

Title: The Exact Renormalization Group - Lecture 2: Exact Renormalization Group Equations

Date: Apr 23, 2008 10:30 AM

URL: <http://pirsa.org/08040020>

Abstract: I will show how to construct very general ERG equations, and will use this as the starting point for a discussion of Polchinski's equation and its cousins. I will introduce diagrammatics and an associated universal calculus, which will be illustrated with a simple calculation.



The Exact Renormalization Group: Introduction & Applications

Lecture 2: ERG Equations

Oliver J. Rosten

Dublin Institute for Advanced Studies

April / May 2008

0801.3287

secs 1, 2 (3)

The Exact Renormalization Group: Introduction & Applications

Lecture 2: ERG Equations

Oliver J. Rosten



Dublin Institute for Advanced Studies

April / May 2008

Useful Literature

- T. R. Morris, "The Exact renormalization group and approximate solutions," *Int. J. Mod. Phys. A* **9** (1994) 2411, [hep-ph/9308265](#).
- T. R. Morris, "Elements of the continuous renormalization group," *Prog. Theor. Phys.* **131** (1998) 395, [hep-th/9802039](#).
- C. Bagnuls and C. Bervillier, "Exact renormalization group equations: An introductory review," *Phys. Rept.* **348** (2001) 91, [hep-th/0002034](#).
- F. Wegner, "The Critical State, General Aspects," in: C. Domb and M. S. Green (Eds.), *Phase Transitions and Critical Phenomena*, Vol VI, Acad. Press, N.-Y., 1976 p. 7.
- T. R. Morris and J. L. Latorre, "Exact scheme independence," *JHEP* **0011** (2000) 004, [hep-th/0008123](#).

Useful Literature

- T. R. Morris, "The Exact renormalization group and approximate solutions," *Int. J. Mod. Phys. A* **9** (1994) 2411, hep-ph/9308265.
- T. R. Morris, "Elements of the continuous renormalization group," *Prog. Theor. Phys.* **131** (1998) 395, hep-th/9802039.
- C. Bagnuls and C. Bervillier, "Exact renormalization group equations: An introductory review," *Phys. Rept.* **348** (2001) 91, hep-th/0002034.
- F. Wegner , "The Critical State, General Aspects," in: C. Domb and M. S. Green (Eds.), *Phase Transitions and Critical Phenomena, Vol VI*, Acad. Press, N.-Y., 1976 p. 7.
- T. R. Morris and J. L. Latorre, "Exact scheme independence," *JHEP* **0011** (2000) 004, hep-th/0008123.

Continued...

- K. Wilson and J. Kogut, "The Renormalization group and the epsilon expansion," Phys. Rept. **12** (1974) 75.
- F. J. Wegner and A. Houghton, "Renormalization group equation for critical phenomena," Phys. Rev. A **8** (1973) 401.
- J. Polchinski, "Renormalization And Effective Lagrangians," Nucl. Phys. **B 231** (1984) 269.
- C. Wetterich, "Exact evolution equation for the effective potential," Phys. Lett. B **301** (1993) 90.
- S. Arnone, A. Gatti, T. R. Morris, and O. J. Rosten, "Exact scheme independence at two loops," Phys. Rev. **D 69** (2004) 065009, hep-th/0309242.

Useful Concepts from Lecture 1

Useful Concepts from Lecture 1

- The ERG relates physics at different scales via a **coarse-graining procedure**
 - There is a huge freedom in the way we coarse-grain (block)
 - Universal physics is independent of the coarse-graining procedure
 - To conveniently see fixed point behaviour etc., we need to rescale to dimensionless units
 - Along Renormalized Trajectories, the action is self-similar

$$S_\Lambda[\varphi] = S[\varphi](g_1(\Lambda), \dots, g_n(\Lambda), \gamma(\Lambda))$$

Useful Concepts from Lecture 1

- The ERG relates physics at different scales via a coarse-graining procedure
- There is a huge freedom in the way we coarse-grain (block)
- Universal physics is independent of the coarse-graining procedure
- To conveniently see fixed point behaviour etc., we need to rescale to dimensionless units
- Along Renormalized Trajectories, the action is self-similar

$$S_\Lambda[\varphi] = S[\varphi](g_1(\Lambda), \dots, g_n(\Lambda), \gamma(\Lambda))$$

Useful Concepts from Lecture 1

- The ERG relates physics at different scales via a coarse-graining procedure
- There is a huge freedom in the way we coarse-grain (block)
- **Universal physics is independent of the coarse-graining procedure**
- To conveniently see fixed point behaviour etc., we need to rescale to dimensionless units
- Along Renormalized Trajectories, the action is self-similar

$$S_\Lambda[\varphi] = S[\varphi](g_1(\Lambda), \dots, g_n(\Lambda), \gamma(\Lambda))$$

Useful Concepts from Lecture 1

- The ERG relates physics at different scales via a coarse-graining procedure
- There is a huge freedom in the way we coarse-grain (block)
- Universal physics is independent of the coarse-graining procedure
- To conveniently see fixed point behaviour etc., we need to **rescale** to dimensionless units
- Along Renormalized Trajectories, the action is self-similar

$$S_\Lambda[\varphi] = S[\varphi](g_1(\Lambda), \dots, g_n(\Lambda), \gamma(\Lambda))$$

Useful Concepts from Lecture 1

- The ERG relates physics at different scales via a coarse-graining procedure
- There is a huge freedom in the way we coarse-grain (block)
- Universal physics is independent of the coarse-graining procedure
- To conveniently see fixed point behaviour etc., we need to rescale to dimensionless units
- Along Renormalized Trajectories, the action is self-similar

$$S_\Lambda[\varphi] = S[\varphi](g_1(\Lambda), \dots, g_n(\Lambda), \gamma(\Lambda))$$

Outline of this Lecture

- 1 General ERG Equations
 - Formulation
 - Other ERG Equations
- 2 Diagrammatics
- 3 One-loop β -function
- 4 Recap

Outline of this Lecture

- 1 General ERG Equations
 - Formulation
 - Other ERG Equations
- 2 Diagrammatics
- 3 One-loop β -function
- 4 Recap

Outline of this Lecture

- 1 General ERG Equations
 - Formulation
 - Other ERG Equations
- 2 Diagrammatics
- 3 One-loop β -function
- 4 Recap

The Wilsonian Effective Action

Start with the partition function

The Wilsonian Effective Action

Start with the partition function

$$Z = \int_{\Lambda_0} \mathcal{D}\Phi e^{-S_{\Lambda_0}[\Phi]} = \int_{\Lambda} \mathcal{D}\Phi e^{-S_{\Lambda}[\Phi]}$$

- The bare scale
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ

The Wilsonian Effective Action

Start with the partition function

$$Z = \int_{\Lambda_0} \mathcal{D}\Phi e^{-S_{\Lambda_0}[\Phi]} = \int_{\Lambda} \mathcal{D}\Phi e^{-S_{\Lambda}[\Phi]}$$

- The bare scale
 - High energy (short distance) scale
 - Modes above this scale are cut off (regularized)
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ

The Wilsonian Effective Action

Start with the partition function

$$Z = \int_{\Lambda_0} \mathcal{D}\Phi e^{-S_{\Lambda_0}[\Phi]} = \int_{\Lambda} \mathcal{D}\Phi e^{-S_{\Lambda}[\Phi]}$$

- The bare scale
 - High energy (short distance) scale
 - Modes above this scale are cut off (regularized)
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ

The Wilsonian Effective Action

Start with the partition function

$$Z = \int_{\Lambda_0} \mathcal{D}\Phi e^{-S_{\Lambda_0}[\Phi]} = \int_{\Lambda} \mathcal{D}\Phi e^{-S_{\Lambda}[\Phi]}$$

- The bare scale
 - High energy (short distance) scale
 - Modes above this scale are cut off (regularized)
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ

The Wilsonian Effective Action

Start with the partition function

$$Z = \int_{\Lambda_0} \mathcal{D}\Phi e^{-S_{\Lambda_0}[\Phi]} = \int_{\Lambda} \mathcal{D}\Phi e^{-S_{\Lambda}[\Phi]}$$

- The bare scale
 - High energy (short distance) scale
 - Modes above this scale are cut off (regularized)
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ
 - The partition function stays the same
 - The effects of the high energy modes must be taken into account
 - The action evolves \Rightarrow Wilsonian effective action

The Wilsonian Effective Action

Start with the partition function

$$Z = \int_{\Lambda_0} \mathcal{D}\Phi e^{-S_{\Lambda_0}[\Phi]} = \int_{\Lambda} \mathcal{D}\Phi e^{-S_{\Lambda}[\Phi]}$$

- The bare scale
 - High energy (short distance) scale
 - Modes above this scale are cut off (regularized)
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ
 - **The partition function stays the same**
 - The effects of the high energy modes must be taken into account
 - The action evolves \Rightarrow Wilsonian effective action

The Wilsonian Effective Action

Start with the partition function

$$Z = \int_{\Lambda_0} \mathcal{D}\Phi e^{-S_{\Lambda_0}[\Phi]} = \int_{\Lambda} \mathcal{D}\Phi e^{-S_{\Lambda}[\Phi]}$$

- The bare scale
 - High energy (short distance) scale
 - Modes above this scale are cut off (regularized)
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ
 - The partition function stays the same
 - The effects of the high energy modes must be taken into account
 - The action evolves \Rightarrow Wilsonian effective action

Implementing a Cutoff

- Implement an overall UV cutoff:

$$\Delta = \frac{1}{p^2} \rightarrow \frac{C(p, \Lambda_0)}{p^2}$$

- Introduce the effective cutoff, Λ

$$C(p, \Lambda_0) = C_{UV}(p, \Lambda) + C_{IR}(p, \Lambda, \Lambda_0)$$

Implementing a Cutoff

- 1 Implement an overall UV cutoff:

$$\Delta = \frac{1}{p^2} \rightarrow \frac{C(p, \Lambda_0)}{p^2}$$

- 2 Introduce the effective cutoff, Λ

$$C(p, \Lambda_0) = C_{UV}(p, \Lambda) + C_{IR}(p, \Lambda, \Lambda_0)$$

Implementing a Cutoff

- 1 Implement an overall UV cutoff:

$$\Delta = \frac{1}{p^2} \rightarrow \frac{C(p, \Lambda_0)}{p^2}$$

- 2 Introduce the effective cutoff, Λ

$$C(p, \Lambda_0) = C_{UV}(p, \Lambda) + C_{IR}(p, \Lambda, \Lambda_0)$$



Implementing a Cutoff

- 1 Implement an overall UV cutoff:

$$\Delta = \frac{1}{p^2} \rightarrow \frac{C(p, \Lambda_0)}{p^2}$$

- 2 Introduce the effective cutoff, Λ

$$C(p, \Lambda_0) = C_{\text{UV}}(p, \Lambda) + C_{\text{IR}}(p, \Lambda, \Lambda_0)$$

- UV cutoff for modes below Λ
- IR cutoff (and overall UV cutoff) for modes above Λ



Implementing a Cutoff

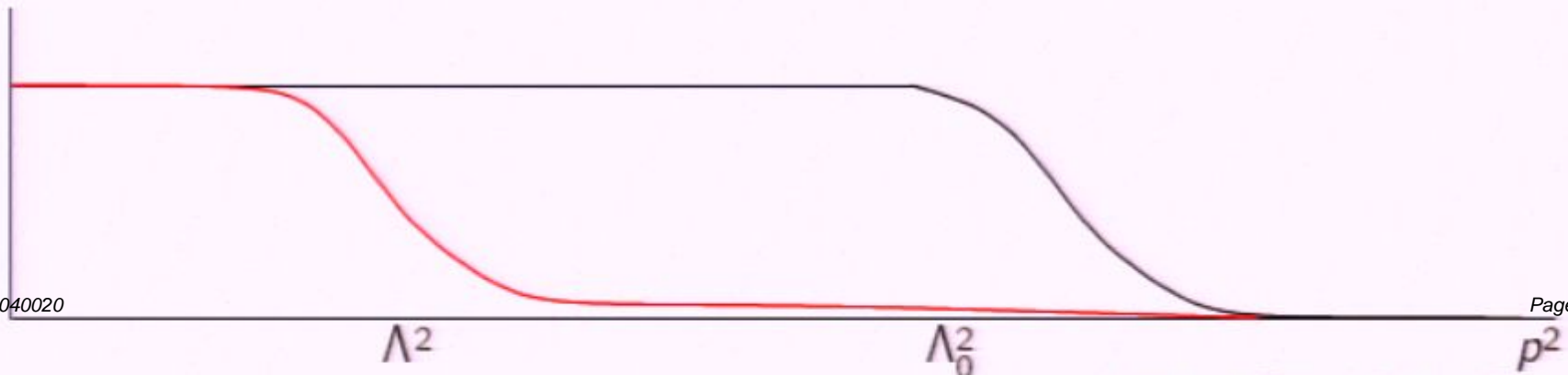
- 1 Implement an overall UV cutoff:

$$\Delta = \frac{1}{p^2} \rightarrow \frac{C(p, \Lambda_0)}{p^2}$$

- 2 Introduce the effective cutoff, Λ

$$C(p, \Lambda_0) = C_{UV}(p, \Lambda) + C_{IR}(p, \Lambda, \Lambda_0)$$

- UV cutoff for modes below Λ
- IR cutoff (and overall UV cutoff) for modes above Λ



Implementing a Cutoff

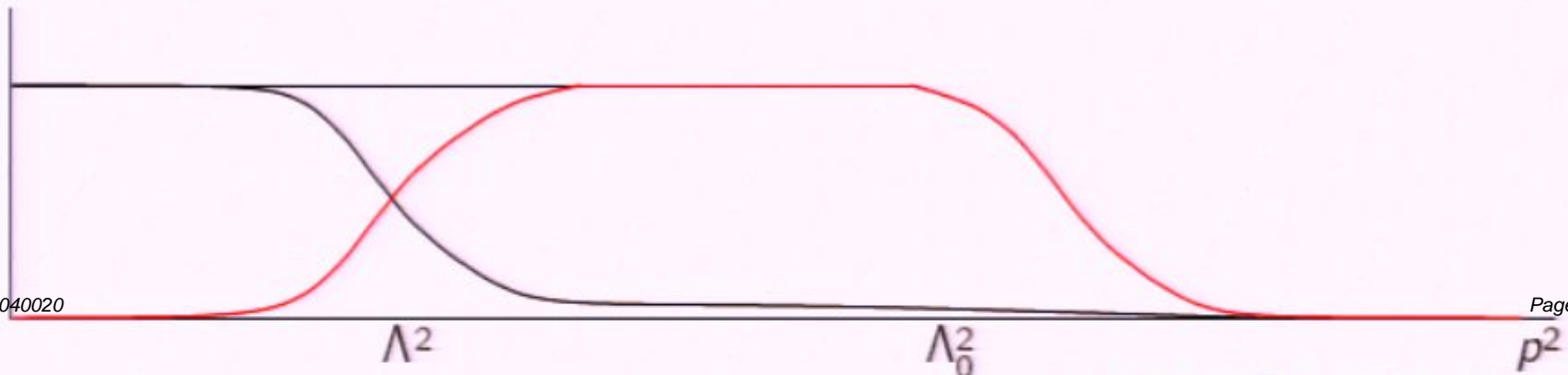
- 1 Implement an overall UV cutoff:

$$\Delta = \frac{1}{p^2} \rightarrow \frac{C(p, \Lambda_0)}{p^2}$$

- 2 Introduce the effective cutoff, Λ

$$C(p, \Lambda_0) = C_{UV}(p, \Lambda) + C_{IR}(p, \Lambda, \Lambda_0)$$

- UV cutoff for modes below Λ
- IR cutoff (and overall UV cutoff) for modes above Λ



Polchinski Equation

Polchinski Equation

An equation telling us how the Wilsonian effective action evolves

$$-\Lambda \partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2} \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} \cdot \hat{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \hat{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi}$$

- Interaction part of Wilsonian effective action

$$S_\Lambda = \frac{1}{2} \varphi \cdot C_{UV}^{-1}(p, \Lambda) p^2 \cdot \varphi + S_\Lambda^{\text{int}}$$

- Any mass term is contained in S_Λ^{int}
- $\hat{\Delta}_{UV} \equiv -\Lambda \partial_\Lambda \Delta_{UV} = -\Lambda \partial_\Lambda \frac{C_{UV}}{p^2}$
- $f \cdot \hat{\Delta}_{UV} \cdot g = \int_p f(-p) \hat{\Delta}_{UV}(p) g(p)$
- Classical term, $a_0[S, \Sigma]$
- Quantum term, $a_1[\Sigma]$

Polchinski Equation

An equation telling us how the Wilsonian effective action evolves

$$-\Lambda \partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2} \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi}$$

- Interaction part of Wilsonian effective action

$$S_\Lambda = \frac{1}{2} \varphi \cdot C_{UV}^{-1}(p, \Lambda) p^2 \cdot \varphi + S_\Lambda^{\text{int}}$$

- Any mass term is contained in S_Λ^{int}

- $\dot{\Delta}_{UV} \equiv -\Lambda \partial_\Lambda \Delta_{UV} = -\Lambda \partial_\Lambda \frac{C_{UV}}{p^2}$

- $f \cdot \dot{\Delta}_{UV} \cdot g = \int_p f(-p) \dot{\Delta}_{UV}(p) g(p)$

- Classical term, $a_0[S, \Sigma]$

- Quantum term, $a_1[\Sigma]$

Polchinski Equation

An equation telling us how the Wilsonian effective action evolves

$$-\Lambda \partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2} \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi}$$

- Interaction part of Wilsonian effective action

$$S_\Lambda = \frac{1}{2} \varphi \cdot C_{UV}^{-1}(p, \Lambda) p^2 \cdot \varphi + S_\Lambda^{\text{int}}$$

- Any mass term is contained in S_Λ^{int}
- $\dot{\Delta}_{UV} \equiv -\Lambda \partial_\Lambda \Delta_{UV} = -\Lambda \partial_\Lambda \frac{C_{UV}}{p^2}$
- $f \cdot \dot{\Delta}_{UV} \cdot g = \int_p f(-p) \dot{\Delta}_{UV}(p) g(p)$
- Classical term, $a_0[S, \Sigma]$
- Quantum term, $a_1[\Sigma]$

Polchinski Equation

An equation telling us how the Wilsonian effective action evolves

$$-\Lambda \partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2} \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi}$$

- Interaction part of Wilsonian effective action

$$S_\Lambda = \frac{1}{2} \varphi \cdot C_{UV}^{-1}(p, \Lambda) p^2 \cdot \varphi + S_\Lambda^{\text{int}}$$

- Any mass term is contained in S_Λ^{int}

- $\dot{\Delta}_{UV} \equiv -\Lambda \partial_\Lambda \Delta_{UV} = -\Lambda \partial_\Lambda \frac{C_{UV}}{p^2}$

- $f \cdot \dot{\Delta}_{UV} \cdot g = \int_p f(-p) \dot{\Delta}_{UV}(p) g(p)$

- Classical term, $a_0[S, \Sigma]$

- Quantum term, $a_1[\Sigma]$

Polchinski Equation

An equation telling us how the Wilsonian effective action evolves

$$-\Lambda \partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2} \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi}$$

- Interaction part of Wilsonian effective action

$$S_\Lambda = \frac{1}{2} \varphi \cdot C_{UV}^{-1}(p, \Lambda) p^2 \cdot \varphi + S_\Lambda^{\text{int}}$$

- Any mass term is contained in S_Λ^{int}

- $\dot{\Delta}_{UV} \equiv -\Lambda \partial_\Lambda \Delta_{UV} = -\Lambda \partial_\Lambda \frac{C_{UV}}{p^2}$

- $f \cdot \dot{\Delta}_{UV} \cdot g = \int_p f(-p) \dot{\Delta}_{UV}(p) g(p)$

- Classical term, $a_0[S, \Sigma]$

- Quantum term, $a_1[\Sigma]$

Polchinski Equation

An equation telling us how the Wilsonian effective action evolves

$$-\Lambda \partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2} \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi}$$

- Interaction part of Wilsonian effective action

$$S_\Lambda = \frac{1}{2} \varphi \cdot C_{UV}^{-1}(p, \Lambda) p^2 \cdot \varphi + S_\Lambda^{\text{int}}$$

- Any mass term is contained in S_Λ^{int}
- $\dot{\Delta}_{UV} \equiv -\Lambda \partial_\Lambda \Delta_{UV} = -\Lambda \partial_\Lambda \frac{C_{UV}}{p^2}$
- $f \cdot \dot{\Delta}_{UV} \cdot g = \int_p f(-p) \dot{\Delta}_{UV}(p) g(p)$

• Classical term, $a_0[S, \Sigma]$

• Quantum term, $a_1[\Sigma]$

Polchinski Equation

An equation telling us how the Wilsonian effective action evolves

$$-\Lambda \partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2} \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi}$$

- Interaction part of Wilsonian effective action

$$S_\Lambda = \frac{1}{2} \varphi \cdot C_{UV}^{-1}(p, \Lambda) p^2 \cdot \varphi + S_\Lambda^{\text{int}}$$

- Any mass term is contained in S_Λ^{int}
- $\dot{\Delta}_{UV} \equiv -\Lambda \partial_\Lambda \Delta_{UV} = -\Lambda \partial_\Lambda \frac{C_{UV}}{p^2}$
- $f \cdot \dot{\Delta}_{UV} \cdot g = \int_p f(-p) \dot{\Delta}_{UV}(p) g(p)$
- **Classical term**, $a_0[S, \Sigma]$

- Quantum term, $a_1[\Sigma]$

Polchinski Equation

An equation telling us how the Wilsonian effective action evolves

$$-\Lambda \partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2} \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta}_{UV} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi}$$

- Interaction part of Wilsonian effective action

$$S_\Lambda = \frac{1}{2} \varphi \cdot C_{UV}^{-1}(p, \Lambda) p^2 \cdot \varphi + S_\Lambda^{\text{int}}$$

- Any mass term is contained in S_Λ^{int}
- $\dot{\Delta}_{UV} \equiv -\Lambda \partial_\Lambda \Delta_{UV} = -\Lambda \partial_\Lambda \frac{C_{UV}}{p^2}$
- $f \cdot \dot{\Delta}_{UV} \cdot g = \int_p f(-p) \dot{\Delta}_{UV}(p) g(p)$
- Classical term, $a_0[S, \Sigma]$
- Quantum term, $a_1[\Sigma]$

$$P_{\alpha} : -\Lambda \partial_{\alpha} S^{Int} = \frac{1}{2} \frac{\delta S^{Int}}{\delta \varphi} \cdot \Delta \frac{\delta S^{Int}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta S^{Int}}{\delta \varphi}$$

$$-\Delta_{\alpha} = \frac{C_{\alpha\beta}(\varphi, \Lambda)}{P^{\alpha}}$$

$$P_{\alpha} = -\Lambda \partial_{\alpha} S^{int} = \frac{1}{2} \frac{\delta S^{int}}{\delta \varphi} \cdot \Delta \frac{\delta S^{int}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta S^{int}}{\delta \varphi}$$

$$\Delta_{\alpha} = \frac{C_{UV}(k, \Lambda)}{P^2} \quad \dot{\Delta} = -\Lambda \partial_{\alpha} \Delta$$

Where Dose the Polchinski Equation Come From?

Direct Derivation

- Partition modes in the functional integral into those above and those below the effective cutoff
- Integrate out the high modes
- See T. R. Morris, "The Exact renormalization group and approximate solutions," Int. J. Mod. Phys. A **9** (1994) 2411, hep-ph/9308265.

Flexible Approach

- Use invariance of the partition function under the flow
- Makes freedom of coarse-graining procedure manifest
- See esp. T. R. Morris and J. L. Latorre, "Exact scheme independence," JHEP **0011** (2000) 004, hep-th/0008123.

Where Dose the Polchinski Equation Come From?

Direct Derivation

- Partition modes in the functional integral into those above and those below the effective cutoff
- Integrate out the high modes
- See T. R. Morris, "The Exact renormalization group and approximate solutions," *Int. J. Mod. Phys. A* **9** (1994) 2411, hep-ph/9308265.

Flexible Approach

- Use invariance of the partition function under the flow
- Makes freedom of coarse-graining procedure manifest
- See esp. T. R. Morris and J. L. Latorre, "Exact scheme independence," *JHEP* **0011** (2000) 004, hep-th/0008123.

Where Dose the Polchinski Equation Come From?

Direct Derivation

- Partition modes in the functional integral into those above and those below the effective cutoff
- Integrate out the high modes
- See T. R. Morris, "The Exact renormalization group and approximate solutions," Int. J. Mod. Phys. A **9** (1994) 2411, hep-ph/9308265.

Flexible Approach

- Use invariance of the partition function under the flow
- Makes freedom of coarse-graining procedure manifest
- See esp. T. R. Morris and J. L. Latorre, "Exact scheme independence," JHEP **0011** (2000) 004, hep-th/0008123.

Where Dose the Polchinski Equation Come From?

Direct Derivation

- Partition modes in the functional integral into those above and those below the effective cutoff
- Integrate out the high modes
- See T. R. Morris, "The Exact renormalization group and approximate solutions," Int. J. Mod. Phys. A **9** (1994) 2411, hep-ph/9308265.

Flexible Approach

- Use invariance of the partition function under the flow
- Makes freedom of coarse-graining procedure manifest
- See esp. T. R. Morris and J. L. Latorre, "Exact scheme independence," JHEP **0011** (2000) 004, hep-th/0008123.

Very General ERGs

Very General ERGs

Formulation

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta \varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- effective scale
- set of fields
- Wilsonian effective action
- partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
- defines our ERG

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta \varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta \varphi(x)}$$

Very General ERGs

Formulation

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta \varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- effective scale
- **set of fields**
- Wilsonian effective action
- partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
- defines our ERG

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta \varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta \varphi(x)}$$

Very General ERGs

Formulation

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta\varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- effective scale
- set of fields
- **Wilsonian effective action**
 - partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
 - defines our ERG

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta\varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta\varphi(x)}$$

Very General ERGs

Formulation

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta\varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- effective scale
- set of fields
- Wilsonian effective action
- **partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow**
- defines our ERG

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta\varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta\varphi(x)}$$

Very General ERGs

Formulation

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta\varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- effective scale
- set of fields
- Wilsonian effective action
- partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
- **defines our ERG**
 - parametrizes blocking procedure
 - huge freedom in precise form—adapt to suit our needs

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta\varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta\varphi(x)}$$

Very General ERGs

Formulation

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta\varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- effective scale
- set of fields
- Wilsonian effective action
- **partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow**
- defines our ERG

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta\varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta\varphi(x)}$$

Very General ERGs

Formulation

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta\varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- effective scale
- set of fields
- Wilsonian effective action
- partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
- **defines our ERG**
 - parametrizes blocking procedure
 - huge freedom in precise form—adapt to suit our needs

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta\varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta\varphi(x)}$$

Very General ERGs

Formulation

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta\varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- effective scale
- set of fields
- Wilsonian effective action
- partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
- defines our ERG
 - parametrizes blocking procedure
 - huge freedom in precise form—adapt to suit our needs

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta\varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta\varphi(x)}$$

Very General ERGs

Formulation

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta \varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- effective scale
- set of fields
- Wilsonian effective action
- partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
- defines our ERG
 - parametrizes blocking procedure
 - huge freedom in precise form—adapt to suit our needs

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta \varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta \varphi(x)}$$

Very General ERGs

Formulation

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta\varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- effective scale
- set of fields
- Wilsonian effective action
- partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
- defines our ERG
 - parametrizes blocking procedure
 - huge freedom in precise form—adapt to suit our needs

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta\varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta\varphi(x)}$$

$$\lambda \int d^D x |\varphi(x)|^4$$



Very General ERGs

Formulation

$$-\Lambda \partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta \varphi(x)} \left(\Psi_x[\varphi] e^{-S[\varphi]} \right)$$

- effective scale
- set of fields
- Wilsonian effective action
- partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
- defines our ERG
 - parametrizes blocking procedure
 - huge freedom in precise form—adapt to suit our needs

Flow Equation

$$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta \varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta \varphi(x)}$$

Details

Details

Choosing Ψ

- Require Ψ incorporates regularization
- Require flow equation is not linear in S

Take
$$\Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$

- ERG kernels: incorporate cutoff functions
- $\Sigma = S - 2\hat{S}$

\hat{S} is the Seed Action

Details

Choosing Ψ

- Require Ψ incorporates regularization
- Require flow equation is not linear in S

Take
$$\Psi_x = \frac{1}{2} \Delta(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$

- ERG kernels: incorporate cutoff functions
- $\Sigma = S - 2\hat{S}$

\hat{S} is the Seed Action

Details

Choosing Ψ

- Require Ψ incorporates regularization
- Require flow equation is not linear in S

$$\text{Take } \Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$

- ERG kernels: incorporate cutoff functions
- $\Sigma = S - 2\hat{S}$

\hat{S} is the Seed Action

Details

Choosing Ψ

- Require Ψ incorporates regularization
- Require flow equation is not linear in S

$$\text{Take } \Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$

- ERG kernels: incorporate cutoff functions
- $\Sigma = S - 2\hat{S}$

\hat{S} is the Seed Action

Details

Choosing Ψ

- Require Ψ incorporates regularization
- Require flow equation is not linear in S

$$\text{Take } \Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$

- **ERG kernels: incorporate cutoff functions**
- $\Sigma = S - 2\hat{S}$

\hat{S} is the Seed Action

Details

Choosing Ψ

- Require Ψ incorporates regularization
- Require flow equation is not linear in S

Take $\Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$

- ERG kernels: incorporate cutoff functions
- $\Sigma = S - 2\hat{S}$

\hat{S} is the Seed Action

Details

Choosing Ψ

- Require Ψ incorporates regularization
- Require flow equation is not linear in S

$$\text{Take } \Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$

- ERG kernels: incorporate cutoff functions
- $\Sigma = S - 2\hat{S}$

\hat{S} is the Seed Action

- A non-universal input which controls the flow
- Same structure and symmetries as S
- Full expansion in terms of fields
- Corresponds to generalizations of Polchinski's equation

Details

Choosing Ψ

- Require Ψ incorporates regularization
- Require flow equation is not linear in S

$$\text{Take } \Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$

- ERG kernels: incorporate cutoff functions
- $\Sigma = S - 2\hat{S}$

\hat{S} is the Seed Action

- A non-universal input which controls the flow
- Same structure and symmetries as S
- Full expansion in terms of fields

Details

Choosing Ψ

- Require Ψ incorporates regularization
- Require flow equation is not linear in S

$$\text{Take } \Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$

- ERG kernels: incorporate cutoff functions
- $\Sigma = S - 2\hat{S}$

\hat{S} is the Seed Action

- A non-universal input which controls the flow
- **Same structure and symmetries as S**
- Full expansion in terms of fields

Details

Choosing Ψ

- Require Ψ incorporates regularization
- Require flow equation is not linear in S

Take $\Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$

- ERG kernels: incorporate cutoff functions
- $\Sigma = S - 2\hat{S}$

\hat{S} is the Seed Action

- A non-universal input which controls the flow
- Same structure and symmetries as S
- **Full expansion in terms of fields**

Details

Choosing Ψ

- Require Ψ incorporates regularization
- Require flow equation is not linear in S

$$\text{Take } \Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$

- ERG kernels: incorporate cutoff functions
- $\Sigma = S - 2\hat{S}$

\hat{S} is the Seed Action

- A non-universal input which controls the flow
- Same structure and symmetries as S
- Full expansion in terms of fields

Properties of Good Flow Equations

Properties of Good Flow Equations

General Constraints

- The flow equation must be UV regularized
- The partition function must be invariant under the flow
- Blocking must only be over a localized patch

Constraints on the Seed Action

Properties of Good Flow Equations

General Constraints

- The flow equation must be UV regularized
- The partition function must be invariant under the flow
- Blocking must only be over a localized patch

Constraints on the Seed Action

Properties of Good Flow Equations

General Constraints

- The flow equation must be UV regularized
- The partition function must be invariant under the flow
- Blocking must only be over a localized patch

Constraints on the Seed Action

Properties of Good Flow Equations

General Constraints

- The flow equation must be UV regularized
- The partition function must be invariant under the flow
- Blocking must only be over a localized patch
 - Long range interactions can only appear after an infinite number of ERG steps, here $\Lambda \rightarrow 0$
 - Violation would lead to infrared singularities in the flow

Constraints on the Seed Action

Properties of Good Flow Equations

General Constraints

- The flow equation must be UV regularized
- The partition function must be invariant under the flow
- Blocking must only be over a localized patch
 - Long range interactions can only appear after an infinite number of ERG steps, here $\Lambda \rightarrow 0$
 - Violation would lead to infrared singularities in the flow

Constraints on the Seed Action

Properties of Good Flow Equations

General Constraints

- The flow equation must be UV regularized
- The partition function must be invariant under the flow
- Blocking must only be over a localized patch
 - Long range interactions can only appear after an infinite number of ERG steps, here $\Lambda \rightarrow 0$
 - Violation would lead to infrared singularities in the flow

Constraints on the Seed Action

Properties of Good Flow Equations

General Constraints

- The flow equation must be UV regularized
- The partition function must be invariant under the flow
- Blocking must only be over a localized patch
 - Long range interactions can only appear after an infinite number of ERG steps, here $\Lambda \rightarrow 0$
 - Violation would lead to infrared singularities in the flow

Constraints on the Seed Action

- It must lead to convergent momentum integrals
- It must have a Taylor expansion in small momenta (quasi-locality)

Properties of Good Flow Equations

General Constraints

- The flow equation must be UV regularized
- The partition function must be invariant under the flow
- Blocking must only be over a localized patch
 - Long range interactions can only appear after an infinite number of ERG steps, here $\Lambda \rightarrow 0$
 - Violation would lead to infrared singularities in the flow

Constraints on the Seed Action

- It must lead to convergent momentum integrals
- It must have a Taylor expansion in small momenta (quasi-locality)

Properties of Good Flow Equations

General Constraints

- The flow equation must be UV regularized
- The partition function must be invariant under the flow
- Blocking must only be over a localized patch
 - Long range interactions can only appear after an infinite number of ERG steps, here $\Lambda \rightarrow 0$
 - Violation would lead to infrared singularities in the flow

Constraints on the Seed Action

- It must lead to convergent momentum integrals
- It must have a Taylor expansion in small momenta (quasi-locality)

The Flow Corresponds to Integrating Out

The Flow Corresponds to Integrating Out

- The flow equation is UV regularized by Λ
- All contributions from momenta larger than q vanish as $q/\Lambda \rightarrow \infty$
- As $\Lambda \rightarrow 0$, contributions from non-vanishing momentum scales vanish
- But the partition function is invariant under the flow!
- Contributions from non-vanishing momentum scales must be incorporated into the effective action
- The flow equation must correspond to integrating out

The Flow Corresponds to Integrating Out

- The flow equation is UV regularized by Λ
- All contributions from momenta larger than q vanish as $q/\Lambda \rightarrow \infty$
- As $\Lambda \rightarrow 0$, contributions from non-vanishing momentum scales vanish
- But the partition function is invariant under the flow!
- Contributions from non-vanishing momentum scales must be incorporated into the effective action
- The flow equation must correspond to integrating out

The Flow Corresponds to Integrating Out

- The flow equation is UV regularized by Λ
- All contributions from momenta larger than q vanish as $q/\Lambda \rightarrow \infty$
- As $\Lambda \rightarrow 0$, contributions from non-vanishing momentum scales vanish
- But the partition function is invariant under the flow!
- Contributions from non-vanishing momentum scales must be incorporated into the effective action
- The flow equation must correspond to integrating out

The Flow Corresponds to Integrating Out

- The flow equation is UV regularized by Λ
- All contributions from momenta larger than q vanish as $q/\Lambda \rightarrow \infty$
- As $\Lambda \rightarrow 0$, contributions from non-vanishing momentum scales vanish
- But the partition function is invariant under the flow!
- Contributions from non-vanishing momentum scales must be incorporated into the effective action
- The flow equation must correspond to integrating out

The Flow Corresponds to Integrating Out

- The flow equation is UV regularized by Λ
- All contributions from momenta larger than q vanish as $q/\Lambda \rightarrow \infty$
- As $\Lambda \rightarrow 0$, contributions from non-vanishing momentum scales vanish
- But the partition function is invariant under the flow!
- Contributions from non-vanishing momentum scales must be incorporated into the effective action
- The flow equation must correspond to integrating out

The Flow Corresponds to Integrating Out

- The flow equation is UV regularized by Λ
- All contributions from momenta larger than q vanish as $q/\Lambda \rightarrow \infty$
- As $\Lambda \rightarrow 0$, contributions from non-vanishing momentum scales vanish
- But the partition function is invariant under the flow!
- Contributions from non-vanishing momentum scales must be incorporated into the effective action
- The flow equation must correspond to integrating out

The Generalized Polchinski Equation

The Generalized Polchinski Equation

- $$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta \varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta \varphi(x)}$$

- $$\Psi_x = \frac{1}{2} \Delta(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$

- $$\Sigma = S - 2\hat{S}$$

- The flow equation is

$$-\Lambda \partial_\Lambda S_\Lambda = \frac{1}{2} \frac{\delta S_\Lambda}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma}{\delta \varphi}$$

- Compare with the Polchinski Equation

$$-\Lambda \partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2} \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} \cdot \Delta \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi}$$

The Generalized Polchinski Equation

- $$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta \varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta \varphi(x)}$$

- $$\Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$

- $$\Sigma = S - 2\hat{S}$$

- The flow equation is

$$-\Lambda \partial_\Lambda S_\Lambda = \frac{1}{2} \frac{\delta S_\Lambda}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

- Compare with the Polchinski Equation

$$-\Lambda \partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2} \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi}$$

The Generalized Polchinski Equation

- $-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta \varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta \varphi(x)}$
- $\Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$
- $\Sigma = S - 2\hat{S}$
- The flow equation is

$$-\Lambda \partial_\Lambda S_\Lambda = \frac{1}{2} \frac{\delta S_\Lambda}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

- Compare with the Polchinski Equation

$$-\Lambda \partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2} \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi}$$

The Generalized Polchinski Equation

- $$-\Lambda \partial_\Lambda S = \int_x \frac{\delta S}{\delta \varphi(x)} \Psi_x - \int_x \frac{\delta \Psi_x}{\delta \varphi(x)}$$
- $$\Psi_x = \frac{1}{2} \dot{\Delta}(x, y) \frac{\delta \Sigma}{\delta \varphi(y)}$$
- $$\Sigma = S - 2\hat{S}$$
- The flow equation is

$$-\Lambda \partial_\Lambda S_\Lambda = \frac{1}{2} \frac{\delta S_\Lambda}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

- Compare with the Polchinski Equation

$$-\Lambda \partial_\Lambda S_\Lambda^{\text{int}} = \frac{1}{2} \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta S_\Lambda^{\text{int}}}{\delta \varphi}$$

Making the link

Making the link

- Define the 'reduced action', S^R :

$$S[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R[\varphi] \quad \hat{S}[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + \hat{S}^R[\varphi]$$

- LHS of flow equation:

$$-\Lambda \partial_\Lambda \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) = -\Lambda \partial_\Lambda S^R[\varphi] - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi$$

- Classical Term:

$$\begin{aligned} \frac{1}{2} \frac{\delta}{\delta \varphi} \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) \cdot \dot{\Delta} \cdot \frac{\delta}{\delta \varphi} \left(S^R - 2\hat{S}^R - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi \right) \\ = -\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi + \dots \end{aligned}$$

- Quantum Term: two-pt contributions only affect vacuum term

Making the link

- Define the 'reduced action', S^R :

$$S[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R[\varphi] \quad \hat{S}[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + \hat{S}^R[\varphi]$$

- LHS of flow equation:

$$-\Lambda \partial_\Lambda \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) = -\Lambda \partial_\Lambda S^R[\varphi] - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi$$

- Classical Term:

$$\begin{aligned} \frac{1}{2} \frac{\delta}{\delta \varphi} \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) \cdot \Delta \cdot \frac{\delta}{\delta \varphi} \left(S^R - 2\hat{S}^R - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi \right) \\ = -\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi + \dots \end{aligned}$$

- Quantum Term: two-pt contributions only affect vacuum term

Making the link

- Define the 'reduced action', S^R :

$$S[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R[\varphi] \quad \hat{S}[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + \hat{S}^R[\varphi]$$

- LHS of flow equation:

$$-\Lambda \partial_\Lambda \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) = -\Lambda \partial_\Lambda S^R[\varphi] - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi$$

- Classical Term:

$$\begin{aligned} \frac{1}{2} \frac{\delta}{\delta \varphi} \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) \cdot \dot{\Delta} \cdot \frac{\delta}{\delta \varphi} \left(S^R - 2\hat{S}^R - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi \right) \\ = -\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi + \dots \end{aligned}$$

- Quantum Term: two-pt contributions only affect vacuum term

Making the link

- Define the 'reduced action', S^R :

$$S[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R[\varphi] \quad \hat{S}[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + \hat{S}^R[\varphi]$$

- LHS of flow equation:

$$-\Lambda \partial_\Lambda \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) = -\Lambda \partial_\Lambda S^R[\varphi] - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi$$

- Classical Term:

$$\begin{aligned} \frac{1}{2} \frac{\delta}{\delta \varphi} \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) \cdot \dot{\Delta} \cdot \frac{\delta}{\delta \varphi} \left(S^R - 2\hat{S}^R - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi \right) \\ = -\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi + \dots \end{aligned}$$

- Quantum Term: two-pt contributions only affect vacuum term

$$P_{\text{ol}}: -\Lambda \partial_\lambda S^{\text{int}} = \frac{1}{2} \frac{\delta S^{\text{int}}}{\delta \varphi} \cdot \Delta \frac{\delta S^{\text{int}}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta S^{\text{int}}}{\delta \varphi}$$

$$\Delta_\lambda = \frac{C_{\text{int}}(\lambda, \lambda)}{P^2}$$

$$\dot{\Delta} = -\Lambda \partial_\lambda \Delta$$

$$\Sigma = S - 2\dot{S} - \Lambda \partial_\lambda S$$

Making the link

- Define the 'reduced action', S^R :

$$S[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R[\varphi] \quad \hat{S}[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + \hat{S}^R[\varphi]$$

- LHS of flow equation:

$$-\Lambda \partial_\Lambda \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) = -\Lambda \partial_\Lambda S^R[\varphi] - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi$$

- Classical Term:

$$\begin{aligned} \frac{1}{2} \frac{\delta}{\delta \varphi} \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) \cdot \dot{\Delta} \cdot \frac{\delta}{\delta \varphi} \left(S^R - 2\hat{S}^R - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi \right) \\ = -\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi + \dots \end{aligned}$$

- Quantum Term: two-pt contributions only affect vacuum term

Making the link

- Define the 'reduced action', S^R :

$$S[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R[\varphi] \quad \hat{S}[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + \hat{S}^R[\varphi]$$

- LHS of flow equation:

$$-\Lambda \partial_\Lambda \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) = -\Lambda \partial_\Lambda S^R[\varphi] - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi$$

- Classical Term:

$$\begin{aligned} \frac{1}{2} \frac{\delta}{\delta \varphi} \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) \cdot \dot{\Delta} \cdot \frac{\delta}{\delta \varphi} \left(S^R - 2\hat{S}^R - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi \right) \\ = -\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi + \dots \end{aligned}$$

- Quantum Term: two-pt contributions only affect vacuum term

Making the link

- Define the 'reduced action', S^R :

$$S[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R[\varphi] \quad \hat{S}[\varphi] = \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + \hat{S}^R[\varphi]$$

- LHS of flow equation:

$$-\Lambda \partial_\Lambda \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) = -\Lambda \partial_\Lambda S^R[\varphi] - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi$$

- Classical Term:

$$\begin{aligned} \frac{1}{2} \frac{\delta}{\delta \varphi} \left(\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi + S^R \right) \cdot \dot{\Delta} \cdot \frac{\delta}{\delta \varphi} \left(S^R - 2\hat{S}^R - \frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \varphi \right) \\ = -\frac{1}{2}\varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \Delta^{-1} \cdot \varphi + \dots \end{aligned}$$

- Quantum Term: two-pt contributions only affect vacuum term

Making the link

Generalised Flow Equation

$$-\Lambda \partial_\Lambda S^R = \frac{1}{2} \frac{\delta S^R}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma^R}{\delta \varphi} - \varphi \cdot \Delta^{-1} \cdot \dot{\Delta} \cdot \frac{\delta \hat{S}^R}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma^R}{\delta \varphi}$$

Relationship to Polchinski Equation

$$P_{\alpha} = -12\lambda S^{3\alpha} = \frac{1}{2} \frac{\delta S^{2\alpha}}{\delta \varphi} \cdot \Delta \frac{\delta S^{2\alpha}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta S^{2\alpha}}{\delta \varphi}$$

$$\Delta_{\alpha} = \frac{C_{\mu\nu}(\lambda)}{P^{\alpha}}$$

$$\dot{\Delta} = -12\lambda \Delta$$

$$\Sigma = S - 2\dot{S}$$

$$-12\lambda S^2 = \frac{1}{2} \frac{\delta S^2}{\delta \varphi} \cdot \Delta \cdot \frac{\delta S^2}{\delta \varphi} - 4 \cdot \Delta \cdot \frac{\delta S^2}{\delta \varphi}$$

$$P_{\alpha} = -\lambda_2 S^{2\alpha} = \frac{1}{2} \frac{\delta S^{2\alpha}}{\delta \varphi} \cdot \Delta \frac{\delta S^{2\alpha}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta S^{2\alpha}}{\delta \varphi}$$

$$\Delta_{\alpha} = \frac{C_{\alpha\alpha}(k, l)}{P^{\alpha}}$$

$$\dot{\Delta} = -\lambda_2 \Delta$$

$$\Sigma = S - 2\hat{S}$$

$$-\lambda_2 S$$

$$-\lambda_2 S^2 = \frac{1}{2} \frac{\delta S^2}{\delta \varphi} \cdot \Delta \cdot \frac{\delta S^2}{\delta \varphi} - 4 \cdot \Delta \cdot \Delta \cdot \frac{\delta S^2}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta S^2}{\delta \varphi}$$

Explicitly Including the Anomalous Dimension

- Fixed point behaviour is most conveniently uncovered by transferring to dimensionless variable
- For generic calculations eg computing the β -function, it is convenient to explicitly include the anomalous dimension
- Under renormalization, the two-point vertex picks up a factor of the field strength renormalization:

$$\varphi \cdot p^2 Z^{-1} \cdot \varphi$$

- So rescale $\varphi \rightarrow \sqrt{Z} \varphi$
- $-\Lambda \partial_\Lambda |_\varphi S \rightarrow \left(-\Lambda \partial_\Lambda |_\varphi + \frac{\gamma}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S$
- $\gamma \equiv \Lambda \partial_\Lambda \ln Z$
- $\frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta}{\delta \varphi} \rightarrow \frac{1}{Z} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta}{\delta \varphi}$
- Annoying!

Explicitly Including the Anomalous Dimension

- Fixed point behaviour is most conveniently uncovered by transferring to dimensionless variable
- For generic calculations eg computing the β -function, it is convenient to explicitly include the anomalous dimension
- Under renormalization, the two-point vertex picks up a factor of the field strength renormalization:

$$\varphi \cdot p^2 Z^{-1} \cdot \varphi$$

- So rescale $\varphi \rightarrow \sqrt{Z} \varphi$
- $-\Lambda \partial_\Lambda |_\varphi S \rightarrow \left(-\Lambda \partial_\Lambda |_\varphi + \frac{\gamma}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S$
- $\gamma \equiv \Lambda \partial_\Lambda \ln Z$
- $\frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta}{\delta \varphi} \rightarrow \frac{1}{Z} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta}{\delta \varphi}$
- Annoying!

Explicitly Including the Anomalous Dimension

- Fixed point behaviour is most conveniently uncovered by transferring to dimensionless variable
- For generic calculations eg computing the β -function, it is convenient to explicitly include the anomalous dimension
- Under renormalization, the two-point vertex picks up a factor of the field strength renormalization:

$$\varphi \cdot p^2 Z^{-1} \cdot \varphi$$

- So rescale $\varphi \rightarrow \sqrt{Z} \varphi$

- $- \Lambda \partial_\Lambda |_\varphi S \rightarrow \left(- \Lambda \partial_\Lambda |_\varphi + \frac{\gamma}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S$

- $\gamma \equiv \Lambda \partial_\Lambda \ln Z$

- $\frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta}{\delta \varphi} \rightarrow \frac{1}{Z} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta}{\delta \varphi}$

- Annoying!

Explicitly Including the Anomalous Dimension

- Fixed point behaviour is most conveniently uncovered by transferring to dimensionless variable
- For generic calculations eg computing the β -function, it is convenient to explicitly include the anomalous dimension
- Under renormalization, the two-point vertex picks up a factor of the field strength renormalization:

$$\varphi \cdot p^2 Z^{-1} \cdot \varphi$$

- So rescale $\varphi \rightarrow \sqrt{Z} \varphi$
- $-\Lambda \partial_\Lambda |_\varphi S \rightarrow \left(-\Lambda \partial_\Lambda |_\varphi + \frac{\gamma}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S$

• $\gamma \equiv \Lambda \partial_\Lambda \ln Z$

• $\frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta}{\delta \varphi} \rightarrow \frac{1}{Z} \frac{\delta}{\delta \varphi} \cdot \tilde{\Delta} \cdot \frac{\delta}{\delta \varphi}$

• Annoying!

Explicitly Including the Anomalous Dimension

- Fixed point behaviour is most conveniently uncovered by transferring to dimensionless variable
- For generic calculations eg computing the β -function, it is convenient to explicitly include the anomalous dimension
- Under renormalization, the two-point vertex picks up a factor of the field strength renormalization:

$$\varphi \cdot p^2 Z^{-1} \cdot \varphi$$

- So rescale $\varphi \rightarrow \sqrt{Z} \varphi$
- $-\Lambda \partial_\Lambda |_\varphi S \rightarrow \left(-\Lambda \partial_\Lambda |_\varphi + \frac{\gamma}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S$
- $\gamma \equiv \Lambda \partial_\Lambda \ln Z$
- $\frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta}{\delta \varphi} \rightarrow \frac{1}{Z} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta}{\delta \varphi}$

• Annoying!

Explicitly Including the Anomalous Dimension

Solution

- Simply drop the $1/Z$ on the RHS of the flow equation!
- The resulting flow equation is perfectly valid
- The partition function is still invariant under the flow
- But the blocking function, Ψ has changed

$$\bullet \left(-\Lambda \partial_\Lambda + \frac{\gamma}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

- First derived in
R. D. Ball, P. E. Haagensen, J. I. Latorre, and E. Moreno,
"Scheme Independence And The Exact Renormalization
Group," *Phys. Lett. B* 347 (1995) 80, hep-th/9411122.

Explicitly Including the Anomalous Dimension

Solution

- Simply drop the $1/Z$ on the RHS of the flow equation!
- The resulting flow equation is perfectly valid
- The partition function is still invariant under the flow
- But the blocking function, Ψ has changed

- $$\left(-\Lambda \partial_\Lambda + \frac{\gamma}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

- First derived in
R. D. Ball, P. E. Haagensen, J. I. Latorre, and E. Moreno,
"Scheme Independence And The Exact Renormalization
Group," *Phys. Lett. B* 347 (1995) 80, hep-th/9411122.

Explicitly Including the Anomalous Dimension

Solution

- Simply drop the $1/Z$ on the RHS of the flow equation!
- The resulting flow equation is perfectly valid
- The partition function is still invariant under the flow
- But the blocking function, Ψ has changed

- $$\left(-\Lambda \partial_\Lambda + \frac{\gamma}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

- First derived in
R. D. Ball, P. E. Haagensen, J. I. Latorre, and E. Moreno,
"Scheme Independence And The Exact Renormalization
Group," *Phys. Lett. B* 347 (1995) 80, hep-th/9411122.

Explicitly Including the Anomalous Dimension

Solution

- Simply drop the $1/Z$ on the RHS of the flow equation!
- The resulting flow equation is perfectly valid
- The partition function is still invariant under the flow
- But the blocking function, Ψ has changed

$$\bullet \left(-\Lambda \partial_\Lambda + \frac{\gamma}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

- First derived in
R. D. Ball, P. E. Haagensen, J. I. Latorre, and E. Moreno,
"Scheme Independence And The Exact Renormalization
Group," *Phys. Lett. B* 347 (1995) 80, hep-th/9411122.

Explicitly Including the Anomalous Dimension

Solution

- Simply drop the $1/Z$ on the RHS of the flow equation!
- The resulting flow equation is perfectly valid
- The partition function is still invariant under the flow
- But the blocking function, Ψ has changed

$$\bullet \left(-\Lambda \partial_\Lambda + \frac{\gamma}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma}{\delta \varphi}$$

- First derived in
R. D. Ball, P. E. Haagensen, J. I. Latorre, and E. Moreno,
"Scheme Independence And The Exact Renormalization
Group," *Phys. Lett. B* 347 (1995) 80, hep-th/9411122.

Explicitly Including the Anomalous Dimension

Solution

- Simply drop the $1/Z$ on the RHS of the flow equation!
- The resulting flow equation is perfectly valid
- The partition function is still invariant under the flow
- But the blocking function, Ψ has changed

$$\bullet \left(-\Lambda \partial_\Lambda + \frac{\gamma}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

- First derived in

R. D. Ball, P. E. Haagensen, J. I. Latorre, and E. Moreno,
"Scheme Independence And The Exact Renormalization
Group," *Phys. Lett. B* 347 (1995) 80, hep-th/9411122.

Explicitly Including the Anomalous Dimension

Solution

- Simply drop the $1/Z$ on the RHS of the flow equation!
- The resulting flow equation is perfectly valid
- The partition function is still invariant under the flow
- But the blocking function, Ψ has changed

- $$\left(-\Lambda \partial_\Lambda + \frac{\gamma}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

- First derived in

R. D. Ball, P. E. Haagensen, J. I. Latorre, and E. Moreno,
"Scheme Independence And The Exact Renormalization
Group," *Phys. Lett.* **B 347** (1995) 80, hep-th/9411122.

Preparation for Perturbation Theory

Preparation for Perturbation Theory

- It is convenient to bring $1/\lambda$ outside the action

- $\varphi \rightarrow \varphi/\sqrt{\lambda}$

- $S \sim \frac{1}{\lambda} S_0 + S_1 + \lambda S_2 + \dots$

- $$\left(-\Lambda \partial_\Lambda + \frac{\bar{\gamma}}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma_\lambda}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma_\lambda}{\delta \varphi}$$

Preparation for Perturbation Theory

- It is convenient to bring $1/\lambda$ outside the action

- $\varphi \rightarrow \varphi/\sqrt{\lambda}$

- $S \sim \frac{1}{\lambda} S_0 + S_1 + \lambda S_2 + \dots$

- Classical action
- One-loop correction
- etc...

- $$\left(-\Lambda \partial_\Lambda + \frac{\tilde{\gamma}}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma_\lambda}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma_\lambda}{\delta \varphi}$$

Preparation for Perturbation Theory

- It is convenient to bring $1/\lambda$ outside the action

- $\varphi \rightarrow \varphi/\sqrt{\lambda}$

- $S \sim \frac{1}{\lambda} S_0 + S_1 + \lambda S_2 + \dots$

- Classical action
- One-loop correction
- etc...

- $$\left(-\Lambda \partial_\Lambda + \frac{\tilde{\gamma}}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma_\lambda}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma_\lambda}{\delta \varphi}$$

Preparation for Perturbation Theory

- It is convenient to bring $1/\lambda$ outside the action

- $\varphi \rightarrow \varphi/\sqrt{\lambda}$

- $S \sim \frac{1}{\lambda} S_0 + S_1 + \lambda S_2 + \dots$

- Classical action
- One-loop correction
- etc...

- $$\left(-\Lambda \partial_\Lambda + \frac{\tilde{\gamma}}{2} \varphi \cdot \frac{\delta}{\delta \varphi} \right) S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma_\lambda}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma_\lambda}{\delta \varphi}$$

- $\tilde{\gamma} = \gamma - \frac{\beta}{\lambda}$

- $\Sigma_\lambda \equiv \lambda \Sigma$

Other ERG Equations

Other ERG Equations

Other Equations for S_Λ

- Wilson's ERG equation:

$$\Psi_p = h(p) \left(\varphi_p - \frac{\delta S}{\delta \varphi_{-p}} \right)$$

- The Wegner-Houghton equation

Other ERG Equations

Other Equations for S_Λ

- Wilson's ERG equation:

$$\Psi_p = h(p) \left(\varphi_p - \frac{\delta S}{\delta \varphi_{-p}} \right)$$

- The Wegner-Houghton equation

Other ERG Equations

Other Equations for S_Λ

- Wilson's ERG equation:

$$\Psi_p = h(p) \left(\varphi_p - \frac{\delta S}{\delta \varphi_{-p}} \right)$$

- The Wegner-Houghton equation
 - Sharp cutoff flow equation
 - The Polchinski equation is ambiguous in this limit

Other ERG Equations

Other Equations for S_Λ

- Wilson's ERG equation:

$$\Psi_p = h(p) \left(\varphi_p - \frac{\delta S}{\delta \varphi_{-p}} \right)$$

- The Wegner-Houghton equation
 - Sharp cutoff flow equation
 - The Polchinski equation is ambiguous in this limit

Other ERG Equations

Other Equations for S_Λ

- Wilson's ERG equation:

$$\Psi_p = h(p) \left(\varphi_p - \frac{\delta S}{\delta \varphi_{-p}} \right)$$

- The Wegner-Houghton equation
 - Sharp cutoff flow equation
 - The Polchinski equation is ambiguous in this limit

The Effective Average Action

The Effective Average Action

- There is a Legendre transformed version of the Polchinski equation

- Integrate over high momentum modes in $Z[J]$ to define $Z_\Lambda[J]$
- Introduce the classical field $\varphi_c \equiv \frac{\delta \ln Z_\Lambda}{\delta J}$
- Define the IR regulated generator of 1PI diagrams

$$S_\Lambda^{\text{int}}[\varphi] = \Gamma_\Lambda^{\text{int}}[\varphi_c] + \frac{1}{2}(\varphi_c - \varphi) \cdot \Delta_{\text{IR}}^{-1} \cdot (\varphi_c - \varphi)$$

- The flow equation for the average effective action is:

$$\frac{\partial \Gamma_\Lambda^{\text{int}}[\varphi_c]}{\partial \Lambda} = \frac{1}{2} \text{tr} \left[\frac{\partial \Delta_{\text{IR}}^{-1}}{\partial \Lambda} \cdot \left(\Delta_{\text{IR}}^{-1} + \frac{\delta^2 \Gamma_\Lambda^{\text{int}}}{\delta \varphi_c \delta \varphi_c} \right)^{-1} \right]$$

- T. R. Morris, Int. J. Mod. Phys. A **9** (1994) 2411.
- For a completely different derivation, see C. Wetterich, Phys. Lett. B **301** (1993) 90.

The Effective Average Action

- There is a Legendre transformed version of the Polchinski equation
 - Integrate over high momentum modes in $Z[J]$ to define $Z_\Lambda[J]$
 - Introduce the classical field $\varphi_c \equiv \frac{\delta \ln Z_\Lambda}{\delta J}$
 - Define the IR regulated generator of 1PI diagrams

$$S_\Lambda^{\text{int}}[\varphi] = \Gamma_\Lambda^{\text{int}}[\varphi_c] + \frac{1}{2}(\varphi_c - \varphi) \cdot \Delta_{\text{IR}}^{-1} \cdot (\varphi_c - \varphi)$$

- The flow equation for the average effective action is:

$$\frac{\partial \Gamma_\Lambda^{\text{int}}[\varphi_c]}{\partial \Lambda} = \frac{1}{2} \text{tr} \left[\frac{\partial \Delta_{\text{IR}}^{-1}}{\partial \Lambda} \cdot \left(\Delta_{\text{IR}}^{-1} + \frac{\delta^2 \Gamma_\Lambda^{\text{int}}}{\delta \varphi_c \delta \varphi_c} \right)^{-1} \right]$$

- T. R. Morris, Int. J. Mod. Phys. A **9** (1994) 2411.
- For a completely different derivation, see C. Wetterich, Phys. Lett. B **301** (1993) 90.

The Effective Average Action

- There is a Legendre transformed version of the Polchinski equation
 - Integrate over high momentum modes in $Z[J]$ to define $Z_\Lambda[J]$
 - Introduce the classical field $\varphi_c \equiv \frac{\delta \ln Z_\Lambda}{\delta J}$
 - Define the IR regulated generator of 1PI diagrams

$$S_\Lambda^{\text{int}}[\varphi] = \Gamma_\Lambda^{\text{int}}[\varphi_c] + \frac{1}{2}(\varphi_c - \varphi) \cdot \Delta_{\text{IR}}^{-1} \cdot (\varphi_c - \varphi)$$

- The flow equation for the average effective action is:

$$\frac{\partial \Gamma_\Lambda^{\text{int}}[\varphi_c]}{\partial \Lambda} = \frac{1}{2} \text{tr} \left[\frac{\partial \Delta_{\text{IR}}^{-1}}{\partial \Lambda} \cdot \left(\Delta_{\text{IR}}^{-1} + \frac{\delta^2 \Gamma_\Lambda^{\text{int}}}{\delta \varphi_c \delta \varphi_c} \right)^{-1} \right]$$

- T. R. Morris, Int. J. Mod. Phys. A 9 (1994) 2411.
- For a completely different derivation, see C. Wetterich, Phys. Lett. B 301 (1993) 90.

The Effective Average Action

- There is a Legendre transformed version of the Polchinski equation
 - Integrate over high momentum modes in $Z[J]$ to define $Z_\Lambda[J]$
 - Introduce the classical field $\varphi_c \equiv \frac{\delta \ln Z_\Lambda}{\delta J}$
 - Define the **IR regulated generator of 1PI diagrams**

$$S_\Lambda^{\text{int}}[\varphi] = \Gamma_\Lambda^{\text{int}}[\varphi_c] + \frac{1}{2}(\varphi_c - \varphi) \cdot \Delta_{\text{IR}}^{-1} \cdot (\varphi_c - \varphi)$$

- The flow equation for the average effective action is:

$$\frac{\partial \Gamma_\Lambda^{\text{int}}[\varphi_c]}{\partial \Lambda} = \frac{1}{2} \text{tr} \left[\frac{\partial \Delta_{\text{IR}}^{-1}}{\partial \Lambda} \cdot \left(\Delta_{\text{IR}}^{-1} + \frac{\delta^2 \Gamma_\Lambda^{\text{int}}}{\delta \varphi_c \delta \varphi_c} \right)^{-1} \right]$$

- T. R. Morris, Int. J. Mod. Phys. A **9** (1994) 2411.
- For a completely different derivation, see C. Wetterich, Phys. Lett. B **301** (1993) 90.

The Effective Average Action

- There is a Legendre transformed version of the Polchinski equation
 - Integrate over high momentum modes in $Z[J]$ to define $Z_\Lambda[J]$
 - Introduce the classical field $\varphi_c \equiv \frac{\delta \ln Z_\Lambda}{\delta J}$
 - Define the **IR regulated generator of 1PI diagrams**

$$S_\Lambda^{\text{int}}[\varphi] = \Gamma_\Lambda^{\text{int}}[\varphi_c] + \frac{1}{2}(\varphi_c - \varphi) \cdot \Delta_{\text{IR}}^{-1} \cdot (\varphi_c - \varphi)$$

- The flow equation for the **average effective action** is:

$$\frac{\partial \Gamma_\Lambda^{\text{int}}[\varphi_c]}{\partial \Lambda} = \frac{1}{2} \text{tr} \left[\frac{\partial \Delta_{\text{IR}}^{-1}}{\partial \Lambda} \cdot \left(\Delta_{\text{IR}}^{-1} + \frac{\delta^2 \Gamma_\Lambda^{\text{int}}}{\delta \varphi_c \delta \varphi_c} \right)^{-1} \right]$$

- T. R. Morris, Int. J. Mod. Phys. A 9 (1994) 2411..
- For a completely different derivation, see C. Wetterich, Phys. Lett. B 301 (1993) 90.

The Effective Average Action

- There is a Legendre transformed version of the Polchinski equation
 - Integrate over high momentum modes in $Z[J]$ to define $Z_\Lambda[J]$
 - Introduce the classical field $\varphi_c \equiv \frac{\delta \ln Z_\Lambda}{\delta J}$
 - Define the IR regulated generator of 1PI diagrams

$$S_\Lambda^{\text{int}}[\varphi] = \Gamma_\Lambda^{\text{int}}[\varphi_c] + \frac{1}{2}(\varphi_c - \varphi) \cdot \Delta_{\text{IR}}^{-1} \cdot (\varphi_c - \varphi)$$

- The flow equation for the average effective action is:

$$\frac{\partial \Gamma_\Lambda^{\text{int}}[\varphi_c]}{\partial \Lambda} = \frac{1}{2} \text{tr} \left[\frac{\partial \Delta_{\text{IR}}^{-1}}{\partial \Lambda} \cdot \left(\Delta_{\text{IR}}^{-1} + \frac{\delta^2 \Gamma_\Lambda^{\text{int}}}{\delta \varphi_c \delta \varphi_c} \right)^{-1} \right]$$

- T. R. Morris, Int. J. Mod. Phys. A **9** (1994) 2411.
- For a completely different derivation, see C. Wetterich, Phys. Lett. B **301** (1993) 90.

The Effective Average Action

- There is a Legendre transformed version of the Polchinski equation
 - Integrate over high momentum modes in $Z[J]$ to define $Z_\Lambda[J]$
 - Introduce the classical field $\varphi_c \equiv \frac{\delta \ln Z_\Lambda}{\delta J}$
 - Define the IR regulated generator of 1PI diagrams

$$S_\Lambda^{\text{int}}[\varphi] = \Gamma_\Lambda^{\text{int}}[\varphi_c] + \frac{1}{2}(\varphi_c - \varphi) \cdot \Delta_{\text{IR}}^{-1} \cdot (\varphi_c - \varphi)$$

- The flow equation for the average effective action is:

$$\frac{\partial \Gamma_\Lambda^{\text{int}}[\varphi_c]}{\partial \Lambda} = \frac{1}{2} \text{tr} \left[\frac{\partial \Delta_{\text{IR}}^{-1}}{\partial \Lambda} \cdot \left(\Delta_{\text{IR}}^{-1} + \frac{\delta^2 \Gamma_\Lambda^{\text{int}}}{\delta \varphi_c \delta \varphi_c} \right)^{-1} \right]$$

- T. R. Morris, Int. J. Mod. Phys. A **9** (1994) 2411.
- For a completely different derivation, see C. Wetterich, Phys. Lett. B **301** (1993) 90.

- 1 General ERG Equations
 - Formulation
 - Other ERG Equations
- 2 Diagrammatics
- 3 One-loop β -function
- 4 Recap

Diagrammatics for the Action

Diagrammatics for the Action

- S has an expansion in terms of fields

$$S = \frac{1}{2} S^{(2)} \varphi\varphi + \frac{1}{4!} S^{(4)} \varphi\varphi\varphi\varphi + \dots$$

- symmetry factor
- vertex coefficient function
- fields
- matching arguments

Diagrammatics for the Action

- S has an expansion in terms of fields

$$S = \frac{1}{2} S^{(2)} \varphi\varphi + \frac{1}{4!} S^{(4)} \varphi\varphi\varphi\varphi + \dots$$

- **symmetry factor**
- vertex coefficient function
- fields
- matching arguments

Diagrammatics for the Action

- S has an expansion in terms of fields

$$S = \frac{1}{2} S^{(2)} \varphi\varphi + \frac{1}{4!} S^{(4)} \varphi\varphi\varphi\varphi + \dots$$

- symmetry factor
- **vertex coefficient function**
- fields
- matching arguments

Diagrammatics for the Action

- S has an expansion in terms of fields

$$S = \frac{1}{2} S^{(2)} \varphi\varphi + \frac{1}{4!} S^{(4)} \varphi\varphi\varphi\varphi + \dots$$

- symmetry factor
- vertex coefficient function
- **fields**
- matching arguments

Diagrammatics for the Action

- S has an expansion in terms of fields

$$S = \frac{1}{2} S^{(2)} \varphi\varphi + \frac{1}{4!} S^{(4)} \varphi\varphi\varphi\varphi + \dots$$

- symmetry factor
- vertex coefficient function
- fields
- **matching arguments**

Diagrammatics for the Action

- S has an expansion in terms of fields

$$S = \frac{1}{2} S^{(2)} \varphi\varphi + \frac{1}{4!} S^{(4)} \varphi\varphi\varphi\varphi + \dots$$

- symmetry factor
- vertex coefficient function
- fields
- matching arguments



Diagrammatics for the Action

- S has an expansion in terms of fields

$$S = \frac{1}{2} S^{(2)} \varphi\varphi + \frac{1}{4!} S^{(4)} \varphi\varphi\varphi\varphi + \dots$$

- **symmetry factor**
- vertex coefficient function
- fields
- matching arguments

Diagrammatics for the Action

- S has an expansion in terms of fields

$$S = \frac{1}{2} S^{(2)} \varphi\varphi + \frac{1}{4!} S^{(4)} \varphi\varphi\varphi\varphi + \dots$$

- symmetry factor
- **vertex coefficient function**
- fields
- matching arguments

Diagrammatics for the Action

- S has an expansion in terms of fields

$$S = \frac{1}{2} S^{(2)} \varphi\varphi + \frac{1}{4!} S^{(4)} \varphi\varphi\varphi\varphi + \dots$$

- symmetry factor
- vertex coefficient function
- **fields**
- matching arguments

Diagrammatics for the Action

- S has an expansion in terms of fields

$$S = \frac{1}{2} S^{(2)} \varphi\varphi + \frac{1}{4!} S^{(4)} \varphi\varphi\varphi\varphi + \dots$$

- symmetry factor
- vertex coefficient function
- fields
- **matching arguments**

Diagrammatics for the Action

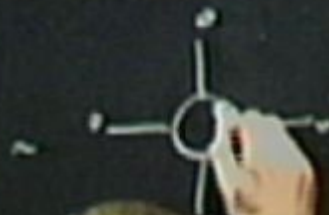
- S has an expansion in terms of fields

$$S = \frac{1}{2} S^{(2)} \varphi\varphi + \frac{1}{4!} S^{(4)} \varphi\varphi\varphi\varphi + \dots$$

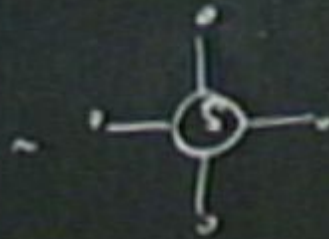
- symmetry factor
- vertex coefficient function
- fields
- matching arguments



$$\lambda \int d^D x |\varphi(x)|^4$$



$$\lambda \int d^D x |\varphi(x)|^4$$



Diagrammatics for the Flow Equation

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

$$-\Lambda \partial_\Lambda \left[\frac{1}{2} \text{---} \left(\text{---} \bigcirc \text{---} \right) \text{---} \right] =$$

(The diagram shows a red circle with a purple 'S' inside, representing a self-energy loop. It is connected to two vertical red lines, one above and one below. The left side of the circle is labeled with a red '1/2' and the right side with a red 'φφ'. The entire diagram is enclosed in large red square brackets.)

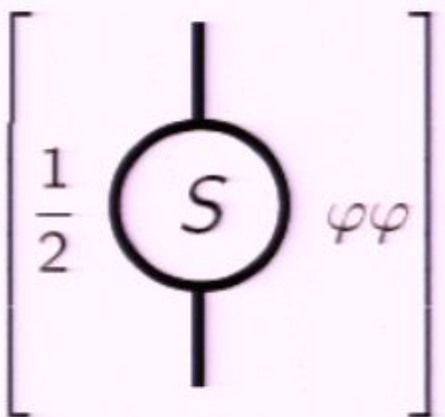
Diagrammatics for the Flow Equation

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

$$-\Lambda \partial_\Lambda \left[\frac{1}{2} \text{---} \left(\text{---} \bigcirc \text{---} \right) \text{---} \right] =$$

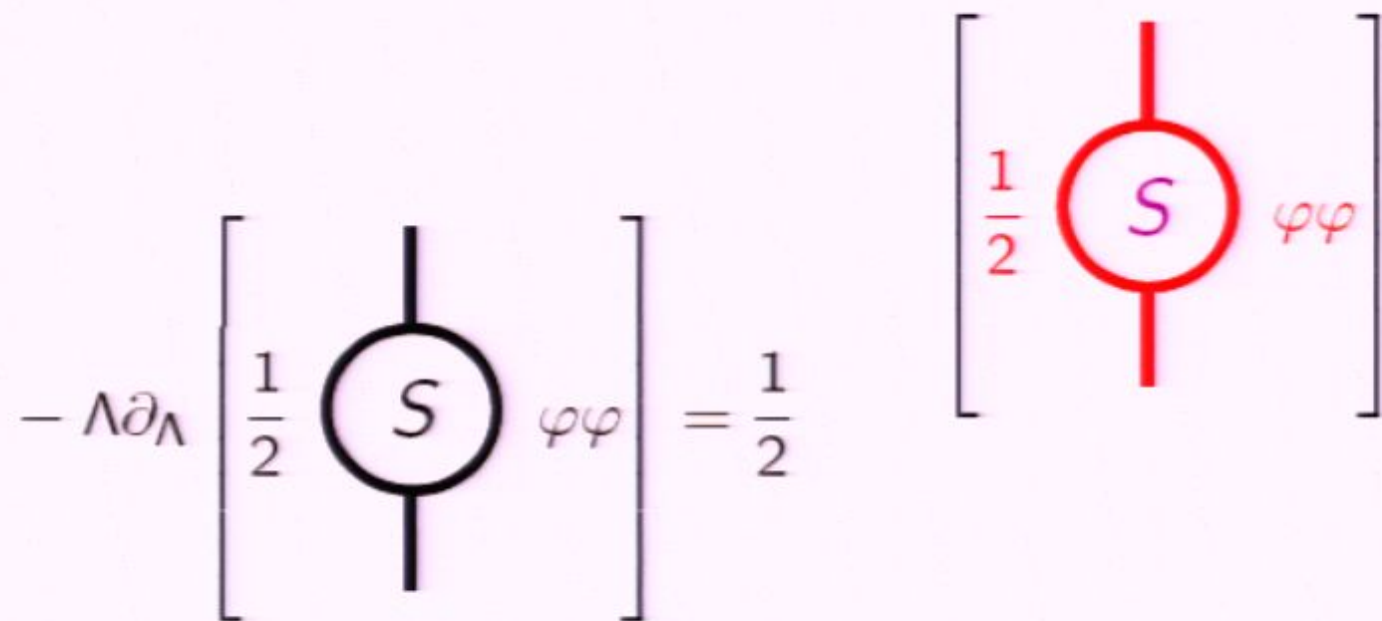
Diagrammatics for the Flow Equation

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$

$$-\Lambda \partial_\Lambda \left[\frac{1}{2} \text{S} \varphi \varphi \right] = \frac{1}{2}$$


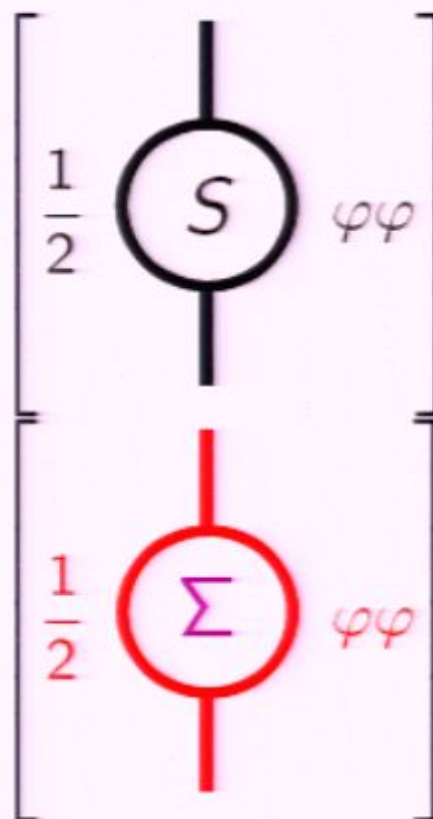
Diagrammatics for the Flow Equation

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma}{\delta \varphi}$$



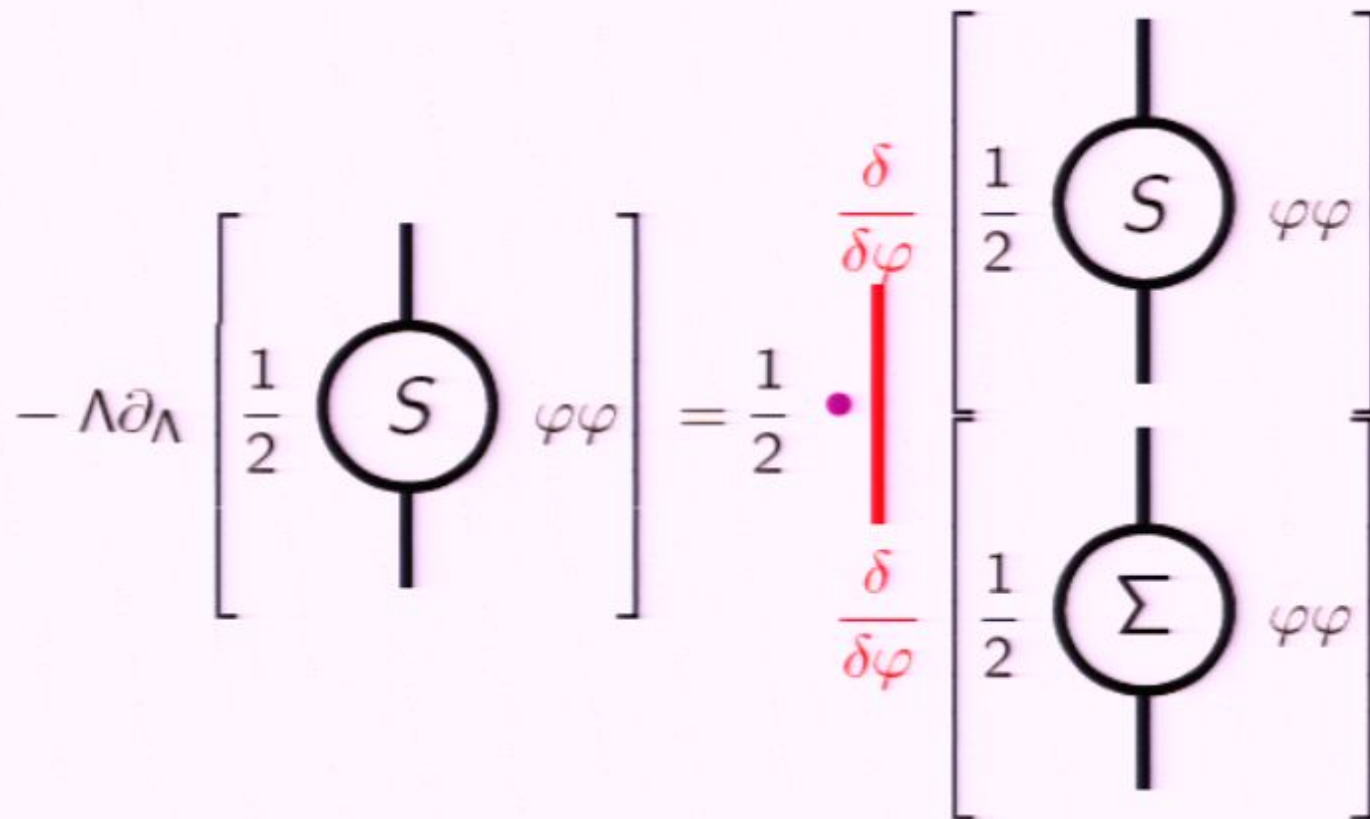
Diagrammatics for the Flow Equation

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma}{\delta \varphi}$$



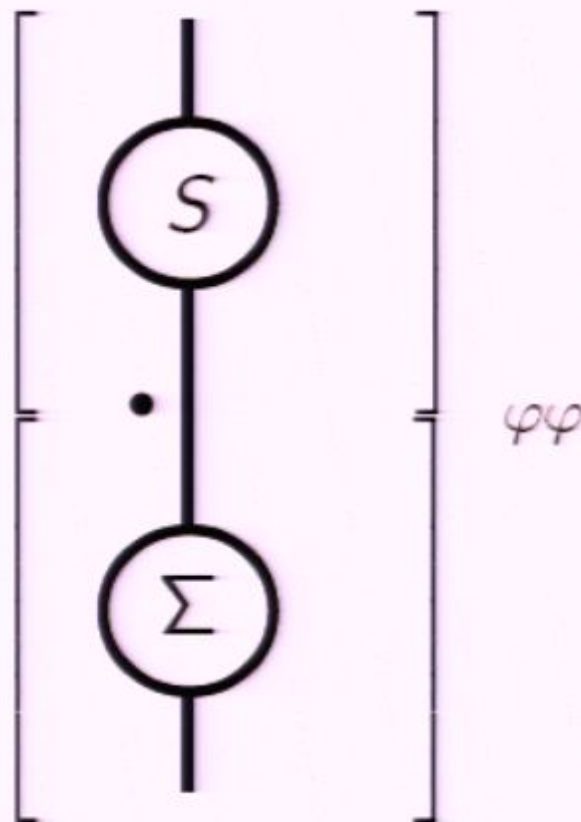
Diagrammatics for the Flow Equation

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$



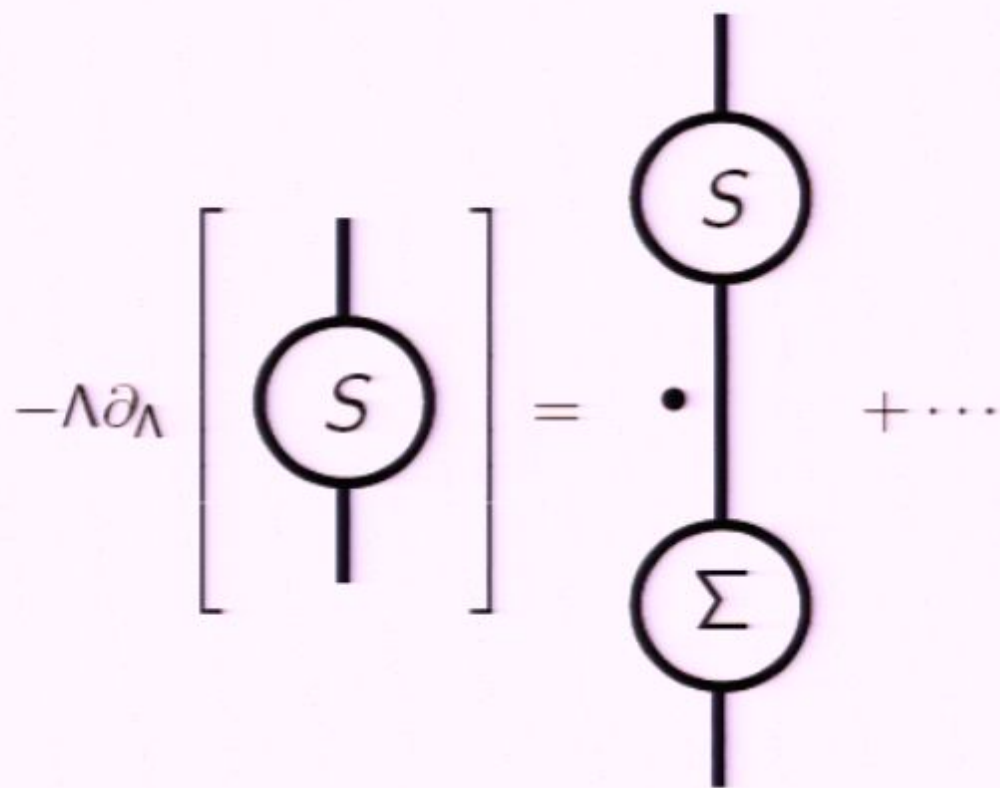
Diagrammatics for the Flow Equation

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta \Sigma}{\delta \varphi}$$



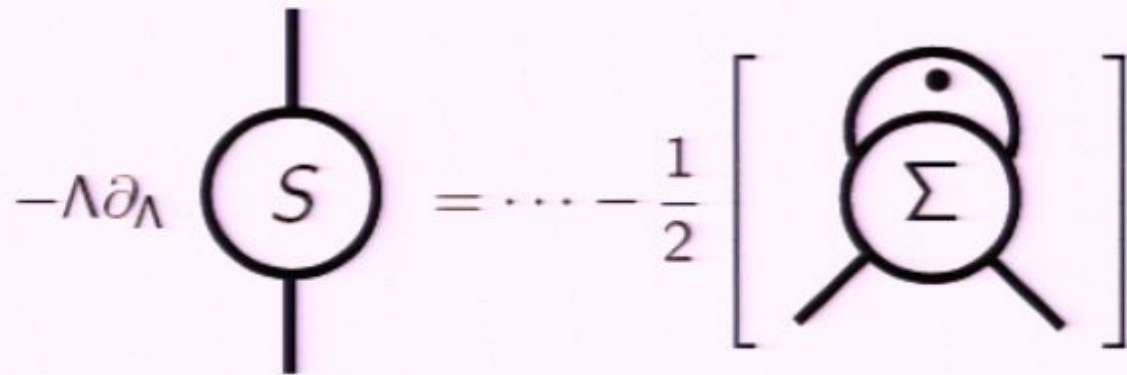
Diagrammatics for the Flow Equation

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta^2 S}{\delta \varphi^2} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$



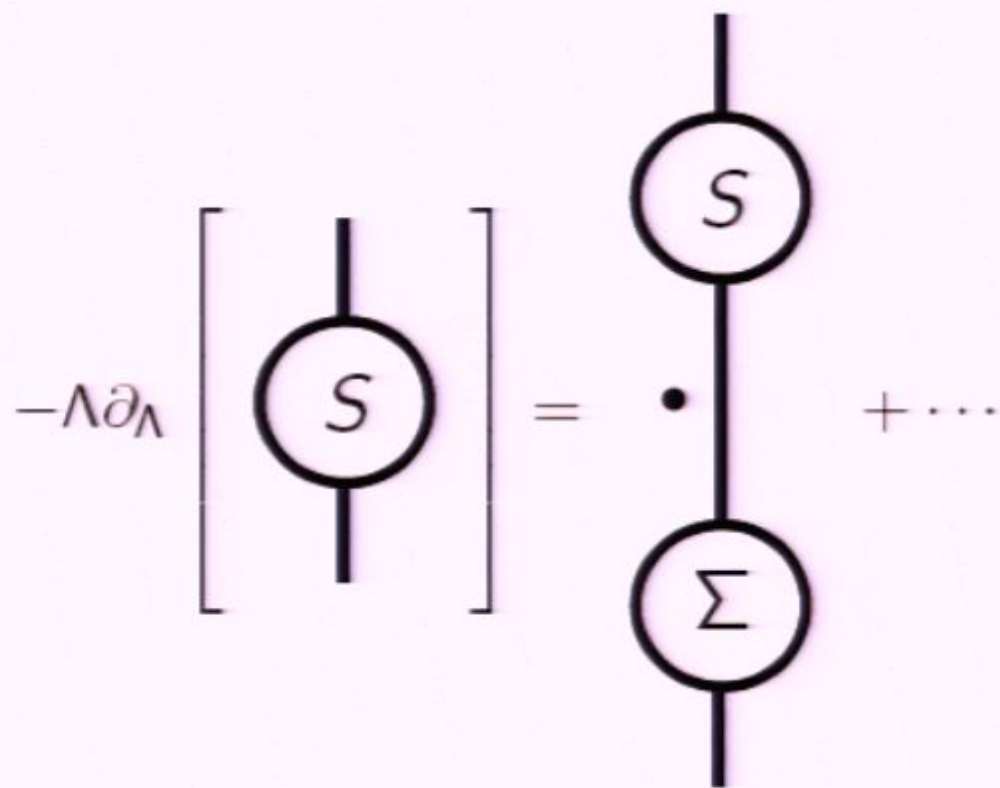
Diagrammatics for the Flow Equation

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$



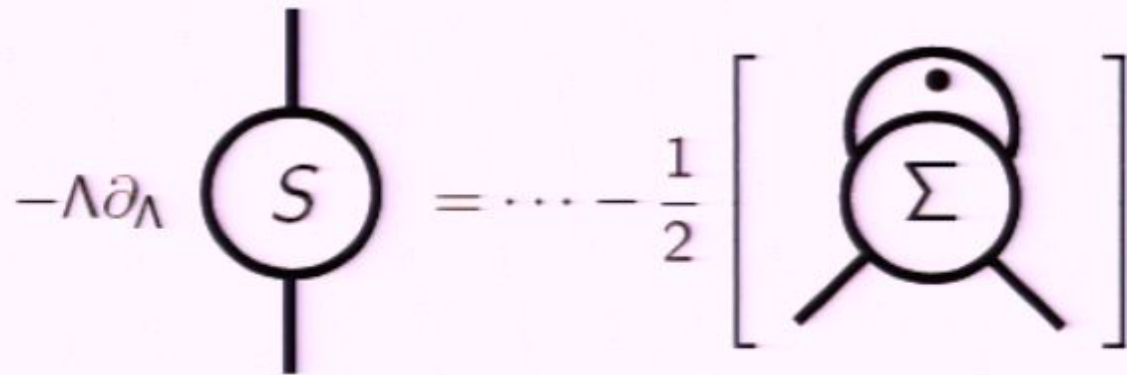
Diagrammatics for the Flow Equation

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$



Diagrammatics for the Flow Equation

$$-\Lambda \partial_\Lambda S = \frac{1}{2} \frac{\delta S}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \dot{\Delta} \cdot \frac{\delta \Sigma}{\delta \varphi}$$



The Full Diagrammatic Flow Equation

The Full Diagrammatic Flow Equation

$$\left(-\Lambda\partial_\Lambda + n\frac{\gamma}{2}\right) \left[\text{Diagram } S \right]^{(n)} = \frac{1}{2} \left[\text{Diagram } \Sigma_\lambda \text{ --- } S - \text{Diagram } \Sigma_\lambda \text{ with bubble} \right]^{(n)}$$

The diagrammatic equation shows the flow of the n -loop diagram S . On the left, the operator $(-\Lambda\partial_\Lambda + n\frac{\gamma}{2})$ acts on the n -loop diagram S . On the right, the flow is given by $\frac{1}{2}$ times the difference of two diagrams: a diagram with a Σ_λ loop and a S loop connected by a vertical line with a central dot, and a diagram with a Σ_λ loop and a bubble (a loop with a central dot) attached to it.

The Full Diagrammatic Flow Equation

$$\left(-\Lambda\partial_\Lambda + n\frac{\gamma}{2}\right) \left[\text{Diagram } S \right]^{(n)} = \frac{1}{2} \left[\begin{array}{c} \text{Diagram } \Sigma_\lambda \\ | \\ \text{Diagram } S \end{array} - \text{Diagram } \Sigma_\lambda \right]^{(n)}$$

The diagrammatic equation shows the flow of the n -point function S . On the left, the operator $(-\Lambda\partial_\Lambda + n\frac{\gamma}{2})$ acts on the n -point function S . On the right, the flow is represented by a sum of two diagrams: a tree-level diagram with a Σ_λ vertex connected to an S vertex, and a one-loop diagram with a Σ_λ vertex and a self-energy loop. The diagrams are enclosed in large square brackets with a superscript (n) .

- The n fields are distributed in all independent ways

The Full Diagrammatic Flow Equation

$$\left(-\Lambda\partial_\Lambda + n\frac{\gamma}{2}\right) \left[\text{Diagram } S \right]^{(n)} = \frac{1}{2} \left[\begin{array}{c} \text{Diagram } \Sigma_\lambda \\ | \\ \text{Diagram } S \end{array} - \text{Diagram } \Sigma_\lambda \right]^{(n)}$$

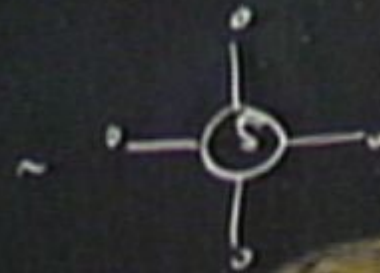
- The n fields are distributed in all independent ways
- $\Sigma_\lambda = \lambda(S - 2\hat{S})$

Flow of Reduced Vertices

Flow of Reduced Vertices

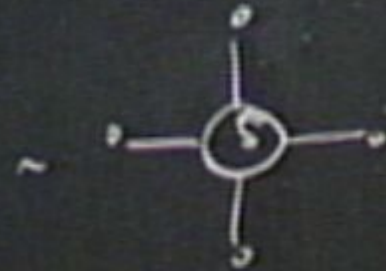
$$\begin{aligned}
 & \left(-\Lambda \partial_\Lambda + n \frac{\gamma}{2} \right) \left[\text{circle with } S^R \right]^{(n)} + \left(n \frac{\gamma}{2} + \frac{\beta}{\lambda^2} \right) \left[\text{circle with } \Delta^{-1} \delta_{n,2} \right]^{(n)} \\
 & = \left[\begin{array}{c} \text{circle with } \Sigma^R \\ | \\ \bullet \\ | \\ \text{circle with } S^R \end{array} - \begin{array}{c} \text{circle with } \hat{S}^R \\ | \\ \bullet \\ | \\ \text{circle with } \Delta^{-1} \end{array} - \frac{\lambda}{2} \begin{array}{c} \text{circle with } \bullet \\ | \\ \text{circle with } \Sigma^R \end{array} \right]^{(n)}
 \end{aligned}$$

$$\lambda \int d^n x |\varphi(x)|^4$$



$$\sim \frac{1}{\lambda} \varphi \cdot \Delta^{-1} \cdot \varphi + \dots$$

$$\lambda \int d^D x |\varphi(x)|^4$$



$$\Lambda_{2,1} \left(\frac{1}{\lambda} \right) \varphi \cdot \Delta^{-1} \cdot \varphi + \dots$$

$$\Lambda_{2,1} \equiv \beta$$

$$P_{\alpha} : -\Lambda \partial_{\lambda} S^{2nd} = \frac{1}{2} \frac{\delta S^{2nd}}{\delta \varphi} \cdot \Delta \frac{\delta S^{2nd}}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta S^{2nd}}{\delta \varphi}$$

$$\Delta_{\alpha} = \frac{C_{UV}(k, \Lambda)}{P^{\alpha}}$$

$$\dot{\Delta} = -\Lambda \partial_{\lambda} \Delta$$

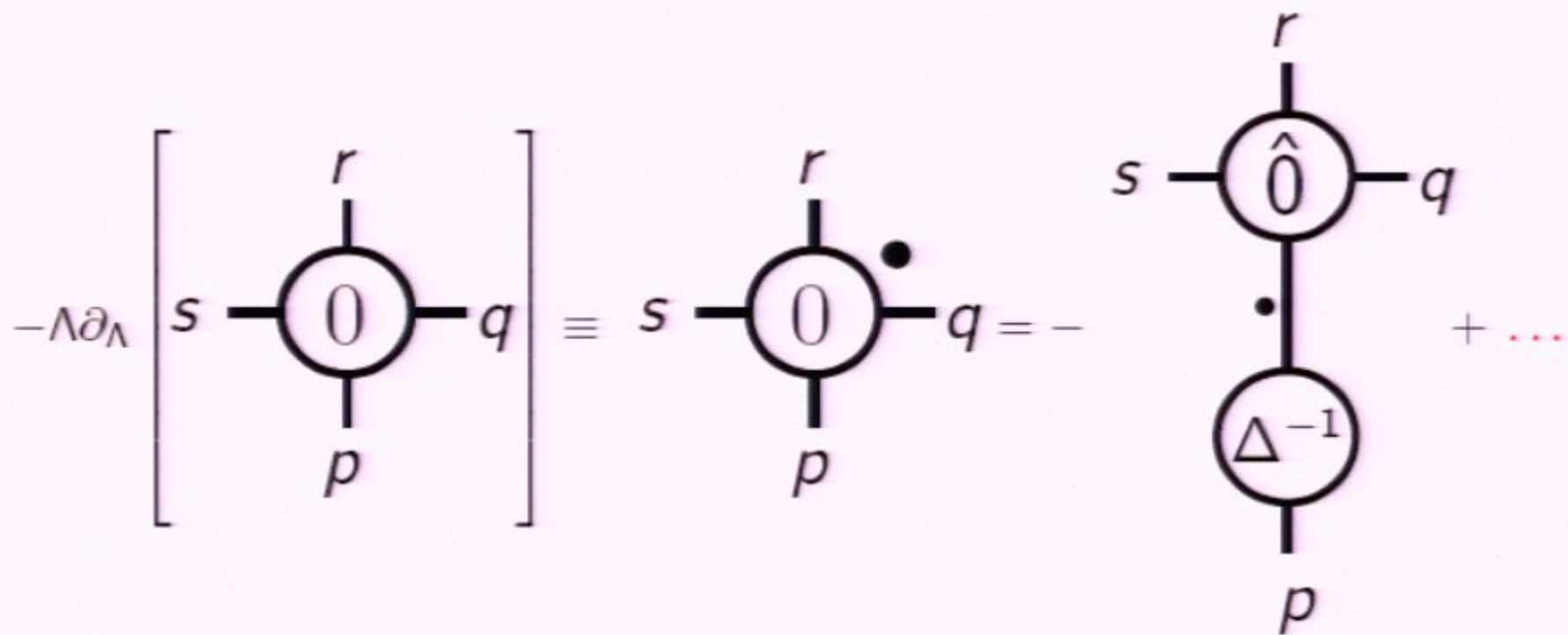
$$\Sigma = S - 2\hat{S} - \Lambda \partial_{\lambda} S$$

$$-\Lambda \partial_{\lambda} S^2 = \frac{1}{2} \frac{\delta S^2}{\delta \varphi} - 4 \cdot \Delta \cdot \Delta \cdot \frac{\delta S^2}{\delta \varphi} - \frac{1}{2} \frac{\delta}{\delta \varphi} \cdot \Delta \cdot \frac{\delta S^2}{\delta \varphi}$$

Flow of Reduced Vertices

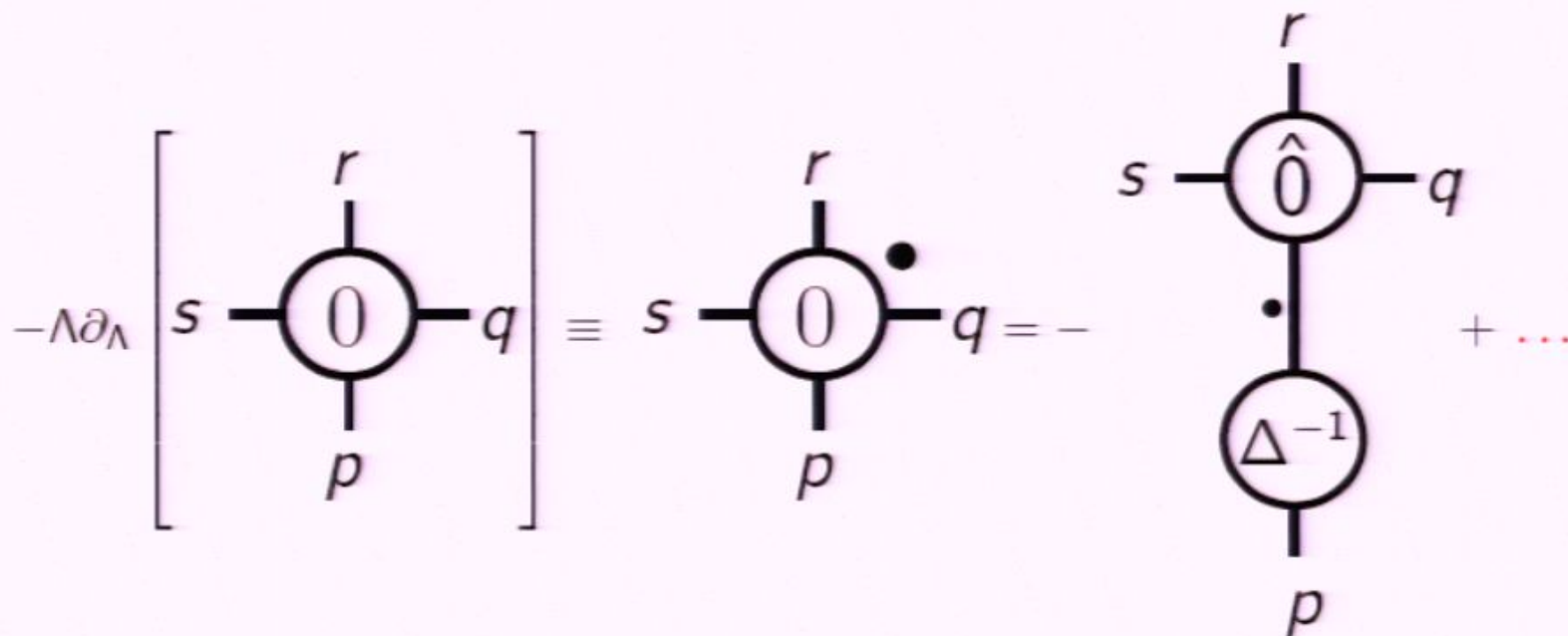
$$\begin{aligned}
 & \left(-\Lambda \partial_\Lambda + n \frac{\gamma}{2} \right) \left[\text{circle with } S^R \right]^{(n)} + \left(n \frac{\gamma}{2} + \frac{\beta}{\lambda^2} \right) \left[\text{circle with } \Delta^{-1} \delta_{n,2} \right]^{(n)} \\
 & = \left[\begin{array}{c} \text{circle with } \Sigma^R \\ | \\ \bullet \\ | \\ \text{circle with } S^R \end{array} - \begin{array}{c} \text{circle with } \hat{S}^R \\ | \\ \bullet \\ | \\ \text{circle with } \Delta^{-1} \end{array} - \frac{\lambda}{2} \begin{array}{c} \text{circle with } \bullet \\ | \\ \text{circle with } \Sigma^R \end{array} \right]^{(n)}
 \end{aligned}$$

Example: Classical 4pt Vertex



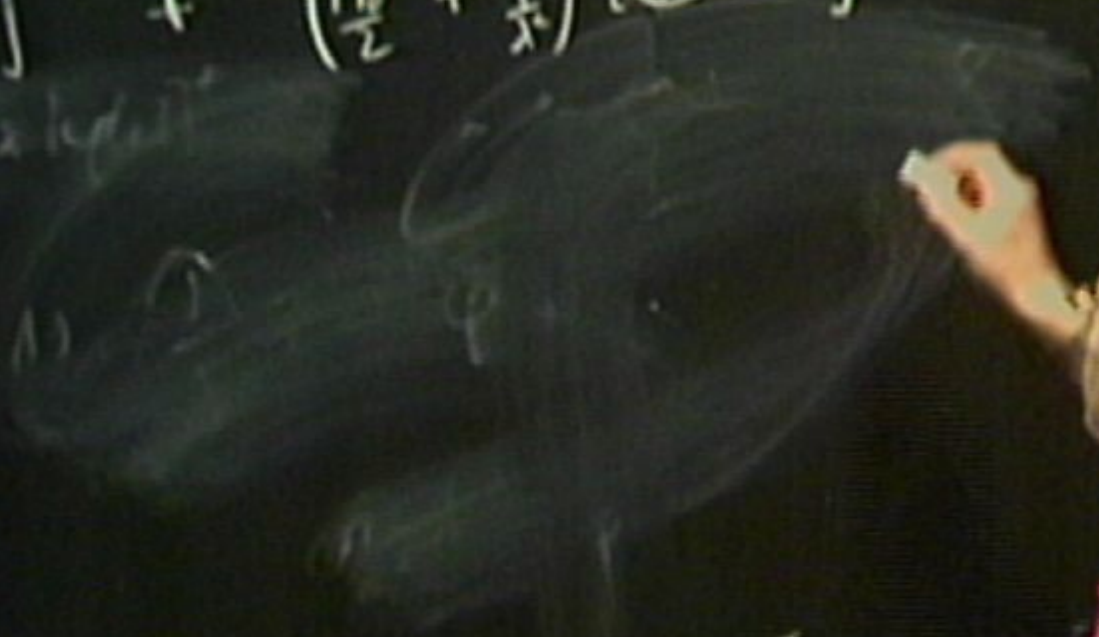
- diagrams with q, r, s on bottom vertex

Example: Classical 4pt Vertex

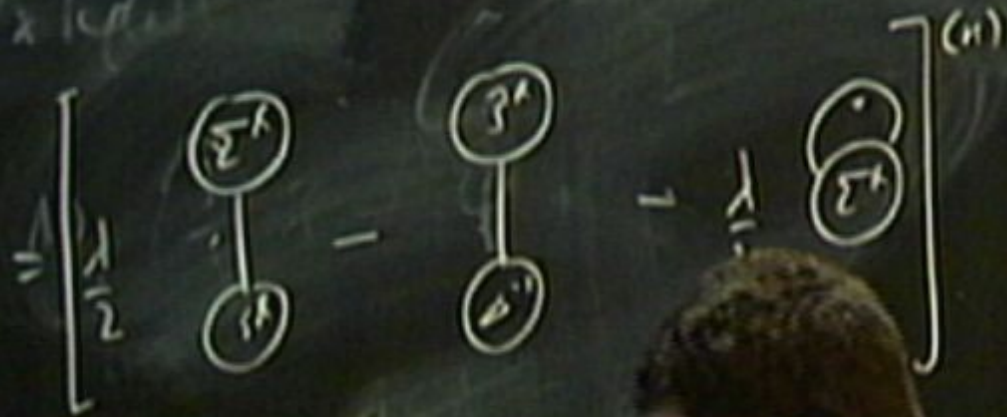


- diagrams with q, r, s on bottom vertex

$$\left(-\lambda_1 + \frac{n\gamma}{2}\right) \left[\begin{pmatrix} 1 \\ \delta^k \end{pmatrix}\right]^{(n)} + \left(\frac{n\gamma}{2} + \frac{\beta}{\lambda^2}\right) \left[\begin{pmatrix} 0 \\ \delta^m \end{pmatrix}\right] \delta_{nk}$$



$$\left(-\lambda_1 + \frac{n\gamma}{2}\right) \left[\begin{array}{c} \textcircled{1} \\ \textcircled{2} \end{array} \right]^{(n)} + \left(\frac{n\gamma}{2} + \frac{\beta}{\lambda_2}\right) \left[\begin{array}{c} \textcircled{1} \\ \textcircled{2} \end{array} \delta_{n2} \right]^{(n)}$$



$$\left(-\lambda_1 + \frac{n\gamma}{2}\right) \left[\begin{array}{c} \textcircled{\gamma^k} \\ \textcircled{\delta^k} \end{array} \right]^{(n)} + \left(\frac{n\gamma}{2} + \frac{\beta}{\lambda_1}\right) \left[\begin{array}{c} \textcircled{\delta^k} \\ \textcircled{\delta_{n-k}} \end{array} \right]^{(n)}$$

$$= \left[\begin{array}{c} \textcircled{\gamma^k} \\ \textcircled{\delta^k} \end{array} - \textcircled{\delta^k} - \textcircled{\delta^k} - \textcircled{\delta^k} \right]^{(n)}$$

$$\left(-\lambda_{11} + \frac{n\gamma}{2}\right) \left[\begin{array}{c} \textcircled{5} \\ \textcircled{4} \end{array} \right]^{(n)} + \left(\frac{n\gamma}{2} + \frac{\beta}{\lambda_{12}}\right) \left[\begin{array}{c} \textcircled{5} \\ \textcircled{4} \end{array} \right]^{(n)}$$



Example: Classical 6pt Vertex

Abstract Notation

$$\left[\overset{\bullet}{\circlearrowleft} 0^R \right]^{(6)} = \left[\frac{1}{2} \overset{\bullet}{\circlearrowleft} \Sigma_0^R \overset{\bullet}{\circlearrowleft} 0^R - \overset{\bullet}{\circlearrowleft} \hat{0}^R \overset{\bullet}{\circlearrowleft} \Delta^{-1} \right]^{(6)}$$

Example: Classical 6pt Vertex

Abstract Notation

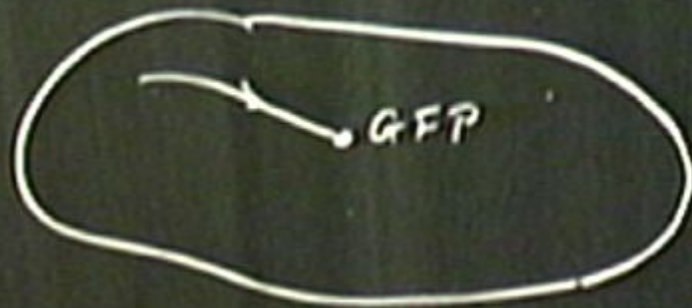
$$\left[\overset{\bullet}{\circlearrowleft} 0^R \right]^{(6)} = \left[\frac{1}{2} \overset{\bullet}{\text{---}} \begin{array}{c} \Sigma_0^R \\ \circlearrowleft 0^R \end{array} - \overset{\bullet}{\text{---}} \begin{array}{c} \hat{0}^R \\ \circlearrowleft \Delta^{-1} \end{array} \right]^{(6)}$$

$$\left(-\lambda \frac{\partial}{\partial \lambda} + n\gamma\right) \left[\begin{array}{c} \circ \\ \gamma^k \end{array} \right]^{(n)} + \left(\frac{n\gamma}{2} + \frac{\beta}{\lambda^2}\right) \left[\begin{array}{c} \circ \\ \delta_{n2} \end{array} \right]^{(n)}$$

$$= \left[\begin{array}{c} \frac{\gamma^k}{2} \\ \circ \\ \gamma^k \end{array} \right]^{(n)} - \left[\begin{array}{c} \gamma^k \\ \circ \\ \delta_{n2} \end{array} \right]^{(n)} - \frac{\gamma^k}{2} \left[\begin{array}{c} \circ \\ \gamma^k \end{array} \right]^{(n)}$$

$$\left[\begin{array}{c} \circ \\ \delta_{n2} \end{array} \right]^{(n)} = \left[\begin{array}{c} \frac{\gamma^k}{2} \\ \circ \\ \delta_{n2} \end{array} \right]^{(n)} - \left[\begin{array}{c} \gamma^k \\ \circ \\ \delta_{n2} \end{array} \right]^{(n)}$$





$\lambda(\lambda)$

$\gamma(\lambda)$

λ/λ

Renormalization Conditions

Two-Point Vertex

- Canonical normalization of the kinetic term
- $S^{(2)}(p) = p^2 + \sigma(\lambda)\Lambda^2 + O(p^4/\Lambda^2)$
- Unit coefficient
- The mass is implicitly set to zero

ERG Kernel

Renormalization Conditions

Two-Point Vertex

- Canonical normalization of the kinetic term
- $S^{(2)}(p) = p^2 + \sigma(\lambda)\Lambda^2 + O(p^4/\Lambda^2)$
- Unit coefficient
- The mass is implicitly set to zero

ERG Kernel

Renormalization Conditions

Two-Point Vertex

- Canonical normalization of the kinetic term
- $S^{(2)}(p) = p^2 + \sigma(\lambda)\Lambda^2 + O(p^4/\Lambda^2)$
- Unit coefficient
- The mass is implicitly set to zero

ERG Kernel

Renormalization Conditions

Two-Point Vertex

- Canonical normalization of the kinetic term
- $S^{(2)}(p) = p^2 + \sigma(\lambda)\Lambda^2 + O(p^4/\Lambda^2)$
- Unit coefficient
- The mass is implicitly set to zero

ERG Kernel



$$\frac{1}{2} \varphi \cdot p^2 \cdot \varphi + O(p^4)$$

$\lambda(\lambda)$
 $\gamma(\lambda)$
 ~~$\lambda(\lambda)$~~



Renormalization Conditions

Two-Point Vertex

- Canonical normalization of the kinetic term
- $S^{(2)}(p) = p^2 + \sigma(\lambda)\Lambda^2 + O(p^4/\Lambda^2)$
- Unit coefficient
- The mass is implicitly set to zero
 - There is no mass scale
 - Only Λ appears, which tends to zero as all modes are integrated out
 - $\sigma(\lambda)$ starts at one-loop and is determined self-consistently

ERG Kernel

Renormalization Conditions

Two-Point Vertex

- Canonical normalization of the kinetic term
- $S^{(2)}(p) = p^2 + \sigma(\lambda)\Lambda^2 + O(p^4/\Lambda^2)$
- Unit coefficient
- The mass is implicitly set to zero
 - There is no mass scale
 - Only Λ appears, which tends to zero as all modes are integrated out
 - $\sigma(\lambda)$ starts at one-loop and is determined self-consistently

ERG Kernel

Renormalization Conditions

Two-Point Vertex

- Canonical normalization of the kinetic term
- $S^{(2)}(p) = p^2 + \sigma(\lambda)\Lambda^2 + O(p^4/\Lambda^2)$
- Unit coefficient
- The mass is implicitly set to zero
 - There is no mass scale
 - Only Λ appears, which tends to zero as all modes are integrated out
 - $\sigma(\lambda)$ starts at one-loop and is determined self-consistently

ERG Kernel

- Choose $\Delta^{-1}(p) = p^2 + O(p^4/\Lambda^2)$
- $\Rightarrow S^{R(2)}(p) \Big|_{p^2} = 0$

Renormalization Conditions

Two-Point Vertex

- Canonical normalization of the kinetic term
- $S^{(2)}(p) = p^2 + \sigma(\lambda)\Lambda^2 + O(p^4/\Lambda^2)$
- Unit coefficient
- The mass is implicitly set to zero
 - There is no mass scale
 - Only Λ appears, which tends to zero as all modes are integrated out
 - $\sigma(\lambda)$ starts at one-loop and is determined self-consistently

ERG Kernel

- **Choose** $\Delta^{-1}(p) = p^2 + O(p^4/\Lambda^2)$
- $\Rightarrow S^{R(2)}(p)|_{p^2} = 0$

Renormalization Conditions

Two-Point Vertex

- Canonical normalization of the kinetic term
- $S^{(2)}(p) = p^2 + \sigma(\lambda)\Lambda^2 + O(p^4/\Lambda^2)$
- Unit coefficient
- The mass is implicitly set to zero
 - There is no mass scale
 - Only Λ appears, which tends to zero as all modes are integrated out
 - $\sigma(\lambda)$ starts at one-loop and is determined self-consistently

ERG Kernel

- Choose $\Delta^{-1}(p) = p^2 + O(p^4/\Lambda^2)$
- $\Rightarrow S^{R(2)}(p)\Big|_{p^2} = 0$



$$\frac{1}{2} \varphi \cdot p^2 \cdot \varphi + O(\varphi^3)$$

$\lambda(\lambda)$
 $\gamma(\lambda)$
 ~~$\lambda(\lambda)$~~

$$S = \frac{1}{2} \varphi \cdot \sigma^i \cdot \varphi + \textcircled{S^R}$$

p^2



Renormalization Conditions

Four-Point Vertex

- Remember that we have
 - rescaled $\varphi \rightarrow \varphi/\sqrt{\lambda}$
 - pulled $1/\lambda$ outside the action
- $S^{(4)}(\underline{0}, \Lambda) = 1$
- This condition is saturated at the classical level:



$$\frac{1}{2} \varphi \cdot p^2 \cdot \varphi + O(p^4)$$

$\lambda(\lambda)$
 $\lambda(\lambda)$
 ~~$\lambda(\lambda)$~~

$$S = \frac{1}{2} \varphi \cdot \square \cdot \varphi + (S^R)$$

p^2



Renormalization Conditions

Four-Point Vertex

- Remember that we have
 - rescaled $\varphi \rightarrow \varphi/\sqrt{\lambda}$
 - pulled $1/\lambda$ outside the action
- $S^{(4)}(\underline{0}, \Lambda) = 1$
- This condition is saturated at the classical level:
 - $S = \frac{1}{\lambda} S_0 + S_1 + \lambda S_2 + \dots$
 - $S_0^{(4)}(\underline{0}, \Lambda) = 1$
 - $S_{>0}^{(4)}(\underline{0}, \Lambda) = 0$

Renormalization Conditions

Four-Point Vertex

- Remember that we have
 - rescaled $\varphi \rightarrow \varphi/\sqrt{\lambda}$
 - pulled $1/\lambda$ outside the action
- $S^{(4)}(\underline{0}, \Lambda) = 1$
- This condition is saturated at the classical level:
 - $S = \frac{1}{\lambda} S_0 + S_1 + \lambda S_2 + \dots$
 - $S_0^{(4)}(\underline{0}, \Lambda) = 1$
 - $S_{>0}^{(4)}(\underline{0}, \Lambda) = 0$

Renormalization Conditions

Four-Point Vertex

- Remember that we have
 - rescaled $\varphi \rightarrow \varphi/\sqrt{\lambda}$
 - pulled $1/\lambda$ outside the action
- $S^{(4)}(\underline{0}, \Lambda) = 1$
- This condition is saturated at the classical level:
 - $S = \frac{1}{\lambda} S_0 + S_1 + \lambda S_2 + \dots$
 - $S_0^{(4)}(\underline{0}, \Lambda) = 1$
 - $S_{>0}^{(4)}(\underline{0}, \Lambda) = 0$

Flow of the 1-loop, Two-Point Vertex

Flow of the 1-loop, Two-Point Vertex

$$\begin{aligned}
 & \left(\text{Diagram: circle with } 1^R \text{ and a dot} \right) + (\beta_1 + \gamma_1) \left[\text{Diagram: circle with } 0^R \text{ and a dot} + \text{Diagram: circle with } \Delta^{-1} \text{ and a dot} \right] \\
 & = 2 \left(\text{Diagram: } \Sigma_1^R \text{ on top, } 0^R \text{ on bottom, with a dot} \right) + 2 \left(\text{Diagram: } \Sigma_0^R \text{ on top, } 1^R \text{ on bottom, with a dot} \right) \\
 & \quad - 2 \left(\text{Diagram: } \hat{1}^R \text{ on top, } \Delta^{-1} \text{ on bottom, with a dot} \right) - \frac{1}{2} \left(\text{Diagram: } \Sigma_0 \text{ with a dot and two external lines} \right)
 \end{aligned}$$

Flow of the 1-loop, Two-Point Vertex

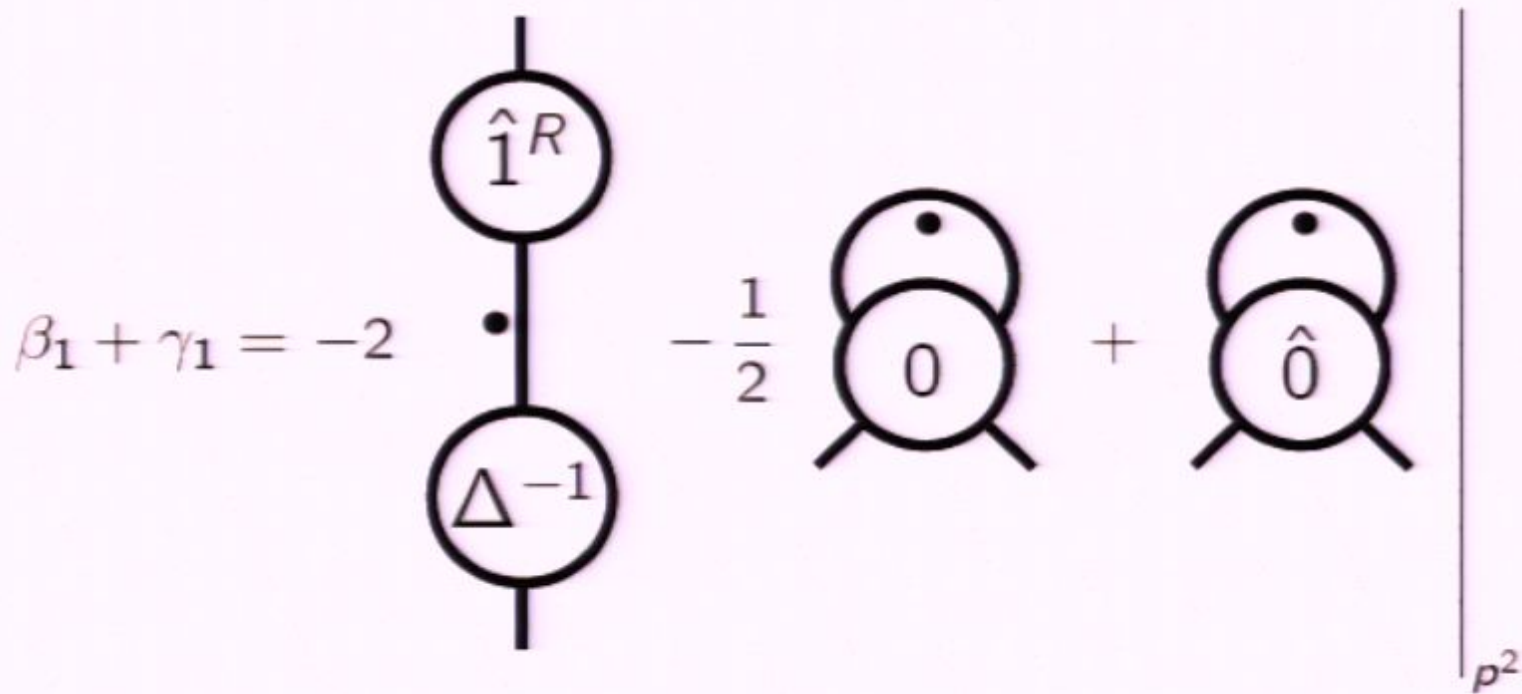
$$\begin{aligned}
 & \text{Diagram with } 1^R \text{ and a dot} + (\beta_1 + \gamma_1) \left[\text{Diagram with } 0^R + \text{Diagram with } \Delta^{-1} \right] \\
 & = 2 \cdot \left[\text{Diagram with } \Sigma_1^R \text{ and } 0^R \right] + 2 \cdot \left[\text{Diagram with } \Sigma_0^R \text{ and } 1^R \right] - 2 \cdot \left[\text{Diagram with } \hat{1}^R \text{ and } \Delta^{-1} \right] - \frac{1}{2} \cdot \left[\text{Diagram with } \Sigma_0 \text{ and a dot} \right]
 \end{aligned}$$

Flow of the 1-loop, Two-Point Vertex

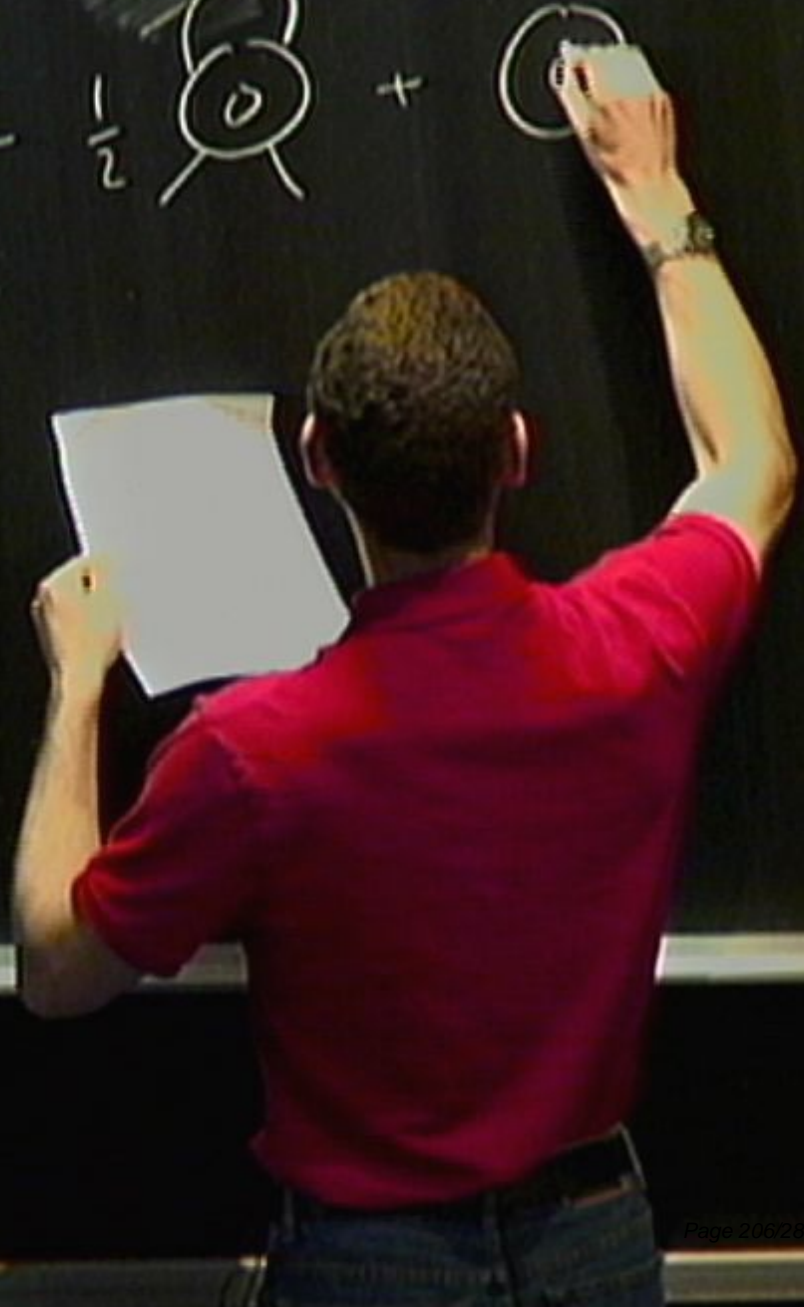
$$\begin{aligned}
 & \cancel{1^R} \bullet + (\beta_1 + \gamma_1) \left[\cancel{0^R} + \Delta^{-1} \right] \\
 & = 2 \left[\cancel{\Sigma_1^R} \bullet \right] + 2 \left[\cancel{\Sigma_0^R} \bullet \right] - 2 \left[\hat{1}^R \bullet \right] - \frac{1}{2} \left[\Sigma_0 \right]
 \end{aligned}$$

The diagrammatic equation shows the flow of the 1-loop, two-point vertex. On the left, a vertex labeled 1^R is crossed out with a red diagonal line. This is followed by a dot and the term $(\beta_1 + \gamma_1)$ multiplied by a bracketed sum of two terms: a crossed-out vertex labeled 0^R and a vertex labeled Δ^{-1} . Below this, the right-hand side is expanded into four terms: $= 2$ times a diagram with a crossed-out Σ_1^R vertex and a 0^R vertex; $+ 2$ times a diagram with a crossed-out Σ_0^R vertex and a 1^R vertex; $- 2$ times a diagram with a $\hat{1}^R$ vertex and a Δ^{-1} vertex; and $- \frac{1}{2}$ times a diagram with a Σ_0 vertex.

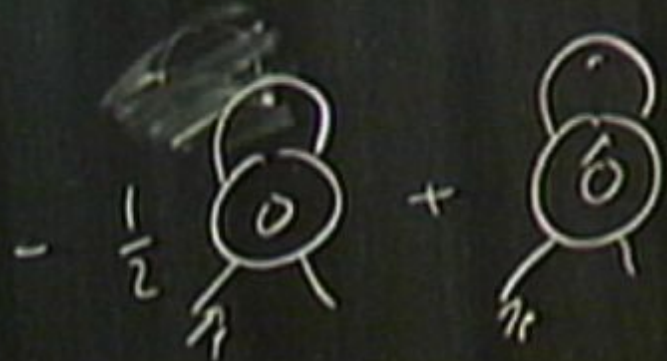
Flow of the 1-loop, Two-Point Vertex



$$\beta_1 + \delta_1 = -2$$



$$\beta_1 + \delta_1 = -2$$



p^2

Diagrammatic Fun & Games

Consider $\left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]^{(n)} - 2 \left[\begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right]^{(n)}$

$$\left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]^{(n)} = \left[\left[\begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right] \cdot \right]^{(n)} - \left[\begin{array}{c} \text{Diagram 5} \\ \text{Diagram 6} \end{array} \right]^{(n)}$$

Diagrammatic Fun & Games

Consider $\left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]^{(n)}$

Diagram 1: Two circles, one above the other. The top circle has a dot at its top. The bottom circle contains the number 0.

Diagram 2: Two circles, one above the other. The top circle has a dot at its top. The bottom circle contains the number 0 with a hat ($\hat{0}$).

$$\left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]^{(n)} = \left[\left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]^{\bullet} - \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array}^{\bullet} \right]^{(n)}$$

Diagram 1: Two circles, one above the other. The top circle has a dot at its top. The bottom circle contains the number 0.

Diagram 2: Two circles, one above the other. The top circle has a dot at its top. The bottom circle contains the number 0.

Diagram 1: Two circles, one above the other. The top circle has a dot at its top. The bottom circle contains the number 0.

Diagram 2: Two circles, one above the other. The top circle has a dot at its top. The bottom circle contains the number 0. A dot is placed on the right side of the bottom circle.

Diagrammatic Fun & Games

Consider $\left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]^{(n)} - 2 \left[\begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right]^{(n)}$

$$\left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]^{(n)} = \left[\left[\begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right] \right]^{(n)} - \left[\text{Diagram 5} \right]^{\Delta(n)}$$

Diagrammatic Fun & Games

Consider $\left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]^{(n)}$

Diagram 1: Two circles, top and bottom, with a dot on the top circle. The bottom circle contains the number 0.

Diagram 2: Two circles, top and bottom, with a dot on the top circle. The bottom circle contains the number $\hat{0}$.

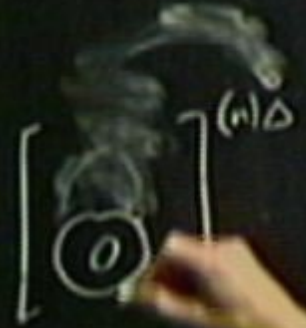
The expression is $\left[\text{Diagram 1} - 2 \text{Diagram 2} \right]^{(n)}$.

$$\left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]^{(n)} = \left[\left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]^{\bullet} - \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array}^{\bullet} \right]^{(n)}$$

Diagram 1: Two circles, top and bottom, with a dot on the top circle. The bottom circle contains the number 0.

Diagram 2: Two circles, top and bottom, with a dot on the bottom circle. The bottom circle contains the number 0.

$$\beta_1 + \delta_1 = -2$$

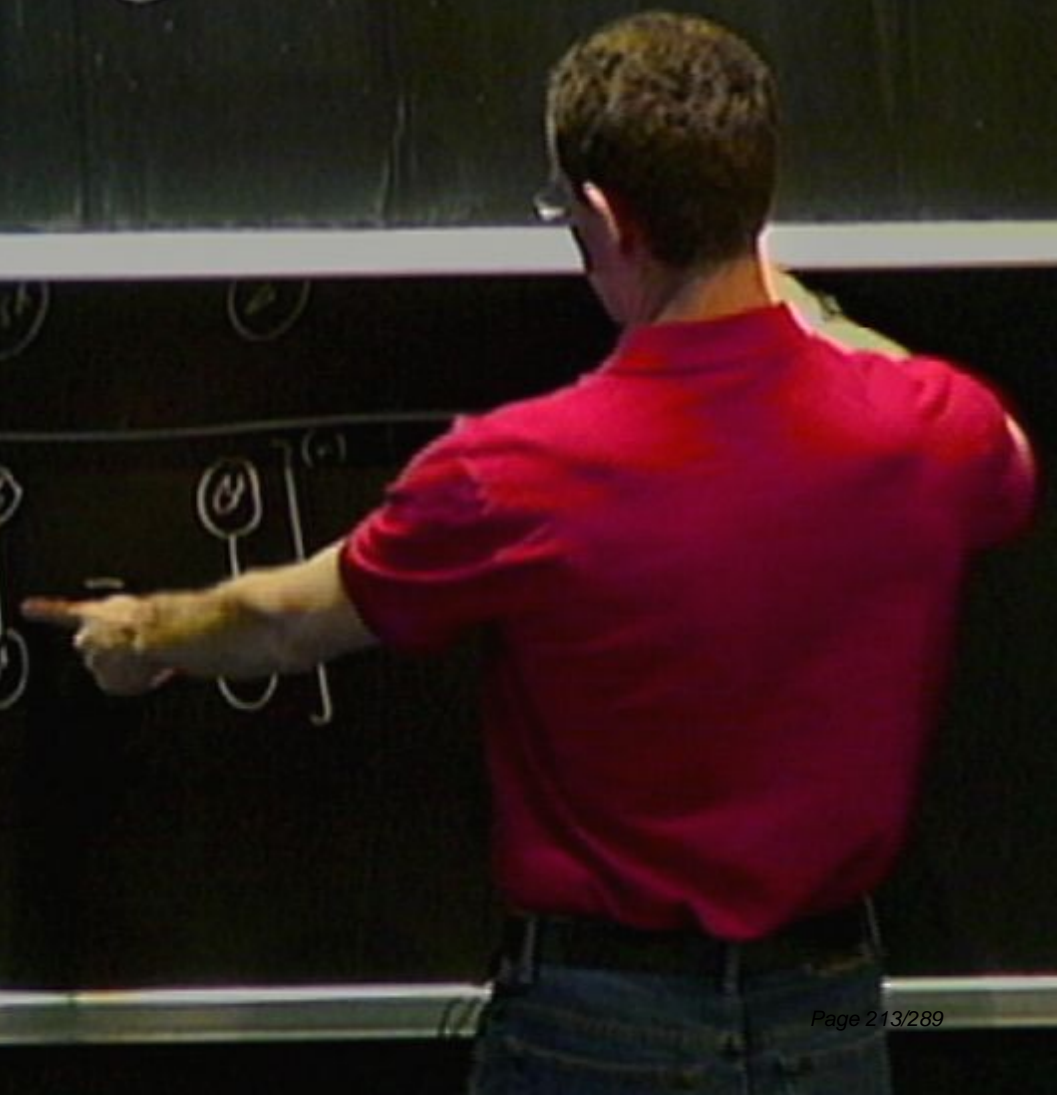


p^2

$$\begin{bmatrix} \sigma_1 \\ 0 \end{bmatrix}^{(n)} = \frac{1}{2} \begin{bmatrix} \sigma_1 \\ \sigma_1 \end{bmatrix}$$

19

$$\begin{bmatrix} \sigma_1 \\ 0 \end{bmatrix}^{(n)} = \frac{1}{2} \begin{bmatrix} \sigma_1 \\ \sigma_1 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_1 \end{bmatrix}^{(n)}$$



$$\begin{bmatrix} \sigma_0 \\ 0 \end{bmatrix}^{(n)\Delta}$$

=

$$\begin{bmatrix} \frac{1}{2} \end{bmatrix}$$



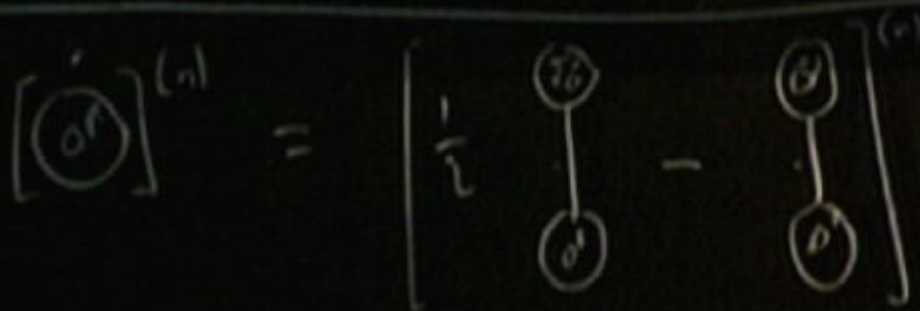
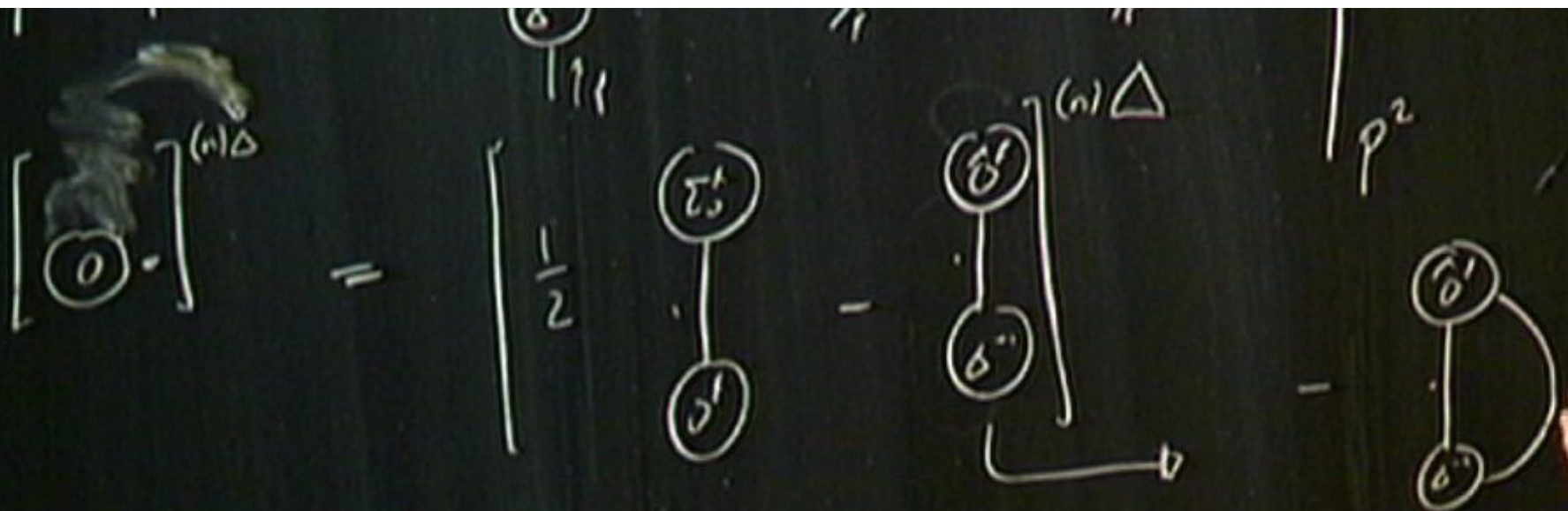
-

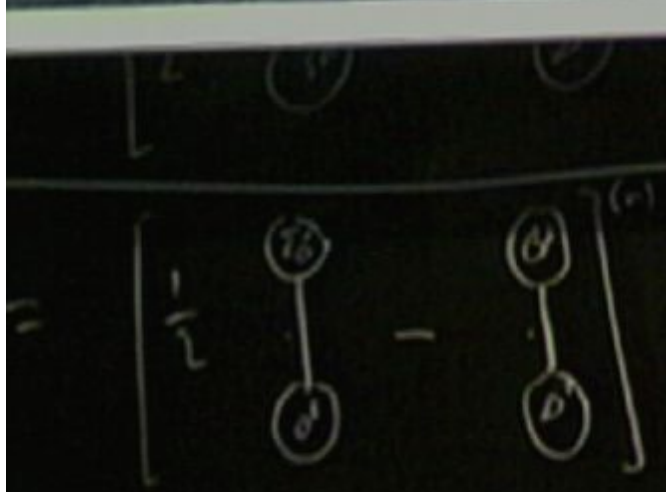
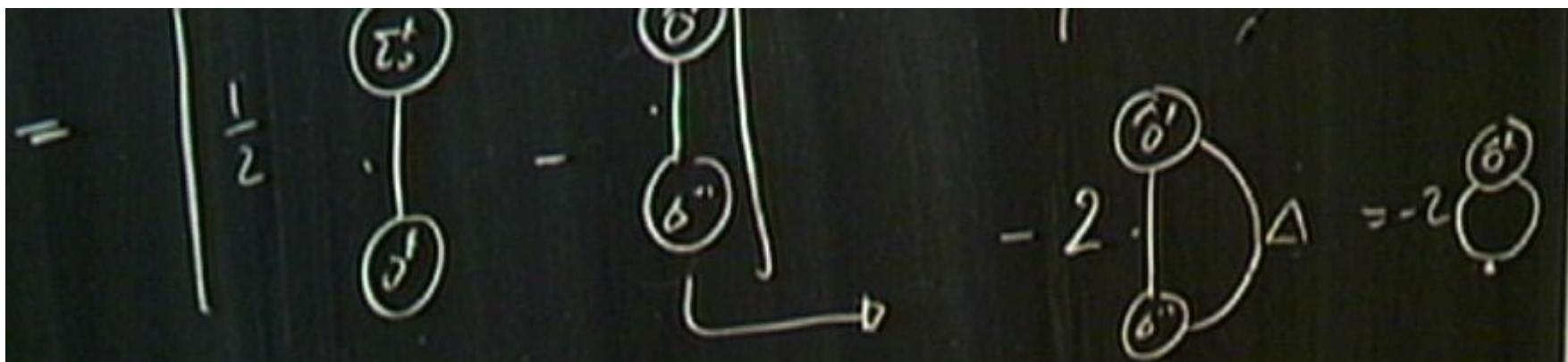


p2

$$\begin{bmatrix} \sigma_0^+ \end{bmatrix}^{(n)}$$

$$= \begin{bmatrix} \frac{1}{2} \end{bmatrix}$$





$$\begin{aligned}
 & \left[\begin{array}{c} \circ \\ \cdot \end{array} \right] = \left| \frac{1}{2} \right| \begin{array}{c} \circ \\ | \\ \circ \end{array} - \begin{array}{c} \cdot \\ | \\ \circ \end{array} \\
 & \left| \equiv -\lambda \lambda \right|
 \end{aligned}$$

$$\left[\begin{array}{c} \cdot \\ \circ \end{array} \right]^{(2)} = \left| \frac{1}{2} \right| \begin{array}{c} \circ \\ | \\ \circ \end{array} - \begin{array}{c} \circ \\ | \\ \circ \end{array}$$

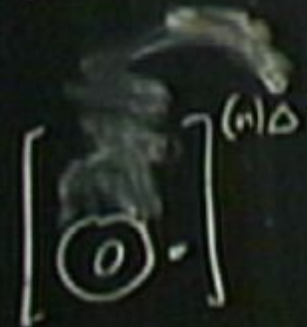
Diagrammatic Fun & Games

Consider $\left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]^{(n)} - 2 \left[\begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right]^{(n)}$

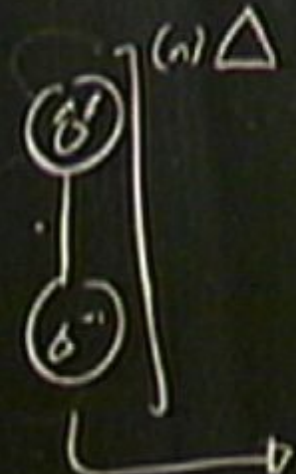
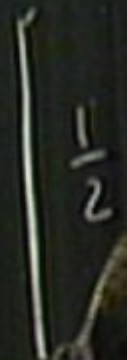
$$\left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right]^{(n)} = \left[\left[\begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} \right] \right]^{(n)} - \left[\text{Diagram 5} \right]^{\Delta(n)}$$

This is one for the blackboard!

$$|s_1 + \delta_1 = -\frac{1}{2}$$



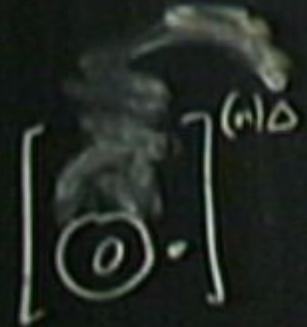
=

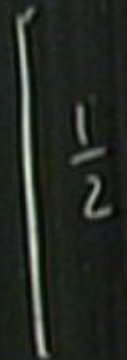


$$| \equiv -\lambda \lambda_n |$$



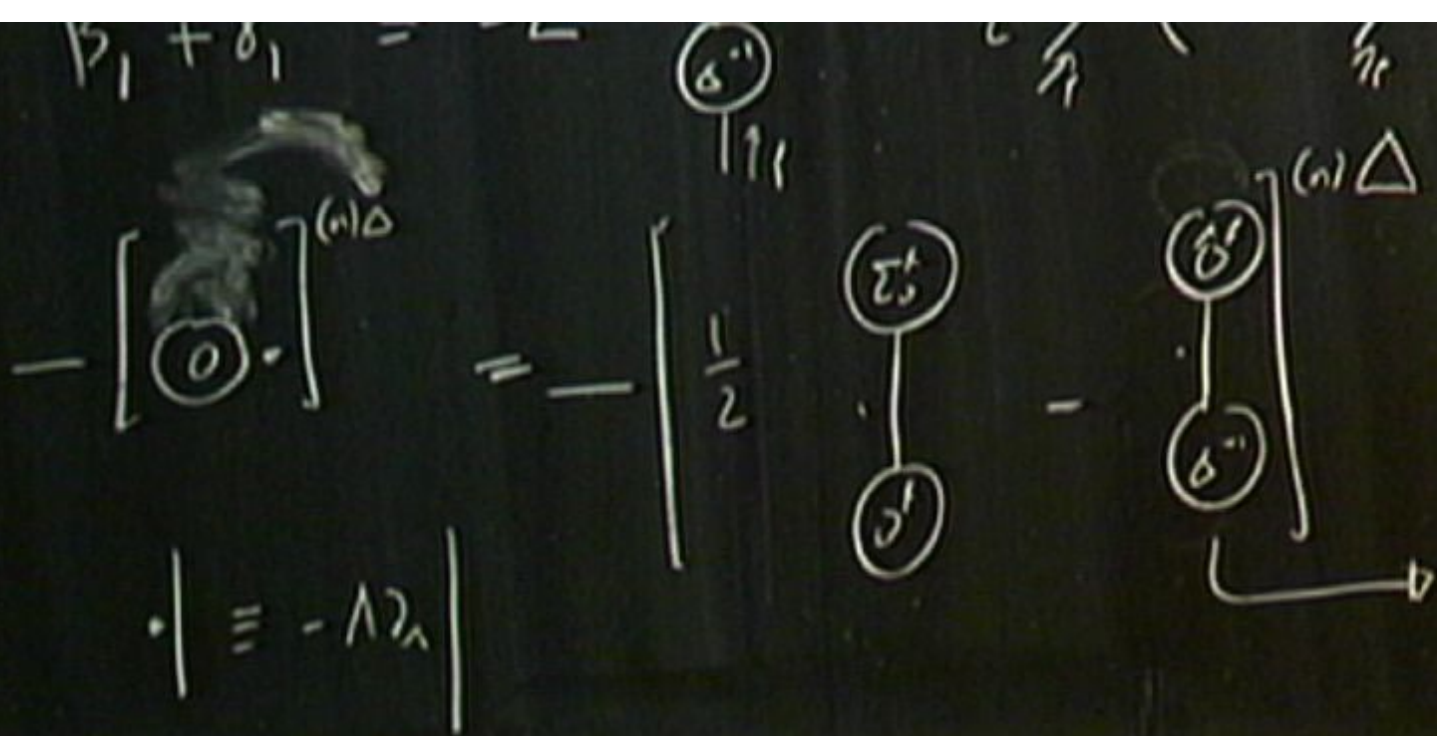
$$|s_1 + \delta_1| = \dots$$

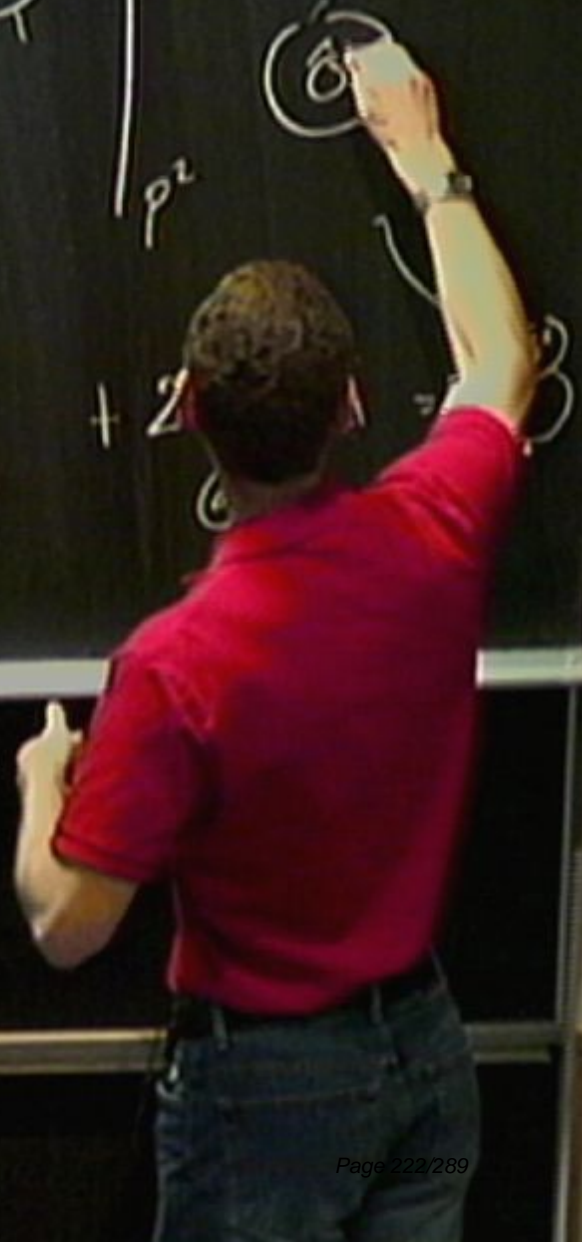
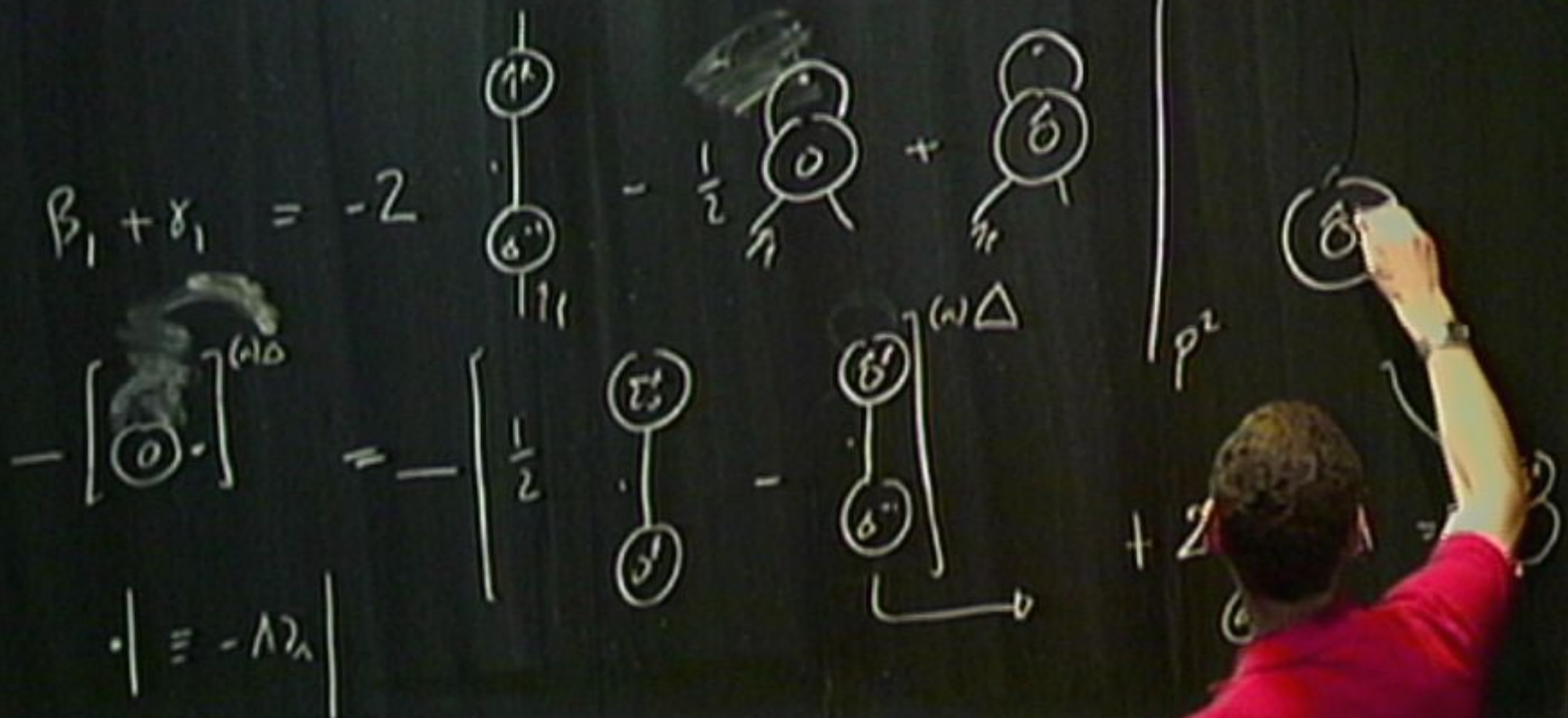


$$=$$


$$| \equiv -\lambda \lambda_n |$$





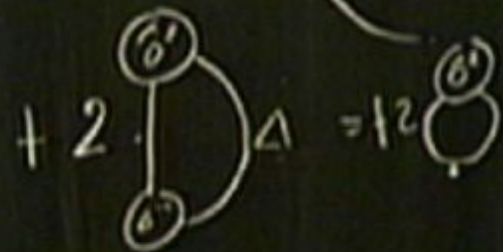
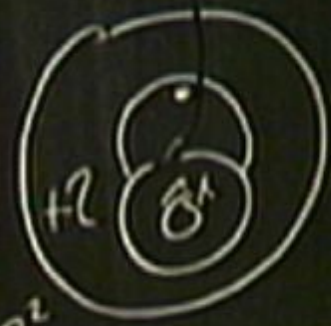
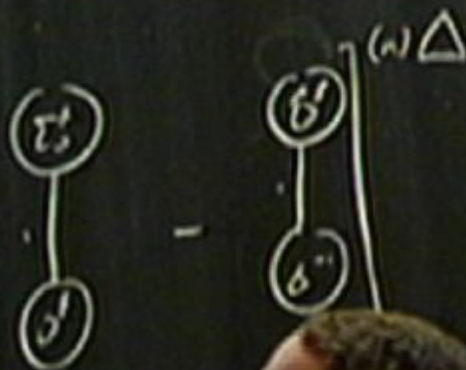


$$\beta_1 + \alpha_1 = -2$$

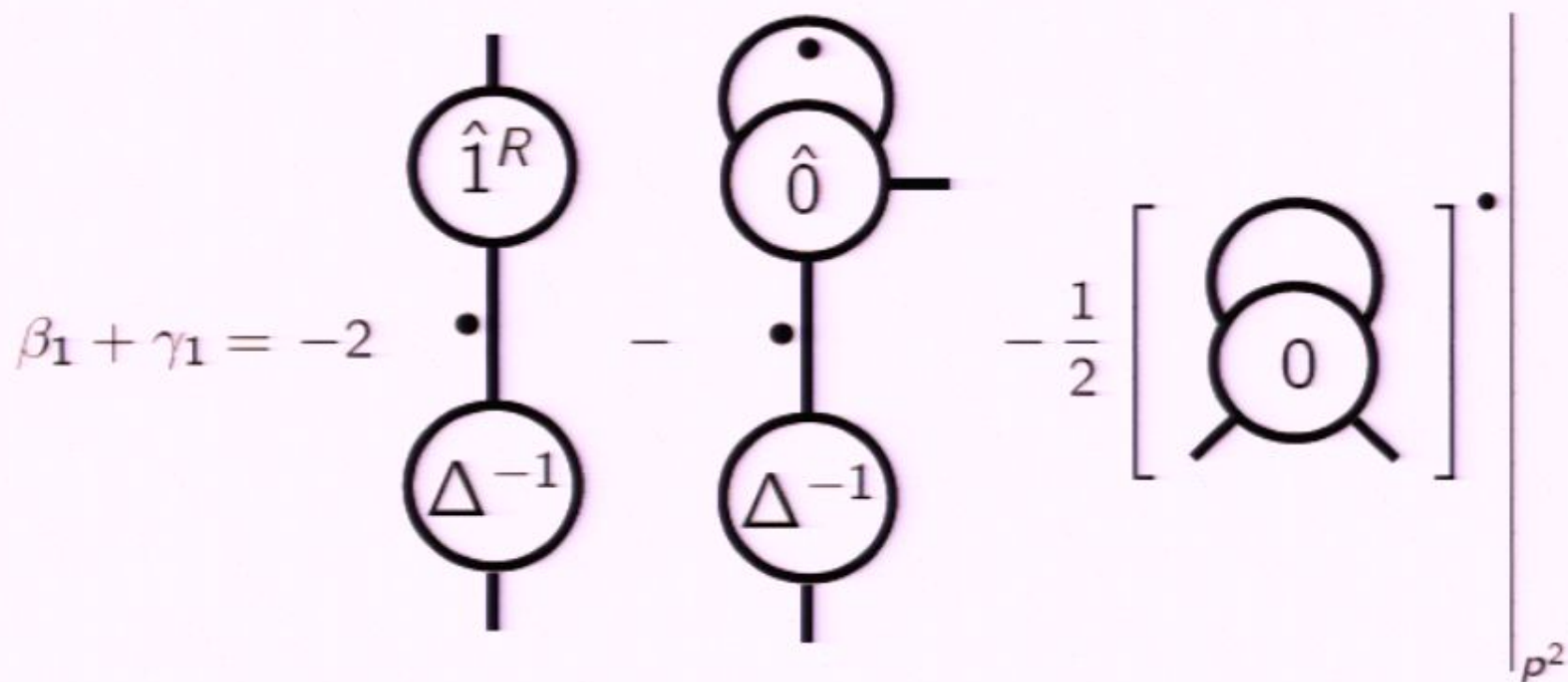
$$-\begin{bmatrix} \cdot \\ \ominus \end{bmatrix}^{(1)\Delta}$$

$$\cdot \equiv -\Lambda_1$$

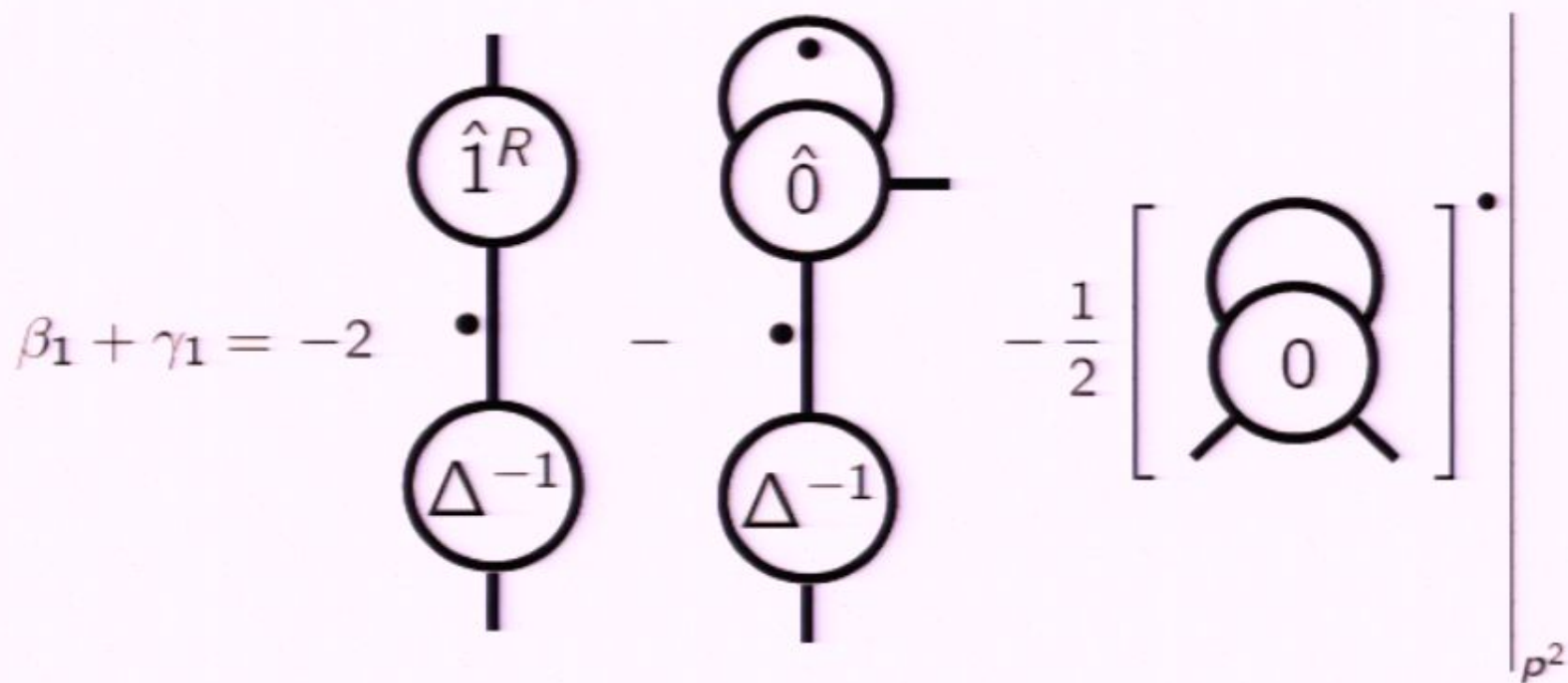
$$= -\frac{1}{2}$$



Flow of the 1-loop, Two-Point Vertex Continued...



Flow of the 1-loop, Two-Point Vertex Continued...



Flow of the Four-Point

Flow of the Four-Point

- Similarly, compute the flow of four-point vertex, but this time with all momenta set to zero
- Use the renormalization condition, $S^{(4)}(\underline{0}, \Lambda) = 1$
- Arrive at a second equation in terms of β_1 and γ_1
- Solve the two simultaneous equations for β_1

Flow of the Four-Point

- Similarly, compute the flow of four-point vertex, but this time with all momenta set to zero
- Use the renormalization condition, $S^{(4)}(\underline{0}, \Lambda) = 1$
- Arrive at a second equation in terms of β_1 and γ_1
- Solve the two simultaneous equations for β_1

Flow of the Four-Point

- Similarly, compute the flow of four-point vertex, but this time with all momenta set to zero
- Use the renormalization condition, $S^{(4)}(\underline{0}, \Lambda) = 1$
- Arrive at a second equation in terms of β_1 and γ_1
- Solve the two simultaneous equations for β_1

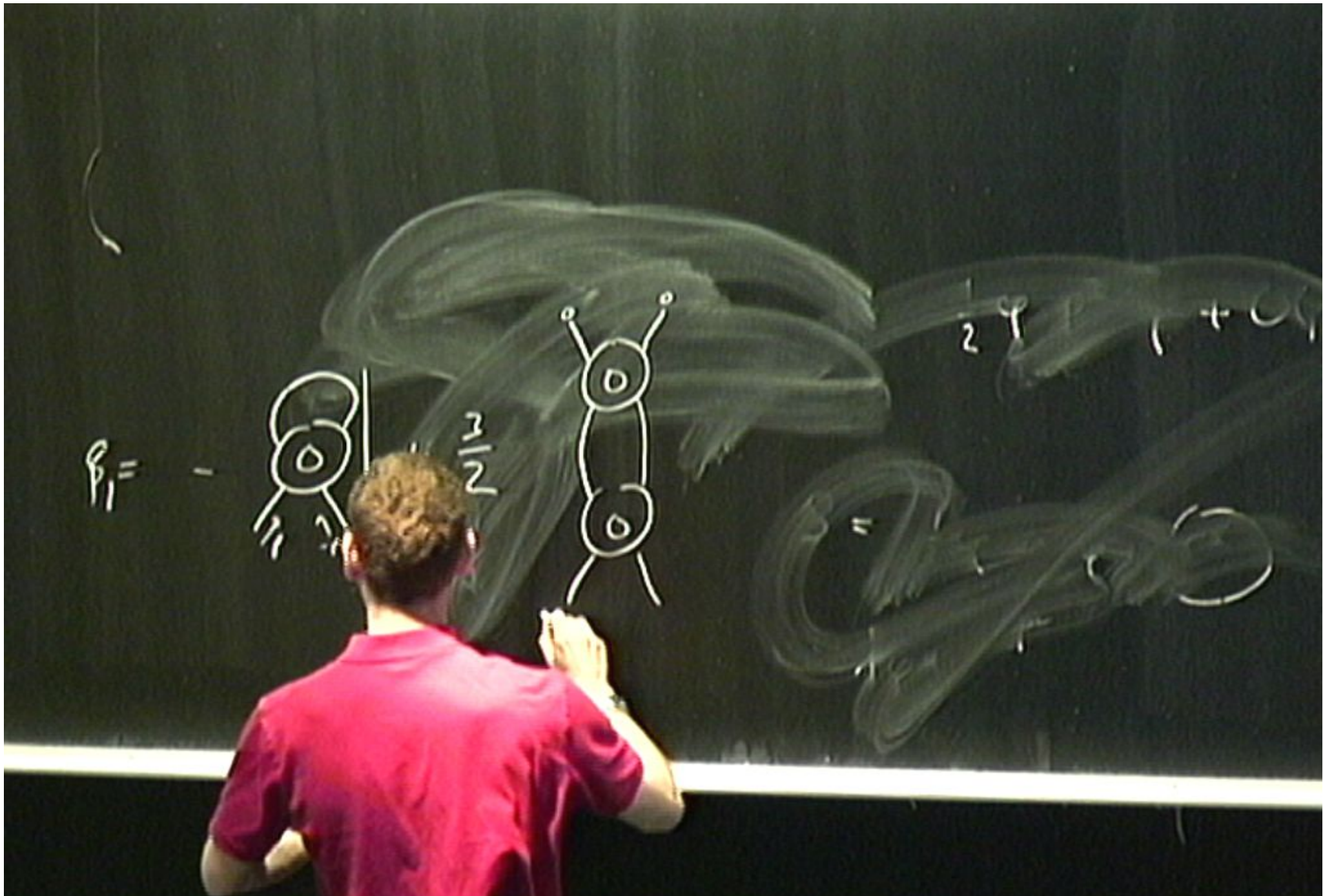
Flow of the Four-Point

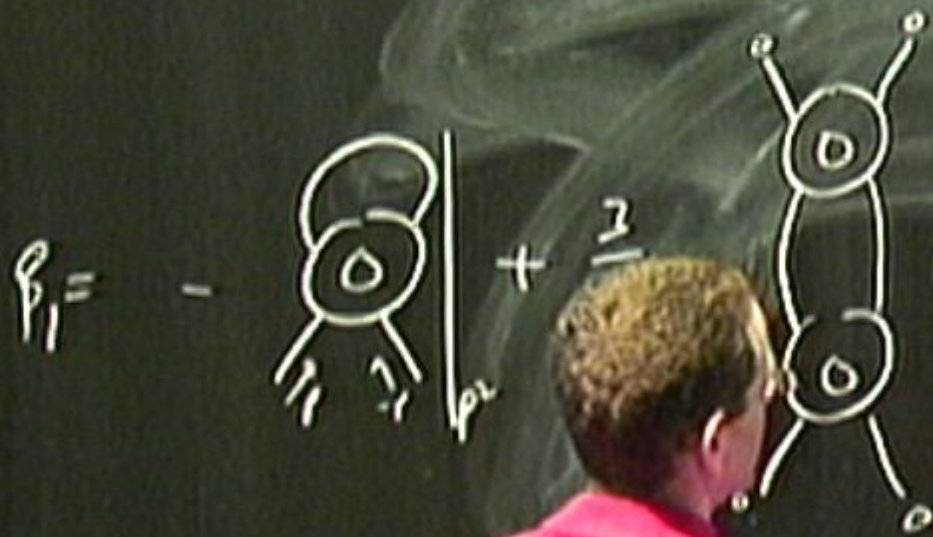
- Similarly, compute the flow of four-point vertex, but this time with all momenta set to zero
- Use the renormalization condition, $S^{(4)}(\underline{0}, \Lambda) = 1$
- Arrive at a second equation in terms of β_1 and γ_1
- Solve the two simultaneous equations for β_1

Evaluating β_1

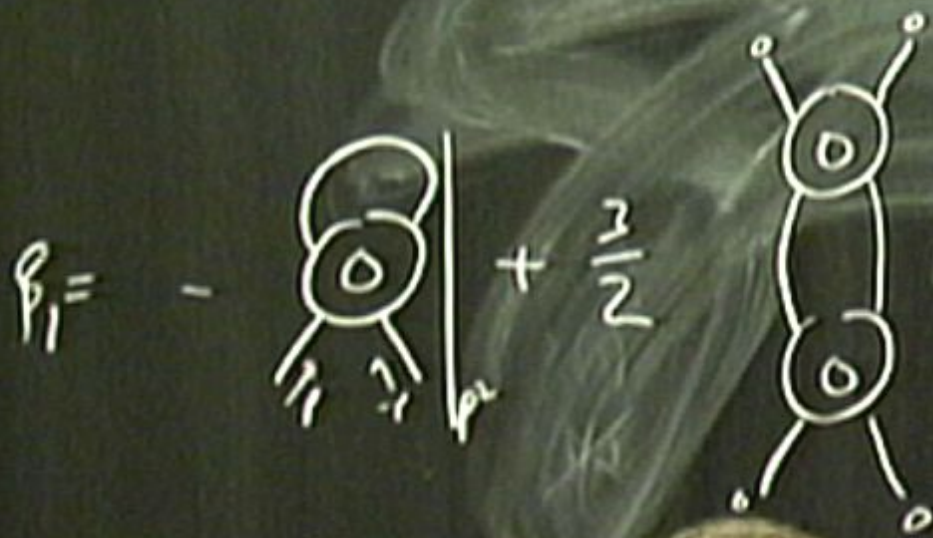
$$\beta_1 = - \left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right] \cdot$$

The diagrammatic equation for β_1 is shown as a sum of two terms enclosed in large square brackets, with a dot at the end of the second term. The first term is a diagram of a circle with a loop on top, two external lines at the bottom labeled p and $-p$, and a '0' inside. A vertical line to its right is labeled p^2 . The second term is a diagram of two circles connected vertically, each with two external lines (top and bottom) labeled '0', and a '0' inside each circle. A coefficient $+\frac{3}{2}$ is placed between the two diagrams.

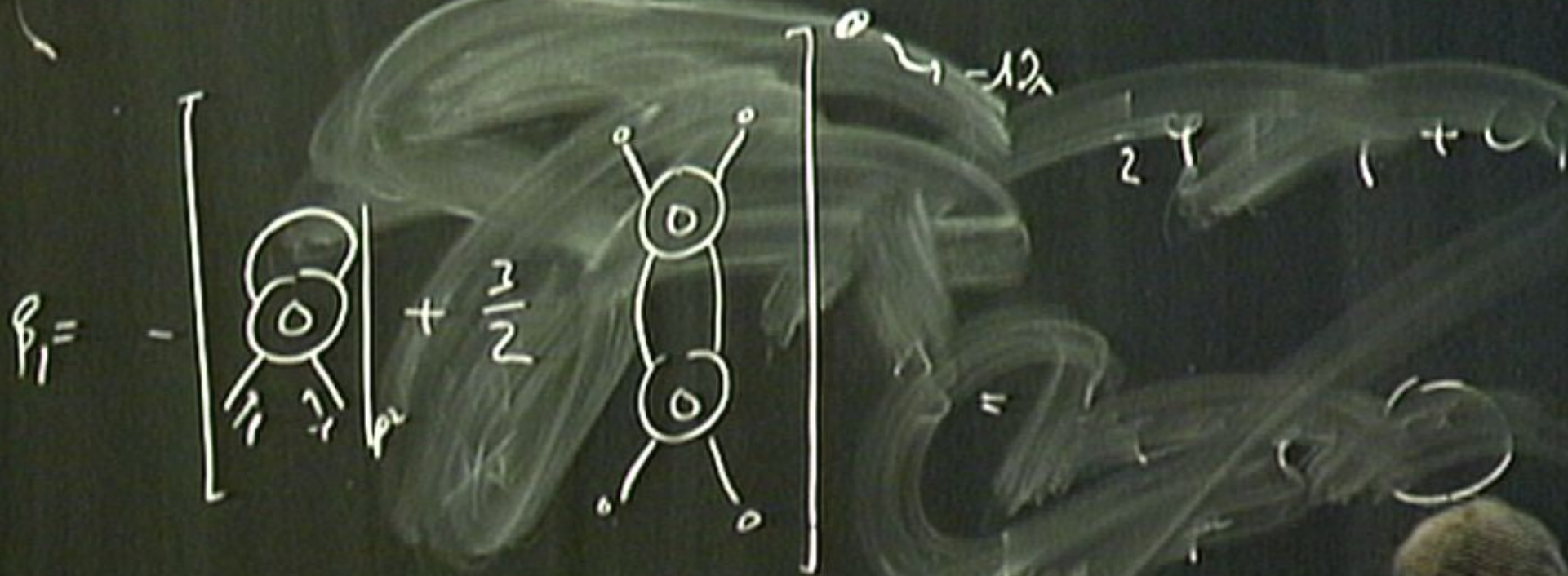


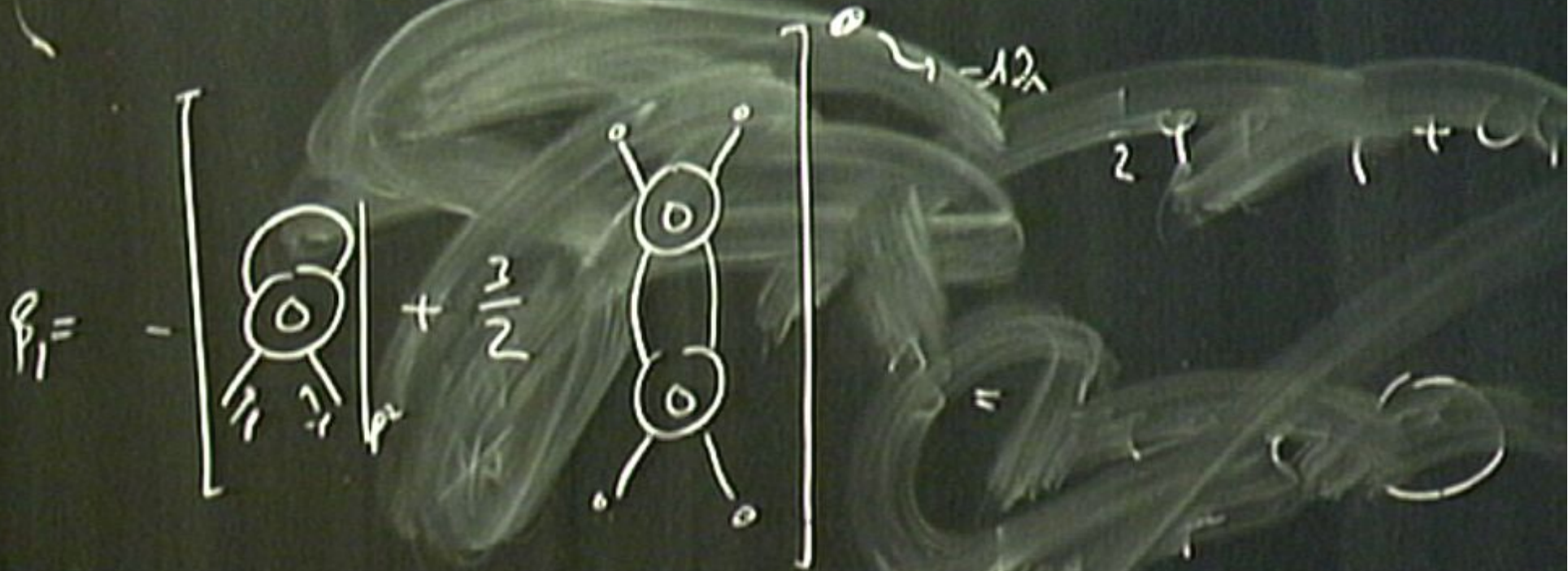


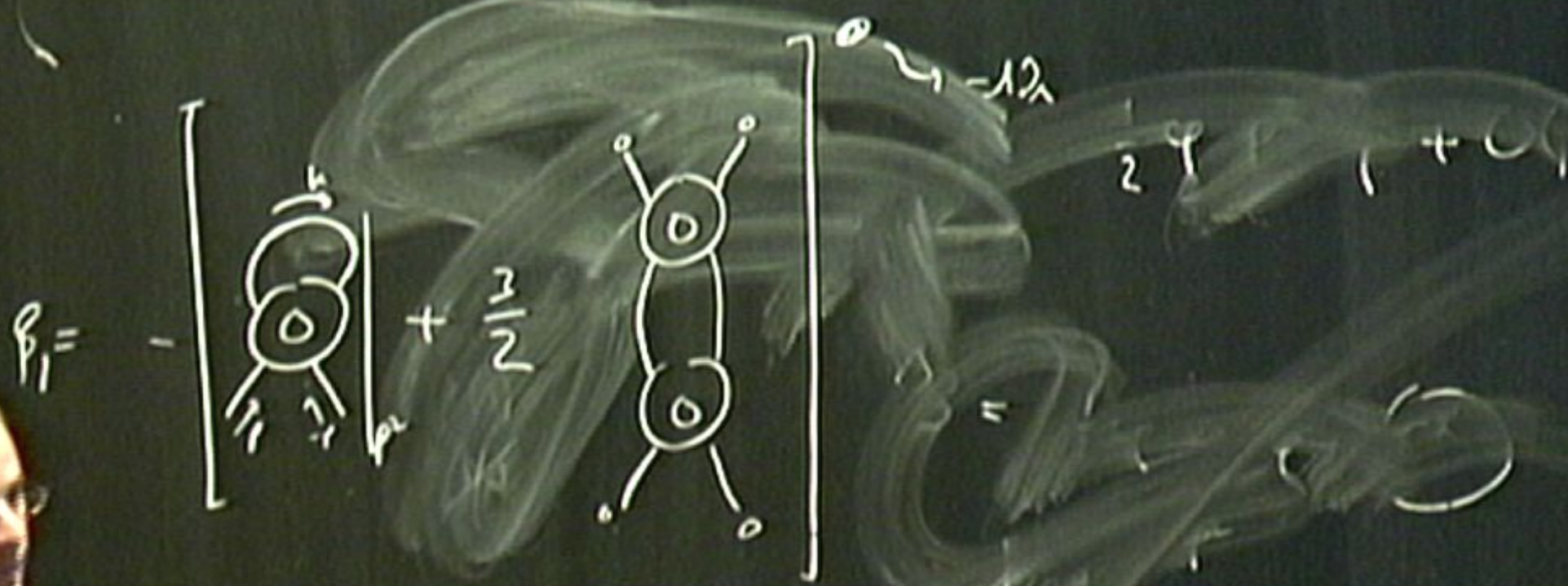
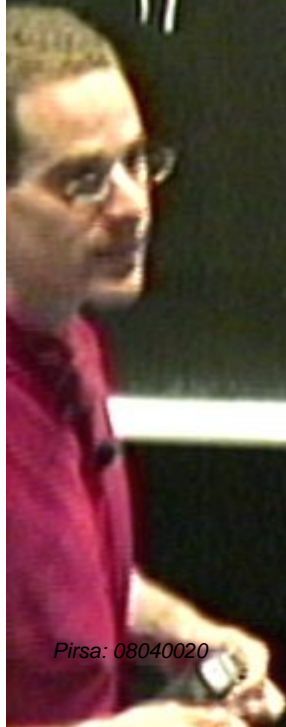
$\frac{1}{2} \gamma + \dots + \dots$



$\frac{1}{2} \gamma + \dots + \dots$





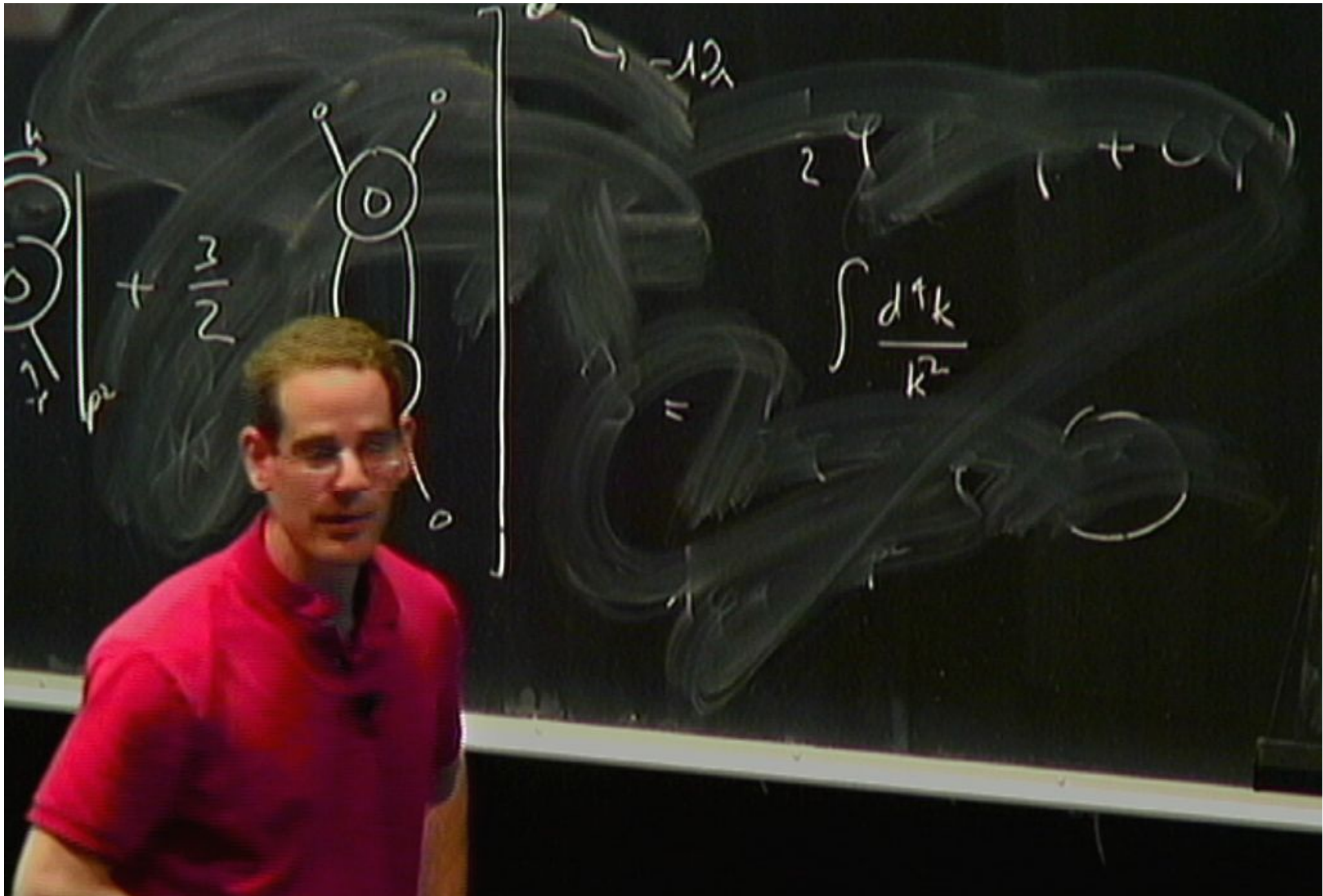


Evaluating β_1

- First diagram:
$$\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, p, -p) \Delta(k) \right] \Big|_{p^2}$$

Evaluating β_1

- First diagram: $\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, p, -p) \Delta(k) \right] \Big|_{p^2}$
- The integral is UV finite by construction
- Since $\Delta(k) \sim 1/k^2 + O(1)$, interchanging $\Lambda \partial_\Lambda$ and \int_k does not introduce any IR divergences



Evaluating β_1

- First diagram: $\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, p, -p) \Delta(k) \right] \Big|_{p^2}$
- The integral is UV finite by construction
- Since $\Delta(k) \sim 1/k^2 + O(1)$, interchanging $\Lambda \partial_\Lambda$ and \int_k does not introduce any IR divergences
- $\Rightarrow -\Lambda \partial_\Lambda \int \frac{d^4 k}{(2\pi)^4} S_0^{(4)}(k, -k, p, -p) \Delta(k) \Big|_{p^2}$

Evaluating β_1

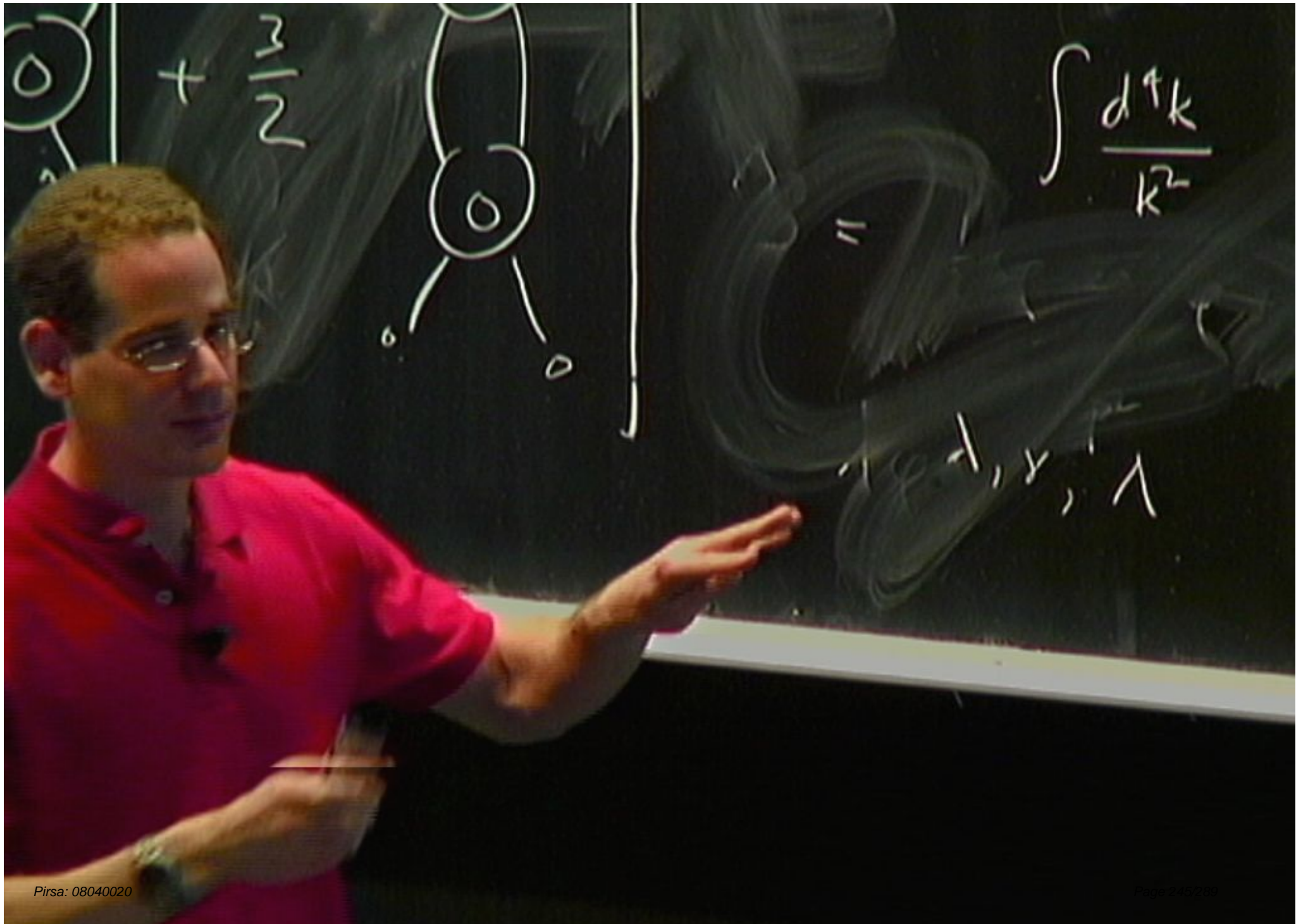
- First diagram: $\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, p, -p) \Delta(k) \right] \Big|_{p^2}$
- The integral is UV finite by construction
- Since $\Delta(k) \sim 1/k^2 + O(1)$, interchanging $\Lambda \partial_\Lambda$ and \int_k does not introduce any IR divergences
- $\Rightarrow -\Lambda \partial_\Lambda \int \frac{d^4 k}{(2\pi)^4} S_0^{(4)}(k, -k, p, -p) \Delta(k) \Big|_{p^2}$

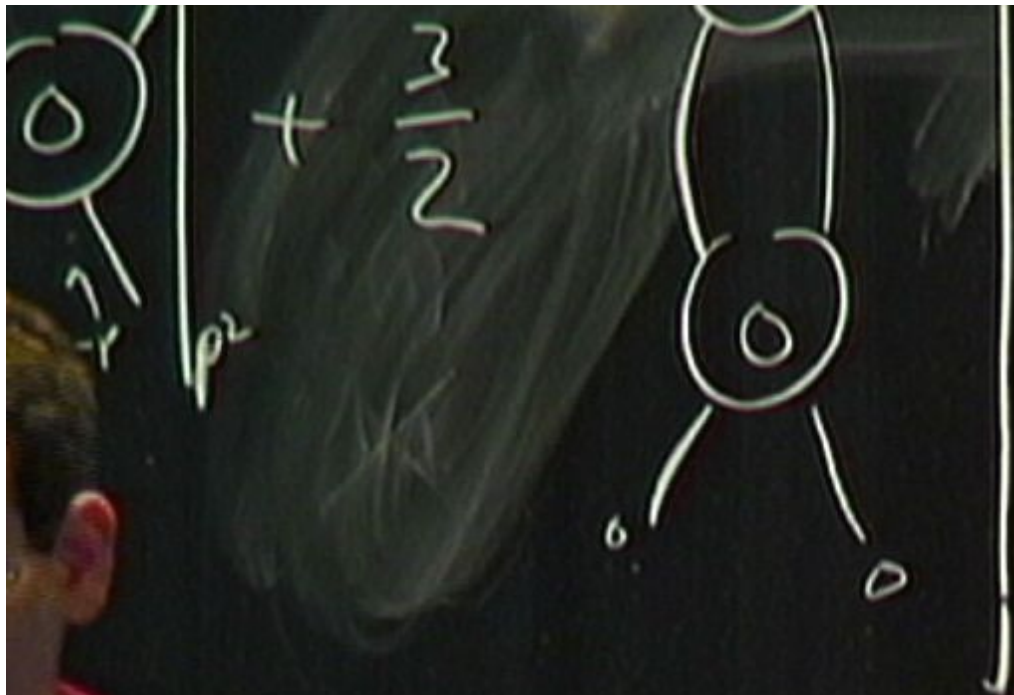
Evaluating β_1

- First diagram: $\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, p, -p) \Delta(k) \right] \Big|_{p^2}$
- The integral is UV finite by construction
- Since $\Delta(k) \sim 1/k^2 + O(1)$, interchanging $\Lambda \partial_\Lambda$ and \int_k does not introduce any IR divergences
- $\Rightarrow -\Lambda \partial_\Lambda \int \frac{d^4 k}{(2\pi)^4} S_0^{(4)}(k, -k, p, -p) \Delta(k) \Big|_{p^2}$
- We have $\Lambda \partial_\Lambda$ of a dimensionless integral

Evaluating β_1

- First diagram: $\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, p, -p) \Delta(k) \right] \Big|_{p^2}$
- The integral is UV finite by construction
- Since $\Delta(k) \sim 1/k^2 + O(1)$, interchanging $\Lambda \partial_\Lambda$ and \int_k does not introduce any IR divergences
- $\Rightarrow -\Lambda \partial_\Lambda \int \frac{d^4 k}{(2\pi)^4} S_0^{(4)}(k, -k, p, -p) \Delta(k) \Big|_{p^2}$
- We have $\Lambda \partial_\Lambda$ of a dimensionless integral
- By perturbative self-similarity (ie no hidden couplings), the answer is zero!





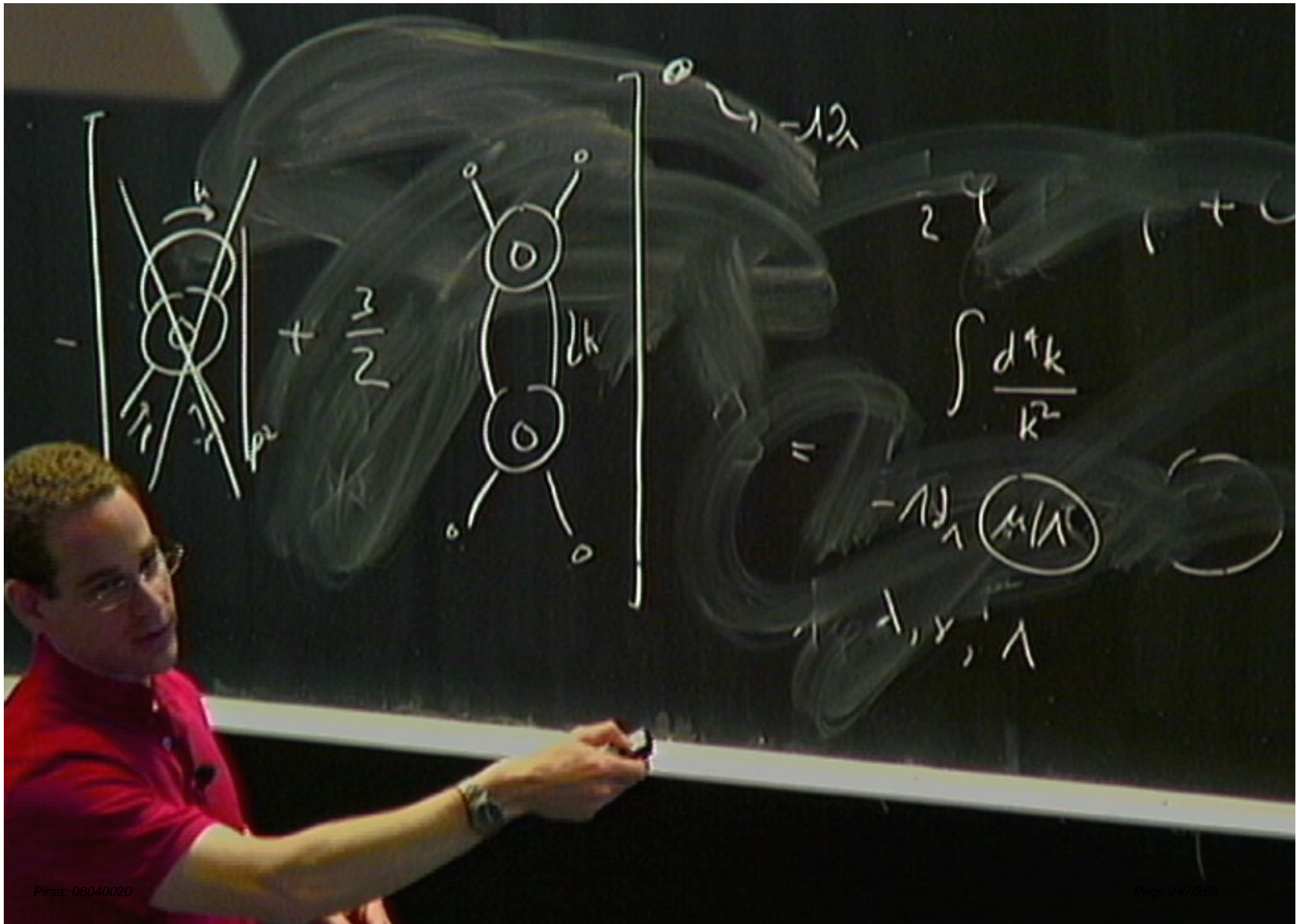
On the right side of the chalkboard, there is a double equals sign (=) followed by a large integral expression:

$$\int \frac{d^4 k}{k^2}$$

Below the integral, there is a minus sign followed by a wavy line and a circled expression:

$$- \text{wavy line} \circlearrowleft \mu/\Lambda$$

At the bottom right, there are some scribbled-out lines and a wavy line with a '1' below it.



Evaluating β_1

- Second Diagram:

$$\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, 0, 0) \Delta^2(k) S_0^{(4)}(0, 0, -k, k) \right] \bullet$$

- UV finite by construction

Evaluating β_1

- Second Diagram:

$$\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, 0, 0) \Delta^2(k) S_0^{(4)}(0, 0, -k, k) \right] \bullet$$

- UV finite by construction
- Interchanging order of differentiation and integration will introduce an IR divergence

Evaluating β_1

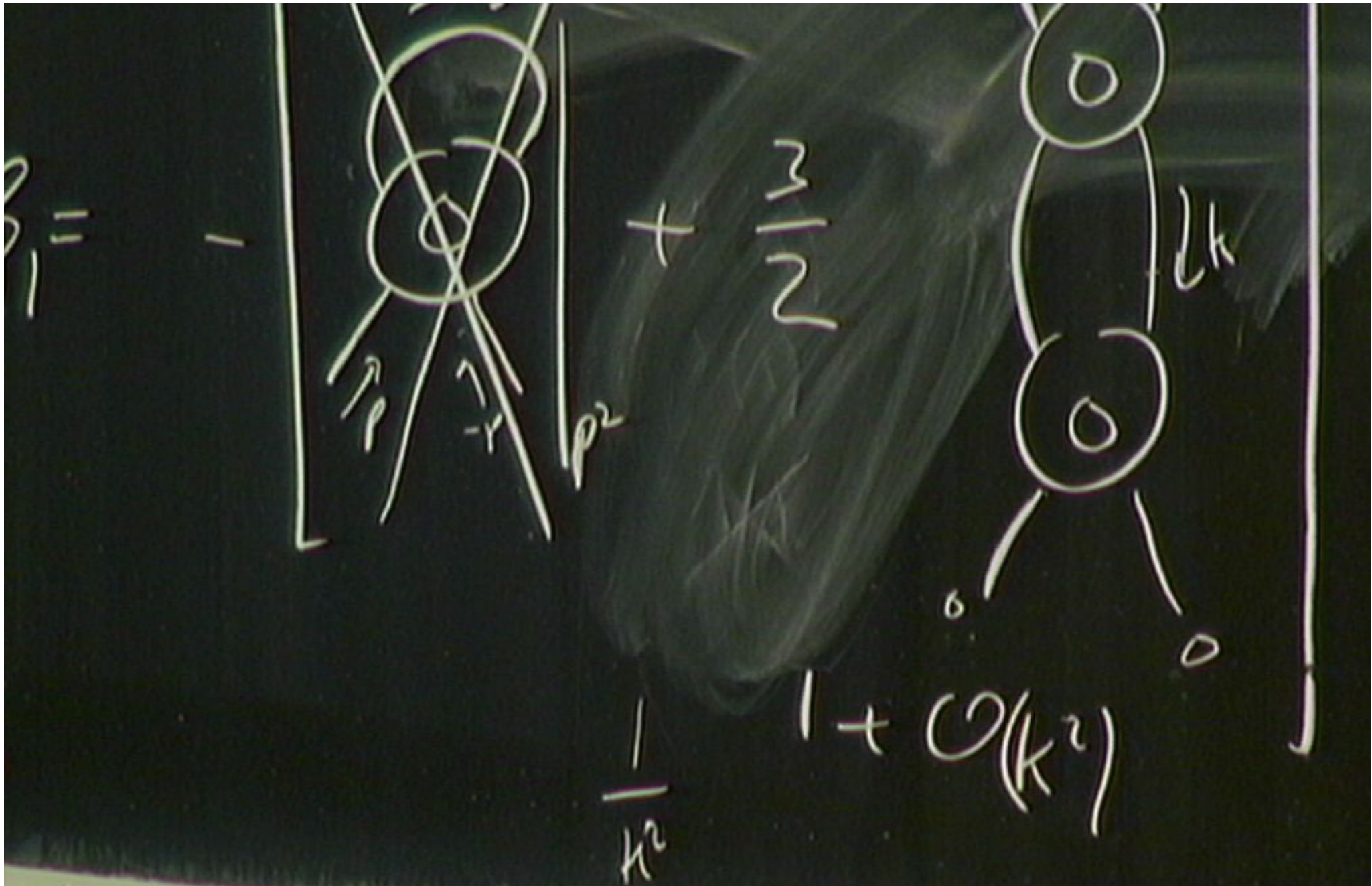
$$\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, 0, 0) \Delta^2(k) S_0^{(4)}(0, 0, -k, k) \right] \bullet$$

Evaluating β_1

$$\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, 0, 0) \Delta^2(k) S_0^{(4)}(0, 0, -k, k) \right] \bullet$$

Evaluating β_1

$$\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, 0, 0) \Delta^2(k) S_0^{(4)}(0, 0, -k, k) \right] \bullet$$
$$\rightarrow -\frac{\Omega_D}{(2\pi)^D} \Lambda \partial_\Lambda \int_0^\Lambda dk \frac{k^{D-1}}{k^4} + O(\epsilon)$$



Evaluating β_1

$$\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, 0, 0) \Delta^2(k) S_0^{(4)}(0, 0, -k, k) \right] \bullet$$
$$\rightarrow -\frac{\Omega_D}{(2\pi)^D} \Lambda \partial_\Lambda \int_0^\Lambda dk \frac{k^{D-1}}{k^4} + O(\epsilon)$$

Evaluating β_1

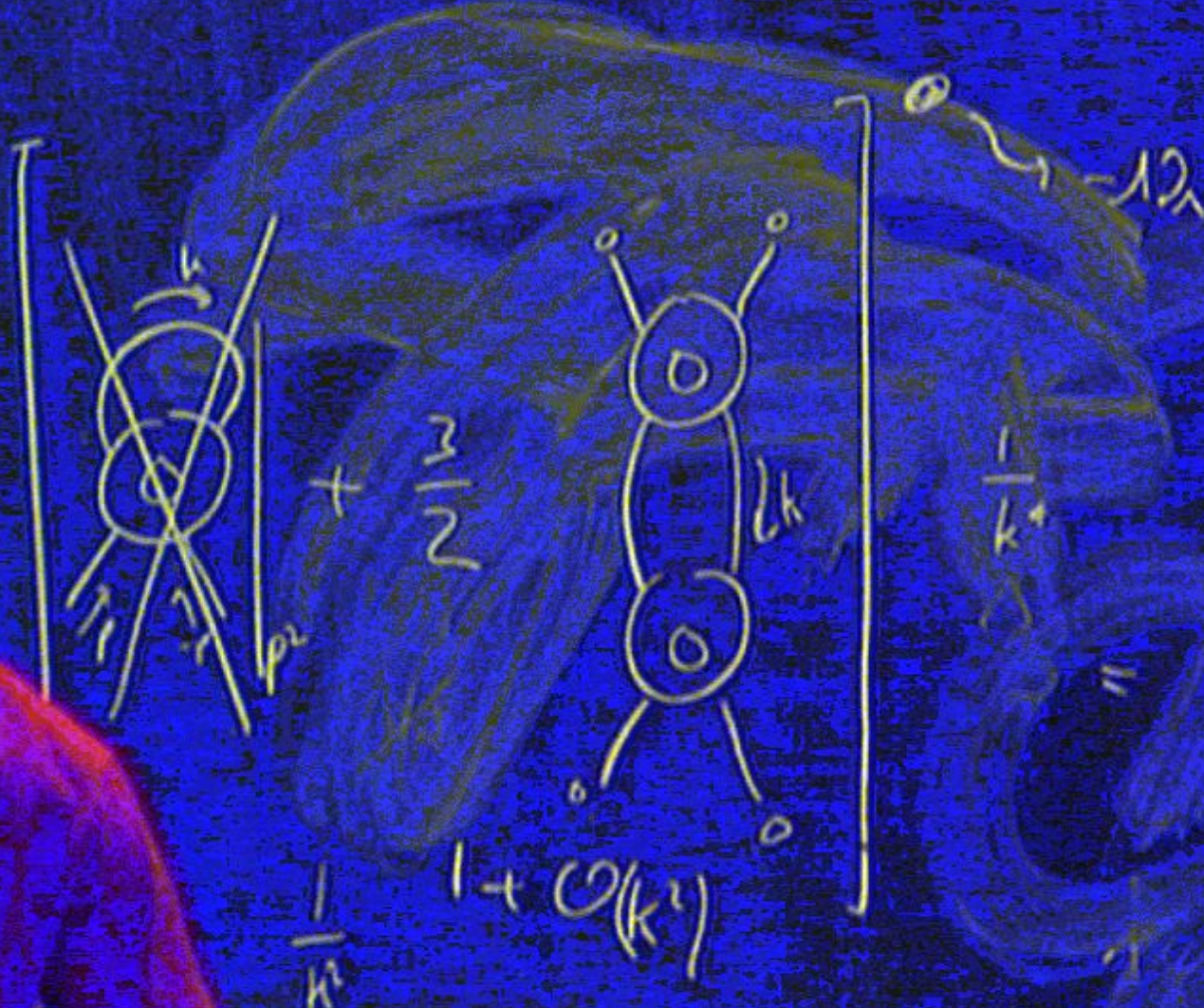
$$\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, 0, 0) \Delta^2(k) S_0^{(4)}(0, 0, -k, k) \right] \bullet$$
$$\rightarrow -\frac{\Omega_D}{(2\pi)^D} \Lambda \partial_\Lambda \int_0^\Lambda dk \frac{k^{D-1}}{k^4} + O(\epsilon)$$

Evaluating β_1

$$\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, 0, 0) \Delta^2(k) S_0^{(4)}(0, 0, -k, k) \right] \bullet$$
$$\rightarrow -\frac{\Omega_D}{(2\pi)^D} \Lambda \partial_\Lambda \int_0^\Lambda dk \frac{k^{D-1}}{k^4} + O(\epsilon)$$
$$\stackrel{k \rightarrow k\Lambda}{=} -\frac{\Omega_D}{(2\pi)^D} \Lambda \partial_\Lambda \Lambda^{2\epsilon} \int_0^1 dk k^{-1+2\epsilon} + O(\epsilon)$$

2ϵ

$\frac{1}{2\epsilon}$



$$\int \frac{d^4 k}{k^4}$$

$$-12 \left(\frac{4}{12} \right)$$

Evaluating β_1

$$\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, 0, 0) \Delta^2(k) S_0^{(4)}(0, 0, -k, k) \right] \bullet$$
$$\rightarrow -\frac{\Omega_D}{(2\pi)^D} \Lambda \partial_\Lambda \int_0^\Lambda dk \frac{k^{D-1}}{k^4} + O(\epsilon)$$
$$\stackrel{k \rightarrow k\Lambda}{=} -\frac{\Omega_D}{(2\pi)^D} \Lambda \partial_\Lambda \Lambda^{2\epsilon} \int_0^1 dk k^{-1+2\epsilon} + O(\epsilon)$$

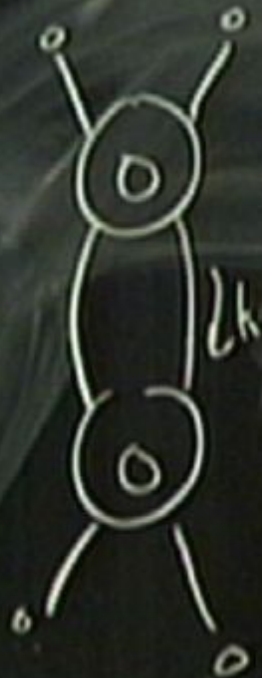
2ϵ

$\frac{1}{2\epsilon}$

$\beta_1 =$



$+\frac{3}{2}$



$\frac{1}{k^2} 1 + \mathcal{O}(k^2)$

$\frac{1}{k^4}$

$$\int \frac{d^4 k}{k^4}$$

$-\Lambda^2$ $\left(\frac{4}{\Lambda^2}\right)$

Evaluating β_1

$$\int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, 0, 0) \Delta^2(k) S_0^{(4)}(0, 0, -k, k) \right] \bullet$$

$$\rightarrow -\frac{\Omega_D}{(2\pi)^D} \Lambda \partial_\Lambda \int_0^\Lambda dk \frac{k^{D-1}}{k^4} + O(\epsilon)$$

$$\stackrel{k \rightarrow k\Lambda}{=} -\frac{\Omega_D}{(2\pi)^D} \Lambda \partial_\Lambda \Lambda^{2\epsilon} \int_0^1 dk k^{-1+2\epsilon} + O(\epsilon)$$

$$= -\frac{\Omega_4}{(2\pi)^4} [k^{2\epsilon}]_0^1 + O(\epsilon)$$

Evaluating β_1

$$\begin{aligned} & \int \frac{d^4 k}{(2\pi)^4} \left[S_0^{(4)}(k, -k, 0, 0) \Delta^2(k) S_0^{(4)}(0, 0, -k, k) \right] \bullet \\ & \rightarrow -\frac{\Omega_D}{(2\pi)^D} \Lambda \partial_\Lambda \int_0^\Lambda dk \frac{k^{D-1}}{k^4} + O(\epsilon) \\ & \stackrel{k \rightarrow k\Lambda}{=} -\frac{\Omega_D}{(2\pi)^D} \Lambda \partial_\Lambda \Lambda^{2\epsilon} \int_0^1 dk k^{-1+2\epsilon} + O(\epsilon) \\ & = -\frac{\Omega_4}{(2\pi)^4} [k^{2\epsilon}]_0^1 + O(\epsilon) \\ & = -\frac{\Omega_4}{(2\pi)^4} + O(\epsilon) \end{aligned}$$

Evaluating β_1

$$\beta_1 = - \left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right] \cdot$$

The diagrammatic equation for β_1 is shown as a sum of two terms enclosed in large square brackets, followed by a dot. The first term is a diagram of a circle with a loop on top, two external lines at the bottom labeled p and $-p$, and a '0' in the center. A vertical line to its right is labeled p^2 . The second term is a diagram of two circles stacked vertically, each with two external lines at the top and bottom labeled '0', and a '0' in the center of each circle. A plus sign and the fraction $\frac{3}{2}$ are placed between the two diagrams.

Evaluating β_1

$$\beta_1 = - \left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right] \cdot$$

The diagrammatic equation for β_1 is shown within large square brackets. On the left is a diagram of a circle with a loop on top, two external lines at the bottom labeled p and $-p$, and a central '0'. To its right is a vertical line labeled p^2 . To the right of this line is a plus sign followed by the fraction $\frac{3}{2}$. To the right of the fraction is a diagram of two circles stacked vertically, each with two external lines at the top and bottom labeled '0', and a central '0'. The entire expression is followed by a dot.

- $\beta_1 = + \frac{3}{16\pi^2}$

- Universality arose from being forced to look at the IR end of the integral

Comment

Comment

- The diagrammatic cancellations of the seed action are completely generic
- These cancellations are the basis of the 'universal diagrammatic calculus' proposed in S. Arnone, A. Gatti, and T. R. Morris, "A proposal for a manifestly gauge invariant and universal calculus in Yang-Mills theory," Phys. Rev. **D 67** (2003) 085004, [hep-th/0209162](https://arxiv.org/abs/hep-th/0209162).
- In a series of works, I completed the calculus and found how to apply it nonperturbatively
- When computing eg the β -function it is now possible to jump straight to the final diagrammatic expression
- The diagrammatic manipulations involved in the proof can be replaced with easy algebra (work in progress)

Comment

- The diagrammatic cancellations of the seed action are completely generic
- These cancellations are the basis of the 'universal diagrammatic calculus' proposed in S. Arnone, A. Gatti, and T. R. Morris, "A proposal for a manifestly gauge invariant and universal calculus in Yang-Mills theory," Phys. Rev. **D 67** (2003) 085004, [hep-th/0209162](#).
- In a series of works, I completed the calculus and found how to apply it nonperturbatively
- When computing eg the β -function it is now possible to jump straight to the final diagrammatic expression
- The diagrammatic manipulations involved in the proof can be replaced with easy algebra (work in progress)

Comment

- The diagrammatic cancellations of the seed action are completely generic
- These cancellations are the basis of the 'universal diagrammatic calculus' proposed in S. Arnone, A. Gatti, and T. R. Morris, "A proposal for a manifestly gauge invariant and universal calculus in Yang-Mills theory," Phys. Rev. **D 67** (2003) 085004, [hep-th/0209162](#).
- In a series of works, I completed the calculus and found how to apply it nonperturbatively
- When computing eg the β -function it is now possible to jump straight to the final diagrammatic expression
- The diagrammatic manipulations involved in the proof can be replaced with easy algebra (work in progress)

Comment

- The diagrammatic cancellations of the seed action are completely generic
- These cancellations are the basis of the 'universal diagrammatic calculus' proposed in S. Arnone, A. Gatti, and T. R. Morris, "A proposal for a manifestly gauge invariant and universal calculus in Yang-Mills theory," Phys. Rev. **D 67** (2003) 085004, [hep-th/0209162](#).
- In a series of works, I completed the calculus and found how to apply it nonperturbatively
- When computing eg the β -function it is now possible to jump straight to the final diagrammatic expression
- The diagrammatic manipulations involved in the proof can be replaced with easy algebra (work in progress)

Comment

- The diagrammatic cancellations of the seed action are completely generic
- These cancellations are the basis of the 'universal diagrammatic calculus' proposed in S. Arnone, A. Gatti, and T. R. Morris, "A proposal for a manifestly gauge invariant and universal calculus in Yang-Mills theory," Phys. Rev. **D 67** (2003) 085004, [hep-th/0209162](#).
- In a series of works, I completed the calculus and found how to apply it nonperturbatively
- When computing eg the β -function it is now possible to jump straight to the final diagrammatic expression
- The diagrammatic manipulations involved in the proof can be replaced with easy algebra (work in progress)

Recap

Recap

- Used blocking freedom to construct general ERGs
- Focussed on cousins of Polchinski's equation but with
- We introduced 'Reduced Vertices'
- The application of the flow equation was illustrated with a computation of β_1
- Commented on the huge simplification of such calculations which is now possible
- Given equal levels of familiarity, perturbation theory is no harder with the ERG than with other methods!

Recap

- Used blocking freedom to construct general ERGs
- Focussed on cousins of Polchinski's equation but with
 - General seed action
 - Convenient incorporation of the anomalous dimension
- We introduced 'Reduced Vertices'

- The application of the flow equation was illustrated with a computation of β_1
- Commented on the huge simplification of such calculations which is now possible
- Given equal levels of familiarity, perturbation theory is no harder with the ERG than with other methods!

Recap

- Used blocking freedom to construct general ERGs
- Focussed on cousins of Polchinski's equation but with
 - General seed action
 - Convenient incorporation of the anomalous dimension
- We introduced 'Reduced Vertices'
- The application of the flow equation was illustrated with a computation of β_1
- Commented on the huge simplification of such calculations which is now possible
- Given equal levels of familiarity, perturbation theory is no harder with the ERG than with other methods!

Recap

- Used blocking freedom to construct general ERGs
- Focussed on cousins of Polchinski's equation but with
 - General seed action
 - Convenient incorporation of the anomalous dimension
- We introduced 'Reduced Vertices'
 - This is simple and obvious
 - But only done quite recently!
- The application of the flow equation was illustrated with a computation of β_1
- Commented on the huge simplification of such calculations which is now possible
- Given equal levels of familiarity, perturbation theory is no harder with the ERG than with other methods!

Recap

- Used blocking freedom to construct general ERGs
- Focussed on cousins of Polchinski's equation but with
 - General seed action
 - Convenient incorporation of the anomalous dimension
- We introduced 'Reduced Vertices'
 - This is simple and obvious
 - But only done quite recently!
- The application of the flow equation was illustrated with a computation of β_1
- Commented on the huge simplification of such calculations which is now possible
- Given equal levels of familiarity, perturbation theory is no harder with the ERG than with other methods!

Recap

- Used blocking freedom to construct general ERGs
- Focussed on cousins of Polchinski's equation but with
 - General seed action
 - Convenient incorporation of the anomalous dimension
- We introduced 'Reduced Vertices'
 - This is simple and obvious
 - But only done quite recently!
- The application of the flow equation was illustrated with a computation of β_1
- Commented on the huge simplification of such calculations which is now possible
- Given equal levels of familiarity, perturbation theory is no harder with the ERG than with other methods!

Recap

- Used blocking freedom to construct general ERGs
- Focussed on cousins of Polchinski's equation but with
 - General seed action
 - Convenient incorporation of the anomalous dimension
- We introduced 'Reduced Vertices'
 - This is simple and obvious
 - But only done quite recently!
- The application of the flow equation was illustrated with a computation of β_1
- Commented on the huge simplification of such calculations which is now possible
- Given equal levels of familiarity, perturbation theory is no harder with the ERG than with other methods!

Recap

- Used blocking freedom to construct general ERGs
- Focussed on cousins of Polchinski's equation but with
 - General seed action
 - Convenient incorporation of the anomalous dimension
- We introduced 'Reduced Vertices'
 - This is simple and obvious
 - But only done quite recently!
- The application of the flow equation was illustrated with a computation of β_1
- Commented on the huge simplification of such calculations which is now possible
- Given equal levels of familiarity, perturbation theory is no harder with the ERG than with other methods!

Recap

- Used blocking freedom to construct general ERGs
- Focussed on cousins of Polchinski's equation but with
 - General seed action
 - Convenient incorporation of the anomalous dimension
- We introduced 'Reduced Vertices'
 - This is simple and obvious
 - But only done quite recently!
- The application of the flow equation was illustrated with a computation of β_1
- Commented on the huge simplification of such calculations which is now possible
- Given **equal levels of familiarity**, perturbation theory is no harder with the ERG than with other methods!

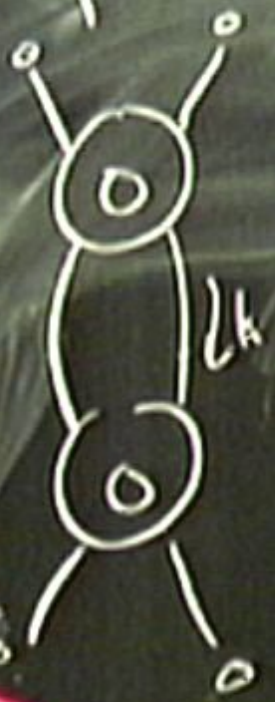
2E

2E

$\frac{1}{8\pi^2}$



+ 3



2h

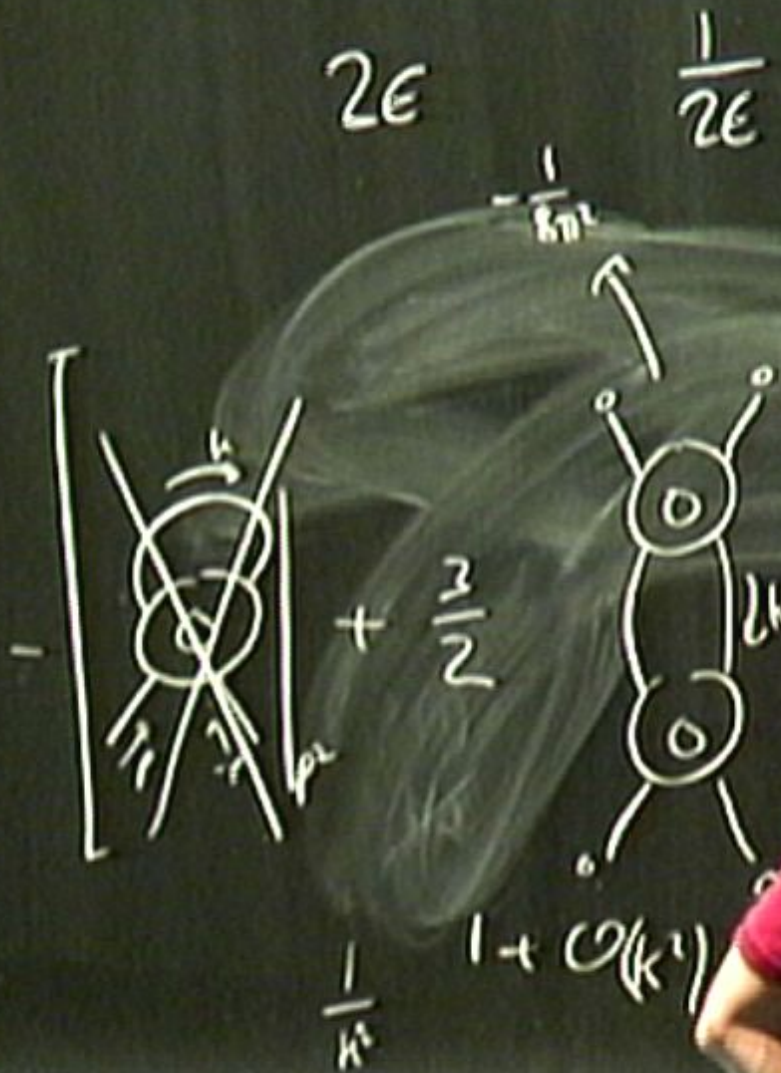
Λ_0

$\Lambda_0 \rightarrow \Lambda_0$

Λ

$S_{1,1}$

G



$t = \ln k$

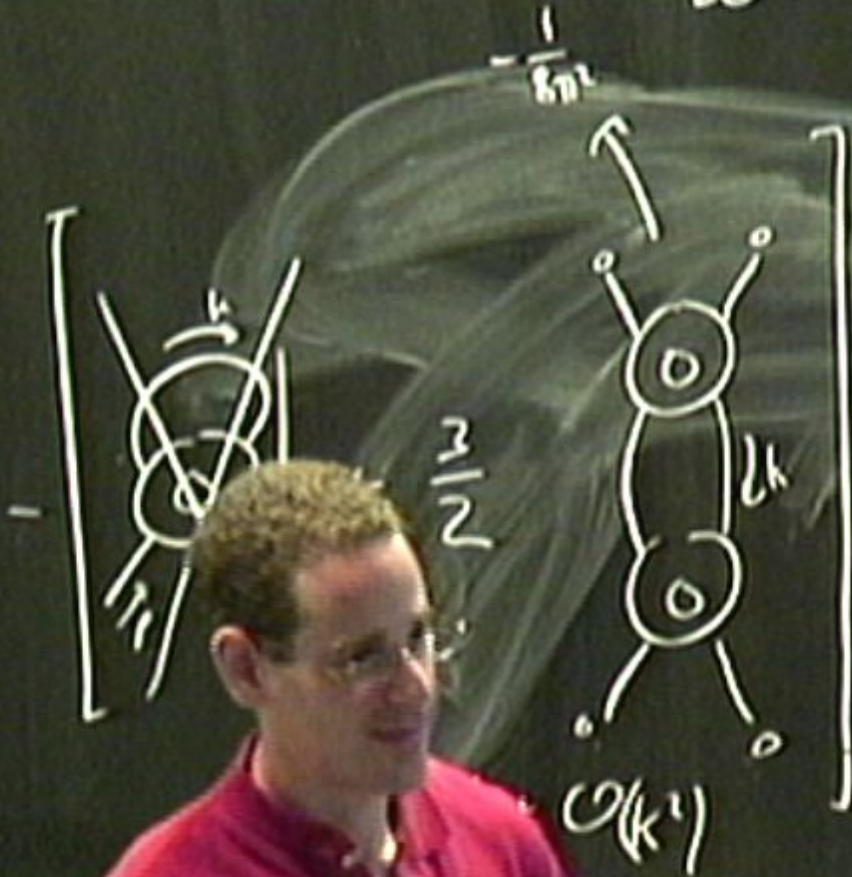
$d_\epsilon S_\nu = 0$

\ln
 \ln
 $S_{1,1}$

Special
 log R/IR
 diverges



$$2\epsilon \quad \frac{1}{2\epsilon}$$



$$t = \ln k$$

$$d_t S_{\nu} = 0$$

ln
10-10

$S_{A, \nu}$

Spinning
log R / MK
dynam

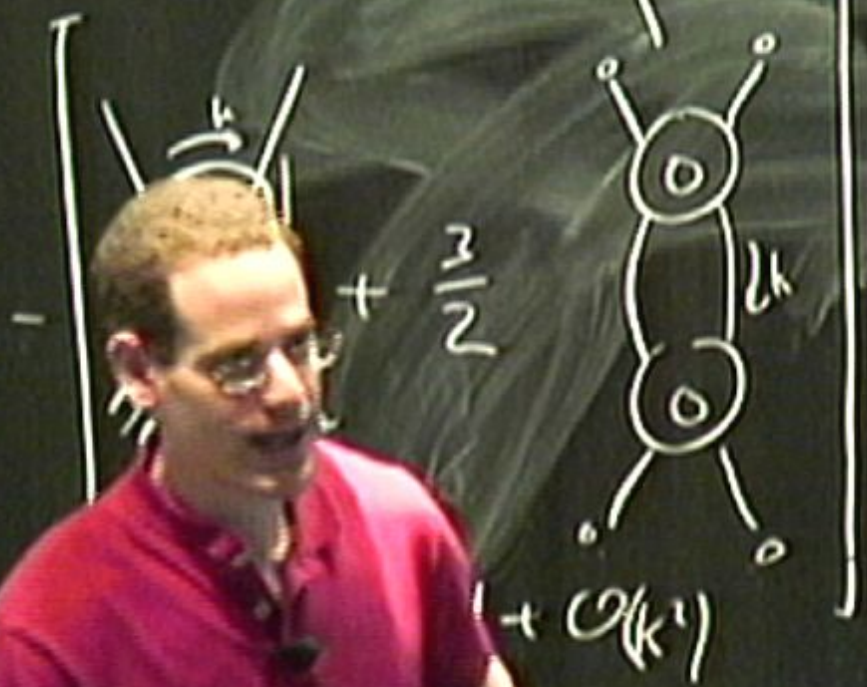
RT

$$S_{\nu} = S(g_{\mu\nu}, \dots)$$



$$2\epsilon \quad \frac{1}{2\epsilon}$$

$$-\frac{1}{8\pi^2}$$



Λ_0

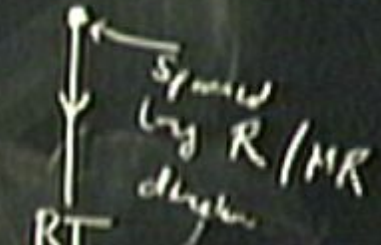
$t = \text{link}$

$$d\epsilon S_{\nu}^2 = 0$$

ln
10-100

Λ

$S_{\Lambda, \nu}$

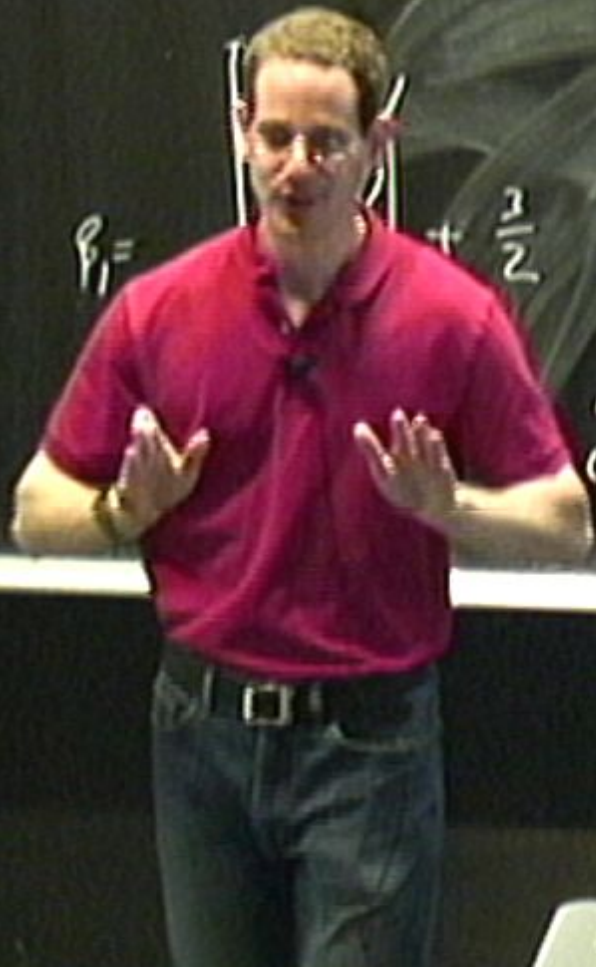


RT

$$S_{\Lambda} = S(g, \Lambda, \dots)$$

Recap

- Used blocking freedom to construct general ERGs
- Focussed on cousins of Polchinski's equation but with
 - General seed action
 - Convenient incorporation of the anomalous dimension
- We introduced 'Reduced Vertices'
 - This is simple and obvious
 - But only done quite recently!
- The application of the flow equation was illustrated with a computation of β_1
- Commented on the huge simplification of such calculations which is now possible
- Given **equal levels of familiarity**, perturbation theory is no harder with the ERG than with other methods!



2ϵ

$\frac{1}{2\epsilon}$

$\frac{Y_k}{2}$ is integer

$\beta_1 =$

$\frac{1}{2}$



Λ

coll. H
 $\partial_L S^2 = 0$

de
time

Λ

S_{eff}
 Λ^2

RT
sing. by R/IR
sing.

$S_k = S(\text{gen.})$

2ϵ $\frac{1}{2\epsilon}$ $\frac{Y_4}{2}$ is integer

$\beta_1 = - \left[\text{Diagram} \right] + \frac{3}{2}$

$\epsilon = \text{LHM}$
 $dL S_2 = 0$

$S_{A,2}$
 $N_{A,2}$

RT
 $S_A = S(\text{gas}, \dots)$

Spiral
 long R / NR
 dangle

$\frac{1}{2\epsilon}$

Useful Literature

- T. R. Morris, "The Exact renormalization group and approximate solutions," *Int. J. Mod. Phys. A* **9** (1994) 2411, hep-ph/9308265.
- T. R. Morris, "Elements of the continuous renormalization group," *Prog. Theor. Phys.* **131** (1998) 395, hep-th/9802039.
- C. Bagnuls and C. Bervillier, "Exact renormalization group equations: An introductory review," *Phys. Rept.* **348** (2001) 91, hep-th/0002034.
- F. Wegner , "The Critical State, General Aspects," in: C. Domb and M. S. Green (Eds.), *Phase Transitions and Critical Phenomena, Vol VI*, Acad. Press, N.-Y., 1976 p. 7.
- T. R. Morris and J. L. Latorre, "Exact scheme independence," *JHEP* **0011** (2000) 004, hep-th/0008123.

2ϵ $\frac{1}{2\epsilon}$

$\frac{1}{k_1}$

$\frac{Y_{\pm}}{2}$ is integer
 δ_{\pm} is anything

$\epsilon = \ln N$

$dL S_{\pm}^{12} = 0$

$\beta_1 =$

$+\frac{3}{2}$

$2h$

$+ O(k^1)$

N_1

$S_{\pm, n}$
 N/A

RT

$S_n = S(\gamma_{rel})$

GFP
 $g(h)$

\rightarrow space
 \rightarrow long
 \rightarrow double

Useful Literature

- T. R. Morris, "The Exact renormalization group and approximate solutions," *Int. J. Mod. Phys. A* **9** (1994) 2411, hep-ph/9308265.
- T. R. Morris, "Elements of the continuous renormalization group," *Prog. Theor. Phys.* **131** (1998) 395, hep-th/9802039.
- C. Bagnuls and C. Bervillier, "Exact renormalization group equations: An introductory review," *Phys. Rept.* **348** (2001) 91, hep-th/0002034.
- F. Wegner , "The Critical State, General Aspects," in: C. Domb and M. S. Green (Eds.), *Phase Transitions and Critical Phenomena*, Vol VI, Acad. Press, N.-Y., 1976 p. 7.
- T. R. Morris and J. L. Latorre, "Exact scheme independence," *JHEP* **0011** (2000) 004, hep-th/0008123.