Title: Symmetry Principles in Physics - Lecture 3

Date: Apr 28, 2008 11:00 AM

URL: http://pirsa.org/08040024

Abstract:

second distinction

"Global Symmetries"

Transformations depend on "constant" parameters

Space-time symmetries related to
boosts
rotations
translations

Global phase in QM

Subject of Noether's first theorem

"Local symmetries"

Transformations depend on functions of x, t

Gauge invariance
General covariance

Subject of Noether's second theorem

The meaning of Noether's theorem

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Bas van Fraassen (1989)
Marc Lange (2007)

traditional

"In the twentieth century we have learned to say that every symmetry yields a conservation law." p. 258

Symmetries "engender" conservation laws. p. 433

van Fraassen, *Laws and Symmetry* 1989

"There are some [quantities] whose constancy is of profound significance, deriving from the fundamental homogeneity and isotropy of space and time ..."

Landau and Lifshitz *Mechanics* 1976

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John Baez 2002

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Katherine Brading & H.R.B. (2003)
H.R.B. & Peter Holland (2004a, b)
Sheldon Smith (2007)

revisionist

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Noether's variational problem

$$S = \int_R \mathcal{L}\, \mathrm{d}^4 x$$

$$\left.\begin{aligned} x^\mu &\to x'^\mu = x^\mu + \delta x^\mu + \cdots \\ \phi_i(x) &\to \phi_i'(x') = \phi_i(x) + \delta\phi_i + \cdots \end{aligned}\right\}$$ *specific*, continuous infinitesimal transformations!

$$\delta S = S[\phi_i', \partial_\mu \phi_i', x'^\mu] - S[\phi_i, \partial_\mu \phi_i, x^\mu]$$
$$= \int_{R'} \mathcal{L}(\phi_i', \partial_\mu \phi_i', x'^\mu)\, \mathrm{d}^4 x' - \int_R \mathcal{L}(\phi_i, \partial_\mu \phi_i, x^\mu)\, \mathrm{d}^4 x$$ to first order

Variational quasi-invariance $\delta S = \int_R \mathrm{d}_\mu(\Lambda^\mu)\, \mathrm{d}^4 x$ for arbitrary region of integration R *(Einstein summation convention)*

⇩

? constraint on dynamics

Noether's variational problem

$$S = \int_R \mathcal{L}\,\mathrm{d}^4x$$

$$\left.\begin{aligned} x^\mu &\to x'^\mu = x^\mu + \delta x^\mu + \cdots \\ \phi_i(x) &\to \phi'_i(x') = \phi_i(x) + \delta\phi_i + \cdots \end{aligned}\right\}$$

specific, continuous infinitesimal transformations!

$$\begin{aligned} \delta S &= S[\phi'_i, \partial_\mu \phi'_i, x'^\mu] - S[\phi_i, \partial_\mu \phi_i, x^\mu] \\ &= \int_{R'} \mathcal{L}(\phi'_i, \partial_\mu \phi'_i, x'^\mu)\,\mathrm{d}^4x' - \int_R \mathcal{L}(\phi_i, \partial_\mu \phi_i, x^\mu)\,\mathrm{d}^4x \end{aligned}$$

to first order

Variational quasi-invariance $\delta S = \int_R \mathrm{d}_\mu(\Lambda^\mu)\,\mathrm{d}^4x$ for arbitrary region of integration R *(Einstein summation convention)*

⇓

? constraint on dynamics

Noether (1918) never used the word "symmetry".

Quasi-invariance

⇩

$$\sum_i E_i^{\mathcal{L}} \delta_0 \phi_i = -\sum_i \mathrm{d}_\mu \left(\frac{\partial \mathcal{L}}{\partial \phi_{i,\mu}} \delta_0 \phi_i + \mathcal{L} \delta x^\mu - \Lambda^\mu \right)$$ Noether condition

where

$$\delta_0 \phi_i = \phi_i'(x) - \phi_i(x)$$

$$E_i^{\mathcal{L}} \equiv \frac{\partial \mathcal{L}}{\partial \phi_i} - \mathrm{d}_\mu \left(\frac{\partial \mathcal{L}}{\partial \phi_{i,\mu}} \right)$$ Euler expression

Noether's first theorem

$$\left.\begin{aligned} \delta x^{\mu} &= \sum_k \omega_k \eta_k^{\mu}(x) \\ \delta_0 \phi_i &= \sum_k \omega_k \xi_{ki}(x) \\ \Lambda^{\mu} &= \sum_k \omega_k \zeta_k^{\mu}(x) \end{aligned}\right\} \Rightarrow$$

$$\sum_i E_i^{\mathcal{L}} \xi_{ki} = \mathrm{d}_{\mu} j_k^{\mu}$$

where

$$j_k^{\mu} = -\sum_i \left(\frac{\partial \mathcal{L}}{\partial \phi_{i,\mu}} \xi_{ki} \right) - \mathcal{L} \eta_k^{\mu} + \zeta_k^{\mu}$$

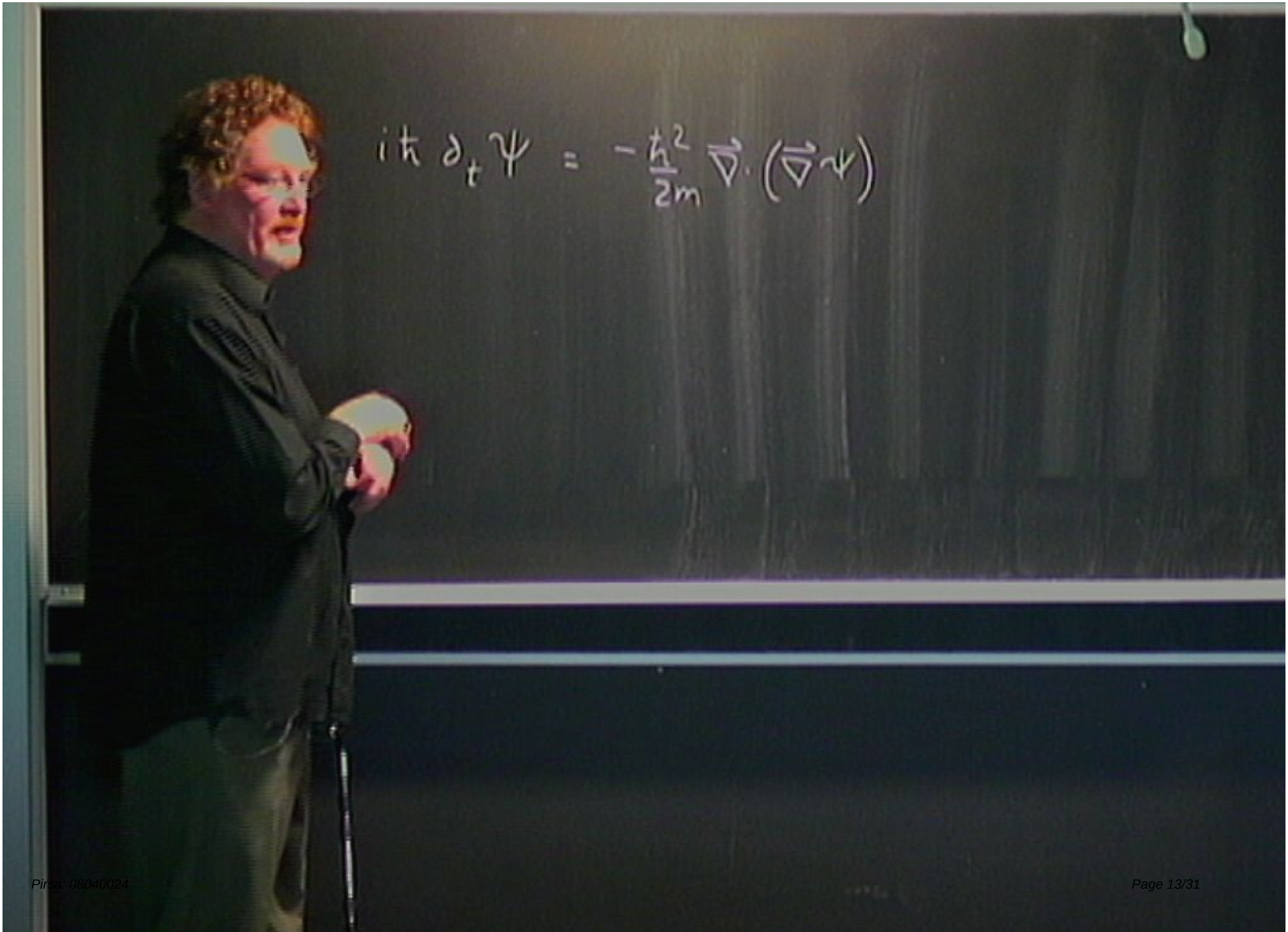
If LHS vanishes. $\mathrm{d}_{\mu} j_k^{\mu} = 0$

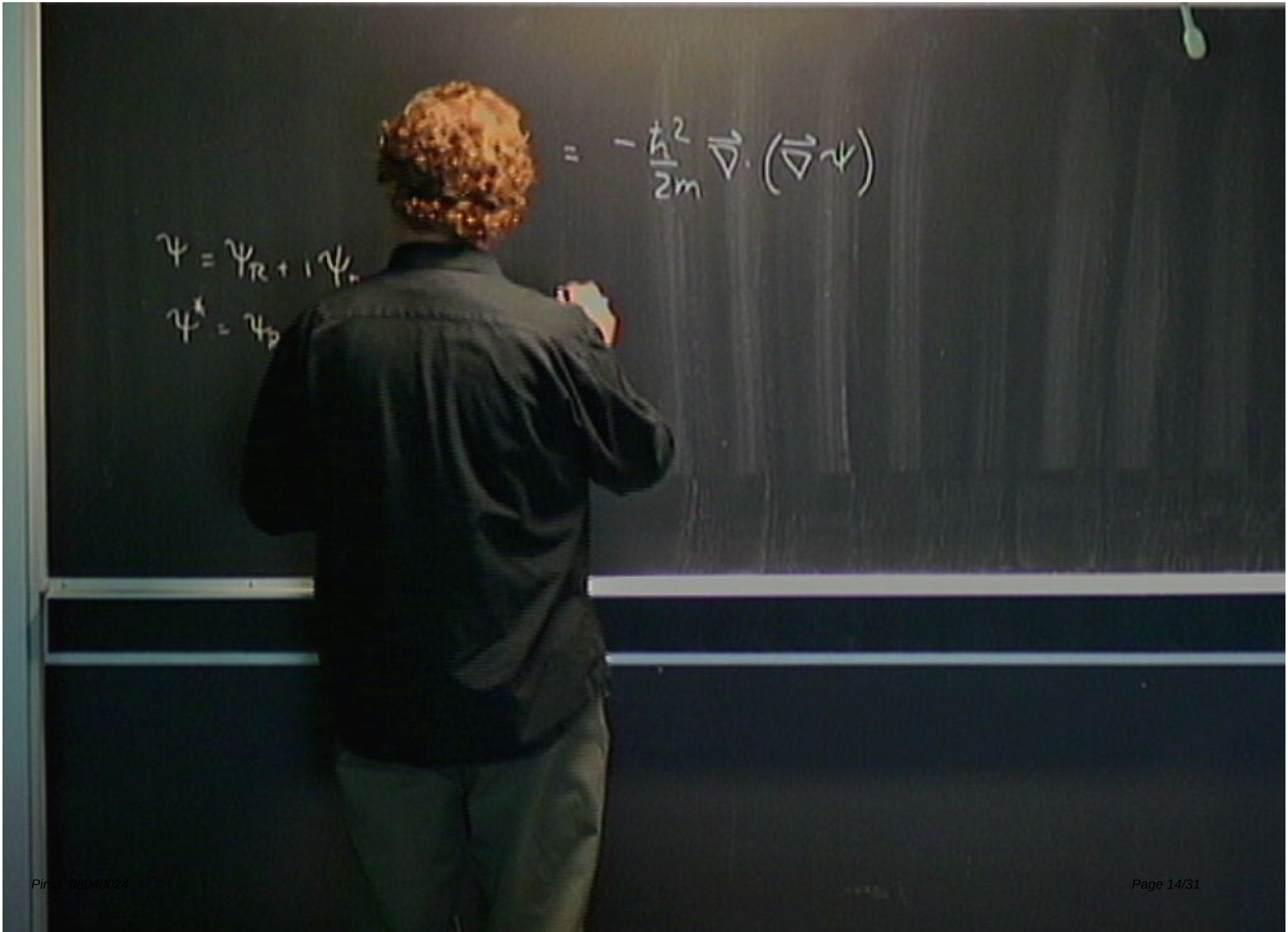
Given suitable boundary conditions. $\frac{\mathrm{d}}{\mathrm{d}t} \int j_k^0 \, \mathrm{d}x \equiv \frac{\mathrm{d}Q_k}{\mathrm{d}t} = 0$

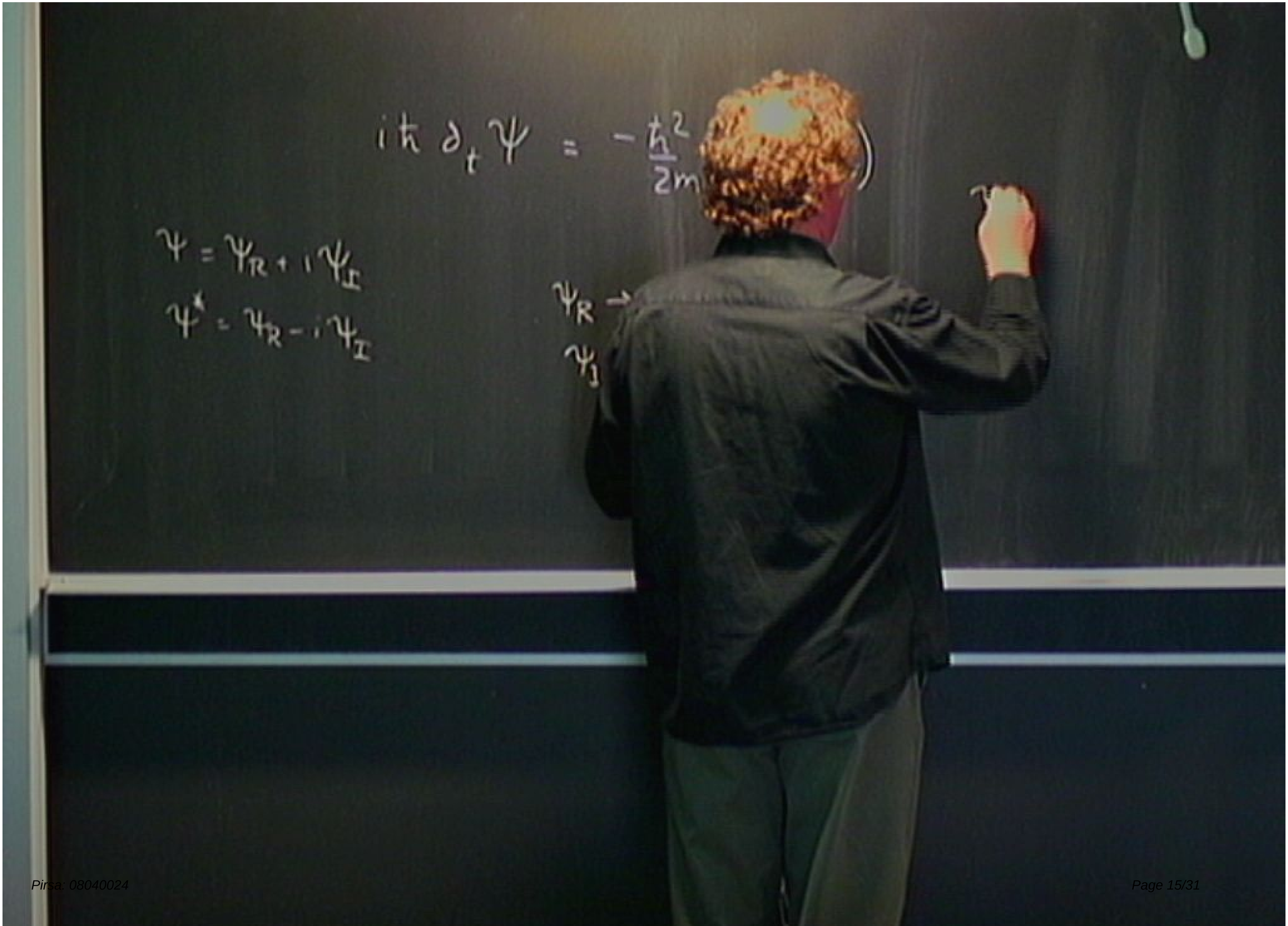
complications & subtleties

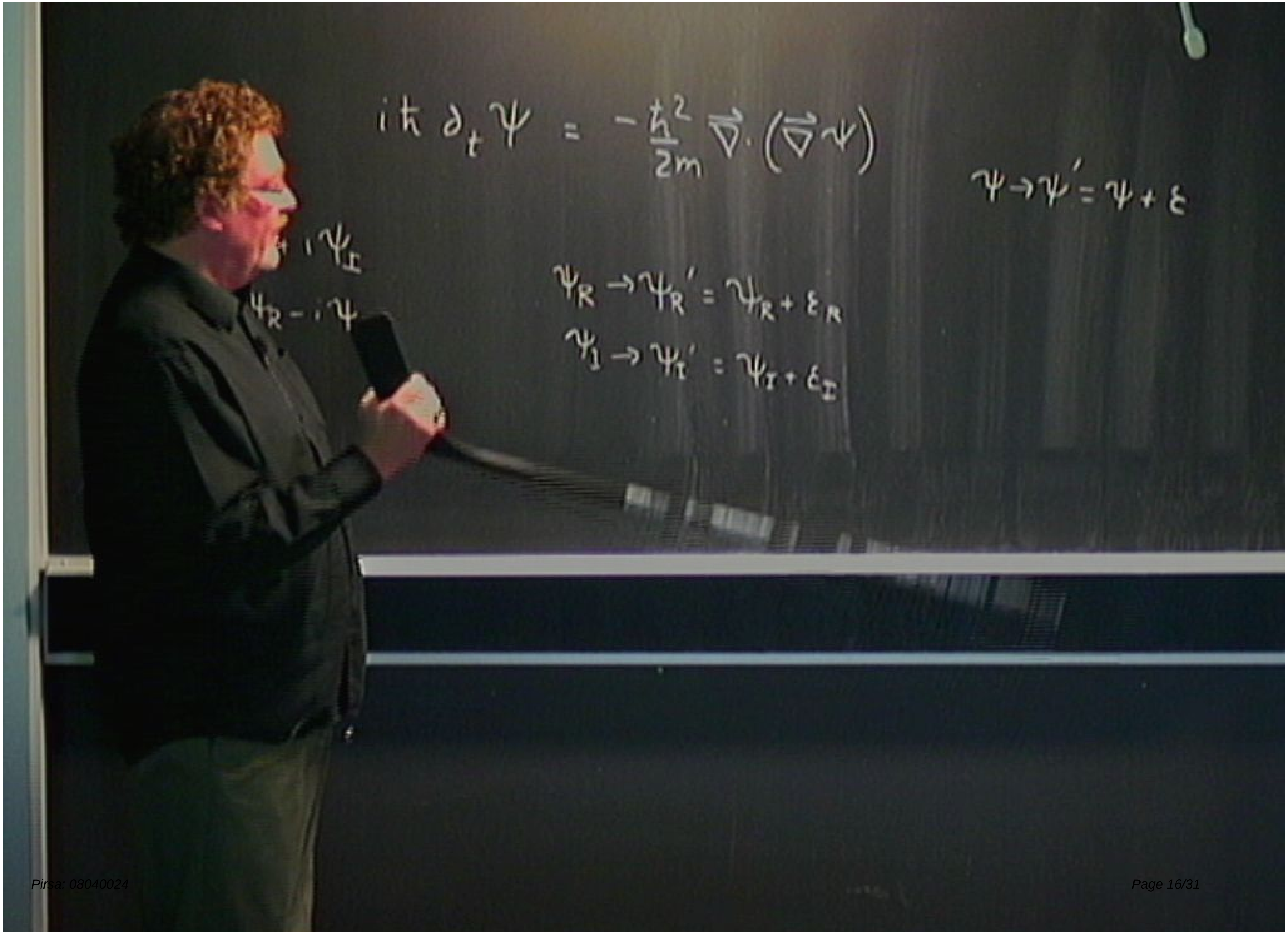
- Not all dynamical systems are susceptible to a Lagrangian formulation. (Wigner 1954)
- Not all symmetries are "Noetherian".
- The symmetry may not lead to a continuity equation, let alone a conserved quantity.
- The continuity equation may not lead to a conserved quantity ("charge").
- The continuity equation may be trivial, i.e. identical to the equations of motion.
- The charge need not be real-valued.
- The Noether symmetry may not carry states into states.
- Which symmetry is associated with a given conserved quantity can depend on the choice of the Lagrangian.

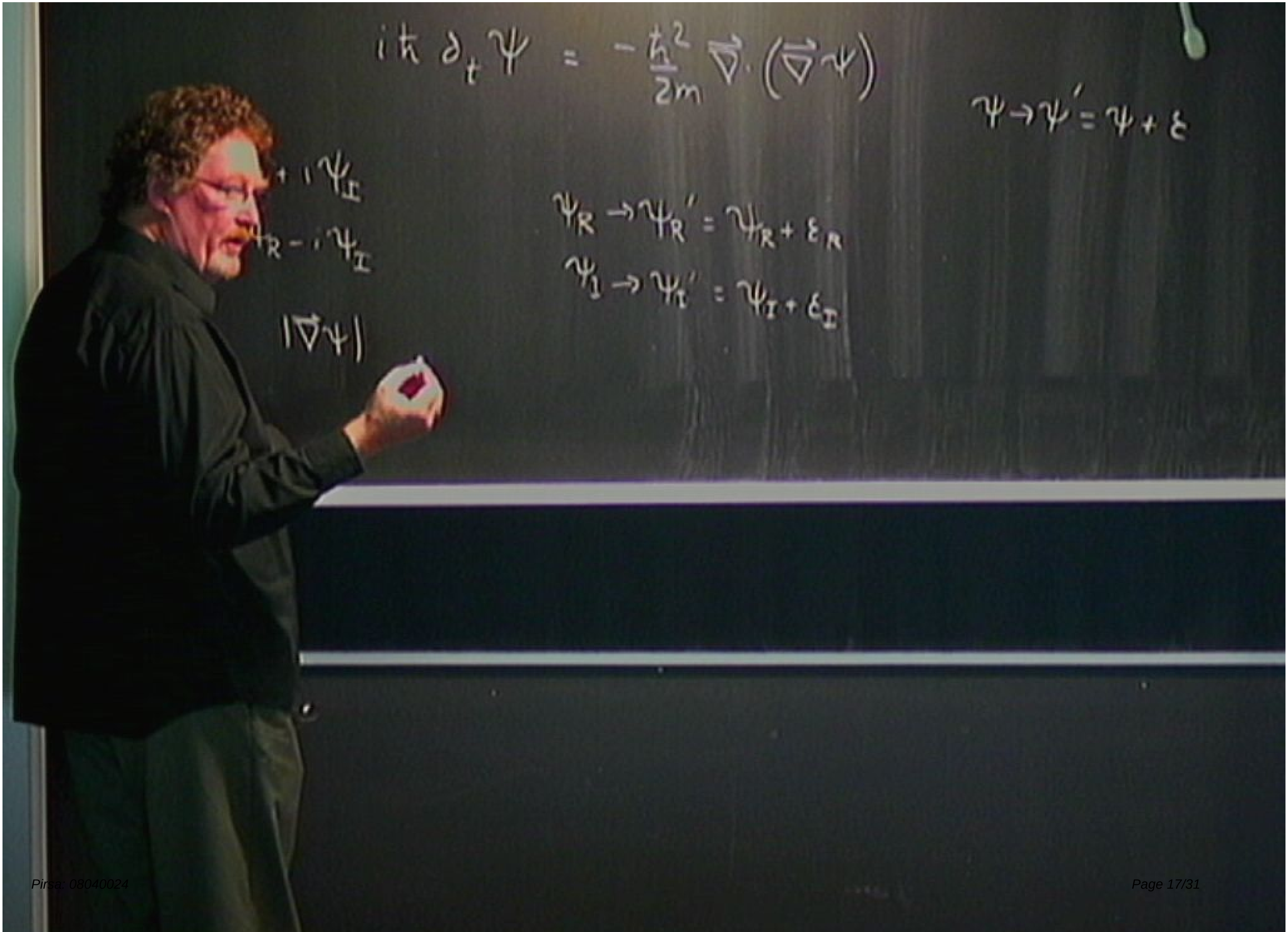
H.R.B. & Holland (2004a, b); Smith (2007)


$i\hbar \partial_t \psi = -\frac{\hbar^2}{2m} \vec{\nabla} \cdot (\vec{\nabla} \psi)$

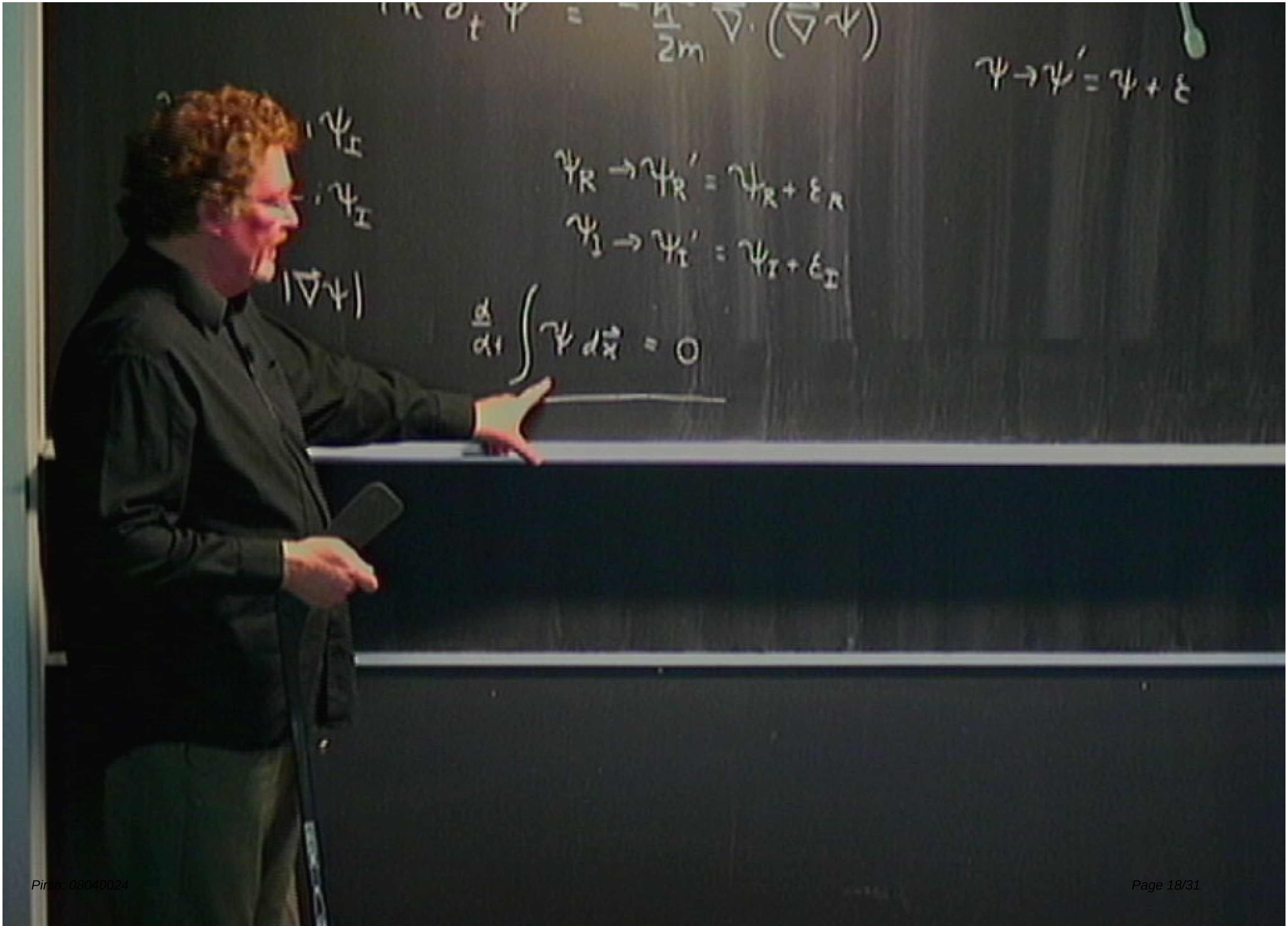

$= -\frac{\hbar^2}{2m}\vec{\nabla}\cdot(\vec{\nabla}\psi)$
$\psi = \psi_R + i\psi_I$
$\psi^* = \psi_R$

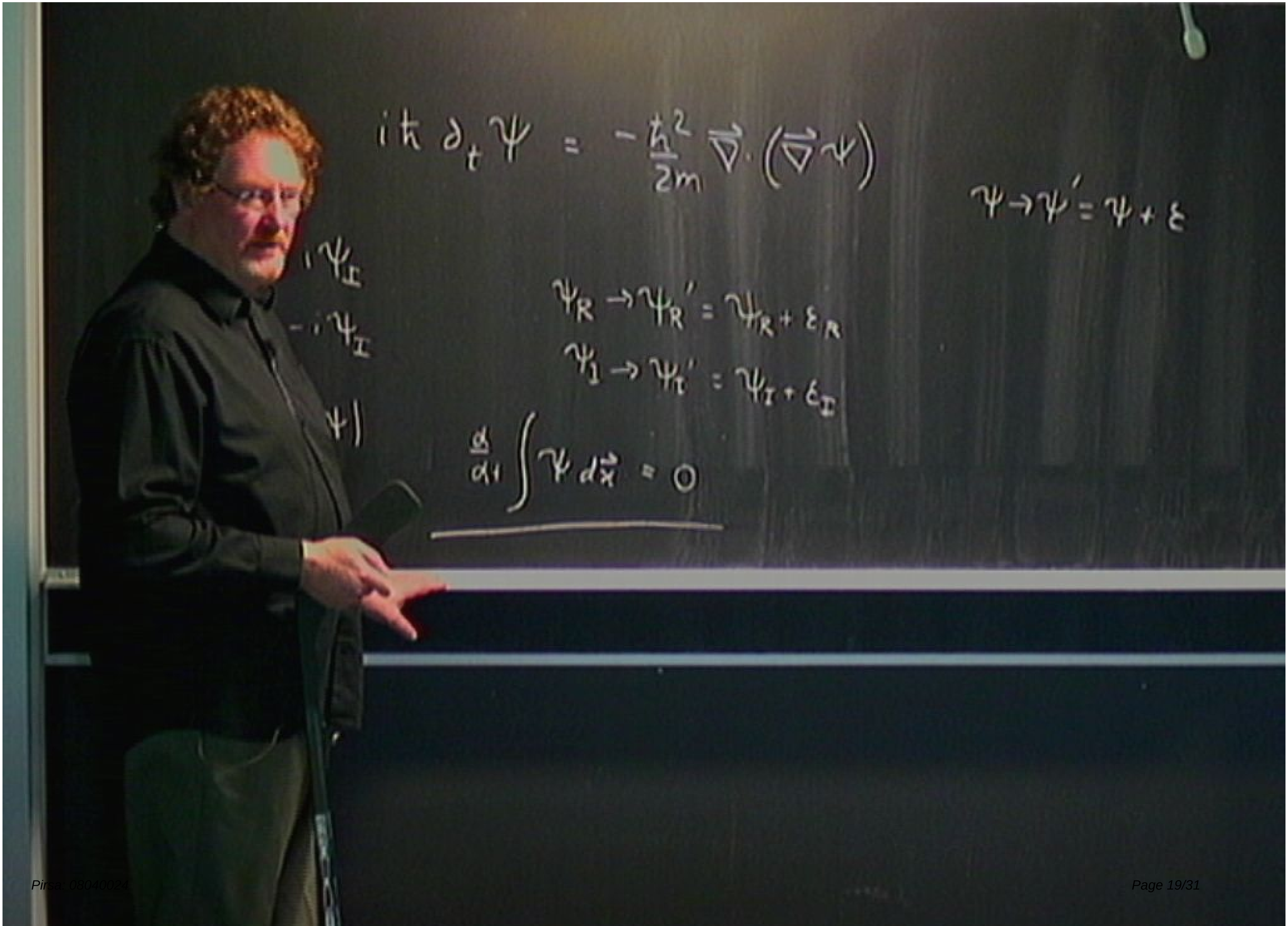

$i\hbar\,\partial_t \psi = -\frac{\hbar^2}{2m}$
$\psi = \psi_R + i\psi_I$
$\psi^* = \psi_R - i\psi_I$
$\psi_R \rightarrow$
ψ_I

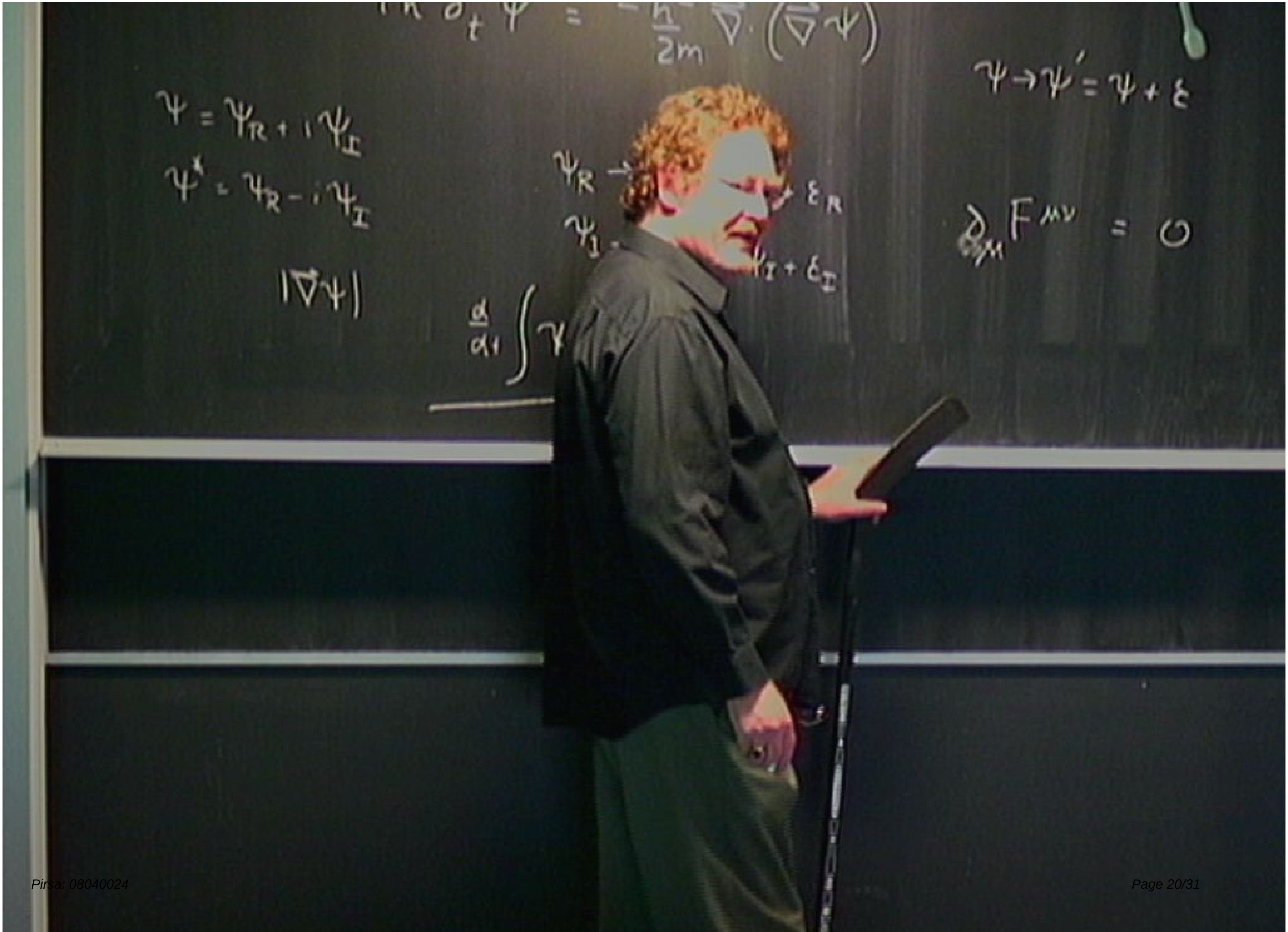

$i\hbar \, \partial_t \psi = -\frac{\hbar^2}{2m} \vec{\nabla} \cdot (\vec{\nabla} \psi)$
$\psi \to \psi' = \psi + \varepsilon$
$\psi_R \to \psi_R' = \psi_R + \varepsilon_R$
$\psi_I \to \psi_I' = \psi_I + \varepsilon_I$

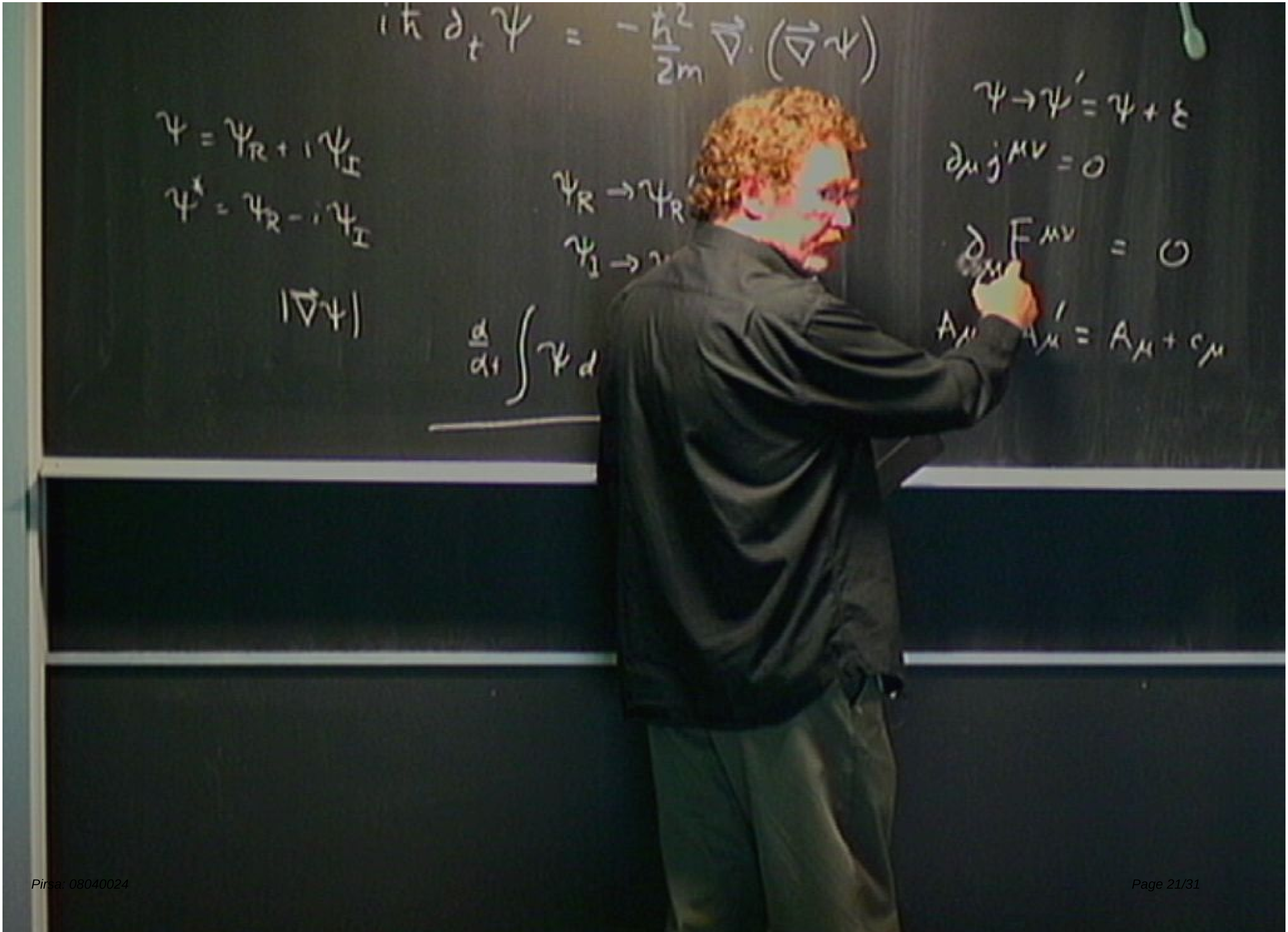


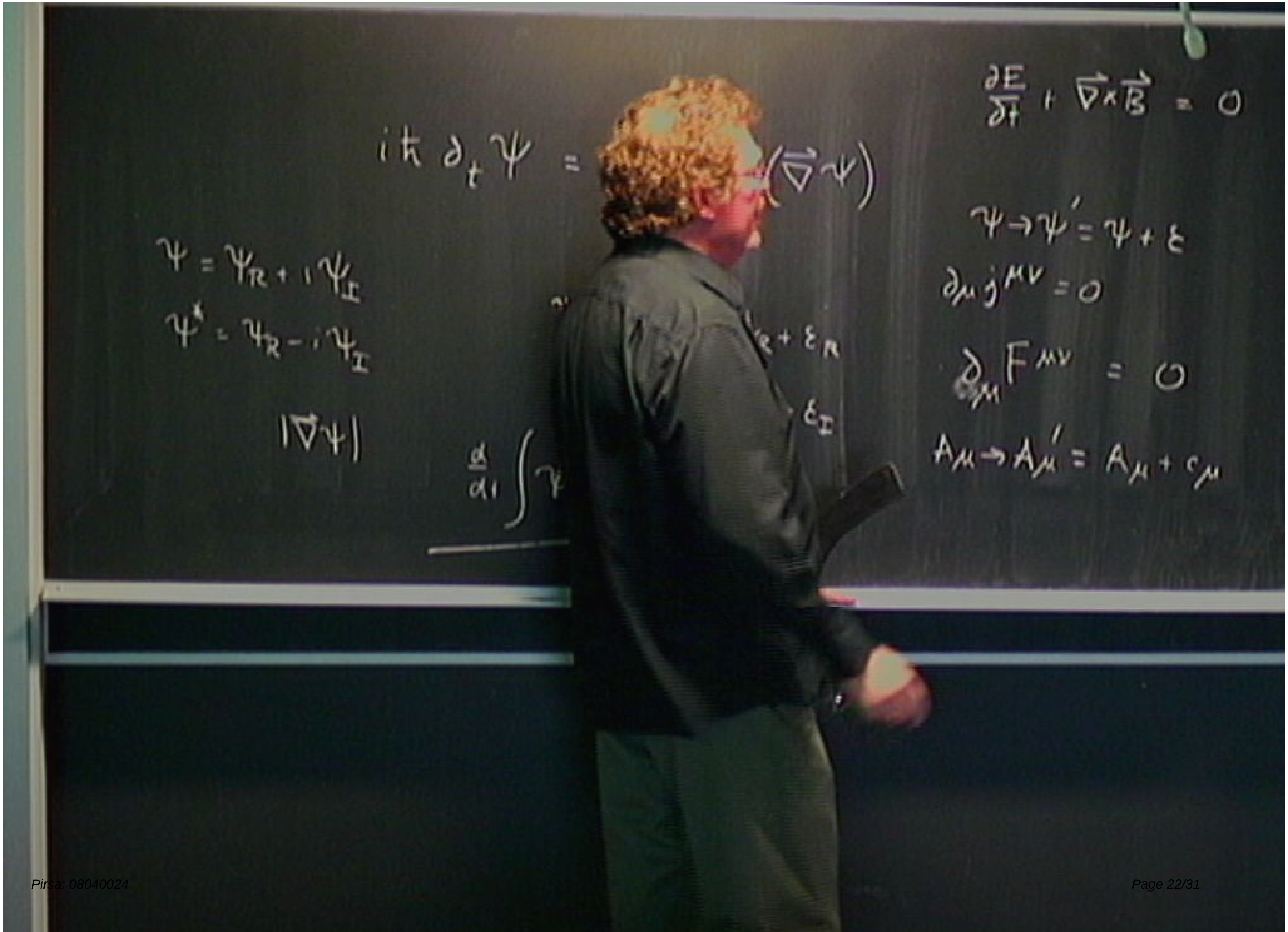
$i\hbar\, \partial_t \Psi = -\frac{\hbar^2}{2m} \vec{\nabla} \cdot (\vec{\nabla} \psi)$
$\psi \rightarrow \psi' = \psi + \varepsilon$
$\psi_R \rightarrow \psi_R' = \psi_R + \varepsilon_R$
$\psi_I \rightarrow \psi_I' = \psi_I + \varepsilon_I$
$|\vec{\nabla} \psi|$

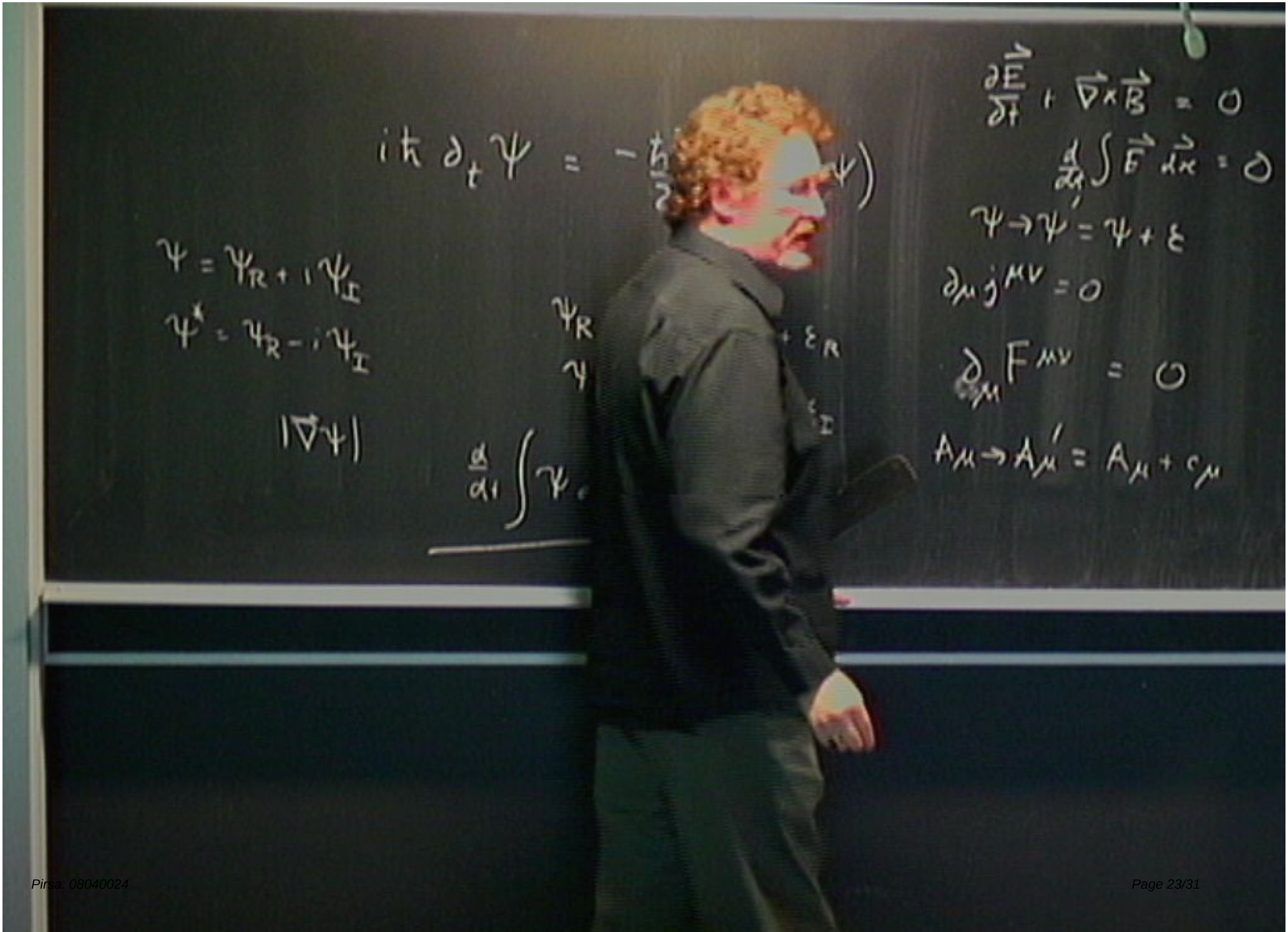

ψ → ψ' = ψ + ε
ψ_R → ψ_R' = ψ_R + ε_R
ψ_I → ψ_I' = ψ_I + ε_I
d/dt ∫ ψ dx⃗ = 0

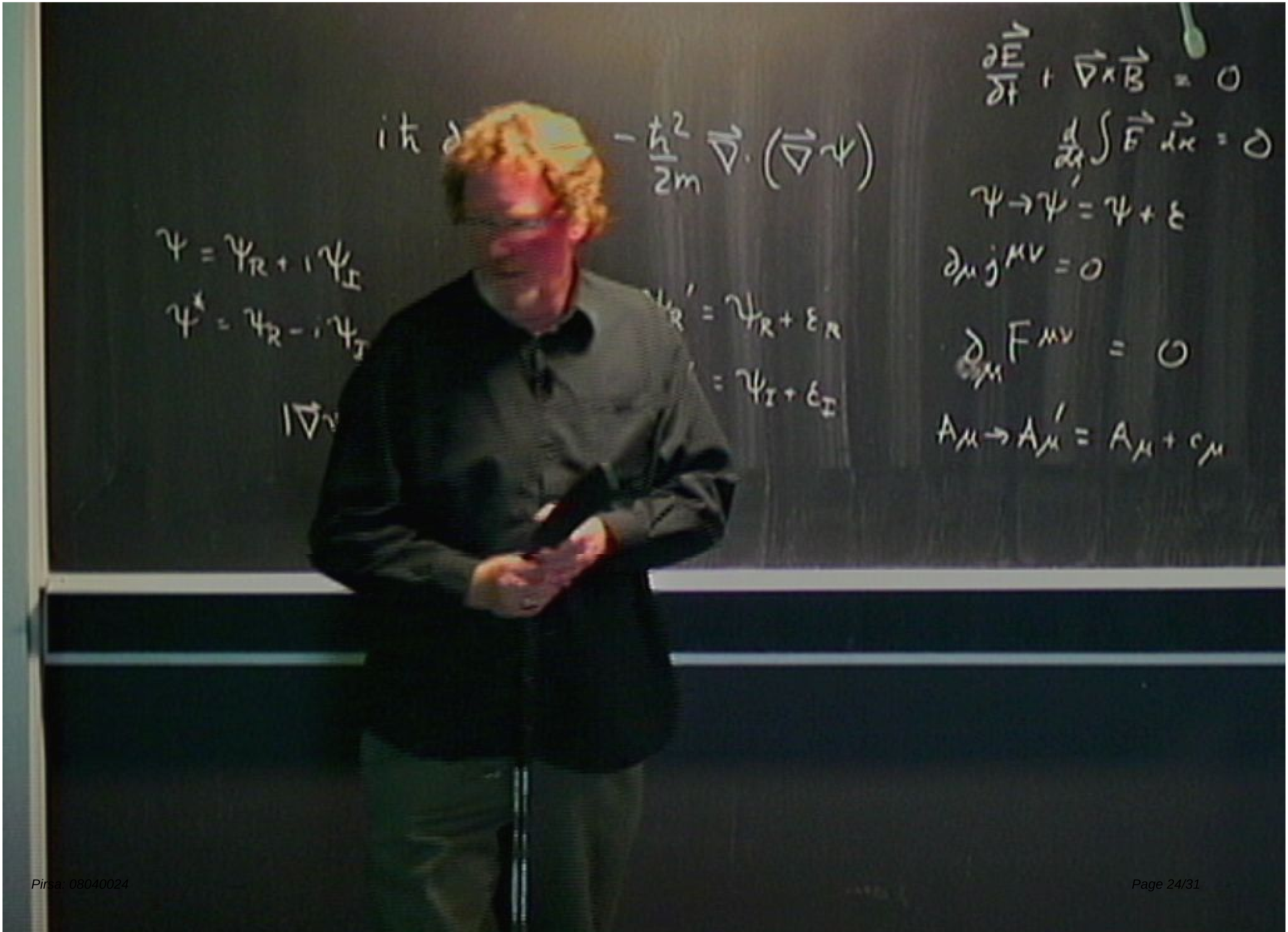

iħ ∂t Ψ = − ħ²/2m ∇⃗·(∇⃗Ψ)
ψ → ψ′ = ψ + ε
ψR → ψR′ = ψR + εR
ψI → ψI′ = ψI + εI
d/dt ∫ ψ dx⃗ = 0


ψ = ψR + i ψI
ψ* = ψR − i ψI
|∇ψ|
ψ → ψ' = ψ + ε
∂μ F^μν = 0


$i\hbar \partial_t \psi = -\frac{\hbar^2}{2m} \vec{\nabla} \cdot (\vec{\nabla} \psi)$
$\psi = \psi_R + i\psi_I$
$\psi^* = \psi_R - i\psi_I$
$|\vec{\nabla}\psi|$
$\psi \rightarrow \psi' = \psi + \varepsilon$
$\partial_\mu j^{\mu\nu} = 0$
$\partial_\mu F^{\mu\nu} = 0$
$A_\mu' = A_\mu + c_\mu$


∂E/∂t + ∇×B = 0
iħ ∂_t ψ =
(∇ψ)
ψ → ψ' = ψ + ε
∂_μ j^μν = 0
∂_μ F^μν = 0
A_μ → A_μ' = A_μ + c_μ
ψ = ψ_R + iψ_I
ψ* = ψ_R − iψ_I
|∇ψ|


∂E/∂t + ∇×B = 0
ψ → ψ' = ψ + ε
∂μ j^μν = 0
∂μ F^μν = 0
Aμ → Aμ' = Aμ + cμ
ψ = ψR + iψI
ψ* = ψR − iψI
|∇ψ|


$\frac{\partial \vec{E}}{\partial t} + \vec{\nabla} \times \vec{B} = 0$
$\frac{d}{dt} \int \vec{E}\, d\vec{x} = 0$
$i\hbar\, \partial$
$- \frac{\hbar^2}{2m} \vec{\nabla} \cdot (\vec{\nabla} \psi)$
$\psi \to \psi' = \psi + \epsilon$
$\psi = \psi_R + i\psi_I$
$\psi^* = \psi_R - i\psi_I$
$\partial_\mu j^{\mu\nu} = 0$
$\psi_R' = \psi_R + \epsilon_R$
$= \psi_I + \epsilon_I$
$\partial_\mu F^{\mu\nu} = 0$
$A_\mu \to A_\mu' = A_\mu + c_\mu$

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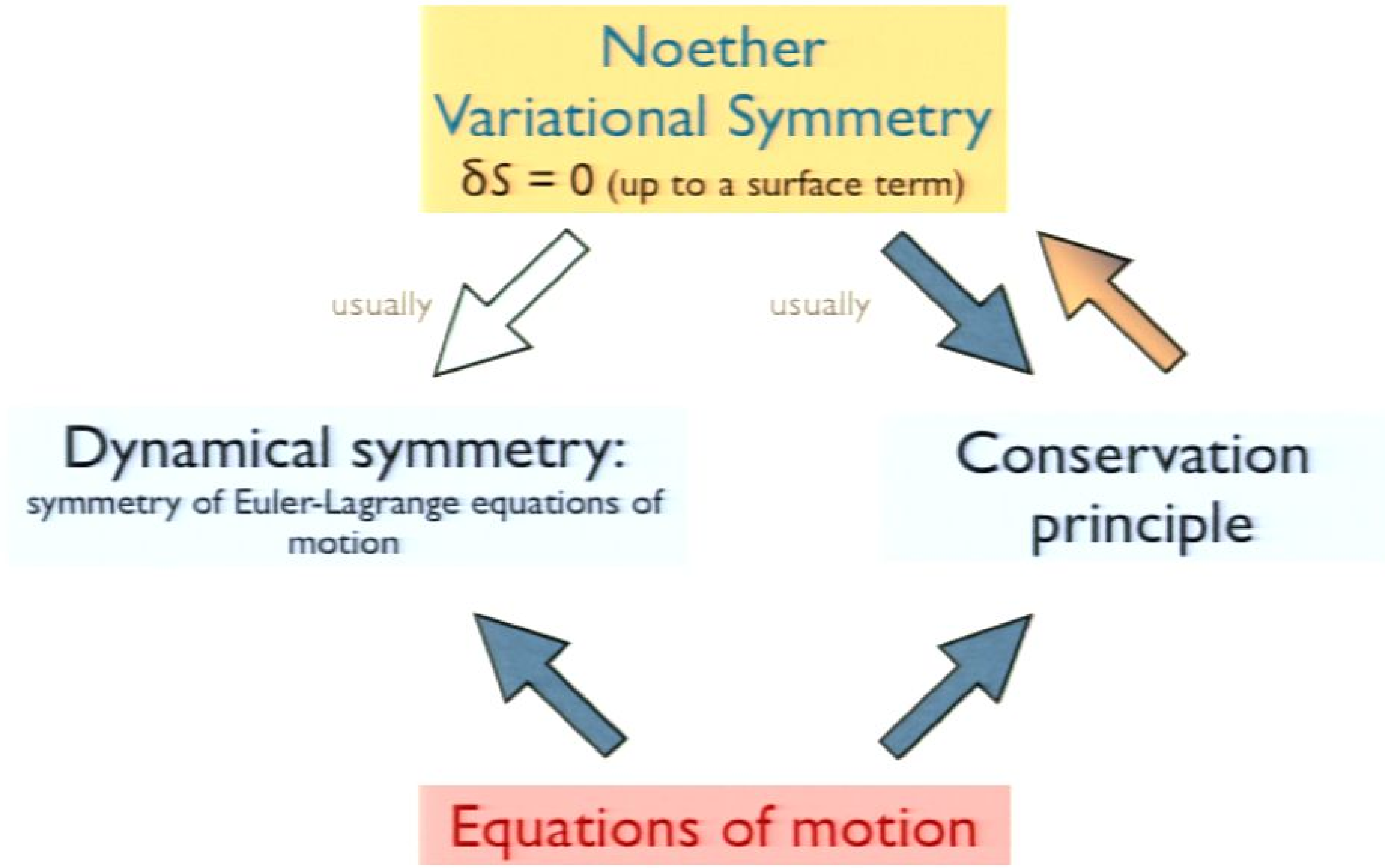
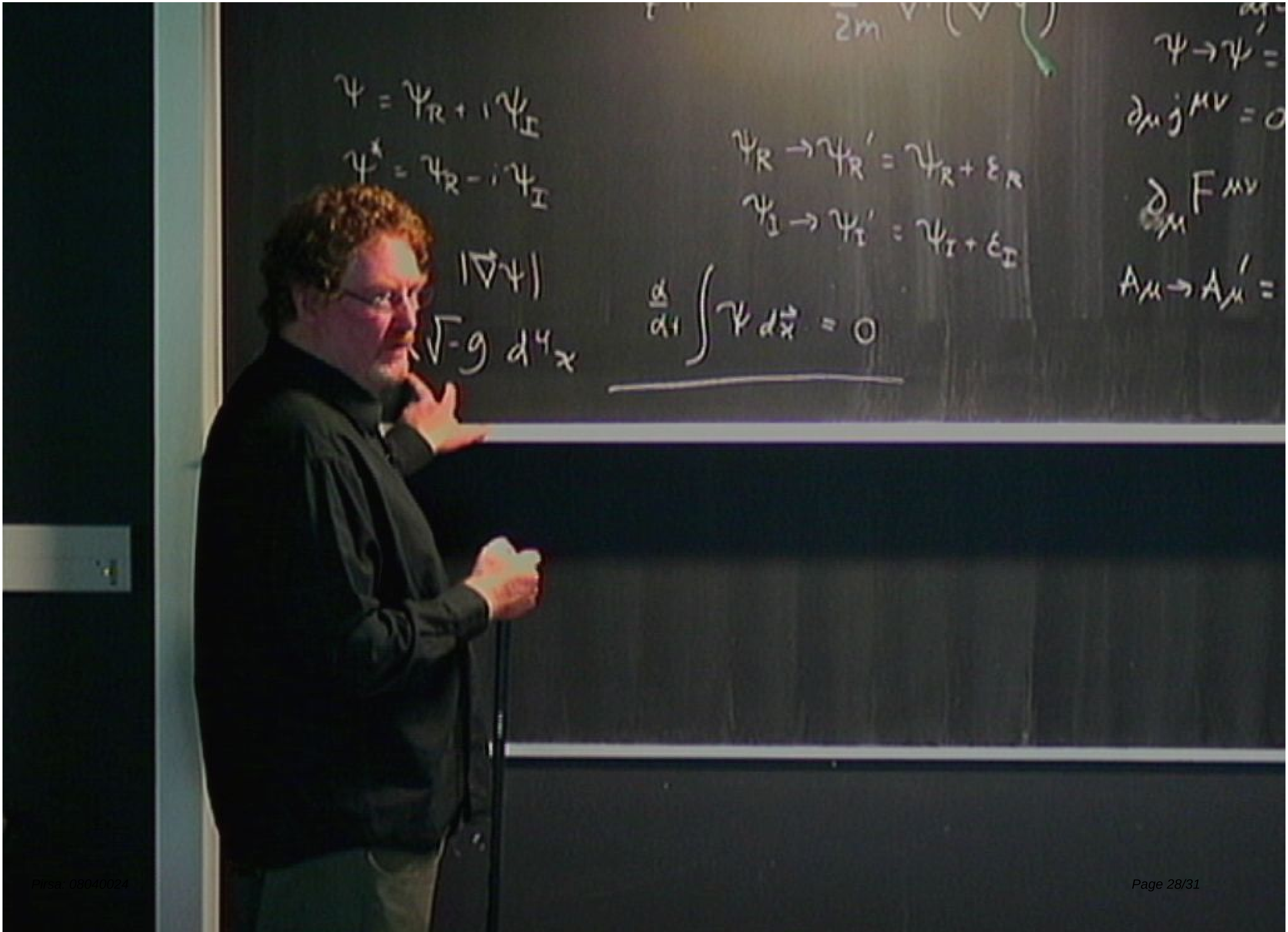

Noether
Variational Symmetry
$\delta S = 0$ (up to a surface term)
usually
usually
Dynamical symmetry:
symmetry of Euler-Lagrange equations of motion
Conservation principle
Equations of motion


$2m$
$\Psi \to \Psi' =$
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$\Psi_I \to \Psi_I' = \Psi_I + \varepsilon_I$
$|\nabla\Psi|$
$A_\mu \to A_\mu' =$
$\frac{d}{dt}\int \Psi \, d\vec{x} = 0$
$\sqrt{-g}\, d^4x$

References

The significance of Noether's first theorem

Marc Lange, "Laws and meta-laws of nature: Conservation laws and symmetries", *Stud. Hist. Phil. Mod. Phys.* **38**(3) 457-481(2007).

Sheldon Smith, "Laws and Symmetries in the Light of the Inverse Problem in Lagrangian Mechanics", unpublished ms 2007

H. R. B. and Peter Holland, "Dynamical versus variational symmetries: understanding Noether's first theorem" *Molecular Physics*, **102**, No. 11-12, 1133-1139 (2004). PITT-PHIL-SCI 2194.

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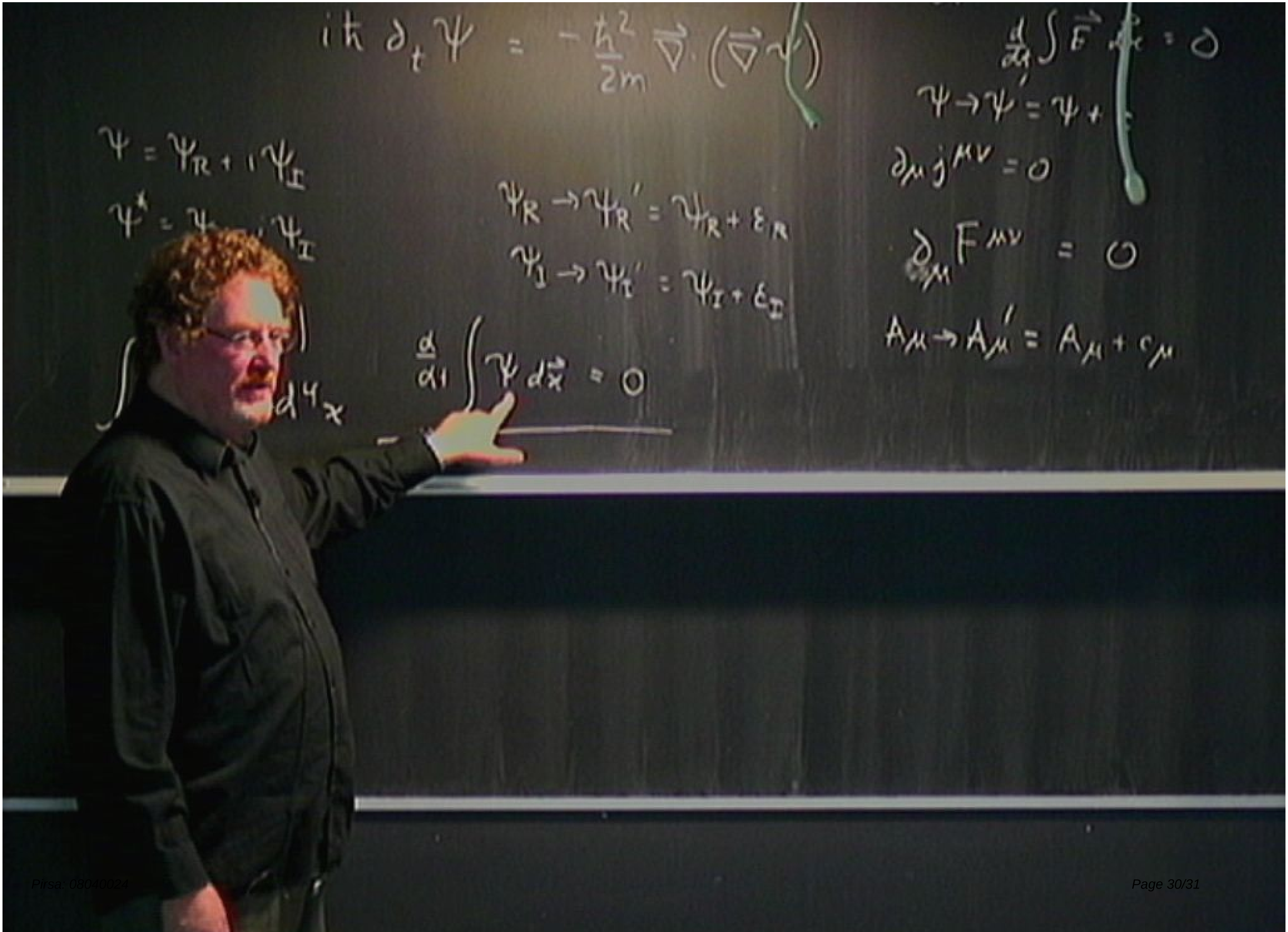
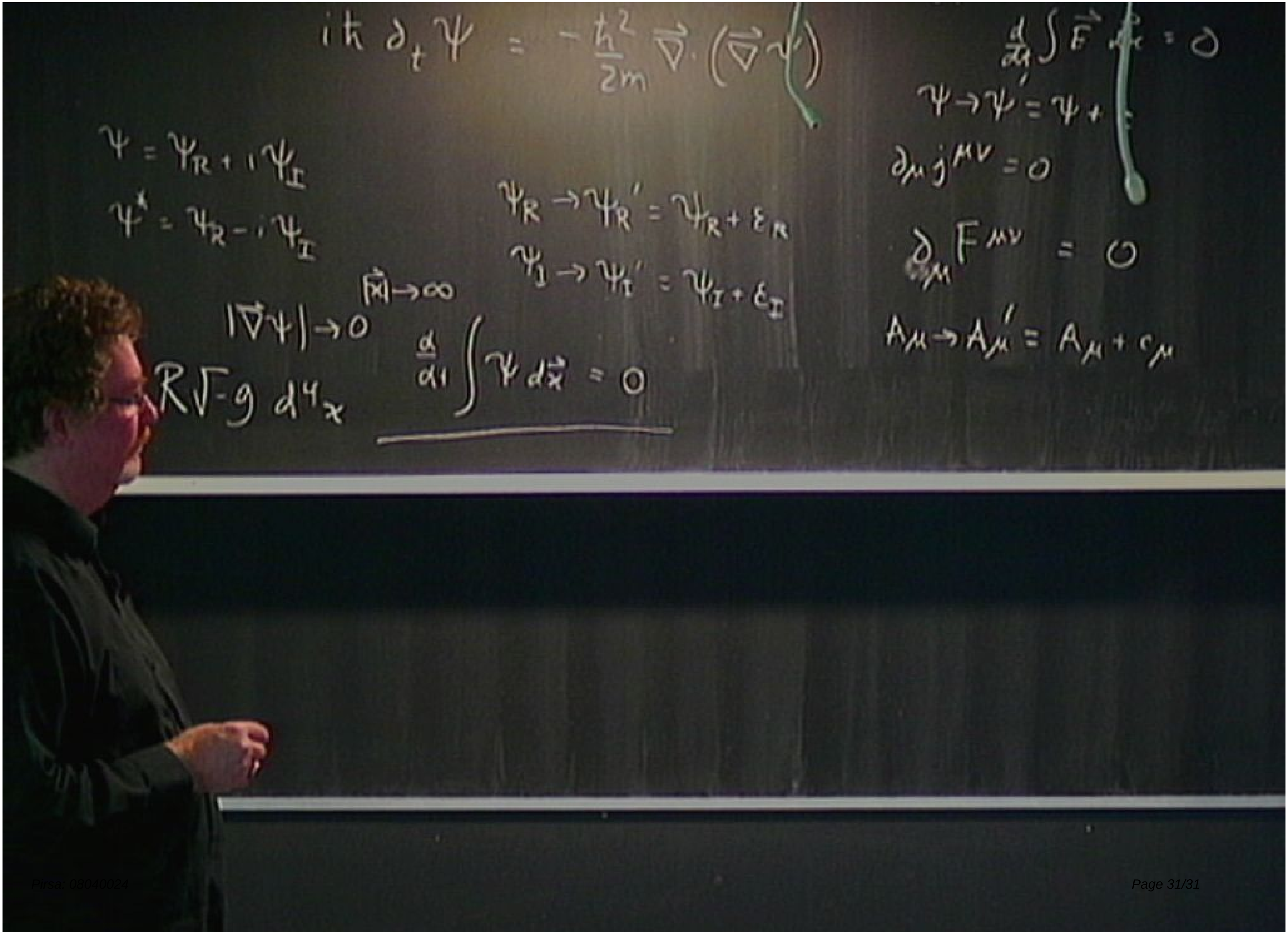
Recent applications

H.R.B. and Peter Holland, "Simple applications of Noether's first theorem in quantum mechanics and electromagnetism", *American Journal of Physics* **72**, 34-39 (2004); quant-ph/0302062

Gauge symmetries

K. Brading and H.R.B. "Gauge symmetry transformations observed?", *British Journal for the Philosophy of Science* **55**, 645-665 (2004). PITT-PHIL-SCI 1436.

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$\psi_R \to \psi_R' = \psi_R + \varepsilon_R$
$\psi_I \to \psi_I' = \psi_I + \varepsilon_I$
$|\vec{x}| \to \infty$
$|\vec{\nabla}\psi| \to 0$
$R\sqrt{-g}\, d^4x$
$\frac{d}{dt}\int \psi\, d\vec{x} = 0$
$\partial_\mu j^{\mu\nu} = 0$
$\partial_\mu F^{\mu\nu} = 0$
$A_\mu \to A_\mu' = A_\mu + c_\mu$