Title: The Exact Renormalization Group - Lecture 1: Wilsonian Renormalization

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Abstract: In this lecture, I will discuss Wilson's picture of renormalization and its relation to the Exact Renormalization Group (ERG). In particular, I will focus on how one can understand, in a physically intuitive way, what it is for a quantum field theory to be nonperturbatively renormalizable.

The Exact Renormalization Group: Introduction & Applications

Lecture 1: Wilsonian Renormalization

Oliver J. Rosten

Dublin Institute for Advanced Studies

April / May 2008

What is the Exact Renormalization Group?

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Physics at Different Scales

- The way we describe physics changes with scale
- The relevant degrees of freedom change with scale
- A few microscopic laws give rich macroscopic behaviour

From Microscopic to Macroscopic

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1. Suppose we understand what happens at small scales
2. If we have a link between small scales and large scales
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- Lecture 1: Wilsonian Renormalization
- Lecture 2: Exact Renormalization Group Equations
- Lecture 3: The Derivative Expansion
- Lecture 4: Gauge Theory

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Useful Literature

- K. Wilson and J. Kogut, "The Renormalization group and the epsilon expansion," Phys. Rept. **12** (1974) 75.
- F. Wegner , "The Critical State, General Aspects," in: C. Domb and M. S. Green (Eds.), Phase Transitions and Critical Phenomena, Vol VI, Acad. Press, N.-Y., 1976 p. 7.
- M. E. Fisher, "Renormalization group theory: Its basis and formulation in statistical physics," Rev. Mod. Phys. **70** (1998) 653.
- J. Cardy, "Scaling and Renormalization in Statistical Physics," CUP, 1996.

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Continued. . .

- T. R. Morris, "Elements of the continuous renormalization group," Prog. Theor. Phys. **131** (1998) 395, hep-th/9802039.
- C. Bagnuls and C. Bervillier, "Exact renormalization group equations: An introductory review," Phys. Rept. **348** (2001) 91, hep-th/0002034.
- J. Berges, N. Tetradis, and C. Wetterich, "Non-perturbative renormalization flow in quantum field theory and statistical physics," Phys. Rept. **363** (2002) 223, hep-ph/0005122.

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1. Broad Uses of the ERG
2. Qualitative Aspects of the ERG
   - The Basic Ideas
   - Relevance, Irrelevance and all that
3. Renormalizability
   - Continuum Limits
   - Examples
   - Sources of Misunderstanding
4. Recap

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- Temperature: T, Magnetization: M, External Field: H

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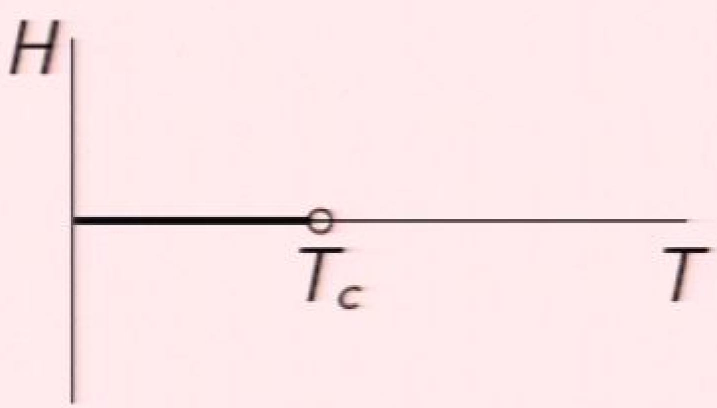
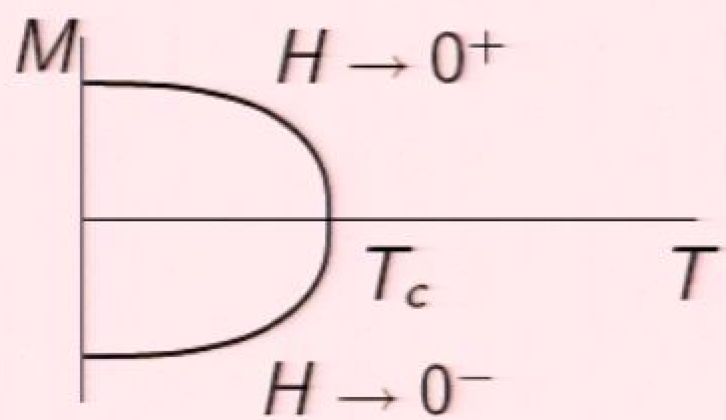
Phase Diagram



M vs T



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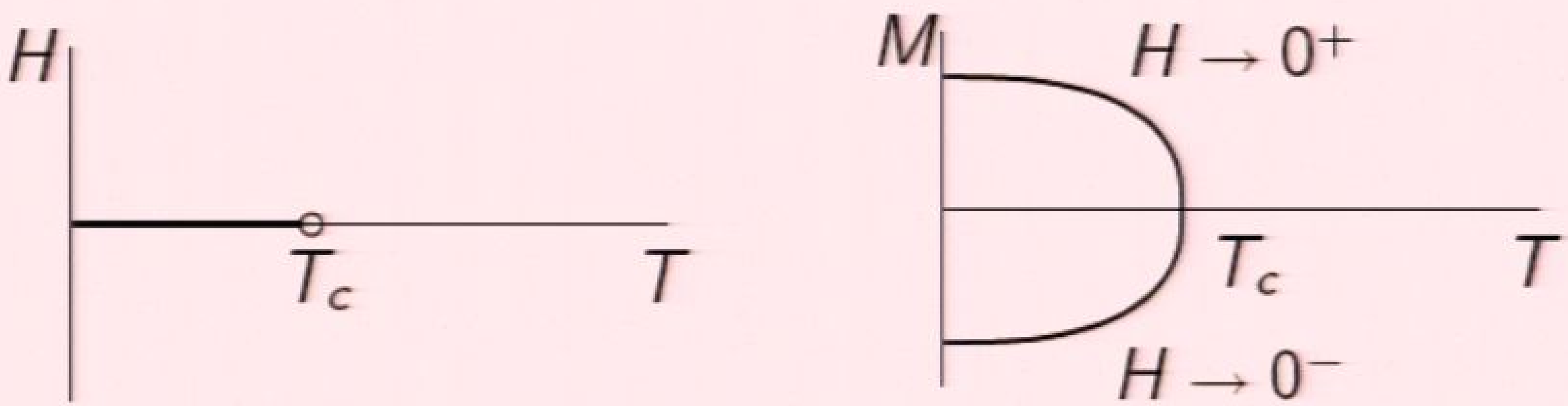

Understanding Critical Phenomena

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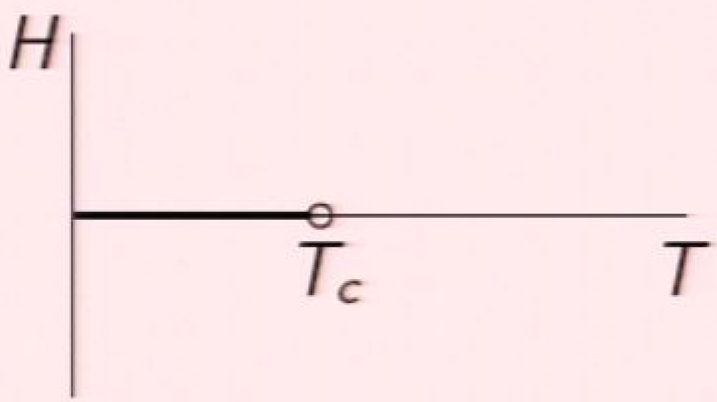

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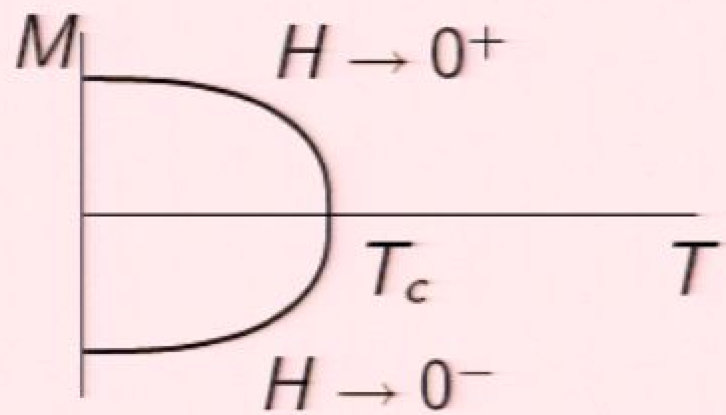

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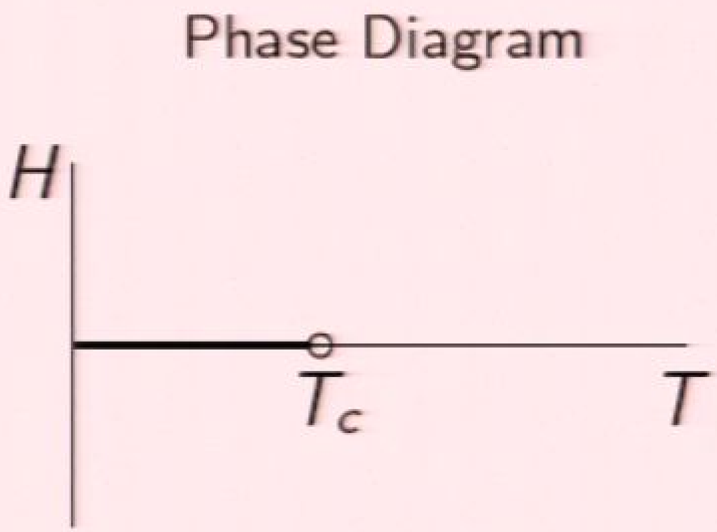

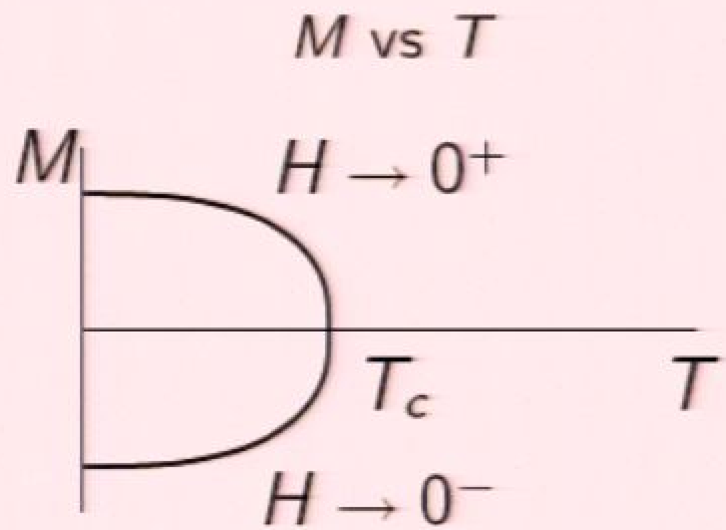

Understanding Critical Phenomena

- Computation of critical exponents eg $M \sim (T_c - T)^{\beta}$
- Universality: why do apparently very different systems have the same critical exponents?

The ERG in QFT

The ERG in QFT

Qualitative Aspects

- Understanding the physical basis of renormalization
  - Asymptotic freedom
  - Asymptotic safety
  - Triviality
- Why perturbative renormalizability does not necessarily imply nonperturbative renormalizability

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- Calculations can be performed directly in terms of renormalized variables
- The ERG admits nonperturbative approximation schemes which preserve renormalizability

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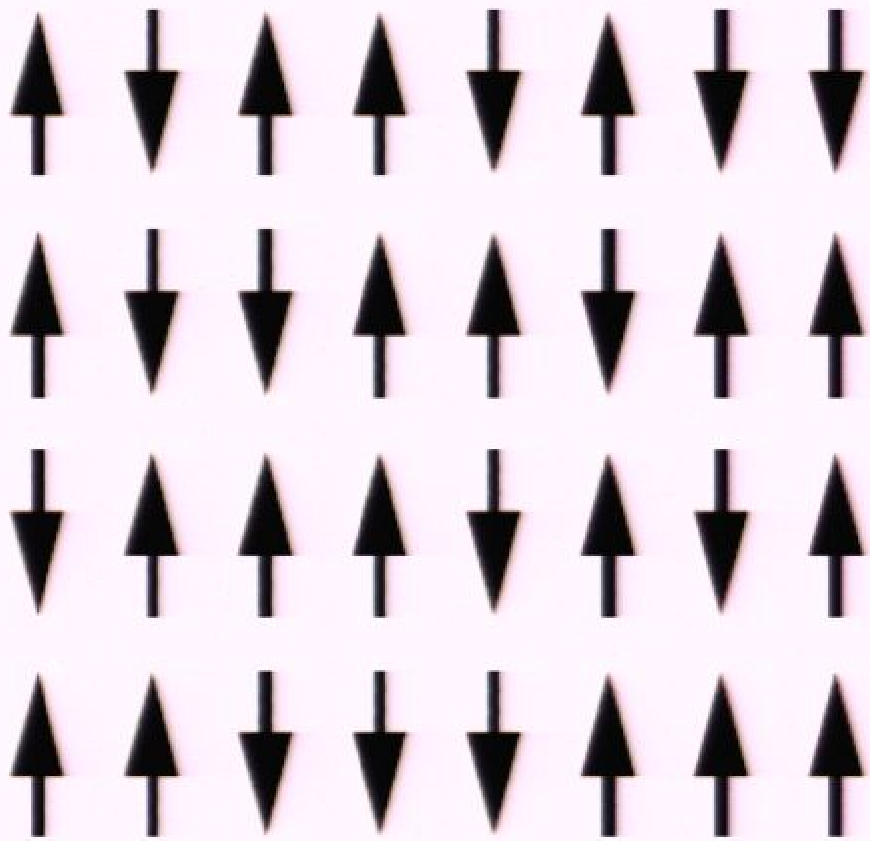

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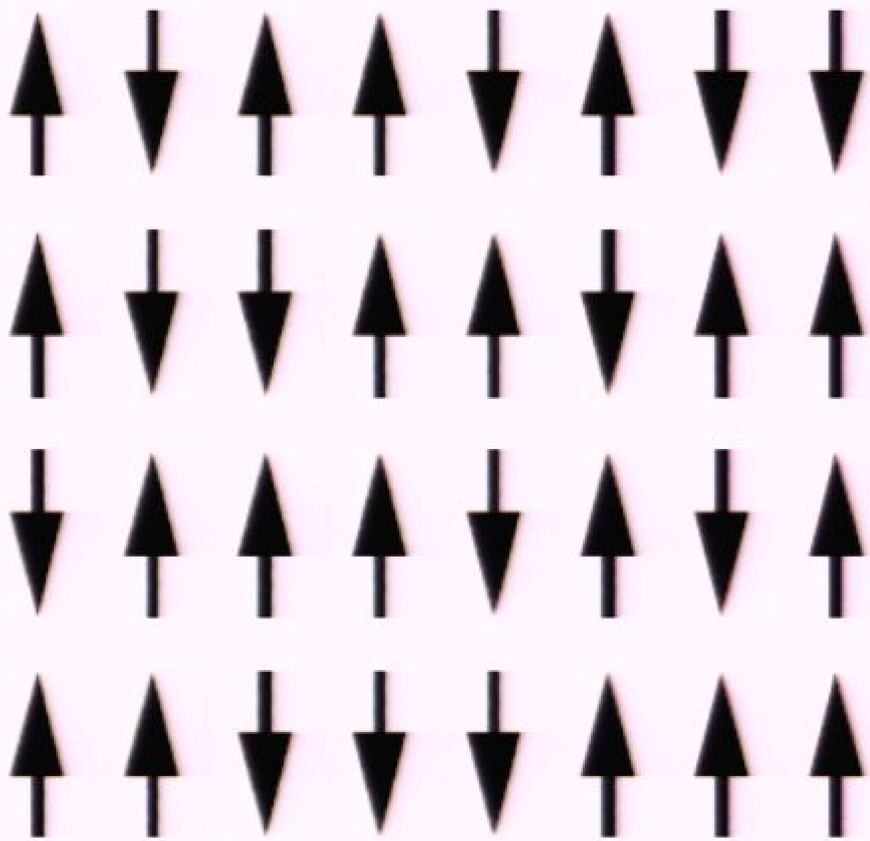
- Consider a lattice of spins
- To go from micro to macro, average over groups of spins
- Rescale



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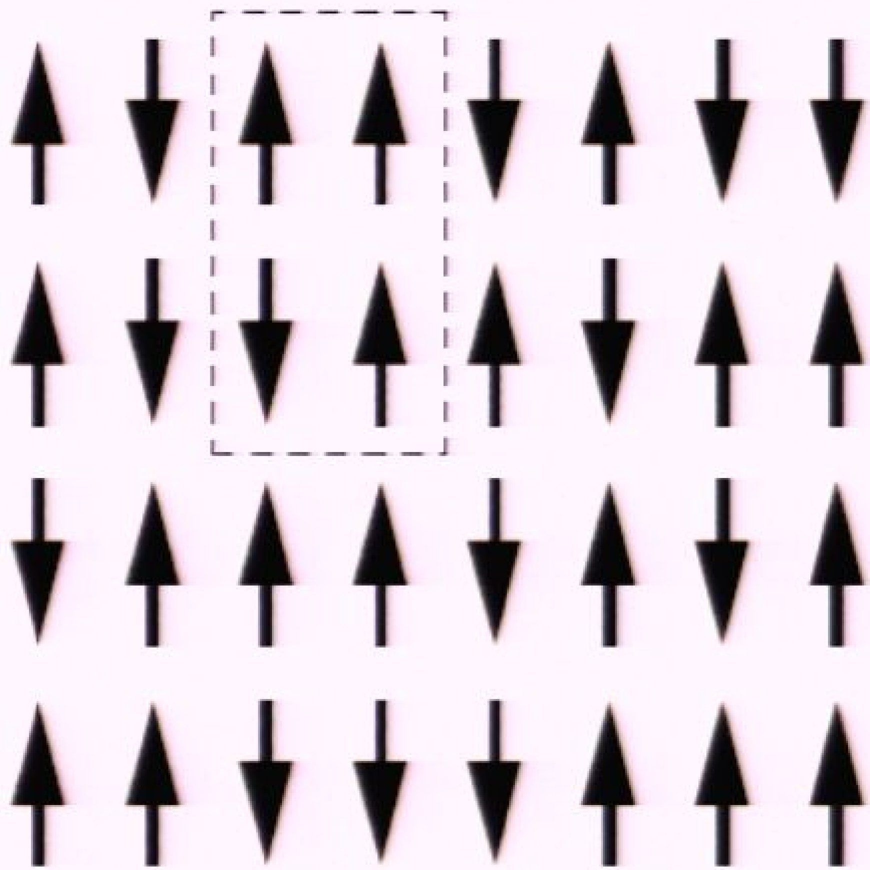

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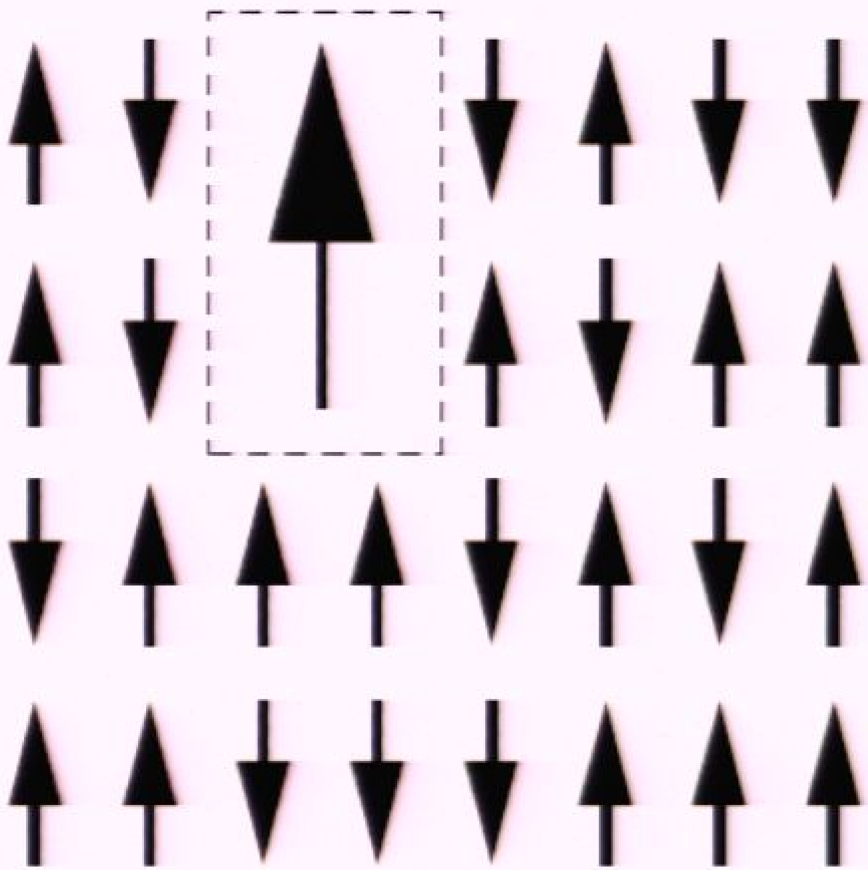

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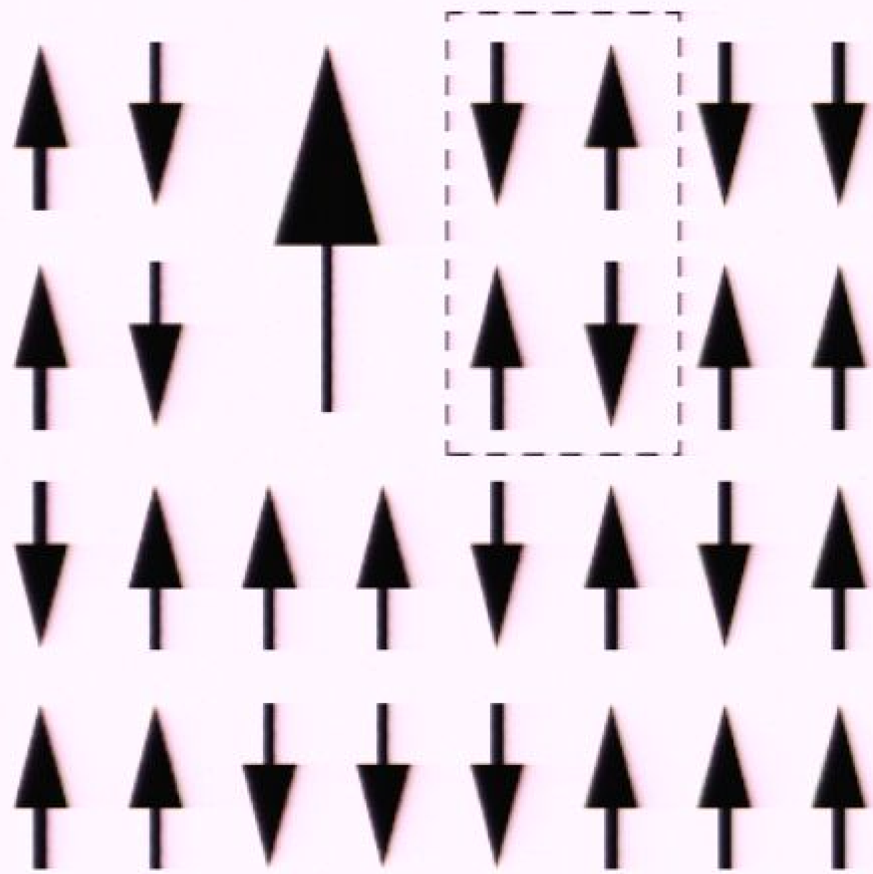

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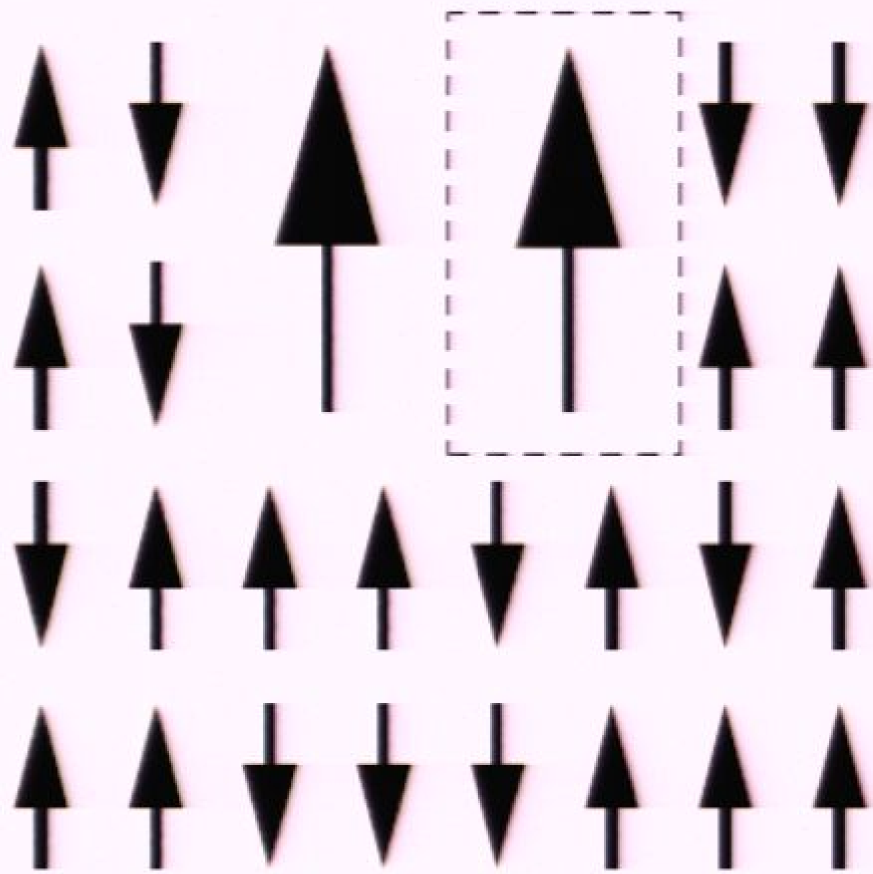

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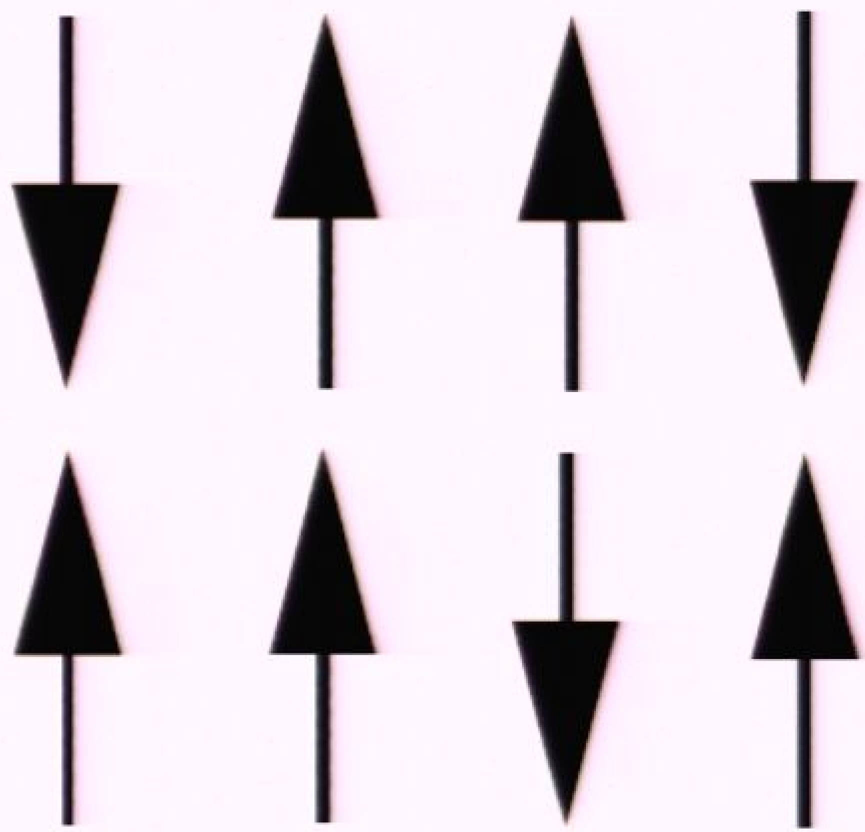

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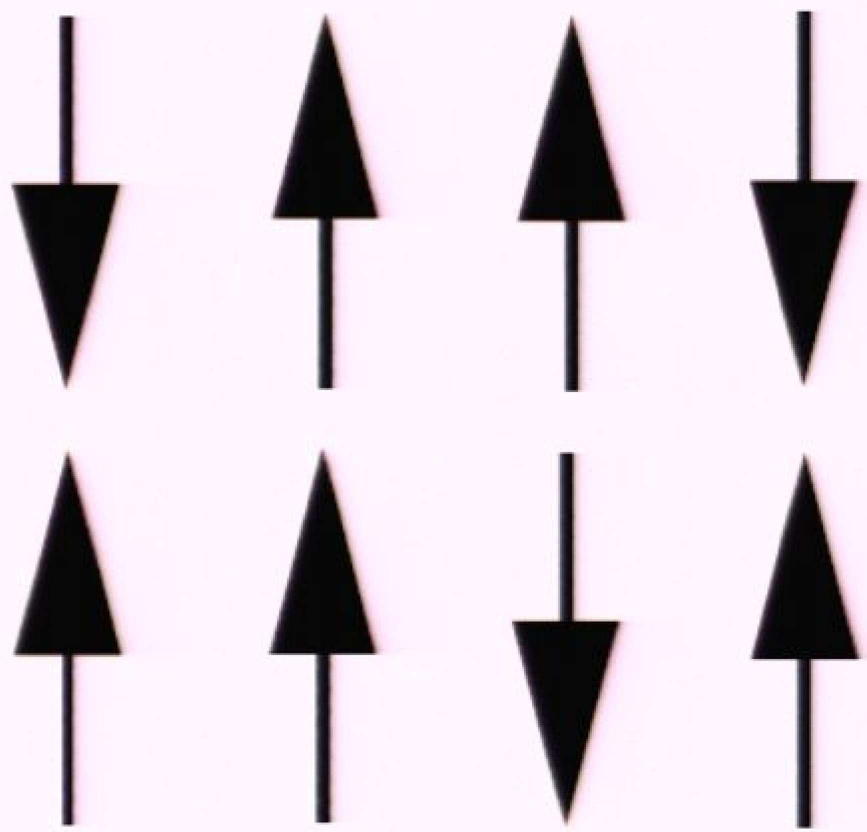

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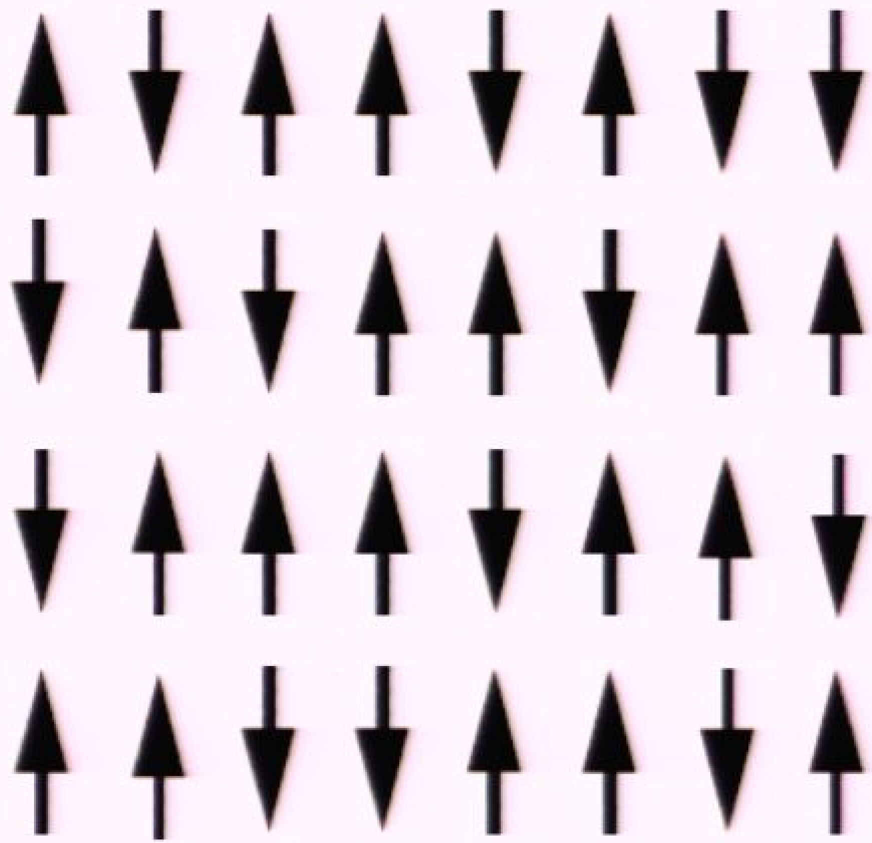

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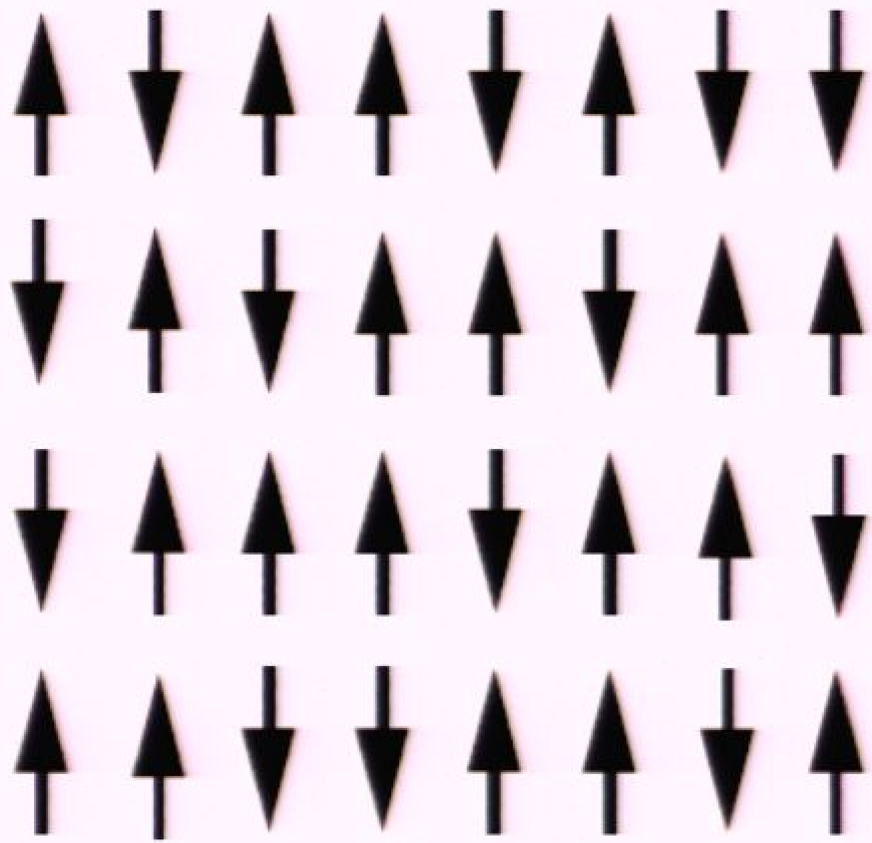

The ERG implements the continuous version of blocking

Flows in Parameter Space

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- The transformation can have fixed points

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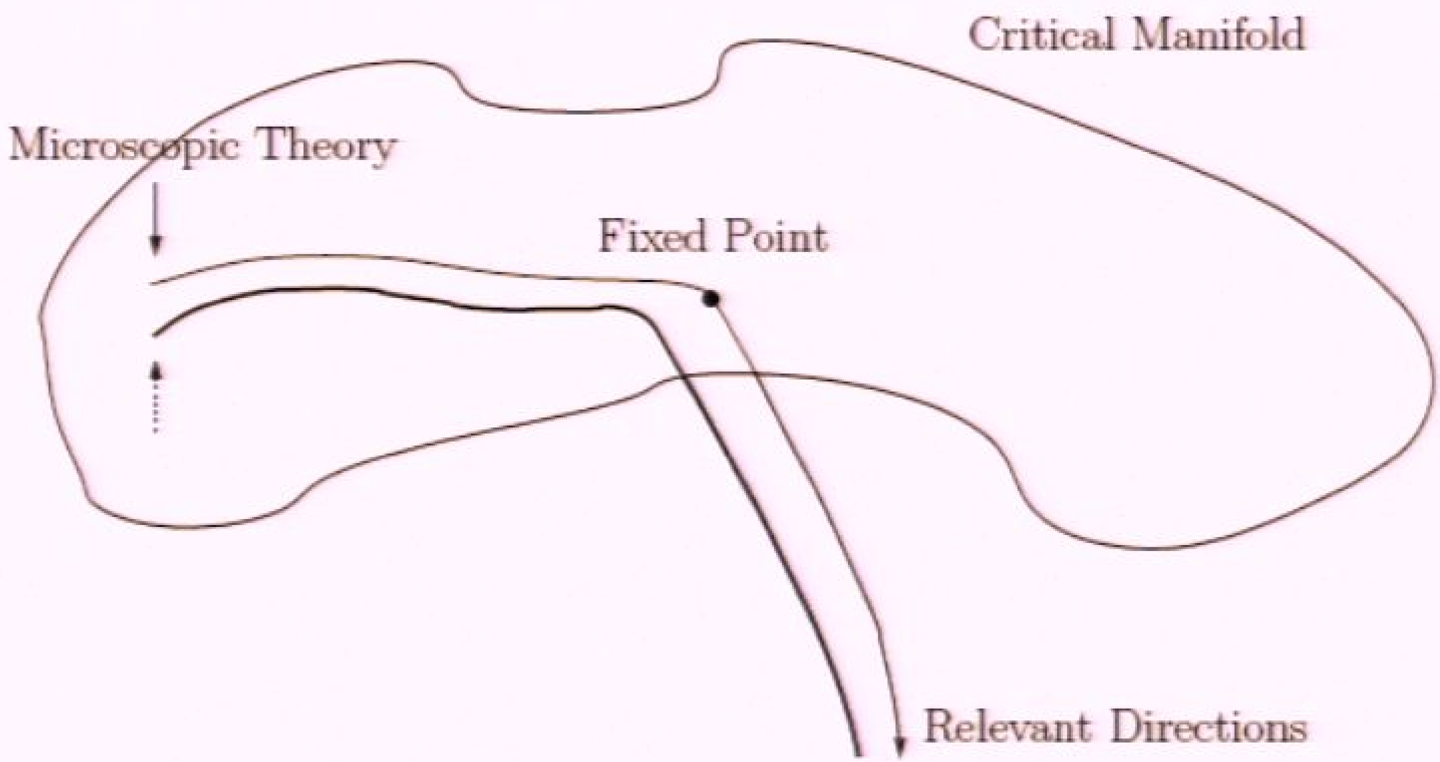

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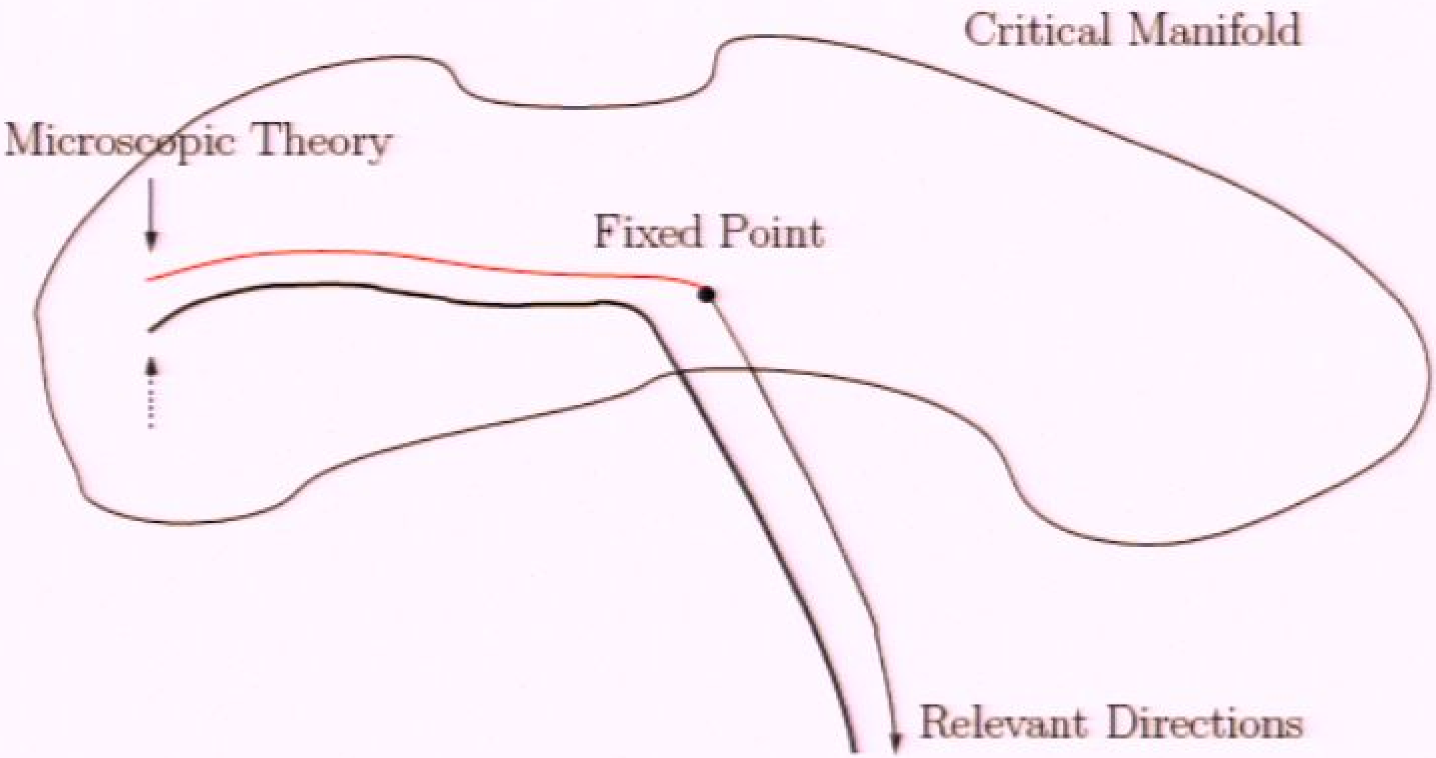

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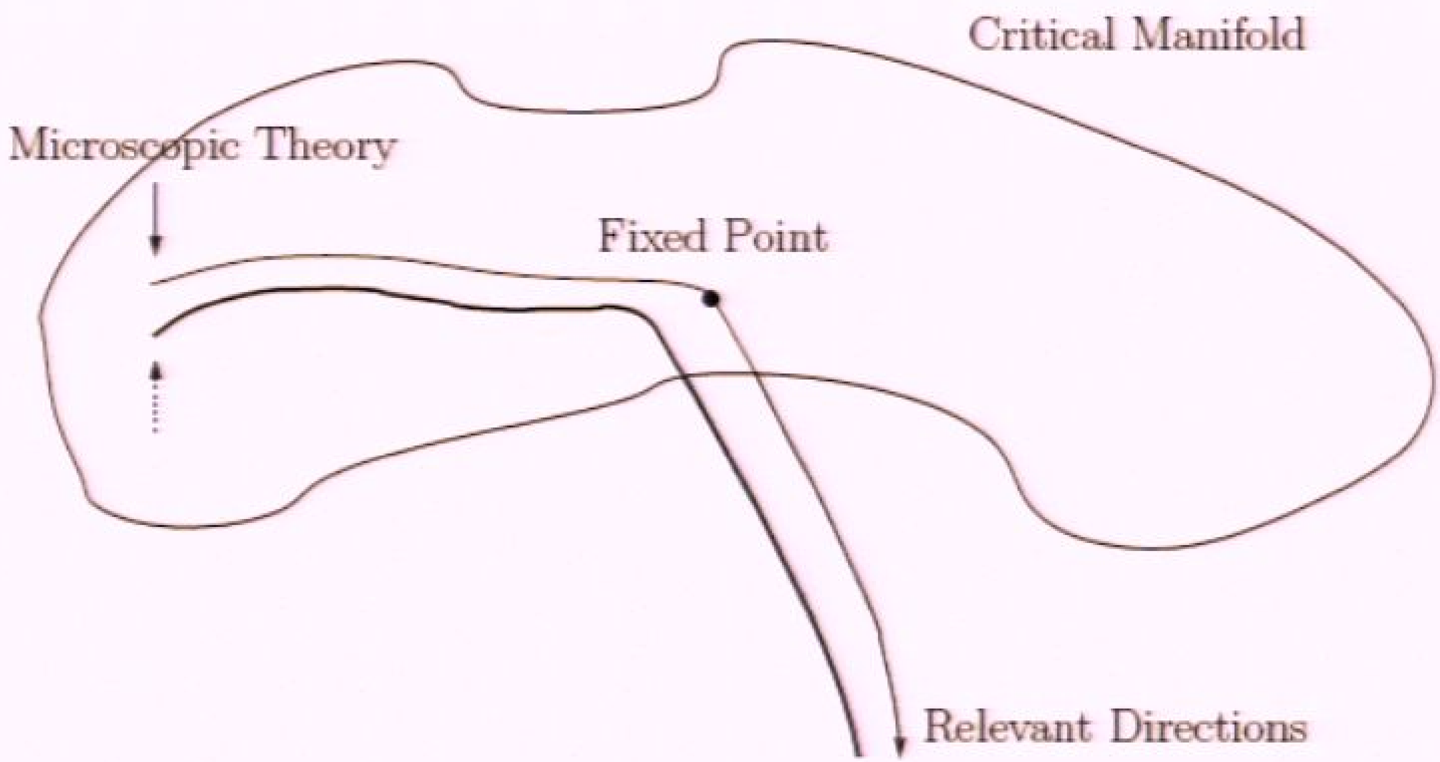

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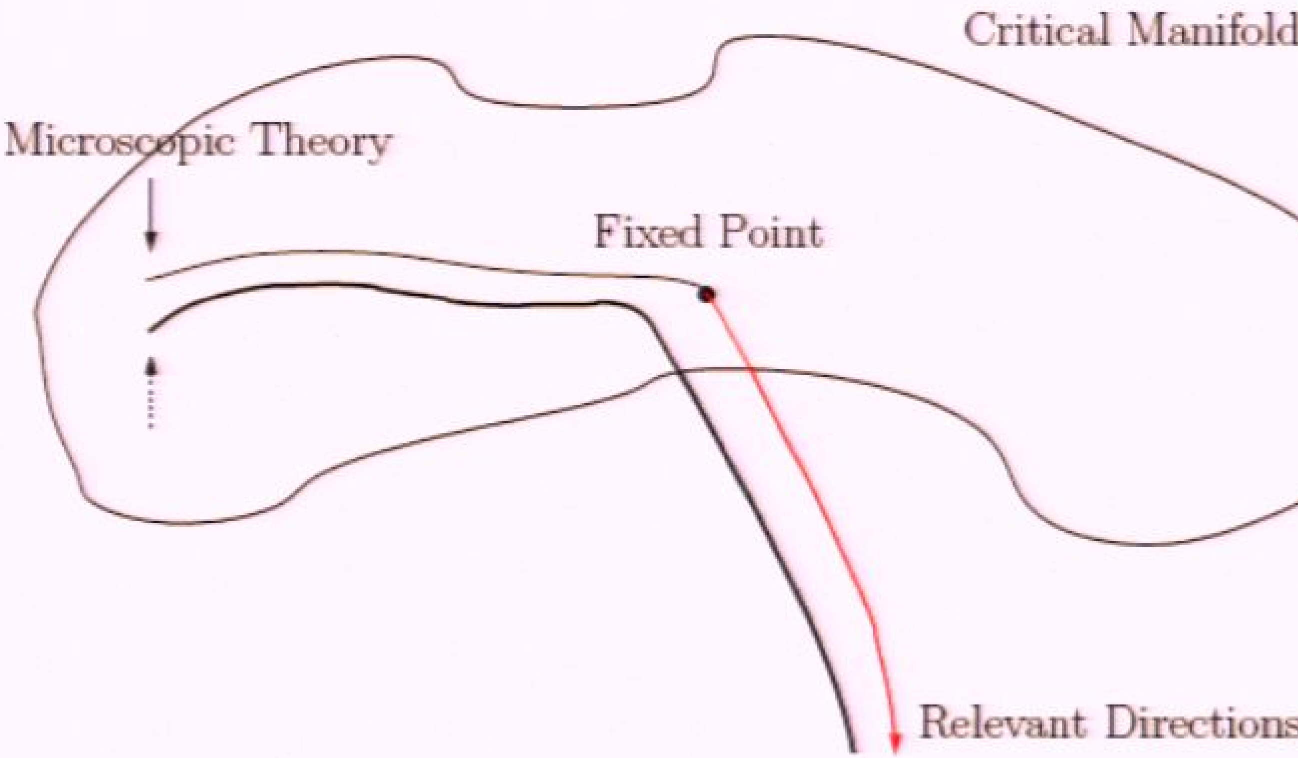

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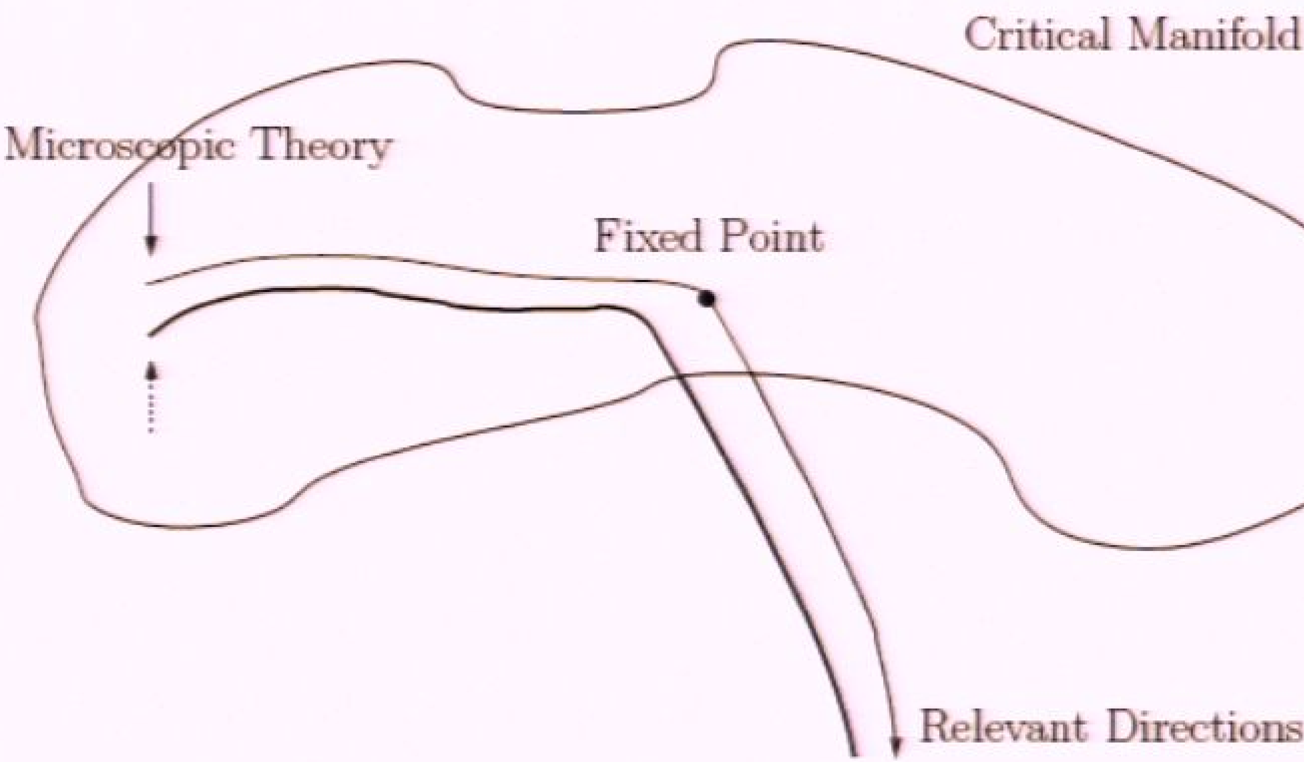
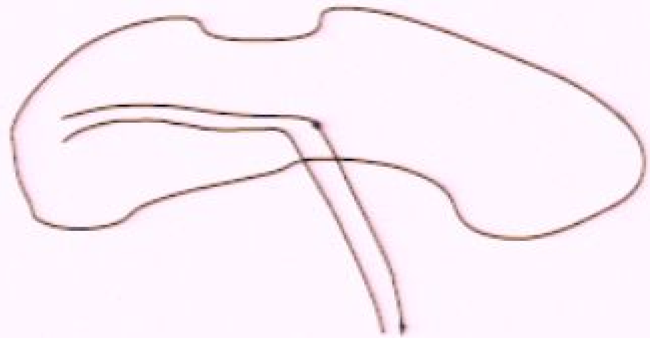


- Trajectories on the critical manifold flow into the fixed point
- The critical manifold is spanned by the irrelevant operators
- Flows along the relevant directions leave the critical surface
- If there are *n* relevant directions, then we must tune *n* quantities to get on to the critical surface

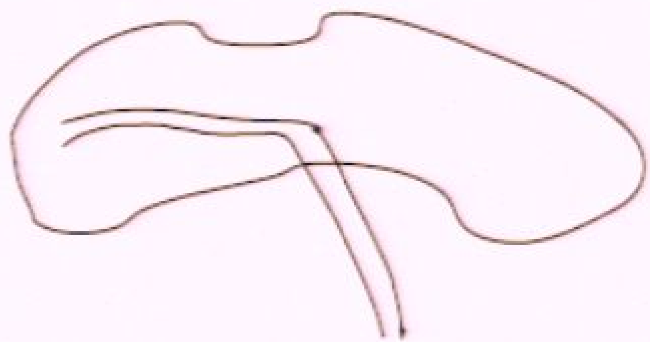
Universality

Universality



- All theories on the critical surface flow towards the fixed point
- Therefore the IR dynamics of these theories is the same
- The theories are in the same universality class
- To determine the critical exponents, move slightly away from the critical surface cf $M \sim (T_c - T)^\beta$
- The IR behaviour is still determined by the fixed point
- The critical exponents are obtained by linearizing the flow around the fixed point
- The critical exponents are universal!

Universality



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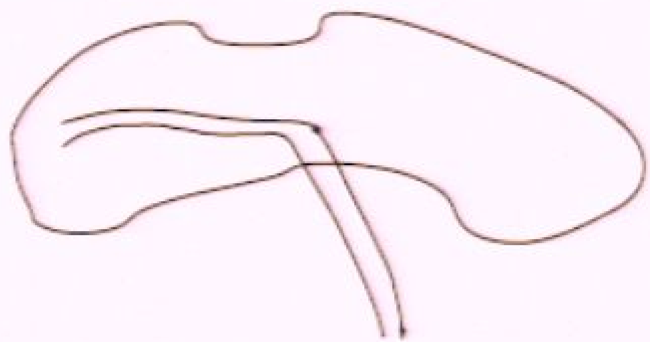

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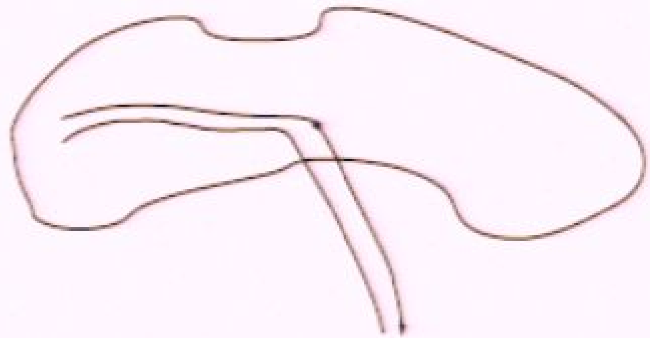

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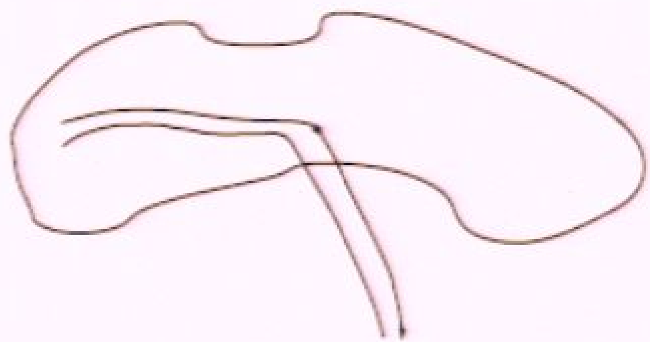

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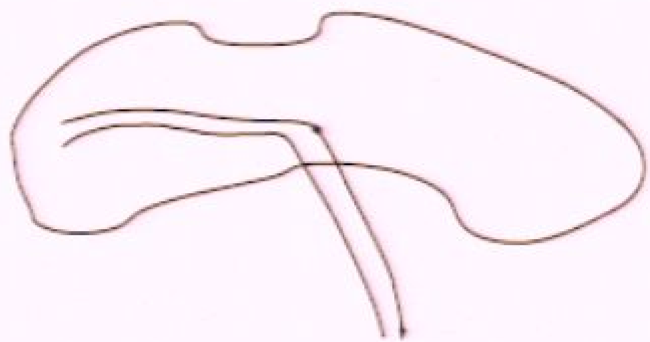

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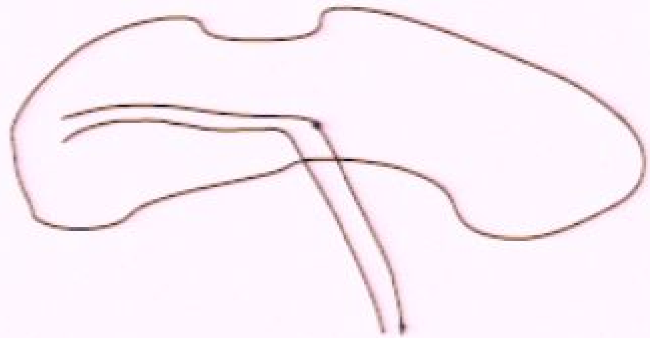

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$$Z = \int_{\Lambda_0} \mathcal{D}\Phi e^{-S_{\Lambda_0}[\Phi]} = \int_{\Lambda} \mathcal{D}\Phi e^{-S_{\Lambda}[\Phi]}$$

- The bare scale
- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ

The Wilsonian Effective Action

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$$Z = \int_{\Lambda_0} \mathcal{D}\Phi e^{-S_{\Lambda_0}[\Phi]} = \int_{\Lambda} \mathcal{D}\Phi e^{-S_{\Lambda}[\Phi]}$$

- The bare scale
  - High energy (short distance) scale
  - Modes above this scale are cut off (regularized)
- The bare (classical) action
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Start with the partition function

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Formulation

$$-\Lambda\partial_\Lambda e^{-S[\varphi]} = \int_x \frac{\delta}{\delta\varphi(x)}\left(\Psi_x[\varphi]e^{-S[\varphi]}\right)$$

- effective scale
- set of fields
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- partition function, $\int \mathcal{D}\varphi e^{-S[\varphi]}$, invariant under the flow
- defines our ERG

Flow Equation

$$-\Lambda\partial_\Lambda S = \int_x \frac{\delta S}{\delta\varphi(x)}\Psi_x - \int_x \frac{\delta \Psi_x}{\delta\varphi(x)}$$

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- Blocking (coarse-graining)
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Implementing Rescaling

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- Remember to take account of anomalous dimensions!
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Linearized ERG Theory

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- At a fixed point we have $\partial_t S_\star = 0$
- Near a fixed point, separate the ERG operator into linear and quadratic parts:

$$\partial_t (S_\star + \Delta S) = \mathcal{L}\Delta S + \mathcal{Q}\Delta S$$

- Separate variables in the linearized equation:

$$\Delta S[\varphi] = \sum_i \alpha_i e^{\lambda_i t} \mathcal{O}_i[\varphi]$$

- Quantization of the λ_i will be discussed in Lecture 3
- Finally, $S_t = S_\star + \sum_i \alpha_i e^{\lambda_i t} \mathcal{O}_i$

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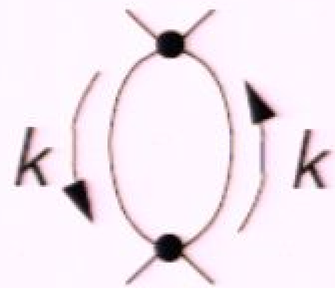

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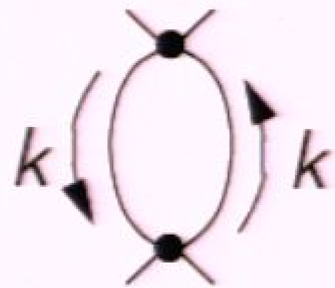

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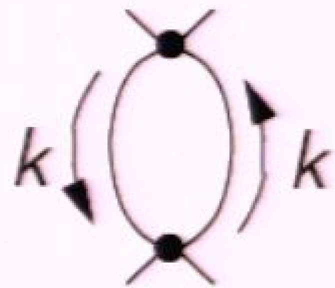

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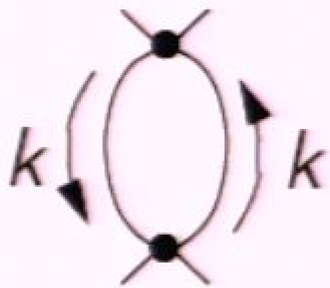

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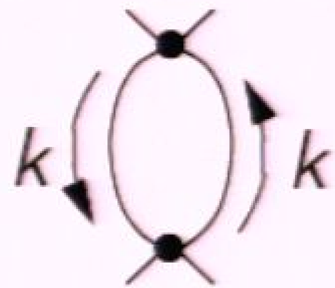

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- With its built in UV cutoff, the ERG is a natural tool to study renormalizability

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The Question

Are there effective actions $S_{\Lambda,\Lambda_0}[\varphi]$ for which we can safely send $\Lambda_0 \to \infty$?

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- Only dimensionless variables appear
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Are there effective actions $S_{\Lambda,\Lambda_0}[\varphi]$ for which we can safely send $\Lambda_0 \to \infty$?

The Simplest Answer

- Rescale all quantities, using Λ
- Only dimensionless variables appear
- Fixed points of the ERG correspond to continuum limits!

$$\partial_t S_\star[\varphi] = 0$$

- $S_\star$ is independent of all scales, including Λ_0
- Trivially, we can send $\Lambda_0 \to \infty$

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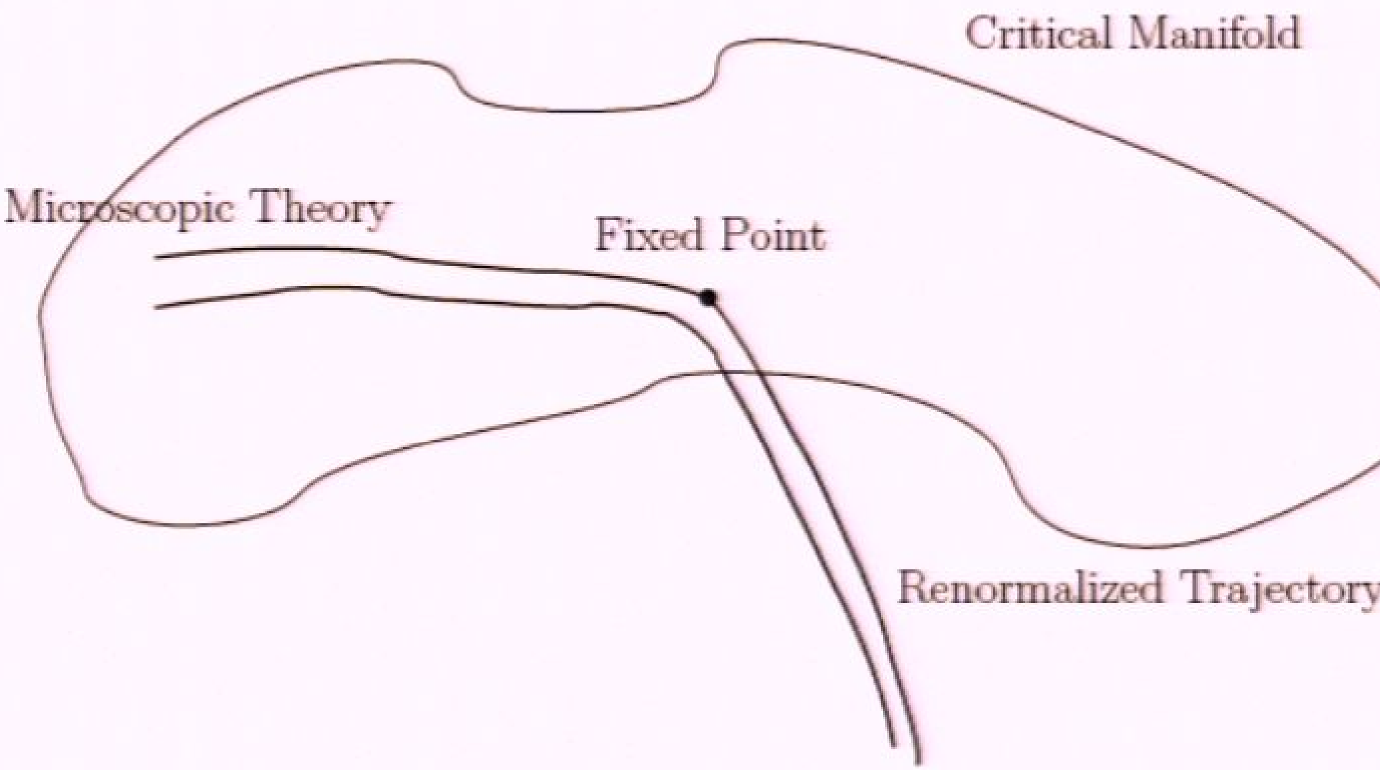

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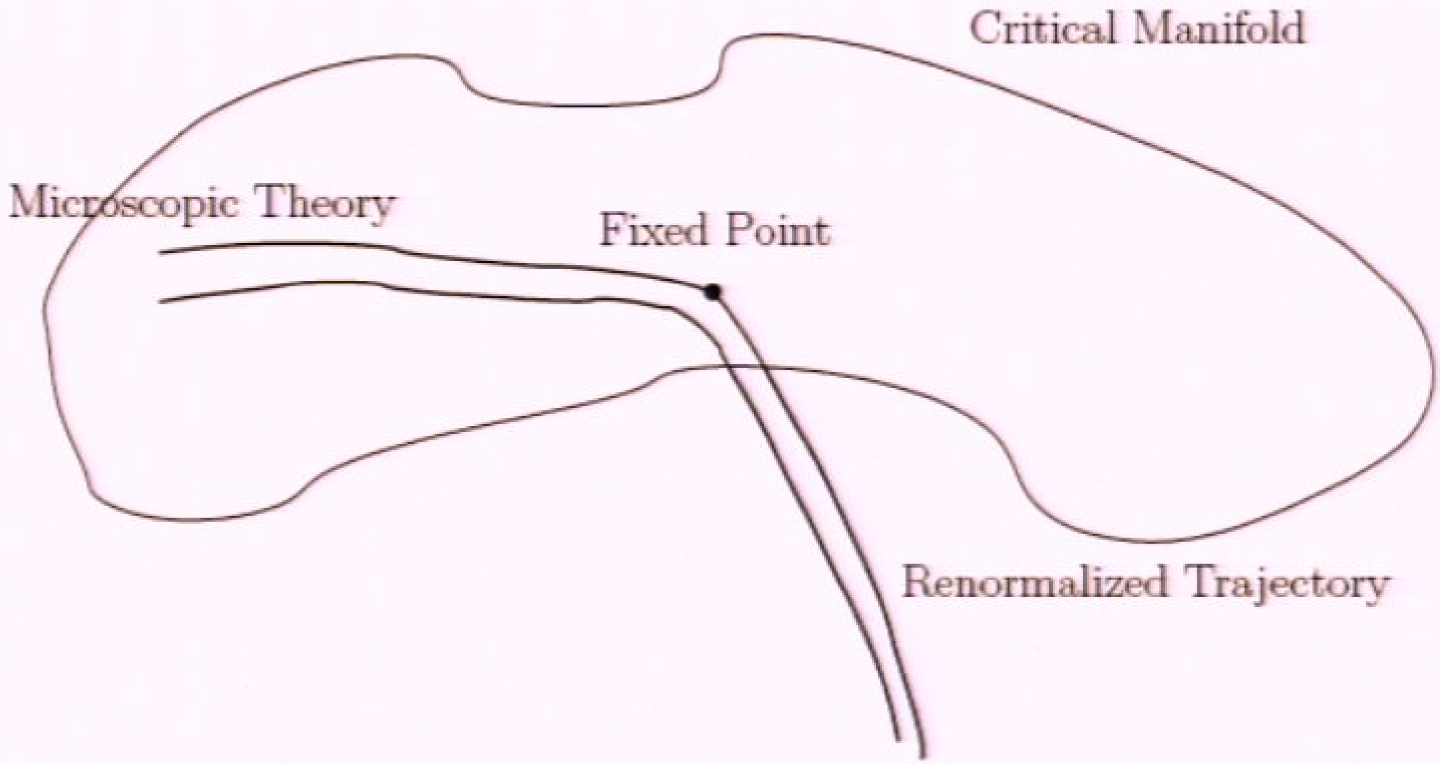

- Tune the trajectory towards the critical surface, as $\Lambda_0 \to \infty$

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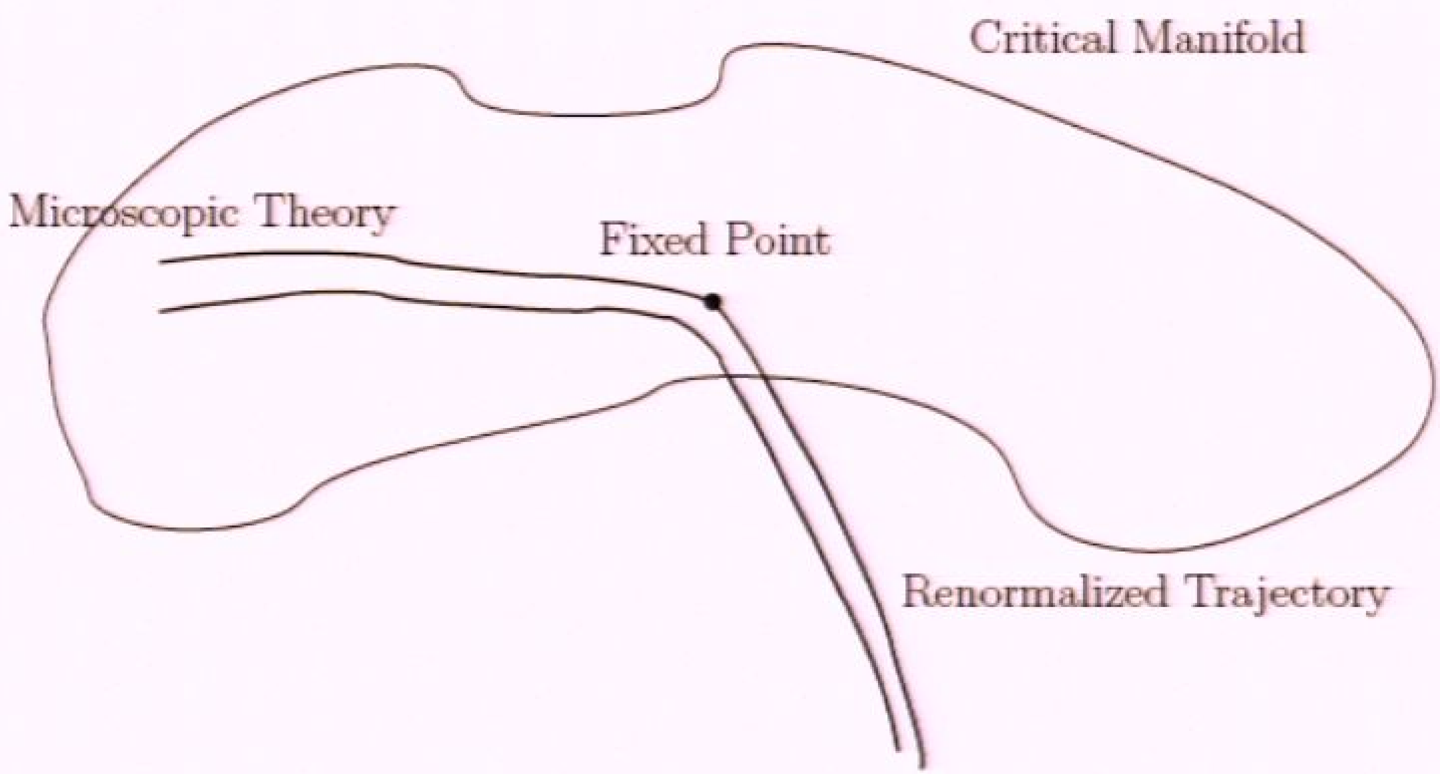

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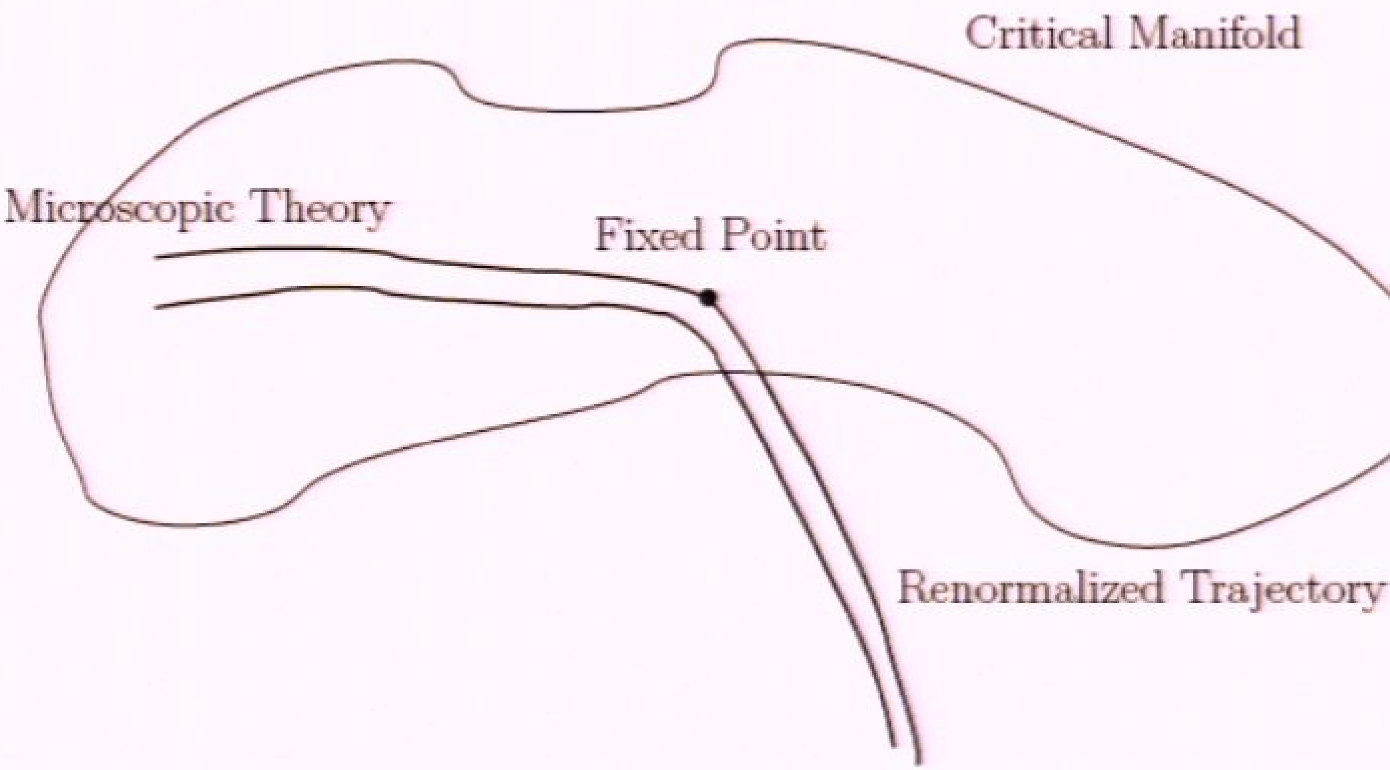

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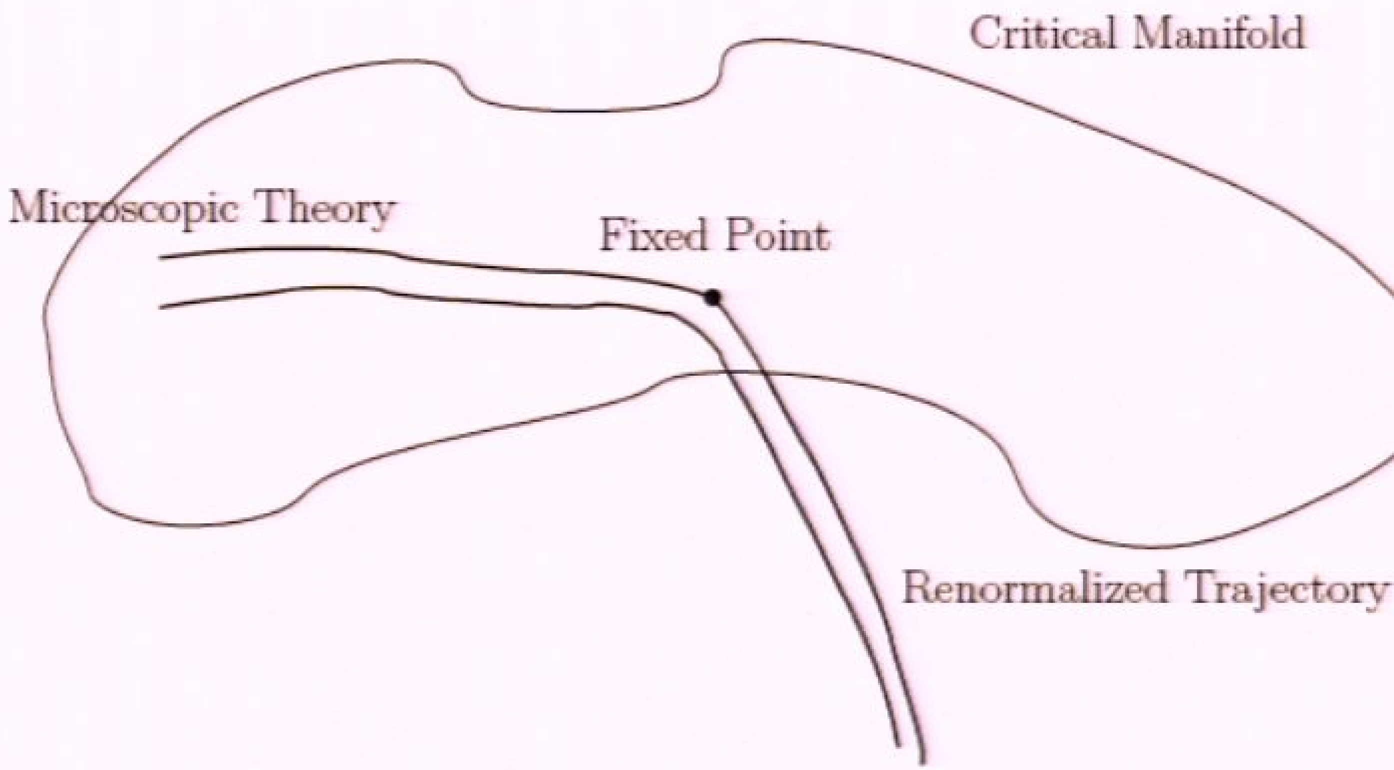

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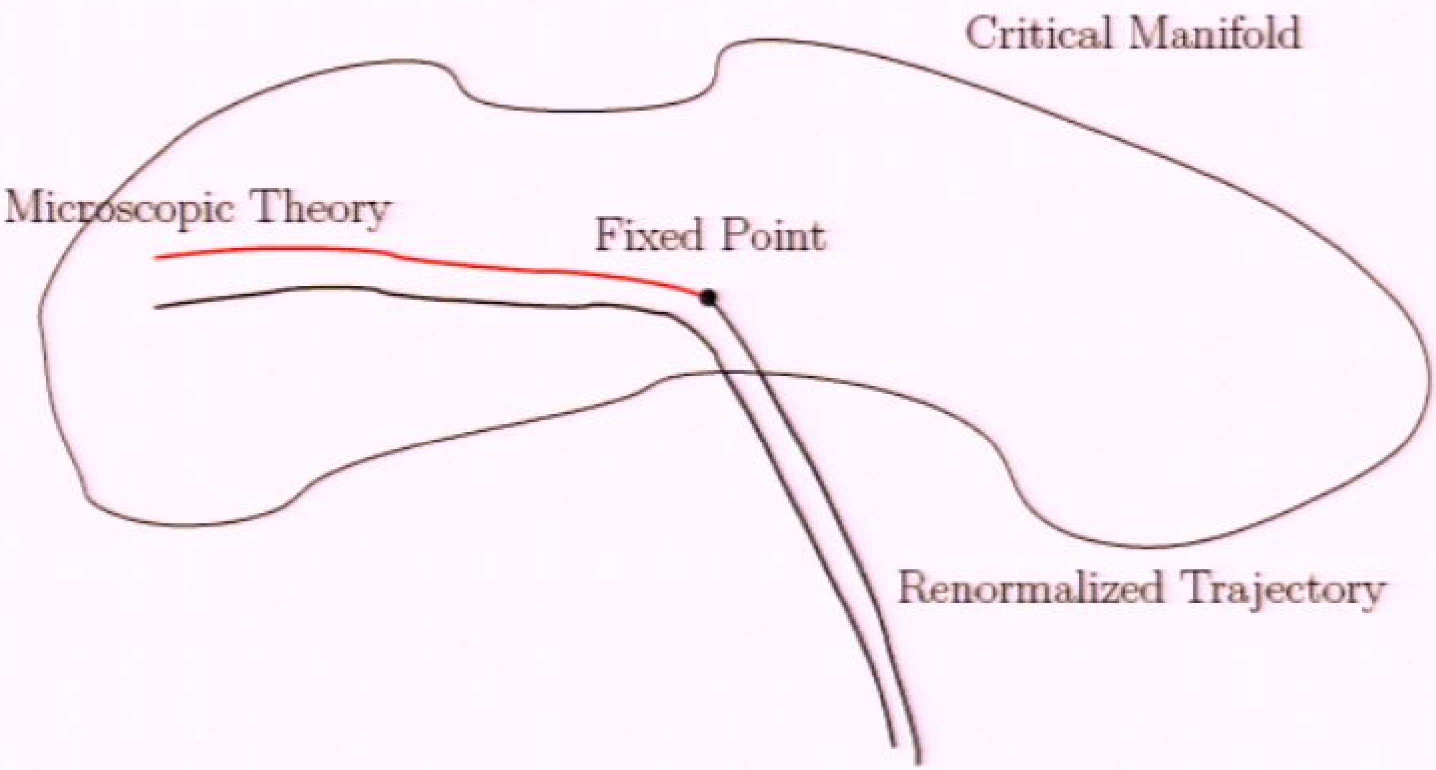

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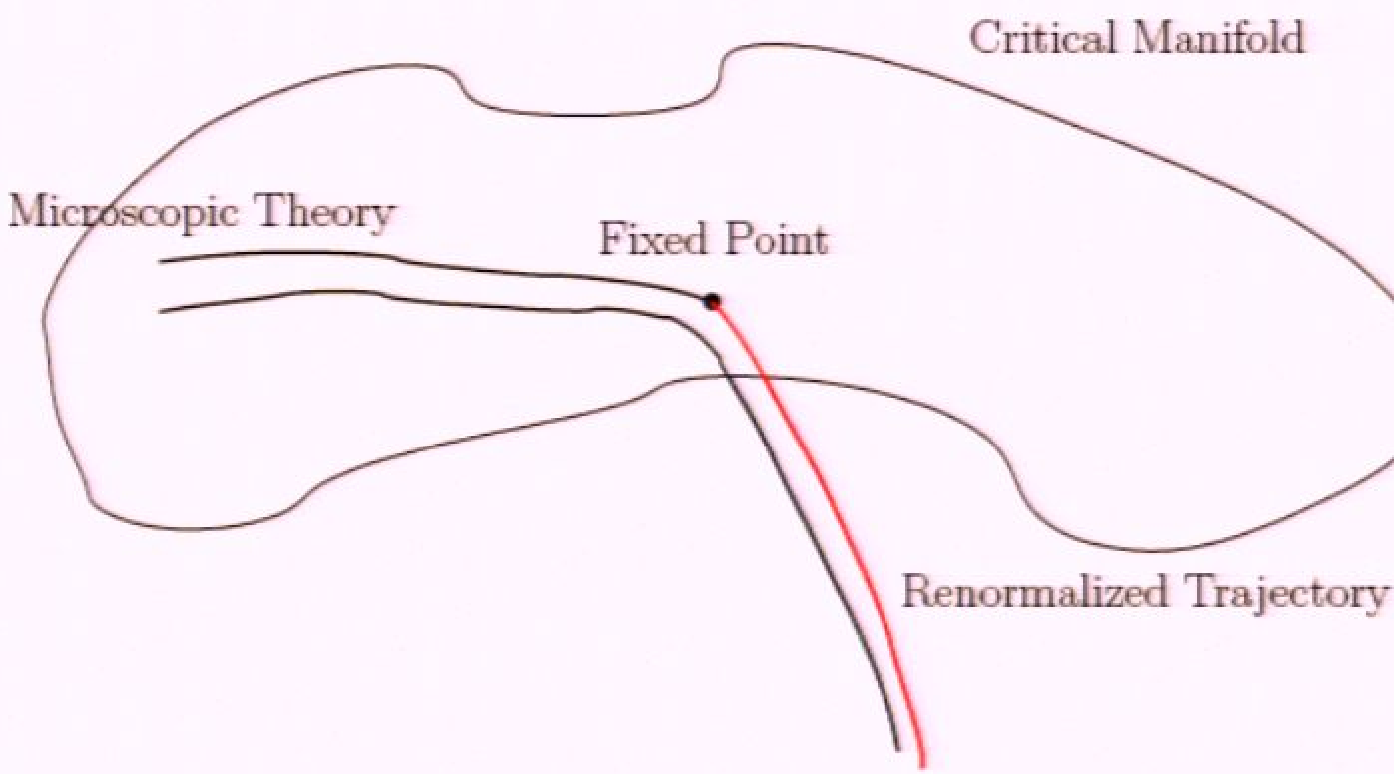

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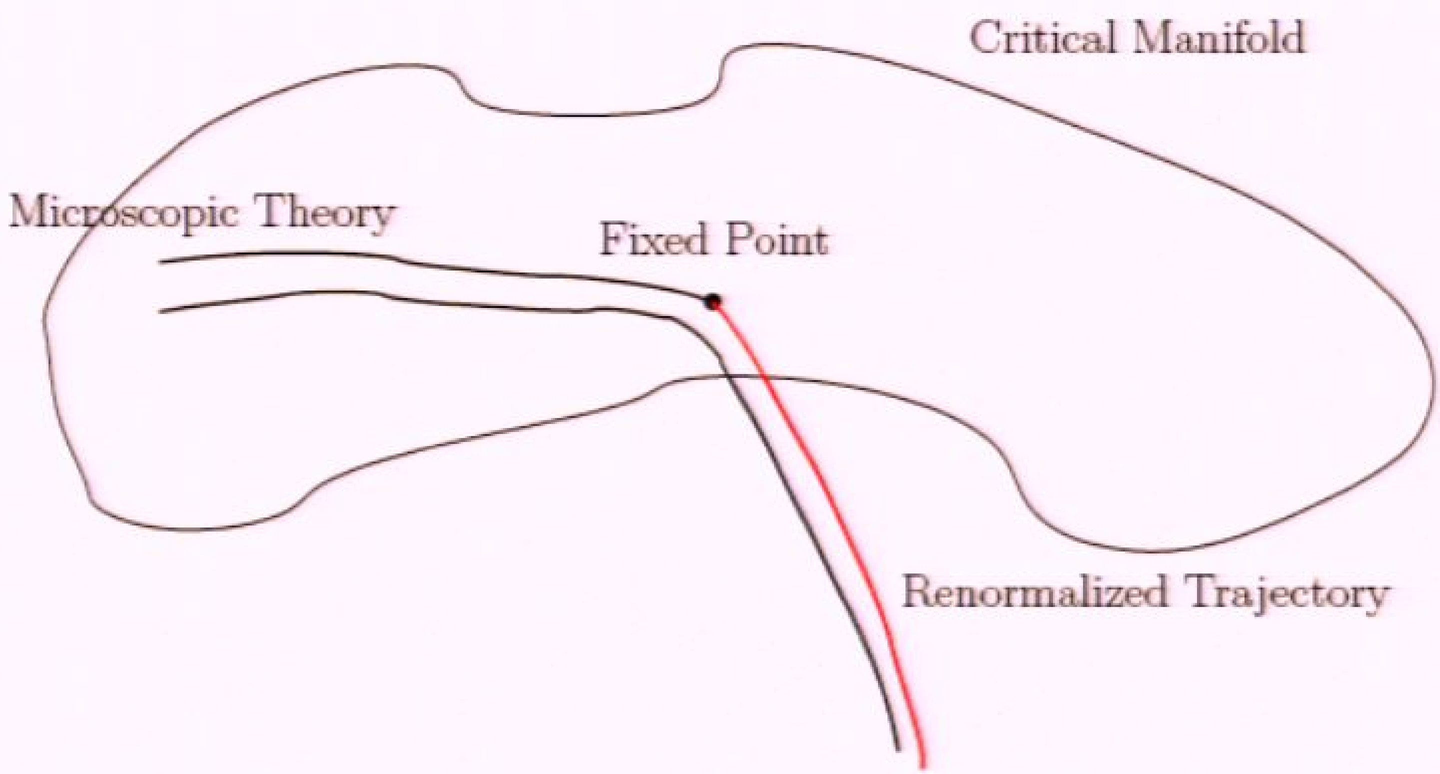

- Tune the trajectory towards the critical surface, as $\Lambda_0 \to \infty$
- The trajectory splits in two:
  - One part sinks into the fixed point
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- Actions on the RT are renormalizable

Renormalized Trajectories

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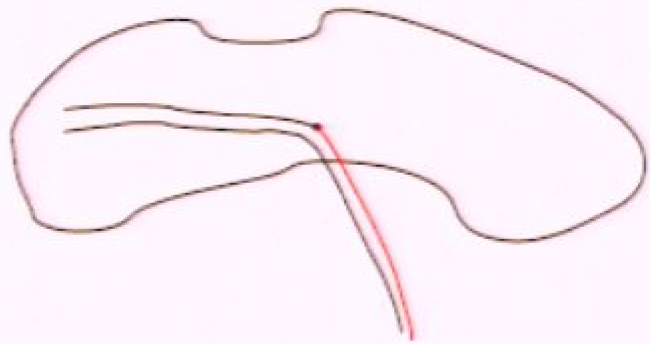

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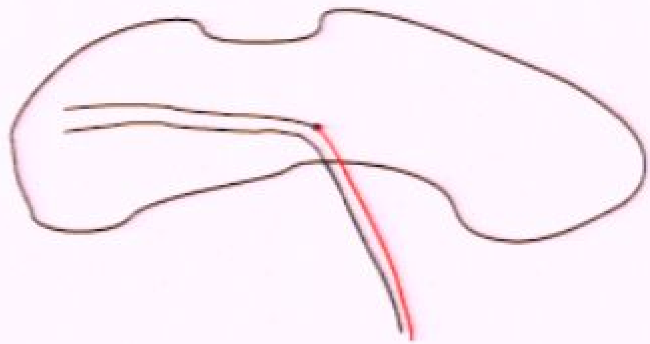

- Linearize ERG about fixed point

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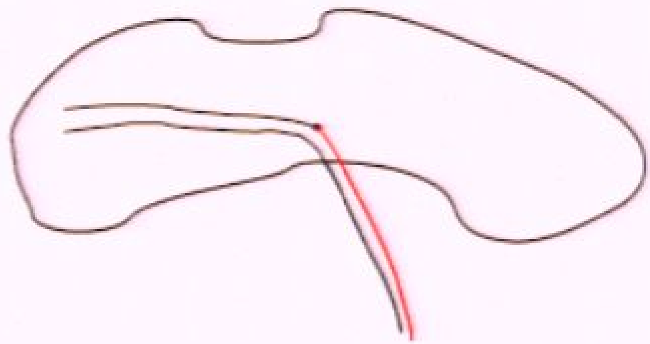

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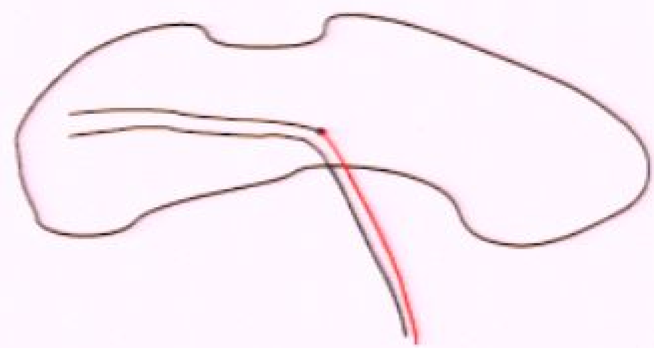

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$$\lim_{t \to -\infty} S_t[\varphi] = S_\star[\varphi] + \sum_{i=1}^{n} \alpha_i e^{\lambda_i t} \mathcal{O}_i$$

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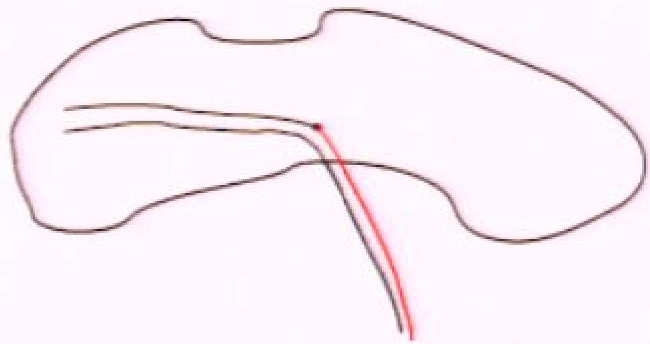

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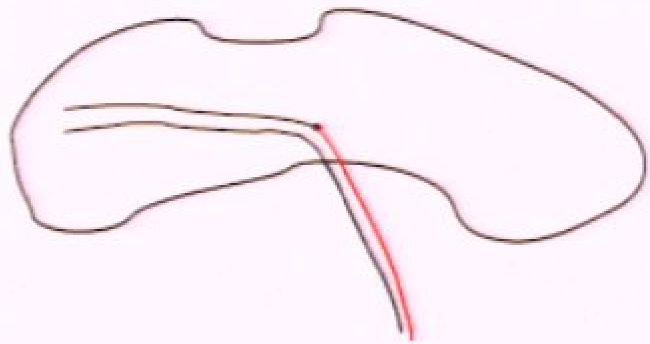

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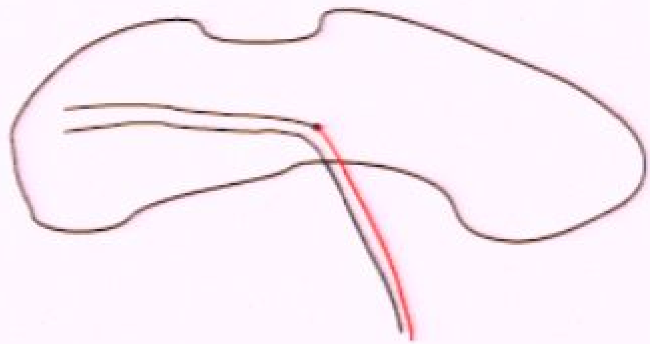

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- If there are marginally relevant terms, include additional terms which sink logarithmically into the fixed point

Renormalized Trajectories II

$$\lim_{\Lambda \to \infty} S_\Lambda[\varphi] = S_\star[\varphi] + \sum_{i=1}^{n} \alpha_i (\mu/\Lambda)^{\lambda_i} \mathcal{O}_i$$

- $S_\Lambda[\varphi] = S_\Lambda[\varphi](\alpha_1, \ldots, \alpha_n)$
- Renormalization Conditions:

- From the functions $g_i(\Lambda), \gamma(\Lambda)$

- Therefore, we can trade Λ and α_i for $g(\Lambda)$ and $\gamma(\Lambda)$
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- Only the Gaussian FP exists
- The mass is relevant
- The four point coupling is marginally irrelevant
- All other couplings are irrelevant
- The only nonperturbatively renormalizable scalar field theories in four dimensions are trivial!

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Scalar Field Theory: Four Dimensions

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- Only the Gaussian FP exists
- The mass is relevant
- The four point coupling is marginally irrelevant
- All other couplings are irrelevant
- The only nonperturbatively renormalizable scalar field theories in four dimensions are trivial!

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M. Beneke, "Renormalons," *Phys. Rept.* **317** (1999) 1, `hep-th/9807443`

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- Consider the divergent series $R \sim \sum_n r_n \alpha^n$
- Divergent series can be useful if they are asymptotic to R in a region, C, of the complex α-plane
- Then there exist numbers K_N such that

$$\left| R(\alpha) - \sum_{n=0}^{N} r_n \alpha^n \right| < K_{N+1} \alpha^{N+1}$$

for all α in C

- For asymptotic series, the truncation error decreases at first, but then increases
- If this happens after a large number of terms, N_*, then the truncation error is exponentially small

$$K_{N_*} \alpha^{N_*} \sim e^{-1/|a|\alpha}$$

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- To improve the approximation to R, the series must be resummed
- Define the Borel transform:

$$R \sim \sum_n r_n \alpha^{n+1} \Rightarrow B[R](t) \equiv \sum_{n=0}^{\infty} r_n \frac{t^n}{n!}$$

- From which the series expansion for R can be reconstructed

$$\tilde{R} = \int_0^\infty dt\, e^{-t/\alpha} B[R](t)$$

- If $B[R](t)$ has no singularities for real positive t (and dies off fast enough)
- Strictly, renormalons refer to singularities in the Borel plane with a particular origin
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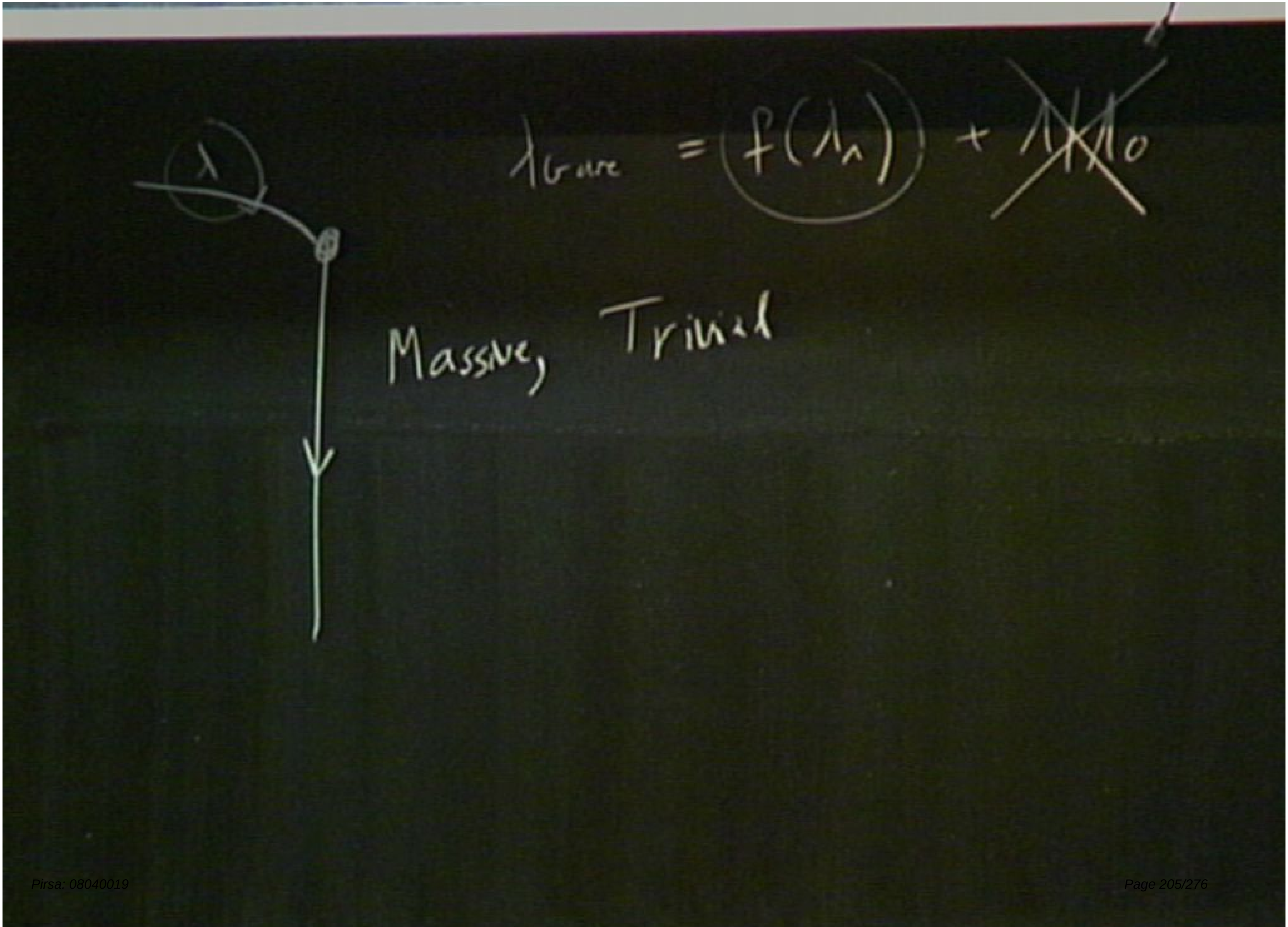
Massive, Trivial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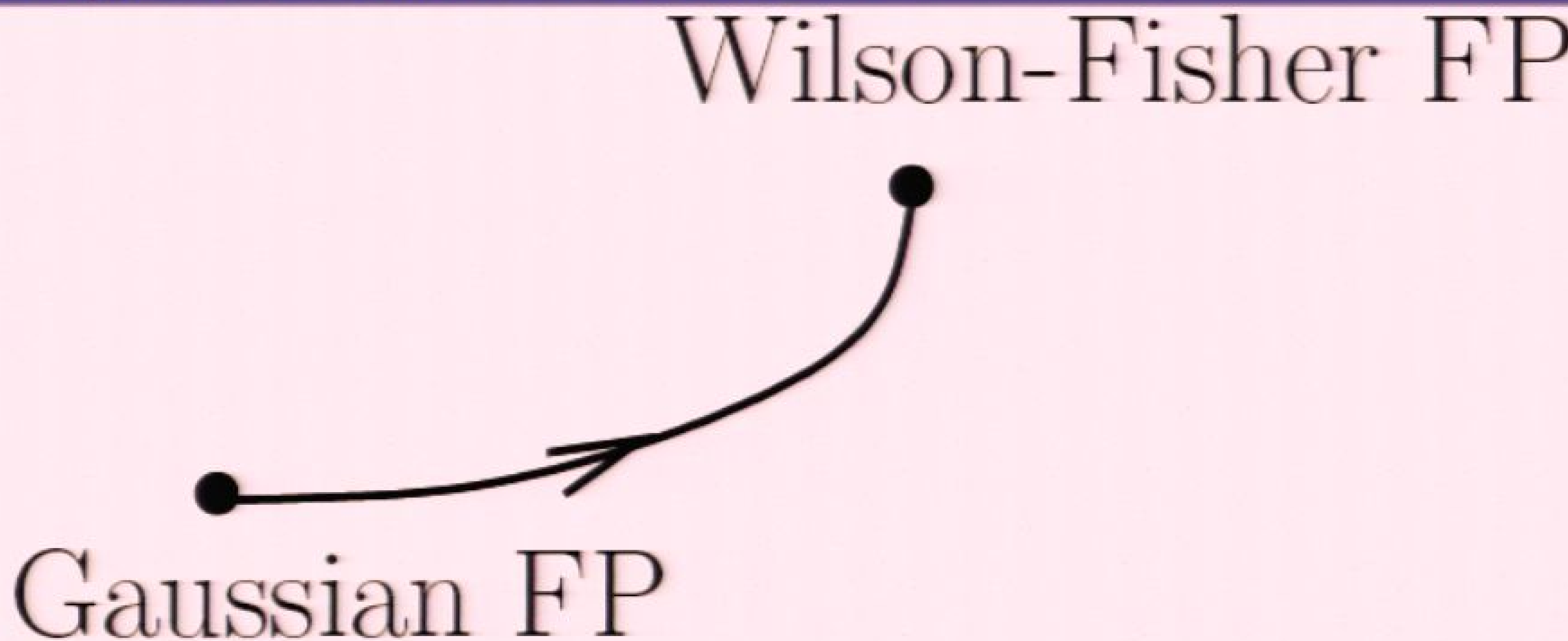
Example of a Continuum Limit in D=3



Asymptotic Freedom etc.

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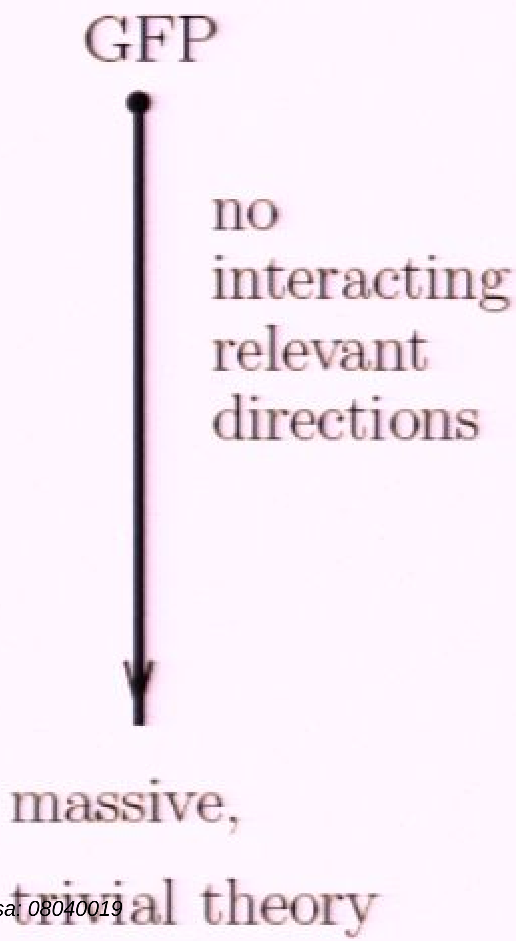
Triviality Asymptotic Freedom Asymptotic Safety



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GFP

no interacting relevant directions

massive, trivial theory

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Triviality	Asymptotic Freedom	Asymptotic Safety
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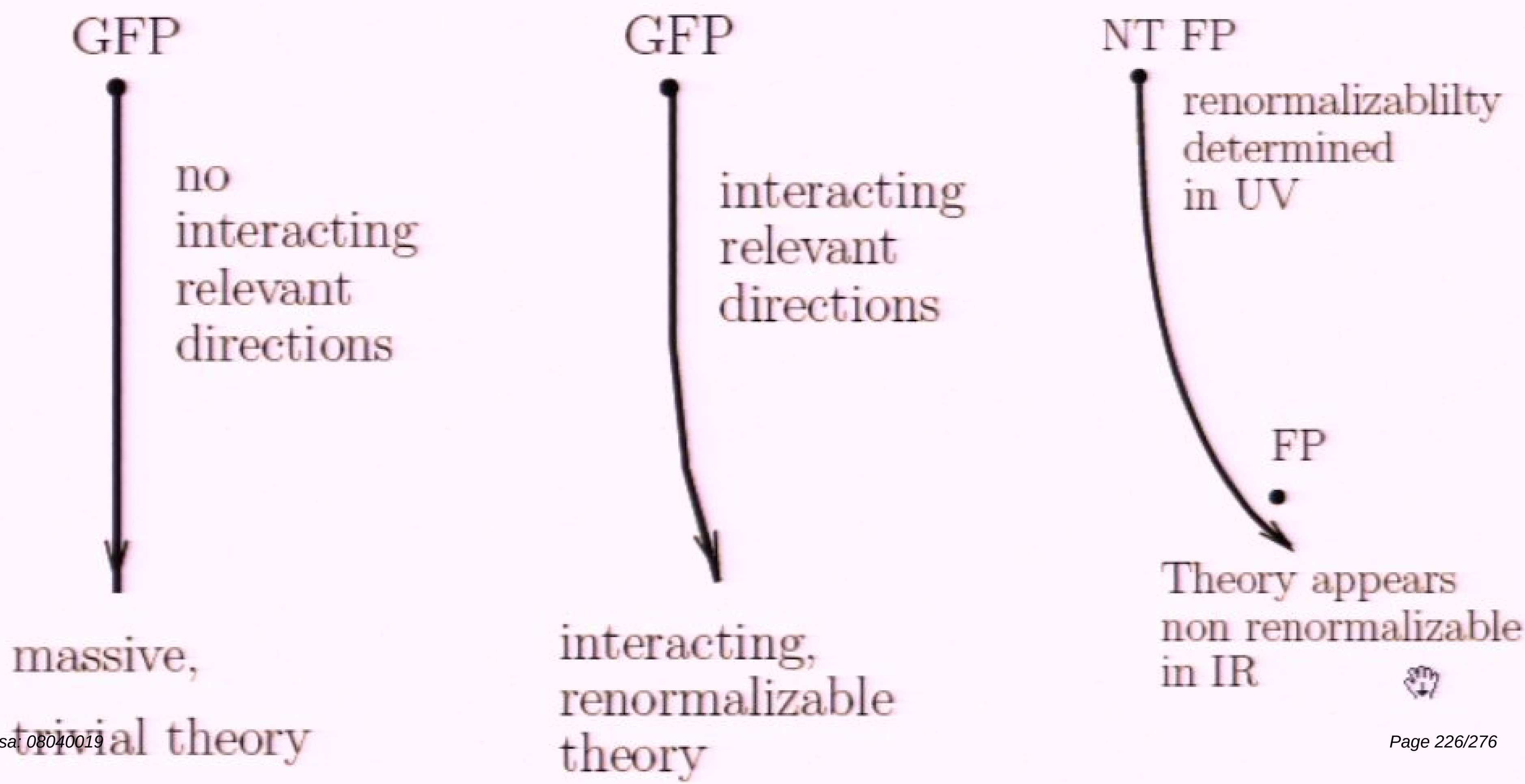

What is the Bare Action?

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Renormalized Trajectories

- Start from a fixed point ($\Lambda = \infty$)
- Choose the integration constants associated with the relevant directions
- Solve for the RT action at all other scales
- Actions along an RT are sometimes called 'Perfect Actions'
- Bare action is sometimes referred to as

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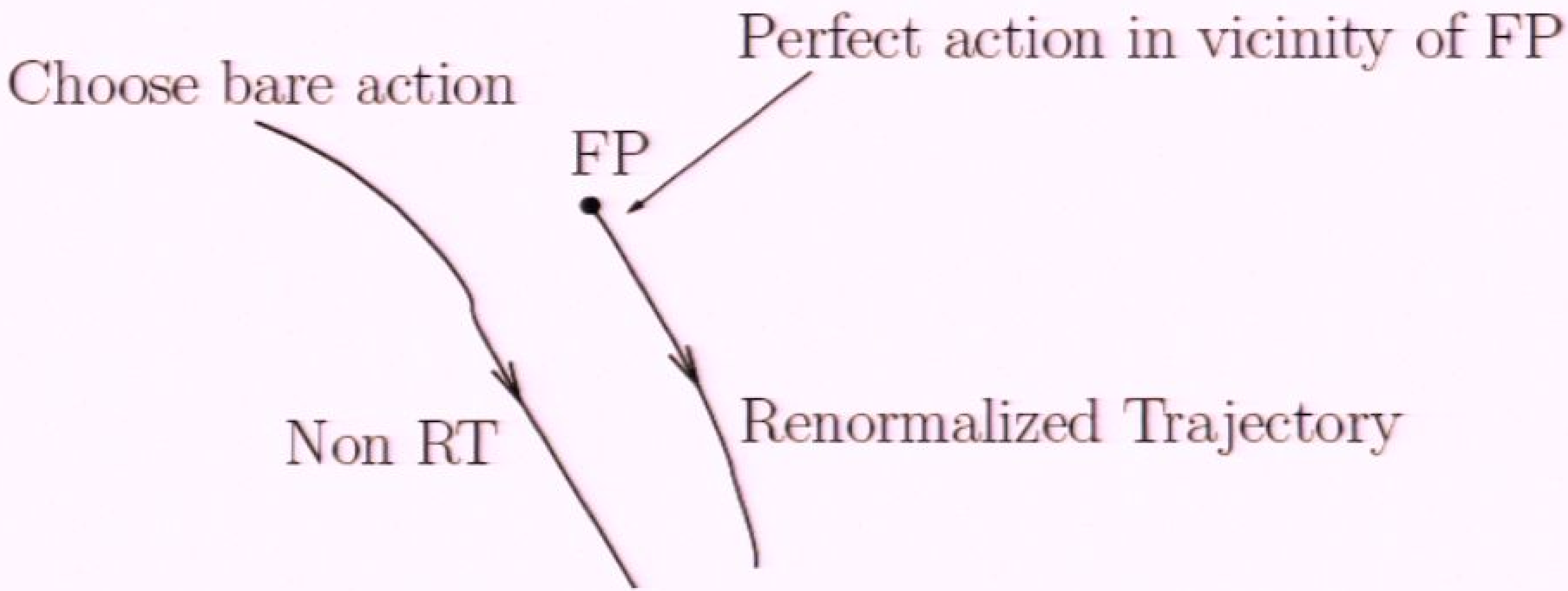

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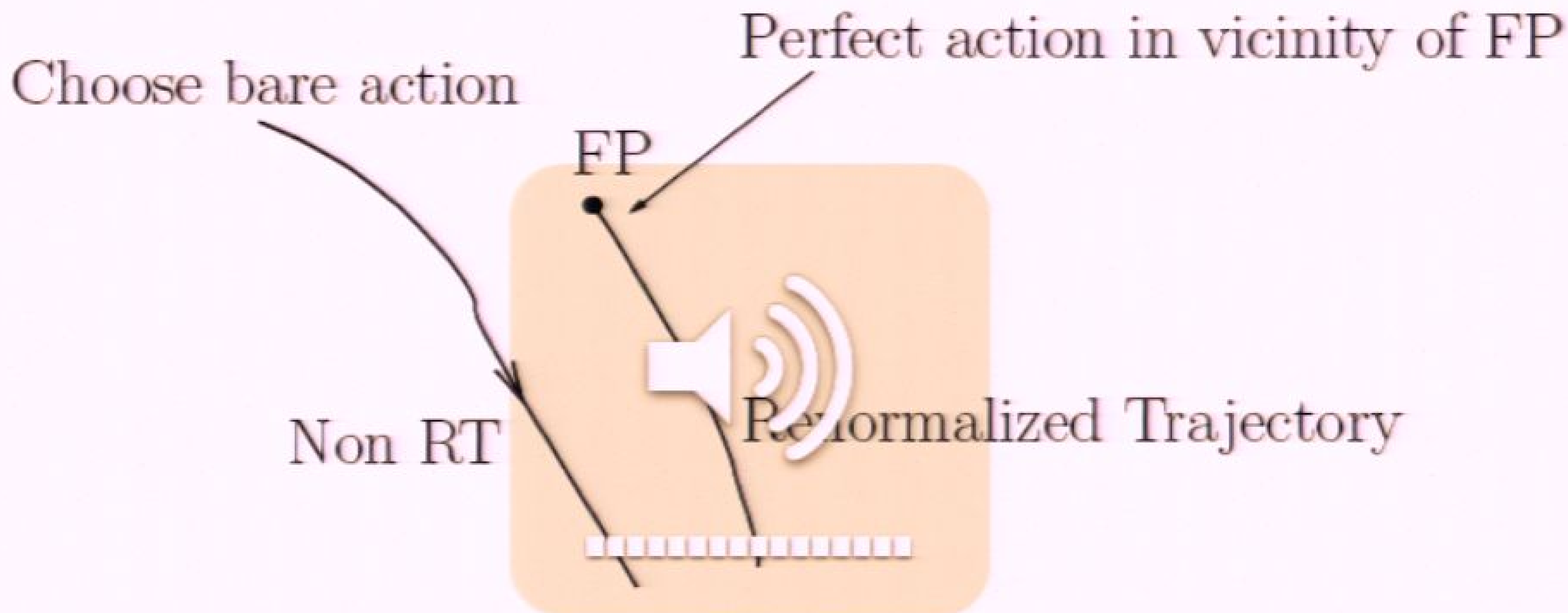

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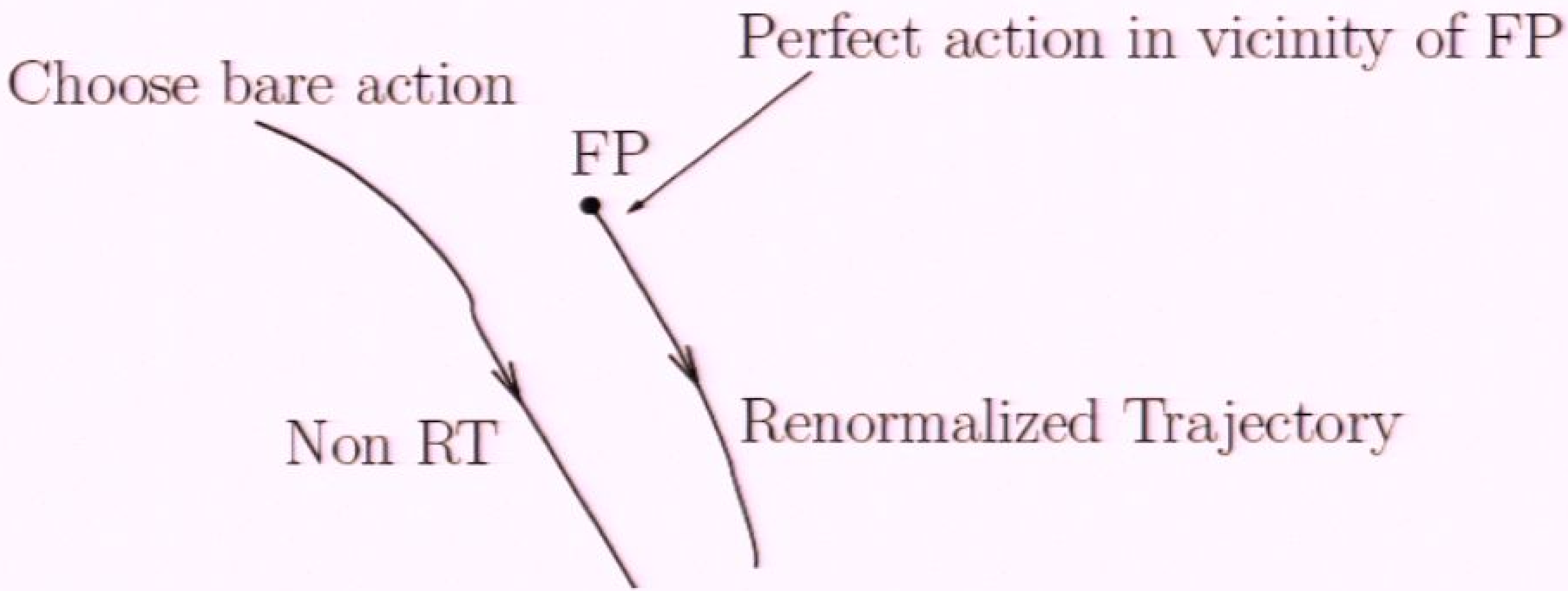

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$$S_\Lambda[\varphi] = S[\varphi]\,(g_1(\Lambda), \ldots, g_n(\Lambda), \gamma(\Lambda))$$

- All scale dependence occurs through $g_i(\Lambda)$ and $\gamma(\Lambda)$
- But this does not mean that eg the non-trivial RT leaving the Gaussian fixed point in $D = 3$ does not develop $c^{(6)}\varphi^6$, $c^{(8)}\varphi^8 \ldots$ interactions
- It just means that $c^{(6)}(\Lambda) = c^{(6)}(g_i, \gamma)$, with $c^{(6)}(\Lambda = \infty) = 0$
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How Much do the Relevant Directions Tell Us?

- Along an RT

$$S_\Lambda[\varphi] = S[\varphi]\,(g_1(\Lambda), \ldots, g_n(\Lambda), \gamma(\Lambda))$$

- All scale dependence occurs through $g_i(\Lambda)$ and $\gamma(\Lambda)$

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- Consider a fixed point with n relevant directions then
- To tune a model on to the critical surface, an experimentalist must 'adjust n knobs'
- Eg for a ferromagnet, we adjust the external magnetic field (to zero) and the temperature
- Tuning these knobs moves our model around in parameter space
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- Under ERG transformations all critical point theories in the same universality class flow into the same fixed point

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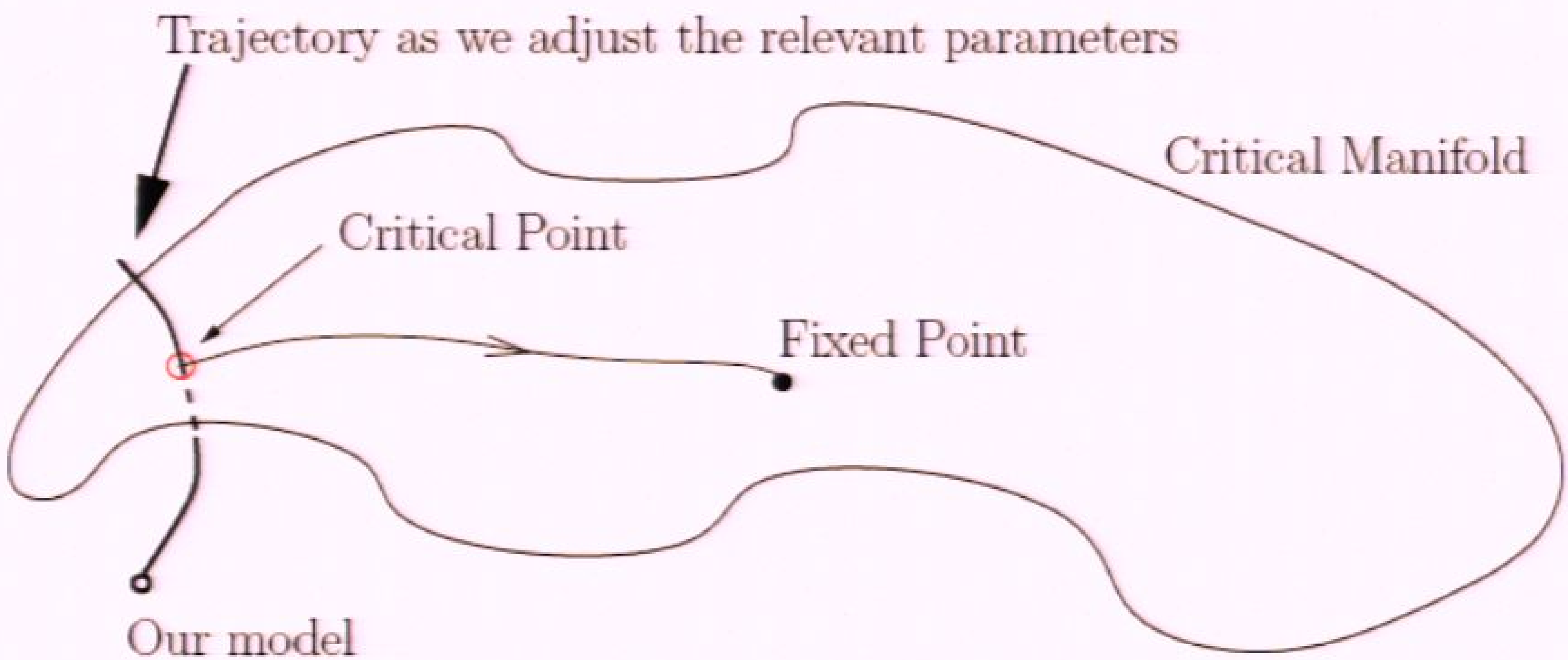

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- The ERG relates physics at different scales via a coarse-graining procedure
- Under an ERG transformation, the effective action evolves
- Including a rescaling step, we find fixed points,
- By linearizing the ERG equation, we define the scaling fields which can be relevant, irrelevant, marginal (or redundant)
- The relevant (and marginally relevant) directions can be used to construct scale dependent continuum limits
- These 'Renormalized Trajectories' (RTs) are self-similar
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Scalar Field Theory: Three Dimensions

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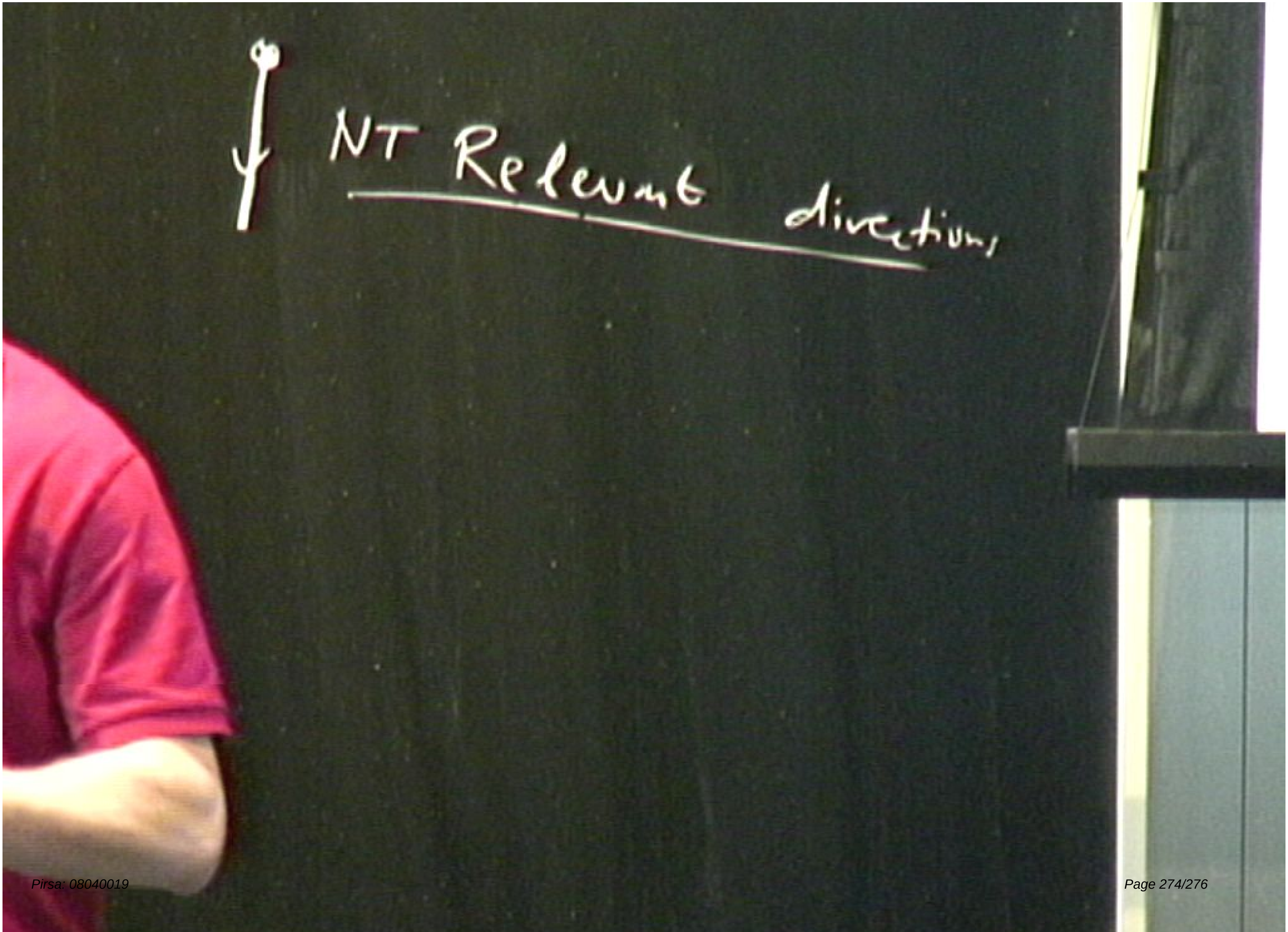
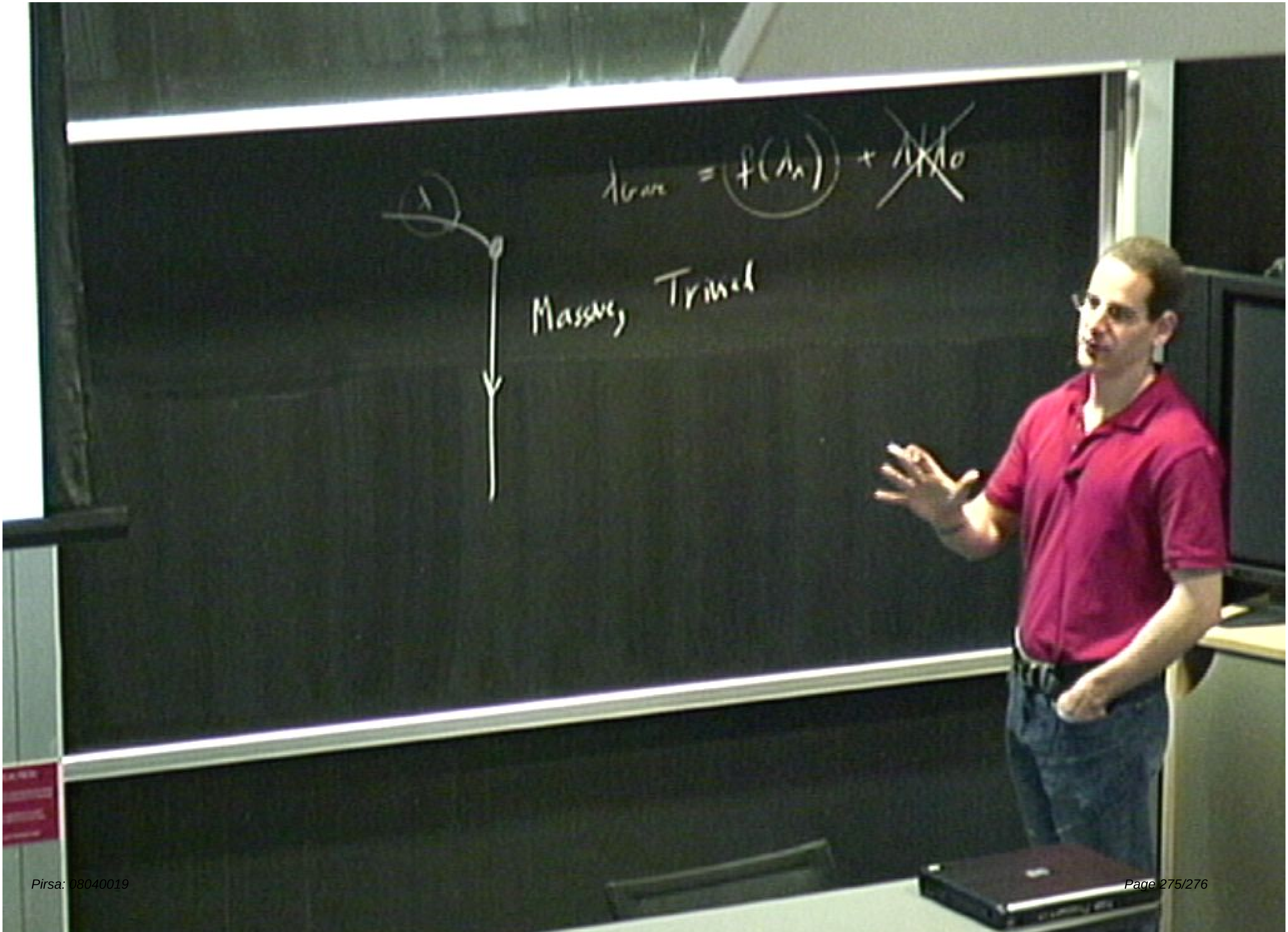
Gaussian Fixed Point

- The mass term is relevant
- The four-point coupling is relevant
- The six-point coupling is marginally irrelevant
- Non-trivial renormalizable theories exist along the $\lambda\varphi^4$ direction!

Wilson-Fisher Fixed Point

- In addition to the Gaussian FP, there is a non-trivial FP
- The W-F FP possesses a single relevant direction
- This can also be used to construct an RT


NT Relevant directions


Massive, Trivial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