

Title: Dynamics in Quantum Hall Effect in Graphene

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Abstract: I present a short review of recent developments both in experiment and theory in Quantum Hall Effect in Graphene. The emphasis is on the interpretation of the dynamics underlying recently experimentally discovered novel plateaus in strong magnetic fields ($B > 20$ T).

Graphene – a one-atom-thick layer of graphite (crystalline carbon).

K. Novoselov, A. Geim et al., Science, 306 (2004) 666

Method to produce: repeated peeling of a piece of graphite with Scotch tape

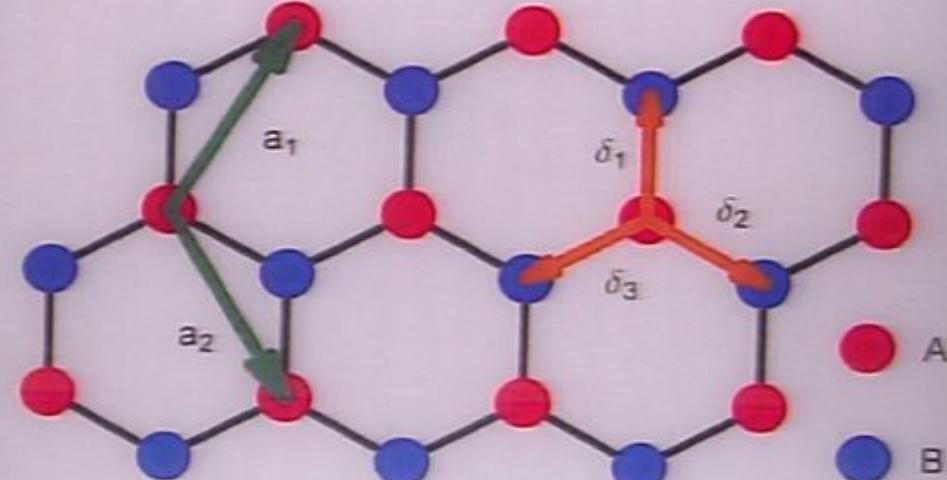
Relativistic like brane world dynamics:

electrons live on a plane;
electromagnetic field is in
3 dim. bulk

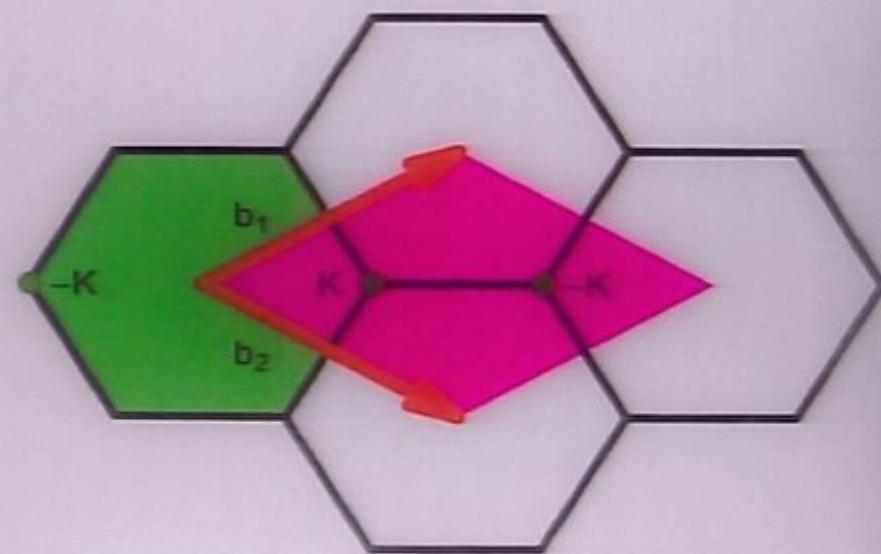
Dispersion relation for quasiparticles:

$$\omega \approx v_F |\vec{k}| ; \quad v_F \approx 10^6 \text{ m/s} ;$$

(P. Wallace, 1947) $v_F/c \approx 1/300$



(b)



Dirac Equation in Graphene

G. Semenoff, PRL, 53 (1984) 2449

$$[ih\gamma^0 \partial_t + ih\sigma_F \gamma^1 \partial_x + ih\sigma_F \gamma^2 \partial_y] \psi(t, \vec{r}) = 0,$$

$s = \pm$ is the spin index playing here the role of flavor;

$$\psi_s = (\psi_{KA}, \psi_{KB}, \psi_{K'A}, \psi_{K'B})$$

$$\gamma^\nu = (\gamma^0, \gamma^1, \gamma^2) = \tilde{\gamma}^3_{KK'} \otimes (\tilde{\gamma}^3, i\tilde{\gamma}^2, -i\tilde{\gamma}^1)_{AB}$$

Lorentz symmetry in 2+1 dimensions, with σ_F playing the role of light velocity

Dynamics underlying new plateaus
with $\nu=0$, $\nu=\pm 1$, $\nu=\pm 4$ observed
in graphene:

Y. Zhang et al., PRL 96 (2006) 136806;
Z. Jiang et al., PRL 99 (2007) 106802

"Old" plateaus:

$$C_{xy} = \frac{e^2}{h}\nu, \quad \nu = \pm 4(n + \frac{1}{2}), \quad n = 0, 1, \dots$$

Experiment: K. Novoselov et al., Nature 438 (2005) 97,
Y. Zhang et al., Nature 438 (2005) 201

Theory: Y. Zhang and T. Ando, PRB 65 (2002) 245410;
V. Gusynin, S. Sharapov, PRL 95 (2005) 146801;
N. Peres, F. Guinea, A. Castro Neto, PRB 73 (2006) 125411

$$E_n = \pm \sqrt{2n\hbar v_F^2 eB/c} \simeq 424\sqrt{n} \sqrt{B/T} \text{ K}$$

$$n = 0, 1, 2, \dots \quad v_F \simeq 10^6 \text{ m/s}$$

LLL with $n=0$, $E_0=0$, is special

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Nonrelativistic Dynamics in Magnetic Field

$$E_n = \frac{n \hbar v_F^2 / eB}{c \cdot mc^2} = n E_i^{(ree)} \cdot E_i / 2mc^2$$

$$E_i^{(ree)} = \sqrt{2\hbar v_F^2 / eB}/c \approx 424 \sqrt{B/T} \text{ K}$$

$$2mc^2 \underset{m=m_e}{\approx} 1 \text{ Mev} \sim 10^{10} \text{ K}$$

$E_i^{(ree)}/2mc^2 \ll 1$ for realistic values of B .

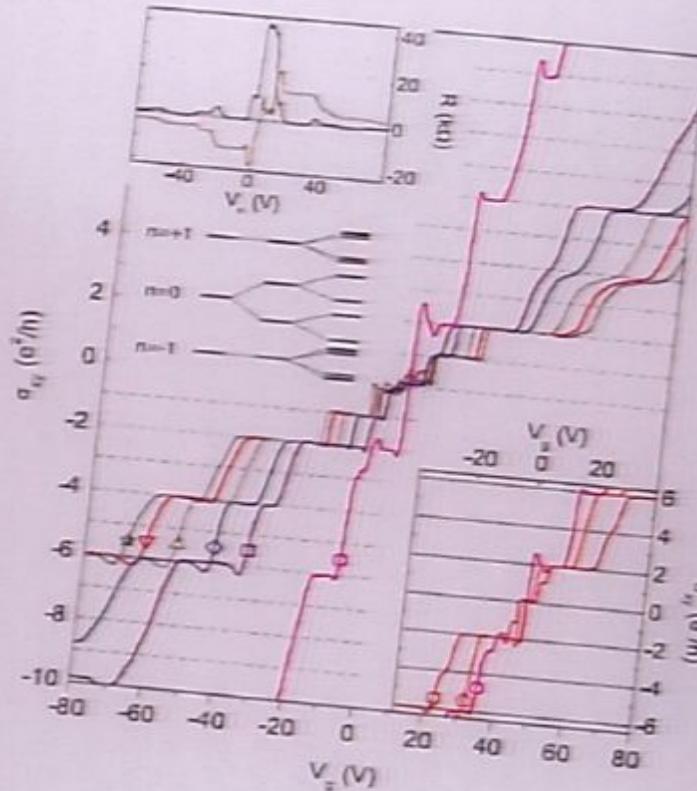


FIG. 2. (color online) σ_{xy} as a function of V_z at different magnetic fields: 9 T (circle), 25 T (square), 30 T (diamond), 37 T (up triangle), 42 T (down triangle), and 45 T (star). All the data sets are taken at $T = 1.4$ K, except for the $B = 9$ T curve, which is taken at $T = 30$ mK. Left upper inset: R_{xx} and R_{yy} for the same device measured at $B = 25$ T. Left lower inset: a schematic drawing of the LLs in low (left) and high (right) magnetic field. Right inset: detailed σ_{xy} data near the Dirac point for $B = 9$ T (circle), 11.5 T (pentagon) and 17.5 T (hexagon) at $T = 30$ mK.

Model: Hamiltonian

$$H = H_0 + H_C - \mu_0 \vec{\psi}_S^\dagger \vec{\psi}_S + \mu_B B \vec{\psi}_S^\dagger \vec{\sigma}^3 \vec{\psi}_S,$$

$$\mu_0 = e\hbar/2mc, g \approx 2, (e>0); B \equiv |\vec{B}|$$

$$H_0 = \partial_F \int d^2 \vec{r} \vec{\psi}_S^\dagger (\gamma^0 \vec{\tau}_x + \gamma^1 \vec{\tau}_y) \vec{\psi}_S,$$

$$\vec{r} = (x, y), \vec{\tau}_I = (\vec{\tau}_x, \vec{\tau}_y) = -i\hbar \vec{\nabla} + e\vec{A}/c,$$

$$\partial_F \approx 10^6 \text{ m/s}, \vec{\psi}_S = (\psi_{KAS}, \psi_{KBS}, \psi_{KB5}, \psi_{KA5}),$$

$$\gamma^\nu = (\gamma^0, \gamma^1, \gamma^2) = \tilde{\tau}_{KK'}^3 \otimes (\tilde{\tau}^3, i\tilde{\tau}^2, -i\tilde{\tau}^1)_{AB},$$

$$\vec{\psi}_S = \vec{\psi}_S^\dagger \gamma^0; \vec{A} \text{ corresponds to } B_\perp.$$

$$H_C = \frac{1}{2} \int d^2 \vec{r} d^2 \vec{r}' \vec{\psi}_S^\dagger(\vec{r}) \vec{\psi}_C(\vec{r}) \vec{\psi}_C(\vec{r}-\vec{r}') \vec{\psi}_S^\dagger(\vec{r}') \vec{\psi}_S(\vec{r})$$

H_0 : Lorentz symmetry in 2+1 dimensions, with ∂_F playing the role of light velocity

Model: Symmetry

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$$H = H_0 + H_c - \mu_0 \psi_s^\dagger \psi_s + \mu_B B \psi_s^\dagger \tilde{\sigma}^3 \psi_s$$

$H_0 + H_c - \mu_0 \psi_s^\dagger \psi_s$ possesses the $U(4)$ symmetry with 16 generators:

$$\frac{\sigma^a}{2} \otimes I_4, \frac{\sigma^a}{2i} \otimes \gamma^3, \frac{\sigma^a}{2} \otimes \gamma^5, \frac{\sigma^a}{2} \otimes \gamma^3 \gamma^5,$$

where I_4 - 4×4 Dirac unit matrix, σ^a - four Pauli matrices (σ^0 is 2×2 unit matrix),

$$\gamma^3 = i \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^5 = i \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \quad \text{and}$$
$$\gamma^3 \gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} - \text{pseudospin matrix}$$

Zeeman term $\mu_B B \psi_s^\dagger \tilde{\sigma}^3 \psi_s$ breaks $U(4)$ down to $U(2)_+ \times U(2)_-$ with 8 generators $I_4 \otimes P_S$, $\gamma^5 \otimes P_S$, $-i\gamma^3 \otimes P_S$, and $\gamma^3 \gamma^5 \otimes P_S$. $S = \pm$, $P_\pm = (I \pm \tilde{\sigma}_3)/2$ are projectors on spin up and down states

Order Parameters

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Goal: searching for solutions of gap equation with spontaneously broken and unbroken $SU(2)_S$ ($U(1)_S$, $S=U$)

Two scenarios for QH effect

a) Quantum Hall Ferromagnetism (QHF)

(K. Namura, R. MacDonald, PRL 96 (2006) 256602;

M. Goerbig, R. Moessner, B. Doucot, PRB 74 (2006) 161407;
J. Alicea and M.P.A. Fisher, PRB 74 (2006) 075422;)

Order parameters: pseudospin densities

$$\langle \psi^\dagger \gamma^3 \gamma^5 P_S \psi \rangle \neq 0 \Rightarrow SU(2)_S \rightarrow U(1)_S$$

with the generator $\gamma^3 \gamma^5 \otimes P_S$, $P_S = \frac{I \pm \sigma^3}{2}$

$\tilde{\mu}_S$ are corresponding chemical potentials. Triplet order parameters

Scenario is based on conventional ferromagnetism

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 $s=\pm$

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Scenario is based on conventional ferromagnetism

b) Magnetic Catalysis (MC)

(V. Gusynin, V. Miransky, S. Sharapov, I. Shovkovy,
PRB 74 (2006) 195429; I. Herbut, PRL 97 (2006)
146401; J. Fuchs, P. Lederer, PRL 98 (2007) 016803)

Order parameters: Dirac mass terms
 $\langle \bar{\psi} \gamma_5 \psi \rangle = \langle \psi^\dagger \gamma_5 \gamma_0 \psi \rangle \neq 0 \Rightarrow$
 $SU(2)_S \rightarrow U(1)_S$

$\tilde{\Delta}_S$ are corresponding Dirac masses (gaps).
Triplet order parameters.

Scenario is based on relativistic 2+1 dimensional dynamics in magnetic field

V. Gusynin, V. Miransky, I. Shovkovy, PRL 73 (1994)
13499

In graphite:

D. Khrestchuk, PRL 87 (2001) 206401;
E. Gorbar, V. Gusynin, V. Miransky, I. Shovkovy, PRB
66 (2002) 045108

Magnetic Catalysis Phenomenon

A constant magnetic field is a strong catalyst of dynamical chiral symmetry breaking, leading to the generation of a Dirac dynamical mass even at the weakest repulsive interactions between fermions

V. Gusynin, V. M. and I. Shorkov, Phys. Rev. Lett. 73, 3499 (1994); Nucl. Phys. B 462, 249 (1996)

Dimensional reduction

$$2+1 \rightarrow 0+1, \quad 3+1 \rightarrow 1+1$$

$$E_0 = 0 \quad LLL$$

Latest developments: QHF and MC
sets of order parameters necessarily
coexist, which implies that they have
the same dynamical origin.

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E. Gorbar, V. Gusynin, V. Miransky, arxiv:
0710.3527 [cond-mat]

E. Gorbar, V. Gusynin, V. Miransky, I. Shovkovy,
(to appear)

General form of inverse quasiparticle propagator:

$$iG_s^{2-1}(v, v') = [(i\hbar \partial_v + \mu_s + \tilde{\mu}_s \gamma^3 \gamma^5) - \eta_F \vec{\gamma}^2 \gamma^5 + \\ + \Delta_s \gamma^3 \gamma^5 - \tilde{\Delta}_s] \gamma^3 (v - v')$$

One more surprise: $\Delta_s = s\alpha(\Omega_s)$ singlet Dirac mass $\Rightarrow \langle \bar{\psi} \gamma^3 \gamma^5 P_S \psi \rangle$. It is odd under time reversal

Δ_s in graphite with no magnetic field:

F. D. M. Haldane, PRL 61 (1988) 2015

Parity anomaly in (2+1)
dimensional QFT: R. Jackiw;
A. Niemi; G. Semenoff

Conclusion

1. It seems QHF and MC scenarios are two sides of the same coin.
2. Generating dynamical Dirac masses in tabletop experiment.
3. Consequences for dynamics of edge states.
4. Potential consequences for pseudo-LNG excitations.
5. "Convergence" of QHF and MC scenarios is a good sign for theory.

Correspondence between order parameters and electron densities

$$\psi = (\psi_{KA}, \psi_{KB}, \psi_{K'A}, \psi_{K'B})$$

(spin is omitted)

$$\mu \rightarrow \langle \psi^+ \psi \rangle = n_{KA} + n_{K'A} + n_{KB} + n_{K'B}$$

$$\tilde{\mu} \rightarrow \langle \psi^+ j_3 j_5 \psi \rangle = n_{KA} - n_{K'A} + n_{KB} - n_{K'B}$$

$$\Delta \rightarrow \langle \bar{\psi} j_3 j_5 \psi \rangle = n_{KA} - n_{K'A} - n_{KB} + n_{K'B}$$

$$\tilde{\Delta} \rightarrow \langle \bar{\psi} \psi \rangle = n_{KA} + n_{K'A} - n_{KB} - n_{K'B},$$

$n_{K(A)A(B)}$ are electron densities at specified valley and sublattice points