Title: Popescu-Rohrlich boxes in quantum measure theory

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Abstract: Quantum measure theory describes quantum theory as a generalization of a classical stochastic process, which may be fruitful for quantum gravity. I will describe the approach, and show that, in the context of an EPRB setup with two distant experimenters, two alternative experiments, and two outcomes per experiment, any set of no signaling probabilities can be realized, albeit by violating a `strong positivity\' condition.

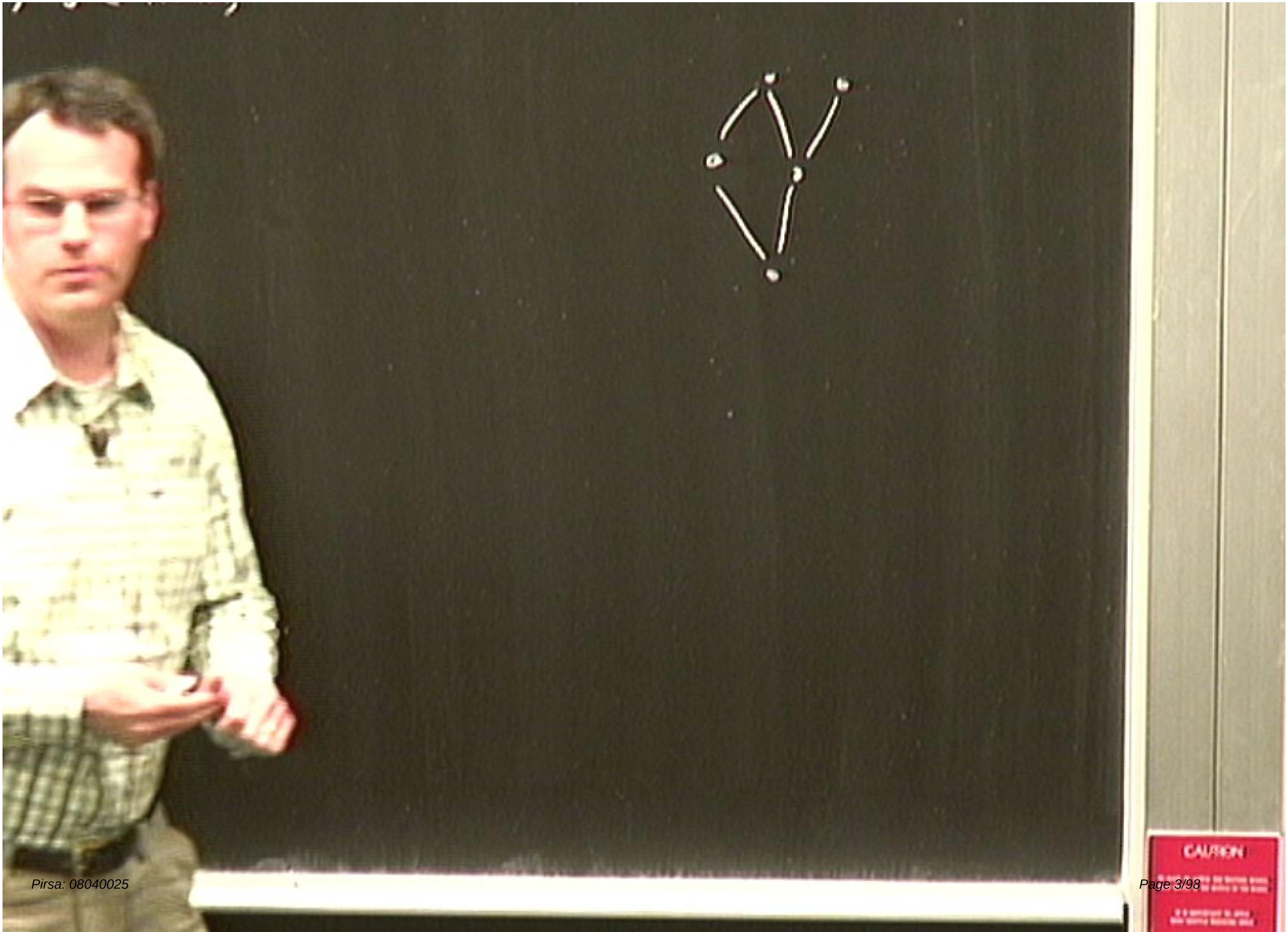
Popescu
Rohrlich

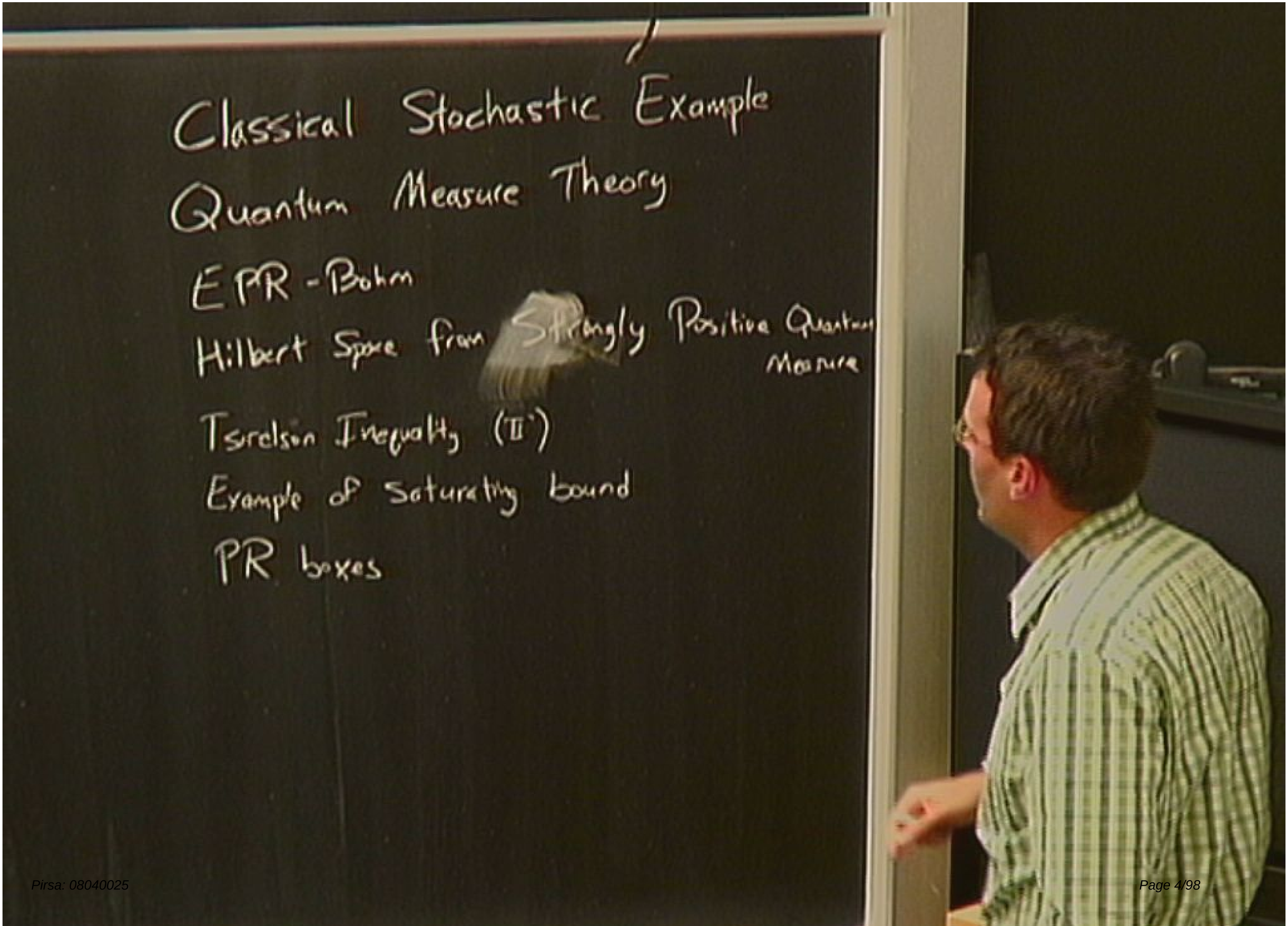
Boxes in Quantum Measure Theory

Matthew Barnett, David Craig, Fay Dowker, Joe Henson, Seth Major, DR, Rafael Sorkin

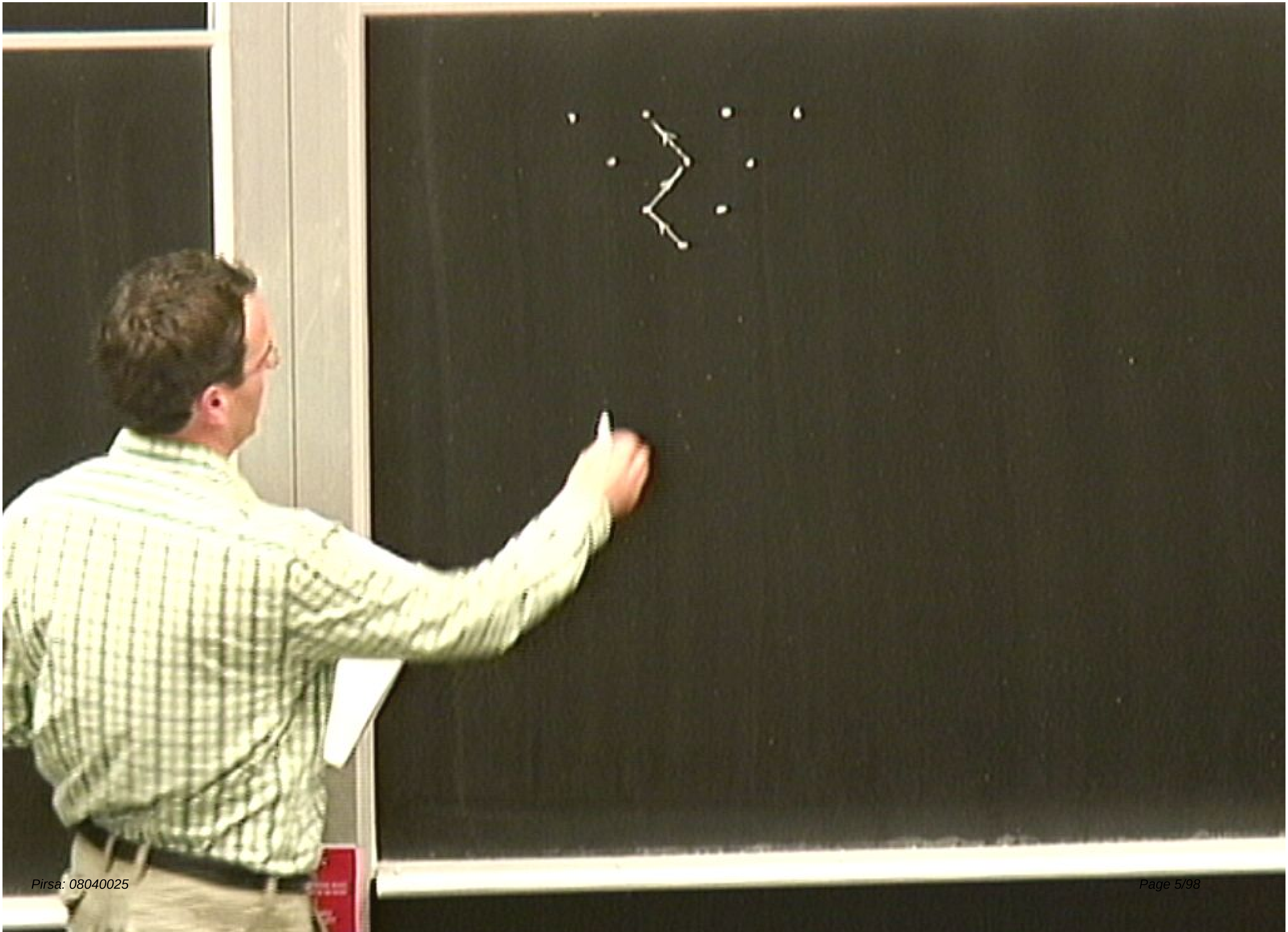
JPA <u>40</u> 501-523 (2007)

JPA <u>40</u> 7255-7264 (2007)

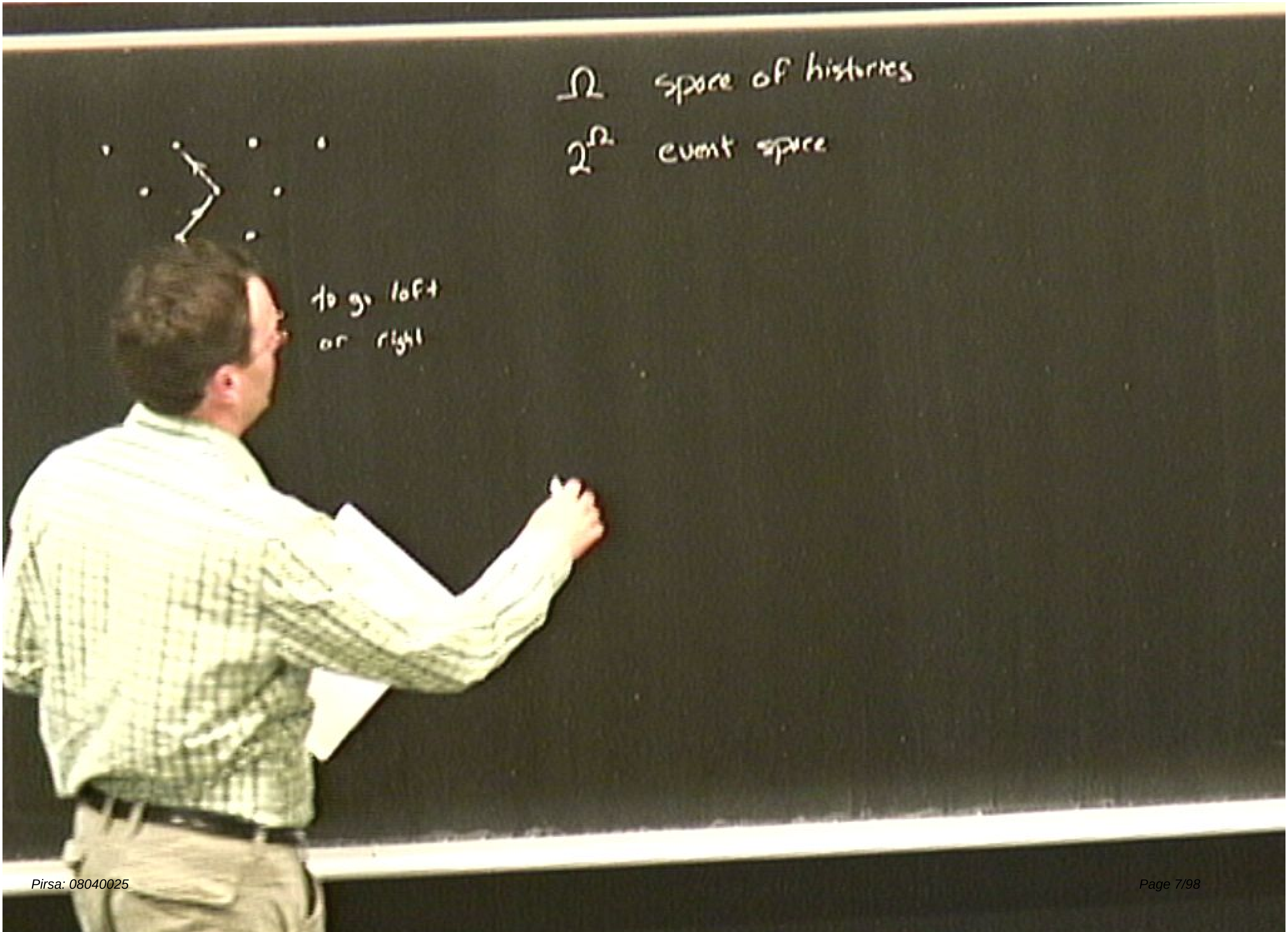




Classical Stochastic Example
Quantum Measure Theory
EPR - Bohm
Hilbert Space from Strongly Positive Quantum Measure
Tsirelson Inequality (II')
Example of saturating bound
PR boxes



Prob $\frac{1}{2}$ to go left
or right


Ω space of histories
2^Ω event space
to go left
or right

Prob $\frac{1}{2}$ to go left
or right

Ω space of histories

2^{Ω} event space

$A \subset \Omega$ is event

$\mathcal{R}$ sigma algebra

Prob $\frac{1}{2}$ to go left or right

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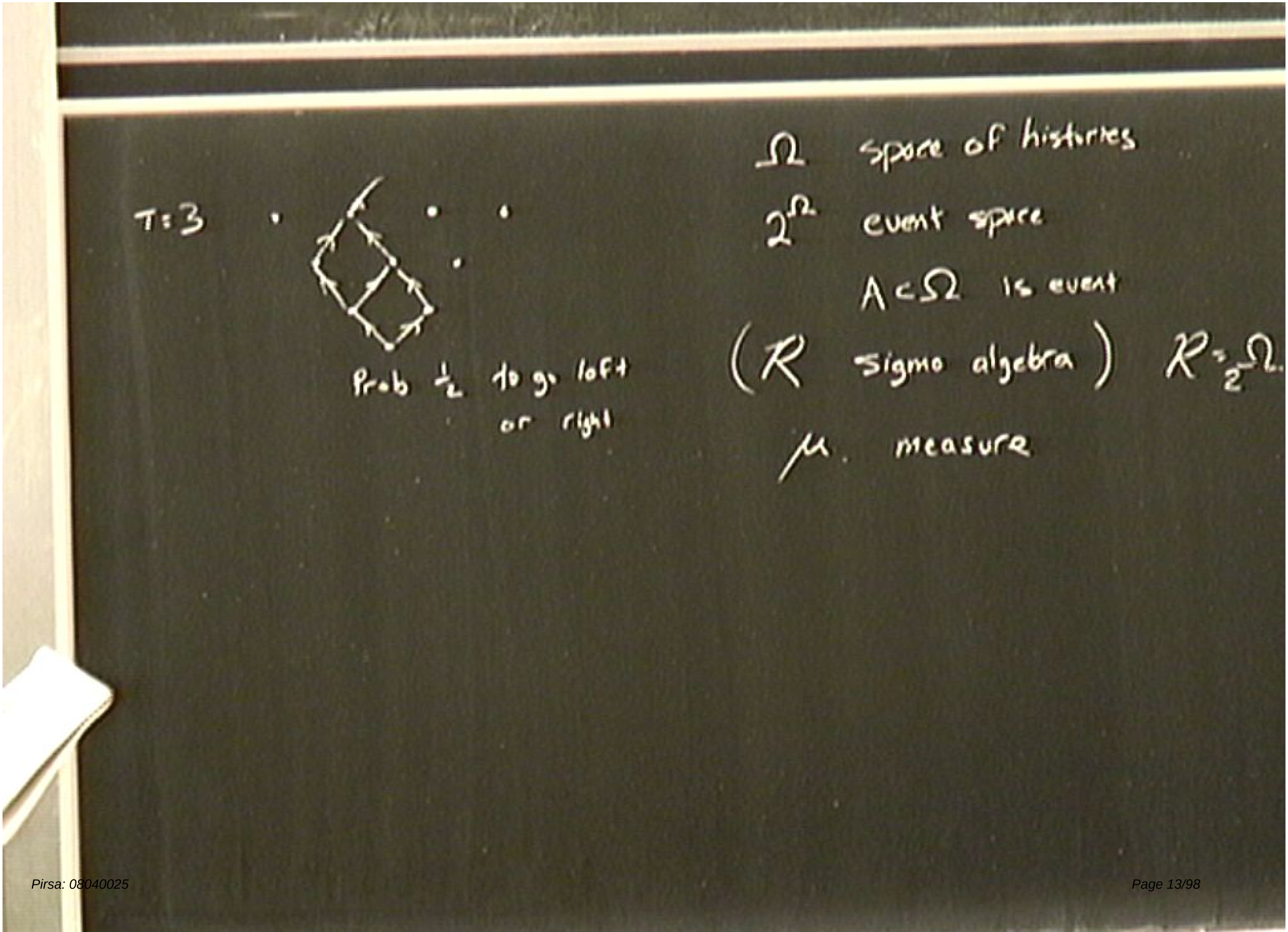
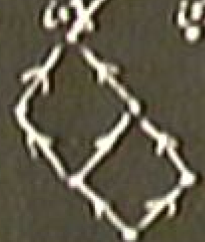

T=3
Prob 1/2 to go left
or right
Ω space of histories
2^Ω event space
A⊂Ω is event
(R sigma algebra) R = 2^Ω
μ. measure

$T=3$ $\quad j=1$ $\quad \cancel{j=2}$ $\quad j=3$ $\quad j=4$



Prob $\frac{1}{2}$ to go left
or right

$\mu(A_i)$

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2^{Ω} event space

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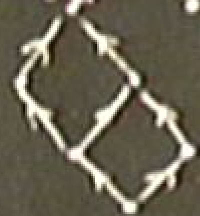

Prob $\frac{1}{2}$ to go left
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$\mu(A_i)$ is binomial distribution

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μ measure

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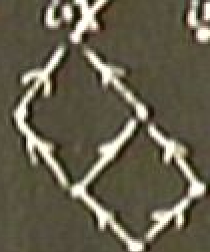

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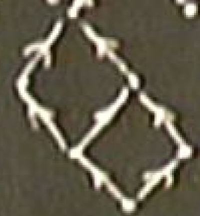
$\left(\mathcal{R} \text{ sigma algebra} \right)$ $\mathcal{R} = 2^{\Omega}$.

μ measure

$\mu : 2^{\Omega} \to [0,1]$

"One true history" of Ω which is realized. $\gamma \in \Omega$

$T = 3$



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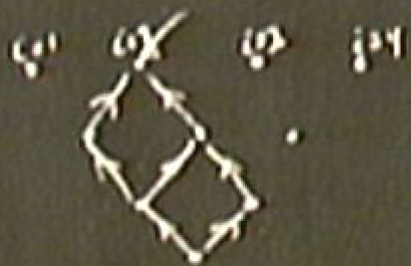

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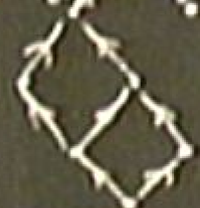

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$$\phi_{\gamma}(A) = \begin{cases} 1 & \gamma \in A \\ 0 & \text{otherwise} \end{cases}$$

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$(\mathcal{R}$ sigma algebra$)$ $\mathcal{R} = 2^{\Omega}$

μ measure $A \subseteq \Omega$ $B \subseteq \Omega$

$A \cap B$ $\times$

$\mu : 2^{\Omega} \to [0,1]$

$A \triangle B$ $+$

is realized. $\gamma \in \Omega$

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3   ξ^1 ξ^2 ξ^3 ξ^4

Prob $\frac{1}{2}$ to go left or right

(A_i) is binomial distribution

true history of Ω which is realized.

$\phi : 2^\Omega \to \mathbb{Z}_2$

$$\phi_\gamma(A) = \begin{cases} 1 & \gamma \in A \\ 0 & \text{otherwise} \end{cases}$$

$A \subseteq \Omega$ $B \subseteq \Omega$

$A \cap B$ $\cdot$

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Claim $\phi_\gamma(A)$ is a homomorphism

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2^Ω event space boolean event algebra

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$A \cap B$ $\cdot$

$A \Delta B$ $+$ 

Claim: $\phi_\gamma(A)$ is a homomorphism

$$\phi(A \Delta B) = \phi(A) + \phi(B)$$

$$\phi(A \cap B) = \phi(A)\, \phi(B)$$

$\subset \Omega$ is event

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$A \cap B$ $\times$

$: [0,1]$ $A \triangle B$ $+$

$\gamma \in \Omega$

Claim: $\phi_\gamma(A)$ is a homomorphism

$\gamma \in A$

otherwise

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$$\phi(A \cap B) = \phi(A)\,\phi(B)$$

T=3 {1} {2} {3} {4}

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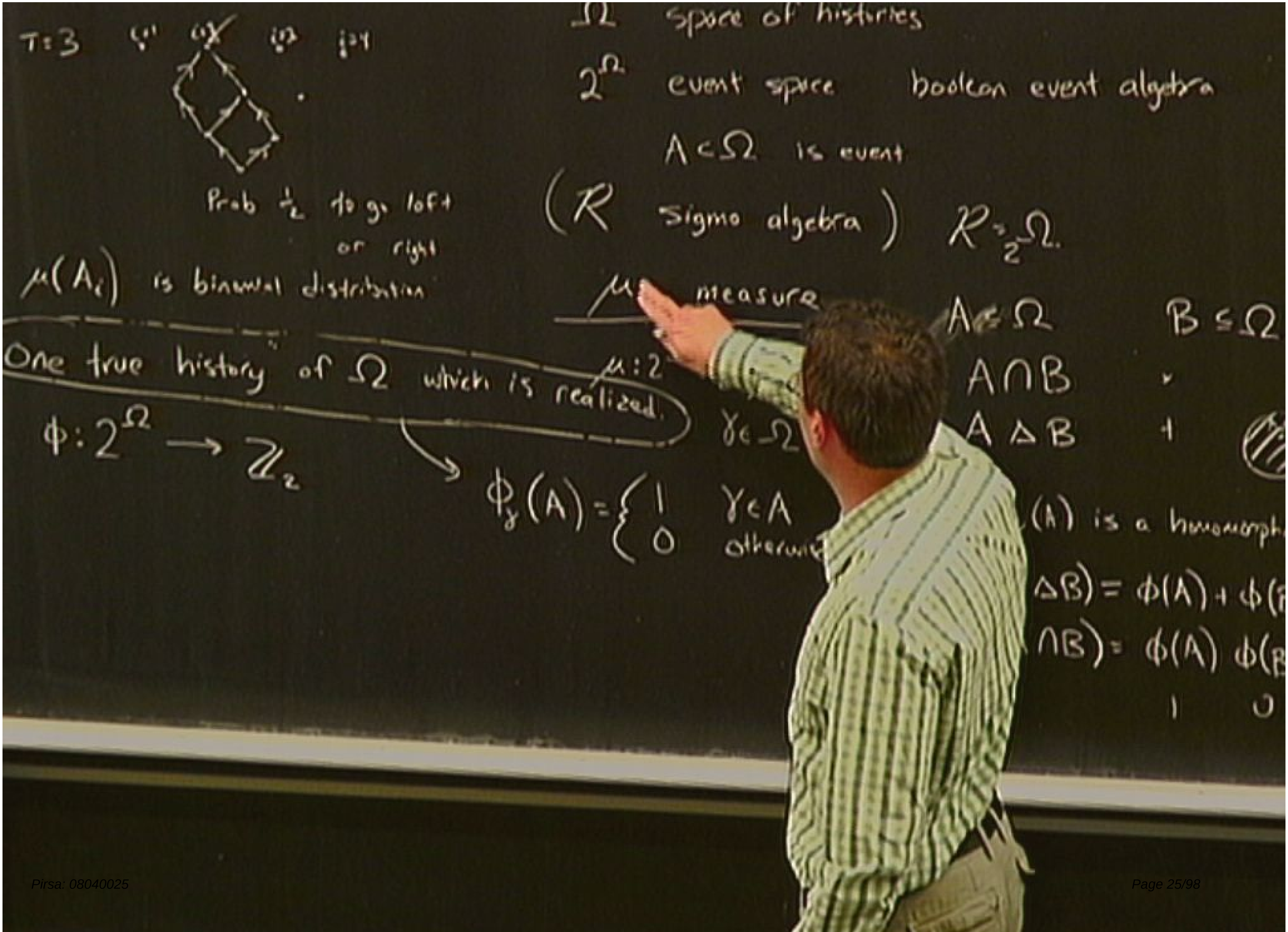
1   0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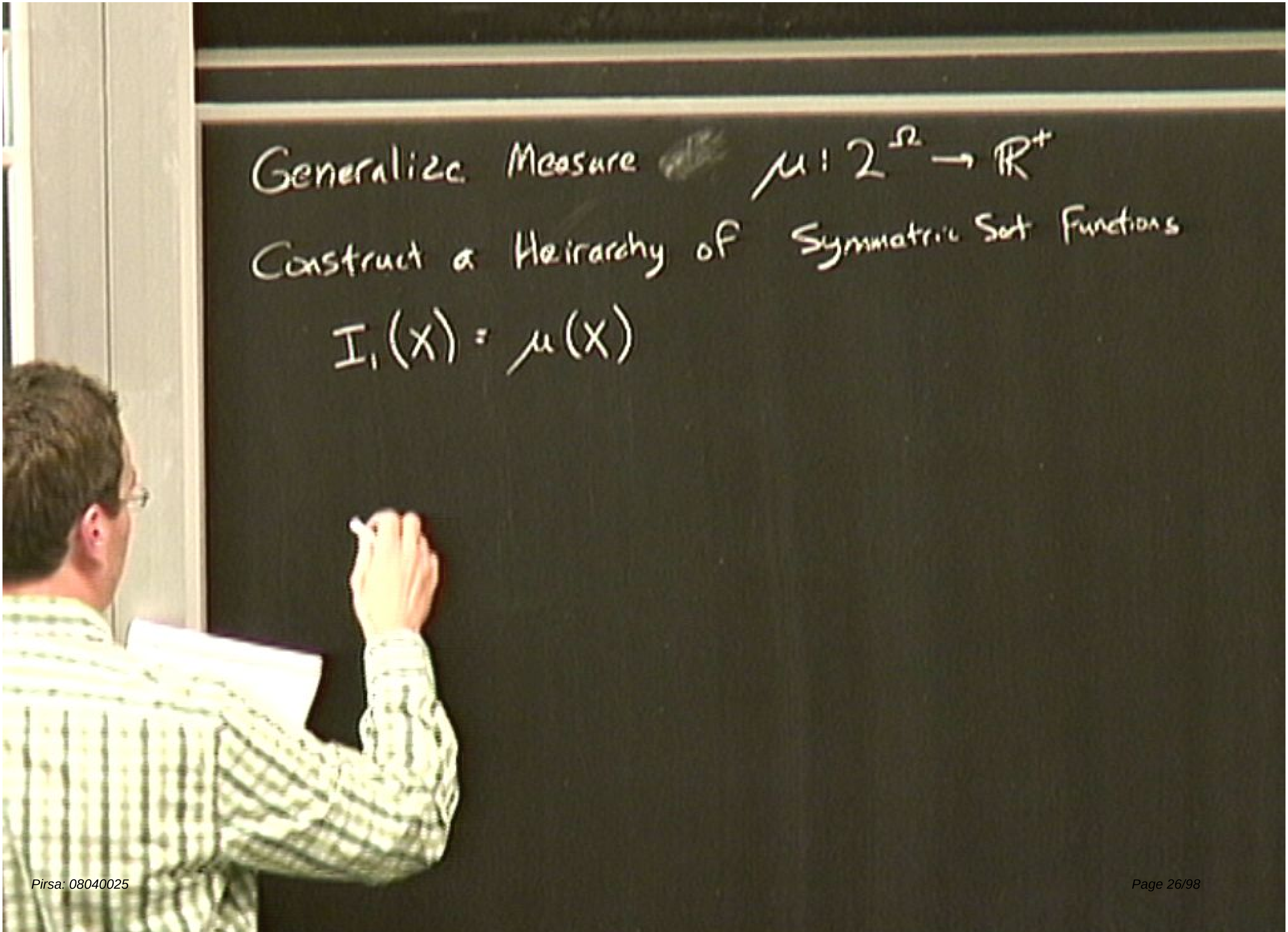
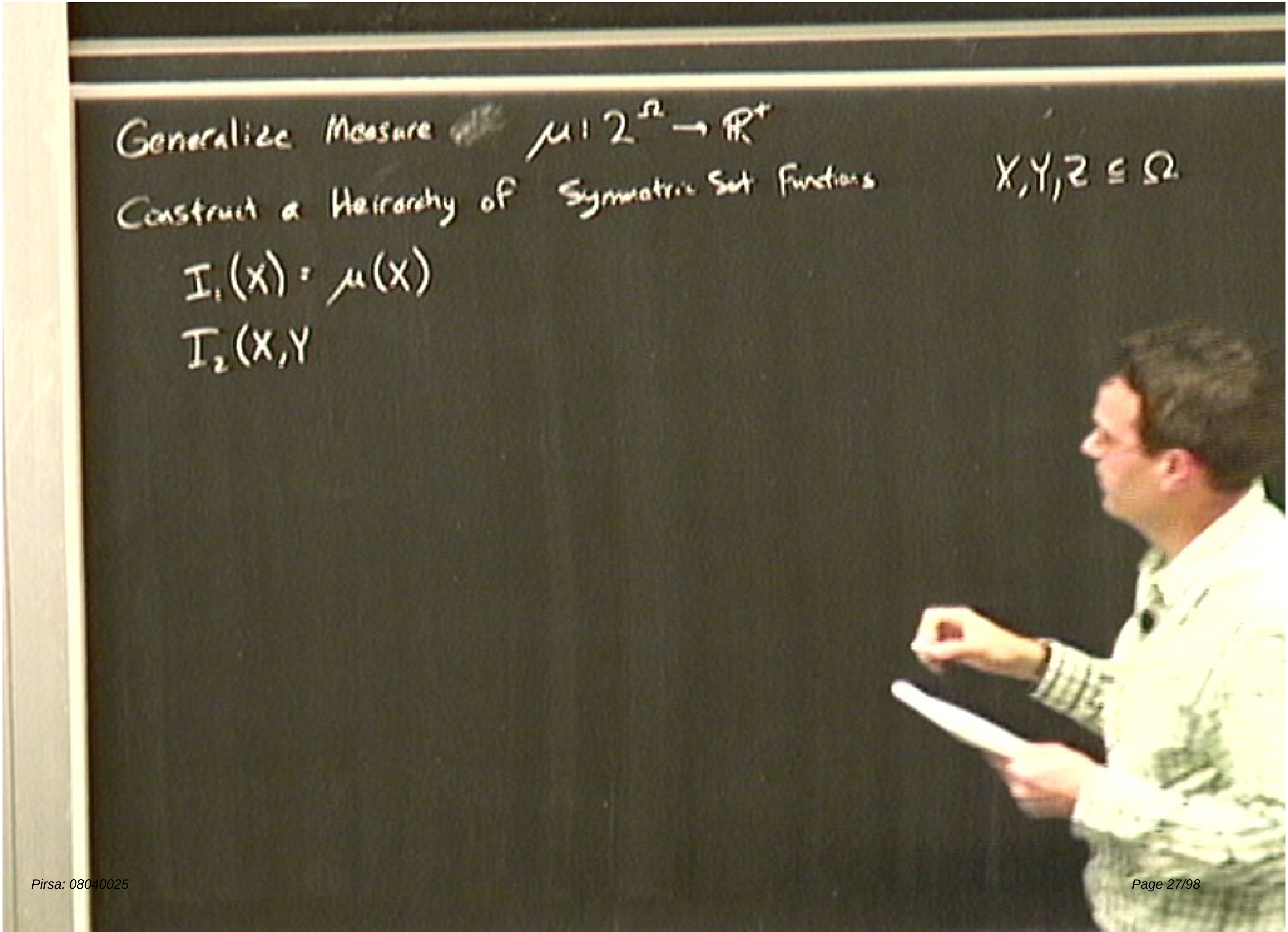
One true history of Ω which is realized.
A∩B
A△B
ϕ: 2^Ω → Z_2
ϕ_γ(A) = { 1 γ∈A ; 0 otherwise


Generalize Measure $\mu : 2^{\Omega} \rightarrow \mathbb{R}^{+}$
Construct a Heirarchy of Symmetric Set Functions
$I_1(X) = \mu(X)$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$I_1(X) = \mu(X)$
$I_2(X, Y$

Generalize Measure $\mu : 2^{\Omega} \to \mathbb{R}^{+}$

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$$I_1(X) = \mu(X)$$

$$I_2(X, Y) = \mu(X \sqcup Y) - \mu(X) - \mu(Y)$$

disjoint union

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disjoint union $X \cap Y = \emptyset$

$$I_3(X,Y,Z) = \mu(X \sqcup Y \sqcup Z) - \mu(X \sqcup Y) - \mu(Y \sqcup Z) - \mu(X \sqcup Z) + \mu(X) + \mu(Y) + \mu(Z)$$

$$I_4(X,Y,Z,W) = \cdots$$

$$\vdots$$

Generalize measure $\mu : \Sigma \to \mathbb{R}$

Construct a Heirarchy of Symmetric Set Functions $X, Y, Z \subseteq \Omega$

$$I_1(X) = \mu(X)$$

$$I_2(X,Y) = \mu(X \sqcup Y) - \mu(X) - \mu(Y)$$

$\sqcup$: disjoint union $X \cap Y = \emptyset$

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Generalized Measure at level k satisfies the condition $I_{k+1} = 0$

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$$I_4(X,Y,Z,W) = \cdots$$

$\vdots$

Generalized Measure at level k satisfies the condition $I_{k+1} = 0$

$\Rightarrow I_\ell = 0 \quad \ell \geq k+1$

$X, Y, Z \subseteq \Omega$

$I_1 = 0$ trivial measure

$I_2 = 0$ probability theory Kolmogorov Sum Rule

$I_3 = 0$ "quantum measure theory"

$$(Y \sqcup Z) - \mu(X \sqcup Z) + \mu(X) + \mu(Y) + \mu(Z)$$

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$\vdots$

Generalized Measure at level k satisfies the condition $I_{k+1} = 0$

$\Rightarrow I_\ell = 0$ for $\ell \geq k+1$

$\mu(Y)$

$I_3 = 0$ "quantum measure theory"

$X \cap Y = \emptyset$

$X \sqcup Y) - \mu(Y \sqcup Z) - \mu(X \sqcup Z) + \mu(X) + \mu(Y) + \mu(Z)$

level k measure allows interference among k-tuples of histories

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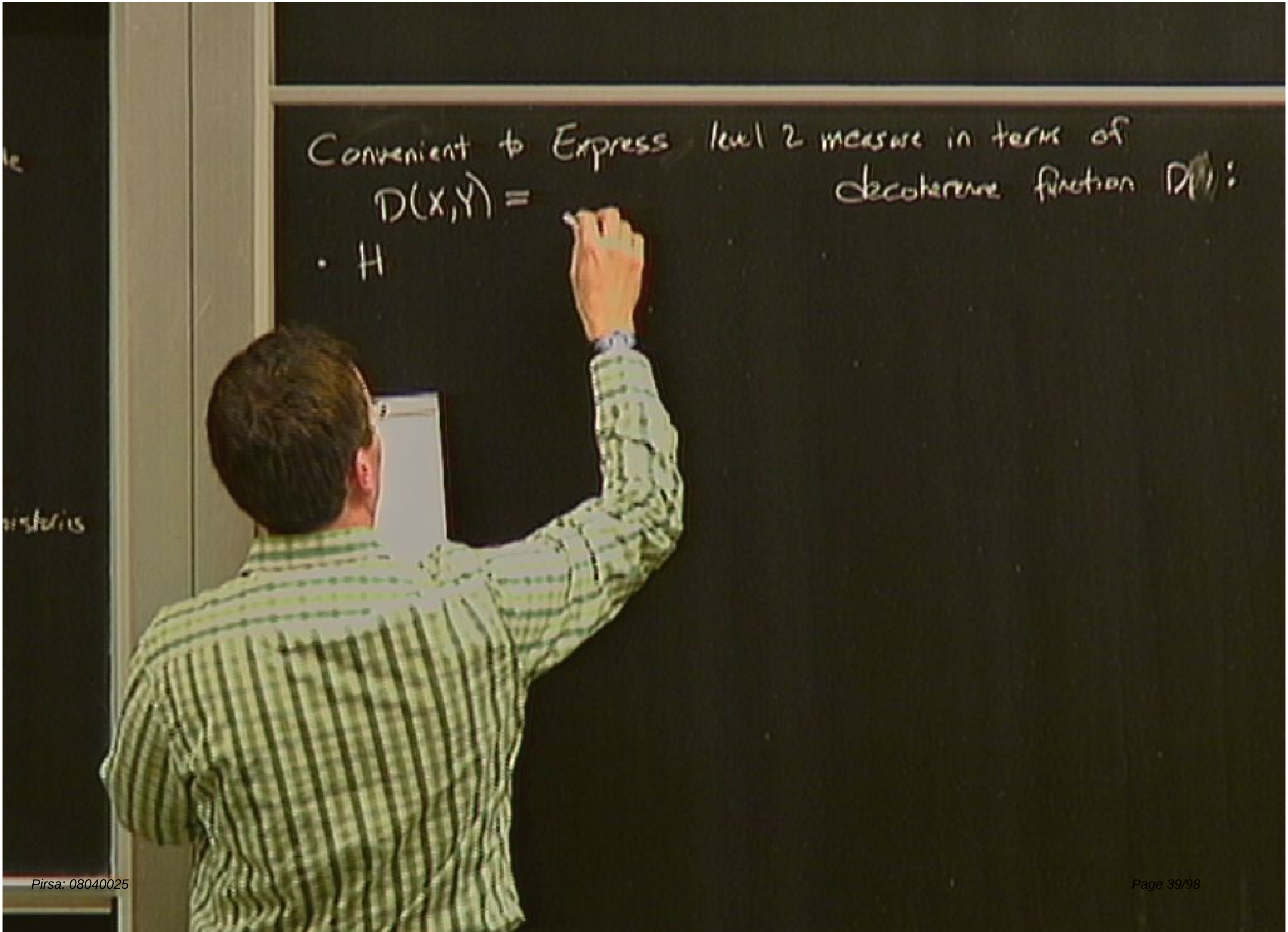
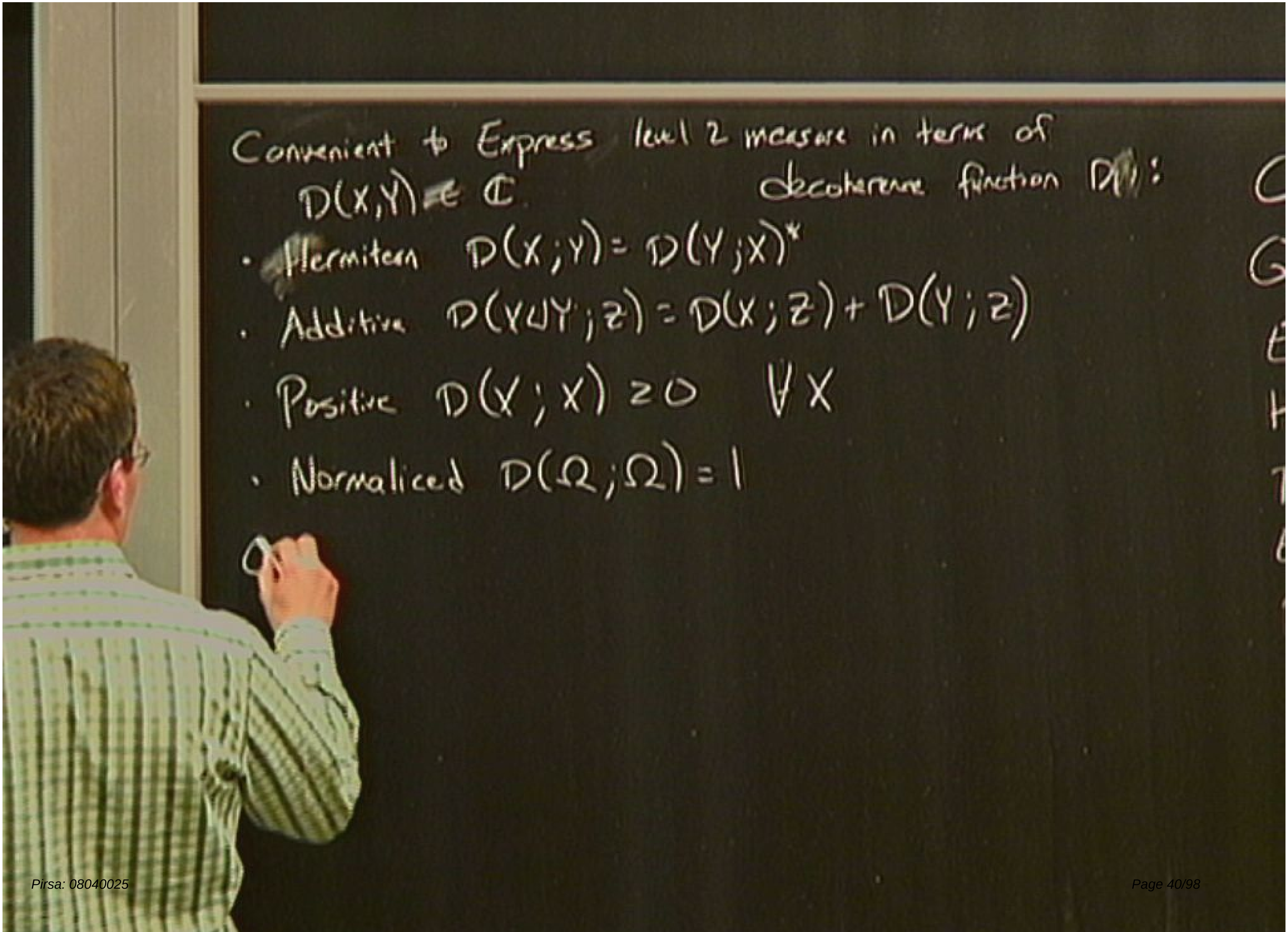

Convenient to Express level 2 measure in terms of
decoherence function D
D(X,Y) =
• H


Convenient to Express level 2 measure in terms of
D(X,Y) ∈ ℂ
decoherence function D(,):
· Hermitean D(X;Y) = D(Y;X)*
· Additive D(X⊔Y;Z) = D(X;Z) + D(Y;Z)
· Positive D(X;X) ≥ 0 ∀X
· Normalized D(Ω;Ω) = 1

Convenient to Express level 2 measure in terms of decoherence function $D(\,;\,)$:

$D(X,Y) \in \mathbb{C}$

- Hermitean $D(X;Y) = D(Y;X)^*$
- Additive $D(X \sqcup Y;Z) = D(X;Z) + D(Y;Z)$
- Positive $D(X;X) \geq 0 \quad \forall X$
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Quantum measure $\mu(X) = D(X;X)$

Convenient to Express level 2 measure in terms of decoherence function $D(\,)$:

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Quantum measure $\mu(X) = D(X;X)$

- Strong Positivity $\forall$ finite collection of (possibly non-disjoint) subsets $X_1, X_2, \dots, X_n$ of Ω

$$M_{ij} = D(X_i, X_j) \text{ is positive semidefinite}$$

Classic

Quant

EPR -

Hilbert

Tsirelson

Example

PR b

Convenient to Express level 2 measure in terms of decoherence function $D(\,;\,)$:

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- Additive $D(X \sqcup Y; Z) = D(X;Z) + D(Y;Z)$
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- Strong Positivity $\forall$ finite collection of (possibly non-disjoint) subsets $X_1, X_2, \ldots, X_n$ of Ω

$M_{ij} = D(X_i;X_j)$ is positive semidefinite

$D(\,;\,)$ from ordinary unitary QM satisfies strong positivity

Classical Stoc

Quantum Measu

EPR - Bohm

Hilbert Space from

Tsirelson Inequality

Example of saturati

PR boxes

Convenient to Express level 2 measure in terms of decoherence function $D(\,)$:

$D(X,Y) \in \mathbb{C}$

- Hermitean $D(X;Y) = D(Y;X)^*$
- Additive ("Weak") $D(X \sqcup Y; Z) = D(X;Z) + D(Y;Z)$
- Positive $D(X;X) \geq 0 \quad \forall X$
- Normalized $D(\Omega;\Omega) = 1$

Quantum measure $\mu(X) = D(X;X)$

- Strong Positivity $\forall$ finite collection of (possibly non-disjoint) subsets $X_1, X_2, \ldots, X_n$ of Ω

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Example of saturati

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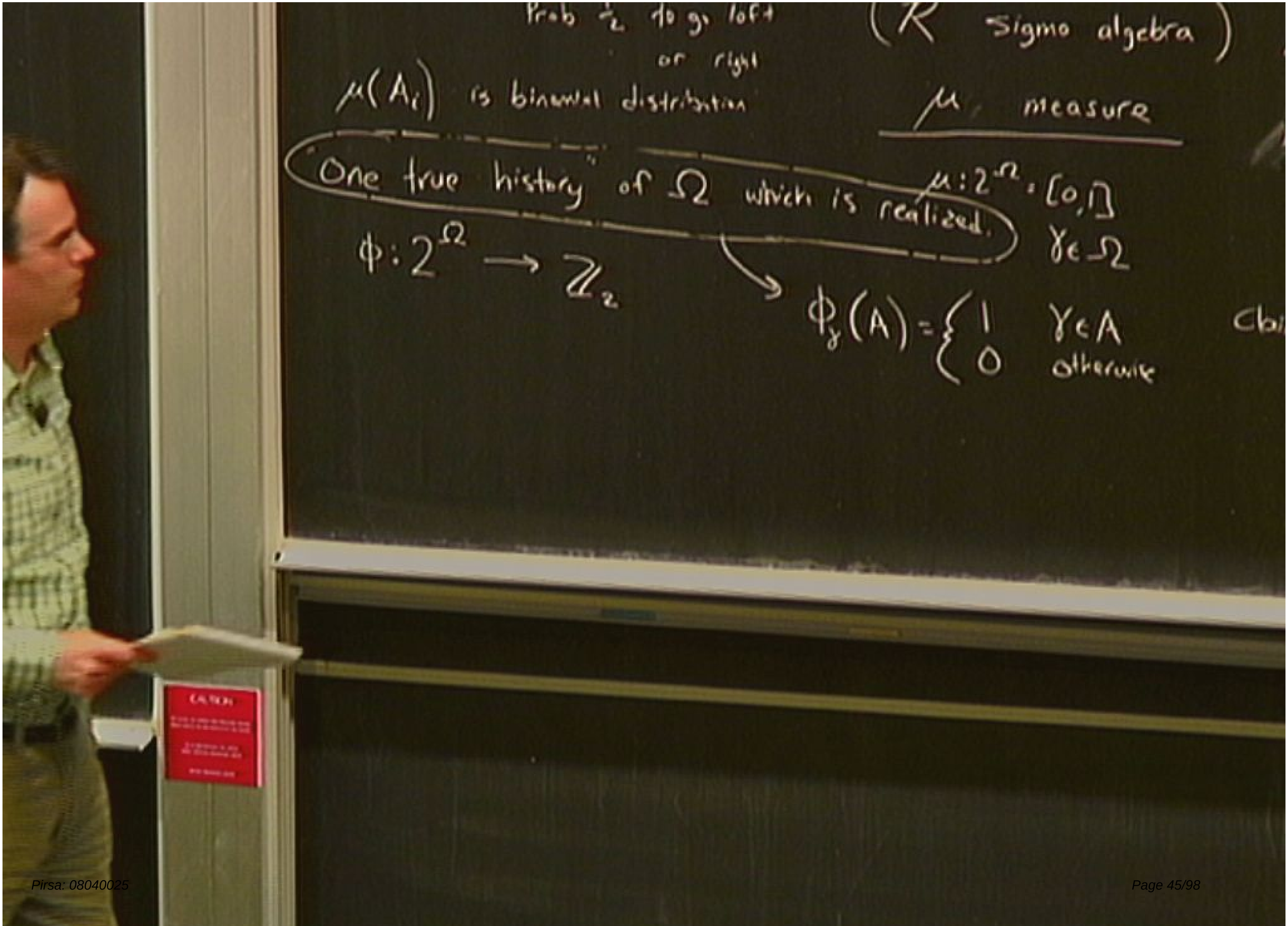
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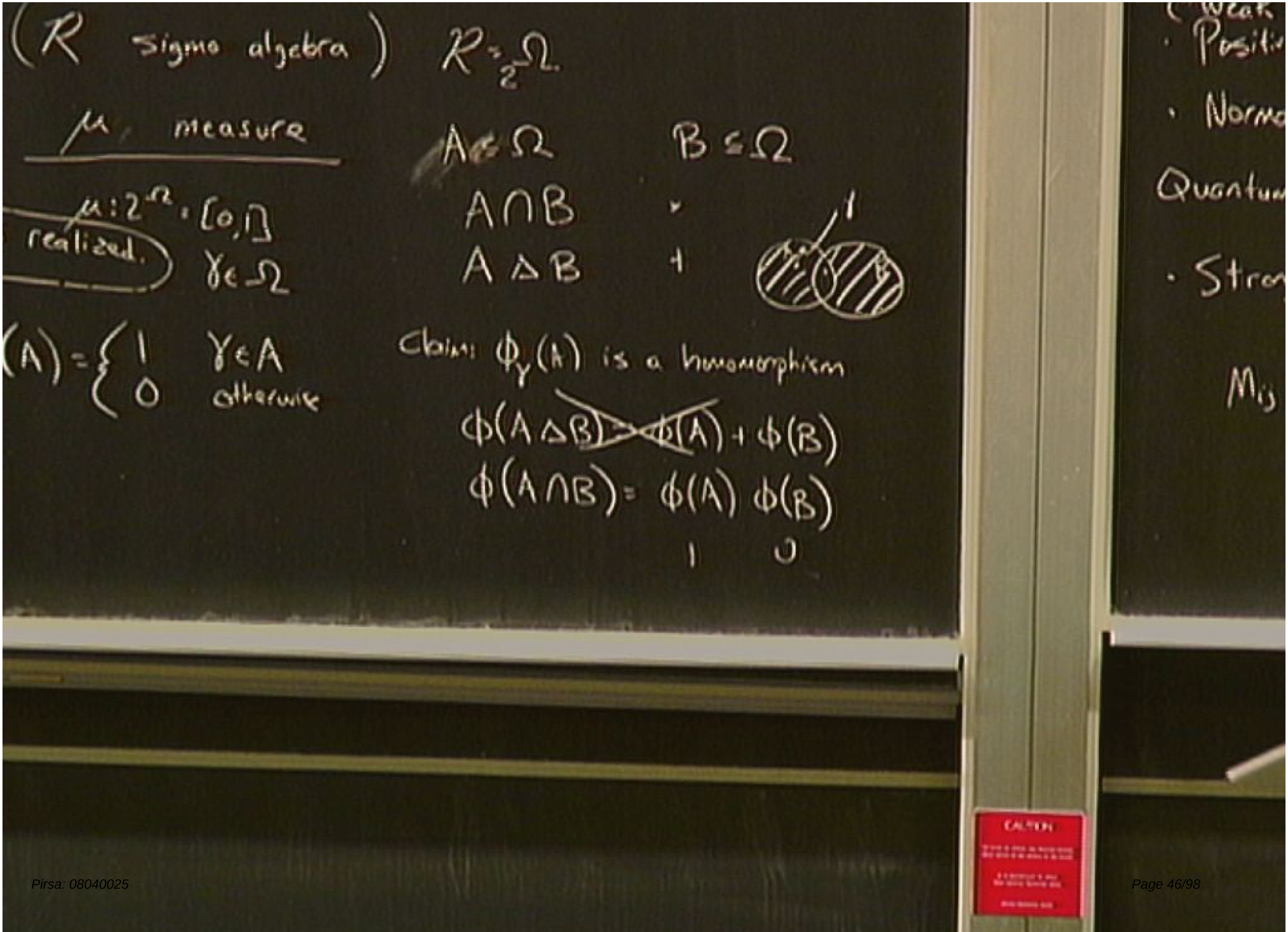
(R sigma algebra) R ⊆ 2^Ω.
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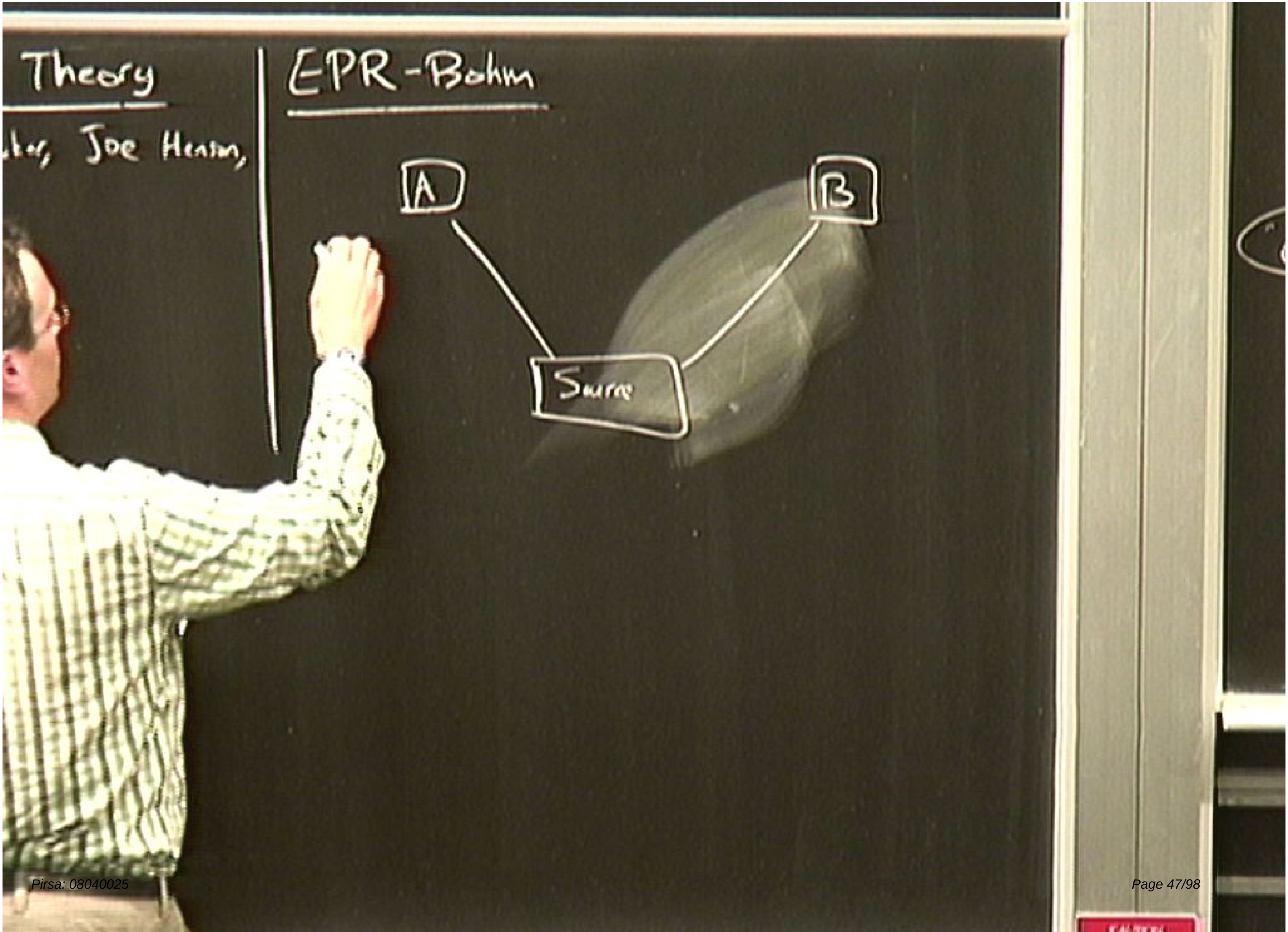
μ: 2^Ω → [0,1]
realized. γ ∈ Ω
(A) = { 1 γ∈A
0 otherwise
A ⊆ Ω B ⊆ Ω
A∩B ×
AΔB +
A B
Claim φ_γ(A) is a homomorphism
φ(AΔB) = φ(A) + φ(B)
φ(A∩B) = φ(A) φ(B)
1 0
Quantum
M_ij


Theory
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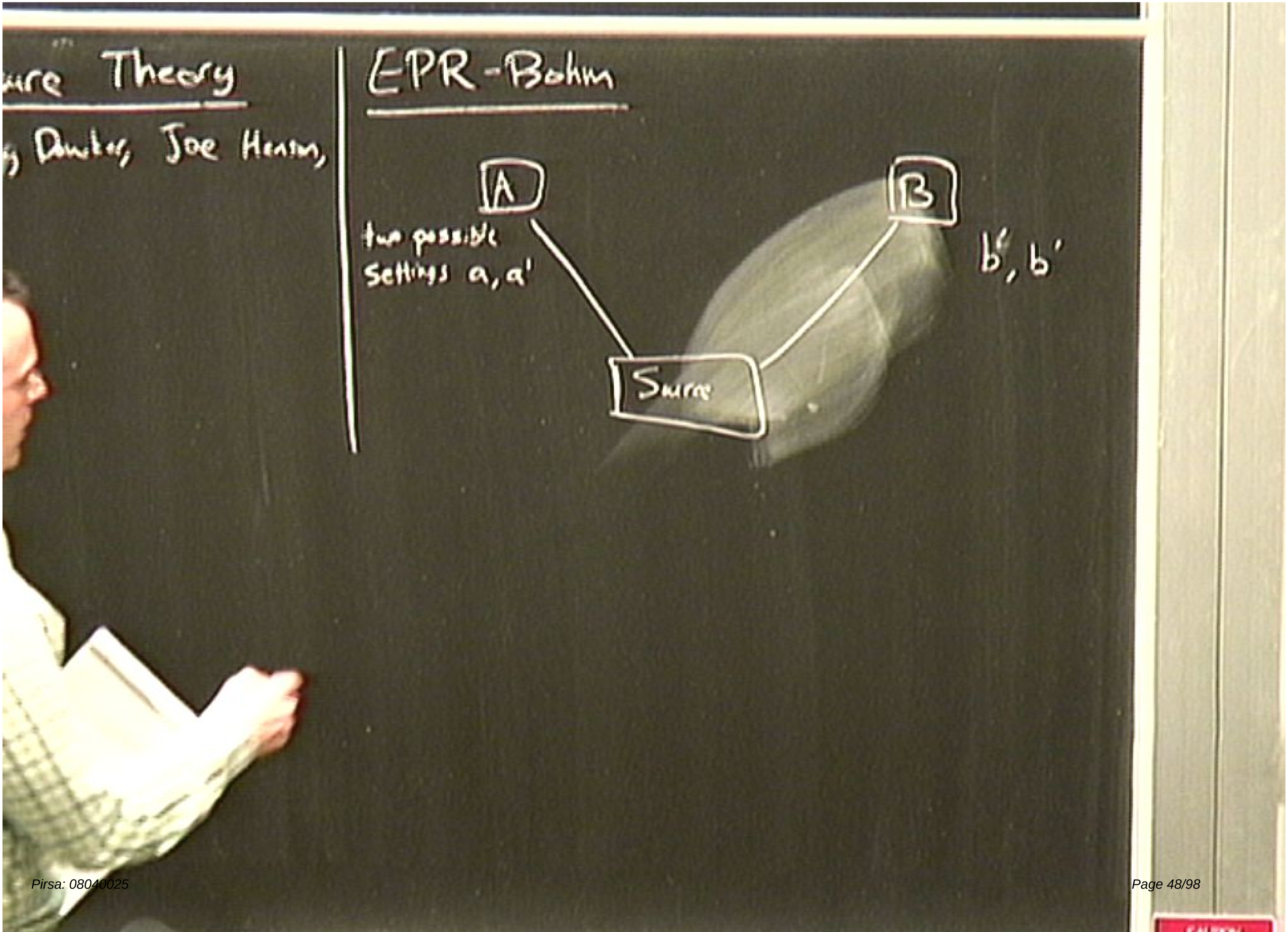
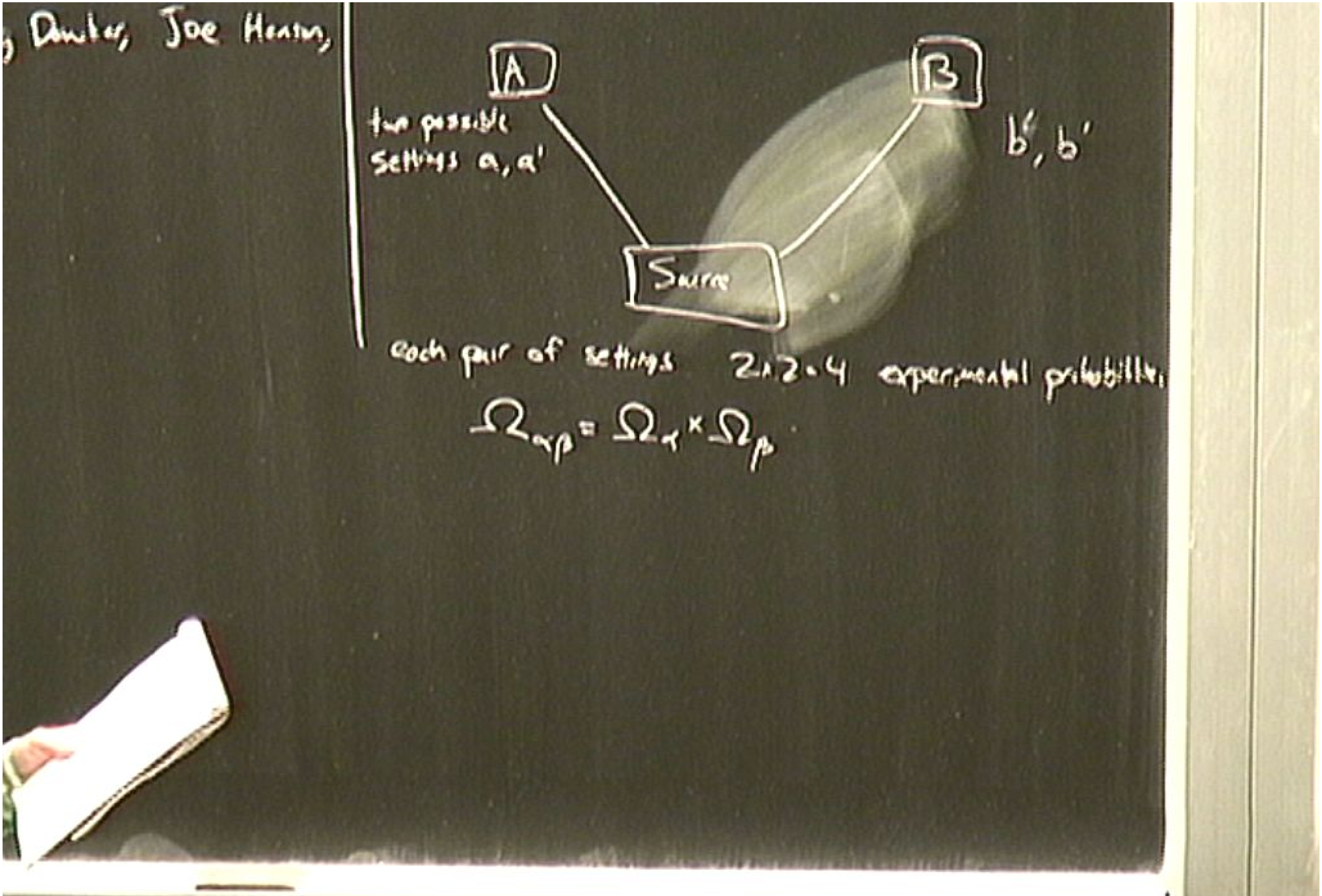
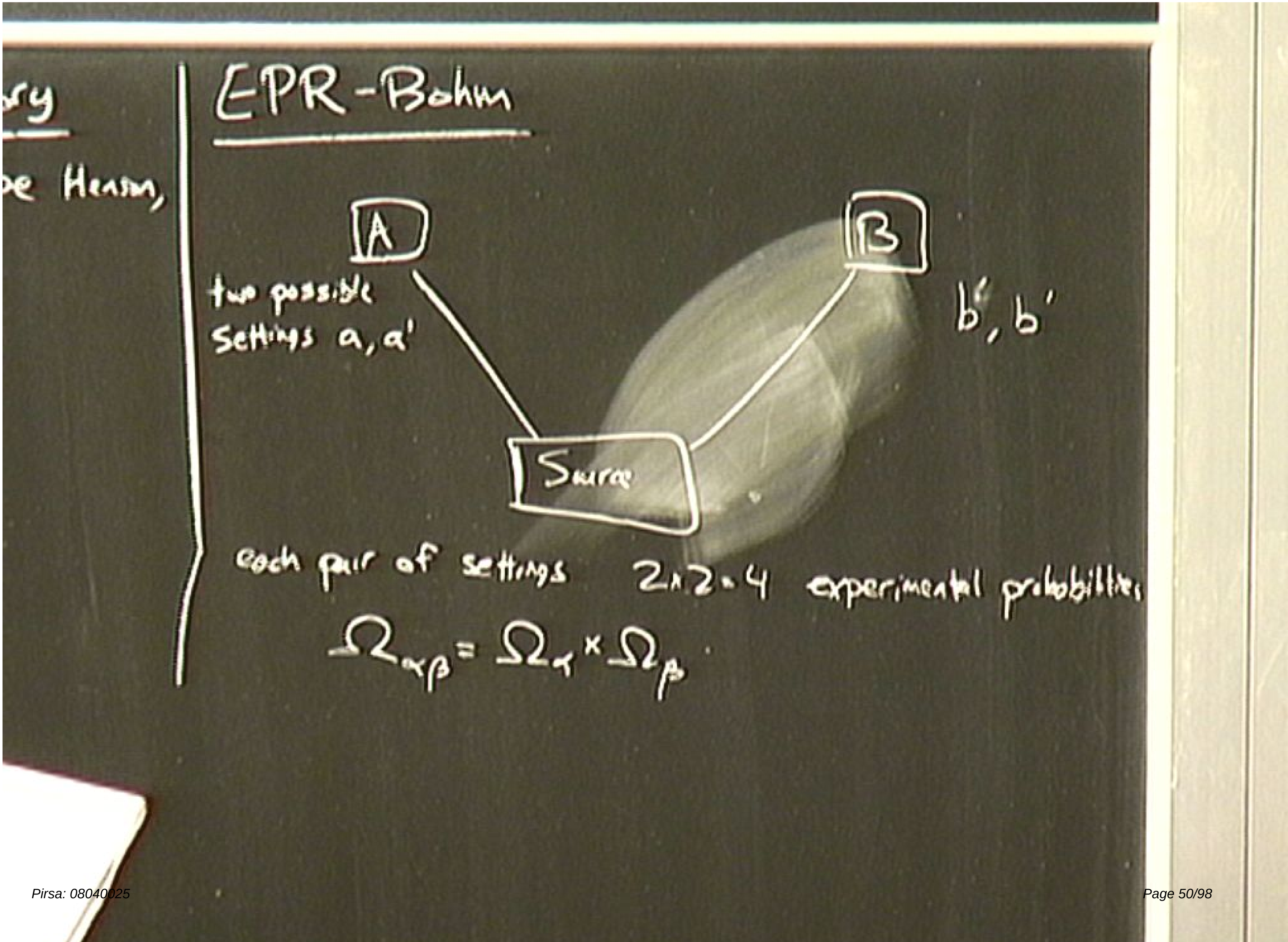
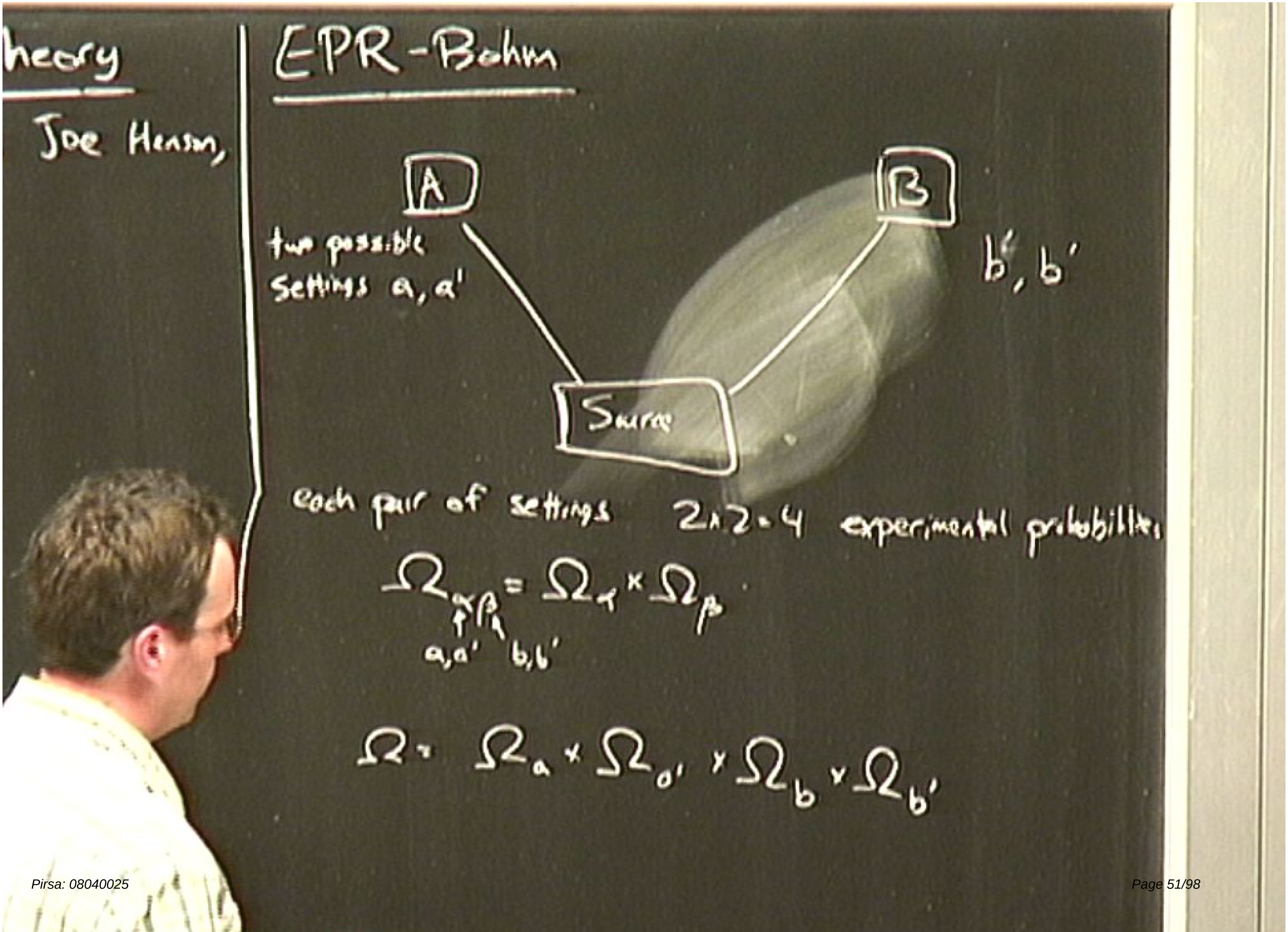
Source


EPR-Bohm
A
B
two possible settings a, a'
b', b'
Source
Joe Henson,
Theory


Dowker, Joe Henson,
A
B
two possible settings a, a'
b', b'
Source
each pair of settings 2×2=4 experimental probabilities
$\Omega_{\alpha\beta} = \Omega_\alpha \times \Omega_\beta$


ry
e Henson,
EPR-Bohm
A
B
two possible settings a, a'
b, b'
Source
each pair of settings 2x2=4 experimental probabilities
Ωαβ = Ωα × Ωβ



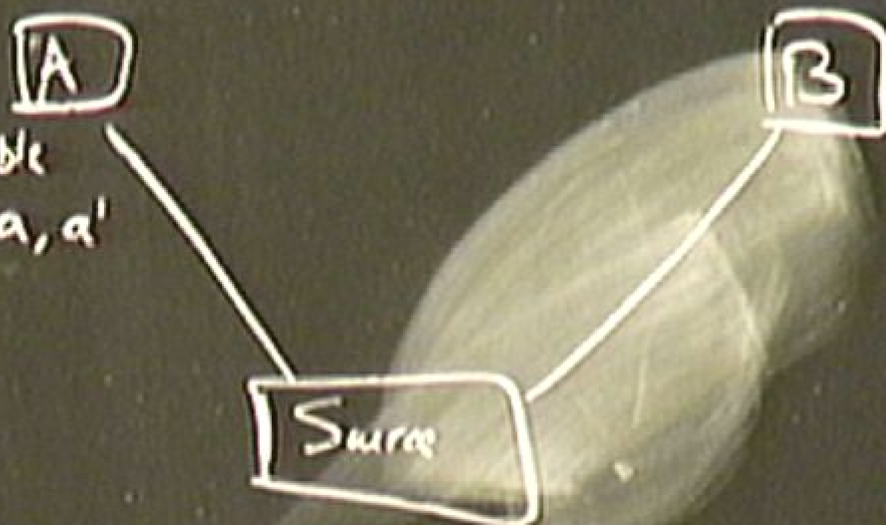
heory
Joe Henson,
EPR-Bohm
A
B
two possible
settings a, a'
b, b'
Source
each pair of settings 2x2=4 experimental probabilities
Ωαβ = Ωα × Ωβ
a,a' b,b'
Ω = Ωa × Ωa' × Ωb × Ωb'

heory

, Joe Henson,

EPR-Bohm

two possible settings a, a'



b, b'

each pair of settings $2 \times 2 = 4$ experimental probabilities

$$\Omega_{\alpha\beta} = \Omega_{\alpha} \times \Omega_{\beta}$$

α: a, a' $\quad$ β: b, b'

joint sample space $\quad \Omega = \Omega_a \times \Omega_{a'} \times \Omega_b \times \Omega_{b'}$

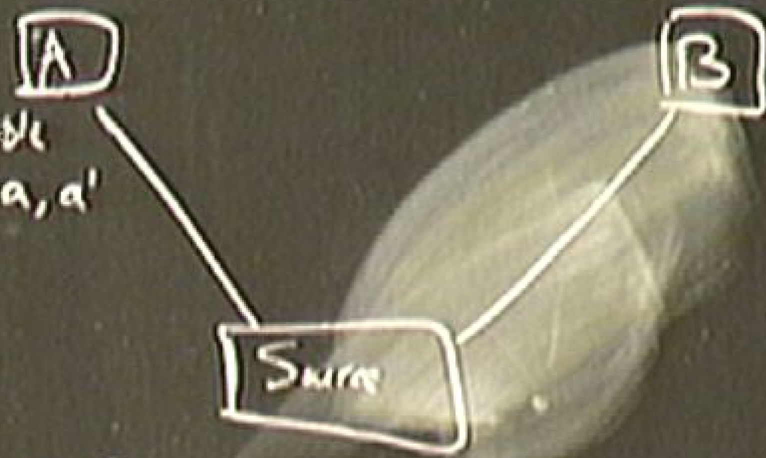
asure Theory

Fay Dowker, Joe Henson,

ilities

EPR-Bohm

two possible settings a, a'

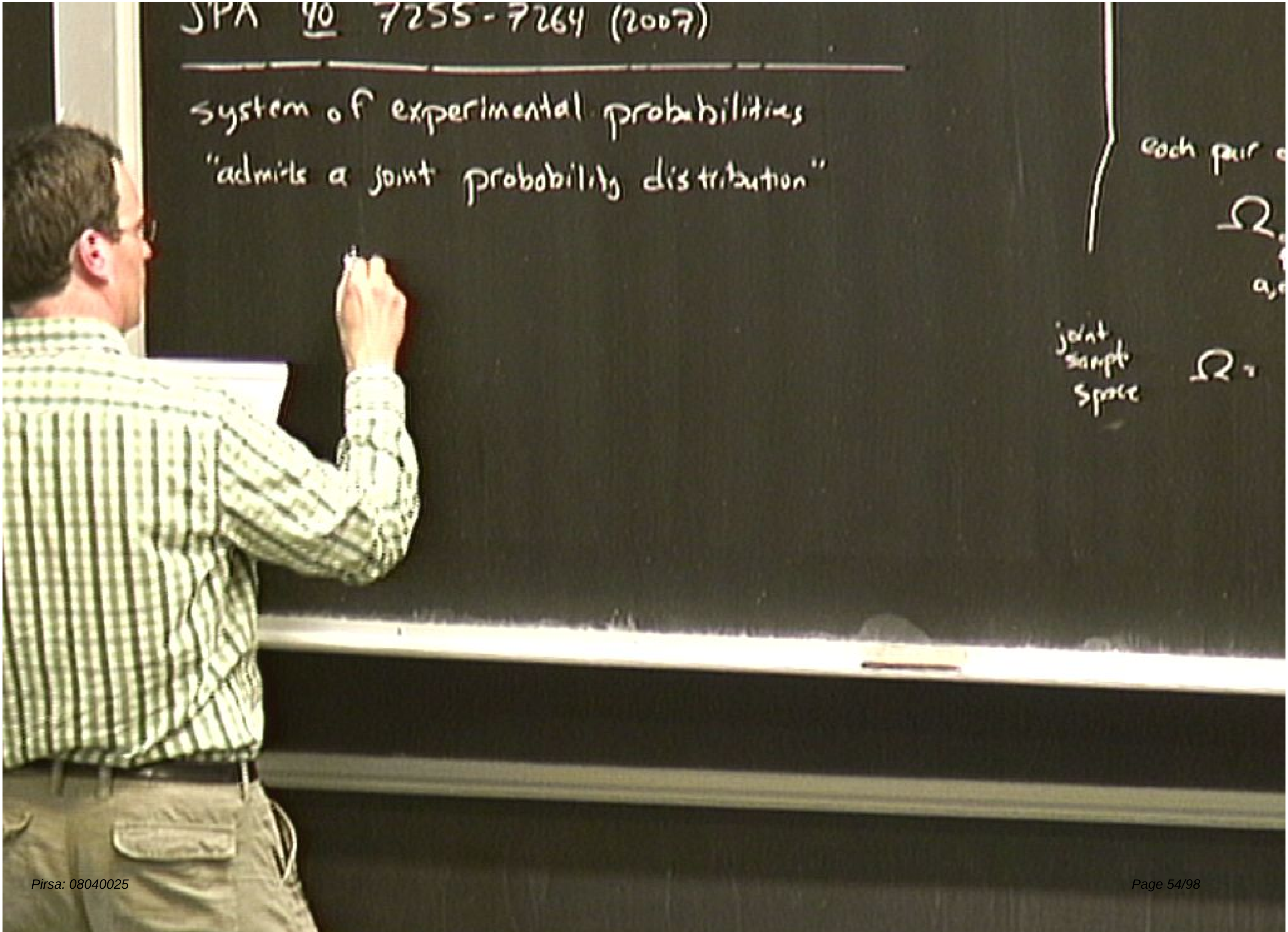


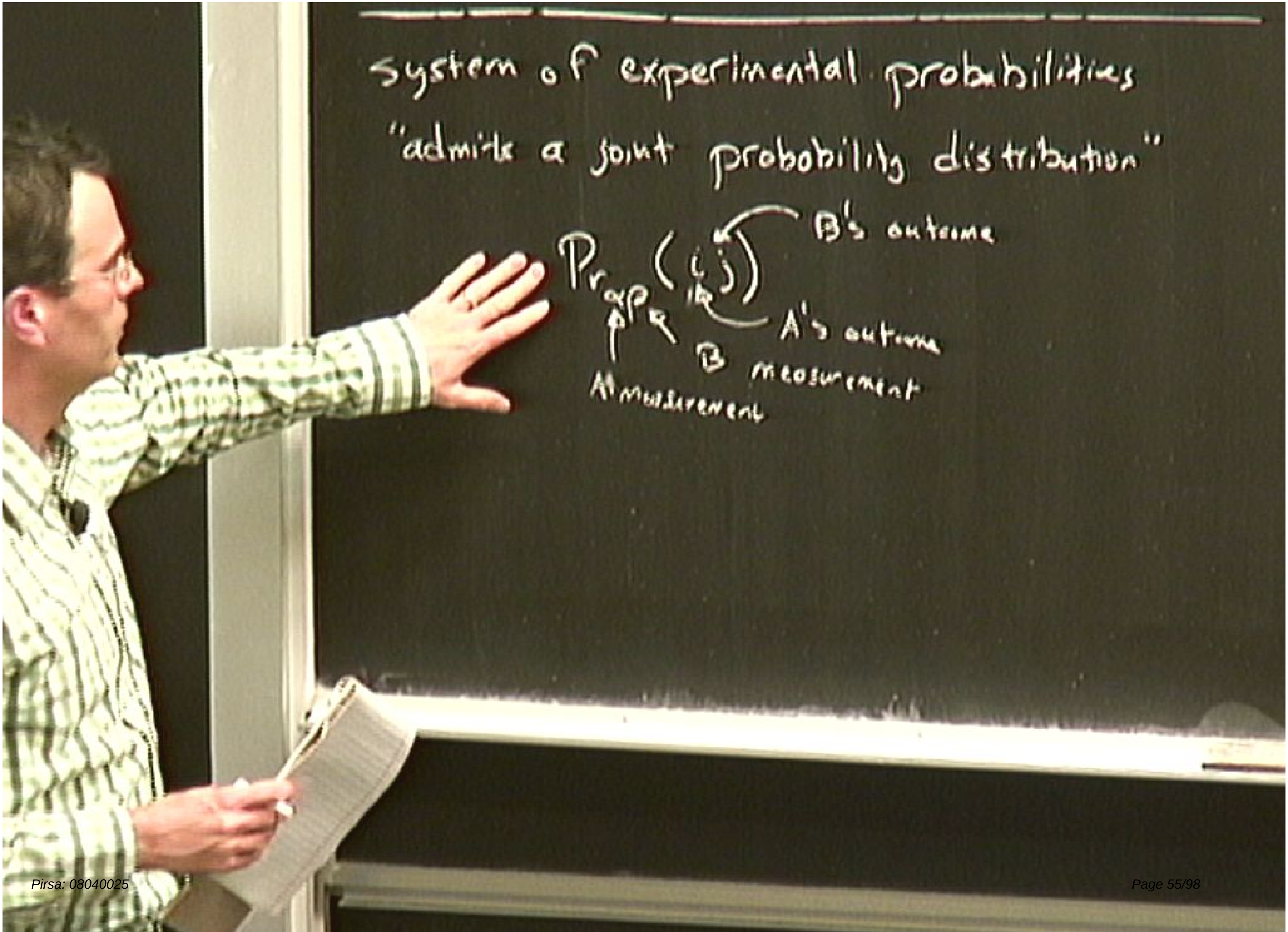
b, b'

each pair of settings 2×2=4 experimental probabilities

$$\underset{a,a'\ \ b,b'}{\Omega_{\alpha\beta}} = \Omega_\alpha \times \Omega_\beta$$

joint sample space $\Omega = \Omega_a \times \Omega_{a'} \times \Omega_b \times \Omega_{b'}$


JPA 40 7255-7264 (2007)
system of experimental probabilities
"admits a joint probability distribution"
each pair
joint
sample
space


system of experimental probabilities
"admits a joint probability distribution"
$Pr_{\alpha\beta R}(i\,j)$
B's outcome
A's outcome
B measurement
A measurement

JPA 40 7255-7264 (2007)

system of experimental probabilities

"admits a joint probability distribution"

$$\Pr_{\substack{exp \\ pq}}(ij)$$

B's outcome

A's outcome

B measurement

A measurement

From this "assumption" one can derive CHSHB inequalities

joint
sam
Sp

Seth Major, DR, Rafael Sorkin

JPA 40 501-523 (2007)

JPA 40 7255-7264 (2007)

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system of experimental probabilities

"admits a joint probability distribution"

$$\Pr_{\alpha\beta}((i,j))$$

B's outcome

A's outcome

A measurement

B measurement

$$\Pr_{ab}((ij)) = \sum_{i'j'} \Pr((ii'jj'))$$

From this assumption

one can derive CHSHB inequalities

two possible settings a, ...

each pair

joint sample space

Ω

$\Omega =$

JPA 40 7255-7264 (2007)

system of experimental probabilities

"admits a joint probability distribution"

B's outcome

$$Pr_{\alpha\beta}(ij)$$

A's outcome

α A measurement

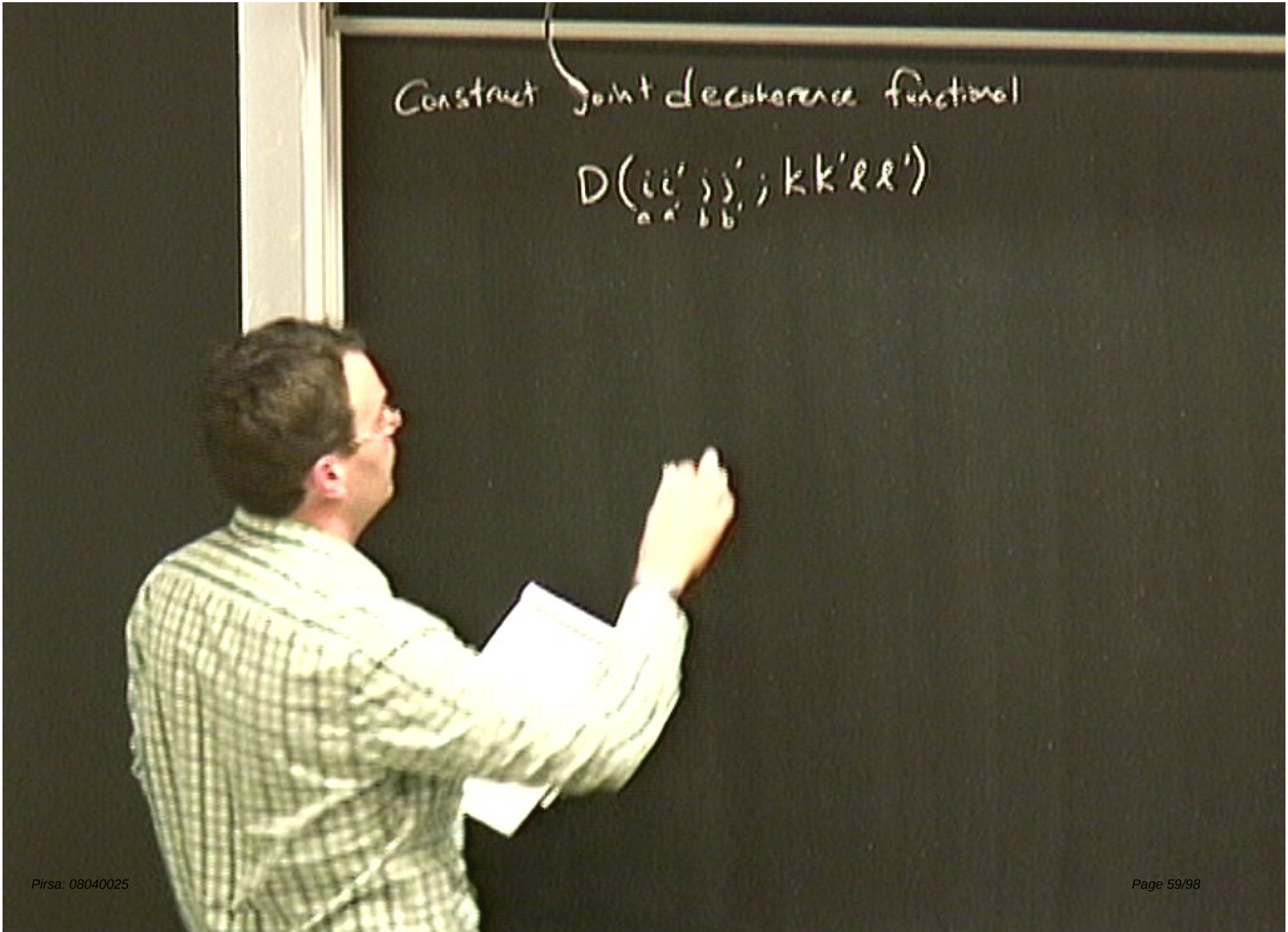
β B measurement

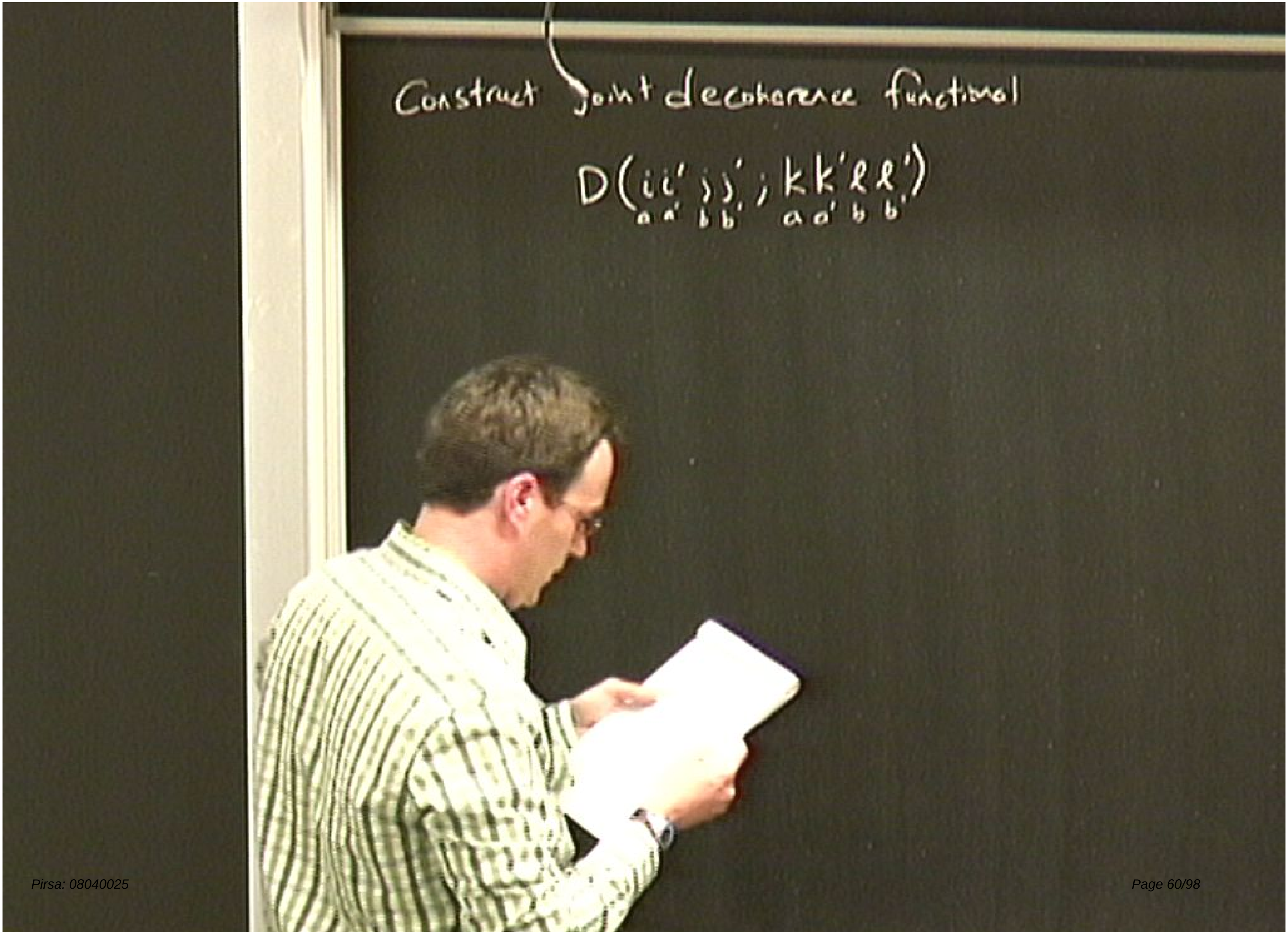
$$Pr_{\alpha\beta}(ij) = \sum_{i'j'} Pr(ii'jj')$$

joint sam sp

From this assumption

one can derive CHSHB inequalities


Construct joint decoherence functional
$D(i_a i'_{a'} j_b j'_{b'} ; k k' \ell \ell')$


Construct joint decoherence functional
$D(i_a i'_{a'} j_b j'_{b'} ; k_a k'_{a'} \ell_b \ell'_{b'})$

Construct joint decoherence functional

$$D(i_a i'_{a'} j_b j'_{b'} ; k_a k'_{a'} \ell_b \ell'_{b'})$$

$$\sum_{i'j'k'\ell'} D(ii'jj';kk'\ell\ell') = \Pr_{ab}(ij)\,\delta_{ik}\,\delta_{j\ell}$$

Construct joint decoherence functional

$$D(\underset{a}{i}\,\underset{a'}{i'}\,\underset{b}{j}\,\underset{b'}{j'};\underset{a}{k}\,\underset{a'}{k'}\,\underset{b}{\ell}\,\underset{b'}{\ell'})$$

$$\sum_{i'j'k'\ell'} D(ii'jj';kk'\ell\ell') = Pr_{ab}(ij)\,\delta_{ik}\,\delta_{j\ell}$$

$$\sum_{i'jk'\ell} D(ii'jj';kk'\ell\ell') = Pr_{ab'}(ij')\,\delta_{ik}\,\delta_{j'\ell'}$$

2 - more

Construct joint decoherence functional

$$D(\underset{a}{i}\,\underset{a'}{i'}\,\underset{b}{j}\,\underset{b'}{j'}\,;\,\underset{a}{k}\,\underset{a'}{k'}\,\underset{b}{\ell}\,\underset{b'}{\ell'})$$

$$\sum_{i'j'k'\ell'} D(ii'jj';kk'\ell\ell') = \Pr_{ab}(ij)\,\delta_{ik}\,\delta_{j\ell}$$

$$\sum_{i'jk'\ell} D(ii'j\ldots\ell') = \Pr_{ab'}(ij')\,\delta_{ik}\,\delta_{j'\ell'}$$

2 -…

co…

$$\ldots = \sum_{ij} ij \Pr_{ab}(ij) \qquad X_{ab'} = \sum_{ij'} ij' \Pr_{ab'}(ij')$$

$$\ldots = \sum_{i'j} i'j \Pr_{a'b}(i'j) \qquad X_{a'b'} = \sum_{i'j'} i'j' \Pr_{a'b'}(i'j')$$

Construct joint decoherence functional

$$D(\underset{a\,a'}{i\,i'}\,\underset{b\,b'}{j\,j'}\,;\,\underset{a\,a'}{k\,k'}\,\underset{b\,b'}{\ell\,\ell'})$$

$$\sum_{i'j'k'\ell'} D(ii'jj';kk'\ell\ell') = Pr_{ab}(ij)\,\delta_{ik}\,\delta_{j\ell}$$

$$\sum_{i'jk'\ell} D(ii'jj';kk'\ell\ell') = Pr_{ab'}(ij')\,\delta_{ik}\,\delta_{j'\ell'}$$

2 -more

correlators

$$X_{ab} = \sum_{ij} ij\,Pr_{ab}(ij) \qquad X_{ab'} = \sum_{ij'} ij'\,Pr_{ab'}(ij')$$

$$X_{a'b} = \sum_{i'j} i'j\,Pr_{a'b}(i'j) \qquad X_{a'b'} = \sum_{i'j'} i'j'\,Pr_{a'b'}(i'j')$$

Construct strictly positive joint decoherence functional

$$D(\underset{a\,a'}{i\,i'}\;\underset{b\,b'}{j\,j'};\underset{a\,a'}{k\,k'}\;\underset{b\,b'}{\ell\,\ell'})$$

$$\sum_{i'j'k'\ell'} D(ii'jj';kk'\ell\ell') = P_{ab}(ij)\,\delta_{ik}\,\delta_{j\ell}$$

$$\sum_{i'jk'\ell} D(ii'jj';kk'\ell\ell') = P_{ab'}(ij')\,\delta_{ik}\,\delta_{j'\ell'}$$

2 - more

correlators $X_{ab} = \sum_{ij} ij\, P_{ab}(ij)$ $\quad X_{ab'} = \sum_{ij'} ij'\, P_{ab'}(ij')$

$X_{a'b} = \sum_{i'j} i'j\, P_{a'b}(i'j)$ $\quad X_{a'b'} = \sum_{i'j'} i'j'\, P_{a'b'}(i'j')$

Assumption that experimental probs admit joint decoherence functional $\Rightarrow$ $|X_{ab} + X_{ab'} + X_{a'b} - X_{a'b'}| \leq 2\sqrt{2}$

$$\sum_{i'jk'l} D(ii'jj';kk'll') = Pr_{ab'}(ij')\,\delta_{ik}\,\delta_{j'l'}$$

2 -more

correlators

$$X_{ab} = \sum_{ij} ij\, Pr_{ab}(ij) \qquad X_{ab'} = \sum_{ij'} ij'\, Pr_{ab'}(ij')$$

$$X_{a'b} = \sum_{i'j} i'j\, Pr_{a'b}(i'j) \qquad X_{a'b'} = \sum_{i'j'} i'j'\, Pr_{a'b'}(i'j')$$

Assumption that experimental probs admit joint decoherence functional $\Rightarrow$

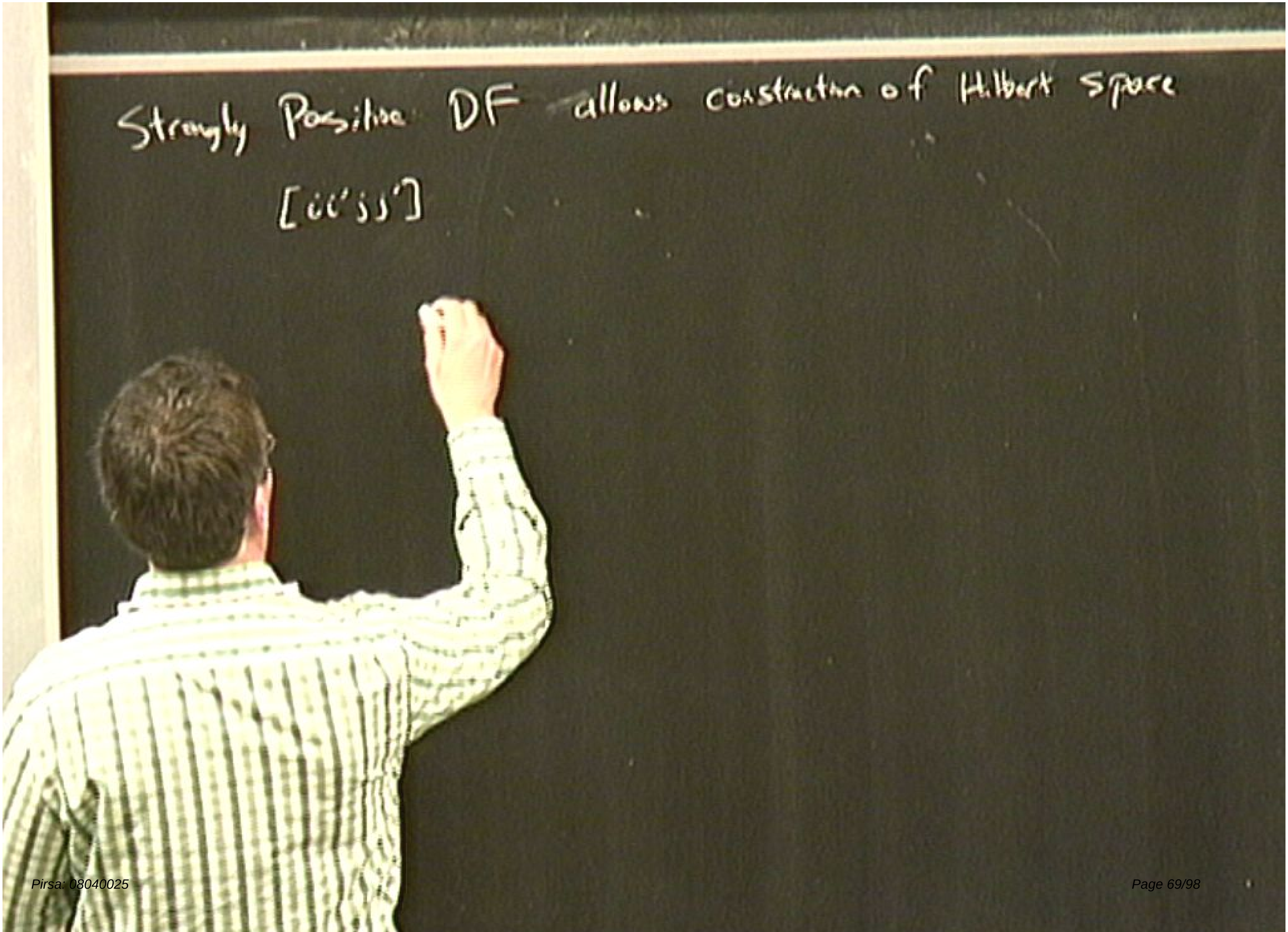
$$Pr_{ab'}(i\,j')$$

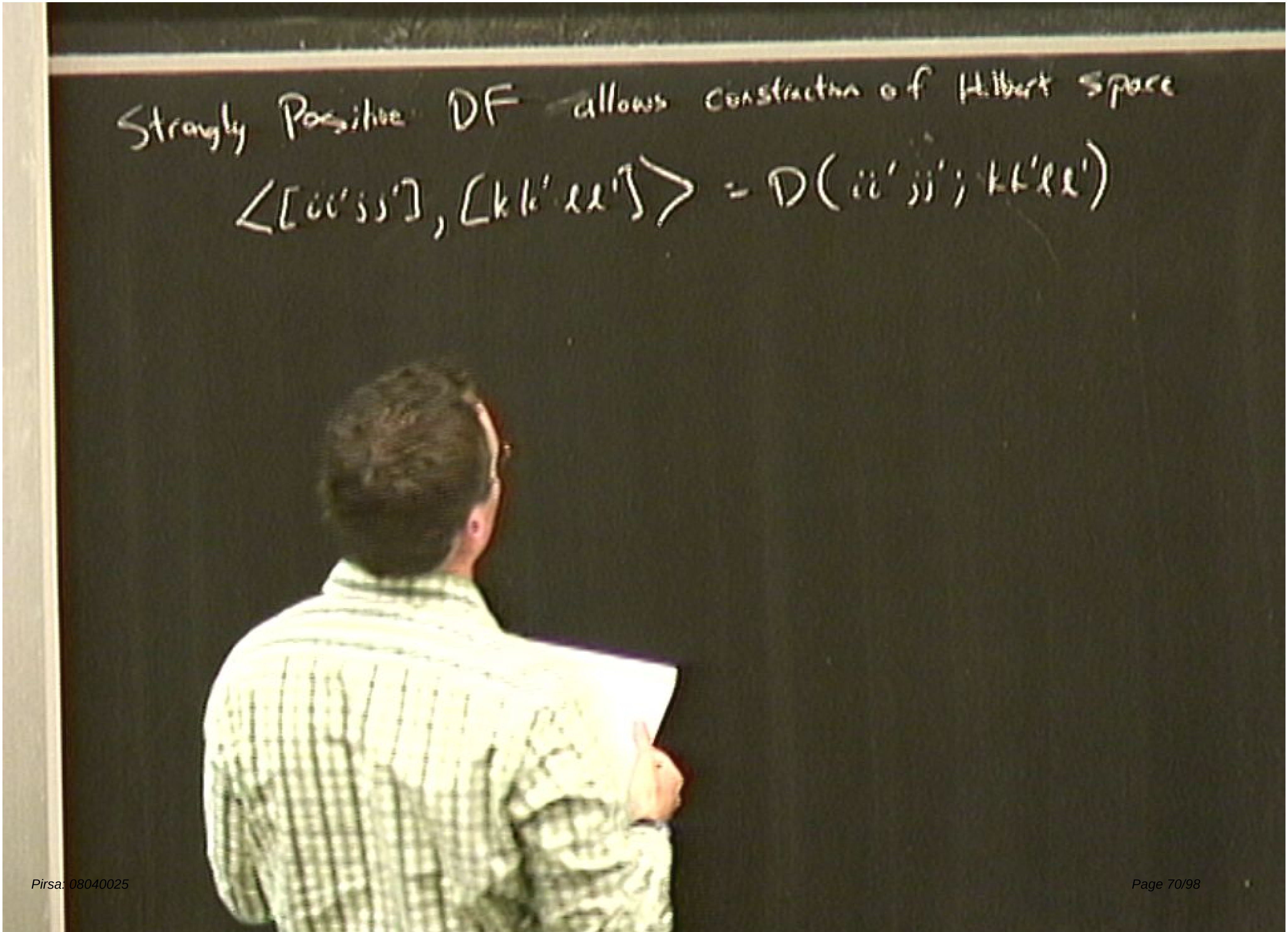
$$Pr_{a'b'}(i''\,j'')$$

functional $\Rightarrow$ $\left| X_{ab} + X_{ab'} + X_{a'b} - X_{a'b'} \right| \leq 2\sqrt{2}$

Strongly Positive DF allows construction of Hilbert space

[αα'ββ']


Strongly Positive DF allows construction of Hilbert space
[αα'ss']


Strongly Positive DF allows construction of Hilbert Space
⟨[ii'jj'], [kk'ℓℓ']⟩ = D(ii'jj'; kk'ℓℓ')

Strongly Positive DF allows construction of Hilbert space

$$\langle [ii'jj'], [kk'll'] \rangle = D(ii'jj'; kk'll')$$

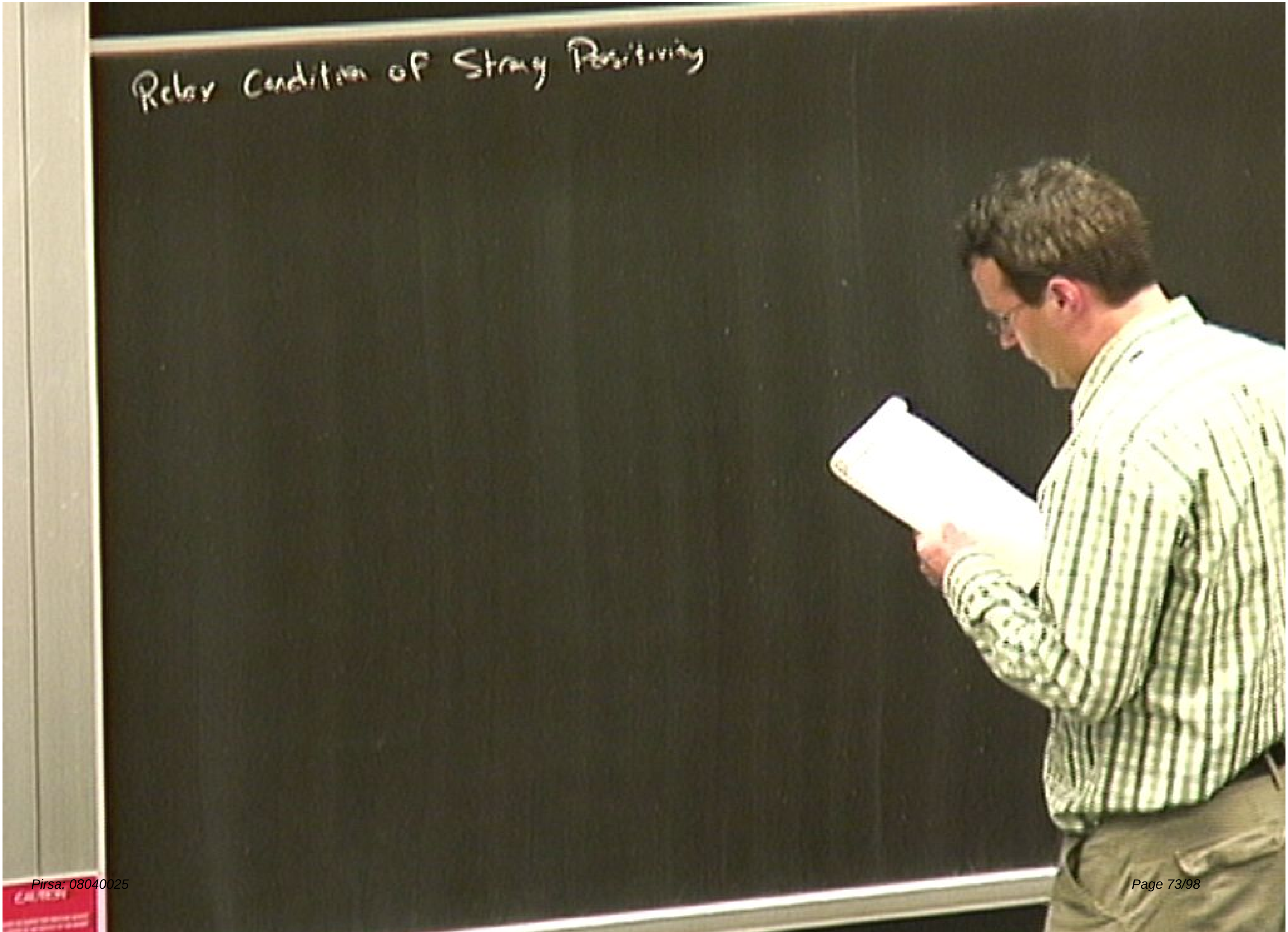
$$\mathcal{H} = \frac{\mathcal{H}_1}{\mathcal{H}_0}$$

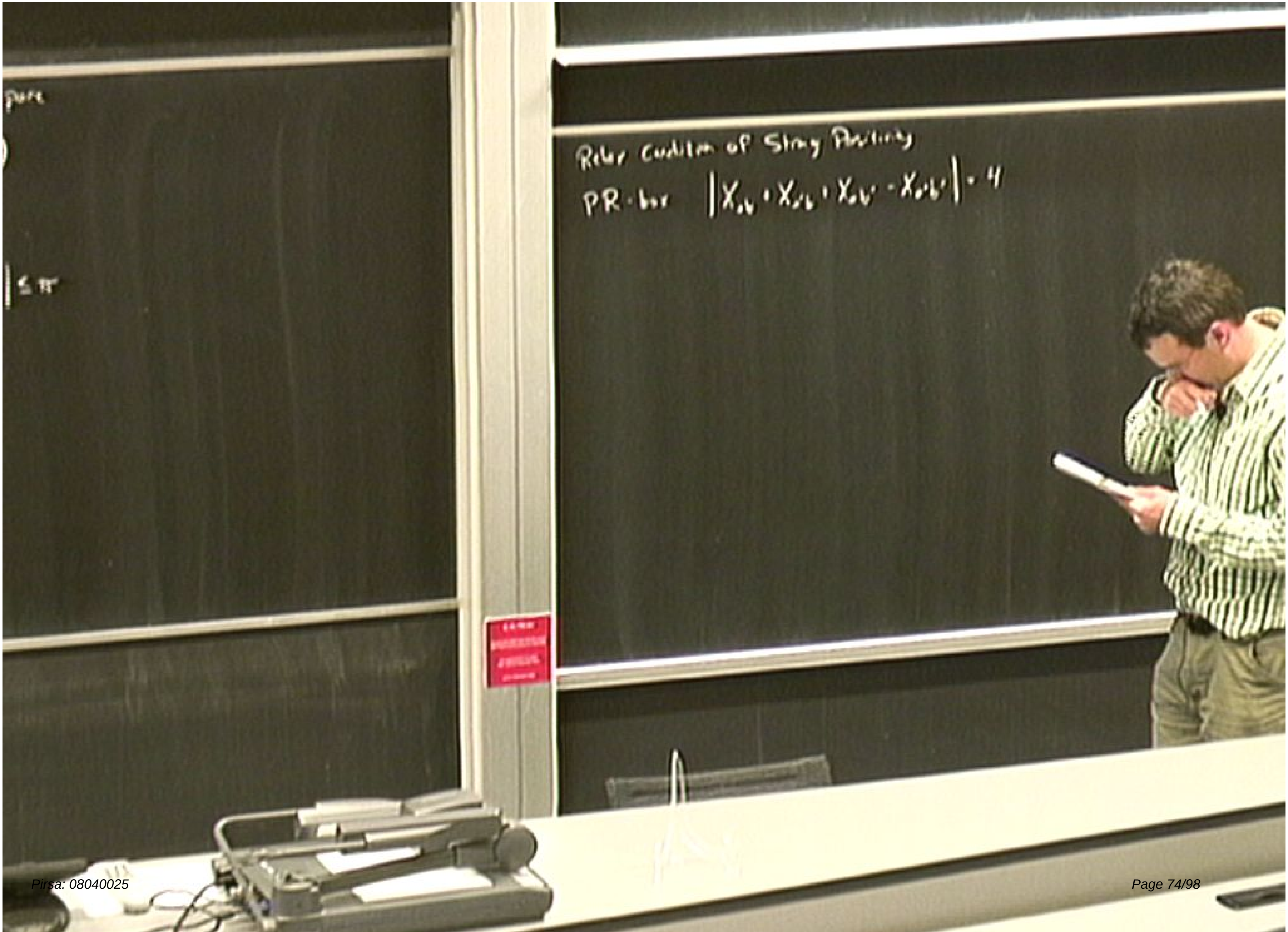
Strongly Positive DF allows construction of Hilbert space

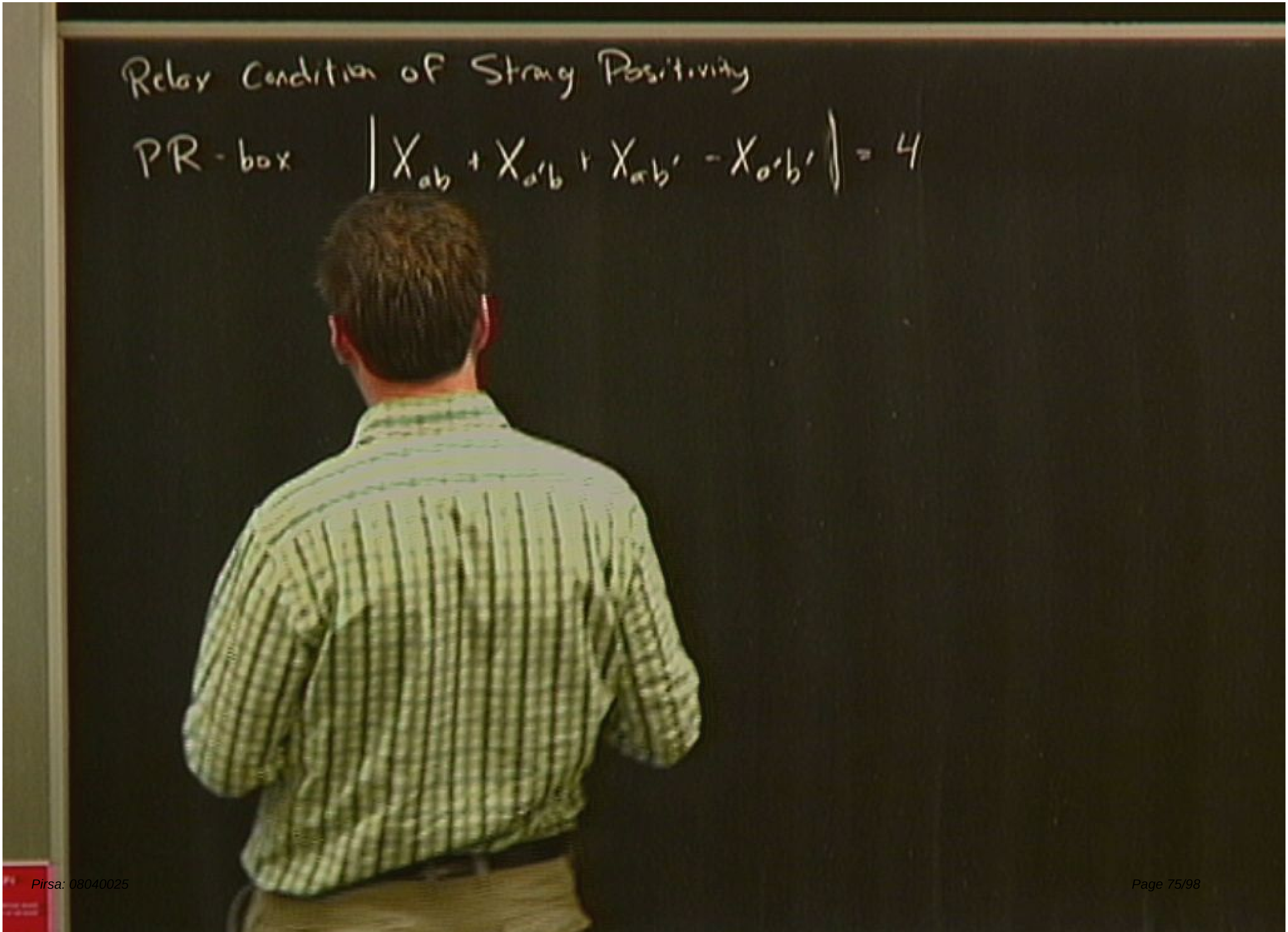
$$\langle [ii'jj'], [kk'\ell\ell'] \rangle = D(ii'jj'; kk'\ell\ell')$$

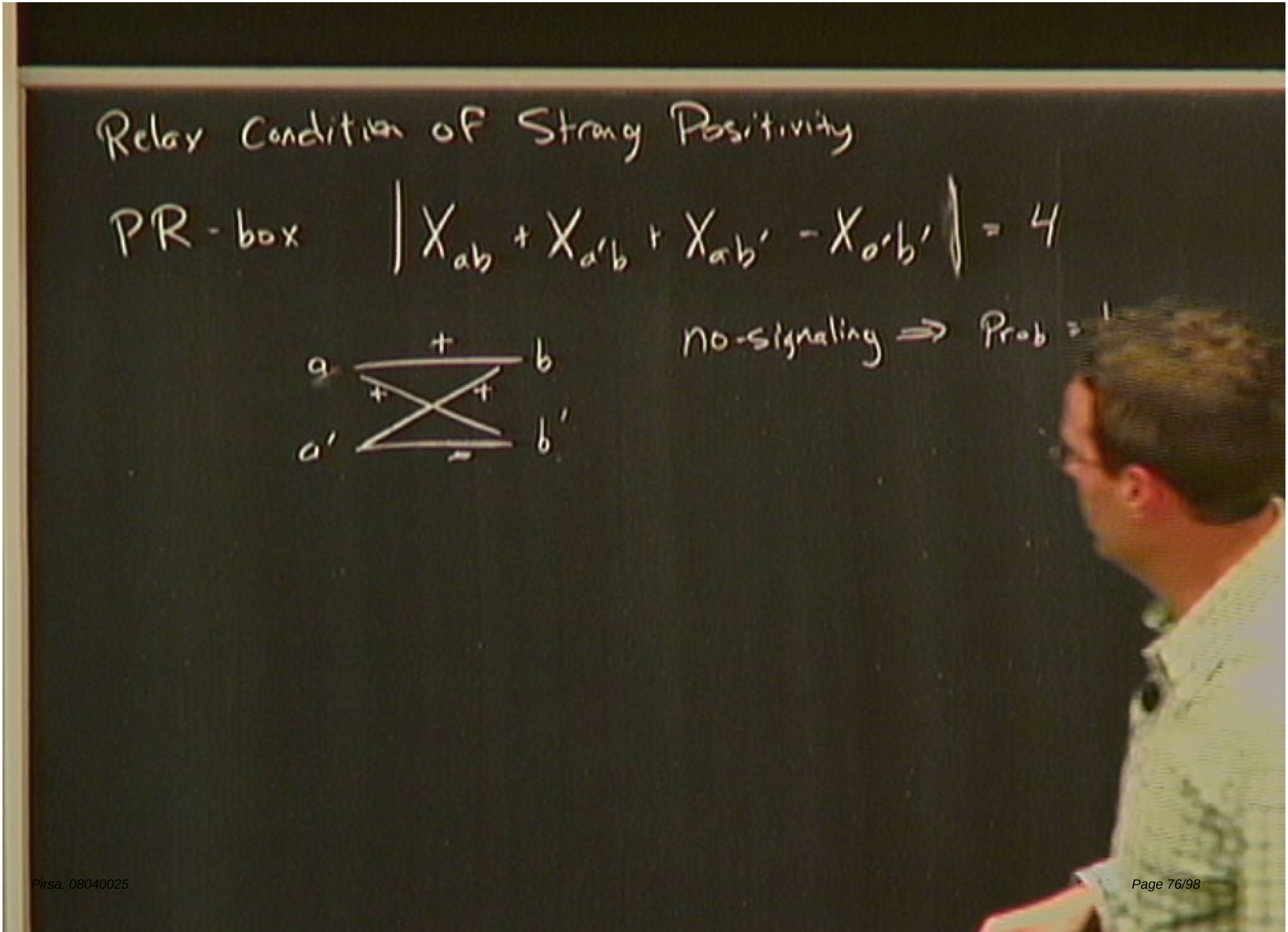
$$\mathcal{H} = \frac{\mathcal{H}_1}{\mathcal{H}_0}$$

$$\left| \operatorname{Arcsin} X_{ab} + \operatorname{Arcsin} X_{a'b} + \operatorname{Arcsin} X_{ab'} - \operatorname{Arcsin} X_{a'b'} \right| \le \pi$$


Relax Condition of Strong Positivity

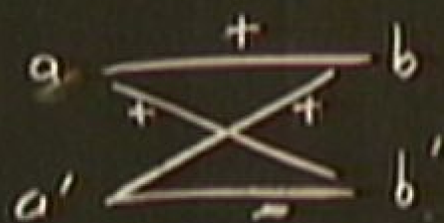

Relax Condition of Strong Positivity
PR-box $|X_{ab} + X_{a'b} + X_{ab'} - X_{a'b'}| = 4$
$\leq \pi$


Relax Condition of Strong Positivity
PR-box $|X_{ab} + X_{a'b} + X_{ab'} - X_{a'b'}| = 4$


Relay Condition of Strong Positivity
PR-box $\left| X_{ab} + X_{a'b} + X_{ab'} - X_{a'b'} \right| = 4$
no-signaling ⟹ Prob = 1
+
a
b
+
+
a'
b'
−

Relay Condition of Strong Positivity

PR-box $\left| X_{ab} + X_{a'b} + X_{ab'} - X_{a'b'} \right| = 4$



no-signaling $\Rightarrow$ Prob $= \frac{1}{2}$

Relax Condition of Strong Positivity

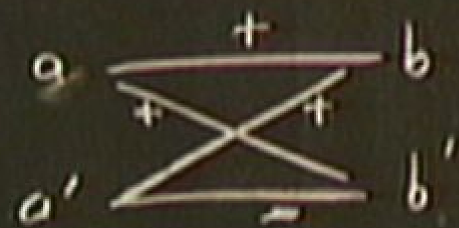
PR-box $\left| X_{ab} + X_{a'b} + X_{ab'} - X_{a'b'} \right| = 4$

no-signaling $\Rightarrow$ Prob $= \frac{1}{2}$

8 PR-box

Relax Condition of Strong Positivity

PR-box $\quad \left| X_{ab} + X_{a'b} + X_{ab'} - X_{a'b'} \right| = 4$



no-signaling $\Longrightarrow$ Prob $= \frac{1}{2}$

8 PR-box

Construct strongly positive joint decoherence functional

$$D(\underset{a\,a'}{i\,i'}\,\underset{b\,b'}{j\,j'};\underset{a\,a'}{k\,k'}\,\underset{b\,b'}{\ell\,\ell'})$$

$$\sum_{i'j'\ell'} D(i\,i'\,j\,j';k\,k'\,\ell\,\ell') = Pr_{ab}(i\,j)\,\delta_{ik}\,\delta_{j\ell}$$

$\Rightarrow$ no-signaling

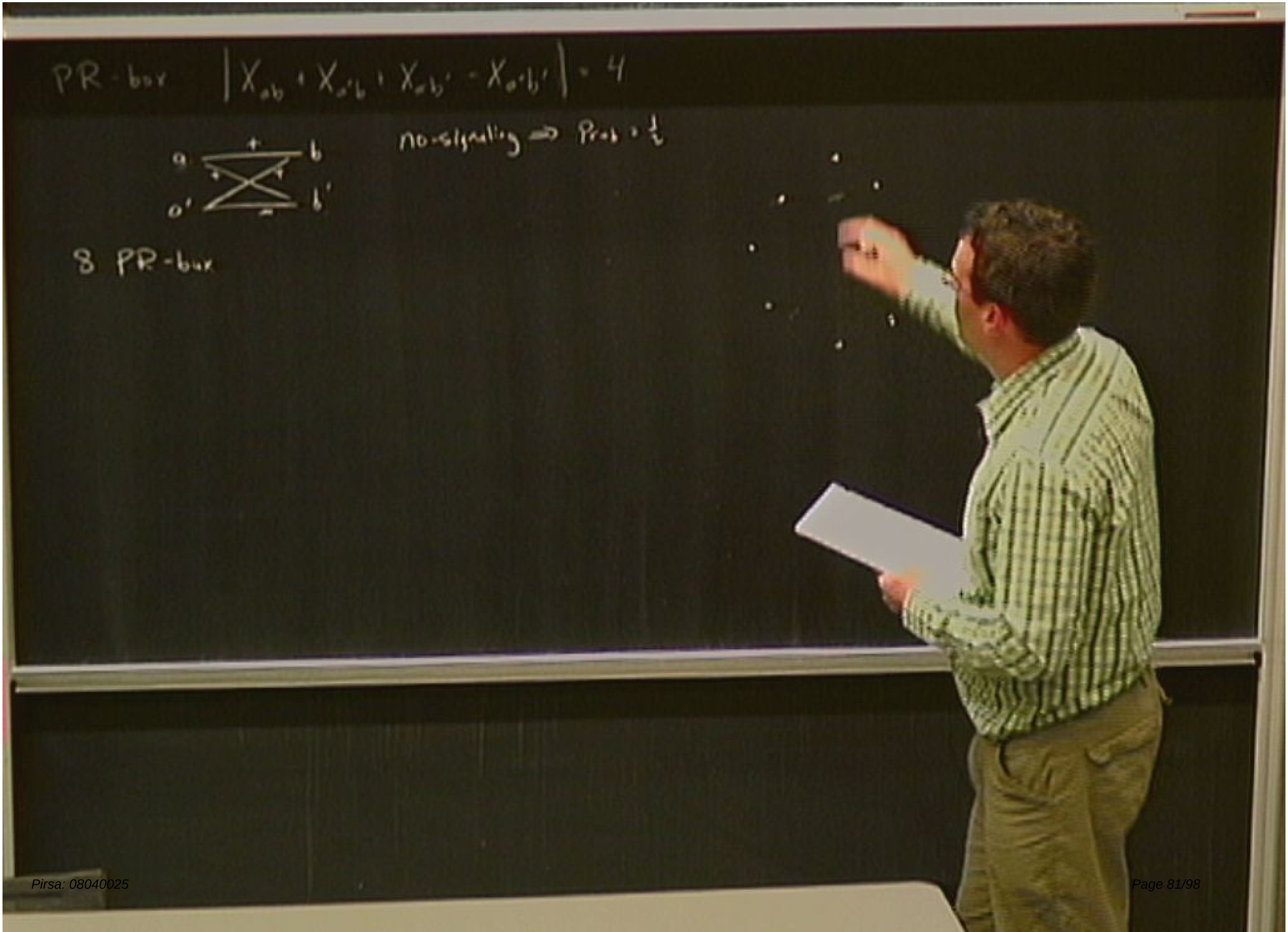
$$\sum_{i'j\,k'\ell} D(i\,i'\,j\,j';k\,k'\,\ell\,\ell') = Pr_{ab'}(i\,j')\,\delta_{ik}\,\delta_{j'\ell'}$$

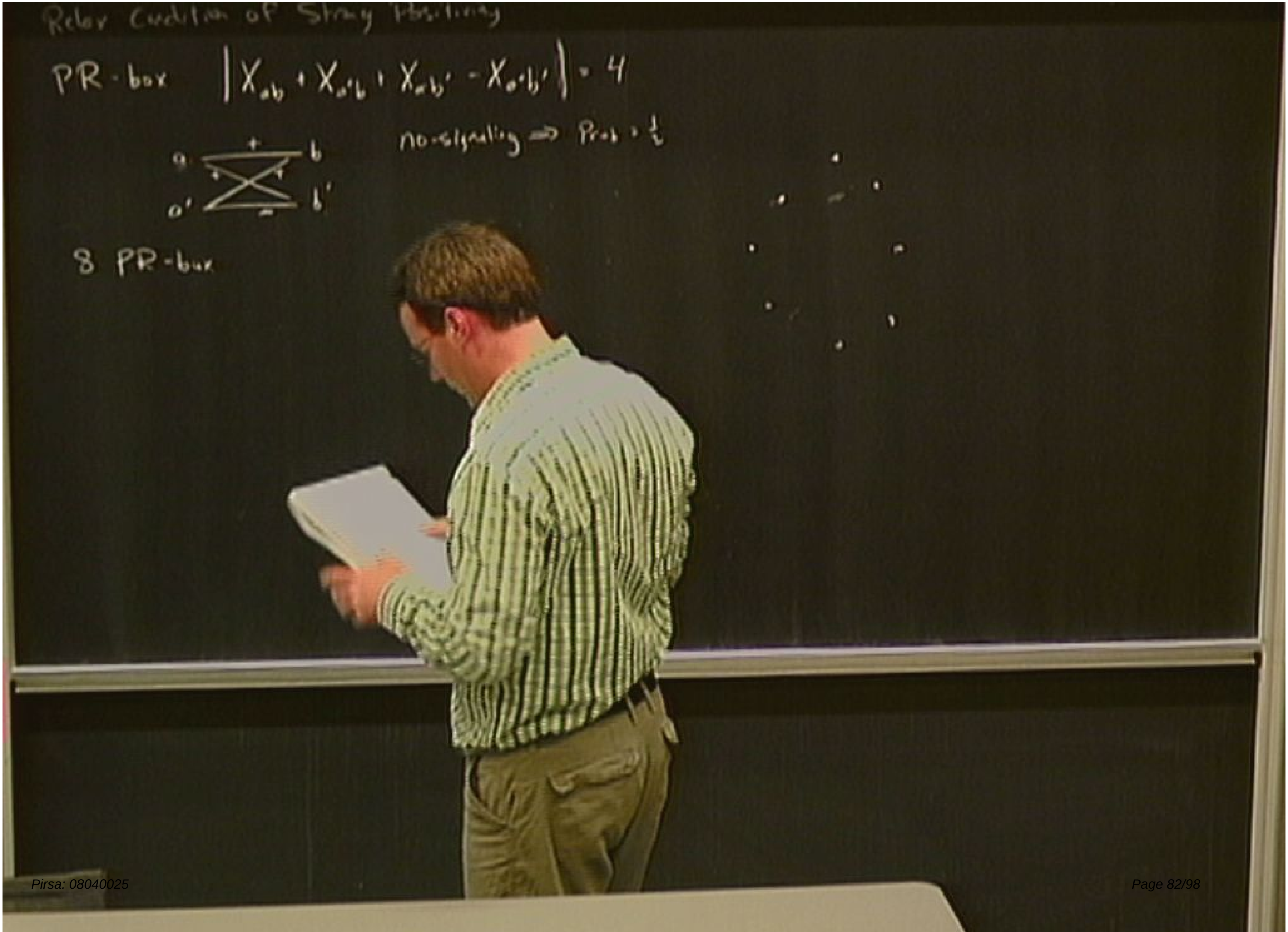
2 - more

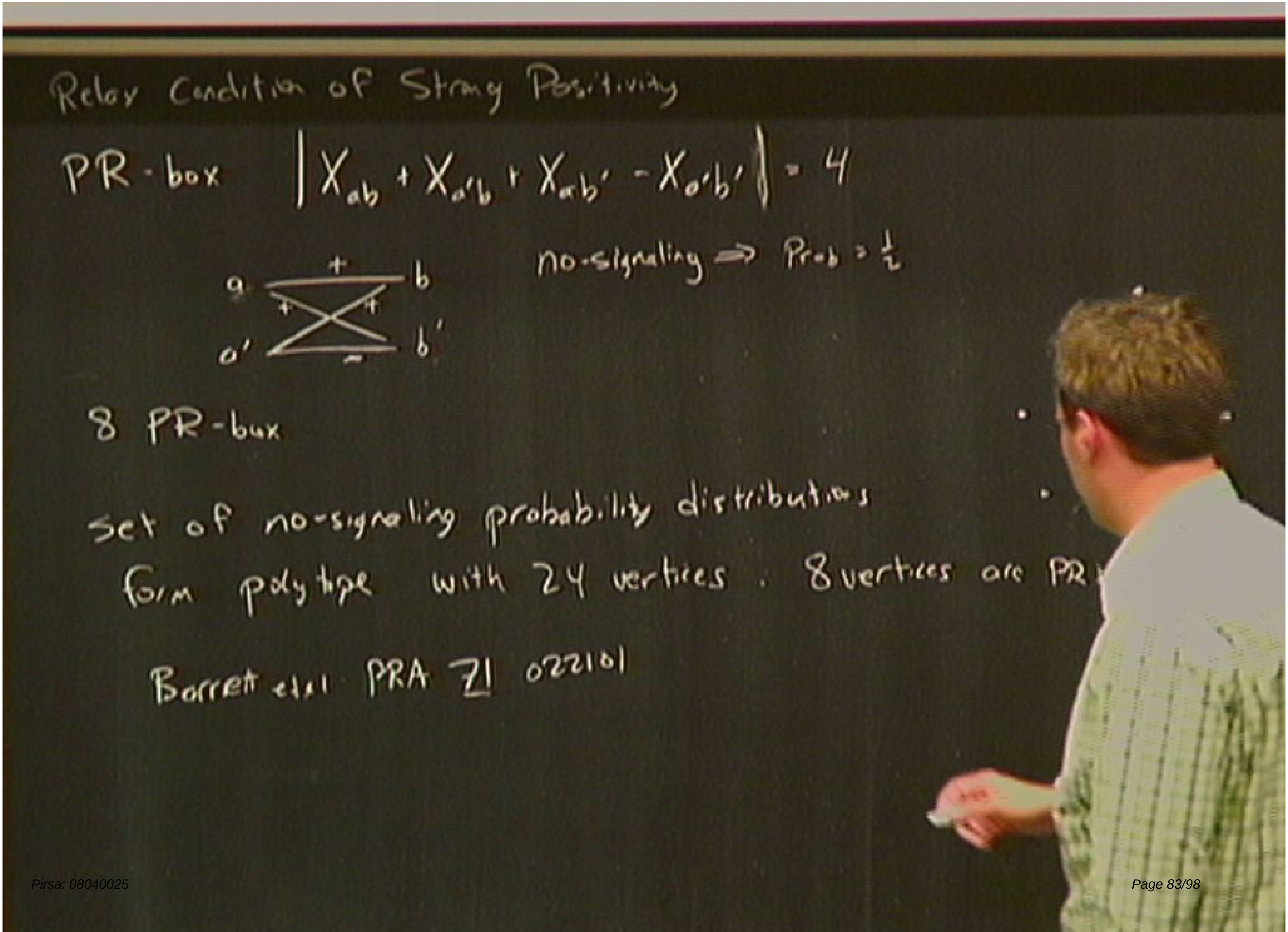
correlators $\quad X_{ab} = \sum_{ij} i\,j\,Pr_{ab}(i\,j) \qquad X_{ab'} = \sum_{ij'} i\,j'\,Pr_{ab'}(i\,j')$

$\qquad X_{a'b} = \sum_{i'j} i'\,j\,Pr_{a'b}(i'\,j) \qquad X_{a'b'} = \sum_{i'j'} i'\,j'\,Pr_{a'b'}(i'\,j')$

Assumption that experimental probs admit joint decoherence functional $\Rightarrow$ |

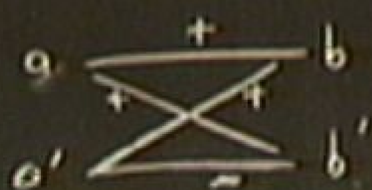

PR-box |X_ab + X_a'b + X_ab' - X_a'b'| = 4
no-signalling ⇒ Prob = 1/2
a
b
a'
b'
+
+
+
-
8 PR-box


Relax Condition of Strong Positivity
PR-box |X_ab + X_a'b + X_ab' - X_a'b'| = 4
no-signalling ⇒ Prob = 1/2
a b a' b'
8 PR-box


Relax Condition of Strong Positivity
PR-box $\left| X_{ab} + X_{a'b} + X_{ab'} - X_{a'b'} \right| = 4$
no-signaling ⟹ Prob = $\frac{1}{2}$
a
b
a'
b'
+
+
+
−
8 PR-box
set of no-signaling probability distributions
form polytope with 24 vertices. 8 vertices are PR
Barrett et al PRA 71 022101

Relax Condition of Strong Positivity

PR-box $\left| X_{ab} + X_{a'b} + X_{ab'} - X_{a'b'} \right| = 4$



no-signaling $\Rightarrow$ Prob $= \frac{1}{2}$

8 PR-box

set of no-signaling probability distributions

form polytope with 24 vertices. 8 vertices are PR box

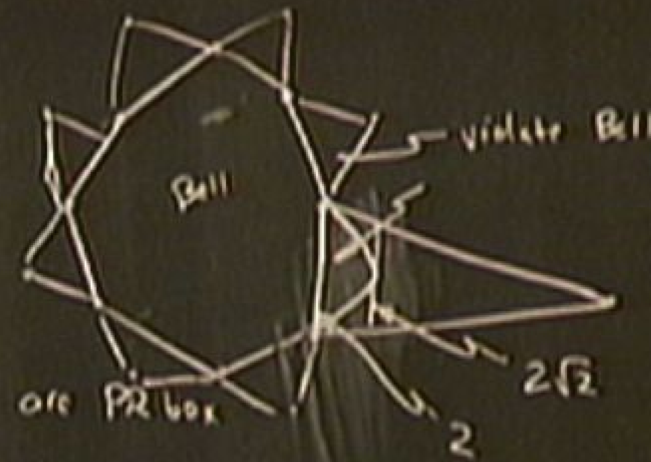
16 vertices deterministic

Barrett et al PRA 71 022101

Relax Condition of Strong Positivity

PR-box $\left| X_{ab} + X_{a'b} + X_{ab'} - X_{a'b'} \right| = 4$

no-signaling ⟹ Prob = $\frac{1}{2}$



8 PR-box

Set of no-signaling probability distributions
form polytope with 24 vertices. 8 vertices are PR box
16 vertices deterministic

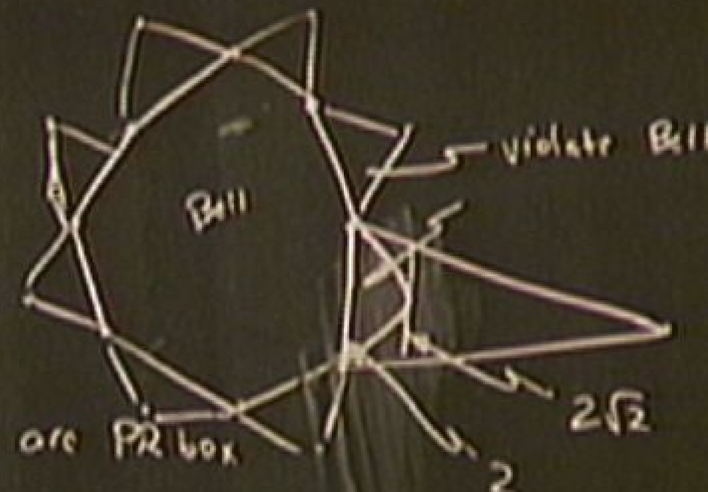
Barrett et al PRA 71 022101

Relax Condition of Strong Positivity

PR-box $\left|X_{ab}+X_{a'b}+X_{ab'}-X_{a'b'}\right|=4$

no-signaling $\Rightarrow$ Prob $=\frac{1}{2}$

8 PR-box



Set of no-signaling probability distributions form polytope with 24 vertices. 8 vertices are PR box

16 vertices deterministic

Barrett et al PRA 71 022101

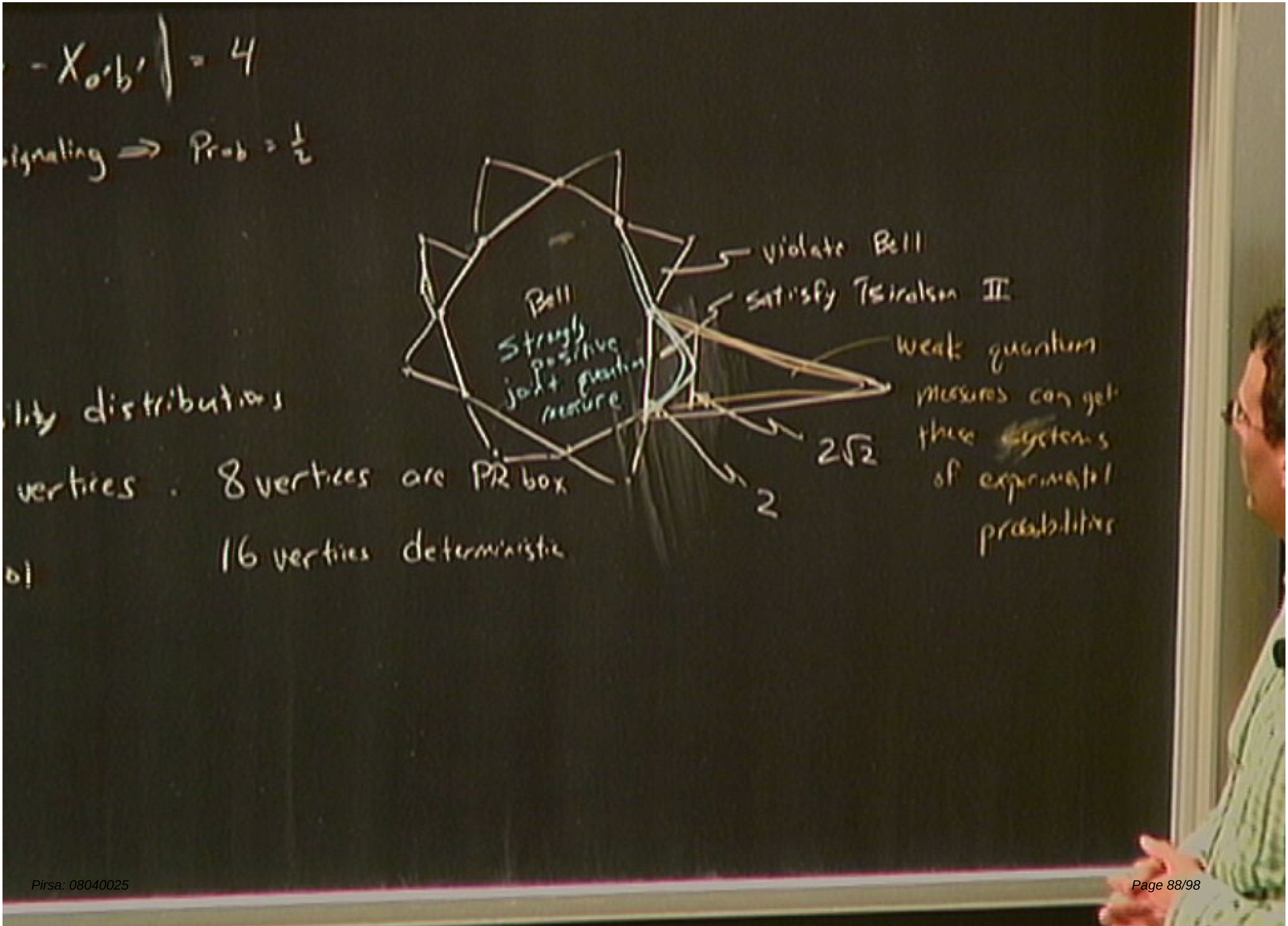
Strongly Positive DF allows construction of Hilbert space

$$\langle [ii'jj'], [kk'\ell\ell'] \rangle = D(ii'jj'; kk'\ell\ell')$$

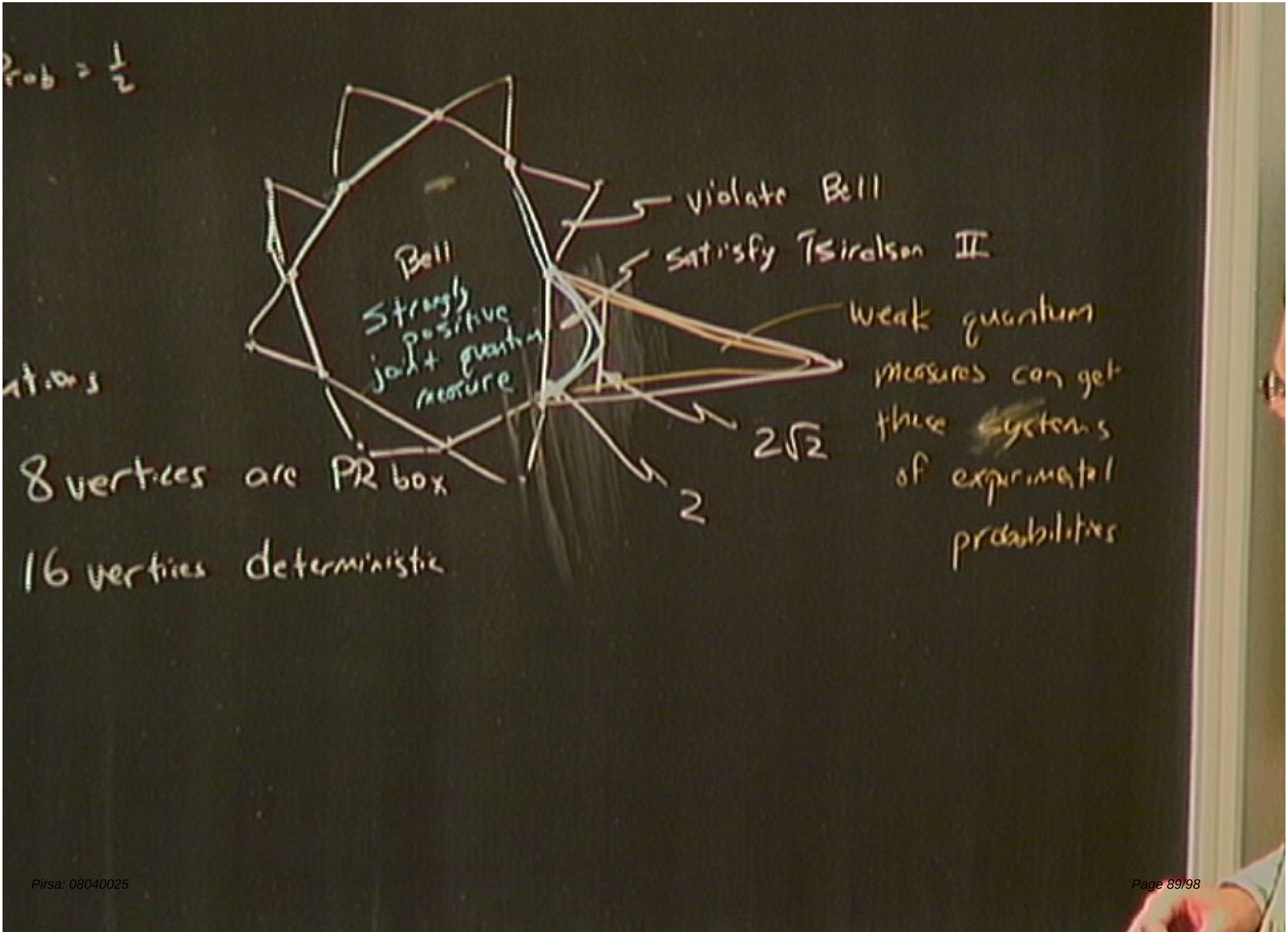
$$\mathcal{H} = \frac{\mathcal{H}_1}{\mathcal{H}_0}$$

$$\left| \operatorname{Arcsin} X_{ab} + \operatorname{Arcsin} X_{a'b} + \operatorname{Arcsin} X_{ab'} - \operatorname{Arcsin} X_{a'b'} \right| \leq \pi$$

"Tsirelson II"



$-X_{a'b'} | = 4$
signaling ⇒ Prob = $\frac{1}{2}$
lity distributions
vertices . 8 vertices are PR box
16 vertices deterministic
Bell
Strongly positive joint quantum measure
violate Bell
satisfy Tsirelson II
weak quantum measures can get these systems of experimental probabilities
$2\sqrt{2}$
2



Bell
Strongly positive joint quantum measure
violate Bell
satisfy Tsirelson II
weak quantum measures can get these systems of experimental probabilities
2√2
2
8 vertices are PR box
16 vertices deterministic

Strongly Positive DF allows construction of Hilbert space

$$\langle [ii'jj'], [kk'll'] \rangle = D(ii'jj'; kk'll')$$

$$\mathcal{H} = \frac{\mathcal{H}_1}{\mathcal{H}_0}$$

$$\left| \operatorname{Arcsin} X_{ab} + \operatorname{Arcsin} X_{a'b} + \operatorname{Arcsin} X_{ab'} - \operatorname{Arcsin} X_{a'b'} \right| \leq \pi$$ "Tsirelson II"

$aa'bb'$

$++++ \quad ++-+ \quad ++-- \quad +--- \quad -+-+ \quad -+-- \quad ---+ \quad ----$

$+++$

$$\left| \mathrm{Arcsin}\, X_{ab} + \mathrm{Arcsin}\, X_{a'b} + \mathrm{Arcsin}\, X_{ab'} - \mathrm{Arcsin}\, X_{a'b'} \right| \le \pi$$

aa'bb'

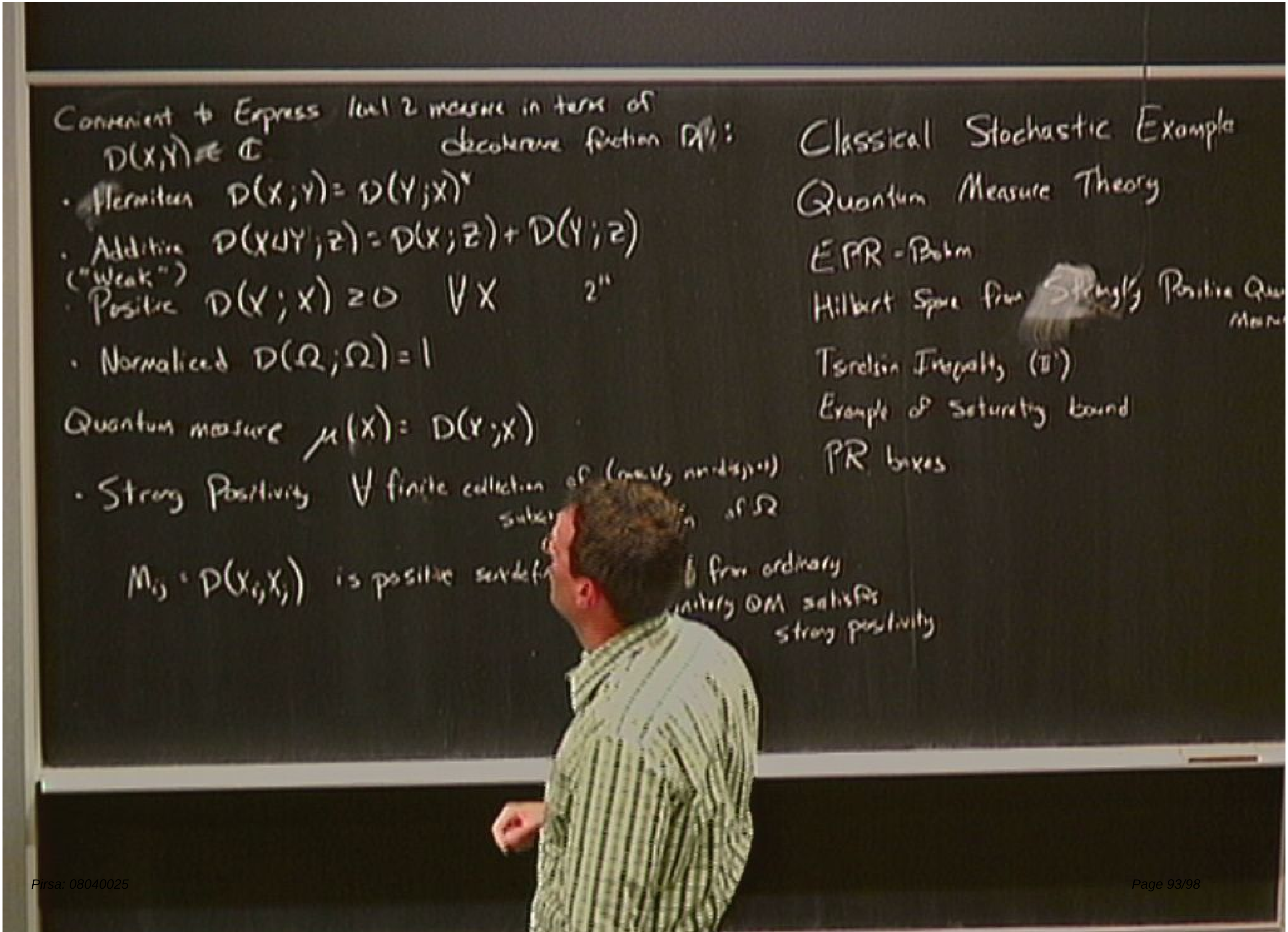
	++++	++-+	++--	+---	-+-+	-+--	---+	----
++++	1/2	-1/4	-1/4					
++-+	1/4	1/2		-1/4	-1/4		-1/4	
++--	1/4						1/4	
+---		1/4			-1/4			
-+-+		1/4		1/4		-1/4	1/4	
-+--						1/2		
---+							1/2	

$$\mathcal{H} = \frac{\cdots}{\mathcal{H}_0}$$

$$\left| \text{Arcsin } X_{ab} + \text{Arcsin } X_{a'b} + \text{Arcsin } X_{ab'} - \text{Arcsin } X_{a'b'} \right| \le \pi$$

aa'bb'

	++++	++-+	++--	+---	-+-+	-+--	---+	----
++++	1/2	-1/4	-1/4					
++-+	1/4	1/2		-1/4	-1/4		-1/4	
++--	1/4							
+---		1/4			-1/4		1/4	
-+-+		1/4		1/4		-1/4	1/4	
-+--						1/2		
---+							1/2	


Convenient to Express level 2 measure in terms of decoherence function
D(X,Y) ∈ ℂ
· Hermitian D(X;Y) = D(Y;X)*
· Additive D(X⊔Y;Z) = D(X;Z) + D(Y;Z)
("Weak")
· Positive D(X;X) ≥ 0 ∀X
· Normalized D(Ω;Ω) = 1
Quantum measure μ(X) = D(X;X)
· Strong Positivity ∀ finite collection of
of Ω
M_ij = D(X_i,X_j) is positive semidefi
from ordinary
QM satisfies
strong positivity
Classical Stochastic Example
Quantum Measure Theory
EPR - Bohm
Hilbert Space from
Positive
Tsirelson Inequality (II')
Example of saturating bound
PR boxes

A_0

$$\left| \mathrm{Arcsin}\, X_{ab} + \mathrm{Arcsin}\, X_{a'b} + \mathrm{Arcsin}\, X_{ab'} - \mathrm{Arcsin}\, X_{a'b'} \right| \le \pi$$

aa'bb'

	++++	++-+	++--	+---	-+-+	-+--	---+	----
++++	1/2	-1/4	-1/4					
++-+	1/4	1/2		-1/4	-1/4		-1/4	
++--	1/4						1/4	
+---		1/4			-1/4			
-+-+		1/4		1/4		-1/4	1/4	
-+--					1/4	1/2		
---+		1/4		-1/4	-1/4		1/2	1/4
----							-1/4	

$X_{a'b'} \Big| \leq \pi$

$aa'bb'$

$- - - + \; - - - -$

$-1/4$

$1/4$

$1/4$

$1/2 \quad 1/4$

$-1/4$

"Tsirelson II" $\Longleftrightarrow$ ordinary quantum model for correlators

· St

$\leq \pi$

aa'bb'

"Tsirelson II" $\Longleftrightarrow$ ordinary quantum model for correlators

$$X_{\alpha\beta} = \langle \psi | S_\alpha S_\beta | \psi \rangle$$

$\frac{1}{4}$

Quantum

· Strong

M_{ij}

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$| \leq \pi$

$aa'bb'$

$\frac{1}{4}$

"Tsirelson II" $\Longleftrightarrow$ ordinary quantum model for correlators

$X_{\alpha\beta} = \langle\psi|S_\alpha S_\beta|\psi\rangle$

"Tsirelson III" $\Longleftrightarrow$ ordinary quantum model for probabilities

$| \leq \pi$

$aa'bb'$

"Tsirelson II" $\Longleftrightarrow$ ordinary quantum model for correlators

$$X_{\alpha\beta} = \langle \psi | S_\alpha S_\beta | \psi \rangle$$

$1/4$

"Tsirelson III" $\Longleftrightarrow$ ordinary quantum model for probabilities,

$$Pr_{\alpha\beta}(i,j) = \langle \psi | P_\alpha^i P_\beta^j | \psi \rangle$$