Title: Stability of Superflow in Ultracold Fermions in Optical Lattices

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Abstract: Motivated by recent observations of superfluidity of ultracold fermions in optical lattices, we investigate the stability of superfluid flow of paired fermions in the lowest band of a strong optical lattice. For fillings close to one fermion per site, we show that superflow breaks down via a dynamical instability leading to a transient density wave. At lower fillings, there is a distinct dynamical instability, where the superfluid stiffness becomes negative; this evolves, with increasing pairing interaction, from the fermion pair breaking instability to the well-known dynamical instability of lattice bosons. Our most interesting finding is the existence of a transition, over a range of fillings close to one fermion per site, from the fermion depairing instability to the density wave instability as the strength of the pairing interaction is increased. By sharp contrast, the ground state in this regime evolves smoothly with increasing interaction analogous to the BCS-BEC crossover.

Stability of superflow for ultracold fermions in optical lattices

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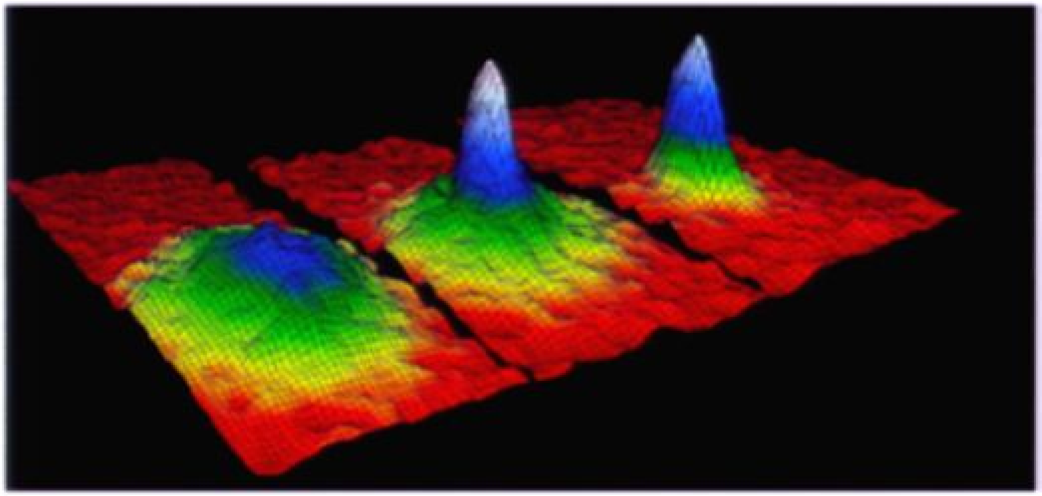
Cold atomic gases



$$n \sim 10^{12} cm^{-3}$$

$$T_c \approx 170 nK$$

Cornell, Wieman, Ketterle, 1995

- New ultralow density, ultralow temperature superfluids.
- Microscopic Hamiltonian is known to high precision.
- Clean and easy to control.
- Only metastable, lifetime of a few seconds.
- Traditional condensed matter experimental probes don't work.

Nonequilibrium dynamics with cold atoms

- In traditional condensed matter measurable nonequilibrium phenomena usually restricted to transport; relaxation times very short.
- Cold atom systems:
  - very low particle density.
  - very low temperature.
  - nearly isolated from environment.
  - ability to change parameters nonadiabatically.

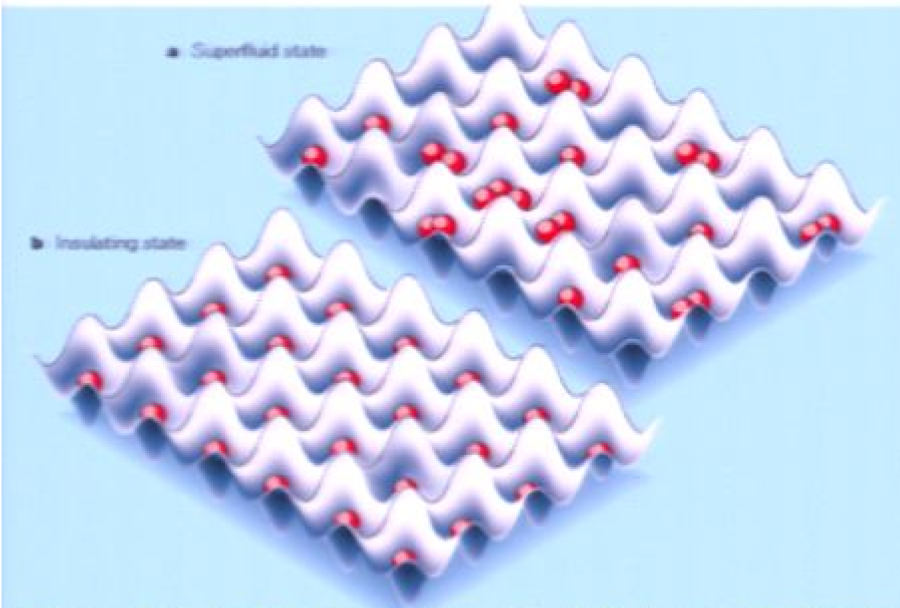
Outline

- Stability of superflow in weakly-interacting superfluids.
- Maximum superflow near superfluid-Mott insulator transition for bosons.
- Stability of superflow of paired fermions in optical lattice.

Weakly-interacting superfluids

- Maximum phase gradient related to important length scales in the superfluid.
- Healing length in a weakly-interacting Bose gas $Q_c \sim \sqrt{m}$
- Cooper pair size in a BCS superconductor $Q_c \sim \Delta / v_F$

Strongly interacting superfluids: Mott transition



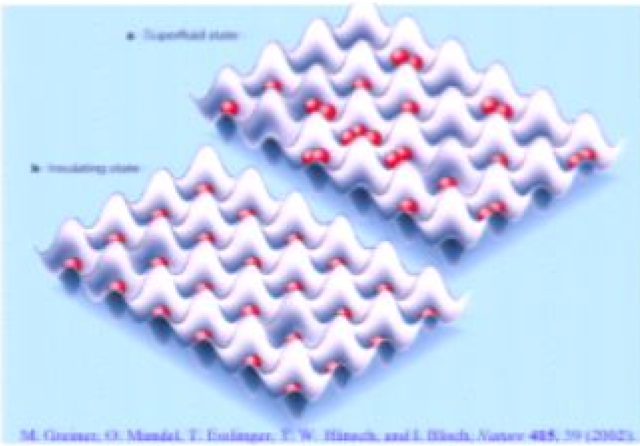
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

$$H = -t\sum_{ij}(b_i^\dagger b_j + h.c.) + U\sum_i n_i(n_i-1) - \mu\sum_i n_i$$

SF-MI transition at a critical value of U/t

One of the few experimental examples of a continuous quantum phase transition.

Superflow stability near Mott transition

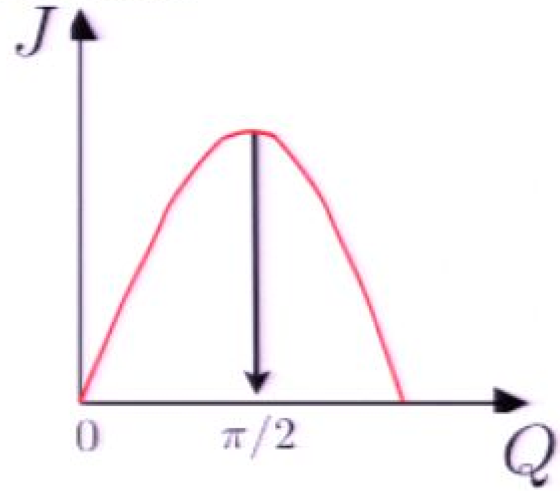


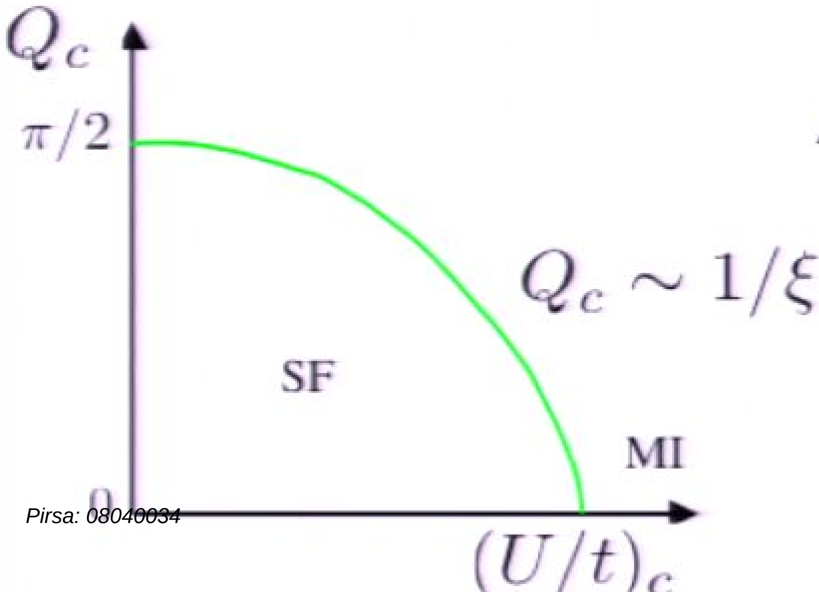
Weakly-interacting lattice bosons

$$J \sim \sin(Q)$$

Wu & Niu, 2001

Fallani et al., 2004





Altman, Polkovnikov, Demler, Halperin, Lukin, 2005

Nonequilibrium dynamical instability of weakly-interacting bosons is smoothly connected to equilibrium SF-MI transition.

Experiment: J. Mun et al., 2007

This talk: dynamical instabilities for fermions in optical lattices

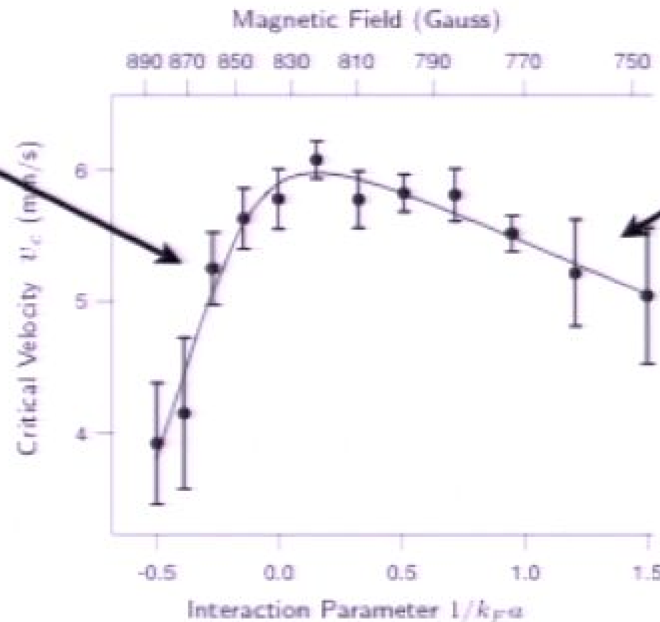
Maximum superflow in ultracold fermion superfluids

- Superflow stability across Feshbach resonance.

Miller et al., 2007 Diener et al., 2007 Stringari et al., 2006

$Q_c \sim \Delta / v_F$

$Q_c \sim \sqrt{m\mu}$

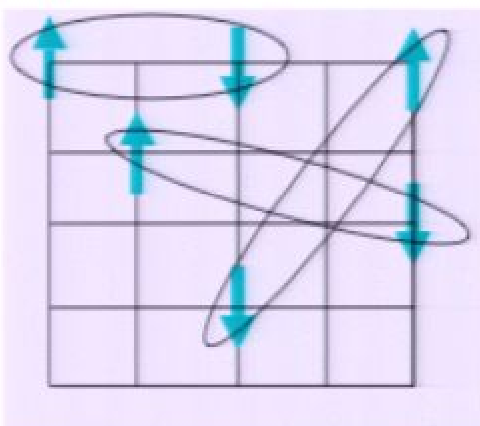


Fermions in optical lattices

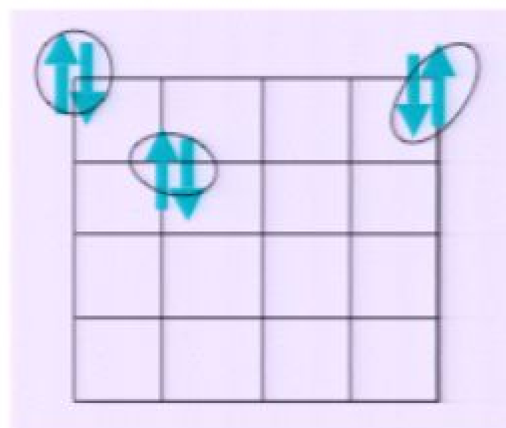
Superfluidity recently seen experimentally by
J.K. Chin et al. ,2006

Simplest model: attractive Hubbard model

$$H = -t\sum_{ij}(c^\dagger_{i\sigma}c_{j\sigma} + h.c.) - \mu\sum_i n_i - U\sum_i n_{i\uparrow}n_{i\downarrow}$$



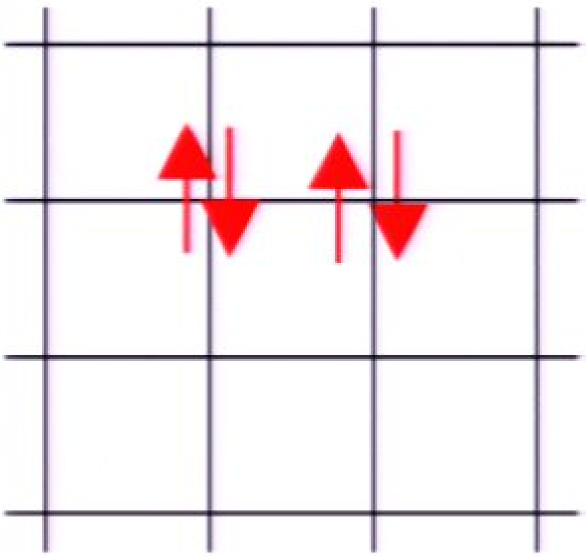
Small U/t, large pairs



Large U/t, small pairs

U/t can be tuned by magnetic field near Feshbach resonance.

Competition between superfluidity and charge order

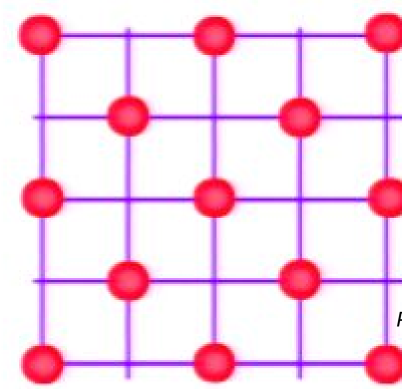


Strong pairing picture: pairs move by virtual ionization, don't want two pairs on nearby sites

$$T_i^+ = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger,\ T_i^- = c_{i\downarrow} c_{i\uparrow},\ T_i^z = \frac{1}{2}(\sum_\sigma c_{i\sigma}^\dagger c_{i\sigma} - 1)$$

$$H_{\mathrm{xxz}} = \sum_{i,j} \frac{J_{ij}}{2} \left[T_i^z T_j^z - \frac{1}{2} \left(T_i^+ T_j^- + h.c. \right) \right] - 2\mu \sum_i T_i^z$$

$$J_{ij} = 4t_{ij}^2/U$$



U(2) pseudospin rotation symmetry at half-filling (for pairs) means that SF is exactly degenerate with CDW (true for nn hopping only)

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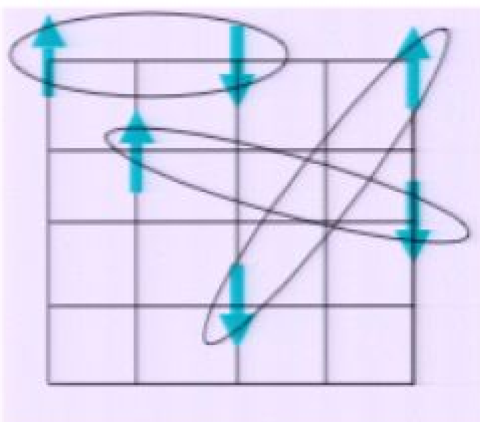

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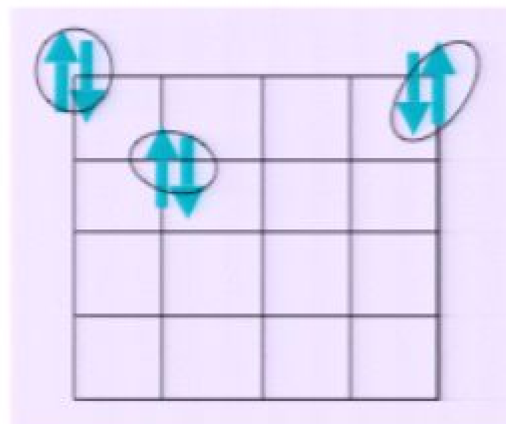

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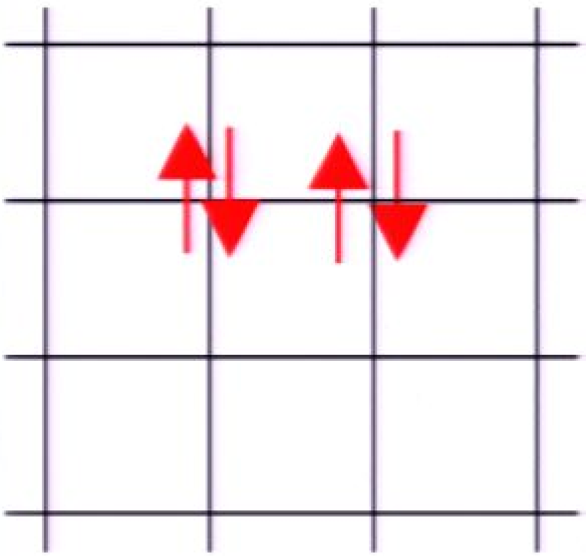

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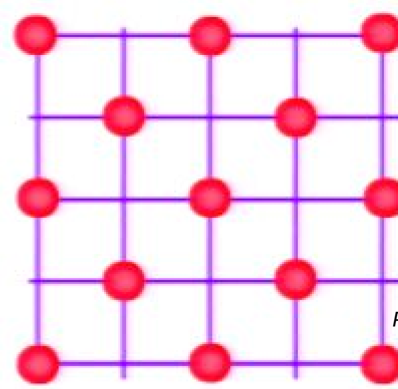

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Stability in the strong pairing limit

Energy functional of a flowing superfluid

$$E \sim \sum_{\mathbf{k}} \left[\rho_{nn}(\mathbf{k})\delta n^*_{\mathbf{k}}\delta n_{\mathbf{k}} + \rho_{\phi\phi}(\mathbf{k})\delta\phi^*_{\mathbf{k}}\delta\phi_{\mathbf{k}} + \rho_{n\phi}(\mathbf{k})\delta n^*_{\mathbf{k}}\delta\phi_{\mathbf{k}} + \rho^*_{n\phi}(\mathbf{k})\delta\phi^*_{\mathbf{k}}\delta n_{\mathbf{k}}\right]$$

Stable minimum if

$$\rho_{nn}(\mathbf{k}),\ \rho_{\phi\phi}(\mathbf{k}) > 0,\ \rho_{nn}(\mathbf{k})\rho_{\phi\phi}(\mathbf{k}) > |\rho_{n\phi}(\mathbf{k})|^2,\ \forall\mathbf{k}$$

Three instabilities: two "dynamical", one "Landau"

Dynamical vs Landau instability

$$E \sim \sum_{\mathbf{k}} \left[\rho_{nn}(\mathbf{k})\delta n^*_{\mathbf{k}}\delta n_{\mathbf{k}} + \rho_{\phi\phi}(\mathbf{k})\delta\phi^*_{\mathbf{k}}\delta\phi_{\mathbf{k}} + \rho_{n\phi}(\mathbf{k})\delta n^*_{\mathbf{k}}\delta\phi_{\mathbf{k}} + \rho^*_{n\phi}(\mathbf{k})\delta\phi^*_{\mathbf{k}}\delta n_{\mathbf{k}}\right]$$

$$\delta n_{\mathbf{k}} \sim p_{1\mathbf{k}} + ip_{2\mathbf{k}} \qquad \delta\phi_{\mathbf{k}} \sim q_{1\mathbf{k}} + iq_{2\mathbf{k}}$$

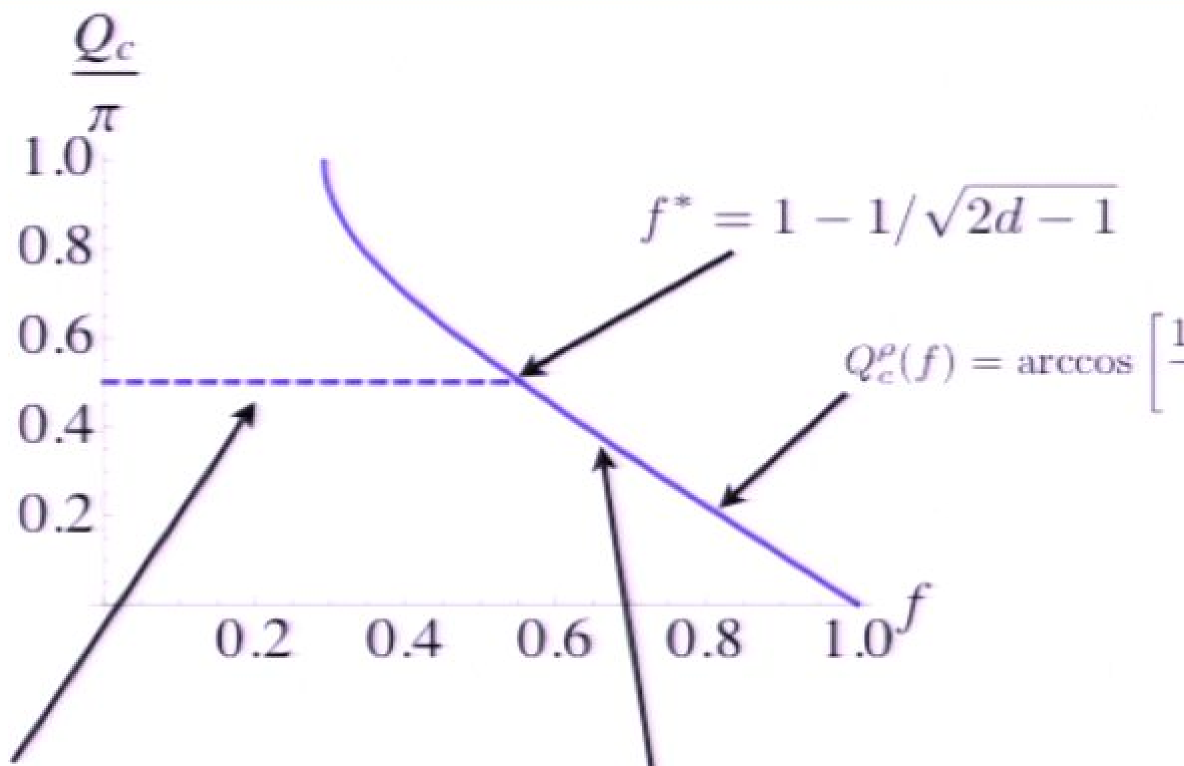
The fluctuation Hamiltonian then looks like Hamiltonian for a set of decoupled rotating Harmonic oscillators

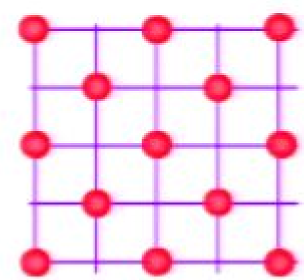
$$\omega_{\mathbf{k}} \sim \sqrt{\rho_{nn}(\mathbf{k})\rho_{\phi\phi}(\mathbf{k})} \pm |\rho_{n\phi}(\mathbf{k})|$$

Landau instability is not an instability of linearized dynamics

Can expect it to occur on much longer time scales at low T compared to dynamical instability

Dynamical instabilities in the strong pairing limit





Vanishing $\rho_{\phi\phi}(\mathbf{k})$

Vanishing $\rho_{nn}(\mathbf{k})$

$\mathbf{k} = (k_x, 0, 0)$

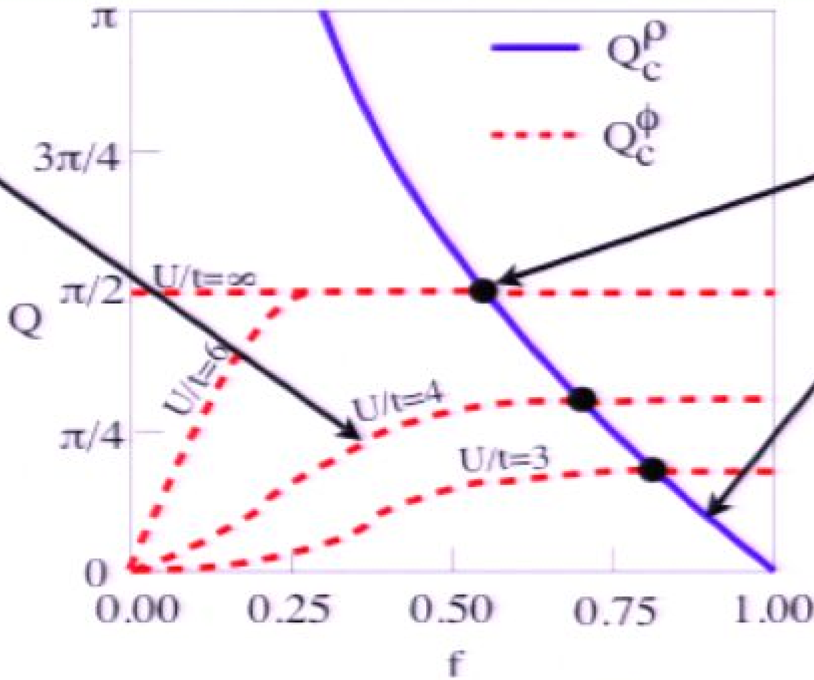
$\mathbf{k} = (\pi, \pi, \pi)$

Interaction between fermion pairs leads to a new dynamical instability, characterized by a diverging density susceptibility.

Away from strong pairing

Depairing = $\rho_{\phi\phi}$ vanishes

$f^* = 1 - 1/\sqrt{2d-1}$



ρ_{nn} vanishes

$Q_c^\phi \sim 1/\xi$ evolves smoothly to $\pi/2$ at increasing U/t

For $f > f^*$ there is a transition from depairing to collective instability

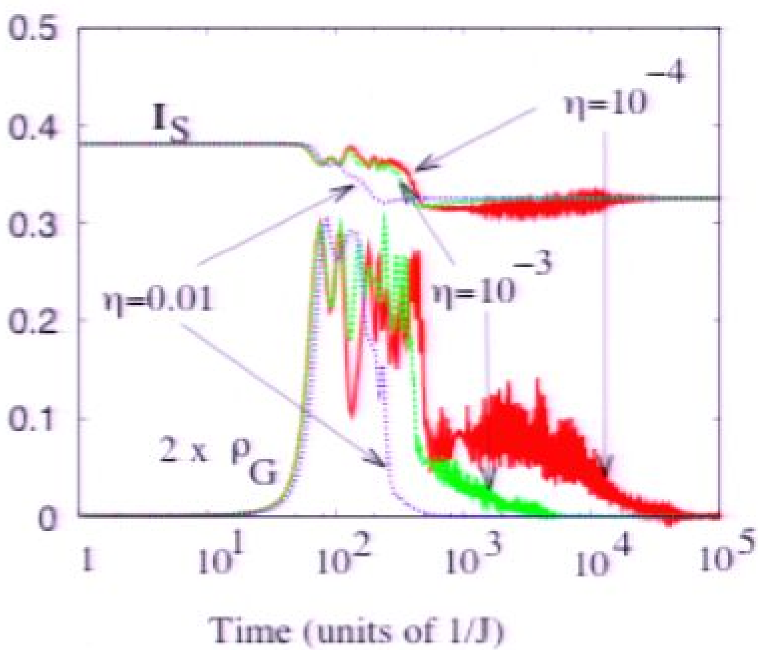
$$Q_c^\rho \sim 1/\xi^{CDW}$$

BCS-BEC crossover for $f > f^*$ is not smooth: singularity in excitation spectrum

Flowing supersolid?

$\rho_{nn}(\mathbf{k})$ vanishes at transition but $\rho_{\phi\phi}(\mathbf{k})$ does not: can expect the state on the other side to be a flowing supersolid.

Turns out not to be true on square and cubic lattice.



Transient CDW, should be possible to observe in light scattering or noise measurements

Conclusions

- Paired fermions in optical lattices have interesting physics, associated with SF-CDW competition.
- This physics can be revealed in maximum superflow measurements: new dynamical instability.
- SF-CDW competition leads to singularities in the excitation spectrum as a function of U/t, which can be seen as transition between two distinct instabilities.

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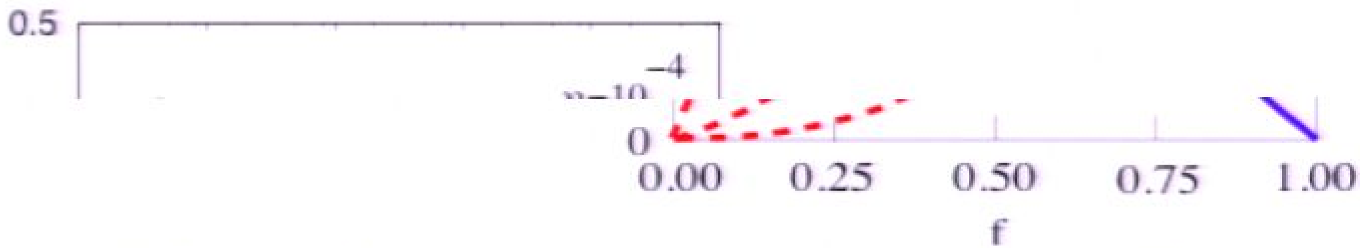

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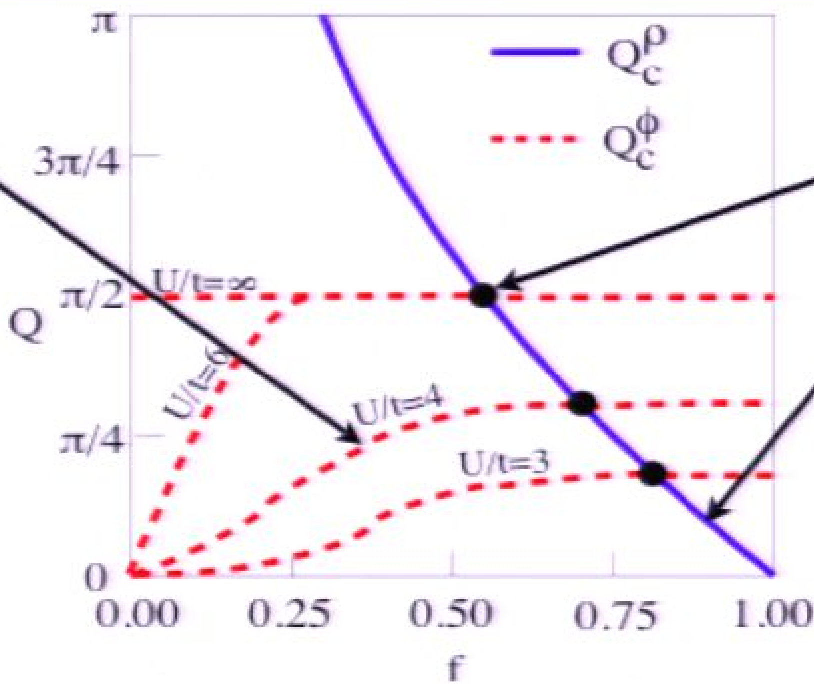

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