Title: Emergent Time in Barbour and Bertotti\'s Timeless Mechanics

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Abstract: The problem of time is studied in a toy model for quantum gravity: Barbour and Bertotti\'s timeless formulation of non-relativistic mechanics. We quantize this timeless theory using path integrals and compare it to the path integral quantization of parameterized Newtonian mechanics, which contains absolute time. In general, we find that the solutions to the timeless theory are energy eigenstates, as predicted by the usual canonical quantization. Nevertheless, the path integral formalism brings new insight as it allows us to precisely determine the difference between the theory with and without time. This difference is found to lie in the form of the constraints imposed on the gauge fixing functions by the boundary conditions. In the stationary phase approximation, the constraints of both theories are equivalent. This suggests that a notion of time can emerge in systems for which the stationary phase approximation is either good or exact. As there are many similarities between this model of classical mechanics and general relativity, these results could provide insight to how time might be emergent in a theory of quantum gravity.

Emergent Time in Barbour and Bertotti's Timeless Mechanics

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The Problem of Time

In General Relativity:

- The Hamiltonian constraint implies the Wheeler-DeWitt equation

$$\hat{H}_{\mathrm{GR}} |\Psi\rangle = 0. \qquad (1)$$

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JBB is a toy model for quantum gravity.

Path integral brings new insight over canonical quantization.

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Classical τ_{BB}	$\Leftarrow$??? $\Rightarrow$	? Quantum τ_{BB} ?
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Lagrangian Formulation of JBB (timeless)

Action

$$S_{JBB} = \int_{\lambda_0}^{\lambda_f} d\lambda \quad 2\sqrt{(T(\lambda))(E - V(\bar{q}))}; \qquad T = \frac{1}{2}m\left(\frac{d\bar{q}}{d\lambda}\right)^2 \tag{3}$$

Note: Product of $\sqrt{}$'s $\Rightarrow$ reparameterization invariance. Depends on image $\bar{q}_i(\Lambda)$.

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Intuition From Classical Theory

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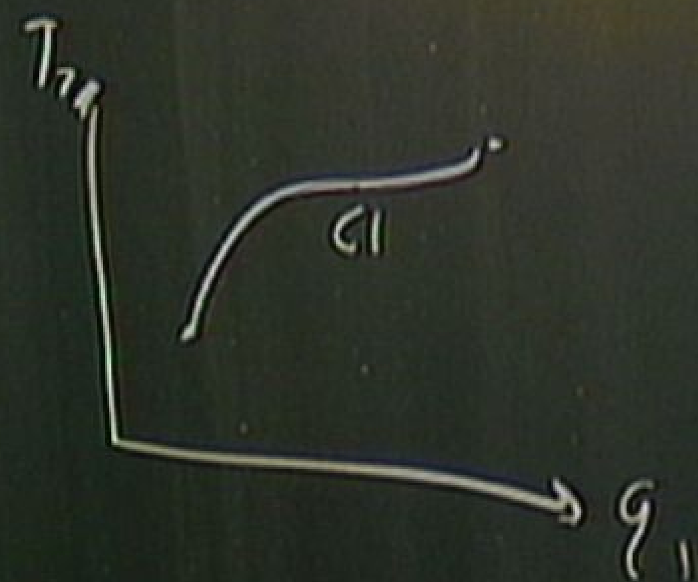

T_{7}
c1
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- Fixed E + arbitrary history $\Rightarrow$ *any* τ_{BB}.
- Path integral sums over *all* histories.
- $\therefore$ the path integral will "average" over all possible τ_{BB}.

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Notice that fixing a gauge means choosing some $N(\lambda)$.
Example: $N(\lambda) = 1 \quad \Rightarrow$ Newton's laws (Newtonian gauge).

This implies the gauge fixing functions:

$$\mathcal{G}(\lambda) = f(q^i, p_i, \lambda) - \frac{m\vec{p}\cdot\dot{\vec{q}}}{p^2} = 0. \tag{8}$$

Lagrangian Formulation of PNM (absolute Time)

The Action

We define the affine parameter λ and replace $\frac{d\vec{q}_i}{dt} \to \frac{d\vec{q}_i}{d\lambda}\frac{d\lambda}{dt}$.

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Setting $t_N = q^0$ gives Newton's laws.

PNM vs JBB

Reminder

$$\dot{q}^0 = \sqrt{\frac{T}{E-V}} (= \tau_{BB}); \qquad \frac{1}{\dot{q}^0}\frac{d}{d\lambda}\left(\frac{1}{\dot{q}^0} m\dot{q}^i\right) = -\frac{\partial V}{\partial q^j}\eta^{ij} \quad (12)$$

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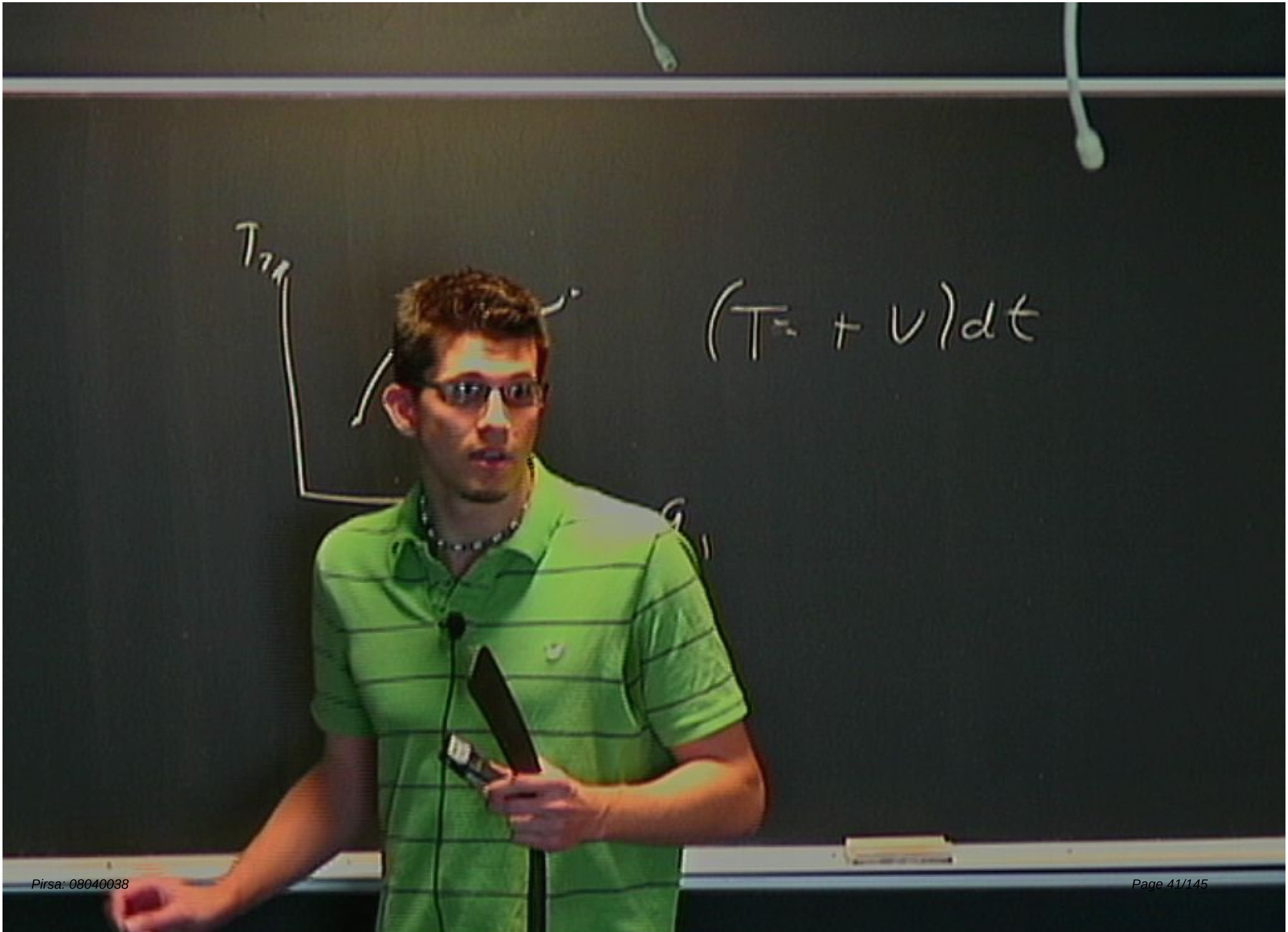
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- Combining the EOM's for PNM gives the EOM's for JBB.

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- In PNM, q^0 is *derived from EOM's* but in JBB τ_{BB} is a *def'n*.
- PNM: t is a free parameter. E is determined by the EOM's.
- JBB: E is a free parameter. τ_{BB} is determined by a def'n.

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This implies the gauge fixing functions:

$$\mathcal{G}(\lambda) = f(q^i, p_i, \lambda) - \dot{q}^0(\lambda) = 0. \tag{14}$$

JBB as a $(0+1)$ Field Theory

$\mathcal{H}$ acting on a Configuration Space point returns a *physically distinguishable* point.

$$q'^i(\lambda_0) = e^{-M\mathcal{H}(\lambda_0)} q^i(\lambda_0) e^{M\mathcal{H}(\lambda_0)}. \tag{15}$$

This is sometimes discussed in the context of canonical quantization in GR. [Kuchař, '92]

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- $\sqrt{}$'s make CS difficult to evaluate.
- Measure turns out to be non-trivial.
- PS path integral allows a comparison between JBB and PNM!!

Phase Space Path Integral for PNM

The Phase Space path integral for gauge theories is defined as: [Faddeev-Popov, '67-'69]

$$k_{\mathrm{PNM}}(q''^{\alpha}, q'^{\alpha}) = \int \mathcal{D}q^{\alpha}\, \mathcal{D}p_{\alpha}\, \mathcal{D}\mathcal{E}\, \mathcal{D}N\, |\{\mathcal{H}, \mathcal{G}\}| \times \exp\left[i \int d\lambda\, \left(p_{\alpha}\dot{q}^{\alpha} - \mathcal{H}_{\mathcal{C}}(q^{\alpha}, p_{\alpha}) - N(\lambda)\mathcal{H}(q^{\alpha}, p_{\alpha}) - \mathcal{E}(\lambda)\mathcal{G}(q^{\alpha}, p_{\alpha}) \right) \right]. \quad (16)$$

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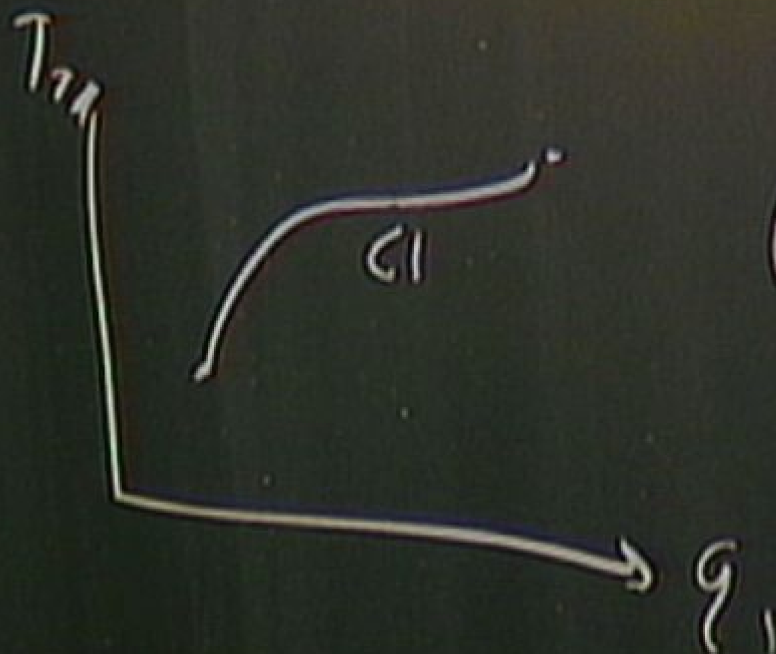

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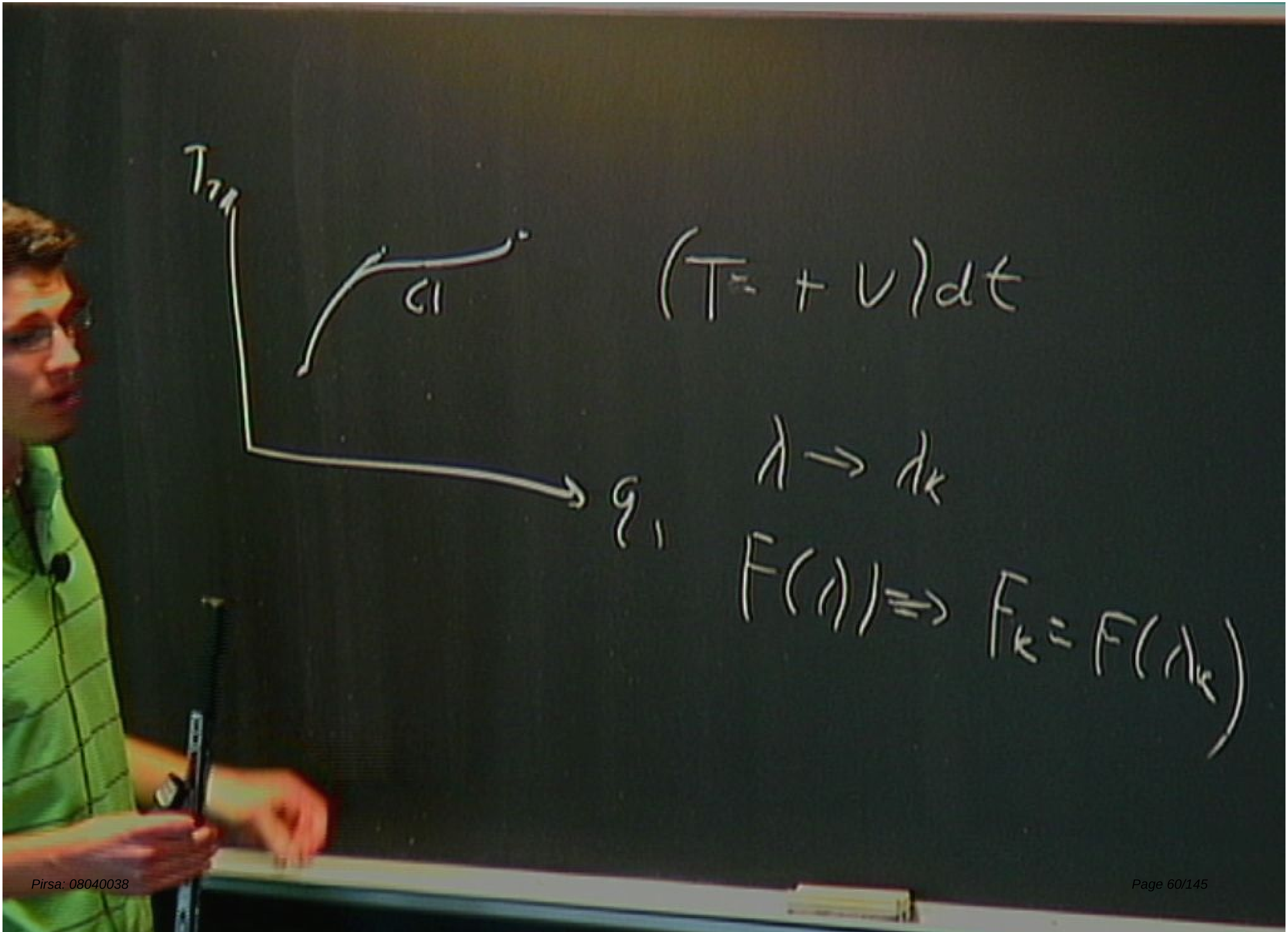
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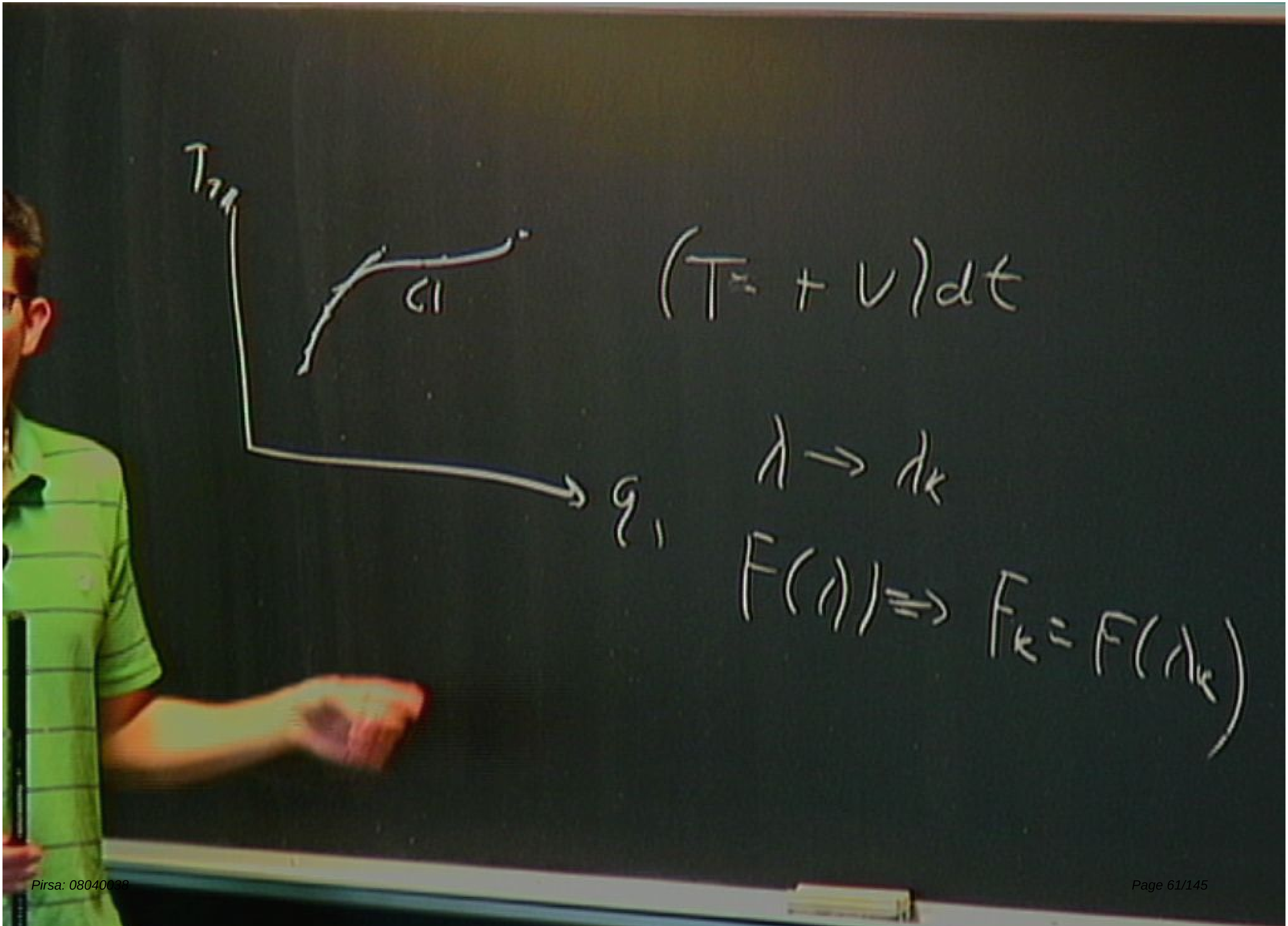
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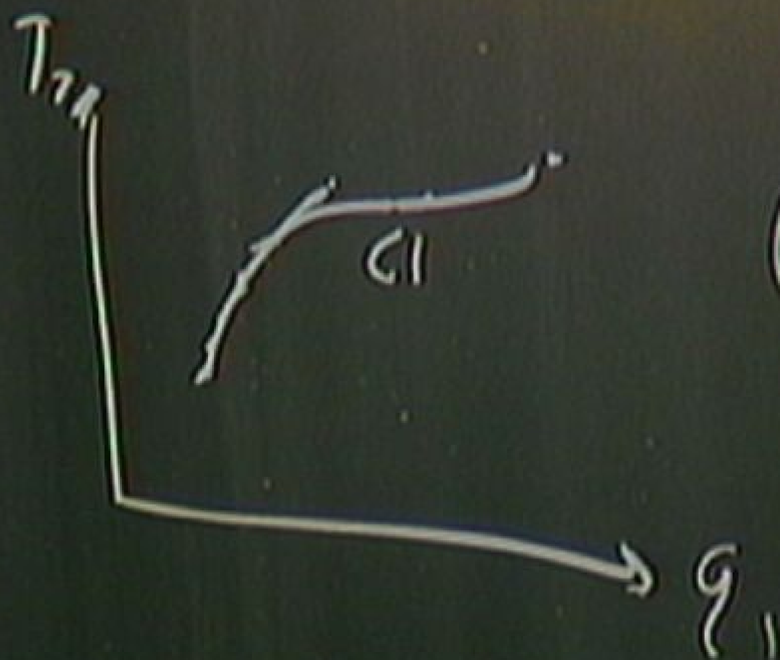

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The analogous constraint in JBB theory is at the heart of issue with time!

The Kernel for JBB

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$$\mathcal{H}^K = \frac{p_K^2}{2m} - E + V^K = 0; \qquad \mathcal{G}_K = f_K(q_K^i, p_i^K) - \frac{m\vec{p}_K \cdot \dot{\vec{q}}_K}{p_K^2} = 0. \qquad (25)$$

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$$k_{\mathrm{JBB}}(\bar{q}'', \bar{q}', E) = \int_{-\infty}^{\infty} \frac{d^3\bar{p}_0}{2\pi} \frac{\Delta\lambda_0 dN_0}{2\pi} \prod_{K=1}^{N-1} \frac{d^3\bar{p}_K}{2\pi} \frac{\Delta\lambda_K dN_K}{2\pi} d^3\bar{q}_K \frac{d\mathcal{E}^K}{2\pi} \underline{[\mathrm{FP}]_{\mathrm{JBB}}}$$

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The Connection Between k_{JBB} and $\tilde{k}_{\text{PNM}}$

Compare $k_{\text{JBB}}(E)$ to $\tilde{k}_{\text{PNM}}(E)$

	$k_{\text{JBB}}(E)$	$\tilde{k}_{\text{PNM}}(E)$
$\mathcal{E}^J$-term	$f_J - \frac{m\vec{p}_J \cdot \dot{\vec{q}}_J}{p_J^2}$	$f_J - N_J$
FP	$\left\lvert\left\{f_M - \frac{m\vec{p}_M \cdot \dot{\vec{q}}_M}{p_M^2}, \frac{\vec{p}_N^{\,2}}{2m} + V^N\right\}\right\rvert$	$\left\lvert\left\{f_M(q^i_K, p^K_i), \frac{\vec{p}_N^{\,2}}{2m} + V^N\right\}\right\rvert$

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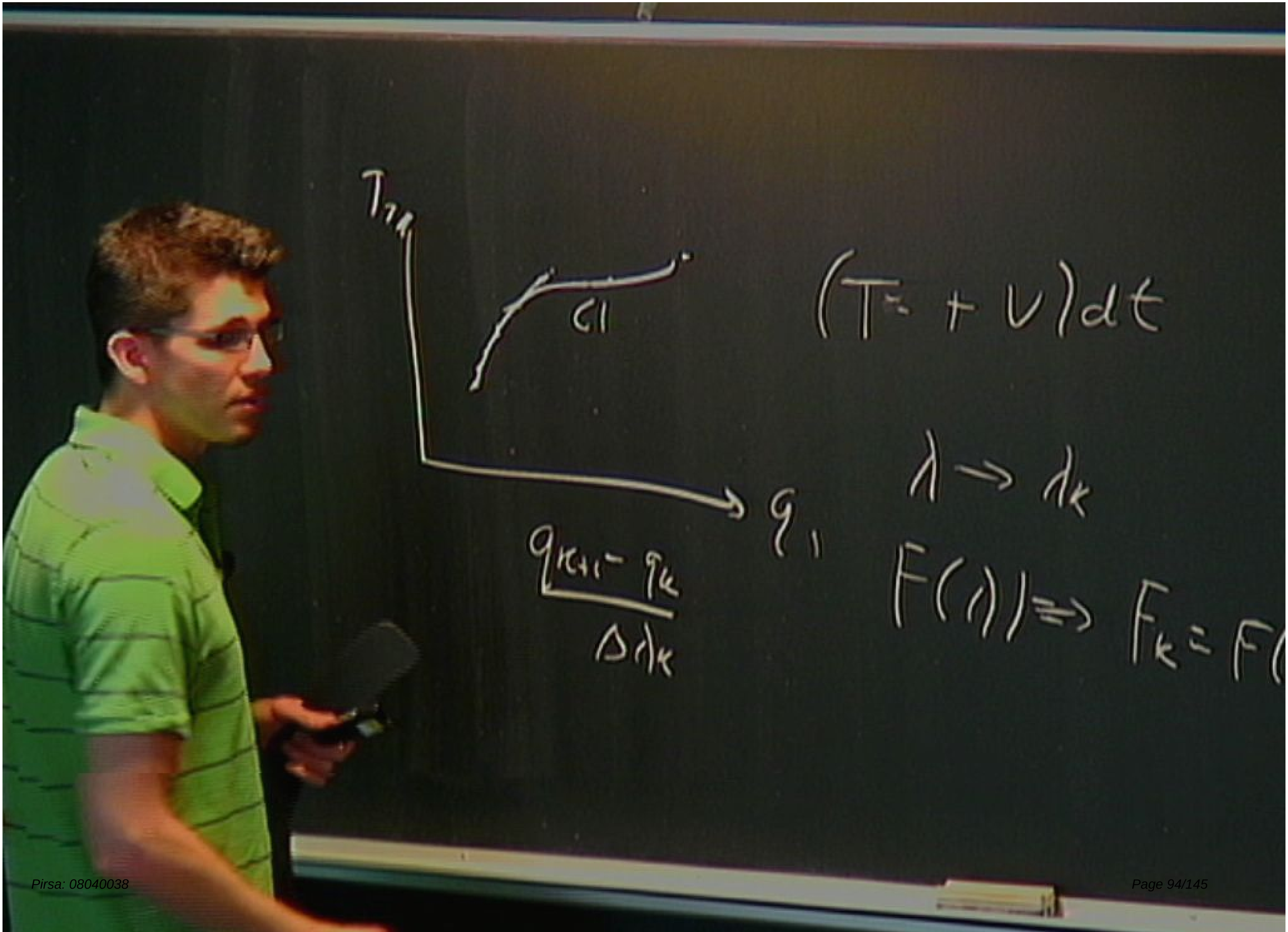

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Boundary Conditions in JBB

Like PNM, the BC's enforce constraints on the f_K.

Here it's more complicated: it's not as easy to separate q^{gauge} from q^{physical}!

If we define: $q^{||} \equiv \frac{\vec{p}_0 \cdot \vec{q}}{p_0}$ then the constraint is:

Constraint on f_K's

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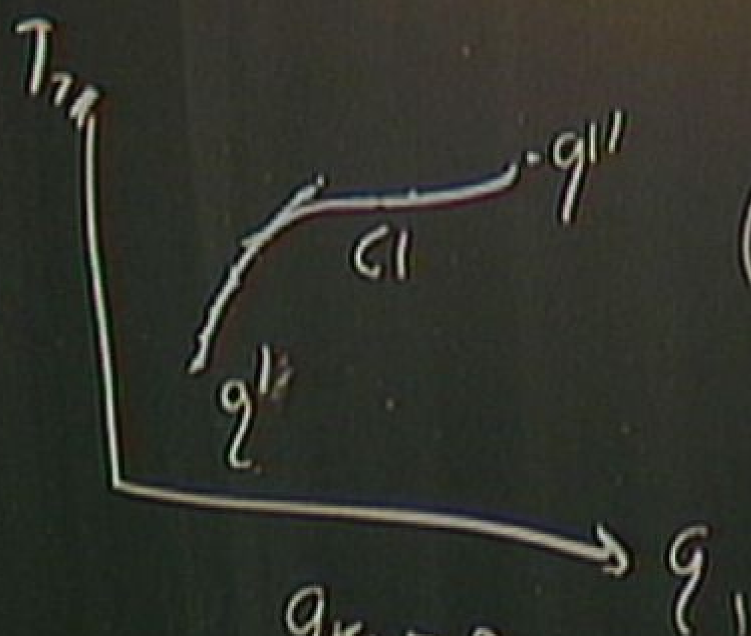
(Comparing this to: $\sum_{J=0}^{N-1} f_J = \tau$ gives us a hint for finding τ in PNM.)



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- Can we define a Schrödinger evolution of "light" subsystems in terms of these quantum clocks?

Thanks

A special thanks to Hans Westman, Rafael Sorkin, Julian Barbour, and Lee Smolin for stimulating discussions and guidance.

Thanks for you attention!

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