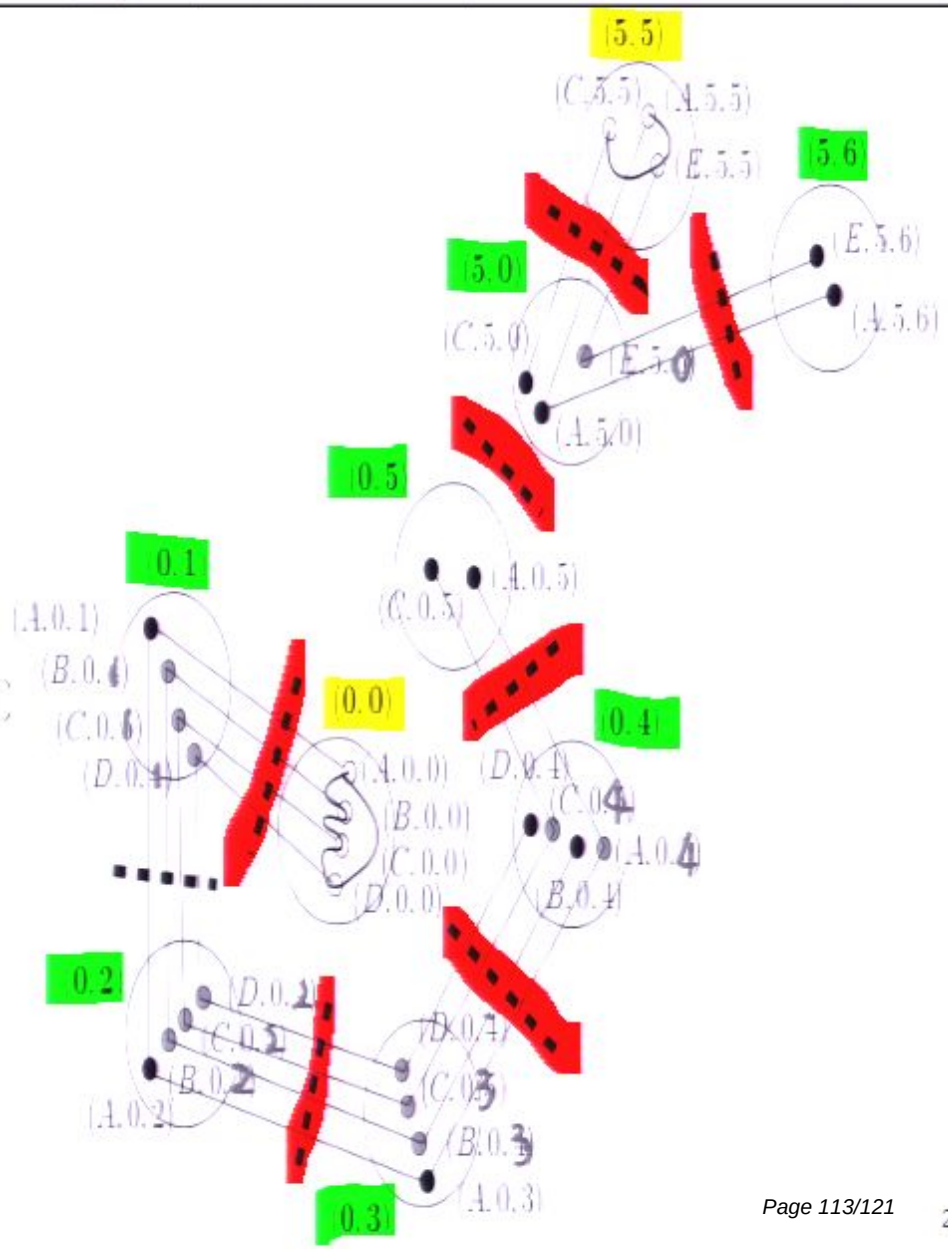
  $$(B, B) = \{(v, B, B) : v \in B\}.$$

  This bag covers all edges between vertex trees.

- For adjacent bags $B_0 B_i$ in $\mathcal{T}$, create

  $$(B_0, B_i) = \{(v, B_0, B_i) : v \in B\}.$$

- "Bridge" the bags (B_0, B_i) and (B_0, B_{i-1}) and the bags (B, B') and (B', B) by a chain of bags.



- $A \cap B = \{u_1, \cdots, u_s\}$, and
- $E_{AB} \stackrel{\text{def}}{=} \{v_i v_i' : 1 \le i \le t\}$, are edges in G'.

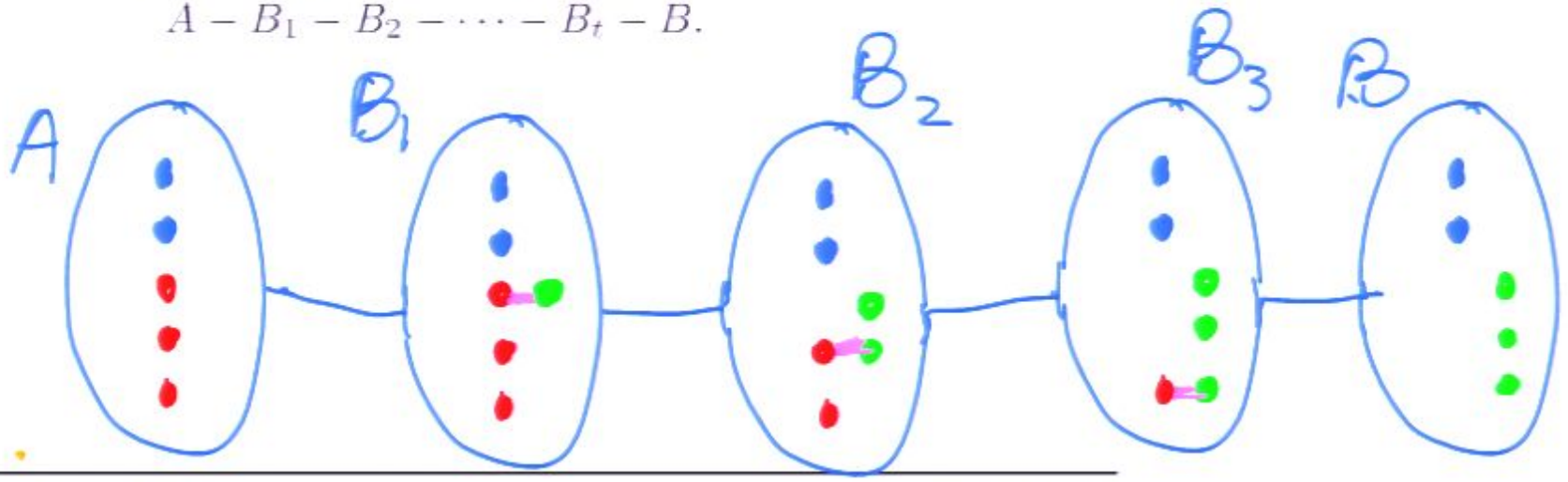
Bridging the two bags to cover E_{AB}:

(Step 1) Create bags $B_1, B_2, \cdots, B_t$, where

$$B_i = (A \cap B) \cup \{v_1', v_2', \cdots, v_{i-1}', v_i', v_i, v_{i+1}, \cdots, v_t\}.$$

(Step 2) Form a path graph of bags

$$A - B_1 - B_2 - \cdots - B_t - B.$$



Concluding the proof

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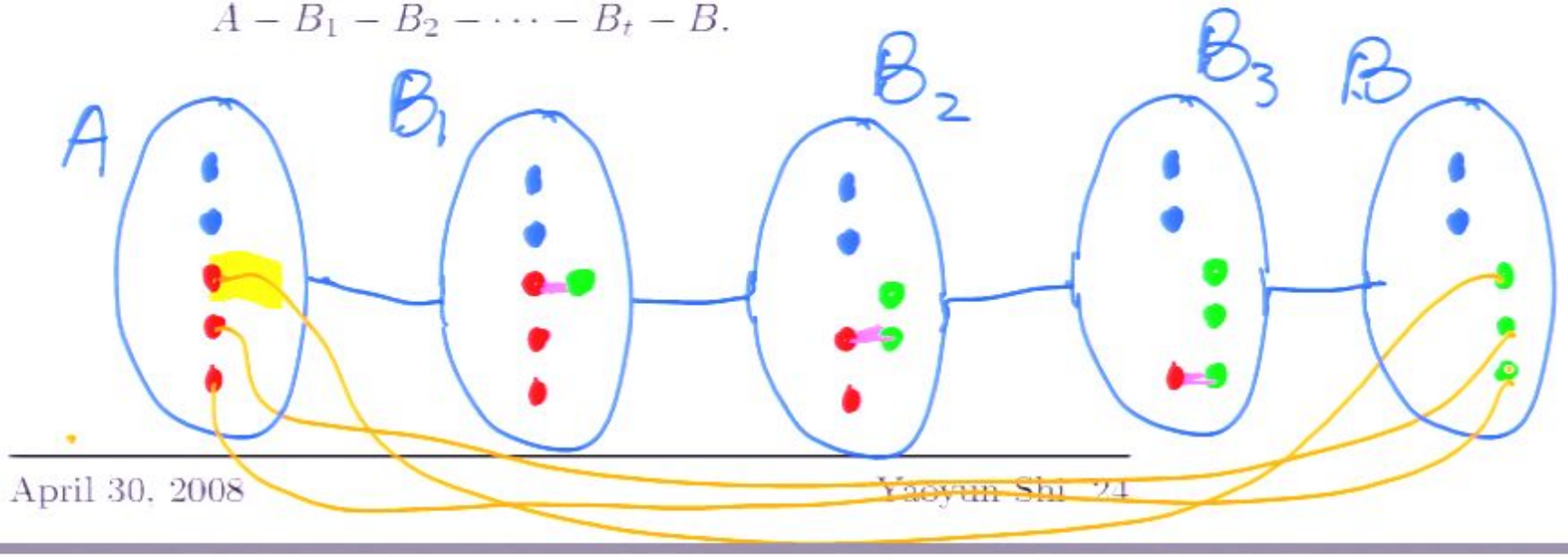

Concluding the proof

Open problems

- Which states are universal for measurement-based computation?
  - A universal state does not need to be a graph state [Gross and Eisert '06...]
  - A definition of universality.
    A family of states $|S_n\rangle$ on n qubits is universal if for any quantum circuit of size m, there exists a measurement-based quantum computation on $|S_n\rangle$, for some $n = poly(m)$, that creates the same state as the final state of the circuit.
- Sharpen the classical upper bound of BQP (the class of functions computable by a polynomial time quantum algorithm).
  - A reduction from a BQP problem to $\exists\forall$ SAT for a constant number of

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