Title: Quantum information, graphs, and statistical mechanics

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Abstract: We give an overview of several connections between topics in quantum information theory, graph theory, and statistical mechanics. The central concepts are mappings from statistical mechanical models defined on graphs, to entangled states of multi-party quantum systems. We present a selection of such mappings, and illustrate how they can be used to obtain a cross-fertilization between different research areas. For example, we show how width parameters in graph theory such as \'tree-width\' and \'rank-width\', which may be used to assess the computational hardness of evaluating partition functions, are intimately related with the entanglement measure \'entanglement width\', which is used to asses to computational power of resource states in quantum information. Furthermore, using our mappings we provide simple techniques to relate different statistical mechanical models with each other via basic graph transformations. These techniques can be used to prove that that there exist models which are \'complete\' in the sense that every other model can be viewed as a special instance of such a complete model via a polynomial reduction. Examples of such complete models include the 2D Ising model in an external field, as well as the zero-field 3D Ising model. Joint work with W. Duer, G. de las Cuevas, R. Huebener and H. Briegel

Quantum computation, graphs and statistical mechanics

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Collaborators: W. Duer, H. Briegel, R. Raussendorf, G. De las Cuevas, R. Huebener

What we are trying to do...

- There exist several connections between statistical mechanics and quantum information theory

  - MPS, PEPS <> ground state approximation (Verstraete, Cirac,..)
  - Quantum algo for Potts model partition function (Arad, Aharonov)
  - Toric code states <> classical Ising model (Kitaev, Bravyi, Raussendorf)
  - Many others – just look around you in the Bob room ;-)

- These connections may (and do) provide conceptual insights
- More importantly, they sometimes allow to solve difficult problems in one model by mapping it to easier problems in another model
- In other words, they are practically useful since they allow to interchange techniques and tools between different areas

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What we are trying to do...

→ Could such fruitful connections also be established to study computational power of quantum computation?

- Our (professional) hopes and dreams: finding connections between stat mech and QIT which teach us something about:

  - Classical simulation of QC, construction of simuleerbare gate sets, simuleerbare resource states, ...
  - Construct new quantum algorithms for stat mech problems, hopefully in simple and systematic way
  - Gain insight in computational complexity of stat mech systems (= 'from QIT to stat mech')

- For some of these dreams we have results, for some we do not

What we are going to do...

IDEA OF THE TALK

Present selection of new mappings:

- Related to circuit model
- Related to teleportation-based QC
- Related to one-way QC

Present selection of their applications

- Simulation of QC
- Quantum algo's
- Complexity of classical models

First class of mappings:

Quantum circuits

⇕

Vertex models (6-vertex, 8-vertex, ...)

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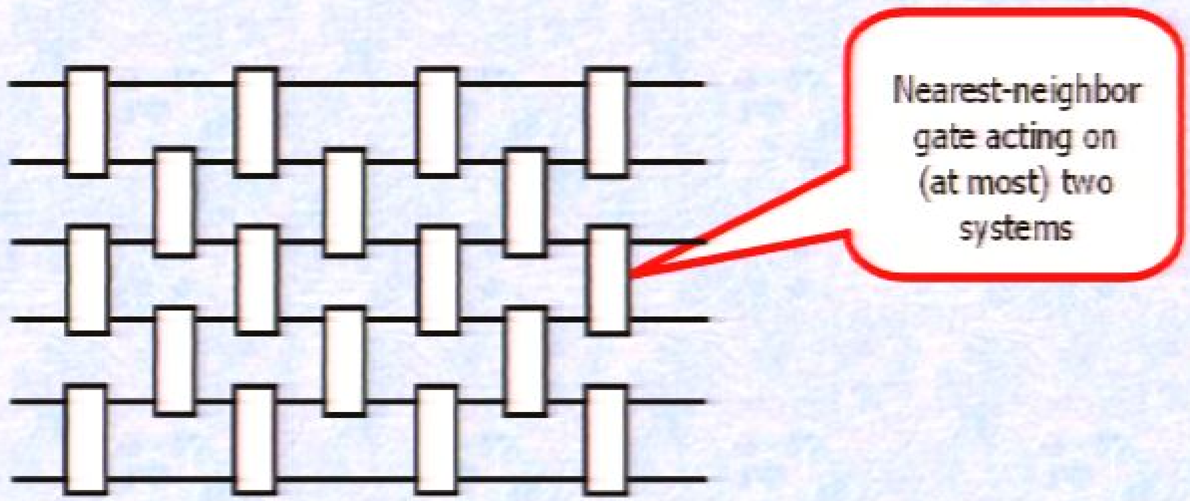
Quantum circuits and vertex models

- Consider a poly-size quantum circuit C as a quantum cellular automaton acting on q-level systems, with gates chosen from some elementary gate set S:

$C \equiv$ 

- We are interested in the hardness of classically computing $\langle x|C|y\rangle$ for arbitrary poly-size quantum circuits C (of above structure) built out of gates from S

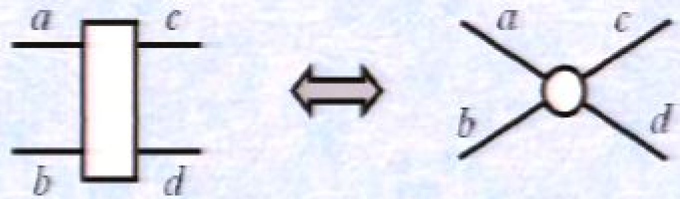
Quantum circuits and vertex models

- $\langle x | C | y \rangle$ = contraction of large number of small tensors

- There exists a fairly standard graphical representation of tensor contractions (see e.g., Markov and Shi, Vidal, many others):

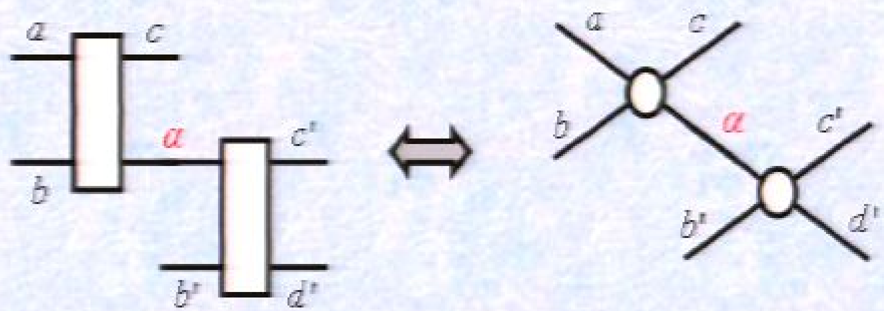
  - Each gate becomes a vertex with 4 incident edges:

$$U^{ab}_{cd} \equiv$$



  - Contraction of two indices = 'gluing together' corresponding edges

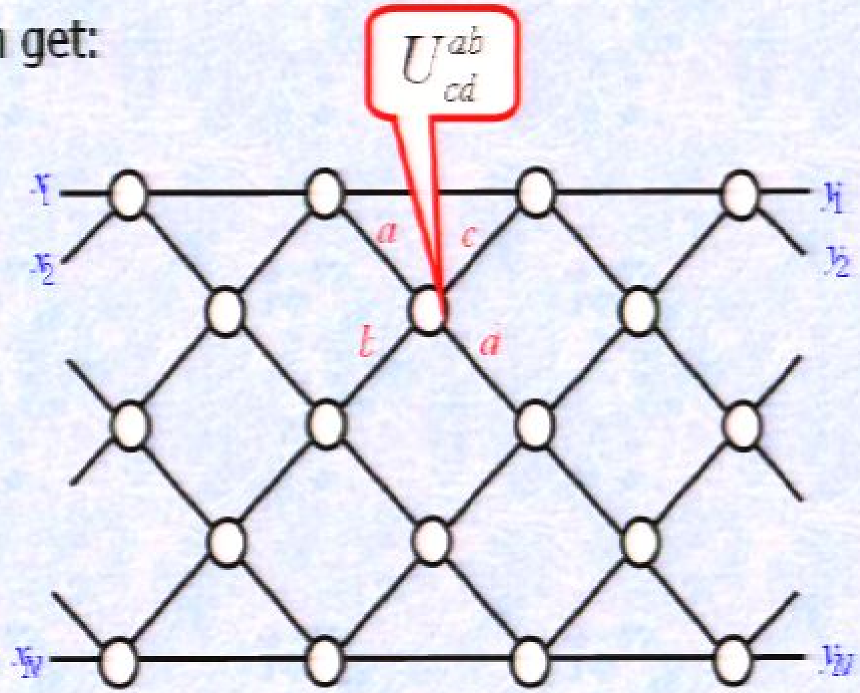
$$\sum_{\alpha} U^{ab}_{c\alpha} U^{\alpha b'}_{c'd'} \equiv$$



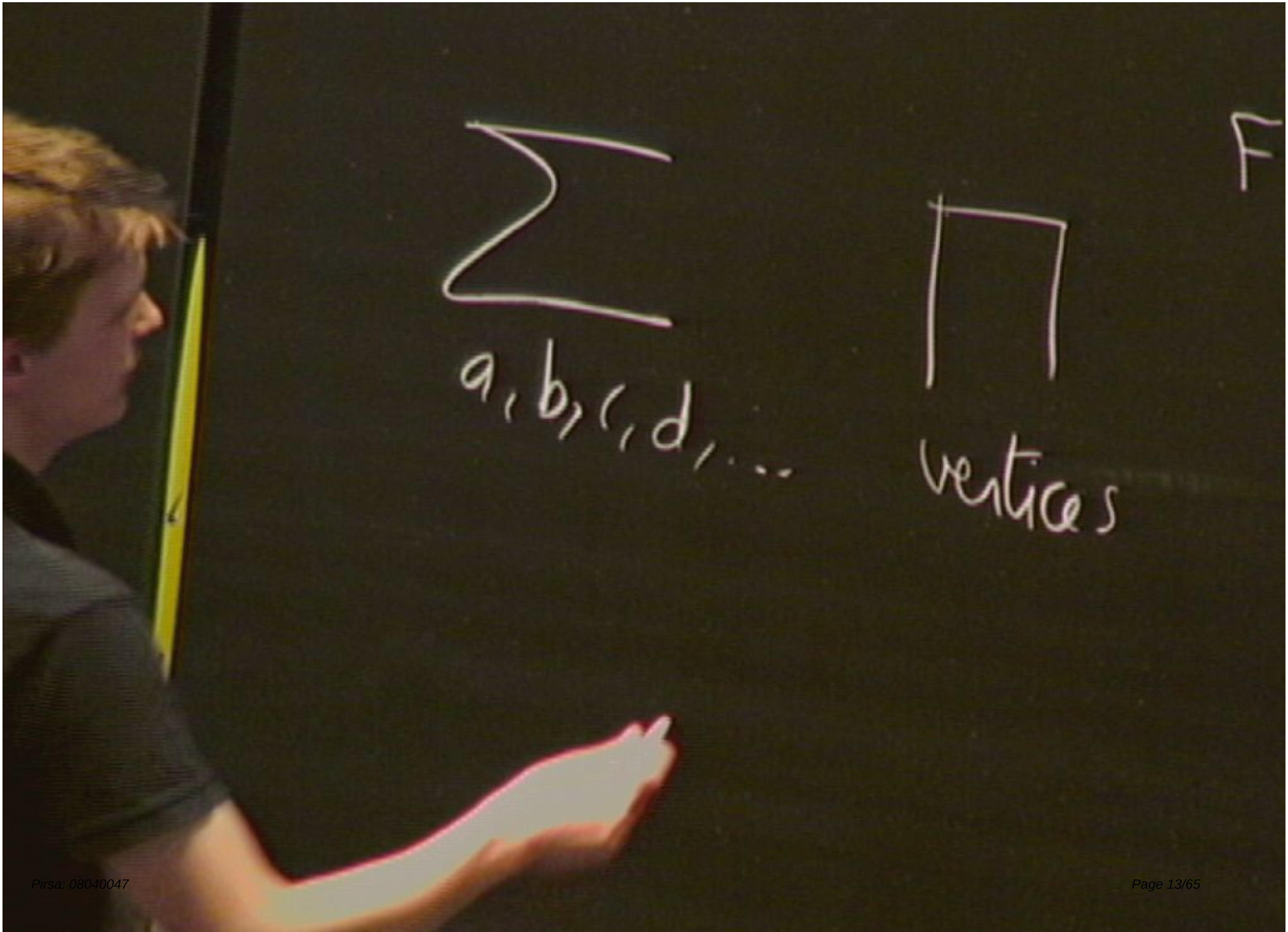
Quantum circuits and vertex models

❑ For the whole circuit we then get:

$$\langle x|C|y\rangle \equiv \sum_{a,b,c,d,\ldots}$$



- At each edge of 2D lattice sits a classical spin which can take q values
- The spins at the left/rights are fixed in boundary conditions x and y
- Each configuration of 4 spins a, b, c, d around a vertex is given a 'weight' U^{ab}_{cd}
- $\langle x|C|y\rangle$ is given as sum, over all spin configurations, of products of U^{ab}_{cd}


Σ
a, b, c, d, ...
∏
vertices
F

$$\sum_{a,b,c,d,\ldots} \prod_{\text{vertices}} W^{ab}_{cd} \qquad F = \sum_{i,j}$$

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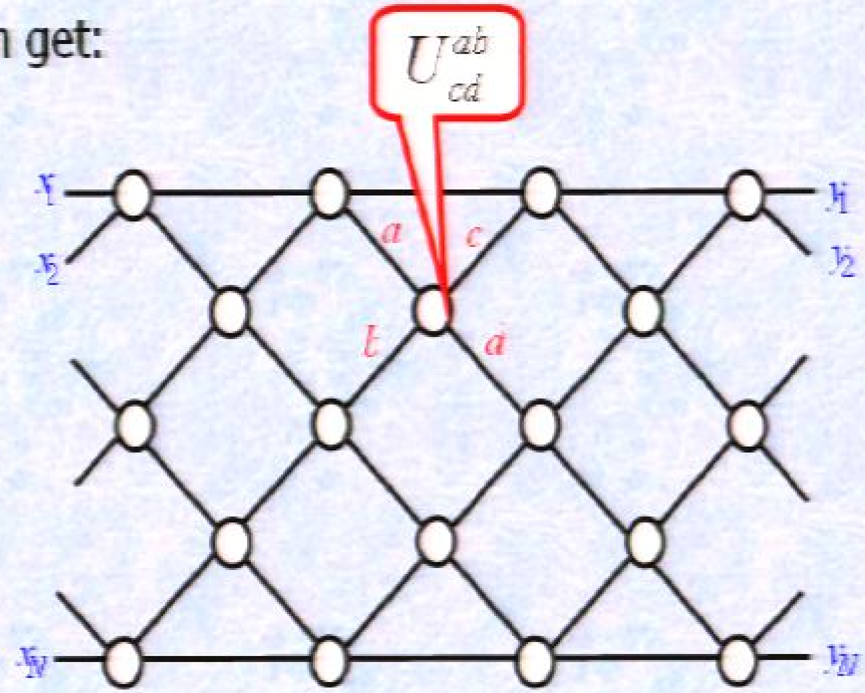

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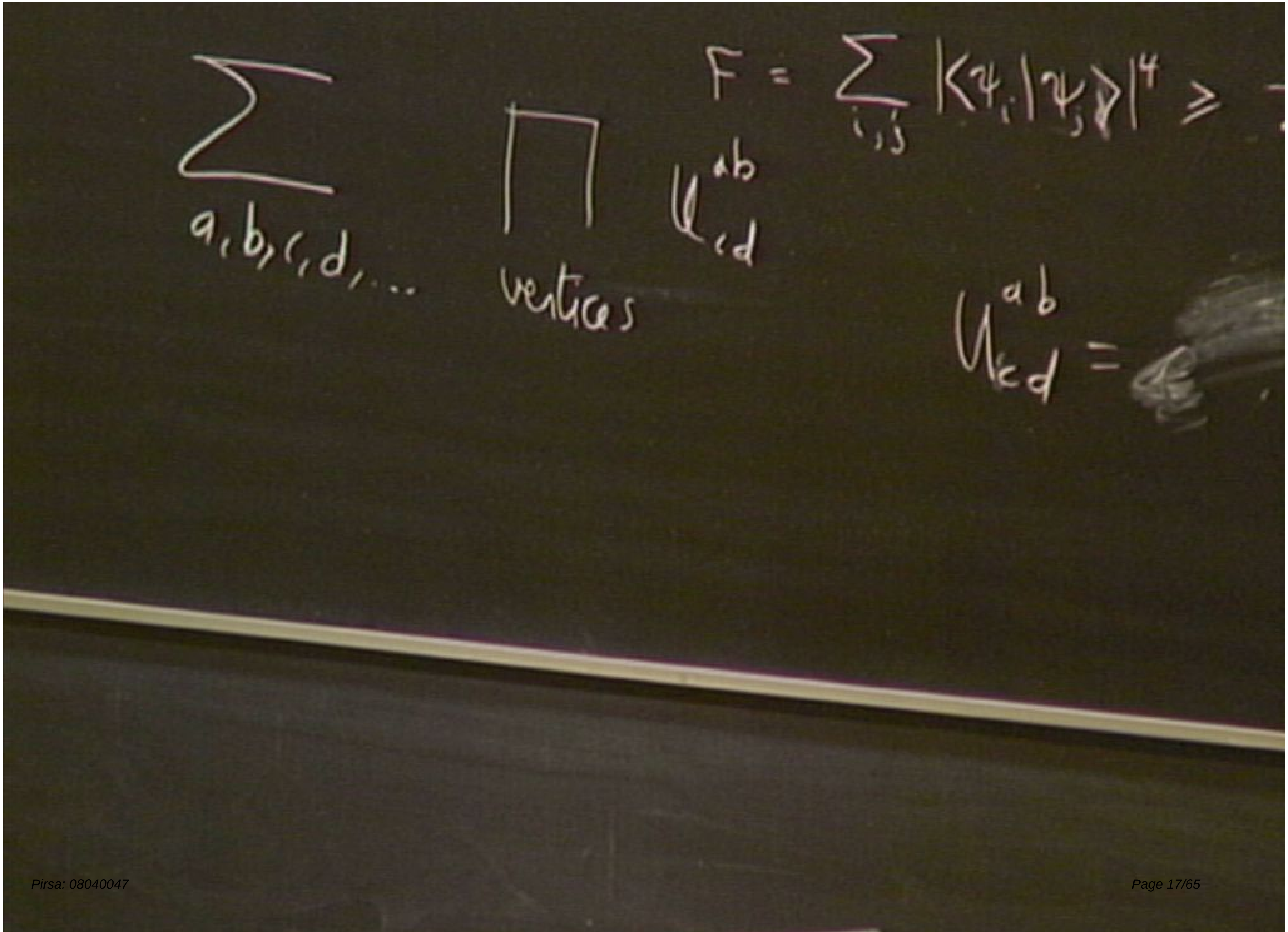
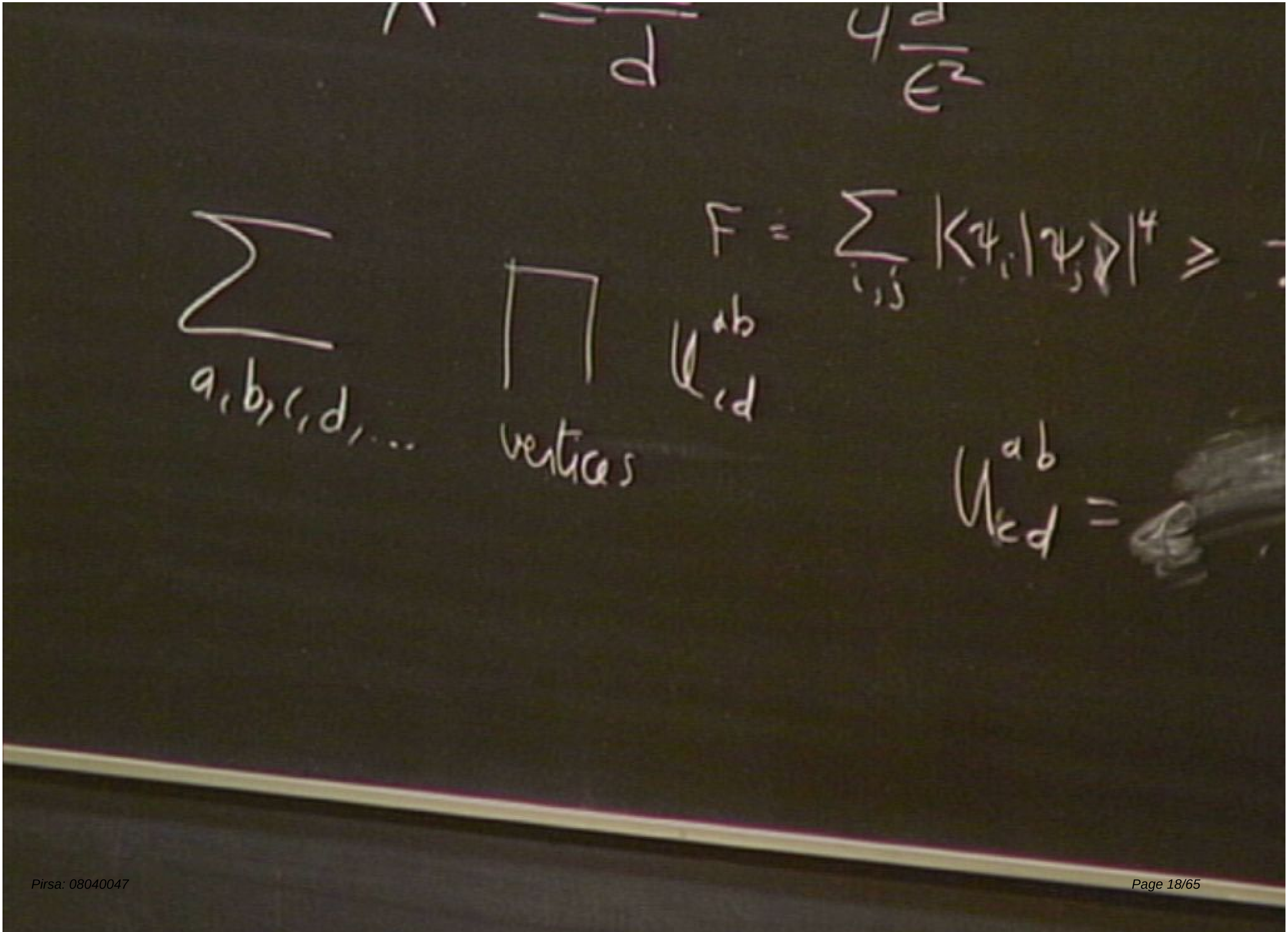
Quantum circuits and vertex models

- This corresponds to a 'vertex model' in stat mech, with $U_{cd}^{ab} \equiv e^{-\beta H(a,b,c,d)}$

Classical spin system where spins live on edges and interactions occur in vertices (see e.g. Baxter)

$\langle x | C | y \rangle$ coincides with partition function of vertex model on (tilted) 2D square lattice with left and right boundary conditions


$\sum_{a,b,c,d,\ldots} \prod_{\text{vertices}} U^{ab}_{cd}$
$F = \sum_{i,j} |\langle \psi_i | \psi_j \rangle|^4 \geq$
$U^{ab}_{cd} =$


d
d
ε²
F = Σ_{i,j} |⟨ψ_i|ψ_j⟩|⁴ ≥
a, b, c, d, ...
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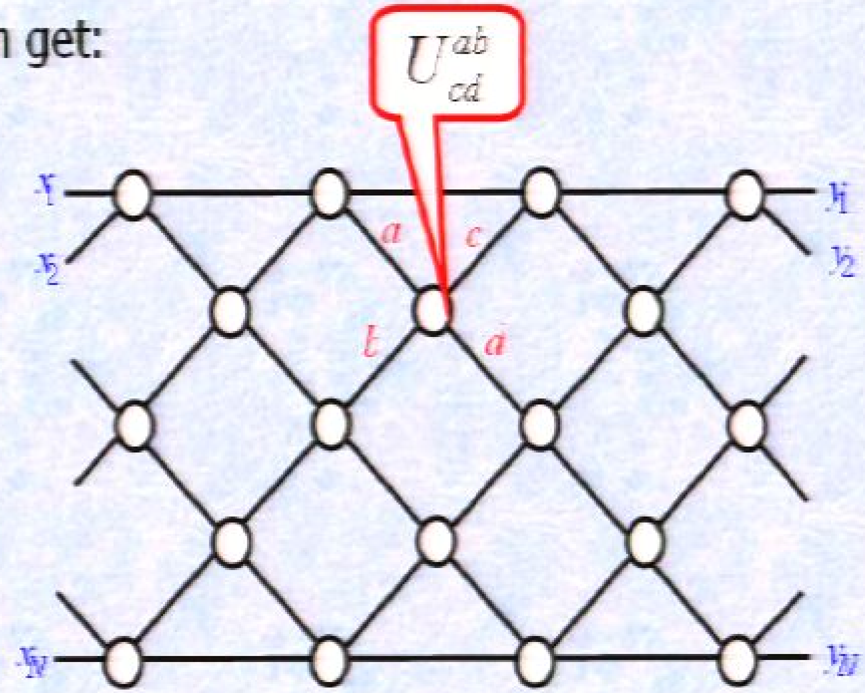

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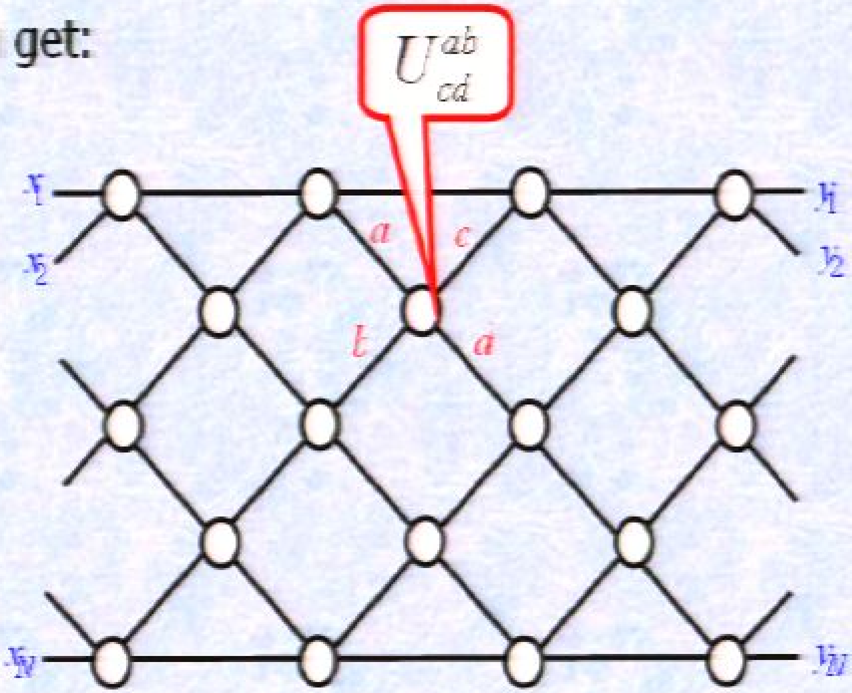

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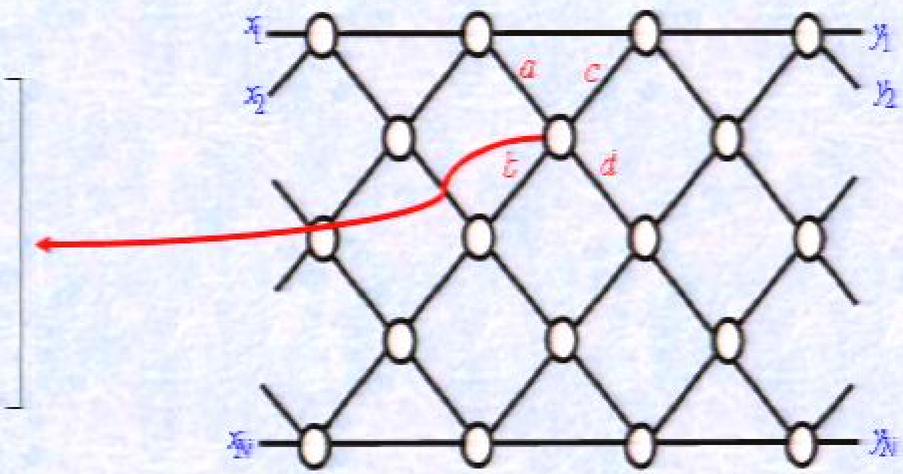
Two standard vertex models

- 2-state spins on a 2D square lattice
- Write Boltzmann weights $W_{cd}^{ab} \equiv e^{-\beta H(a,b,c,d)}$ as a 4x4 matrix
- Eight-vertex model:

$$\left[W_{cd}^{ab}\right] = \begin{bmatrix} \varpi_1 & 0 & 0 & \varpi_2 \\ 0 & \varpi_3 & \varpi_4 & 0 \\ 0 & \varpi_5 & \varpi_6 & 0 \\ \varpi_7 & 0 & 0 & \varpi_8 \end{bmatrix}$$



- Six-vertex model: $\varpi_2 = 0 = \varpi_7$

Two immediate conclusions

- [1] exactly solved vertex models → simulable gate sets
- [2] quantum algorithms to approximate partition functions in certain (complex) parameter regimes

Simuleerbare gate sets from solvable models

- Via mapping: evaluation of $\langle x|C|y\rangle$ corresponds to evaluation of partition function of vertex model

- If corresponding vertex model is 'solved' (i.e., it is possible to compute partition function efficiently) then corresponding matrix elements can be computed efficiently → simuleerbare gate sets from solvable models

- However, watch out for
  - Translation invariance
  - Complex couplings
  - Finite lattice size
  - Boundary conditions

- Nevertheless, certain solvable models can immediately be translated into simulable gate sets

Simuleerbare gate sets from solvable models

- For example, the eight-vertex model

$$\left[W_{cd}^{ab}\right]=\begin{bmatrix}\varpi_1 & 0 & 0 & \varpi_2\\ 0 & \varpi_3 & \varpi_4 & 0\\ 0 & \varpi_5 & \varpi_6 & 0\\ \varpi_7 & 0 & 0 & \varpi_8\end{bmatrix}$$

  is known to be solvable for finite dimensions, complex non-translation-invariant couplings, whenever the condition

$$\varpi_1\varpi_8-\varpi_2\varpi_7=\varpi_3\varpi_6-\varpi_4\varpi_5$$

  is fulfilled. The solution is given by mapping to free fermions.

- This recovers simulability of Valiant's Matchgates (+ connection to free fermions by Terhal and Divincenzo, see also Bravyi, and Jozsa and Miyake)

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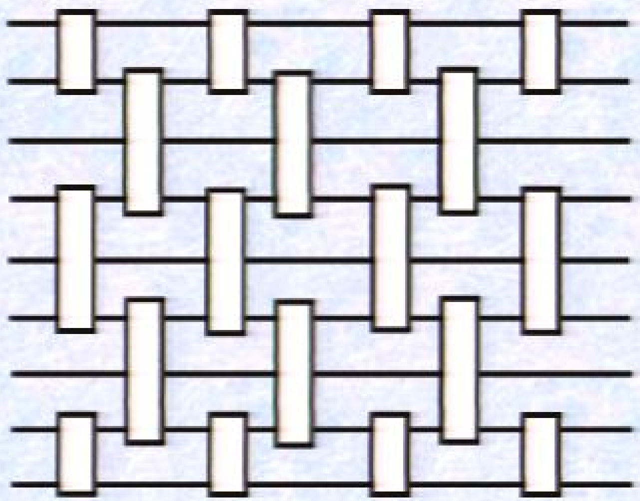
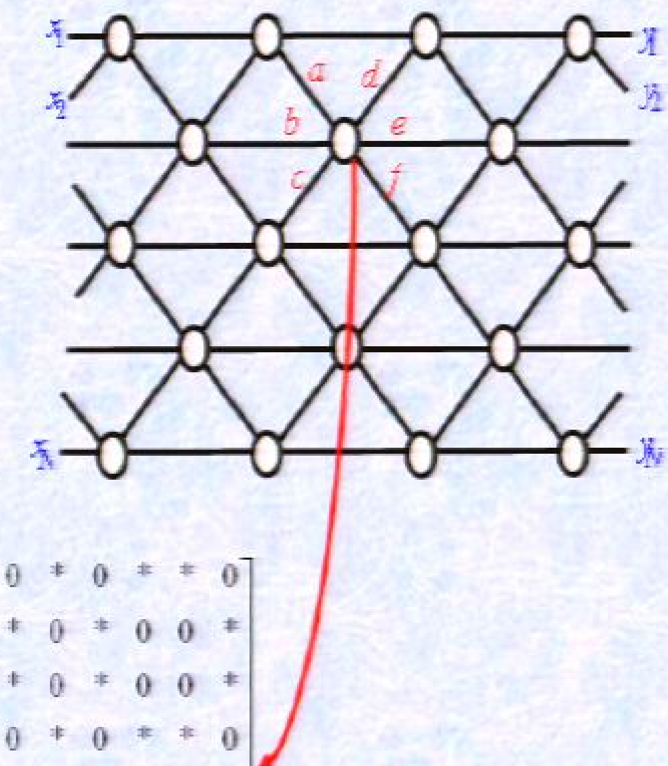
Simuleerbare gate sets from solvable models

- Beyond Valiant: consider 32-vertex model on triangular lattice
- Also solved for certain parameters (again: mapping to free fermions)





$$\left[W^{abc}_{def}\right] = \begin{bmatrix} * & 0 & 0 & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & 0 & 0 & * \\ 0 & * & * & 0 & * & 0 & 0 & * \\ * & 0 & 0 & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & 0 & 0 & * \\ * & 0 & 0 & * & 0 & * & * & 0 \\ * & 0 & 0 & * & 0 & * & * & 0 \end{bmatrix}$$

Quantum algorithms for vertex models

- Immediate quantum algorithm for $Z = \langle x|C|y\rangle$ whenever Boltzmann weights are chosen such that C is unitary circuit: just do Hadamard test

- This yields 'Additive approximation': algo returns number c such that (with all but exponentially small probability) one has

$$|c - \langle x|C|y\rangle| \leq \frac{1}{\mathrm{poly}(N)}$$

- Easy to get BQP-complete problems: just take vertex model corresponding to universal gate set

- For example: (inhomogeneous) six-vertex model on 2D square lattice is BQP-complete

$$[W^{ab}_{cd}] = \begin{bmatrix} \varpi_1 & 0 & 0 & 0 \\ 0 & \varpi_3 & \varpi_4 & 0 \\ 0 & \varpi_5 & \varpi_6 & 0 \\ \end{bmatrix}$$

→ use encoded universality of exchange interaction (Kempe, Divincenzo et al)

Pros and cons of these quantum algorithms:

- PRO:
  - Very simple, general, systematic mapping from classical model to quantum algo
  - BQP-complete algo's may teach us something about computational complexity of classical models
- CON:
  - Quantum algo only provides (additive) approximation
  - It seems difficult to get algo's in 'physical regime': complex couplings naturally pop up due to unitarity. This problem also occurs in related work, so might be a tough one to circumvent..

2nd class of mappings:

Teleportation-based QC

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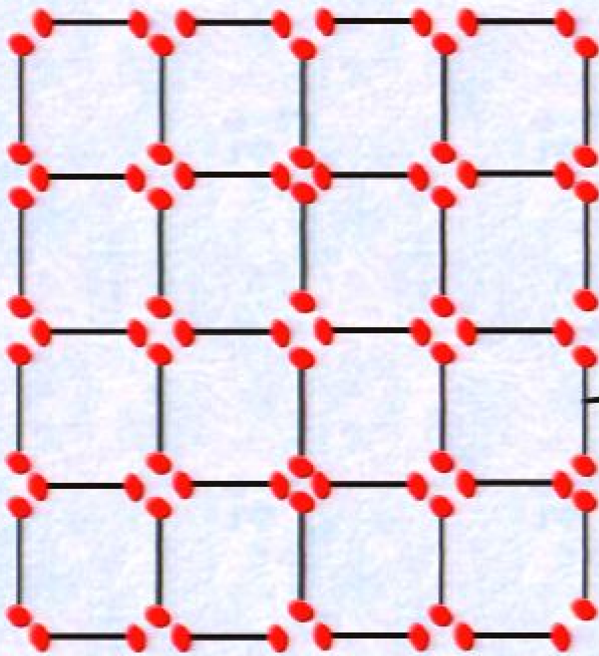
Edge models (Ising, Potts ...)

Edge models and teleportation

- Consider a 2D network of two-qubit states



where every ●—● is an arbitrary two-qubit state:

$$|\psi\rangle = \psi_{00}|00\rangle + \psi_{01}|01\rangle + \psi_{10}|10\rangle + \psi_{11}|11\rangle$$

state may be different at each edge

- Suppose a GHZ measurement is performed on every site of the lattice

GHZ measurement on 4 sites

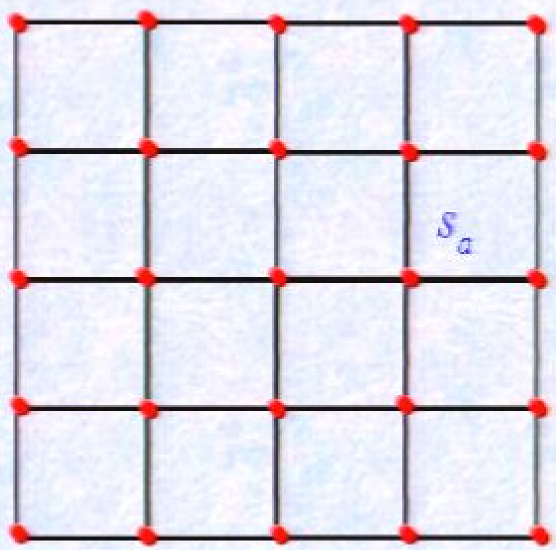
$$\left\{ |0\rangle^{\otimes 4} \pm |1\rangle^{\otimes 4},\ \ldots \right\}$$

Edge models and teleportation

- We now consider those unitary operations which can be implemented via these GHZ measurements on the states $|\psi\rangle$

  - For which $|\psi\rangle$ are these operations classically simuleerbaar?

- ...Enters the link with statistical mechanics: Ising and Potts-type models



-- consider spin system (edge model)

-- on every site lives 2-state spin $s_a = 0,1$

-- over every edge there is interaction

-- Boltzmann weights given by

$$\psi_{st} = e^{-\beta H(s,t)} \qquad s,t = 0,1$$

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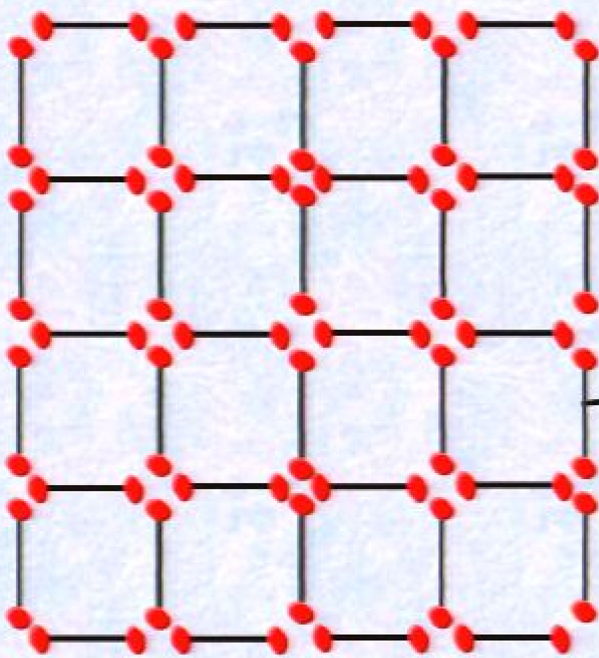

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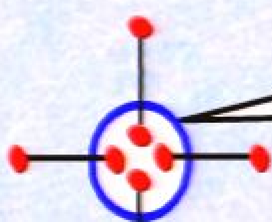

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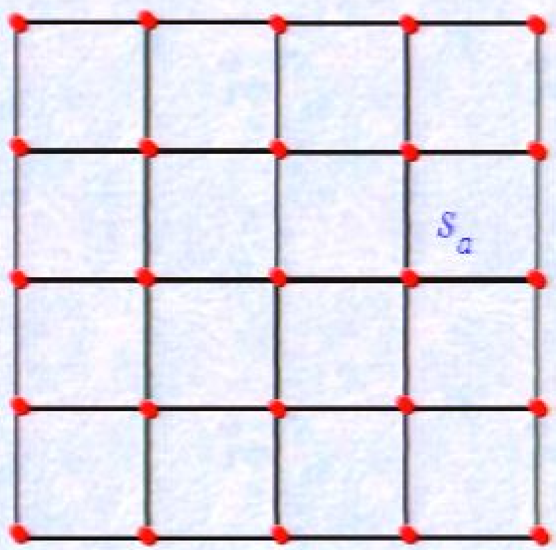

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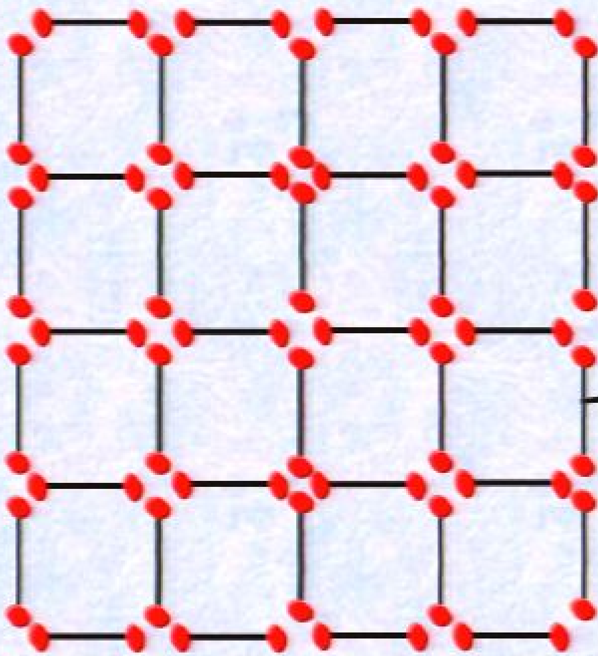

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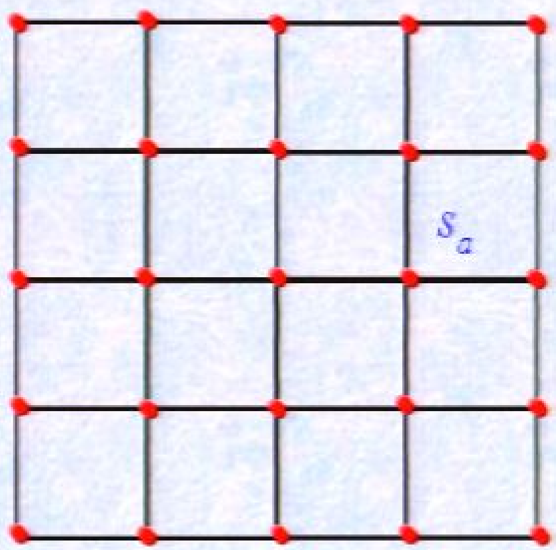

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- We can then write the partition function of this model as

$$Z = \left(\bigotimes_{\text{all sites}} \langle \text{GHZ} | \right) \left(\bigotimes_{\text{all edges}} | \psi \rangle \right)$$

(one branch of) GHZ measurement

'resource' of two-qubit states

- First Observation (roughly):
  - suppose we choose the $|\psi\rangle$'s to correspond to a solvable model.
    E.g. 2D Ising model without external field: $\psi_{00} = \psi_{11}$ and $\psi_{01} = \psi_{10}$
  - Take a unitary operation U which can be implemented via GHZ measurements on such $|\psi\rangle$'s. Then

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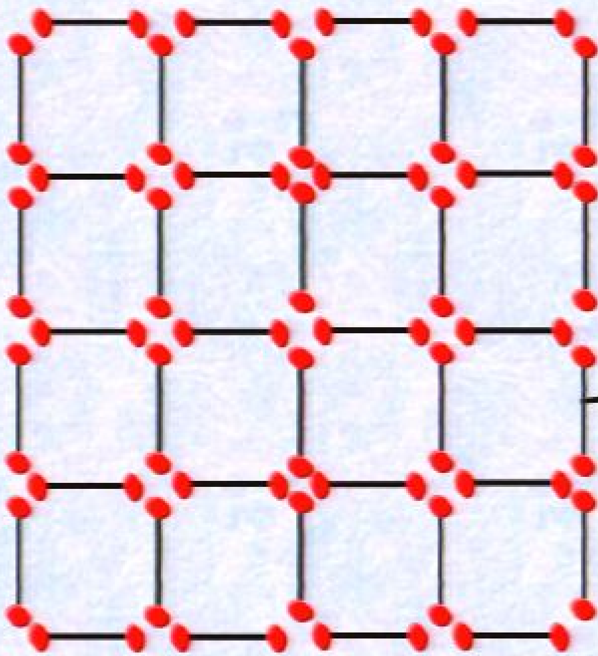

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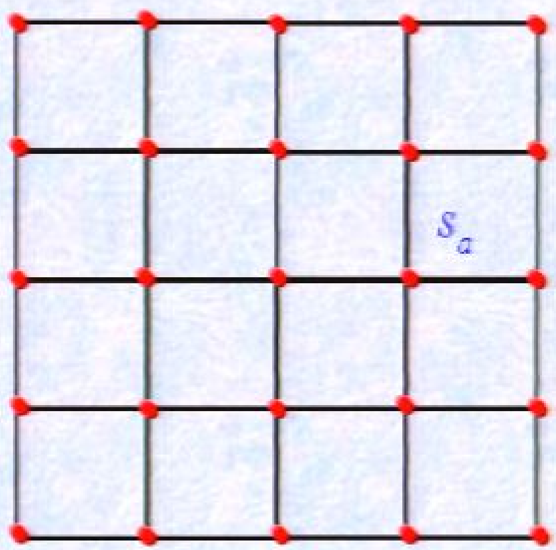

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Simulable gate sets from solvable models

- Example: 2D Ising model without external field: $\psi_{00} = \psi_{11}$ and $\psi_{01} = \psi_{10}$
- With these $|\psi\rangle$'s, the following (possibly non-unitary) gates can be implemented:

$$aI + bX \qquad \text{and} \qquad aI \otimes I + bZ \otimes Z \quad \text{(n.n. qubits)}$$

- Since partition function of 2D Ising can be efficiently evaluated, we can conclude that

$$Z = \left(\bigotimes \langle \mathrm{GHZ} | \right) \left(\bigotimes | \psi \rangle \right) \propto \langle 0 | U | 0 \rangle$$

can be computed efficiently for any circuit U built out of the above gates

- Thus, as for the case of vertex models: solvable models lead to simulable gates

Simulable gate sets from solvable models

- Example: 2D Ising model without external field: $\psi_{00} = \psi_{11}$ and $\psi_{01} = \psi_{10}$
- With these $|\psi\rangle$'s, the following (possibly non-unitary) gates can be implemented:

$$aI + bX \quad \text{and} \quad aI \otimes I + bZ \otimes Z \;\; \text{(n.n. qubits)}$$

- Since partition function of 2D Ising can be efficiently evaluated, we can conclude that

$$Z = \left(\bigotimes \langle \mathrm{GHZ}|\right)\left(\bigotimes |\psi\rangle\right) \propto \langle 0|U|0\rangle$$

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Quantum algorithms

- Second observation (also roughly):
  - Recall again that $Z = \left(\bigotimes \langle \mathrm{GHZ}|\right)\left(\bigotimes |\psi\rangle\right) \propto \langle 0|U|0\rangle$
  - Using Hadamard test, we can efficiently provide additive approximation of matrix element of unitary circuit
  - This yields quantum algo to approximate partition function of edge models in those couplings where the corresponding circuit is unitary
  - Again: BQP-complete algorithms are obtained by producing universal gate sets. Example: Ising-type spin glass on 2D square lattice (with complex temperature)
- All pros and cons of quantum algo's for vertex models are also present here...

3rd class of mappings:

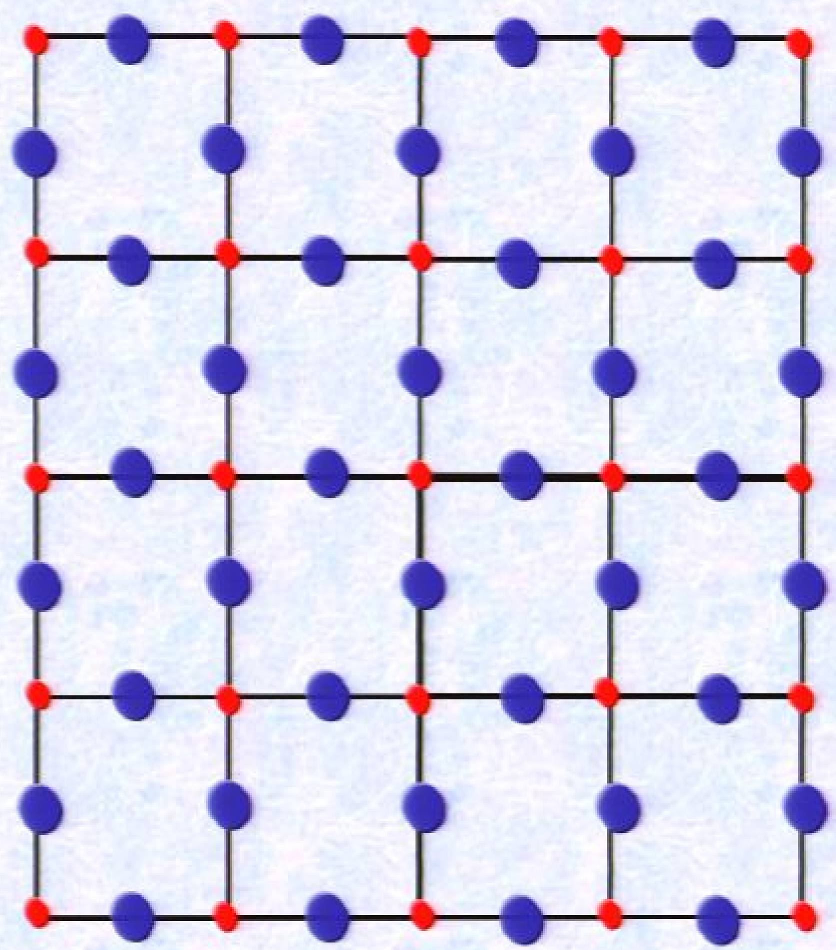
One-way QC

⇕

Edge models (Ising, Potts...)

Mapping spin models to graph states

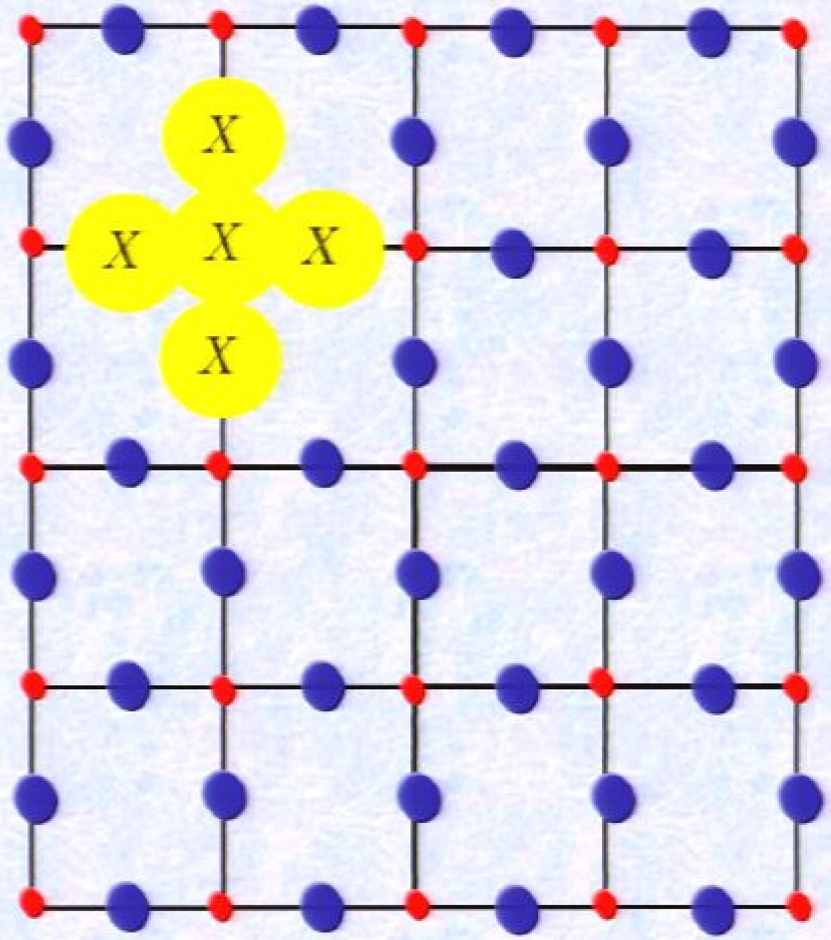
- Start with a graph G
- Edge-qubits
- Vertex-qubits



Mapping spin models to graph states

- Start with a graph G
- Edge-qubits ●
- Vertex-qubits ●
- 1 stabilizer operator per vertex:

$$V_a = X^{(a)} \prod_{e:a\in e} X^{(e)}$$



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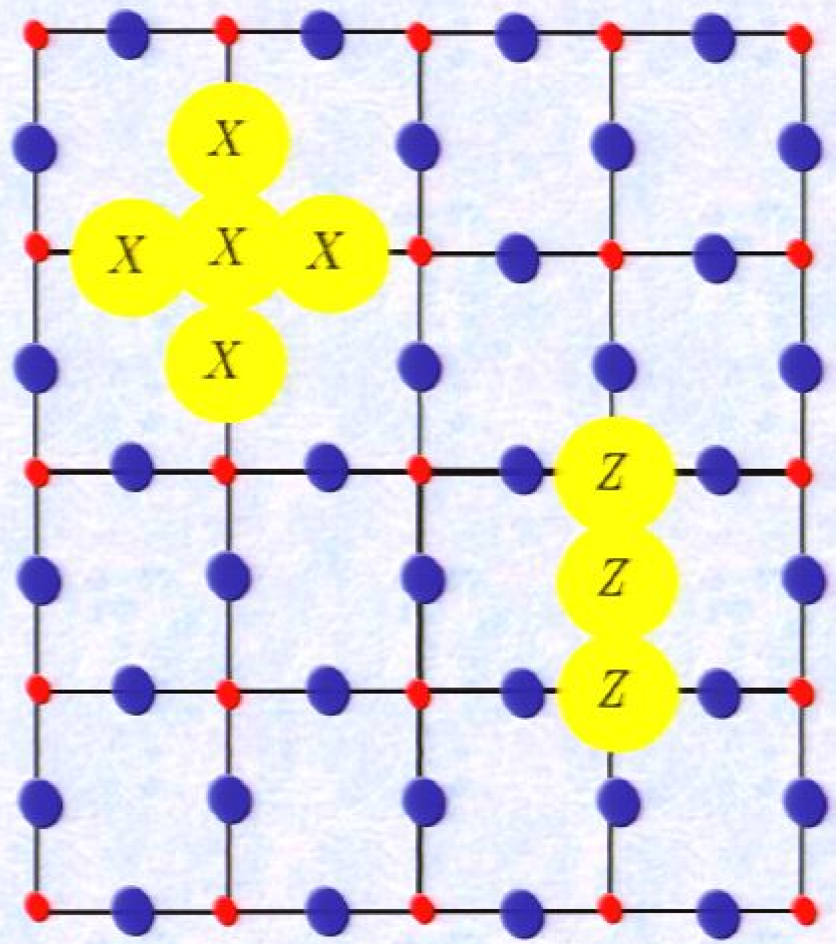

The mapping

- Ising model with external fields
- $|\alpha_{ab}\rangle = e^{\beta J_{ab}}|0\rangle + e^{-\beta J_{ab}}|1\rangle$ Also: $|\alpha_a\rangle = e^{\beta h_a}|0\rangle + e^{-\beta h_a}|1\rangle$
- Then: $$Z_G \propto \langle \varphi_G | \bigotimes_{ab} |\alpha_{ab}\rangle \bigotimes_a |\alpha_a\rangle$$
- Partition function = overlap between $|\varphi_G\rangle$ and product state

Interaction PATTERN | Interaction STRENGTH

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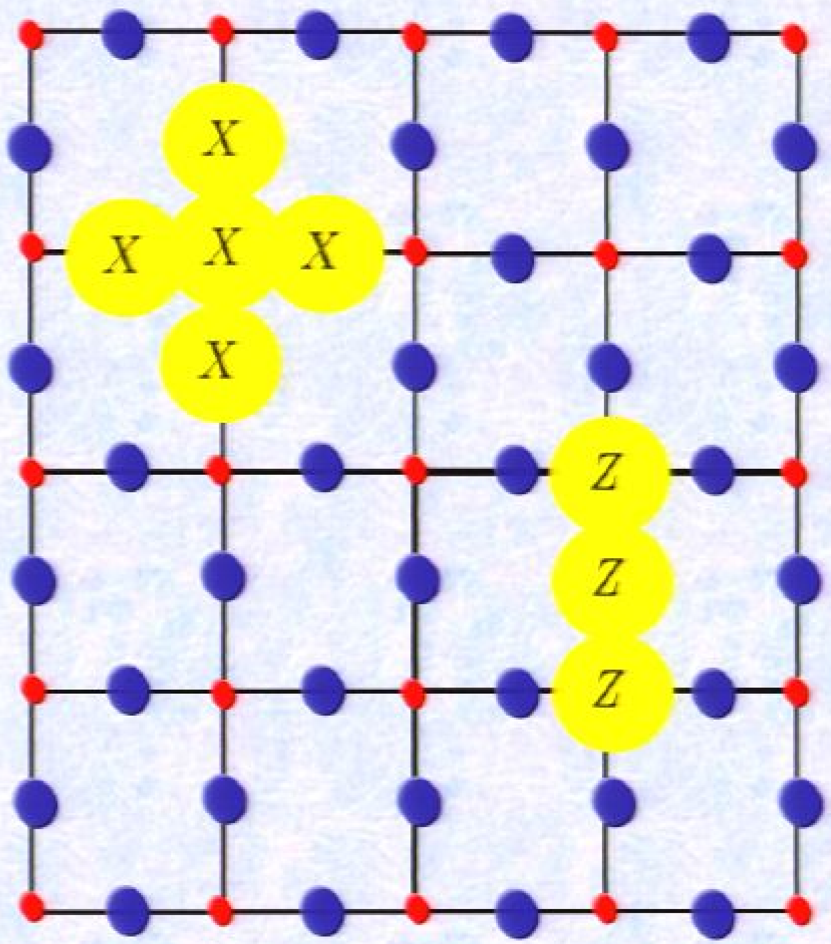

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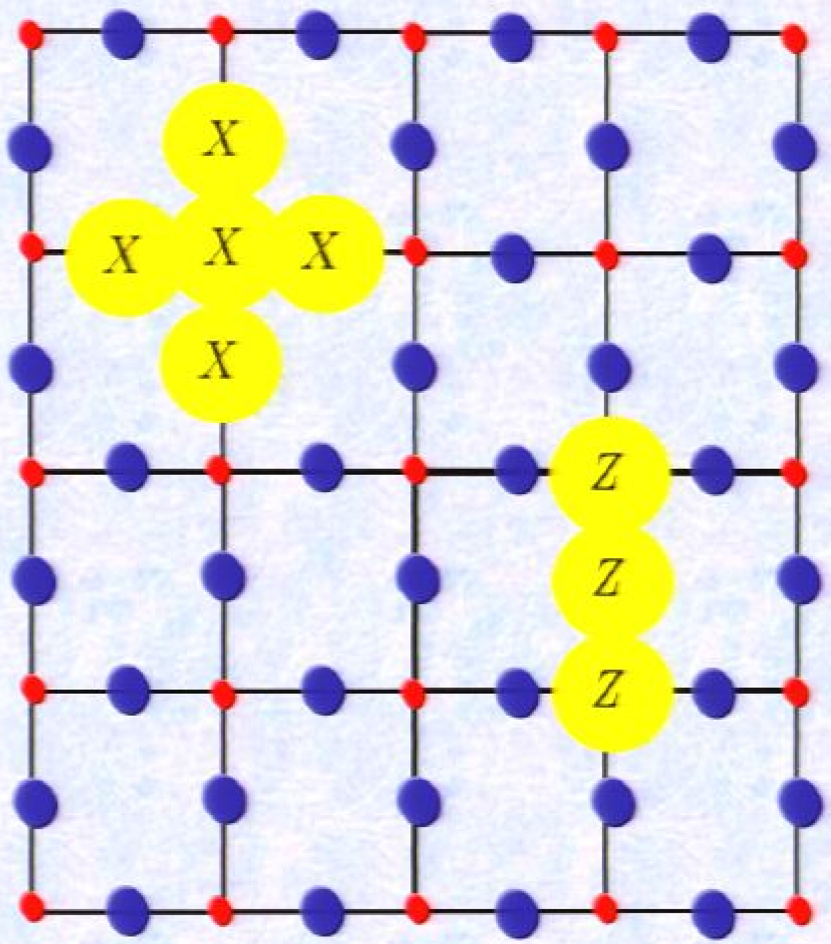

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Interaction PATTERN | Interaction STRENGTH

Examples

- 1D Ising model with open BCs → No field: product state

  With field: 1D cluster state

- 1D Ising model with periodic BCs → No field: GHZ state

  With field: 1D ring cluster state

- 2D Ising model → No field: toric code state

  With field: 2D cluster state

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Simulable resources from solved models

- Mappings connect computational power of resource state with solvability of corresponding spin model
- Solvable model (1D, 2D without field) gives rise to simulatable resource state: e.g. planar code state, see Bravyi and Raussendorf,
- Intractable model probably/possibly gives rise to powerful resource state (e.g. 2D with field)

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Ising model		Resource state
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Without field: solvable With field: NP-hard	2D	Planar code state: simulatable 2D cluster state: UNIVERSAL

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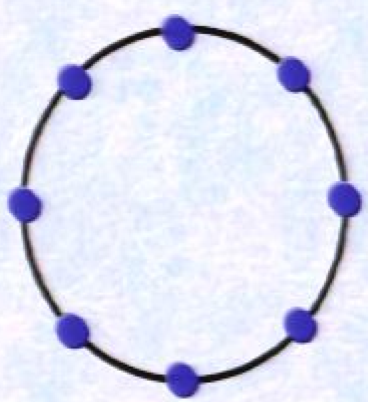

No field: toric code state

- 2D Ising model →

With field: 2D cluster state

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Conclusions and outlook

- Natural mappings between quantum computation (circuits, teleportation, MQC) and classical statistical mechanics

- Mappings are general but simple

- Solvable models lead to simulable quantum computers

- Easy way to get quantum algorithms

- Main challenges:
  - Yang-Baxter equation
  - Quantum algorithms in physical parameter regime
  - Relation to existing quantum algo's (Jones, Potts)
  - Make up our minds whether it is "simulable", "simulatable" or something else....