Title: The Cosmic String Inverse Problem

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Abstract: TBA

The Cosmic String Inverse Problem

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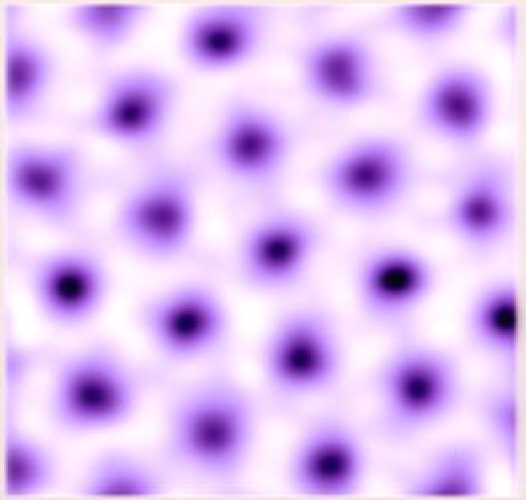
JP & Jorge Rocha, hep-ph/0606205
JP & Jorge Rocha, gr-qc/0702055
JP, arXiv:0707.0888
Florian Dubath, JP & Jorge Rocha, arXiv:0711.0994
JP, arXiv:0803.0557

PI, April 10, 2008

One-dimensional structures are common in physics (e.g. flux tubes in superconductor, seen end-on):



Under the right circum-stances, these can form in the early universe, and expand with it:

There are many potential cosmic strings from string compactifications:

- The fundamental string themselves
- D-strings
- Higher-dimensional D-branes, with all but one direction wrapped.
- Solitonic strings and branes in ten dimensions
- Solitons involving compactification moduli
- Magnetic flux tubes (classical solitons) in the effective 4-d theory: the classic cosmic strings.
- Electric flux tubes in the 4-d theory.

A network of any of these might form in an appropriate phase transition in the early universe, and then expand with the universe.

- What are the current bounds, and prospects for improvement?

- To what extent can we distinguish different kinds of cosmic string?

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The cosmic string inverse problem:

Observations → Microscopic models

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• To what extent can we distinguish different kinds of cosmic string?

The cosmic string inverse problem:

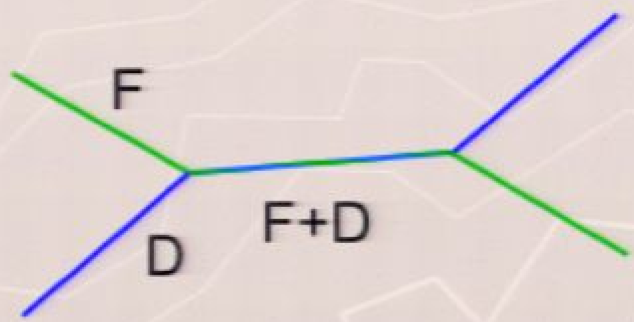
There is an intermediate step:

Observations → Macroscopic parameters → Microscopic models

Macroscopic parameters:

- Tension μ
- Reconnection probability P:

- Light degrees of freedom: just the oscillations in 3+1, or additional bosonic or fermionic modes?
- Long-range interactions: gravitational only, or axionic or gauge as well?
- One kind of string, or many?
- Multistring junctions?

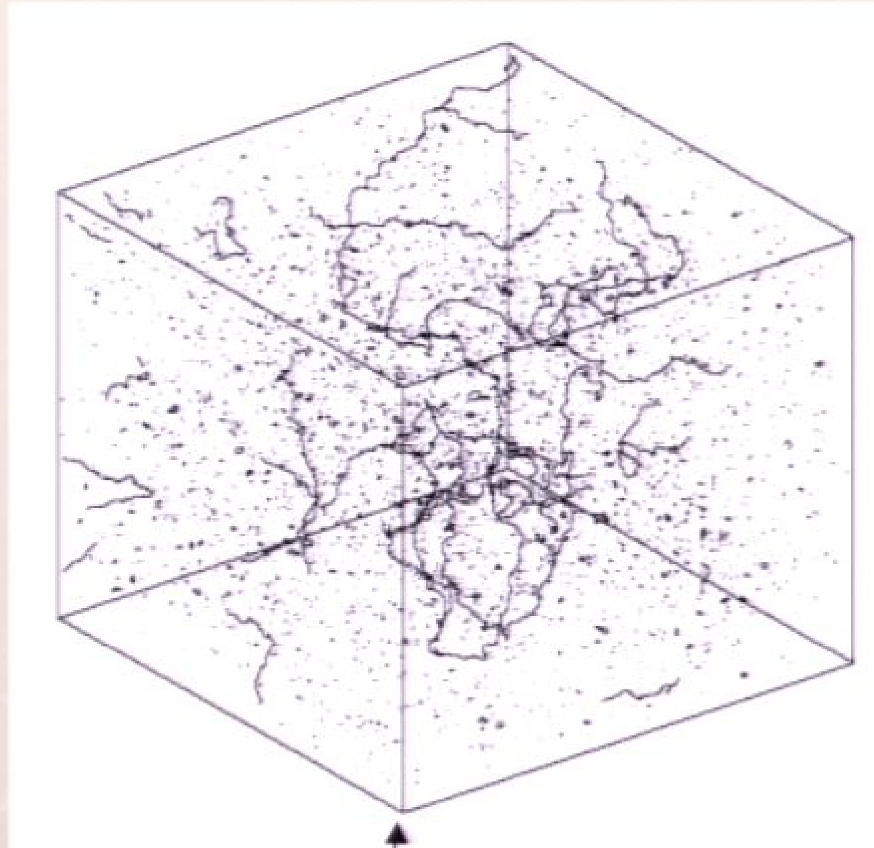


Vanilla Cosmic Strings:

- $P = 1$
- No extra light degrees of freedom
- No long-range interactions besides gravity
- One kind of string
- No junctions

Even for these, there are major uncertainties.

A simulation of vanilla strings (radiation era, box size ~ *.5t*).

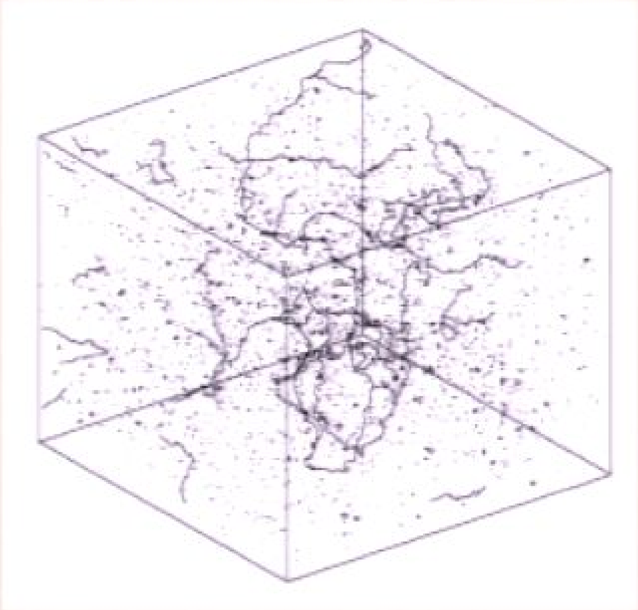


Simple arguments suggest that *t* ~ Hubble length ~ horizon length is the only relevant scale.

If so, simulations (Albrecht & Turok, Bennett & Bouchet, Allen & Shellard, ~ 1989) would have readily given a quantitative understanding.

However, one sees kinks and loops on shorter scales (BB). Limitations: UV cutoff, expansion time. Analytic approachs limited by nonlinearities, fairly crude.

Estimates of the sizes at which loops are produced range over more than *fifty* orders of magnitude, in a completely well-posed, classical problem.



Since the problem is a large ratio of scales, shouldn't some approach like the RG work?

Not exactly like the RG: since the comoving scale increases more slowly than t, structure flows from long distance to short.

However, with the aid of recent simulations, we have perhaps understood what the relevant scales are, and why.

Outline:

- Review of network evolution*
- Signatures of vanilla strings*
- A model of short distance structure
- The current picture

*Good references:
Vilenkin & Shellard, *Cosmic Strings and Other Topological Defects*; Hindmarsh & Kibble, hep-ph/9411342.

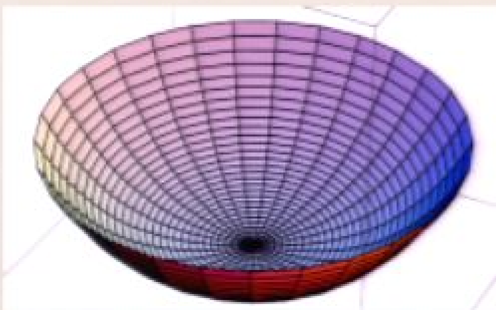
I. Review of Vanilla Network Evolution

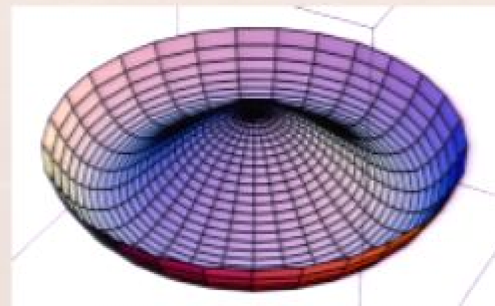
Processes:

1. Formation of initial network in a phase transition.
2. Stretching of the network by expansion of the universe.
3. Long string intercommutation.
4. Long string smoothing by gravitational radiation.
5. Loop formation by long string self-intercommutation.
6. Loop decay by gravitational radiation.

1. Network creation

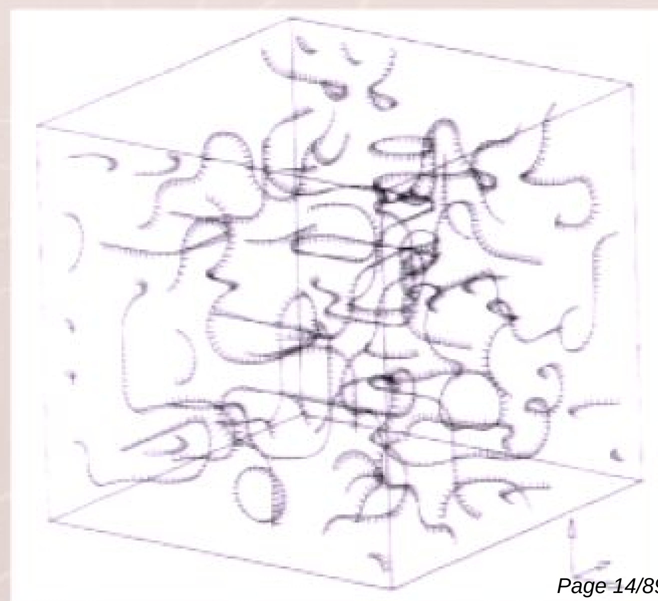
String solitons exist whenever a $U(1)$ is broken, and they are actually produced whenever a $U(1)$ *becomes* broken during the evolution of the universe (Kibble):





Phase is uncorrelated over distances > horizon. $O(50\%)$ of string is in infinite random walks.

(Dual story for other strings).



1.5 Stability

We must assume that the strings are essentially stable against breakage and axion domain wall confinement (this is model dependent).

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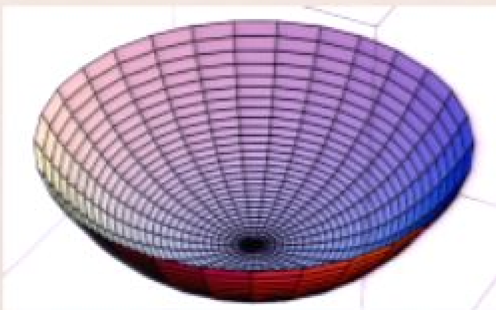

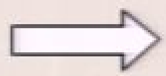

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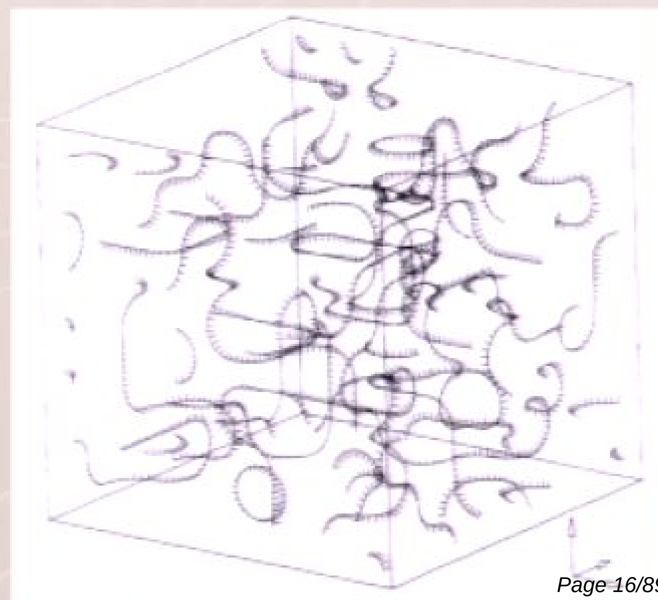

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2. Expansion

FRW metric: $ds^2 = -dt^2 + a(t)^2 d\mathbf{x} \cdot d\mathbf{x}$

String EOM: $\ddot{\mathbf{x}} + 2\,\frac{\dot{a}}{a}\,(1 - \dot{\mathbf{x}}^2)\,\dot{\mathbf{x}} = \frac{1}{\epsilon}\left(\frac{\mathbf{x}'}{\epsilon}\right)'$

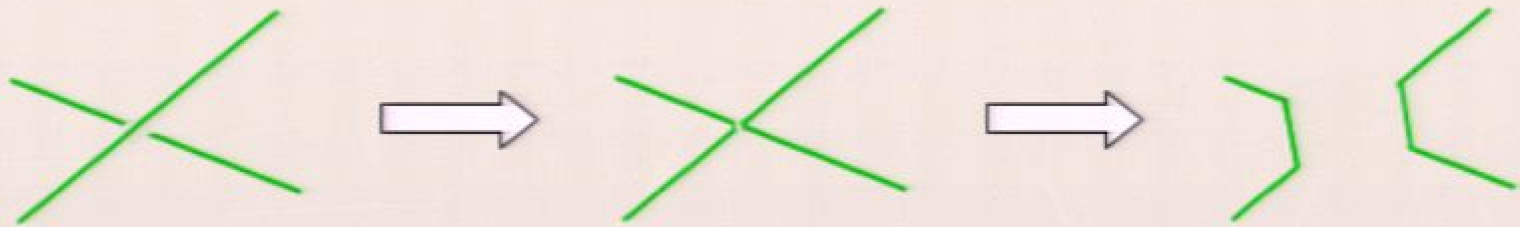
gauge $\dot{\mathbf{x}} \cdot \mathbf{x}' = 0$ $\quad \epsilon \equiv \left(\frac{\mathbf{x}'^2}{1 - \dot{\mathbf{x}}^2}\right)^{1/2}$

L/R form: define unit vectors: $\mathbf{p}_\pm \equiv \dot{\mathbf{x}} \pm \frac{1}{\epsilon}\mathbf{x}'$

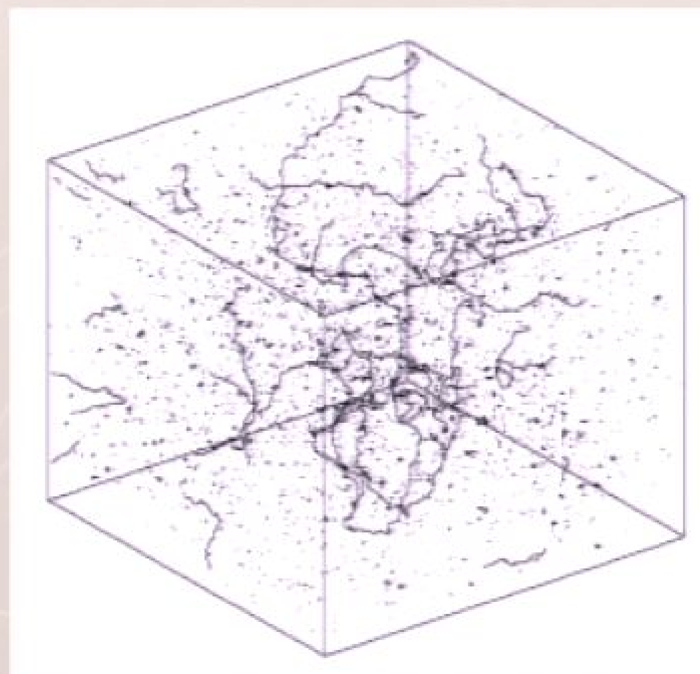
Then: $\dot{\mathbf{p}}_\pm \mp \frac{1}{\epsilon}\mathbf{p}'_\pm = -\frac{\dot{a}}{a}\left[\mathbf{p}_\mp - (\mathbf{p}_+ \cdot \mathbf{p}_-)\,\mathbf{p}_\pm\right]$

(Comoving expansion above horizon scale, oscillation and redshifting below horizon scale.

3. Long string intercommutation



Produces L- and R-moving kinks. Expansion of the universe straightens these slowly, but more enter the horizon (BB).



4. Long string gravitational radiation

This smooths the long strings at distances less than some scale l_G.

Simple estimate gives $l_G = \Gamma G\mu t$, with $\Gamma \sim 50$.

Subtle suppression when L- and R-moving wave-lengths are very different,

so in fact $l_G = \Gamma(G\mu)^{1+2\chi}$, with χ to be explained later (Siemens & Olum; … & Vilenkin; JP & Rocha).

5. Loop formation by long string self-intersection

6. Loop decay by gravitational radiation

Dimensionally, for a loop of length l, the rate of gravitational wave emission is

$$\dot{E} = \Gamma G \mu^2$$

A loop of initial length l_i (energy μl_i) decays in time

$$\tau = l_i / \Gamma G \mu$$

A loop of size $l_i = \Gamma G \mu t$ lives around a Hubble time

Review:

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(Simulations replace grav. rad. with a rule that removes loops after a while)

Scaling hypothesis:

All statistical properties of the network are constant when viewed on scale t (Kibble).

If only expansion were operating, the long string separation would grow as $a(t)$. With scaling, it grows more rapidly, as t, so the various processes must eliminate string at the maximum rate allowed by causality.

Simulations, models, indicate that the scaling solution is an attractor under broad conditions (m & r) (more string → more intercommutions → more kinks → more loops → less string). Washes out initial conditions.

Estimates of loop formation size

$0.1\,t$: original expectation, and some recent work (Vanchurin, Olum & Vilenkin)

$10^{-3}\,t$: other recent work (Martins & Shellard)

$\Gamma G\mu\,t$: still scales, but dependent on gravitational wave smoothing (Bennett & Bouchet)

$\Gamma(G\mu)^{1+2\chi}\,t$: corrected gravitational wave smoothing (Siemens, Olum & Vilenkin; JP & Rocha)

τ_{string}: the string thickness - a fixed scale, not $\propto t$ (Vincent, Hindmarsh & Sakellariadou)

II. Gravitational Signatures

Vanilla strings have only gravitational long-range interactions, so we look for gravitational signatures:

1. Dark matter.
2. Effect on CMB and galaxy formation.
3. Lensing.
4. Gravitational wave emission.

Key parameter: $G\mu$. This is the typical gravitational perturbation produced by string. In brane inflation models,

$$10^{-12} < G\mu < 10^{-6}$$

Normalized by $\delta T/T$. (Jones, Stoica, Tye)

1. Dark Matter

Scaling implies that the density of string is a constant times μ/t^2. This is the same time-dependence as the dominant (matter or radiation) energy density, so these are proportional, with a factor of $G\mu$.

Simulations:

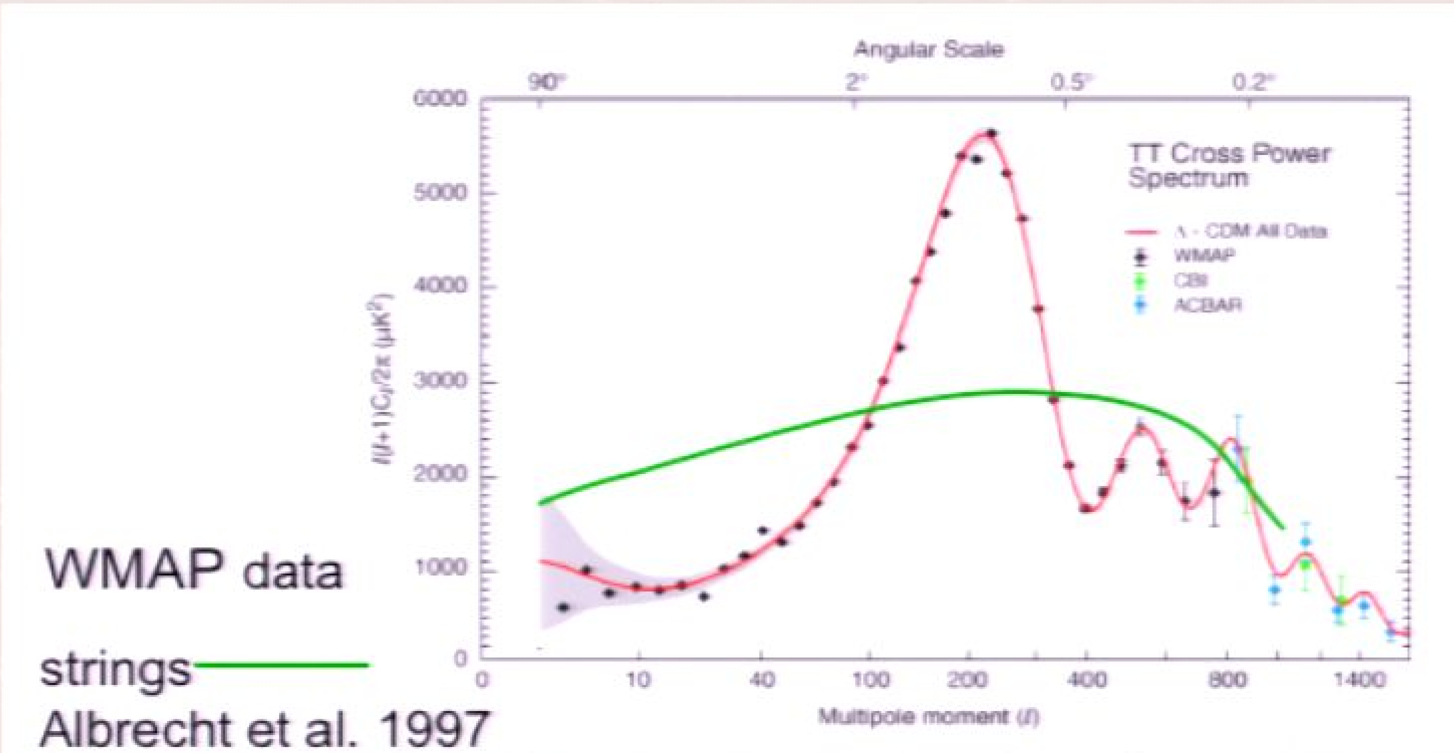
$$\rho_{\text{string}}/\rho_{\text{matter}} = 70\, G\mu$$

$$\rho_{\text{string}}/\rho_{\text{radiation}} = 400\, G\mu$$

Too small to be the dark matter.

2. Perturbations of CMB:

These come primarily from the long strings, which are fairly well understood.* Scaling implies a scale-invariant perturbations, which could have been the origin of structure, but the power spectrum is wrong:



Bound from power spectrum: $G\mu < 2 \times 10^{-7}$

(Pogosian, Wasserman & Wyman; Seljak & Slosar).

Bound from nongaussianity: $G\mu < 6 \times 10^{-7}$ (Jeong & Smoot).

Bound from Doppler distortion of black body:

$G\mu < 3.3 \times 10^{-7}$

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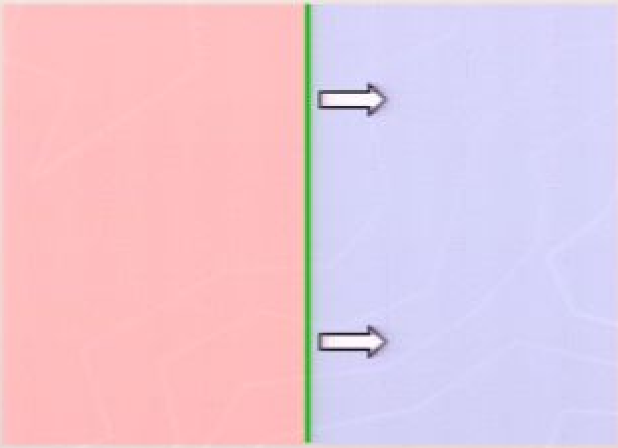

Improved future bounds from polarization, non-gaussianity.

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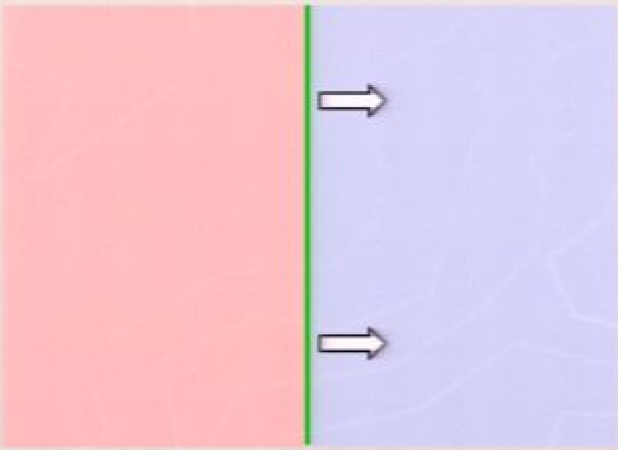

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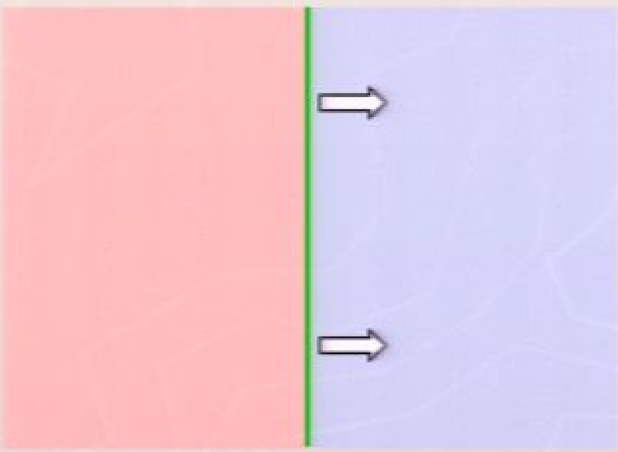

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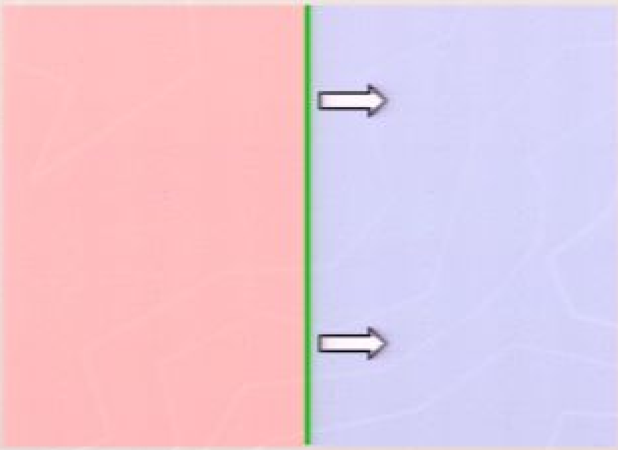

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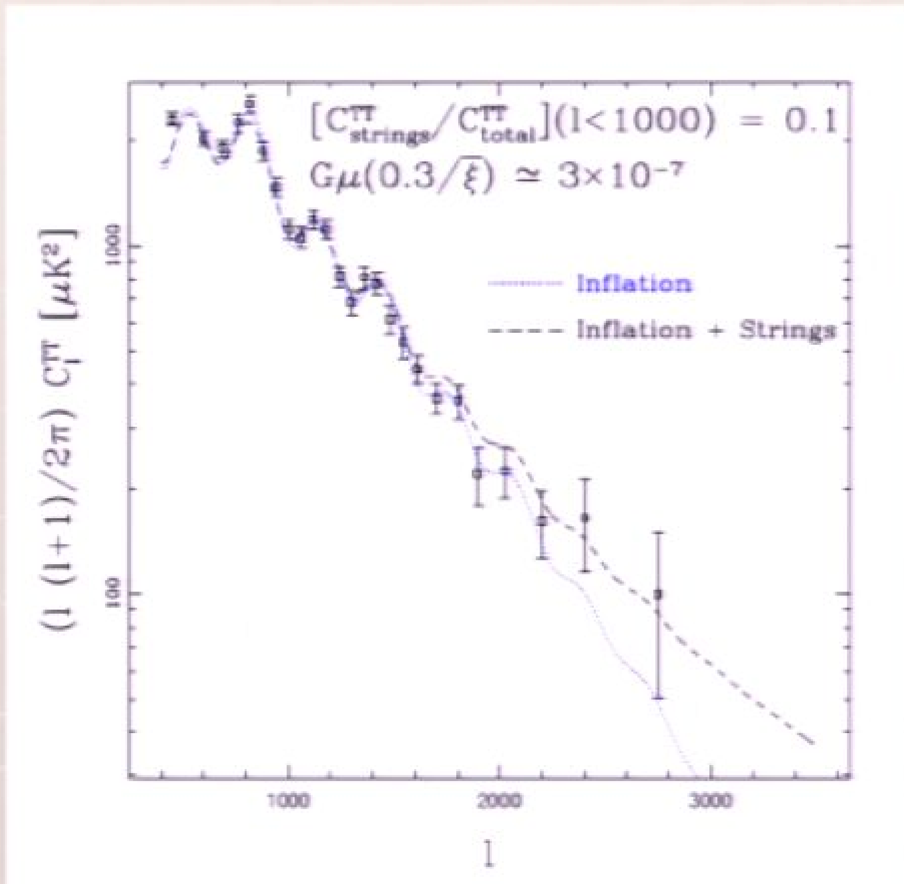
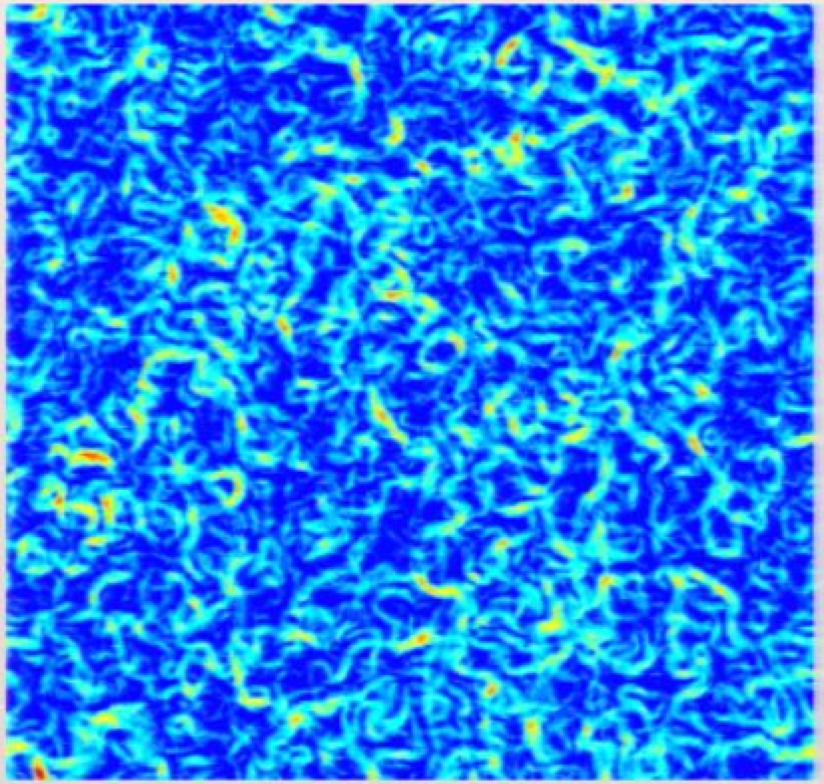
Measurement of power spectrum at large multipoles is particularly sensitive to strings (Pogosian, Tye, Wasserman, Wyman, 0804.0810):

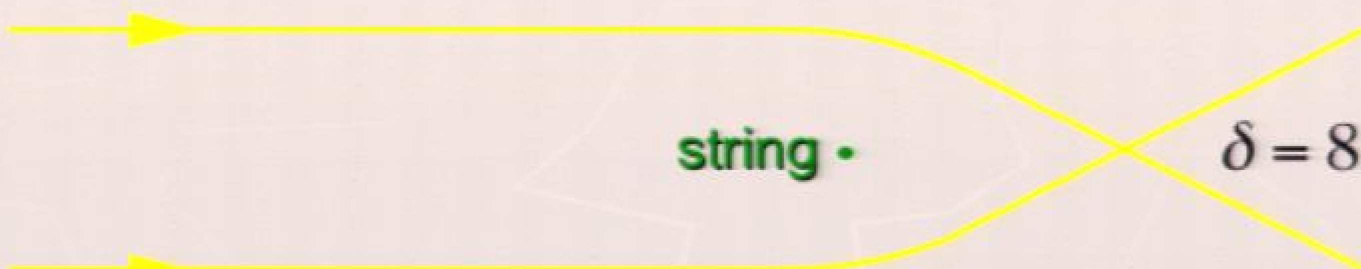


Small scale CMB surveys (e.g. ACT): easily down to $G\mu = 2 \times 10^{-7}$, better with statistics (Fraisse, Ringeval, Spergel and Bouchet, 0708.1162)

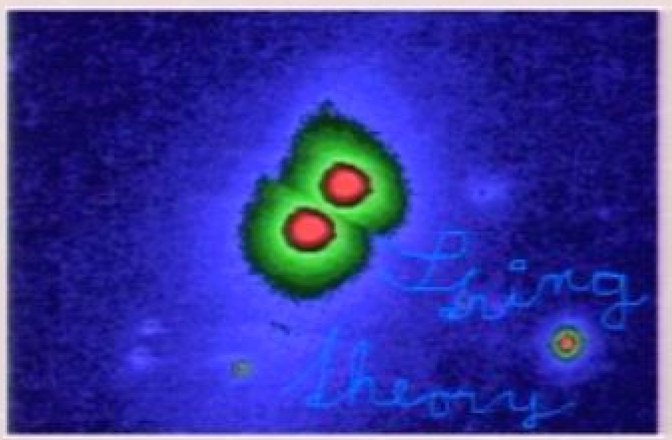


3. Lensing:

(By long strings or loops)



$$\delta = 8\pi G\mu$$



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A string with tension $G\mu = 2 \times 10^{-7}$ has a deficit angle 1 arc-sec; lens angle depends on geometry and velocity, can be a bit larger or smaller. Recent survey (Christiansen, Albin, James, Goldman, Maruyama, Smoot): $G\mu < 3 \times 10^{-7}$.

Only a tiny fraction of the sky is lensed, so one needs a large survey - radio might reach 10^{-9} (Mack, Wesley & King); also, HST, with a clever search strategy, might reach 10^{-9} (Gasparini, Marshall, Treu, Morganson, Dubath).

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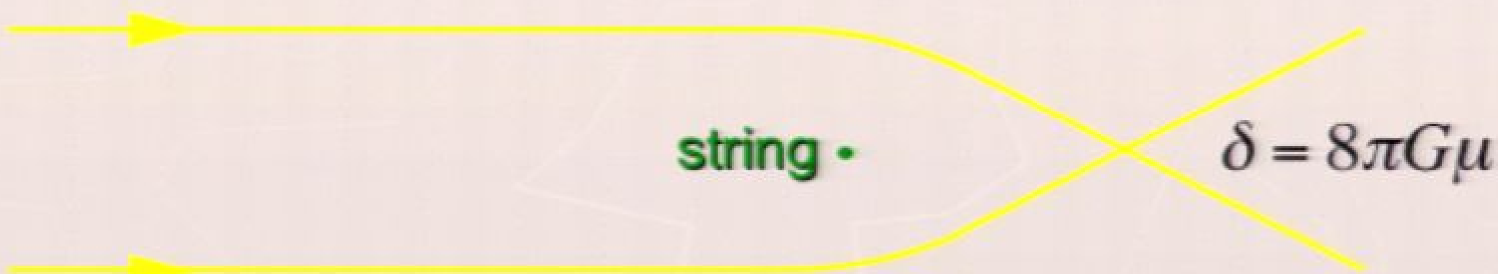

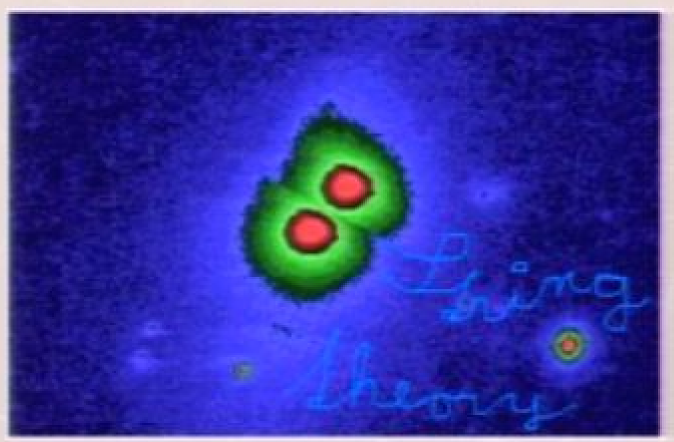

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4. Gravitational radiation

Primarily from loops (they have higher frequencies).

There is interesting radiation both from the low harmonics of the loop and the high harmonics; will consider the low harmonics first.

period = $l/2$ $\quad \nu = 2n/l$

Most of the energy goes into the low harmonics…

Follow the energy:

Long strings ← density known from simulations

↓ ← rate set by scaling

Loops ← red-shift like matter

↓ ← decay at $t_d - t_f = l/\Gamma G\mu$

Stochastic gravitational radiation

If $l/t_f \equiv \alpha > \Gamma G\mu$ then energy density is enhanced by $(\alpha/\Gamma G\mu)^{1/2}$ during the radiation era (relevant to LIGO, LISA, and to pulsars down to $G\mu \sim 10^{-10}$).

Pulsar bounds: the observed regularity of pulsar signals limits the extent to which the spacetime through which they pass can be fluctuating. Significant recent improvement (PPTA, Jenet, et al.):

$$\nu_0 \frac{d\Omega_{\rm GW}}{d\nu_0} < 4 \times 10^{-8}$$

(Will improve substantially.)

Energy balance gives:

$$\nu_0 \frac{d\Omega_{\rm GW}}{d\nu_0} = 0.0035\gamma^{-3/2}(\alpha G\mu)^{1/2} \; *$$

Bound: $G\mu < 1.3 \times 10^{-10}\alpha^{-1}\gamma^3$

γ = initial boost of loop, ~1 for large loop.

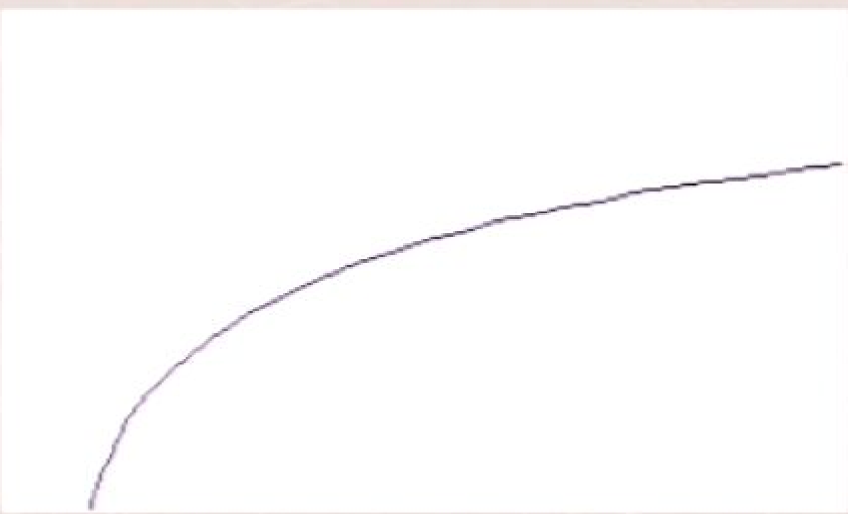
E.g. $\alpha = 0.1$, $\gamma = 1$ gives $G\mu < 1.3 \times 10^{-9}$, but much smaller α gives a weak bound.

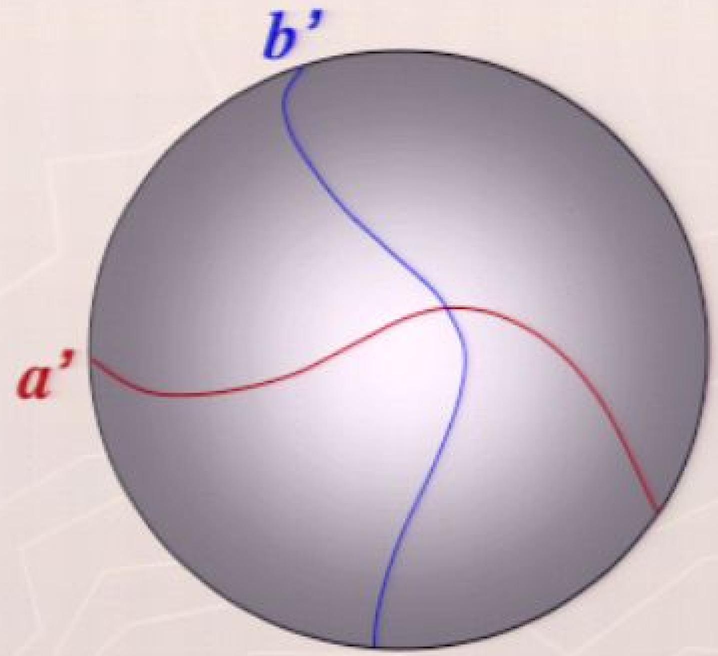
*factor of 0.25 due to vacuum energy, = 16 in $G\mu$

High harmonics: kinks give $\omega^{-5/3}$ spectrum, cusps give $\omega^{-4/3}$.

Cusp (Turok): in conformal gauge, $\boldsymbol{x}(u,v) = \boldsymbol{a}(u) + \boldsymbol{b}(v)$, with $\boldsymbol{a}'$ and $\boldsymbol{b}'$ unit vectors.

When these intersect the string has a cusp





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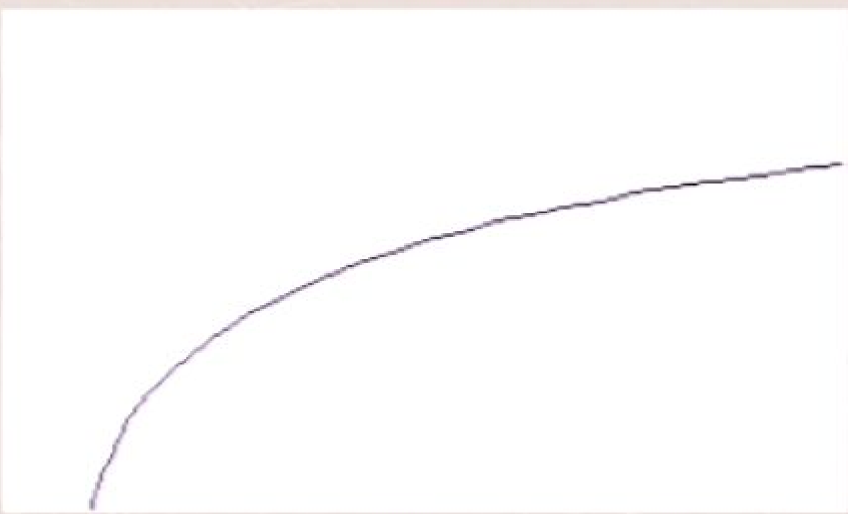

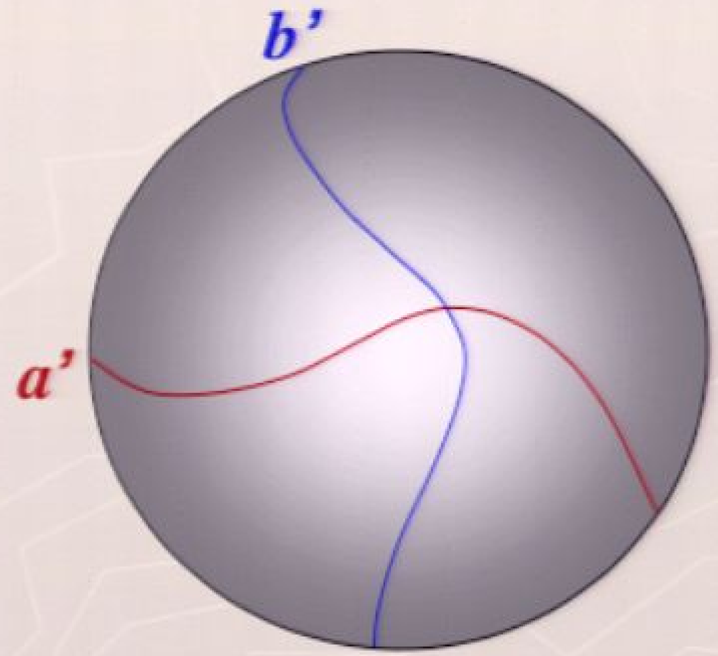

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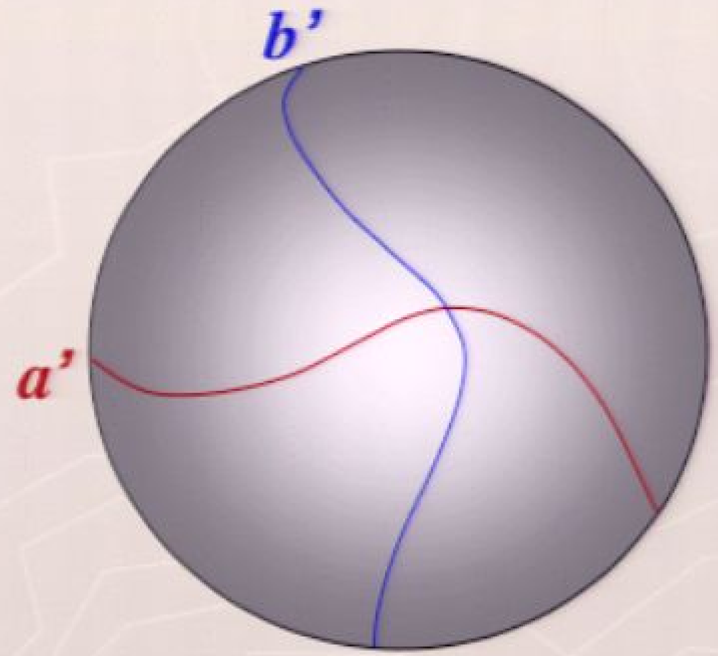

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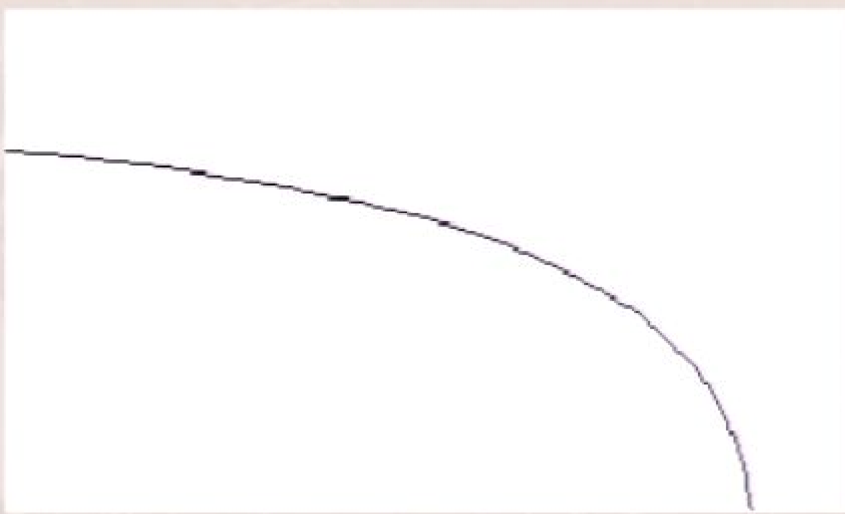

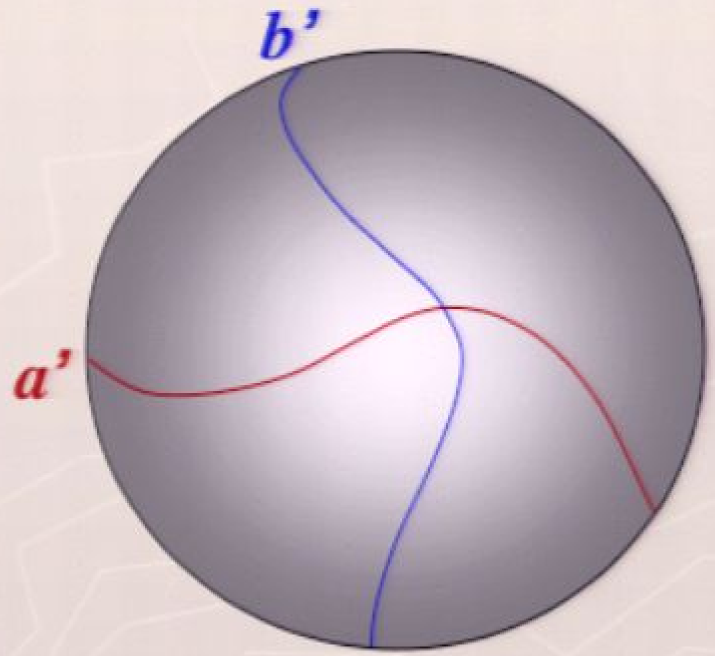

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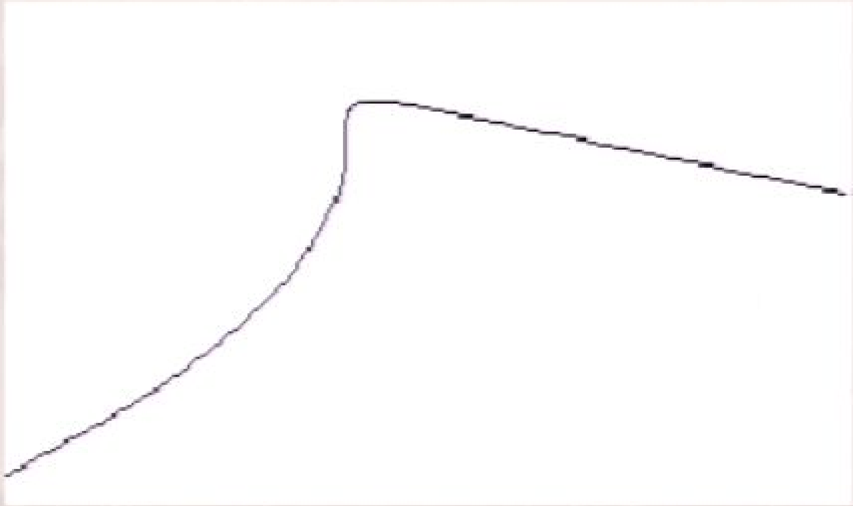

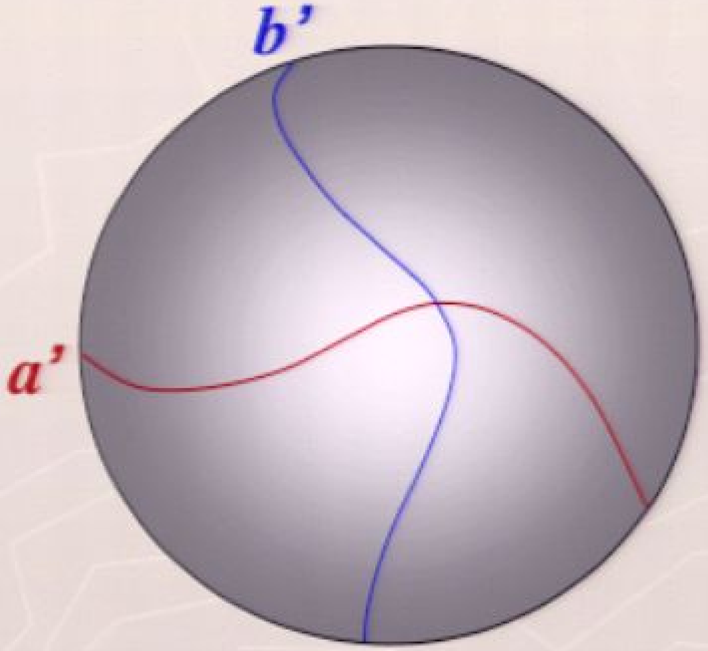

High harmonics: kinks give $\omega^{-5/3}$ spectrum, cusps give $\omega^{-4/3}$.

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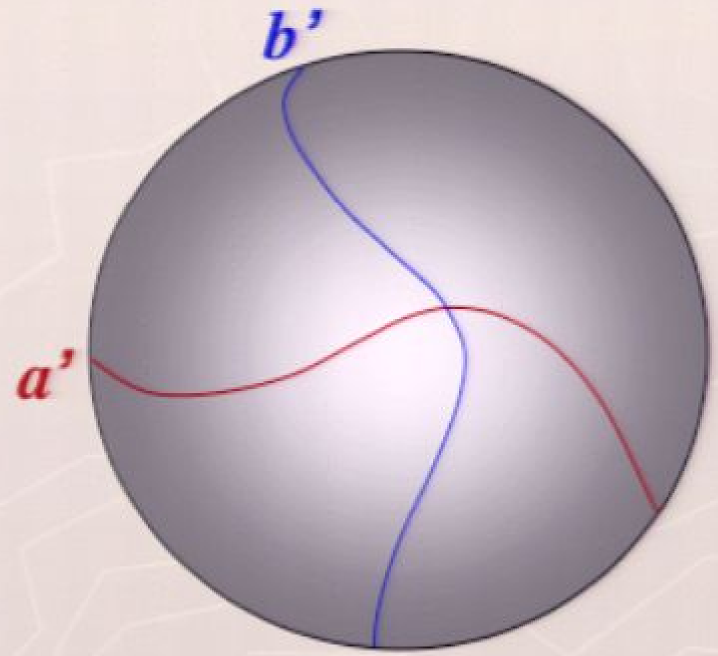

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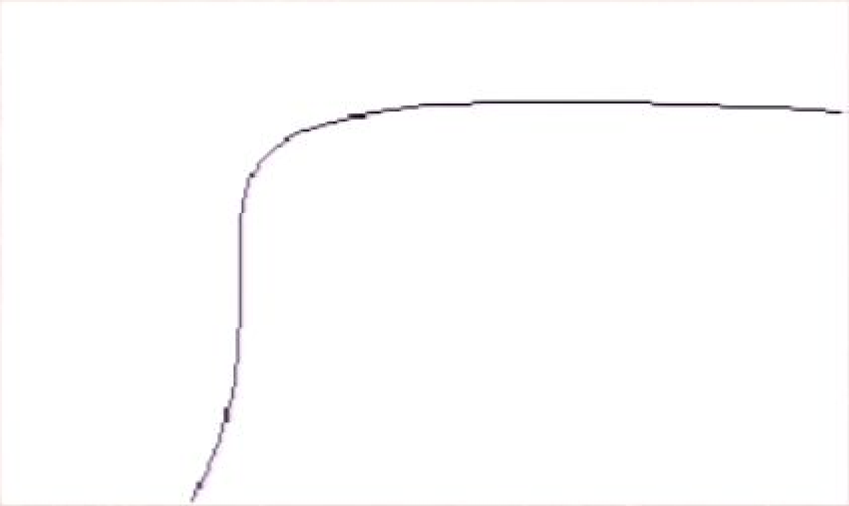

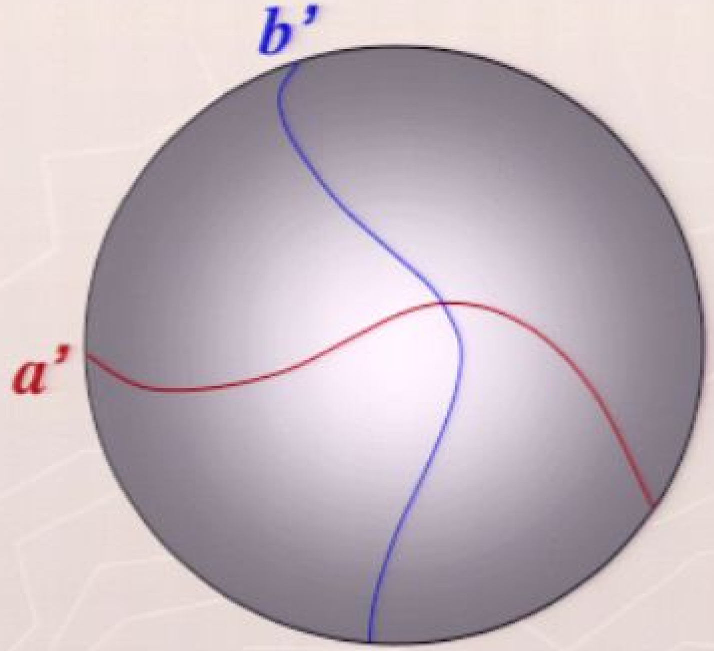

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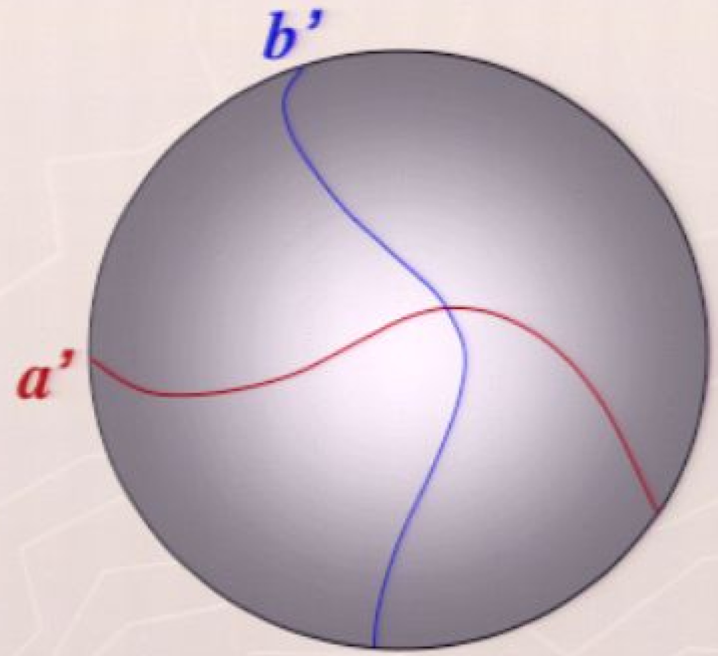

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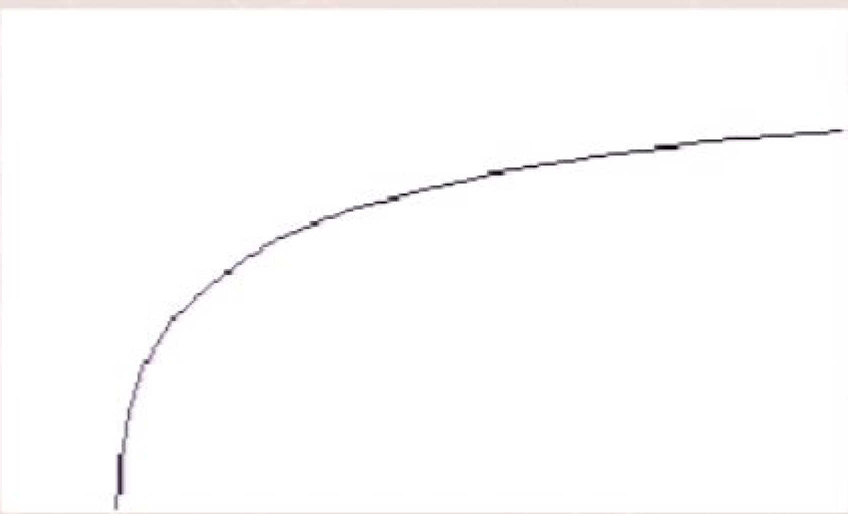

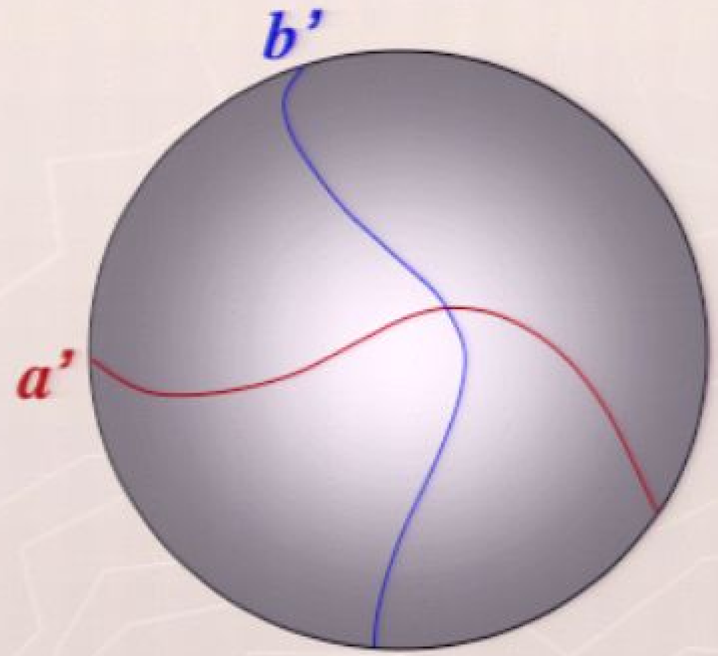

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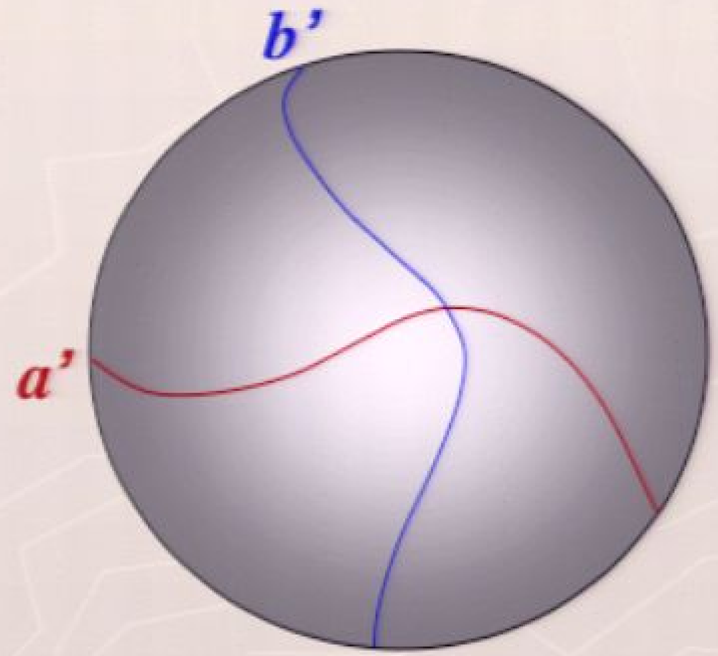

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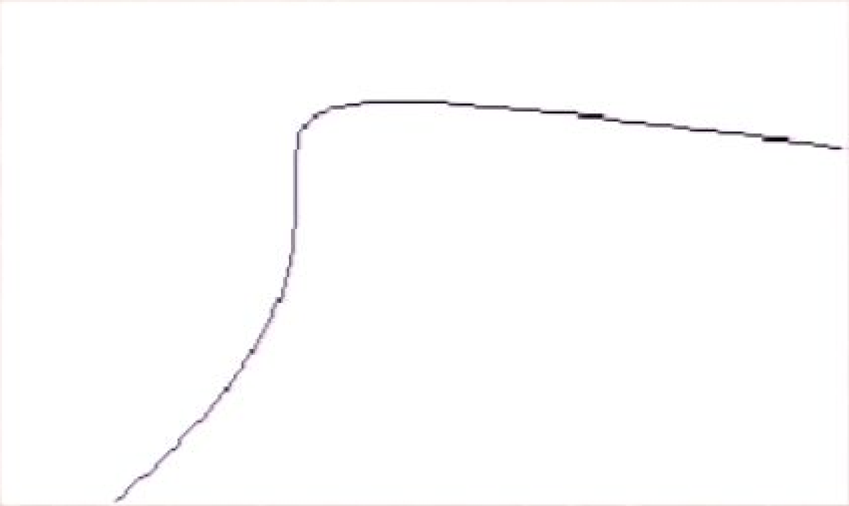

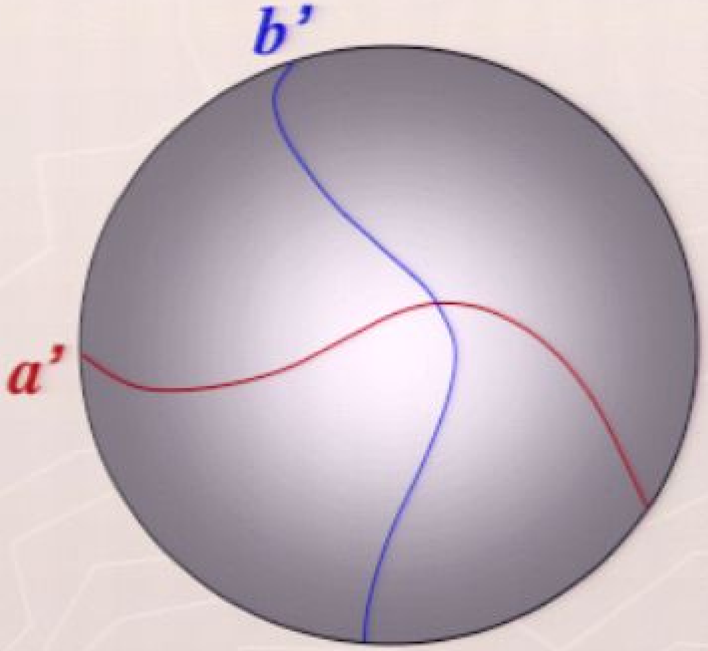

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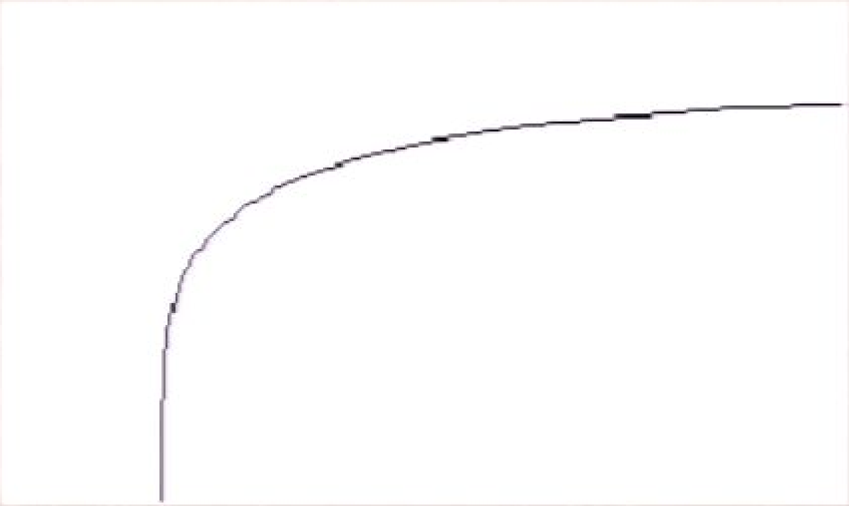

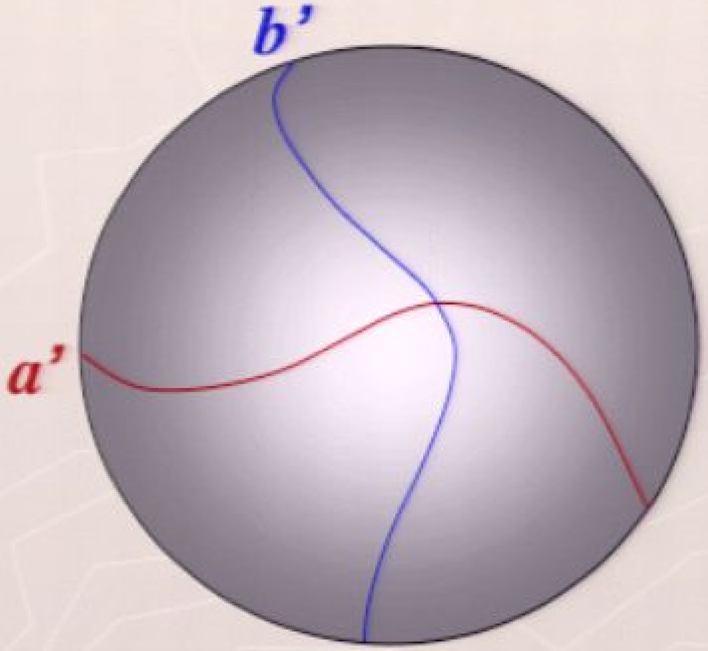

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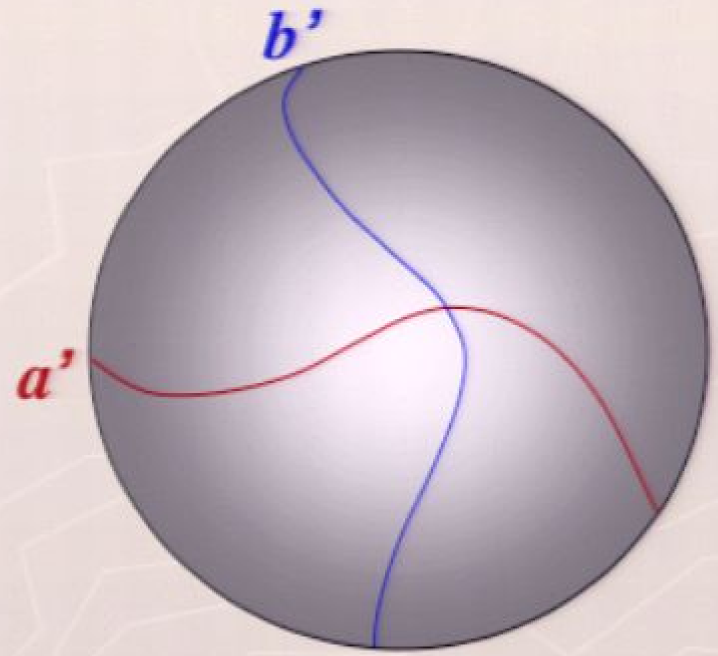

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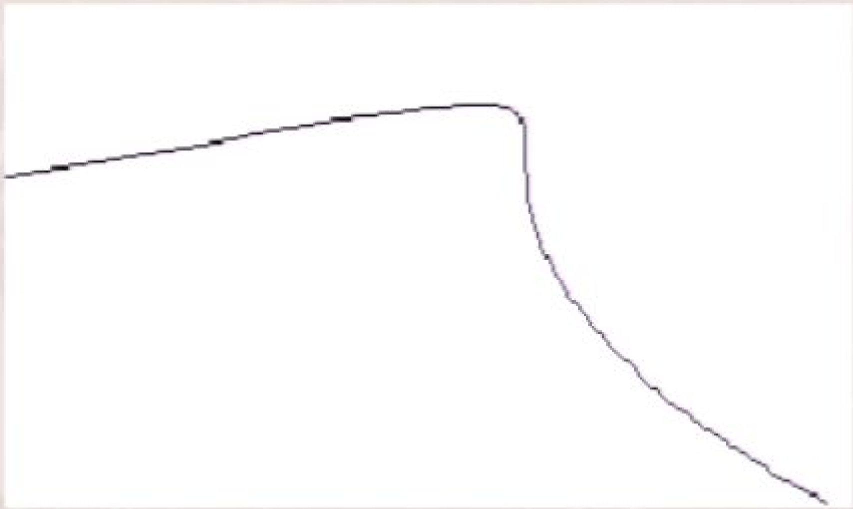

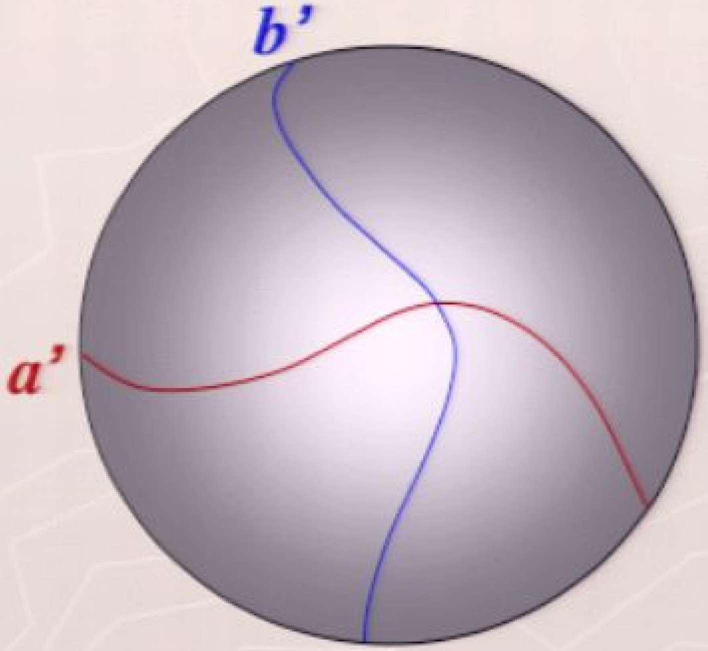

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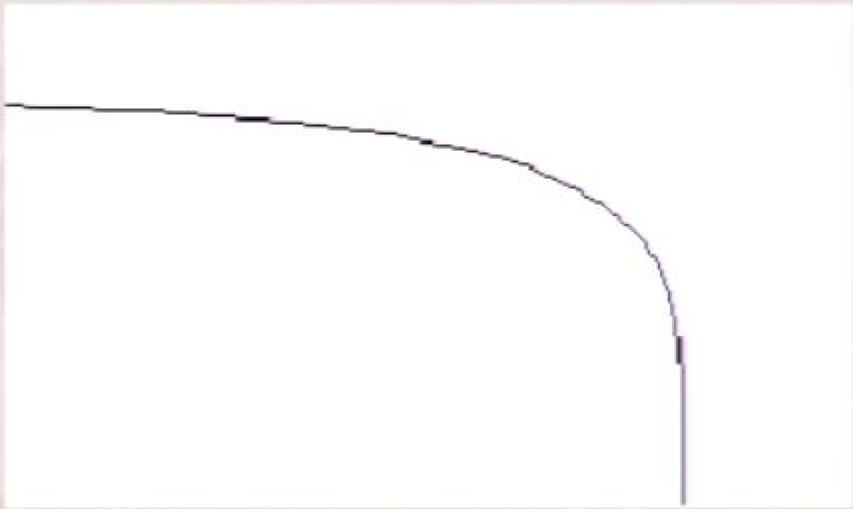
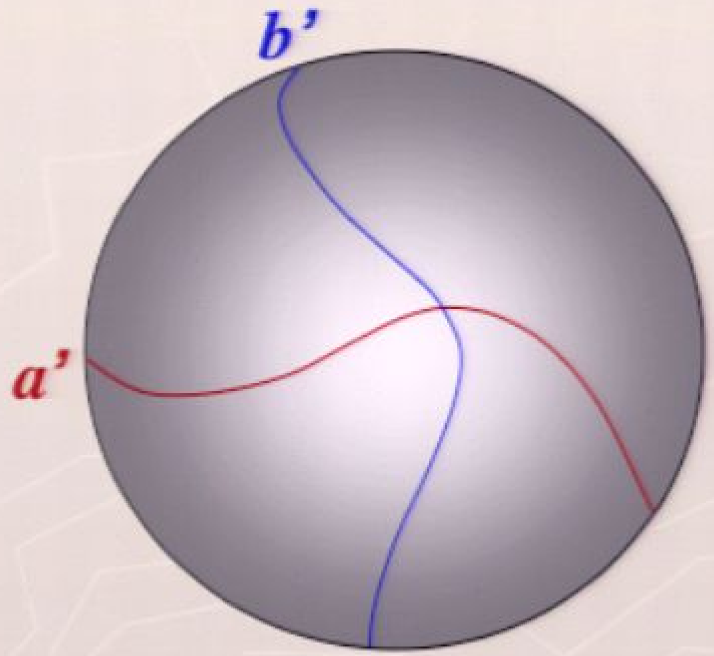
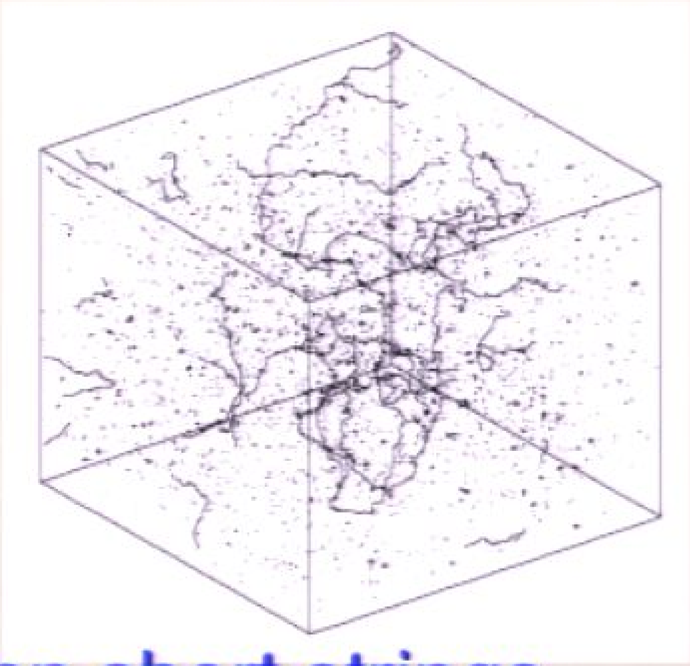


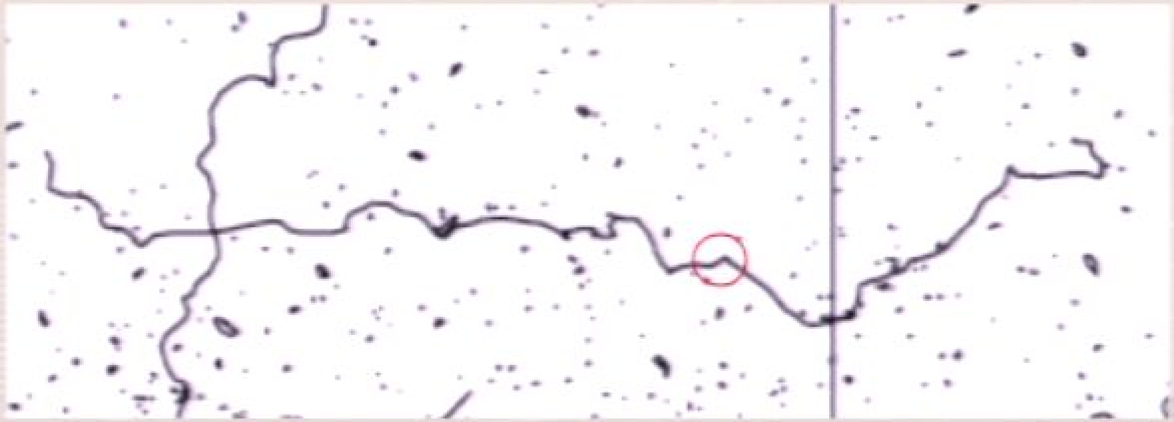


Initial calculations (Damour & Vilenkin) suggested that these might be visible at LIGO I, and likely at Advanced LIGO. More careful analysis (Siemens, Creighton, Maor, Majumder, Cannon, Read) suggests that we may have to wait until LISA. Large α helps, but probably not enough.

III. A Model of Short Distance Structure



1. Small scale structure on short strings



Strategy: consider the evolution of a small (right- or left-moving) segment on a long string.

Evolution of a short segment, length l.
Possible effects:

1. Evolution via Nambu-FRW equation

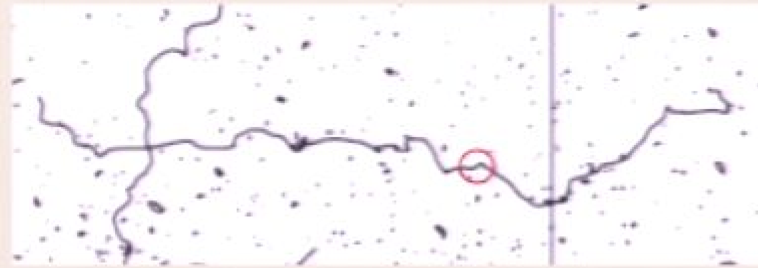
2. Long-string intercommutation

3. Incorporation in a larger loop

4. Emission of a loop of size l or smaller?

5. Gravitational radiation.

Evolution of a short segment, length l. Possible effects:



1. Evolution via Nambu-FRW equation
2. Long-string intercommutation

   very small probability, $\propto l$
3. Incorporation in a larger loop

   controlled by longer-scale configuration, will not change mean ensemble at length l^*
4. Emission of a loop of size l or smaller

   ignore? not self-consistent, but again controlled by longer-scale physics
5. Gravitational radiation

   ignore until we get to small scales

Nambu-FRW equations

$$\dot{\mathbf{p}}_\pm \mp \frac{1}{\epsilon}\mathbf{p}'_\pm = -\frac{\dot{a}}{a}\left[\mathbf{p}_\mp - (\mathbf{p}_+ \cdot \mathbf{p}_-)\,\mathbf{p}_\pm\right]$$

Separate segment into mean and (small) fluctuation:

$$\mathbf{p}_\pm(\tau,\sigma) = \mathbf{P}_\pm(\tau) + \mathbf{w}_\pm(\tau,\sigma) - \frac{1}{2}\mathbf{P}_\pm(\tau) w_\pm^2(\tau,\sigma) + \ldots$$

where $P_\pm^2 = 1 \text{ and } \mathbf{P}_\pm \cdot \mathbf{w}_\pm = 0$

Then $\dot{\mathbf{w}}_+ - \frac{1}{\epsilon}\mathbf{w}'_+ = -(\mathbf{w}_+ \cdot \dot{\mathbf{P}}_+)\,\mathbf{P}_+ + \frac{\dot{a}}{a}(\mathbf{P}_+ \cdot \mathbf{P}_-)\,\mathbf{w}_+$

just precession

over Hubble times, encounter many opposite-moving segments, so average.

$$P_+ \cdot P_- = 2\overline{v}^2 - 1 \qquad w_{+,-} \propto a^{2\overline{v}^2 - 1}$$

$w_{+,-} \propto a^{2\bar{v}^2 - 1}$

In flat spacetime, virial theorem gives $\bar{v}^2 = 1/2$, but redshifting reduces this to 0.41 (radiation era) and 0.35 (matter era), from simulations. For $a = t^r$,

$$\langle [\mathbf{w}_+(\sigma,\tau) - \mathbf{w}_+(\sigma',\tau)]^2 \rangle = t^{-2r(1-2\bar{v}^2)} f(\sigma - \sigma')$$

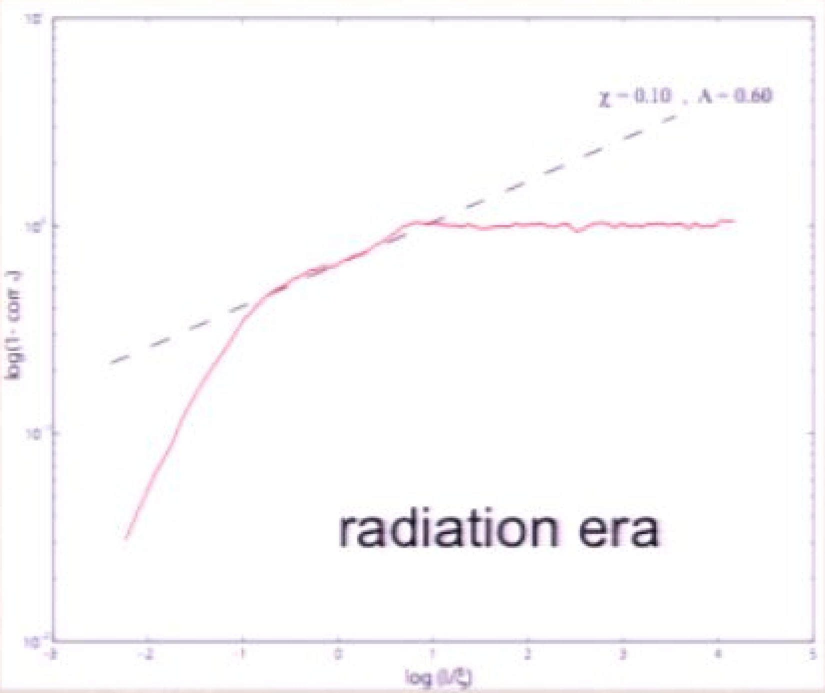
Initial condition when segment approaches horizon scale, gives

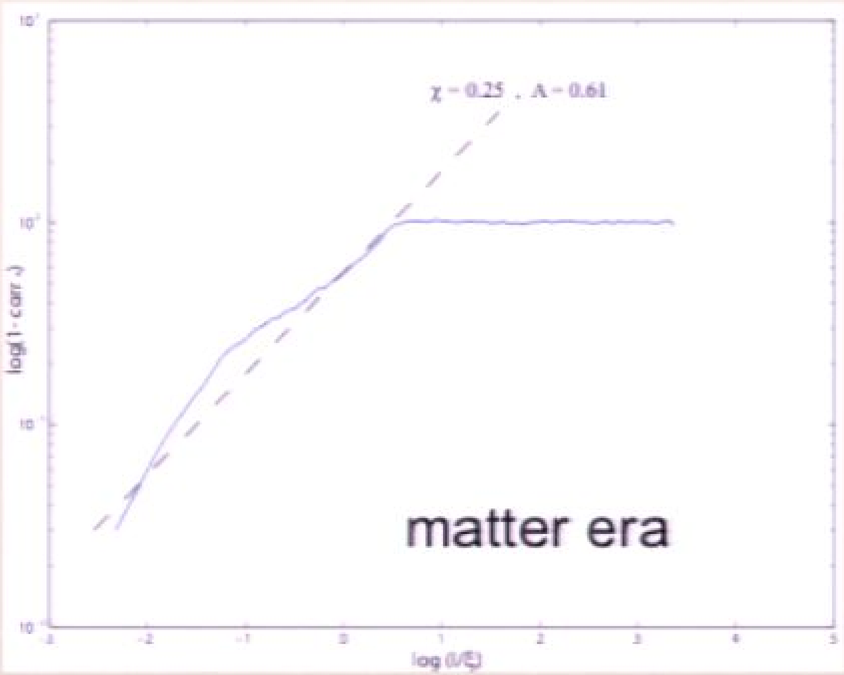
$$f(\sigma - \sigma') = 2A|\sigma - \sigma'|^{2\chi} \,, \quad \chi = \frac{r(1-2\bar{v}^2)}{1 - r(1-2\bar{v}^2)}$$

$$\langle [\mathbf{w}_+(\sigma,\tau) - \mathbf{w}_+(\sigma',\tau)]^2 \rangle = 2A(l/t)^{2\chi}$$

$$\chi_{\mathrm{m}} = 0.25 \text{ and } \chi_{\mathrm{r}} = 0.10$$

Compare with simulations (Martins & Shellard):





Random walk at long distance. Discrepancy at short distance - but expansion factor is only 3.

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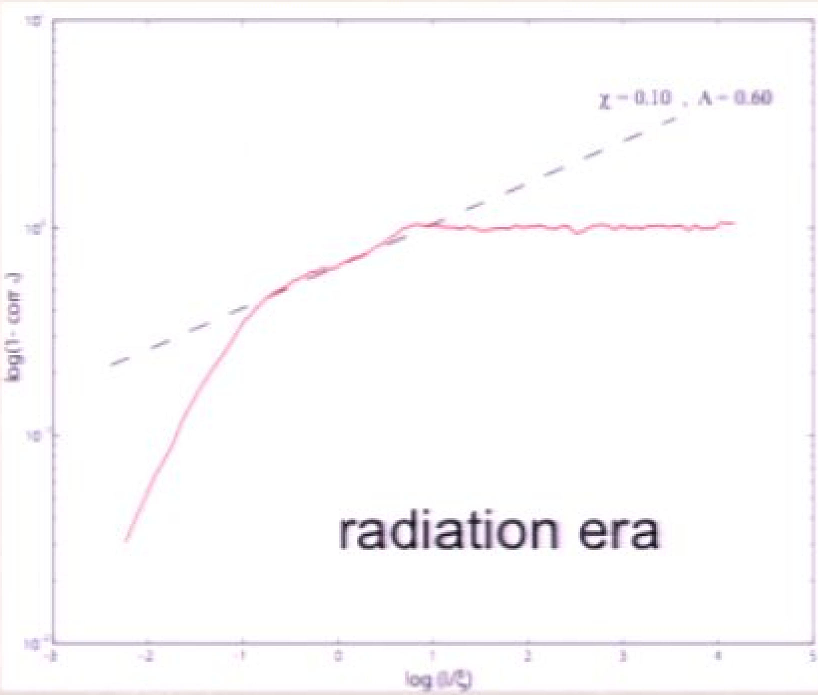

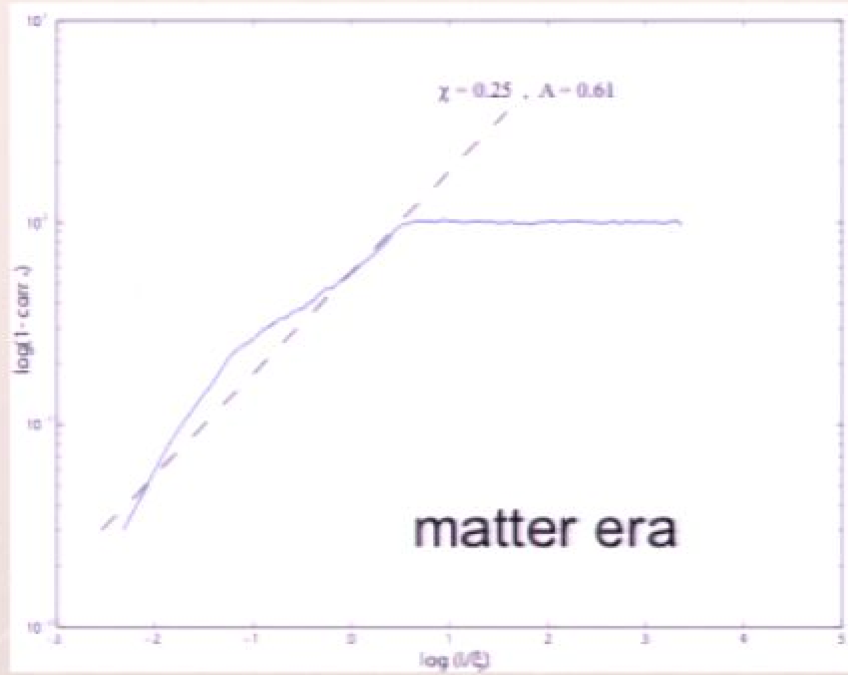

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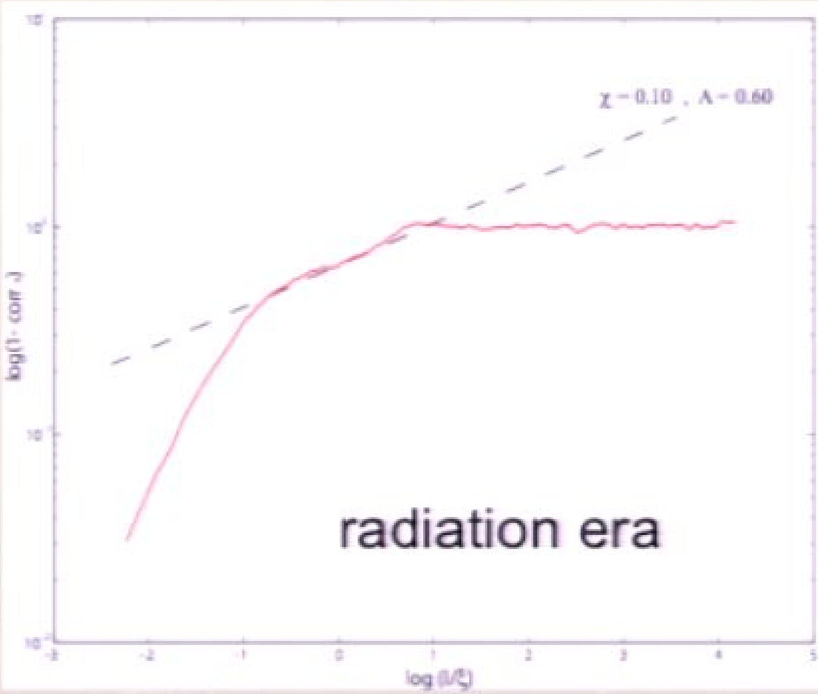
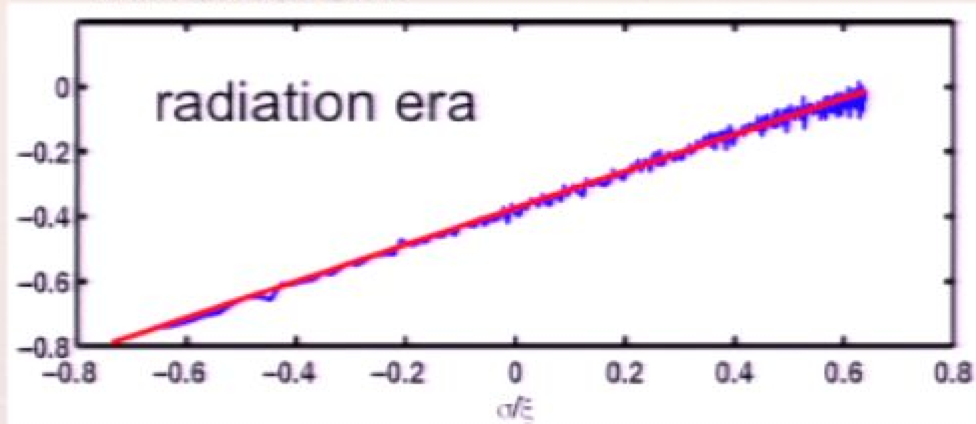
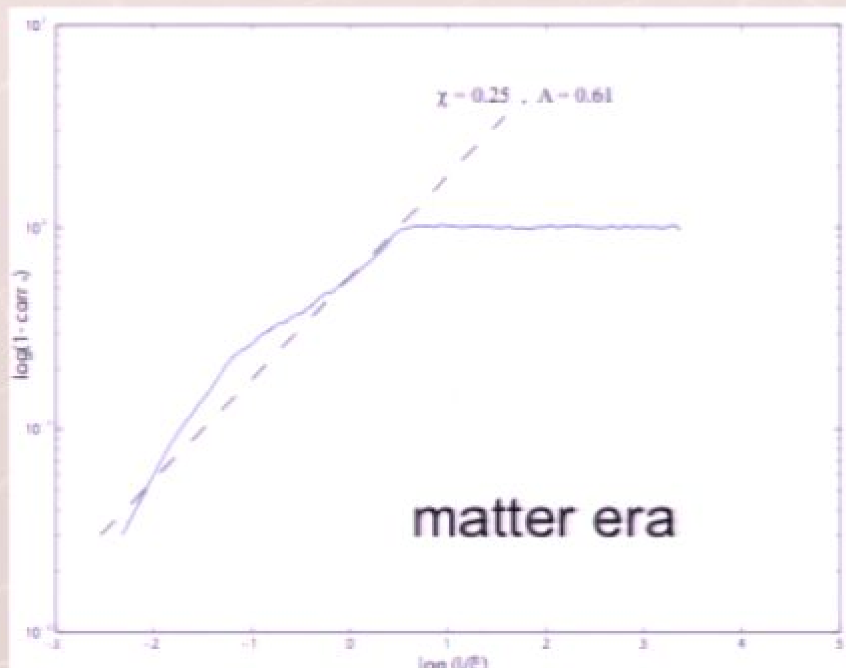


Hindmarsh:





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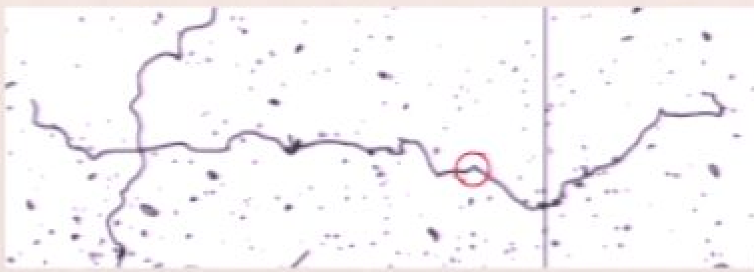
Lensing: fractal dimension is $1 + O([l/t]^{2\chi})$.



Gives 1% difference between images.

2. Loop formation

Loops form whenever string self-intersects. This occurs when $\Delta \mathbf{x} = \int \mathbf{x}' = 0$ on some segment, i.e.

$$\mathbf{L}_+(u,l) = \mathbf{L}_-(v,l)$$

$$\mathbf{L}_+(u,l) = \int_u^{u+l} du\, \mathbf{p}_+(u) \ , \quad \mathbf{L}_-(v,l) = \int_v^{v+l} dv\, \mathbf{p}_-(v)$$

Rate per unit u, v, l:

$$\langle \det J\, \delta^3(\mathbf{L}_+(u,l) - \mathbf{L}_-(v,l)) \rangle \ , \quad J = \frac{\partial^3(\mathbf{L}_+(u,l) - \mathbf{L}_-(v,l))}{\partial u\, \partial v\, \partial l}$$

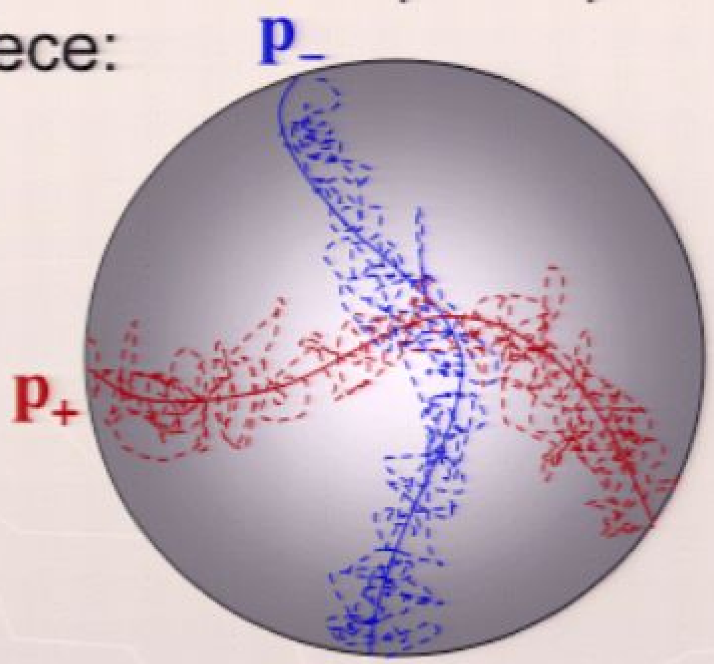
Components of $L_+ - L_-$ are of order $l, l, l^{1+2\chi}$.
Columns of J are of order $l^\chi, l^\chi, l^{2\chi}$.
Rate ~ $l^{-3+2\chi}$.

Rate of loop emission ~ $l^{-3+2\chi}$.
Rate of *string* emission ~ $l^{-2+2\chi}$.

Rate per world-sheet area = $\int dl\, l^{-2+2\chi}$: this diverges at the lower end for $\chi < 0.5$, even though the string is becoming smoother there.

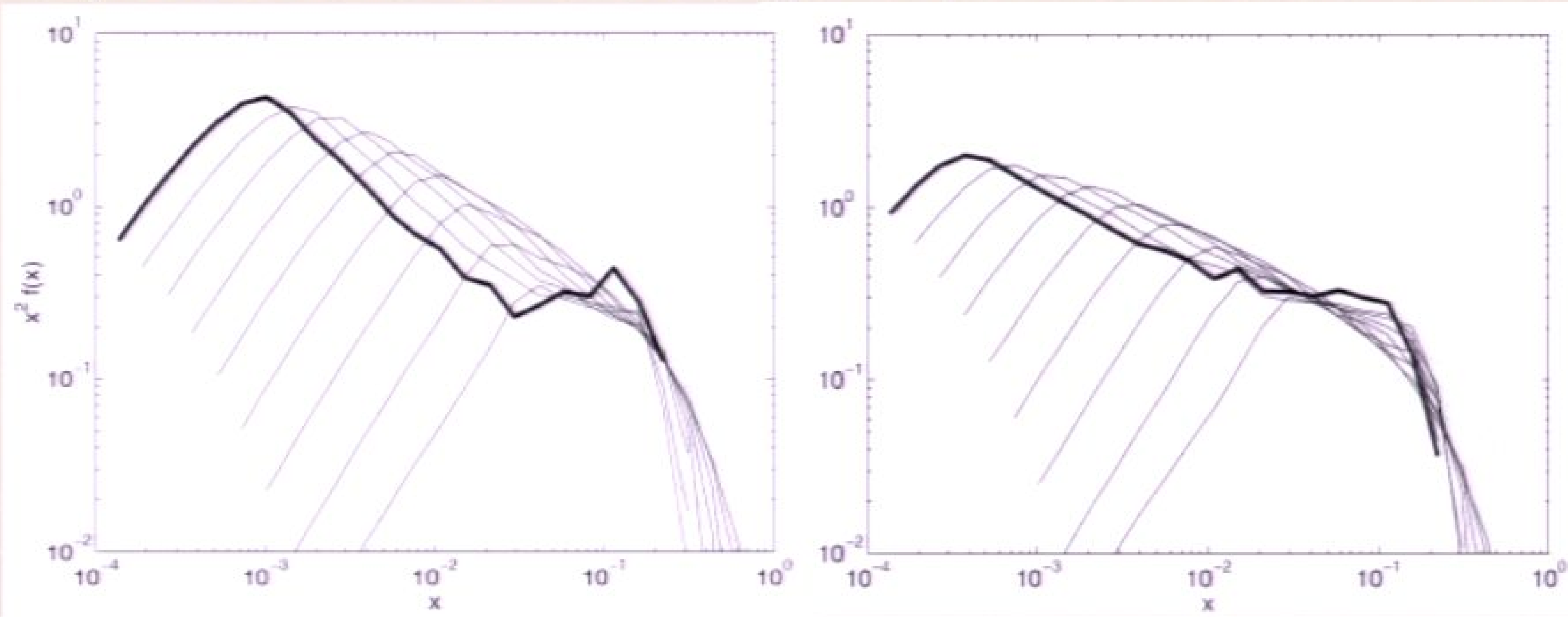
Total string conservation saturates at $l \sim 0.1t$, but rapid loop formation occurs internally to the loops - this suggests a complicated fragmentation process.

Resolving the divergence: separate the motion into a long-distance `classical' piece plus short-distance fluctating piece:



Loops form near the cusps of the long-distance piece. *All sizes form at the same time.* Get loop production function $l^{-2+2\chi}$, but with cutoff at gravitational radiation scale, and reduced normalization.

Recent simulations (Vanchurin, Olum, Vilenkin) use volume-expansion trick to reach larger expansion factors. Result:



radiation era

Two peaks, one near the horizon and one near the UV cutoff. VOV interpret the latter as a transient, but this is the one we found.

What about the large loops?

Scorecard on loop formation size

10-20% $0.1\,t$: original expectation, and some recent work (Vanchurin, Olum & Vilenkin)

$10^{-3}\,t$: other recent work (Martins & Shellard)

$\Gamma G\mu\,t$: still scales, but dependent on gravitational wave smoothing (Bennett & Bouchet)

$\Gamma(G\mu)^{1+2\chi}\,t$: corrected gravitational wave smoothing (Siemens, Olum & Vilenkin; JP & Rocha)

80-90%

τ_{string}: the string thickness - a fixed scale, not $\propto t$ (Vincent, Hindmarsh & Sakellariadou)

Two-peak distribution - effect on bounds:

	Low harmonics	High harmonics
Large loops:	Current (pulsar): 2×10^{-7} PPTA: 10^{-9} Advanced LIGO: 10^{-10} SKA, LISA: 10^{-11}	Advanced LIGO: ?? LISA: 10^{-13}
Small loops:		Advanced LIGO: ?? LISA: 10^{-10}

The inverse problem:

Observation of low harmonics of large loops probably allows measurement of $G\mu$ only (through absolute normalization) - *if the networks are understood perfectly.*

Slightly less vanilla strings: $P \neq 1$: Normalization $\propto$ P^{-1}? P^{-2}? $P^{-0.6}$? : degenerate with $G\mu$ for low harmonics (in pulsar range, slope of spectrum may have independent dependence on μ).

Observation of high harmonics gives several independent measurements: measure $G\mu$, P, look for less vanilla strings.

The inverse problem

Gravitational wave detection probably has the greatest reach in $G\mu$, but is statistical.

Down to $G\mu \sim 10^{-9}$, lensing and small scale CMB anisotropies might show individual strings, which would be much more useful in determining their nature (e.g., one might see a 3-string junction).

A remaining puzzle:

Simulations of the Nambu action (zero-thickness strings)* and the full field theory action (thick strings) give persistently different results (3x less string, and slower strings, for the field theory action), even when the thickness is small compared to all other scales.

Which is right??

*Albrecht & Turok; Allen & Shellard; Bennett & Bouchet; Martins & Shellard; Vanchurin, Olum & Vilenkin; Ringeval, Sakellariadou & Bouchet
**Vincent, Hindmarsh & Sakellariadou; Bevis, Hindmarsh, Kunz & Urrestilla

Conclusions

- Long-standing problem perhaps nearing solution

- Observations will probe most or all of brane inflation range

- If so, there is prospect to distinguish different string models, maybe not until LISA.

- Precise understanding of string networks will require a careful meshing of analytic and numerical methods.

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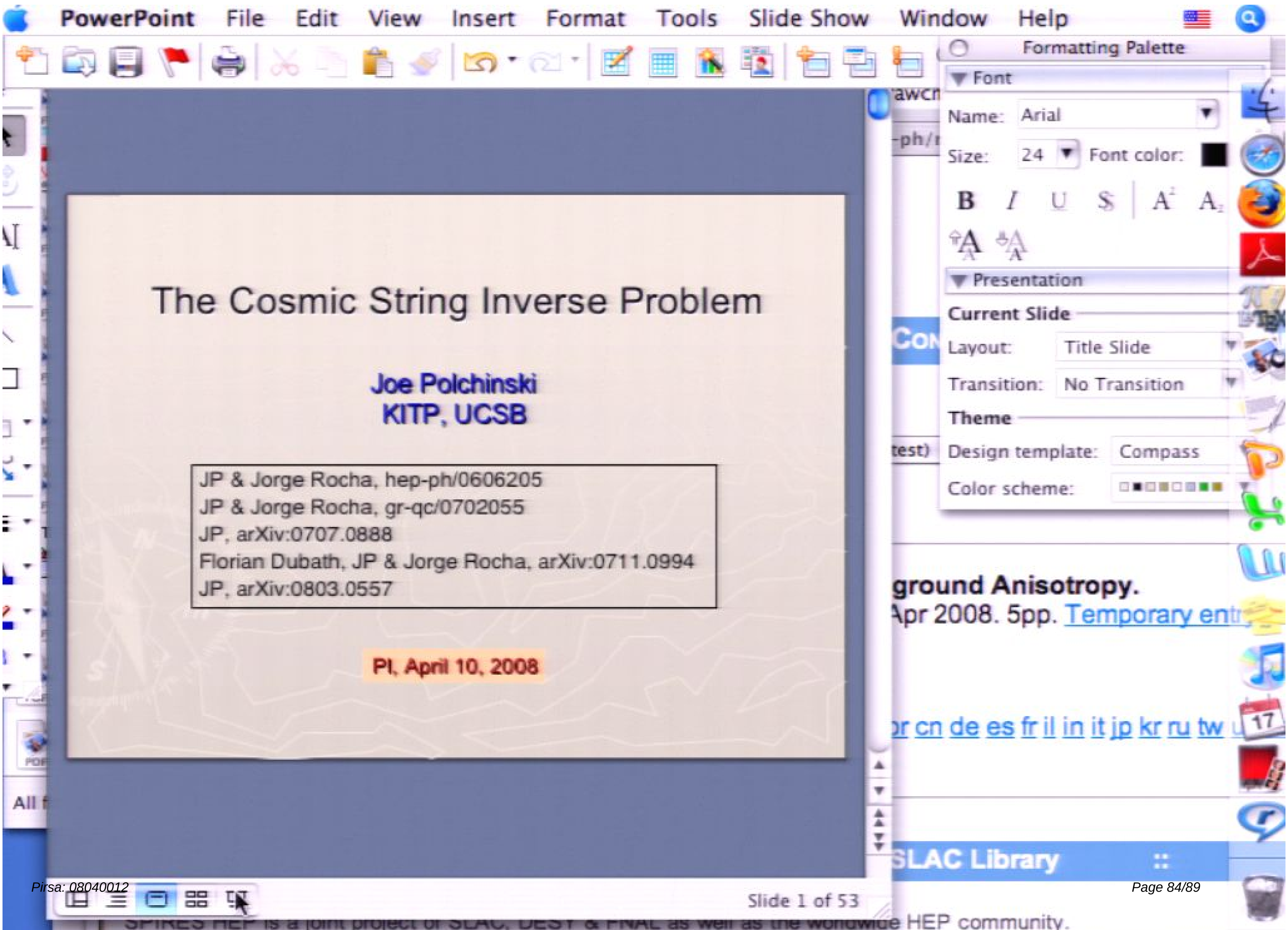
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Color scheme:
The Cosmic String Inverse Problem
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JP & Jorge Rocha, hep-ph/0606205
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JP, arXiv:0707.0888
Florian Dubath, JP & Jorge Rocha, arXiv:0711.0994
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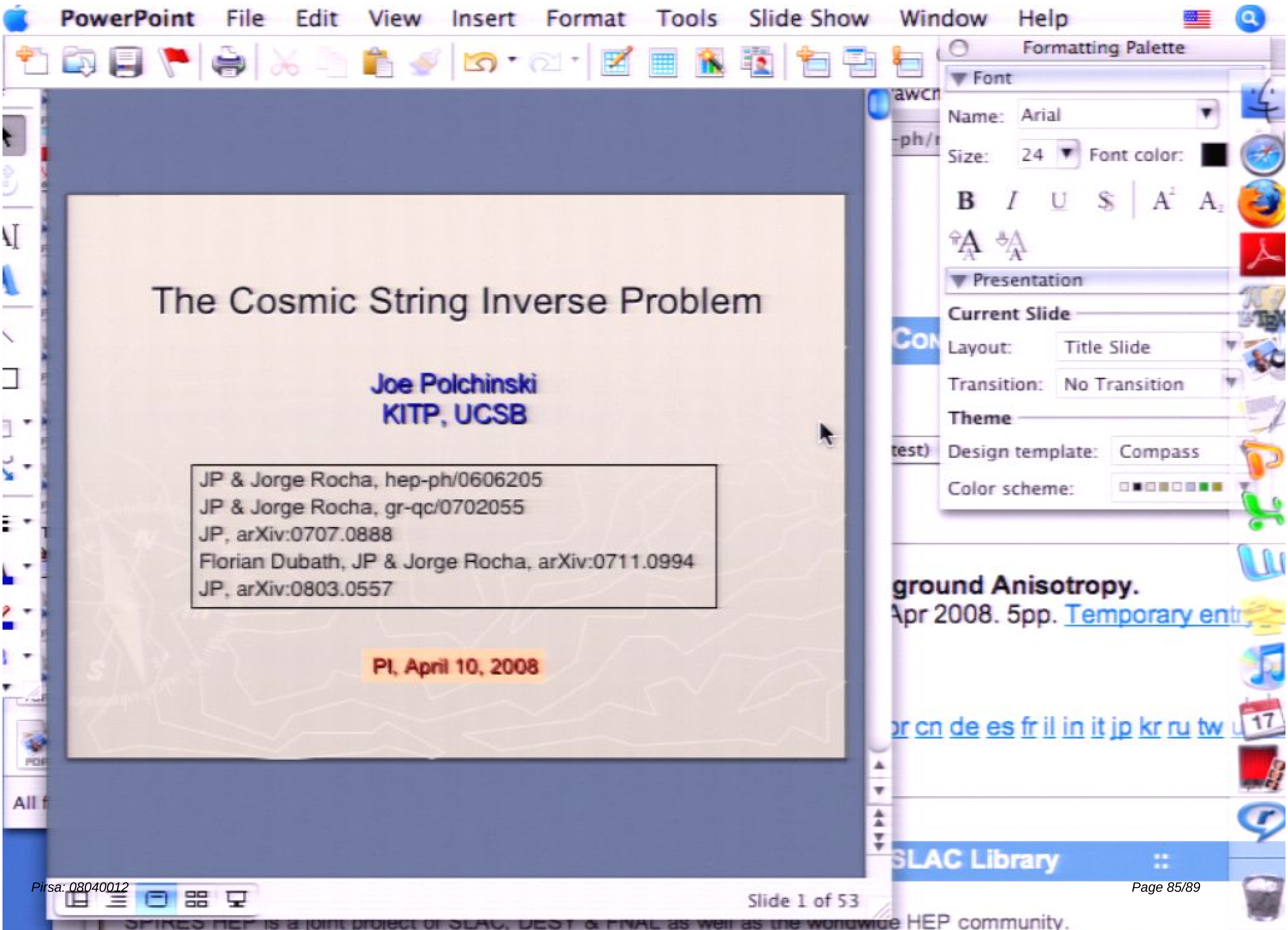
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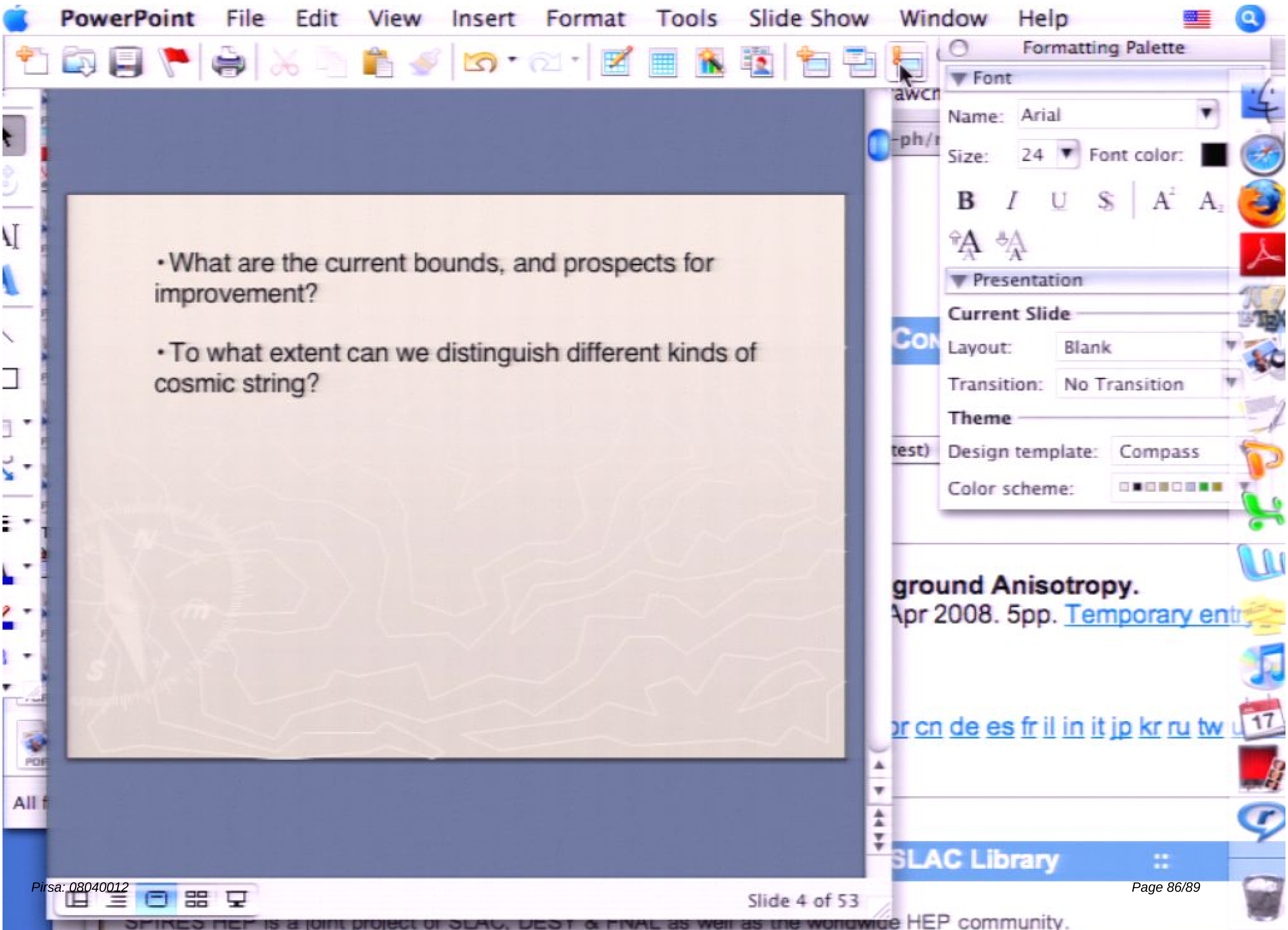
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Formatting Palette
Font
Name: Arial
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Presentation
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•What are the current bounds, and prospects for improvement?
•To what extent can we distinguish different kinds of cosmic string?
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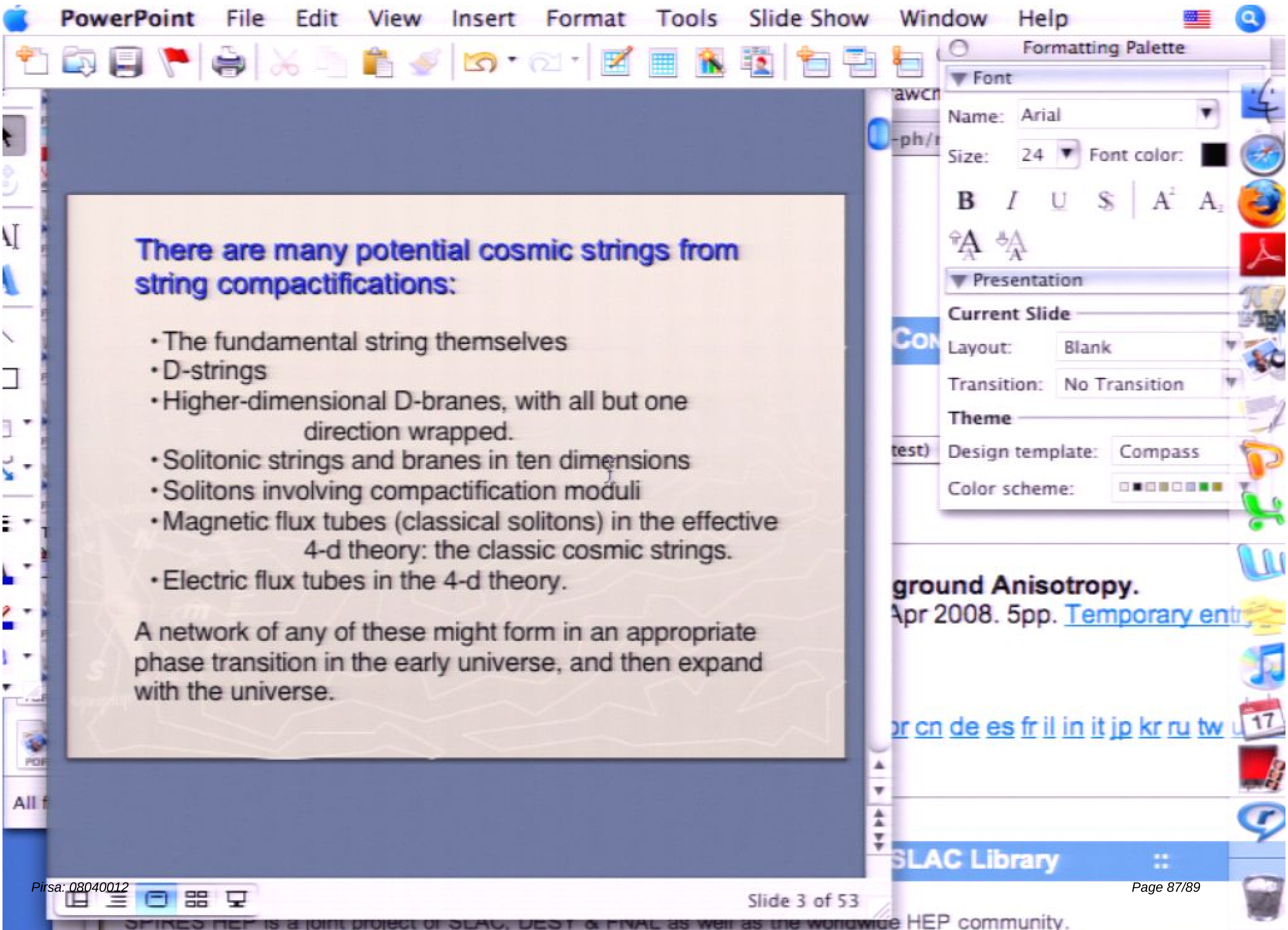
ground Anisotropy.
Apr 2008. 5pp. Temporary entr
SLAC Library
SPIRES HEP is a joint project of SLAC, DESY & FNAL as well as the worldwide HEP community.


There are many potential cosmic strings from string compactifications:
• The fundamental string themselves
• D-strings
• Higher-dimensional D-branes, with all but one direction wrapped.
• Solitonic strings and branes in ten dimensions
• Solitons involving compactification moduli
• Magnetic flux tubes (classical solitons) in the effective 4-d theory: the classic cosmic strings.
• Electric flux tubes in the 4-d theory.
A network of any of these might form in an appropriate phase transition in the early universe, and then expand with the universe.
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Macroscopic parameters:

- Tension μ
- Reconnection probability P:

- Light degrees of freedom: just the oscillations in 3+1, or additional bosonic or fermionic modes?
- Long-range interactions: gravitational only, or axionic or gauge as well?
- One kind of string, or many?
- Multistring junctions?