Title: Relativistic Dynamics of Graphene

Date: Apr 16, 2008 02:00 PM

URL: http://pirsa.org/08040014

Abstract: Graphene, a single atomic layer of graphite, was created only a few years ago. It is a remarkable system, whose law energy effective theory has a lot in common with relativistic 2 + 1 dimensional ones. Graphene allows tabletop experiments for observing nonperturbative relativistic phenomena, most notably spontaneous chiral symmetry breaking both in vacuum and in an external magnetic field. The latter is in turn crucial for the dynamics of Quantum Hall effect in this system. I will review the basic physics and the latest results obtained both in the experimental studies and the theory of graphene.

Relativistic Dynamics in Graphene

Graphene — a one-atom-thick layer of graphite (crystalline carbon).

K. Novoselov, A. Geim et al., Science, 306 (2004) 666

Method to produce: repeated peeling of a piece of graphite with Scotch tape

Relativistic like brane world dynamics:
electrons live on a plane;
electromagnetic field is in 3 dim. bulk

Dispersion relation for quasiparticles:
$\omega \simeq v_F |\vec{k}|$; $v_F \simeq 10^6$ m/s ;
$v_F/c \simeq 1/300$

(P. Wallace, 1947)

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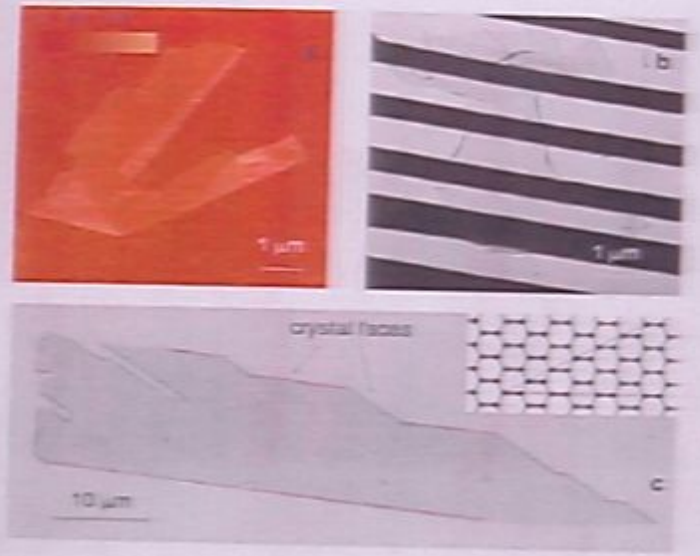
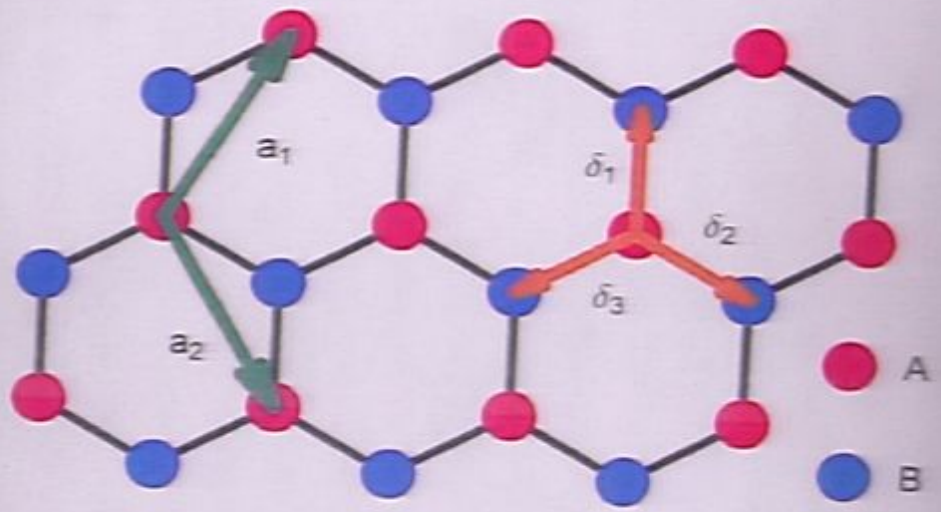
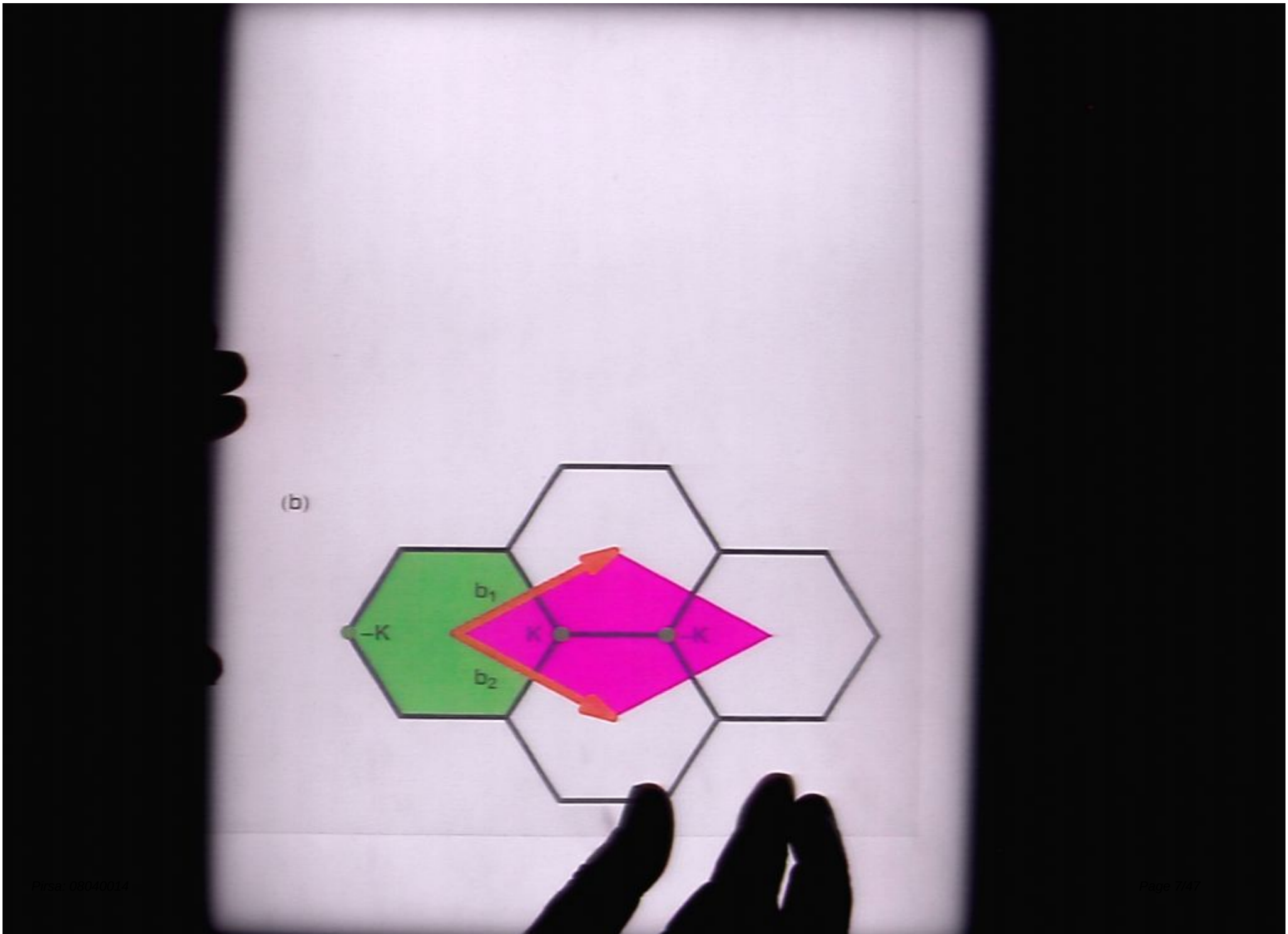



a_1
a_2
δ_1
δ_2
δ_3
A
B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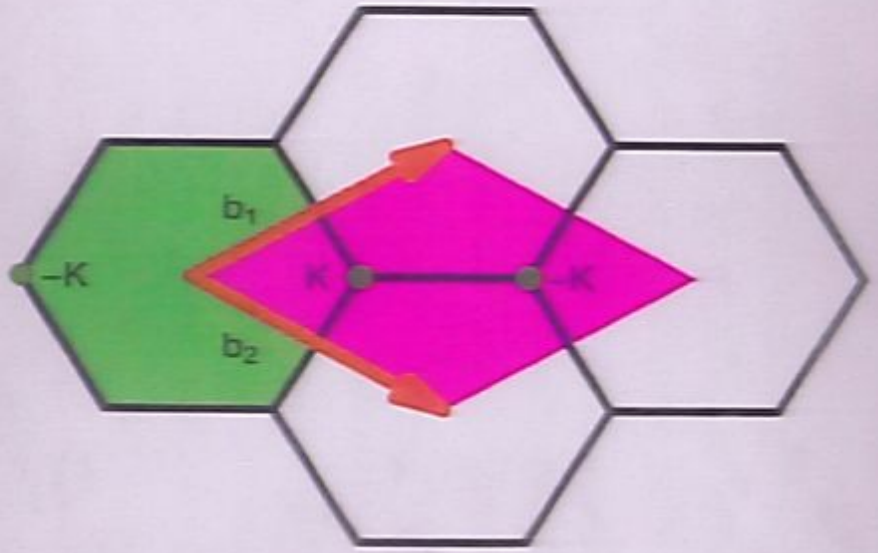
(b)



Dirac Equation in Graphene

G. Semenoff, PRL, 53 (1984) 2449

$$[i\hbar\gamma^0\partial_t + i\hbar v_F\gamma^1\partial_x + i\hbar v_F\gamma^2\partial_y]\Psi_s(t,\vec{r}) = 0$$

$s = \pm$ is the spin index playing here the role of flavor;

$$\Psi_s = (\Psi_{KA}, \Psi_{KB}, \Psi_{K'B}, \Psi_{K'A})_s$$

$$\gamma^\nu = (\gamma^0, \gamma^1, \gamma^2) = \tilde{\tilde{\tau}}^3_{KK'} \otimes (\tau^3, i\tau^2, -i\tau^1)_{AB}$$

Lorentz symmetry in 2+1 dimensions, with v_F playing the role of light velocity

Extended chiral symmetry:

$\{\gamma^3, \gamma^\nu\} = 0$, $\{\gamma^5, \gamma^\nu\} = 0$; $\nu = 0, 1, 2$

$U(4)$ symmetry with 16 generators:

$\frac{\sigma^a}{2} \otimes I_4$, $\frac{\sigma^a}{2i} \otimes \gamma^3$, $\frac{\sigma^a}{2} \otimes \gamma^5$, $\frac{\sigma^a}{2} \otimes \gamma^3\gamma^5$,

where I_4 – 4×4 Dirac unit matrix,

$\gamma^3 = i\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$, $\gamma^5 = i\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$, $\gamma^3\gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$

σ^a – four Pauli matrices acting on spin

(σ^0 – 2×2 unit matrix).

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Adding Dirac mass term
$\tilde{\Delta}\bar{\psi}\psi \equiv \tilde{\Delta}\psi^{\dagger}\gamma^{0}\psi$, $\tilde{\Delta} \equiv m^{2}v_{F}^{2}$,
would break the $U(4)$ down
to $U(2)_{a} \times U(2)_{b}$ with generators
$\frac{\sigma^{a}}{2} \otimes I_{4}$, $\frac{\sigma^{a}}{2} \otimes \gamma^{3}\gamma^{5}$

Chiral (excitonic) condensate:
$\langle \bar{\psi}\psi \rangle = n_{KA} + n_{K'A} - n_{KB} - n_{K'B}$,
$n_{KA} = \langle \psi_{KA}^{+}\psi_{KA} \rangle$, etc — electron densities at specified valleys, K and K', and sublattices, A and B.
$\langle \bar{\psi}\psi \rangle \neq 0 \Longrightarrow$ density imbalance between A and B sublattices (charge density wave).

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Could a Dirac mass be generated dynamically?

More about dynamics in graphene

Because $v_F \neq c$, including electromagnetic interactions leads to Lorentz symmetry breaking. In particular, because $v_F/c \simeq \frac{1}{300} \ll 1$, static Coulomb interactions dominate. However, scale (dilation) symmetry is still (classilly) exact.

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Effective coupling constant

$$g \equiv \frac{e^2}{\hbar v_F \varepsilon} = \frac{c}{v_F \varepsilon} \frac{e^2}{\hbar c} =$$

$$\simeq \frac{c}{v_F \varepsilon} \frac{1}{137} \simeq \frac{300}{\varepsilon 137} \sim 1 \text{ for}$$

$\varepsilon \sim 2-4$.

Theoretical analysis:

E. Gorbar, V. Gusynin, V. M, and I. Shovkovy, Phys. Rev. B 66 (2002) 045108

$g_{cr} \simeq 2.3$

D. Khveshchenko and H. Leal, Nucl. Phys. B 687 (2004) 323

$g_{cr} \simeq 1.1$

Experiment: there is no gap in graphene.

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But it is not the end of the story: In the rest of this talk, I will argue (show?) that a Dirac mass, actually two different types of Dirac masses, have been already observed in graphene in a strong magnetic field.

Magnetic catalysis phenomenon

A constant magnetic field is a strong catalyst of dynamical chiral symmetry breaking, leading to the generation of a fermion dynamical mass even at the weakest attractive interaction between fermions.

V. Gusynin, V. M., and I. Shovkovy, Phys. Rev. Lett. 73, 3499 (1994). Nucl. Phys. B 462, 249 (1996)

D = 3+1 and D = 2+1 cases.

Extension to D = d+1, d > 3 in

E. Gorbar, Phys. Lett. B 491, 305 (2000)

The LLL fermion propagator:

$$S(x,y) = \exp\left[\frac{ieB}{2}\epsilon^{ab}x^a y^b\right] \cdot \tilde{S}(x-y); \quad a,b = 1,2$$

Schwinger phase

$$\tilde{S}(k) = 2i\, e^{\frac{-k_\perp^2}{|eB|}} \frac{k^0\gamma^0 - k^3\gamma^3 + m}{k_0^2 - k_3^2 - m^2} \cdot \frac{1}{2}\left(1 - i\gamma^1\gamma^2 \operatorname{sign}(eB)\right)$$

projector

$3+1 \longrightarrow 1+1$

$2+1 \longrightarrow 0+1$

Dynamics underlying new plateaus with $\nu = 0$, $\nu = \pm 1$, $\nu = \pm 4$ observed in graphene:

Y. Zhang et al., PRL 96 (2006) 136806;
Z. Jiang et al., PRL 99 (2007) 106802

"Old" plateaus:

$$\sigma_{xy} = \frac{e^2}{h}\nu, \quad \nu = \pm 4(n+\tfrac{1}{2}), \quad n = 0, 1, \ldots$$

Experiment: K. Novoselov et al., Nature 438 (2005) 197,
Y. Zhang et al., Nature 438 (2005) 201

Theory: Y. Zheng and T. Ando, PRB 65 (2002) 245420;
V. Gusynin, S. Sharapov, PRL 95 (2005) 146801;
N. Peres, F. Guinea, A. Castro Neto, PRB 73 (2006) 125411

$$E_n = \pm\sqrt{2n\hbar v_F^2 |eB|/c} \simeq 424\sqrt{n}\sqrt{B[T]}\ K$$

$n = 0, 1, 2, \ldots$ $\quad v_F \simeq 10^6$ m/s

LLL with $n=0$, $E_0 = 0$, is special

Nonrelativistic Dynamics in Magnetic Field

$$E_n \simeq \frac{n\hbar v_F^2 |eB|}{c \cdot mc^2} \equiv n L(B) \cdot L(B)/mc^2,$$

$$L(B) \equiv \sqrt{\hbar v_F^2 |eB|/c} \simeq 300\sqrt{B[T]}\ K$$

$$mc^2 \simeq 0.5\ MeV \sim 10^{10}\ K$$

$L(B)/mc^2 \ll 1$ for realistic values of $B \lesssim 50\ T$

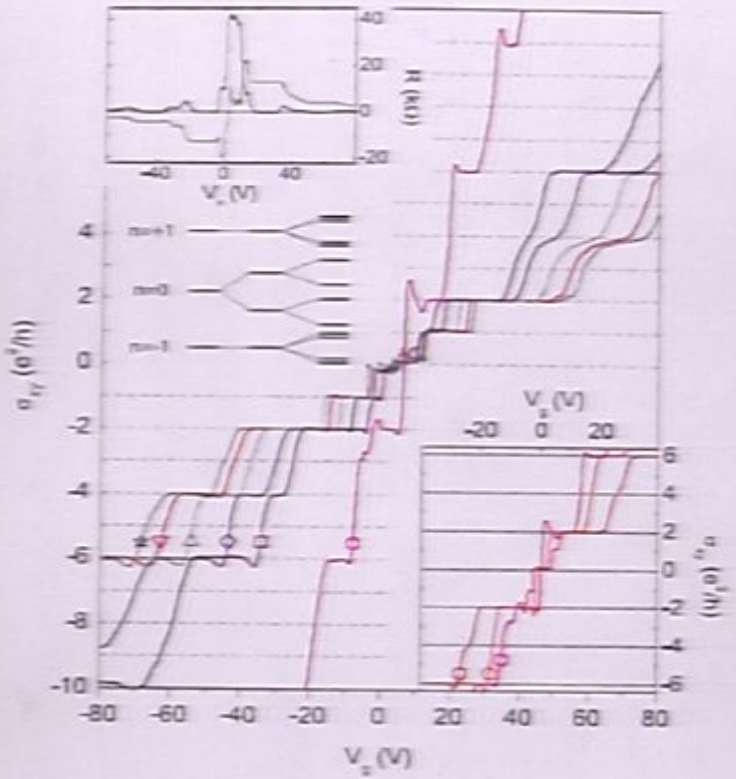
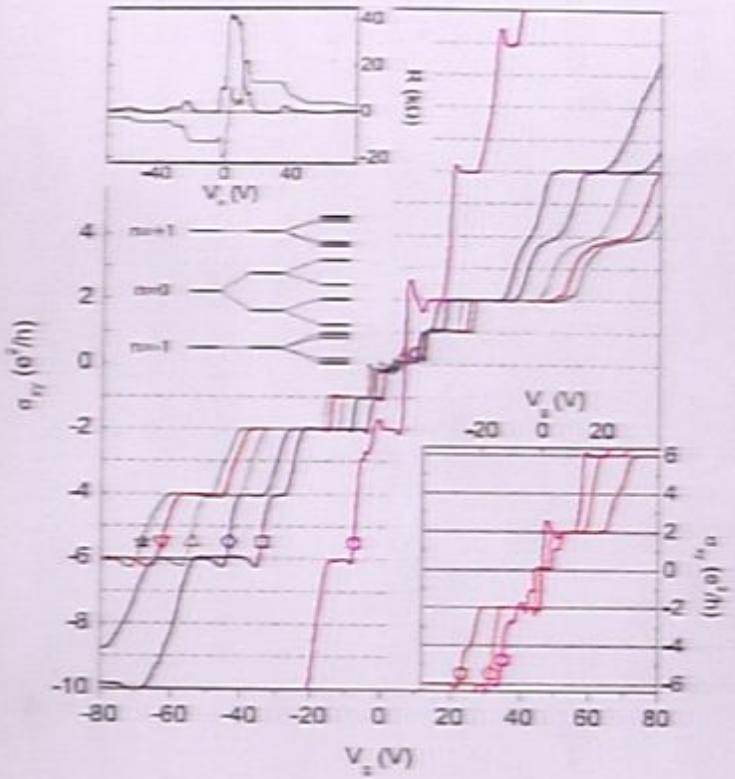


FIG 2. (color online) σ_{xy} as a function of V_g at different magnetic fields: 9 T (circle), 25 T (square), 30 T (diamond), 37 T (up triangle), 42 T (down triangle), and 45 T (star). All the data sets are taken at $T = 1.4$ K, except for the $B = 9$ T curve, which is taken at $T = 30$ mK. Left upper inset: R_{xy} and R_{xx} for the same device measured at $B = 25$ T. Left lower inset: a schematic drawing of the LLs in low (left) and high (right) magnetic field. Right inset: detailed σ_{xy} data near the Dirac point for B = 9 T (circle), 11.5 T (pentagon) and 17.5 T (hexagon) at $T = 30$ mK.

$\nu = \pm 4(n+\frac{1}{2})$

$\pm 2, \pm 6, \pm 10, \ldots$

NT Reversal direction



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Model: Hamiltonian

$$H = H_0 + H_C - \mu_0 \psi_S^\dagger \psi_S + \mu_B B \psi_S^\dagger \sigma^3 \psi_S ,$$

$$\mu_B = e\hbar/2mc , \quad g_L \simeq 2, \ (e>0) ; \quad B \equiv |\vec{B}|$$

$$H_0 = v_F \int d^2\vec{r}\, \overline{\psi_S} \left(\gamma^1 \Pi_x + \gamma^2 \Pi_y\right) \psi_S ,$$

$$\vec{r} = (x, y), \quad \vec{\Pi} = (\Pi_x, \Pi_y) = -i\hbar\vec{\nabla} + e\vec{A}/c ,$$

$$v_F \simeq 10^6 \ m/s , \quad \psi_S = \left(\psi_{KAS}, \psi_{KBS}, \psi_{K'BS}, \psi_{K'AS}\right),$$

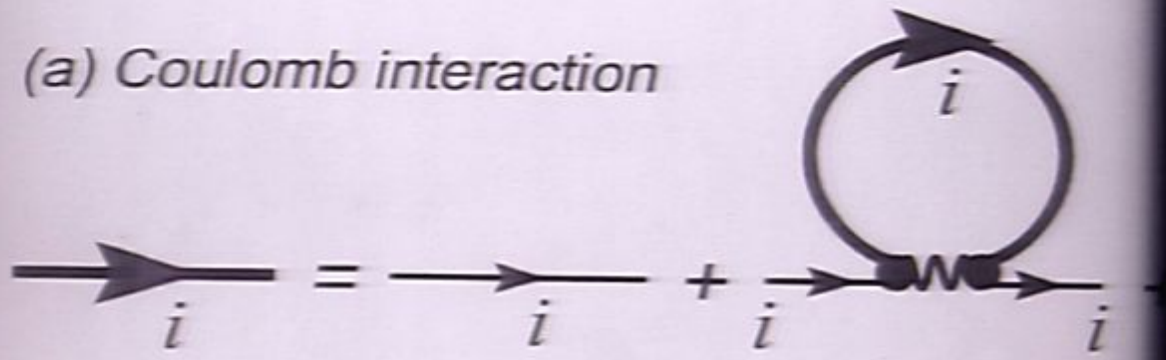
$$\gamma^0 = (\gamma^0, \gamma^1, \gamma^2) = \tilde{\tau}^3_{KK'} \otimes (\tau^3, i\tau^2, -i\tau^1)_{AB} ,$$

$$\overline{\psi_S} = \psi_S^\dagger \gamma^0 ; \quad \vec{A} \text{ corresponds to } B_\perp .$$

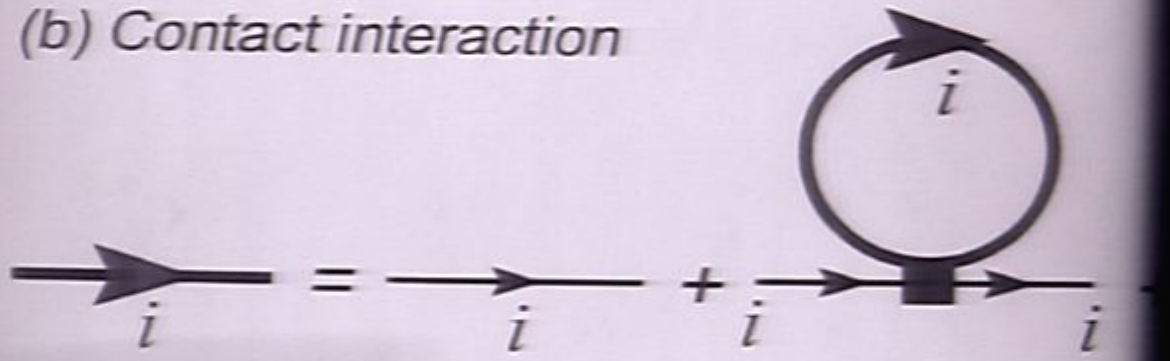
$$H_C = \frac{1}{2} \int d^2\vec{r}\, d^2\vec{r}' \, \psi_S^\dagger(\vec{r}) \psi_S(\vec{r}) U_C(\vec{r}-\vec{r}') \psi_{S'}^\dagger(\vec{r}') \psi_{S'}(\vec{r}') .$$

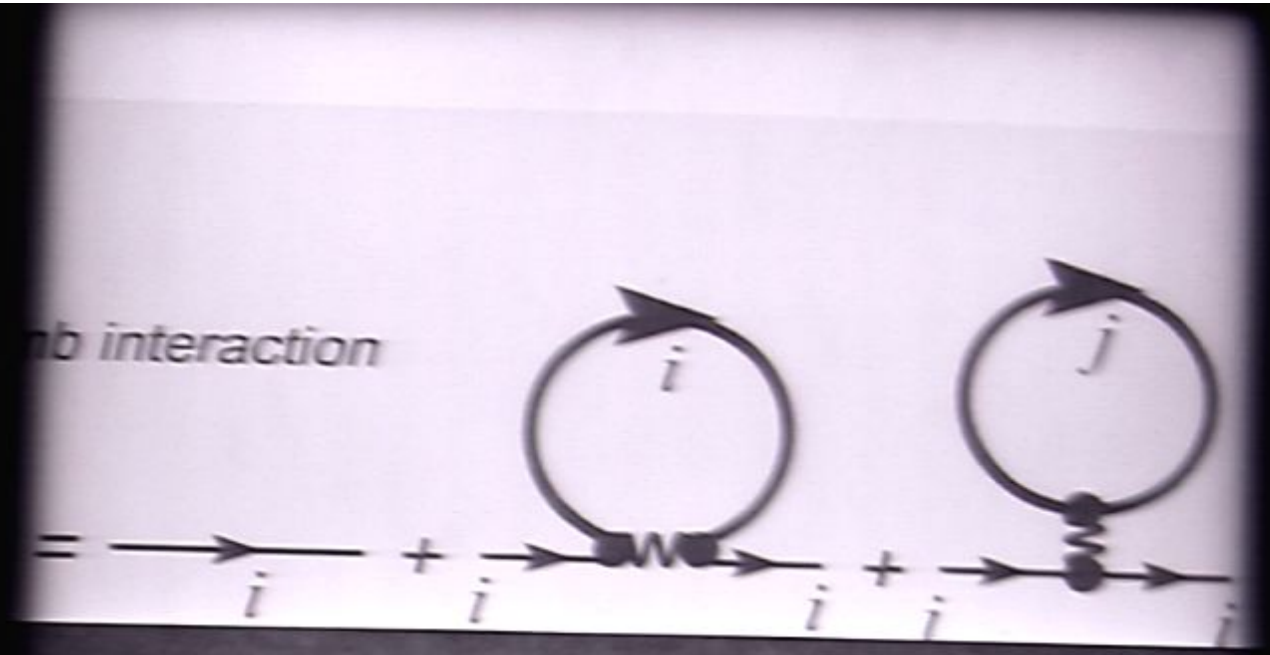
H_0: Lorentz symmetry in 2+1 dimensions, with v_F playing the role of light velocity

(a) Coulomb interaction



(b) Contact interaction




b interaction
i
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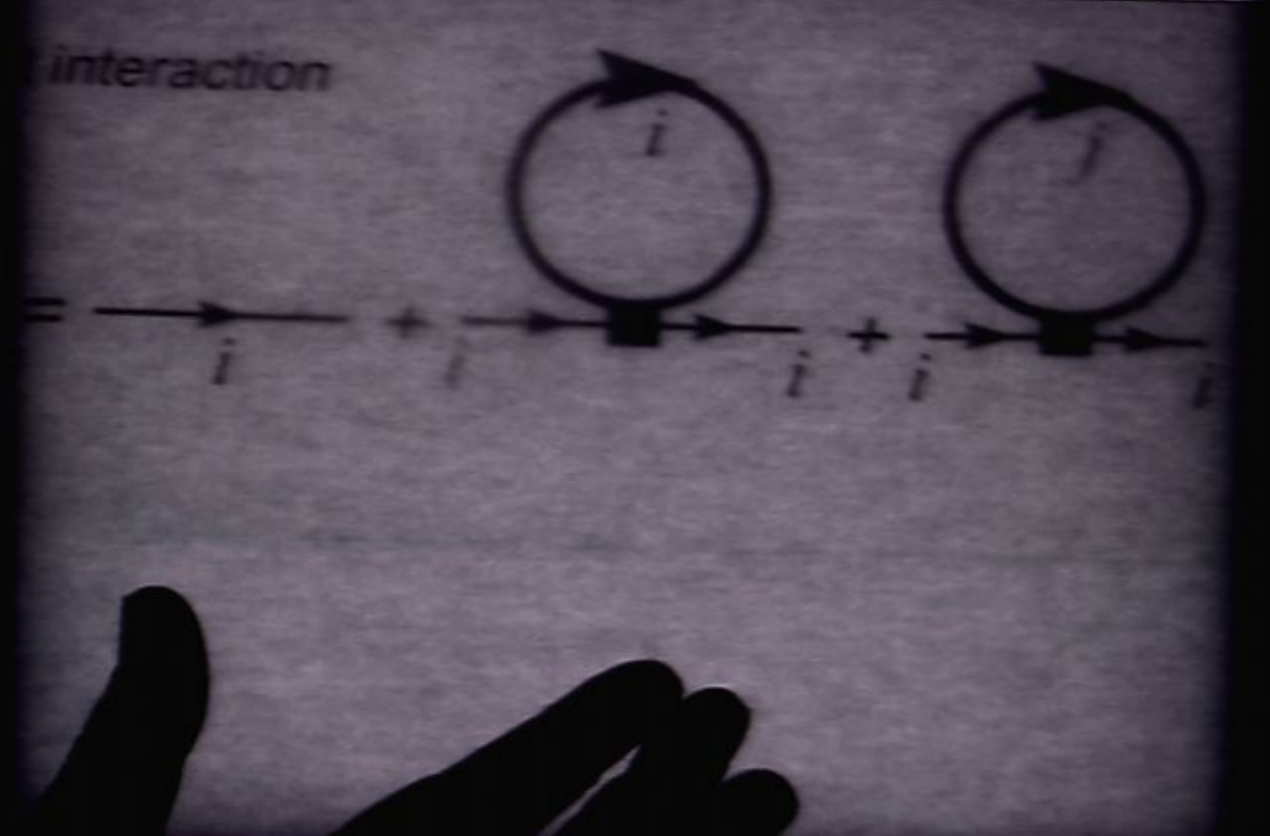
i


interaction
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+
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i
i
i
i

Model: Gap Equation

$G(x,x')$ – propagator of Dirac quasiparticles;
$x \equiv (x_0, \vec{r})$, $x_0 \equiv t$.

Hartree-Fock (mean field) approximation:

$$G^{-1}(x,x') = S^{-1}(x,x') + ik\gamma^0 G(x,x)\gamma^0 \delta(t-t')U_c(\vec{r}-\vec{r}') - ik\gamma^0 \mathrm{tr}\,[\gamma^0 G(x,x)]\delta(t-t')U_c(\vec{r}-\vec{r}')$$

direct interactions — exchange interactions

$$iS^{-1}(x,x') = \left[(ik\partial_t + \mu_0 - \mu_B B\sigma^3)\gamma^0 - v_F \vec{\gamma}\vec{\nabla}\right]\delta^3(x-x')$$

– bare inverse quasiparticle propagator

Approximation: $U_c(\vec{r}) \to G_{int}\delta^3(\vec{r})$

$$G^{-1}(x,x') = S^{-1}(x,x') + ik G_{int}\delta^0 G(x,x)\gamma^0 \delta^3(x-x') - ik G_{int}\delta^0 \mathrm{tr}\,[\gamma^0 G(x,x)]\delta^3(x-x')$$

$\Lambda \sim L(B) \equiv \sqrt{\hbar/eB_\perp}/k_F$, $\lambda \equiv G_{int} N(\mu_0, \hbar^2 k_F^2)$

Λ – London scale; λ – dimensionless coupling constant

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direct interactions — exchange interactions

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$$\Lambda \sim L(B) \equiv \sqrt{\hbar v_F / eB_\parallel} \ll \ell_t \,, \quad \lambda \equiv G_{int} N(\dots)$$

Landau scale — dimensionless coupling constant

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$$\Lambda \sim L(B) \equiv \sqrt{\hbar|eB_\perp|}\,v_F/c\,, \quad \lambda \equiv G_{int}N(0)_{3D}\hbar^2 v_F^2$$

Landau scale — dimensionless coupling constant

... ψ_s passes the $u(4)$ symmetry with 16 generators:

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where I_4 – 4×4 Dirac unit matrix, σ^a – four Pauli matrices (σ^0 is 2×2 unit matrix),

$\gamma^3 = i\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$, $\gamma^5 = i\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$, and

$\gamma^3\gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$ – pseudospin matrix

Zeeman term $\mu_B B \psi_s^+ \sigma^3 \psi_s$ breaks $u(4)$ down to $u(2)_+ \times u(2)_-$ with 8 generators $I_4 \otimes P_s$, $\gamma^5 \otimes P_s$, $-i\gamma^3 \otimes P_s$, and $\gamma^3\gamma^5 \otimes P_s$; $s = \pm$, $P_\pm = (1 \pm \sigma_3)/2$ are projectors on spin up and down states

Order Parameters

Goal: searching for solutions of gap equation with spontaneously broken and unbroken $SU(2)_s$, $U(2)_s$, $s=\pm$

Two scenarios for QH effect

a) Quantum Hall Ferromagnetism (QHF)

(K. Nomura, A. MacDonald, PRL 96 (2006) 256602; M. Goerbig, R. Moessner, B. Doucot, PRB 74 (2006) 161407; I. Alicea and M.P.A. Fisher, PRB 74 (2006) 075422;)

Order parameters: pseudospin densities

$\langle \psi^+ \gamma^3\gamma^5 P_s \psi \rangle \neq 0 \Rightarrow SU(2)_s \to U(1)_s$

with the generator $\gamma^3\gamma^5 \otimes P_s$, $P_\pm = \frac{I \pm \sigma^3}{2}$

$\tilde{\mu}_s$ are corresponding chemical potentials. Triplet order parameters

Scenario is based on conventional ferromagnetism

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b) Magnetic Catalysis (MC)

(V. Gusynin, V. Miransky, S. Sharapov, I. Shovkovy, PRB 74 (2006) 195429; I. Herbut, PRL 97 (2006) 146401; J. Fuchs, P. Lederer, PRL 98 (2007) 016803)

Order parameters: Dirac mass terms

$\langle \bar{\psi} P_s \psi \rangle \equiv \langle \psi^+ \gamma^0 P_s \psi \rangle \neq 0 \Longrightarrow$

$SU(2)_s \to U(1)_s$

$\tilde{\Delta}_s$ are corresponding Dirac masses (gaps).

Triplet order parameters.

Scenario is based on relativistic 2+1 dimensional dynamics in magnetic field

V. Gusynin, V. Miransky, I. Shovkovy, PRL 73 (1994) 3499

In graphite:

D. Khveshchenko, PRL 87 (2001) 206401;

E. Gorbar, V. Gusynin, V. Miransky, I. Shovkovy, PRB 66 (2002) 045108

Latest developments: QHF and MC sets of order parameters necessarily coexist, which implies that they have the same dynamical origin.

E. Gorbar, V. Gusynin, V. Miransky, arXiv: 0710.3527 [cond-mat]

E. Gorbar, V. Gusynin, V. Miransky, I. Shovkovy, (to appear)

General form of inverse quasiparticle propagator:

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One more surprise: Δ_S — $SU(2)_S$ singlet Dirac mass $\Rightarrow \langle \bar{\psi}\gamma^3\gamma^5 P_S \psi \rangle$. It is odd under time reversal

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Dispersion Relations

$$iG_s^{-1}(x,x') = \left[\left(i\hbar\partial_t + \mu_s + \tilde{\mu}_s\gamma^3\gamma^5\right)\gamma^0 - v_F\vec{\Pi}\vec{\gamma} + \right.$$
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LLL

$$\omega_s^{(0)} = -\mu_s + G\left[\tilde{\mu}_s \operatorname{sign}(eB_z) + \tilde{\Delta}_s\right] + \Delta_s \operatorname{sign}(eB_z)$$

Higher LLs ($|n|\geq 1$)

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LLL

$$\omega_s^{(\sigma)} = -\mu_s + \sigma\left[\tilde{\mu}_s \operatorname{sign}(eB_z) + \tilde{\Delta}_s\right] + \Delta_s \operatorname{sign}(eB_z)$$

Higher LLs ($|n|\geq 1$)

$$\omega_{ns}^{(\sigma)} = -\mu_s + \sigma\tilde{\mu}_s + \operatorname{sign}(n)\sqrt{2\hbar|neB_z|v_F^2/c} + \left(\tilde{\Delta}_s + \sigma\Delta_s\right)^2$$

Conclusion

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4. Potential consequences for (pseudo-)NG excitations.
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