Title: Nodal Quasiparticles and Spin and Charge Order in the Cuprate Superconductors

Date: Apr 24, 2008 09:00 AM

URL: http://pirsa.org/08040031

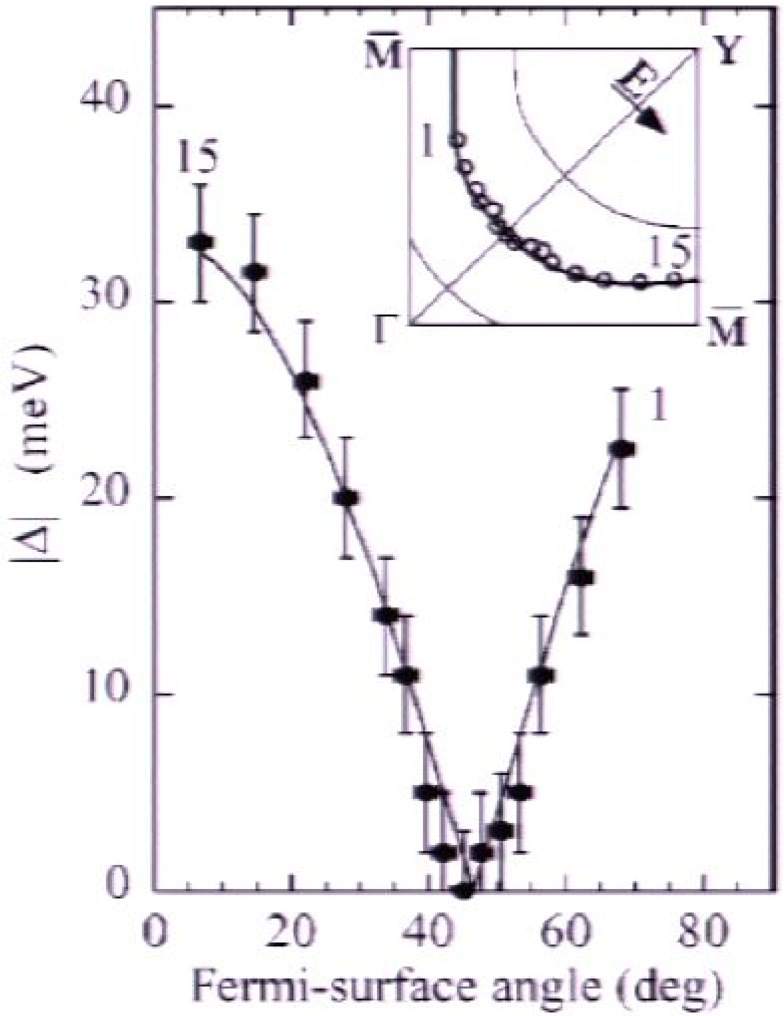
Abstract: I will discuss the interplay between the fermionic nodal quasiparticles of a d-wave superconductor and the various spin and charge orders that have been observed in the cuprate superconductors. Fluctuations of a composite \'nematic\' order are identified as the dominant source of inelastic scattering which broadens the quasiparticle spectral function.

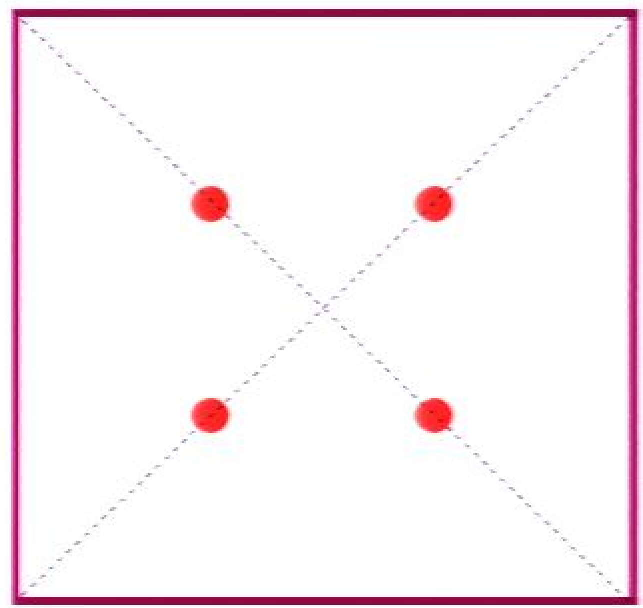
Electronic quasiparticles and competing orders in the cuprate superconductors

Andrea Pelissetto, Rome
Ettore Vicari, Pisa

Yejin Huh
Subir Sachdev

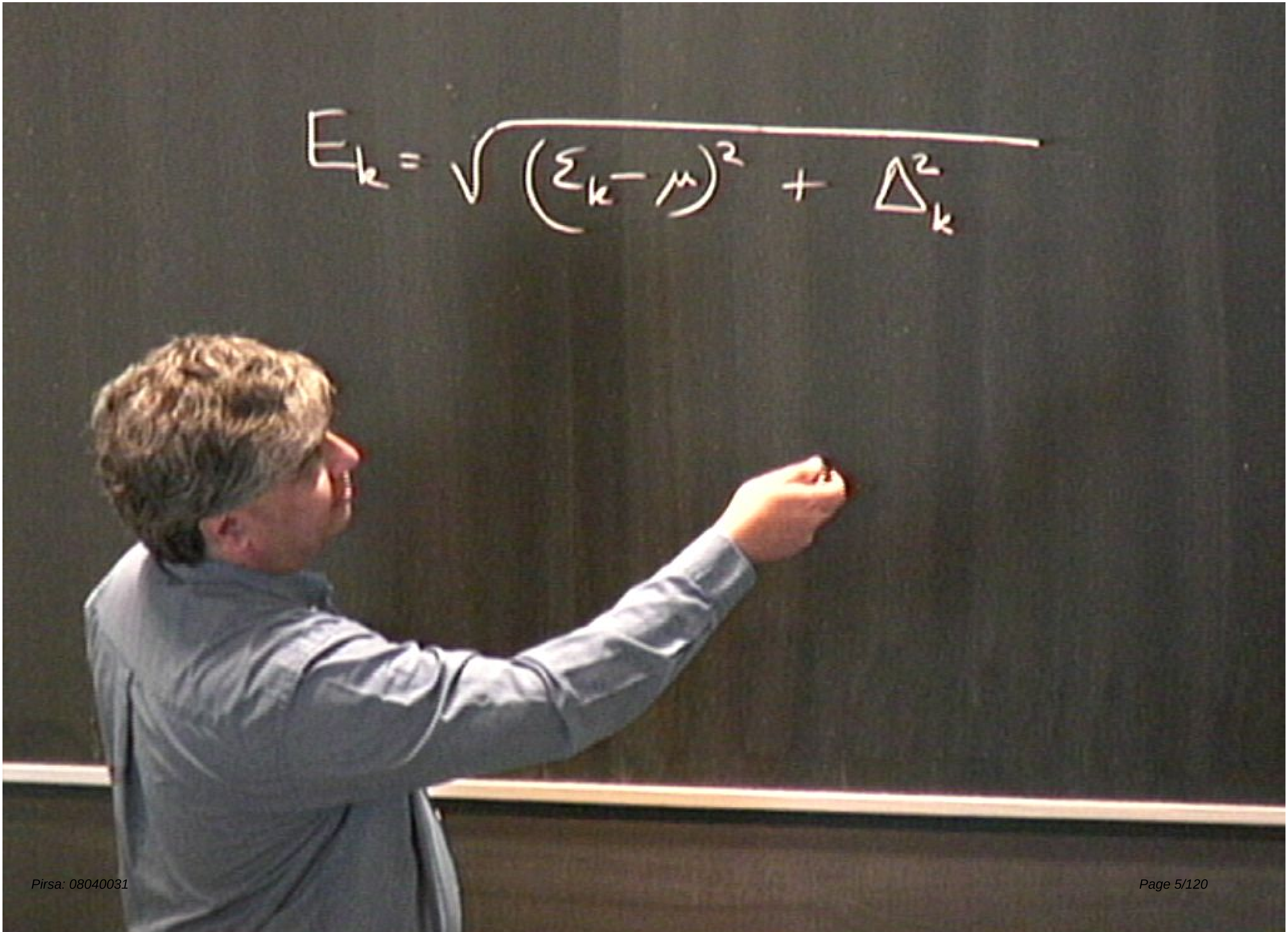
Gapless nodal quasiparticles in *d*-wave superconductors



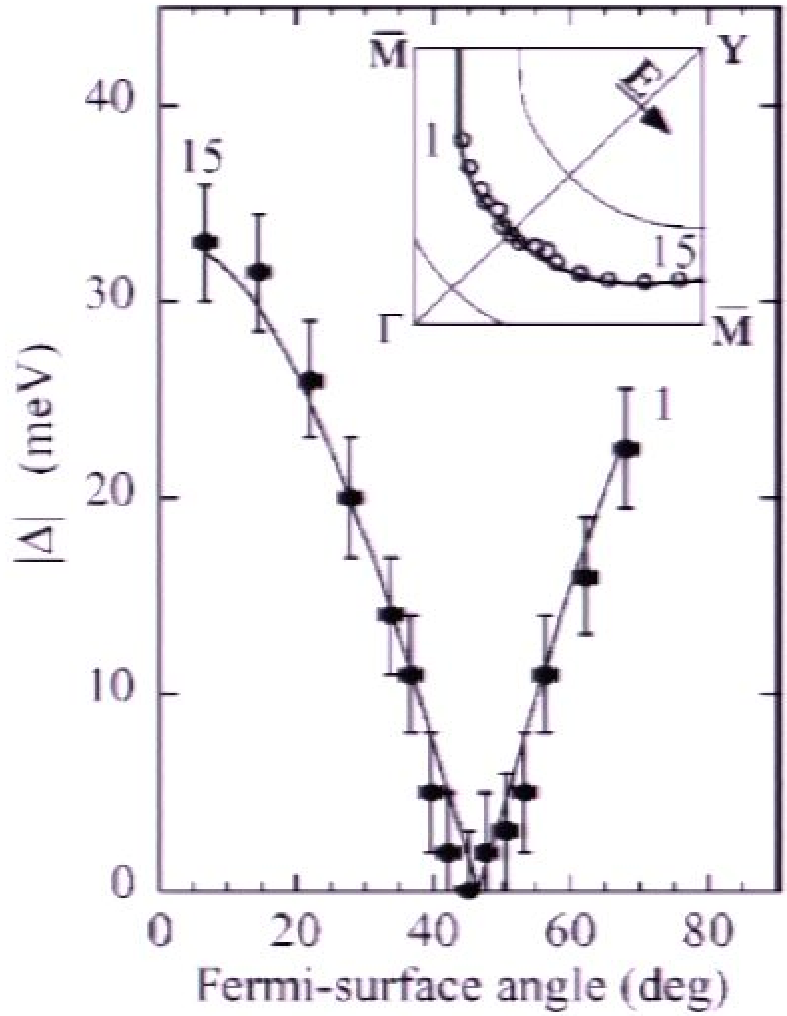


Brillouin zone

FIG. 46. Superconducting gap measured at 13 K on Bi2212 (T_c=87 K) plotted vs the angle along the normal-state Fermi surface (see sketch of the Brillouin zone), together with a *d*-wave fit. From Ding, Norman, *et al.*, 1996.


$E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$

Gapless nodal quasiparticles in *d*-wave superconductors



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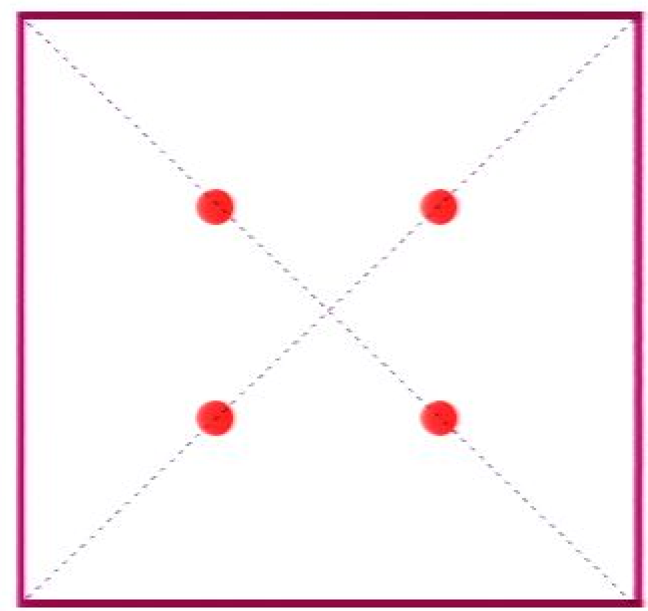

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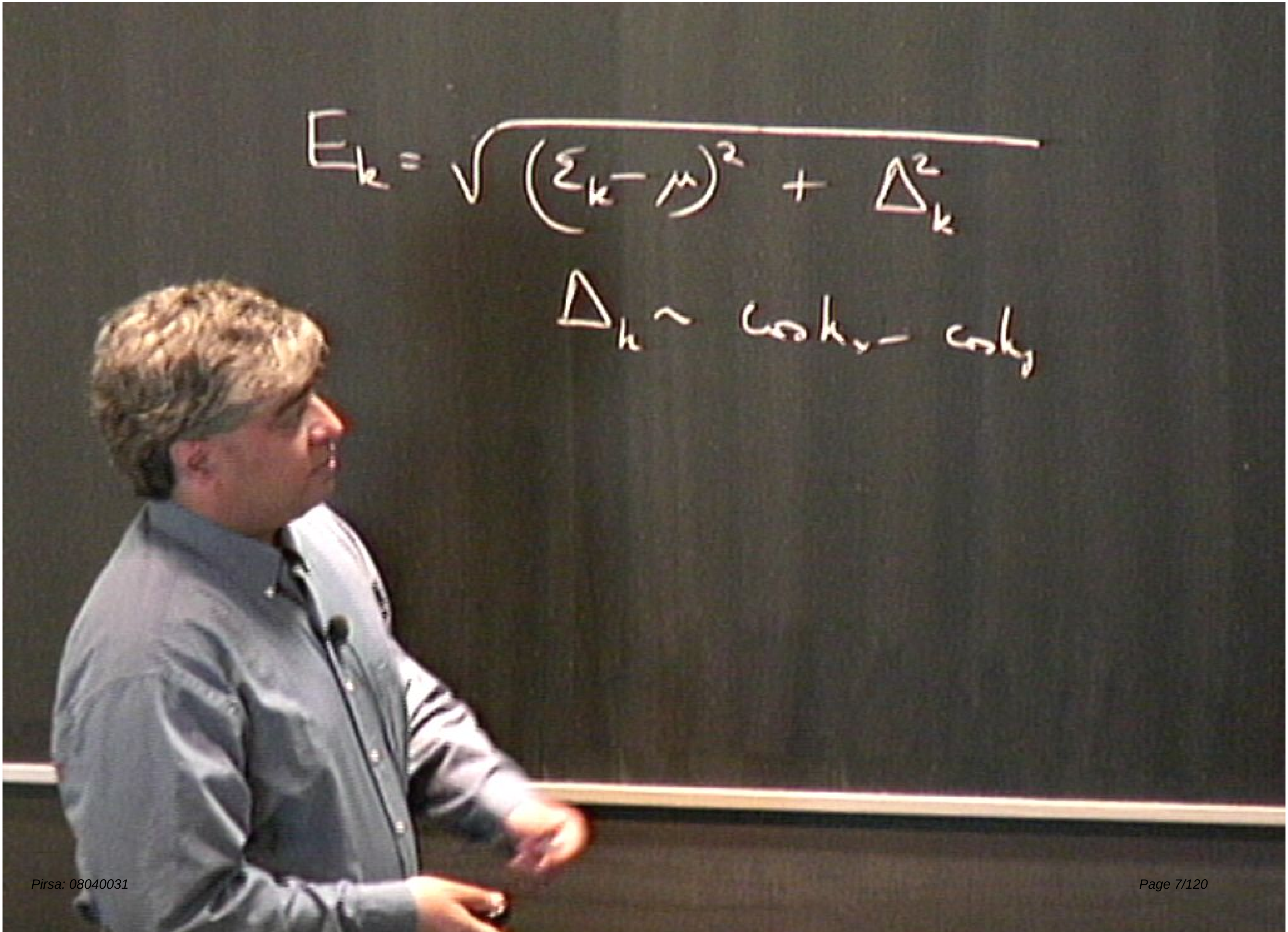
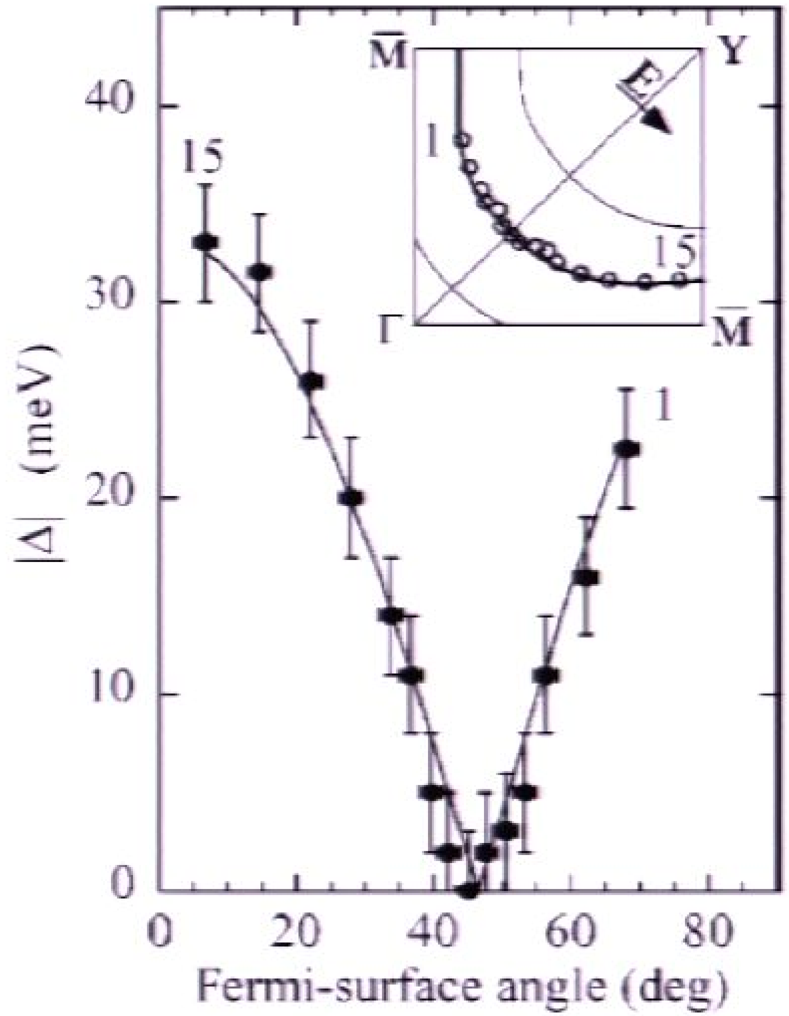

$E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2}$
$\Delta_k \sim \cos k_x - \cos k_y$

Gapless nodal quasiparticles in d-wave superconductors



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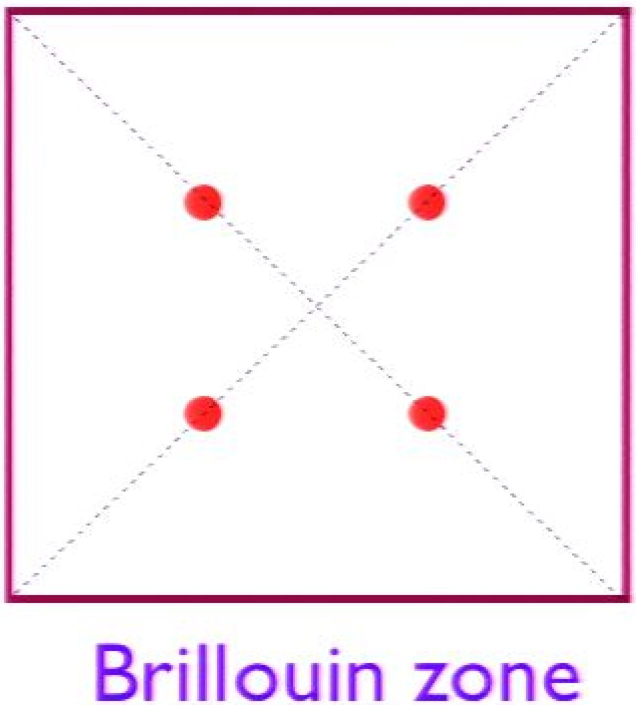

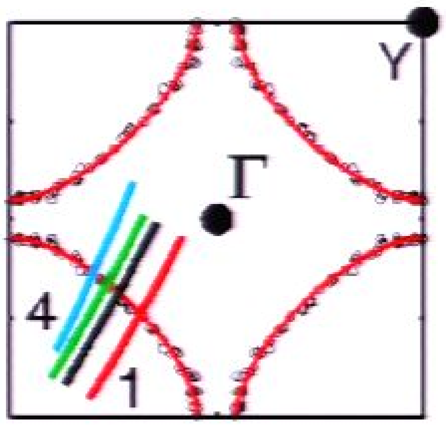

Photoemission spectra of $La_{2-x}Sr_xCuO_4$

x=0.145

J. Chang et al. (PSI Mesot group), arXiv:0708.2782

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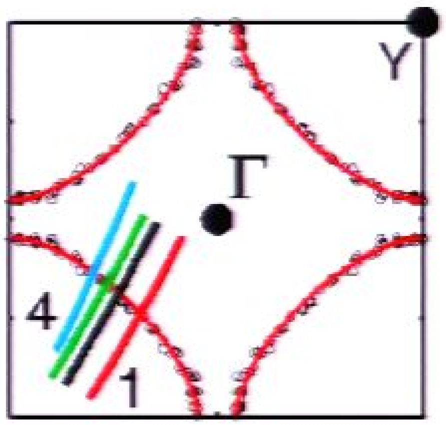

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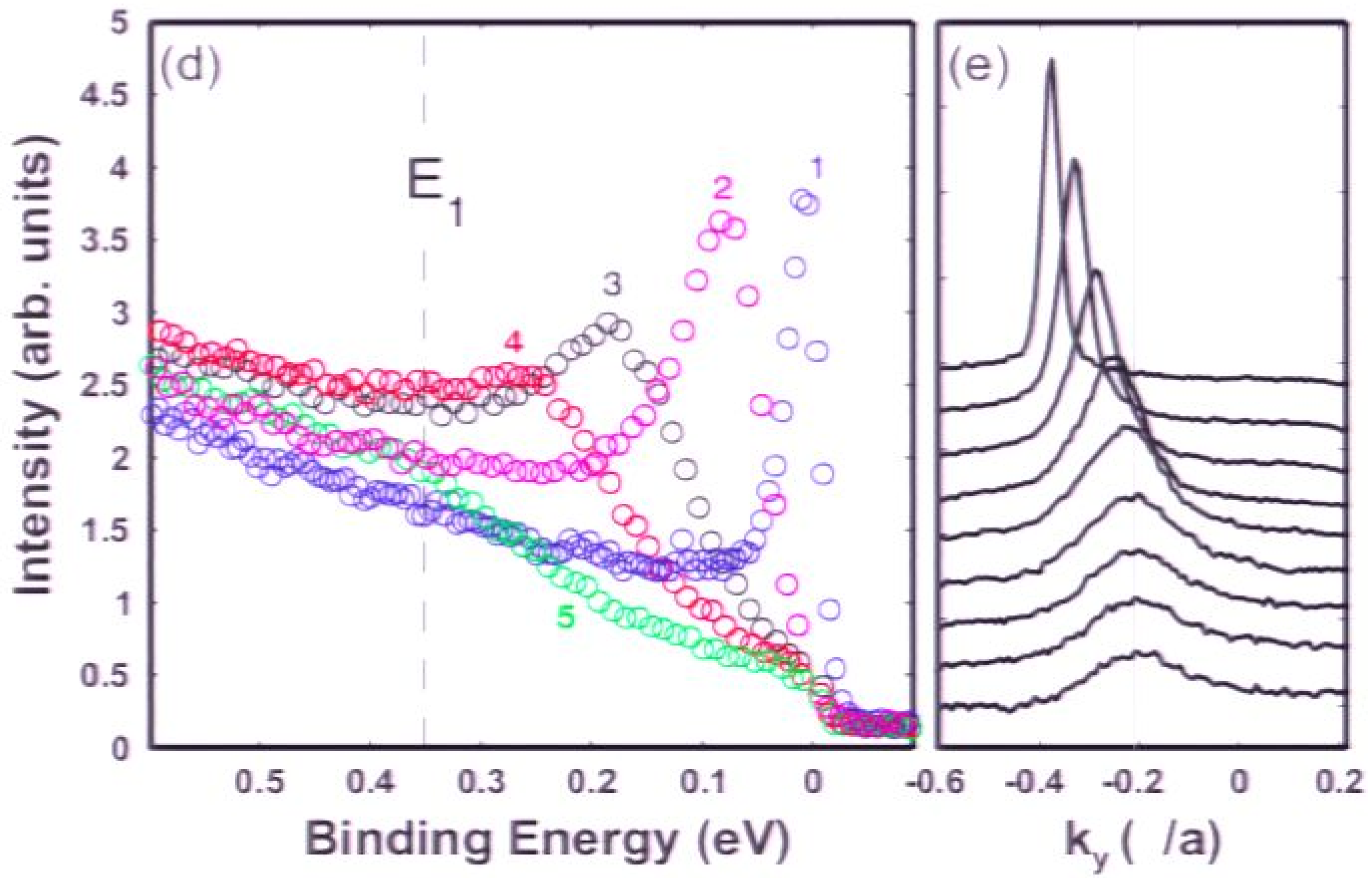

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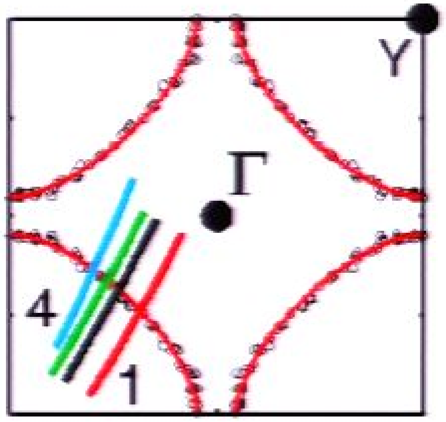

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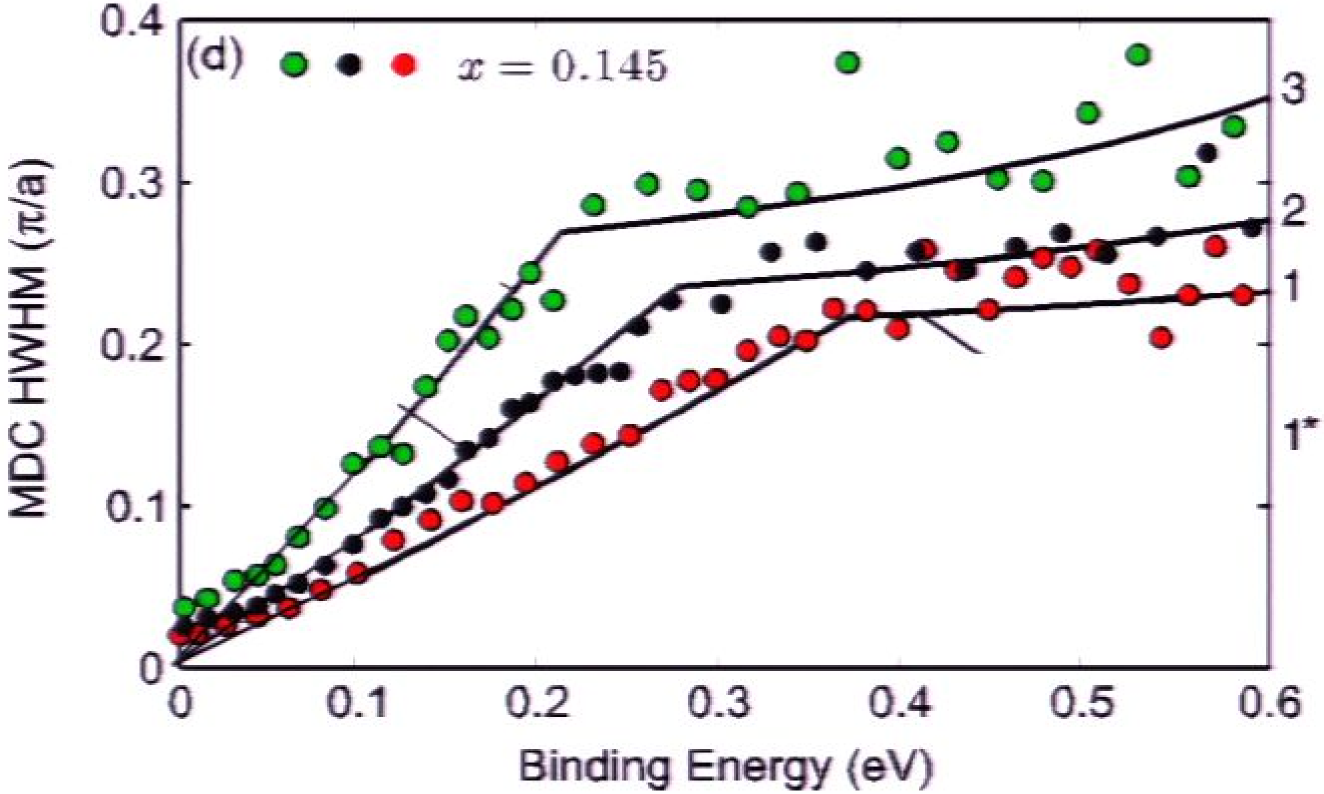

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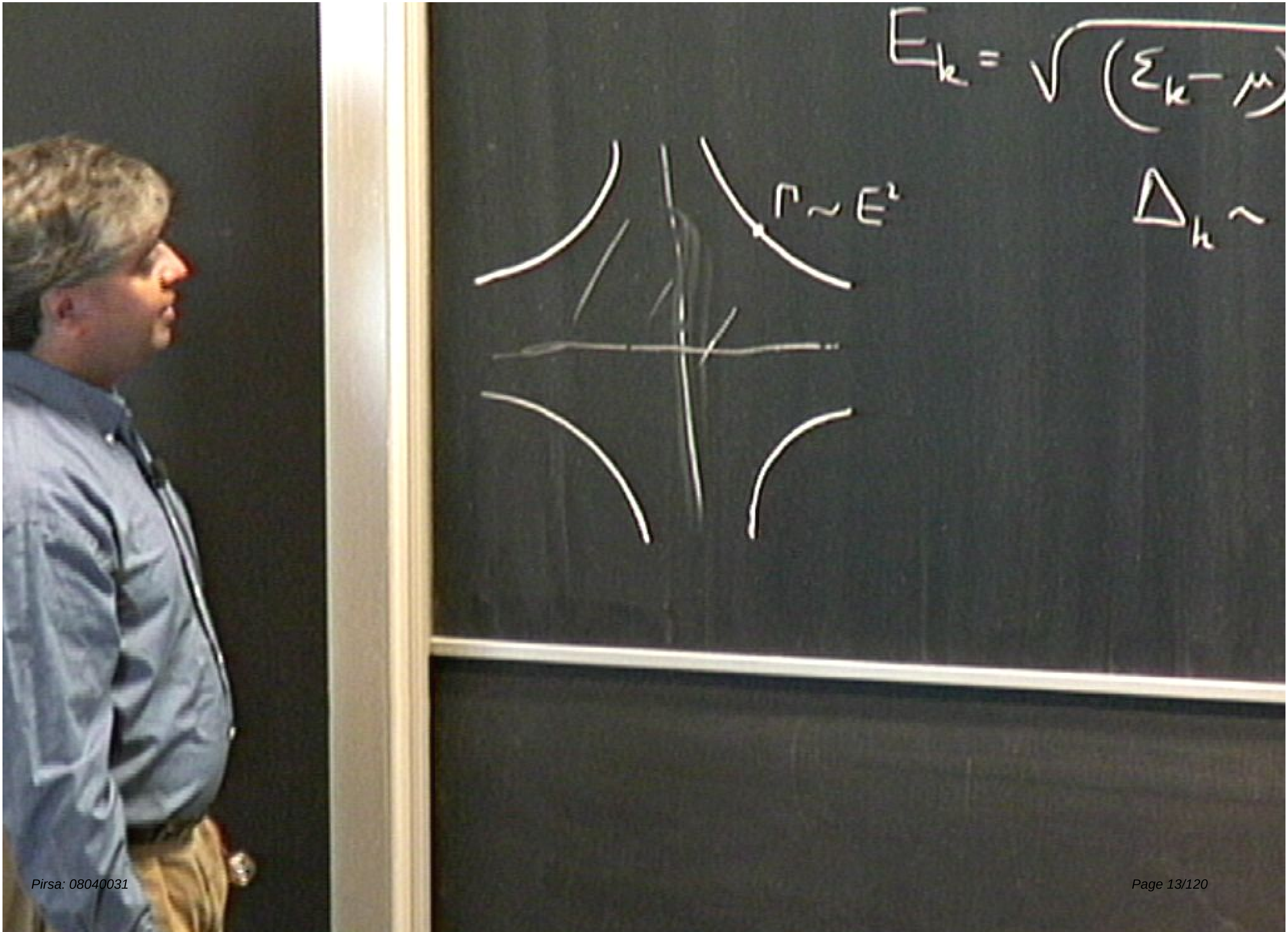
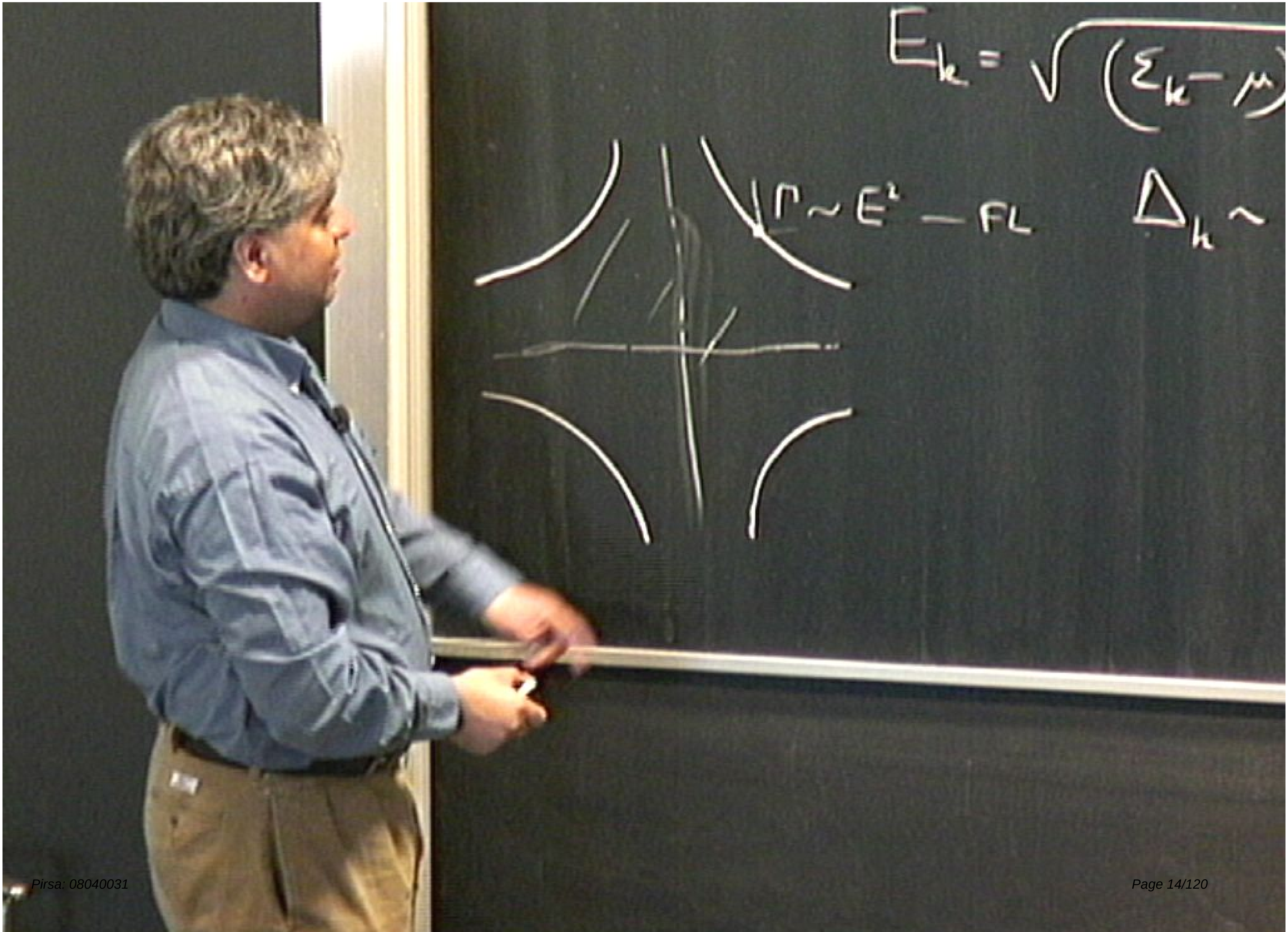

$E_k = \sqrt{(\varepsilon_k - \mu)}$
$\Gamma \sim E^2$
$\Delta_k \sim$


$E_k = \sqrt{(\epsilon_k - \mu)}$
$\Gamma \sim E^2$ — FL
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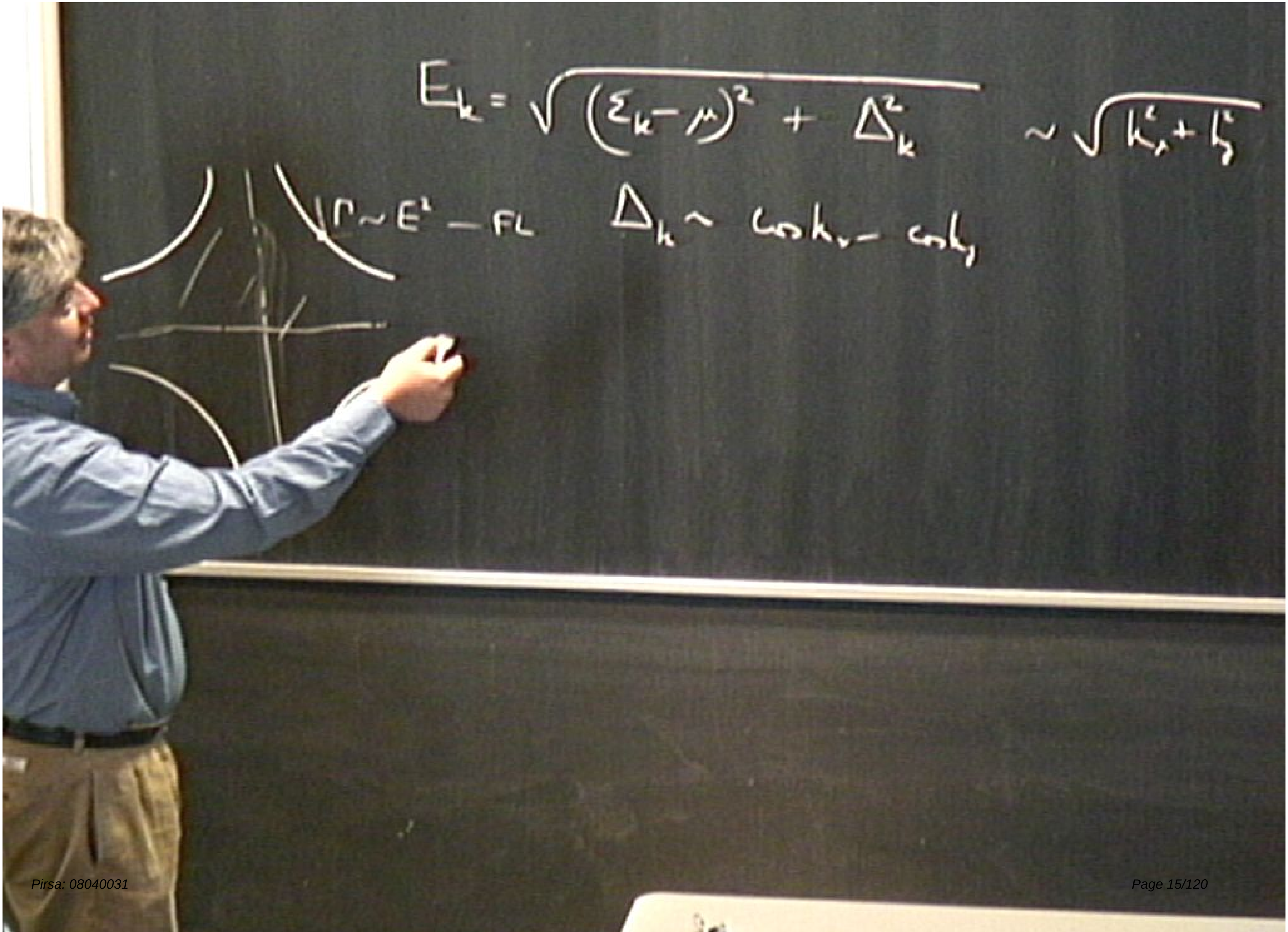
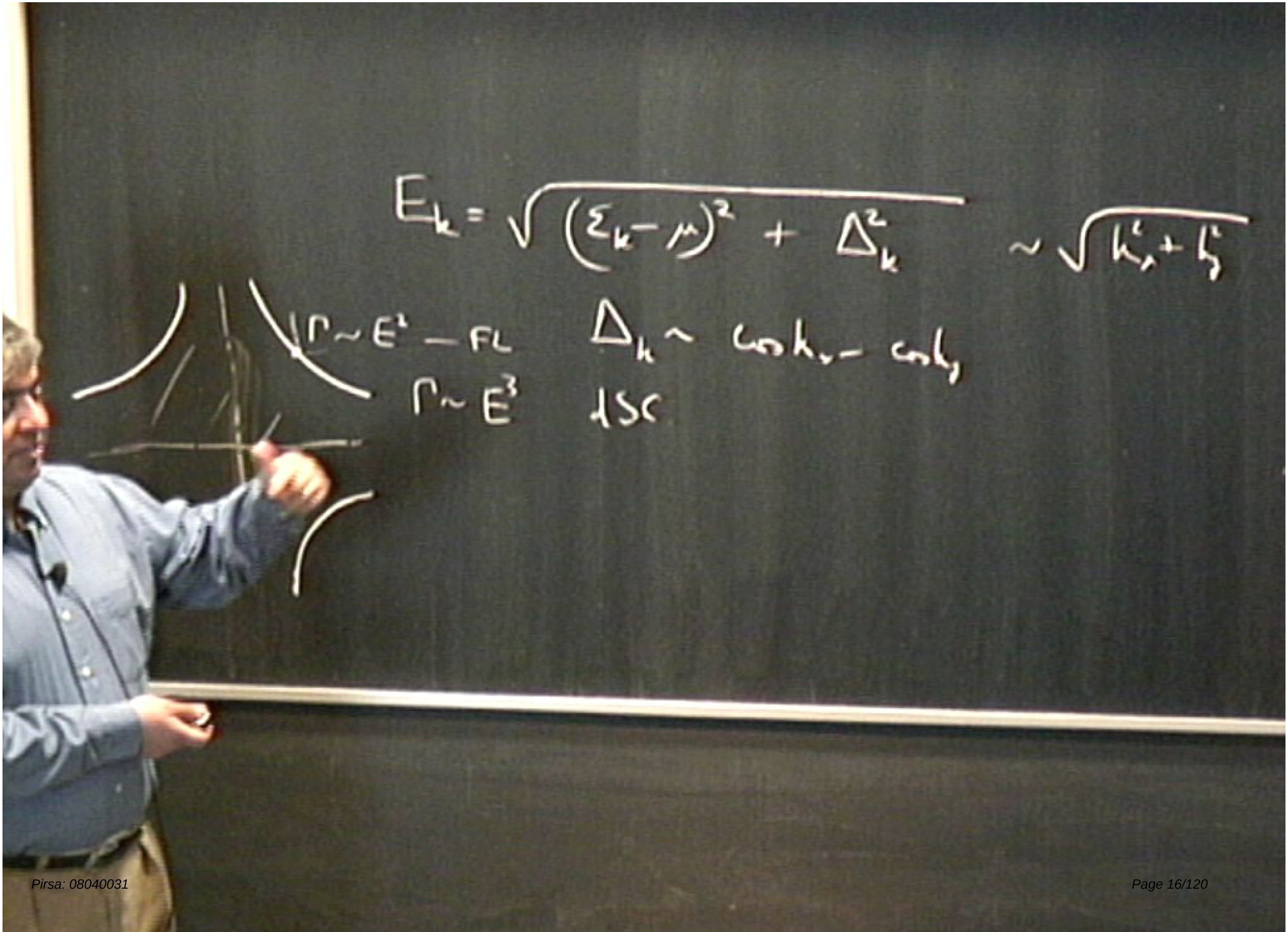

$E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta_k^2} \sim \sqrt{k_x^2 + k_y^2}$
$\Gamma \sim E^2$ — FL
$\Delta_k \sim \cos k_x - \cos k_y$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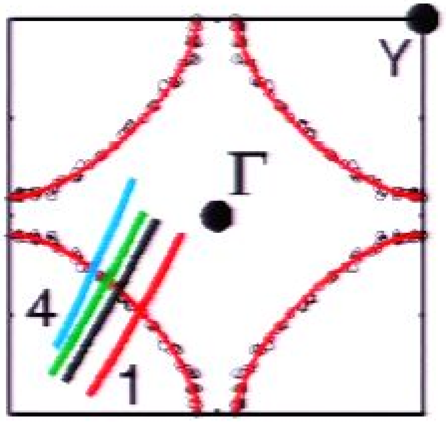
$\Gamma \sim E^2$ — FL
$\Delta_k \sim \cos k_x - \cos k_y$
$\Gamma \sim E^3$
dSC



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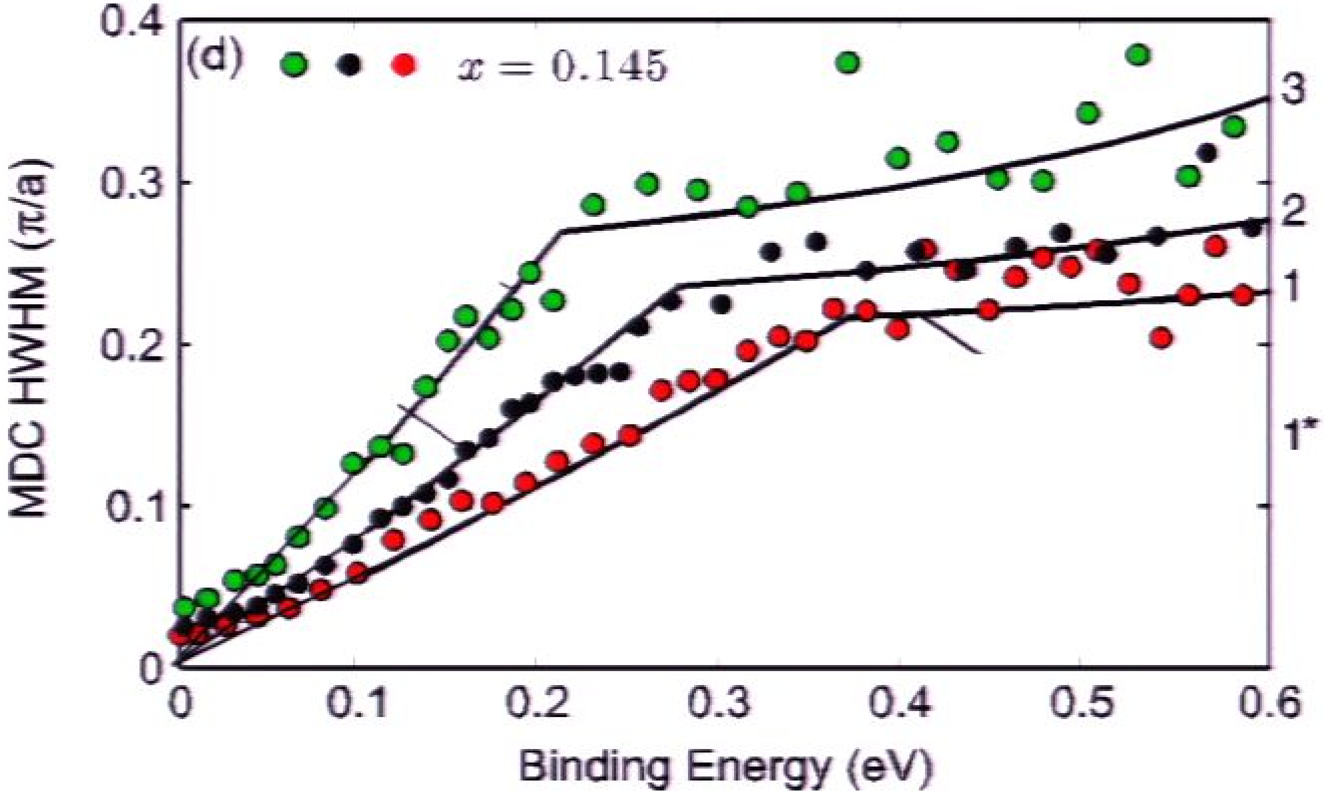

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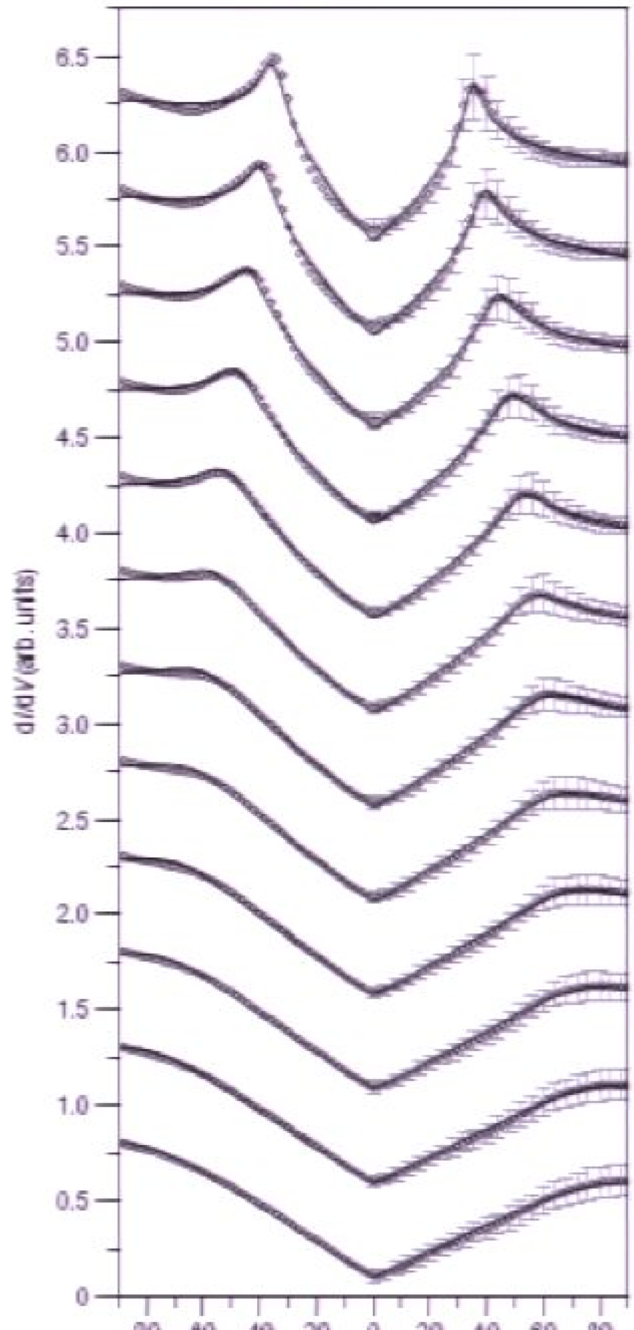
Scanning tunneling microscopy of BSCCO

$$N(E,\Gamma_2) = A \times \mathrm{Re}\left(\left\langle \frac{E + i\Gamma_2(E)}{\sqrt{(E+i\Gamma_2(E))^2 - \Delta(k)^2}} \right\rangle_{fs}\right) + I$$

Good fit with

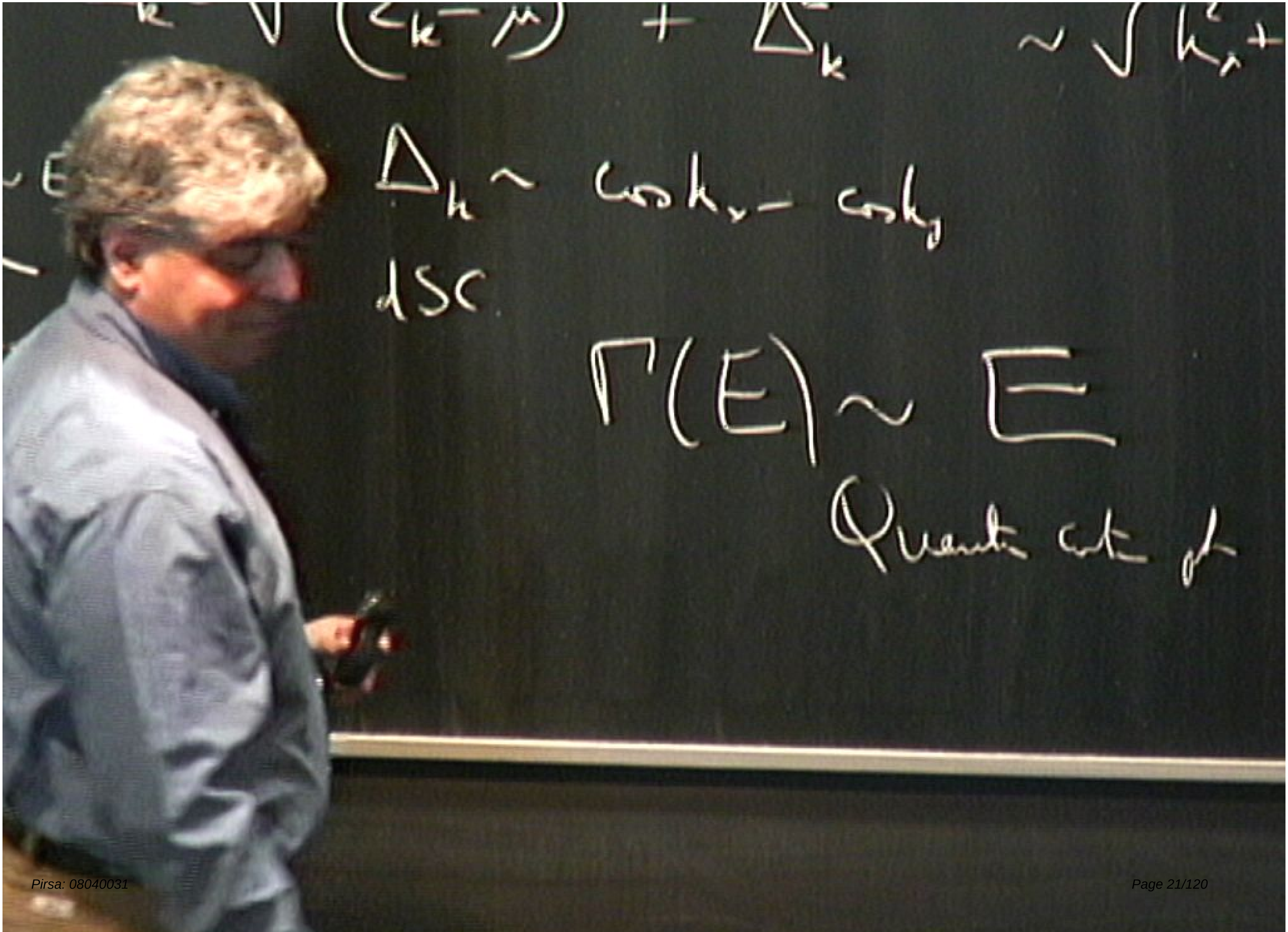
$$\Gamma_2(E) = \alpha E$$

J. W. Alldredge, Jinho Lee, K. McElroy, M. Wang, K. Fujita, Y. Kohsaka, C. Taylor, H. Eisaki, S. Uchida, P. J. Hirschfeld, J. C. Davis



Nature Physics **4**, 319 - 326 (01 Apr 2008)

Needed:
Quantum critical point with nodal quasiparticles part of the critical theory


$\Delta_k \sim \cos k_x - \cos k_y$
dSC
$\Gamma(E) \sim E$

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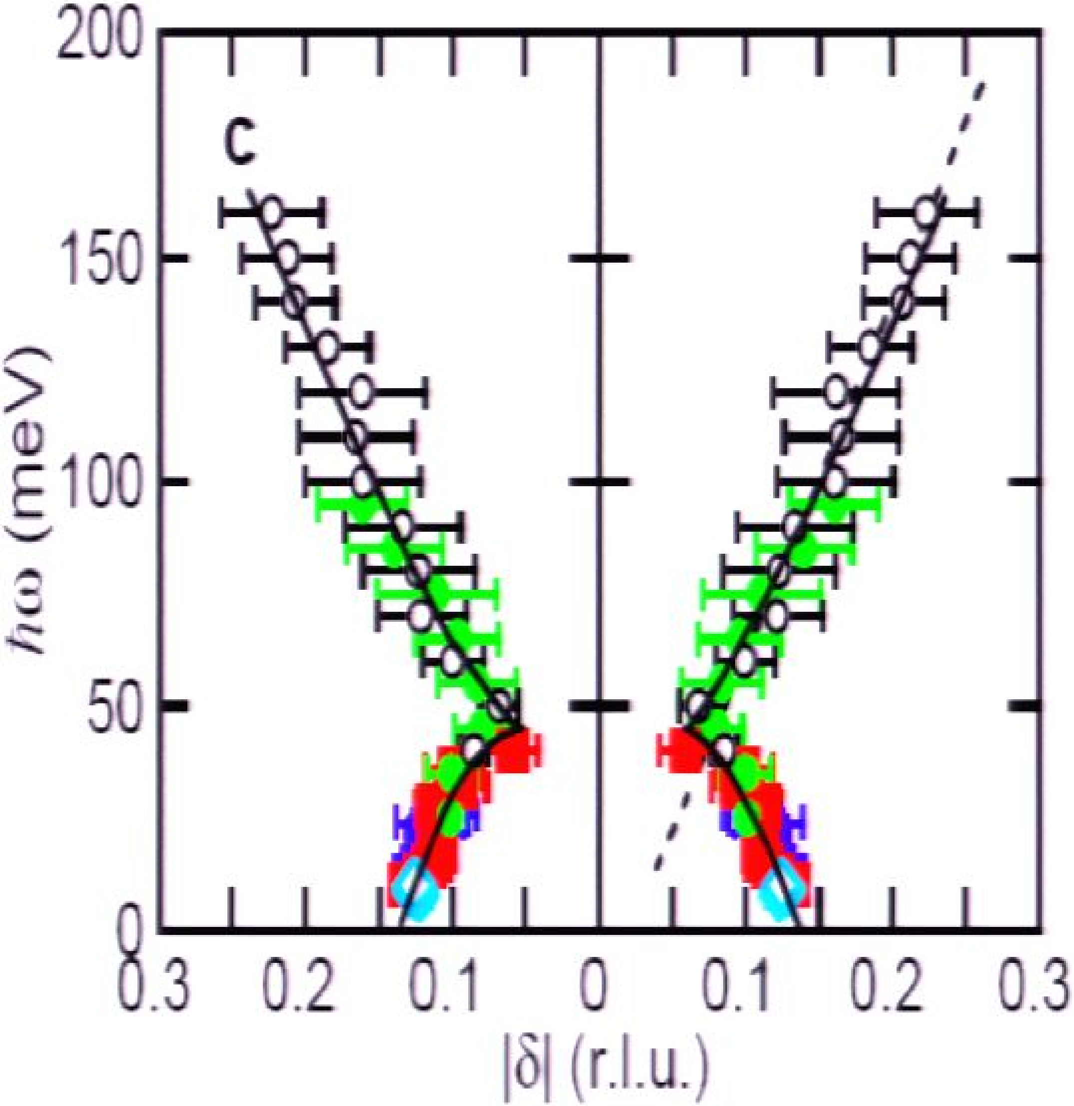
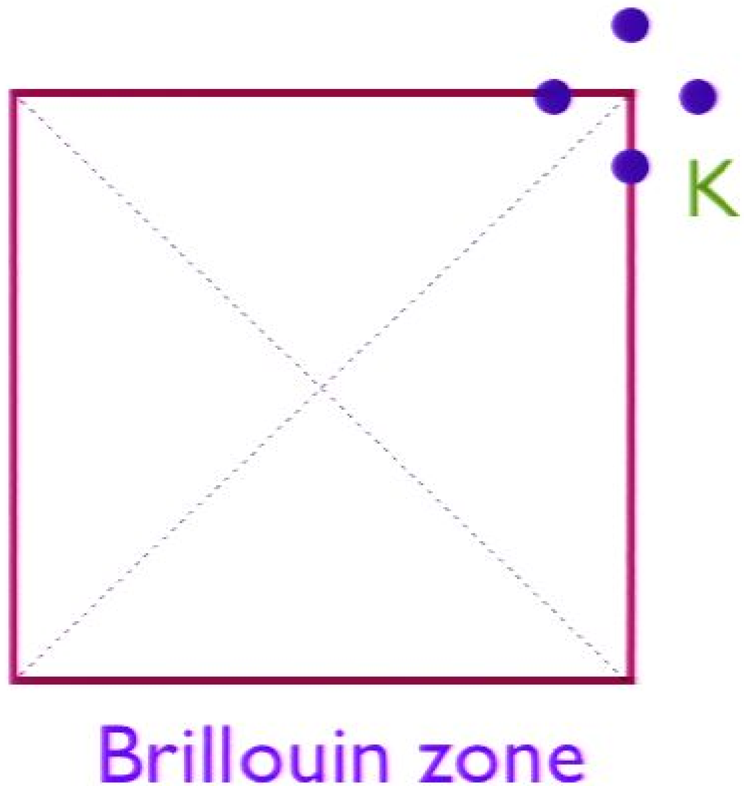
Neutron Scattering-LSCO





Vignolle *et al.*, Nature Phys. 07
Christensen *et al.*, PRL 04
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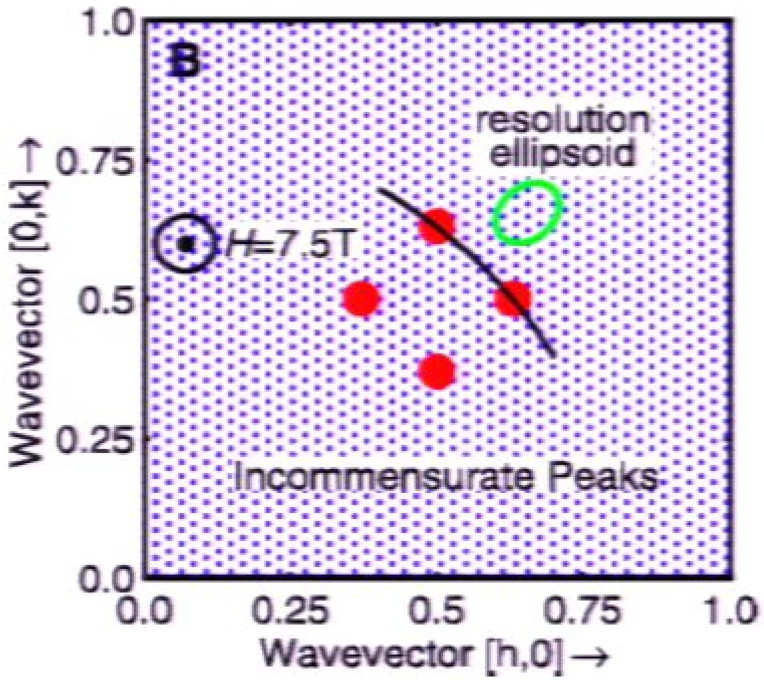
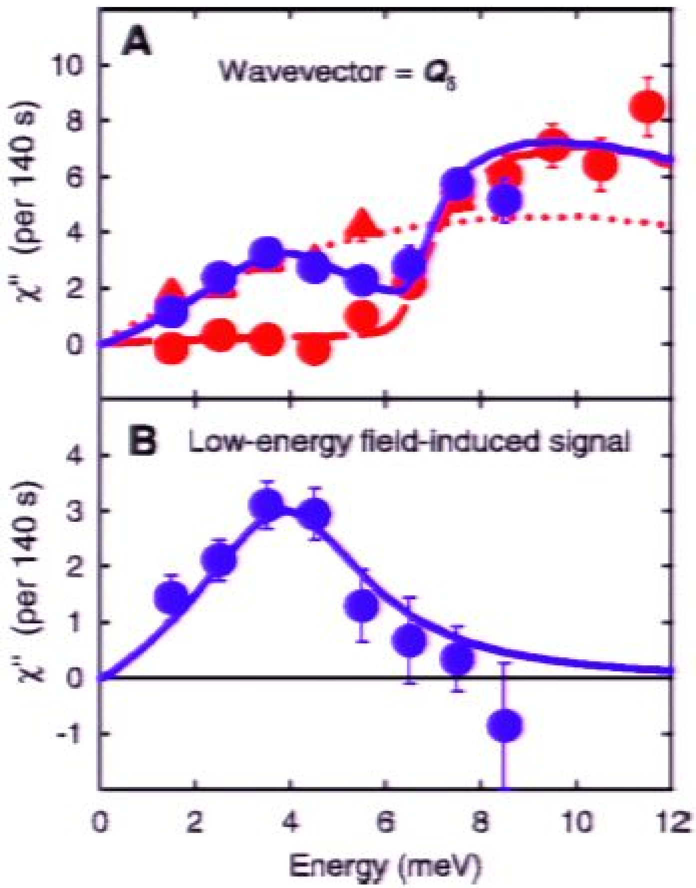
Neutron scattering measurements of dynamic spin correlations of the superconductor (SC) in a magnetic field

B. Lake, G. Aeppli, K. N. Clausen, D. F. McMorrow, K. Lefmann, N. E. Hussey, N. Mangkorntong, M. Nohara, H. Takagi, T. E. Mason, and A. Schröder, *Science* **291**, 1759 (2001).





Peaks at $(0.5,0.5)\pm(0.125,0)$

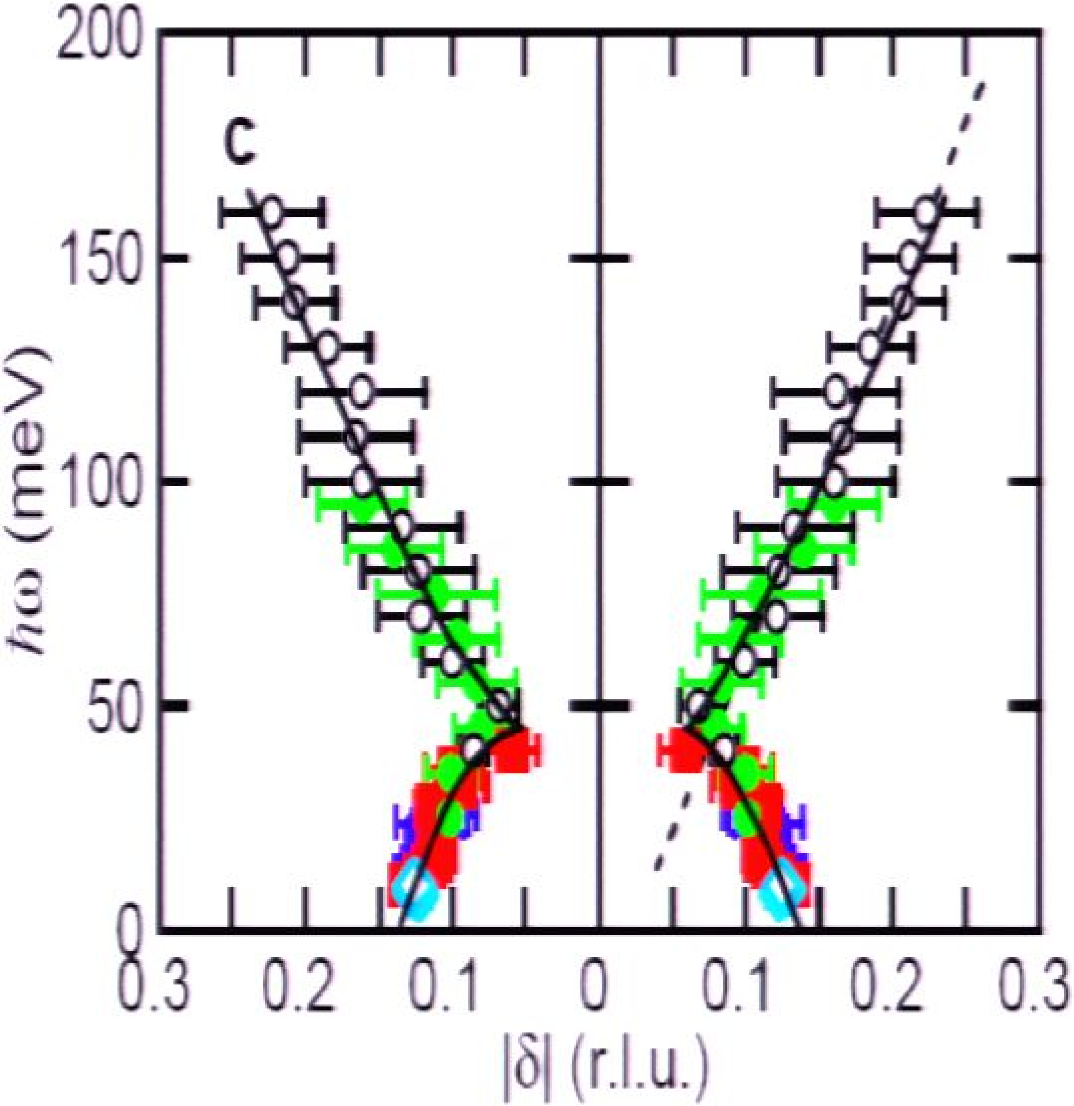
and $(0.5,0.5)\pm(0,0.125)$

$\Rightarrow$ dynamic SDW of period 8

Neutron scattering off $La_{2-\delta}Sr_\delta CuO_4$ ($\delta = 0.163$, ***SC phase***)

at low temperatures in H=0 (red dots) and H=7.5T (blue dots)

Neutron Scattering-LSCO



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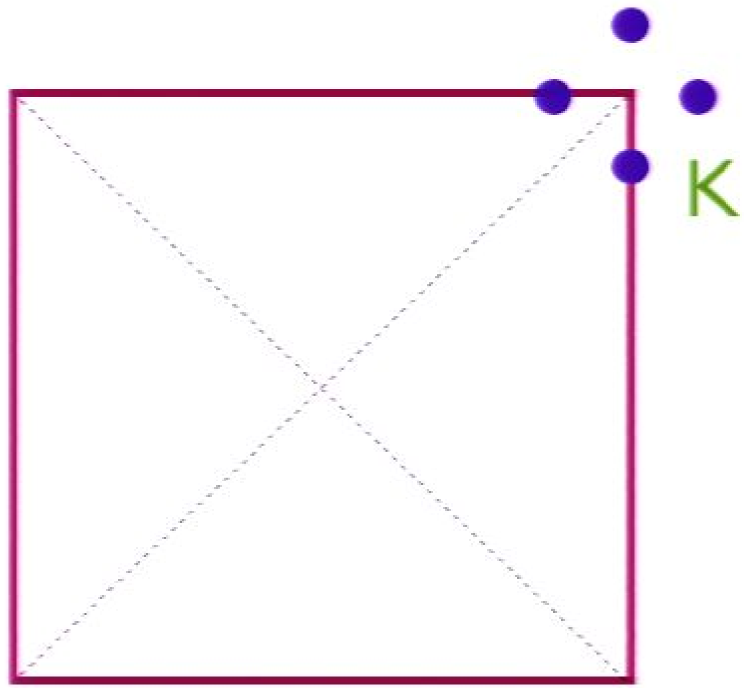

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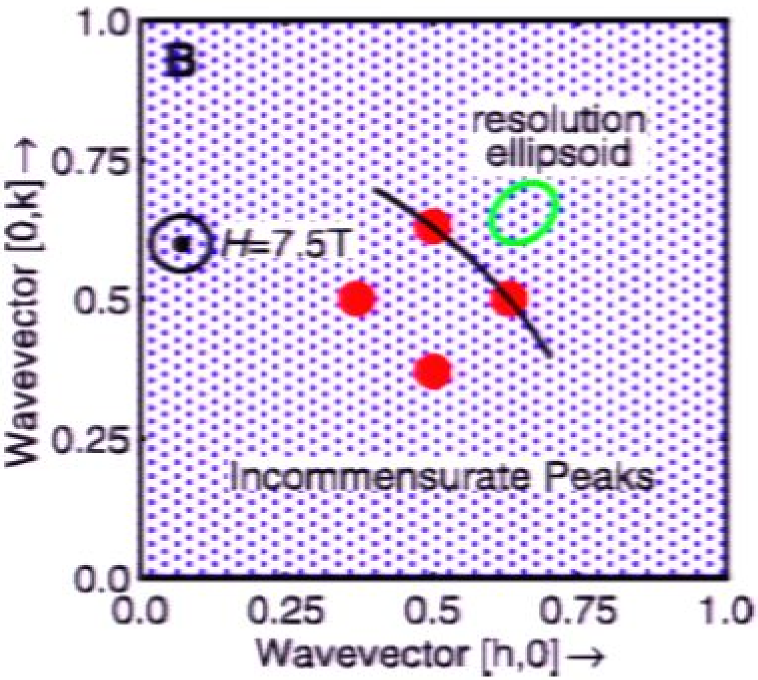

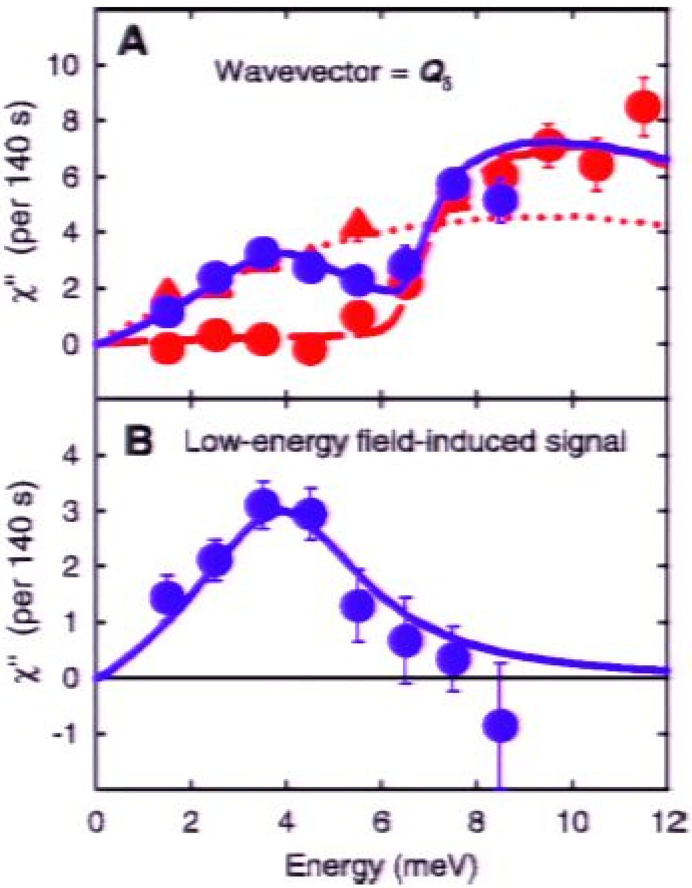

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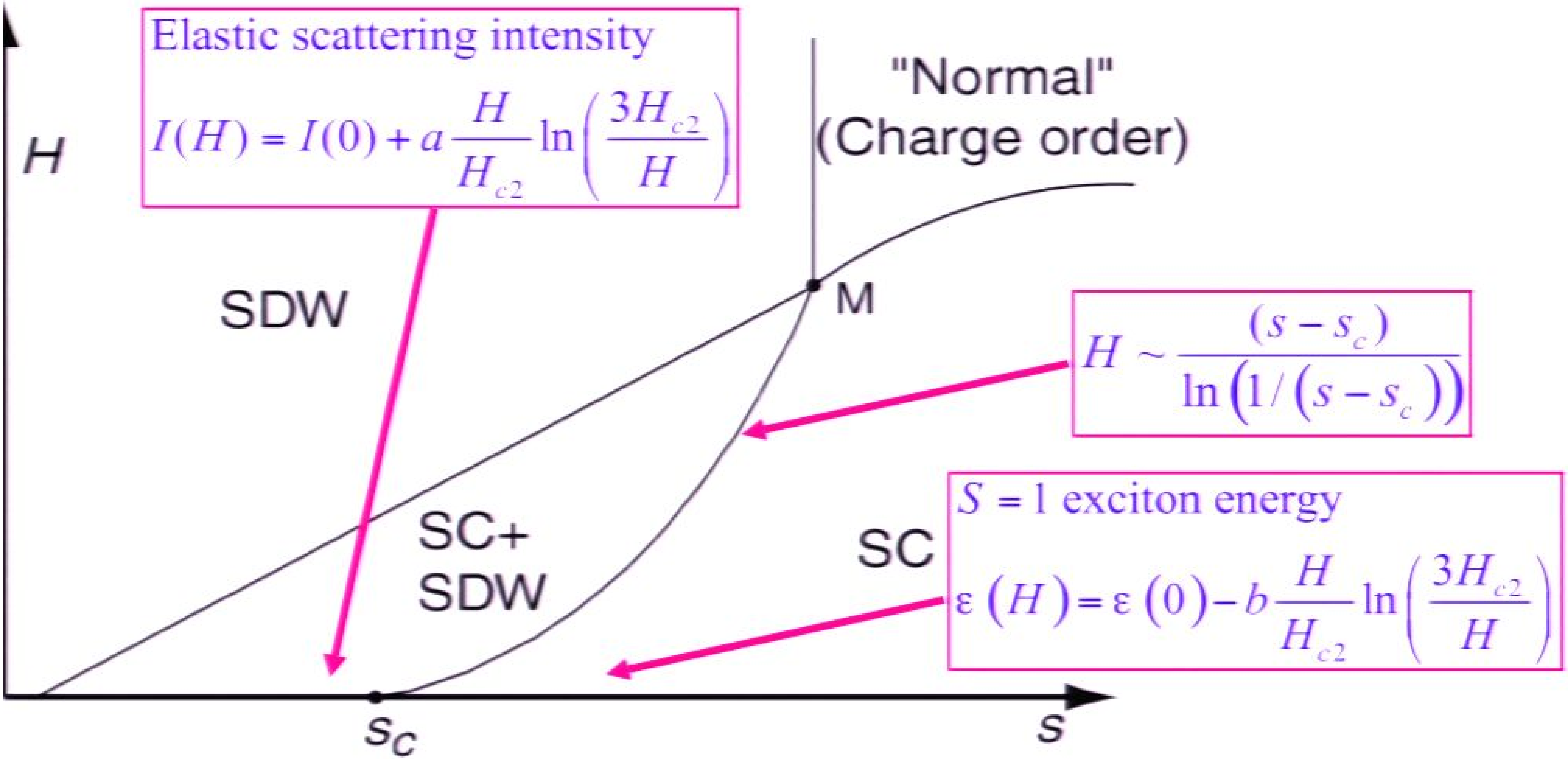
$T=0$



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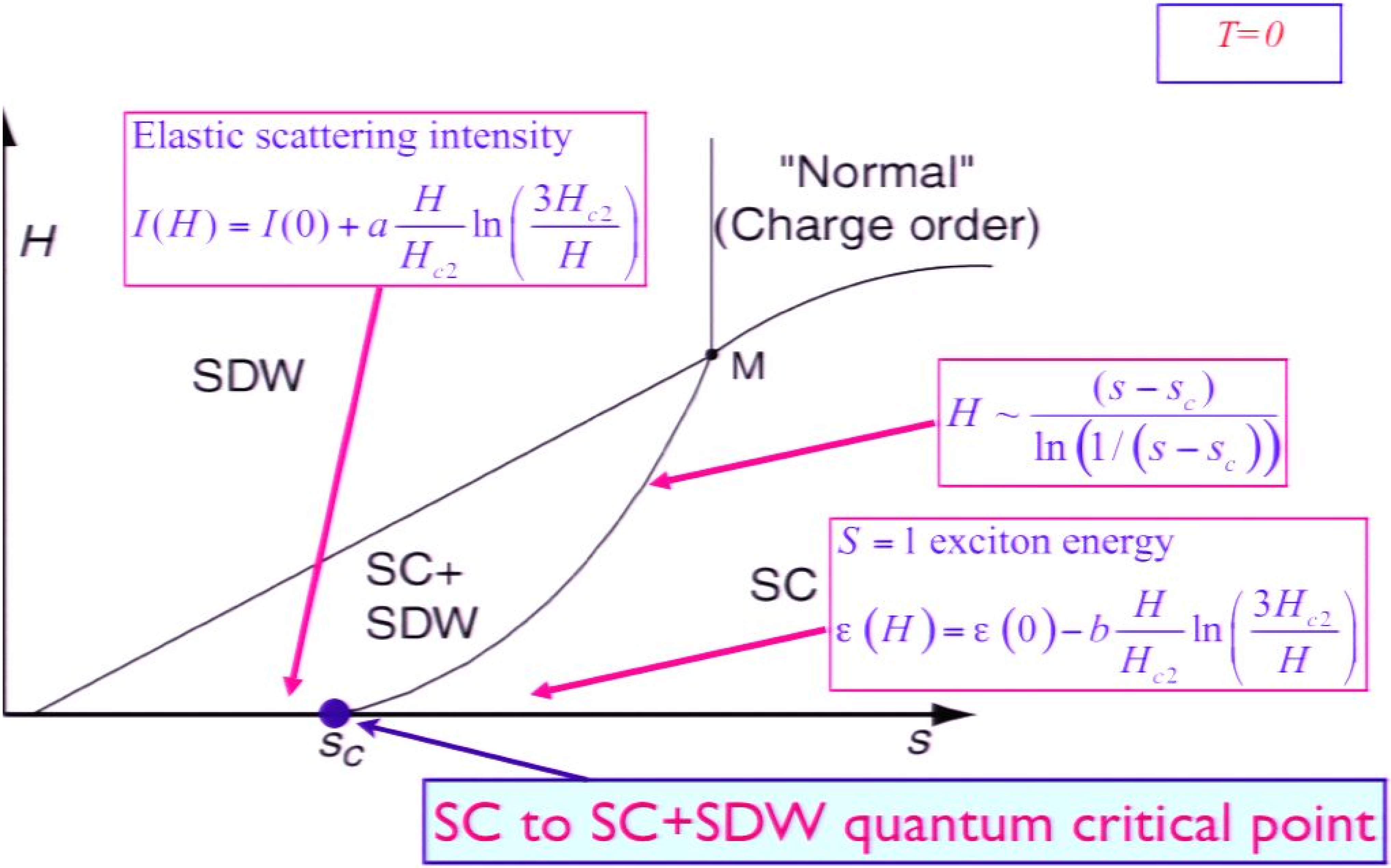

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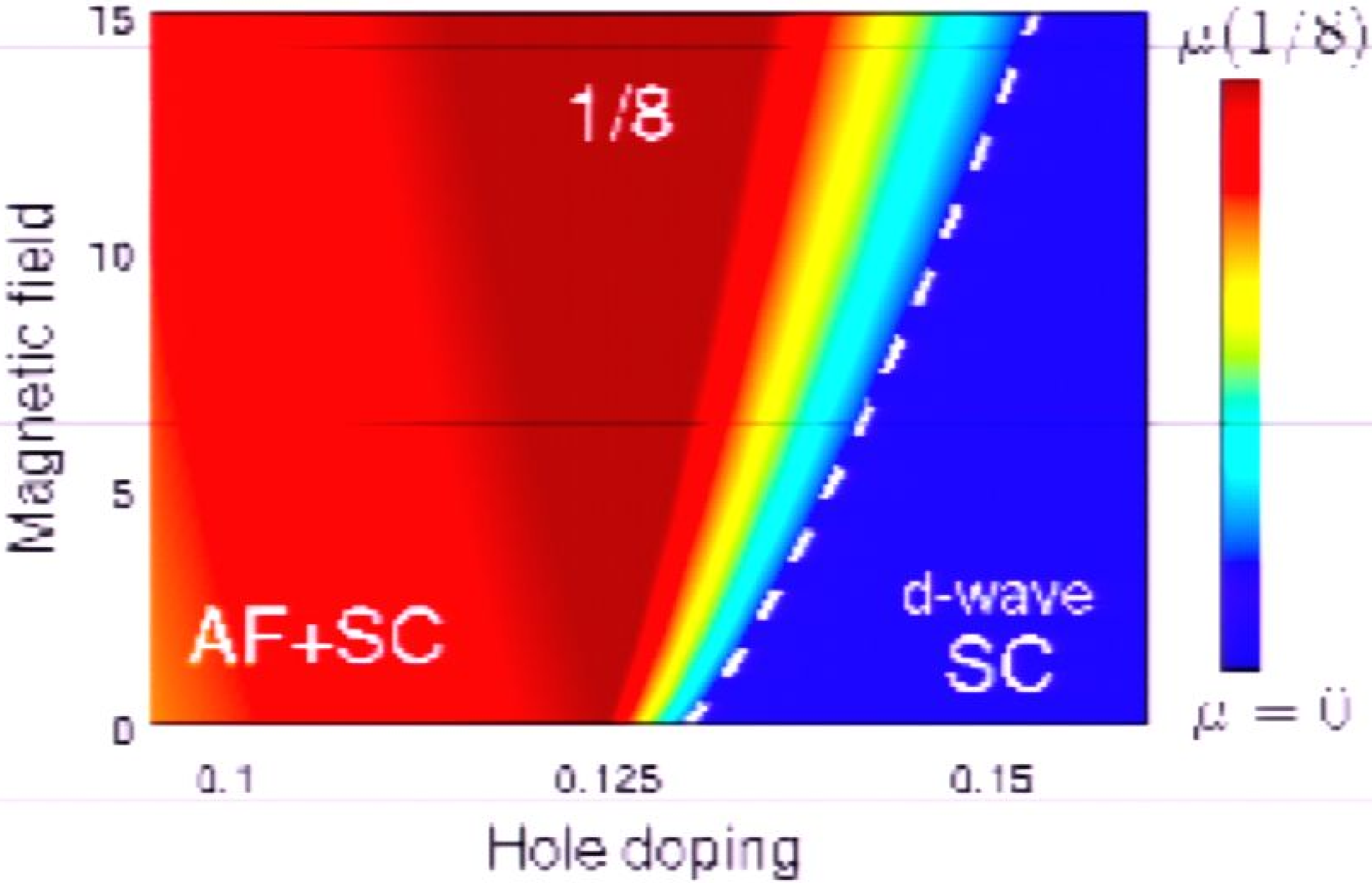

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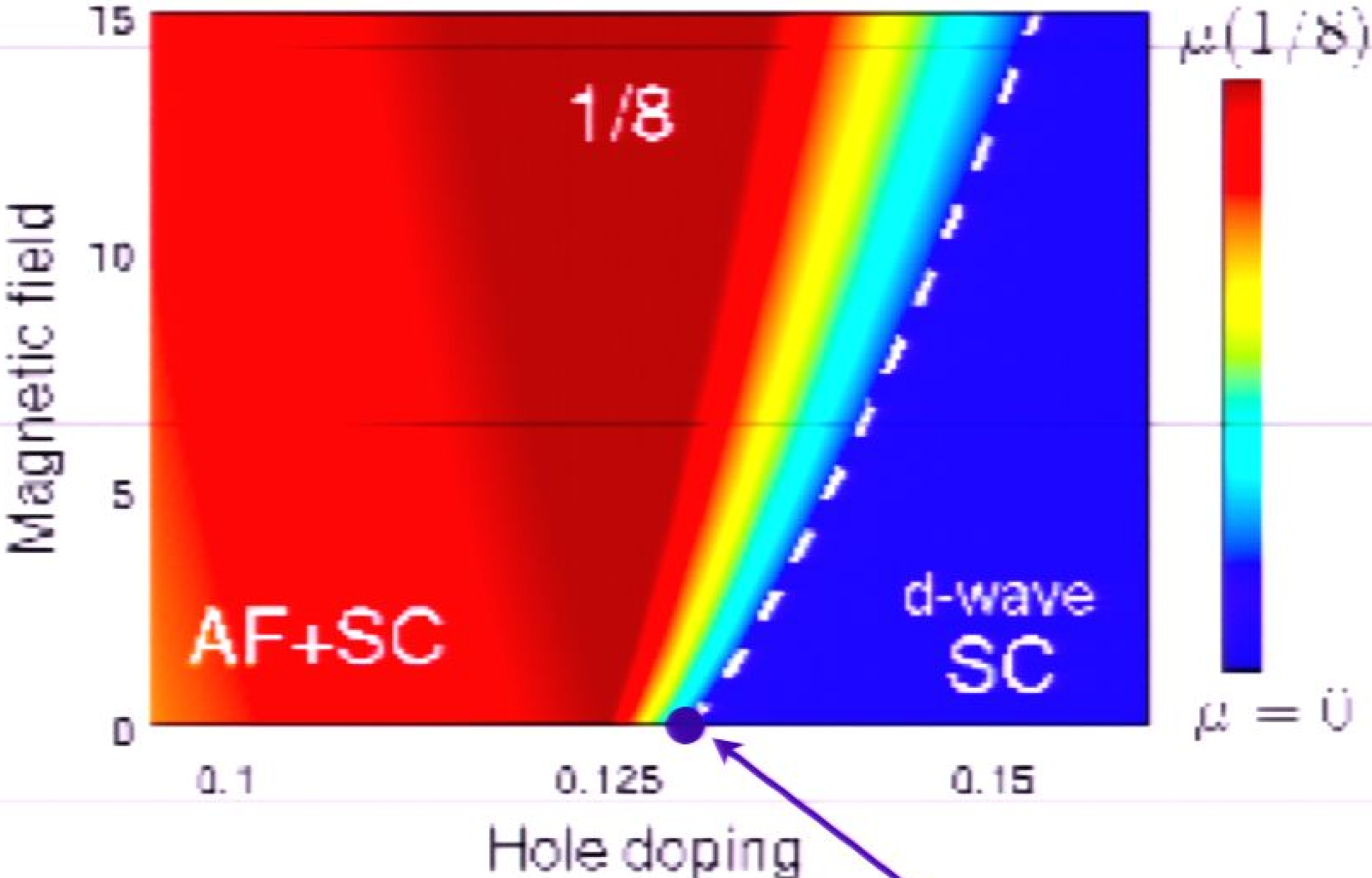

SC to SC+SDW quantum critical point

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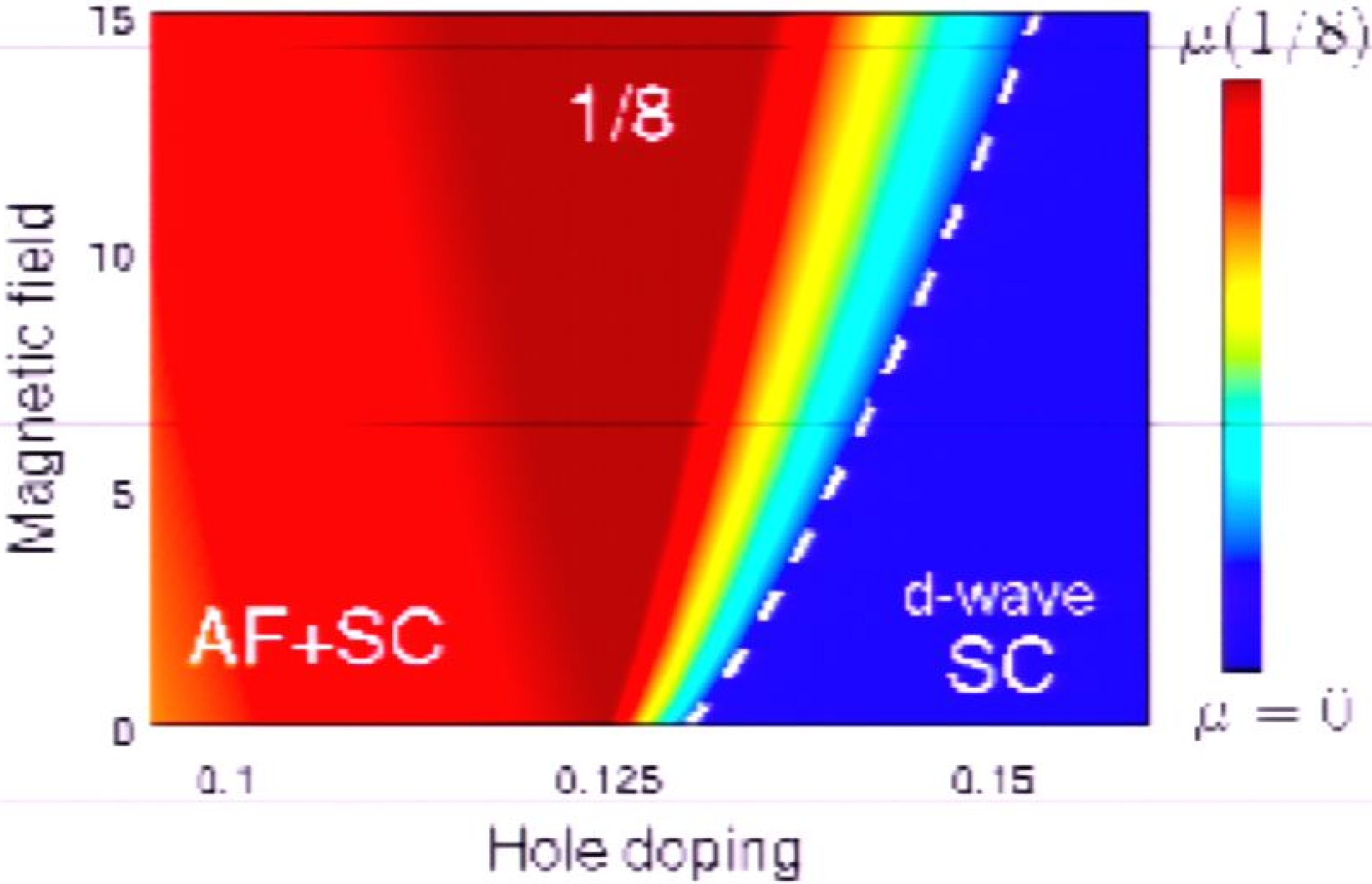

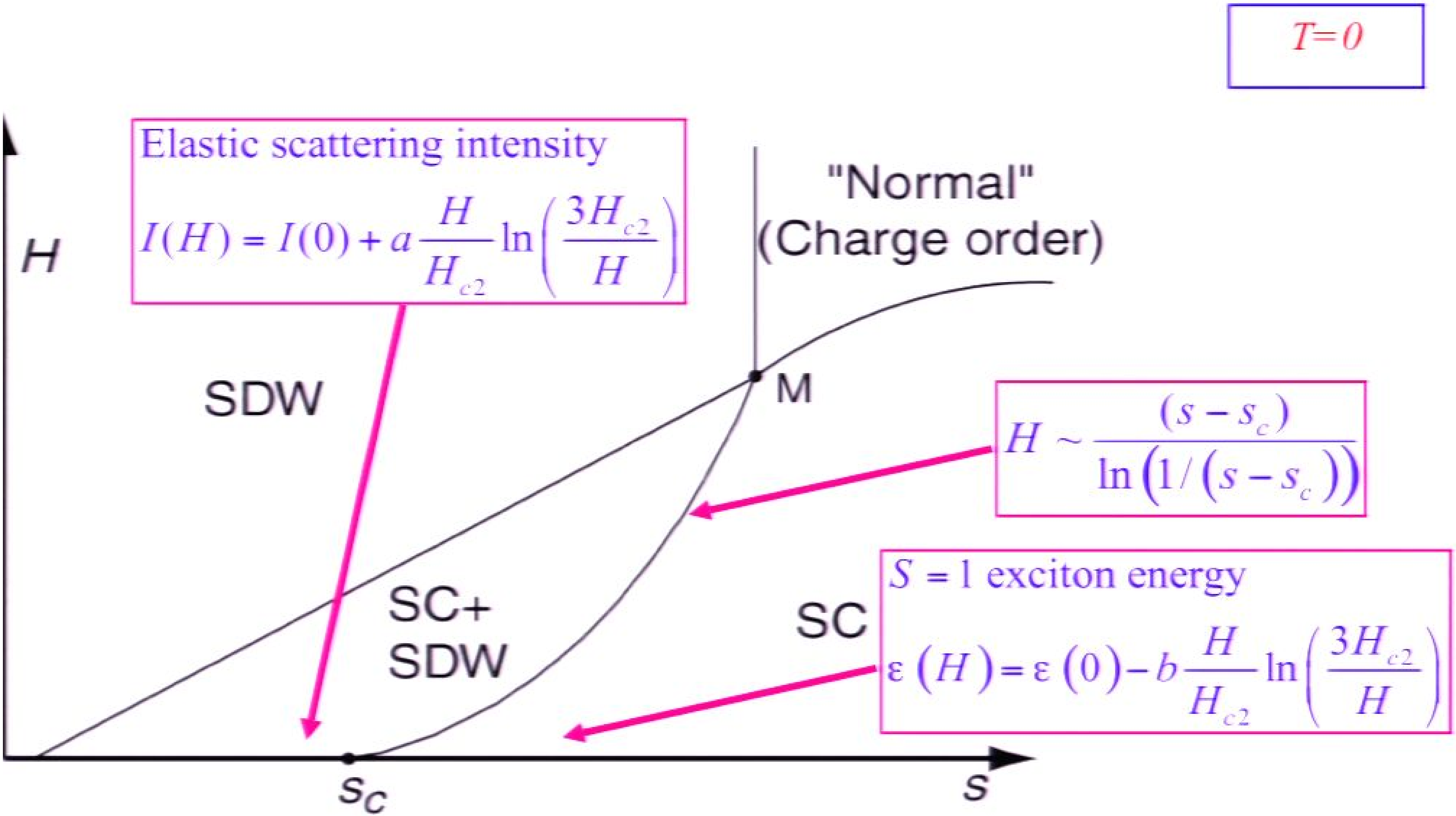

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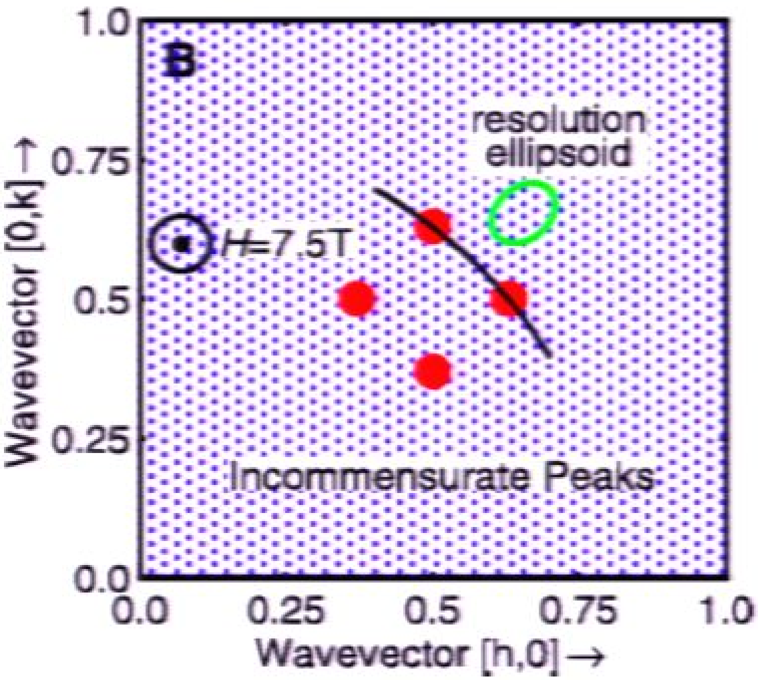

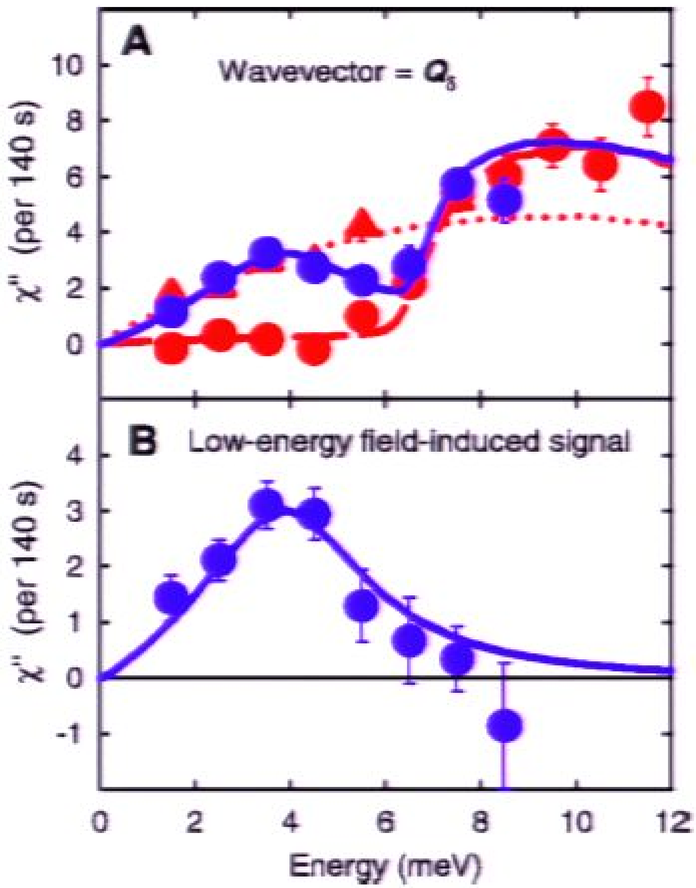

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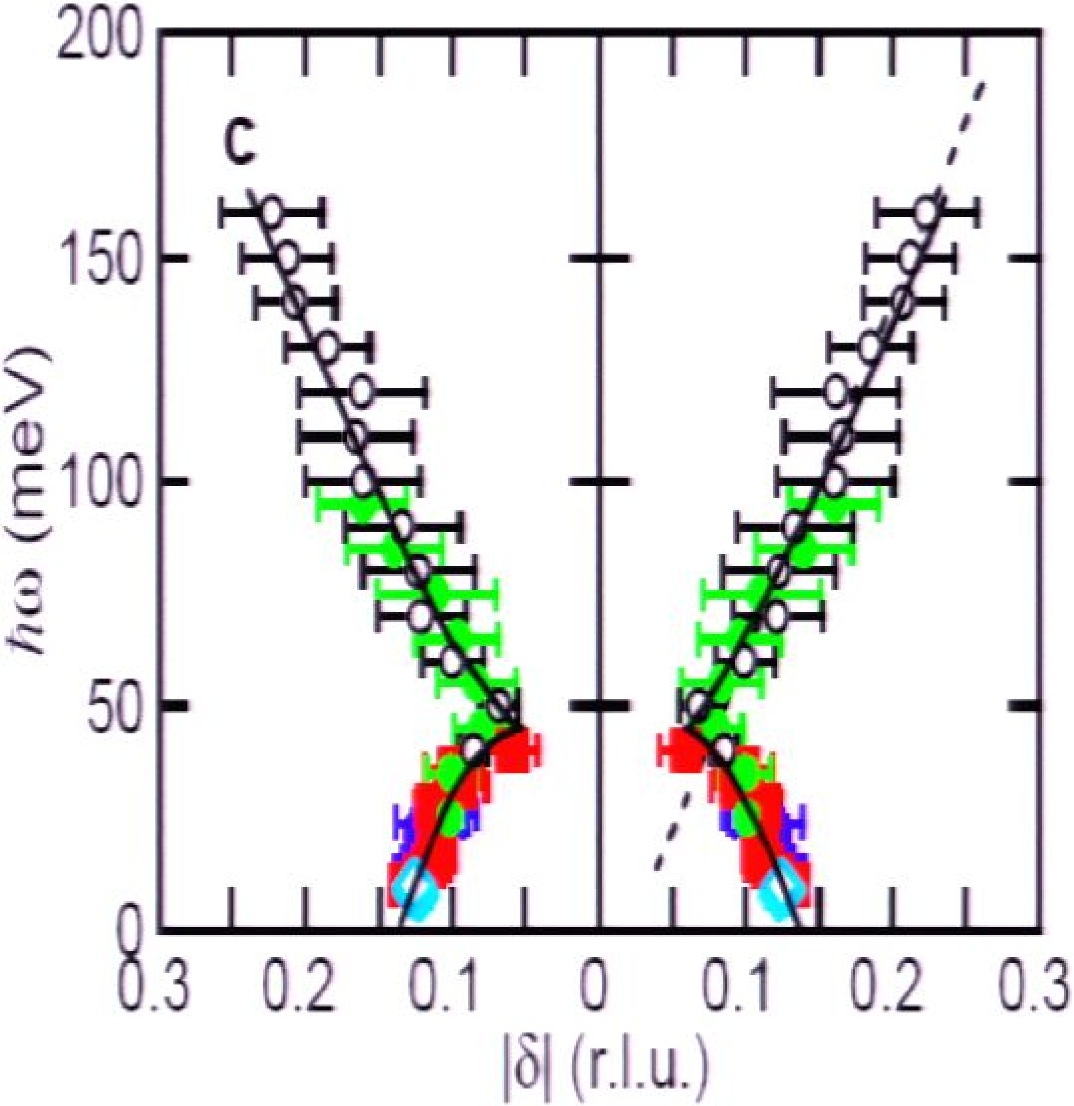

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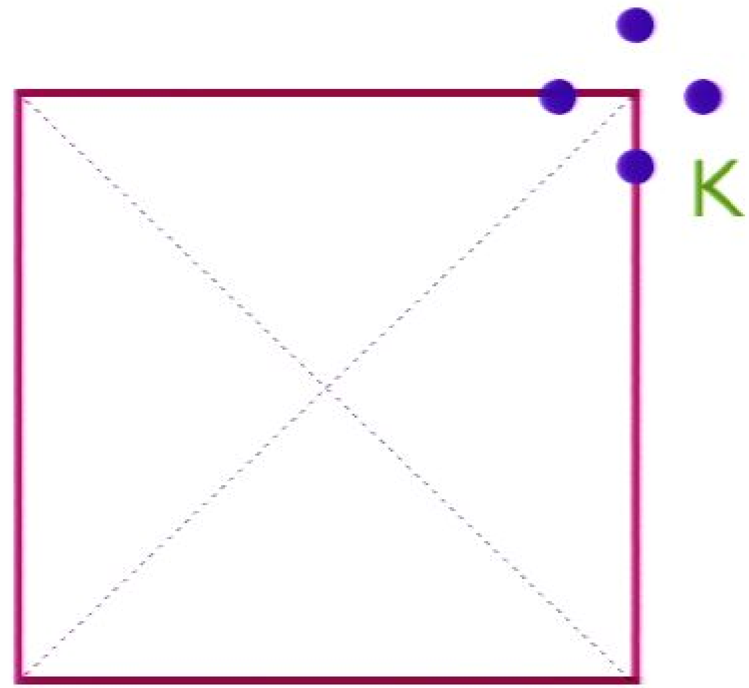
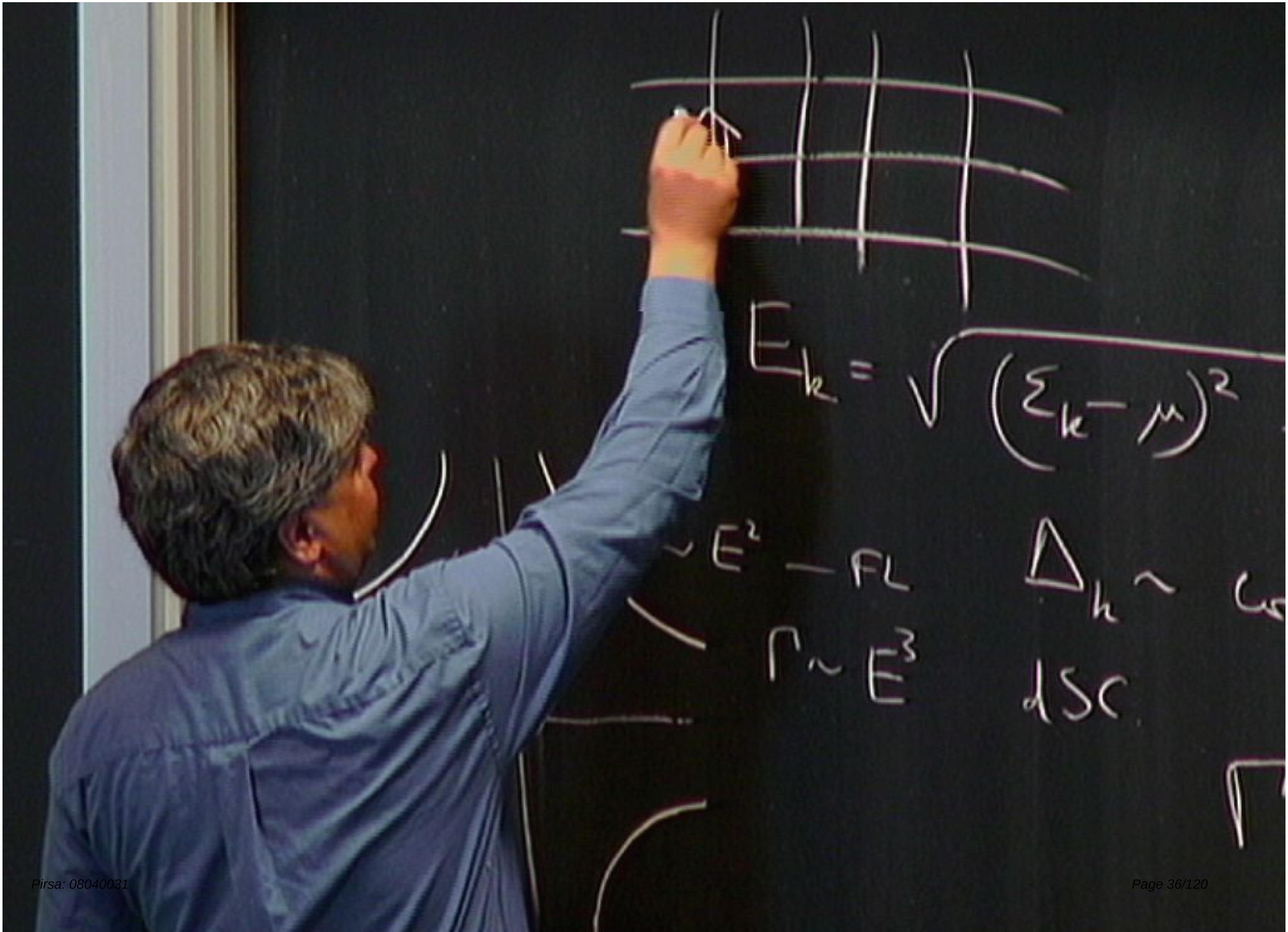


Brillouin zone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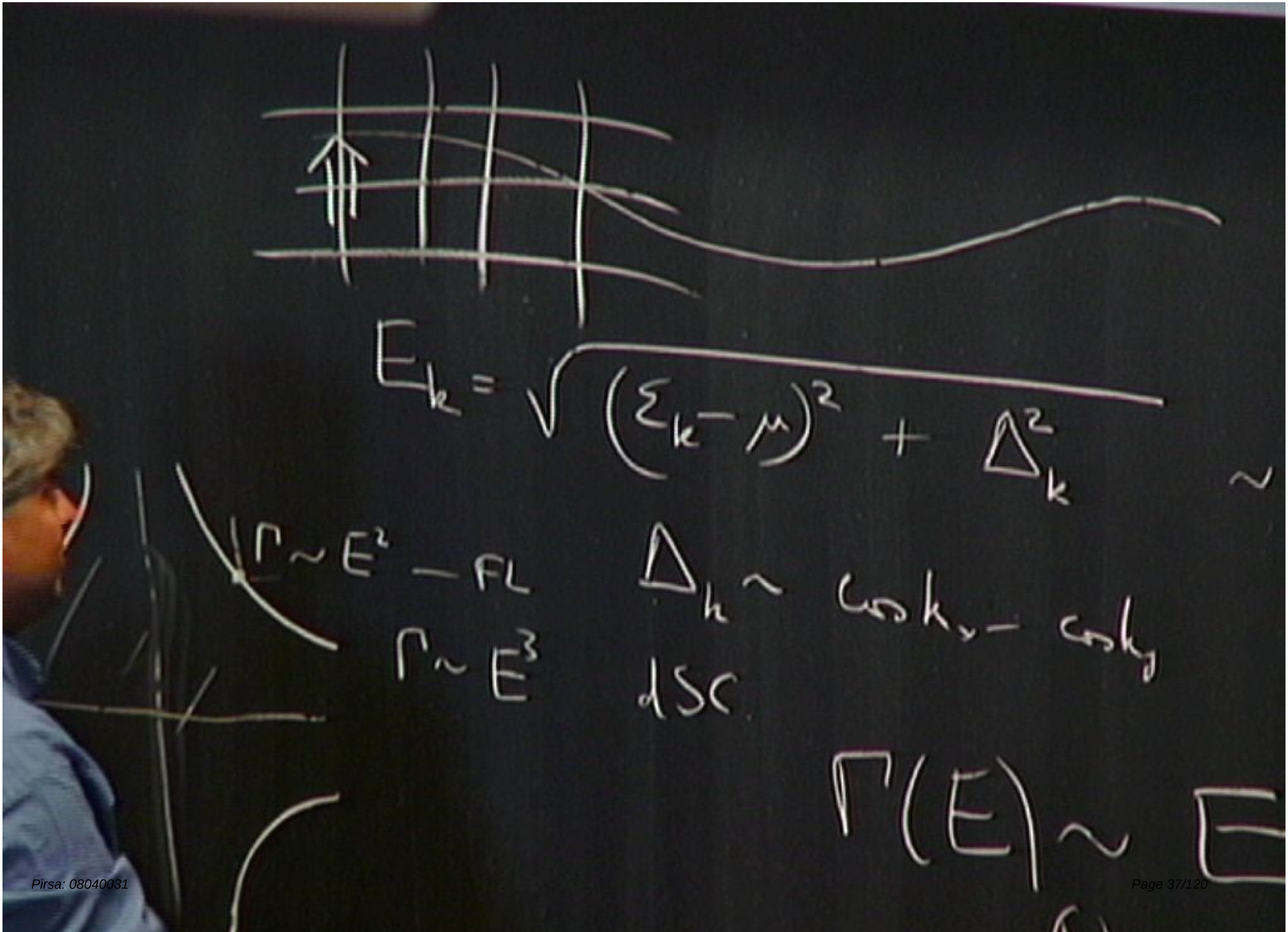
$\sim E^2$ — FL
$\Delta_k \sim$
$\Gamma \sim E^3$
dSC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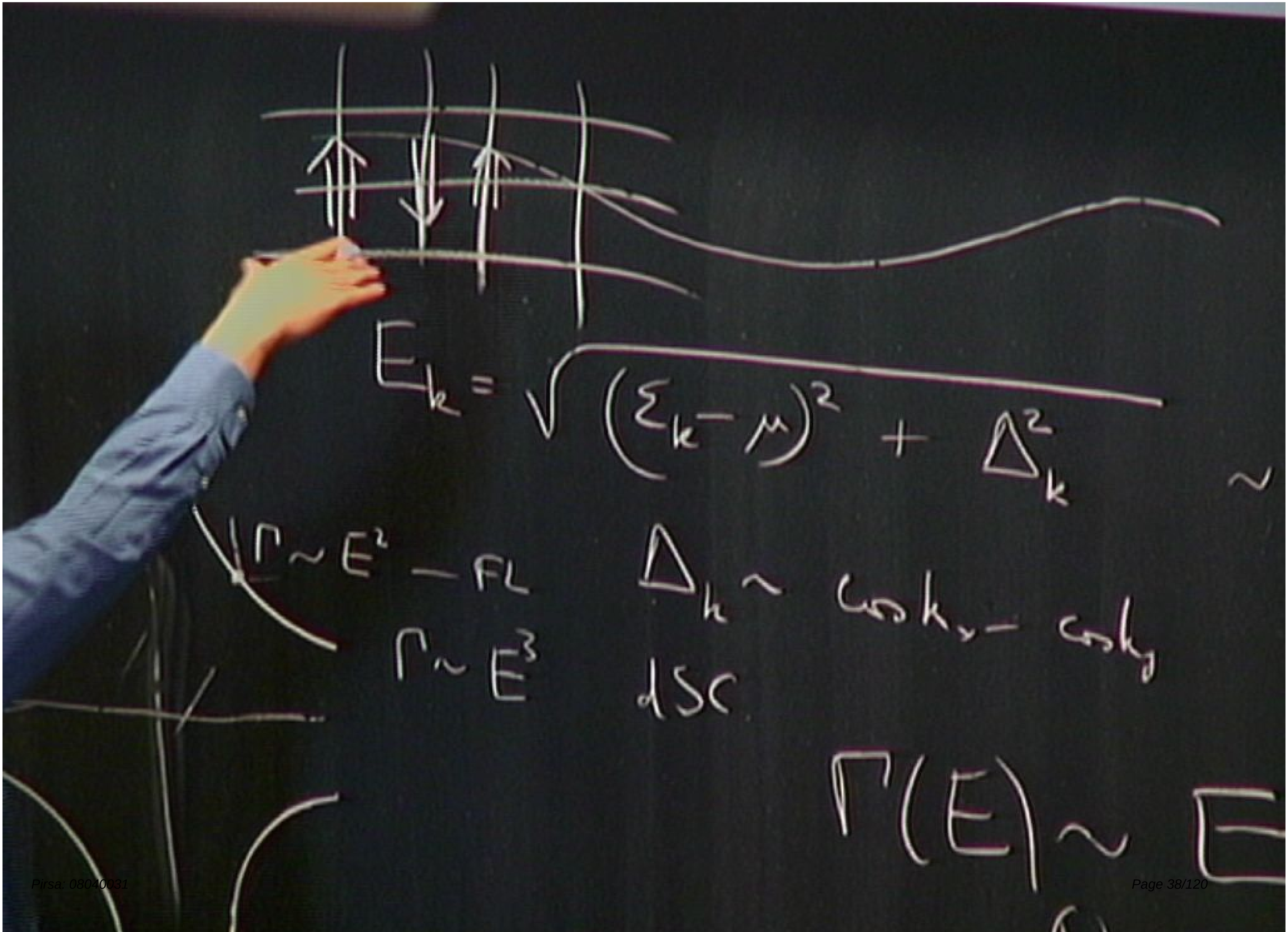
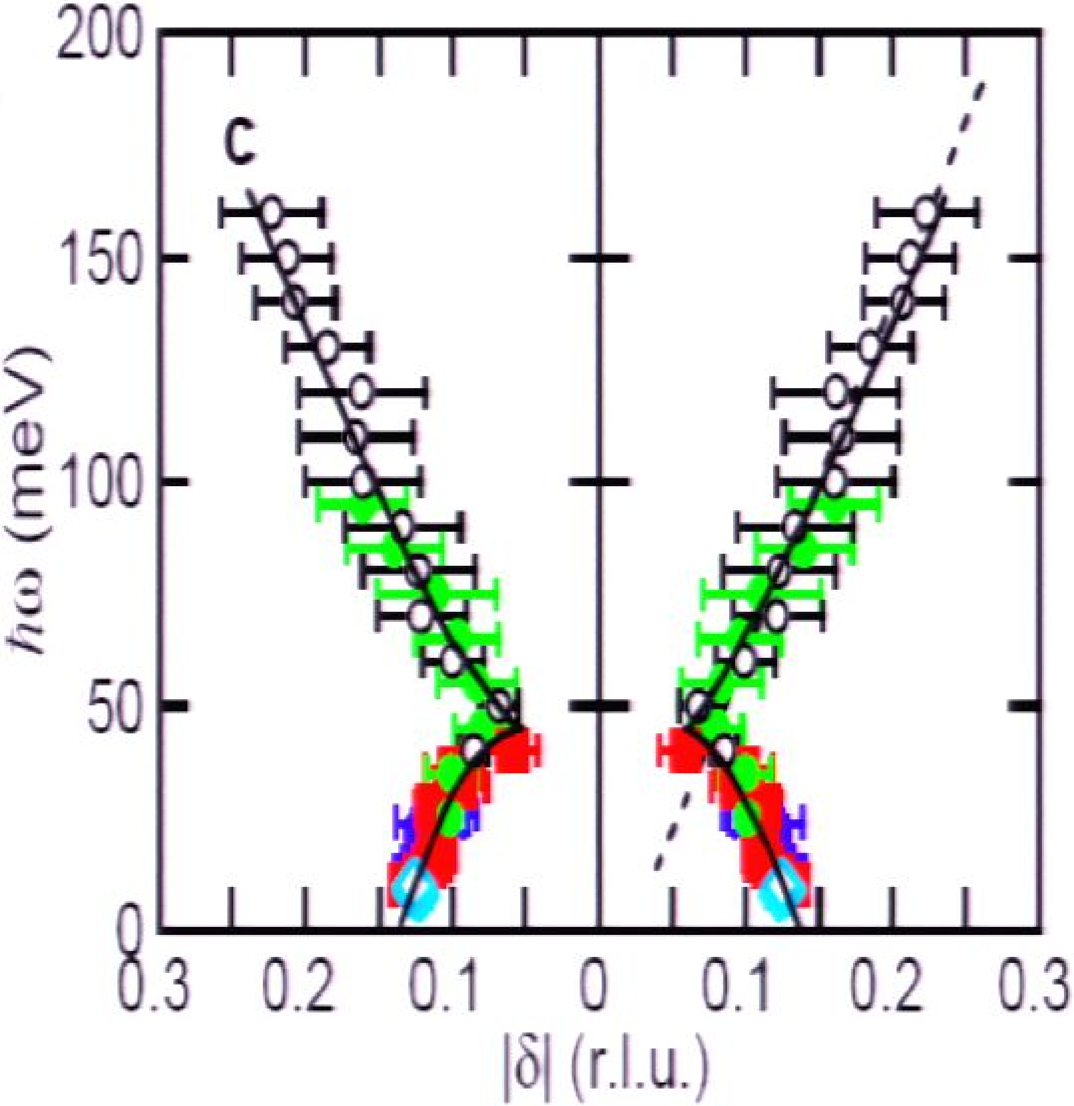
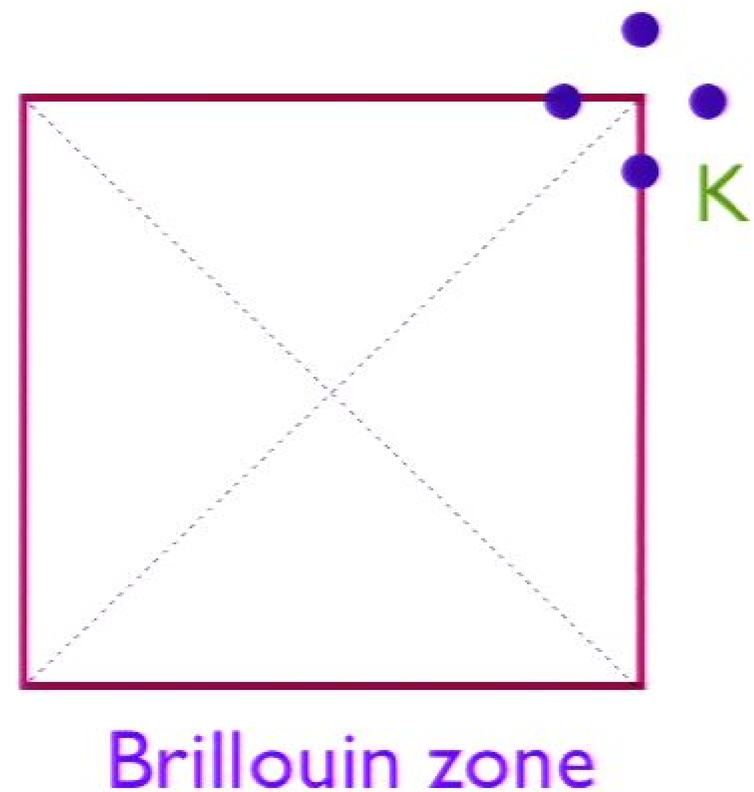
$\Gamma \sim E^3$ dSC
$\Gamma(E) \sim E$



E_k = √((ε_k − μ)² + Δ_k²)
Γ ~ E² — FL
Δ_k ~ cos k_x − cos k_y
Γ ~ E³ dSC
Γ(E) ~ E

Neutron Scattering-LSCO





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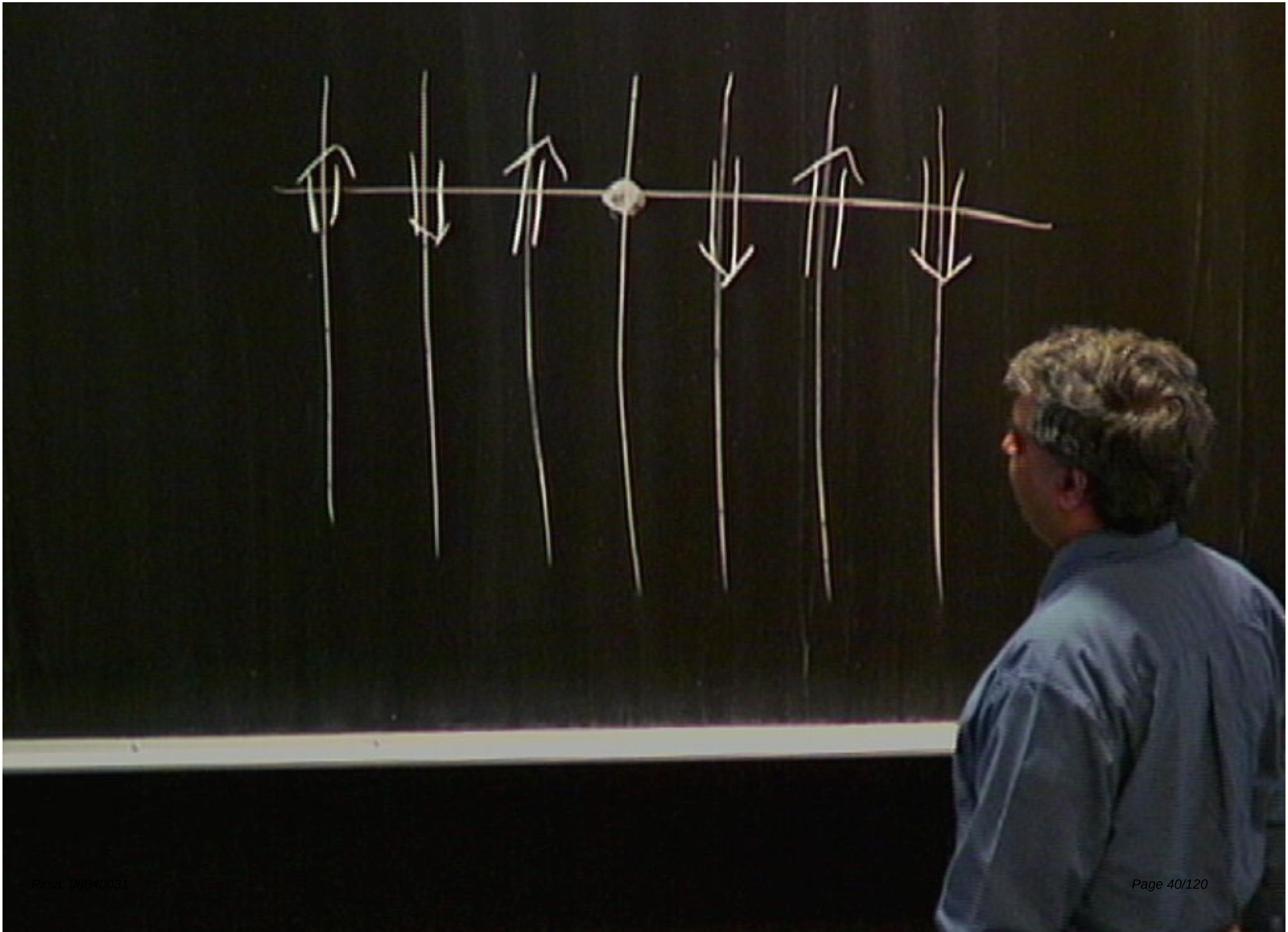

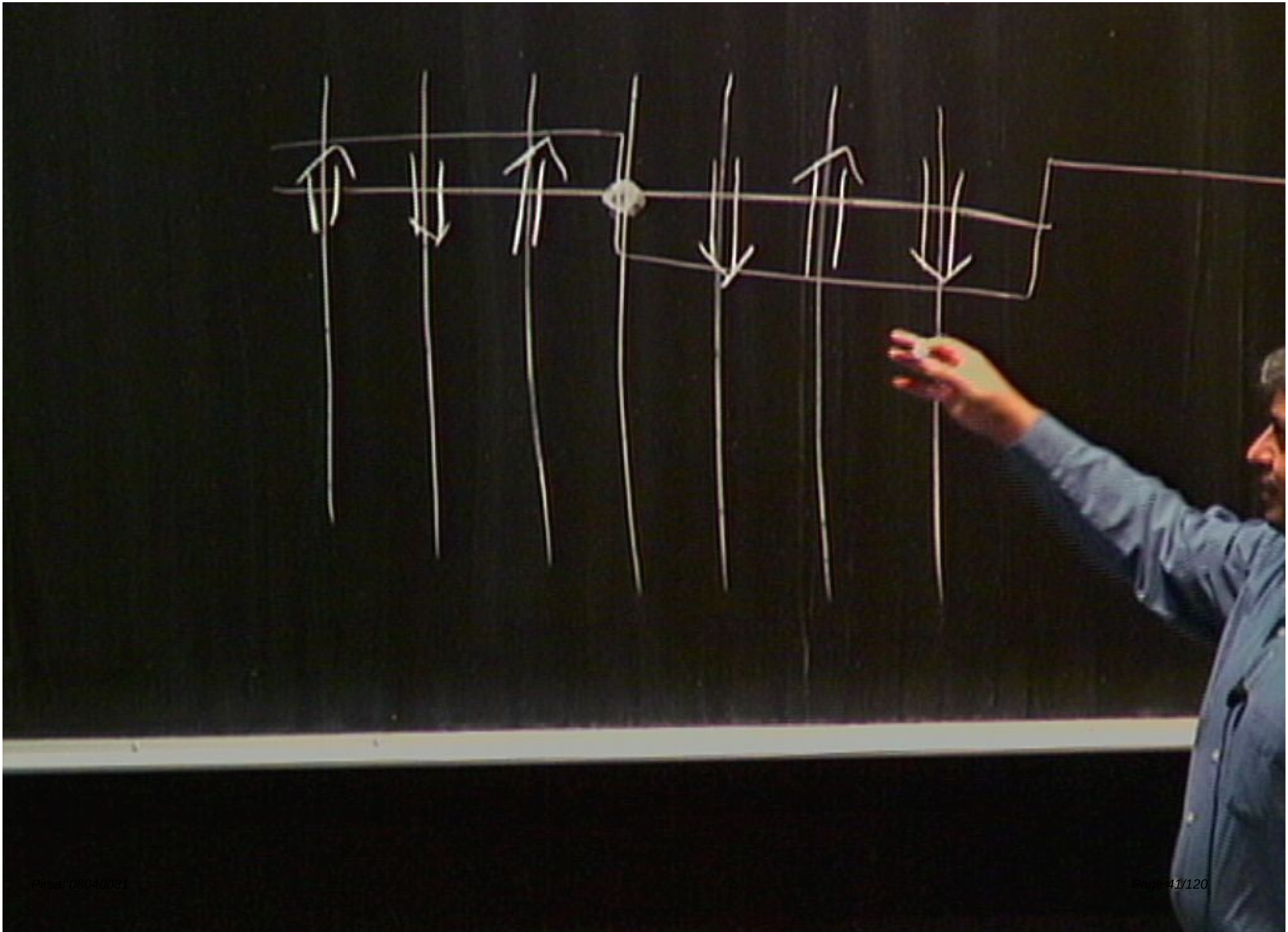

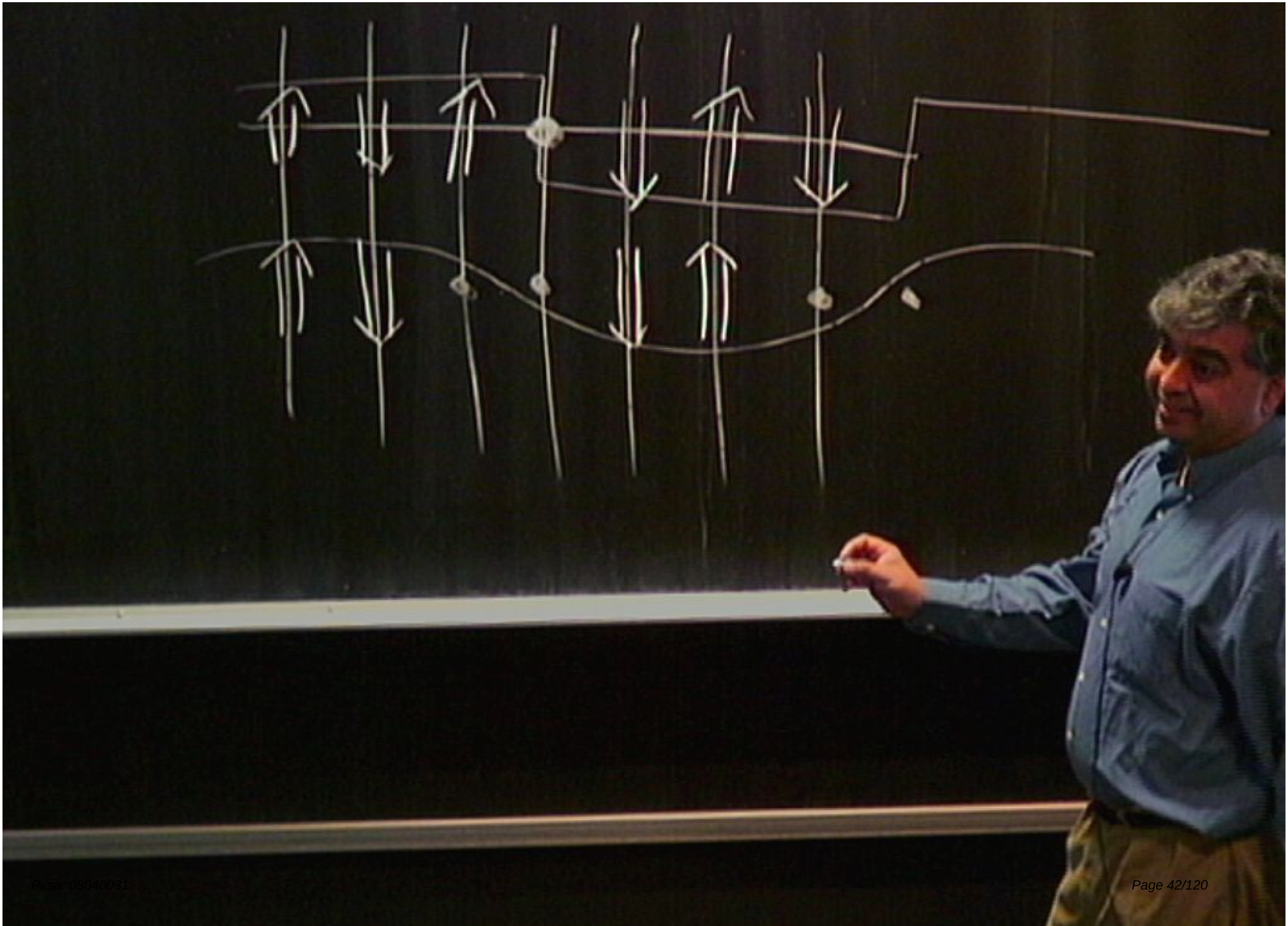

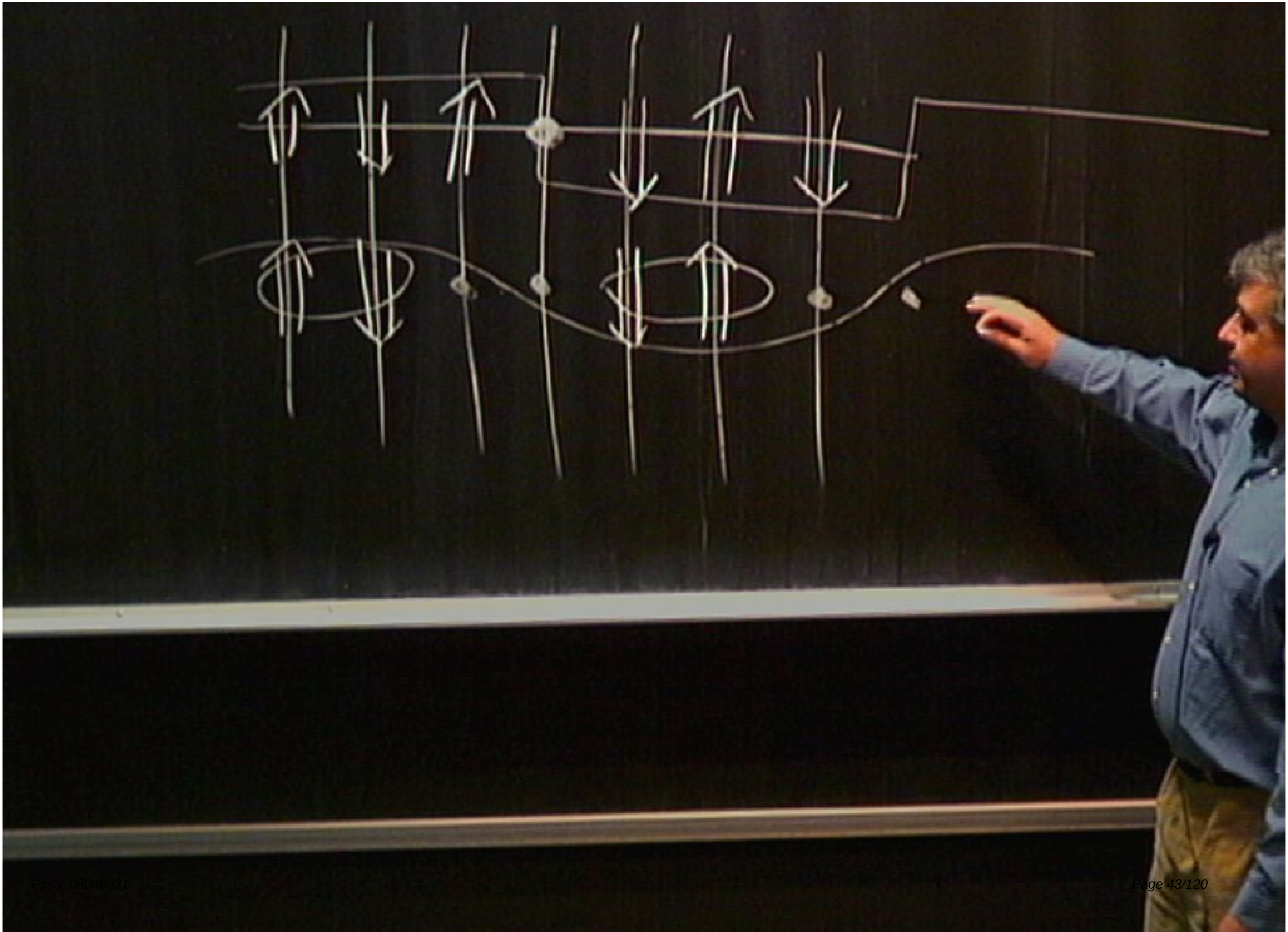

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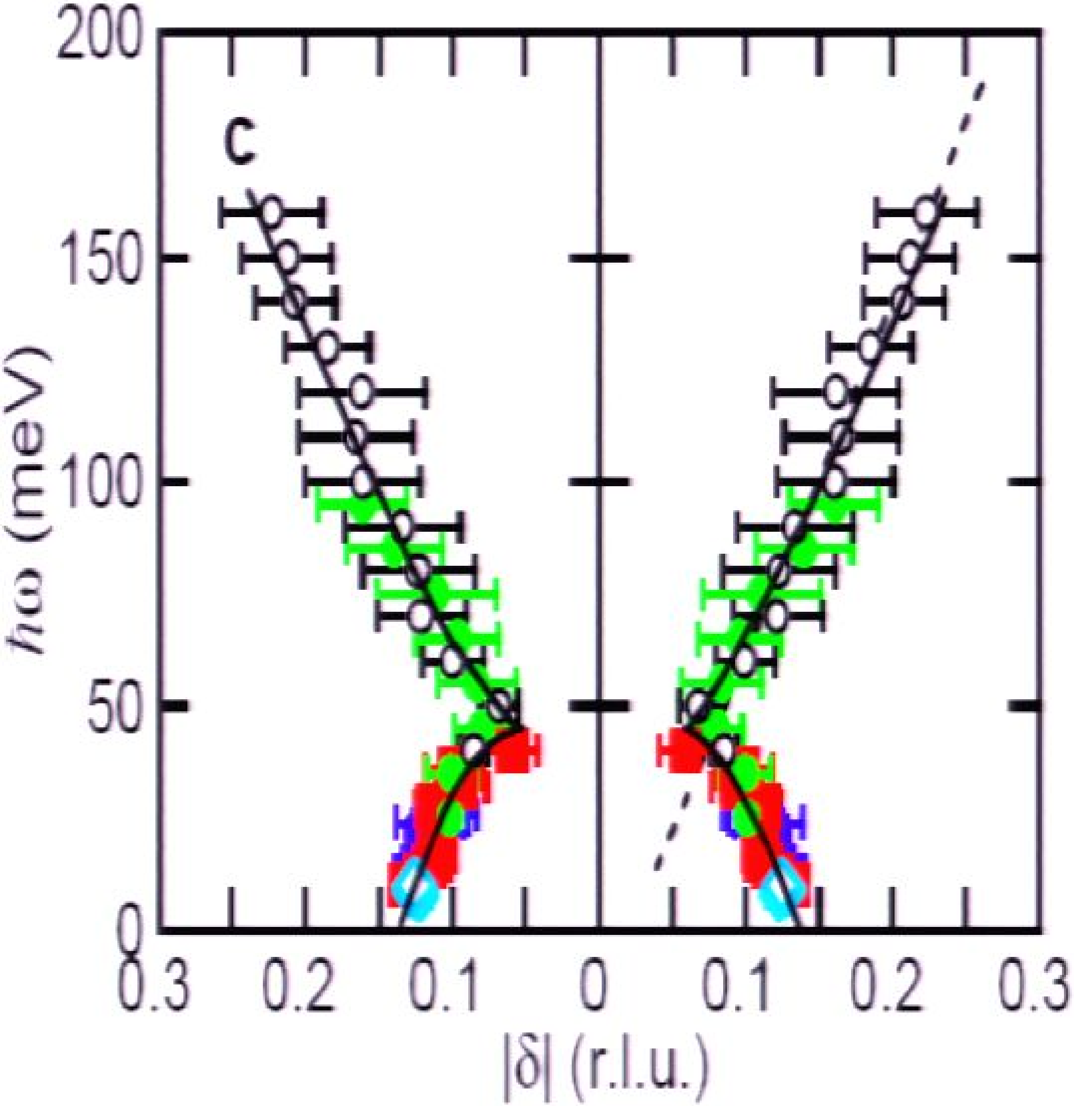

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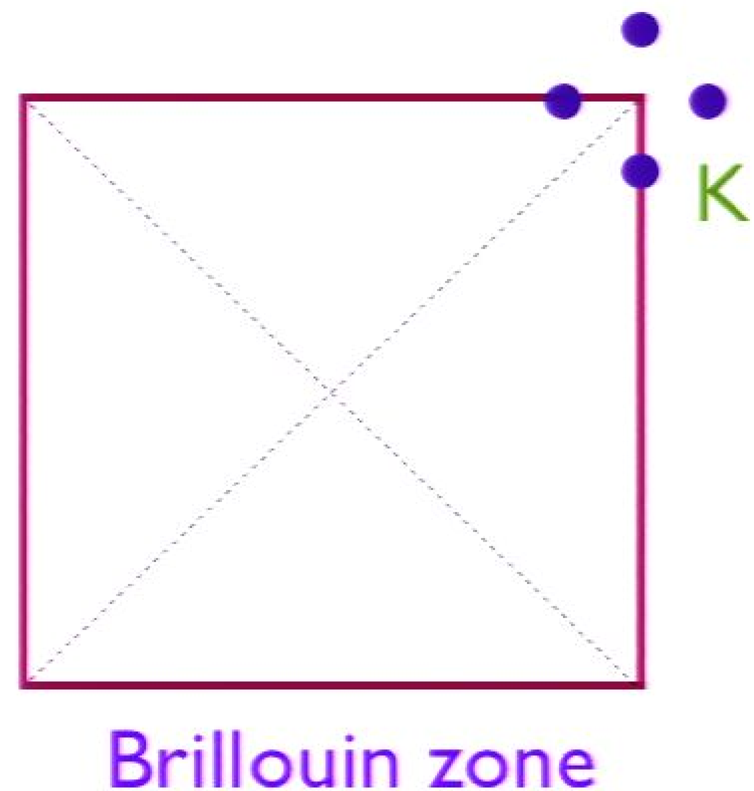

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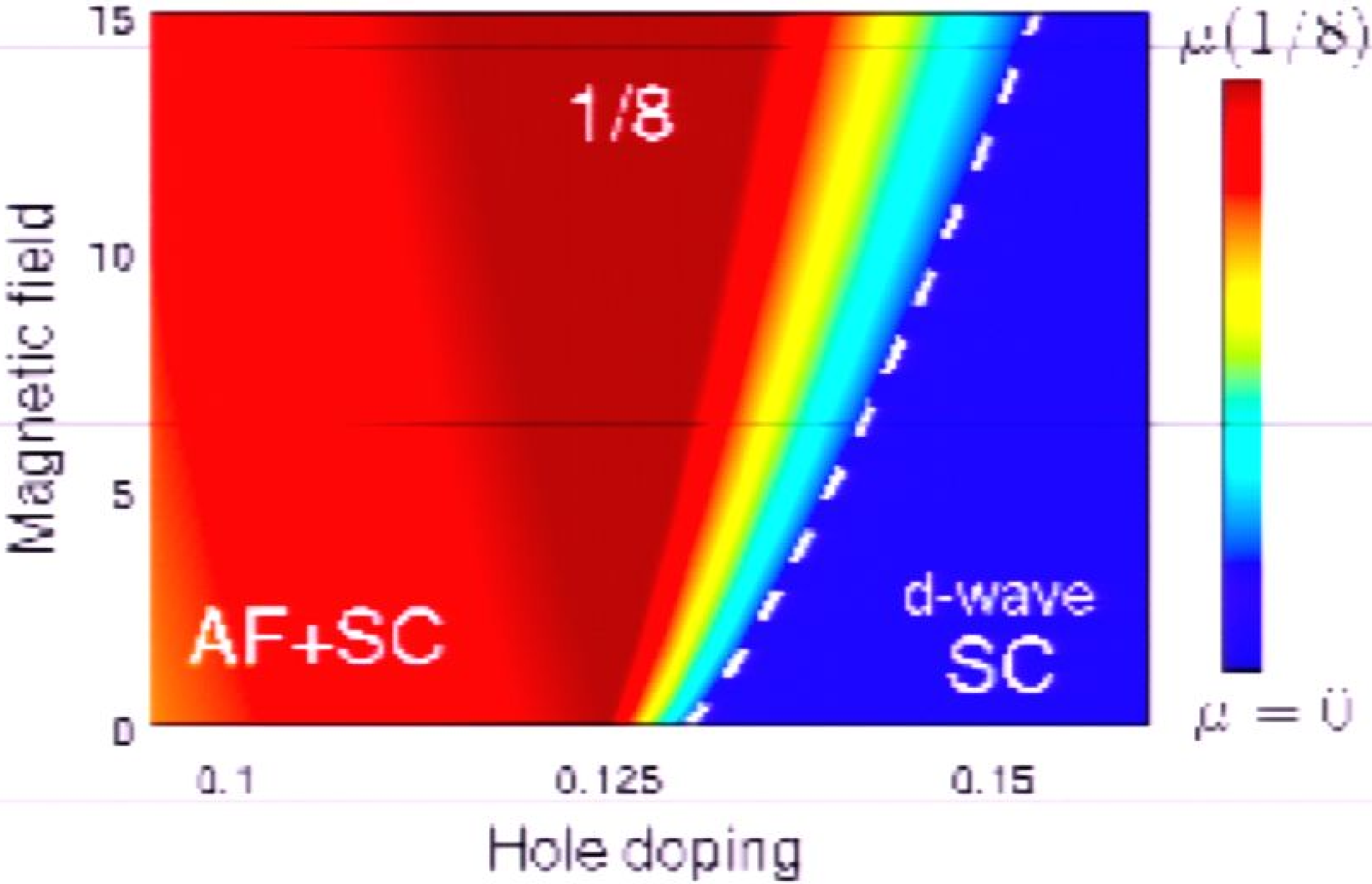

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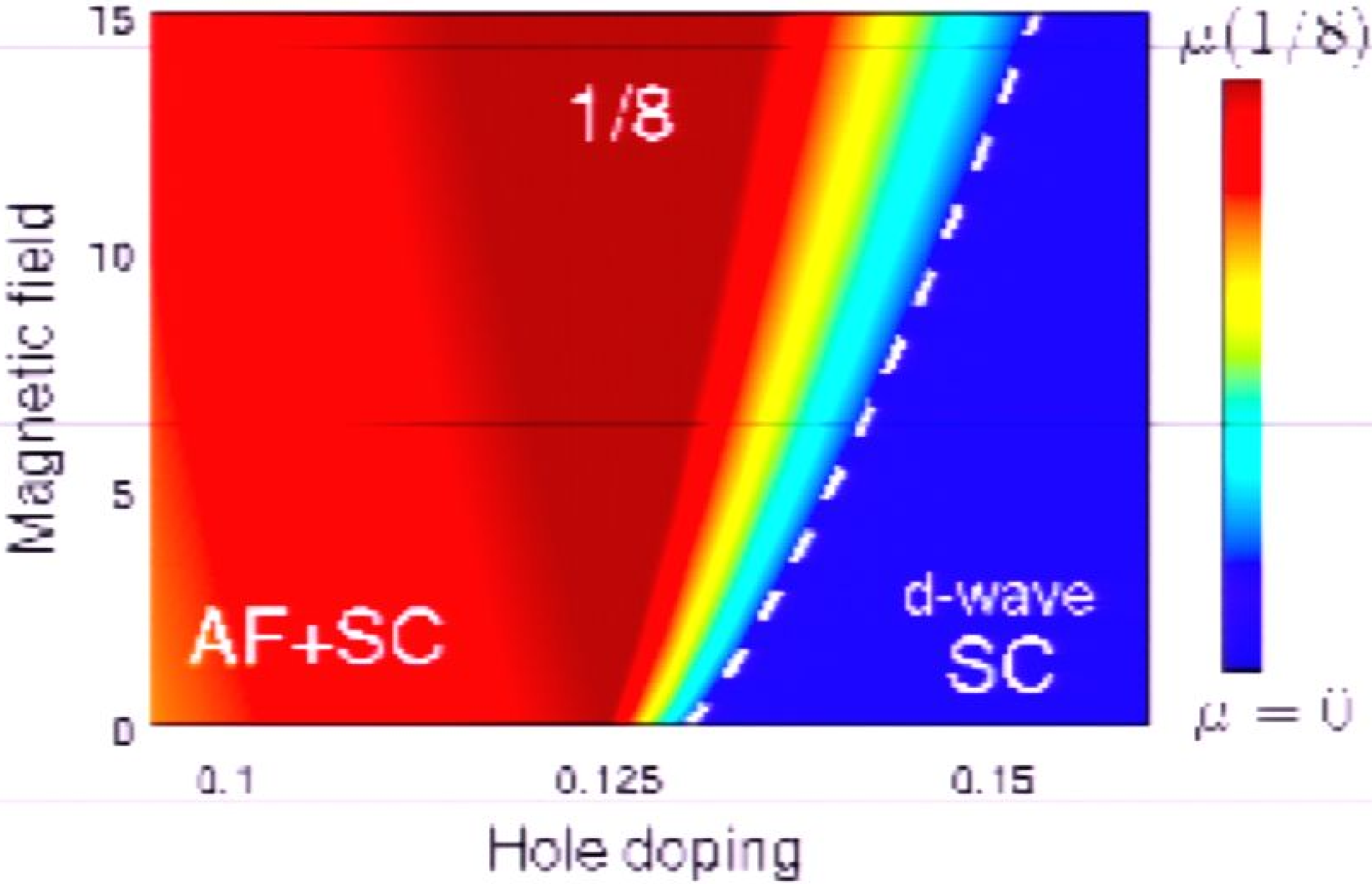

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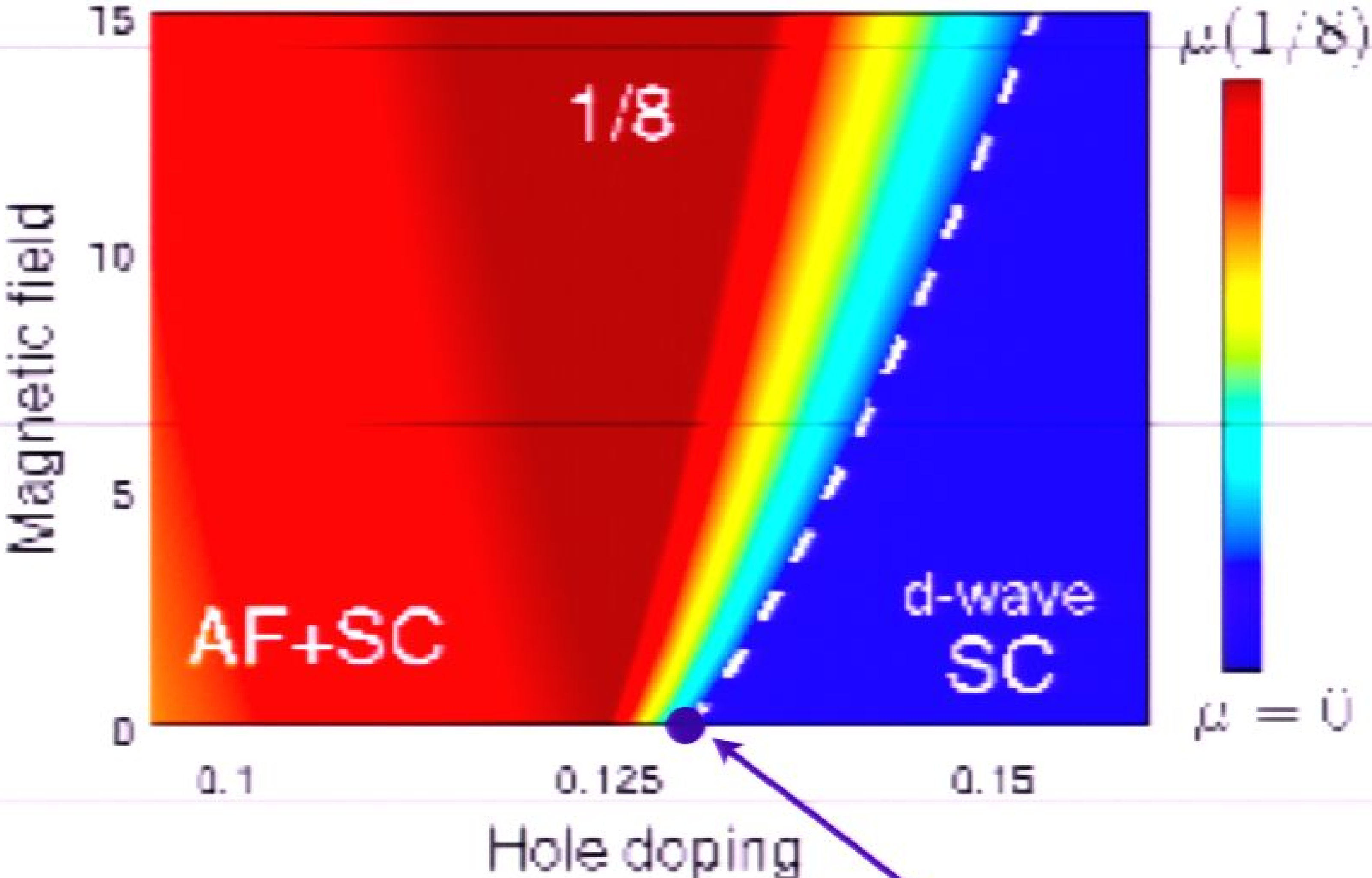
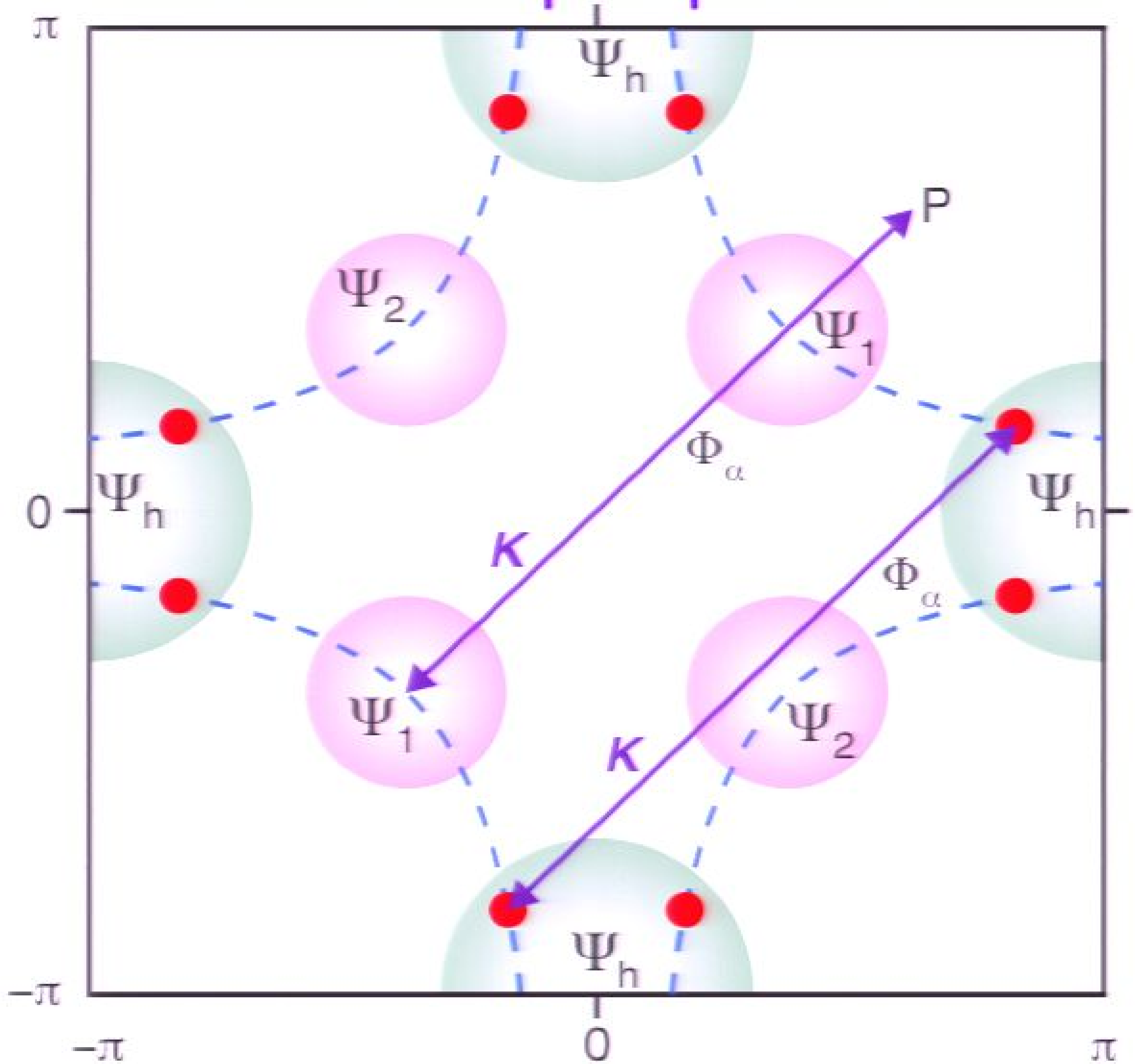
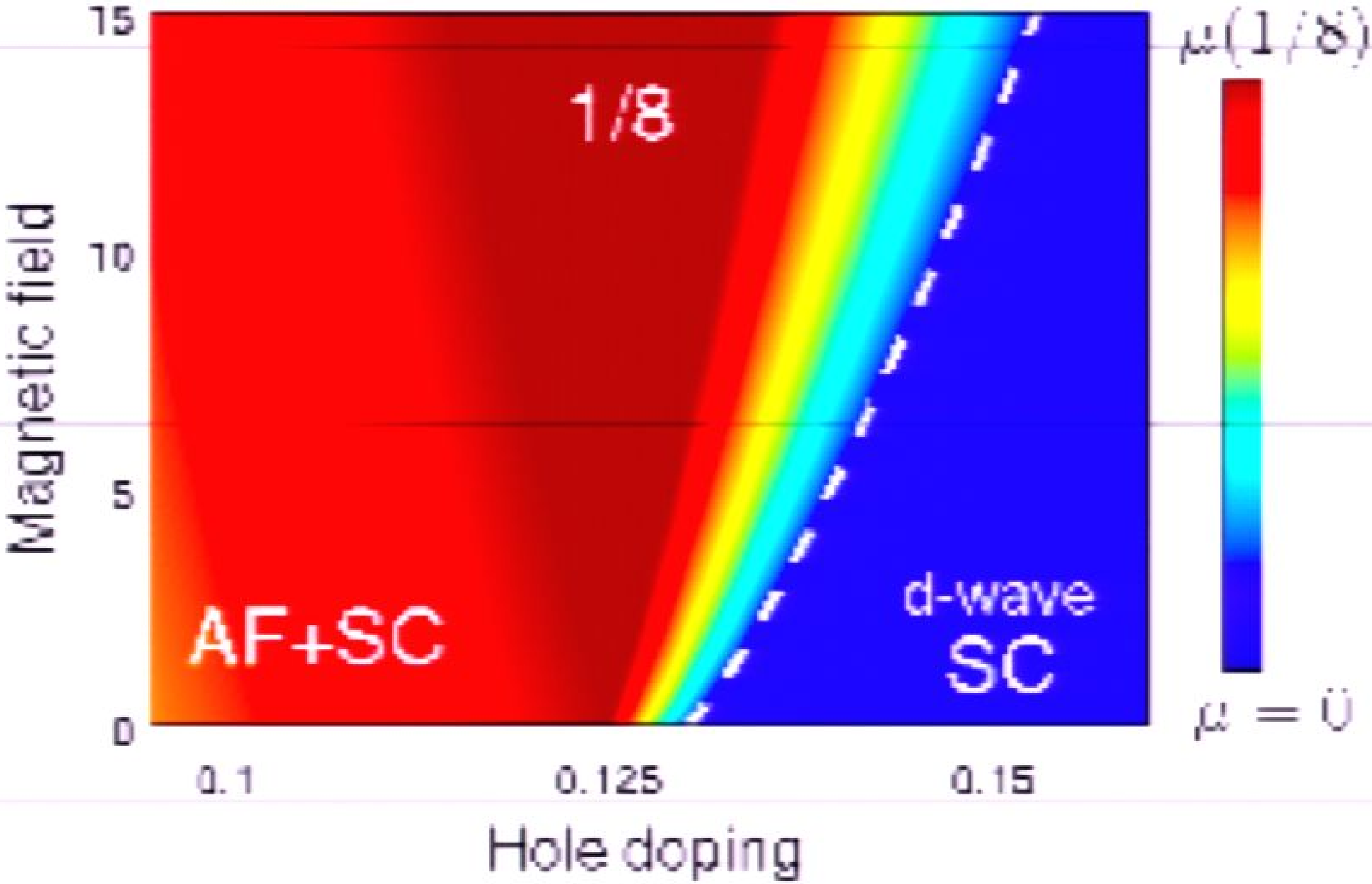


SC to SC+SDW quantum critical point

Coupling between SDW order and nodal quasiparticles



J. Chang et al. (PSI Mesot group), arXiv:0712.2181



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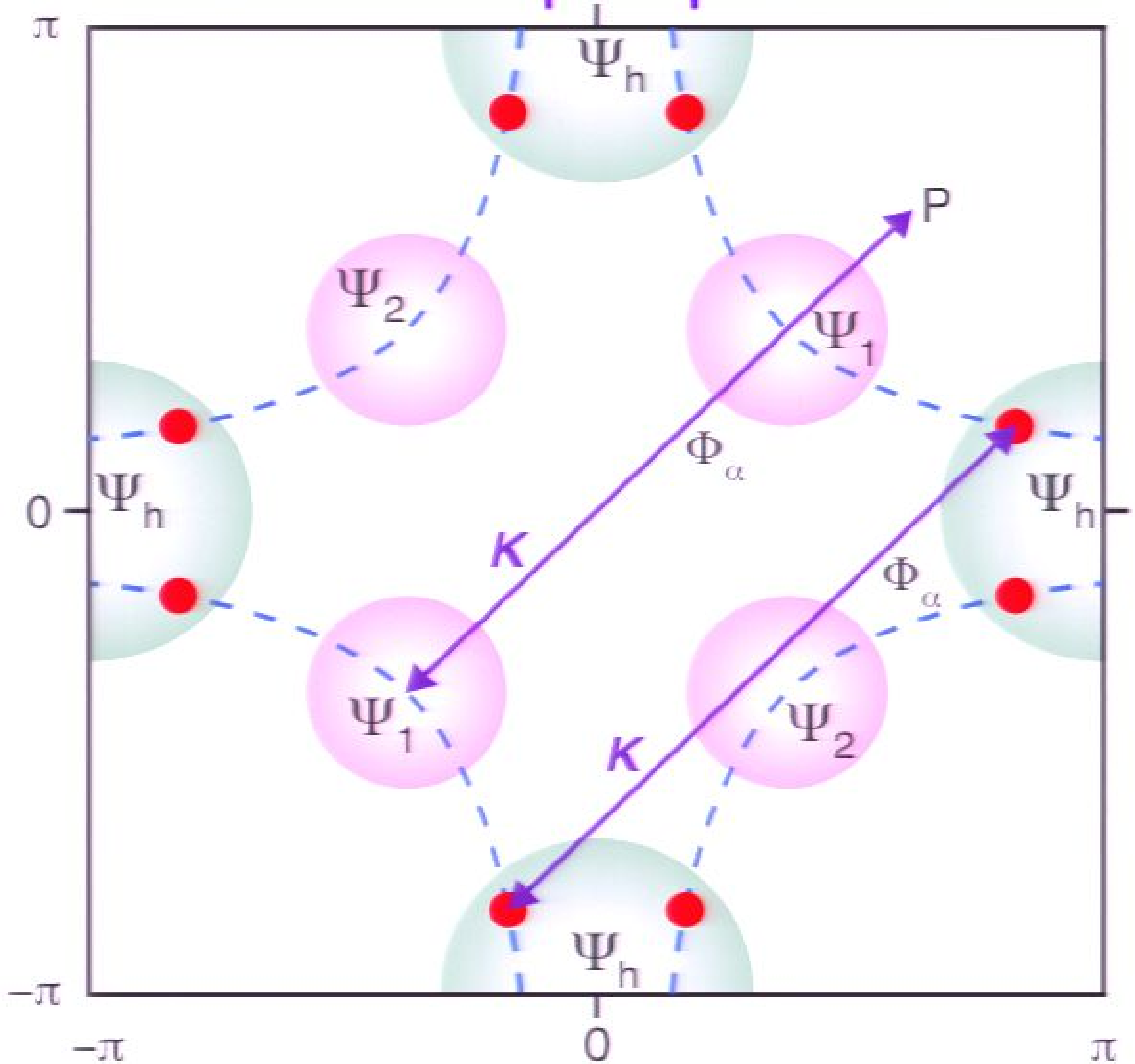

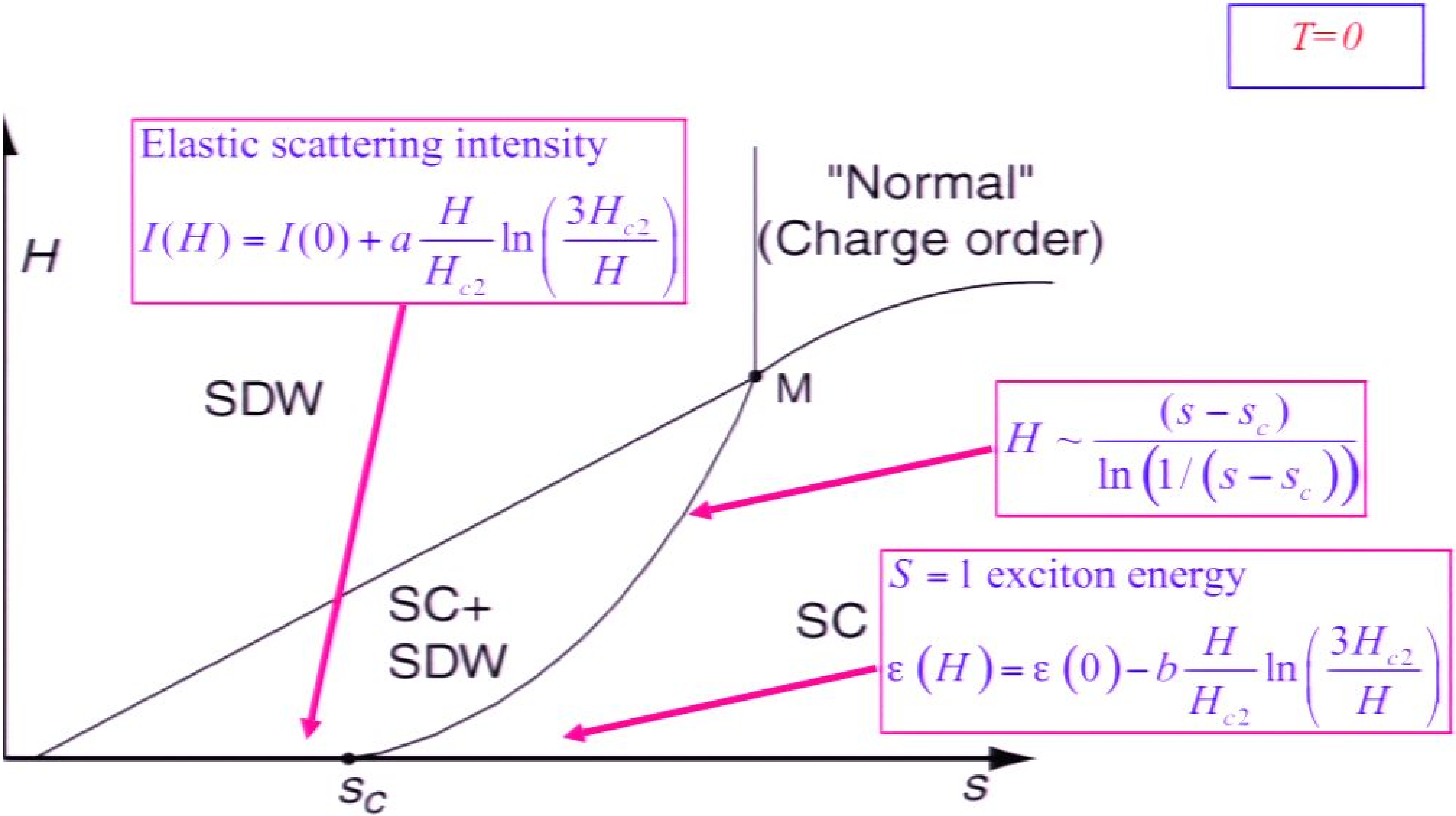

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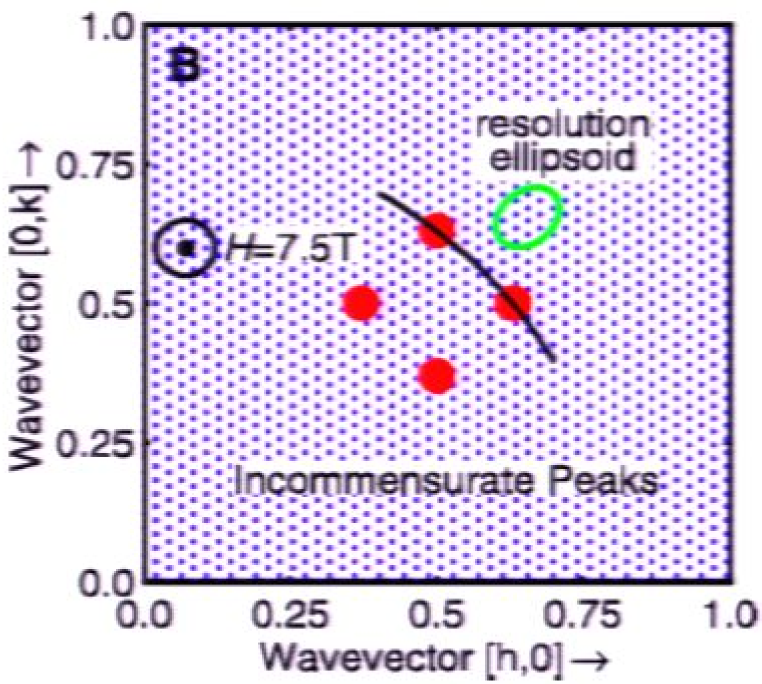

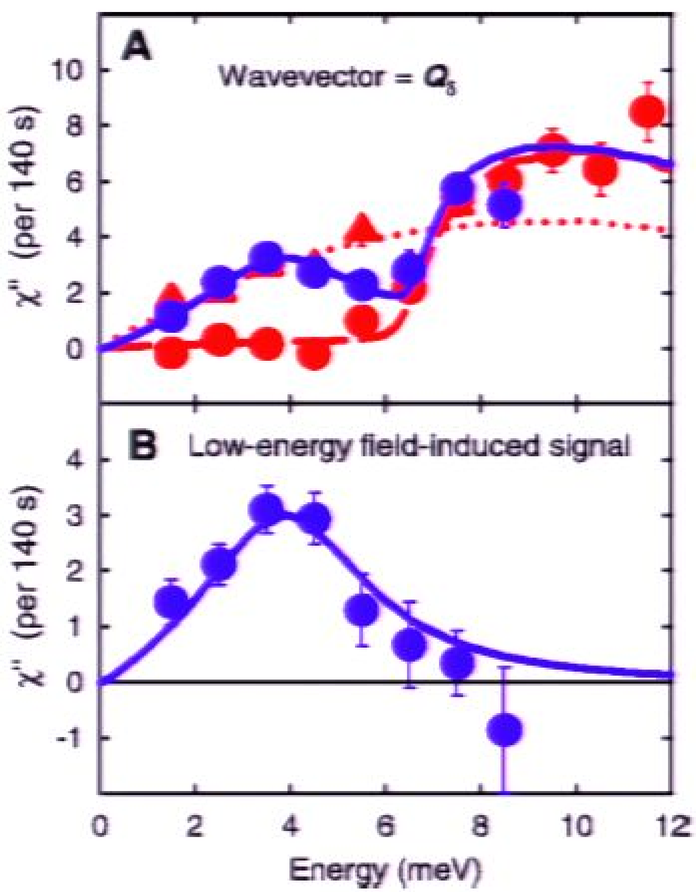

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$\Rightarrow$ dynamic SDW of period 8

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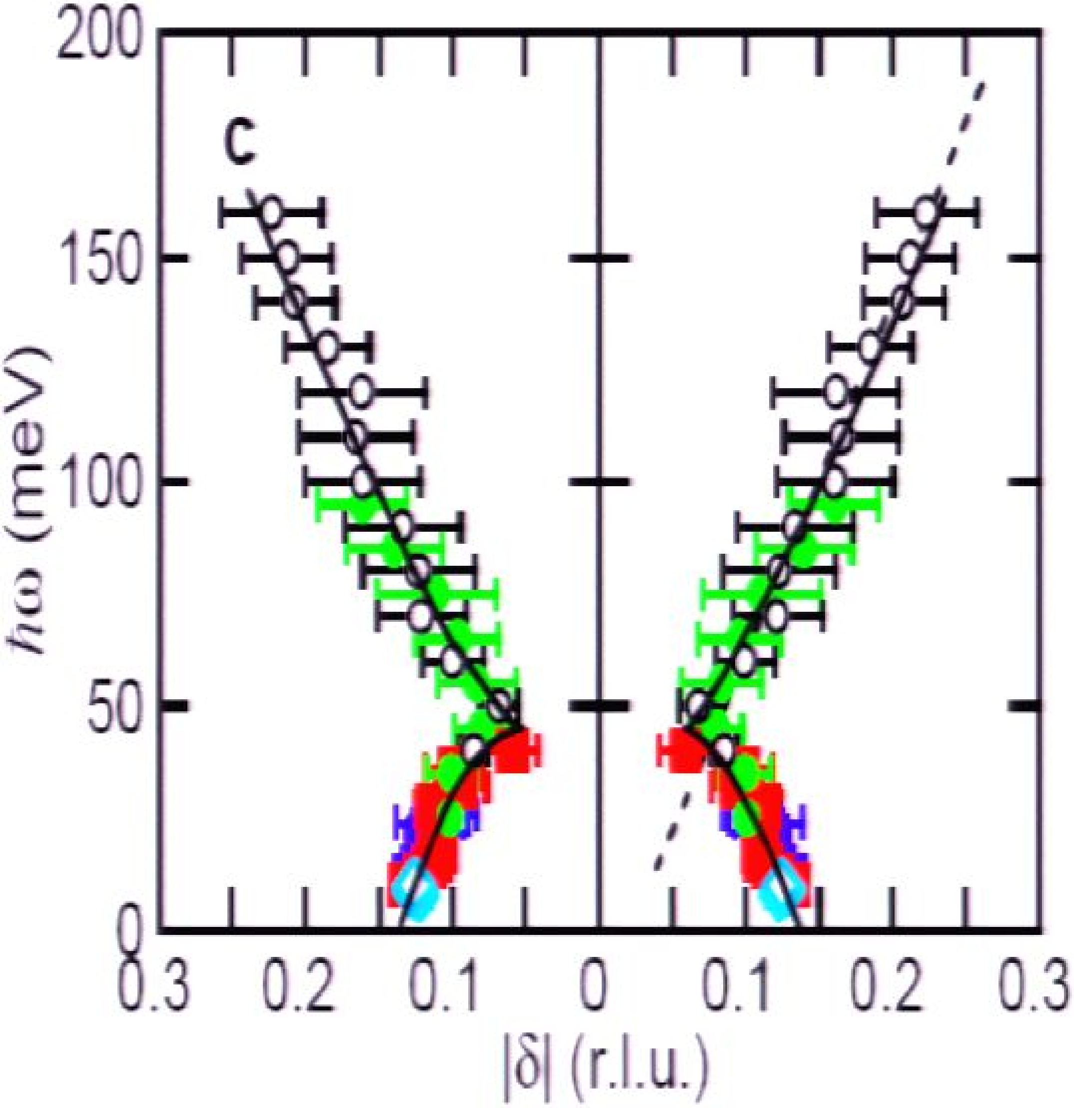

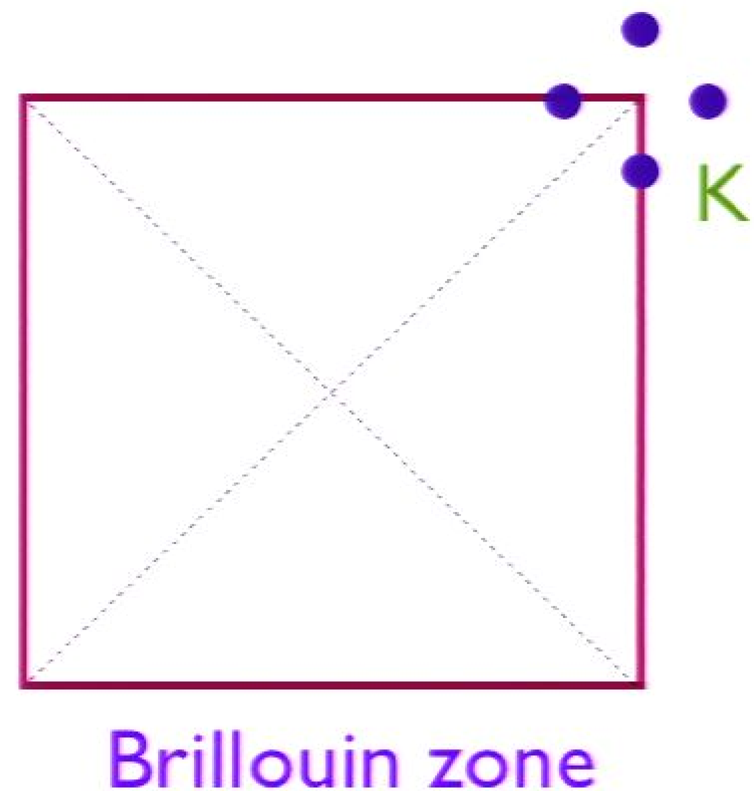

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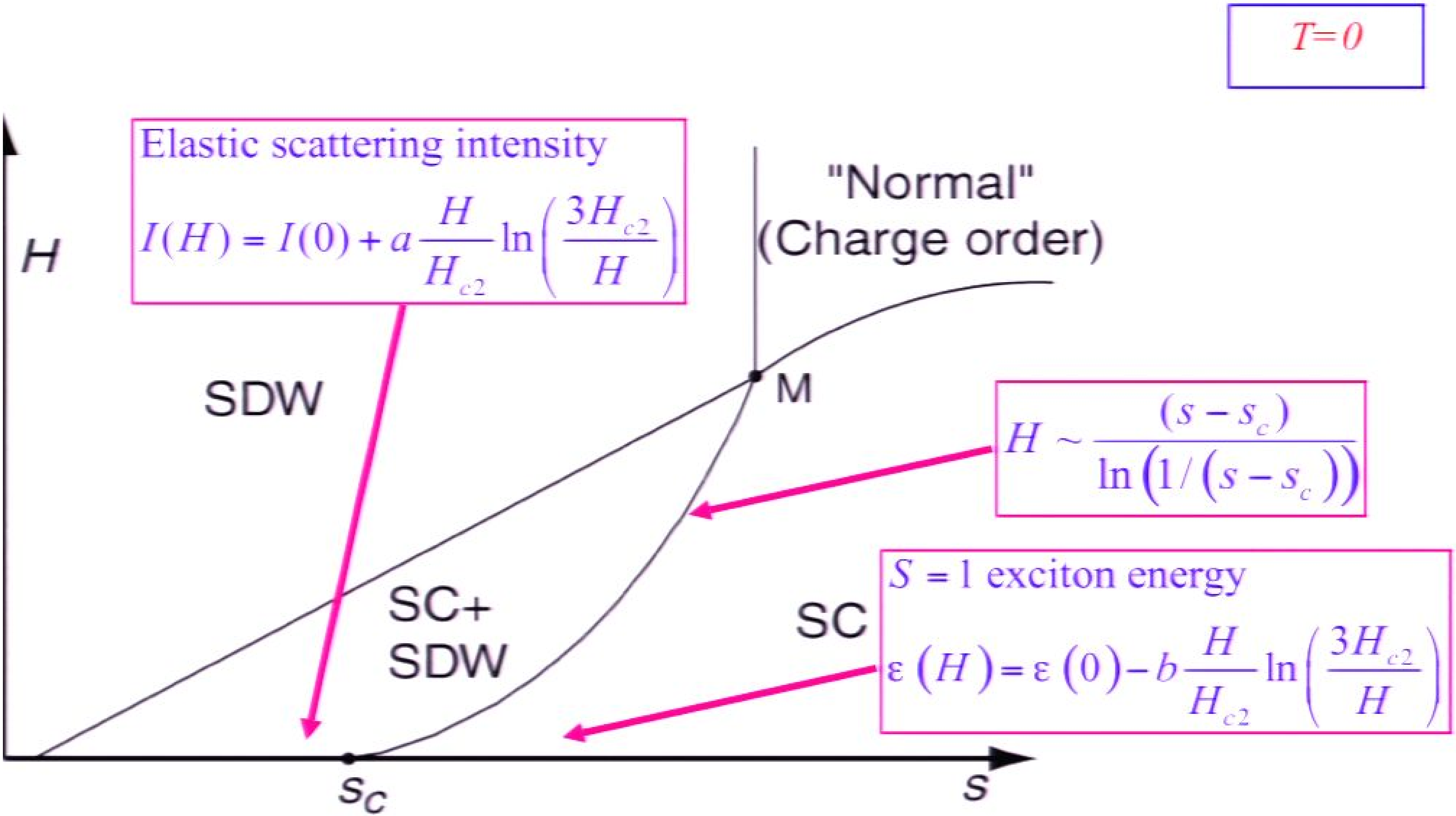

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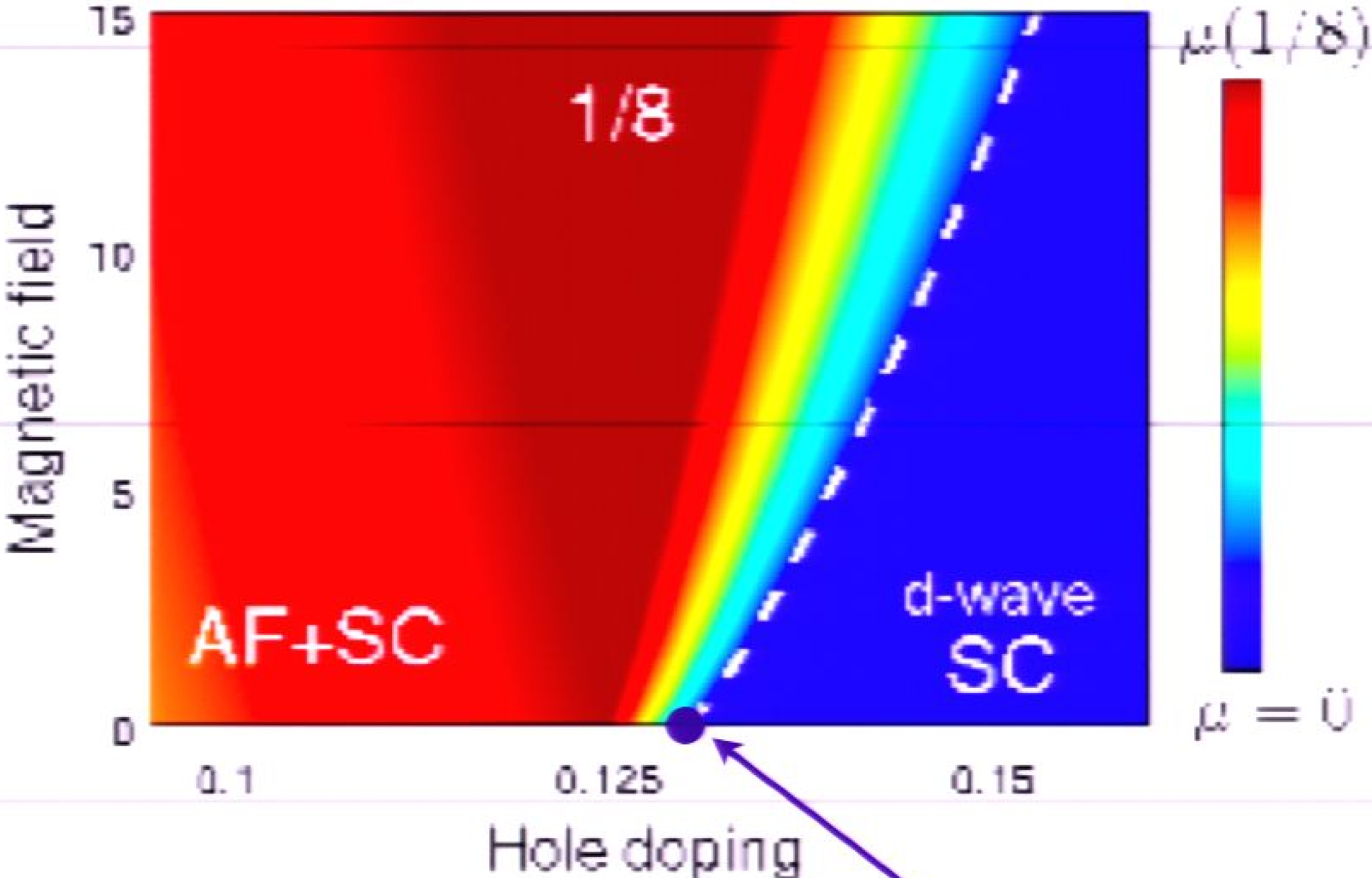

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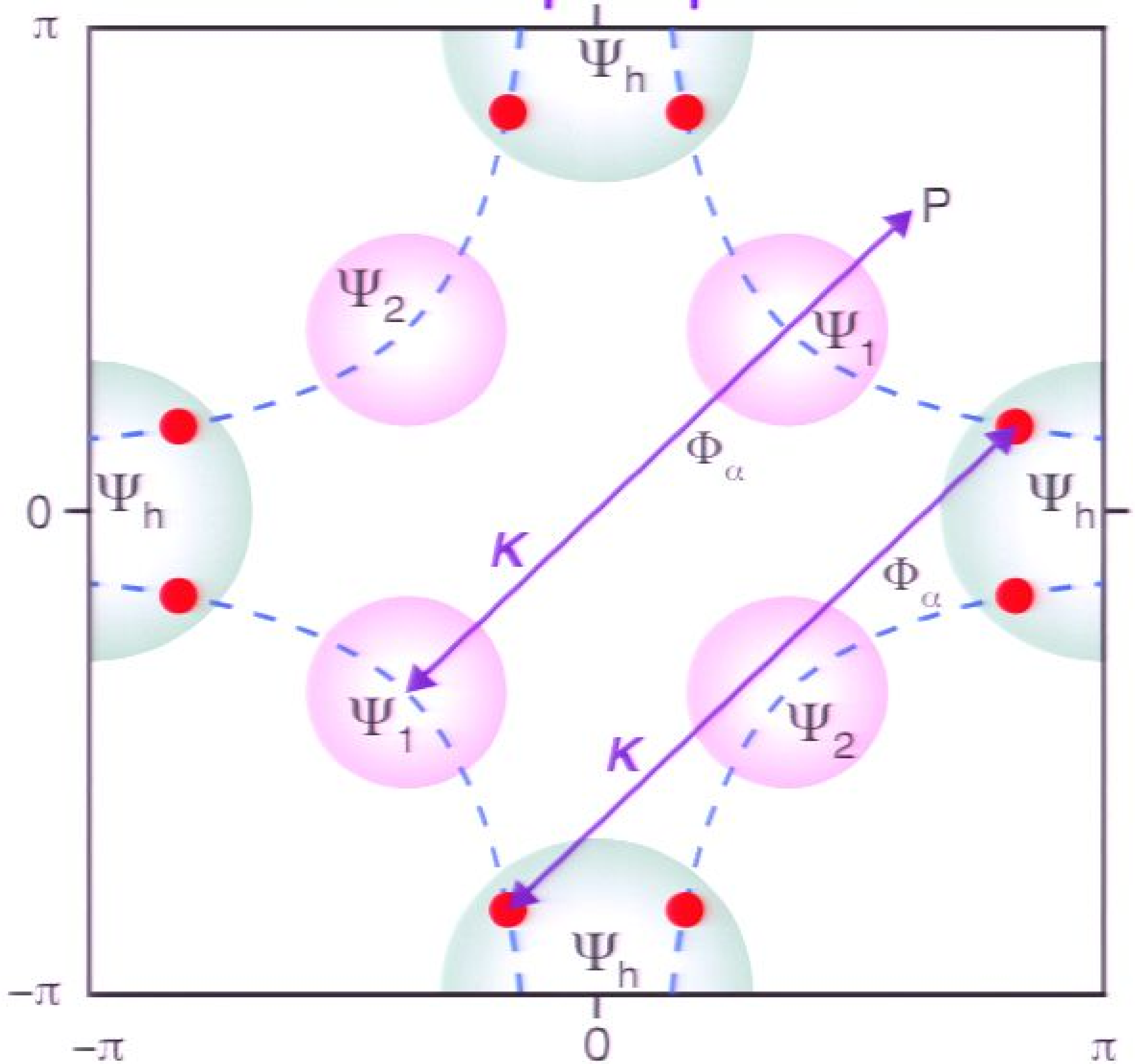

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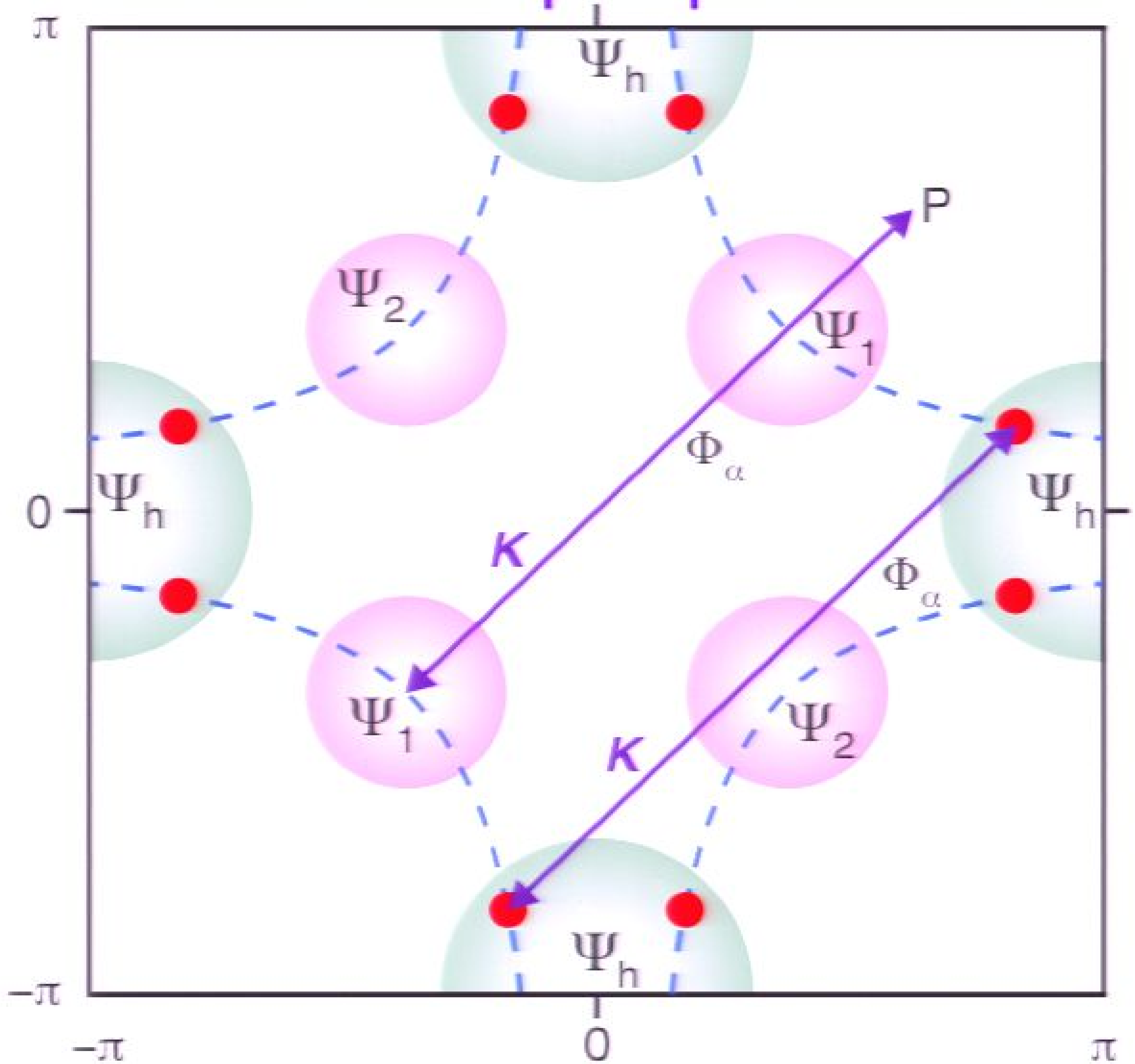
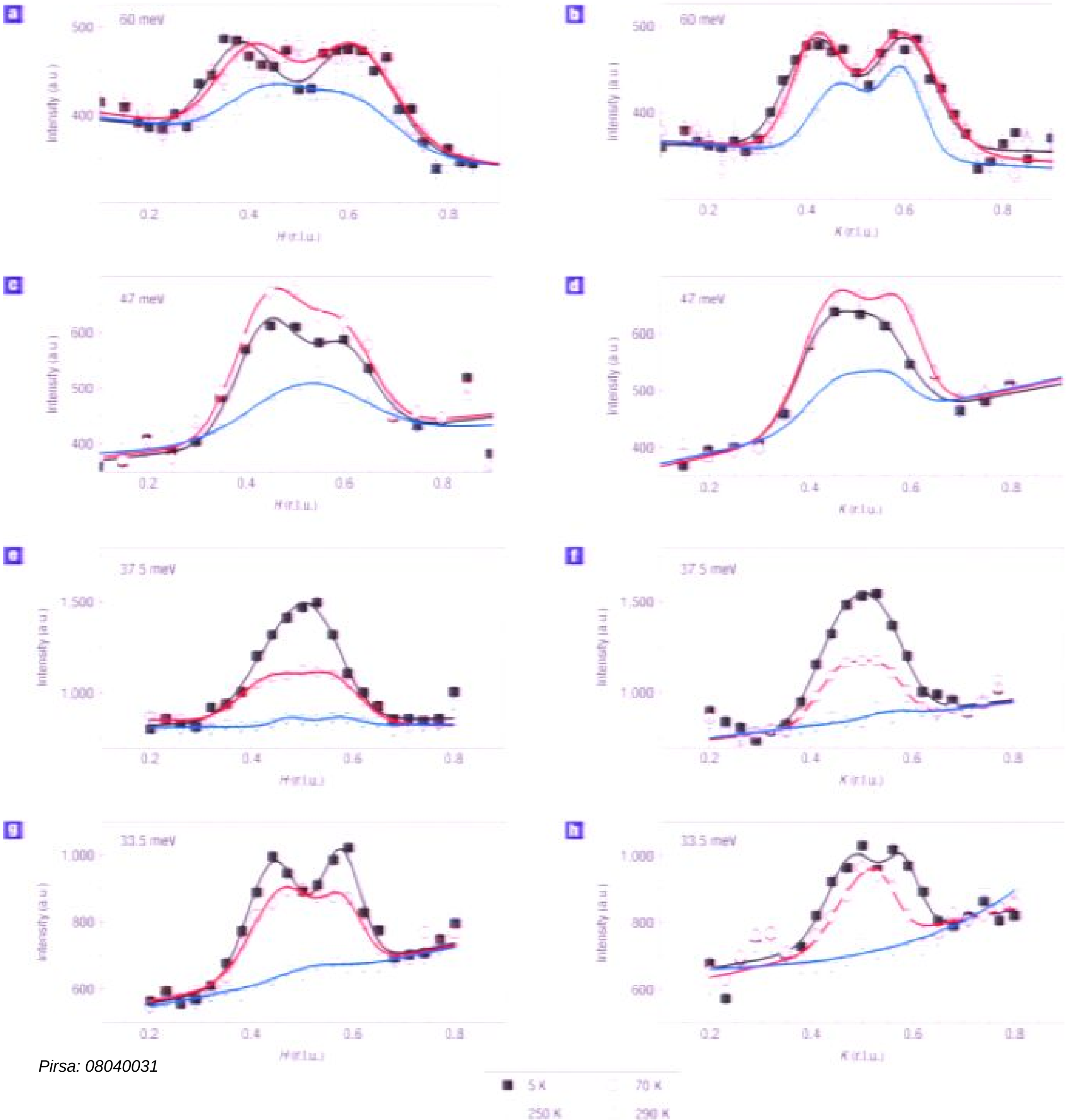


Wavevector mismatch suggests SDW order and nodal quasiparticles are decoupled



Nematic order in YBCO

V. Hinkov, P. Bourges, S. Pailhès, Y. Sidis, A. Ivanov, C. D. Frost, T. G. Perring, C. T. Lin, D. P. Chen & B. Keimer *Nature Physics* **3**, 780 - 785 (2007)

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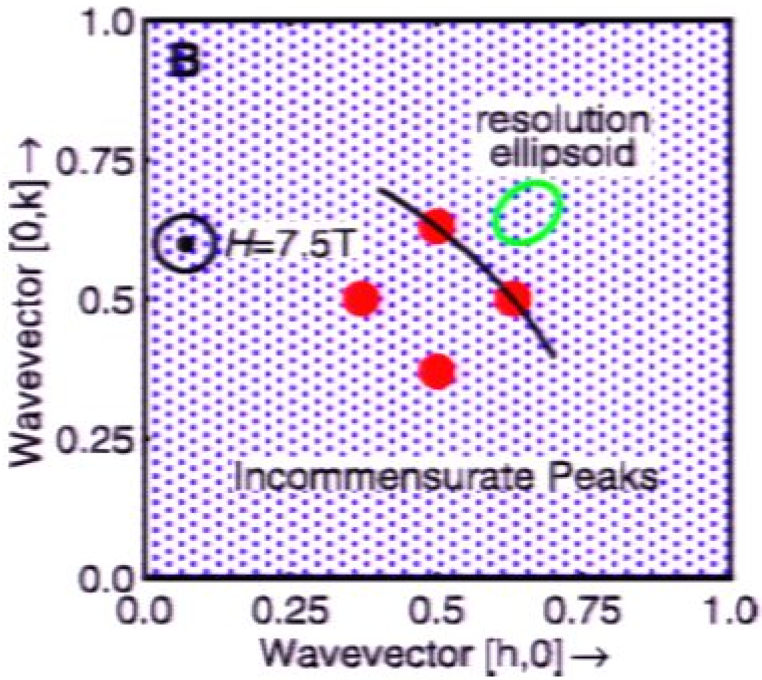

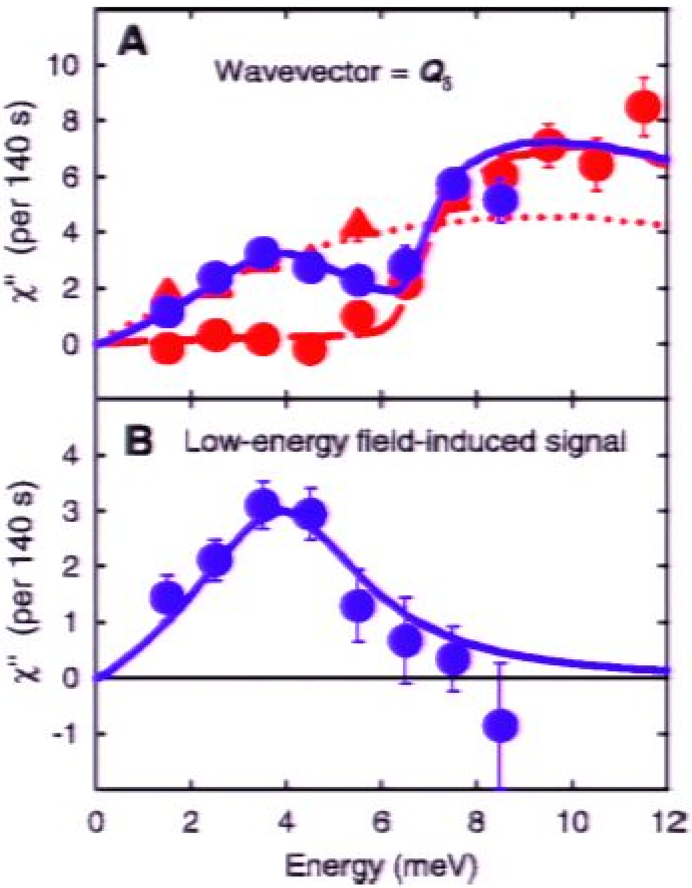

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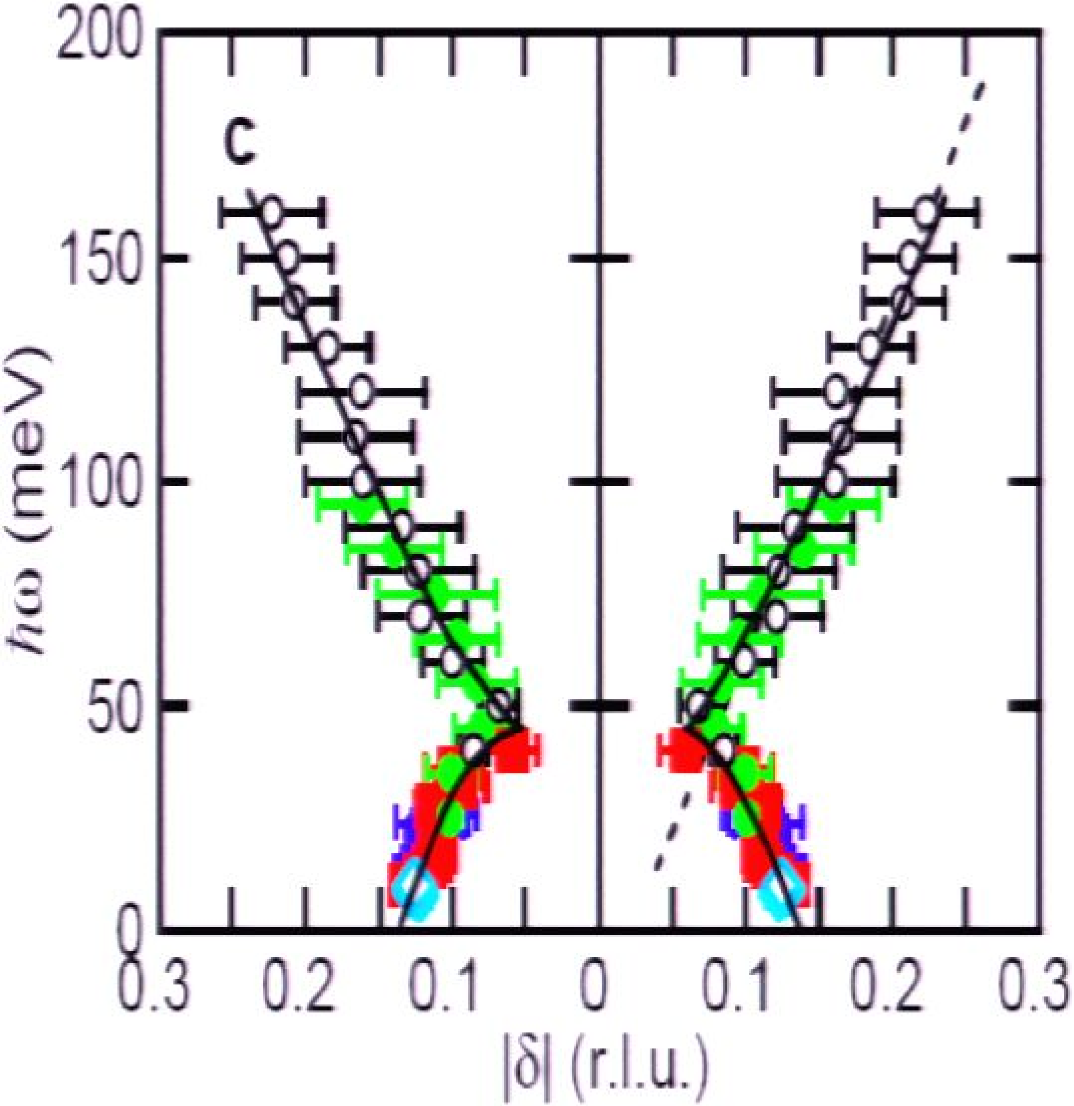

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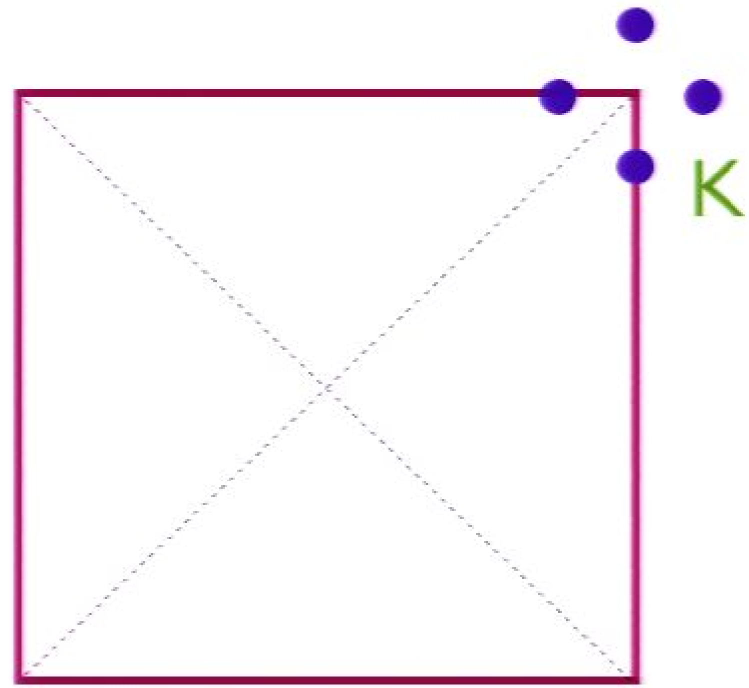
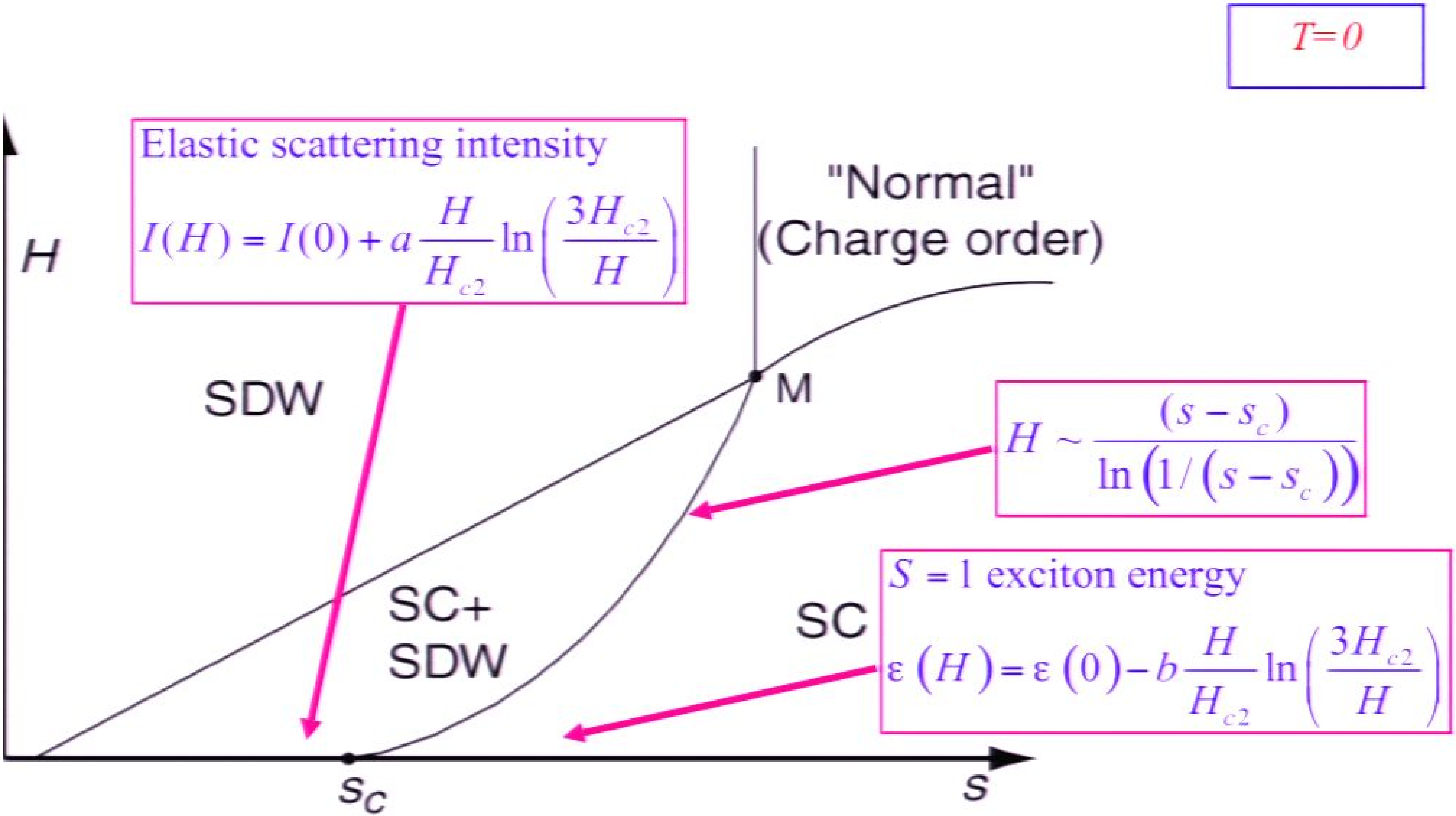


Brillouin zone



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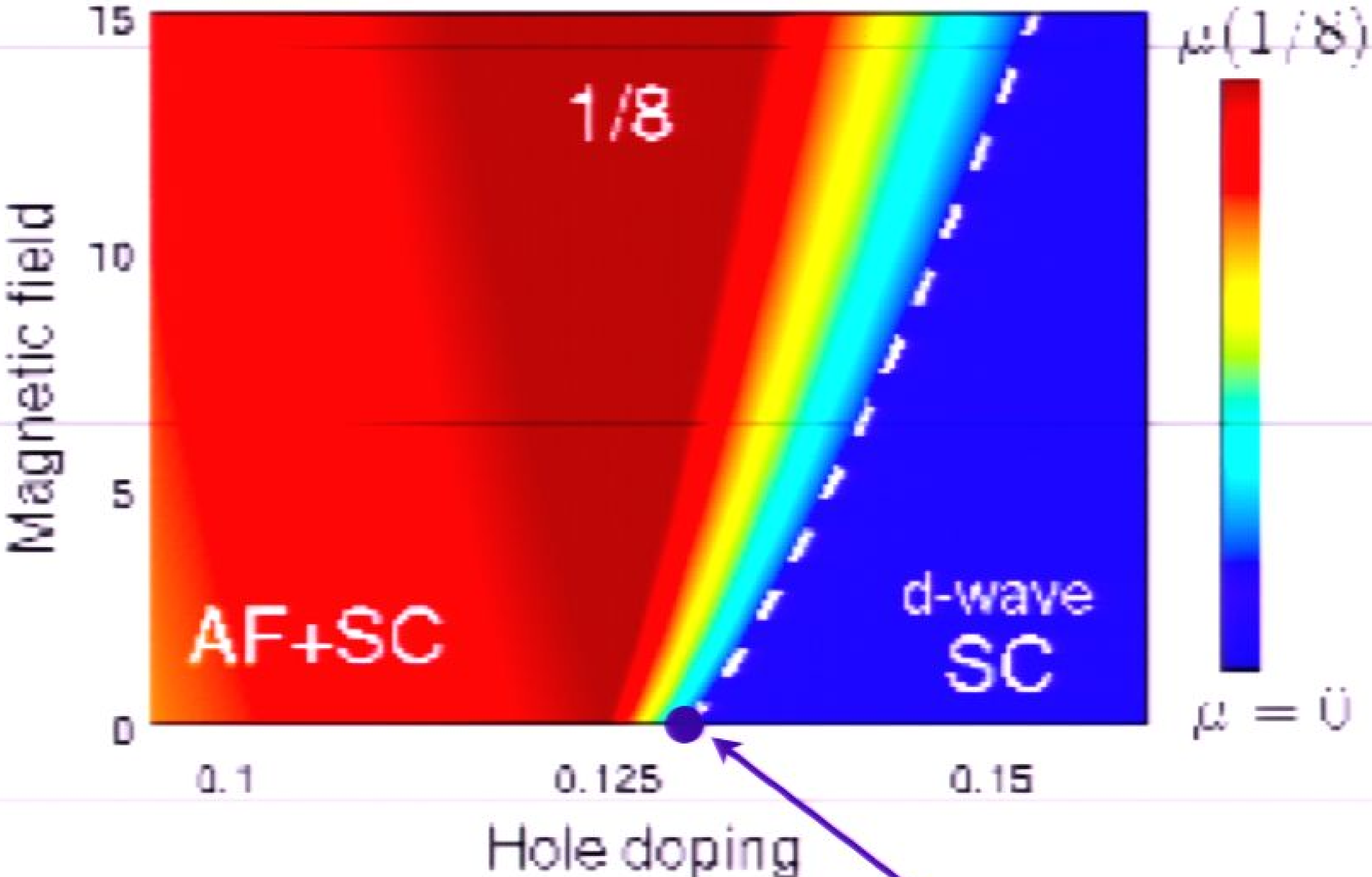

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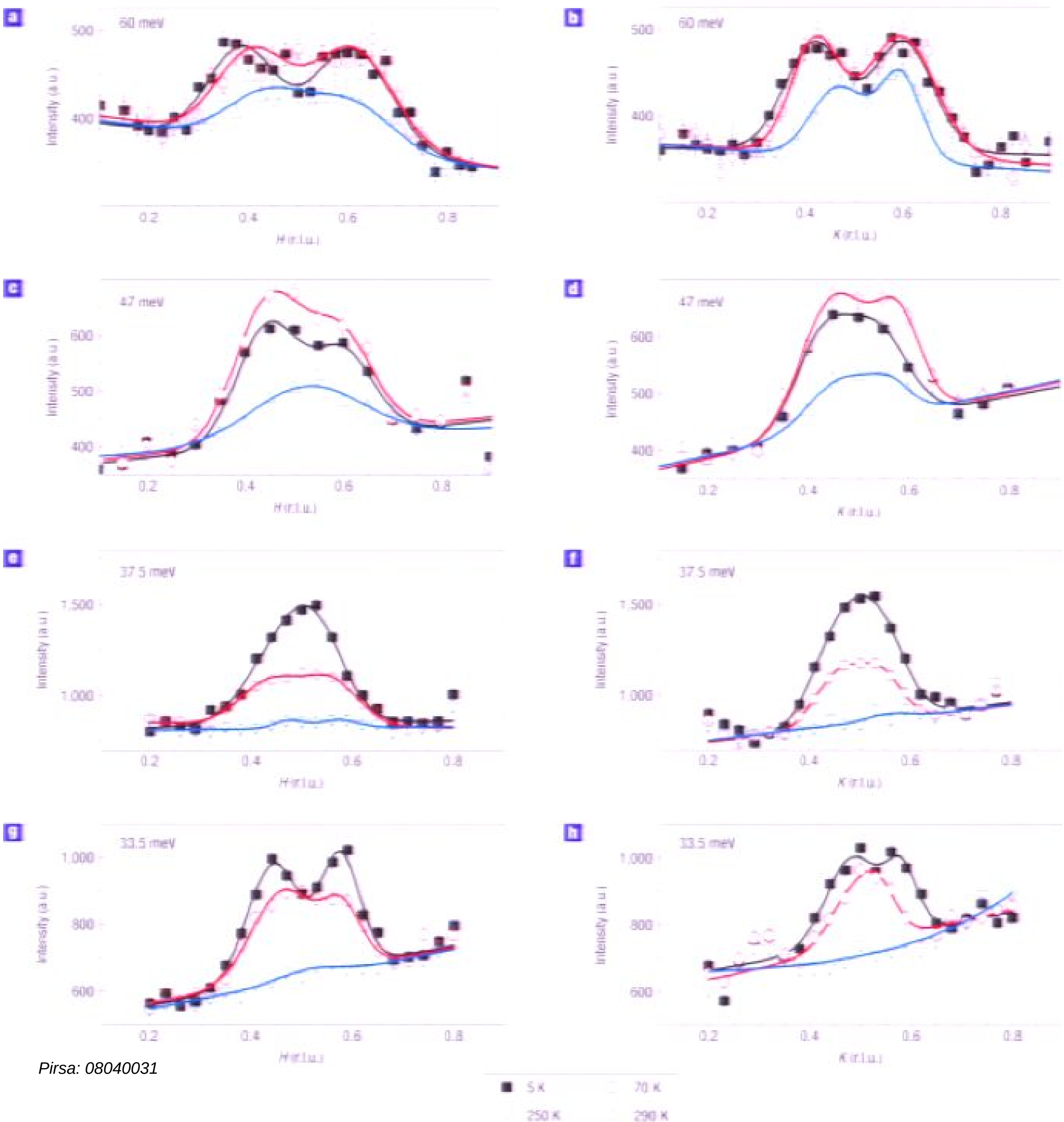

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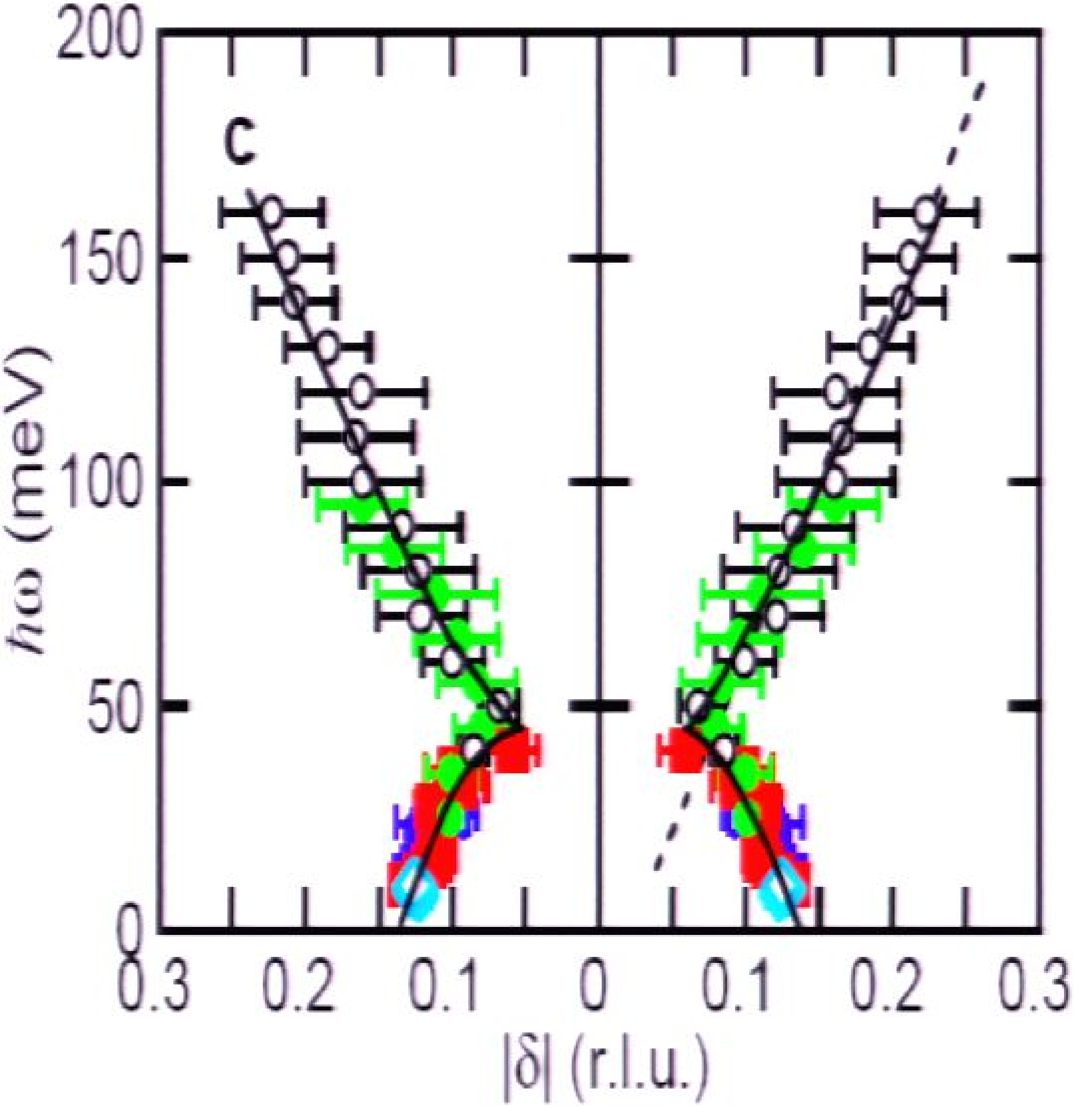

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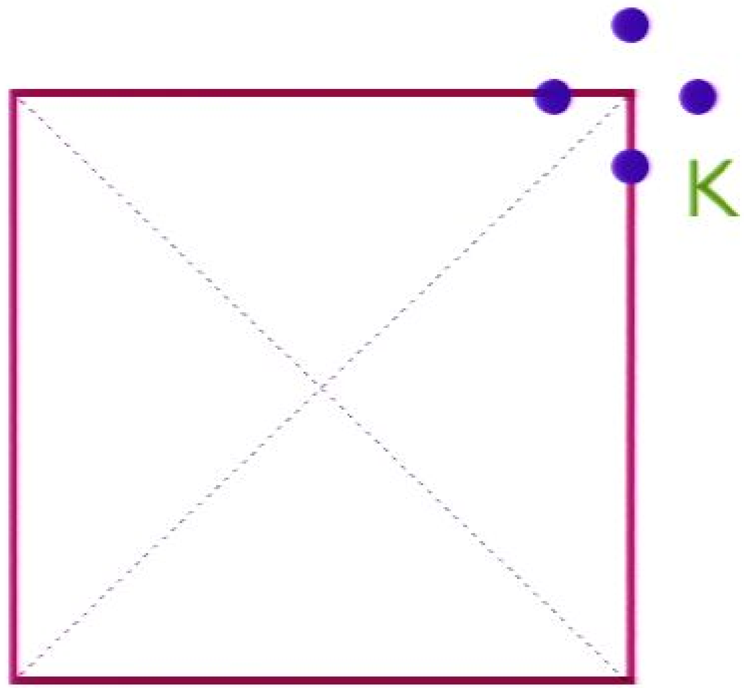

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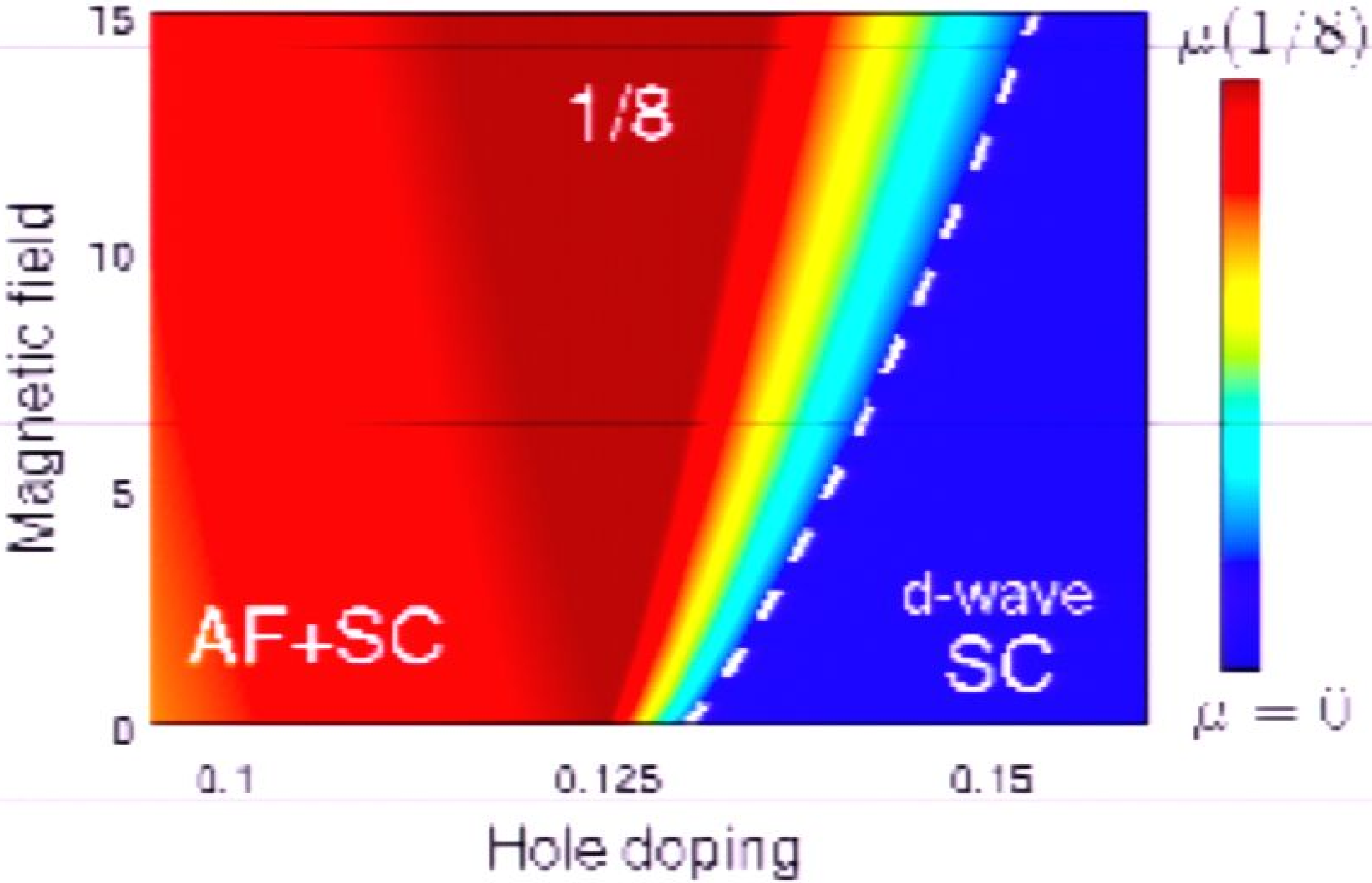

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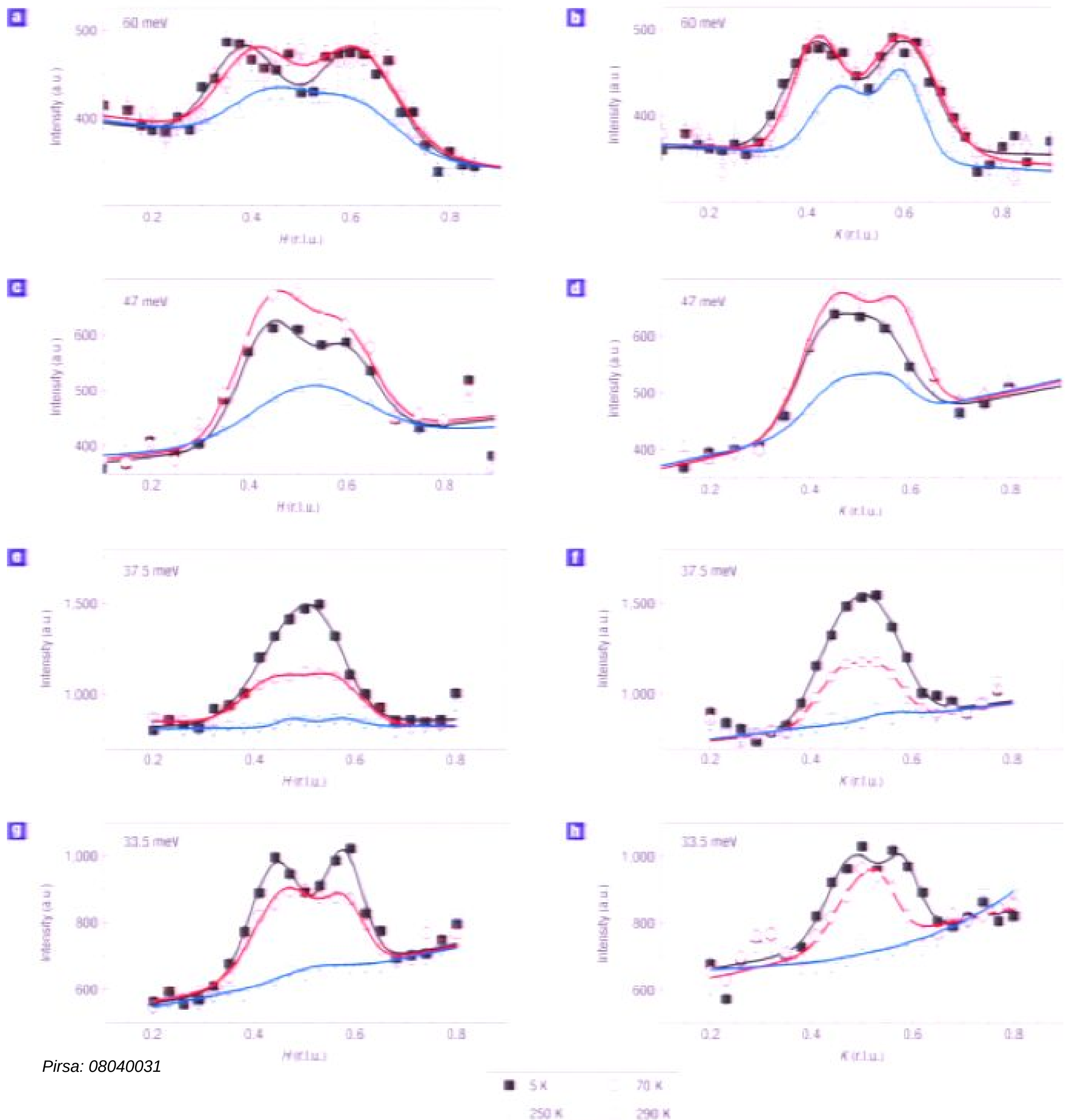

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Outline

1. Theory of SC to SC+SDW transition
   Emergent O(4) symmetry

2. Nodal quasiparticles at the O(4) critical point
   Unique selection of quasiparticle coupling to (composite) nematic order

3. Theory of the onset of nematic order in a d-wave superconductor
   Infinite anisotropy fixed point

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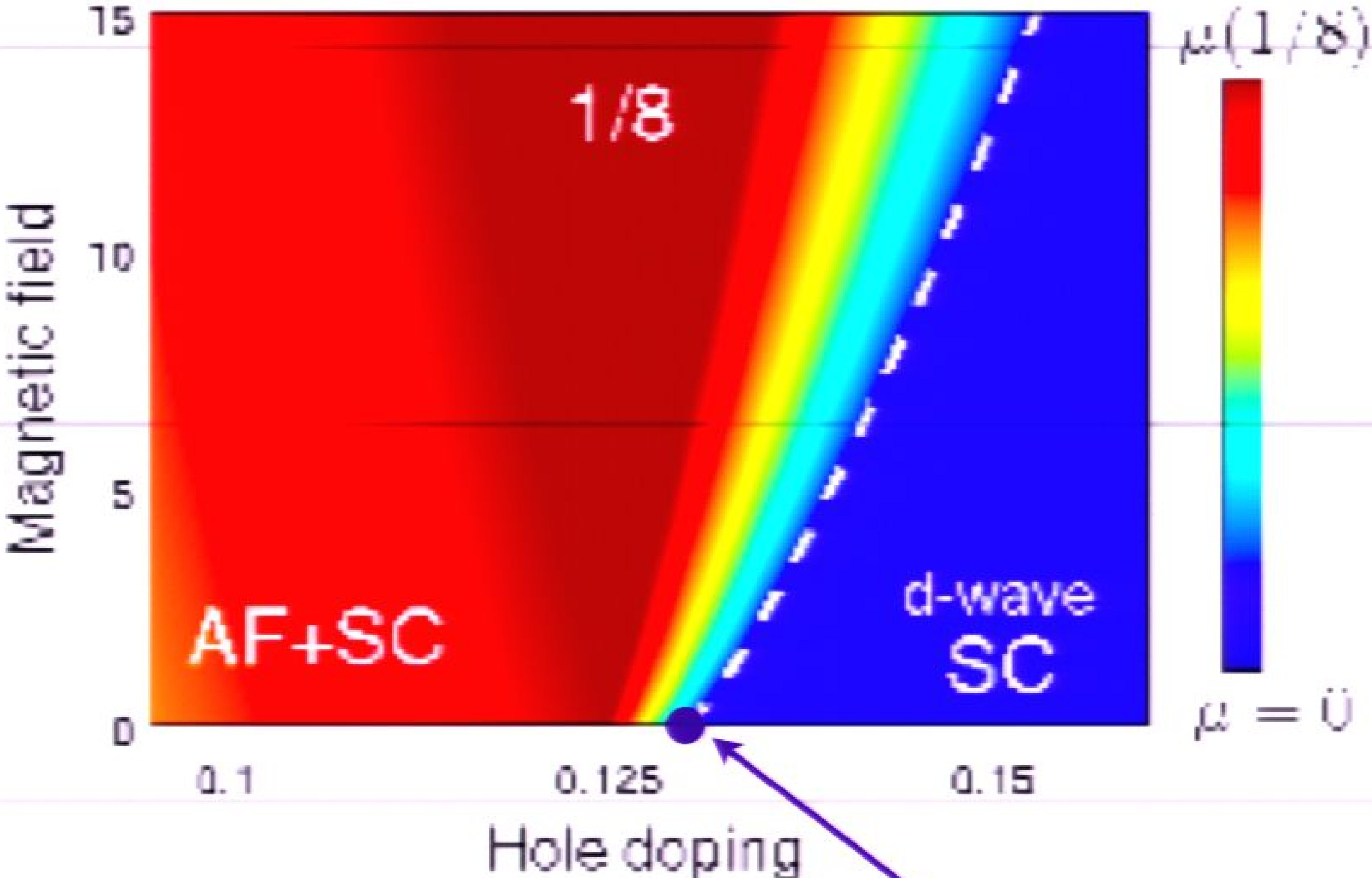
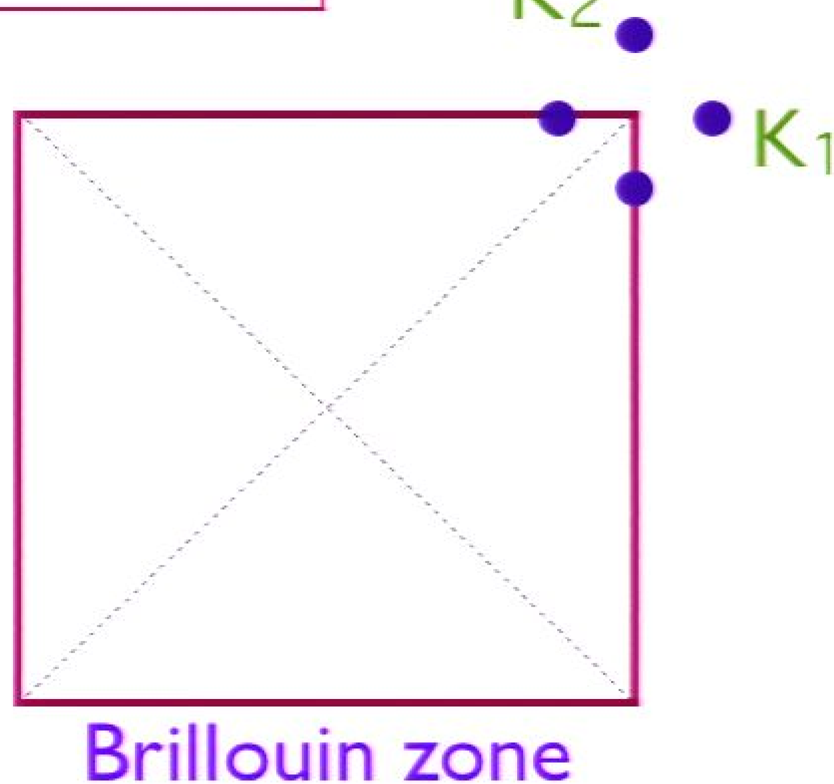


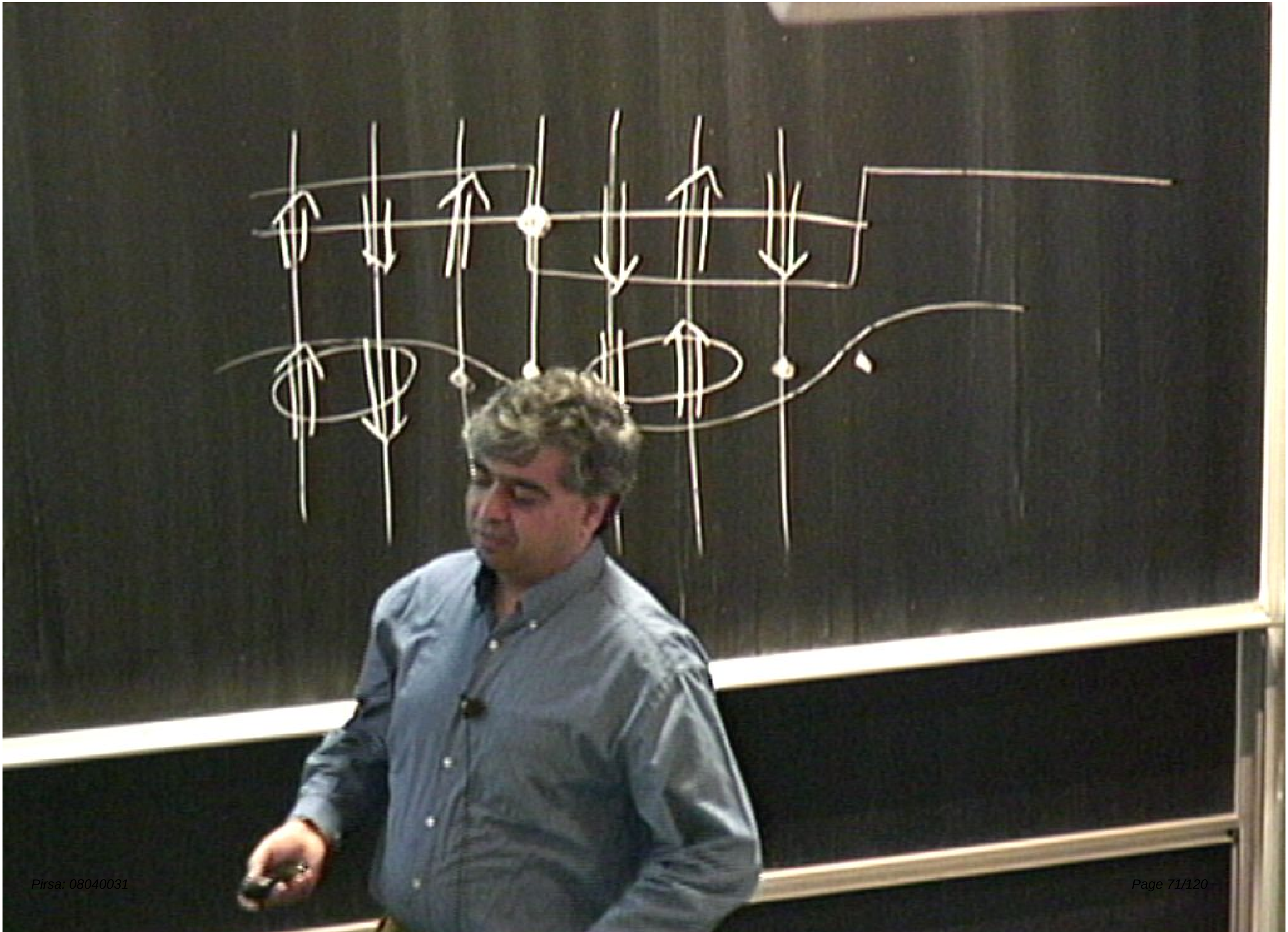
SC to SC+SDW quantum critical point

SDW order parameters



$$S_i(\mathbf{r},\tau) = \mathrm{Re}\left[e^{i\mathbf{K}_1\cdot\mathbf{r}}\,\Phi_{1i}(\mathbf{r},\tau) + e^{i\mathbf{K}_2\cdot\mathbf{r}}\,\Phi_{2i}(\mathbf{r},\tau)\right].$$

$$\mathbf{K}_1 = \left(\frac{2\pi}{a}\right)\left(\frac{1}{2}-\vartheta, \frac{1}{2}\right) \quad , \quad \mathbf{K}_2 = \left(\frac{2\pi}{a}\right)\left(\frac{1}{2}, \frac{1}{2}-\vartheta\right),$$



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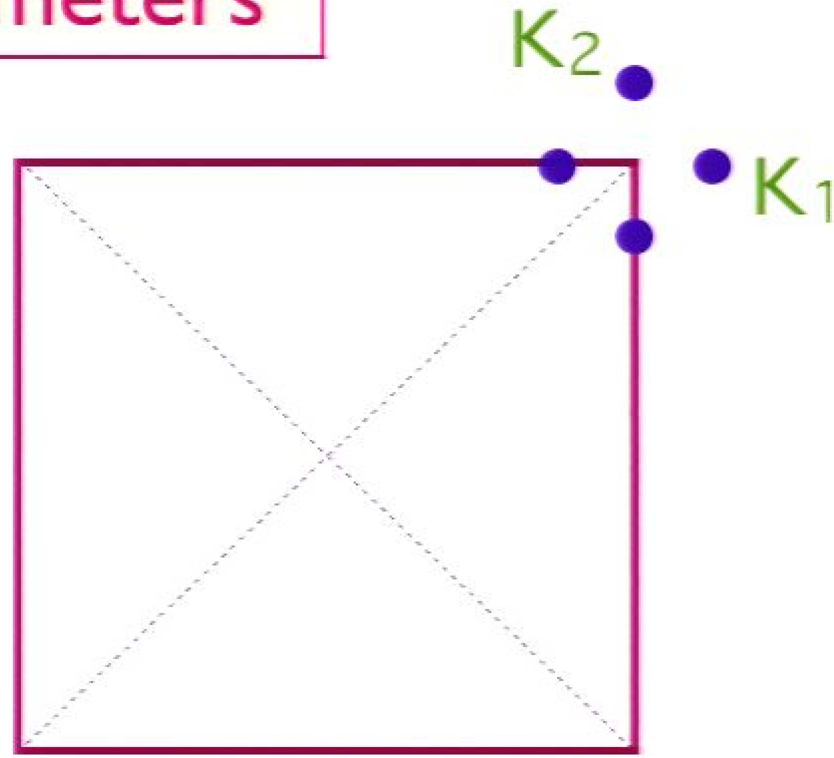

Brillouin zone

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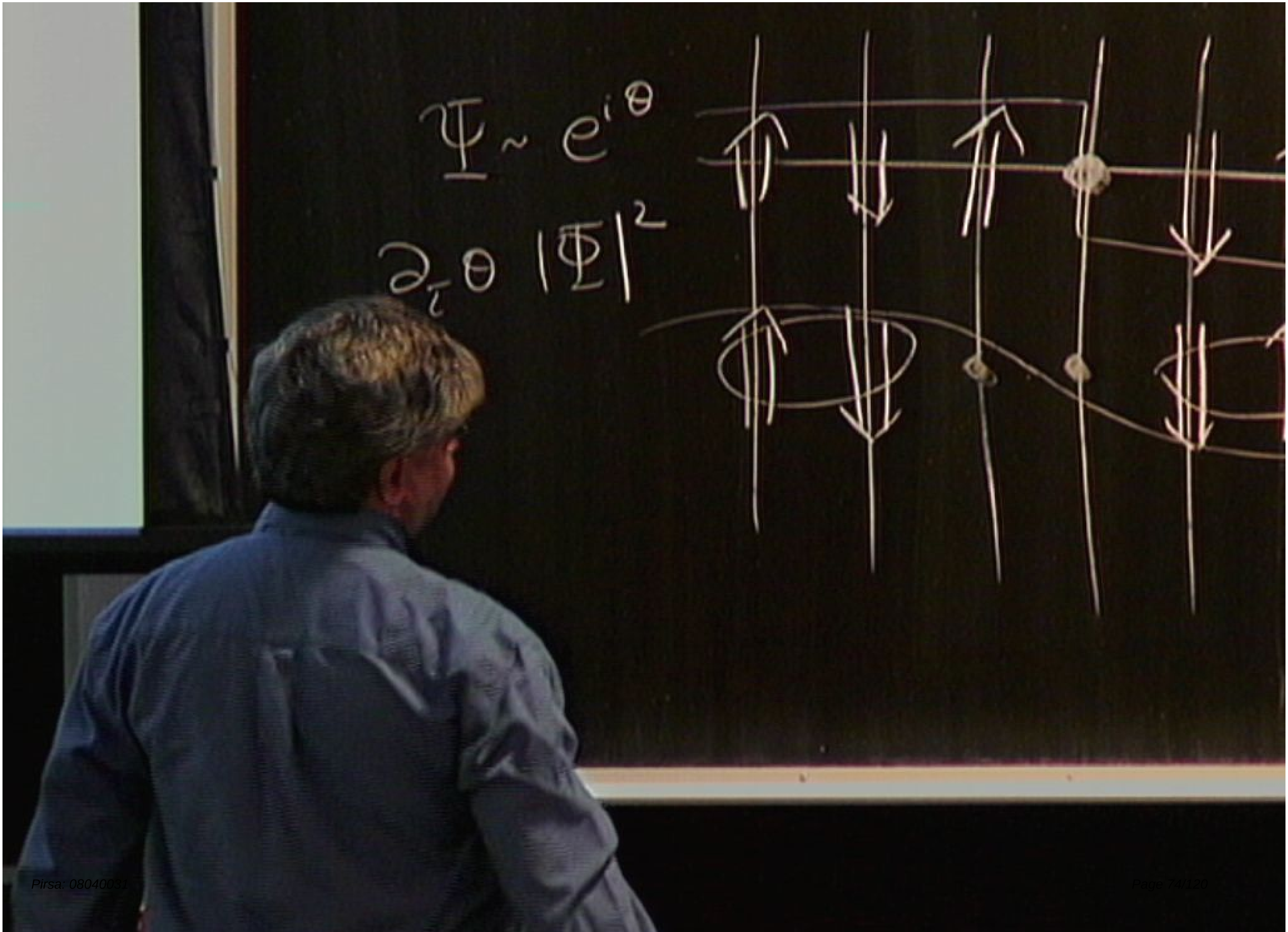
SDW field theory

$$\mathcal{L}_\Phi = |\partial_\tau \Phi_1|^2 + v_1^2 |\partial_x \Phi_1|^2 + v_2^2 |\partial_y \Phi_1|^2$$
$$+|\partial_\tau \Phi_2|^2 + v_2^2 |\partial_x \Phi_2|^2 + v_1^2 |\partial_y \Phi_2|^2 + r(|\Phi_1|^2 + |\Phi_2|^2)$$
$$+\frac{u_1}{2}(|\Phi_1|^4 + |\Phi_2|^4) + \frac{u_2}{2}(|\Phi_1^2|^2 + |\Phi_2^2|^2)$$
$$+w_1 |\Phi_1|^2 |\Phi_2|^2 + w_2 |\Phi_1 \cdot \Phi_2|^2 + w_3 |\Phi_1^* \cdot \Phi_2|^2$$

Most general theory invariant under spin rotation, square lattice space group, and time-reversal symmetries

	T_x	T_y	R	I	$\mathcal{T}$
Φ_{1i}	$-e^{-i\vartheta}\Phi_{1i}$	$-\Phi_{1i}$	Φ_{2i}	Φ_{1i}^*	$-\Phi_{1i}$
Φ_{2i}	$-\Phi_{2i}$	$-e^{-i\vartheta}\Phi_{2i}$	Φ_{1i}^*	Φ_{2i}^*	$-\Phi_{2i}$


Ψ ~ e^{iθ}
∂_t θ |Ψ|^2

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Symmetries:

$$\mathrm{U}(1) \otimes \mathrm{U}(1) \otimes Z_4 \otimes \mathrm{O}(3)$$

x-translations

y-translations

lattice rotations

spin rotations

SDW field theory

Stable fixed point in a 6-loop RG analysis:

$w_1^* = u_1^* - u_2^*$, $w_2^* = w_3^* = u_2^*$, $v_1^* = v_2^*$

O(4)⊗O(3) invariant theory for φ_{ai}, with $a = 1 \ldots 4$ an O(4) index, and $i = 1 \ldots 3$ an O(3) index, and

$$\Phi_{1i} = \varphi_{1i} + i\varphi_{2i}, \quad \Phi_{2i} = \varphi_{3i} + i\varphi_{4i}.$$

M. De Prato, A. Pelissetto, and E. Vicari
Phys. Rev. B **74**, 144507 (2006).

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Properties of O(4)⊗O(3) fixed point:

The following 9 order parameters have divergent fluctuations at the spin-density wave ordering transition with the same exponent $\overline{\gamma}$:

- The real 'nematic' order parameter, $\phi \equiv \sum_i \left(|\Phi_{1i}|^2 - |\Phi_{2i}|^2\right)$ which measures breaking of Z_4 symmetry
- Charge density waves at $2\mathbf{K}_1$ and $2\mathbf{K}_2$: $\sum_i \Phi_{1i}^2$ and $\sum_i \Phi_{2i}^2$
- Charge density waves at $\mathbf{K}_1 \pm \mathbf{K}_2$: $\sum_i \Phi_{1i}\Phi_{2i}$ and $\sum_i \Phi_{1i}^*\Phi_{2i}$

At the quantum critical point, the susceptibilities of these orders *all* diverge as $\chi \sim T^{-\overline{\gamma}}$, with

$$\overline{\gamma} = \begin{cases} 0.90(36) & \mathrm{MZM}, 6\text{ loops} \\ 0.80(54) & d = 3\ \overline{\mathrm{MS}}, 5\text{ loops} \end{cases}$$

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

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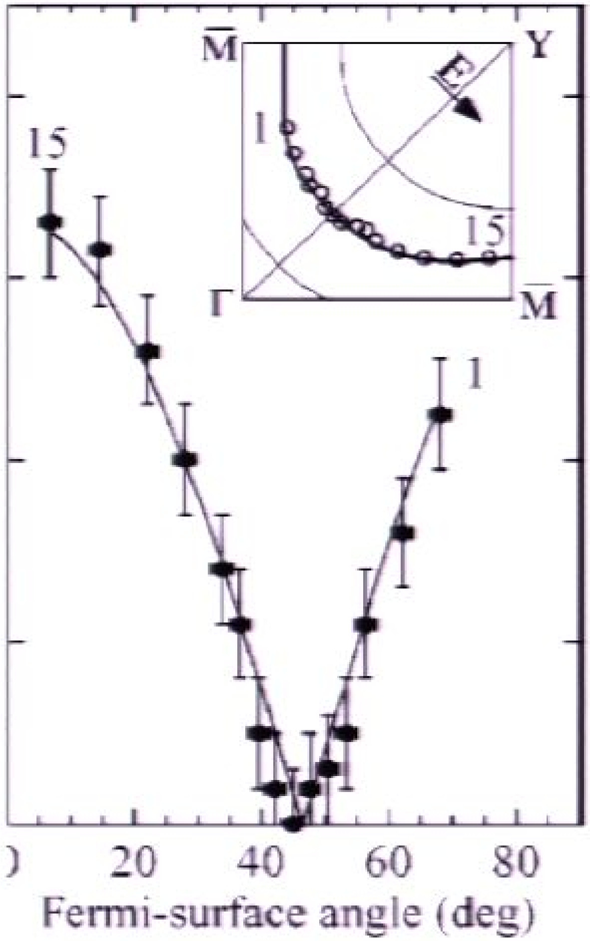
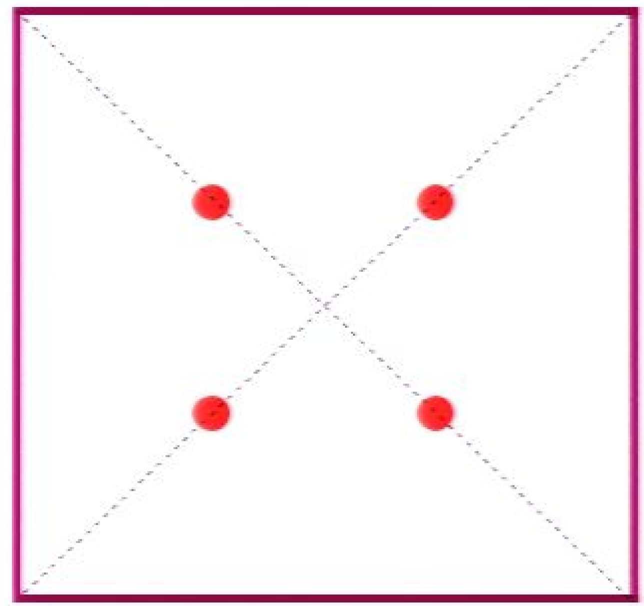


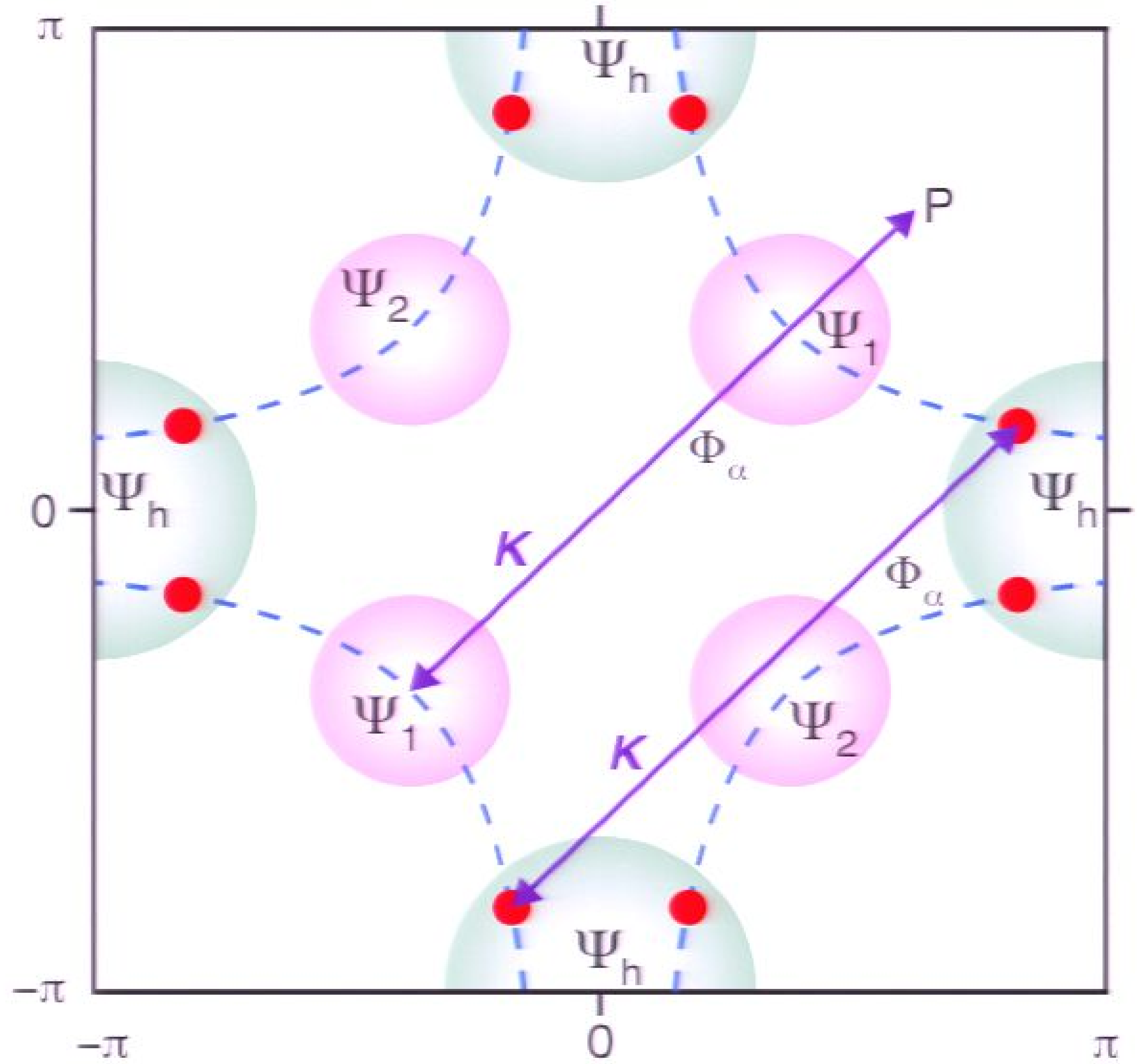


Brillouin zone

$$\mathcal{L}_\Psi = \Psi_1^\dagger \left(\partial_\tau - i\frac{v_F}{\sqrt{2}}(\partial_x + \partial_y)\tau^z - i\frac{v_\Delta}{\sqrt{2}}(-\partial_x + \partial_y)\tau^x \right) \Psi_1$$

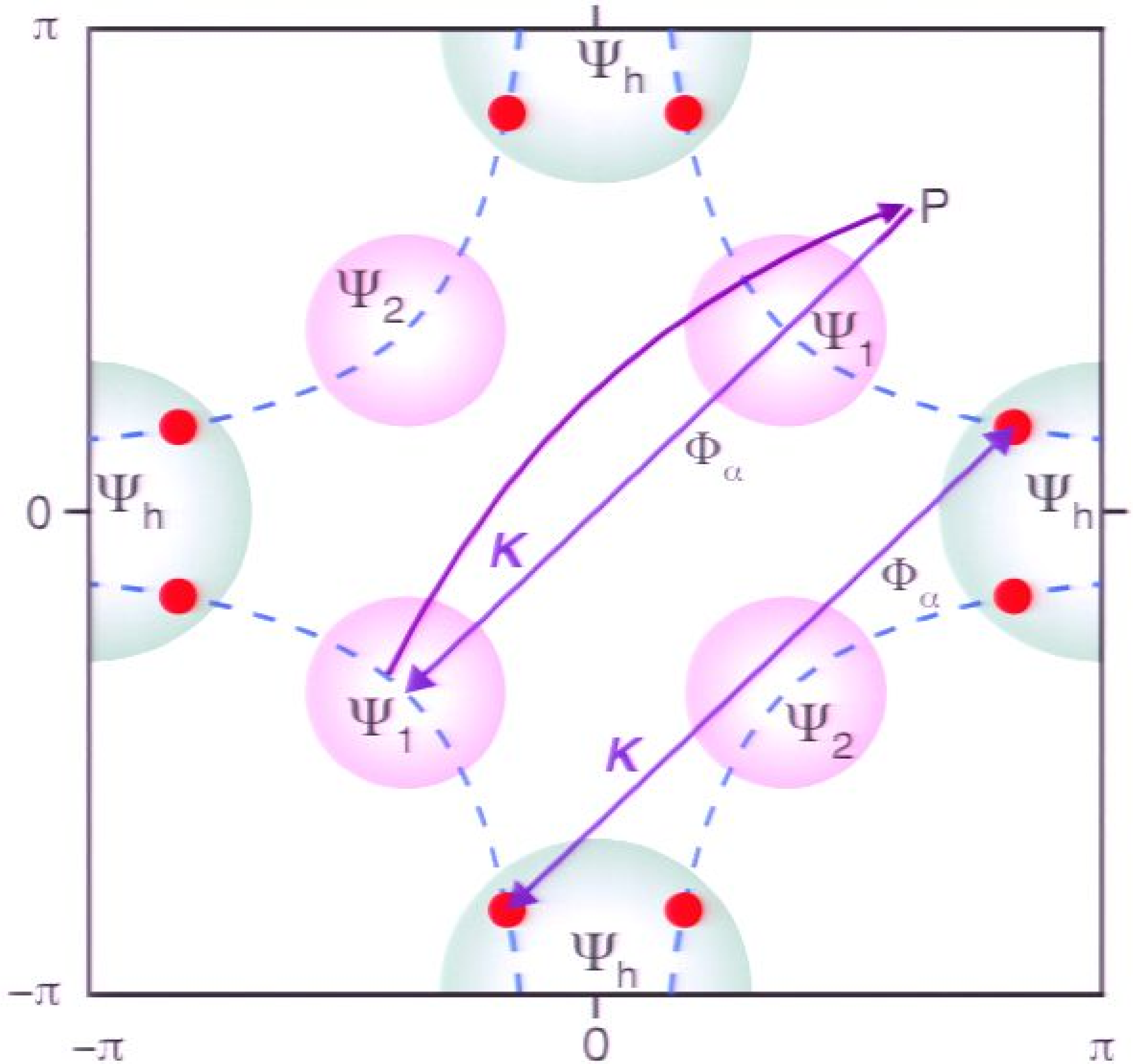
$$+\Psi_2^\dagger \left(\partial_\tau - i\frac{v_F}{\sqrt{2}}(-\partial_x + \partial_y)\tau^z - i\frac{v_\Delta}{\sqrt{2}}(\partial_x + \partial_y)\tau^x \right) \Psi_2.$$

Coupling of quasiparticles to SDW order



No "Yukawa" coupling $\Phi\Psi^\dagger\Psi$

Coupling of quasiparticles to SDW order



Possible higher order coupling ~ $|\Phi|^2 \Psi^\dagger \Psi$

Coupling of quasiparticles to SDW order

Higher - order couplings allowed by symmetry:

$$\mathcal{L}_1 = \lambda_1 \left(|\Phi_1|^2 + |\Phi_2|^2\right) \left(\Psi_1^\dagger \tau^z \Psi_1 + \Psi_2^\dagger \tau^z \Psi_2\right)$$

Energy-energy coupling

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

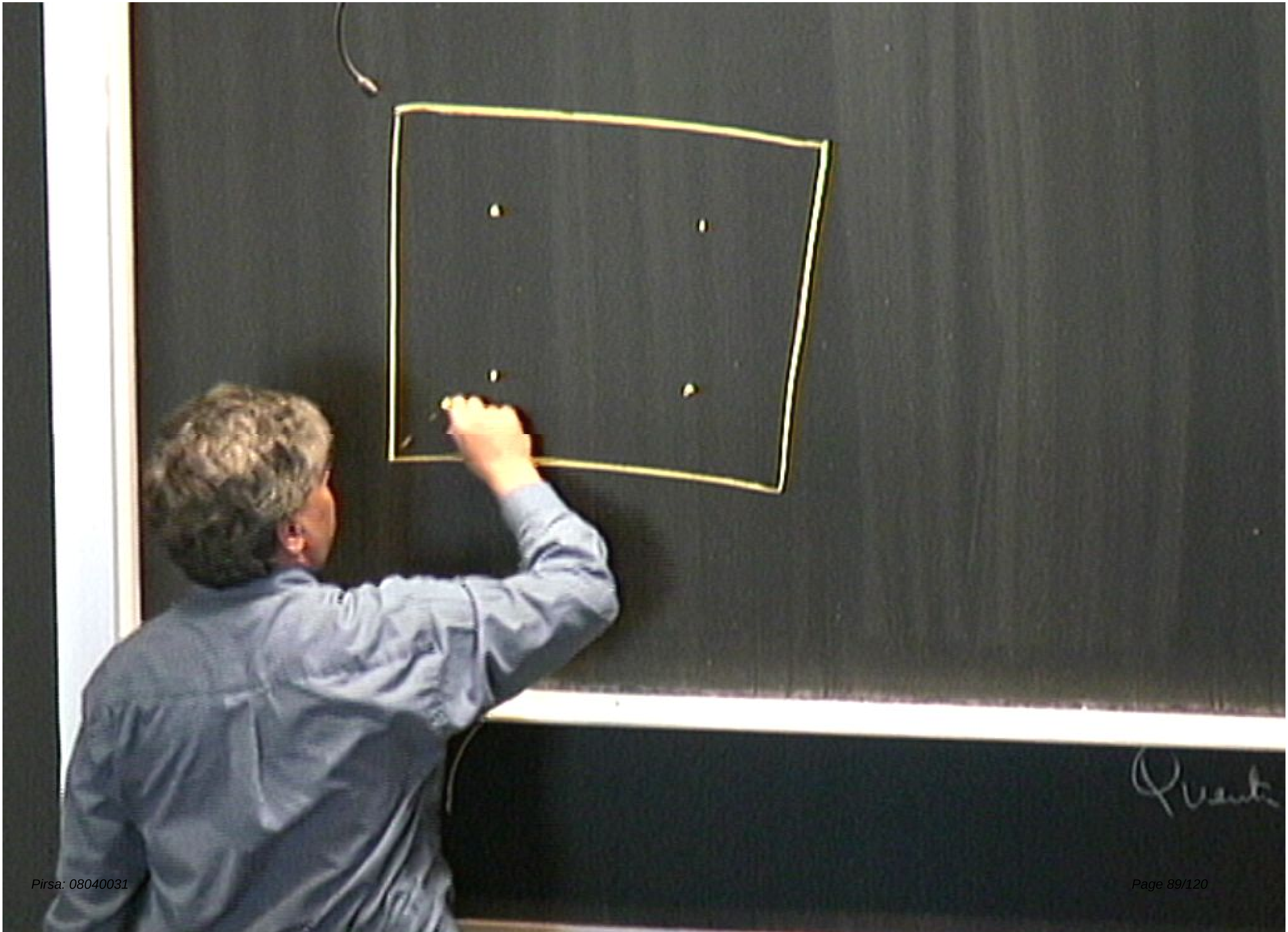
Coupling of quasiparticles to SDW order

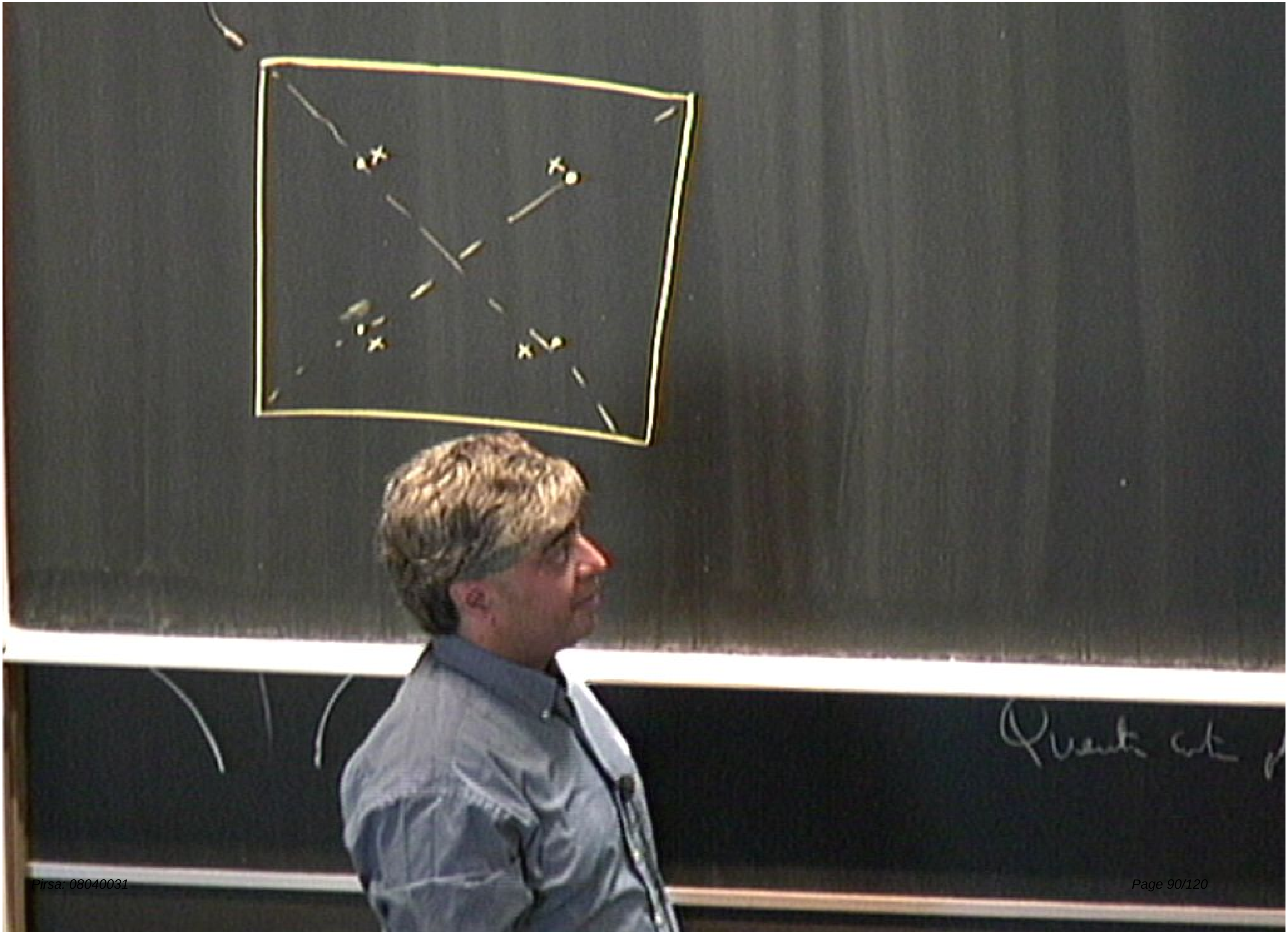
Higher - order couplings allowed by symmetry:

$$\mathcal{L}_2 = \lambda_2 \left(|\Phi_1|^2 - |\Phi_2|^2 \right) \left(\Psi_1^\dagger \tau^x \Psi_1 + \Psi_2^\dagger \tau^x \Psi_2 \right)$$

Nematic coupling

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199





Coupling of quasiparticles to SDW order

Higher - order couplings allowed by symmetry:

$$\mathcal{L}_2 = \lambda_2 \left(|\Phi_1|^2 - |\Phi_2|^2 \right) \left(\Psi_1^\dagger \tau^x \Psi_1 + \Psi_2^\dagger \tau^x \Psi_2 \right)$$

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$$
\begin{aligned}
\mathcal{L}_3 = \epsilon_{ijk} \Big[\\
& \left(\Phi_{1j}^* \Phi_{1k} + \Phi_{2j}^* \Phi_{2k}\right) \left(-\lambda_3 \Psi_2^\dagger \tau^x \sigma^i \Psi_2 + \lambda_3' \Psi_1^\dagger \tau^z \sigma^i \Psi_1 \right) \\
& + \left(\Phi_{1j}^* \Phi_{1k} - \Phi_{2j}^* \Phi_{2k}\right) \left(\lambda_3 \Psi_1^\dagger \tau^x \sigma^i \Psi_1 - \lambda_3' \Psi_2^\dagger \tau^z \sigma^i \Psi_2 \right) \Big].
\end{aligned}
$$

Spiral spin order coupling

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

Coupling of quasiparticles to SDW order

Scaling dimensions of these couplings:

$$
\begin{aligned}
\dim[\lambda_1] &= \frac{1}{\nu} - 2 = -0.9(2) \\
\dim[\lambda_2] &= \frac{(\overline{\gamma} - 1)}{2} = \begin{cases} -0.05(18) & \text{MZM, 6 loops} \\ -0.10(27) & d = 3\ \overline{\text{MS}}, 5 \text{ loops} \end{cases} \\
\dim[\lambda_3, \lambda_3'] &= \begin{cases} -0.84(8) & \text{MZM, 6 loops} \\ -0.76(8) & d = 3\ \overline{\text{MS}}, 5 \text{ loops} \end{cases}
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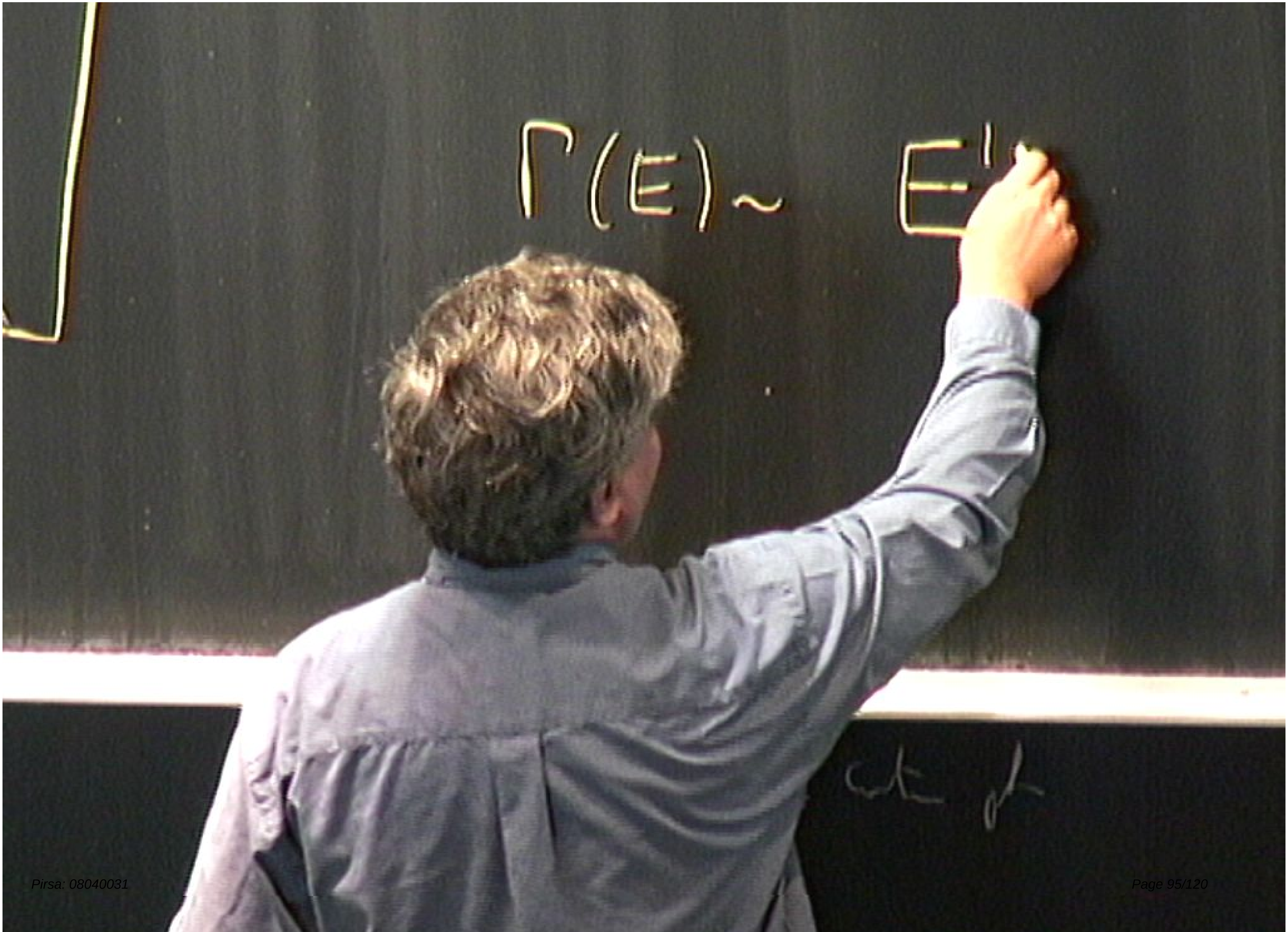
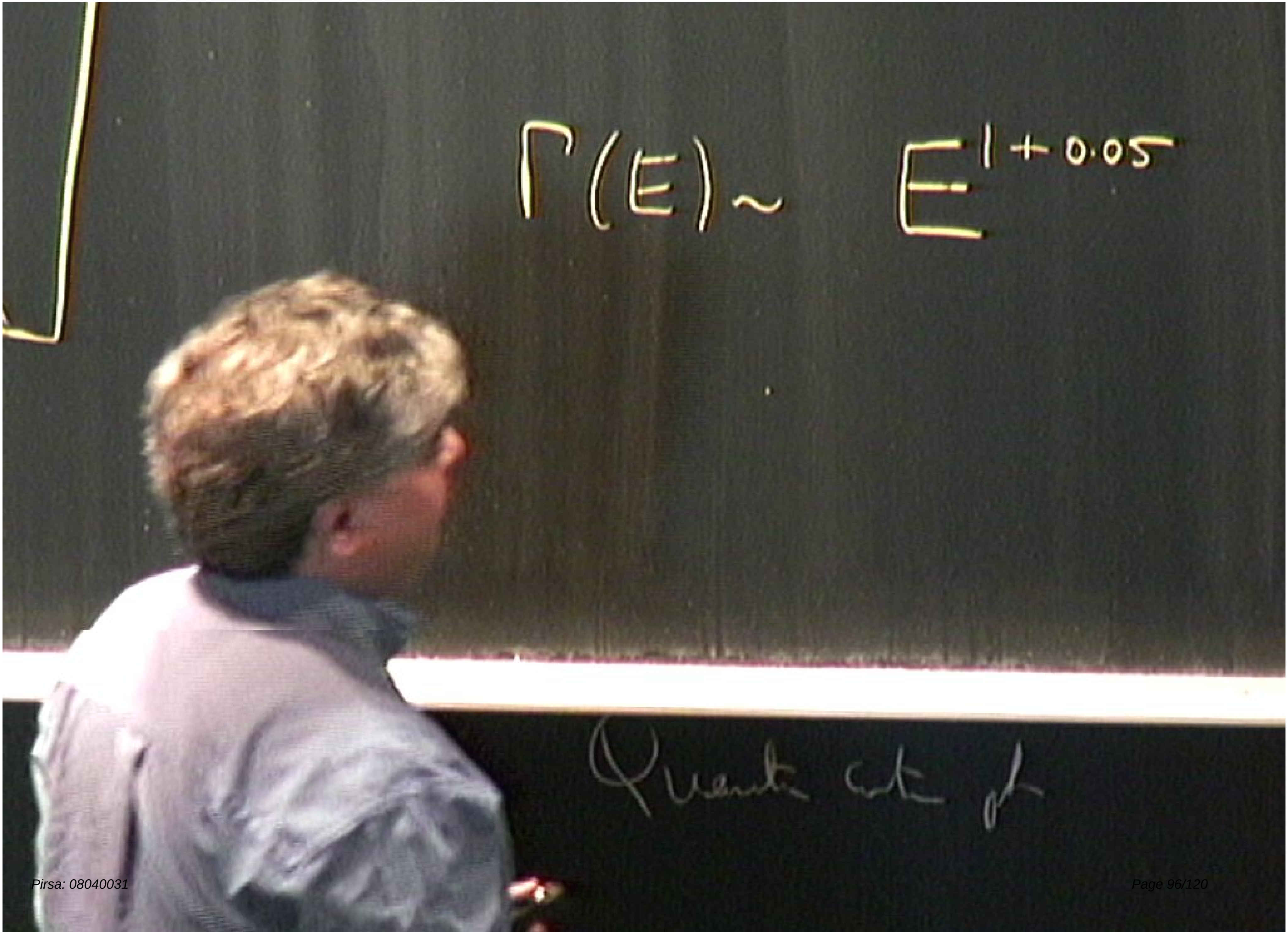
A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199


Γ(E)~ E


$\Gamma(E) \sim E^{1+0.05}$

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Coupling of quasiparticles to SDW order

Coupling of nematic order is nearly marginal:

Quantum-critical features appear in fermion spectrum via coupling to nematic fluctuations of spin density wave order.

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

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Outline

1. Theory of SC to SC+SDW transition
   Emergent O(4) symmetry

2. Nodal quasiparticles at the O(4) critical point
   Unique selection of quasiparticle coupling to (composite) nematic order

3. Theory of the onset of nematic order in a d-wave superconductor
   Infinite anisotropy fixed point

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   Infinite anisotropy fixed point

Ignore SDW orders $\Phi_{1,2i}$ and focus directly on nematic order $\phi = \sum_i \left(|\Phi_{1i}|^2 - |\Phi_{2i}|^2\right)$

$$S_\phi^0 = \int d^2x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla \phi)^2 + \frac{r}{2}\phi^2 + \frac{u_0}{24}\phi^4\right]$$

Ising theory for nematic ordering

$$\begin{aligned} S_\Psi &= \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_{1a}^\dagger \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_{1a} \\ &+ \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_{2a}^\dagger \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x\right) \Psi_{2a}. \end{aligned}$$

Free nodal quasiparticles

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

Ignore SDW orders $\Phi_{1,2i}$ and focus directly on nematic order $\phi = \sum_i \left(|\Phi_{1i}|^2 - |\Phi_{2i}|^2\right)$

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda_0 \phi \left(\Psi_{1a}^\dagger \tau^x \Psi_{1a} + \Psi_{2a}^\dagger \tau^x \Psi_{2a}\right)\right],$$

Yukawa coupling is now permitted and is strongly relevant

RG analysis close to 3 dimensions yields runaway flow to strong coupling

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

Expansion in number of fermion spin components N_f

The nematic action is dominated by the non-analytic fermion loop contribution

$$S_\phi = \frac{N_f}{2} \int \frac{d\omega}{2\pi} \int \frac{d^2k}{4\pi^2} |\phi(k,\omega)|^2 G_\phi^{-1}(k,\omega) K^{-1}(k^2/\Lambda^2)$$

where G_ϕ is given by a fermion loop integral, K is a smooth ultraviolet cutoff function, which suppresses fluctuations above the momentum scale Λ.

$$G_\phi^{-1}(k,\omega) = r + \frac{\lambda_0^2}{8 v_F v_\Delta} \left[\frac{\omega^2 + v_F^2 k_x^2}{\sqrt{\omega^2 + v_F^2 k_x^2 + v_\Delta^2 k_y^2}} + (x \leftrightarrow y) \right]$$

Similar results apply to terms to all orders in ϕ

Renormalization group analysis

The $1/N_f$ expansion has only one coupling constant at criticality: v_Δ/v_F.

The RG has the structure:

$$\begin{aligned} \text{dynamic critical exponent}: \ z &= 1 + \frac{1}{N_f} F_1(v_\Delta/v_F) \\ \text{fermion anomalous dimension}: \ \eta_f &= \frac{1}{N_f} F_2(v_\Delta/v_F) \\ \text{RG flow equation}: \ \frac{d(v_\Delta/v_F)}{d\ell} &= \frac{1}{N_f} F_3(v_\Delta/v_F) \end{aligned}$$

where we have computed the functions $F_{1,2,3}(v_\Delta/v_F)$.

Renormalization group analysis

The RG flow is to $v_\Delta/v_F \to 0$ with

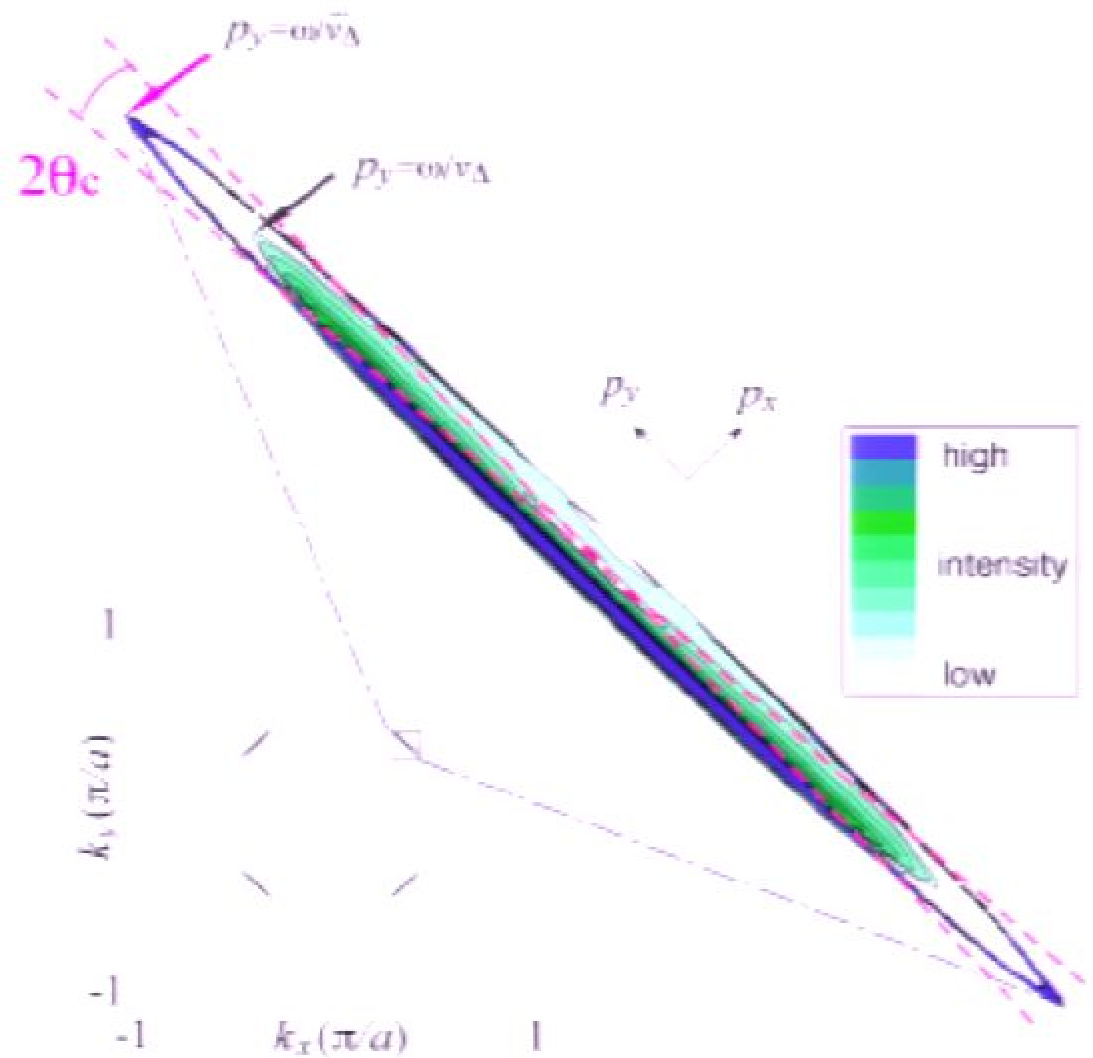
$$\frac{d(v_\Delta/v_F)}{d\ell} = -\frac{4}{\pi^2 N_f}(v_\Delta/v_F)^2 \ln\left(\frac{0.46987}{(v_\Delta/v_F)}\right)$$

This implies that at the critical point, as $T \to 0$ we have the asymptotic result

$$\frac{v_\Delta}{v_F} = \frac{\pi^2 N_f}{4}\frac{1}{\ln\left(\frac{\Lambda}{T}\right)\ln\left[\frac{0.1904}{N_f}\ln\left(\frac{\Lambda}{T}\right)\right]},$$

and a more precise result is obtained by numerically integrating the RG equation.

Fermion spectral functions



(a) $A(\vec{p}, -9meV)$

Eun-Ah Kim, Michael J. Lawler, Paul Oreto, Subir Sachdev, Eduardo Fradkin, Steven A. Kivelson, arXiv:0705.4099

Conclusions

v_Δ / v_F

v_Δ / v_F

v_Δ / v_F

Conclusions

1. Theories for damping of nodal quasiparticles in LSCO

2. SDW t

functions v_Δ/v_F

v_Δ/v_F

v_Δ/v_F

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v_Δ/v_F

v_Δ/v_F

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$$v_\Delta / v_F$$

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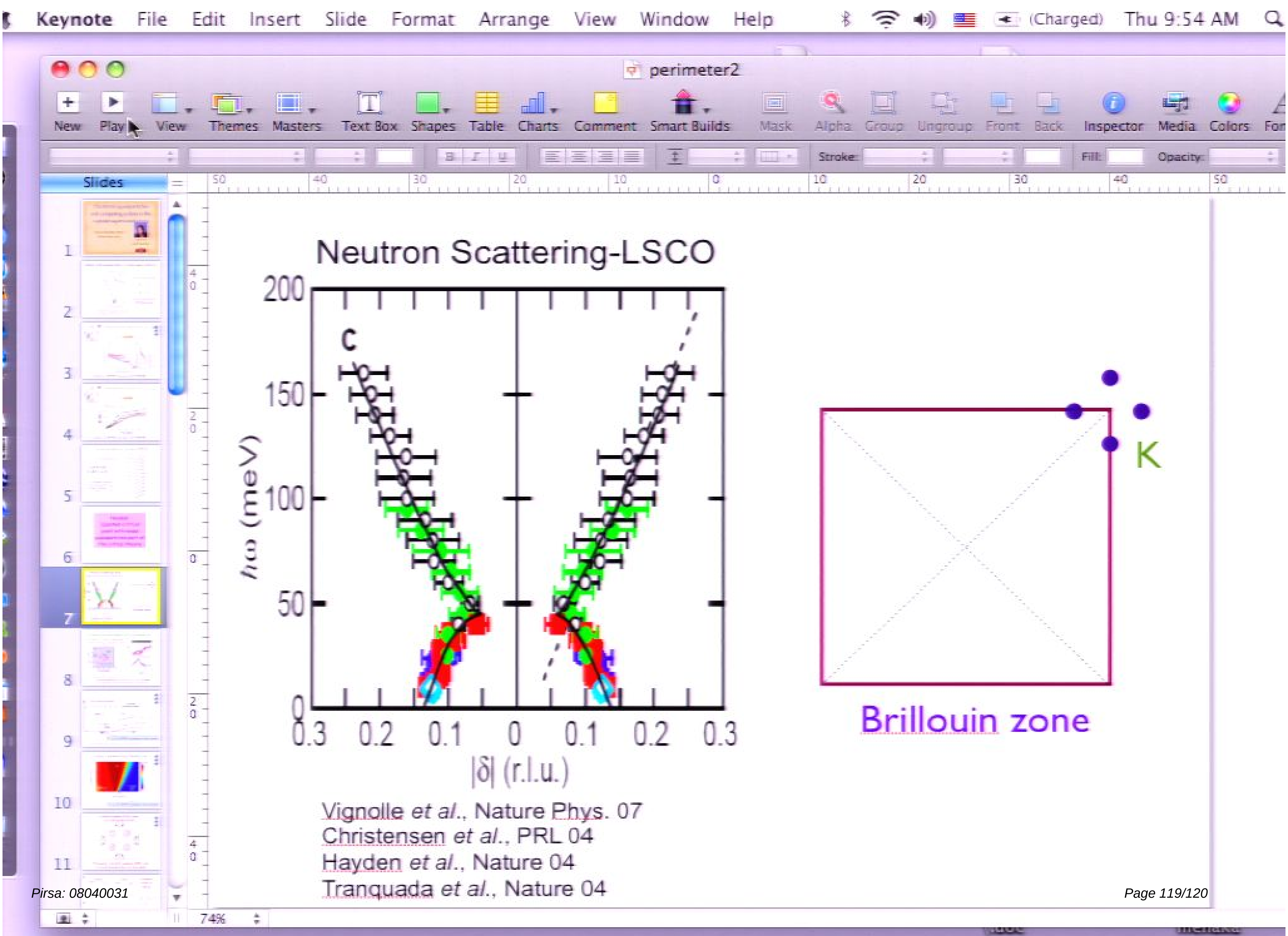
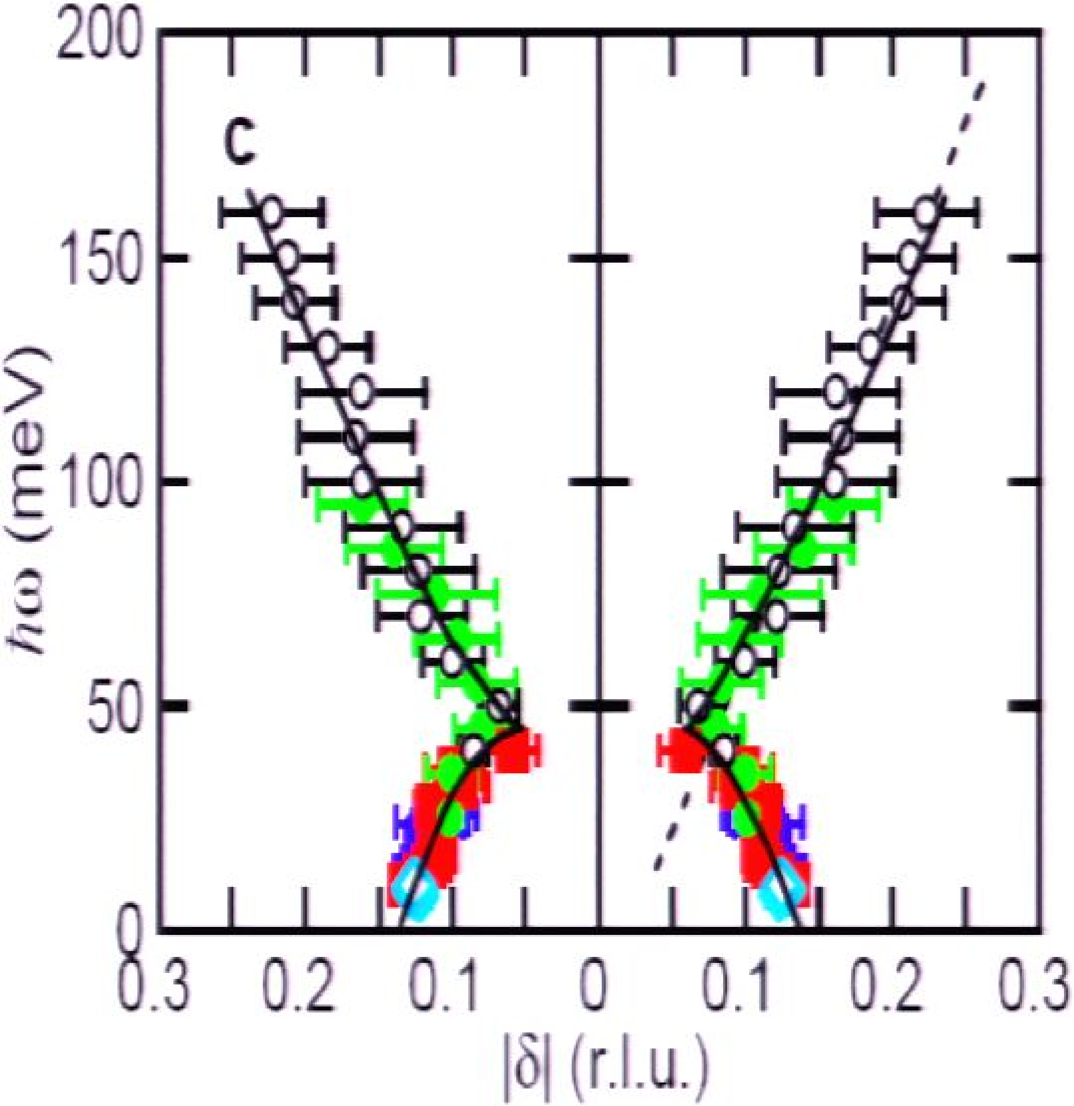

Neutron Scattering-LSCO
c
200
150
100
50
0
ħω (meV)
0.3 0.2 0.1 0 0.1 0.2 0.3
|δ| (r.l.u.)
Vignolle et al., Nature Phys. 07
Christensen et al., PRL 04
Hayden et al., Nature 04
Tranquada et al., Nature 04
K
Brillouin zone

Neutron Scattering-LSCO



Vignolle *et al.*, Nature Phys. 07
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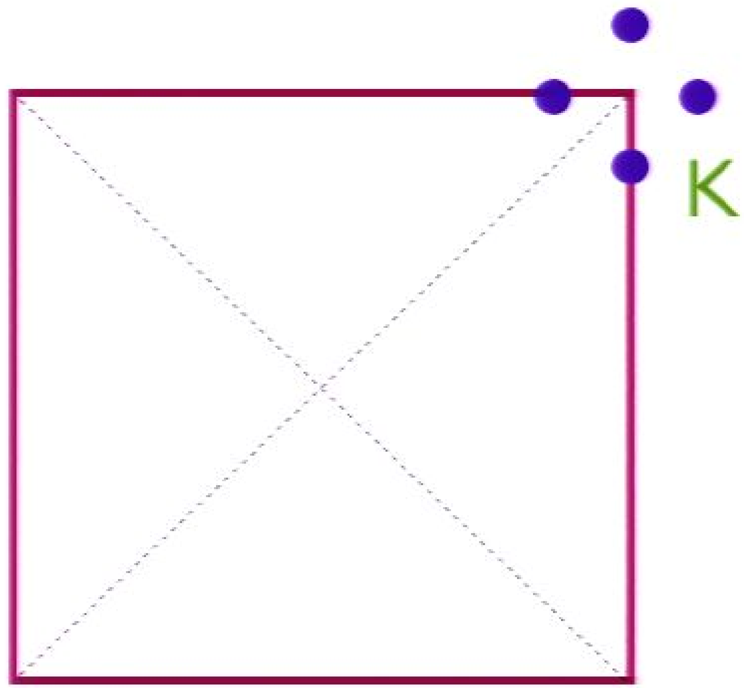

Brillouin zone