Title: Intersecting branes from supergravity

Date: Apr 22, 2008 11:00 AM

URL: http://pirsa.org/08040016

Abstract: Geometries produced by brane intersections preserving eight supercharges are constructed. Typical examples of such configurations are given by fundamental strings ending on D branes and by brane webs. Consistency conditions of supergravity are shown to impose certain requirements on the locations of the sources, and these restrictions are found to be in a perfect agreement with results of the probe analysis. This agreement serves as a nontrivial test of the duality between open and closed strings. Some applications to AdS/CFT correspondence are also discussed.

Intersecting branes from supergravity

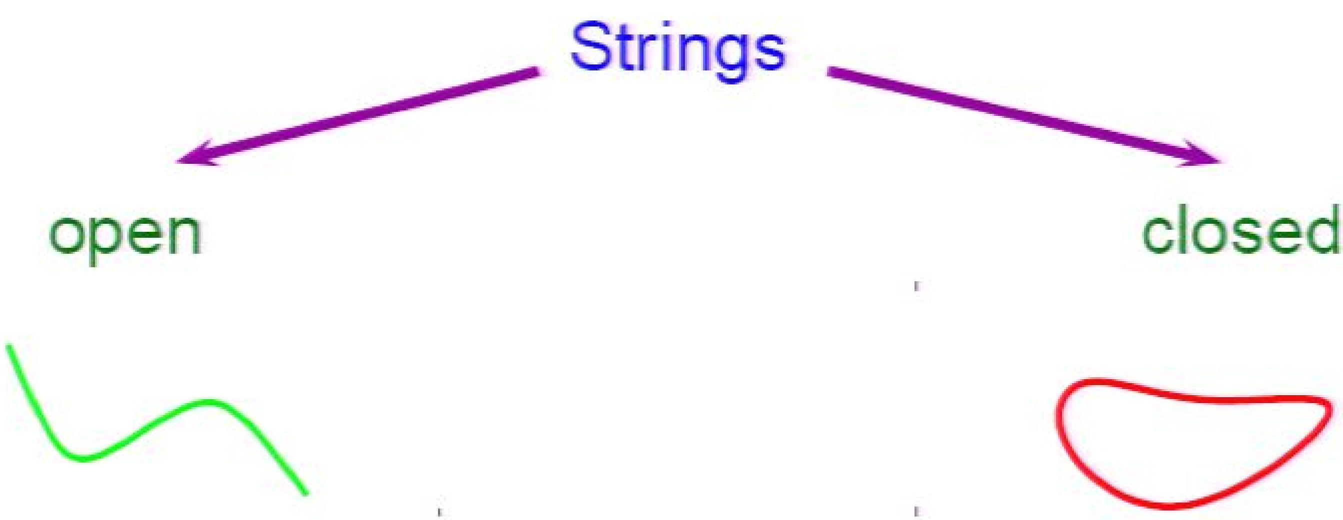
Oleg Lunin

University of Chicago

arXiv:0706.3396

0802.0735

Branes in string theory

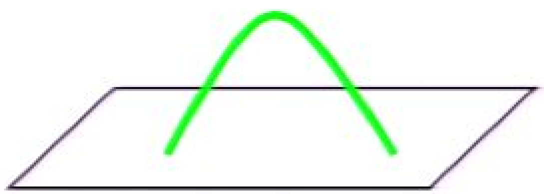


Branes in string theory

Strings

open

closed



Dai–Leigh–Polchinski '89

- D–brane

Branes in string theory

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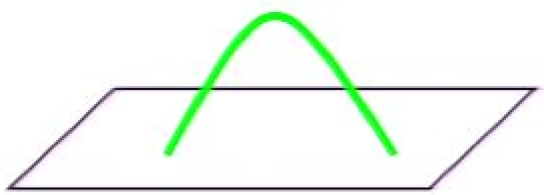

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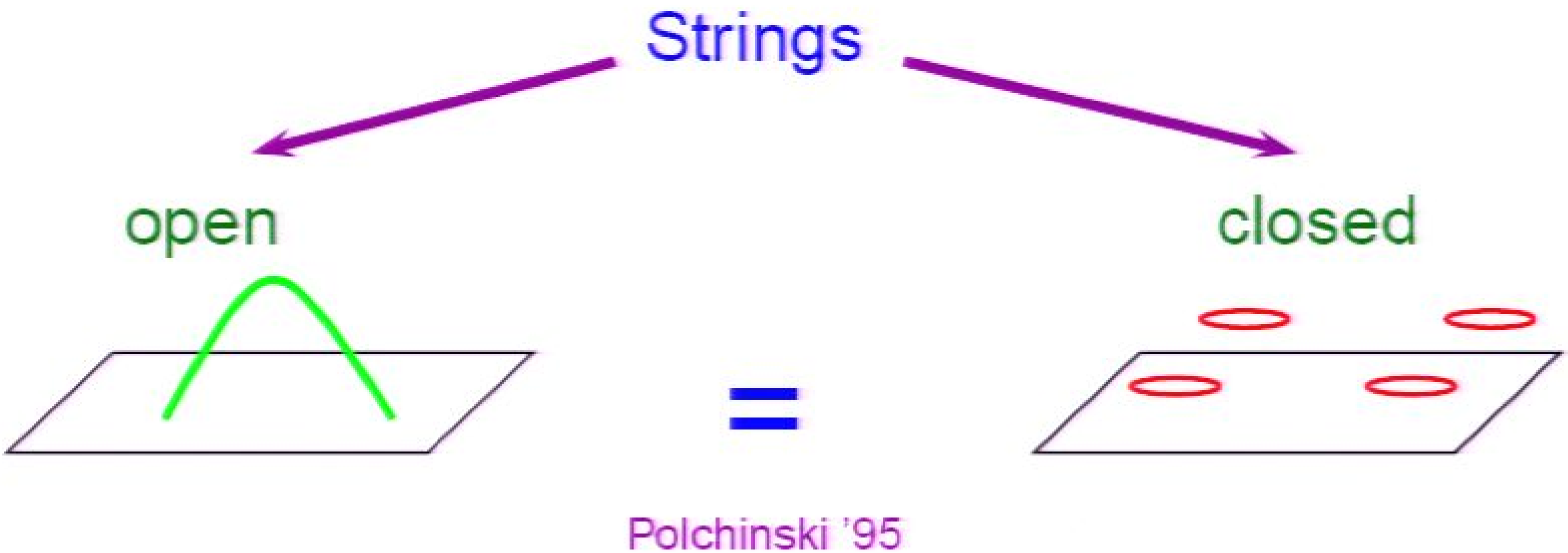
Horowitz–Strominger '91

- D–brane
- black brane (geometry)

Branes in string theory



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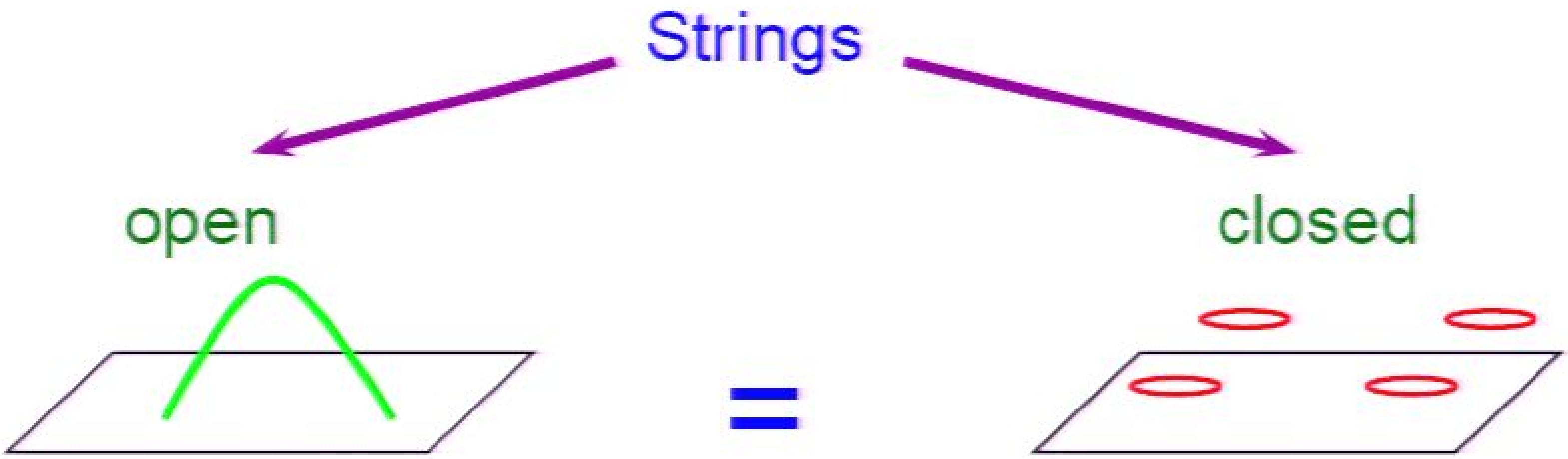

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- effective action: DBI

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- supergravity

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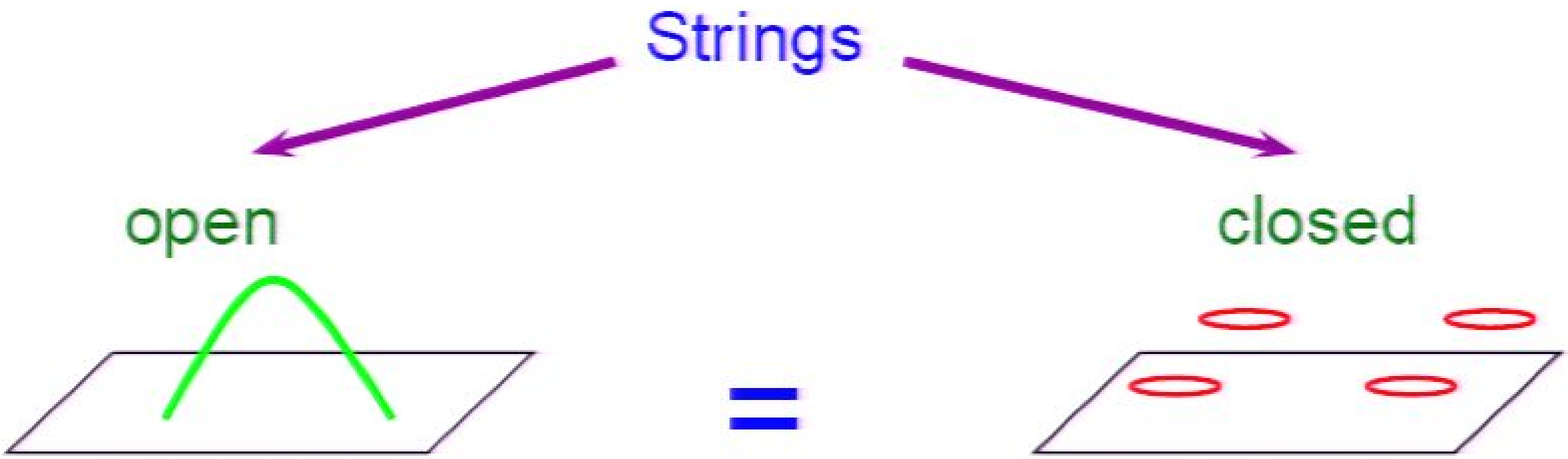

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- large symmetry

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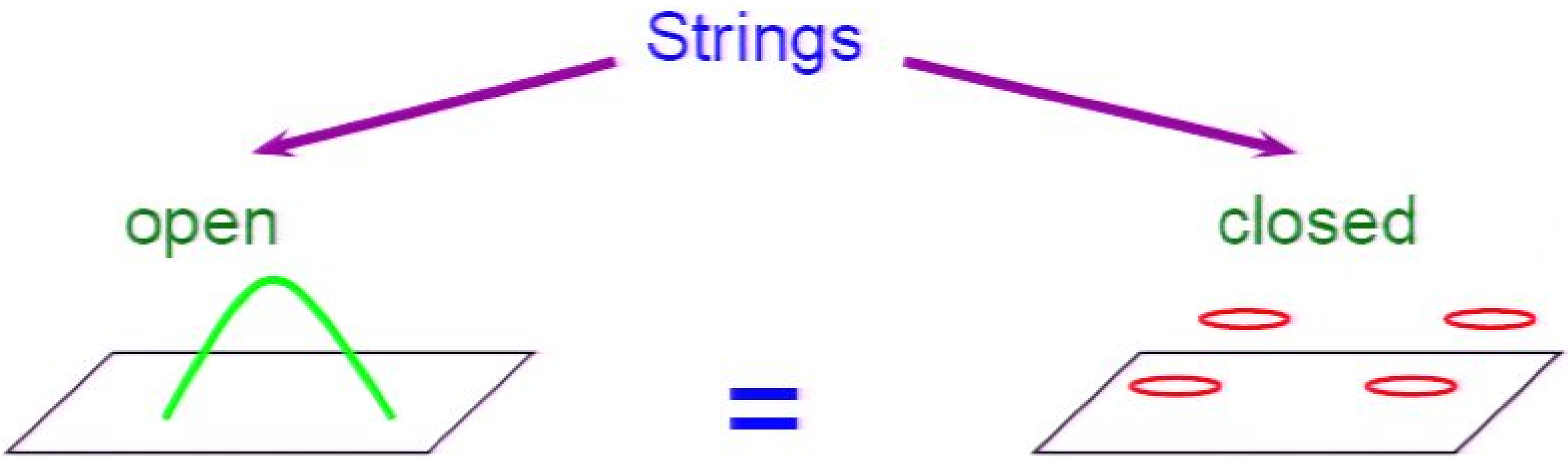

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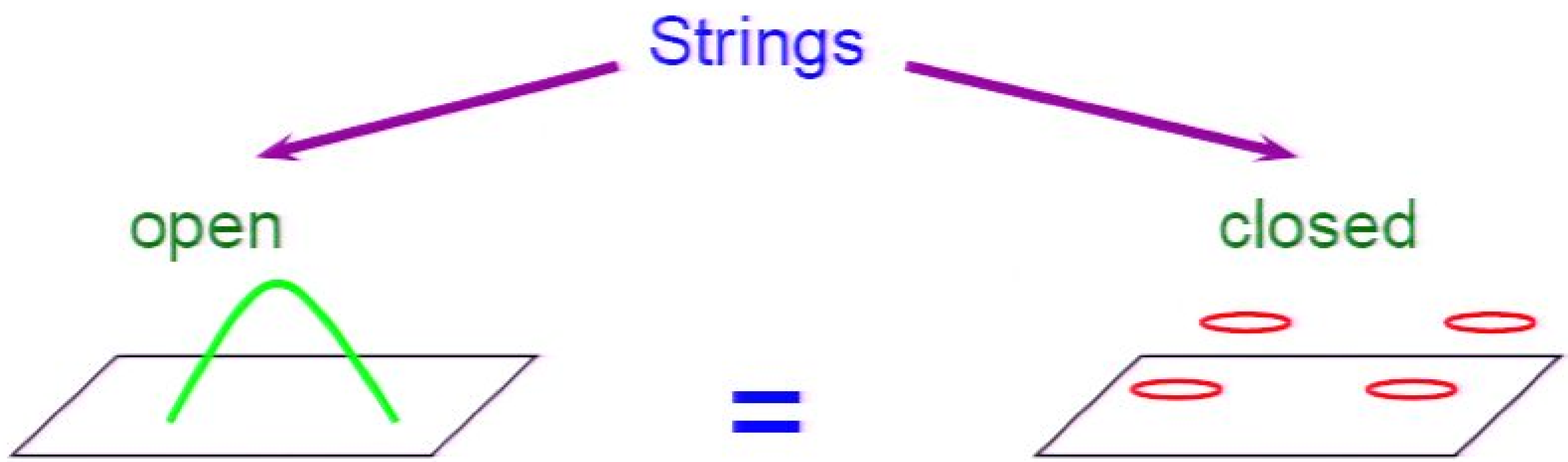

- D–brane
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- flat branes
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  - shapes from dynamics

- black brane (geometry)
- supergravity
- large symmetry
- special solutions
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The uses of branes

- Applications to gauge theory
  - low energy dynamics: super–Yang–Mills
  - intersecting branes: colors and flavors
    Hanany–Witten '96; Witten '97
  - geometric picture for Seiberg duality

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black hole

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black hole $\longrightarrow$ thermal state $\downarrow$ member of the ensemble

Strominger, Vafa '96

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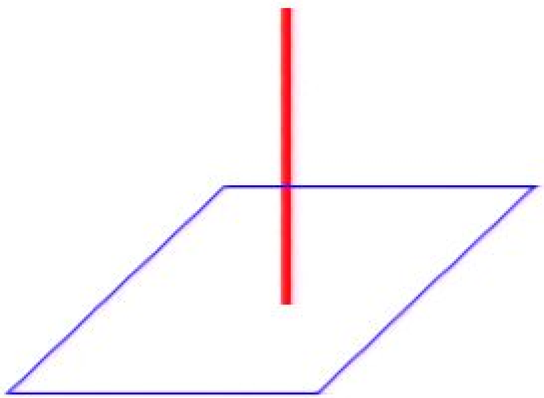
- Gauge/gravity duality
  - field theory = string theory
  - strong/weak coupling complementarity Maldacena '97

Outline

- Motivation
  - quantum gravity, gauge dynamics
  - understanding shapes on the gravity side
- Probe approximation
- Solutions in supergravity
  - technique for constructing the geometries
  - local description and consistency conditions
- "DBI/SUGRA correspondence"
- Generalizations and solutions in M theory
- Brane webs and "bubbling geometries"
- Open questions

BIons in flat space

- Strings ending on a brane:



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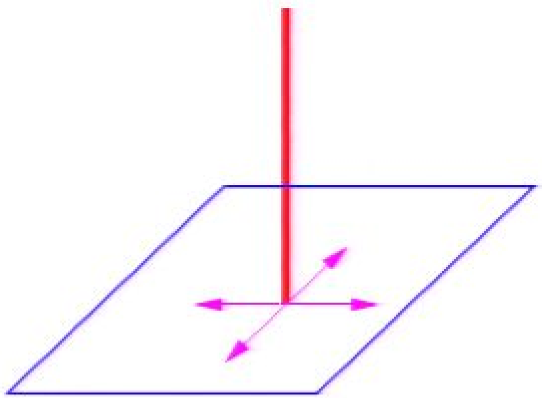

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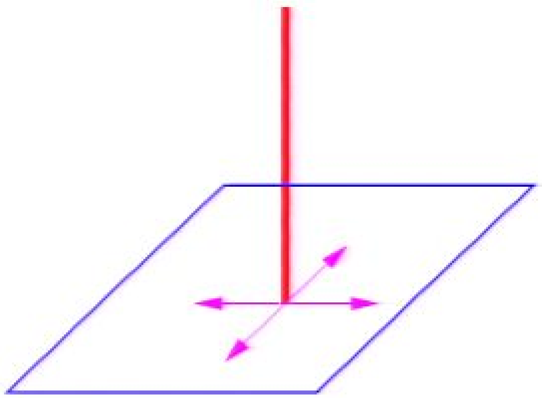

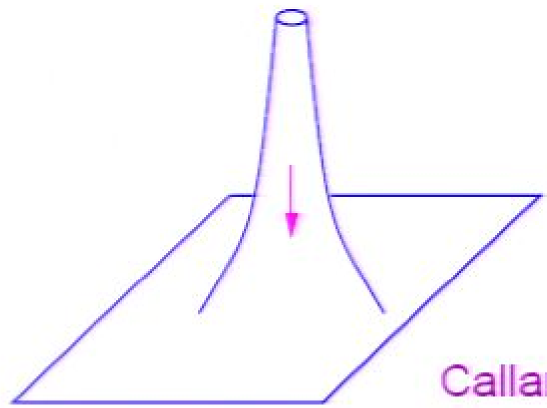

Callan–Maldacena '97

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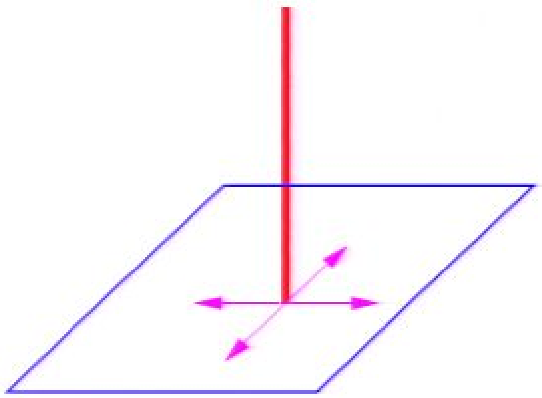

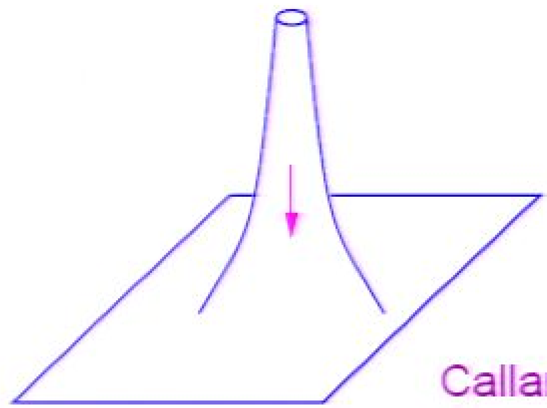

Callan–Maldacena '97

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$$S_{DBI} = -T \int d^{p+1}\xi \sqrt{-\mathrm{det}(G + 2\pi\alpha' F)}$$

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BIons in curved space

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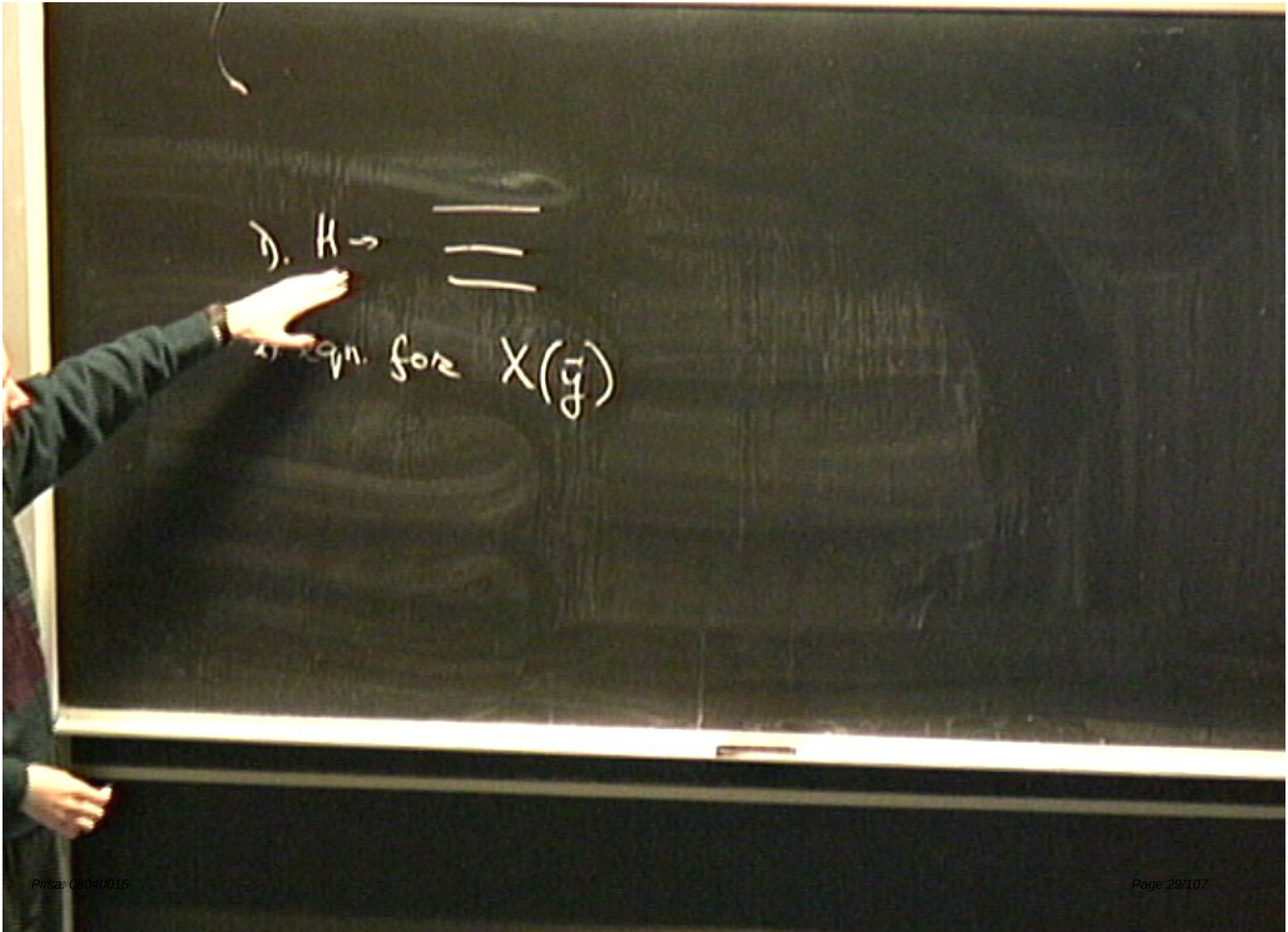
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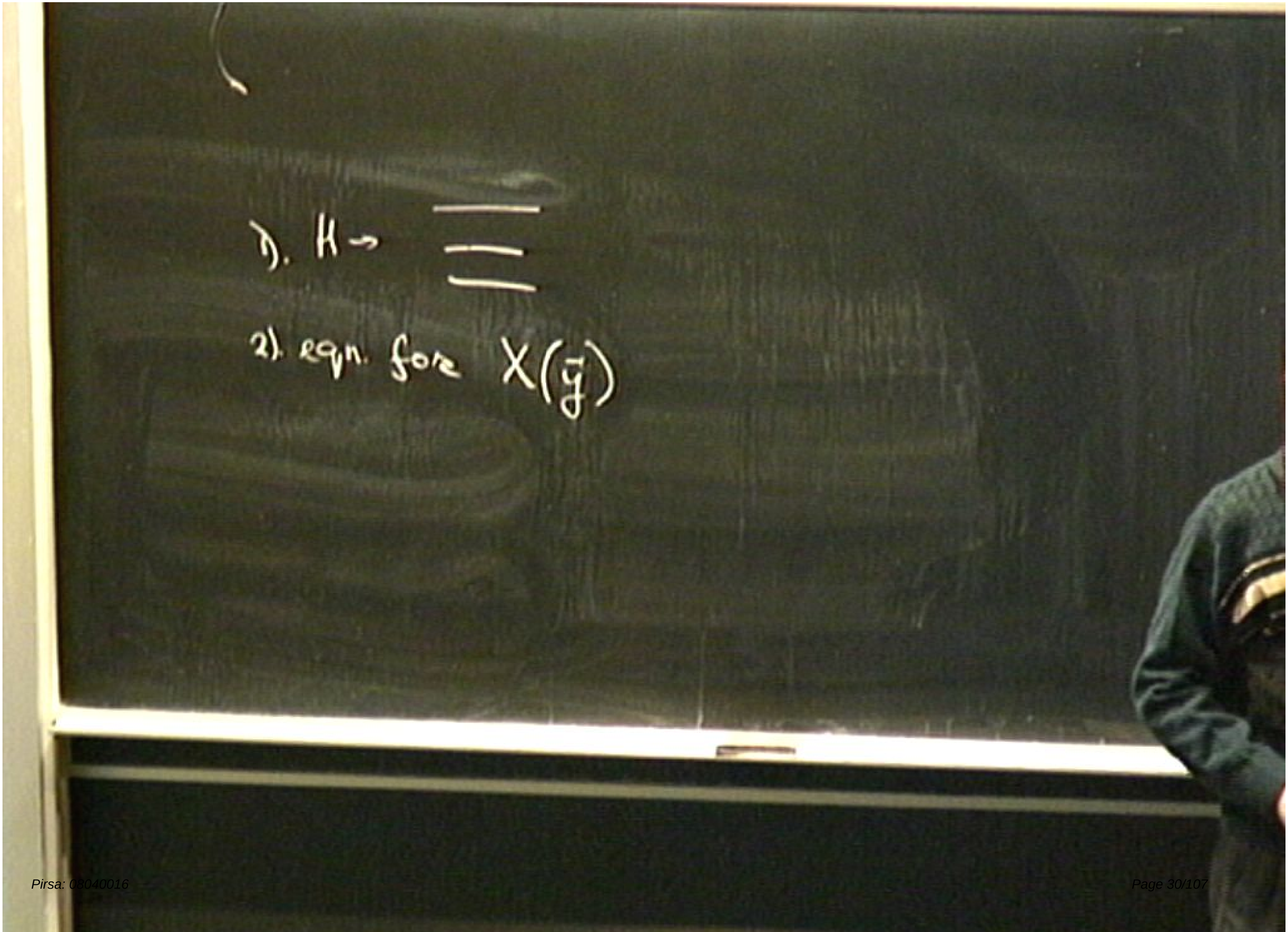
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  - $SO(3) \times SO(5) \times U(1)_t$ isometries
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Structure of the geometry

- Local description
  - two functions of 9 variables $(w, \vec{x}_3, \vec{y}_5)$:

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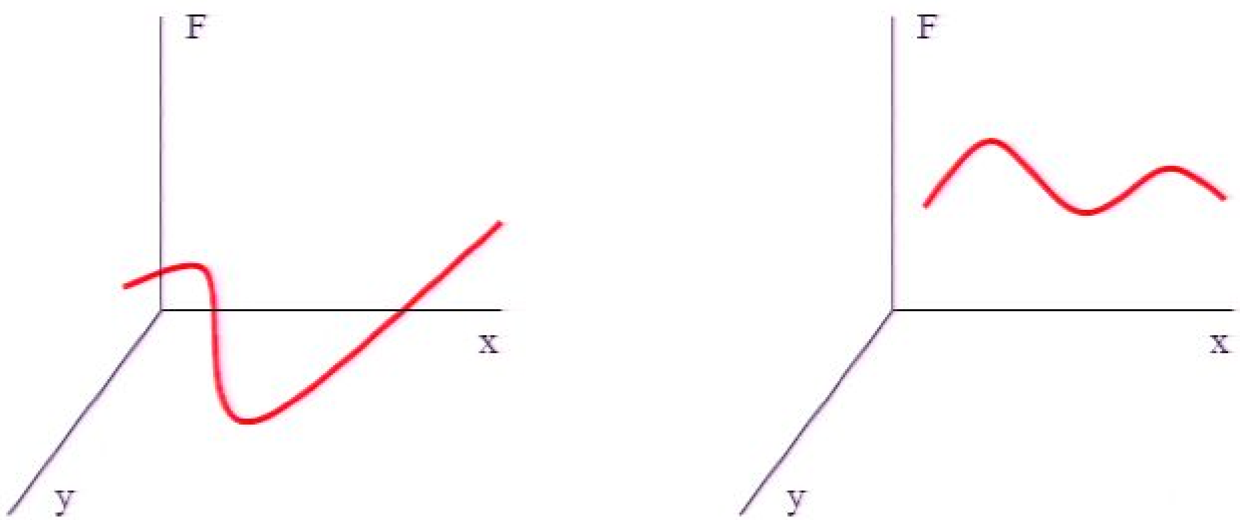

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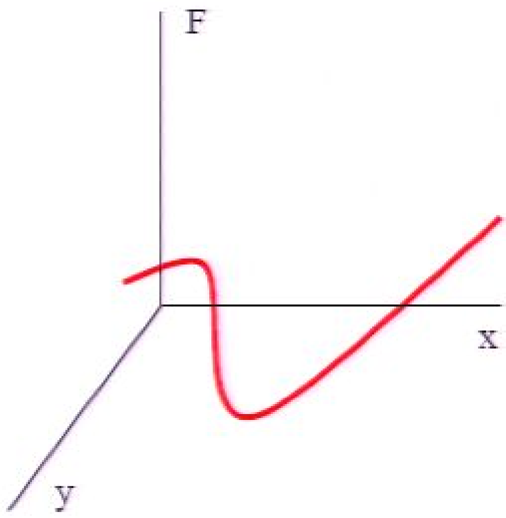

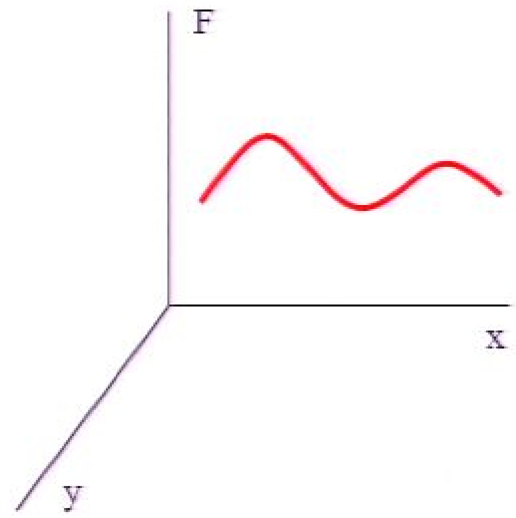

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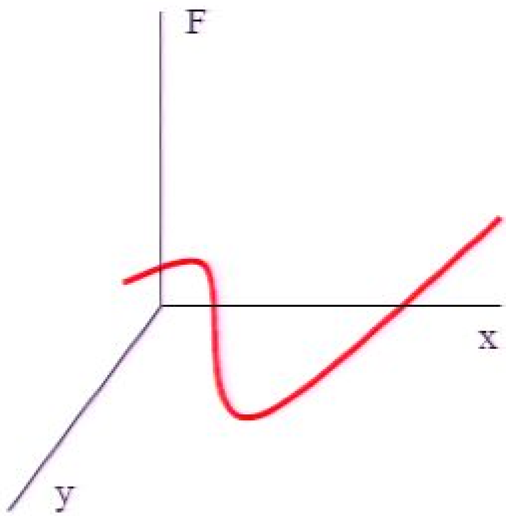

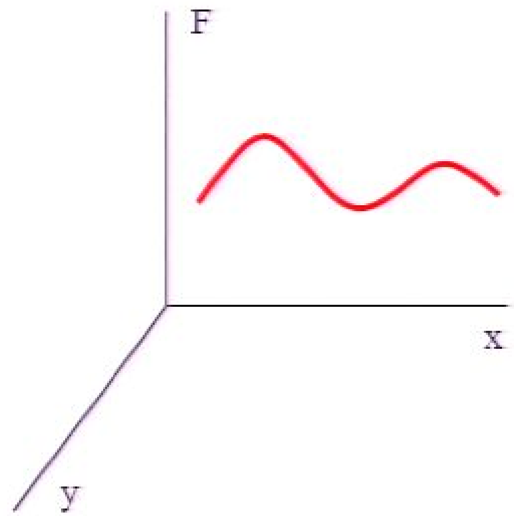

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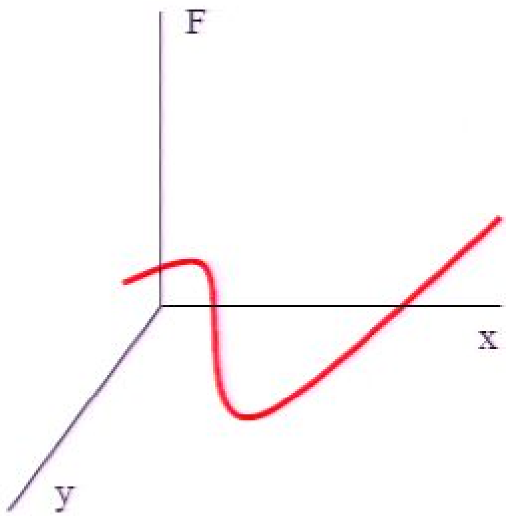

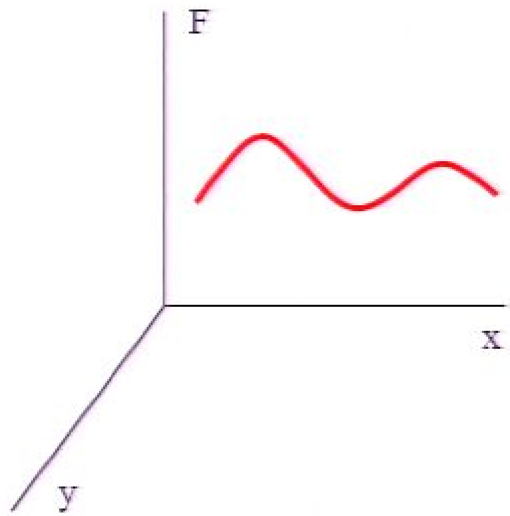

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Summary of the solution

- Metric

$$ds^2 = e^H \left[-e^{3\phi/2}dt^2 + e^{-\phi/2}d\mathbf{x}_3^2\right] + e^{-H-\phi/2}d\mathbf{y}_5^2$$
$$+e^{-H+3\phi/2}(\partial_w F dw + \partial_{\mathbf{y}} F d\mathbf{y})^2, \qquad e^{2H} = \partial_w F$$

- Fluxes

$$F_5 = -\frac{1}{4\cdot 4!}d\left[e^{-2H}\varepsilon_{ijklm}\partial^{y_m}F dy^{ijkl}\right] + dual,$$
$$H_3 = d\left[e^{2\phi}(\partial_w F dw + \partial_{\mathbf{y}} F d\mathbf{y})\right]dt, \quad F_3 = \frac{1}{2}d(\varepsilon_{ijk}\partial^k F dx^{ij}).$$

- Coupled PDEs for F, e^{ϕ}
- 1/4–BPS: two projectors for the spinor

$$\varepsilon = \exp\left[\frac{1}{4}(H + \frac{3\phi}{2})\right]\varepsilon_0 : \quad \Gamma_w\Gamma_{45678}\varepsilon_0 = -i\varepsilon_0, \quad \Gamma_w\Gamma_{123}\varepsilon_0^* = i\varepsilon_0$$

Boundary conditions

- D–branes must follow harmonic profiles
  - D3 brane: $w = \tilde{w}[\vec{y}, F - f(\vec{x})]$
  - D5 brane: $F = \tilde{F}[\vec{x}, w - g(\vec{y})]$
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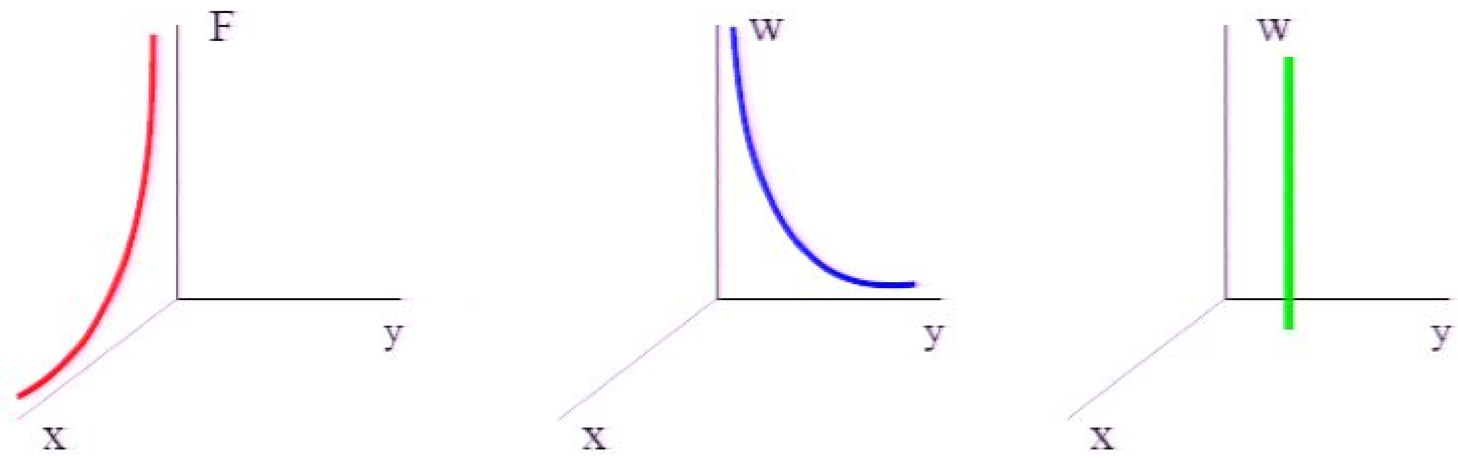

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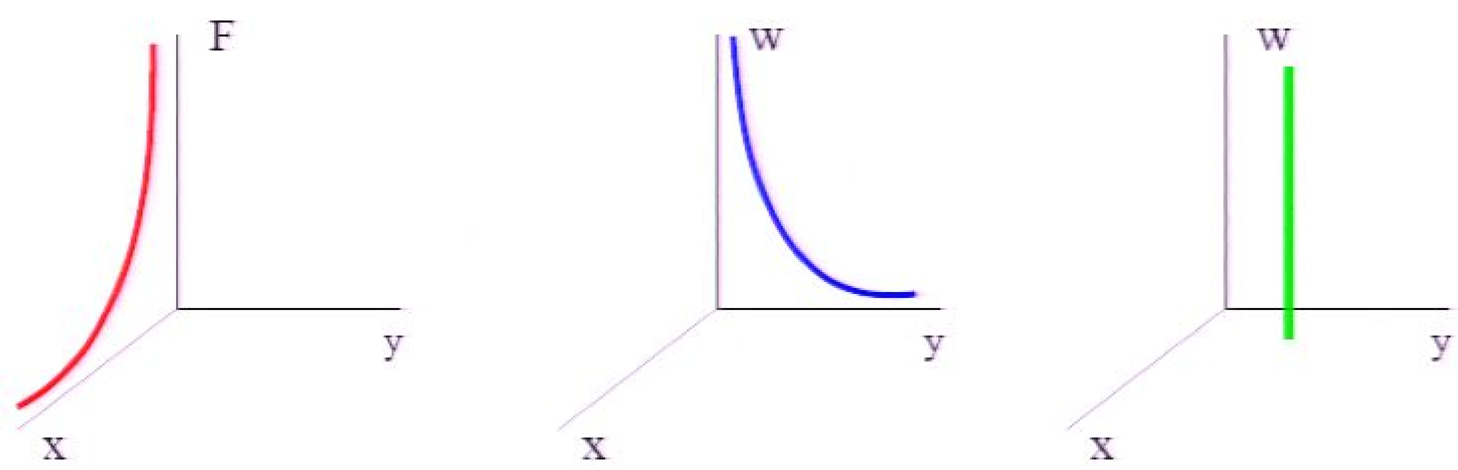

- Vicinity of the brane: Poisson eqn for $\tilde{w}$, $\tilde{F}$ or $e^{-2\phi}$
- Unique solution in perturbation theory

Supergravity vs DBI

- Profiles of D3 branes from SUGRA:

$$\Delta_{\mathbf{x}} f = 0$$

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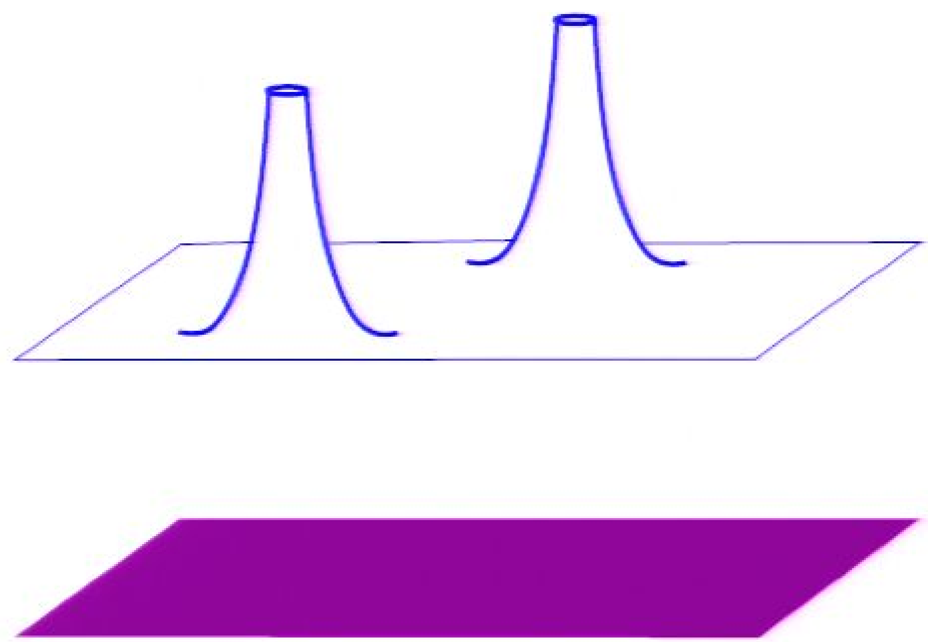

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$$-(1 + (\nabla X)^2)\partial_X H + H\nabla^2 X + 2\nabla H \nabla X = 0$$

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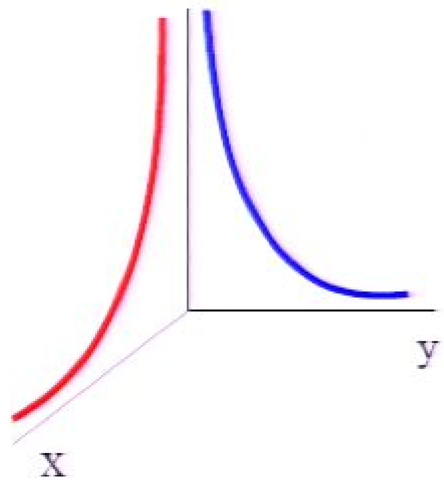
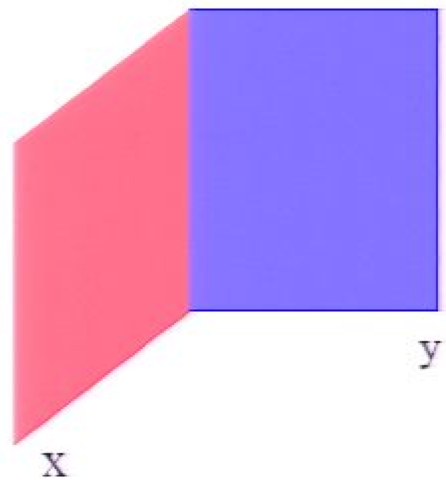
Explicit solutions

- Smeared D branes





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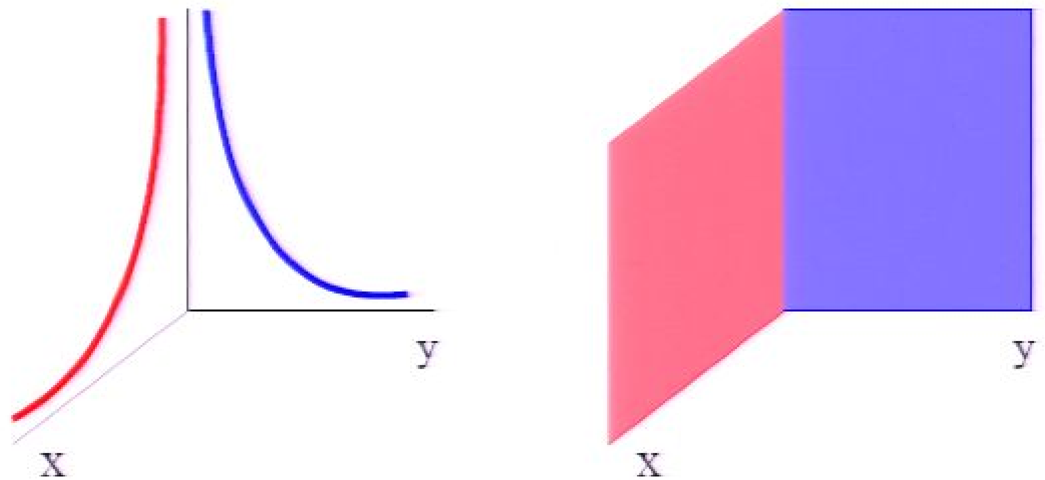

Explicit solutions

- Smeared D branes: harmonic functions

$$q(x), \quad p(y), \quad q\Delta_y(q^{-1}e^{-2\phi}) + p\Delta_x(q^{-1}e^{-2\phi}) = 0$$

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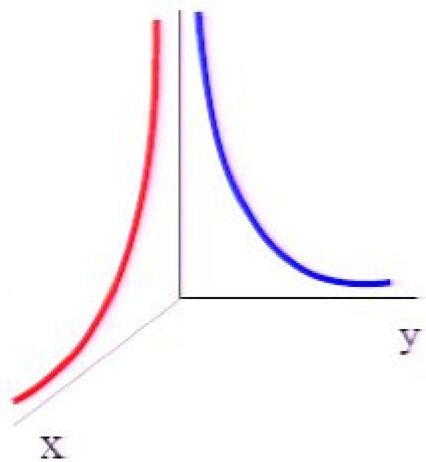

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  - D3 brane: $\partial_x = 0$: dual of NCYM_{3+1}
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Hashimoto–Itzhaki, Maldacena–Russo '99

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- Near–horizon limits
  - 1/4–BPS states in $\text{AdS}_5\times\text{S}^5$/linear dilaton geometry
  - special solutions: 1/2–BPS states in $\text{AdS}_5\times\text{S}^5$
    - Wilson lines in $\mathcal{N} = 4$ SYM

Wilson lines and AdS/CFT

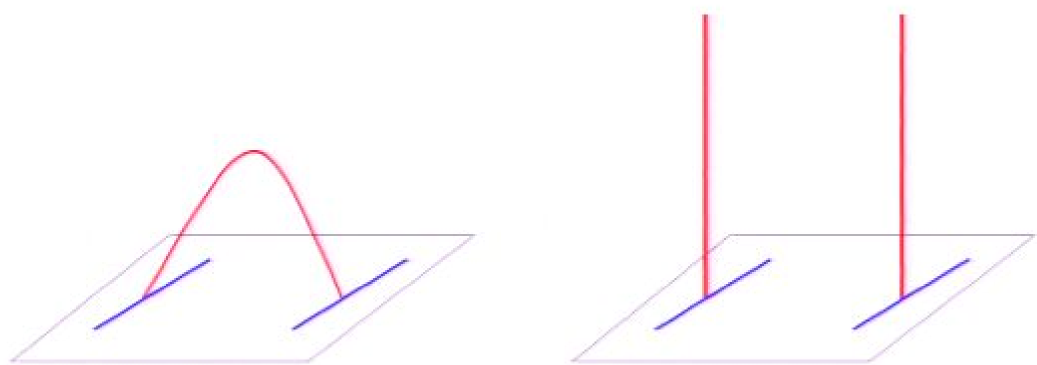
- Wilson line in field theory

$$W(\mathcal{C}) = \frac{1}{d_R} \text{Tr}_R P e^{i \int_{\mathcal{C}} A}$$

- Dual description
  - fund. rep: string ending on a contour

Rey, Yee; Maldacena '98

$$\langle W(\mathcal{C}) \rangle = e^{-(S - S_0)}$$



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  - heavy states ($\Delta \sim N^2$): gravitational backreaction

Wilson lines and AdS/CFT

- Supersymmetric Wilson line in field theory

$$W = \frac{1}{d_R}\text{Tr}Pe^{i\int(A_\mu \dot{x}^\mu + i\Phi|\dot{x}|)dt}$$

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- Structure of the geometries
  - symmetry: $SO(2,1) \times SO(3) \times SO(5)$
  - solutions in terms of harmonic $\Phi(x, y)$
  - regularity at $y = 0$: $\partial_y \Phi = \pm\frac{\pi}{4}$

    OL '06

1/2–BPS states

- Wilson lines: Young tableaux, branes & geometry

$\Delta = 0$

$AdS_5 \times S^5$

1/2–BPS states

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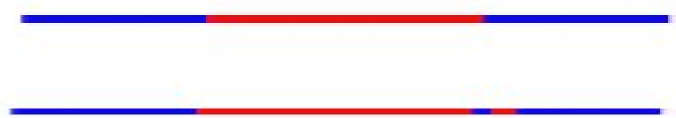
$\Delta \sim 1$ $\qquad$ string



1/2–BPS states

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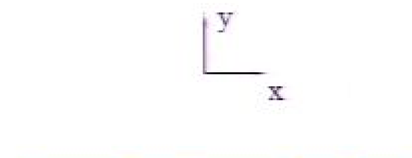

- Boundary condition $\rightarrow \Phi(x, y) \rightarrow$ geometry

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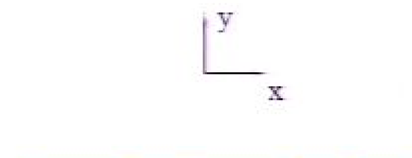

- Boundary condition $\rightarrow \Phi(x, y) \rightarrow$ geometry
- Relation to the 1/4–BPS F1–D3–D5 system
  - extra (super)symmetries
  - no sources
  - fluxes are supported by non–trivial topology

1/4–BPS intersections

- Intersections in IIB string theory
  - harmonic rule

$$(D5_{12345}, D5_{16789}, P_1)\ (D3_{123}, D7_{1456789}, P_1)\ (D1_1, P_1)$$

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- Intersections in M theory
  - U dualities: localized M2/M5/M5 intersections
  - agreement with probe analysis (PST action)
  - enhanced (super)symmetry near the branes

1/2–BPS geometries in M theory

- Properties of the geometries
  - dual description: defects in the CFTs
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OL '07

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OL '07

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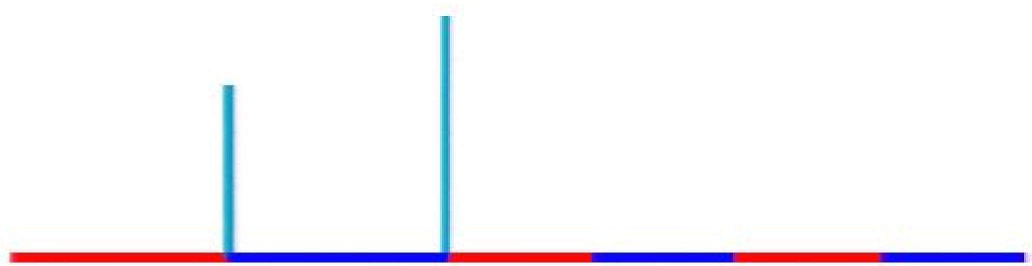
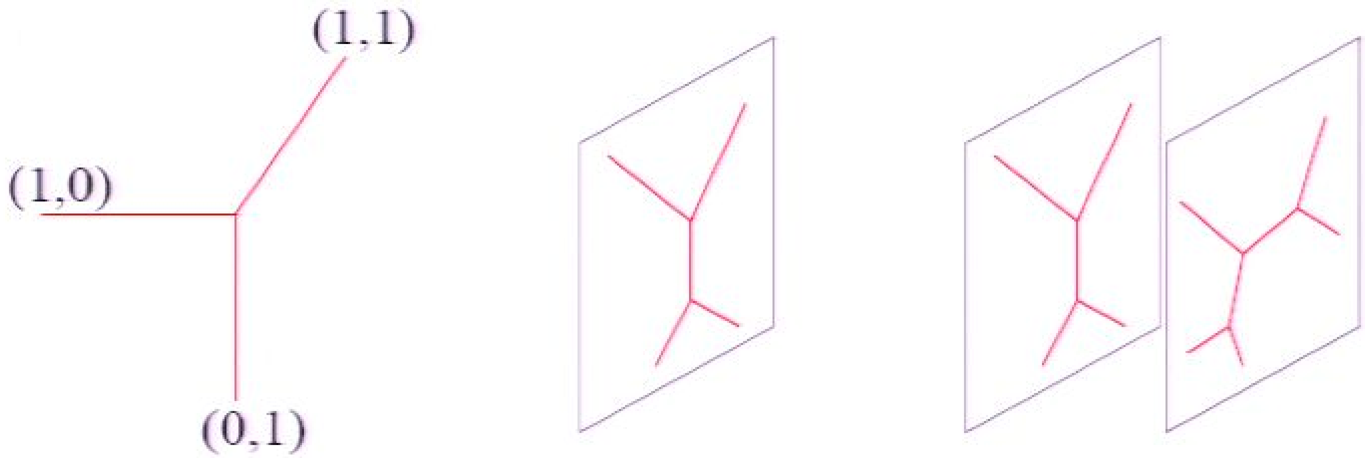


Brane webs

- String webs
  - probes: straight lines, orientation and dilaton
  - gravity solution: consistent boundary conditions



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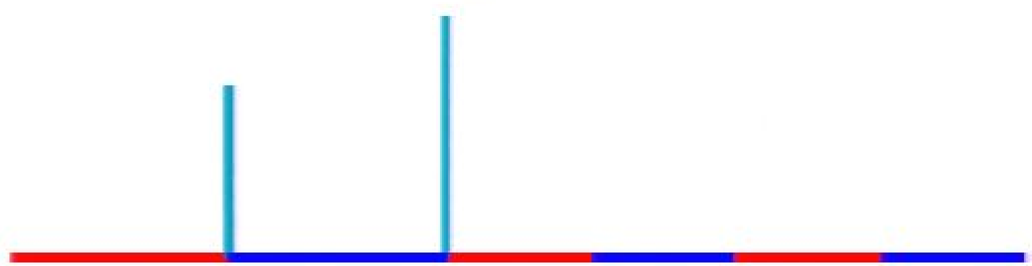

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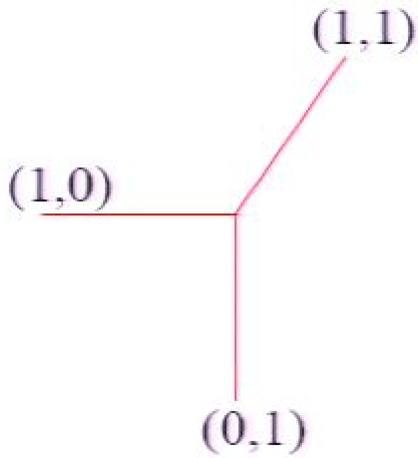

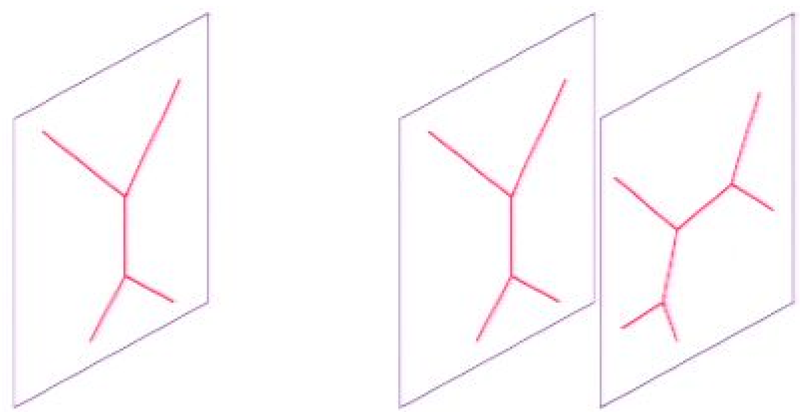

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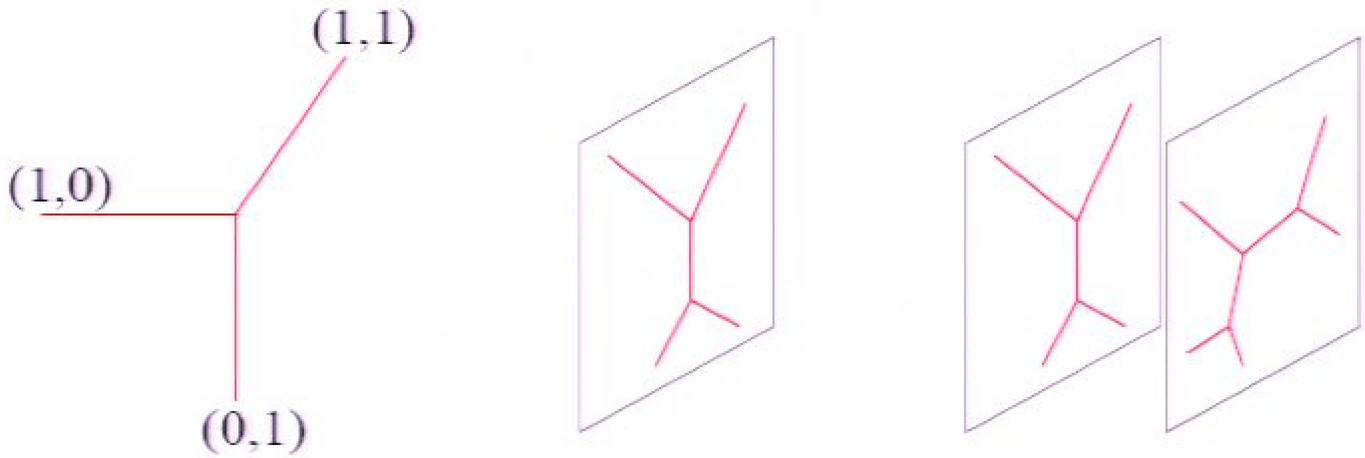

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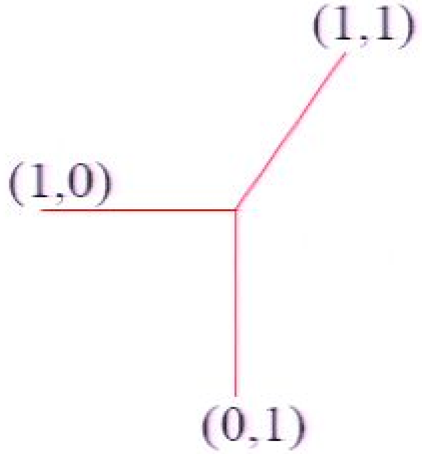

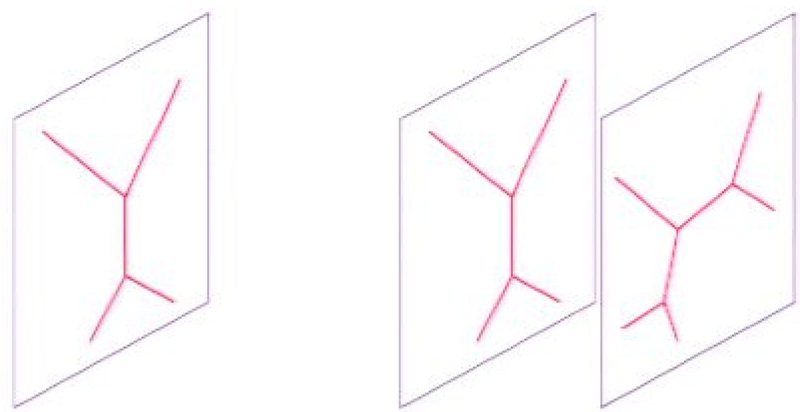

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  - 1/4–BPS geometries in $AdS_p \times S^q$
  - regular metrics: droplets in 2D Kahler space
  - degenerate limits and giant gravitons

OL '08

1/4–BPS bubbling solutions

- Field theory
  - two adjoint scalars $X = \Phi_1 + i\Phi_2$, $Y = \Phi_3 + i\Phi_4$.
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  - 2D Kahler space, $y = R_3 R_1$
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  - interesting topological structure

Structure of the geometry

- Local description
  - Monge–Ampere type equation in $4+1$ dimensions

$$\det h_{a\bar{b}} = -\frac{y^3}{8}\partial_y \left[y^{-1} \exp\{y^{-1}\partial_y K\}\right]$$

  - Kahler potential in 4D $\rightarrow$ geometry in 10D

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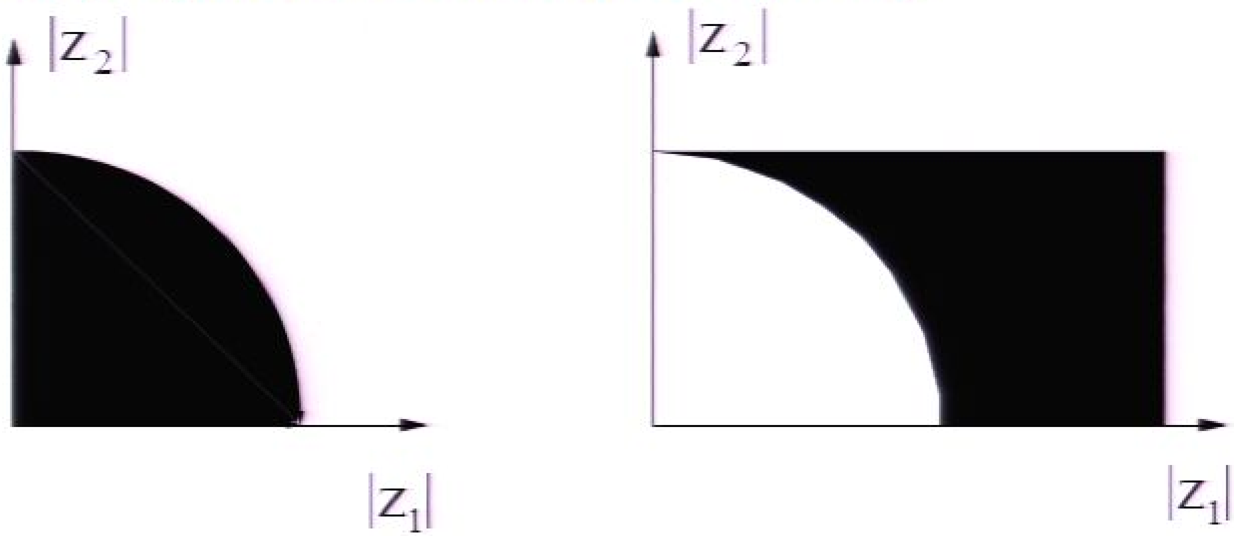

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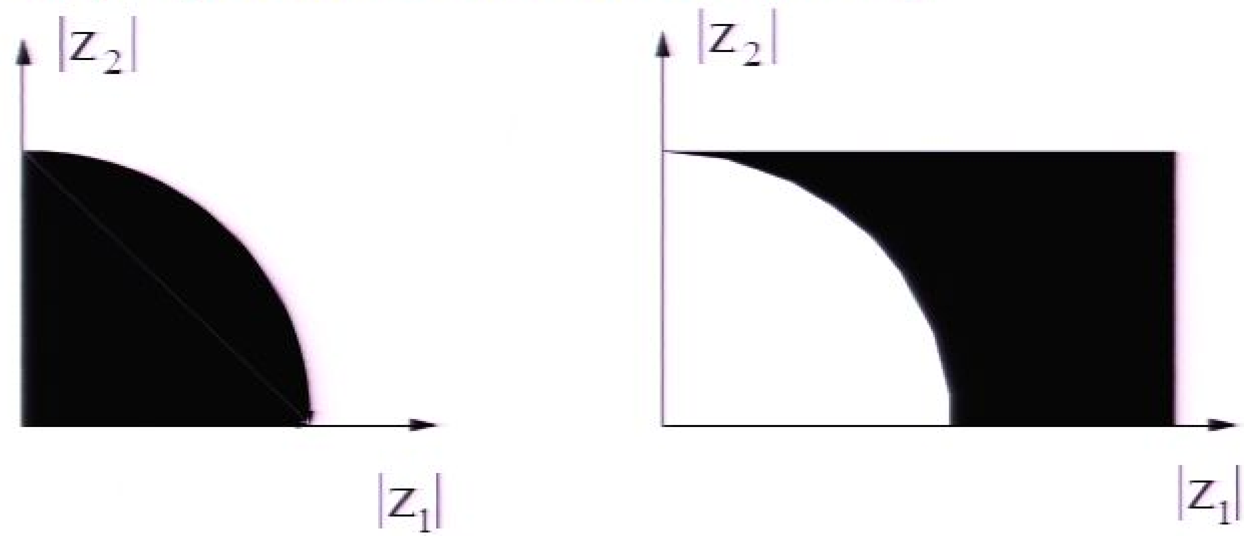

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- Degenerate droplets $\rightarrow$ holomorphic curves

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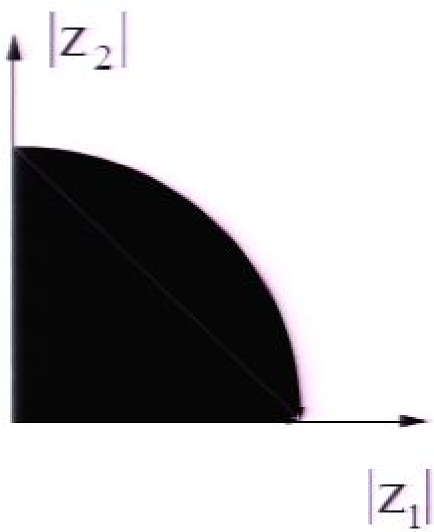

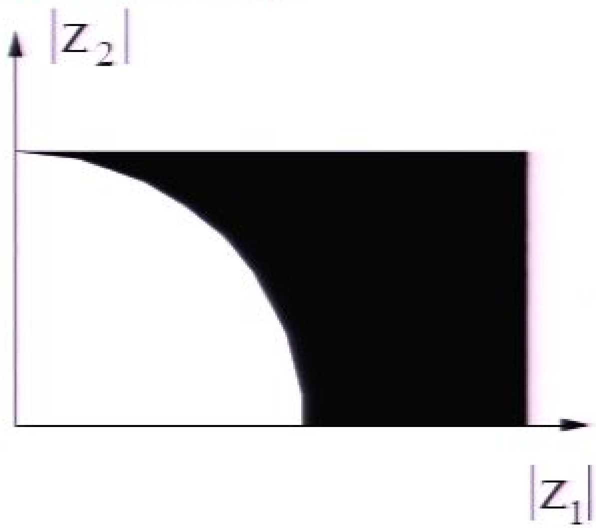

Structure of the geometry

- Local description
  - Monge–Ampere type equation in $4+1$ dimensions
  - Kahler potential in 4D $\rightarrow$ geometry in 10D
- Boundary conditions
  - generic point at $y=0$: $Z=\pm\frac{1}{2}$
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  - regularity at the domain walls: shapes of the droples are restricted

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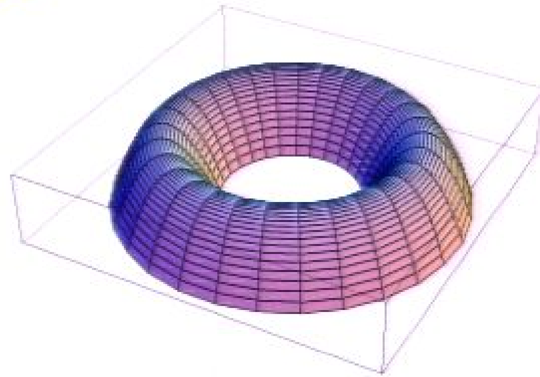
- Relation to 1/2–BPS case



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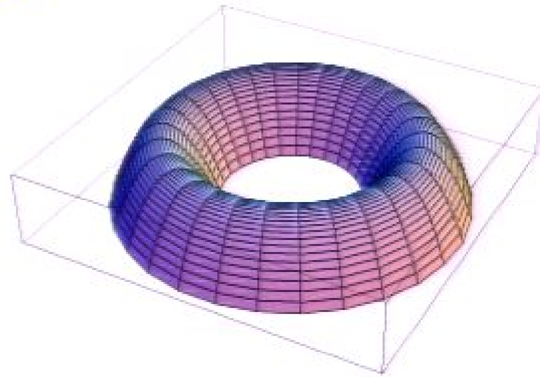
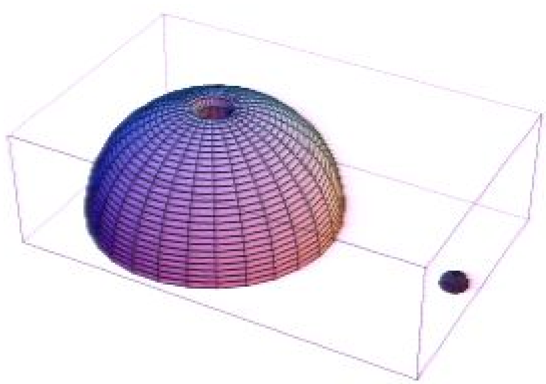
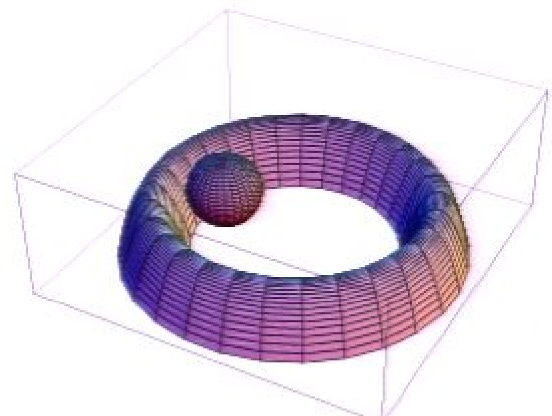


- Generic boundary conditions





Outlook

- Shapes of branes are determined dynamically
  - open strings: solutions of DBI
  - closed strings: consistency of SUGRA
- Explicit example: 1/4–BPS D3/D5/F1 system
  - complete gravity solution, only $U(1)$ isometry
  - perfect agreement between DBI and SUGRA
- Other solutions
  - systems related by U–dualities
  - M2–M5 intersections: PST action and gravity
  - brane webs and holomorphic surfaces
- Open questions
  - extension to cases with lower SUSY