Title: Nonlocal Inflation

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Abstract: Many string theorists and cosmologists have recently turned their attention to building and testing string theory models of inflation. One of the main goals is to find novel features that could distinguish stringy models from their field theoretic counterparts. This is difficult because, in most examples, string theory is used to derived an effective theory operating at energies well below the string scale. However, since string theory provides a complete description of dynamics also at higher energies, it may be interesting to construct inflationary models which take advantage of this distinctive feature. I will discuss recent progress in this direction using p-adic string theory - a toy model of the bosonic string for which the full series of higher dimensional operators is known explicitly - as a playground for studying string cosmology to all order in $alpha\'$. The p-adic string is a nonlocal theory containing derivatives of all orders and this structure is also ubiquitous in string field theory. After discussing the difficulties (such as ghosts and classical instabilities) that arise in working with higher derivative theories I will show how to construct generic inflationary models with infinitely many derivatives. Novel features include the possibility of realizing slow roll inflation with a steep potential and large nongaussian signatures in the CMB.

References

1. p-adic Inflation, N. Barnaby, T. Biswas & J. Cline, JHEP **0704**, 056; arXiv:hep-th/0612230.
2. Large Nongaussianity from Nonlocal Inflation, N. Barnaby & J. Cline, JCAP **0707**, 017; arXiv:0704.3426.
3. Dynamics with Infinitely Many Derivatives: The Initial Value Problem, N. Barnaby & N. Kamran, JHEP **0802**, 008; arXiv:0709.3968.
4. Predictions for Nongaussianity from Nonlocal Inflation, N. Barnaby & J. Cline; arXiv:0802.3218.
5. Work in progress, N. Barnaby, T. Biswas, J. Cline.

Outline

1. **(Nonlocal) Inflation from String Theory**
2. Ghosts and Instabilities
3. Dynamics with Infinitely Many Derivatives
4. Nonlocal Inflation
5. Predictions for Nongaussianity

String Theory Cosmology

- ⋆ Construction of string theory models of inflation has attracted considerable interest.
- ⋆ Motivations:[a]
  1. String theorist: rare potential observational window into stringy physics.
  2. Cosmologist: Opportunity to address open problem with inflationary paradigm.

[a] Burgess (2006).

Inflation from String Theory

- Considerable prgress in constructing stringy inflation models (especially wrt moduli stabilization).
- A number of plausible scenarios:
  - Brane-antibrane (KKLMMT, Baumann et al. (2006) Burgess et al. (2001), ...)
  - D3/D7 (Dasgupta et al. (2002), ...)
  - Moduli inflation (Racetrack inflation, Roulette inflation, ...)
  - DBI inflation (Silverstein & Tong (2004), ...)
  - Tachyonic inflation (Cremades et al. (2005), ...)
  - . . .
- Model building: crucial to look for distinctly stringy features or observational signatures...

Inflation from String Theory

- ⋆ Can we distinguish between string theory models and field theoretic counterparts?
- ⋆ Most examples use string theory to derive a low energy effective action describing dynamics well below m_s:

$$\mathcal{L}_{\rm kklmmt} \cong -\frac{1}{2}(\partial\phi)^2 - \left[V_0 + \frac{m^2}{2}\phi^2 - \frac{A}{\phi^4} + \cdots\right]$$

- ⋆ String theory provides a complete description of dynamics also a higher energies.
  - – Can we construct models which takes advantage of this?
- ⋆ Daunting, since QFT description should be supplemented by infinitely many higher dimensional operators...

Towards UV Complete Inflation

- Consider inflation in toy model where the full series of high dimension operators is known explicitly:

$$\mathcal{L}_{\text{osft}} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}\phi^2 + \frac{K^{-3}}{3}\left[K^{-\Box}\phi\right]^3$$

  - Known examples are nonlocal...

- PROS:
  - Distinctive, novel dynamics/predictions.
  - Might hope to extract some generic, qualitative features of UV complete inflation.

- CONS:
  - Necessarily working with toy models.
  - Difficult to construct "realistic" scenarios (moduli stabilization, SM sector, ...)

Example: p-adic String Theory

- Toy model of the bosonic string tachyon.[a]
- World-sheet coordinates of the string are restricted to the field of p-adic numbers.
- All amplitudes of the lowest state can be computed exactly and one can determine a simple field-theoretic Lagrangian which reproduces them:

$$\mathcal{L} = \frac{m_s^4\, p^2}{g_s^2\,(p-1)}\left[-\frac{1}{2}\phi p^{-\frac{\Box}{2m_s^2}}\phi + \frac{1}{p+1}\phi^{p+1}\right]$$

- Derived for p a prime number but the theory can be sensibly continued to other values.
- Contains infinitely many derivatives: $e^{-\Box} = 1 - \Box + \cdots$

[a] Brekke, Freund, Olson & Witten (1987).

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(Non)Local Limit

- The field equation for the p-adic scalar is:

$$p^{-\frac{\Box}{2m_s^2}}\phi = \phi^p$$

- Infinite order in derivatives, can be re-cast as an integral equation.[a]
- In the limit $p \to 1$ this equation becomes local:[b]

$$\Box\phi = 2m_s^2\phi \ln \phi$$

- For $p \gg 1$ the nonlocal structure plays an important role in the dynamics.
  - Limit $p \gg 1$ will be most intesting for cosmology...

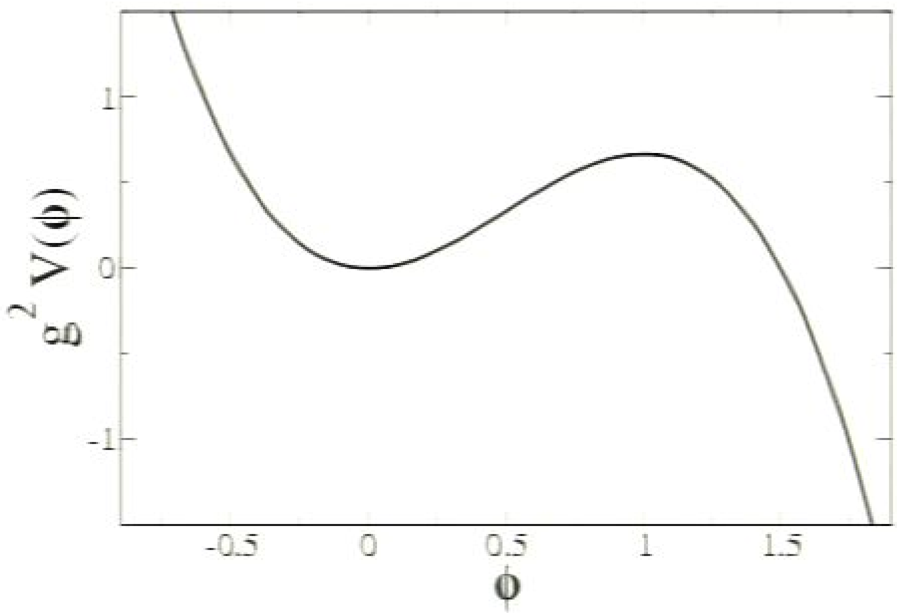
[a] Zwiebach (2002).

[b] Gerasimov & Shatashvili (2000).

p-adic Tachyon Condensation

$$V(\phi) = \frac{m_s^4}{g_p^2}\left[\frac{1}{2}\phi^2 - \frac{1}{p+1}\phi^{p+1}\right]$$

- ⋆ $\phi = 1$ (and $\phi = -1$ for odd p) is the unstable maximum: D25 brane
- ⋆ $\phi = 0$ is the true vacuum: no brane, open strings



- ⋆ Rolling the tachyon from $\phi = 1$ to $\phi = 0$ gives a time dep description of brane decay.

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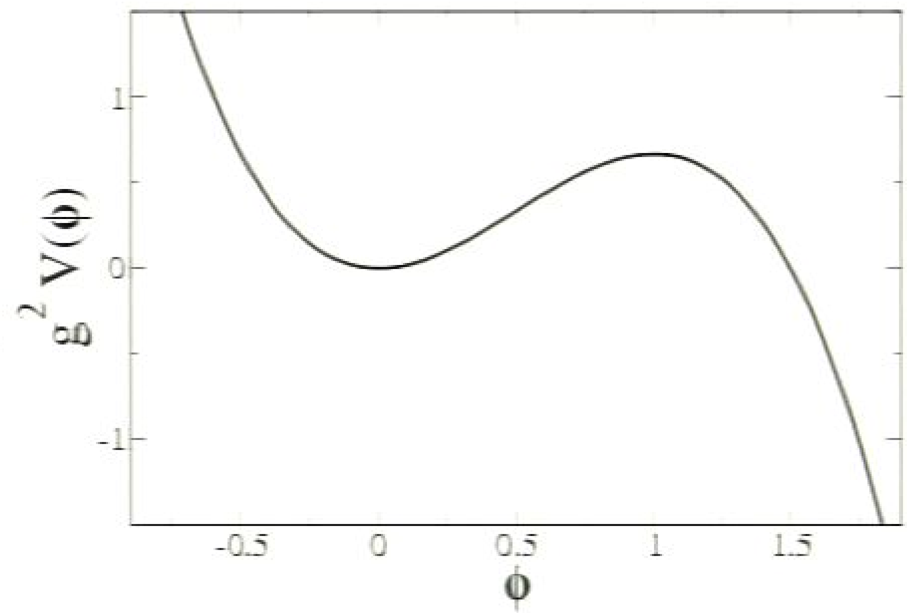

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More Examples

- Nonlocal theories of the form

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - V(\phi)$$

with nontrivial $F(z)$ arise in:

- String field theory.
- p-adic strings, strings quantized on a random lattice.[a]
- Brane-world constructions.[b]
- Unparticle effective actions.

- Similar nonlocal theories arise in:
  - QFT with a minimal length scale[c] (eg LQG, DSR).
  - Noncommutative geometry.

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[a] Biswas, Grisaru & Siegel (2005).

[b] de Rham (2007).

[c] Hossenfelder (2007).

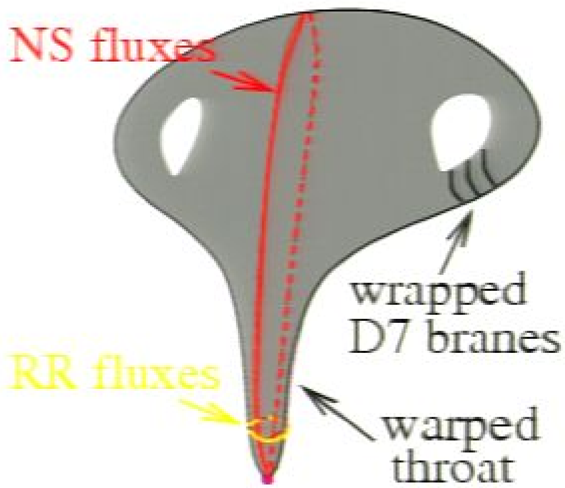
Nonlocal Inflation

- Can embed inflation into general class of nonlocal theories:

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - V(\phi)$$

- Novelties:
  - Can realize slow roll inflation with a very steep potential.
  - Scenario is predictive, generically have $f_{NL} \gg 1$.

- Note: flat potentials surprisingly hard to obtain in realistic settings. (KKLMMT; Baumann et al. (2007); Burgess, Cline, Firouzjahi, Leblond, Shandera, Tye...)



Problems/Complications

- ⋆ Difficulties of working with higher derivative theories are well known:[a]
  - Instabilities, ghosts, ···
  - Difficulties in setting up IVP.
- ⋆ Any application to physics must address fundamental issues:
  - When can nonlocal theories be ghost-free?[b]
  - Can one make rigorous sense of the IVP in infinite order theories?[c]
- ⋆ Before discussing cosmology need to make a detour to discuss formalism...

[a] Woodard (1989).

[b] NB. Biswas, Cline, Prokushkin (2008).

[c] NB. Kamran (2007).

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2. **Ghosts and Instabilities**
3. Dynamics with Infinitely Many Derivatives
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Finite High Derivative Corrections

- ⋆ Addition of finite higher derivative terms always leads to trouble...
- ⋆ Example: Lee & Wick (1969) model

$$\mathcal{L}_{LW} = \frac{1}{2}\phi\Box\phi - \frac{1}{2M^2}\phi\Box^2\phi - \frac{1}{2}m^2\phi^2 + \cdots$$

assume $M^2 \gg m^2$.

- – Generalization to SM solves the hierarchy problem.[a]
- – Predictions for LHC...

- ⋆ Classical EOM requires four initial data:

$$\left(\Box - \frac{1}{M^2}\Box^2 - m^2\right)\phi = 0$$

[a] Grinstein, O'Connel & Wise (2007)

Lee-Wick Theory

⋆ Propagator has two poles ⇒ two physical states!

$$G(p^2) \propto \frac{1}{-p^2 - p^4/M^2 - m^2} \sim \underbrace{\frac{1}{-p^2 - m^2}}_{\text{reg, mass } m} - \underbrace{\frac{1}{-p^2 - M^2}}_{\text{ghost, mass } M}$$

⋆ Ghost = excitation with wrong-sign kinetic term

Multi-Particle Decomposition

⋆ Can see explicitly by introducing auxiliary fields $\chi_{1,2}$:

$$\chi_1 = \left(\frac{\Box}{M^2} - 1\right)\phi$$

$$\chi_2 = \left(\frac{\Box}{m^2} - 1\right)\phi$$

⋆ Lagrangian decomposes as

$$\mathcal{L}_{LW} \cong -\left[\frac{1}{2}(\partial\chi_1)^2 + \frac{m^2}{2}\chi_1^2\right] + \left[\frac{1}{2}(\partial\chi_2)^2 + \frac{M^2}{2}\chi_2^2\right] + \mathcal{L}_{\rm int}$$

⋆ Wrong sign kinetic term. So what?

What's Wrong with Ghosts?

- Hamiltonian is unbounded from below!

$$\mathcal{H}_{LW} = +\left[\frac{1}{2}\dot{\chi_1}^2 + \frac{m^2}{2}\chi_1^2\right] - \left[\frac{1}{2}\dot{\chi_2}^2 + \frac{M^2}{2}\chi_2^2\right] + \mathcal{H}_{\rm int}$$

- Unstable: dynamics drives system to become arbitrarily excited.
- This is a classical pathology (QFT can be made unitary).
- Note: taking M^2 larger only makes things worse!
- Only way to salvage the theory is by imposing auxiliary constraints.[a]

[a] Cutkosky et al. (1969).

Ostrogradski Theorem

- Problem VERY general. Has nothing to do with QFT...
- Consider:

$$L = L\left[q(t), \dot{q}(t), \cdots, q^{(n)}(t)\right]$$

- EL equation is $2n$-th order ODE, requires $2n$ IC

$$\sum_{i=0}^{n}\left(-\frac{d}{dt}\right)^{i}\frac{\partial L}{\partial q^{(i)}} = 0$$

- Hamiltonian depends on $2n$ canonical coordinates Q_i, P_i $(i = 1, \cdots, n)$.
- Theorem: Hamiltonian is unbounded from below with respect to $n-1$ momenta.

$$F = ma + \underbrace{g\dot{a}}_{=0!}$$

Counting Initial Data

- Generalization to field theory:

$$S = S\left[\phi, \Box\phi, \cdots, \Box^N\phi\right]$$

$$\begin{aligned} N &= \text{(num poles in propagator)} \\ &= \text{(num physical states)} \\ &= \frac{1}{2}\text{ (num initial data)} \\ &= \frac{1}{2}\text{ (dim phase space)} \end{aligned}$$

- Theorem: Finite N theory is always sick if $N > 1$!
- Larger N just makes things worse. What about $N = \infty$?

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Infinite Order Theories

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - V(\phi)$$

$$F(z) = \sum_{n=0}^{\infty} a_n z^n$$

- EOM is infinite order:

$$F(\Box)\phi = V'(\phi)$$

- Infinite order PDEs fundamentally different from $N \gg 1$.
- Stability intimately related to initial data counting.
  - How many initial data are necessary?
- Require a formal treatment of IVP infinite order PDEs.[a]

[a] NB & Kamran (2007).

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Pseudo-differential Operators

- First step is to make rigorous sense of $f(\partial_t)$ operators.
- Plausible definition:

$$f(\partial_t)\phi(t) \equiv \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}\phi^{(n)}(t)$$

- Problems:
  - Only works for analytic $f(s)$.
  - Subtelties associated with convergence of the series.
- Mathematicians use:

$$\phi(t) = \frac{1}{2\pi i}\oint_C ds\, e^{st}\tilde{\phi}(s)$$

$$f(\partial_t)\phi(t) \equiv \frac{1}{2\pi i}\oint_C ds\, e^{st} f(s)\tilde{\phi}(s)$$

Pseudo-differential Operators

$$f(\partial_t)\phi(t) \equiv \frac{1}{2\pi i}\oint_C ds\, e^{st} f(s)\tilde{\phi}(s)$$

- PROS:
  - General, works even with poles, branch cuts, etc.
  - Simultaneously handle many types of nonlocality.
- CONS:
  - For nonanalytic case, choice of C may be subtle.
  - Generalization to curved space nontrivial. (dS is the exception.)
- With infinite C and analytic $f(s)$ reproduces series defn.
- Could take C as part of the definition of $f(\partial_t)$. Different C yield different theories...

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Infinite Order Equations

Trivial example ($f(s)$ analytic with isolated zeroes):

$$f(\partial_t)\phi(t) = 0$$
$$\frac{1}{2\pi i}\oint_C ds \left[e^{st} f(s)\tilde{\phi}(s)\right] = 0$$

- ⋆ Require that the integrand is analytic inside C.
- ⋆ Most general $\tilde{\phi}(s)$ has poles at the zeroes of $f(s)$.
- ⋆ General solution ($P_i =$ polynomial):

$$\phi(t) = \sum_i P_i(t)e^{s_i t}$$

- ⋆ NOTE: no. of IC $=$ no. zeroes of $f(s)$ (by multiplicity)

The Initial Value Problem

- Consider a nonlocal theory:

$$\mathcal{L} = \frac{1}{2}\phi\, F(\Box)\, \phi$$

- Propagator: $G(p^2) \sim F(-p^2)^{-1}$ has N poles.
- General solution contains $2N$ free coefficients:

$$\phi(t, \mathbf{x}) = \sum_{i=1}^{N} \int \frac{d^3k}{(2\pi)^{3/2}} \left[a_k^{(i)} \phi_k^{(i)}(t) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right]$$

  describes N physical states.

- Theorem: Poles of the propagator exhaustively count initial data.[a]
- Ostrogradski construction doesn't apply.

[a] NB & Kamran (2007)

Multi-Particle Decomposition

- Every theory with meromorphic $F(z)$ can be decomposed as:[a]

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\phi F(\Box)\phi - V_{\text{int}}(\phi) \\ &= \sum_i \epsilon_i \phi_i \underbrace{\Gamma(\Box)}_{\text{no zeroes}} (\Box - m_i^2)\phi_i + \mathcal{L}_{\text{int}} \end{aligned}$$

- Stability of i-th state related to the sign of ϵ_i.
  - Sign of ϵ_i fixed by $G(p^2)$ at the pole m_i^2
- Can systematically determine every potentially ghost-free theory with meromorphic kinetic function.
  - Single pole theories.
  - Multi-pole theories with nonanalytic $F(z)$.

[a] Biswas, Barnaby & Cline, in prog.

What if ghosts ARE present?

- Exorcism: could project out ghost-like excitations by alternative choice of C.

$$f(\partial_t)\phi(t) \equiv \frac{1}{2\pi i}\oint_C ds\, e^{st} f(s)\tilde{\phi}(s)$$
$$\rightarrow \frac{1}{2\pi i}\oint_{C'} ds\, e^{st} f(s)\tilde{\phi}(s)$$

  - C is an infinite contour
  - C' excludes the unwanted poles

- Constrains dynamics to a slice of phase space.
- Very general: applies to infinite order, nonanalytic, etc...

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Example 1: Lee-Wick Theory

- Is the exorcised theory just a local theory? No.
- Example: Lee-Wick theory

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi + \mathcal{L}_{\text{int}}$$

$$F(z) = z - \frac{z^2}{2M^2} - m^2$$

- Zeroes: $F(+\omega_i^2) = 0$

$$\omega_{1,2}^2 = \frac{M^2}{2}\left[1 \pm \sqrt{1 - 4\frac{m^2}{M^2}}\right]$$

  – $e^{\pm i\omega_2}$ correspond to high frequency ghost excitations
  – $e^{\pm i\omega_1}$ correspond to low frequency positive energy states.

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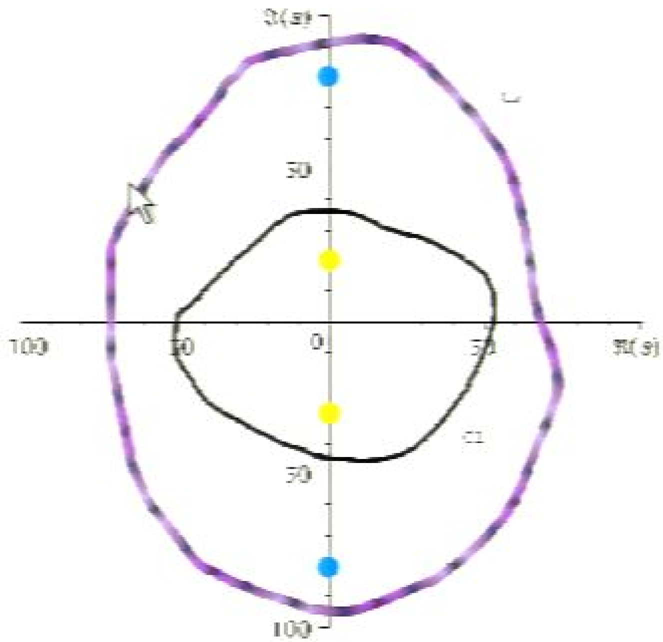


$$F(-\partial_t^2)\phi(t) \to \frac{1}{2\pi i}\oint_{C'} ds\, e^{st} F(-s^2)\tilde{\phi}(s)$$

- ⋆ Illustrate effect in perturbation theory about the free solution: $\phi(t) = \delta_1\phi + \delta_2\phi + \cdots$
- ⋆ Free solutions ($\mathcal{L}_{\text{int}} = 0$):

$$\delta_1\phi(t) = Ae^{i\omega_1 t} + Be^{-i\omega_1 t}$$

contour redefinition trivially omits $e^{\pm i\omega_2 t}$ modes.

Exorcising Lee-Wick

⋆ At higher order ($\mathcal{L}_{\rm int} \neq 0$) nonlinearity arises through source terms:

$$F(\Box)\delta_2\phi(t) = J(t)$$

with $J(t) \sim (\delta_1\phi)^2$.

⋆ For $J(t) = Ae^{i\omega_s t}$ have:

$$\begin{aligned}\delta_2\phi(t) &= \frac{1}{2\pi i}\oint_{C'} ds\, e^{st}\frac{\tilde{J}(s)}{F(-s^2)} \\ &= \frac{1}{2\pi i}\oint_{C'} ds\, e^{st}\frac{-M^2}{\omega_2^2-\omega_1^2}\frac{A}{s-i\omega_s}\left[\underbrace{\frac{1}{s^2+\omega_1^2}}_{\rm normal} - \underbrace{\frac{1}{s^2+\omega_2^2}}_{\rm ghost}\right]\end{aligned}$$

⋆ Ghost term contributes if $\omega_s \in C'$.

Pumping the Lee-Wick Oscillator

- Inhomogeneous solution:

$$\delta_2\phi(t) = \frac{AM^2}{(\omega_s^2 - \omega_1^2)(\omega_2^2 - \omega_s^2)} e^{i\omega_s t} + \cdots$$

- In the single derivative theory ($M^2 \to \infty$) have:

$$\delta_2\phi(t) = \frac{A}{(\omega_s^2 - m^2)} e^{i\omega_s t} + \cdots$$

- Physically inequivalent to a first order system.
  - Resonance frequency changed.
  - Can take $\omega_s \to i \times \infty$ while keeping $\omega_s \in C'$, then $\phi(t) \sim |\omega_s|^{-4}$ rather than $|\omega_s|^{-2}$.
- More dramatic effect for less trivial $F(z)$...

Multi-Particle Decomposition

- Previously introduced independent fields

$$\chi_i = \left(\frac{\Box}{\omega_i^2} - 1\right)\phi$$
$$\mathcal{L} = \sum_i \epsilon_i \chi_i (\Box - m_i^2)\chi_i + \mathcal{L}_{\rm int}$$

with $i = 1, 2$.

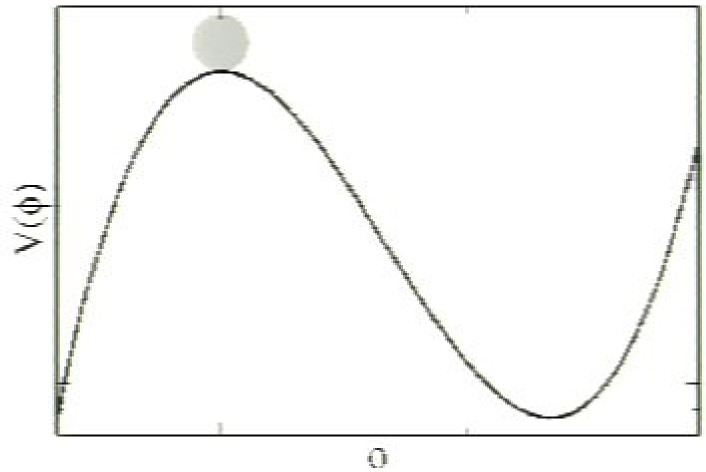
- In the exorcised theory:
  - Both χ_i are nonzero.[a]
  - They are no longer independent.

[a] At the interacting level. Have $\chi_2 = 0$ in the free theory.

Example 2: p-adic String Theory

- p-adic theory linearized about the false vacuum ($\phi = 1$):

$$\left(p^{-\frac{1}{2}m_s^{-2}\Box} - p\right)\delta\phi = 0$$



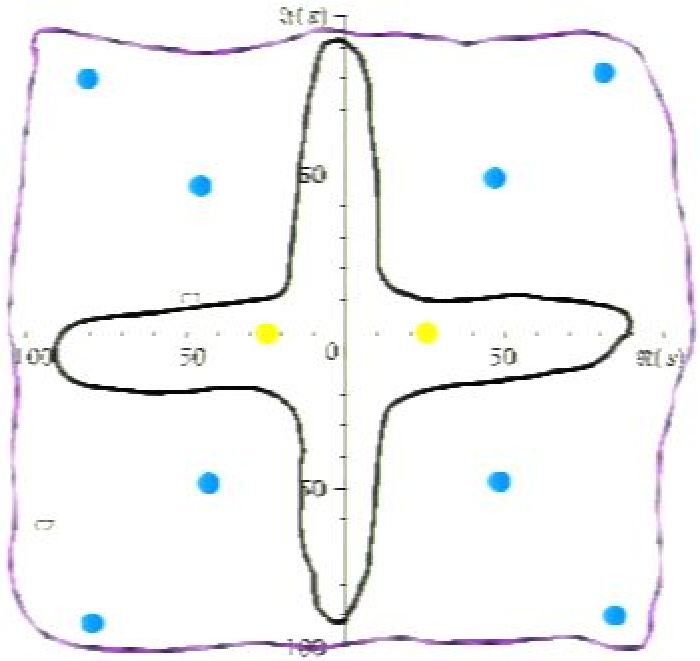
- The propagator is:

$$G(k^2) \sim \frac{1}{p^{k^2/(2m_s^2)} - p}$$

- Infinite tower of excitations: $m_n^2 = -2m_s^2 + \frac{4\pi i m_s^2}{\ln p}n$.
- $n = 0$ is the open string tachyon, $n = \pm 1, \pm 2, \cdots$ infinite tower of ghost modes

Exorcising p-adic strings

- Redefine the theory by deforming C.



- Similar to a UV cut-off, however still have access to $|s| \to \infty$.
- Consistently projects out ghosts in perturbation theory.
  - Can be applied nonlinearly also.

Multi-Particle Decomposition

- Previously introduced independent fields

$$\chi_i = \left(\frac{\Box}{\omega_i^2} - 1\right)\phi$$

$$\mathcal{L} = \sum_i \epsilon_i \chi_i (\Box - m_i^2)\chi_i + \mathcal{L}_{\text{int}}$$

with $i = 1, 2$.

- In the exorcised theory:
  - Both χ_i are nonzero.[a]
  - They are no longer independent.

---

[a]At the interacting level. Have $\chi_2 = 0$ in the free theory.

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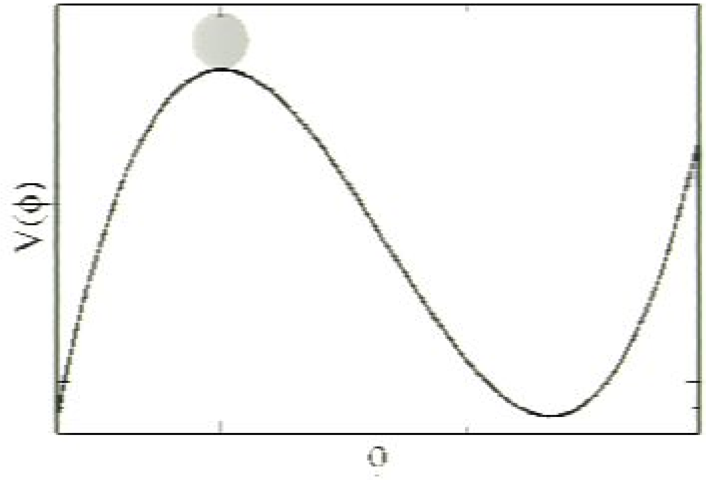

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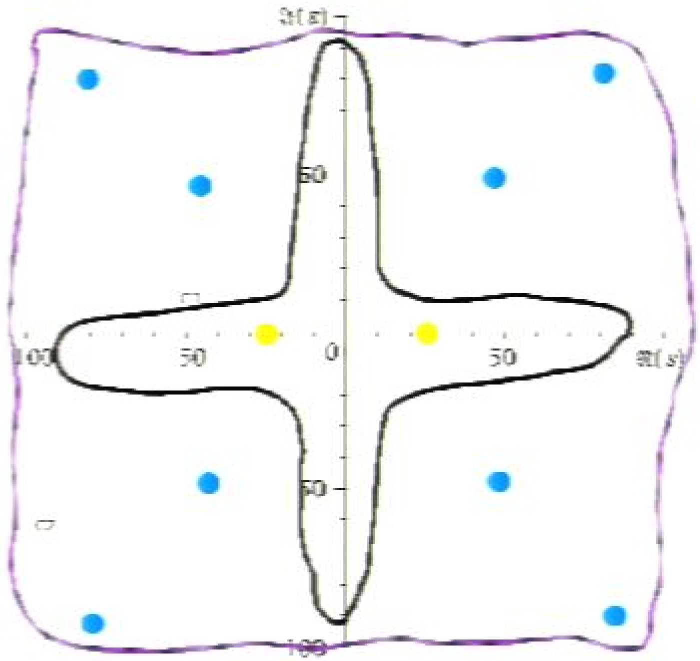

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2. Ghosts and Instabilities
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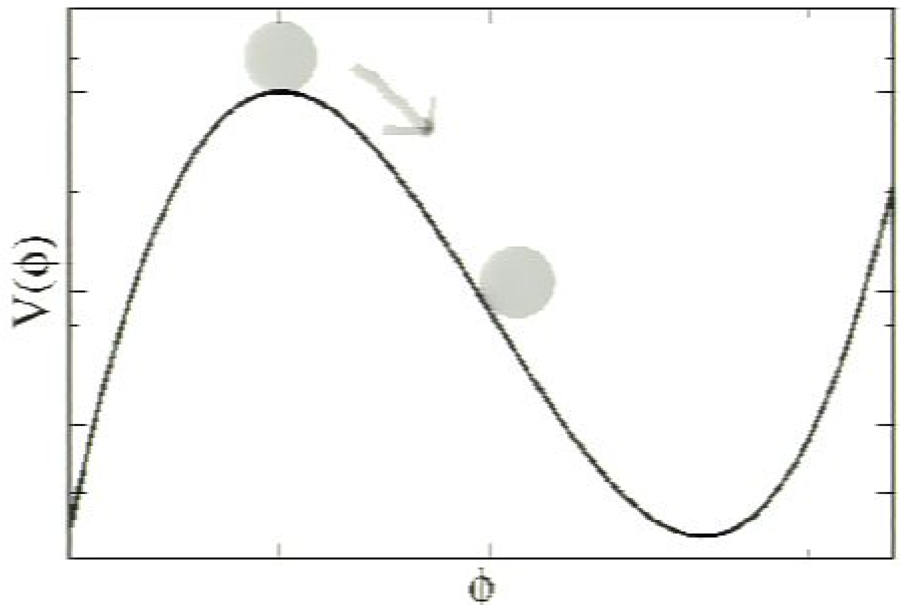
4. **Nonlocal Inflation**
5. Predictions for Nongaussianity

Nonlocal Hill-Top Inflation

- ⋆ Seek inflation in theories of the form:

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - U(\phi)$$

$$U(\phi) = U_0 - \frac{\mu^2}{2}\phi^2 + \frac{g}{3}\phi^3 + \cdots$$



- ⋆ Seek inflationary solution rolling away from $\phi = 0$.
- ⋆ In string theory examples corresponds to inflation during brane decay.

Naive Derivative Truncation

- Naively expect that during slow roll high derivative corrections are negligible:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\phi F(\Box)\phi - U(\phi)\\ &\cong -\frac{1}{2}(\partial\phi)^2 - U_0 + \frac{\mu^2}{2}\phi^2 + \mathcal{O}(\Box^2) + \cdots\end{aligned}$$

- Expect that inflation is only possible when $|\eta| \sim M_p^2|U''/U| \ll 1 \Rightarrow \mu^2 \ll H^2$.
- Naive picture is not always correct: can still obtain slow roll even when $M_p^2|U''/U| \gg 1$!
- Most models of string cosmology follow this approach...

Nonlocal Dynamics

Near the top of the potential ($\phi = 0$) have:

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - \left(U_0 - \frac{\mu^2}{2}\phi^2 + \cdots\right)$$

- Equation of motion:

$$F(\Box)\phi = -\mu^2\phi$$

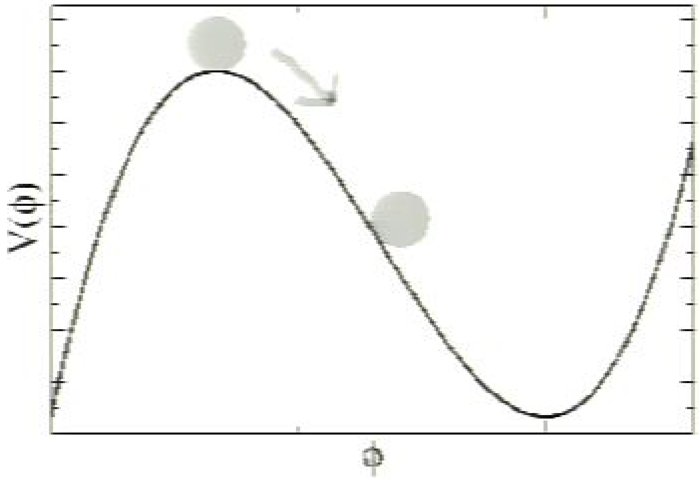
- Can obtain solution by taking:

$$\Box\phi = -\omega^2\phi \quad \text{if} \quad F\left(-\omega^2\right) = -\mu^2$$

- Dual to a local theory with mass ω.
- The effective mass, ω^2, can be small even naive mass μ^2 is large!

Stretching the Inflaton Potential

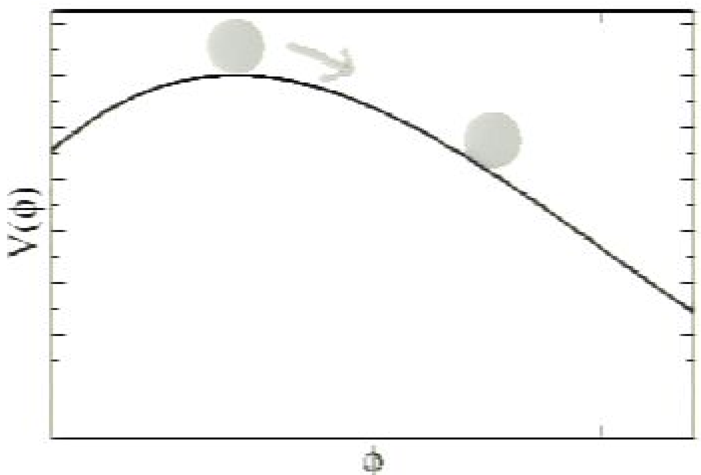
$$\mathcal{L} = \left[\frac{1}{2}\phi F(\Box)\phi - \left(U_0 - \frac{\mu^2}{2}\phi^2 + \cdots\right)\right]$$



$$\mathcal{L} = \frac{1}{2}\phi\Box\phi - U(\phi) + \mathcal{O}(\Box^2)$$

$$U(\phi) = U_0 - \frac{\mu^2}{2}\phi^2 + \cdots$$

Steep potential, higher derivative terms slow the rolling.



$$\mathcal{L}_{\rm dual} = \frac{1}{2}\varphi\Box\varphi - V(\varphi)$$

$$V(\varphi) = U_0 - \frac{\omega^2}{2}\varphi^2 + \cdots$$

Effective potential in dual local theory is stretched.[a]

[a] Lidsey (2007).

Example: p-adic Inflation

Explicit example in p-adic string theory:[a]

$$\mathcal{L} = \frac{m_s^4}{g_p^2}\left[\frac{1}{2}\phi\left(1 - p^{-\frac{\Box}{2m_s^2}}\right)\phi - U(\phi)\right]$$

$$U(\phi) = \underbrace{\frac{p-1}{2(p+1)}}_{\equiv U_0} - \underbrace{\frac{p-1}{2}}_{\mu^2 \equiv p-1}\phi^2 + \cdots$$

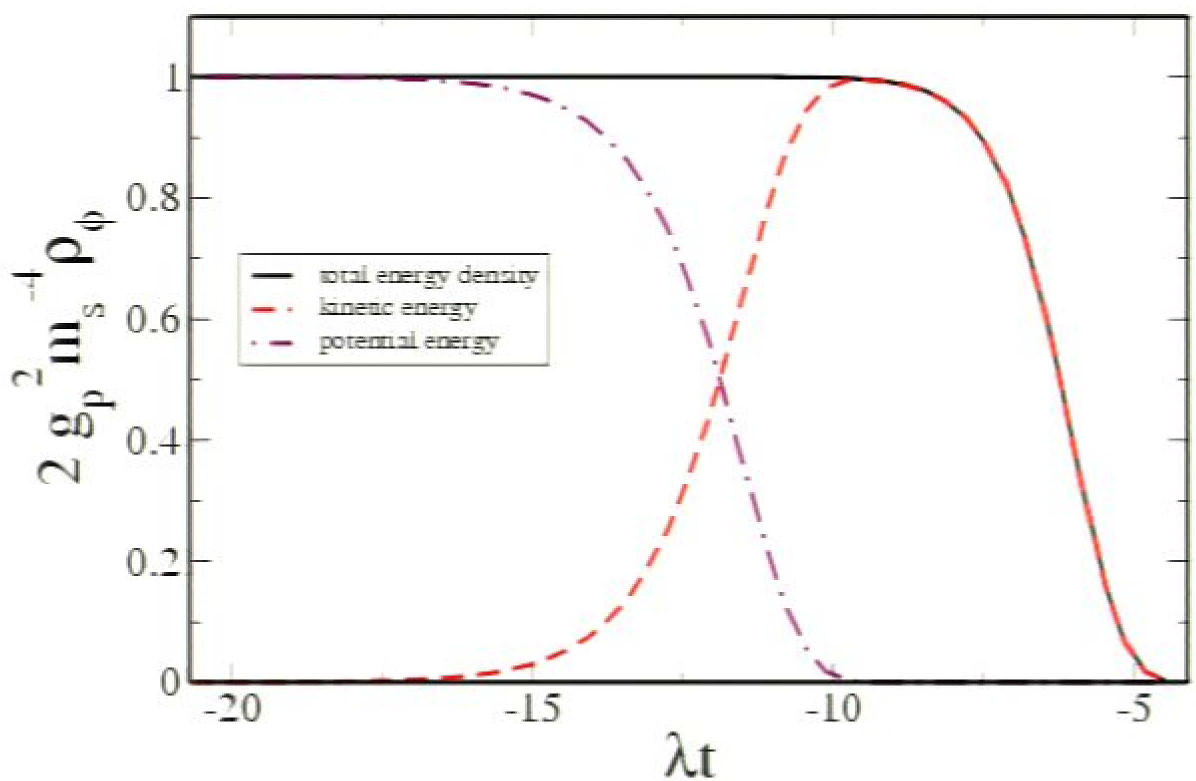
- ⋆ Naively don't expect slow roll since $\mu^2 \sim p \gg 1$ but effective mass, $\omega^2 = -2m_s^2$, insensitive to p.
- ⋆ COBE normalization constrains $g_s/\sqrt{p} \sim 10^{-7}$ so for $g_s \sim 1$ have $p \gg 1$. $\Rightarrow$ Strongly nonlocal!
- ⋆ Note: $H > m_s$, horizon is inside the scale of nonlocality during inflation!

[a] NB. Biswas & Cline (2006).

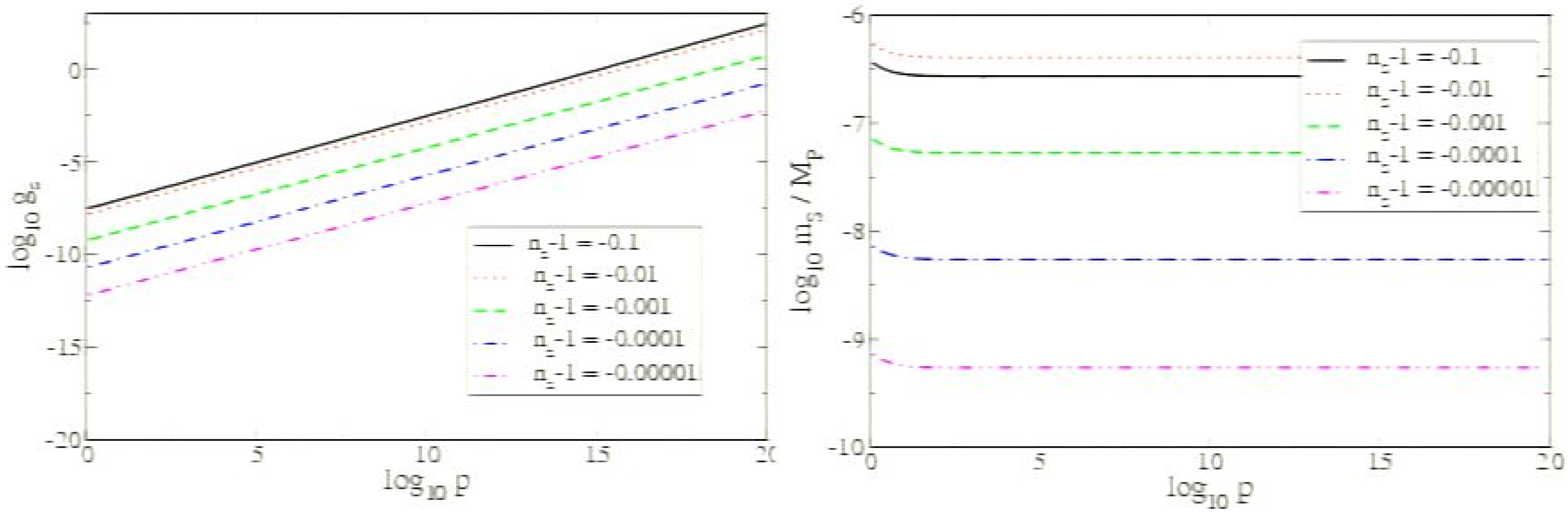
p-adic Inflation

- ⋆ Dynamics checked using an indep, nonperturbative formalism, $\Box \cong -3H\partial_t \;\Rightarrow$

$$\phi(t+\alpha) = \phi(t)^p$$



Predictions for the CMB



- ⋆ Spectral index: $n_s < 1$.
- ⋆ String scale bounded as $m_s \lesssim 10^{-6} M_p$.
- ⋆ Tensor modes undetectably small: $r \lesssim 0.006$.
- ⋆ Nongaussianity...

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Including Interactions

Include the cubic term in the action:

$$\mathcal{L} = \frac{1}{2}\phi F(\Box)\phi - U(\phi)$$

$$U(\phi) = U_0 - \frac{\mu^2}{2!}\phi^2 + \frac{g}{3!}\phi^3 + \cdots$$

- ⋆ For $g \neq 0$ the correspondence between local and nonlocal theories breaks down.
- ⋆ Expect $\langle\phi^3\rangle \propto f_{NL} \propto g$ so for large g the nongaussianity could be large.
- ⋆ In conventional models $g \gg 1$ would spoil inflaton but this need not be true in nonlocal theories!
- ⋆ In p-adic inflation:

$$|g| \sim p^2 \gg 1 \quad \text{for} \quad p \lesssim 10^{13}$$

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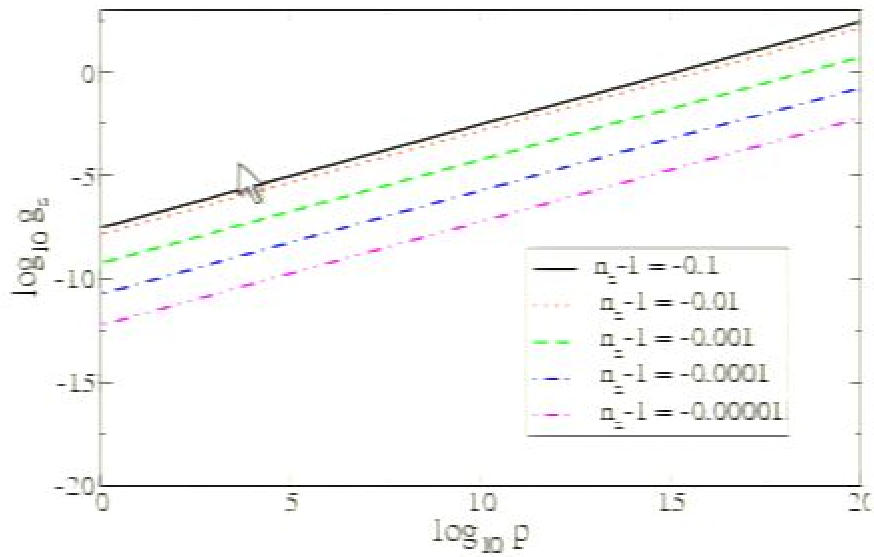

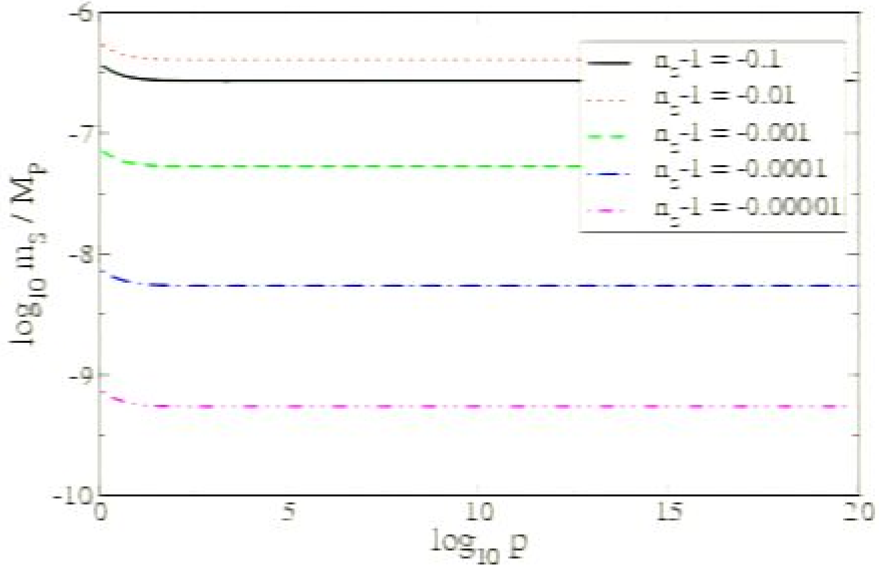

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Field Redefinitions

For ghost-free theory:

$$\mathcal{L} = \frac{1}{2}\phi\Gamma(\Box)(\Box + \omega^2)\phi - U_0 - \frac{g}{3!}\phi^3 + \cdots$$

(where $\Gamma(z)$ has no zeroes).

- Nonlocal field redef $\varphi = \Gamma(\Box)^{1/2}\phi$ gives

$$\mathcal{L} = \frac{1}{2}\varphi(\Box + \omega^2)\varphi - U_0 - \frac{g}{3!}\left(\Gamma(\Box)^{-1/2}\varphi\right)^3 + \cdots$$

- Canonical kinetic structure, nonlocality in the interactions.
- Appropriate starting point to match onto standard perturbation theory calculation.

Perturbed Field Equations

Canonical field equation:

$$(\Box + \omega^2)\varphi = \frac{g}{2}\Gamma(\Box)^{-1/2}\left[\Gamma(\Box)^{-1/2}\varphi\right]^2$$

⋆ Gaussian perturbations:

$$(\Box + \omega^2)\delta_1\varphi \cong 0$$

insensitive to nonlocality.

⋆ Second order:

$$(\Box + \omega^2)\delta_2\varphi \cong \frac{g}{2}\Gamma(\Box)^{-1/2}\left[\Gamma(\Box)^{-1/2}\delta_1\varphi\right]^2 + \cdots$$

⋆ Nonlocal structure in source term mimics a large cubic coupling V''', leads to $f_{NL} \gg 1$

Nonlinearity Parameter

- Calculation is simplest using Seery, Malik & Lyth (2008) formalism.
- Results:

$$f_{NL} = \frac{5}{6} \underbrace{\xi_{\text{eff}}}_{\propto g} \left[N_\star + \frac{3}{\sum_i k_i^3} \left(k_t \sum_{i<j} k_i k_j - \frac{4}{9} k_t^3 \right) \right] + \cdots$$

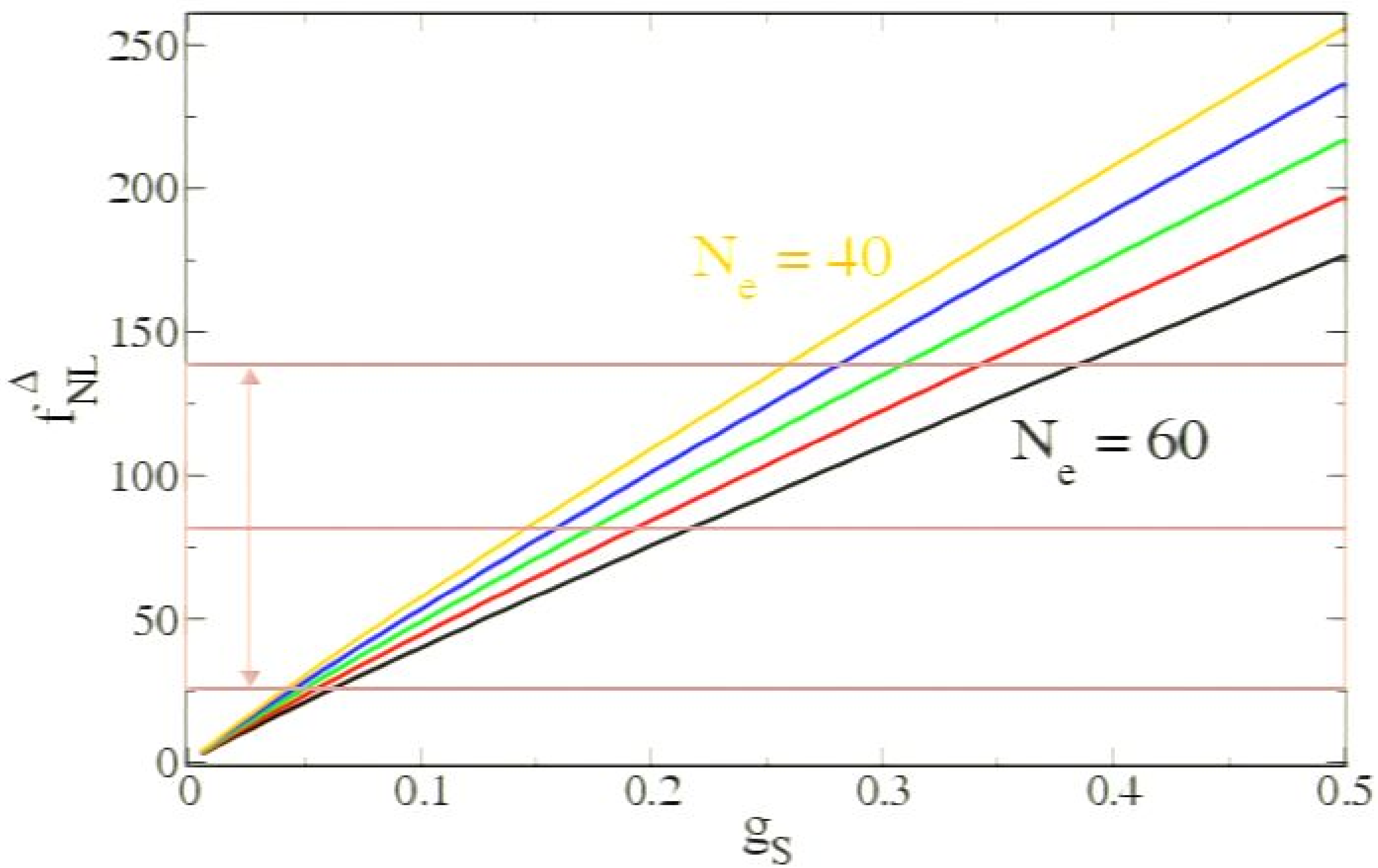
- For p-adic inflation have

$$f_{NL}^{\triangle} \sim 10^{-3} \frac{\sqrt{p}}{\ln p}$$

for $p \gg 1$. (Recall: $p \sim 10^{13}$ for $g_s \sim 1$.)

- In the local limit $p \to 1$ have $f_{NL} \sim n_s - 1$.

p-adic Inflation



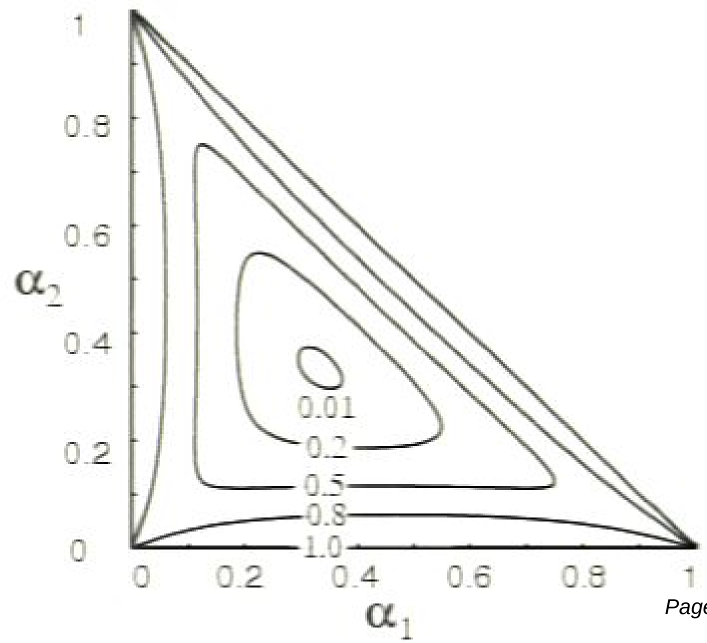
- ⋆ For natural values $g_s \sim 0.1 - 0.3$ reproduce central value for Yadav & Wandelt detection.

Comparison to DBI Models

⋆ Formally very different from DBI:
  – High powers of $\Box$ rather than $(\partial\phi)^2$ leads to infinite order EOM, distinctive dynamics.
  – Sound speed $c_s = 1$ rather than $c_s \ll 1$.
  – Inflation is coming from brane decay rather than motion down a warped throat, $\cdots$

⋆ Dynamics: inflation is NOT fast roll; $\ddot{\phi} \ll H\dot{\phi}$, $\dot{\phi}^2 \ll H^2 M_p^2$.

⋆ Observationally: shape of NG makes p-adic model distinguishable.



Conclusions

- ⋆ Study of high derivative theories is well-motivated.
- ⋆ Provides a playground to study string cosmology to all orders in α'.
- ⋆ Leads to novel cosmological behaviours:
  - Slow roll with a steep potential.
  - Predictive: $f_{NL} \gg 1$.
- ⋆ Rich mathematical structure...
- ⋆ Outlook: can we realize similar phenomena in more realistic string theories?
- ⋆ Effect relies on UV completion: CMB as a probe of distinctly stringy phenomena!

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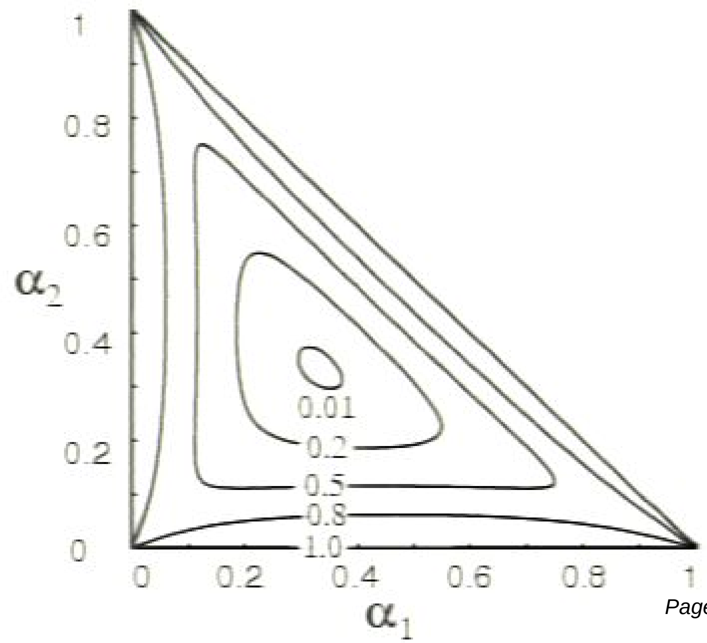

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