Title: The Exact Renormalization Group - Lecture 2: Exact Renormalization Group Equations

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Abstract: I will show how to construct very general ERG equations, and will use this as the starting point for a discussion of Polchinski\'s equation and its cousins. I will introduce diagrammatics and an associated universal calculus, which will be illustrated with a simple calculation.

The Exact Renormalization Group: Introduction & Applications

Lecture 2: ERG Equations

Oliver J. Rosten

Dublin Institute for Advanced Studies

April / May 2008

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Secs 1, 2 (3)

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- C. Bagnuls and C. Bervillier, "Exact renormalization group equations: An introductory review," Phys. Rept. **348** (2001) 91, hep-th/0002034.
- F. Wegner , "The Critical State, General Aspects," in: C. Domb and M. S. Green (Eds.), Phase Transitions and Critical Phenomena, Vol VI, Acad. Press, N.-Y., 1976 p. 7.
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Continued...

- K. Wilson and J. Kogut, “The Renormalization group and the epsilon expansion,” Phys. Rept. **12** (1974) 75.
- F. J. Wegner and A. Houghton, “Renormalization group equation for critical phenomena,” Phys. Rev. A **8** (1973) 401.
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Useful Concepts from Lecture 1

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- The ERG relates physics at different scales via a coarse-graining procedure
- There is a huge freedom in the way we coarse-grain (block)
- Universal physics is independent of the coarse-graining procedure
- To conveniently see fixed point behaviour etc., we need to rescale to dimensionless units
- Along Renormalized Trajectories, the action is self-similar

$$S_\Lambda[\varphi] = S[\varphi](g_1(\Lambda), \ldots, g_n(\Lambda); \gamma(\Lambda))$$

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1. General ERG Equations
   - Formulation
   - Other ERG Equations
2. Diagrammatics
3. One-loop β-function
4. Recap

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$$Z = \int_{\Lambda_0} \mathcal{D}\Phi e^{-S_{\Lambda_0}[\Phi]} = \int_{\Lambda} \mathcal{D}\Phi e^{-S_{\Lambda}[\Phi]}$$

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Implementing a Cutoff

1. Implement an overall UV cutoff:

$$\Delta = \frac{1}{p^2} \to \frac{C(p, \Lambda_0)}{p^2}$$

2. Introduce the effective cutoff, Λ

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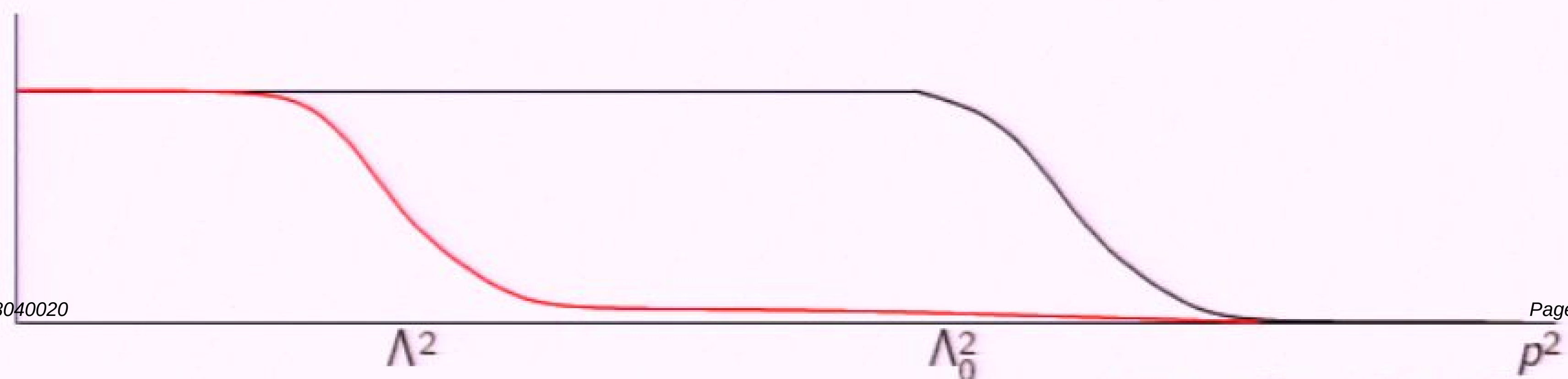

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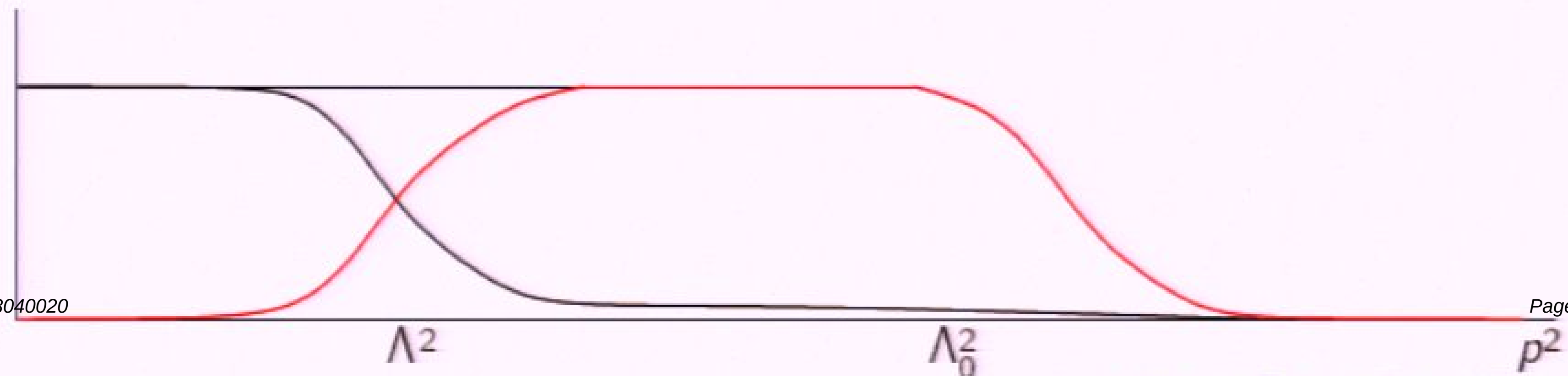

Polchinski Equation

Polchinski Equation

An equation telling us how the Wilsonian effective action evolves

$$-\Lambda\partial_\Lambda S_\Lambda^{\rm int} = \frac{1}{2}\frac{\delta S_\Lambda^{\rm int}}{\delta\varphi}\cdot\dot{\Delta}_{\rm UV}\cdot\frac{\delta S_\Lambda^{\rm int}}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\dot{\Delta}_{\rm UV}\cdot\frac{\delta S_\Lambda^{\rm int}}{\delta\varphi}$$

- Interaction part of Wilsonian effective action

$$S_\Lambda = \frac{1}{2}\varphi\cdot C_{\rm UV}^{-1}(p,\Lambda)p^2\cdot\varphi + S_\Lambda^{\rm int}$$

- Any mass term is contained in $S_\Lambda^{\rm int}$
- $\dot{\Delta}_{\rm UV} \equiv -\Lambda\partial_\Lambda\Delta_{\rm UV} = -\Lambda\partial_\Lambda\frac{C_{\rm UV}}{p^2}$
- $f\cdot\dot{\Delta}_{\rm UV}\cdot g = \int_p f(-p)\dot{\Delta}_{\rm UV}(p)g(p)$
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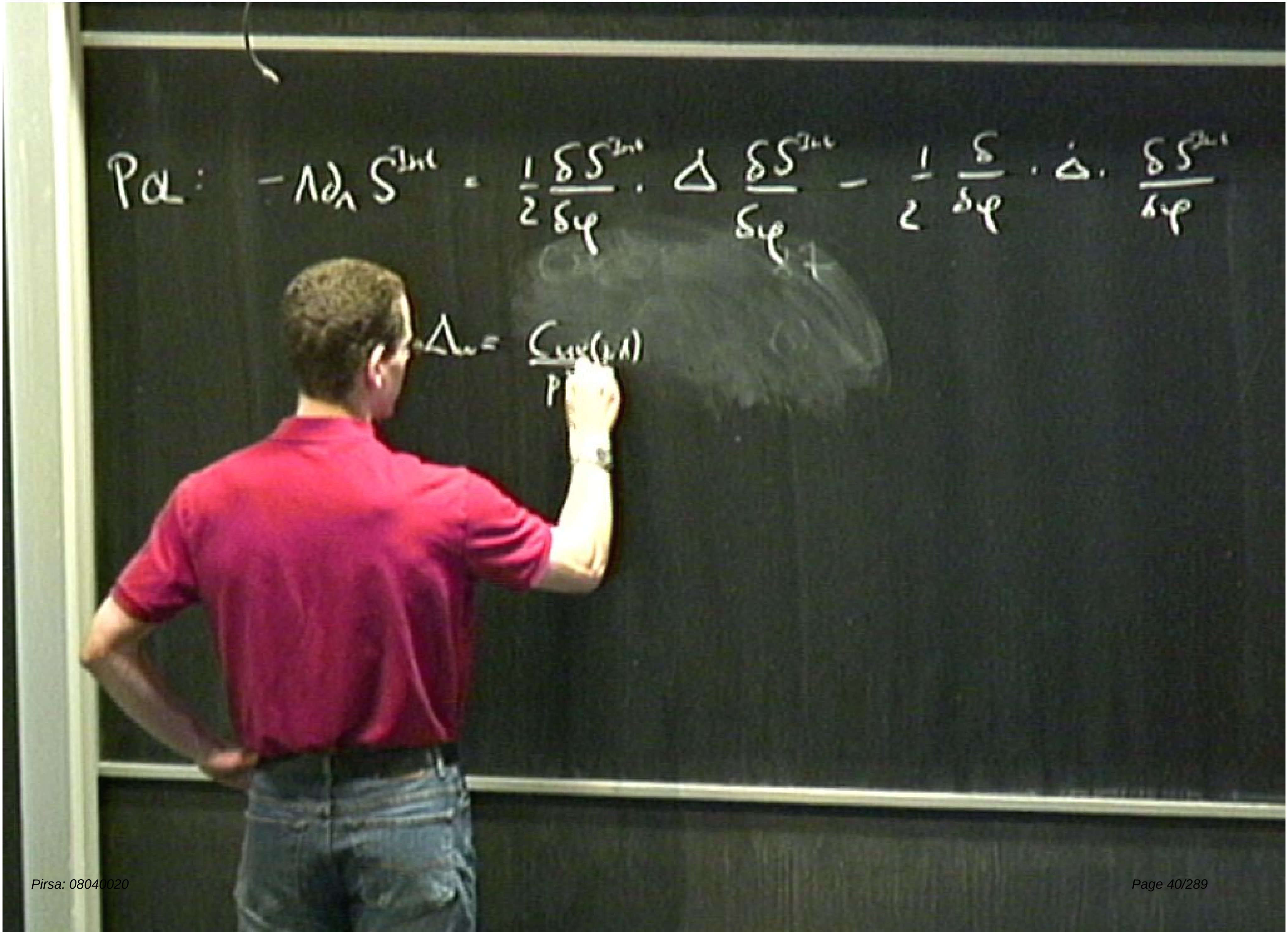
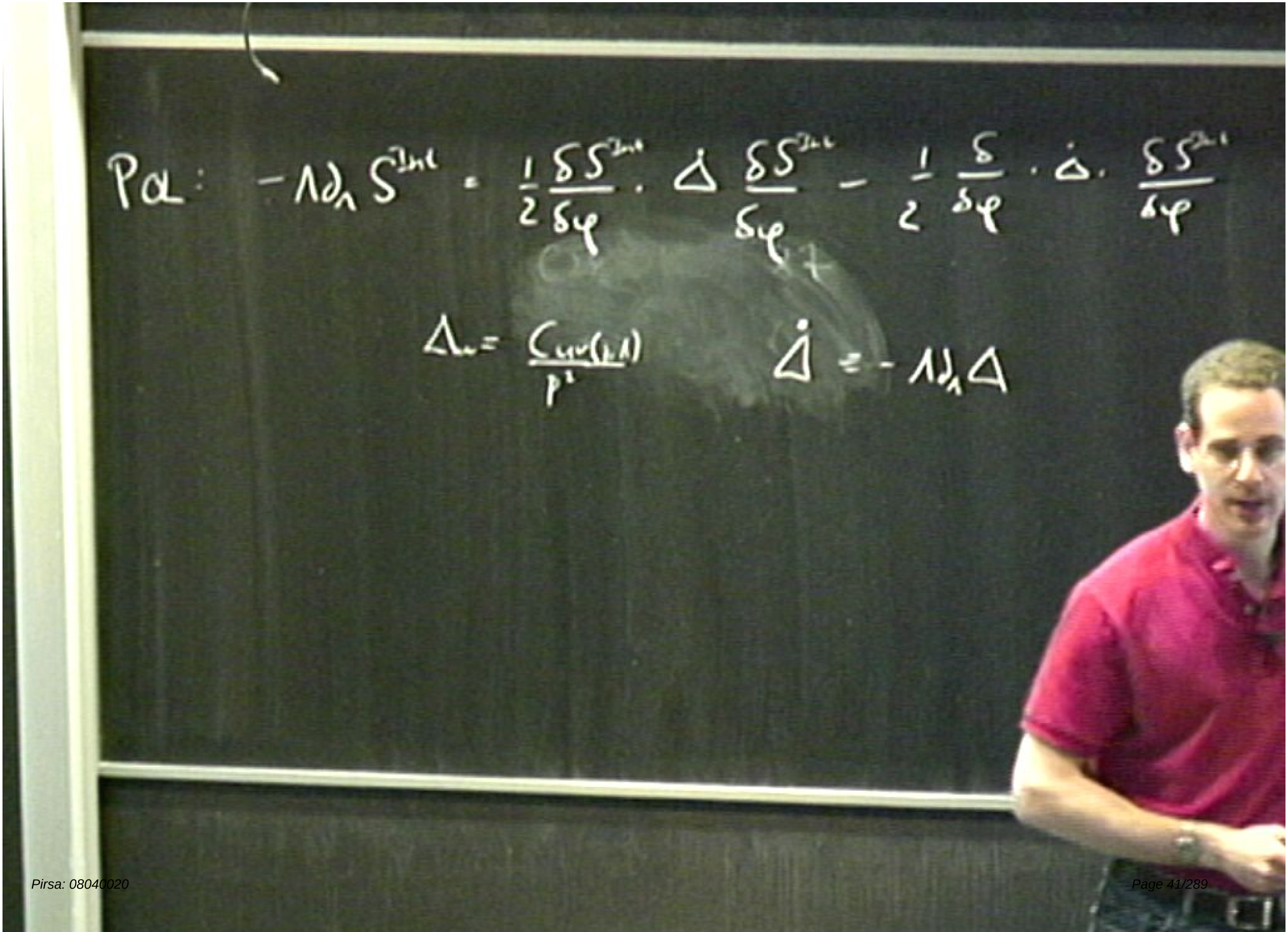

PCL: $-\Lambda\partial_\Lambda S^{int} = \frac{1}{2}\frac{\delta S^{int}}{\delta\varphi}\cdot\Delta\frac{\delta S^{int}}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta S^{int}}{\delta\varphi}$


PoL: $-\Lambda\partial_\Lambda S^{int} = \frac{1}{2}\frac{\delta S^{int}}{\delta\varphi}\cdot\dot{\Delta}\,\frac{\delta S^{int}}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta S^{int}}{\delta\varphi}$
$\Delta_{\mu\nu} = \frac{C_{UV}(p,\Lambda)}{p^2}$
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Where Dose the Polchinski Equation Come From?

Direct Derivation

- Partition modes in the functional integral into those above and those below the effective cutoff
- Integrate out the high modes
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Flexible Approach

- Use invariance of the partition function under the flow
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$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta \varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- effective scale
- set of fields
- Wilsonian effective action
- partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
- defines our ERG

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta \varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta \varphi(x)}$$

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    - parametrizes blocking procedure
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$$-\Lambda\partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta\varphi(x)}\left(\Psi_x[\varphi]e^{-S[\varphi]}\right)$$

- effective scale
- set of fields
- Wilsonian effective action
- partition function, $\int \mathcal{D}\varphi\, e^{-S[\varphi]}$, invariant under the flow
- defines our ERG
  - parametrizes blocking procedure
  - huge freedom in precise form—adapt to suit our needs

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$$-\Lambda\partial_\Lambda S = \int_x \frac{\delta S}{\delta\varphi(x)}\Psi_x - \int_x \frac{\delta\Psi_x}{\delta\varphi(x)}$$

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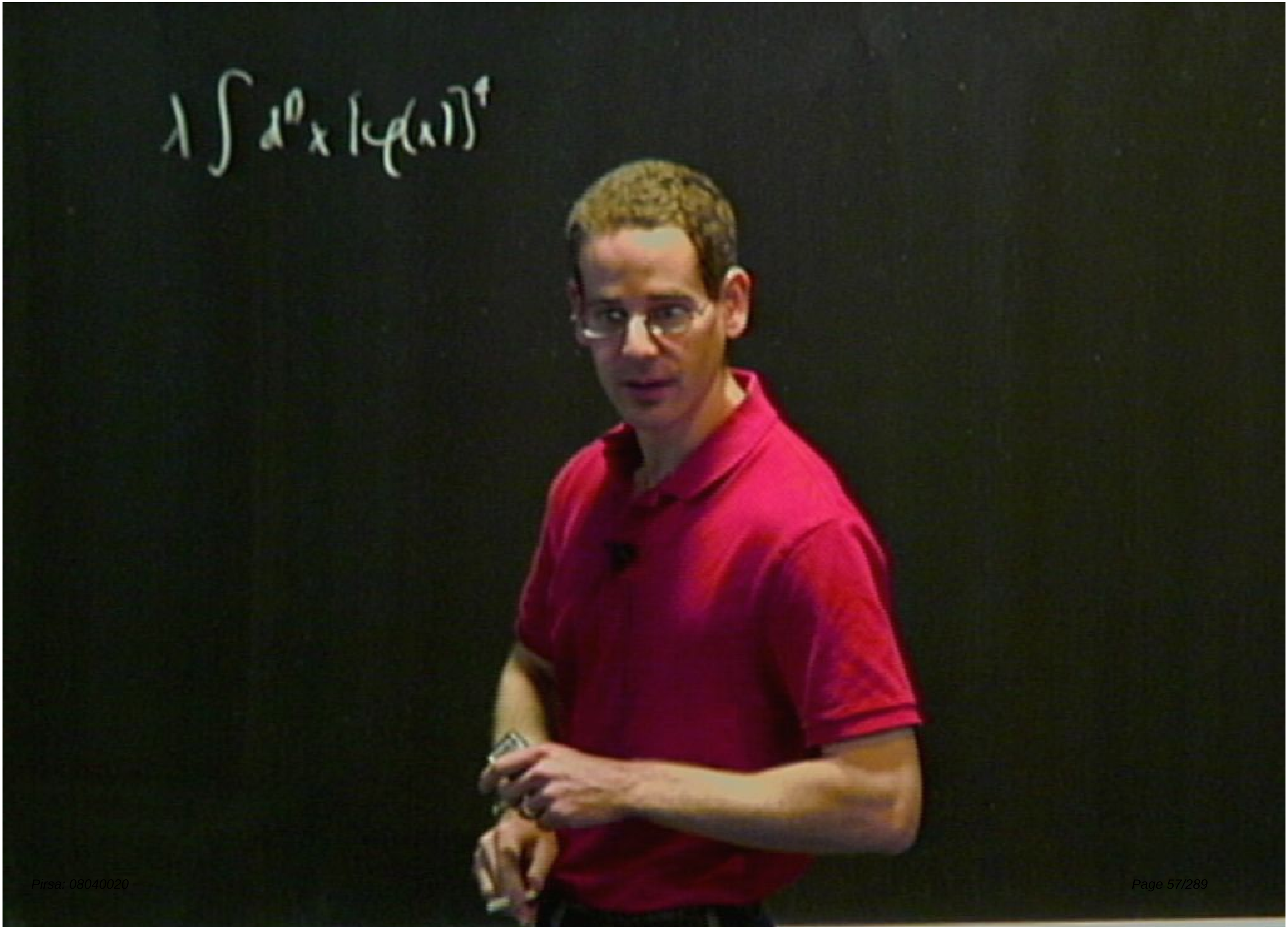
$\lambda \int d^n x \, [\varphi(x)]^4$

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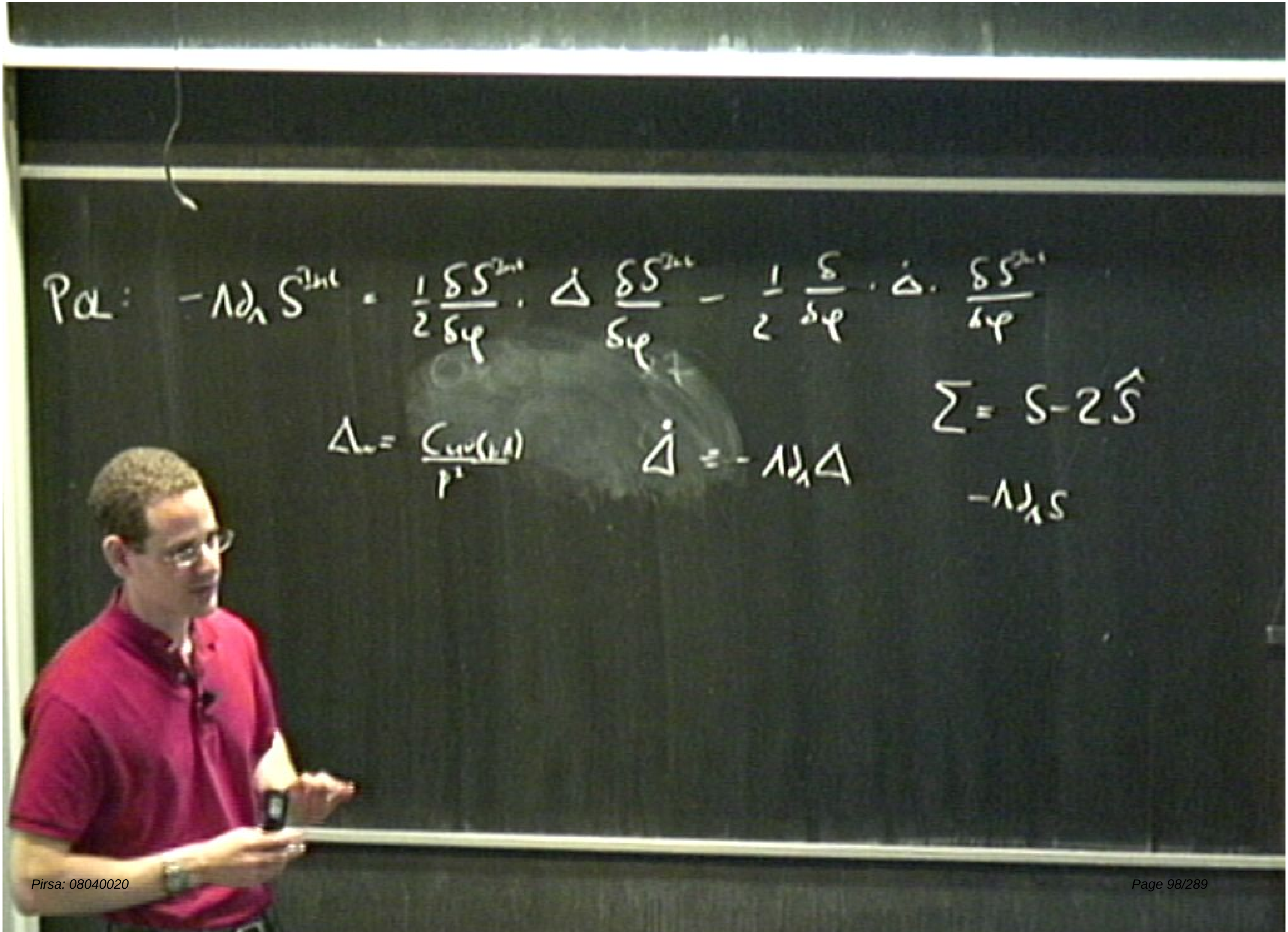
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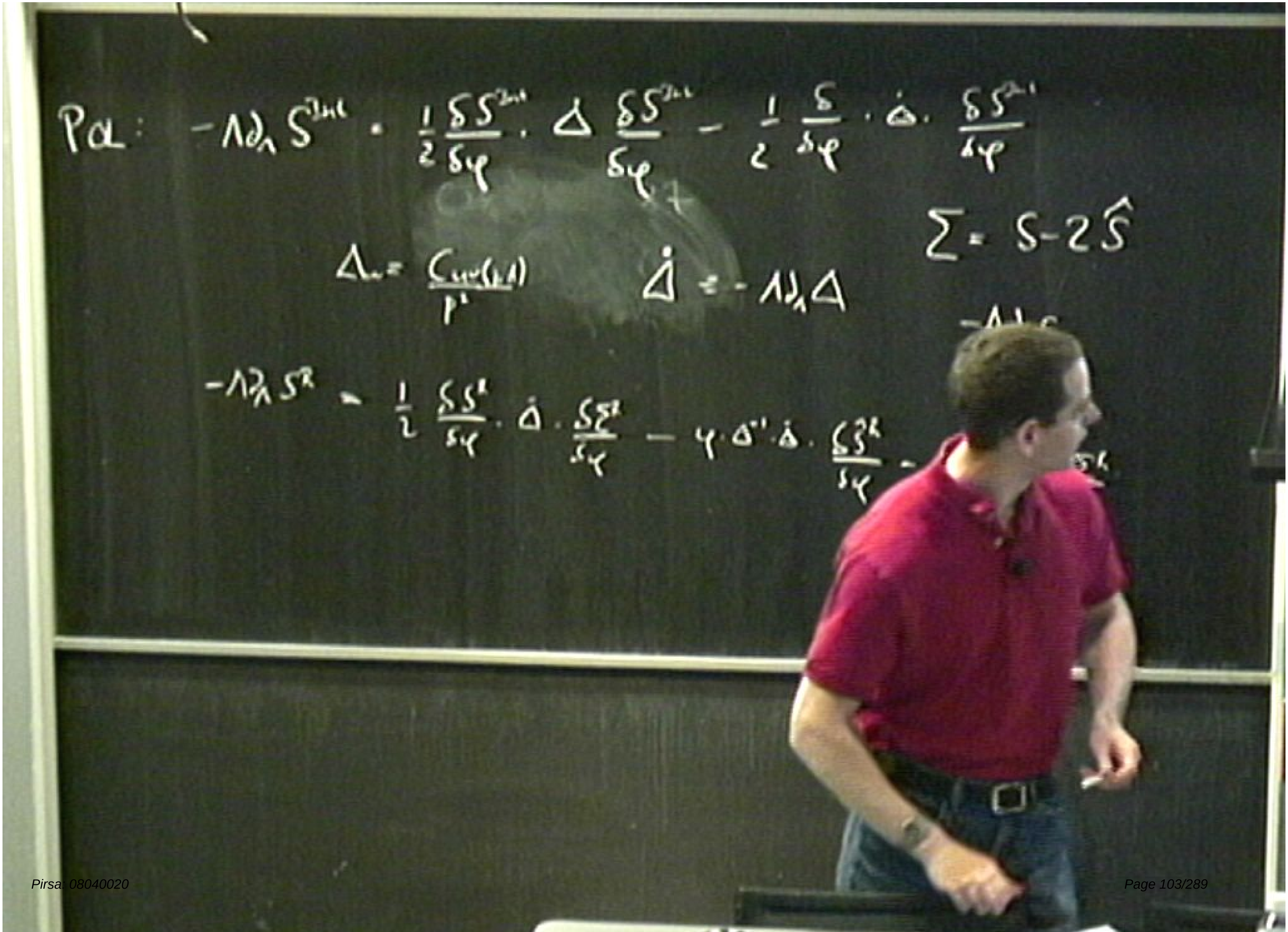
Relationship to Polchinski Equation


Pol:
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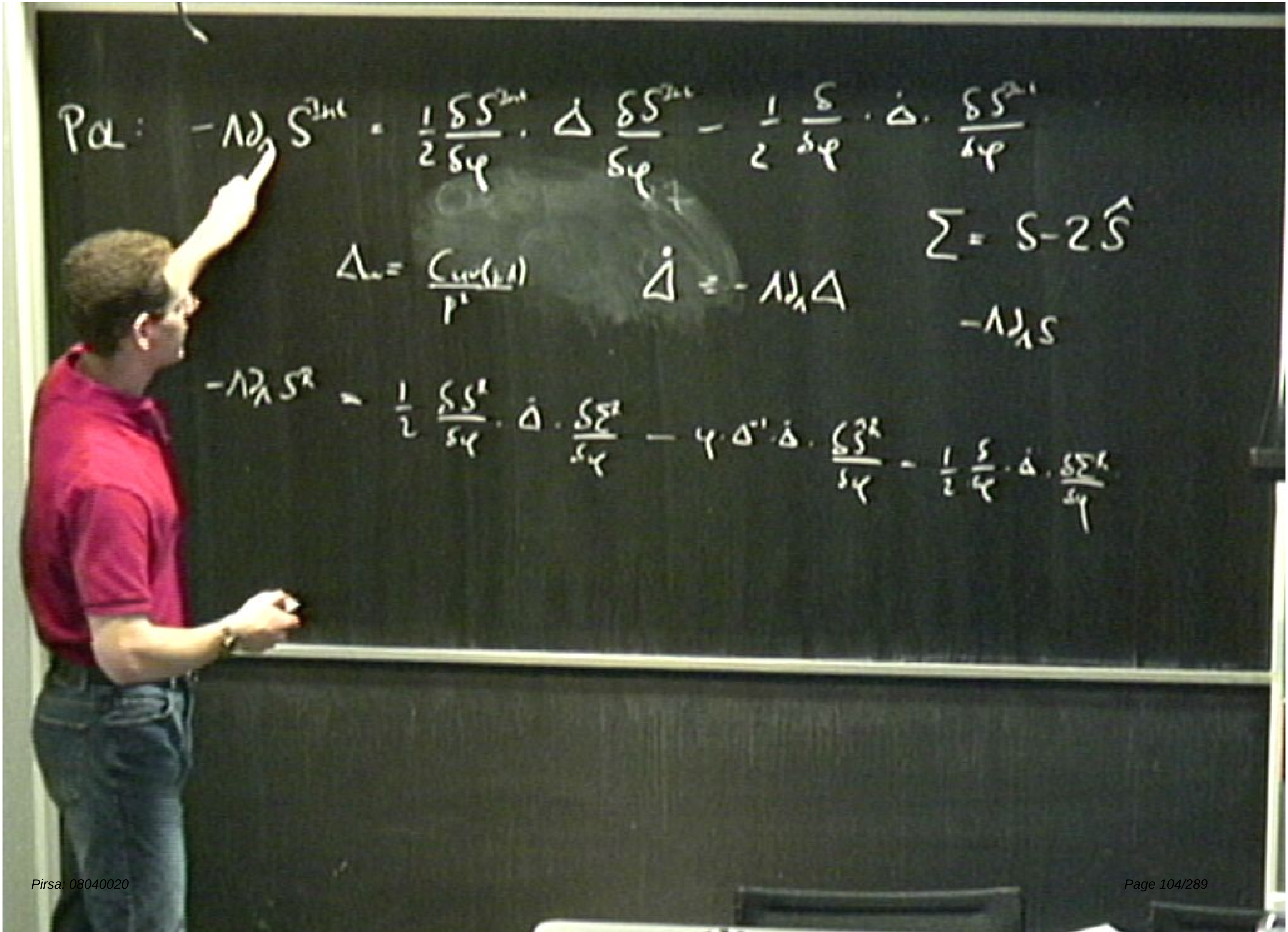
PaL: $-\Lambda\partial_\Lambda S^{int} = \frac{1}{2}\frac{\delta S^{int}}{\delta\varphi}\cdot\dot{\Delta}\,\frac{\delta S^{int}}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta S^{int}}{\delta\varphi}$
$\Sigma = S - 2\hat{S}$
$\Delta = \frac{C_{UV}(p,\Lambda)}{p^2}$
$\dot{\Delta} = -\Lambda\partial_\Lambda\Delta$
$-\Lambda\partial_\Lambda S$
$-\Lambda\partial_\Lambda S^R = \frac{1}{2}\frac{\delta S^R}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta\Sigma^R}{\delta\varphi} - \varphi\cdot\Delta^{-1}\cdot\dot{\Delta}\cdot\frac{\delta S^R}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta\Sigma^R}{\delta\varphi}$

Explicitly Including the Anomalous Dimension

- Fixed point behaviour is most conveniently uncovered by transferring to dimensionless variable
- For generic calculations eg computing the β-function, it is convenient to explicitly include the anomalous dimension
- Under renormalization, the two-point vertex picks up a factor of the field strength renormalization:

$$\varphi \cdot p^2 Z^{-1} \cdot \varphi$$

- So rescale $\varphi \to \sqrt{Z}\varphi$
- $-\Lambda\partial_\Lambda|_\varphi S \to \left(-\Lambda\partial_\Lambda|_\varphi + \frac{\gamma}{2}\varphi \cdot \frac{\delta}{\delta\varphi}\right) S$
- $\gamma \equiv \Lambda\partial_\Lambda \ln Z$
- $\frac{\delta}{\delta\varphi} \cdot \Delta \cdot \frac{\delta}{\delta\varphi} \to \frac{1}{Z}\frac{\delta}{\delta\varphi} \cdot \Delta \cdot \frac{\delta}{\delta\varphi}$
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- Simply drop the $1/Z$ on the RHS of the flow equation!
- The resulting flow equation is perfectly valid
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- $$\left(-\Lambda\partial_\Lambda + \frac{\gamma}{2}\varphi\cdot\frac{\delta}{\delta\varphi}\right)S = \frac{1}{2}\frac{\delta S}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta\Sigma}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta\Sigma}{\delta\varphi}$$
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- $\varphi \to \varphi/\sqrt{\lambda}$
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  - $\tilde{\gamma} = \gamma - \frac{\beta}{\lambda}$
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- Wilson's ERG equation:

$$\Psi_p = h(p)\left(\varphi_p - \frac{\delta S}{\delta \varphi_{-p}}\right)$$

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  - Integrate over high momentum modes in $Z[J]$ to define $Z_\Lambda[J]$
  - Introduce the classical field $\varphi_c \equiv \dfrac{\delta \ln Z_\Lambda}{\delta J}$
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1. General ERG Equations
   - Formulation
   - Other ERG Equations

2. Diagrammatics

3. One-loop β-function

4. Recap

Diagrammatics for the Action

Diagrammatics for the Action

- S has an expansion in terms of fields

$$S = \frac{1}{2} S^{(2)} \varphi\varphi + \frac{1}{4!} S^{(4)} \varphi\varphi\varphi\varphi + \cdots$$

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- vertex coefficient function
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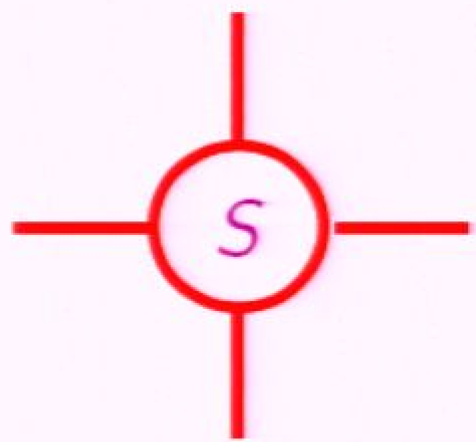

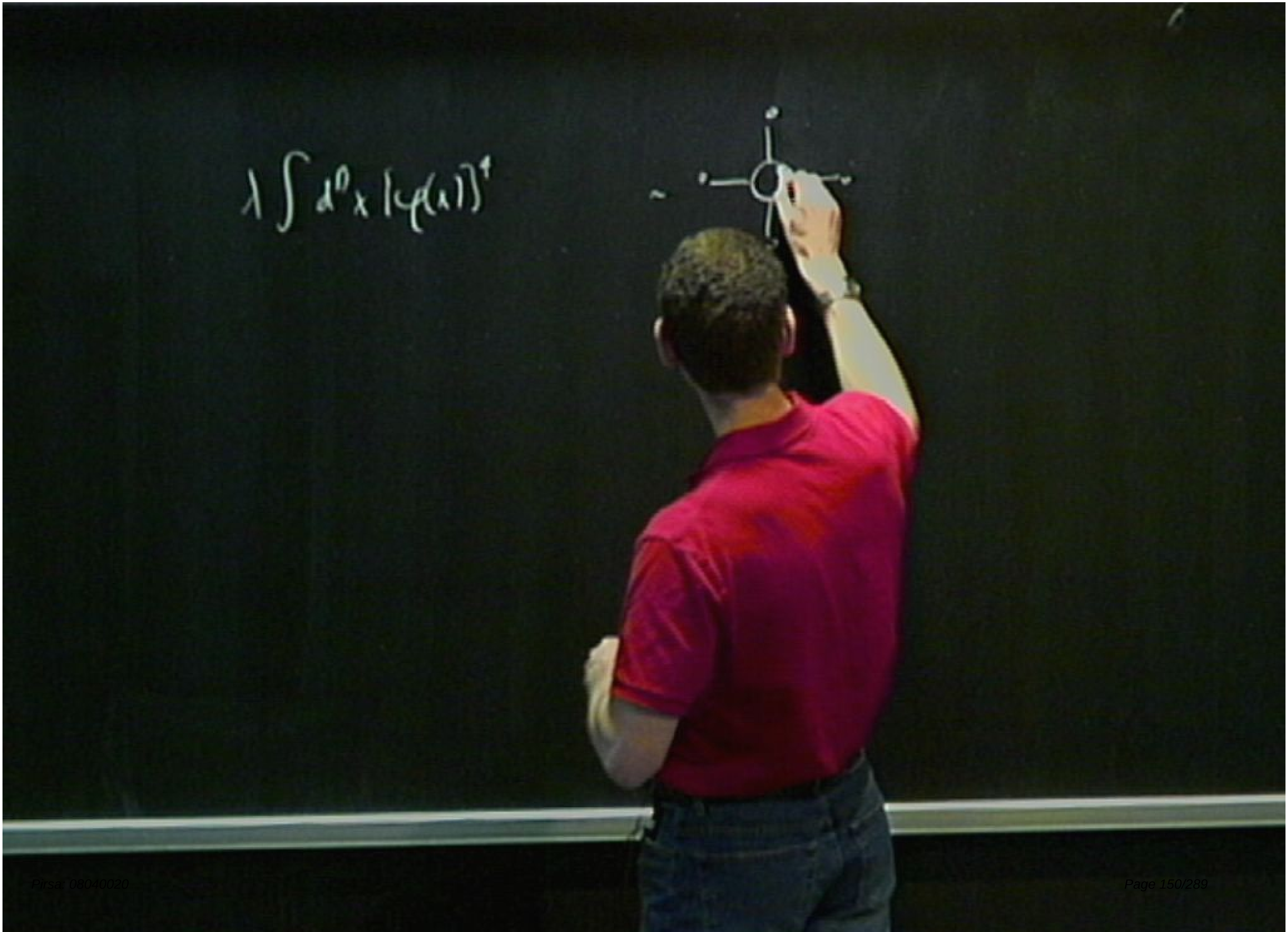
$\lambda \int d^D x \, |\varphi(x)|^4$

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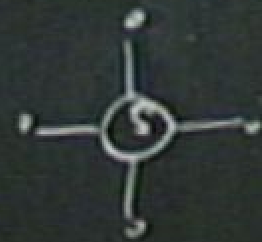

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$$\left.\varphi\varphi\right] =$$

Diagrammatics for the Flow Equation

$$-\Lambda\partial_\Lambda S = \frac{1}{2}\frac{\delta S}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta\Sigma}{\delta\varphi}$$

$-\Lambda\partial_\Lambda \Big[\frac{1}{2}$  $\varphi\varphi \Big] =$

Diagrammatics for the Flow Equation

$$-\Lambda\partial_\Lambda S = \frac{1}{2}\frac{\delta S}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta\Sigma}{\delta\varphi} - \frac{1}{2}\frac{\delta}{\delta\varphi}\cdot\dot{\Delta}\cdot\frac{\delta\Sigma}{\delta\varphi}$$

$-\Lambda\partial_\Lambda \left[\frac{1}{2} \right.$ 

$\left. \varphi\varphi \right] = \frac{1}{2}$

Diagrammatics for the Flow Equation

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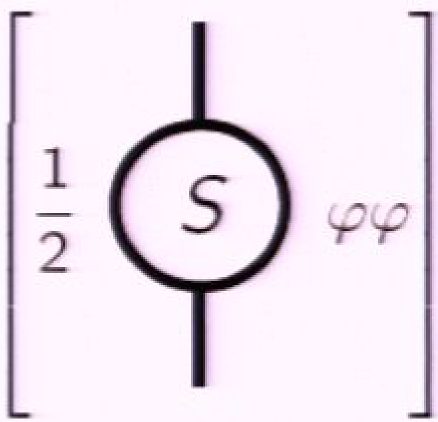
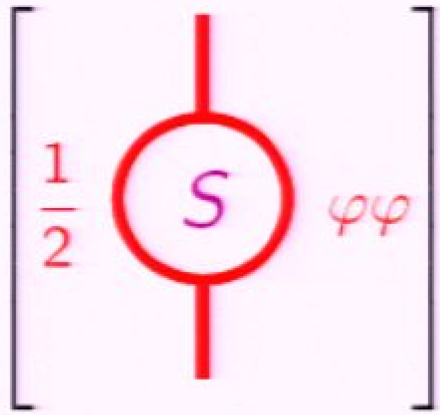
$-\Lambda\partial_\Lambda$ 

$= \frac{1}{2}$ 

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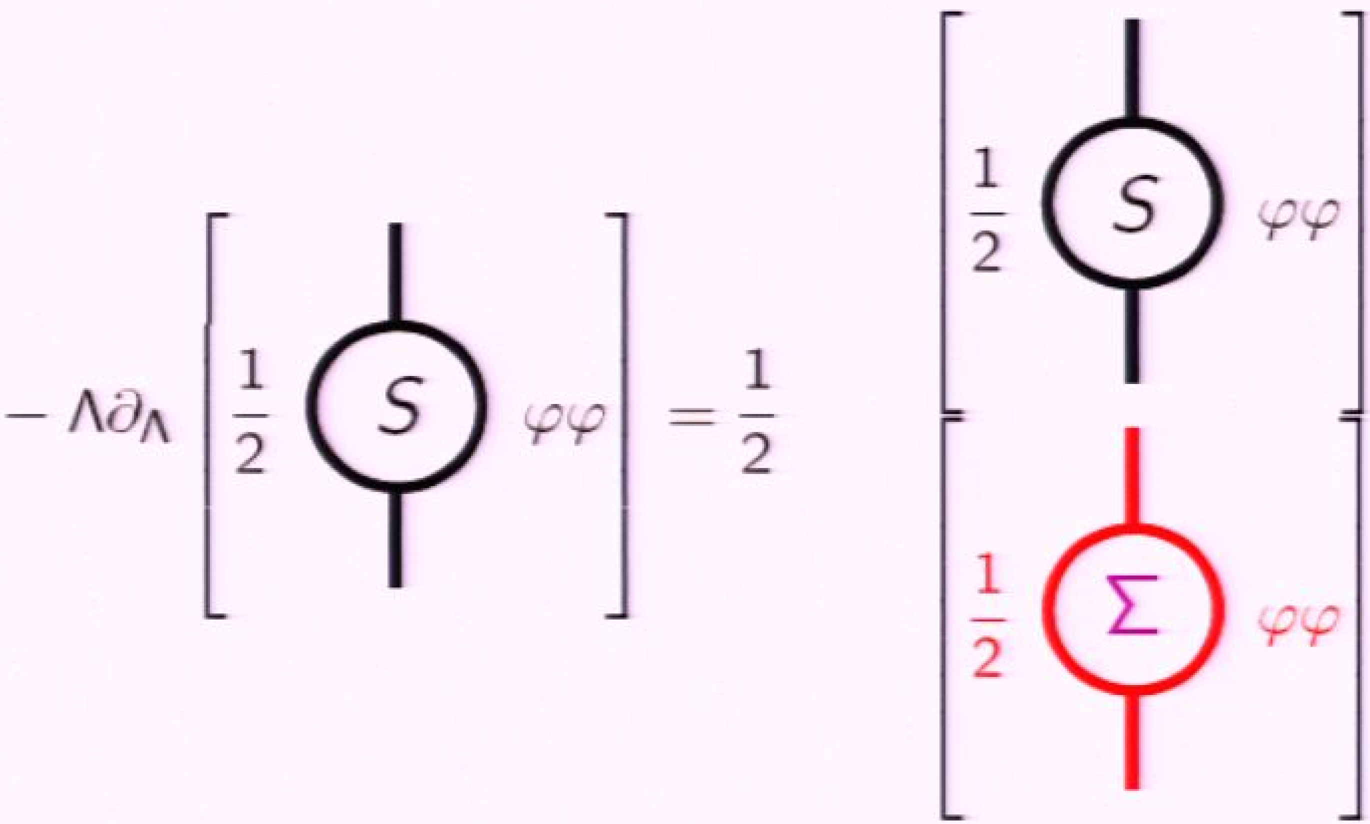

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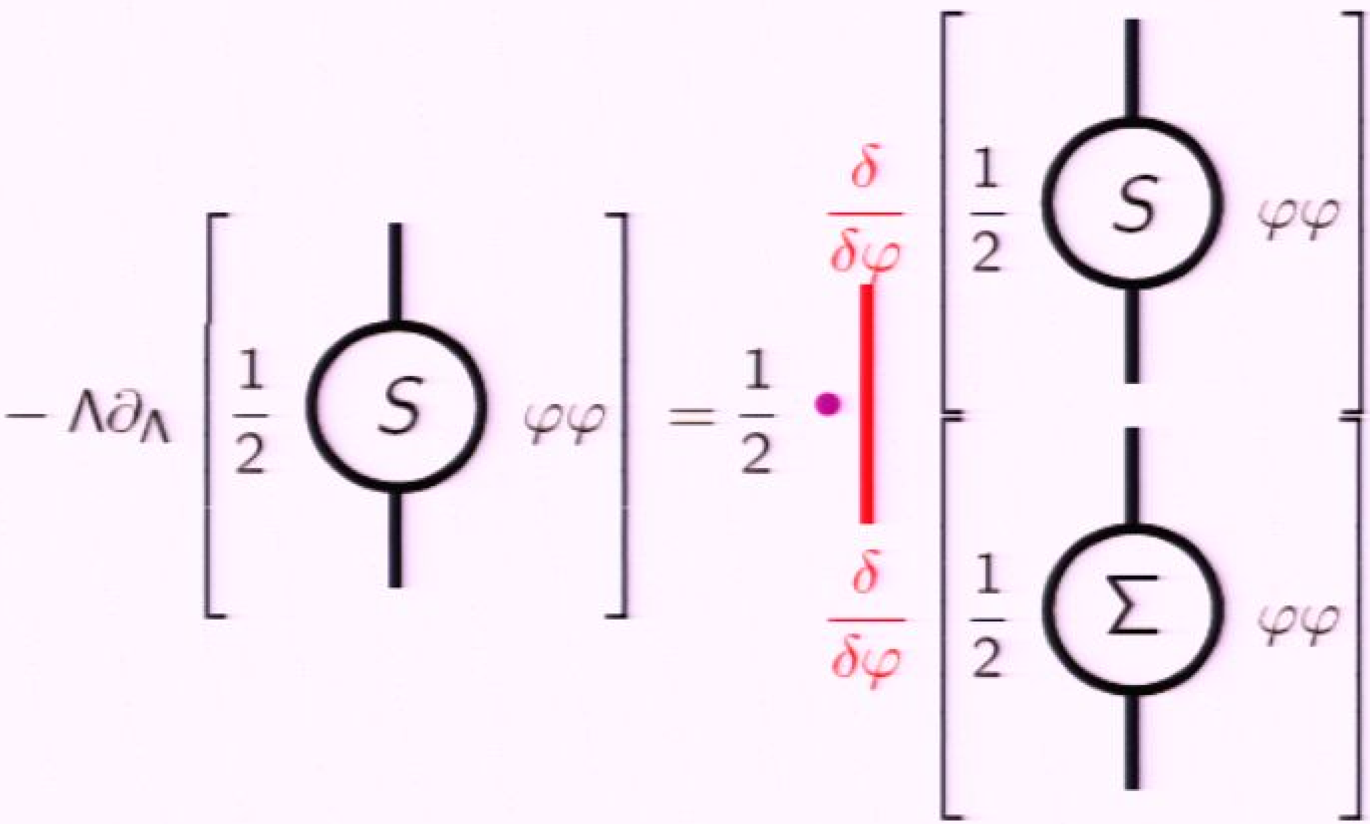

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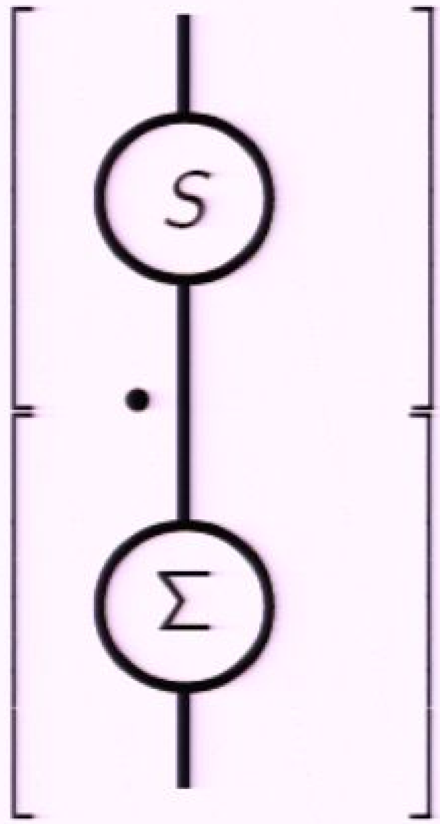
$-\Lambda\partial_\Lambda \left[\frac{1}{2}\right.$  $\left.\varphi\varphi\right] = \frac{1}{2}$  $\varphi\varphi$

Diagrammatics for the Flow Equation

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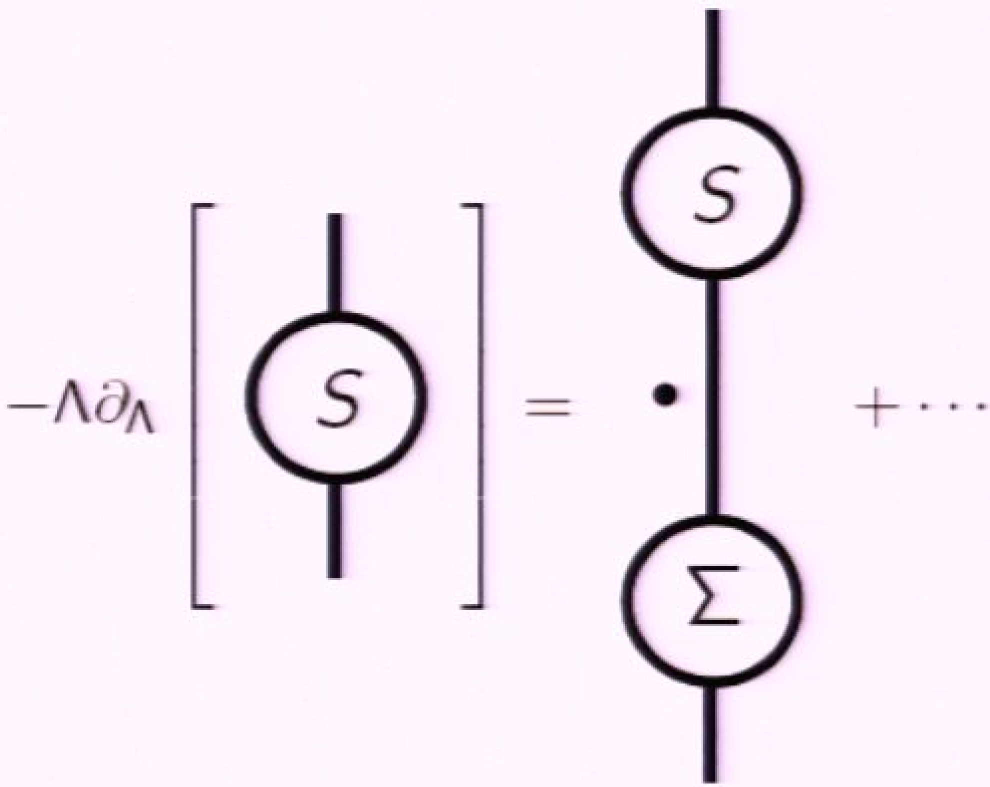

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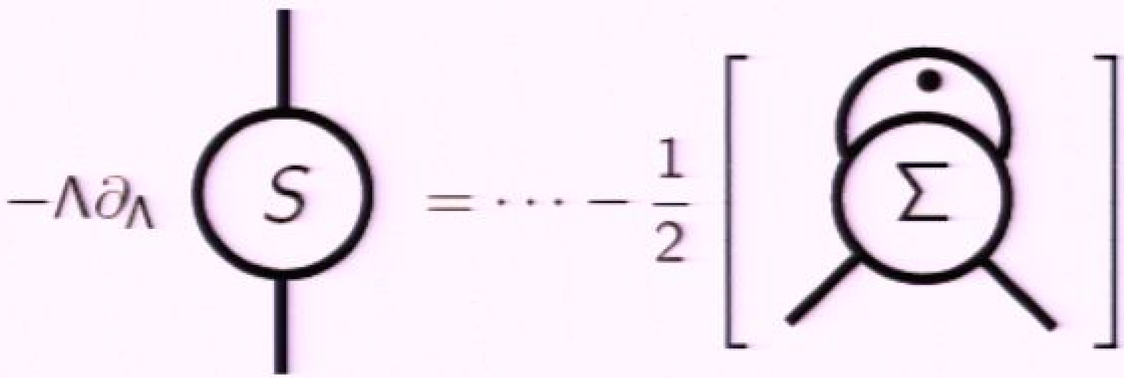

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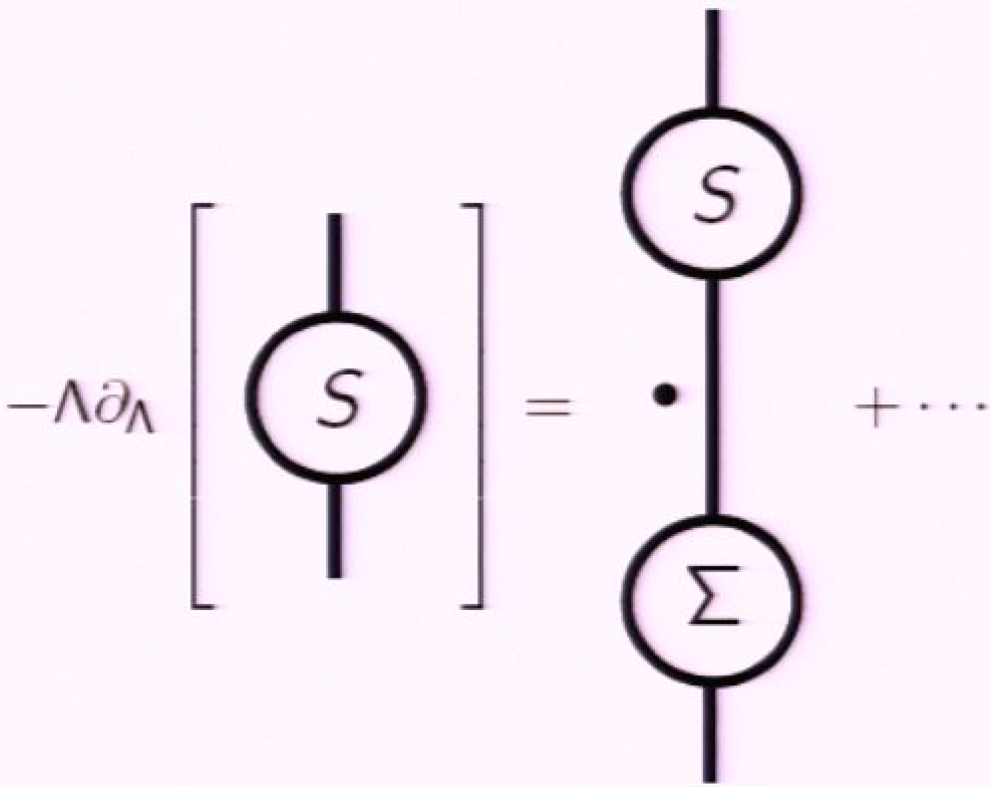

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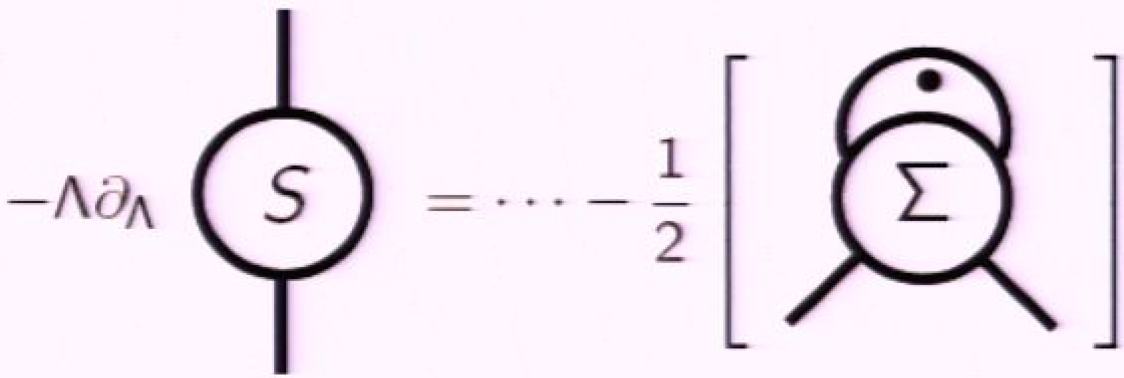

The Full Diagrammatic Flow Equation

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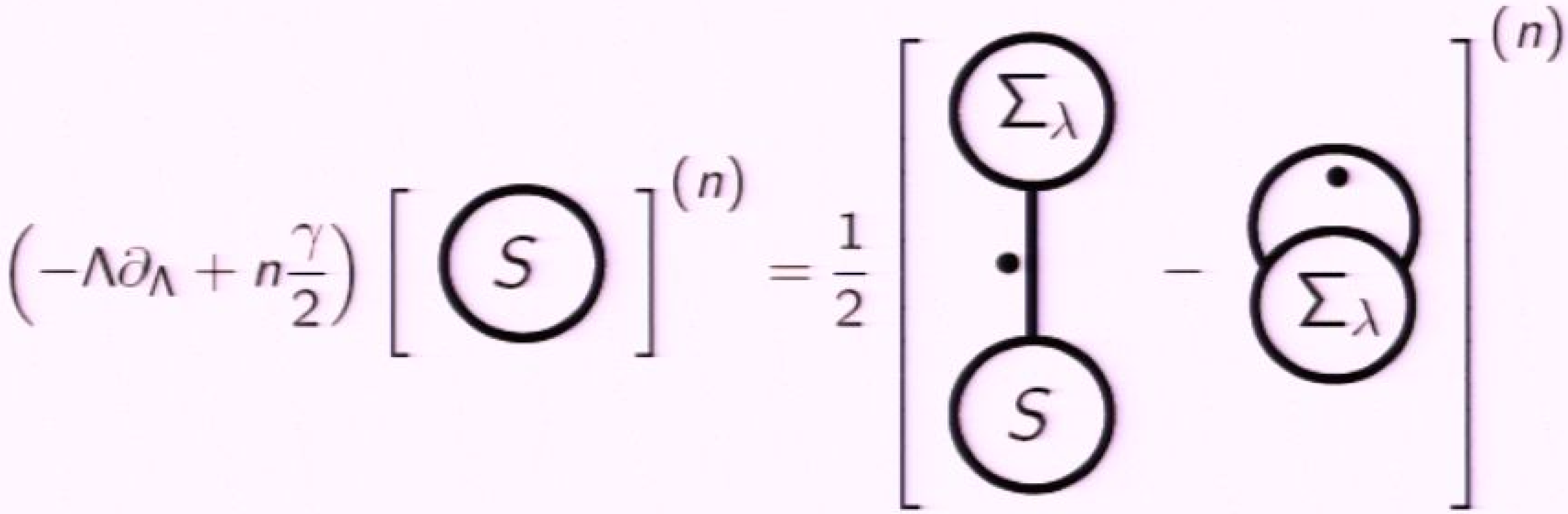
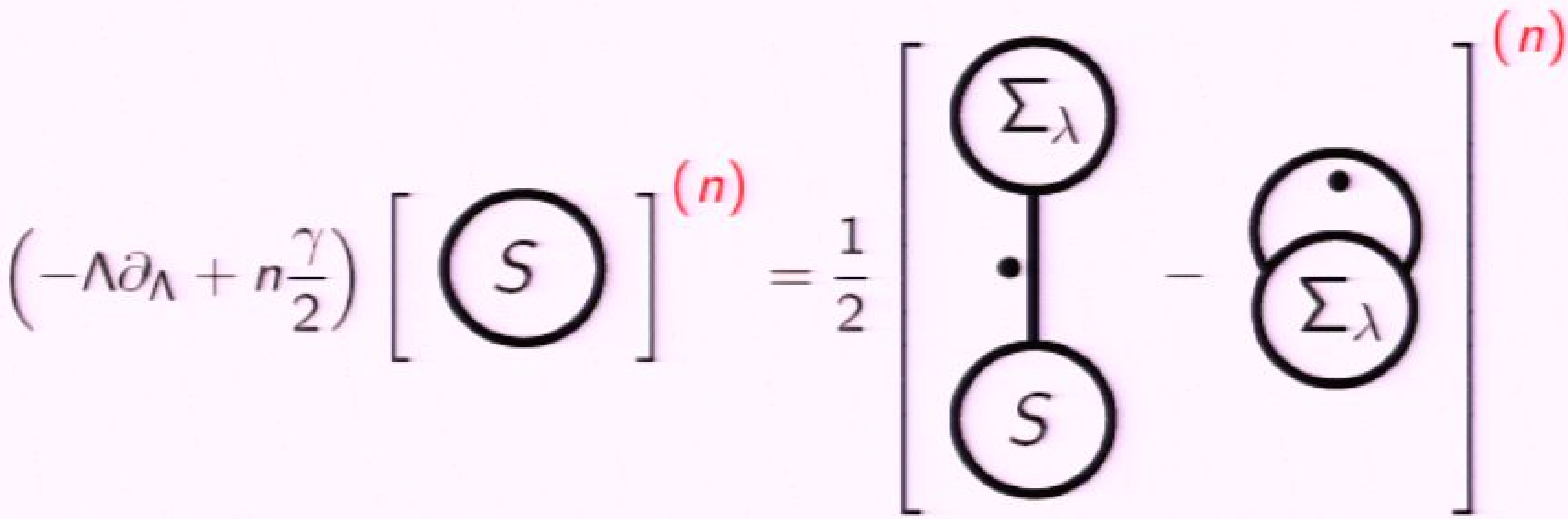
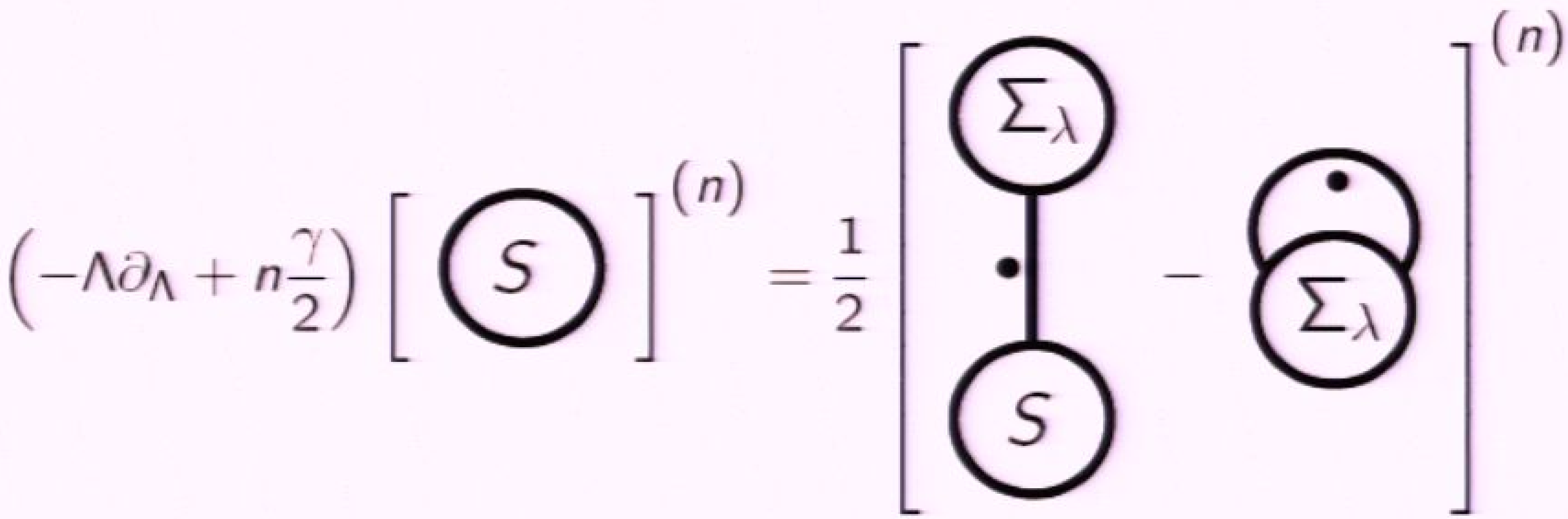


The Full Diagrammatic Flow Equation



- The *n* fields are distributed in all independent ways

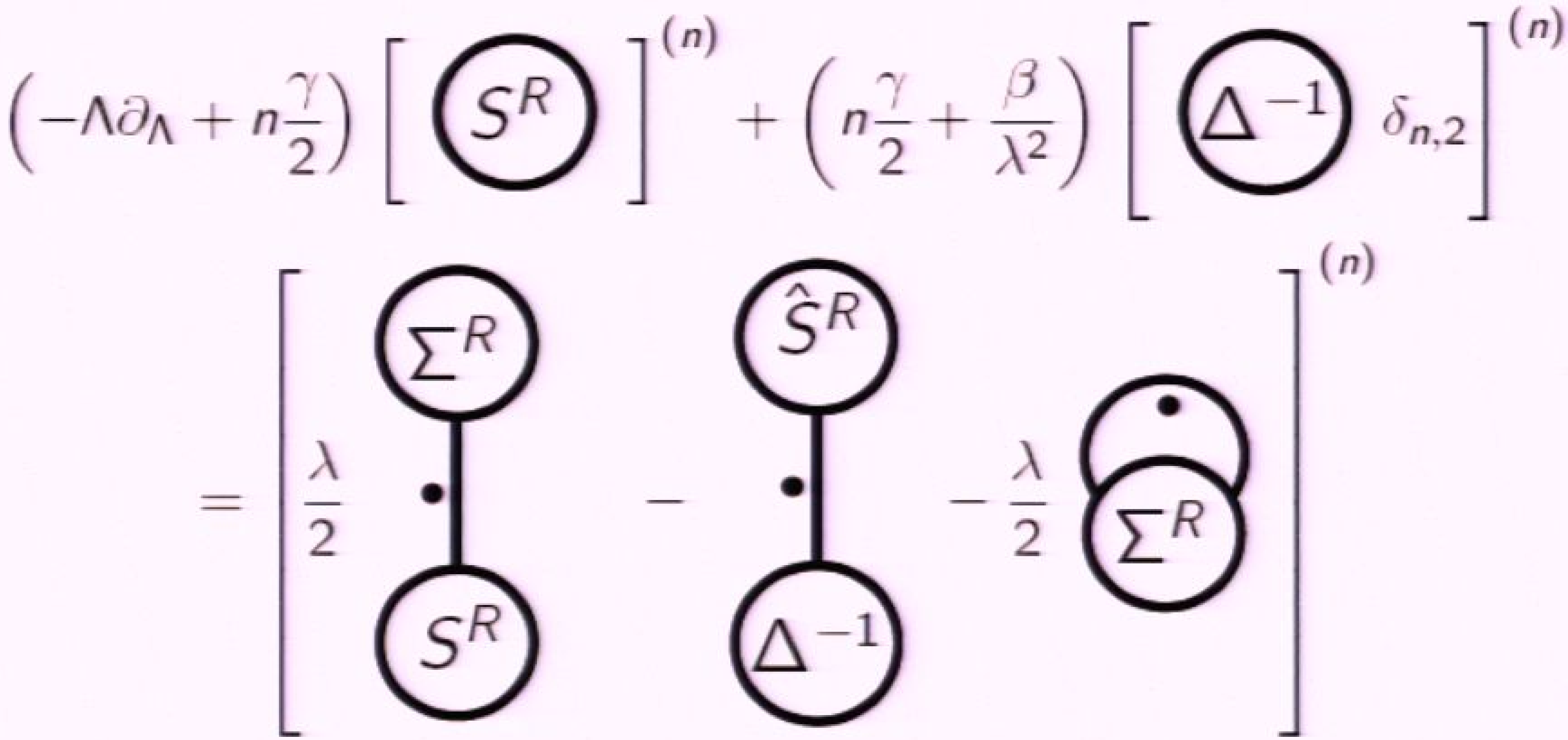
The Full Diagrammatic Flow Equation

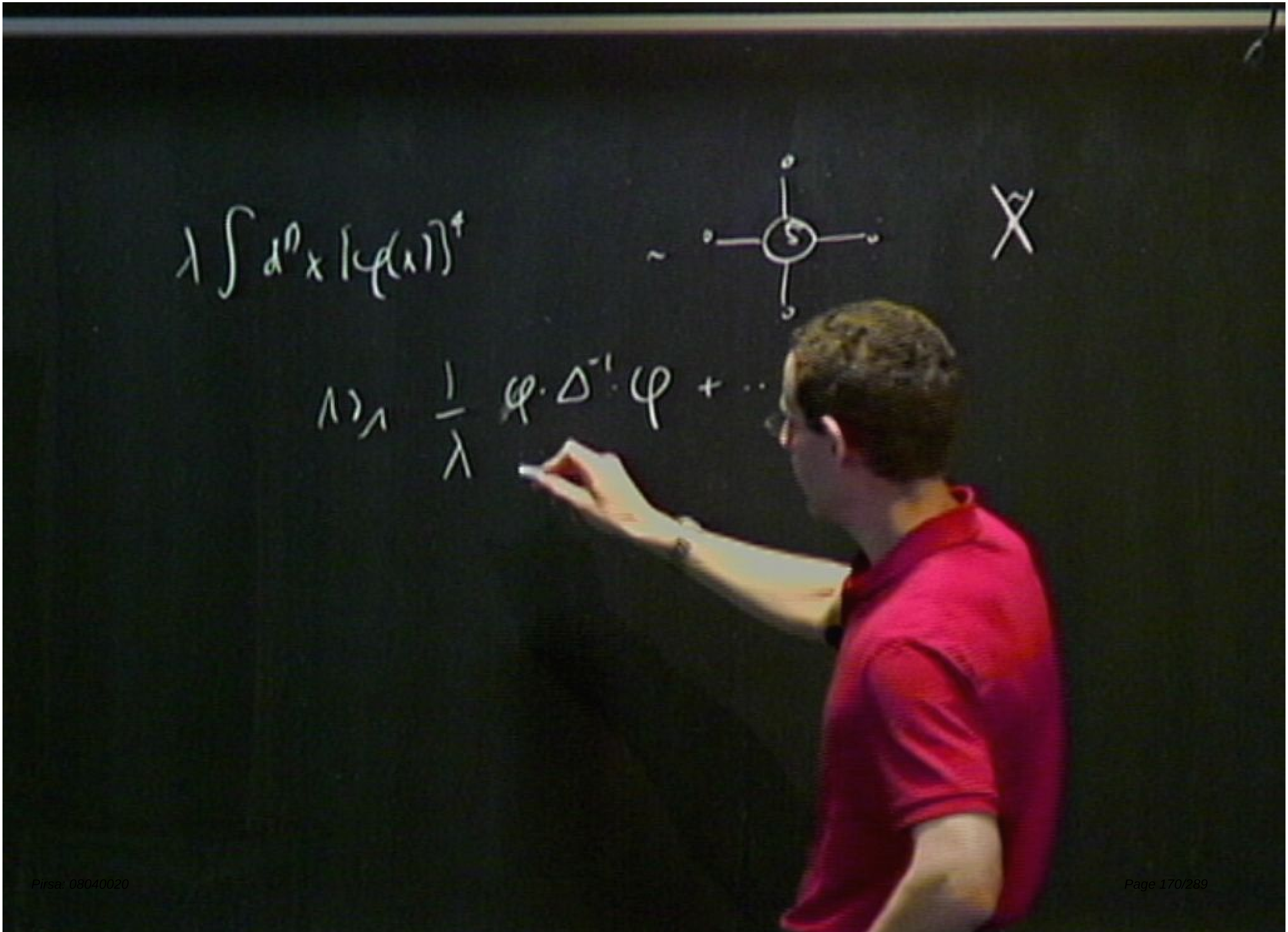


- The n fields are distributed in all independent ways
- $\Sigma_\lambda = \lambda(S - 2\hat{S})$

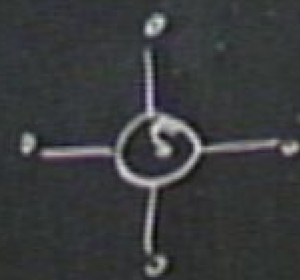
Flow of Reduced Vertices

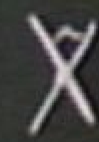
Flow of Reduced Vertices




$\lambda \int d^n x\, [\varphi(x)]^4 \sim$
$\lambda \gg 1 \quad \frac{1}{\lambda}\, \varphi \cdot \Delta^{-1} \cdot \varphi + \cdots$

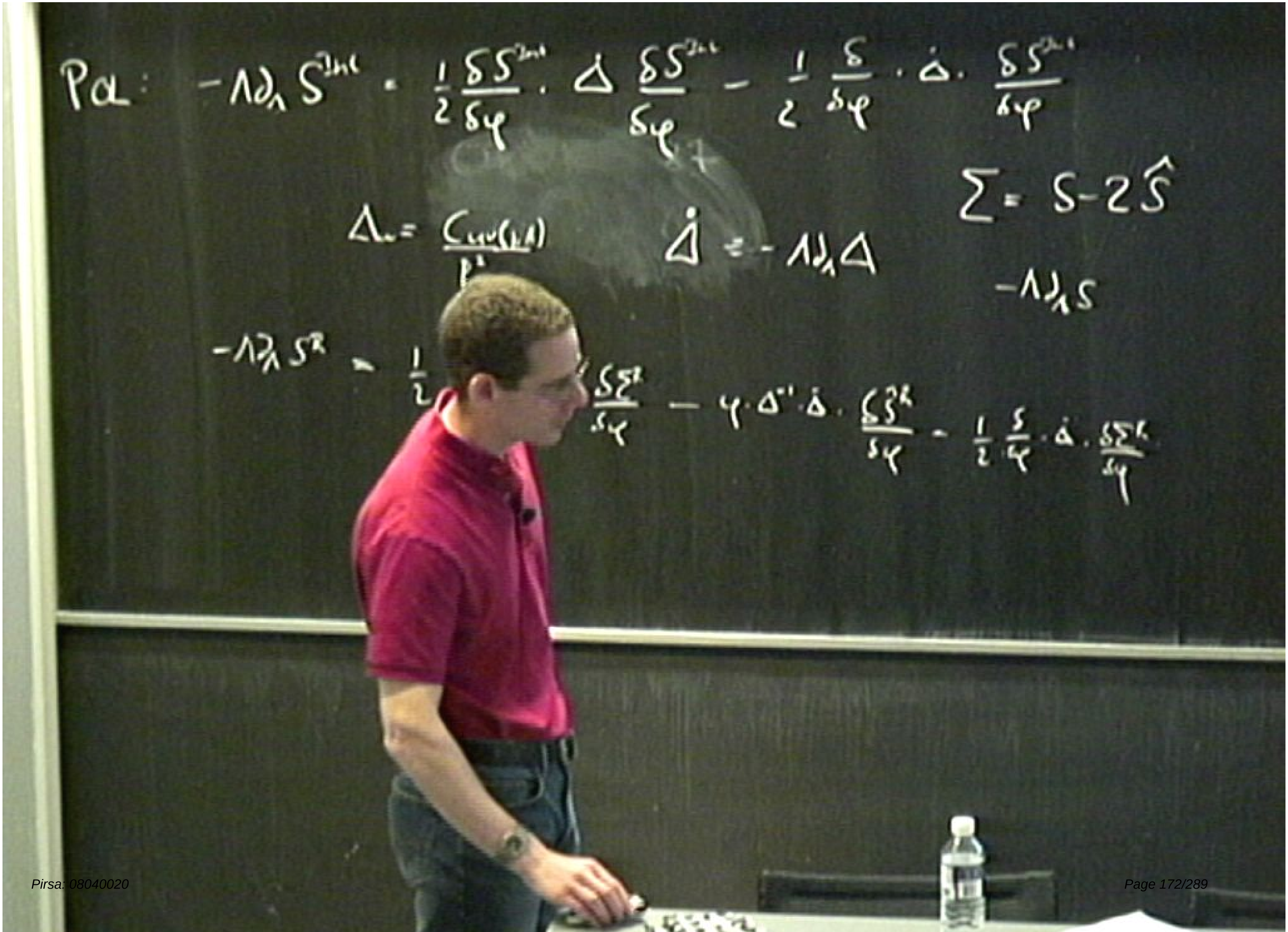
$$\lambda \int d^n x \, [\varphi(x)]^4 \sim$$



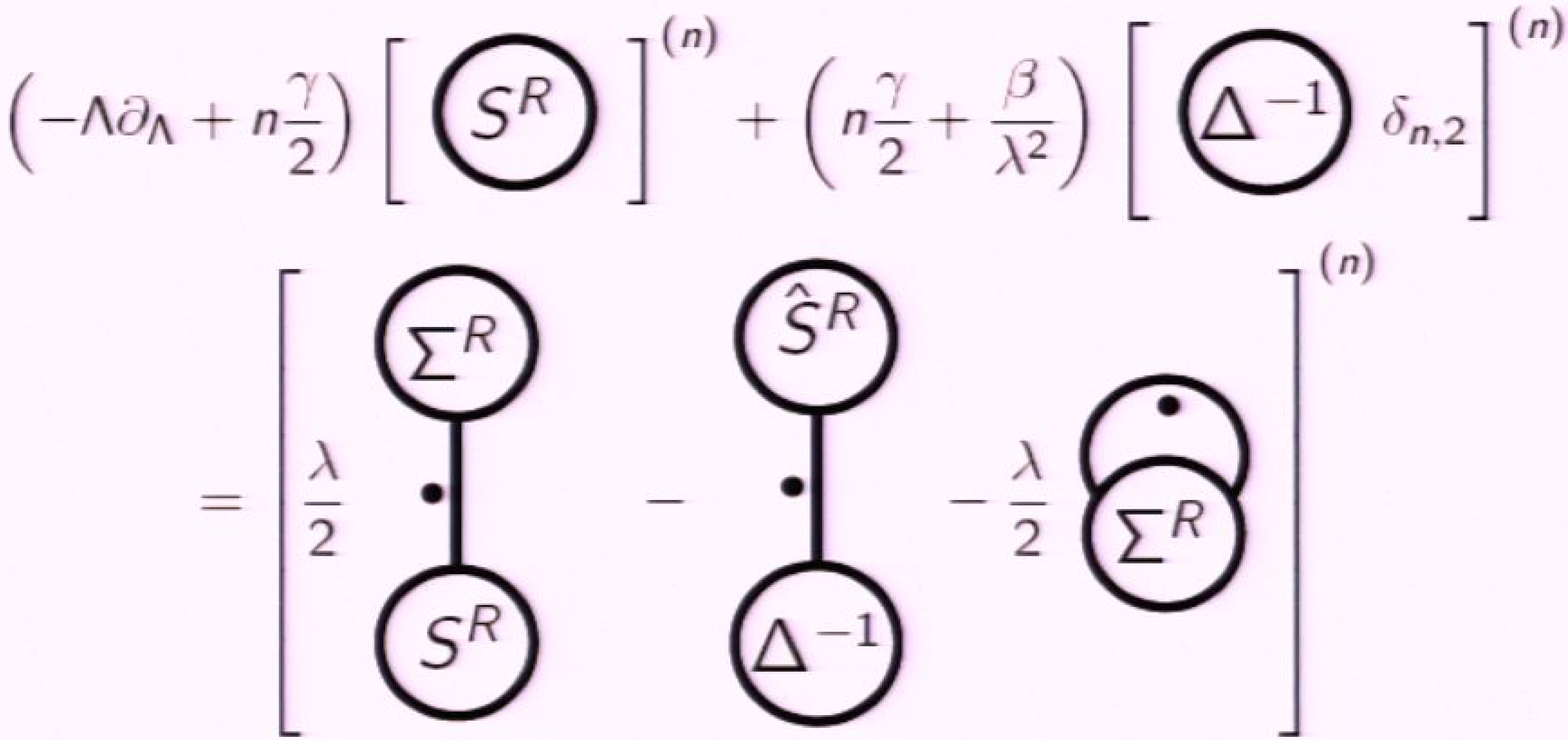


$$N_\lambda \, \frac{1}{\lambda} \, \varphi \cdot \Delta^{-1} \cdot \varphi + \cdots$$

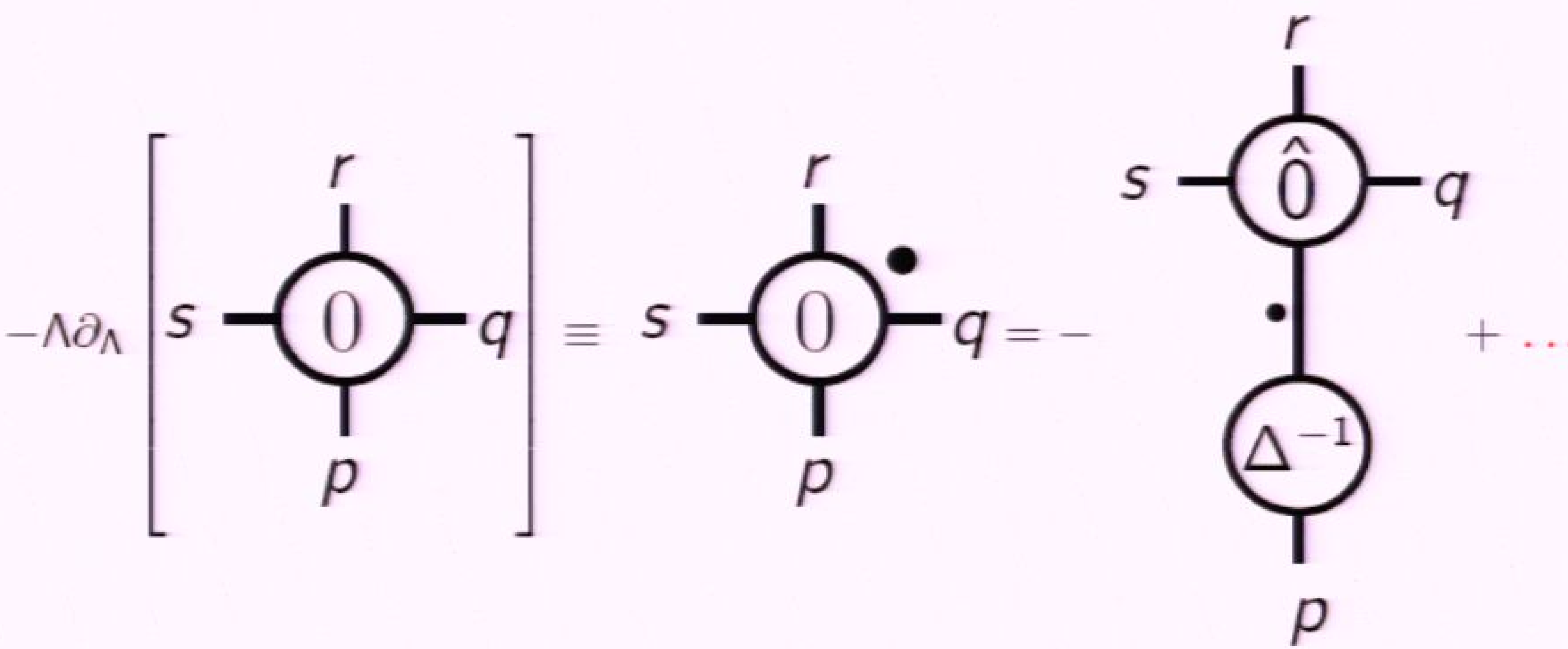
$$N_\lambda \lambda \equiv \beta$$


PoL: −Λ∂_Λ S^int = 1/2 δS^int/δφ · Δ δS^int/δφ − 1/2 δ/δφ · Δ̇ · δS^int/δφ
Σ = S − 2Ŝ
Δ̇ = −Λ∂_Λ Δ
−Λ∂_Λ S

Flow of Reduced Vertices

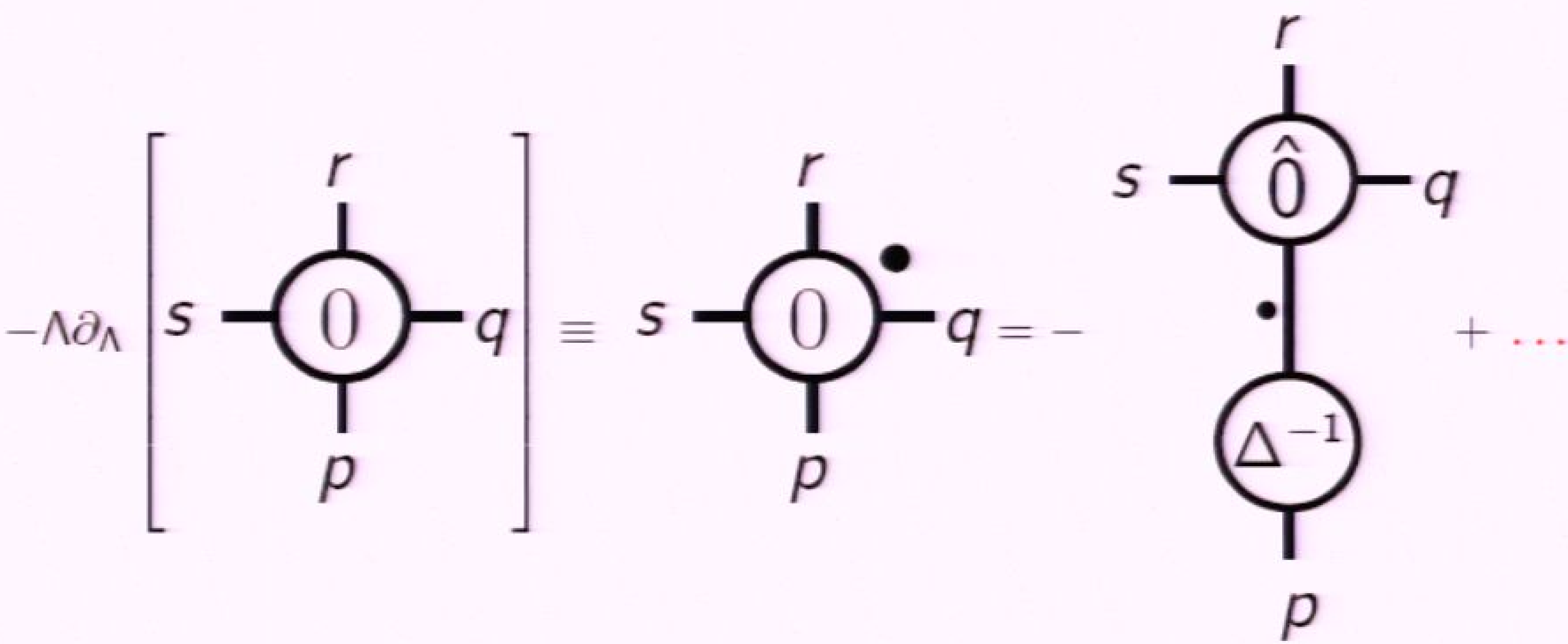


Example: Classical 4pt Vertex

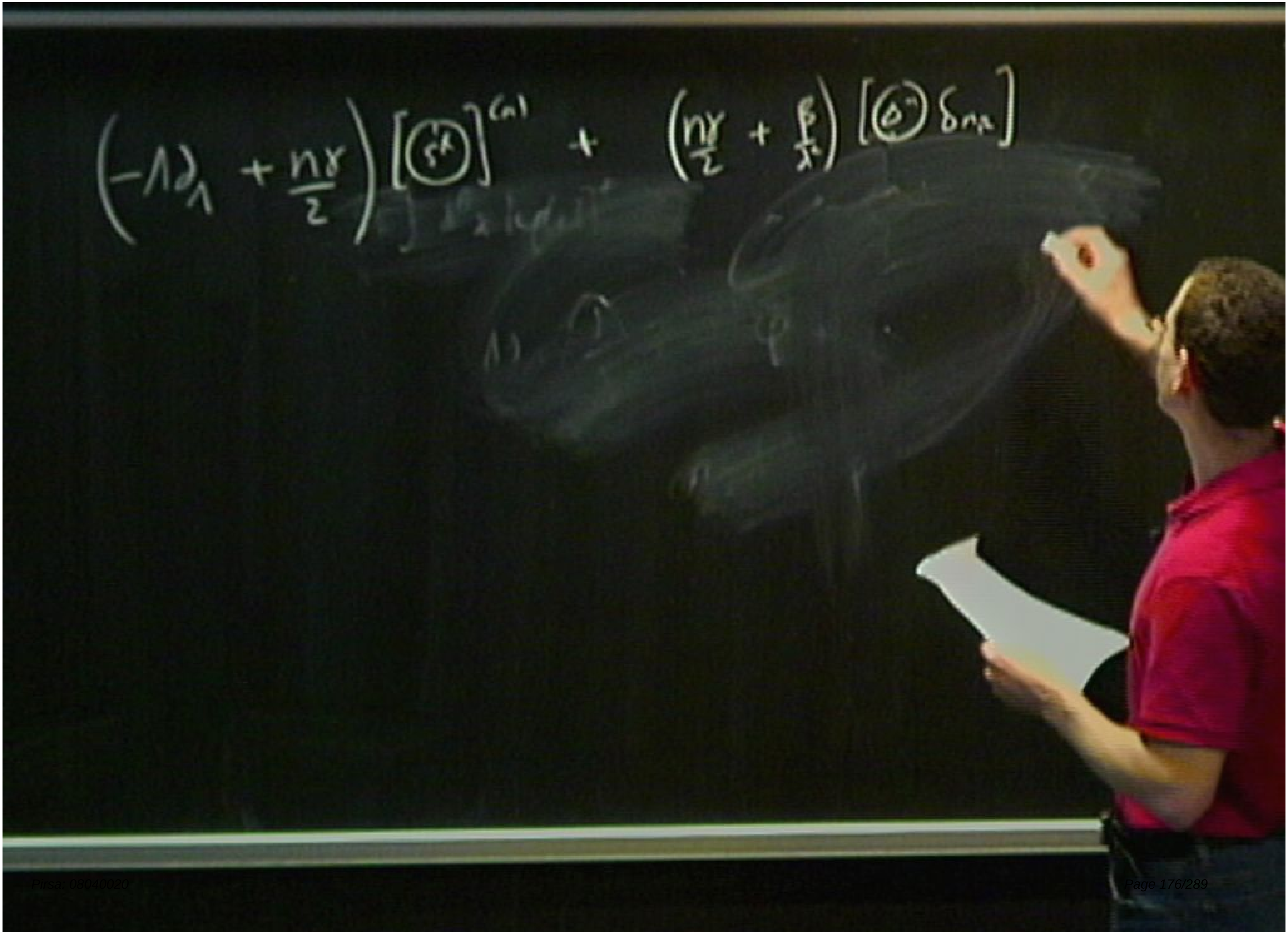


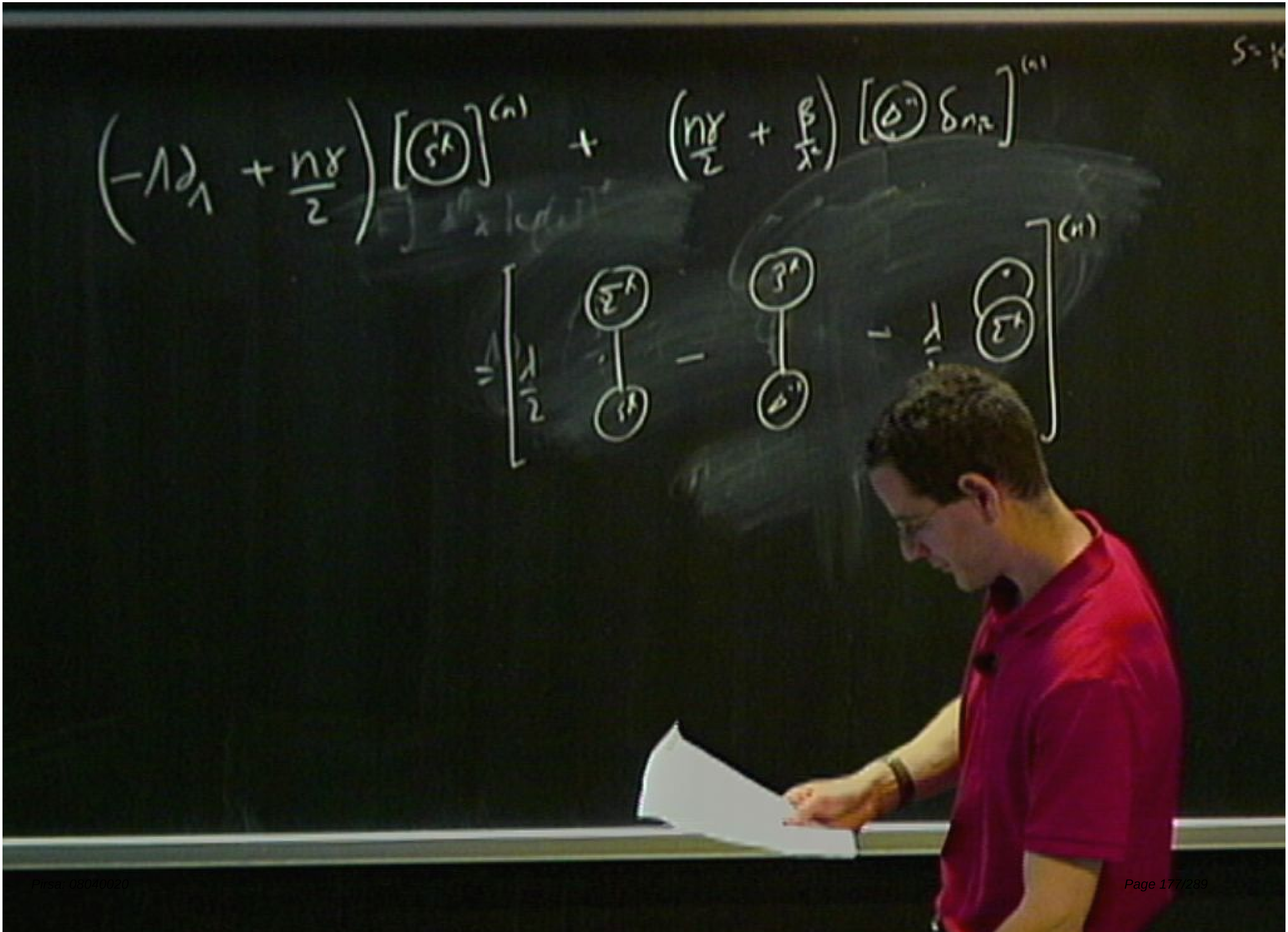
- diagrams with q, r, s on bottom vertex

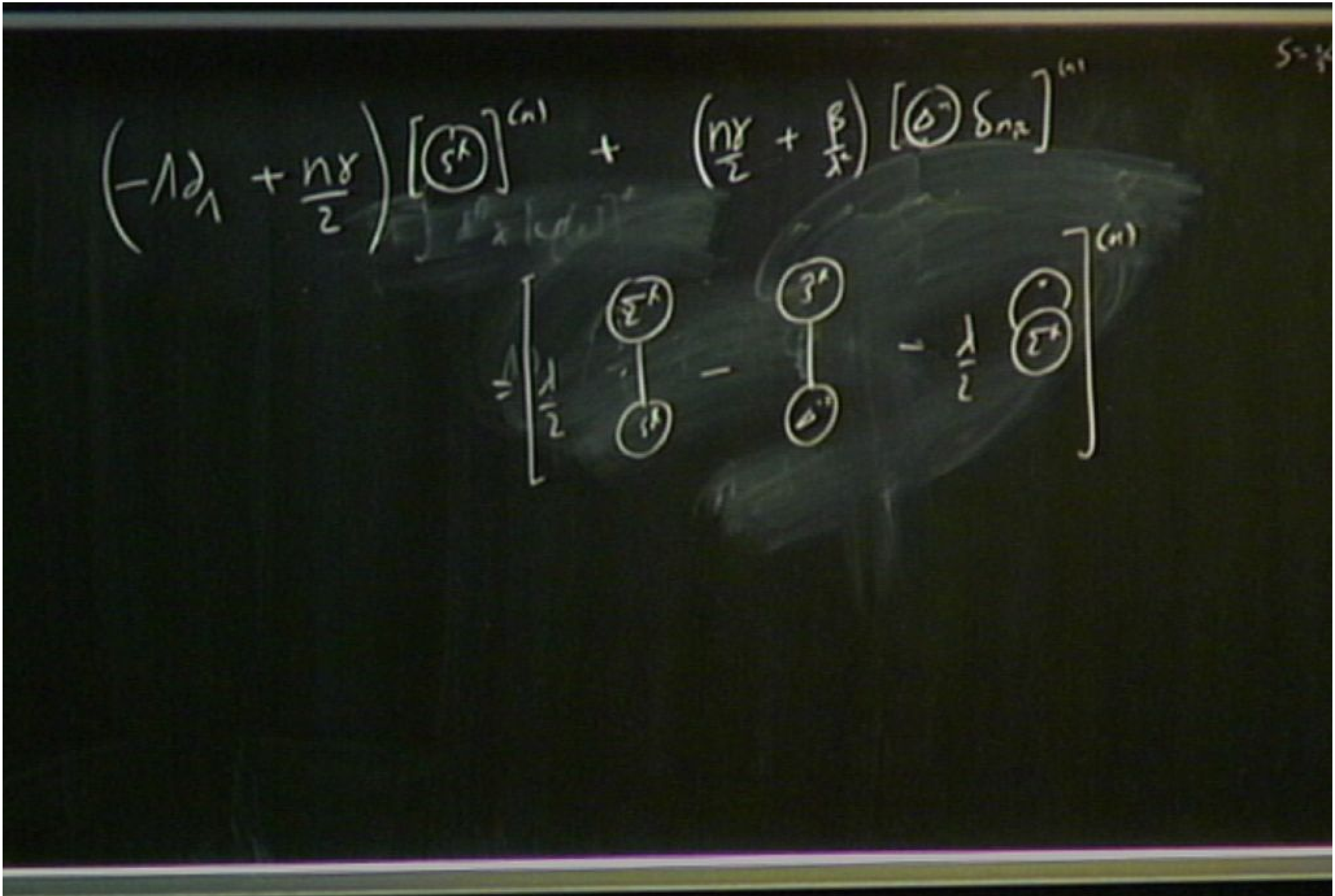
Example: Classical 4pt Vertex



- diagrams with q, r, s on bottom vertex


$\left(-\Lambda\partial_\Lambda + \frac{n\gamma}{2}\right)\left[\ \right]^{(n)} + \left(\frac{n\gamma}{2} + \frac{\beta}{\lambda^2}\right)\left[\ \delta_{n,2}\right]$

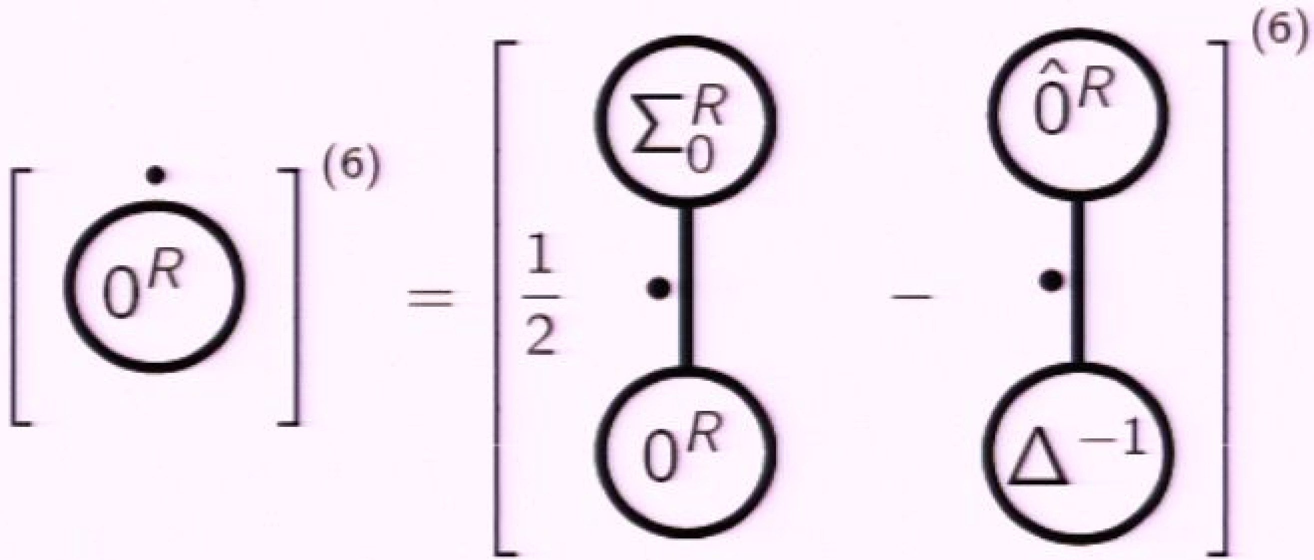




$$\left(-\Lambda\partial_\Lambda + \frac{n\gamma}{2}\right)\left[\,\circledS^\Lambda\,\right]^{(n)} + \left(\frac{n\gamma}{2} + \frac{\beta}{\lambda^2}\right)\left[\,\bigcirc\,\delta_{n2}\right]^{(n)}$$

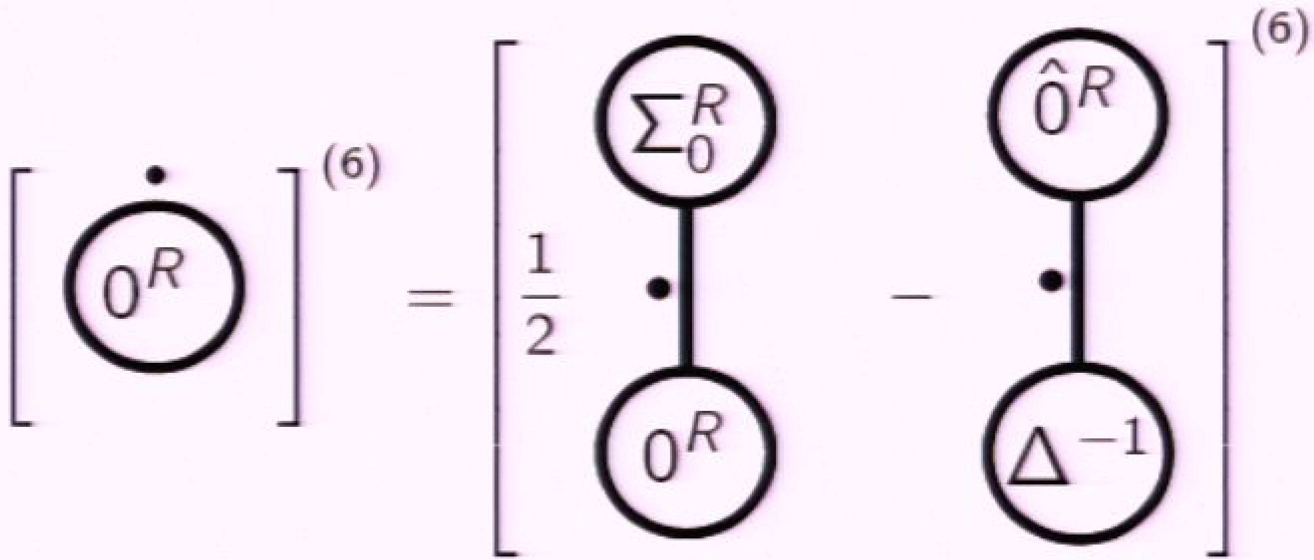
Example: Classical 6pt Vertex

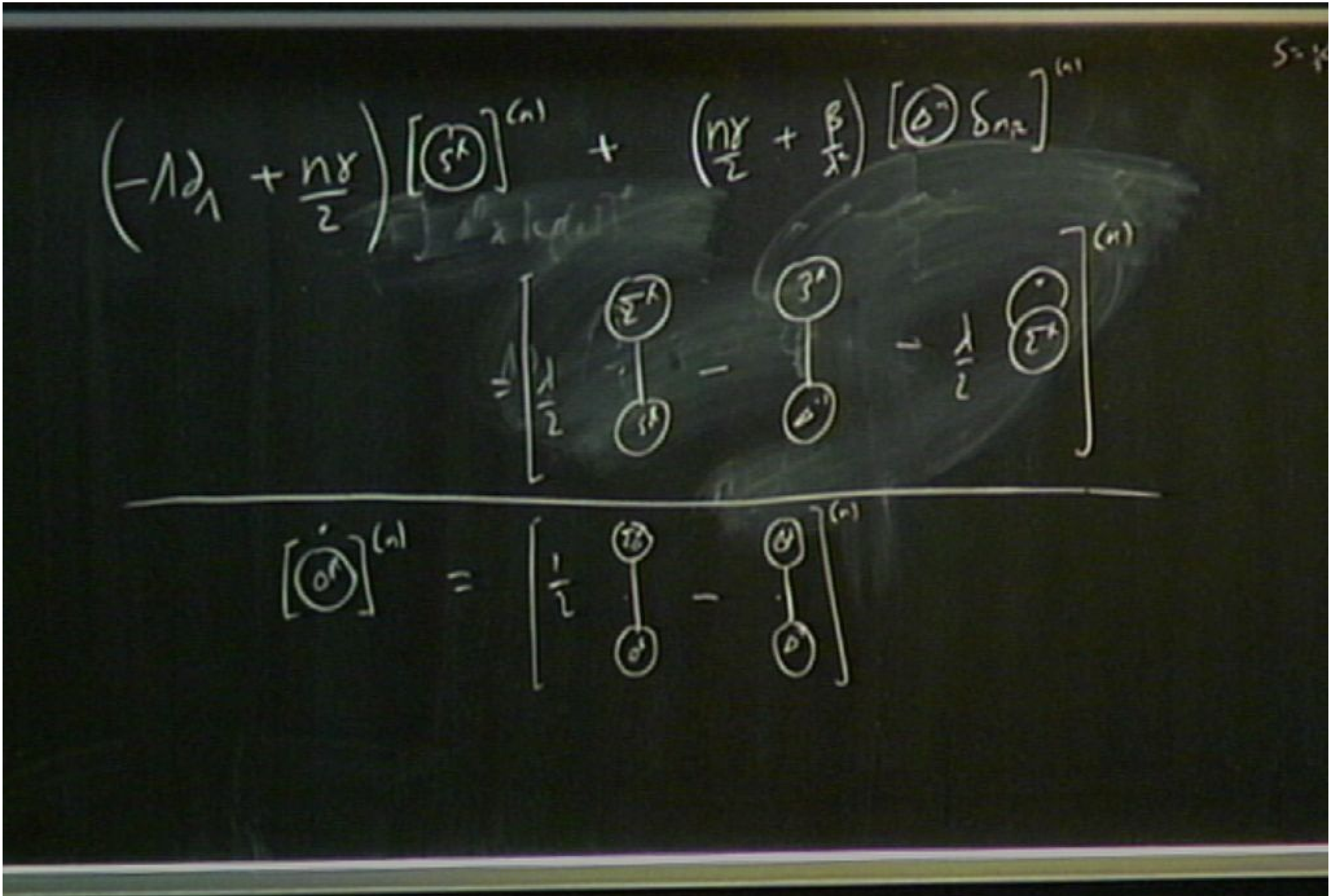
Abstract Notation

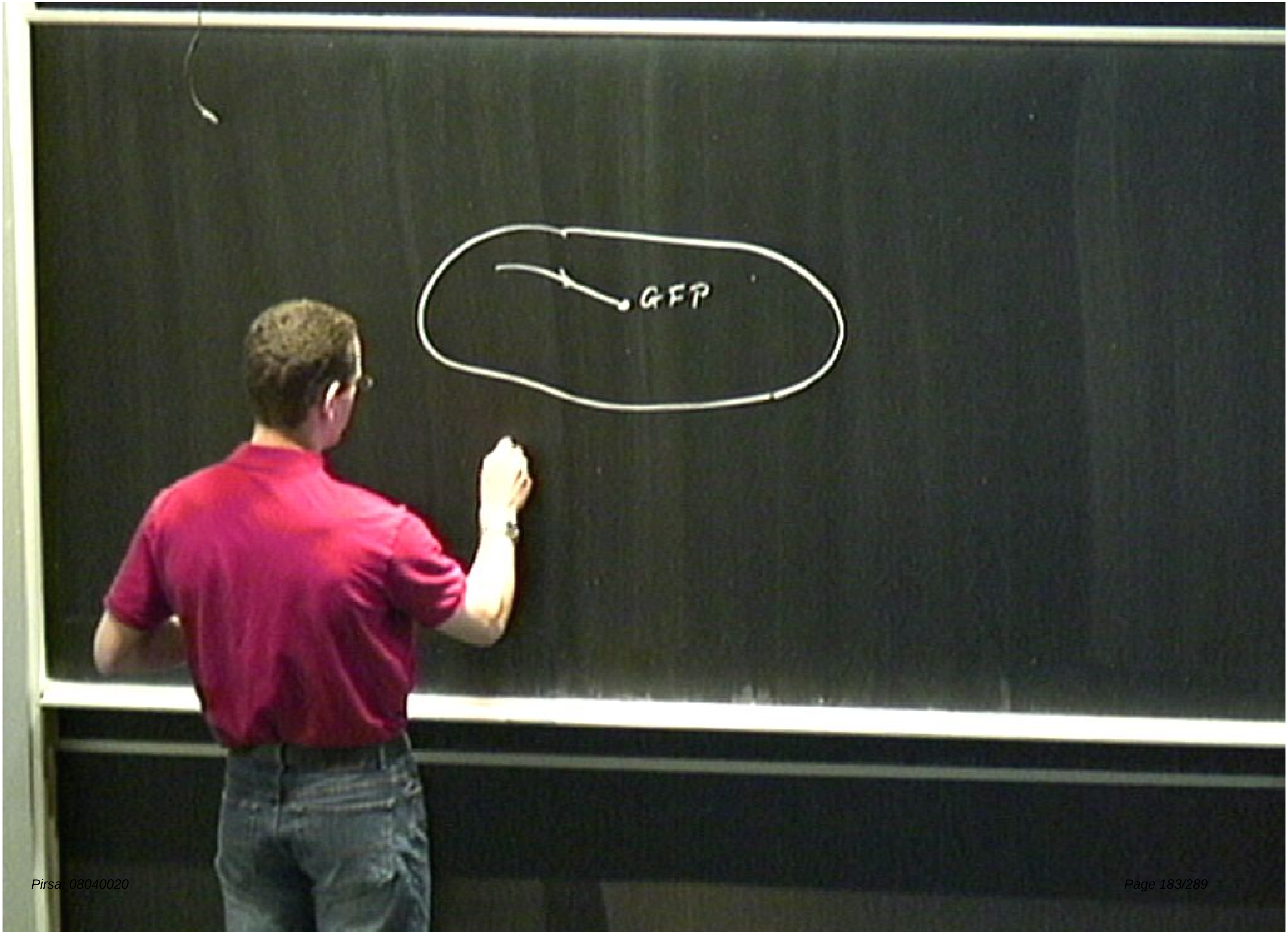


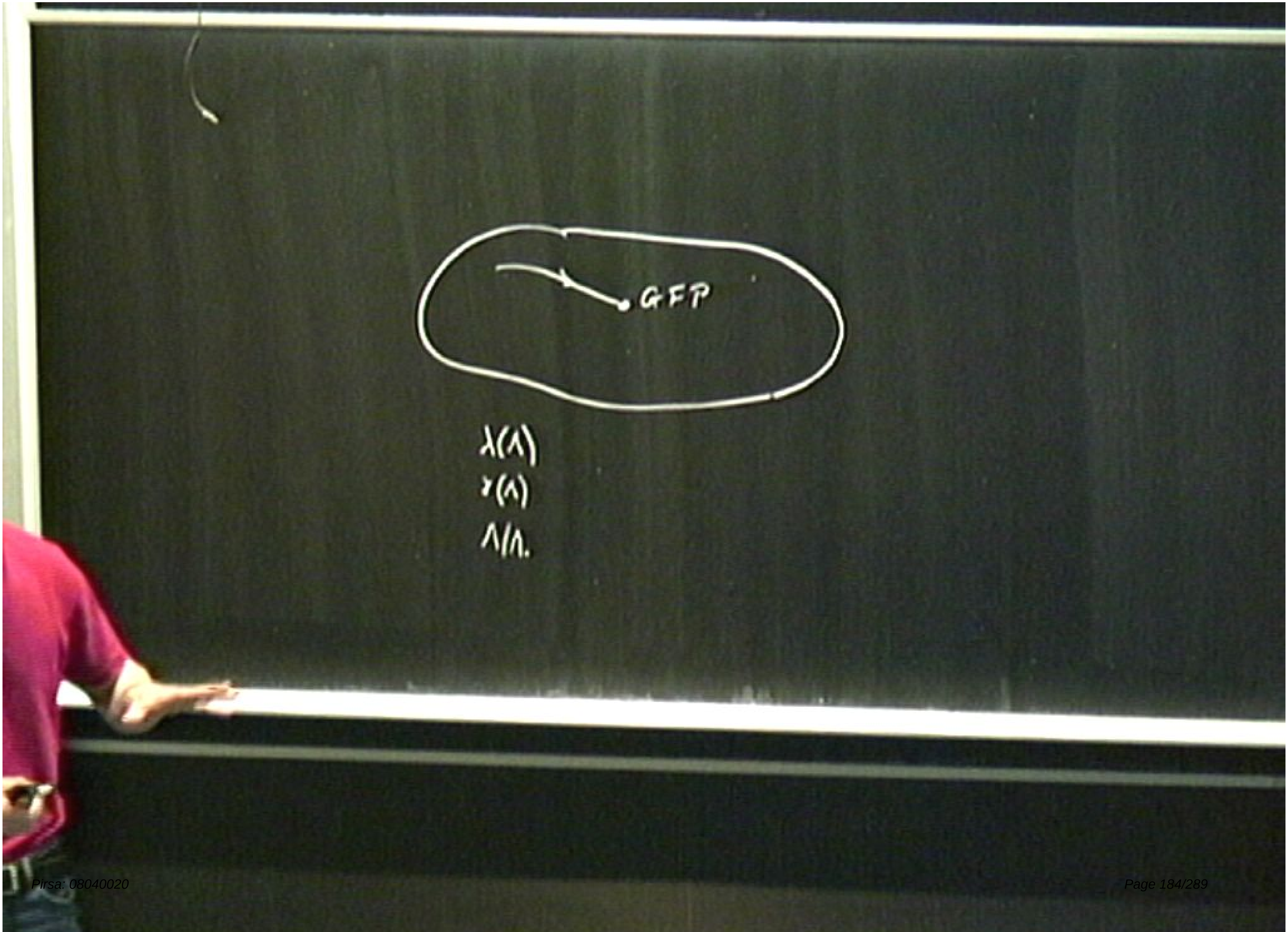
Example: Classical 6pt Vertex

Abstract Notation






GFP


GFP
λ(Λ)

Renormalization Conditions

Two-Point Vertex

- Canonical normalization of the kinetic term
- $S^{(2)}(p) = p^2 + \sigma(\lambda)\Lambda^2 + \mathrm{O}\left(p^4/\Lambda^2\right)$
- Unit coefficient
- The mass is implicitly set to zero

ERG Kernel

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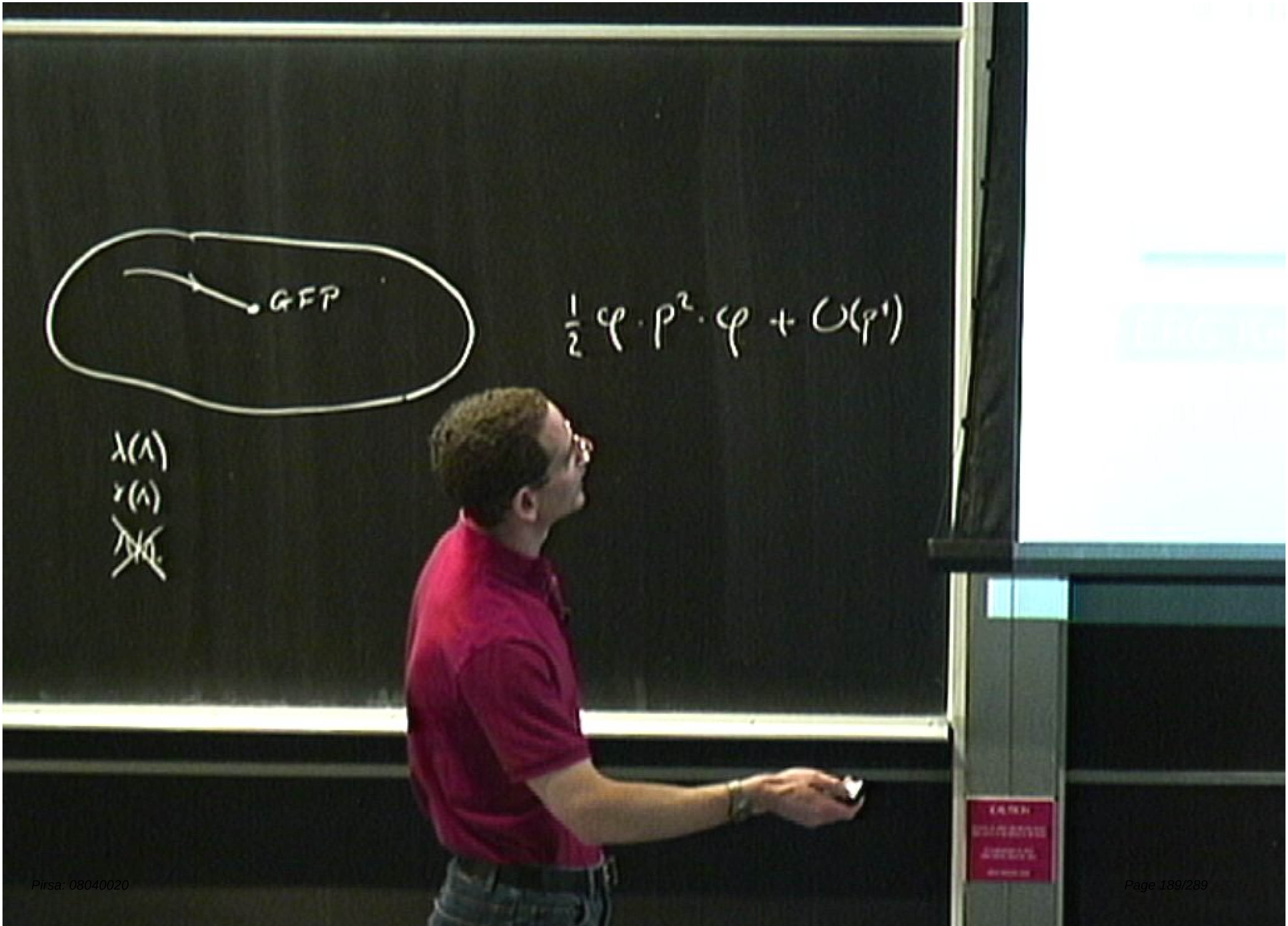
GFP
1/2 φ · p² · φ + O(φ⁴)
λ(Λ)

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  - Only Λ appears, which tends to zero as all modes are integrated out
  - $\sigma(\lambda)$ starts at one-loop and is determined self-consistently

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- $\Rightarrow S^{\mathrm{R}(2)}(p)\Big|_{p^2} = 0$

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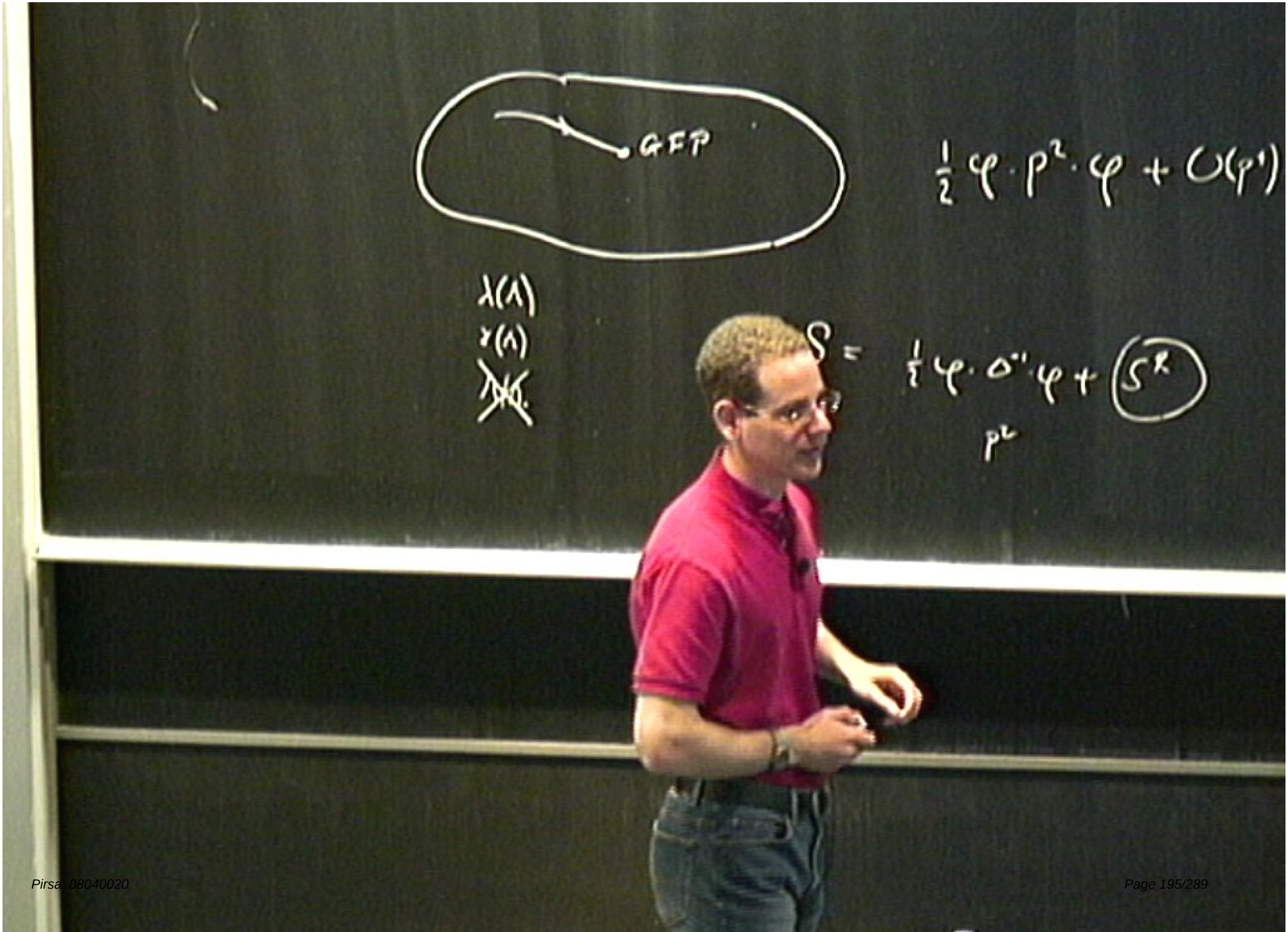
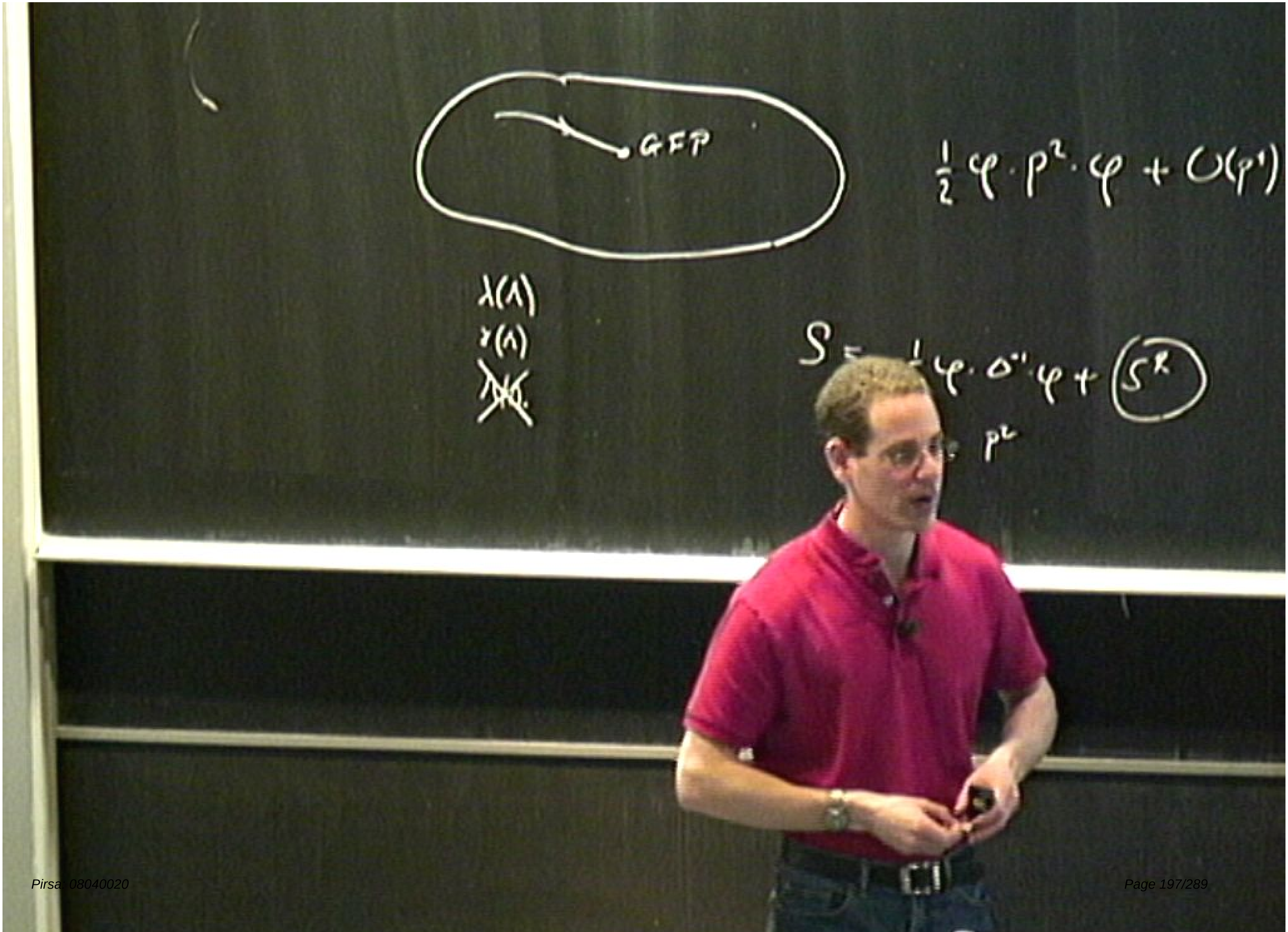

GFP
½ φ · p² · φ + O(φ⁴)
λ(Λ)
γ(Λ)
S = ½ φ · Δ⁻¹ · φ + S^R
p²

Renormalization Conditions

Four-Point Vertex

- Remember that we have
  - rescaled $\varphi \to \varphi/\sqrt{\lambda}$
  - pulled $1/\lambda$ outside the action
- $S^{(4)}(\underline{0}, \Lambda) = 1$
- This condition is saturated at the classical level: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  - $S_0^{(4)}(\underline{0}, \Lambda) = 1$
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Flow of the 1-loop, Two-Point Vertex

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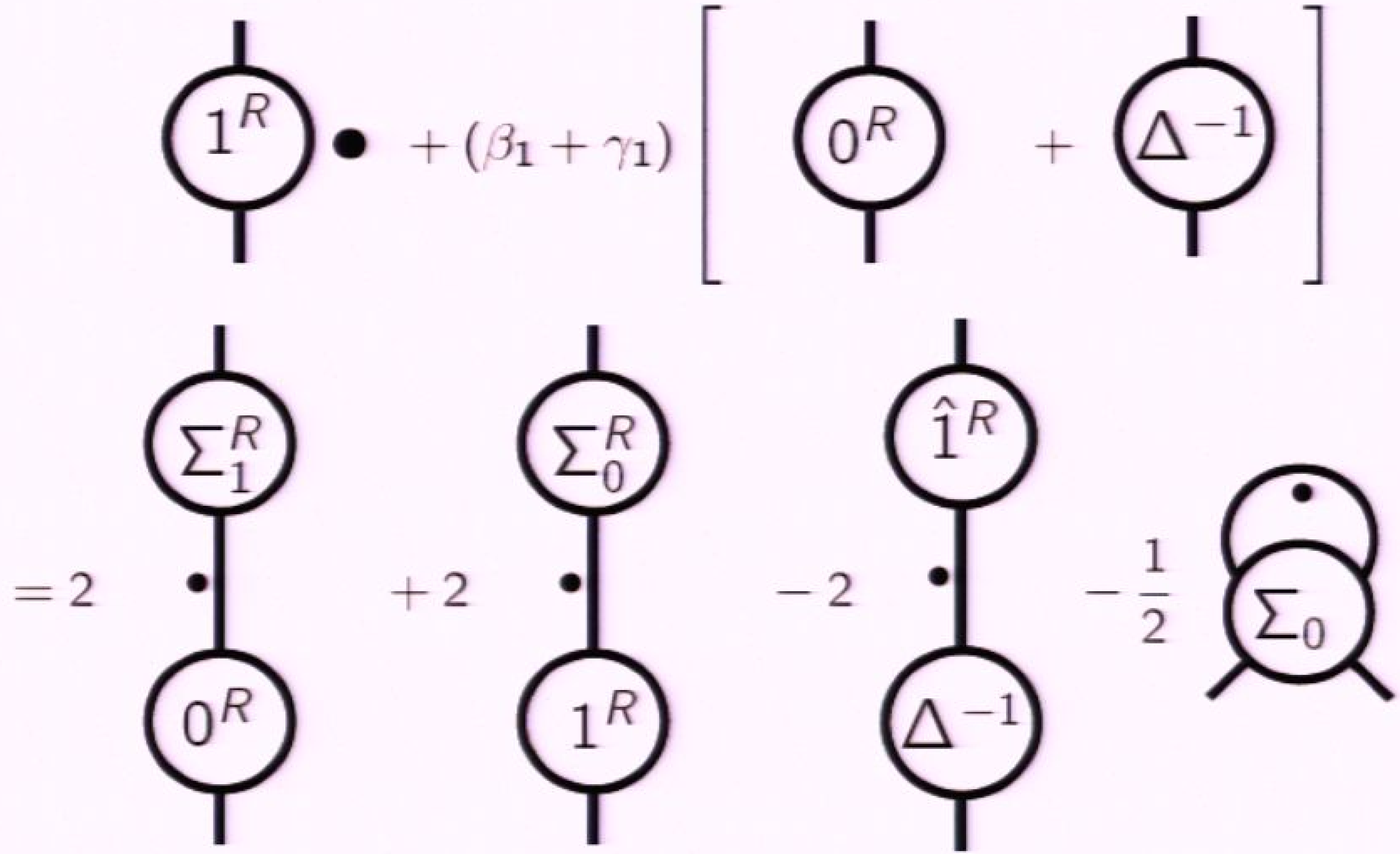
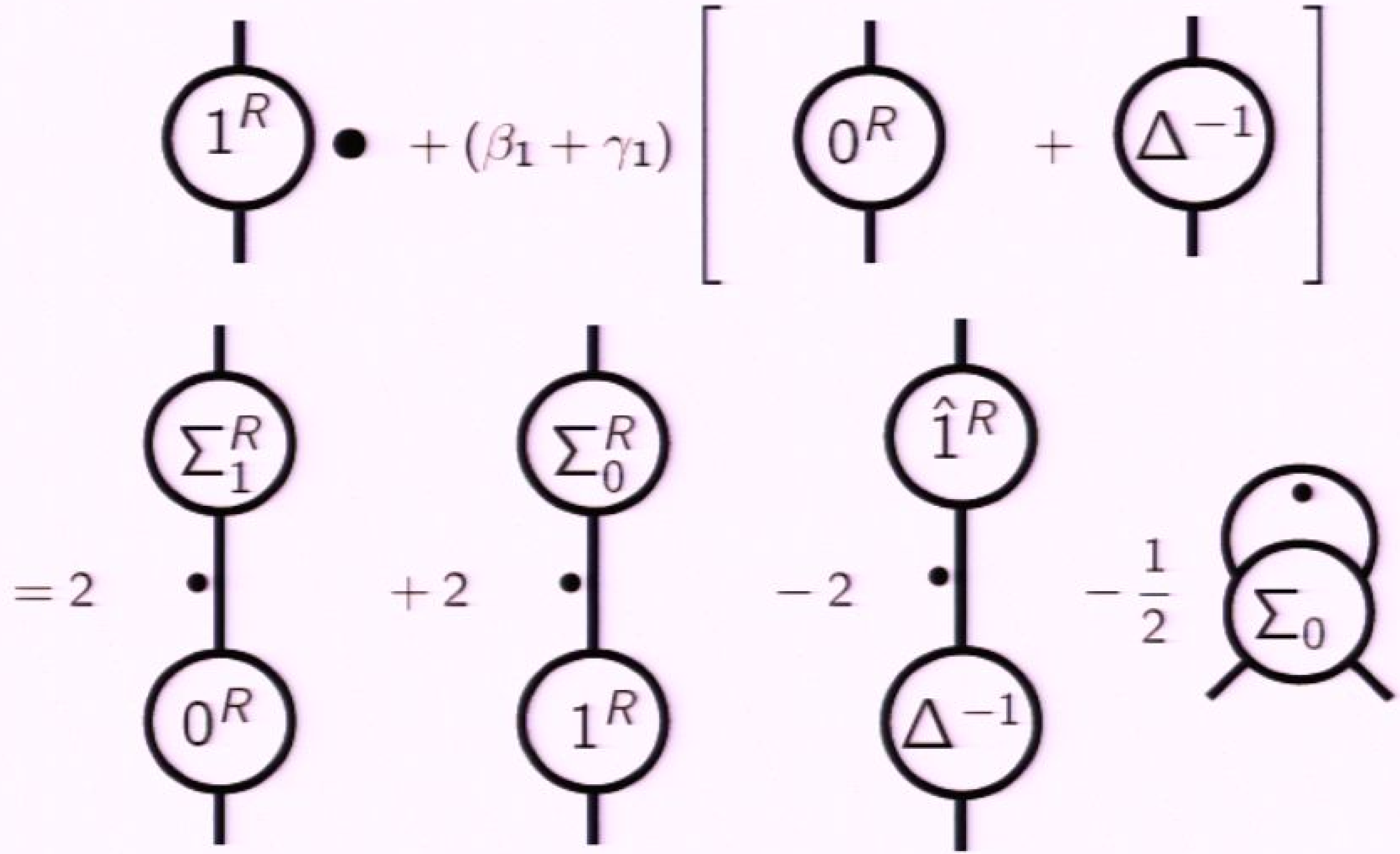
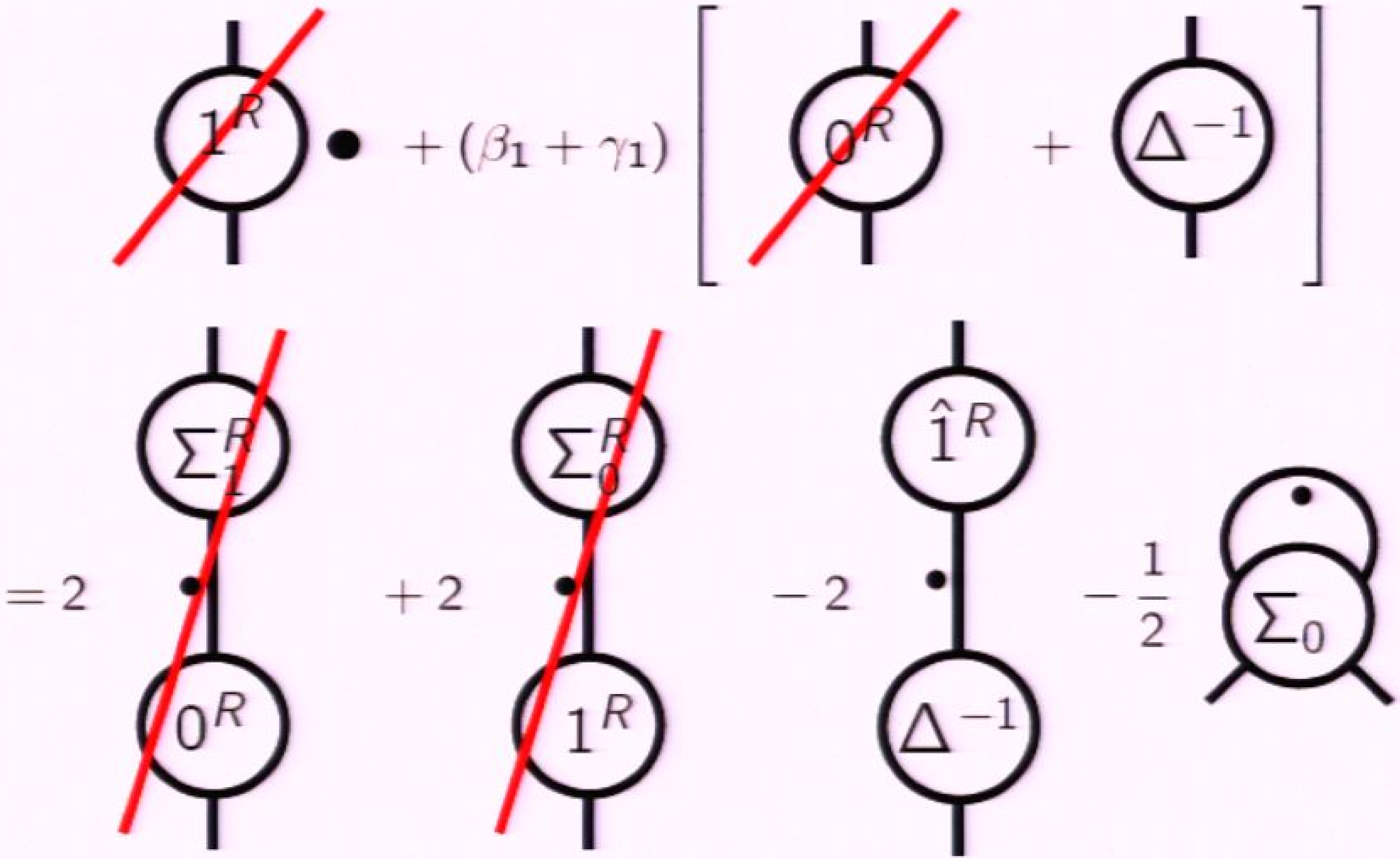


Flow of the 1-loop, Two-Point Vertex



- At $\mathrm{O}\left(p^2\right)$, recalling $S^{\mathrm{R}(2)}(p)\big|_{p^2} = 0$

Flow of the 1-loop, Two-Point Vertex



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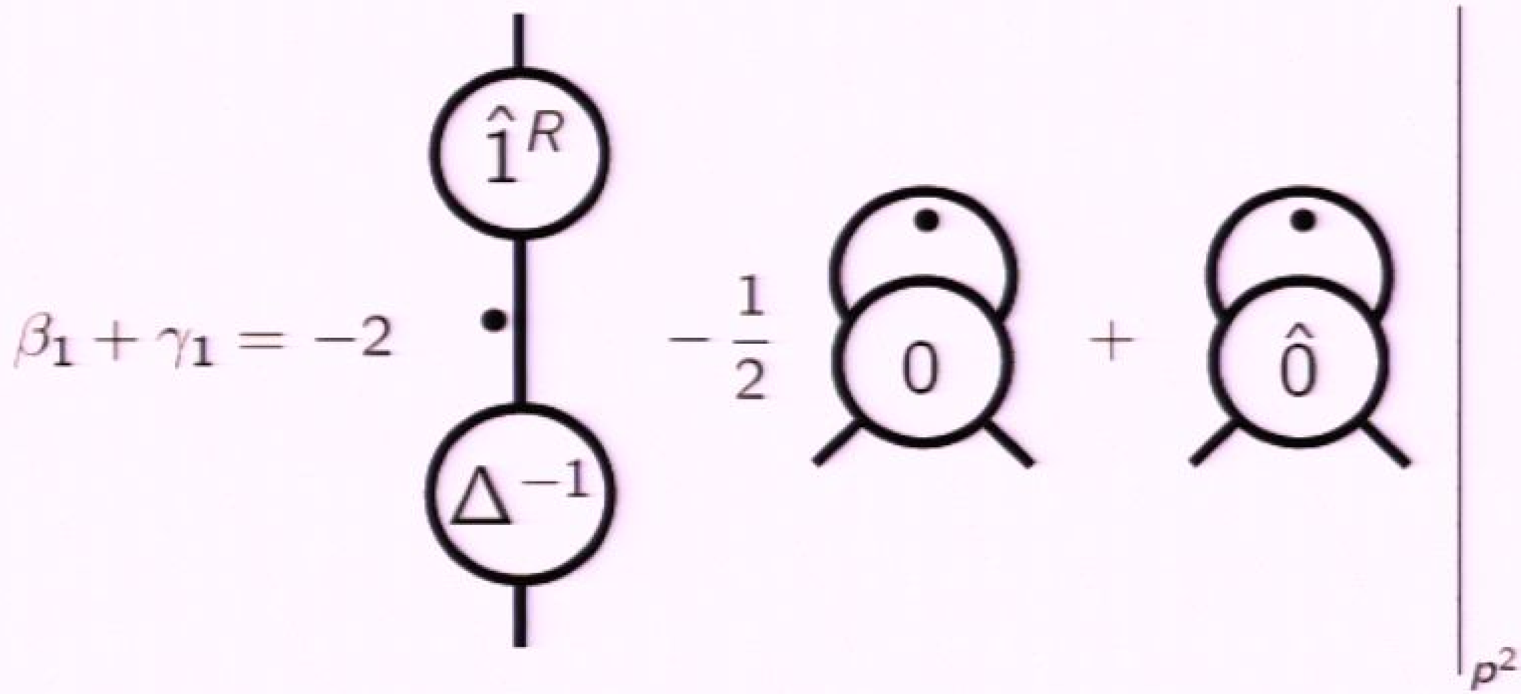
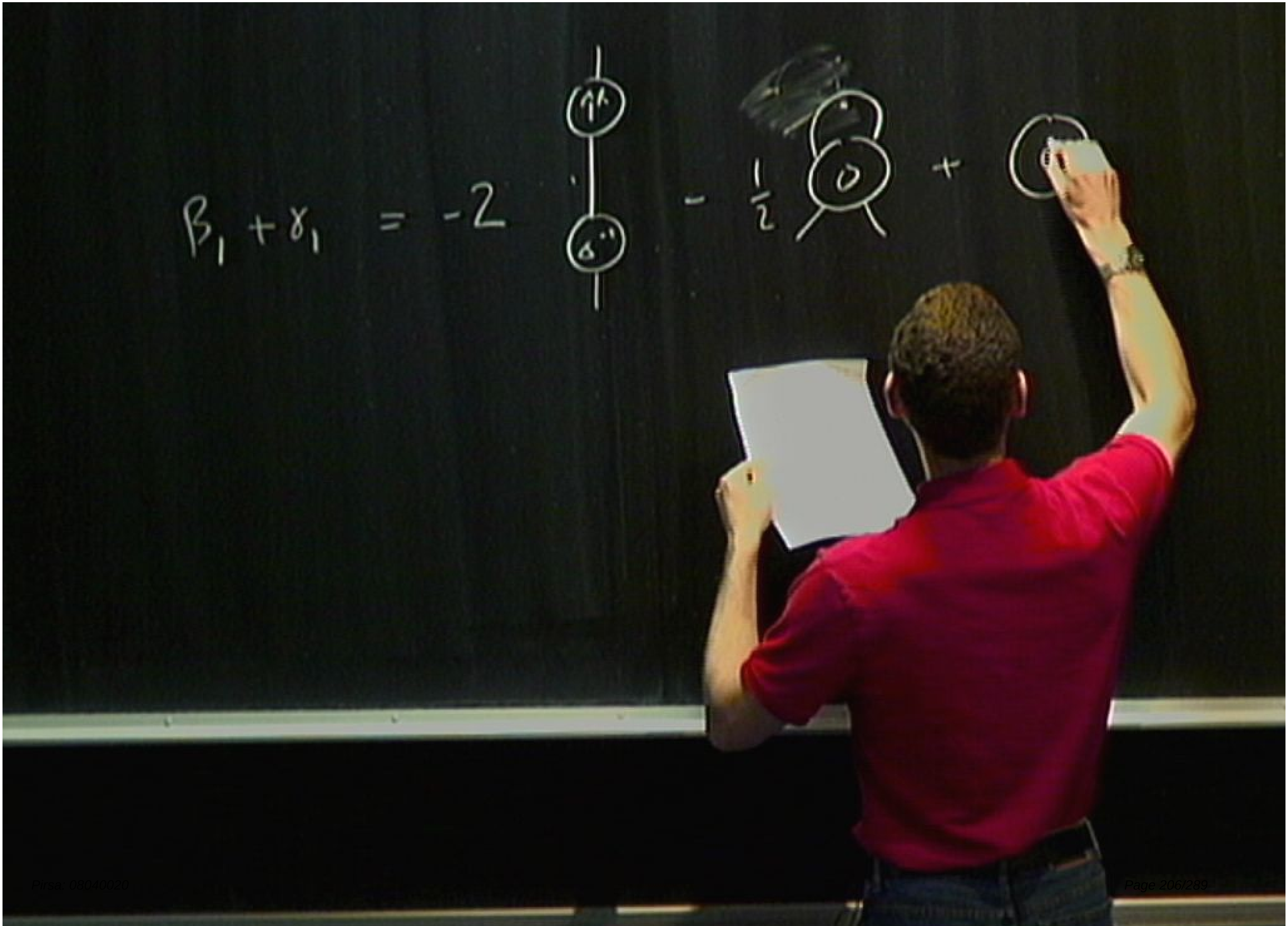
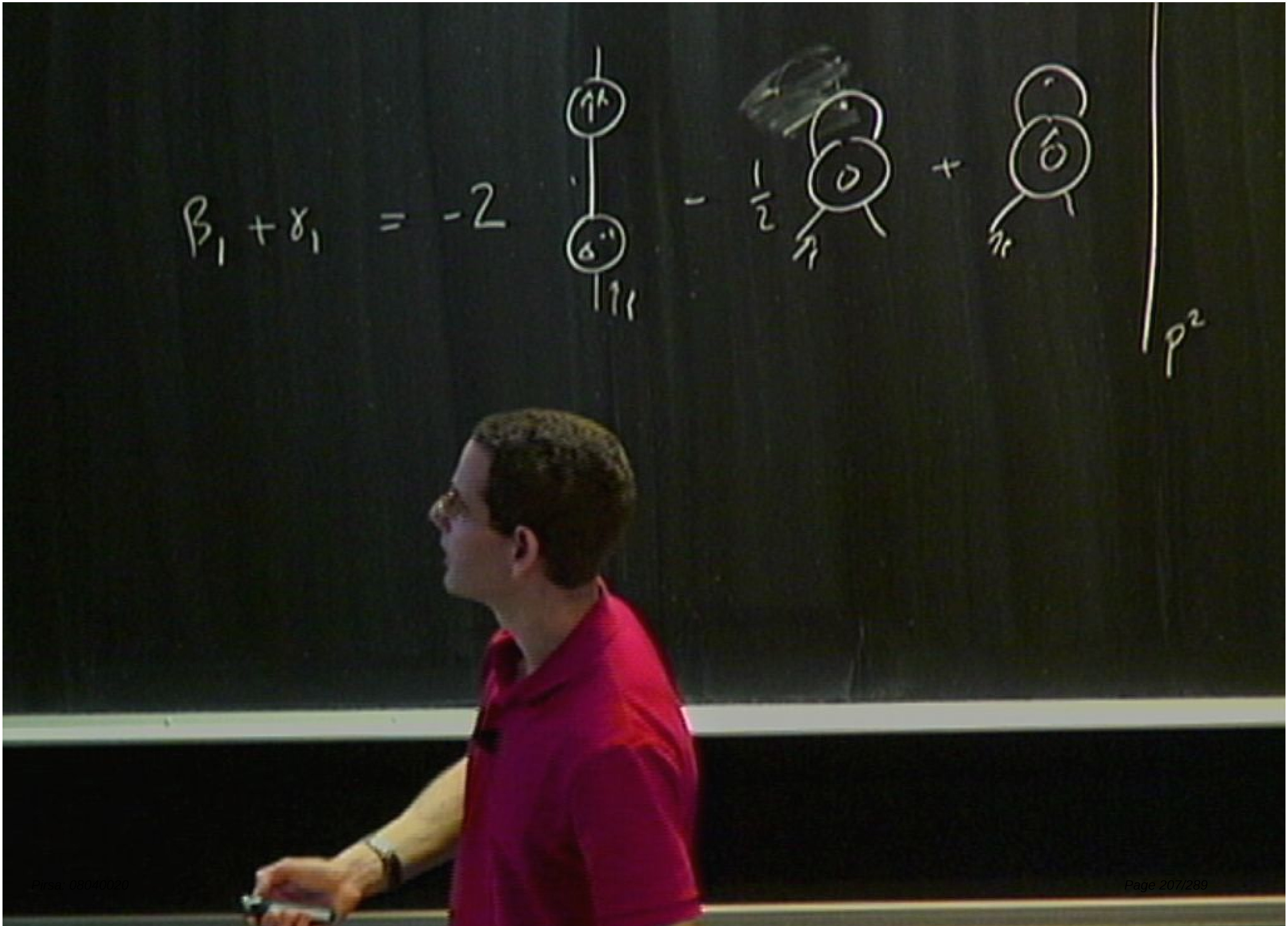
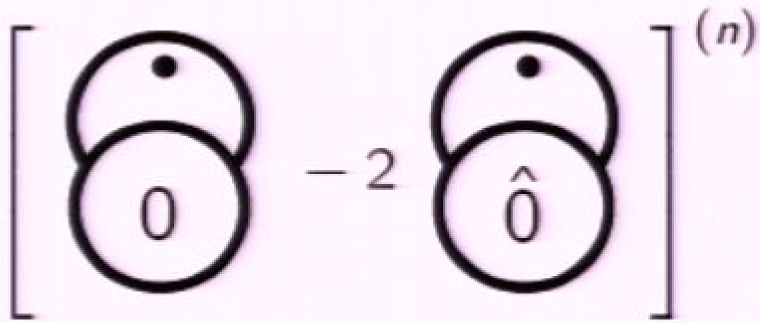



β1 + γ1 = -2
- 1/2
+


β1 + γ1 = -2
-
1/2
+
p2

Diagrammatic Fun & Games

Consider 

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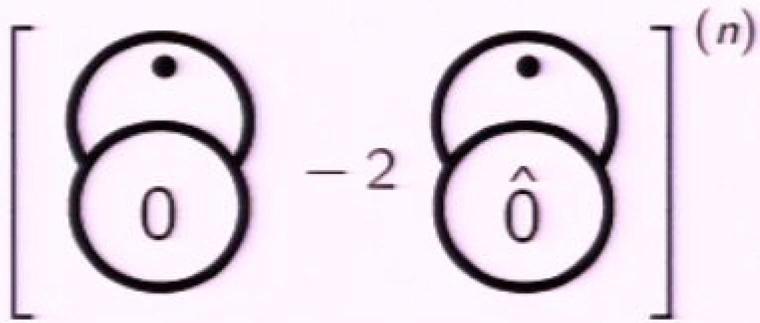
Consider 

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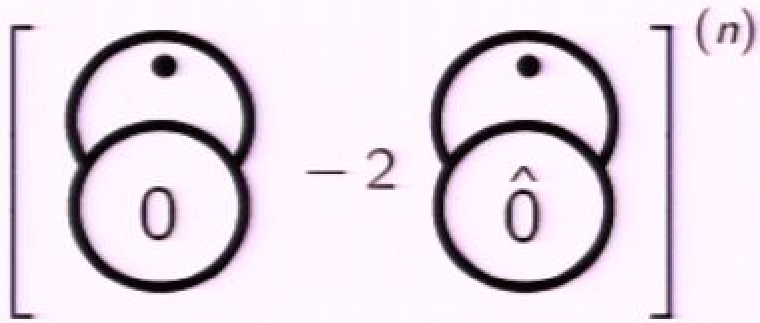
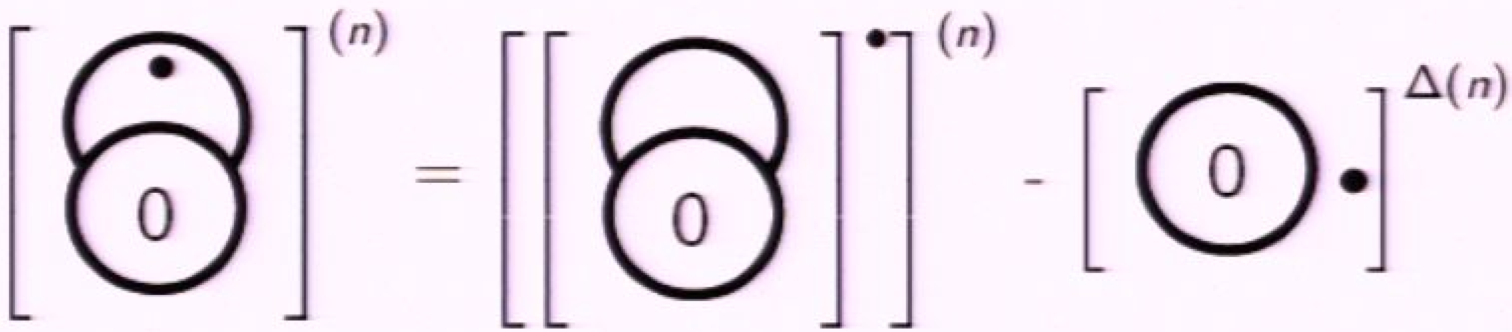
Consider 



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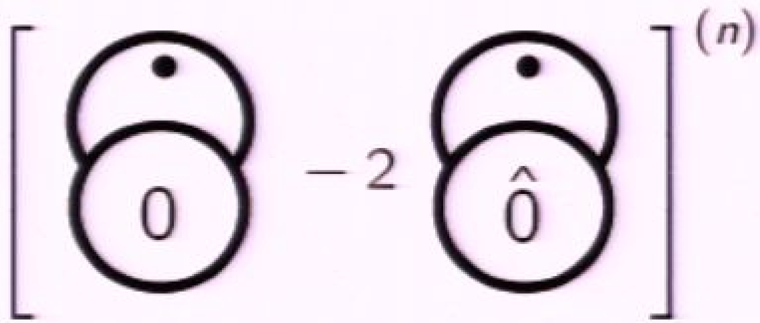
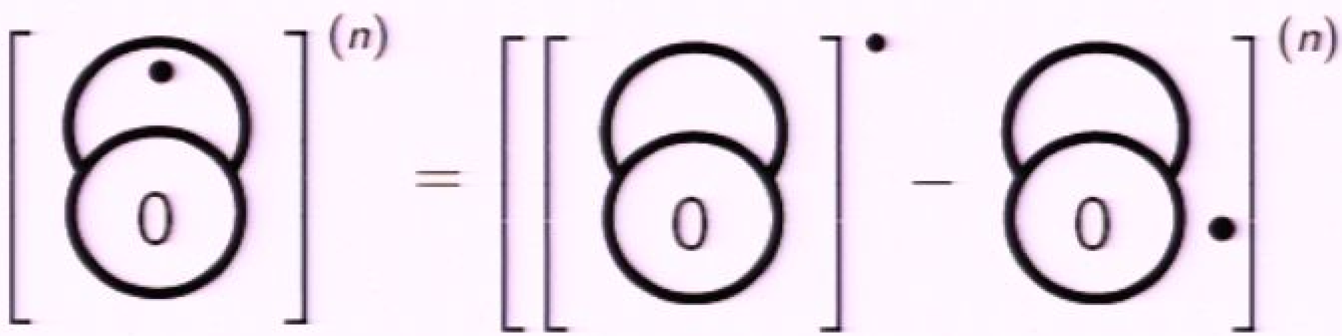
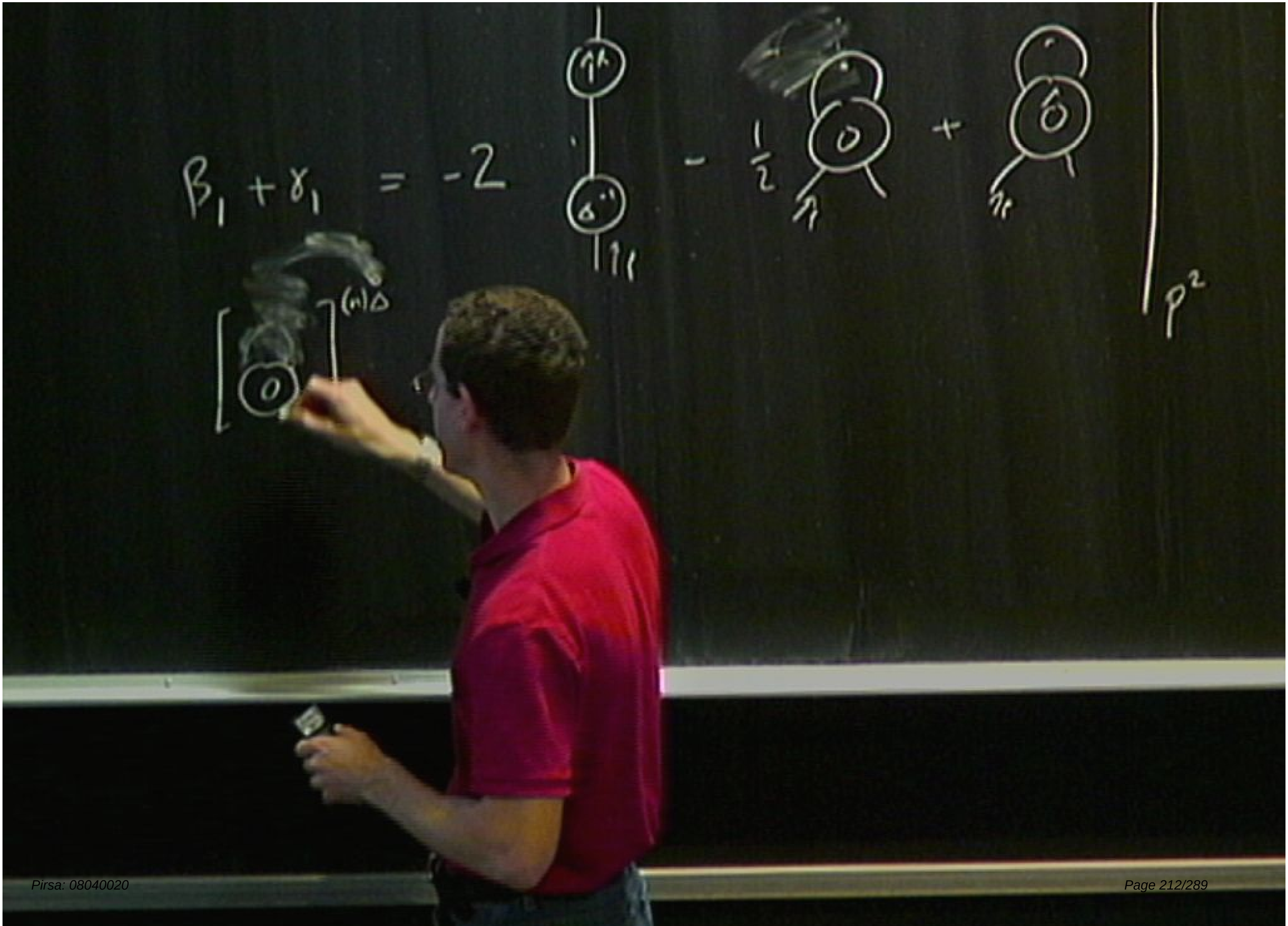
Consider 

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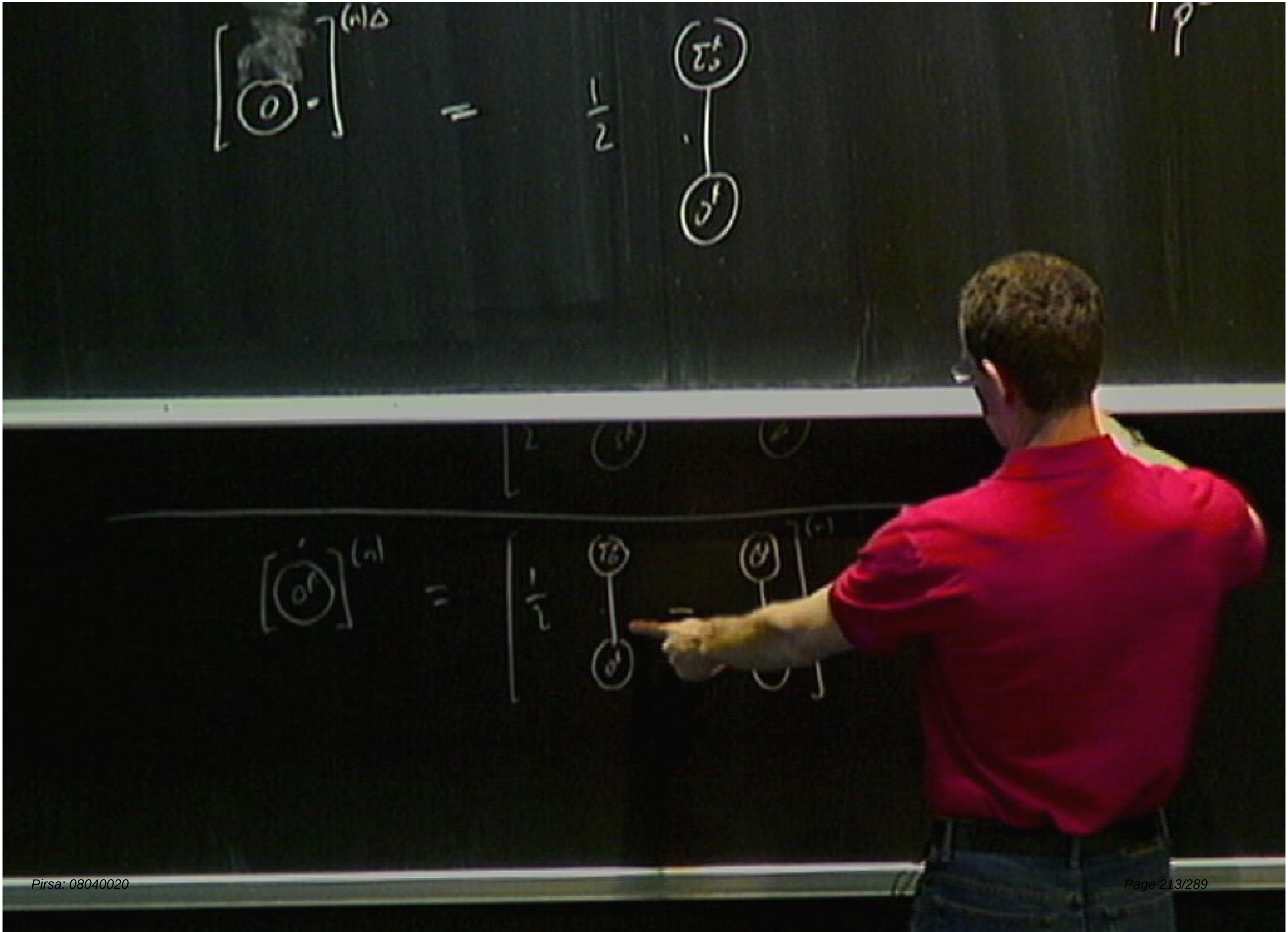
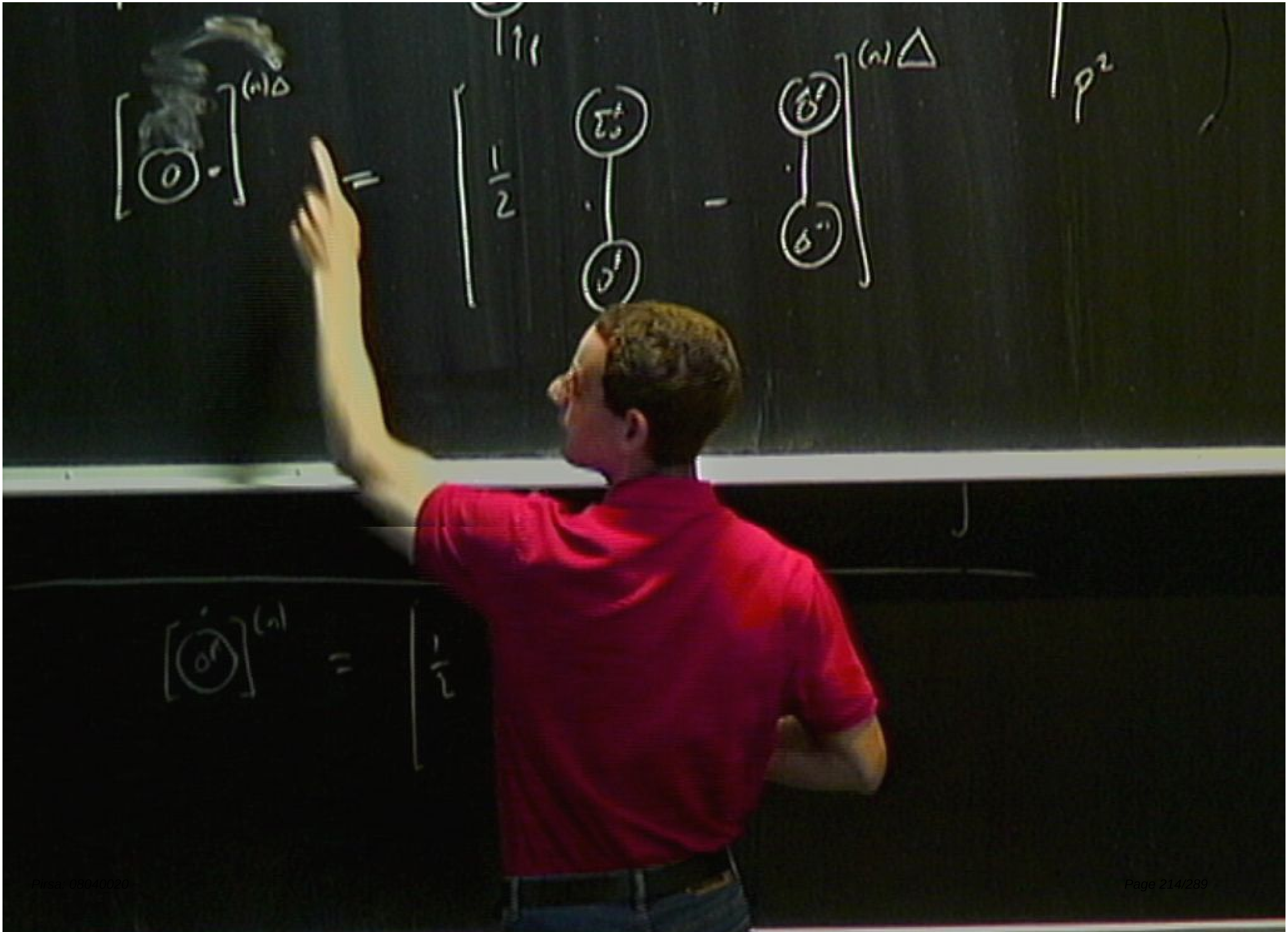
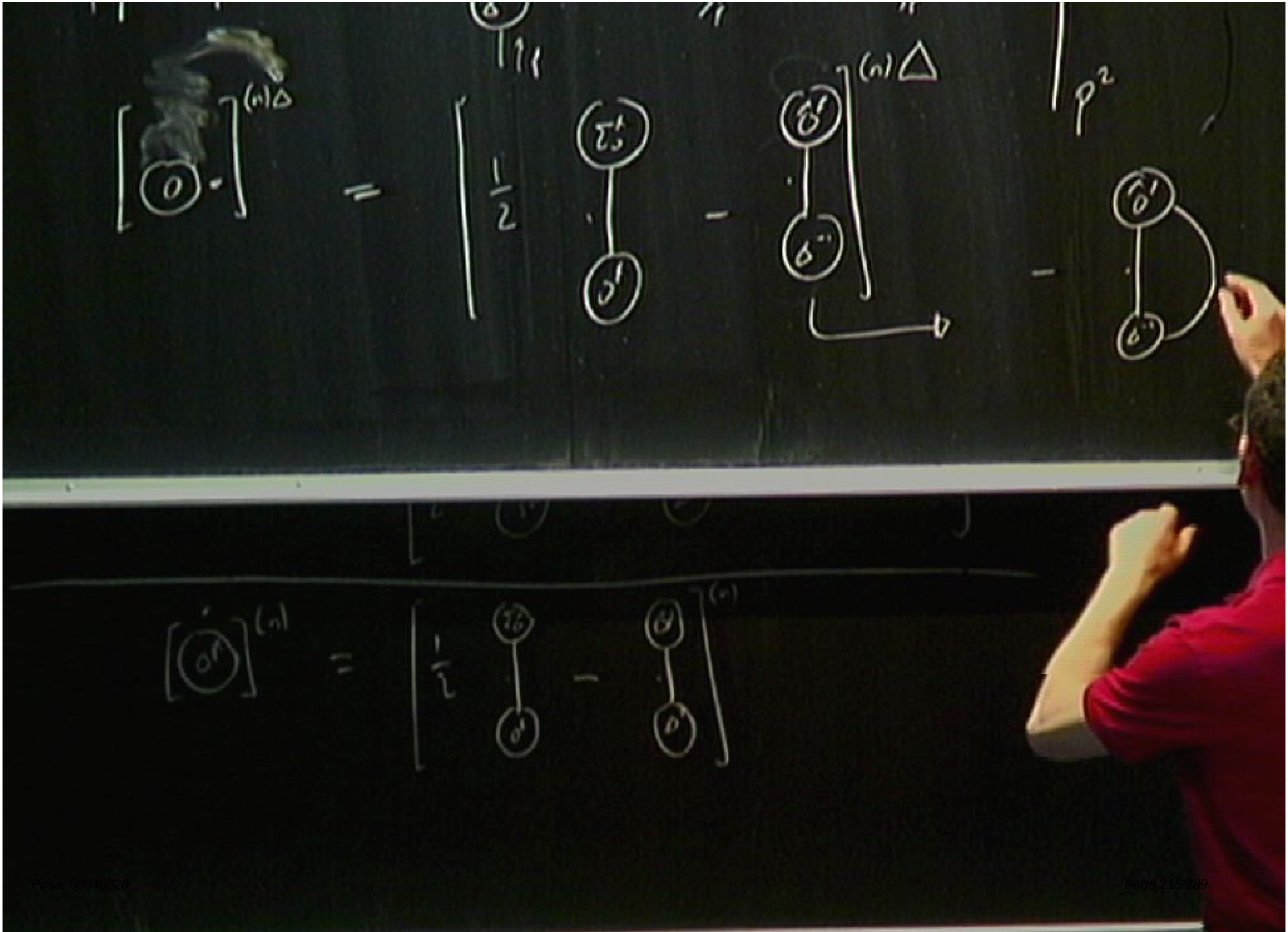
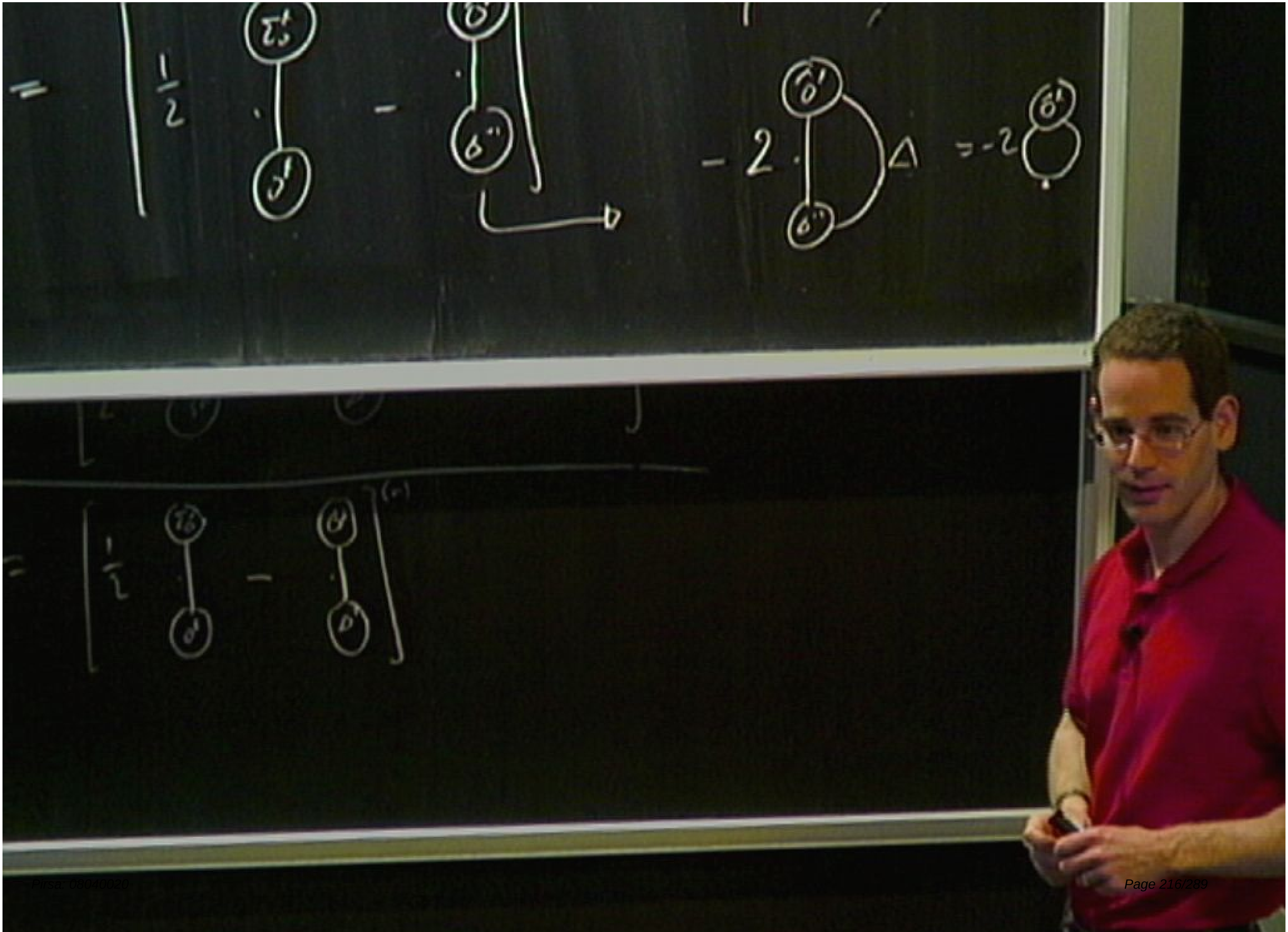
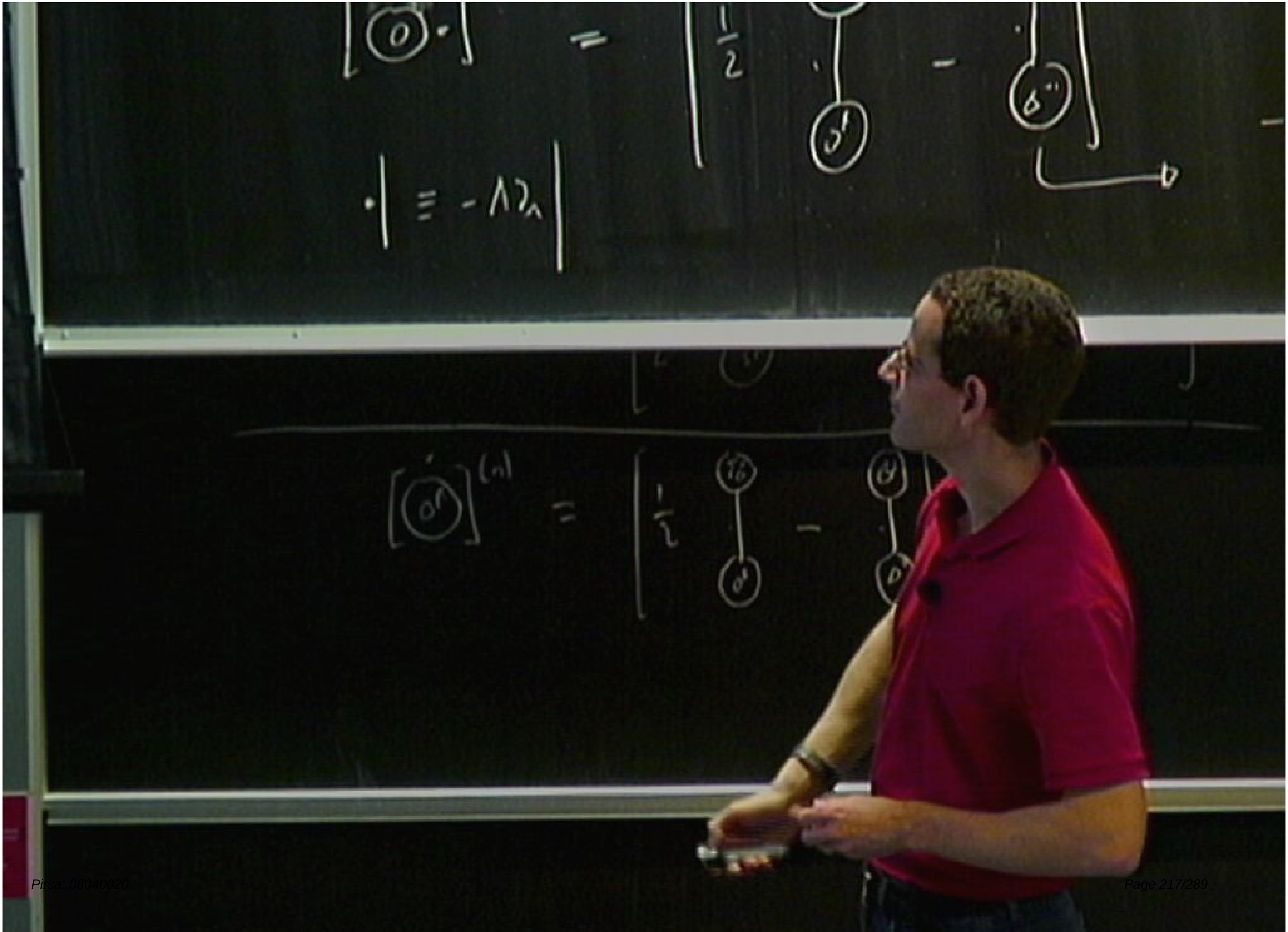
- 1/2
+
p2



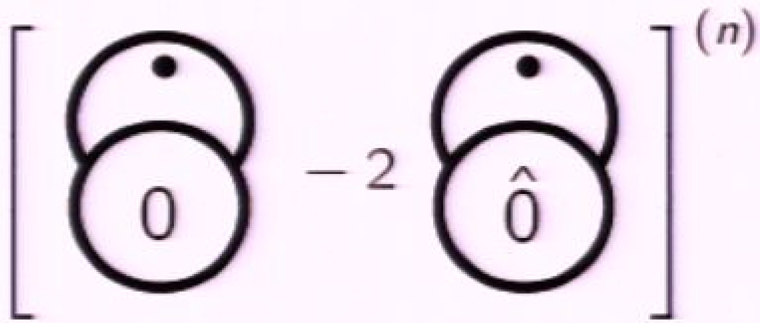


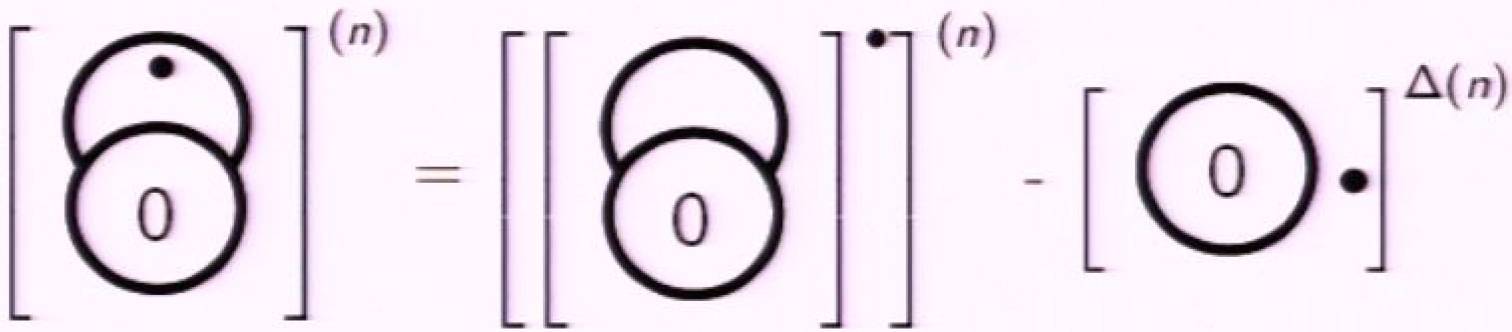




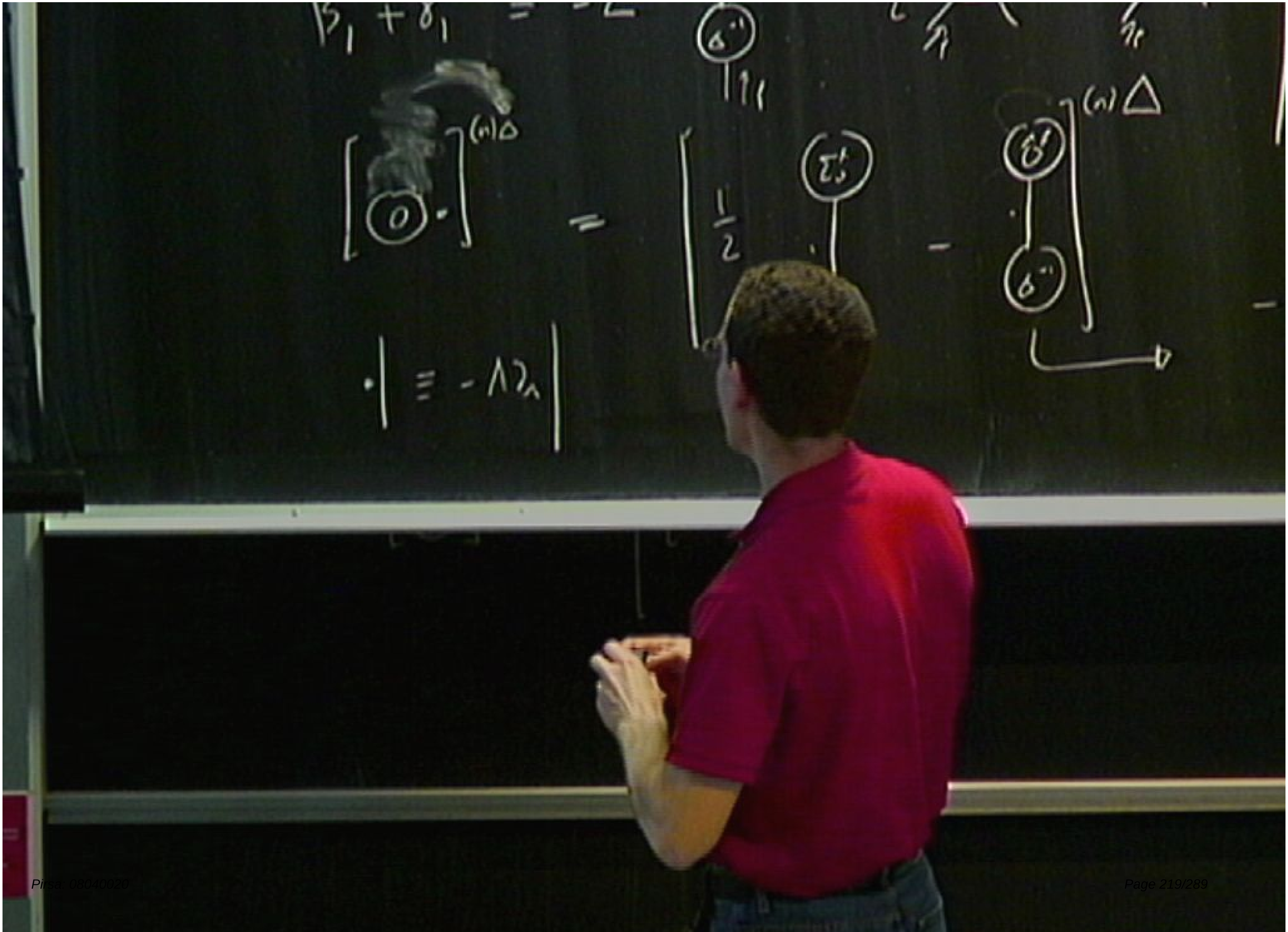


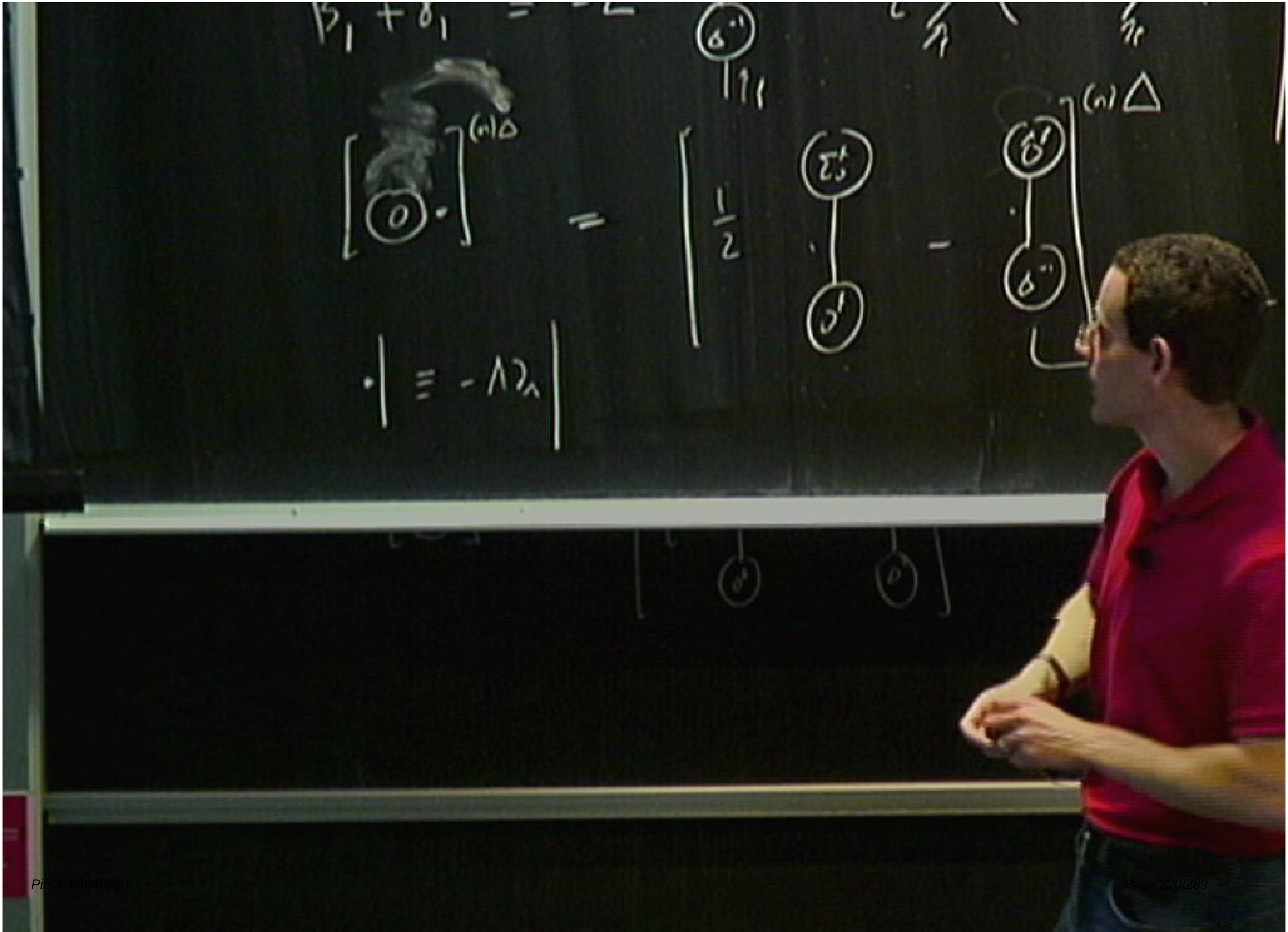
Diagrammatic Fun & Games

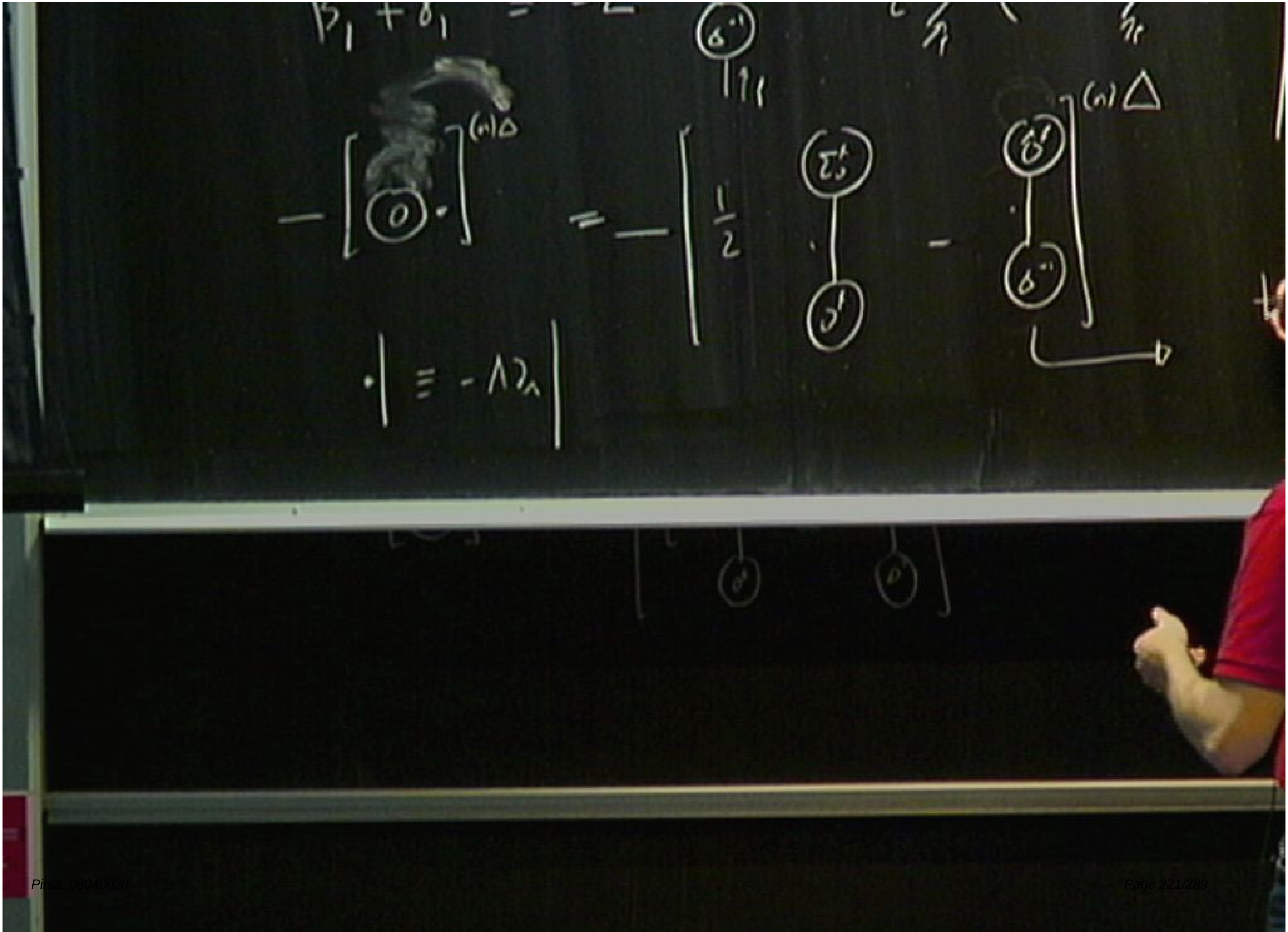
Consider


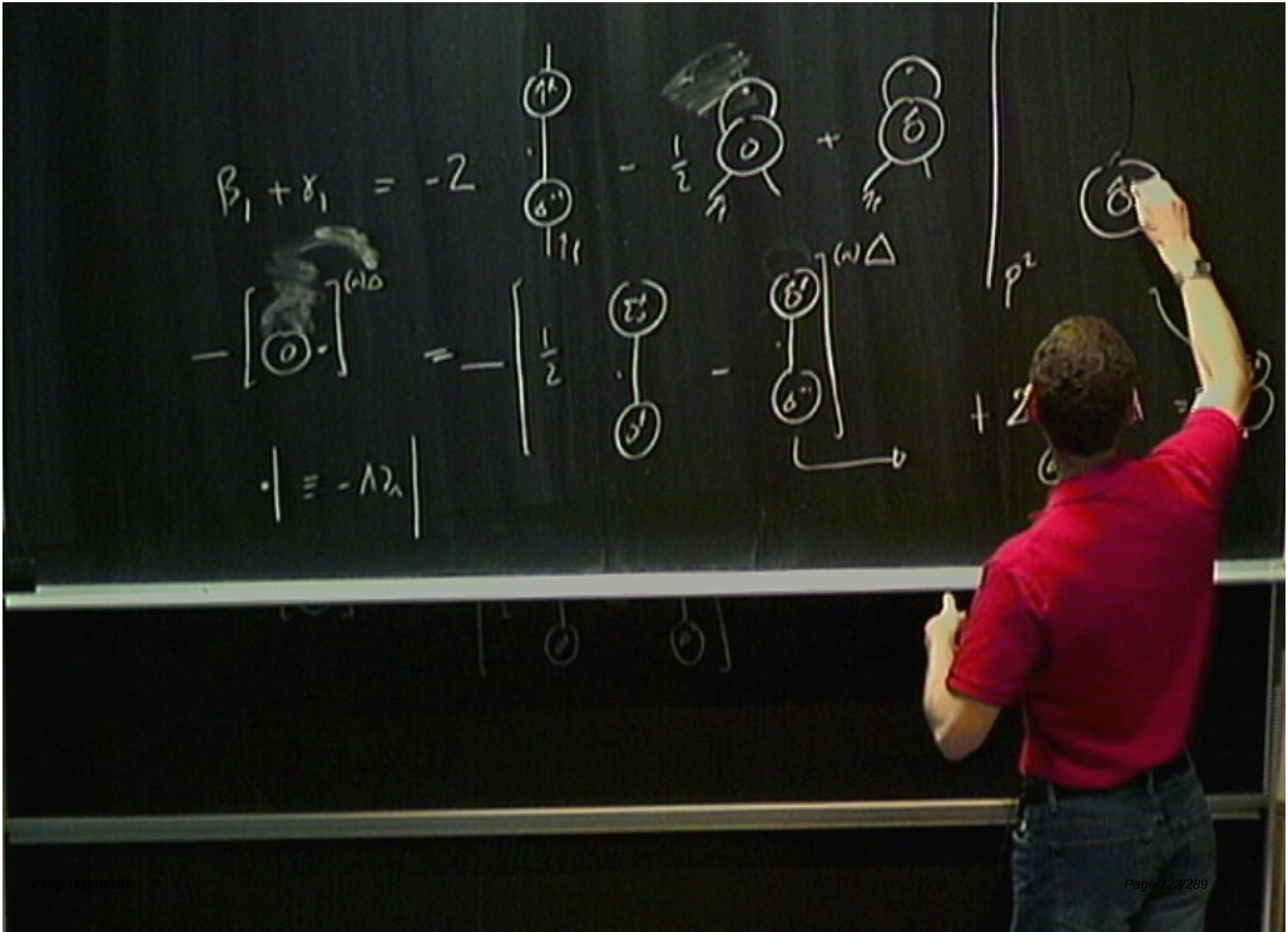


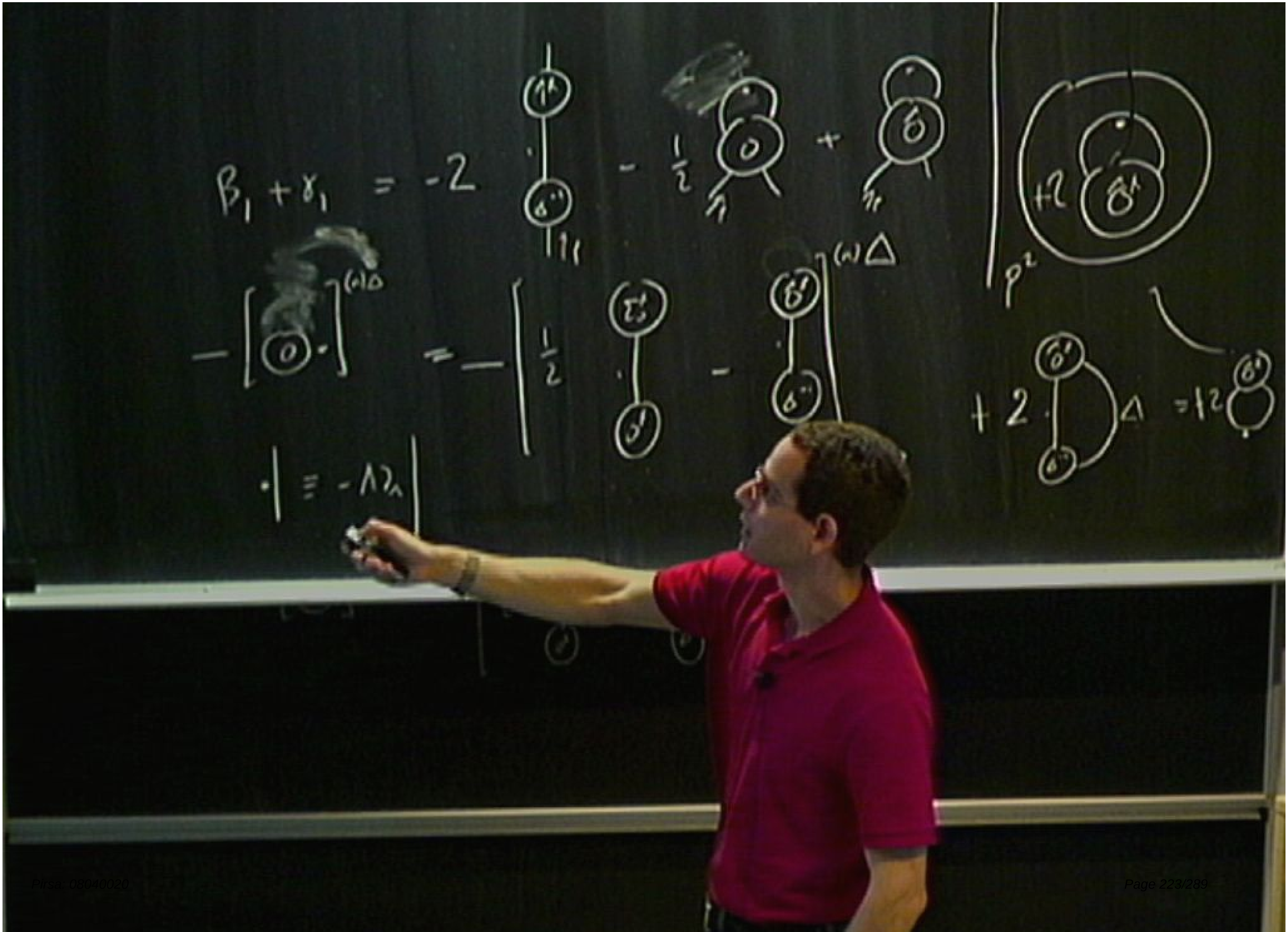
This is one for the blackboard!





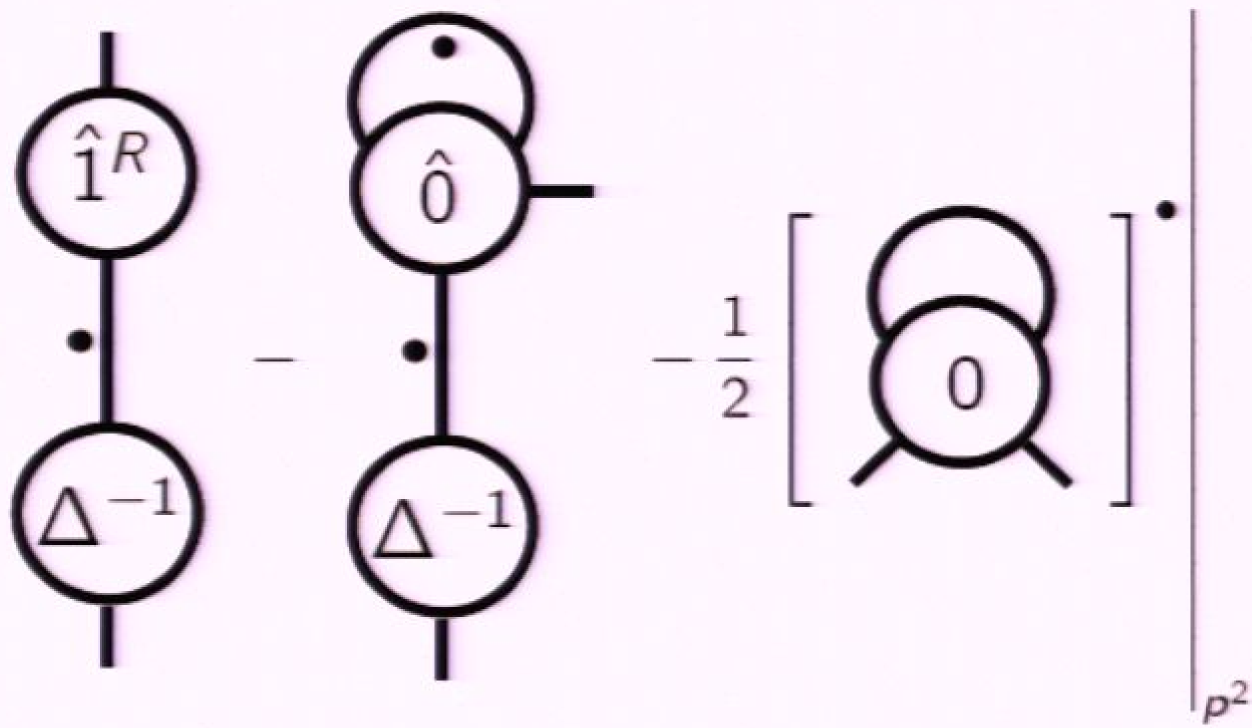



β1 + γ1 = -2


β1 + γ1 = -2
= -Λλ_Λ

Flow of the 1-loop, Two-Point Vertex Continued...

$\beta_1 + \gamma_1 = -2$



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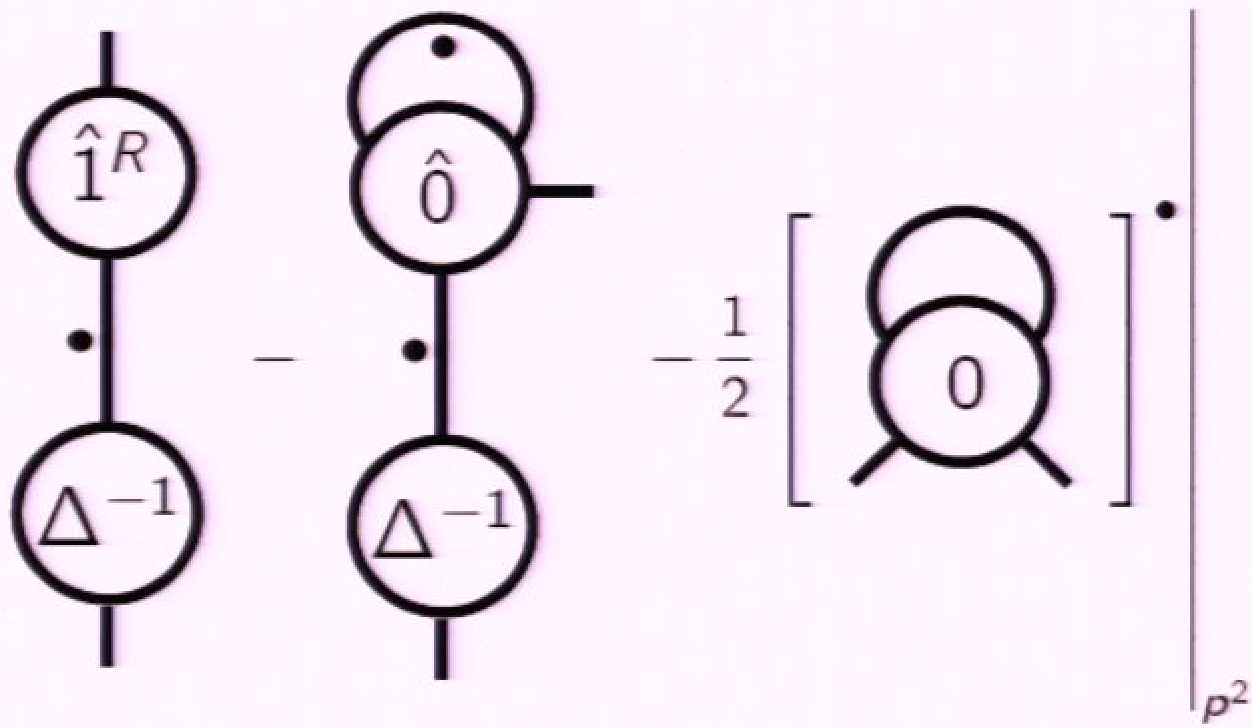

Flow of the Four-Point

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- Use the renormalization condition, $S^{(4)}(\underline{0}, \Lambda) = 1$
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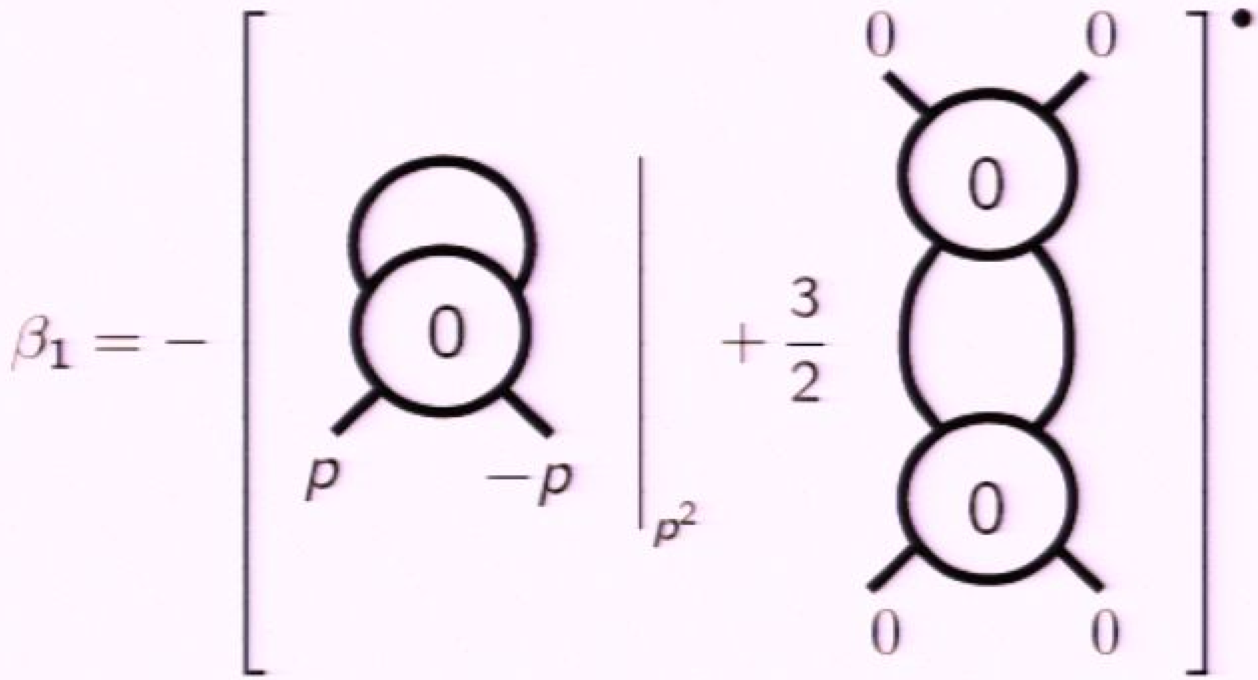

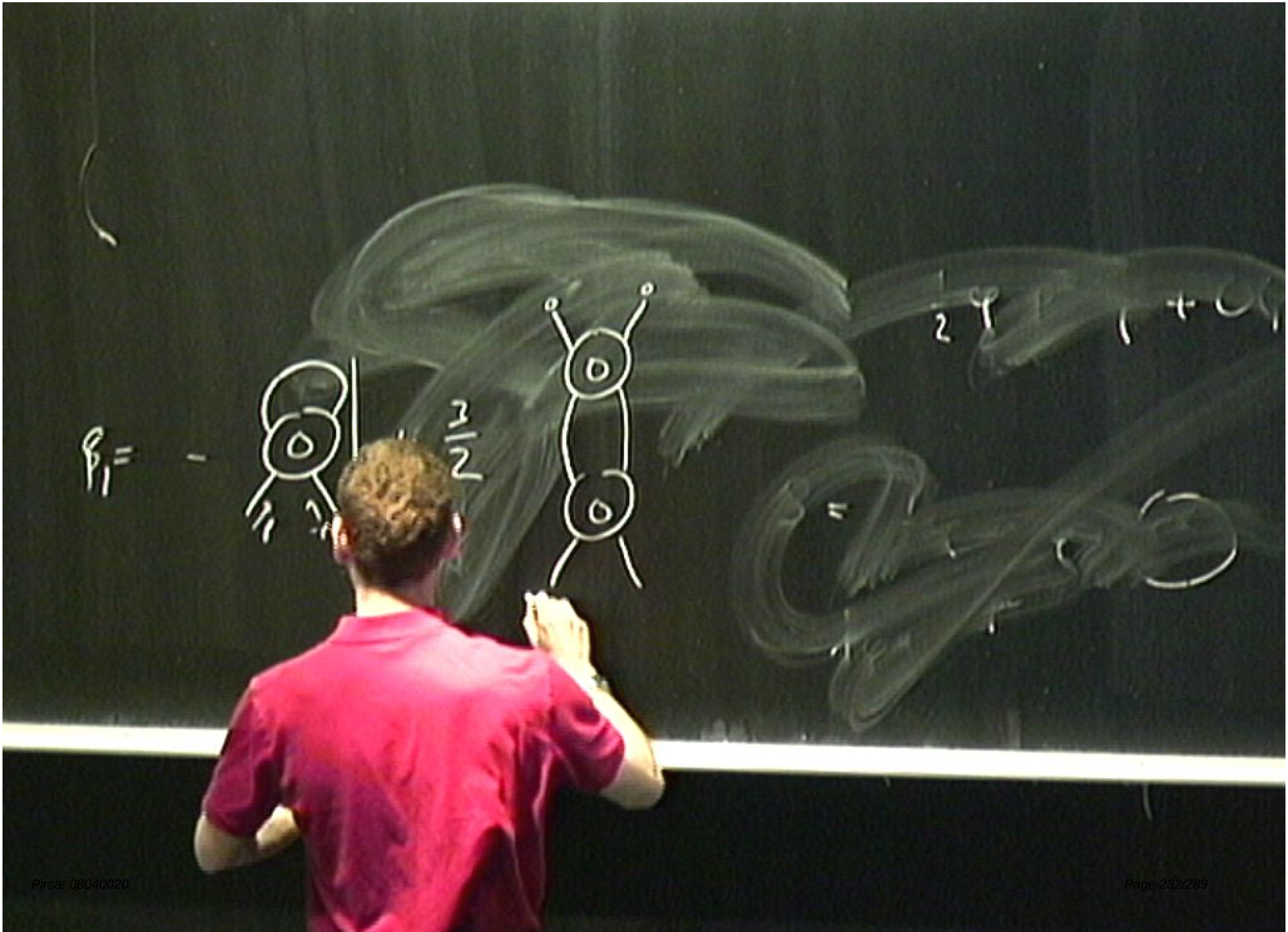

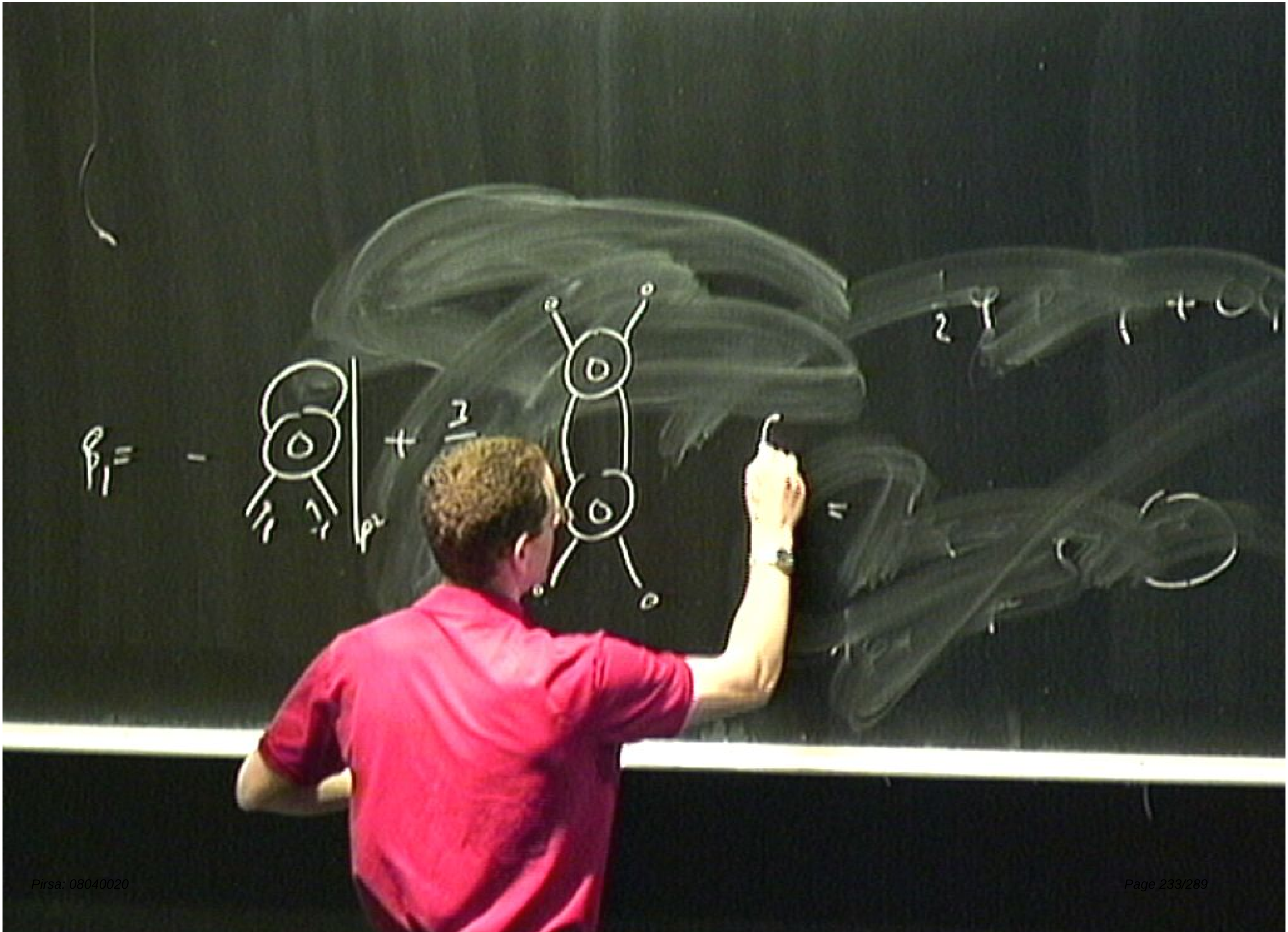

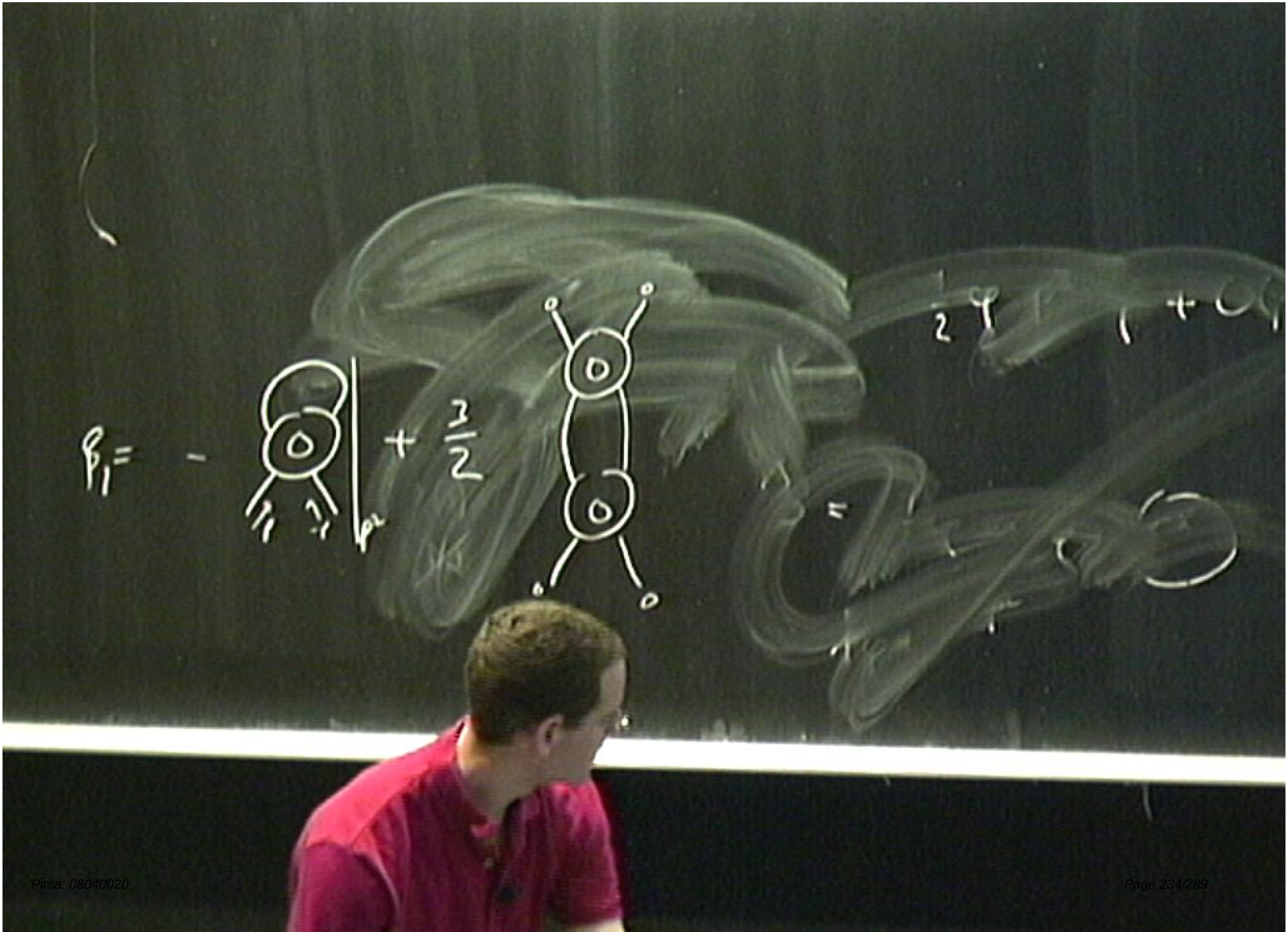
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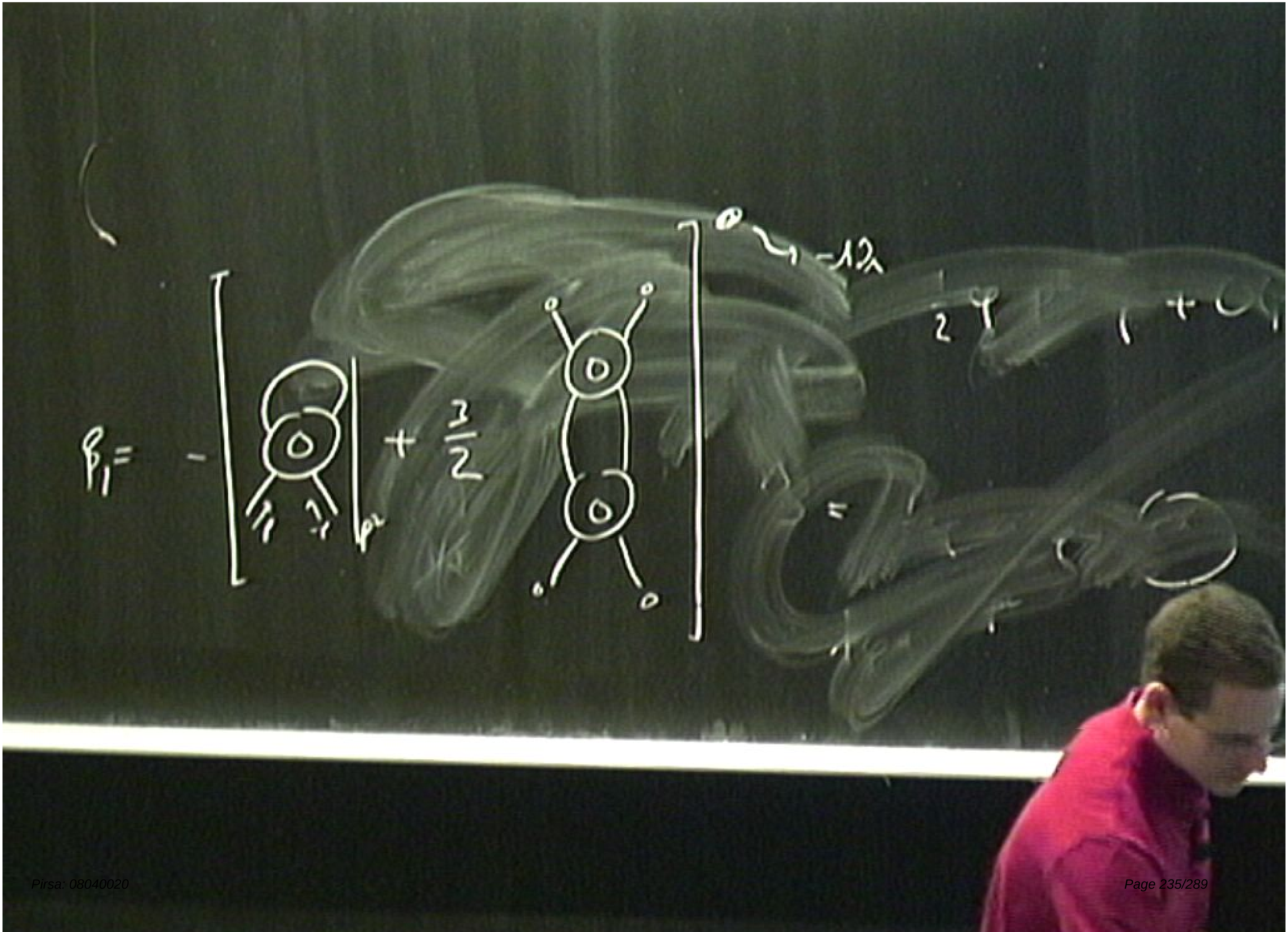
+ 3/2



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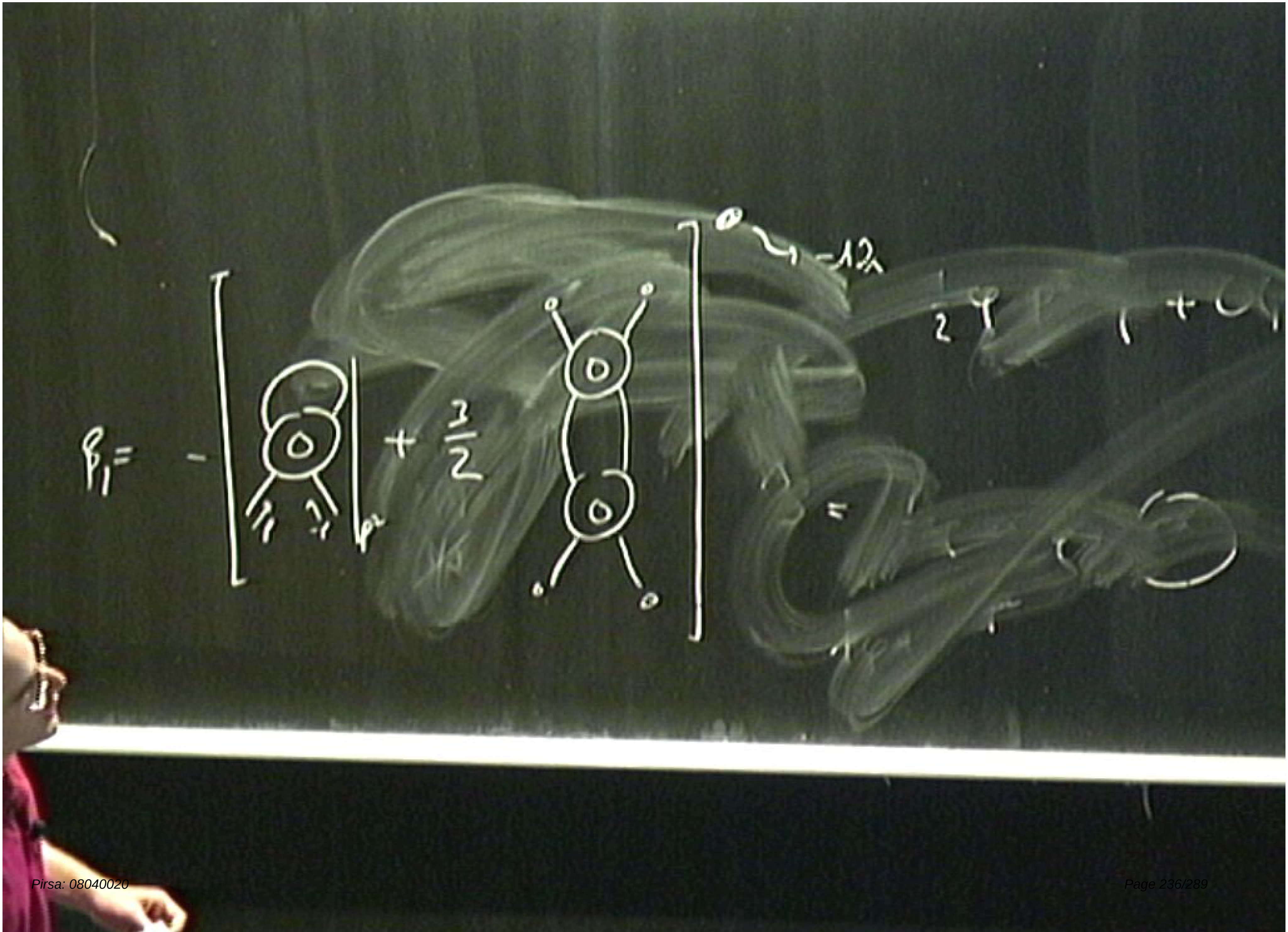
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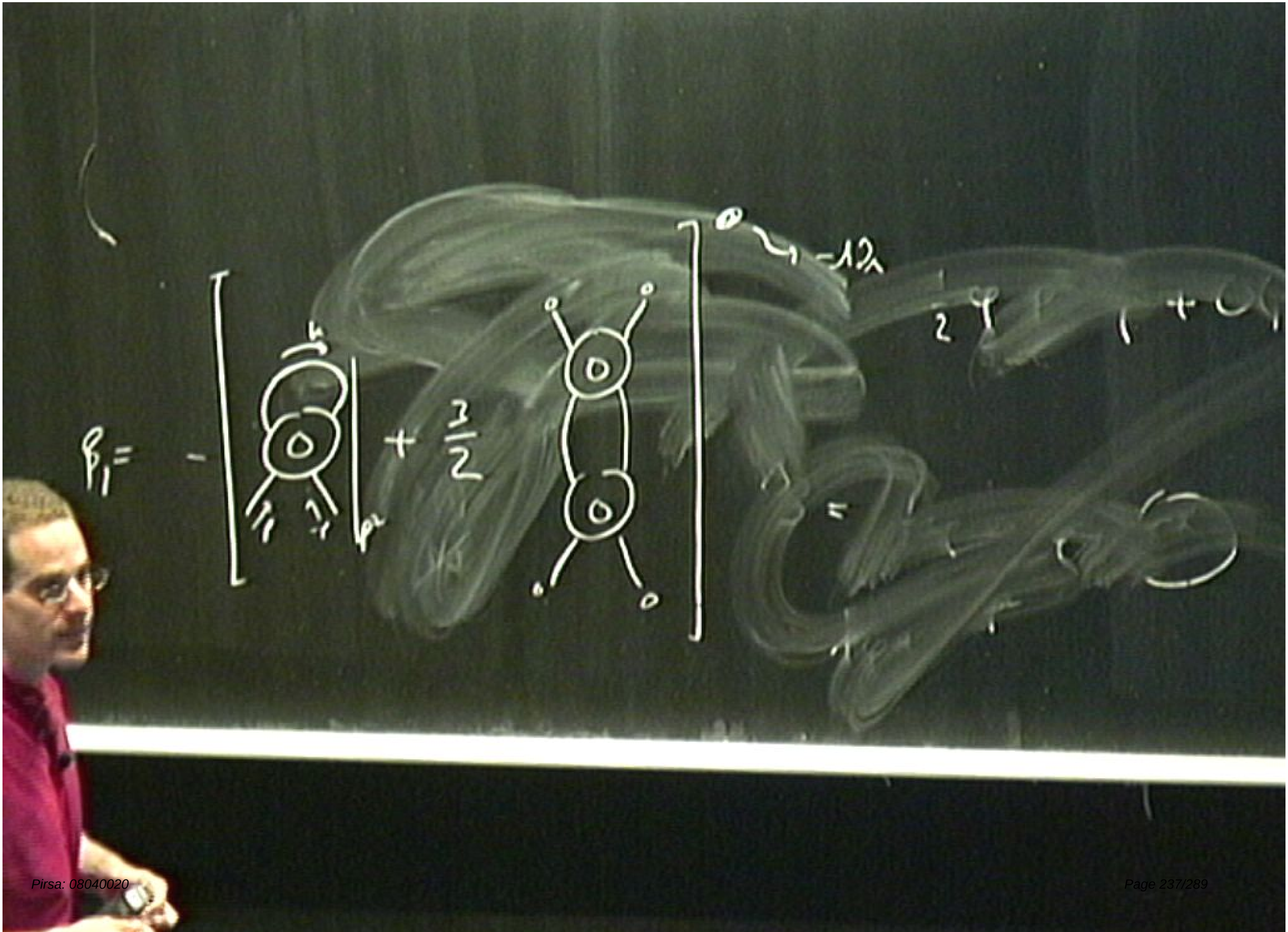
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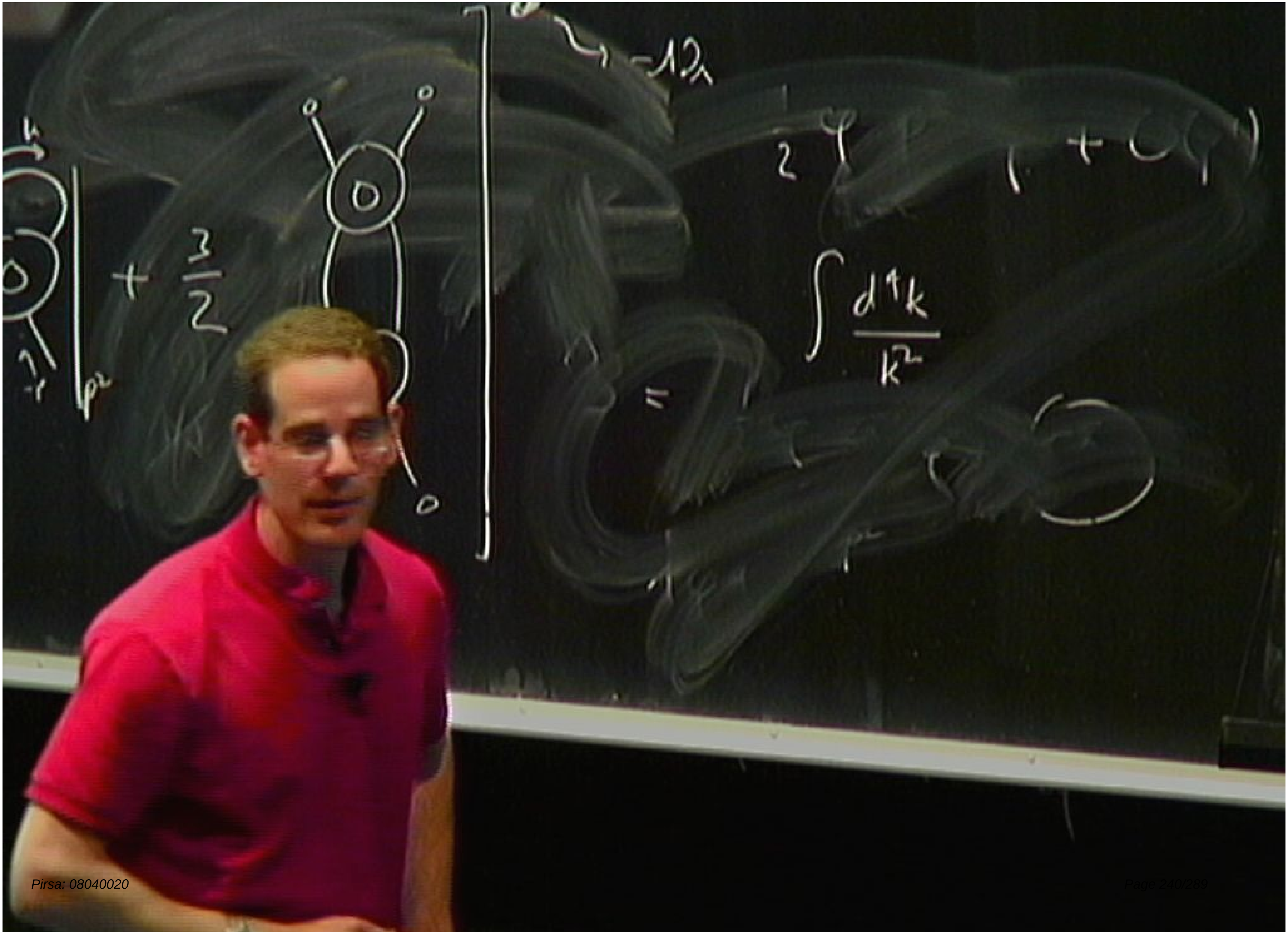

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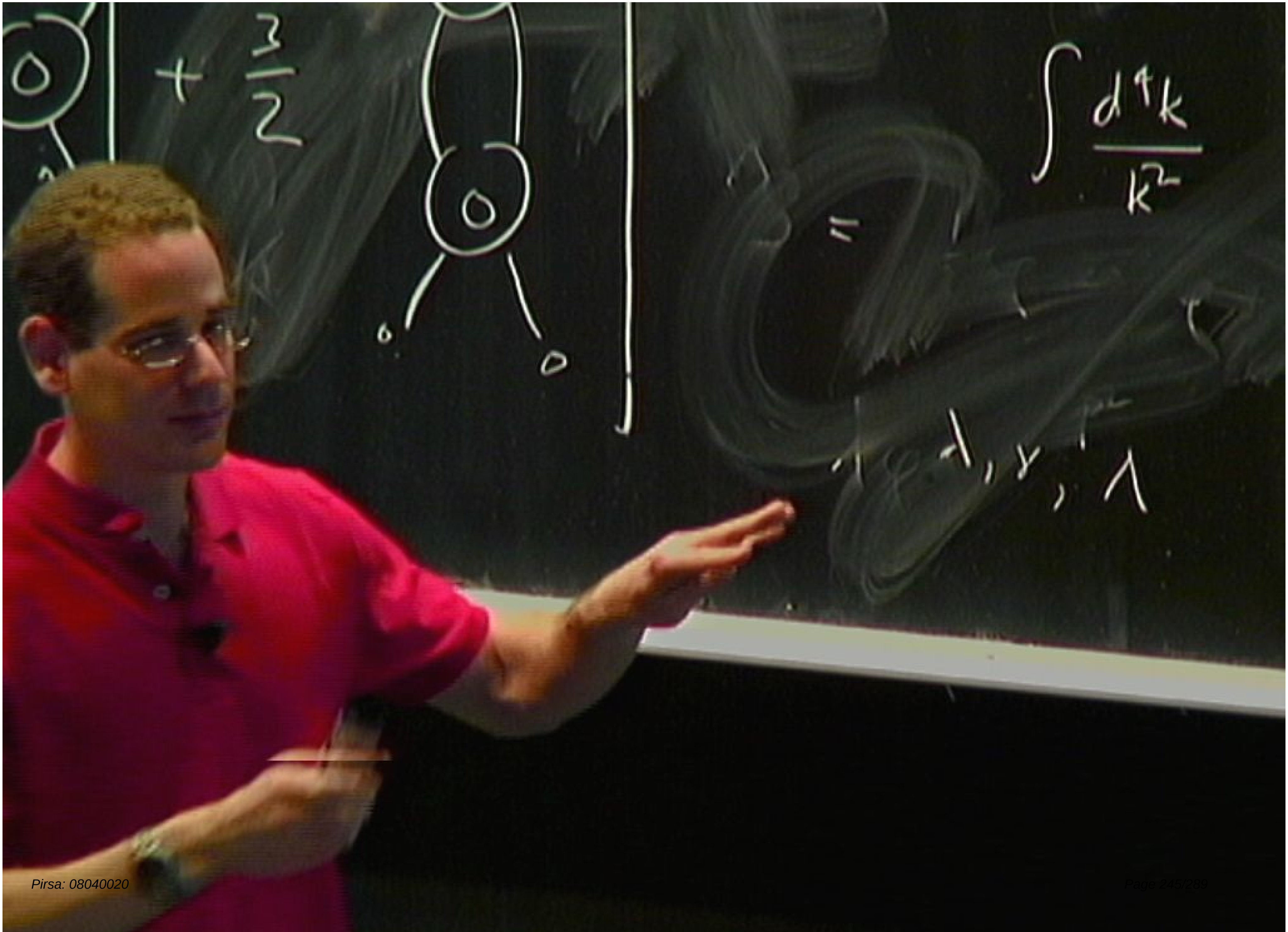
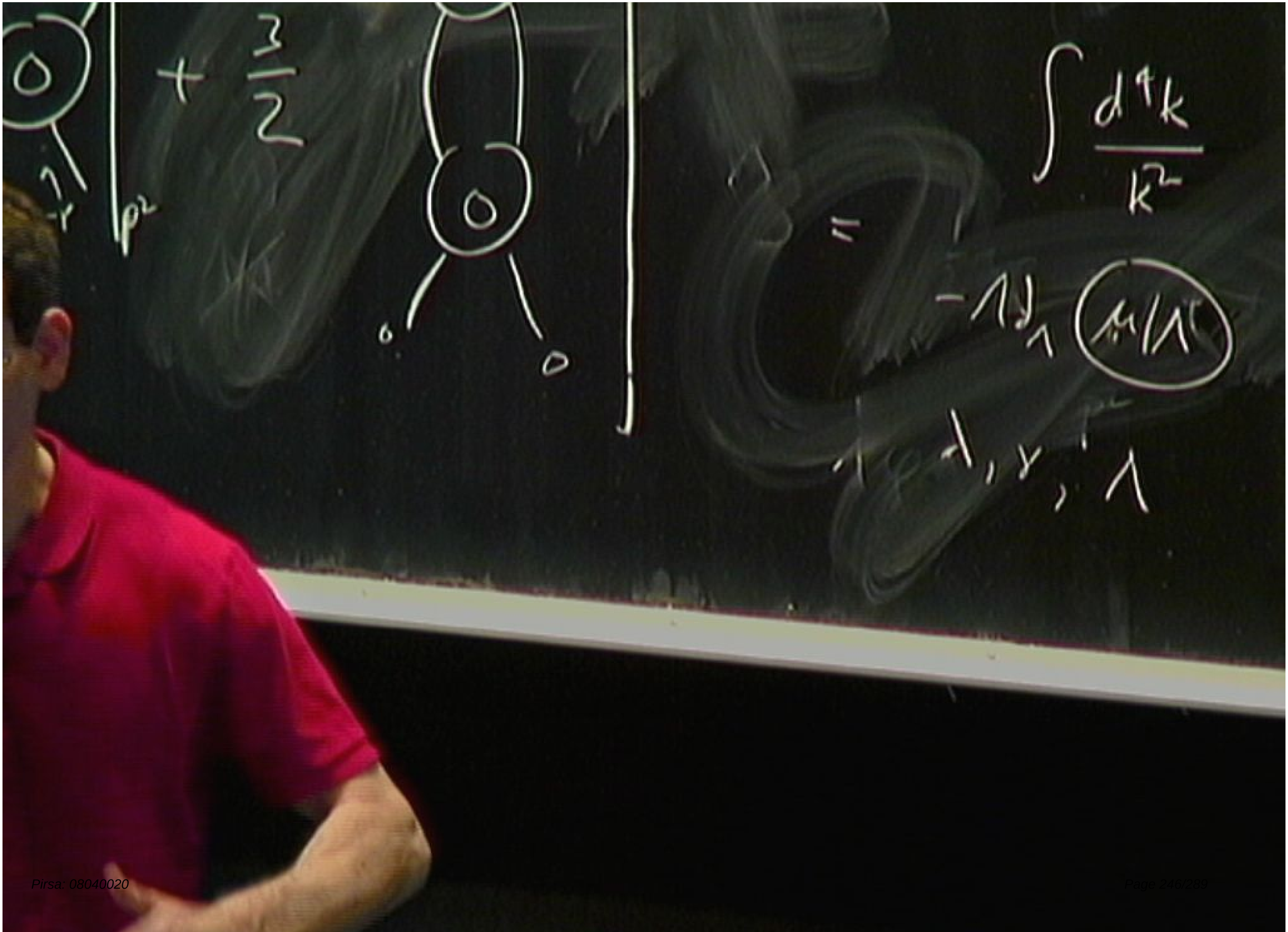
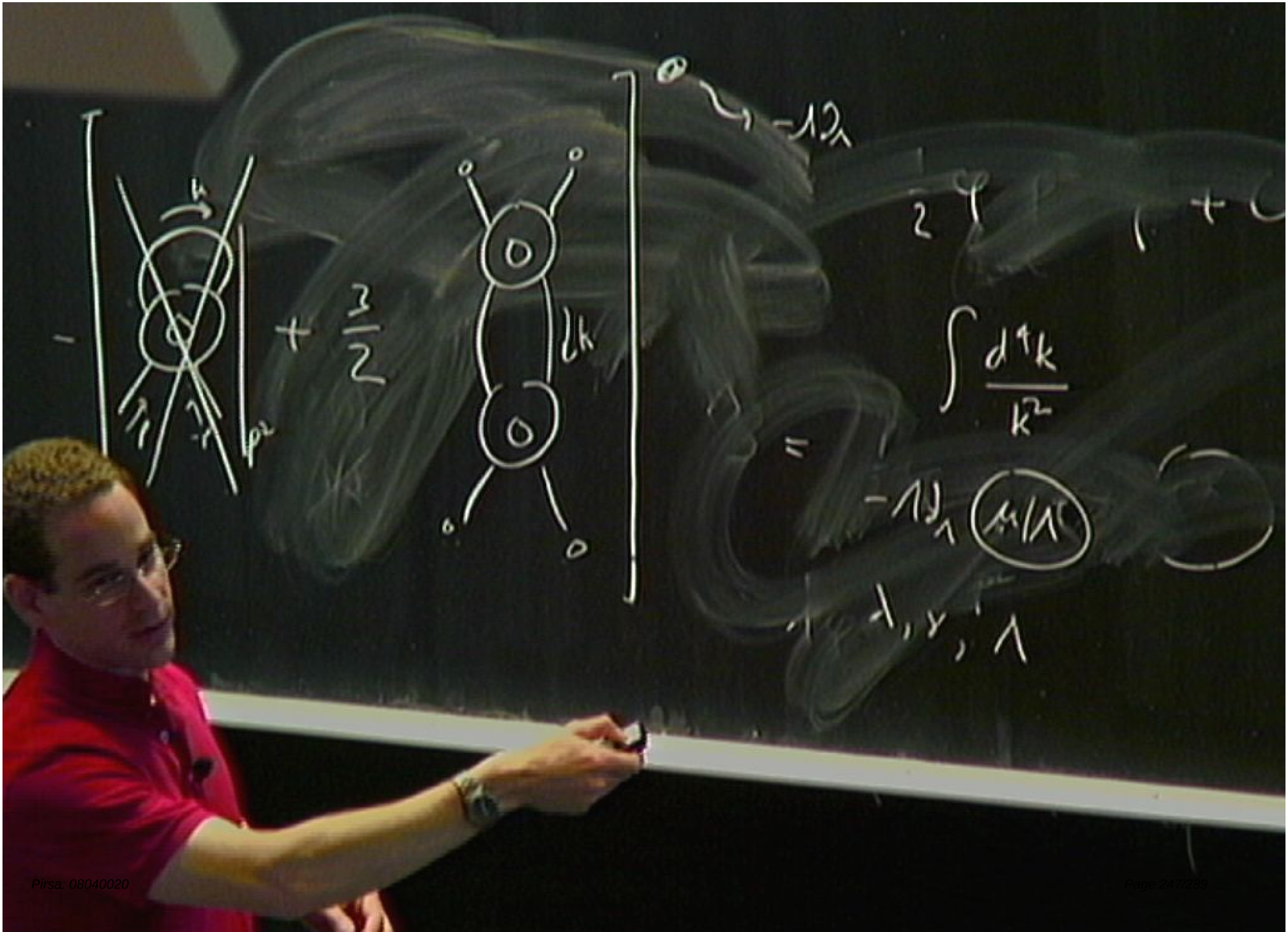
- We have $\Lambda\partial_\Lambda$ of a dimensionless integral
- By perturbative self-similarity (ie no hidden couplings), the answer is zero!




+ 3/2
∫ d⁴k / k²
=
−Λ∂_Λ



+ 3/2
∫ d^4k / k^2

Evaluating β_1

- Second Diagram:
$$\int \frac{d^4k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, 0, 0) \Delta^2(k) S_0^{(4)}(0, 0, -k, k) \right]^{\bullet}$$
- UV finite by construction

Evaluating β_1

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- UV finite by construction
- Interchanging order of differentiation and integration will introduce an IR divergence

Evaluating β_1

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Evaluating β_1

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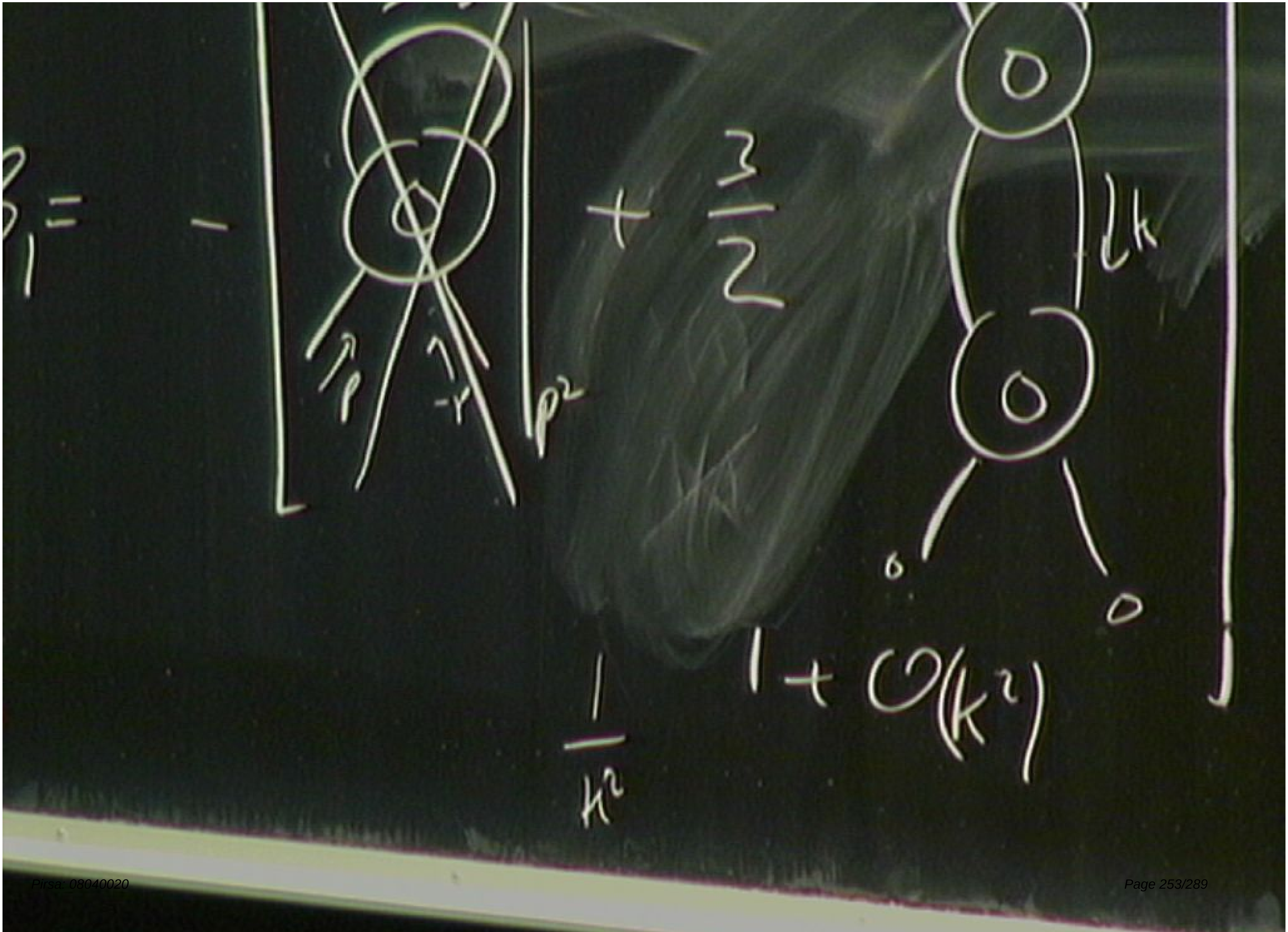
$$\rightarrow -\frac{\Omega_D}{(2\pi)^D} \Lambda\partial_\Lambda \int_0^\Lambda dk \frac{k^{D-1}}{k^4} + \mathrm{O}(\epsilon)$$


$\beta_1 = -$
p
$-r$
p^2
$+ \frac{3}{2}$
lk
$\frac{1}{k^2}$
$1 + O(k^2)$

Evaluating β_1

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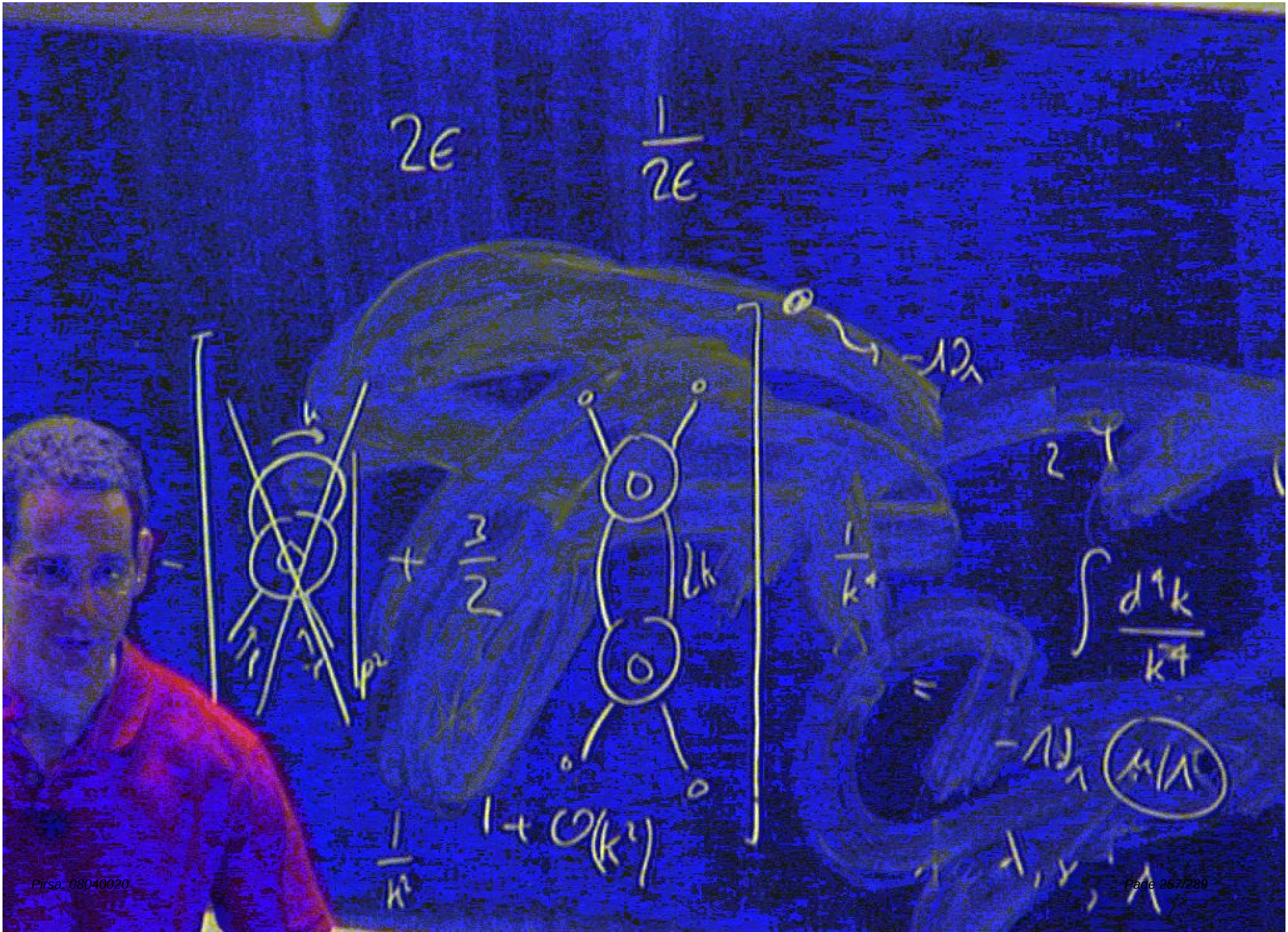
$$\stackrel{k\to k\Lambda}{=} -\frac{\Omega_D}{(2\pi)^D} \Lambda\partial_\Lambda \Lambda^{2\epsilon} \int_0^1 dk k^{-1+2\epsilon} + \mathrm{O}(\epsilon)$$


2ε
1/2ε
3/2
dk
1/k^4
d^4k/k^4
1+O(k^2)
1/k^2
λ, y, Λ

Evaluating β_1

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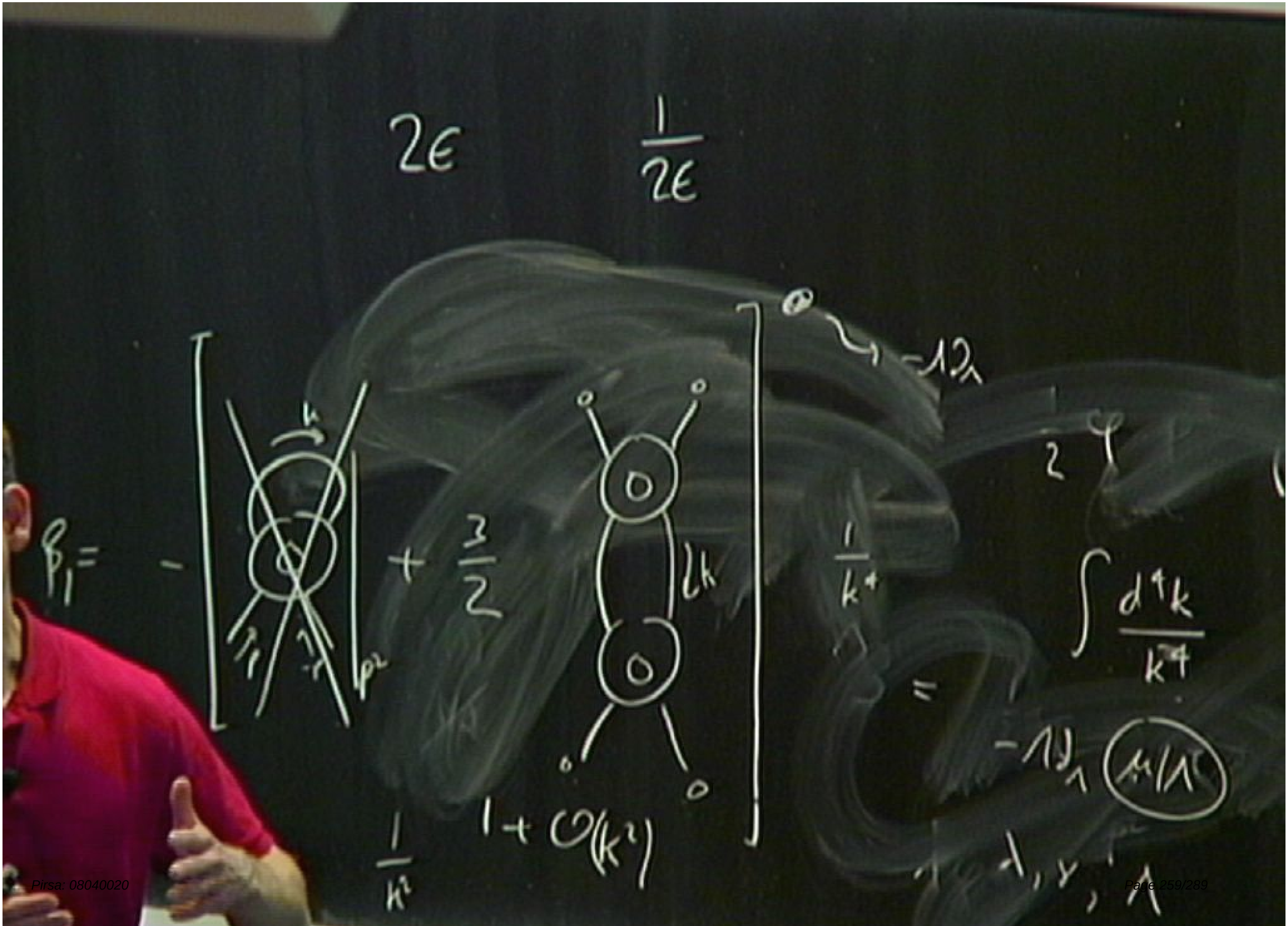
2ε
1/2ε
β1 =
3/2
1/k⁴
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μ/Λ

Evaluating β_1

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Evaluating β_1

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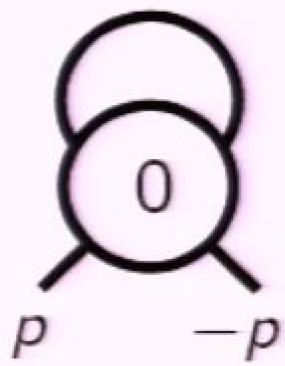
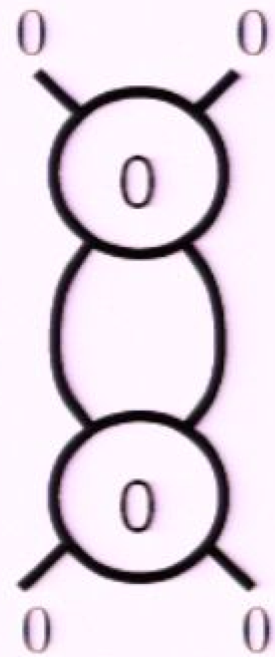
Evaluating β_1

$$\beta_1 = -\Bigg[$$

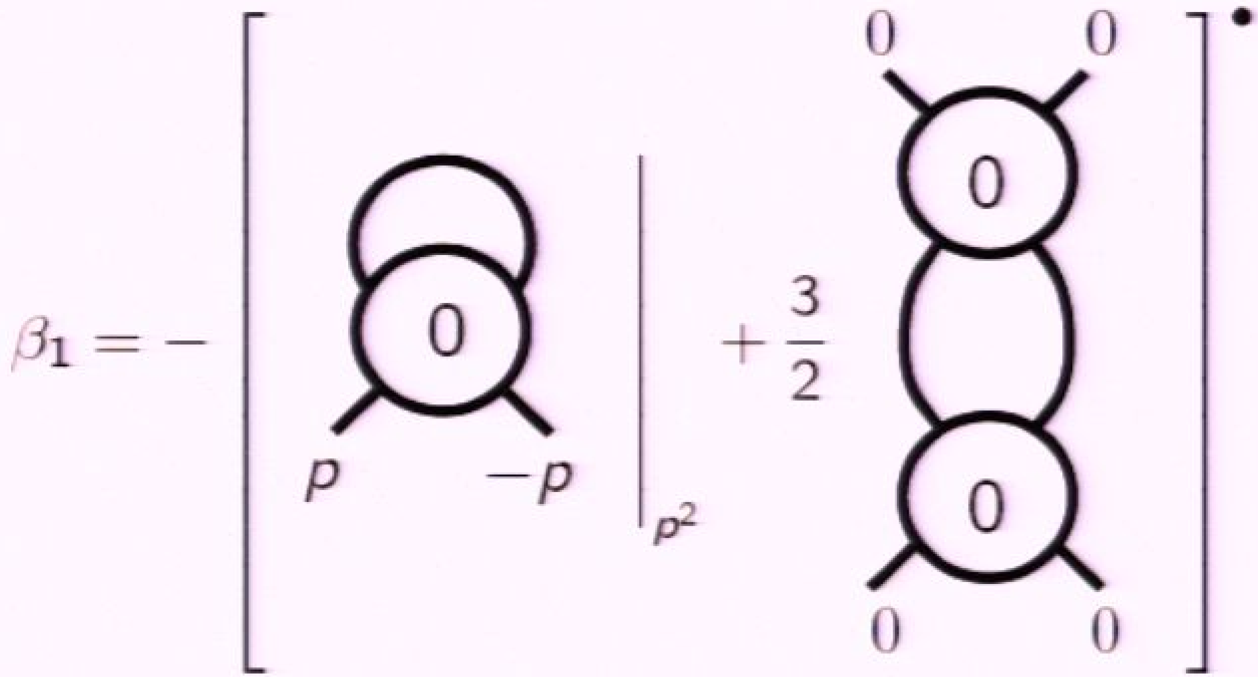


$$\Bigg|_{p^2} + \frac{3}{2}$$



$$\Bigg]^{\bullet}$$

Evaluating β_1



- $\beta_1 = +\dfrac{3}{16\pi^2}$
- Universality arose from being forced to look at the IR end of the integral

Comment

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- The diagrammatic cancellations of the seed action are completely generic
- These cancellations are the basis of the 'universal diagrammatic calculus' proposed in S. Arnone, A. Gatti, and T. R. Morris, "A proposal for a manifestly gauge invariant and universal calculus in Yang-Mills theory," Phys. Rev. **D 67** (2003) 085004, `hep-th/0209162`.
- In a series of works, I completed the calculus and found how to apply it nonperturbatively
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- We introduced 'Reduced Vertices'
- The application of the flow equation was illustrated with a computation of β_1
- Commented on the huge simplification of such calculations which is now possible
- Given equal levels of familiarity, perturbation theory is no harder with the ERG than with other methods!

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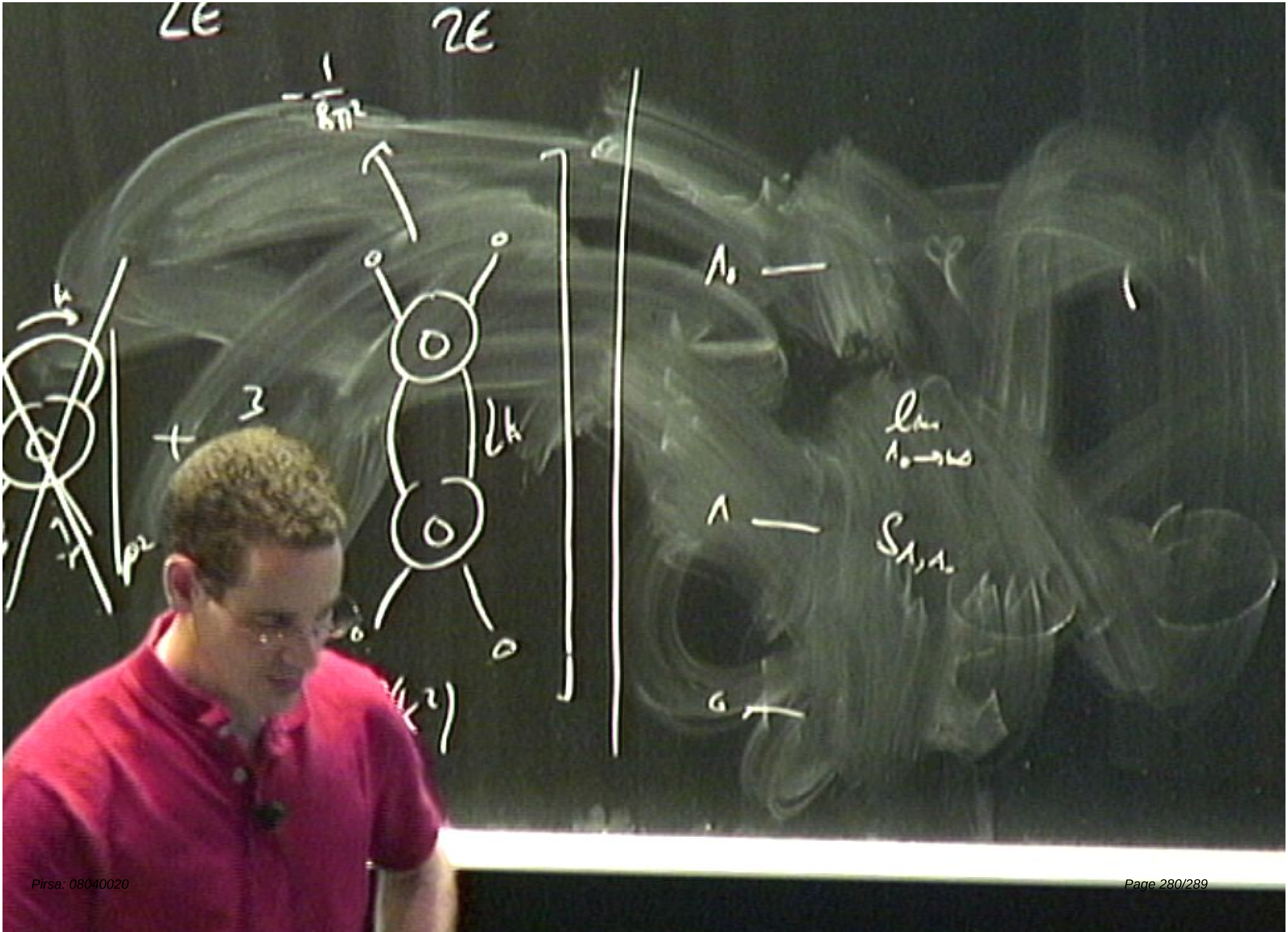
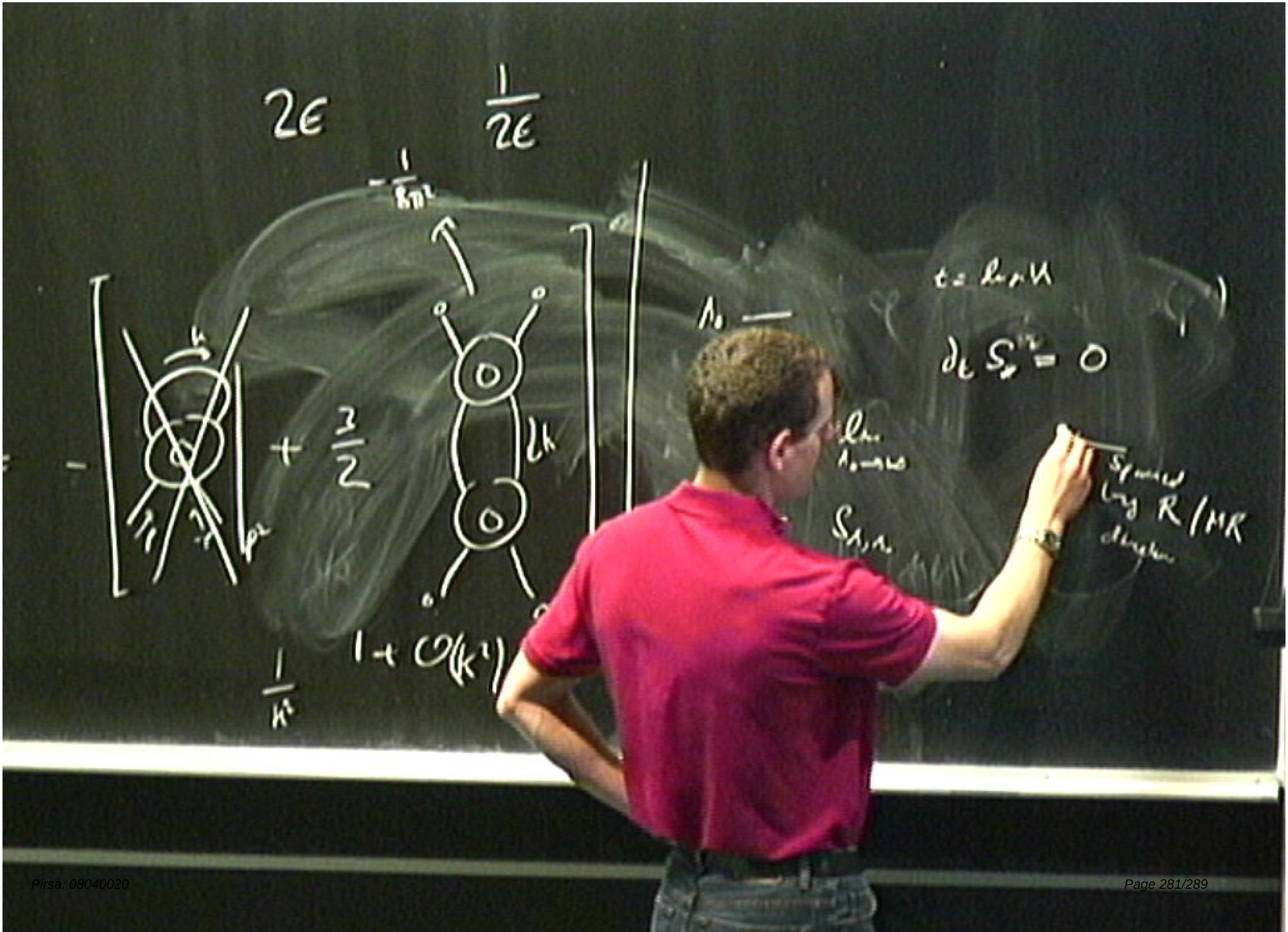
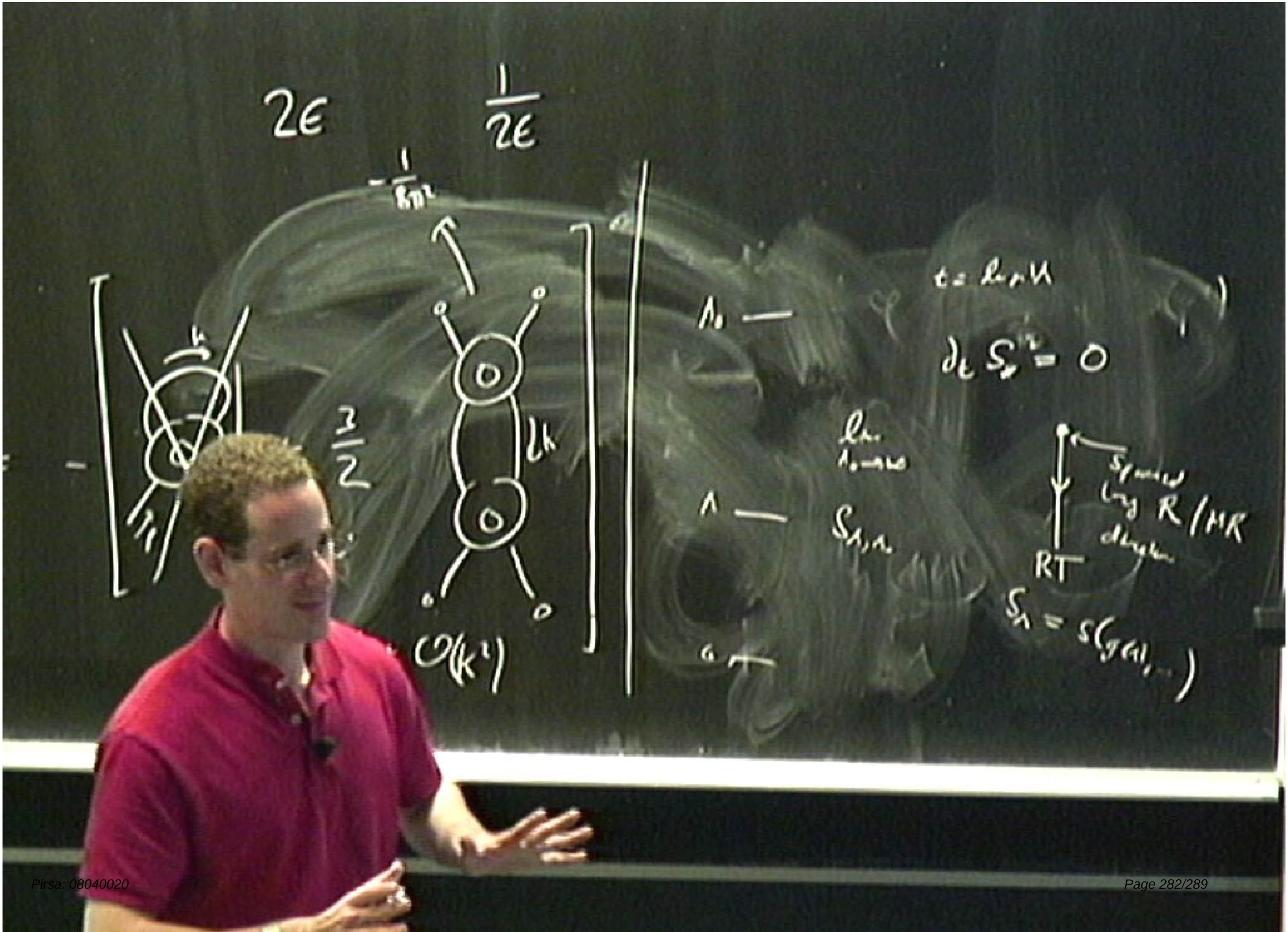
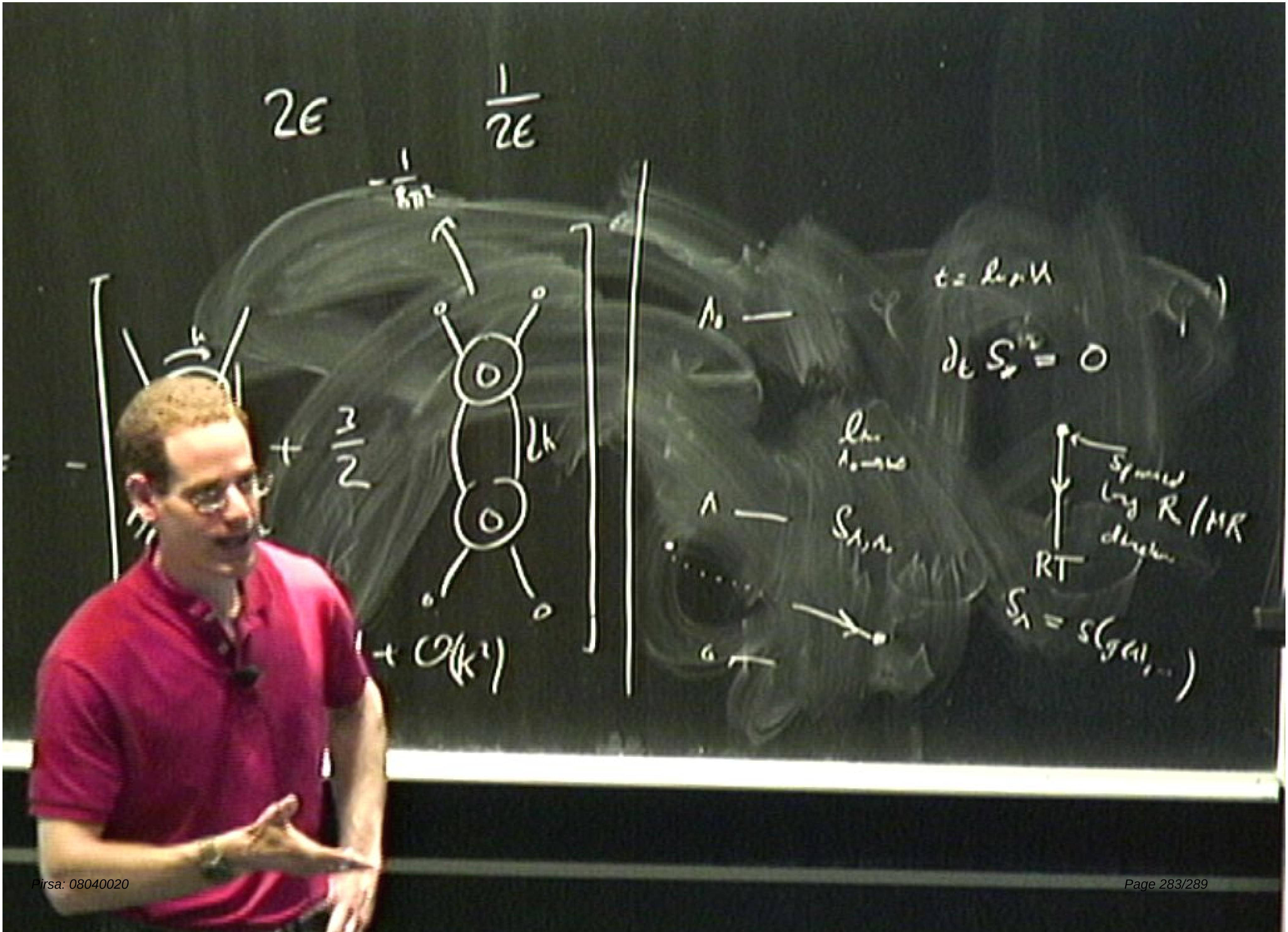

2E
Λ₀
Λ




2ε
1
2ε
3
2
RT


2ε
1/2ε
+ 3/2
+ O(k²)
∂_t S = 0
RT
S_Λ = s(g(Λ), ...)

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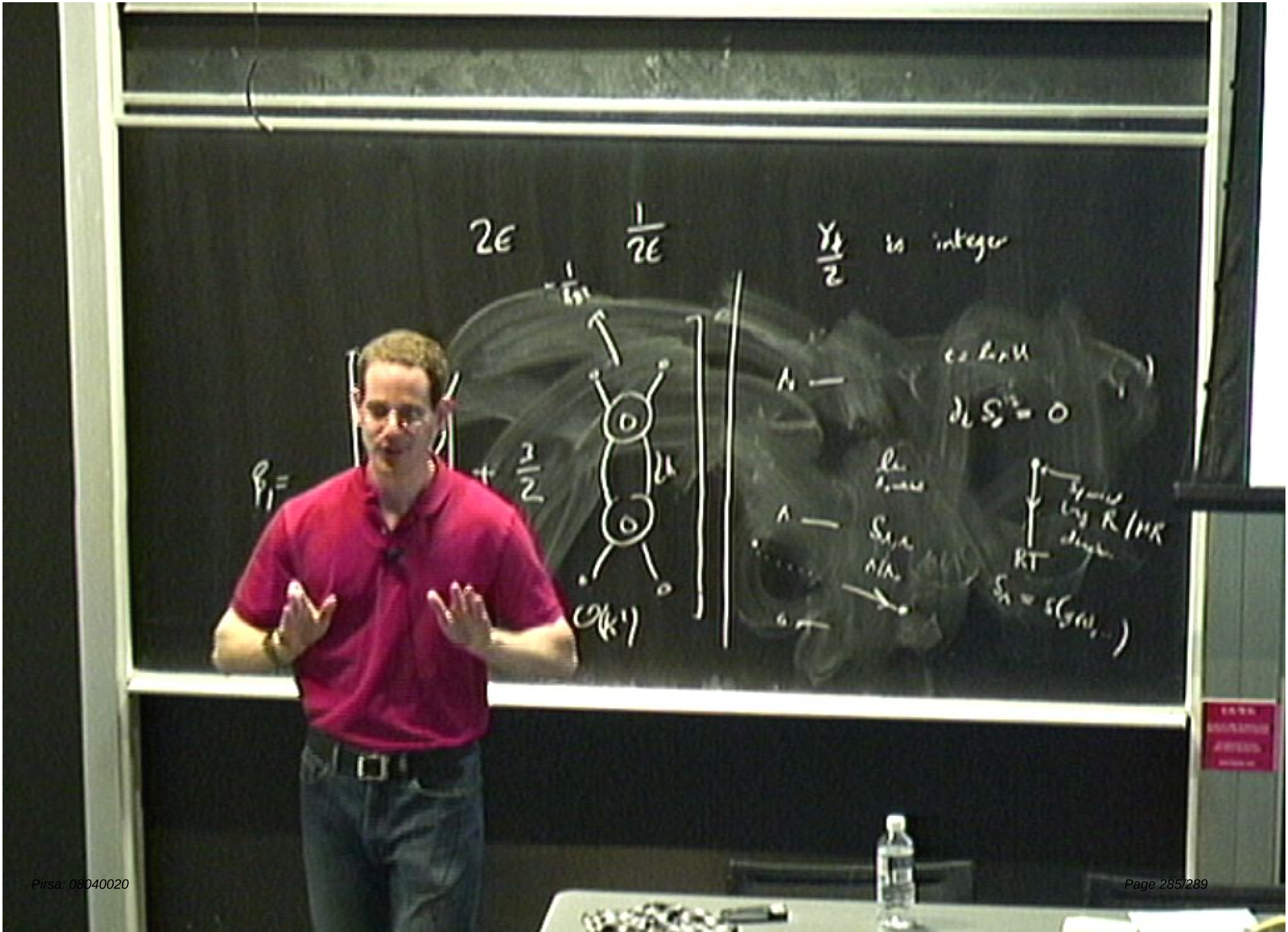
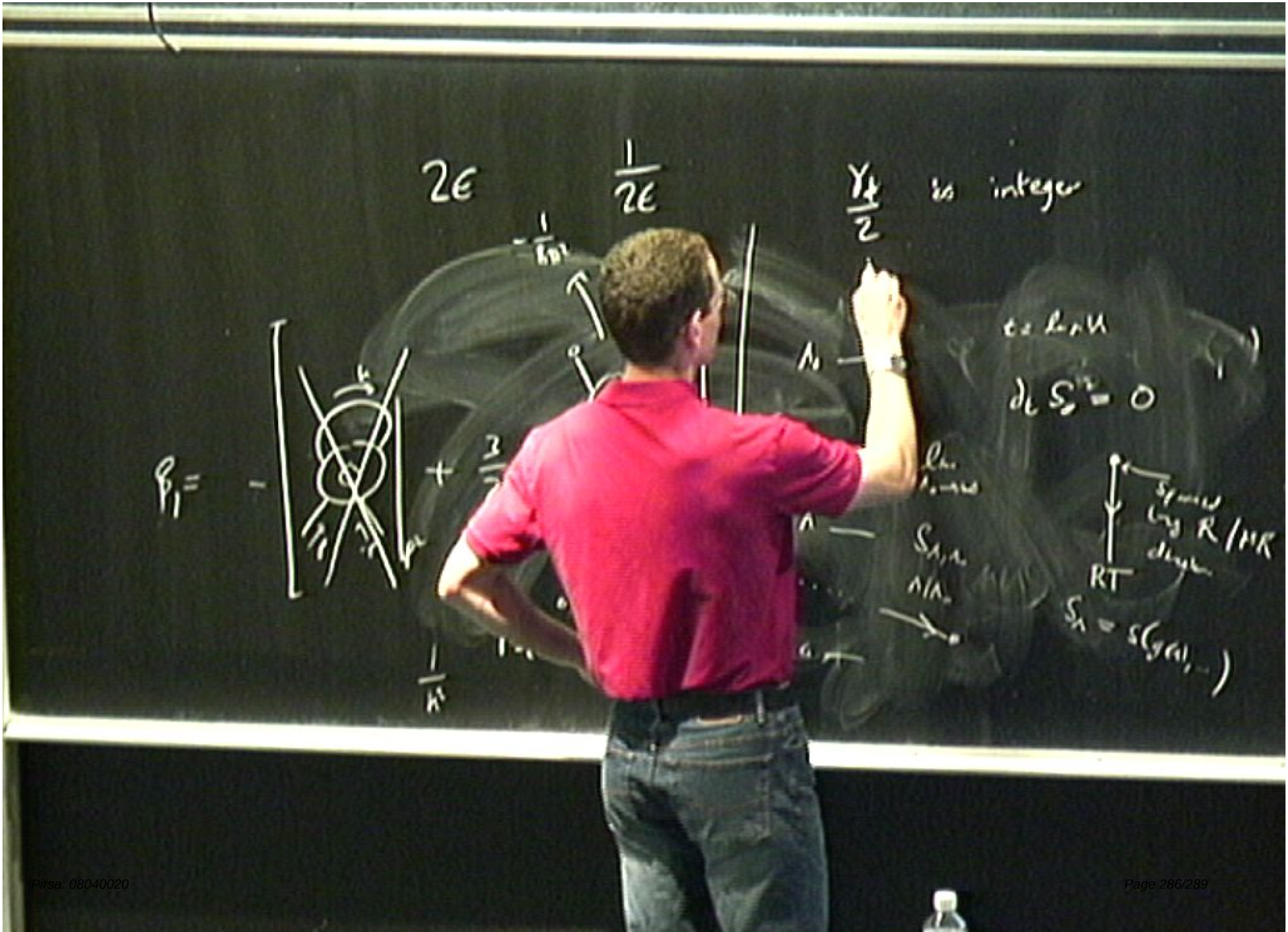

2ε
integer
RT



2ε
1/2ε
is integer
RT

Useful Literature

- T. R. Morris, "The Exact renormalization group and approximate solutions," Int. J. Mod. Phys. A **9** (1994) 2411, hep-ph/9308265.
- T. R. Morris, "Elements of the continuous renormalization group," Prog. Theor. Phys. **131** (1998) 395, hep-th/9802039.
- C. Bagnuls and C. Bervillier, "Exact renormalization group equations: An introductory review," Phys. Rept. **348** (2001) 91, hep-th/0002034.
- F. Wegner , "The Critical State, General Aspects," in: C. Domb and M. S. Green (Eds.), Phase Transitions and Critical Phenomena, Vol VI, Acad. Press, N.-Y., 1976 p. 7.
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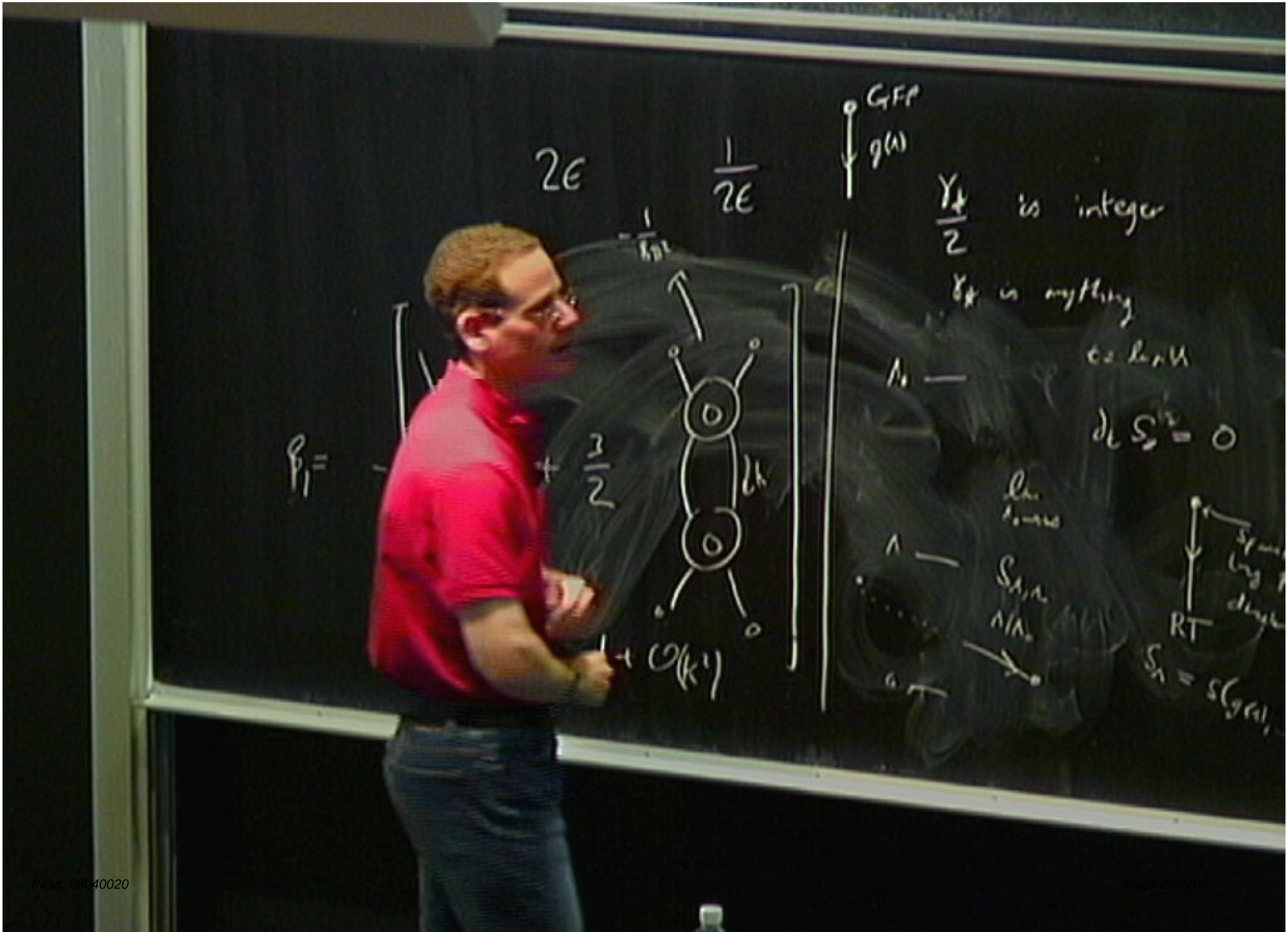
GFP
is integer
RT

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