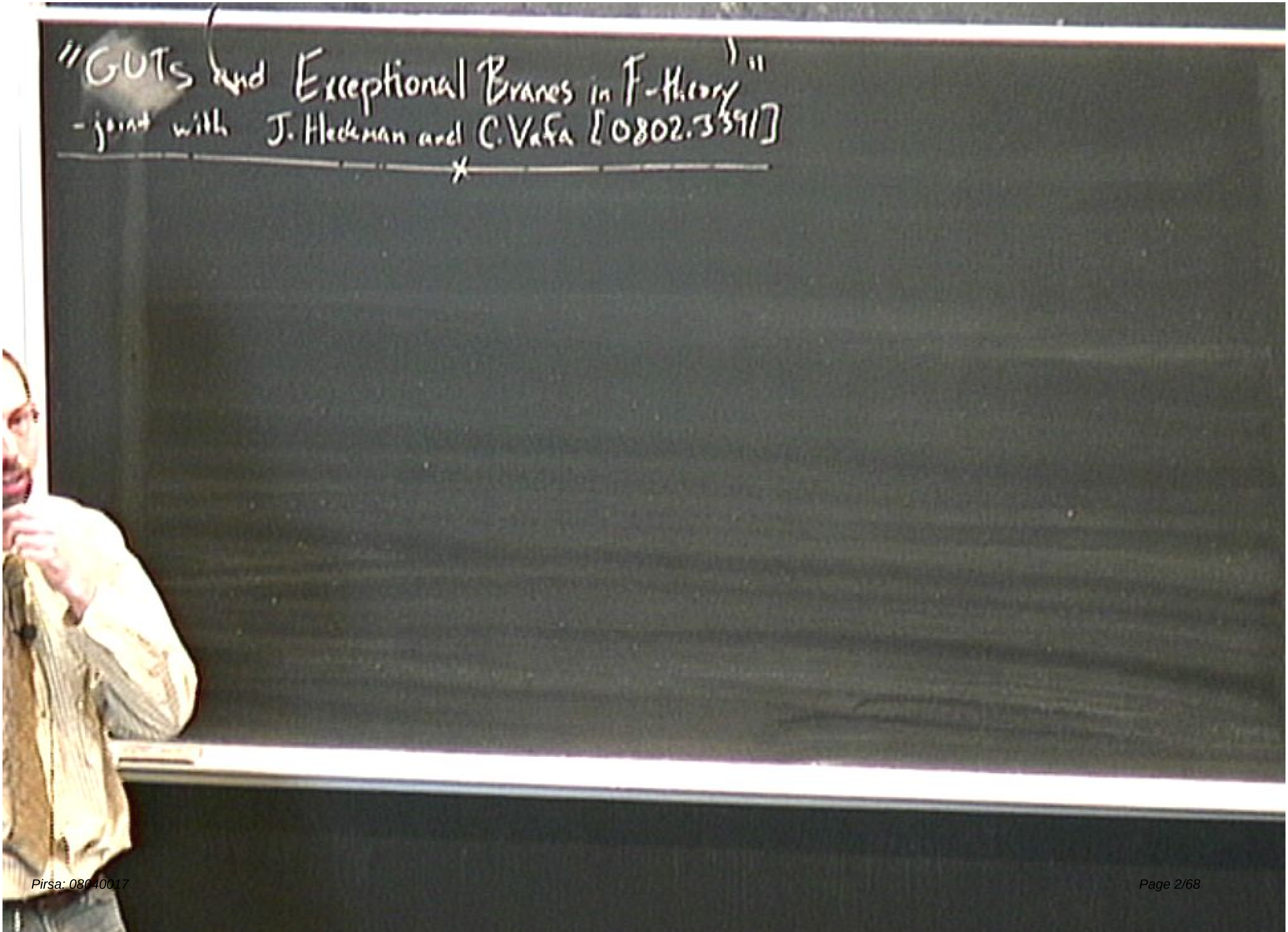
Title: GUTs and Exceptional Branes in F-Theory

Date: Apr 15, 2008 11:00 AM

URL: http://pirsa.org/08040017

Abstract: F-theory compactifications on Calabi-Yau fourfolds provide one way to obtain N=1 supersymmetric grand unified models of particle physics from string theory. With this motivation in mind, I will consider F-theory on a particularly simple class of local Calabi-Yau fourfolds, for which a low-energy analysis based upon topological gauge theory is valid. For these models, I will explain how the geometry of the fourfold determines the spectrum of massless charged matter and the effective superpotential in four dimensions. If time permits, I will also discuss the relation between certain surface operators in gauge theory and brane recombination in F-theory.


"GUTs and Exceptional Branes in F-theory"
- joint with J. Heckman and C. Vafa [0802.3391]

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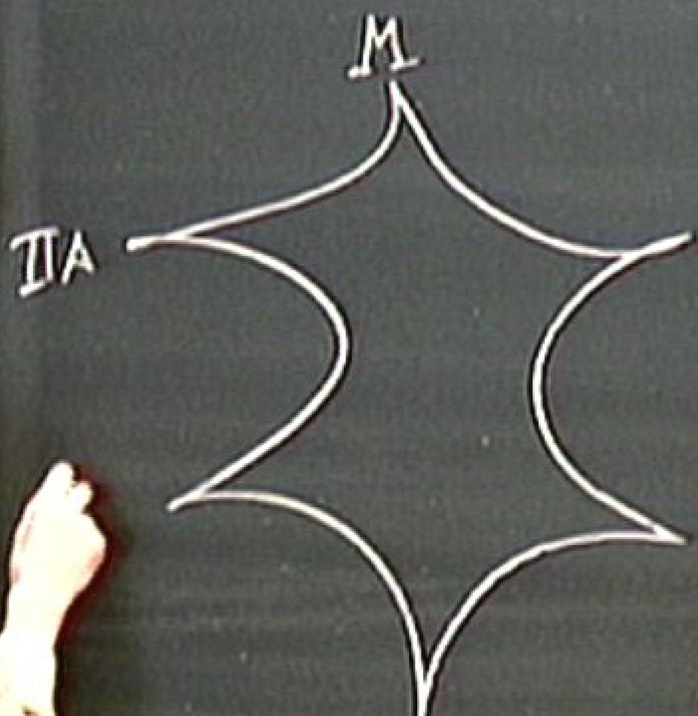

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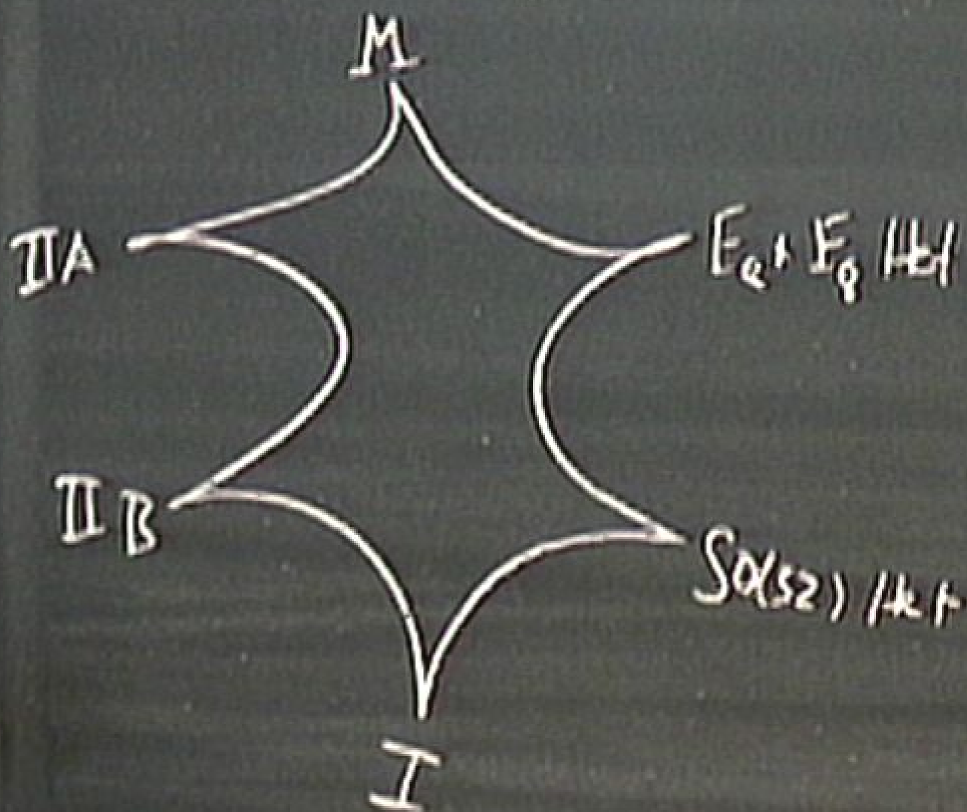

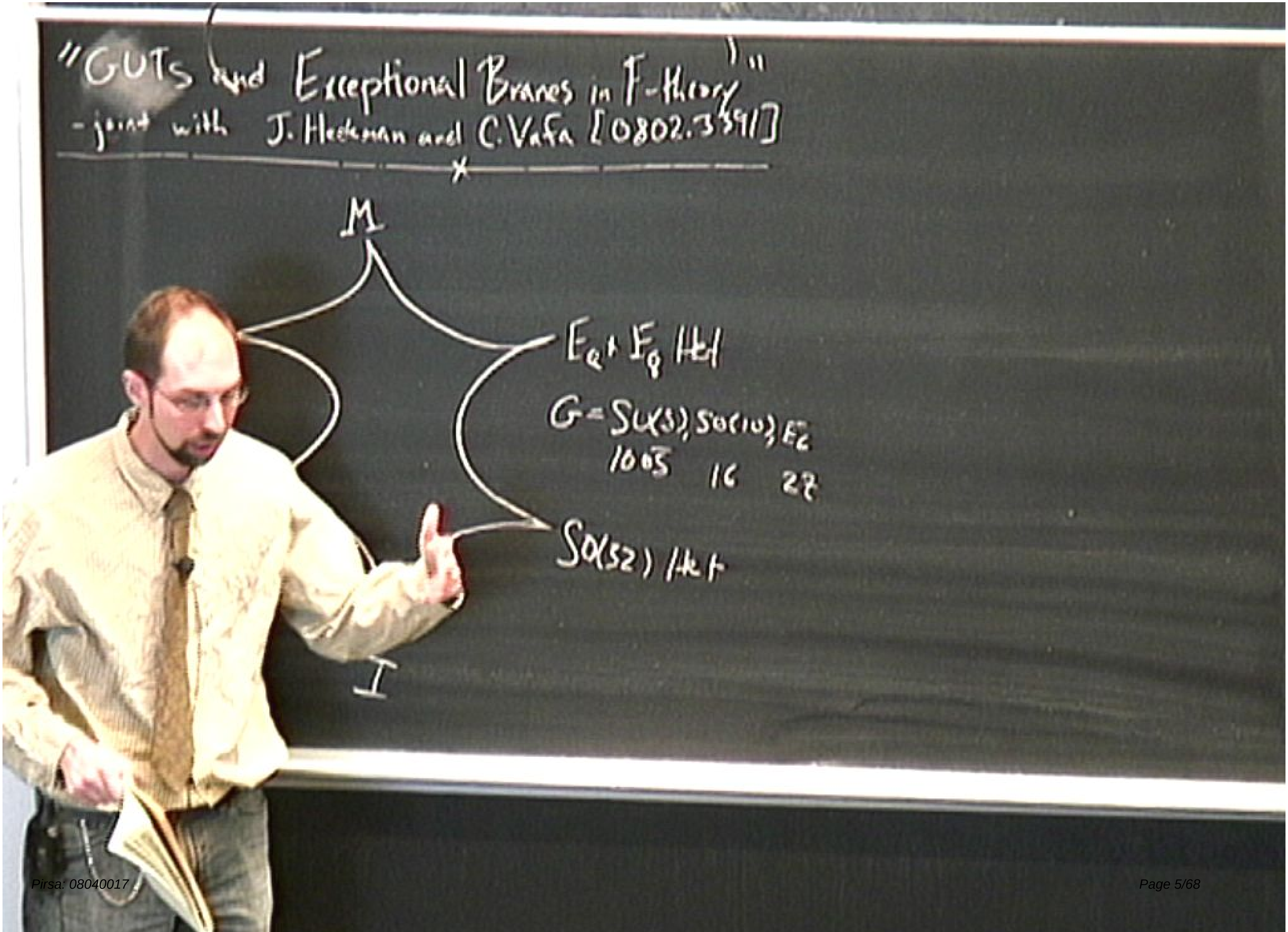
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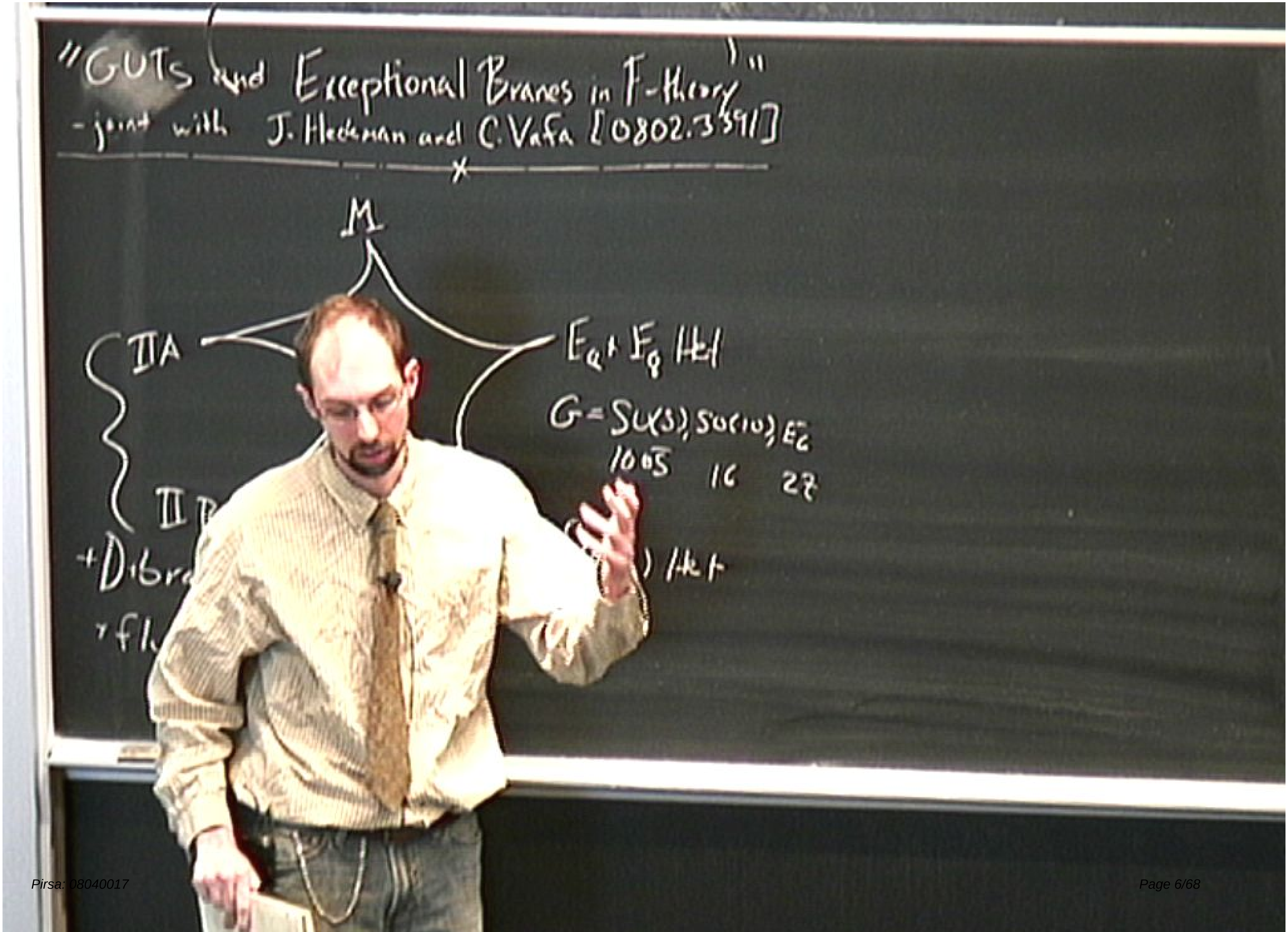
M
E8 × E8 Het
G = SU(5), SO(10), E6
10⊕5̄ 16 27
SO(32) Het
I


"GUTs and Exceptional Branes in F-theory"
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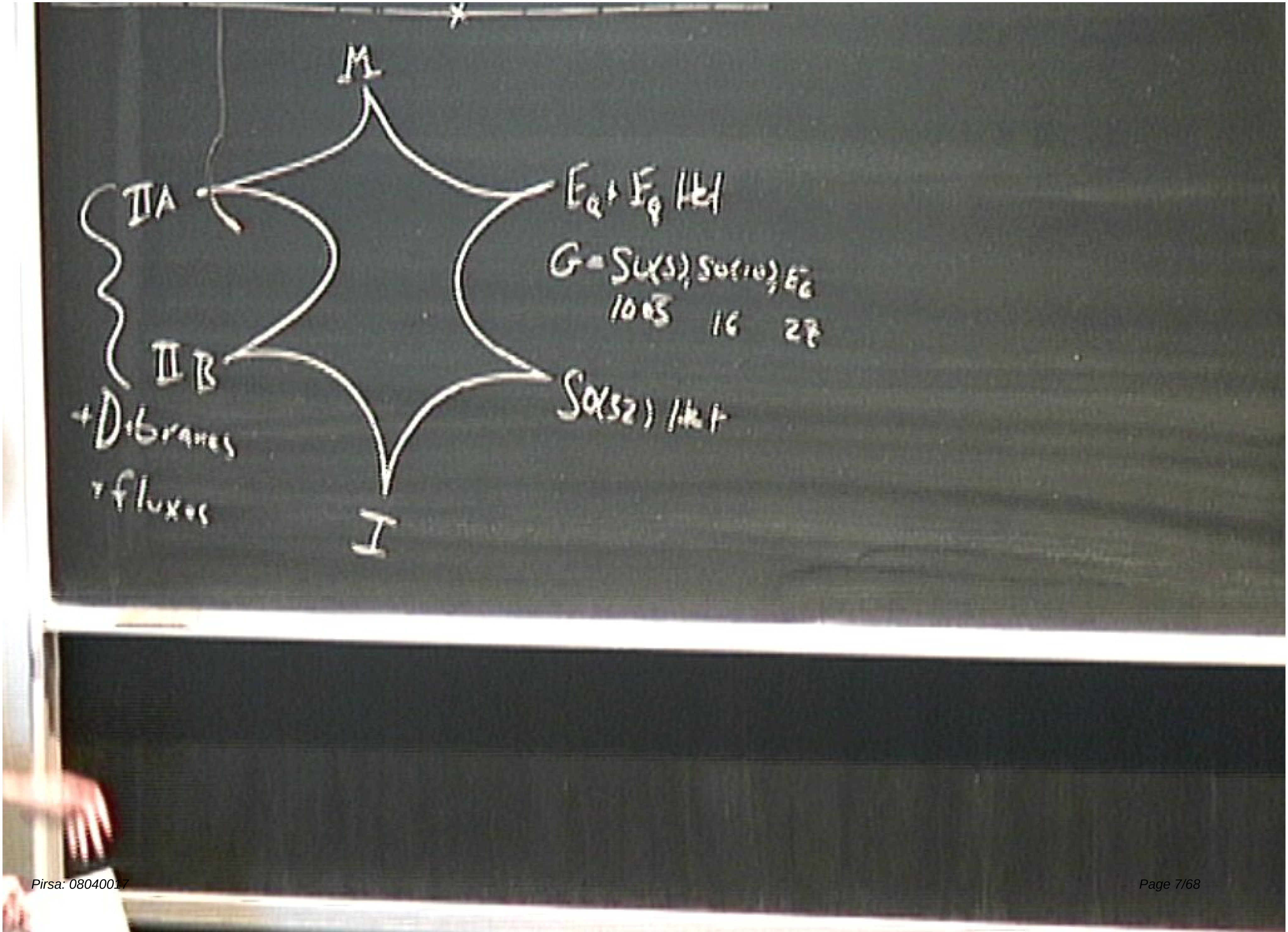
M
IIA
IIB
$E_8 \times E_8$ Het
$G = SU(5), SO(10), E_6$
$10 \oplus \bar{5}$ 16 27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IIB
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$10 \oplus \bar{5}$ 16 27
$SO(32)$ het
+ D-branes
+ fluxes

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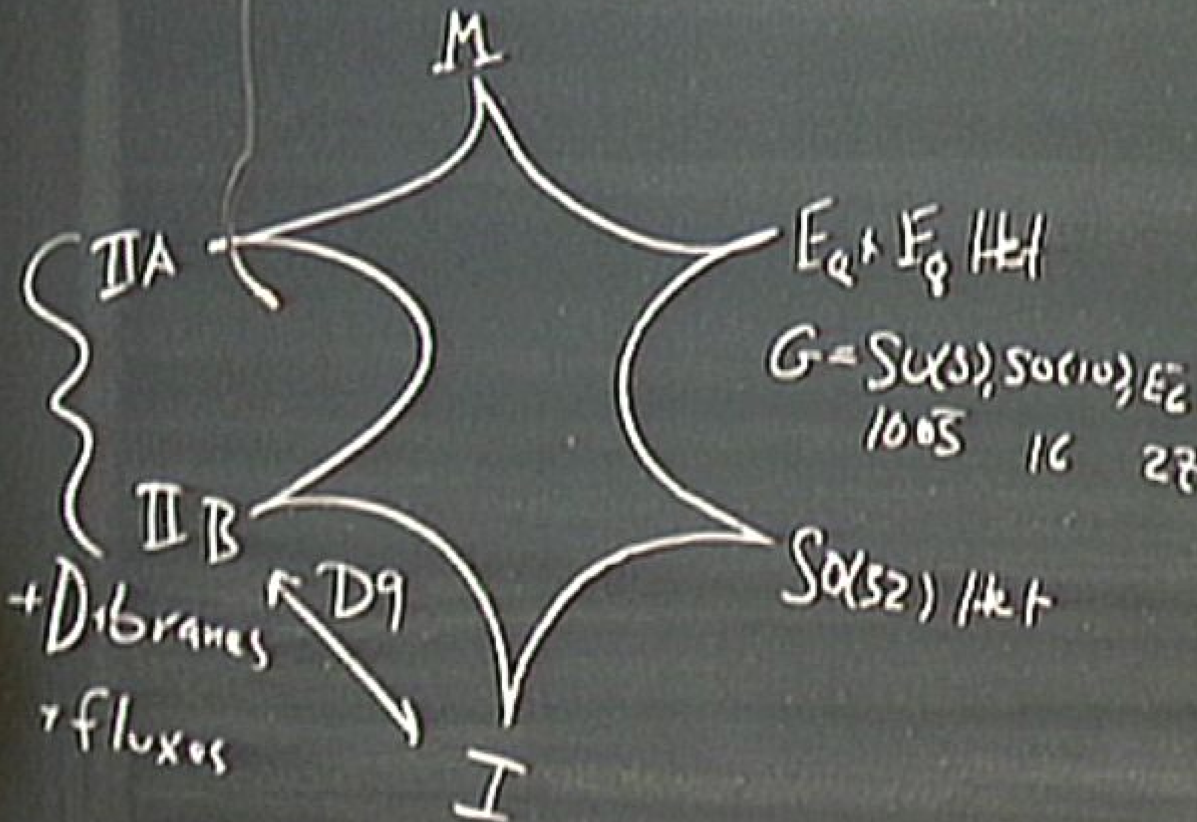

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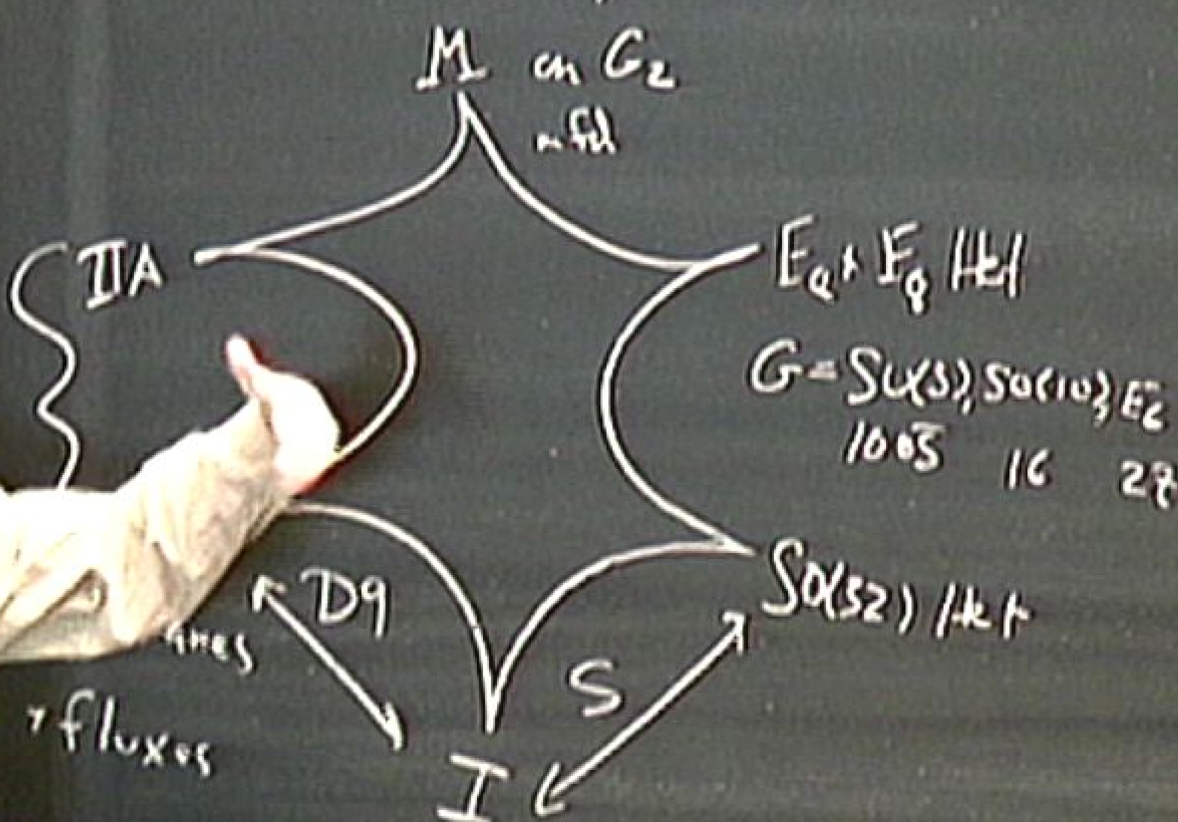

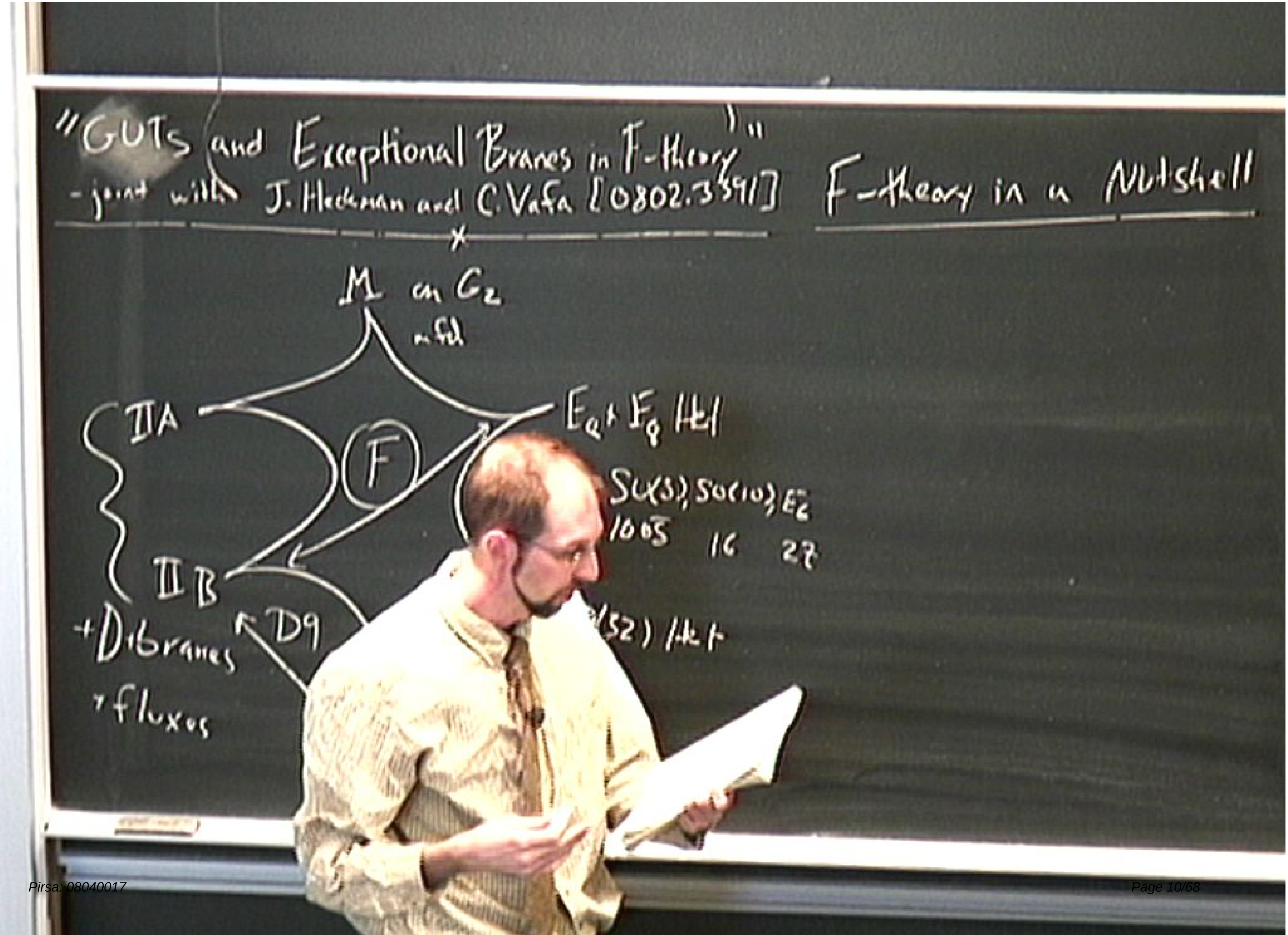
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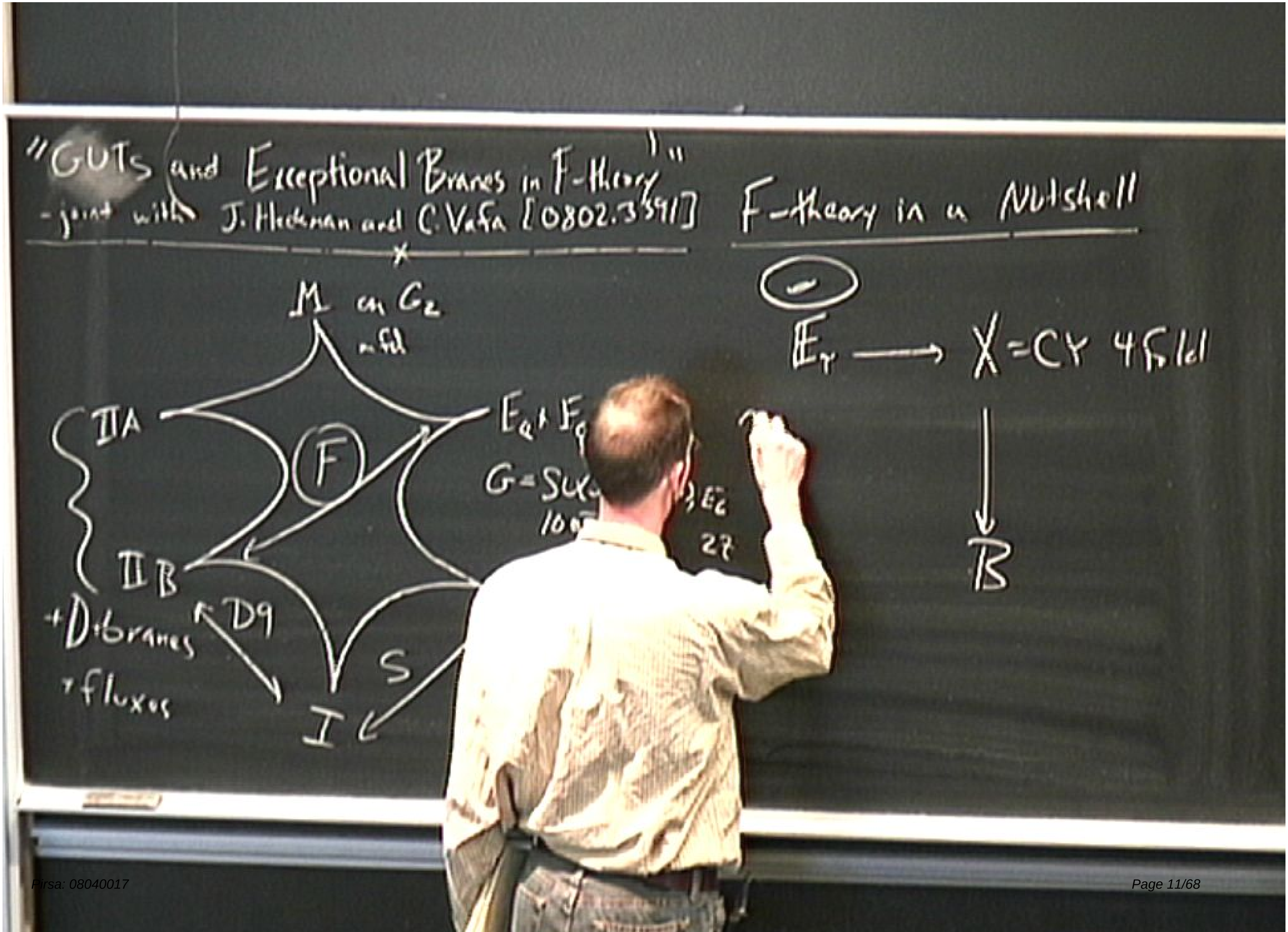
F-theory in a Nutshell
M on G2 mfld
IIA
E8 × E8 Het
F
SU(5), SO(10), E6
10⊕5̄ 16 27
IIB
+ D-branes
+ fluxes
D9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IIA
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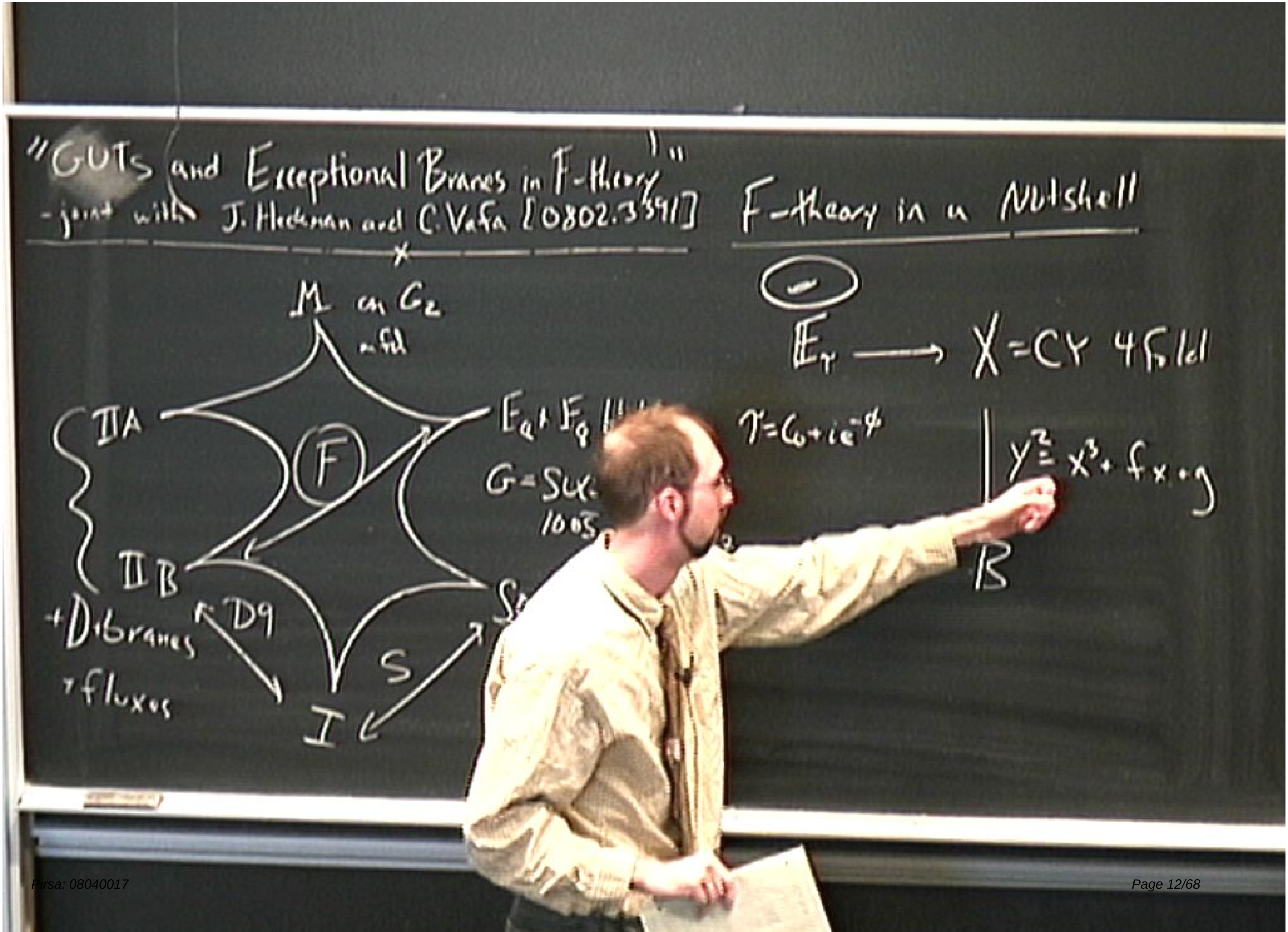
+D-branes
+fluxes
D9
S
I
X=CY 4fold
B
27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F-theory in a Nutshell
M on G_2 mfd
IIA
IIB
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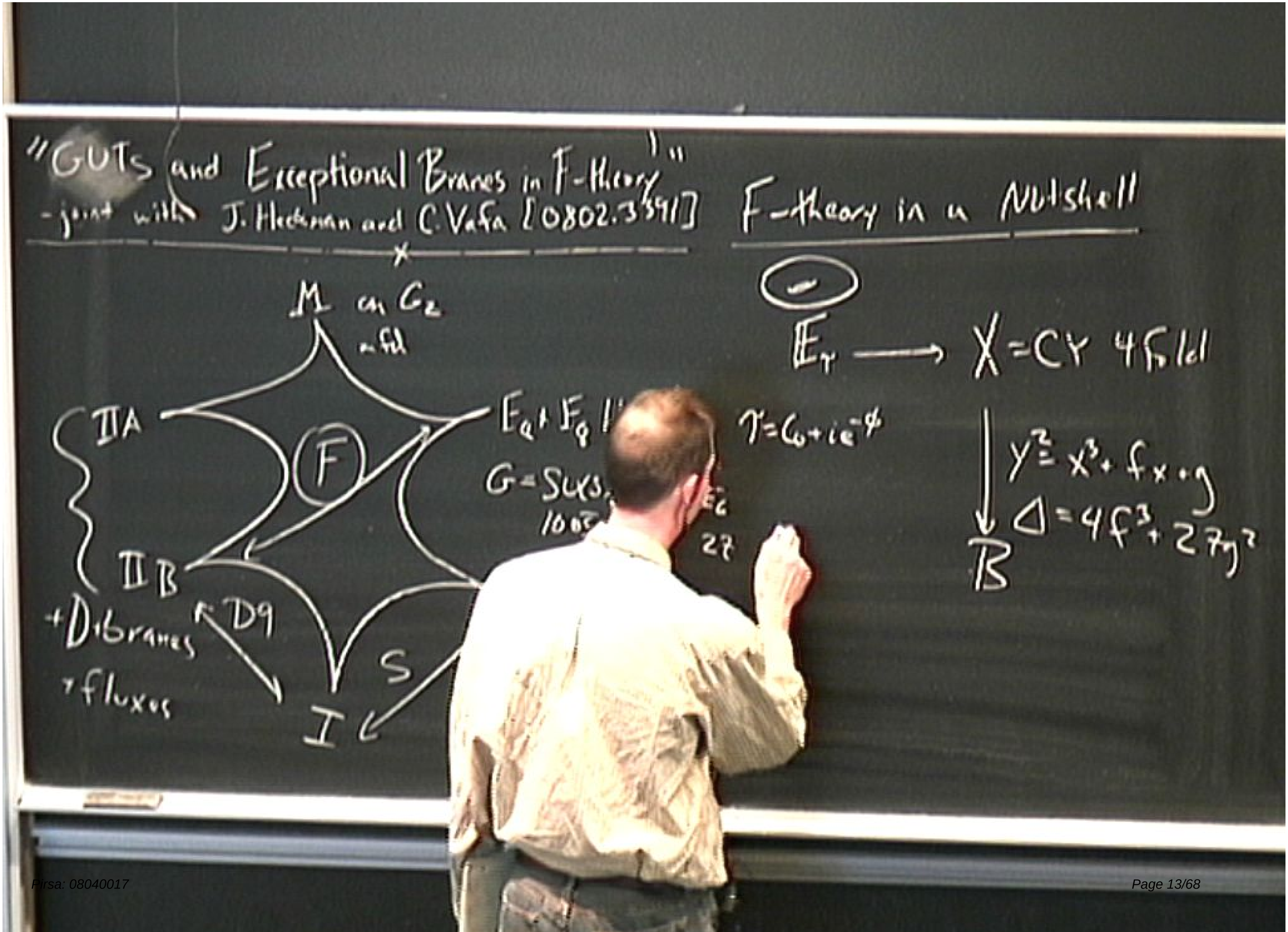
S
I
D9
+ D-branes
+ fluxes
$E_8 \times E_8$
$\tau = C_0 + ie^{-\phi}$
$\mathbb{E}_\tau \longrightarrow X = CY$ 4fold
$y^2 = x^3 + fx + g$
B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IIA
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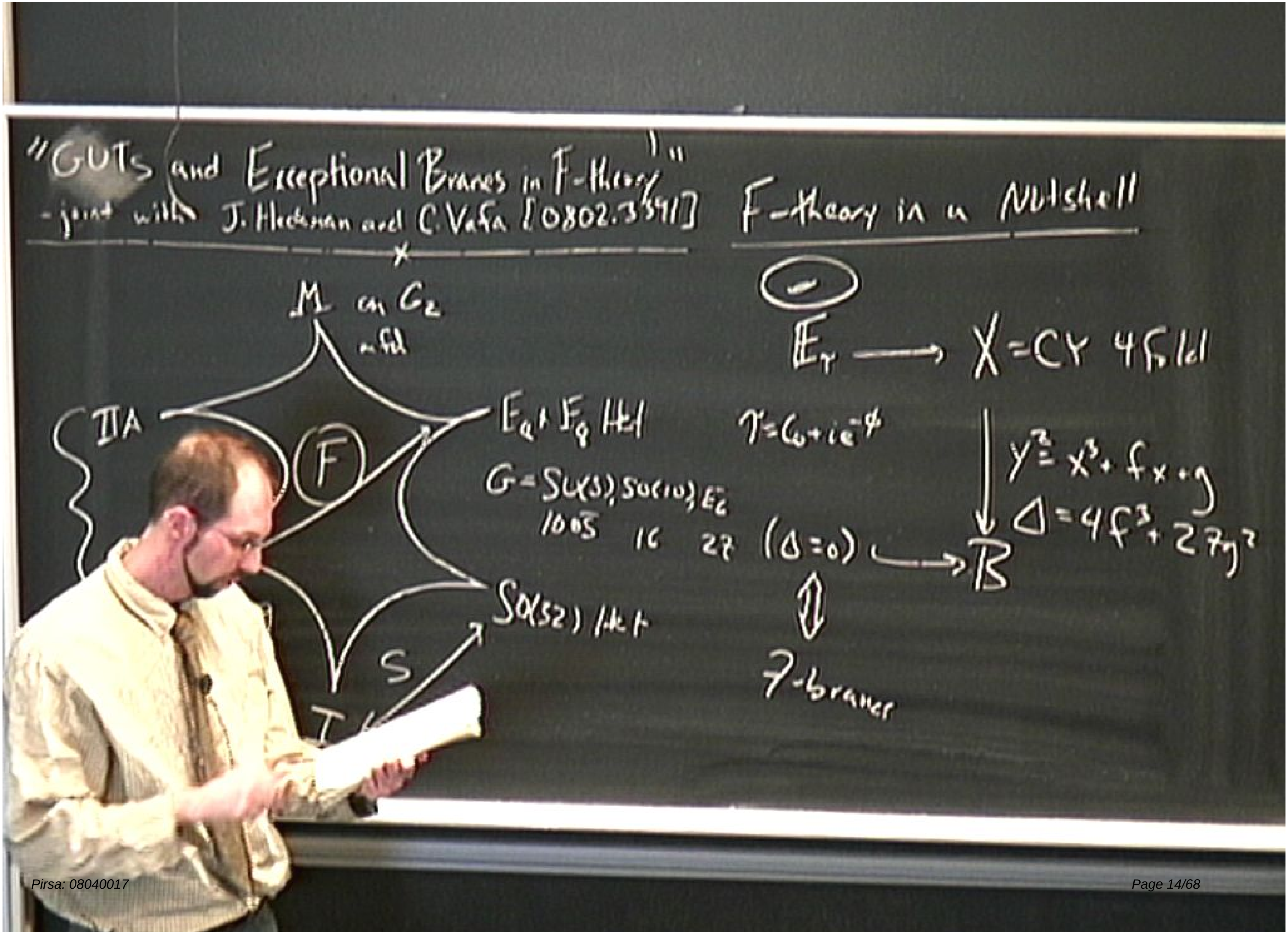
D9
+D-branes
+fluxes
E8 × E8
G = SU(3)
27
X = CY 4fold
B
y² = x³ + fx + g
Δ = 4f³ + 27g²
τ = C0 + ie^-φ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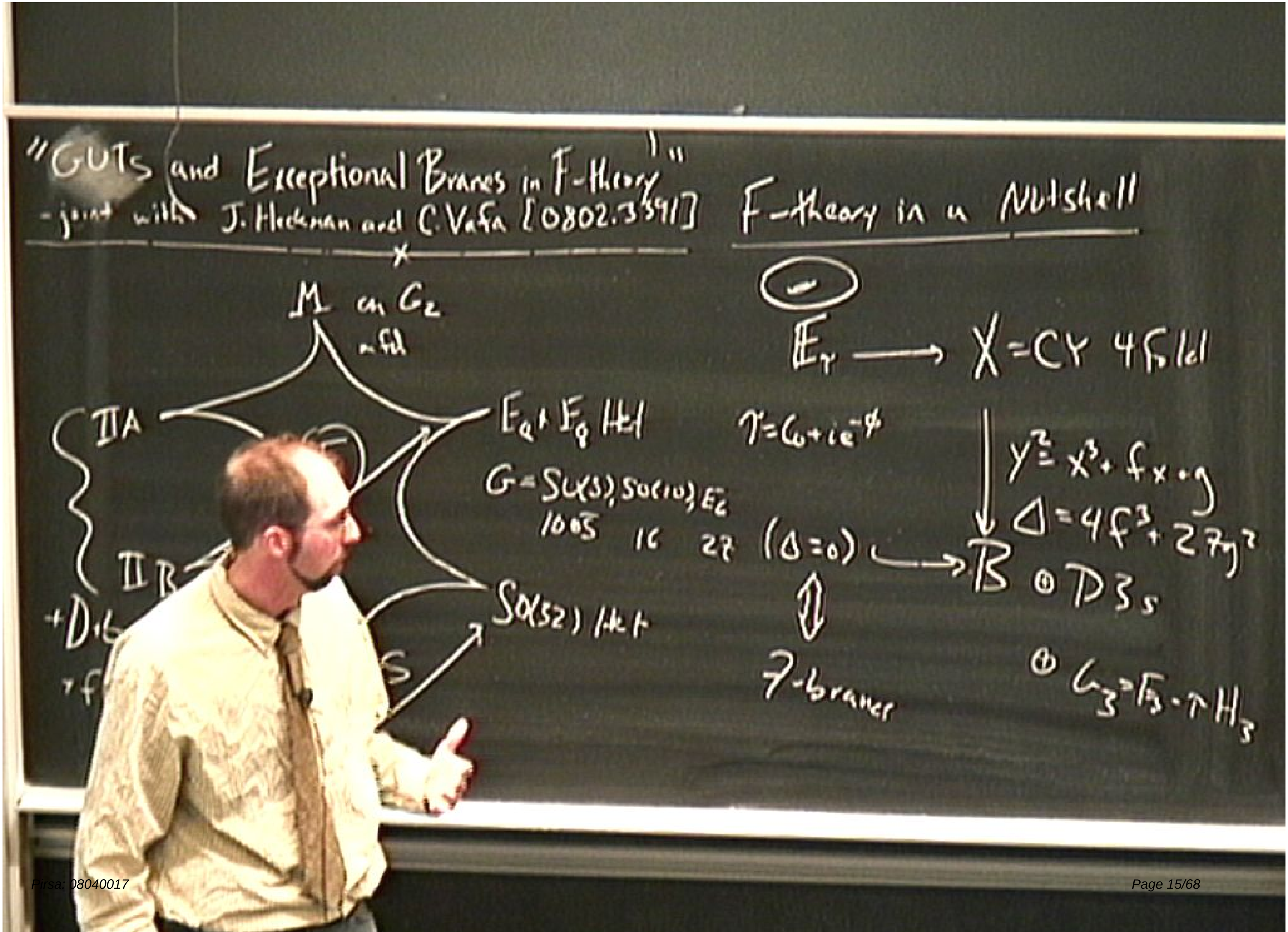
SO(32) Het
S
I
τ = C0 + ie^{-φ}
E_τ → X = CY 4fold
y² = x³ + fx + g
Δ = 4f³ + 27g²
(Δ = 0) → B
7-branes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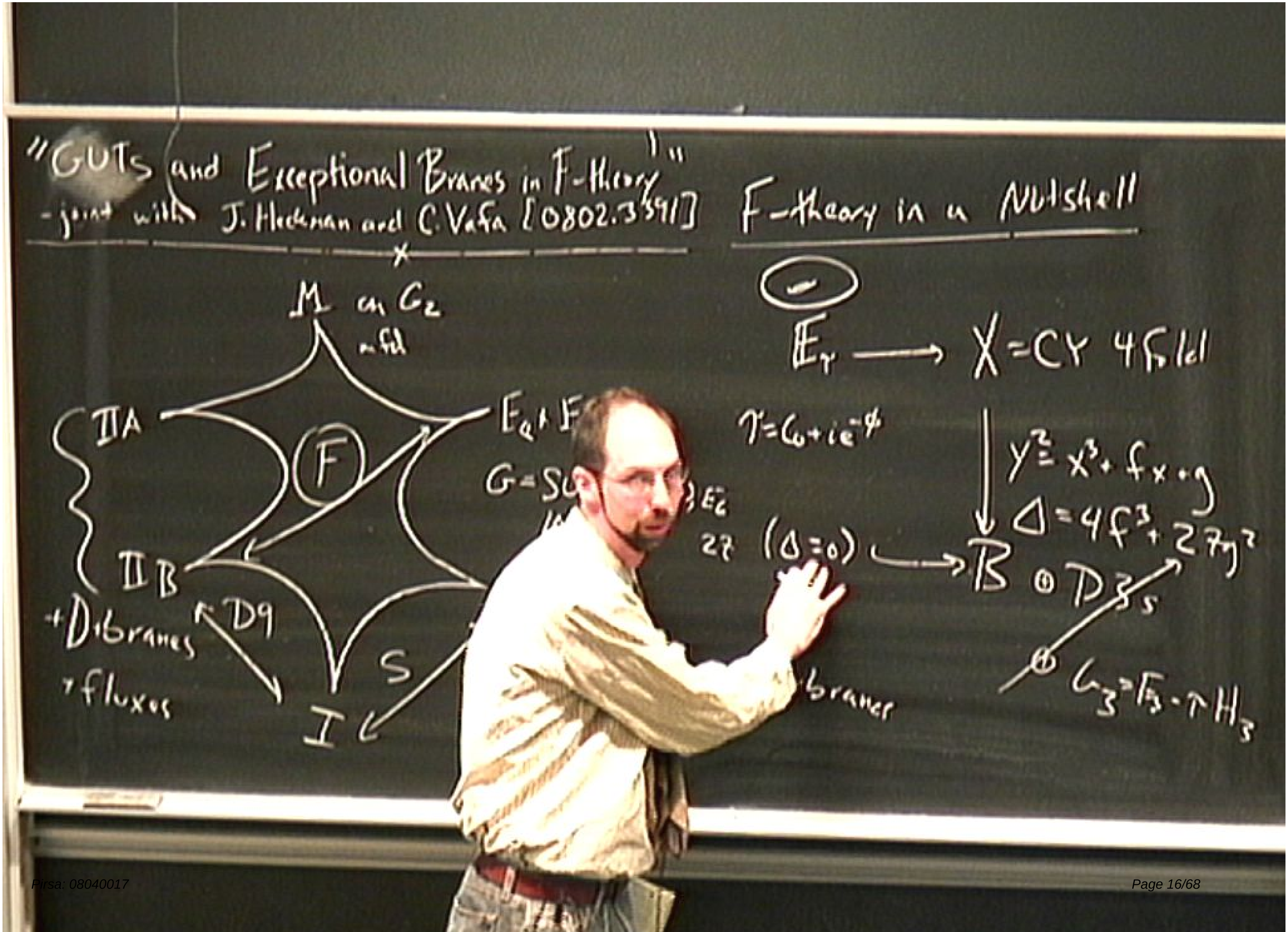
τ = C0 + i e^-φ
X = CY 4 fold
y² = x³ + f x + g
Δ = 4f³ + 27g²
(Δ = 0)
B
⊕ D3s
⊕ G3 = F3 - τ H3
7-branes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$\Delta = 4f^3 + 27g^2$
$(\Delta = 0)$
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$G_3 = F_3 - \tau H_3$
branes

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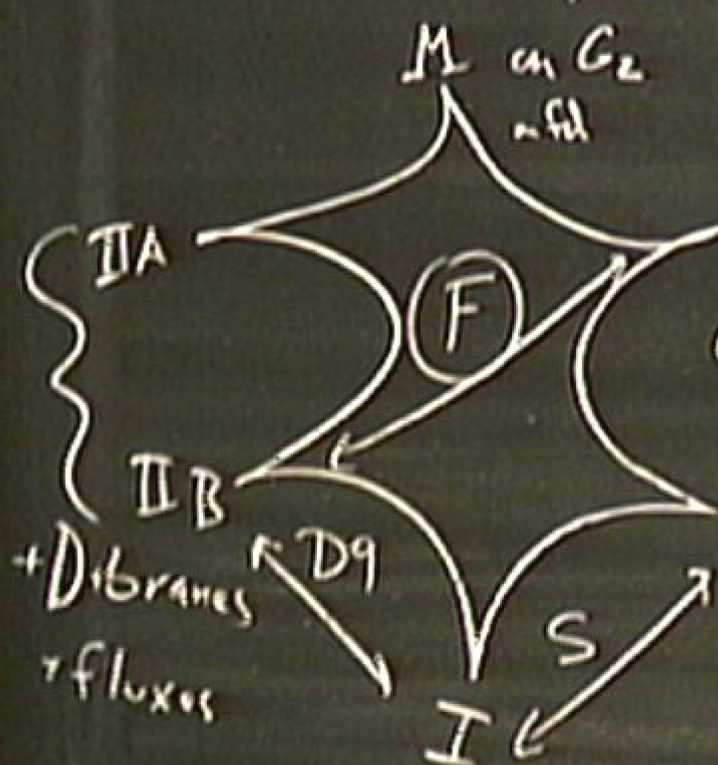


$G = SU(5), SO(10), E_6$

$10 \oplus \bar{5}$ 16   27

F-theory in a Nutshell

$$\mathbb{E}_\tau \longrightarrow X = CY\ 4\text{fold}$$

$$\tau = C_0 + i e^{-\phi}$$

$$y^2 = x^3 + f x + g$$

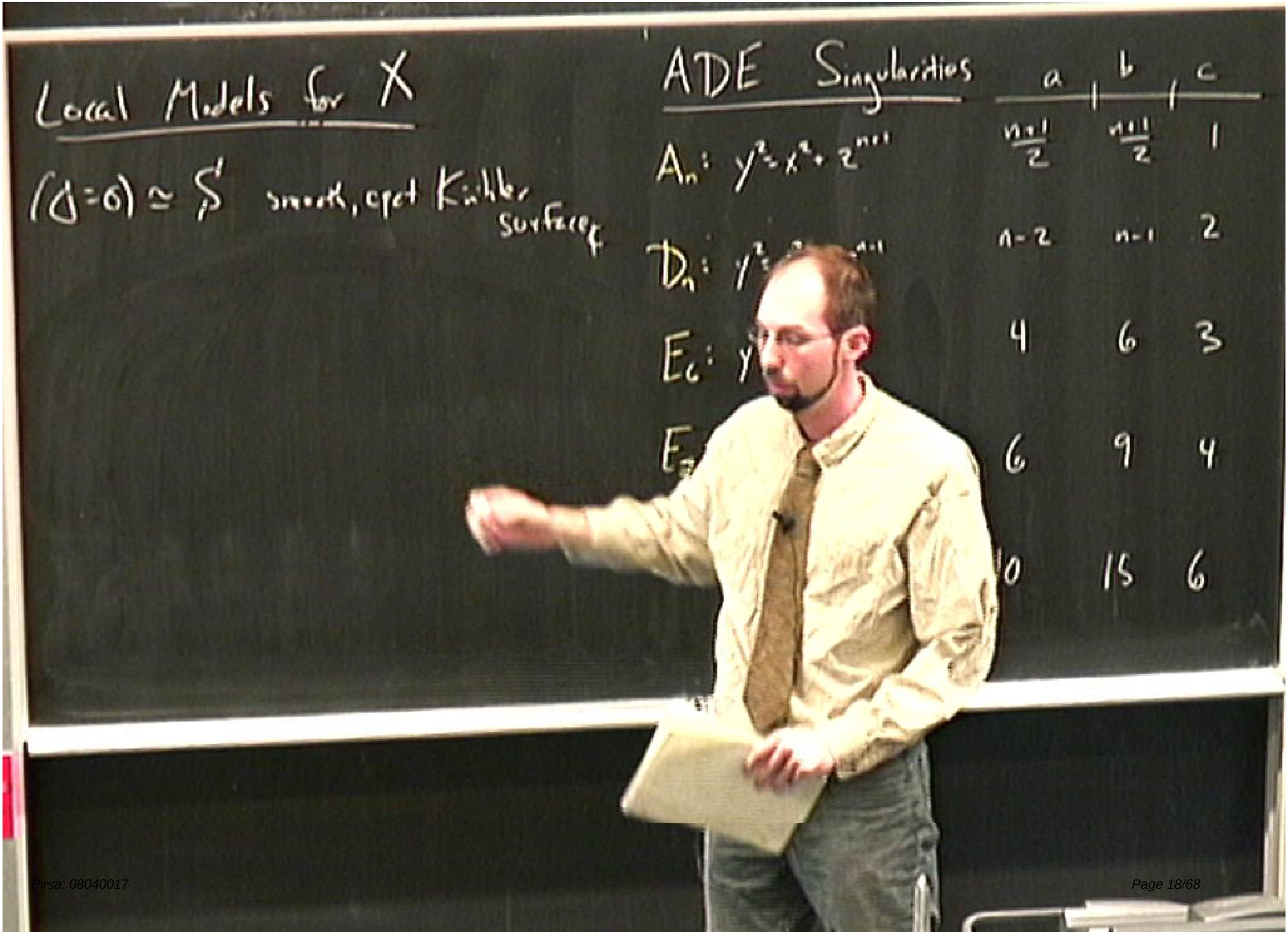
$$\Delta = 4f^3 + 27g^2$$

$$(\Delta = 0) \hookrightarrow B \oplus D3s$$

$\Updownarrow$

7-branes

$$\oplus\ G_3 = F_3 - \tau H_3$$


Local Models for X
(Δ=0) ≃ S smooth, cpct Kähler surface
ADE Singularities
a b c
A_n: y²=x²+z^{n+1}
D_n
E_6
4 6 3
6 9 4
15 6

Local Models for X

$(\Delta = 0) \simeq S$ smooth, cpct Kähler surface.

ADE Singularities

		a	b	c
A_n:	$y^2 = x^2 + z^{n+1}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	1
D_n:	$y^2 = x^2 z + z^{n-1}$	$n-2$	$n-1$	2
E_6:	$y^2 = x^3 + z^4$	4	6	3
E_7:	$y^2 = x^3 + x z^3$	6	9	4
E_8:	$y^2 = x^3 + z^5$	10	15	6

Local Models for X

$(\Delta = 0) \simeq S$ smooth, cpct Kähler surface

K3 fibration

$$\begin{array}{ccc} Y_{ADE} & \longrightarrow & X \\ & & \downarrow \\ & & S \end{array}$$

ADE Singularities

		a	b	c
A_n:	$y^2 = x^2 + z^{n+1}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	1
D_n:	$y^2 = x^2 z + z^{n-1}$	$n-2$	$n-1$	2
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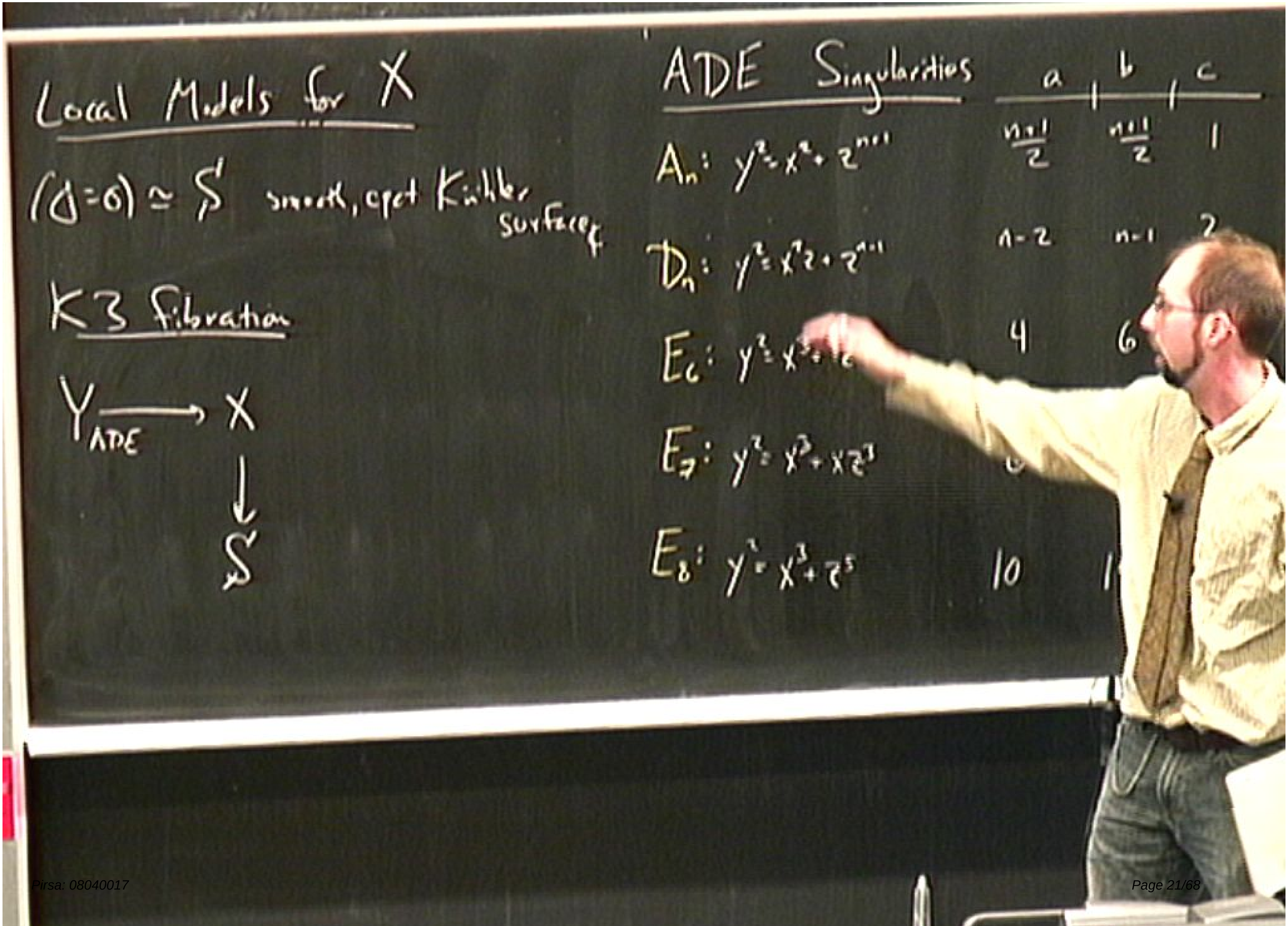
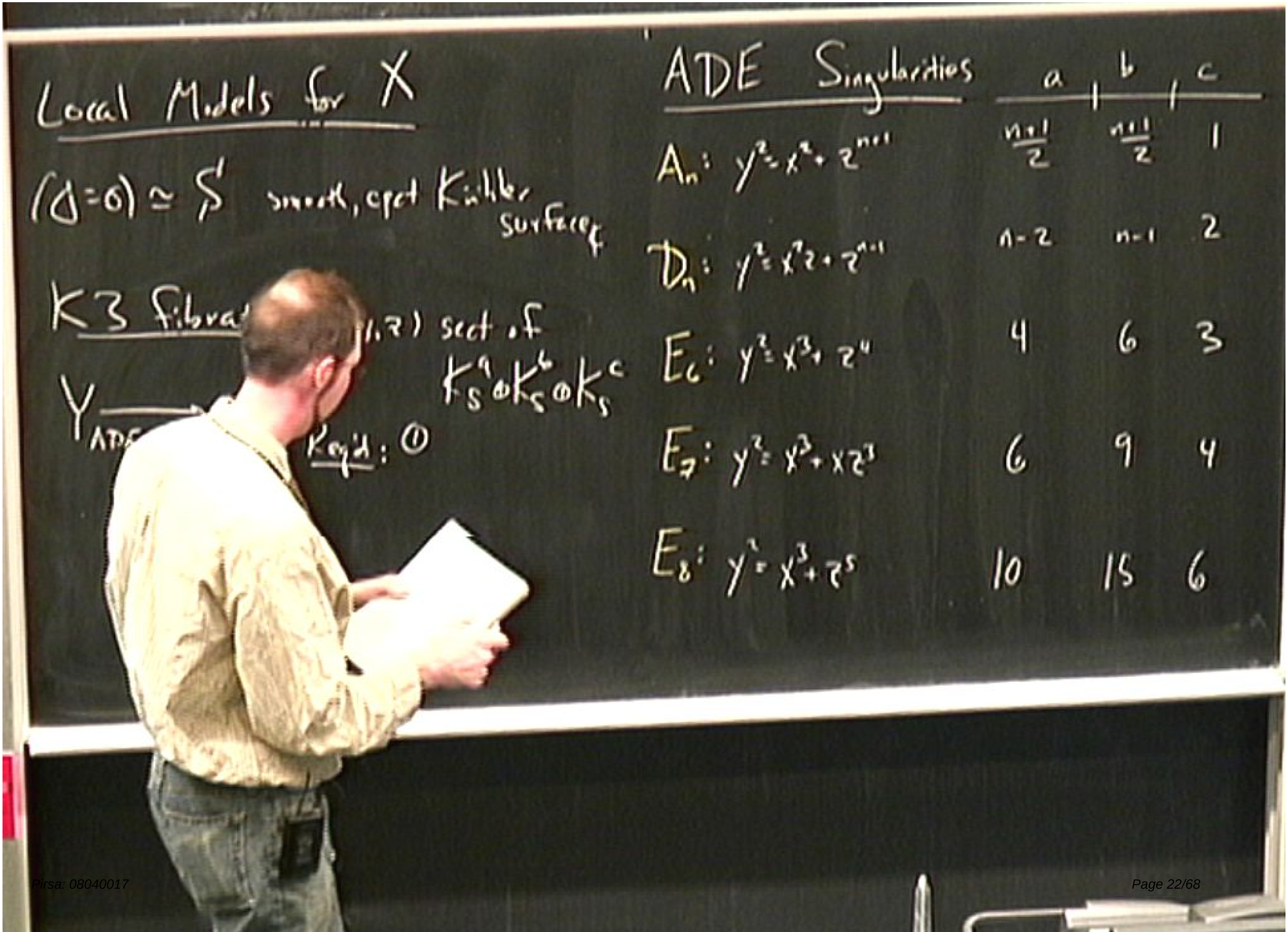
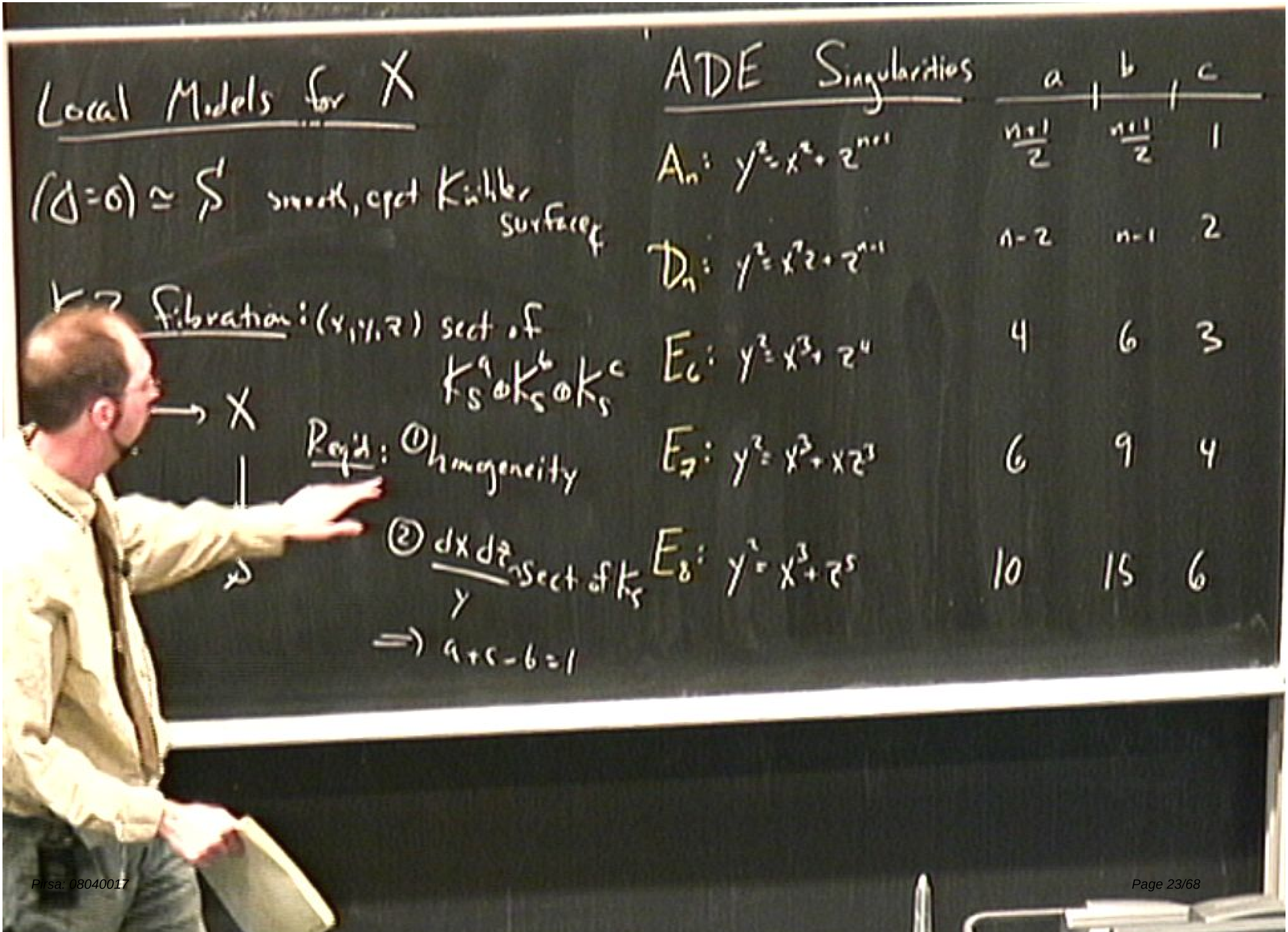
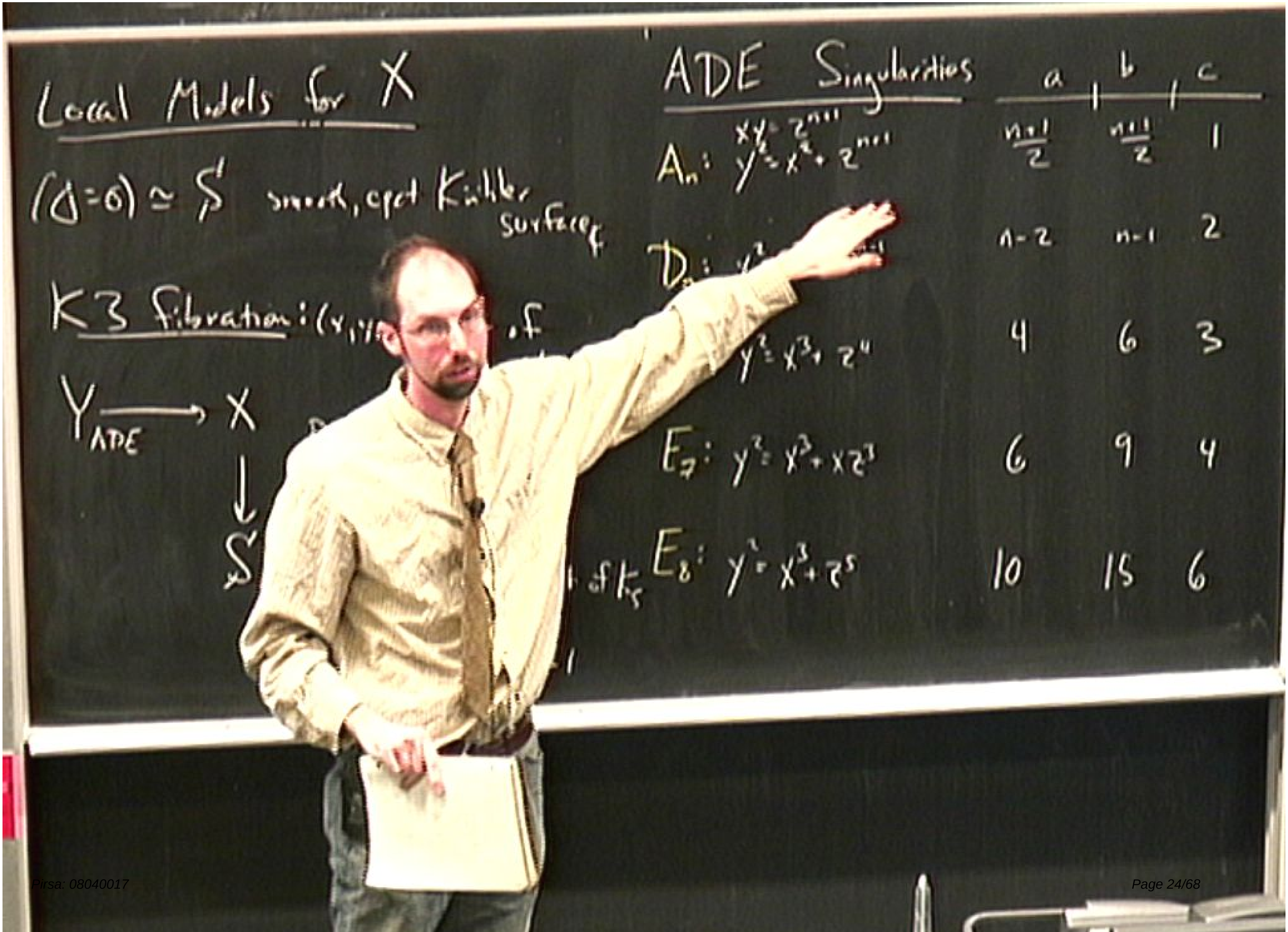

Local Models for X
(Δ=0) ≃ S smooth, cpct Kähler surface
K3 fibration
Y_ADE → X
S
ADE Singularities
a b c
A_n: y^2 = x^2 + z^{n+1}
(n+1)/2 (n+1)/2 1
D_n: y^2 = x^2 z + z^{n-1}
n-2 n-1 2
E_6: y^2 = x^3
4 6
E_7: y^2 = x^3 + x z^3
E_8: y^2 = x^3 + z^5
10


Local Models for X
(Δ=0) ≃ S smooth, cpct Kähler surface
K3 fibrat… (1,2) sect of
K_S^a ⊕ K_S^b ⊕ K_S^c
Y_ADE
Req'd: ①
ADE Singularities
a b c
A_n: y² = x² + z^(n+1) (n+1)/2 (n+1)/2 1
D_n: y² = x²z + z^(n-1) n−2 n−1 2
E_6: y² = x³ + z⁴ 4 6 3
E_7: y² = x³ + xz³ 6 9 4
E_8: y² = x³ + z⁵ 10 15 6


Local Models for X
(Δ=0) ≃ S smooth, cpct Kähler surface
fibration: (x,y,z) sect of
K_S^a ⊕ K_S^b ⊕ K_S^c
X
Req'd: ① homogeneity
② dx dz / y sect of K_S
⇒ a+c−b=1
ADE Singularities
a | b | c
A_n: y² = x² + z^(n+1)   (n+1)/2   (n+1)/2   1
D_n: y² = x²z + z^(n−1)   n−2   n−1   2
E_6: y² = x³ + z⁴   4   6   3
E_7: y² = x³ + xz³   6   9   4
E_8: y² = x³ + z⁵   10   15   6



Local Models for X
(Δ=0) ≃ S smooth, cpct Kähler surface
K3 fibration
ADE Singularities
a b c
xy = z^{n+1}
A_n: y^2 = x^2 + z^{n+1}
(n+1)/2 (n+1)/2 1
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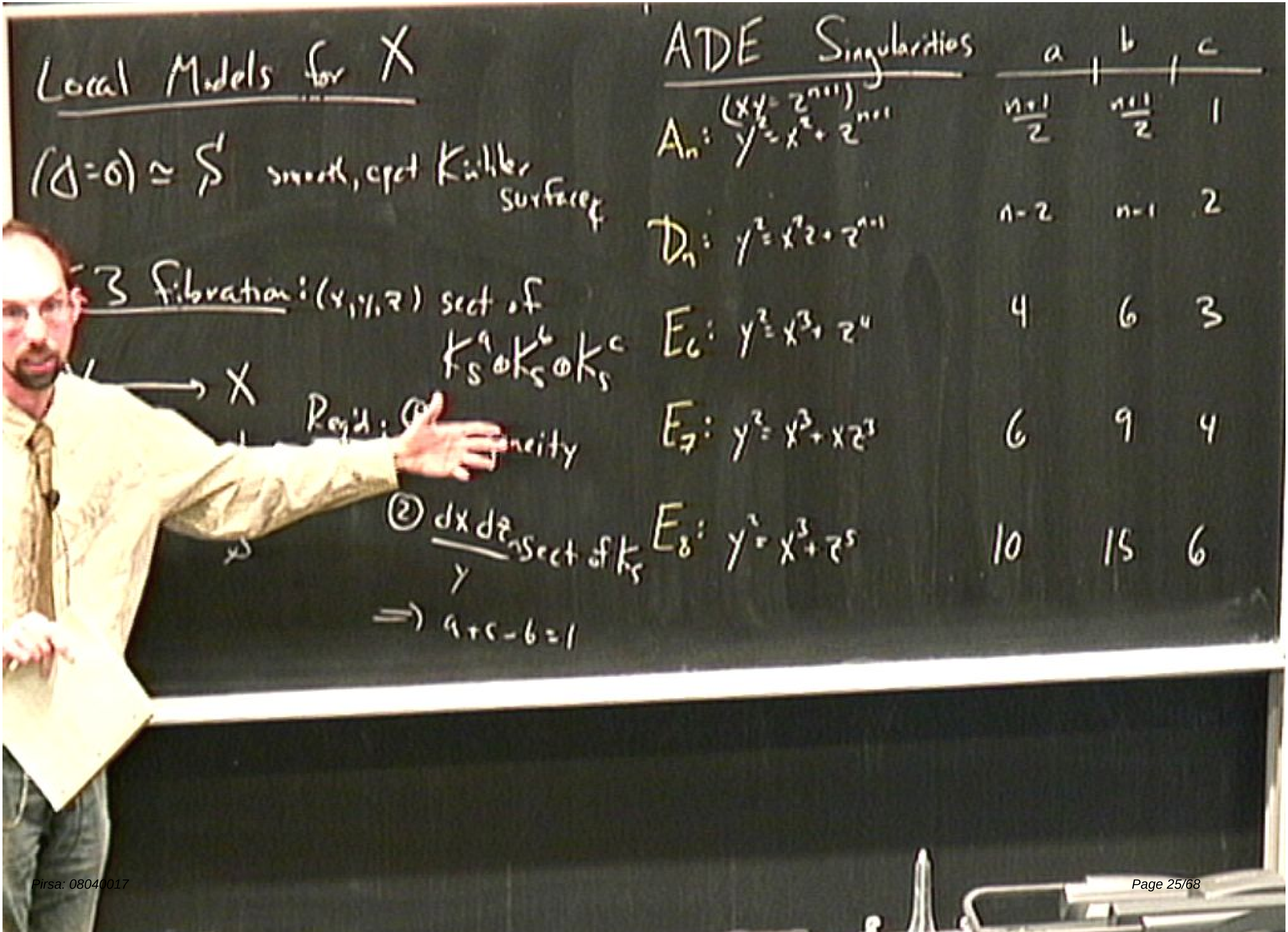
10 15 6


Local Models for X
(Δ=0) ≃ S smooth, cpct Kähler surface
K3 fibration: (x,y,z) sect of
K_S^a ⊕ K_S^b ⊕ K_S^c
Y → X
Req'd:
② dx dz / y ~ sect of K_S
⇒ a+c-b=1
ADE Singularities
a
b
c
A_n: (xy = z^{n+1}) y^2 = x^2 + z^{n+1}
(n+1)/2
(n+1)/2
1
D_n: y^2 = x^2 z + z^{n-1}
n-2
n-1
2
E_6: y^2 = x^3 + z^4
4
6
3
E_7: y^2 = x^3 + xz^3
6
9
4
E_8: y^2 = x^3 + z^5
10
15
6

Local Models for X

$(\Delta = 0) \simeq S$ smooth, cpct Kähler surface

K3 fibration: (x, y, z) sect of $K_S^a \oplus K_S^b \oplus K_S^c$

$$\begin{array}{ccc} Y_{ADE} & \longrightarrow & X \\ & & \downarrow \\ & & S \end{array}$$

Req'd: ① homogeneity

② $\frac{dx\,dz}{y}$ sect of K_S

$\Rightarrow a + c - b = 1$

ADE Singularities

	a	b	c
A_n: $y^2 = x^2 + z^{n+1}$ $(xy = z^{n+1})$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	1
D_n: $y^2 = x^2 z + z^{n-1}$	$n-2$	$n-1$	2
E_6: $y^2 = x^3 + z^4$	4	6	3
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Local Models for X

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Req'd: ① homogeneity

② $\frac{dx\, dz}{y} \sim$ sect. of K_S

$\Rightarrow a + c - b = 1$

ADE Singularities

	a	b	c
A_n: $y^2 = x^2 + z^{n+1}$ $(xy = z^{n+1})$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	1
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Local Models for X

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K3 fibration: (x, y, z) sect of $K_S^a \oplus K_S^b \oplus K_S^c$

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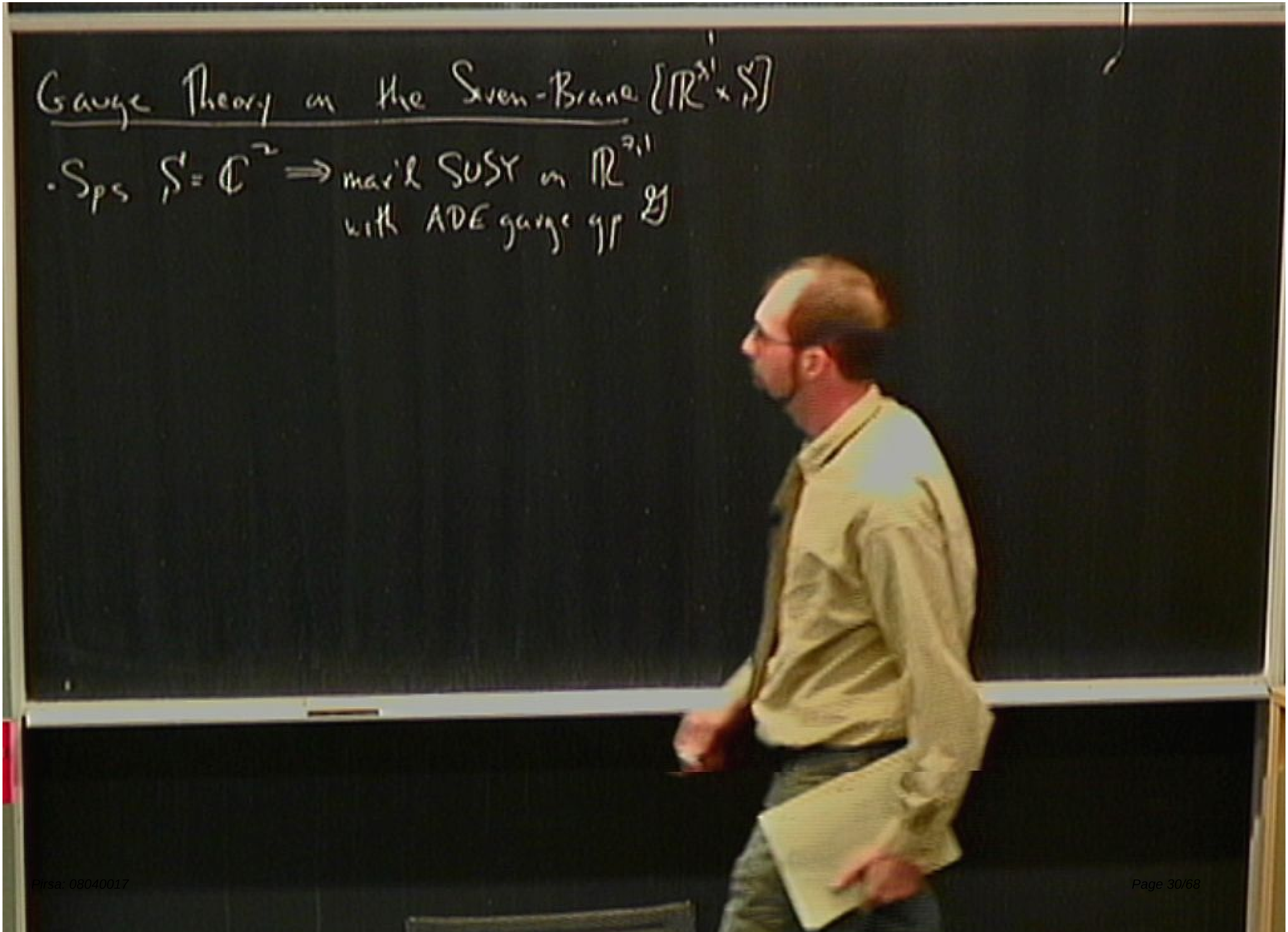
E_8: $y^2 = x^3 + z^5$	10	15	6

Gauge Theory on the Seven-Brane $[\mathbb{R}^{3,1} \times S]$


Gauge Theory on the Seven-Brane [ℝ^{7,1} × S]
· Sps S = ℂ^2 ⟹ max'l SUSY on ℝ^{7,1}
with ADE gauge gp 𝒢

Gauge Theory on the Seven-Brane $[\mathbb{R}^{3,1} \times S]$

$\cdot$ Sps $S = \mathbb{C}^2 \Rightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$
with ADE gauge gp $\mathcal{G}$

Bosons

Fermions

A

Gauge Theory on the Seven-Brane $[\mathbb{R}^{3,1} \times S]$

- Sps $S = \mathbb{C}^2 \Rightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$ with ADE gauge gp $\mathcal{G}$

Bosons

A connection on a $\mathcal{G}$-bdl P over S

Fermions

with ADE gauge gp $\mathcal{G}$

<u>Bosons</u>

<u>Fermions</u>

A connection on a $\mathcal{G}$-bdl P over S

$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$

Gauge Theory on the Seven-Brane $[\mathbb{R}^{3,1} \times S]$

- Sps $S = \mathbb{C}^2 \Longrightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$ with ADE gauge gp G

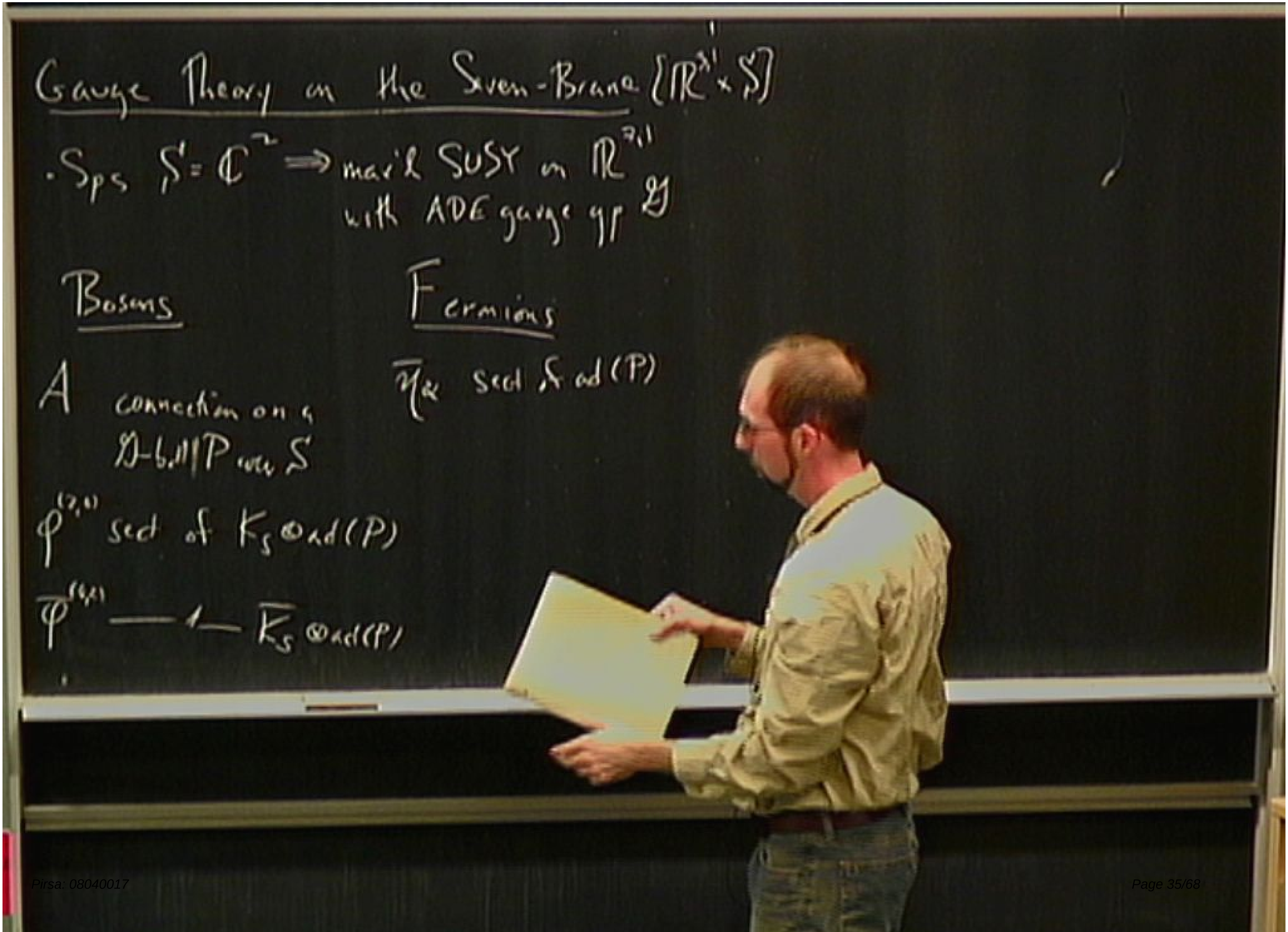
Bosons

A connection on a G-bdl P over S

$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$

$\overline{\varphi}^{(0,2)}$ — " — $\overline{K_S} \otimes ad(P)$

Fermions


Gauge Theory on the Seven-Brane [$\mathbb{R}^{3,1} \times S$]
• Sps $S = \mathbb{C}^2 \Rightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$
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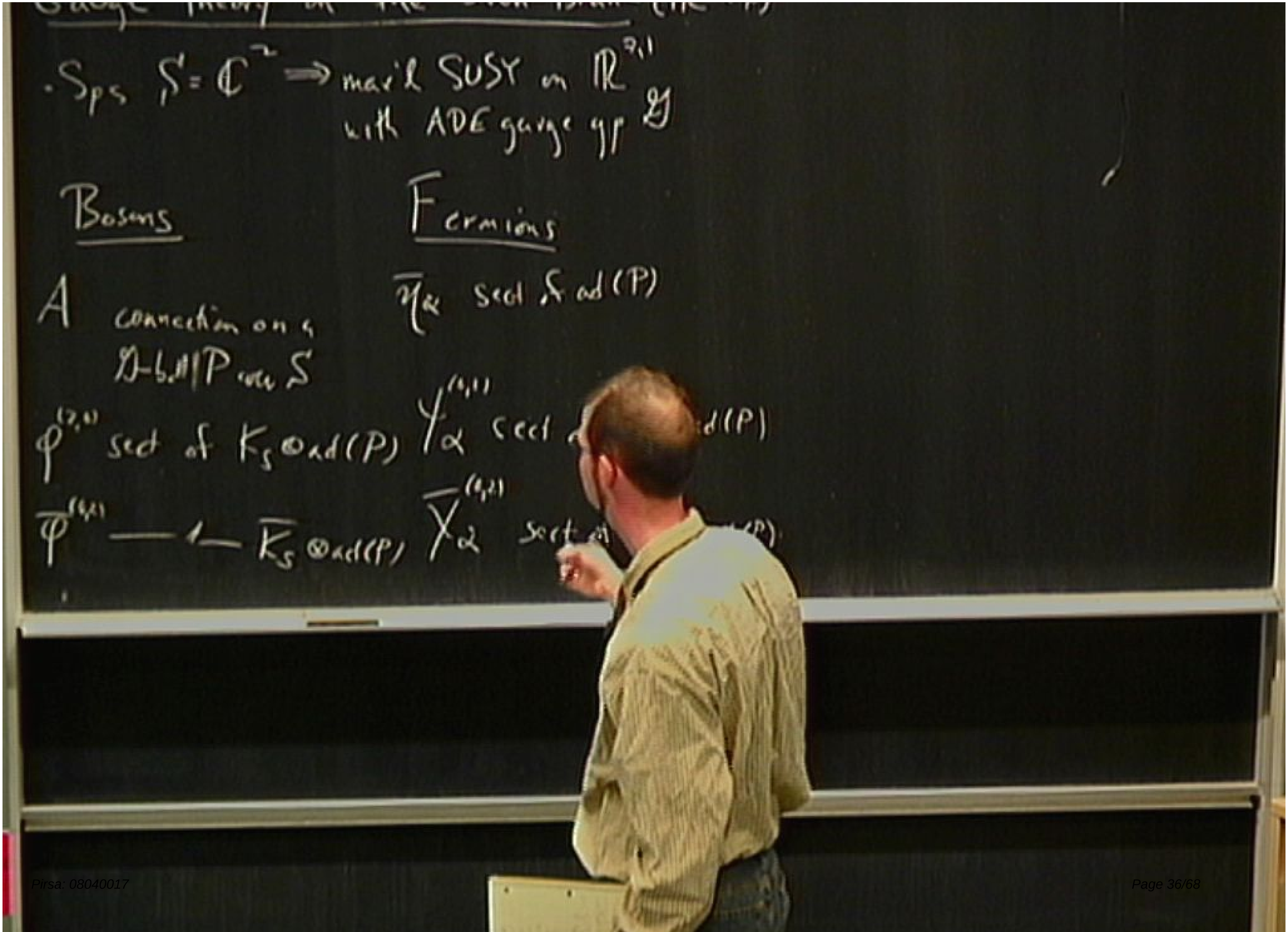
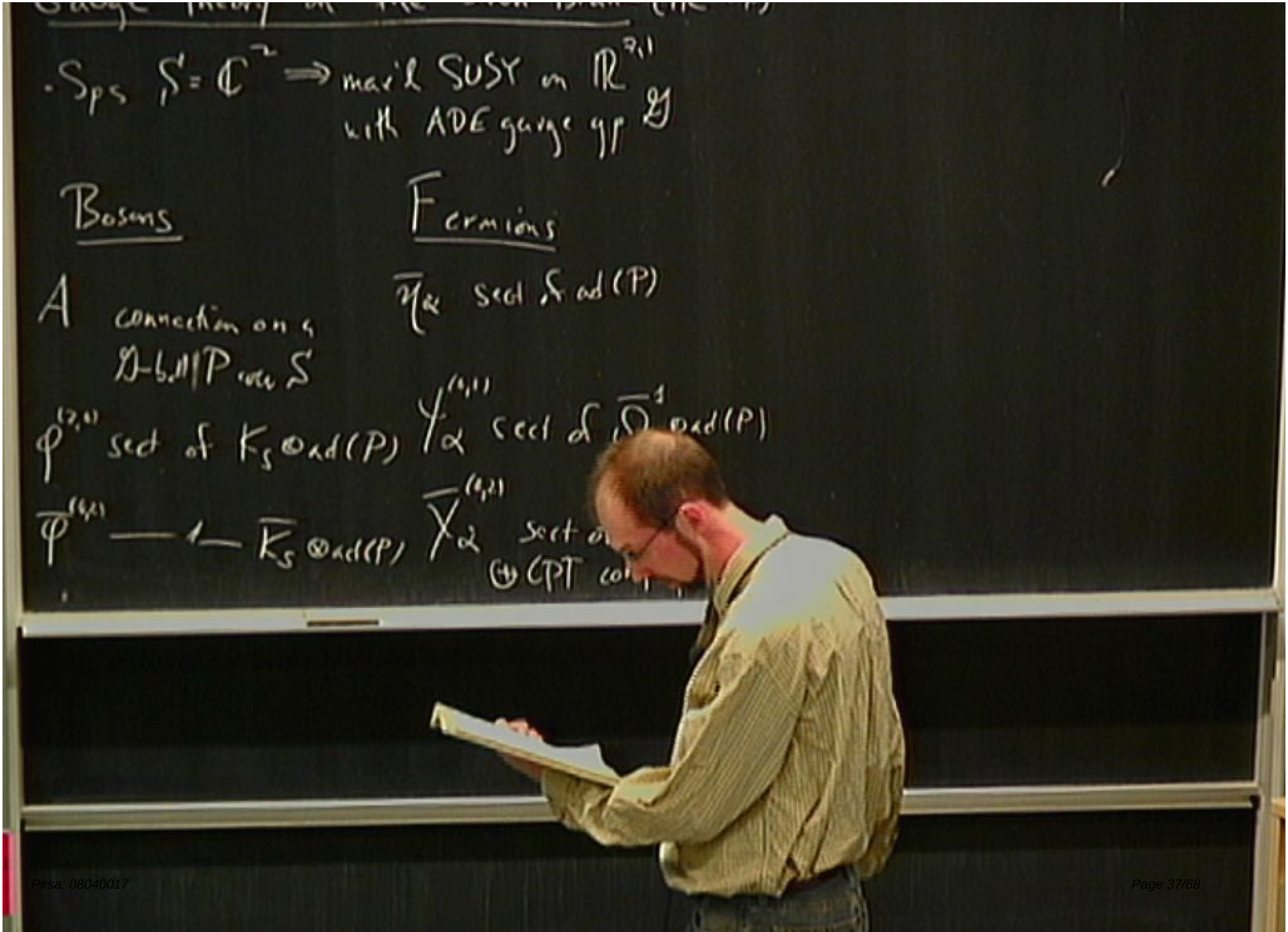
Bosons
Fermions
A connection on a $\mathcal{G}$-bdl P over S
$\bar{\eta}_\alpha$ sect of ad(P)
$\varphi^{(2,0)}$ sect of $K_S \otimes$ ad(P)
$\bar{\varphi}^{(0,2)}$ — " — $\bar{K}_S \otimes$ ad(P)



• Sps S = ℂ² ⟹ max'l SUSY on ℝ^{2,1}
with ADE gauge gp G
Bosons
Fermions
A connection on a G-bdl P over S
η̄_α sect of ad(P)
φ^{(2,0)} sect of K_S ⊗ ad(P)
ψ_α^{(1,1)} sect
φ̄^{(0,2)}
K̄_S ⊗ ad(P)
χ̄_α^{(0,2)} sect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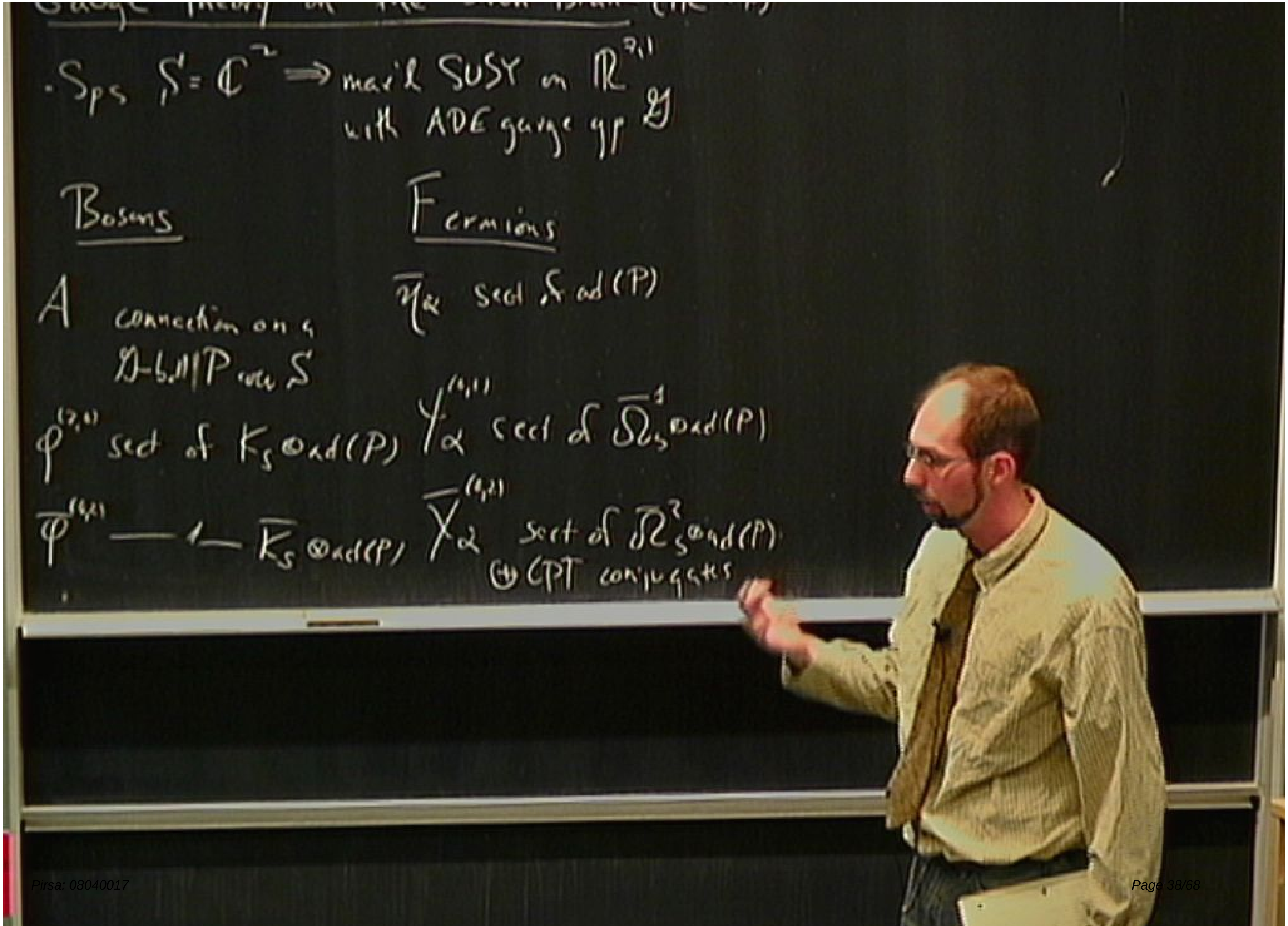
ψ_α^(1,1) sect of Ω̄¹ ⊗ ad(P)
φ̄^(0,2) — " — K̄_S ⊗ ad(P)
χ̄_α^(0,2) sect o
⊕ CPT co



. Sps $S = \mathbb{C}^2 \Rightarrow$ max'l SUSY on $\mathbb{R}^{2,1}$
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A connection on a G-bdl P over S
$\bar{\eta}_\alpha$ sect of ad(P)
$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$
$\psi_\alpha^{(0,1)}$ sect of $\bar{\Omega}^1_S \otimes ad(P)$
$\bar{\varphi}^{(0,2)}$ —— 1 —— $\bar{K}_S \otimes ad(P)$
$\bar{\chi}_\alpha^{(0,2)}$ sect of $\bar{\Omega}^2_S \otimes ad(P)$
$\oplus$ CPT conjugates

Gauge Theory on the Seven-Brane [$\mathbb{R}^{3,1}\times S$]

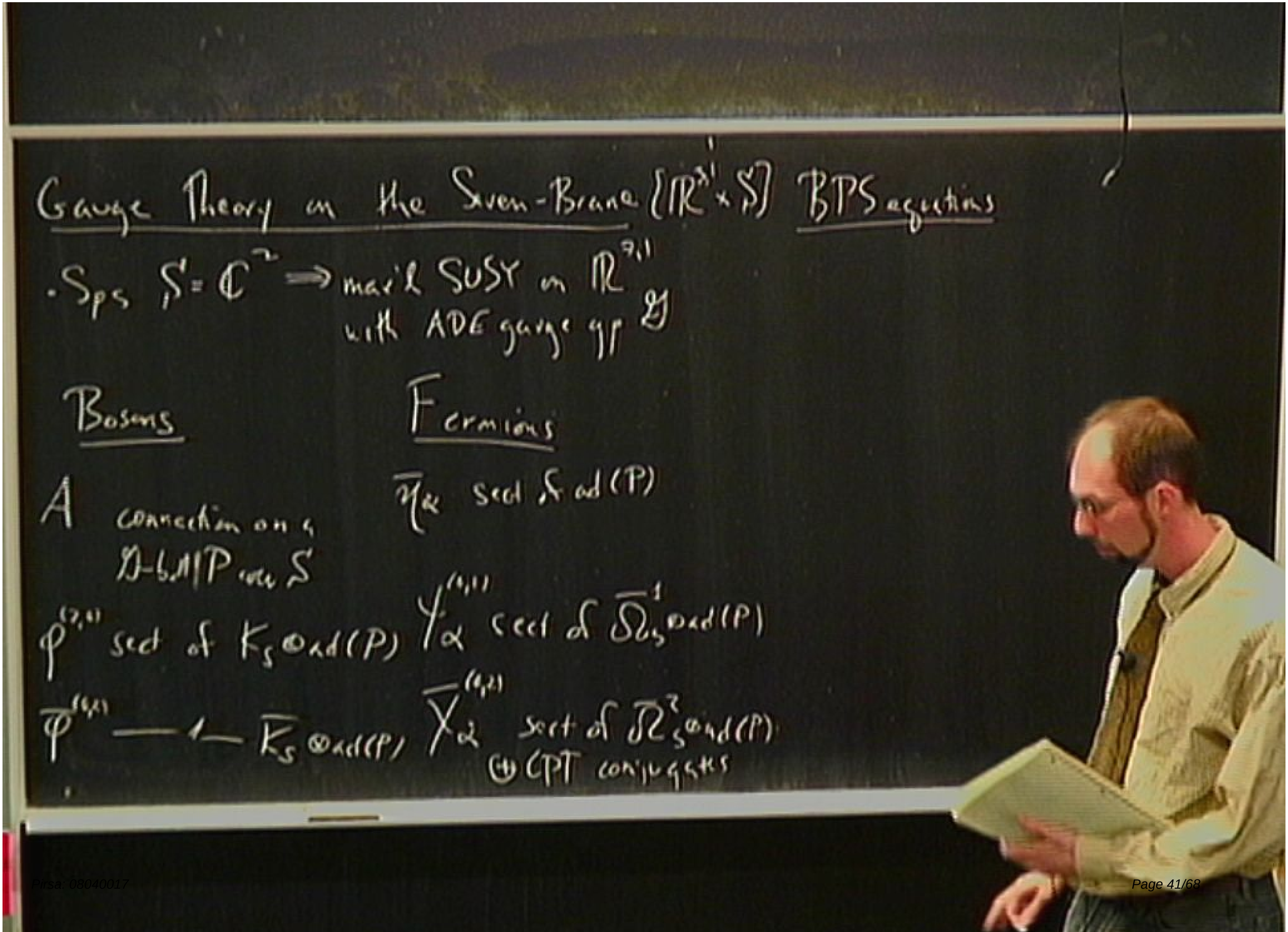
• Sps $S = \mathbb{C}^2 \Longrightarrow$ max'l SUSY on $\mathbb{R}^{3,1}$
with ADE gauge gp $\mathcal{G}$

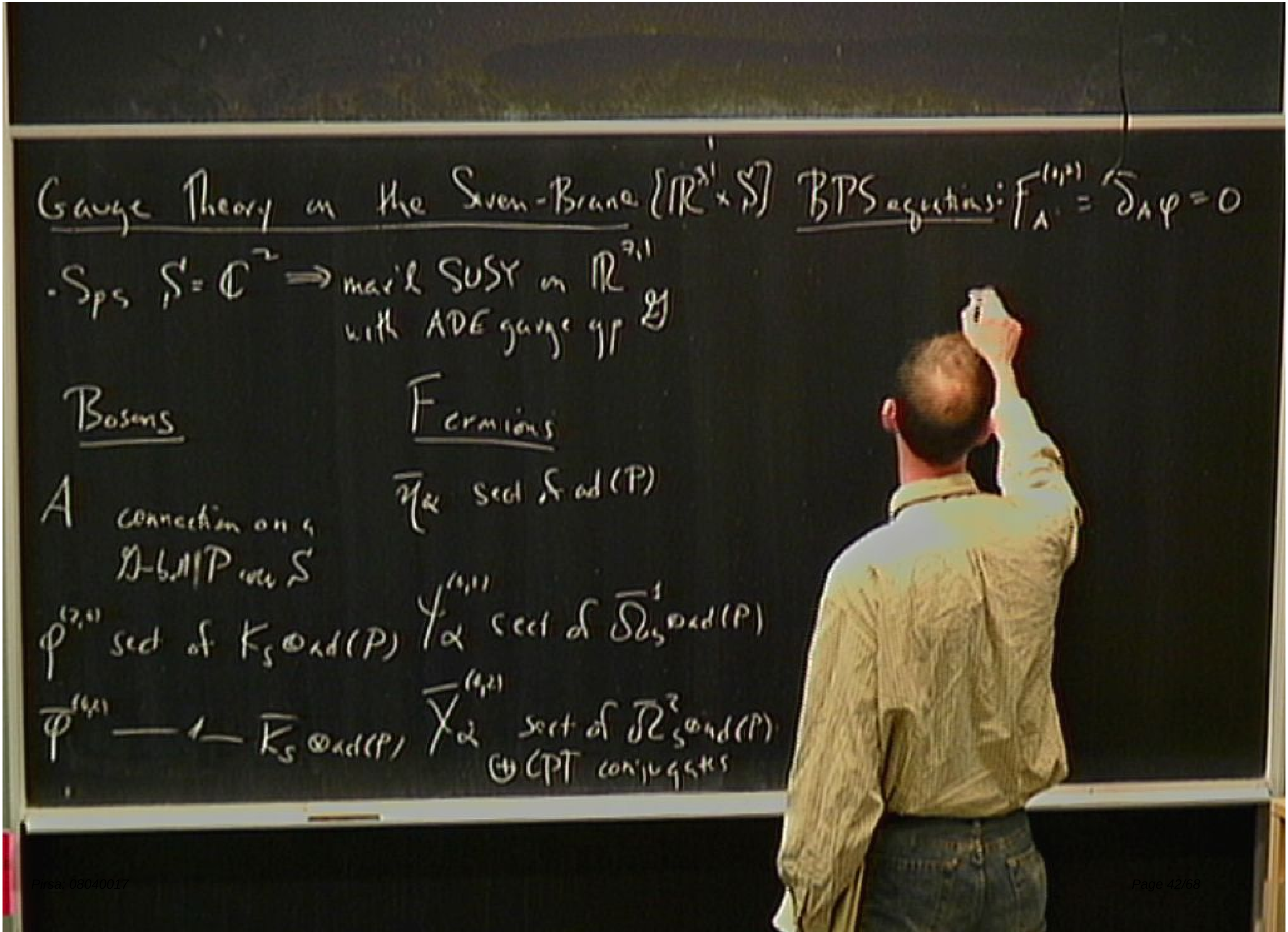
Bosons	Fermions
A connection on a $\mathcal{G}$-bdl P over S	$\bar\eta_\alpha$ sect of ad(P)
$\varphi^{(2,0)}$ sect of $K_S \otimes \mathrm{ad}(P)$	$\psi^{(0,1)}_\alpha$ sect of $\bar\Omega^1_S \otimes \mathrm{ad}(P)$
$\bar\varphi^{(0,2)}$ — " — $\bar K_S \otimes \mathrm{ad}(P)$	$\bar\chi^{(0,2)}_\alpha$ sect of [illegible] (P) ⊕ CPT conjugates

Gauge Theory on the Seven-Brane [$\mathbb{R}^{3,1} \times S$]

· Sps $S = \mathbb{C}^2 \Rightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$ with ADE gauge gp $\mathcal{G}$

Bosons	Fermions
A connection on a $\mathcal{G}$-bdl P over S	$\bar{\eta}_\alpha$ sect of ad(P)
$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$	$\psi_\alpha^{(0,1)}$ sect of $\bar{\Omega}^1_S \otimes ad(P)$
$\bar{\varphi}^{(0,2)}$ — " — $\bar{K}_S \otimes ad(P)$	$\bar{\chi}_\alpha^{(0,2)}$ sect of $\bar{\Omega}^2_S \otimes ad(P)$
	$\oplus$ CPT conjugates


Gauge Theory on the Seven-Brane [$\mathbb{R}^{3,1} \times S$] BPS equations
• Sps $S = \mathbb{C}^2 \Rightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$
with ADE gauge gp G
Bosons
Fermions
A connection on a G-bdl P over S
$\bar{\eta}_\alpha$ sect of ad(P)
$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$
$\psi^{(1,0)}_\alpha$ sect of $\bar{\Omega}^1_S \otimes ad(P)$
$\bar{\varphi}^{(0,2)}$ — " — $\bar{K}_S \otimes ad(P)$
$\bar{\chi}^{(0,2)}_\alpha$ sect of $\bar{\Omega}^2_S \otimes ad(P)$
⊕ CPT conjugates


Gauge Theory on the Seven-Brane [R^{3,1} × S]
BPS equations: F_A^{(1,1)} = ∂̄_A φ = 0
· Sps S = C^2 ⟹ max'l SUSY on R^{7,1}
with ADE gauge gp G
Bosons
Fermions
A connection on a G-bdl P over S
η̄_α sect of ad(P)
φ^{(2,0)} sect of K_S ⊗ ad(P)
ψ_α^{(0,1)} sect of Ω̄^1_S ⊗ ad(P)
φ̄^{(0,2)} — " — K̄_S ⊗ ad(P)
χ̄_α^{(0,2)} sect of Ω̄^2_S ⊗ ad(P)
⊕ CPT conjugates

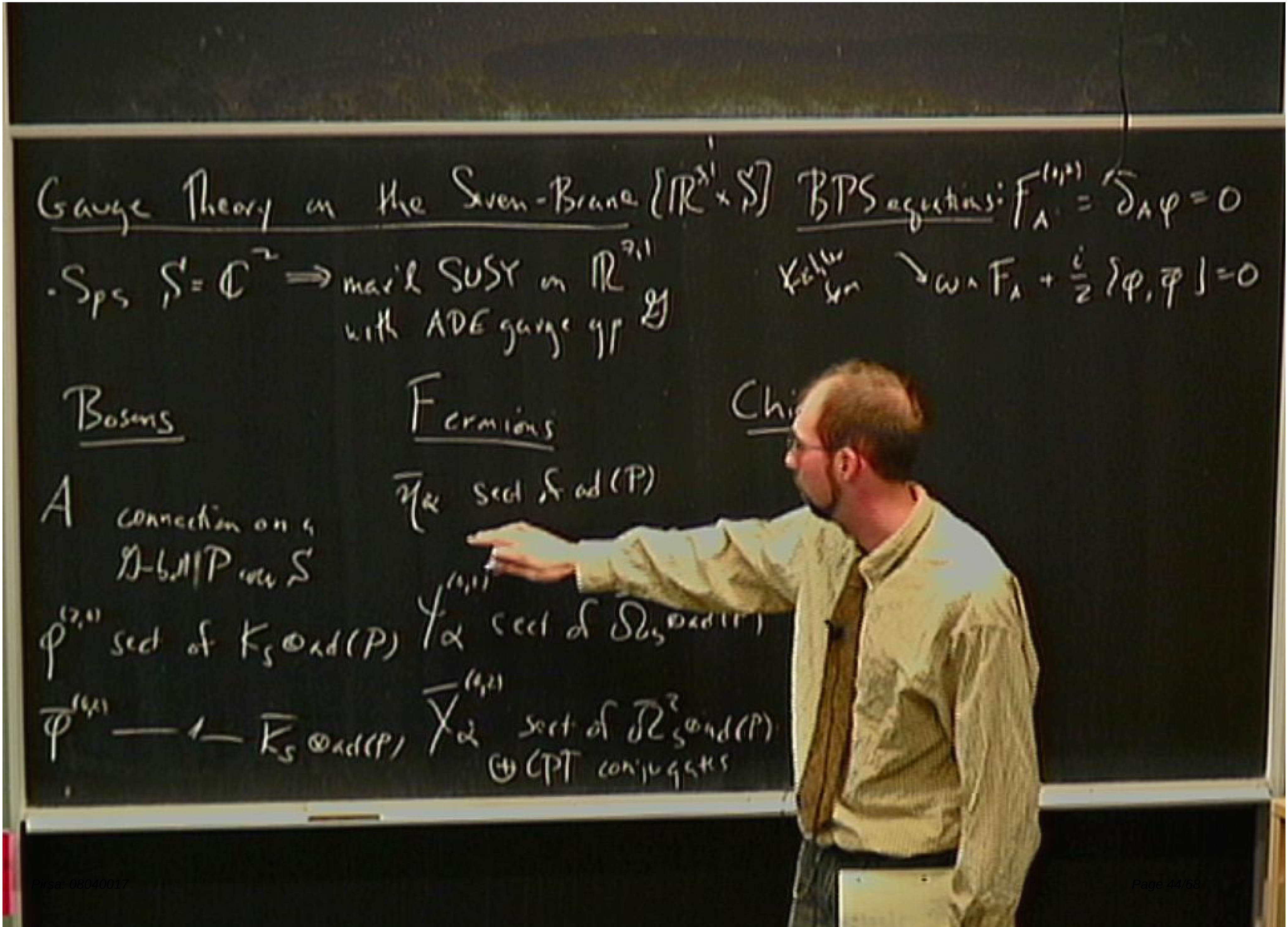
Gauge Theory on the Seven-Brane $[\mathbb{R}^{3,1}\times S]$

BPS equations: $F_A^{(0,2)} = \bar\partial_A \varphi = 0$

Kähler form $\omega \wedge F_A + \frac{i}{2}[\varphi, \bar\varphi] = 0$

- Sps $S = \mathbb{C}^2 \Longrightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$ with ADE gauge gp G

Bosons	Fermions
A connection on a G-bdl P over S	η_α sect of $ad(P)$
$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$	$\psi_\alpha^{(0,1)}$ sect of $\bar\Omega^1_S \otimes ad(P)$
$\bar\varphi^{(0,2)}$ — " — $\bar K_S \otimes ad(P)$	$\bar\chi_\alpha^{(0,2)}$ sect of $\bar\Omega^2_S \otimes ad(P)$
	$\oplus$ CPT conjugates


Gauge Theory on the Seven-Brane $[\mathbb{R}^{3,1} \times S]$
BPS equations: $F_A^{(0,2)} = \bar{\partial}_A \varphi = 0$
• Sps $S = \mathbb{C}^2 \Longrightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$
with ADE gauge gp $\mathcal{G}$
Kähler form
$\omega \wedge F_A + \frac{i}{2}[\varphi, \bar{\varphi}] = 0$
Bosons
Fermions
Chi
A connection on a $\mathcal{G}$-bdl P over S
$\bar{\eta}_\alpha$ sect of ad(P)
$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$
$\psi_\alpha^{(1,0)}$ sect of $\Omega_S \otimes ad(P)$
$\bar{\varphi}^{(0,2)}$ — " — $\bar{K}_S \otimes ad(P)$
$\bar{\chi}_\alpha^{(0,2)}$ sect of $\bar{\Omega}^2_S \otimes ad(P)$
⊕ CPT conjugates

Gauge Theory on the Seven-Brane $[\mathbb{R}^{3,1} \times S]$

• Sps $S = \mathbb{C}^2 \Longrightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$ with ADE gauge gp $\mathcal{G}$

BPS equations: $F_A^{(0,2)} = \bar\partial_A \varphi = 0$

Kähler form: $\omega \wedge F_A + \frac{i}{2}[\varphi, \bar\varphi] = 0$

Bosons

A connection on a $\mathcal{G}$-bdl P over S

$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$

$\bar\varphi^{(0,2)}$ — " — $\bar K_S \otimes ad(P)$

Fermions

$\bar\eta_{\dot\alpha}$ sect of $ad(P)$

$\psi_\alpha^{(0,1)}$ sect of $\bar\Omega_S^1 \otimes ad(P)$

$\bar\chi_{\dot\alpha}^{(0,2)}$ sect of $\bar\Omega_S^2 \otimes ad(P)$

⊕ CPT conjugates

Chirality: $\#_R - \#_{\bar R} = \int_S c_1(S) \cdot c_1(\mathcal{V}_R)$

Gauge Theory on the Seven-Brane [$\mathbb{R}^{3,1}\times S$]

• Sps $S = \mathbb{C}^2 \Longrightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$ with ADE gauge gp G

Bosons

A connection on a G-bdl P over S

$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$

$\bar{\varphi}^{(0,2)}$ —— " —— $\bar{K}_S \otimes ad(P)$

Fermions

$\bar{\eta}_\alpha$ sect of $ad(P)$

ψ_α sect of

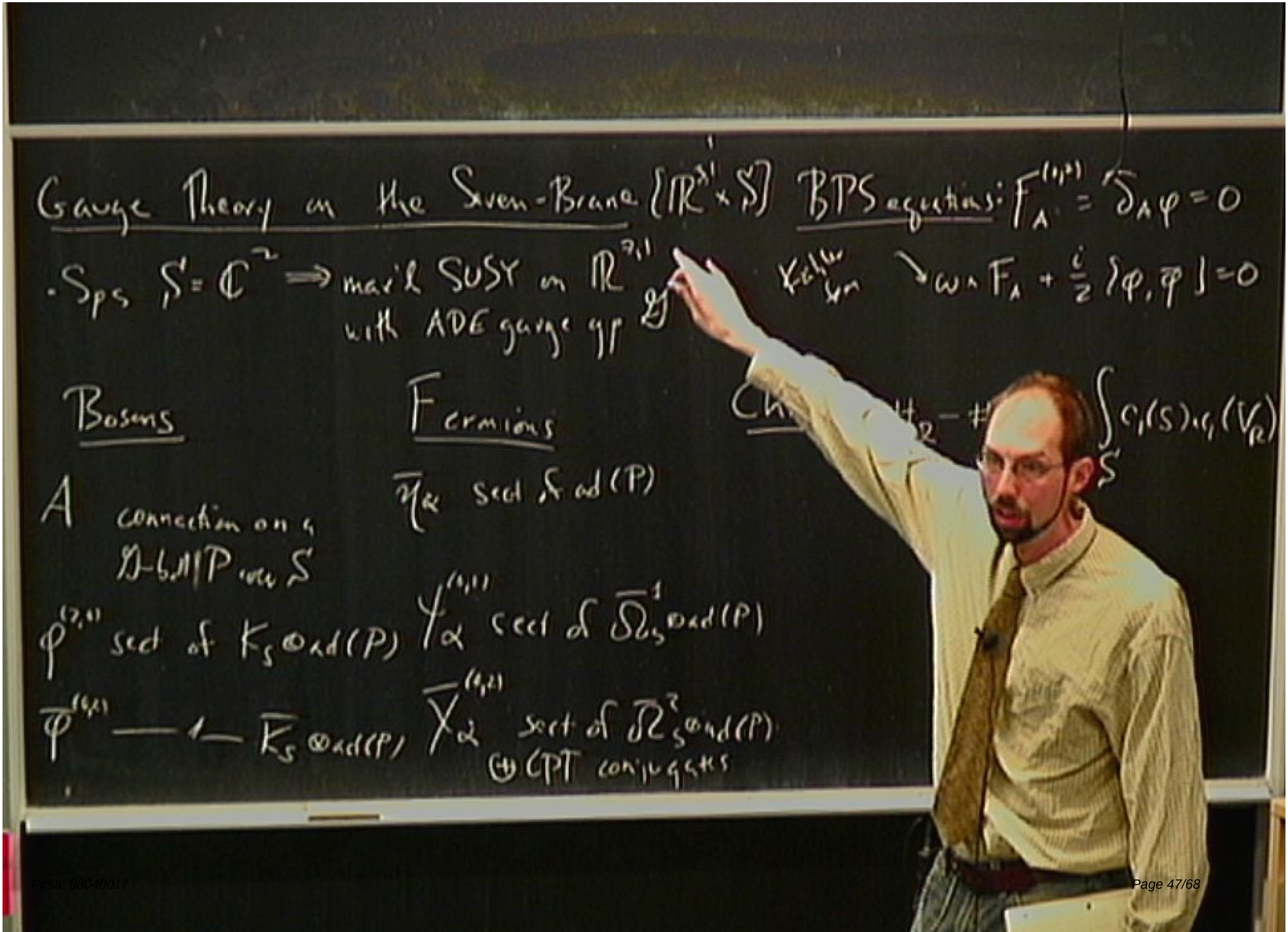
$\bar{\chi}_\alpha^{(0,2)}$ sect of $\bar{\Omega}^2_S \otimes ad$

⊕ CPT conjugates

BPS equations: $F_A^{(0,2)} = \bar{\partial}_A \varphi = 0$

Kähler form $\omega \wedge F_A + \frac{i}{2}[\varphi, \bar{\varphi}] = 0$

C $\#_R - \#_{\bar{R}} = -\int_S c_1(S)\, c_1(V_R)$


Gauge Theory on the Seven-Brane [$\mathbb{R}^{3,1} \times S$]
BPS equations: $F_A^{(0,2)} = \bar\partial_A \varphi = 0$
· Sps $S = \mathbb{C}^2 \Longrightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$
with ADE gauge gp G
Kähler form $\omega \wedge F_A + \frac{i}{2}[\varphi, \bar\varphi] = 0$
Bosons
Fermions
$\#_R - \# \int_S c_1(S) \cdot c_1(V_R)$
A connection on a G-bdl P over S
$\bar\eta_\alpha$ sect of ad(P)
$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$
$\psi_\alpha^{(1,0)}$ sect of $\bar\Omega_S^1 \otimes ad(P)$
$\bar\varphi^{(0,2)}$ — " — $\bar K_S \otimes ad(P)$
$\bar\chi_\alpha^{(0,2)}$ sect of $\bar\Omega_S^2 \otimes ad(P)$
⊕ CPT conjugates

Gauge Theory on the Seven-Brane [$\mathbb{R}^{3,1} \times S$]

BPS equations: $F_A^{(0,2)} = \bar{\partial}_A \varphi = 0$

Kähler form $\omega \wedge F_A + \frac{i}{2}[\varphi, \bar{\varphi}] = 0$

• Sps $S = \mathbb{C}^2 \Longrightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$ with ADE gauge gp $\mathfrak{g}$

Chirality: $\#_R - \ldots \int_S c_1(S) \cdot c_1(V_R)$

$I_S = \int_{\mathbb{R}^{3,1} \times S} d^4x \, [\bar{\eta}_{\dot{\alpha}} - \ldots$

Bosons	Fermions
A connection on a G-bdl P over S	$\bar{\eta}_{\dot{\alpha}}$ sect of ad(P)
$\varphi^{(2,0)}$ sect of $K_S \otimes \mathrm{ad}(P)$	$\psi_\alpha^{(0,1)}$ sect of $\bar{\Omega}_S^1 \otimes \mathrm{ad}(P)$
$\bar{\varphi}^{(0,2)}$ — " — $\bar{K}_S \otimes \mathrm{ad}(P)$	$\bar{\chi}_\alpha^{(0,2)}$ sect of $\bar{\Omega}_S^2 \otimes \mathrm{ad}(P)$
	⊕ CPT conjugates

Gauge Theory on the Seven-Brane $[\mathbb{R}^{3,1} \times S]$

BPS equations: $F_A^{(0,2)} = \bar{\partial}_A \varphi = 0$

Kähler form $\searrow \omega \wedge F_A + \frac{i}{2}[\varphi, \bar{\varphi}] = 0$

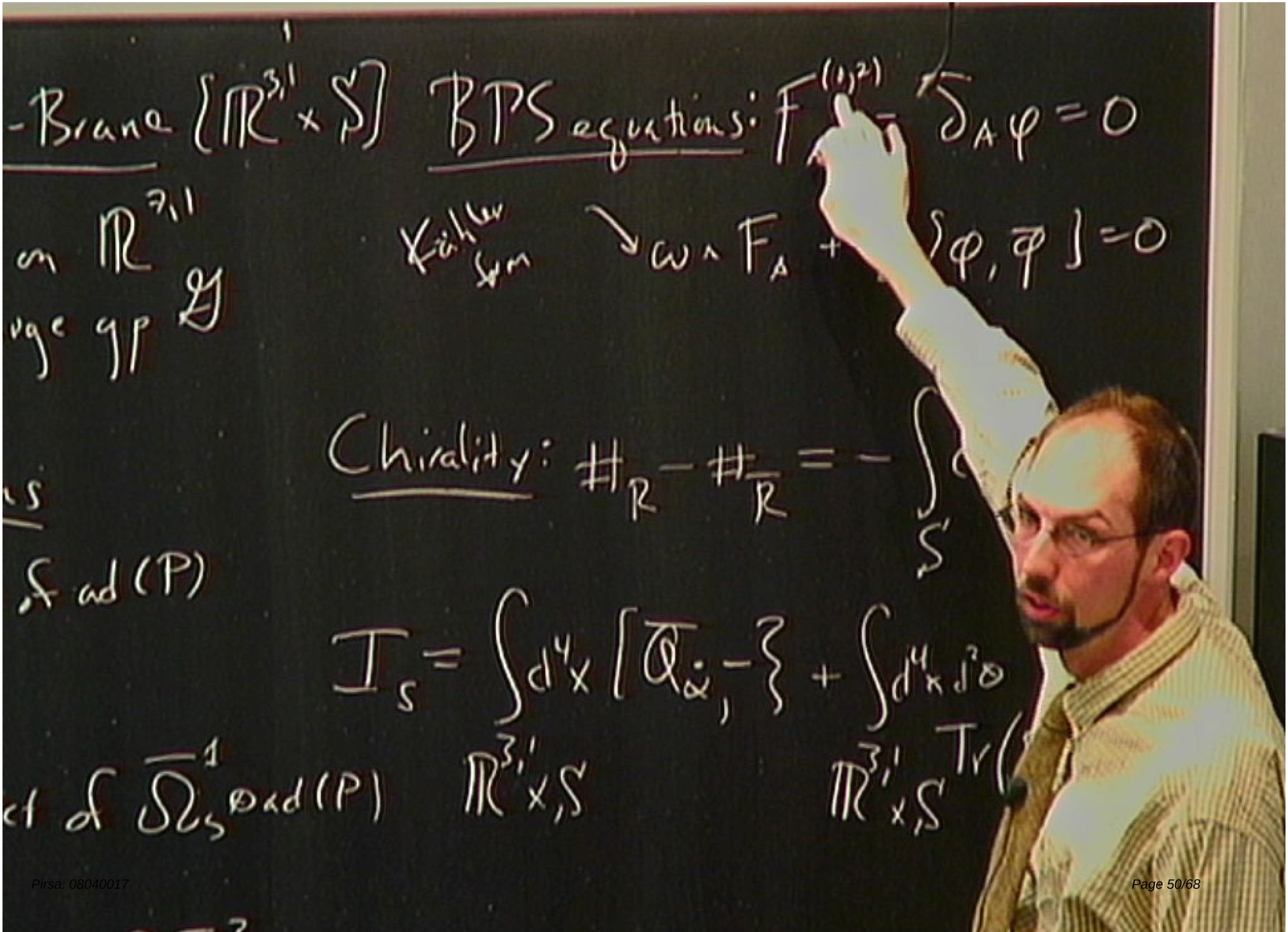
- Sps $S = \mathbb{C}^2 \Longrightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$ with ADE gauge gp $\mathfrak{g}$

Chirality: $\#_R - \#_{\bar{R}} = -\int_S c_1(S) \cdot c_1(V_R)$

$$I_S = \int_{\mathbb{R}^{3,1} \times S} d^4x \, [\bar{\eta}_{\dot{\alpha}} \ldots\} + \int_{\mathbb{R}^{3,1} \times S} d^4x \, d^2\theta \, \mathrm{Tr}(\mathbb{F}_A^{(0,2)} \wedge \Phi)$$

Bosons	**Fermions**
A connection on a G-bdl P over S	$\bar{\eta}_{\dot{\alpha}}$ sect of ad(P)
$\varphi^{(2,0)}$ sect of $K_S \otimes \mathrm{ad}(P)$	$\psi_\alpha^{(0,1)}$ sect of $\bar{\Omega}^1_S \otimes \mathrm{ad}(P)$
$\bar{\varphi}^{(0,2)}$ — " — $\bar{K}_S \otimes \mathrm{ad}(P)$	$\bar{\chi}_{\dot{\alpha}}^{(0,2)}$ sect of $\bar{\Omega}^2_S \otimes \mathrm{ad}(P)$

(+) CPT conjugates


-Brane [$\mathbb{R}^{3,1} \times S$]
BPS equations: $F^{(0,2)} - \bar{\partial}_A \varphi = 0$
Kähler sym
$\omega \wedge F_A + [\varphi, \bar{\varphi}] = 0$
on $\mathbb{R}^{7,1}$
ge gp G
Chirality: $\#_R - \#_{\bar{R}} = -\int_S$
of ad(P)
$I_S = \int_{\mathbb{R}^{3,1}\times S} d^4x \, [\bar{Q}_{\dot{\alpha}}, -\} + \int_{\mathbb{R}^{3,1}\times S} d^4x \, d^2\theta \, \mathrm{Tr}($
of $\bar{\Omega}^1_S \otimes \mathrm{ad}(P)$

Gauge Theory on the Seven-Brane

- Sps $S = \mathbb{C}^2 \Rightarrow$ max'l SUSY on $\mathbb{R}^7$ with ADE gauge gp

Bosons

A connection on a G-bdl P over S

$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$

$\phi^{(0,0)}$

Fermions

$\bar{\eta}_\alpha$ sect of ad (P

$\psi_\alpha^{(0,1)}$ sect of $\bar{\Omega}$

$\bar{\chi}^{(0,2)}$

Gauge Theory on the Seven-Brane $[\mathbb{R}^{3,1}\times S]$

- Sps $S = \mathbb{C}^2 \Longrightarrow$ max'l SUSY on $\mathbb{R}^{3,1}$ with ADE gauge gp $\mathcal{G}$

Bosons

A connection on a $\mathcal{G}$-bdl P over S

$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$

$\bar{\varphi}^{(0,2)}$ — " — $\bar{K}_S \otimes ad(P)$

Fermions

$\bar{\eta}_{\dot\alpha}$ sect of $ad(P)$

$\psi_\alpha^{(1,0)}$ sect of $\bar{\Omega}^1_S \otimes ad(P)$

$\bar{\chi}_{\dot\alpha}^{(0,2)}$ sect of $\bar{\Omega}^2_S \otimes ad(P)$

$\oplus$ CPT conjugates

BPS equations: $F_A^{(0,2)} = \bar{\partial}_A \varphi = 0$

Kähler form $\omega \wedge F_A + \frac{i}{2}[\varphi, \bar{\varphi}] = 0$

Chirality: $\#_R - \#_{\bar{R}} = -\int_S c_1(S) \wedge c_1(\ldots$

$I_S = \int_{\mathbb{R}^{3,1}\times S} d^4x \, [\bar{Q}_{\dot\alpha}, -\} + \int_{\mathbb{R}^{3,1}\times S} d^4x \, d^2\theta \, \mathrm{Tr}(\mathbb{F}_A^{(1,1)} \wedge \Phi)$

Gauge Theory on the Seven-Brane ($\mathbb{R}^{3,1} \times S$)

• Sps $S = \mathbb{C}^2 \Longrightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$
with ADE gauge gp G

Bosons

A connection on a G-bdl P over S

$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$

$\bar{\varphi}^{(0,2)}$ — " — $\bar{K}_S \otimes ad(P)$

Fermions

$\bar{\eta}_{\dot\alpha}$ sect of $ad(P)$

$\psi_\alpha^{(1,0)}$ sect of $\bar{\Omega}_S^1 \otimes ad(P)$

$\bar{\chi}_{\dot\alpha}^{(0,2)}$ sect of $\bar{\Omega}_S^2 \otimes ad(P)$

⊕ CPT conjugates

BPS equations: $F_A^{(0,2)} = \bar\partial_A \varphi = 0$

Kähler form $\omega \wedge F_A + \frac{i}{2}[\varphi, \bar\varphi] = 0$

Chirality: $\#_R - \#_{\bar{R}} = -\int_S c_1(S)\, c_1(V_R)$

$$I_S = \int_{\mathbb{R}^{3,1}\times S} d^4x\, [\bar{Q}_{\dot\alpha}, -\} + \underbrace{\int_{\mathbb{R}^{3,1}\times S} d^4x\, d^2\theta\, \mathrm{Tr}(\mathbb{F}_A^{(1,0)} \wedge \Phi)}$$

Gauge Theory on the Seven-Brane ($\mathbb{R}^{3,1} \times S$)

- Sps $S = \mathbb{C}^2 \Rightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$ with ADE gauge gp $\mathcal{G}$

BPS equations: $F_A^{(0,2)} = \bar\partial_A \varphi = 0$

Kähler-Yang-Mills: $\omega \wedge F_A + \frac{i}{2}[\varphi, \bar\varphi] = 0$

Bosons

A connection on a $\mathcal{G}$-bdl P over S

$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$

$\bar\varphi^{(0,2)}$ — " — $\bar K_S \otimes ad(P)$

Fermions

$\bar\eta_{\dot\alpha}$ sect of $ad(P)$

$\psi_\alpha^{(0,1)}$ sect of $\bar\Omega_S^1 \otimes ad(P)$

$\bar\chi_{\dot\alpha}^{(0,2)}$ sect of $\bar\Omega_S^2 \otimes ad(P)$

$\oplus$ CPT conjugates

Chirality: $\#_R - \#_{\bar R} = -\int_S c_1(S)\, c_1(V_R)$

$$I_S = \int d^4x\, [\bar Q_{\dot\alpha}, -\} + \int d^4x\, d^2\theta \int_{\mathbb{R}^{3,1}\times S} \mathrm{Tr}(\mathbb{F}_A^{(0,2)} \wedge \Phi)$$

$$\mathrm{Tr}(\psi[\varphi, \psi]) + \cdots$$

Geometric Interpretation of φ

- Adjunction $\Rightarrow K_S \simeq K_B\big|_S \oplus N_{S/B}$

Geometric Interpretation of φ

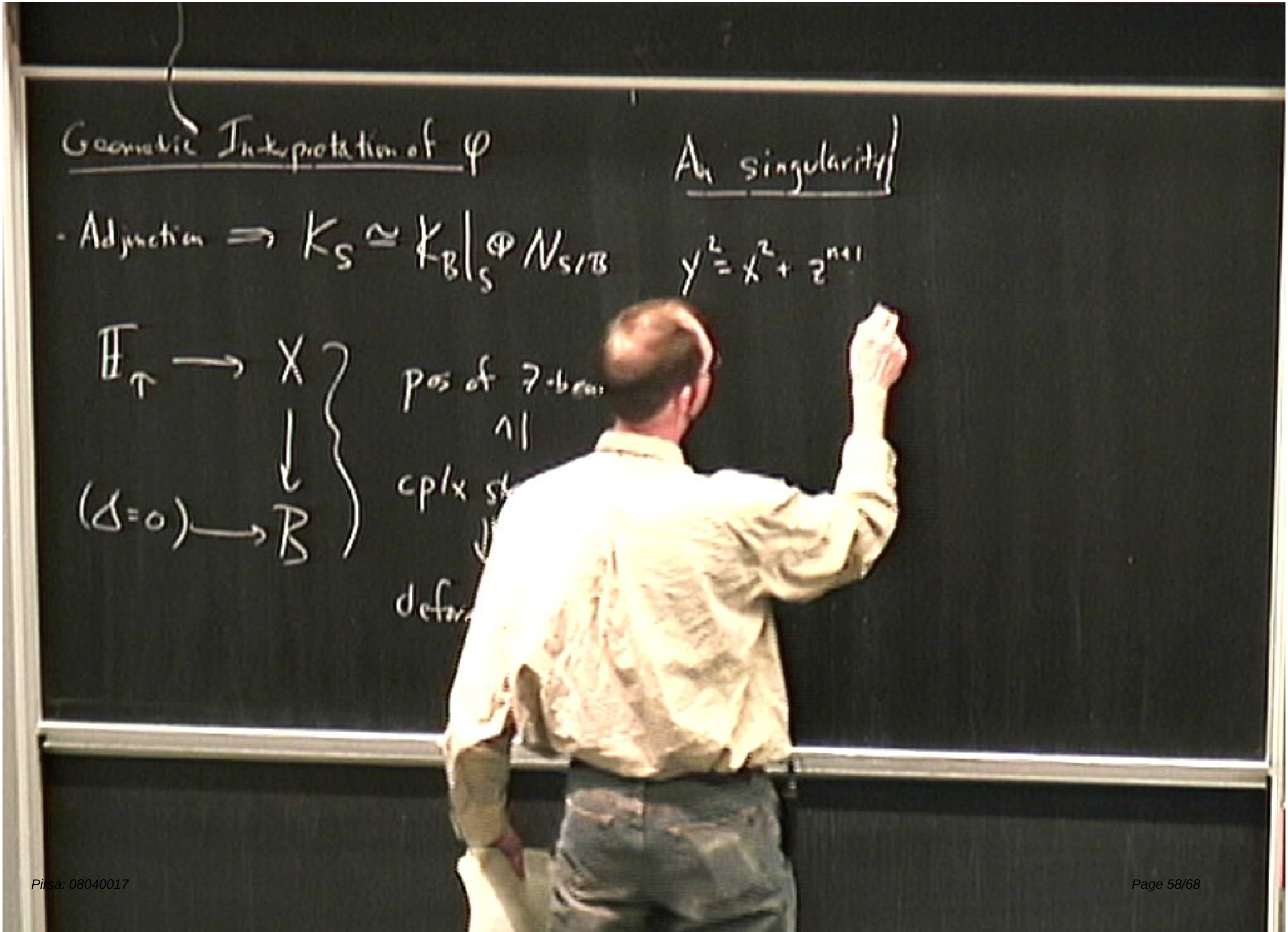
- Adjunction $\Rightarrow K_S \simeq K_B\big|_S \oplus N_{S/B}$

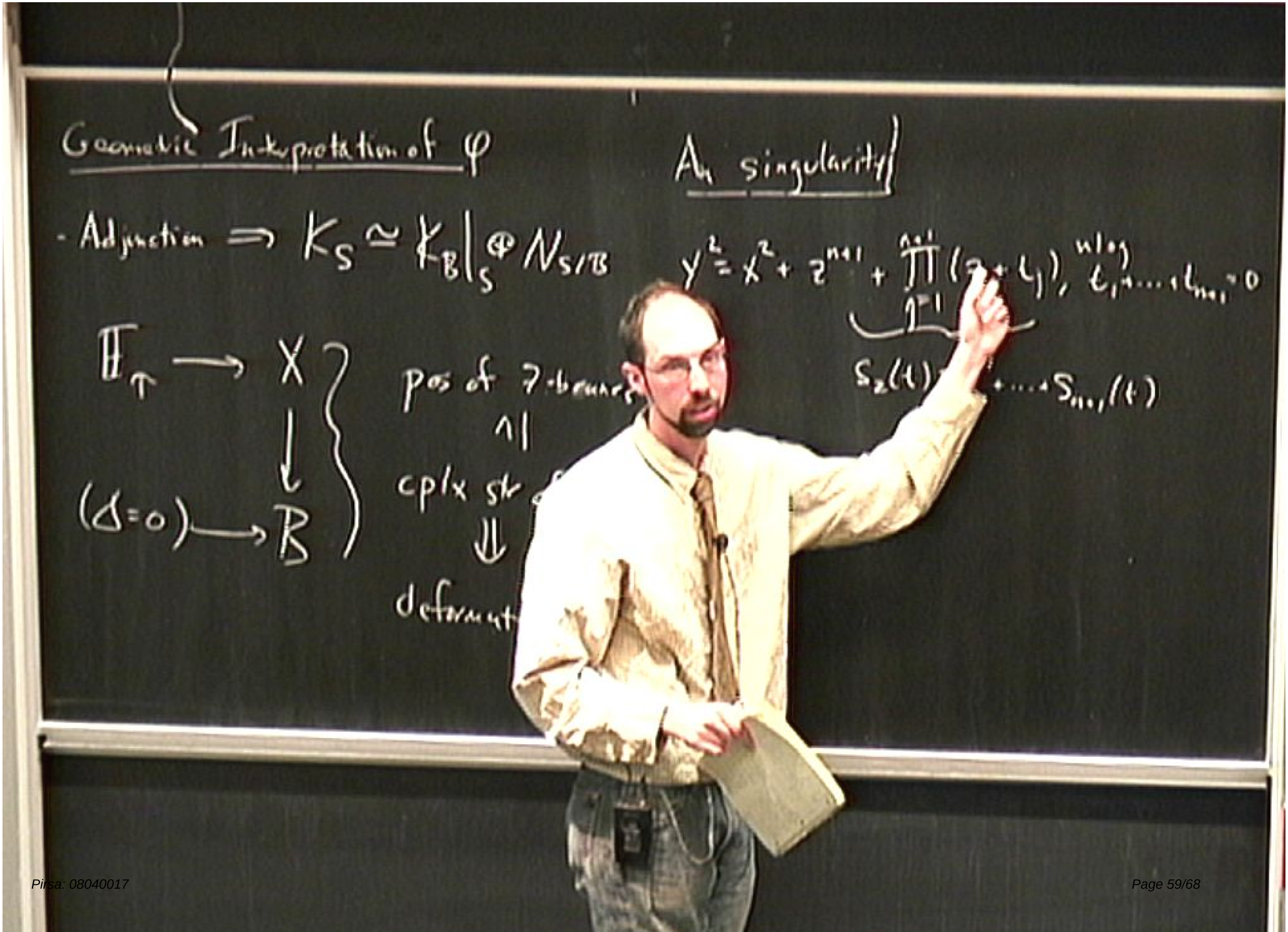
$$\left.\begin{array}{ccc} [\text{illegible}] & \longrightarrow & X \\ & & \downarrow \\ (\Delta = 0) & \longrightarrow & B \end{array}\right\} \text{pos of 7-branes}$$

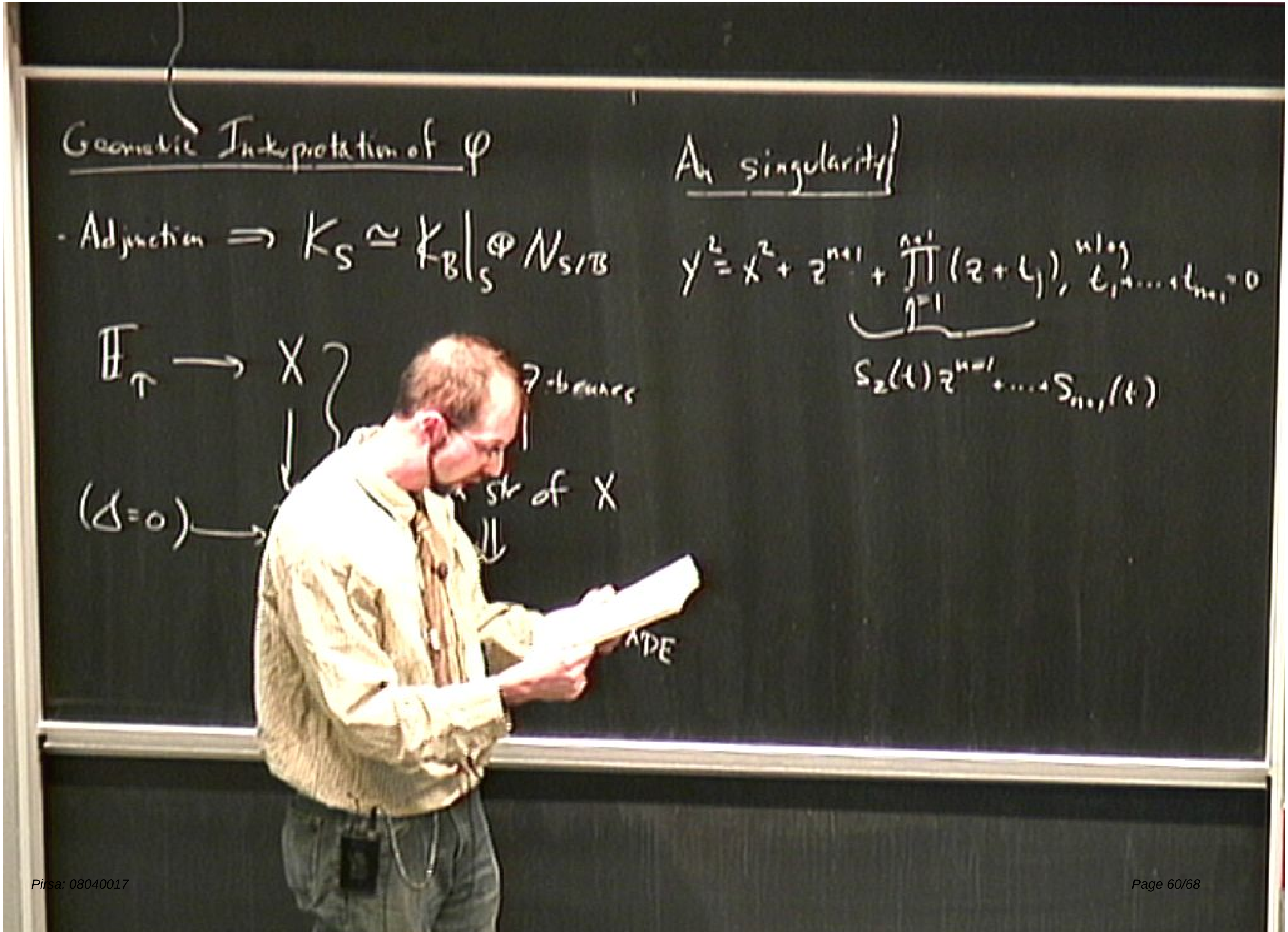
Geometric Interpretation of φ

- Adjunction $\Rightarrow K_S \simeq K_B\big|_S \oplus N_{S/B}$

$$\begin{array}{ccc} \mathbb{F}_T & \longrightarrow & X \\ & & \downarrow \\ (\Delta = 0) & \longrightarrow & B \end{array} \left.\vphantom{\begin{array}{c} a \\ b \\ c \end{array}}\right\} \begin{array}{c} \text{pos of 7-branes} \\ \wedge| \\ \text{cplx str of } X \end{array}$$


Geometric Interpretation of φ
Adjunction $\Rightarrow K_S \simeq K_B|_S \oplus N_{S/B}$
$\mathbb{F}_T \longrightarrow X$
$(\Delta=0) \longrightarrow B$
pos of 7-bran
$\wedge|$
cp/x st
defor
A_n singularity
$y^2 = x^2 + z^{n+1}$


Geometric Interpretation of φ
A_n singularity
Adjunction $\Rightarrow K_S \simeq K_B|_S \oplus N_{S/B}$
pos of 7-branes
$\mathbb{F}_T \longrightarrow X$
$(\Delta = 0) \longrightarrow B$


Geometric Interpretation of φ
Adjunction ⇒ $K_S \simeq K_B|_S \otimes N_{S/B}$
7-branes
str of X
ADE
A_n singularity
$y^2 = x^2 + z^{n+1} + \prod_{i=1}^{n+1}(z+t_i)$
$s_2(t)z^{n-1} + \ldots + s_{n+1}(t)$

Geometric Interpretation of φ

- Adjunction $\Rightarrow K_S \simeq K_B\big|_S \oplus N_{S/B}$

$$\begin{array}{ccc} \mathbb{E}_\tau & \longrightarrow & X \\ & & \downarrow \\ (\Delta=0) & \longrightarrow & B \end{array}$$

pos of 7-bran[es]

$\wedge\|$

cplx str of [...]

$\Downarrow$

deformat[ions]

A_n singularity t coords a CSA of $SU(n+1)$

$$y^2 = x^2 + \underbrace{z^{n+1} + \prod_{i=1}^{n+1} (z+t_i)}_{s_2(t) z^{n-1} + \cdots + s_{n+1}(t)}, \quad t_1 + \cdots + t_{n+1} = 0$$

Geometric Interpretation of φ

- Adjunction $\Rightarrow K_S \simeq K_B|_S \oplus N_{S/B}$

$$\begin{array}{ccc} \mathbb{E}_\tau & \longrightarrow & X \\ & & \downarrow \\ (\Delta = 0) & \longrightarrow & B \end{array} \quad \left.\right\}$$

pos of 7-branes

$\cong$

cplx str of X

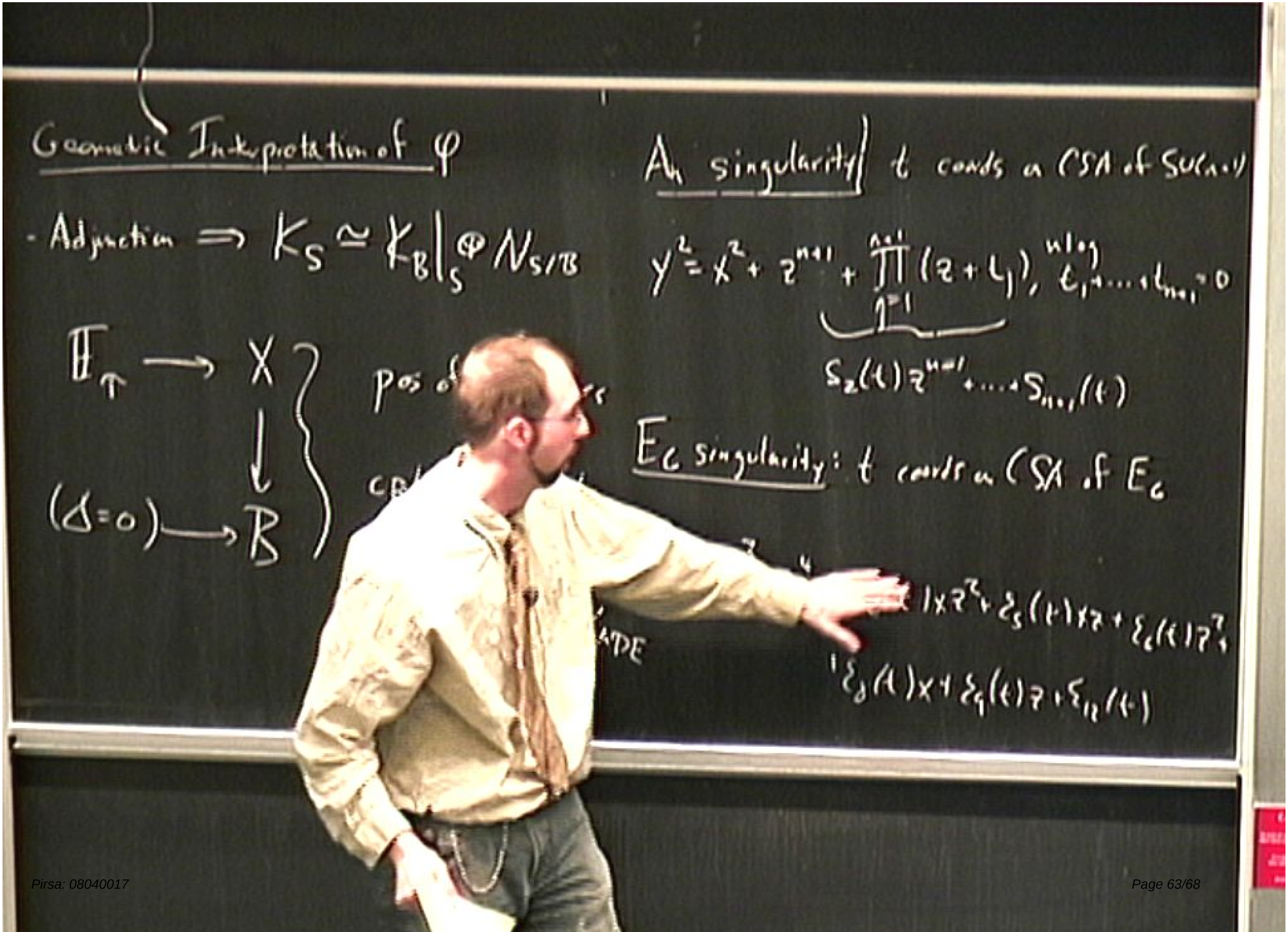
$\Downarrow$

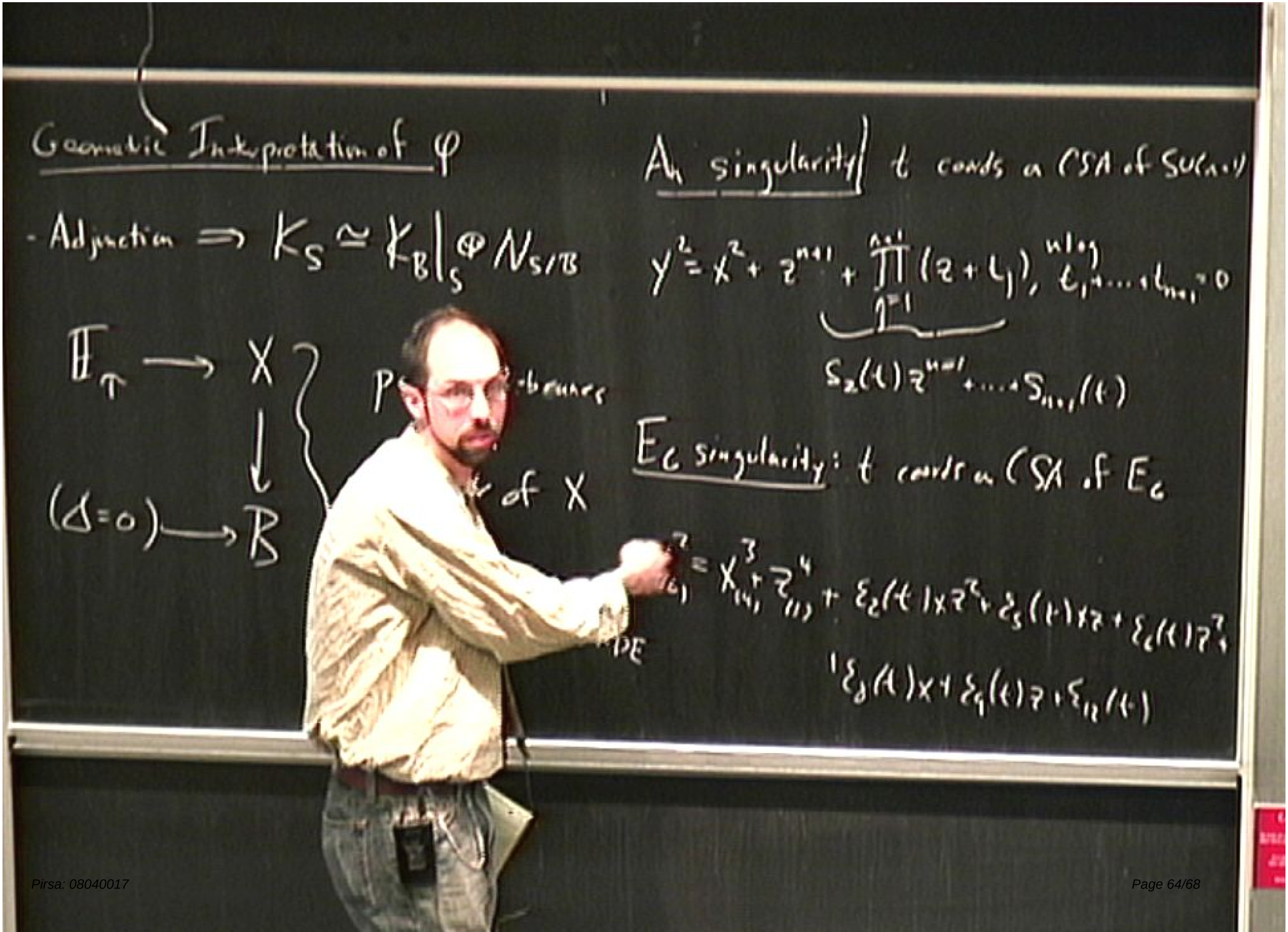
deformations of Y_{ADE}

A_n singularity

t coords on CSA of $SU(n+1)$

$$y^2 = x^2 + z^{n+1} + \underbrace{\prod_{i=1}^{n+1}(z + t_i)}_{s_2(t) z^{n-1} + \dots + s_{n+1}(t)}, \quad \overset{\text{wlog}}{t_1 + \dots + t_{n+1} = 0}$$


Geometric Interpretation of φ
- Adjunction ⇒ K_S ≃ K_B|_S ⊕ N_S/B
X
B
(Δ=0)
pos of
ADE
A_n singularity | t coords on CSA of SU(n+1)
y² = x² + z^(n+1) + ∏_(j=1)^(n+1) (z + t_j), t_1 + … + t_(n+1) = 0
s_2(t) z^(n−1) + … + s_(n+1)(t)
E_6 singularity: t coords on CSA of E_6
x z² + ε_5(t) x z + ε_6(t) z² +
ε_8(t) x + ε_9(t) z + ε_12(t)


Geometric Interpretation of φ
Adjunction ⇒ K_S ≃ K_B|_S ⊕ N_S/B
E_τ → X
(Δ=0) → B
of X
A_n singularity: t coords on CSA of SU(n+1)
y² = x² + z^(n+1) + ∏ (z + t_i), t_1 + ... + t_(n+1) = 0
s_2(t) z^(n-1) + ... + s_(n+1)(t)
E_6 singularity: t coords on CSA of E_6
= x³ + z⁴ + ε_2(t) x z² + ε_5(t) x z + ε_6(t) z³
+ ε_8(t) x + ε_9(t) z + ε_12(t)

Geometric Interpretation of φ

- Adjunction $\Rightarrow K_S \simeq K_B\big|_S \oplus N_{S/B}$

$$\begin{array}{ccc} \mathbb{E}_{\uparrow} & \longrightarrow & X \\ & & \downarrow \\ (\Delta=0) & \longrightarrow & B \end{array}$$

pos of 7-branes

$\wedge|$

cplx str of X

$\Downarrow$

deformations o...

A_n singularity: t coords on CSA of $SU(n+1)$

$$y^2 = x^2 + z^{n+1} + \underbrace{\prod_{j=1}^{n+1}(z+t_j)}, \quad t_1+\dots+t_{n+1}=0$$

$S_2(t)z^{n-1}+\dots+S_{n+1}(t)$

E_6 ...y: t coords on CSA of E_6

z^4 ... $+ \varepsilon_2(t)xz^2 + \varepsilon_5(t)xz + \varepsilon_6(t)z^2 +$

$\varepsilon_8(t)x + \varepsilon_9(t)z + \varepsilon_{12}(t)$

Geometric Interpretation of φ

· Adjunction $\Rightarrow K_S \simeq K_B|_S \oplus N_{S/B}$

$$\begin{tikzcd} \mathbb{E}_\tau \arrow[r] & X \arrow[d] \\ (\Delta=0) \arrow[r] & B \end{tikzcd}$$

pos of 7-branes

$\cong$

cplx str of X

$\Downarrow$

deformations of Y_{ADE}

A_n singularity: t coords on CSA of $SU(n+1)$

$$y^2 = x^2 + z^{n+1} + \underbrace{\prod_{i=1}^{n+1}(z+t_i)}_{s_2(t)z^{n-1}+\dots+s_{n+1}(t)}, \quad t_1+\dots+t_{n+1}=0$$

E_6 singularity: t coords on CSA of E_6

$$y_{(6)}^2 = x_{(4)}^3 + z_{(3)}^4 + \epsilon_2(t)xz^2 + \epsilon_5(t)xz + \epsilon_6(t)z^2 + \epsilon_8(t)x + \epsilon_9(t)z + \epsilon_{12}(t)$$

Gauge Theory on the Seven-Brane $[\mathbb{R}^{3,1} \times S]$

· Sps $S = \mathbb{C}^2 \Rightarrow$ max'l SUSY on $\mathbb{R}^{7,1}$ with ADE gauge gp $\mathcal{G}$

BPS equations: $F_A^{(0,2)} = \bar{\partial}_A \varphi = 0$

Kähler form $\searrow \omega \wedge F_A + \frac{i}{2}[\varphi, \bar{\varphi}] = 0$

Bosons

A connection on a $\mathcal{G}$-bdl P over S

$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$

$\bar{\varphi}^{(0,2)}$ — " — $\bar{K}_S \otimes ad(P)$

Fermions

$\bar{\eta}_{\dot\alpha}$ sect of $ad(P)$

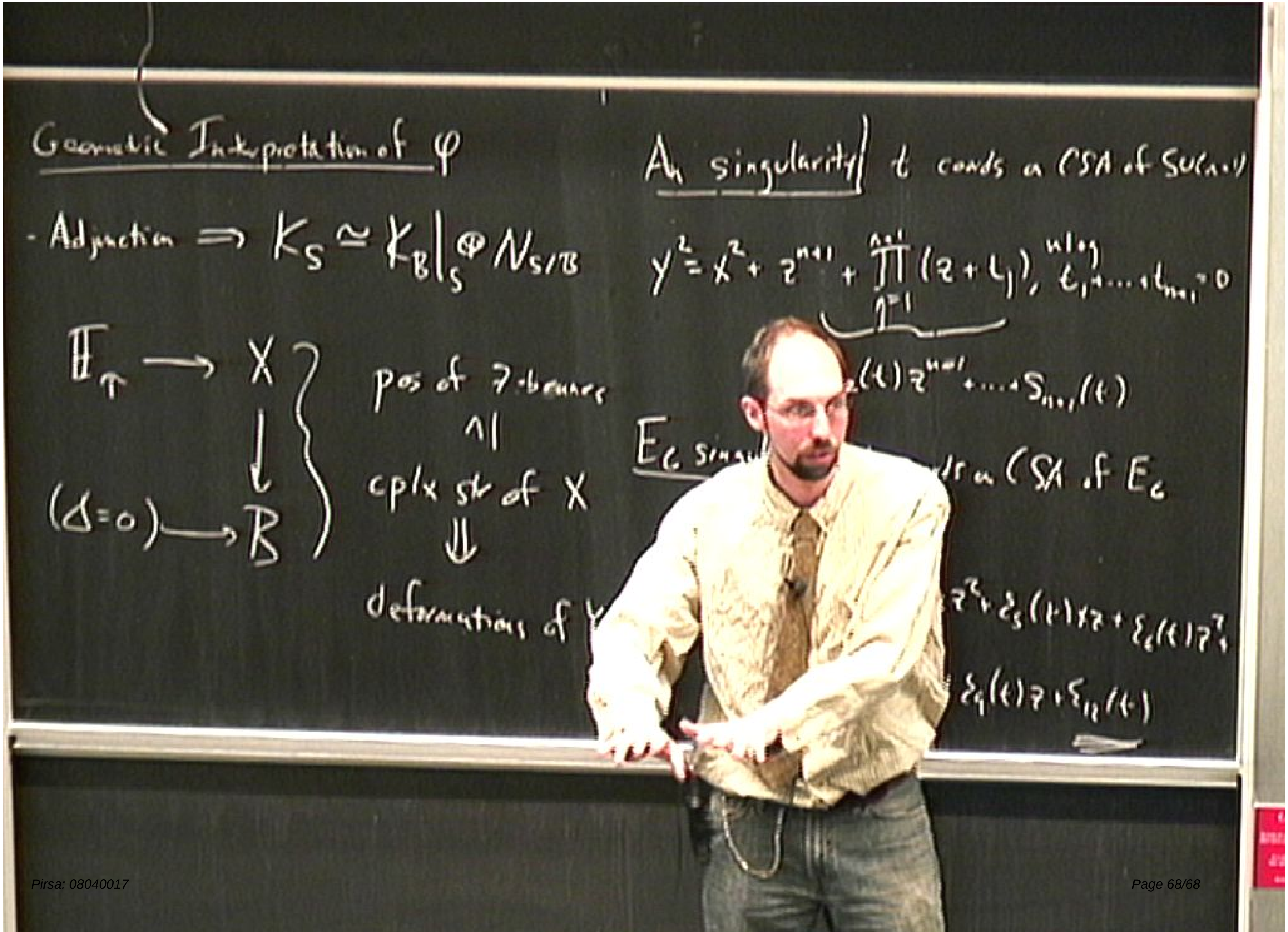
$\psi_\alpha^{(1,0)}$ sect of $\bar{\Omega}_S^1 \otimes ad(P)$

$\bar{\chi}_{\dot\alpha}^{(0,2)}$ sect of $\bar{\Omega}_S^2 \otimes ad(P)$

(+) CPT conjugates

Chirality: $\#_R - \#_{\bar{R}} = -\int_S c_1(S)\, c_1(V_R)$

$$I_S = \int_{\mathbb{R}^{3,1} \times S} d^4x\, [\bar{\eta}_{\dot\alpha}, -] + \int_{\mathbb{R}^{3,1} \times S} d^4x\, d^2\theta\, \underbrace{Tr(\mathbb{F}_A^{(1,1)} \wedge \Phi)}_{Tr(\psi [\varphi, \psi]) + \cdots}$$


Geometric Interpretation of φ
- Adjunction ⇒ K_S ≅ K_B|_S ⊕ N_S/B
pos of 7-branes
cplx str of X
deformations of Y
A_n singularity
t coords on CSA of SU(n+1)
y^2 = x^2 + z^{n+1} + ∏ (z + t_i), t_1 + ... + t_{n+1} = 0
E_6 singularity
CSA of E_6