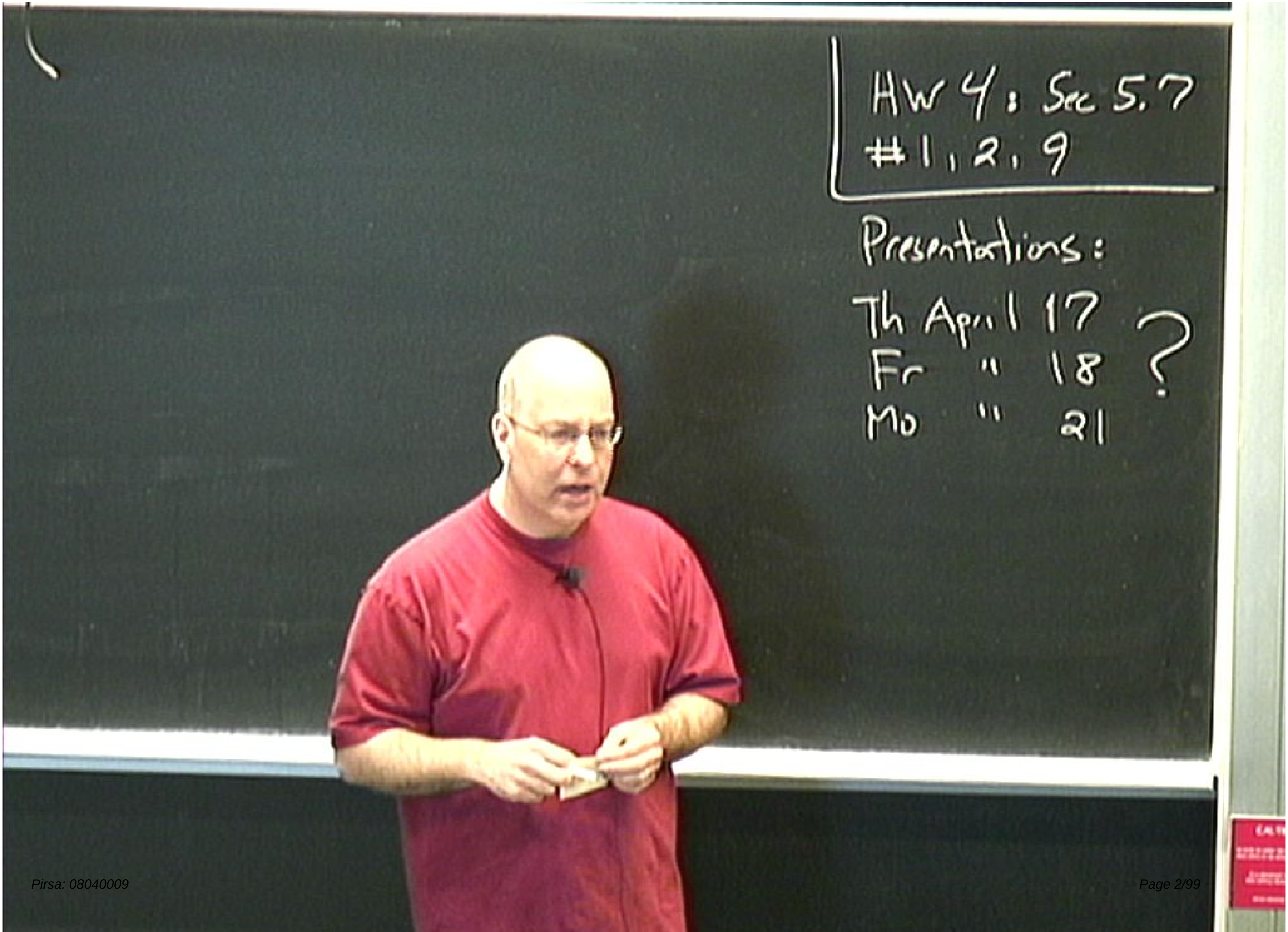
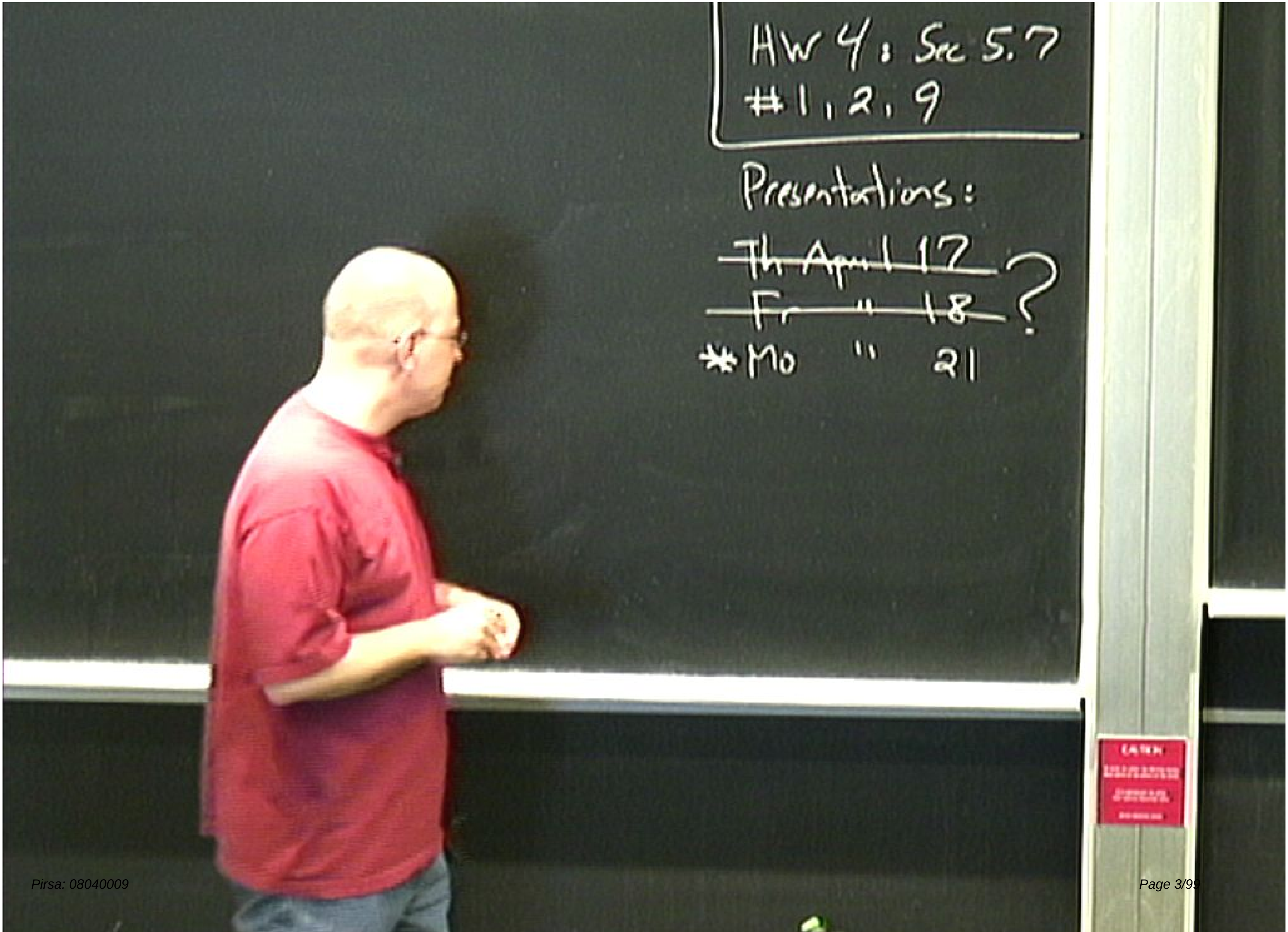
Title: Advanced General Relativity - Lecture 11A

Date: Apr 02, 2008 10:30 AM

URL: http://pirsa.org/08040009

Abstract: Advanced General Relativity


HW 4: Sec 5.7
#1, 2, 9
Presentations:
Th April 17
Fr " 18
Mo " 21
?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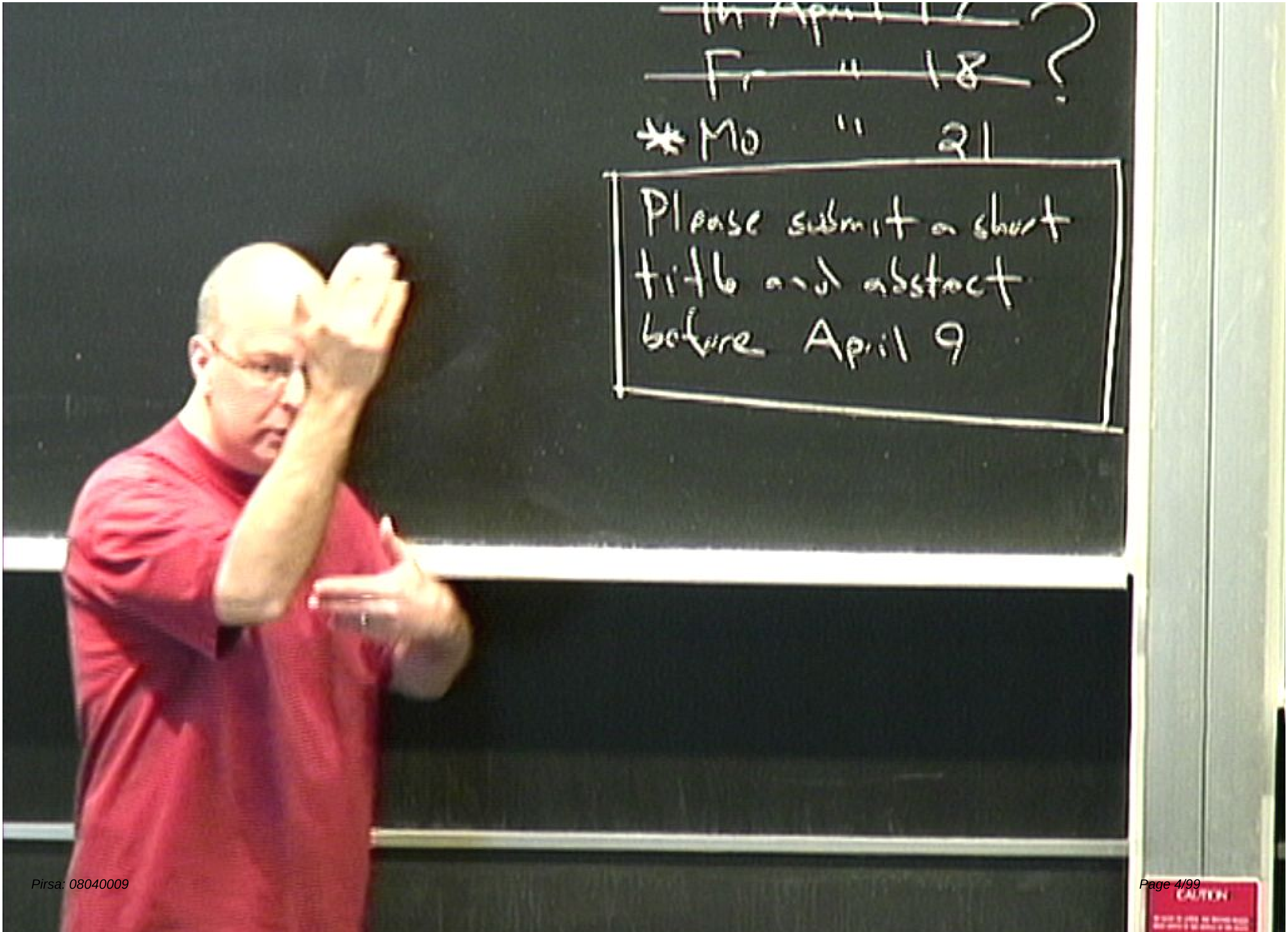
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Please submit a short title and abstract before April 9

HW 4: Sec 5.7
#1, 2, 9

Presentations:

~~Th April 17~~ ?

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$\boxed{\text{* Mo " 21}}$

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SCHWARZSCHILD

$$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2$$

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$$\left.\begin{aligned} U &= t - r^* \\ V &= t + r^* \end{aligned}\right\} \quad r^* = \int \frac{dr}{f(r)}$$

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SCHWARZSCHILD

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$$\kappa = \frac{1}{2} f'(r_0)$$

$$= \frac{M}{r_0^2} = \frac{1}{4M}$$

= surface gravity

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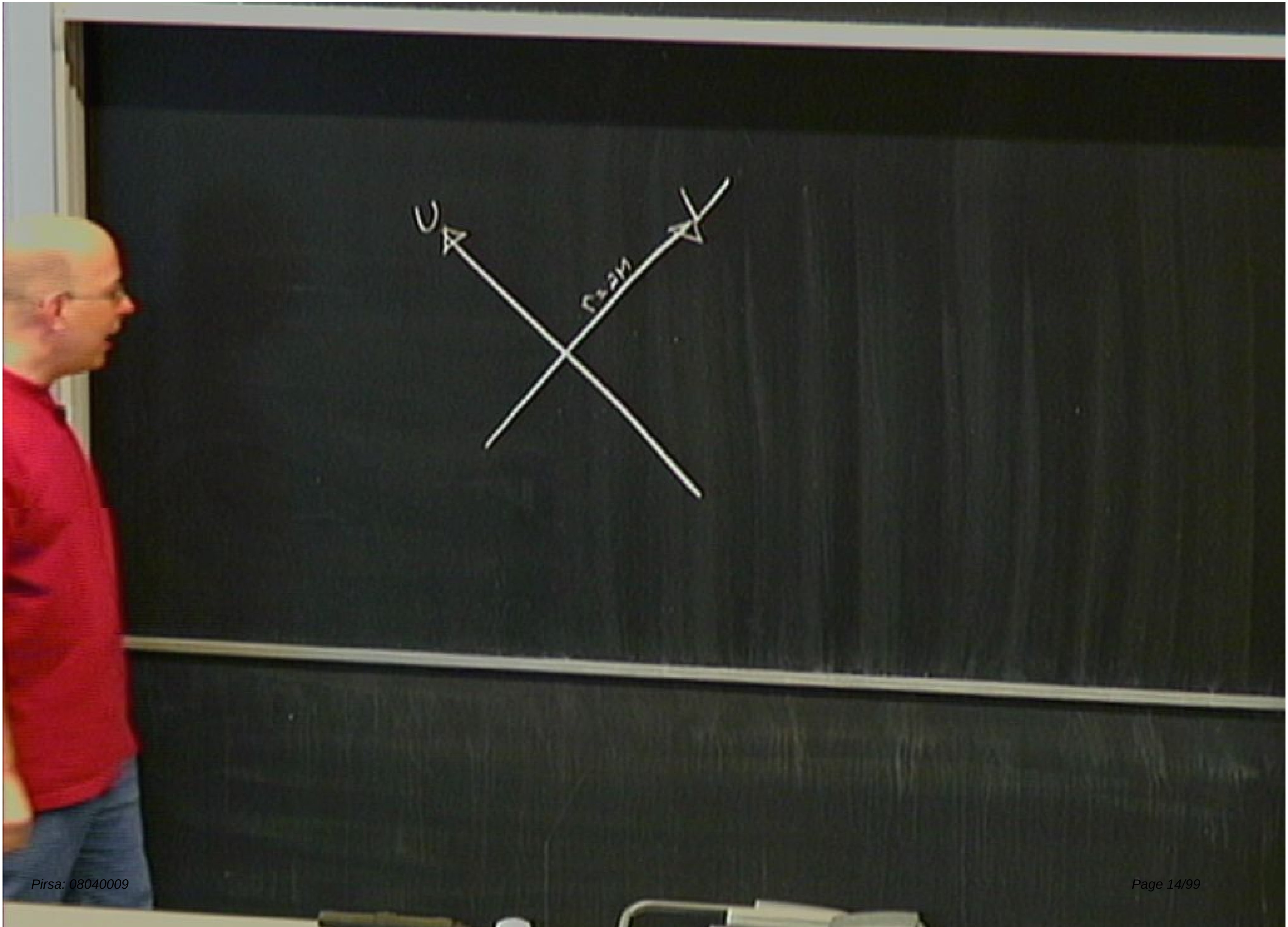
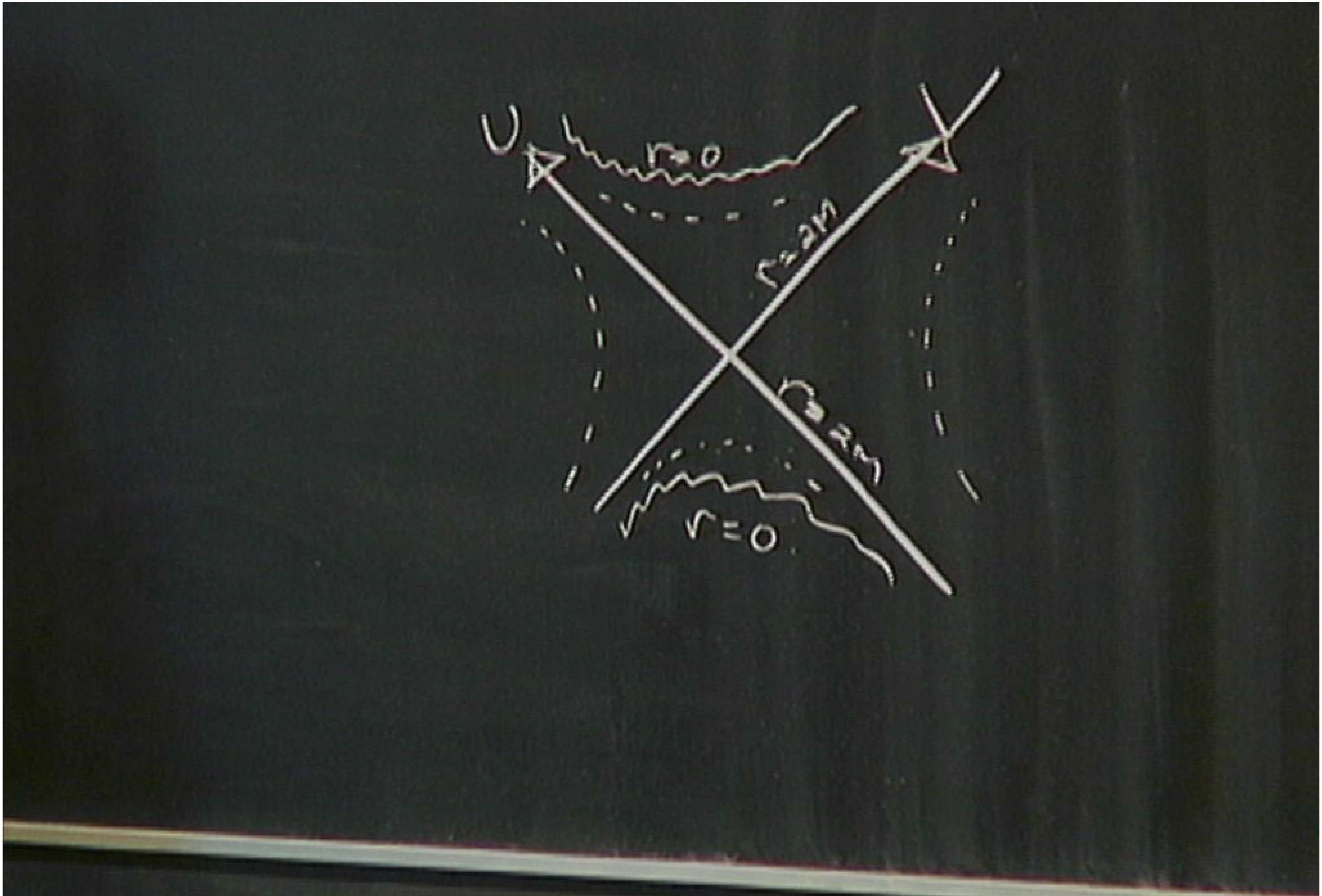
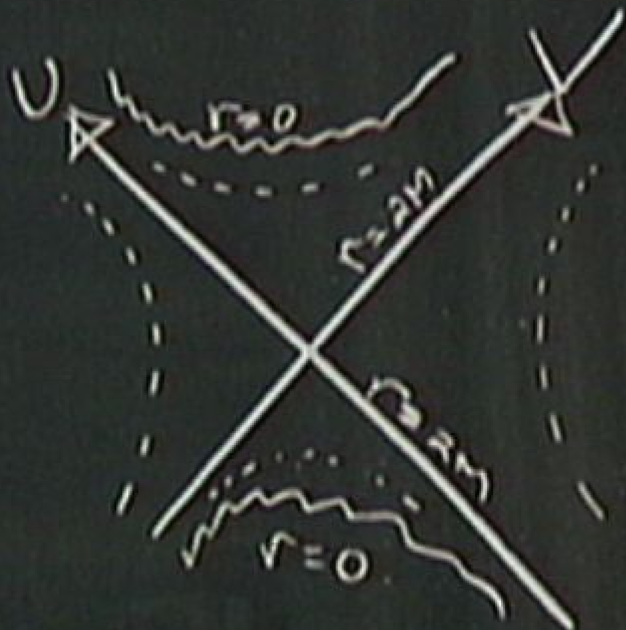

U
V

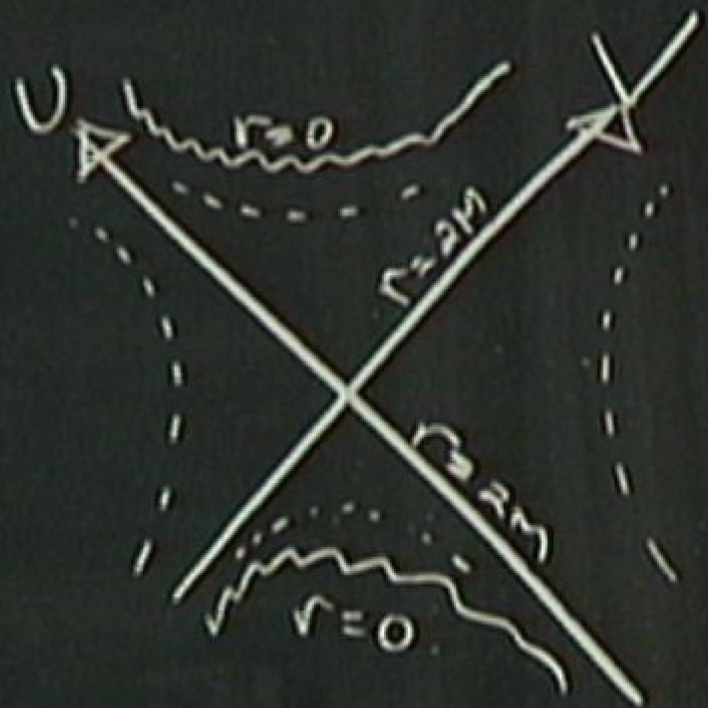


U
V
r=0
r=2M
r=2M
r=0



Penrose-Carter diagram:

$\tilde{U} = \arctan U$



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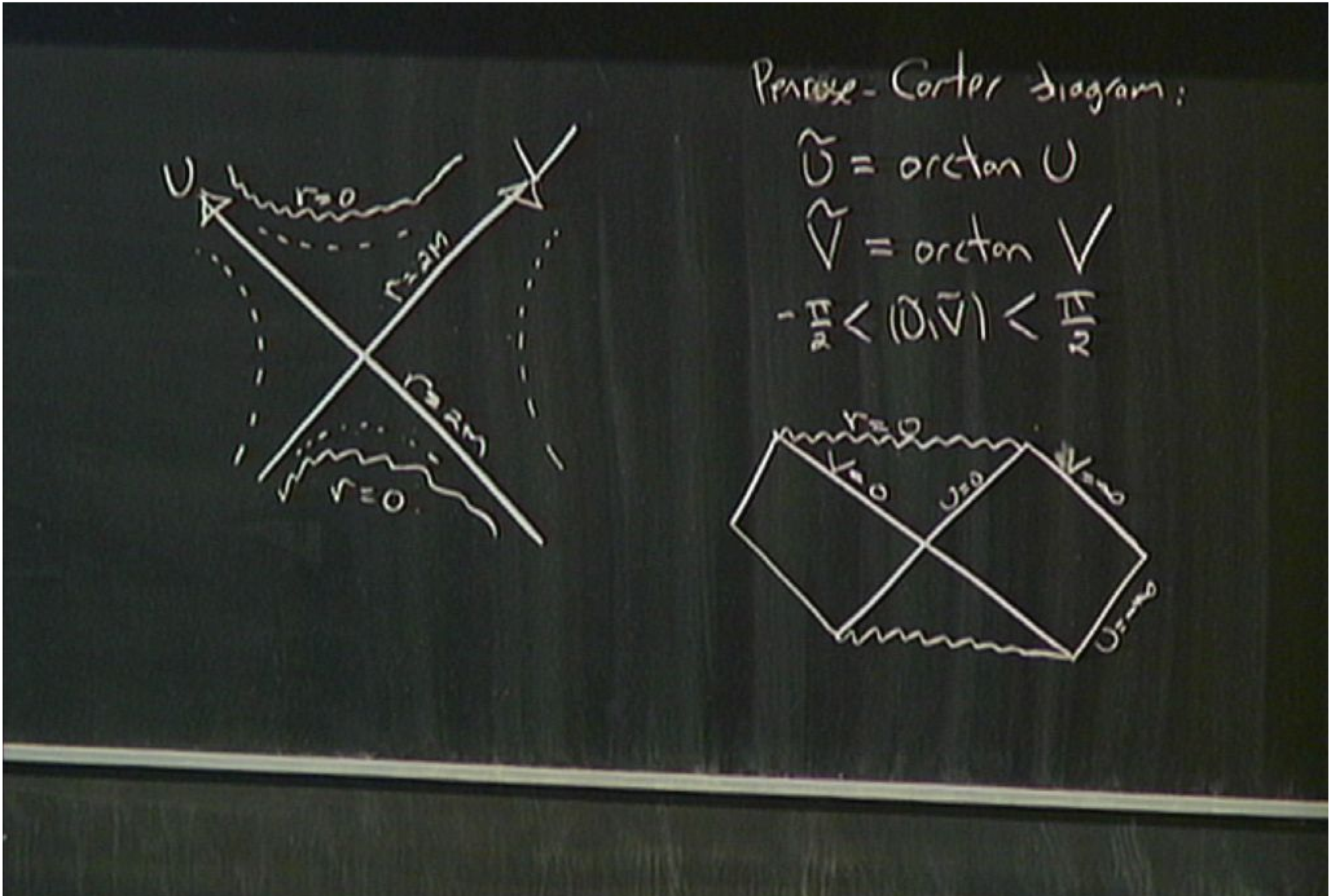
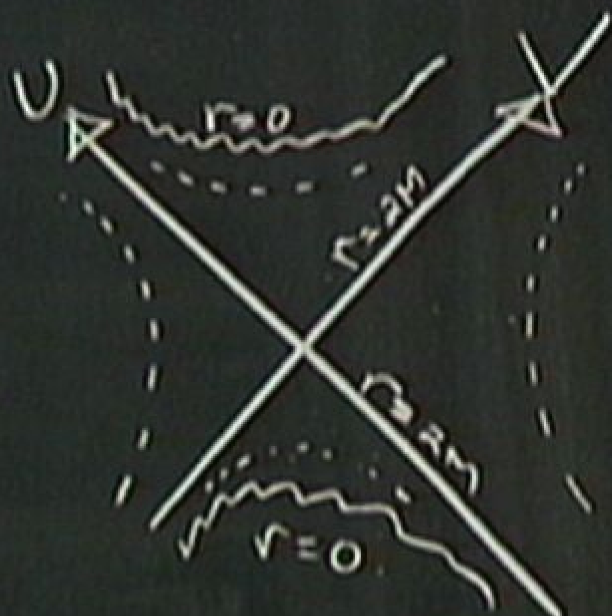
$-\frac{\pi}{2} < (\tilde{U}, \tilde{V}) < \frac{\pi}{2}$


U
V
r=0
r=2M
r=2M
r=0
Penrose-Carter diagram:
$\tilde{U} = \arctan U$
$\tilde{V} = \arctan V$
$-\frac{\pi}{2} < (\tilde{U}, \tilde{V}) < \frac{\pi}{2}$
r=0
V=0
U=0
V=∞
U=∞



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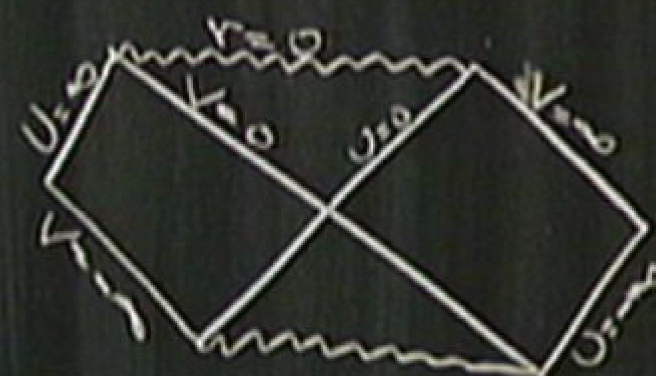

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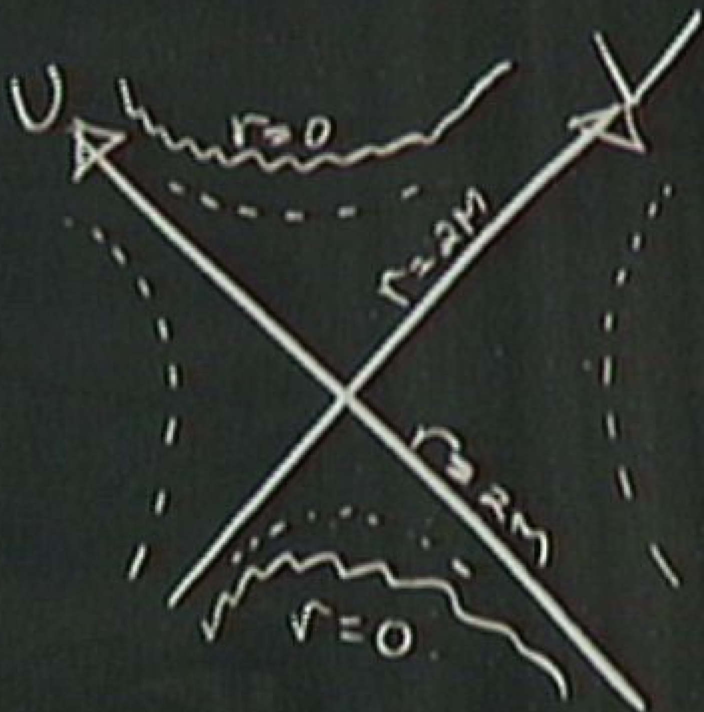
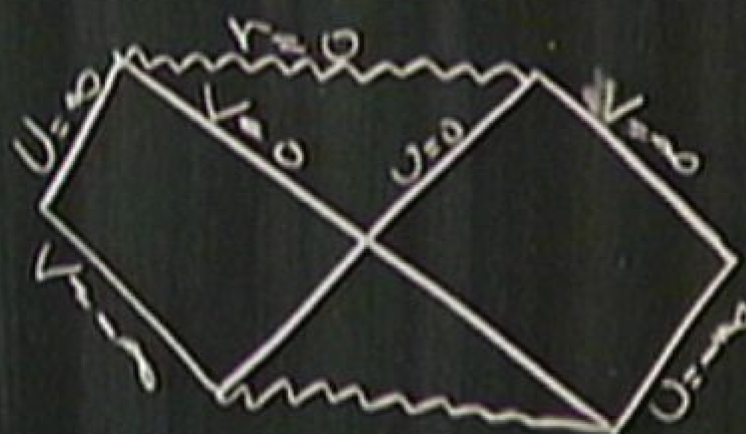
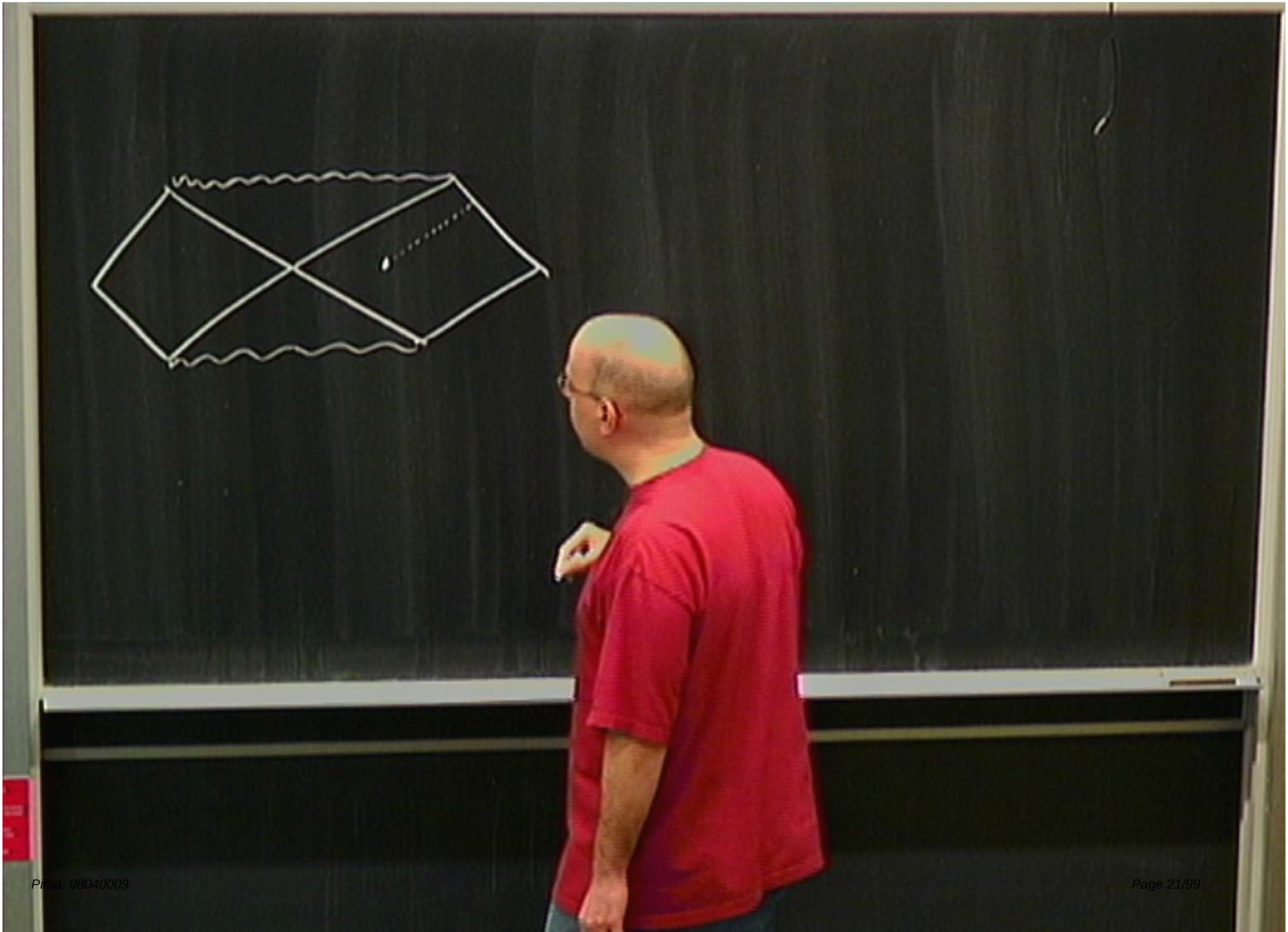
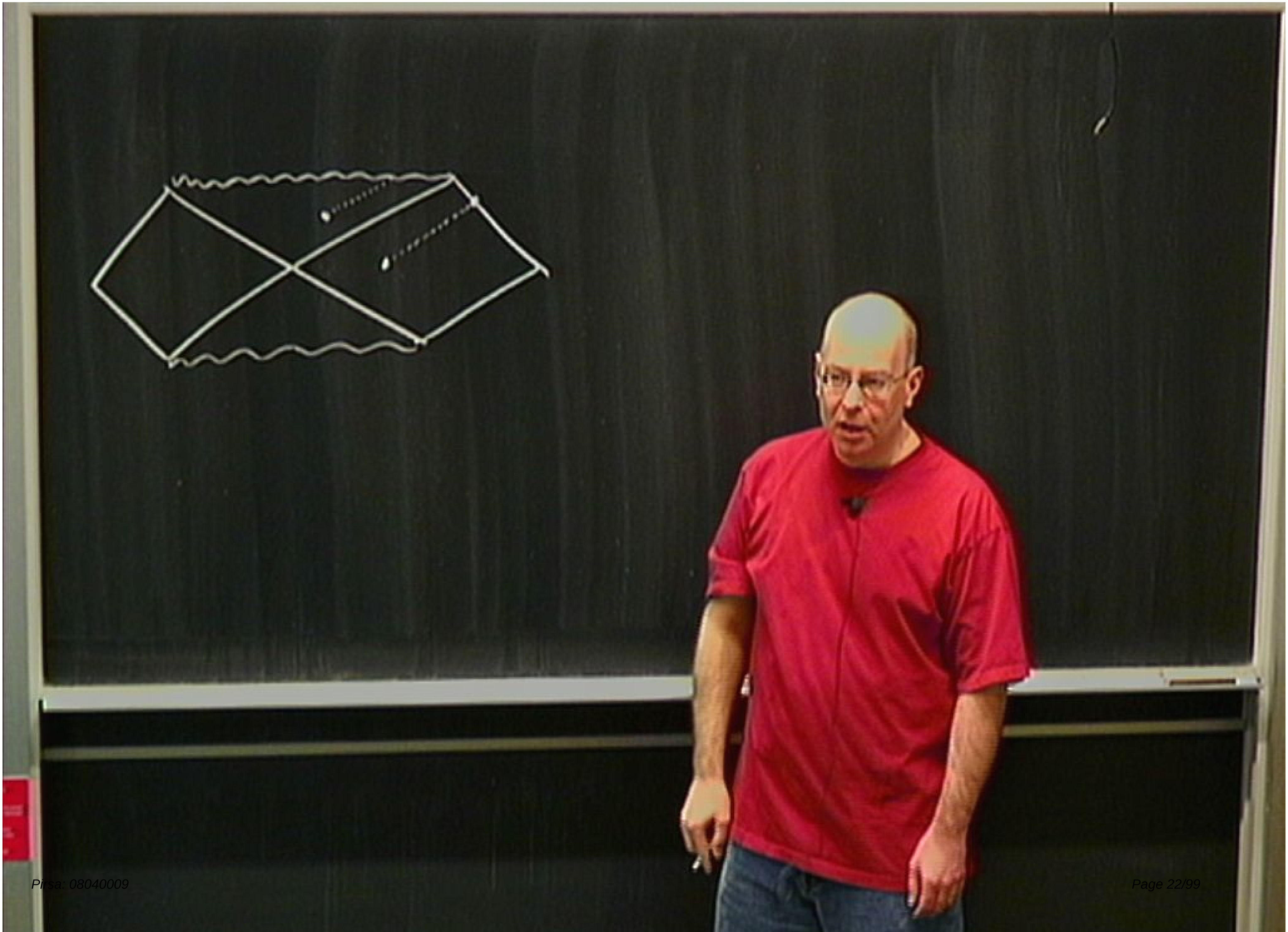
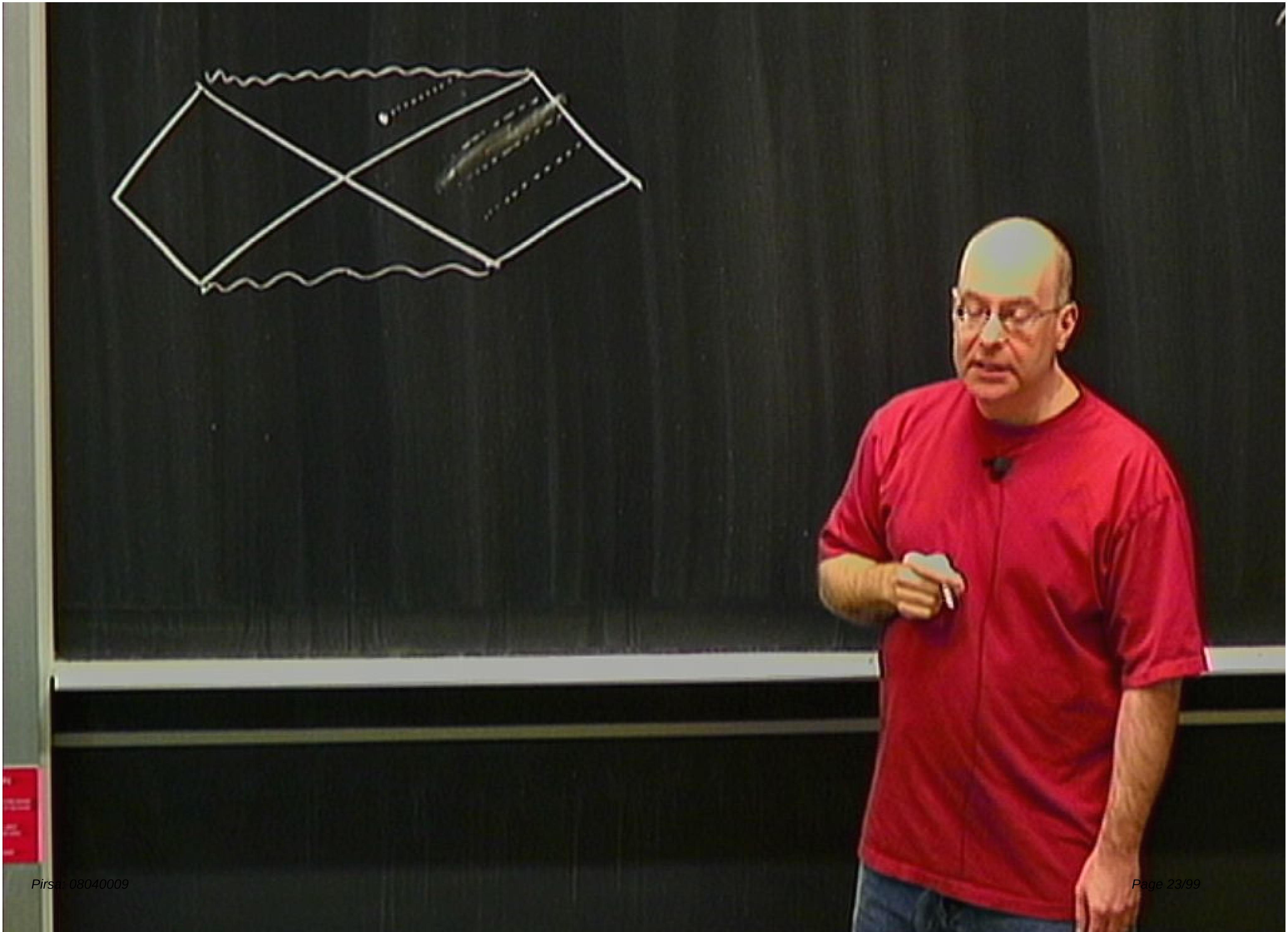
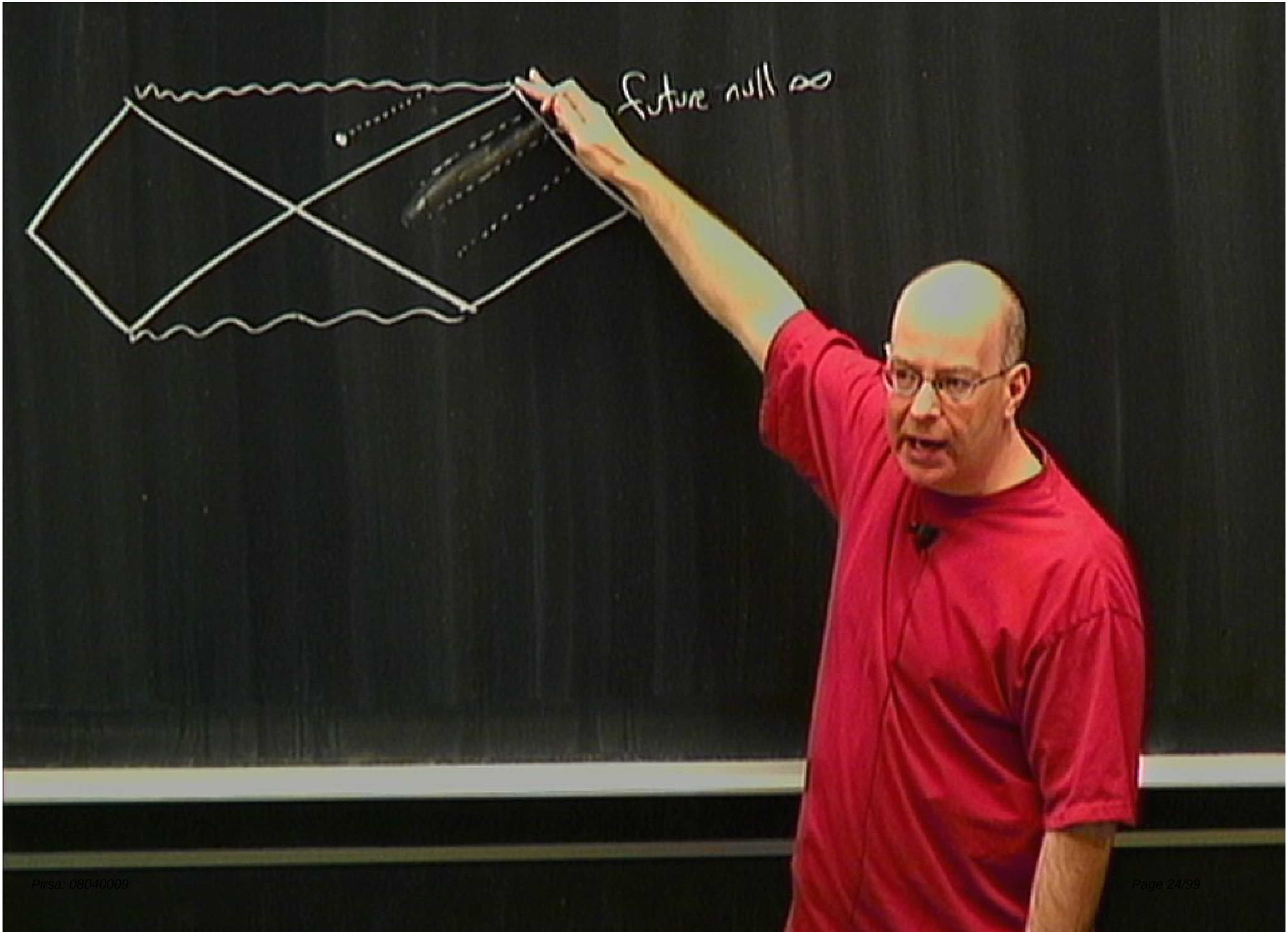


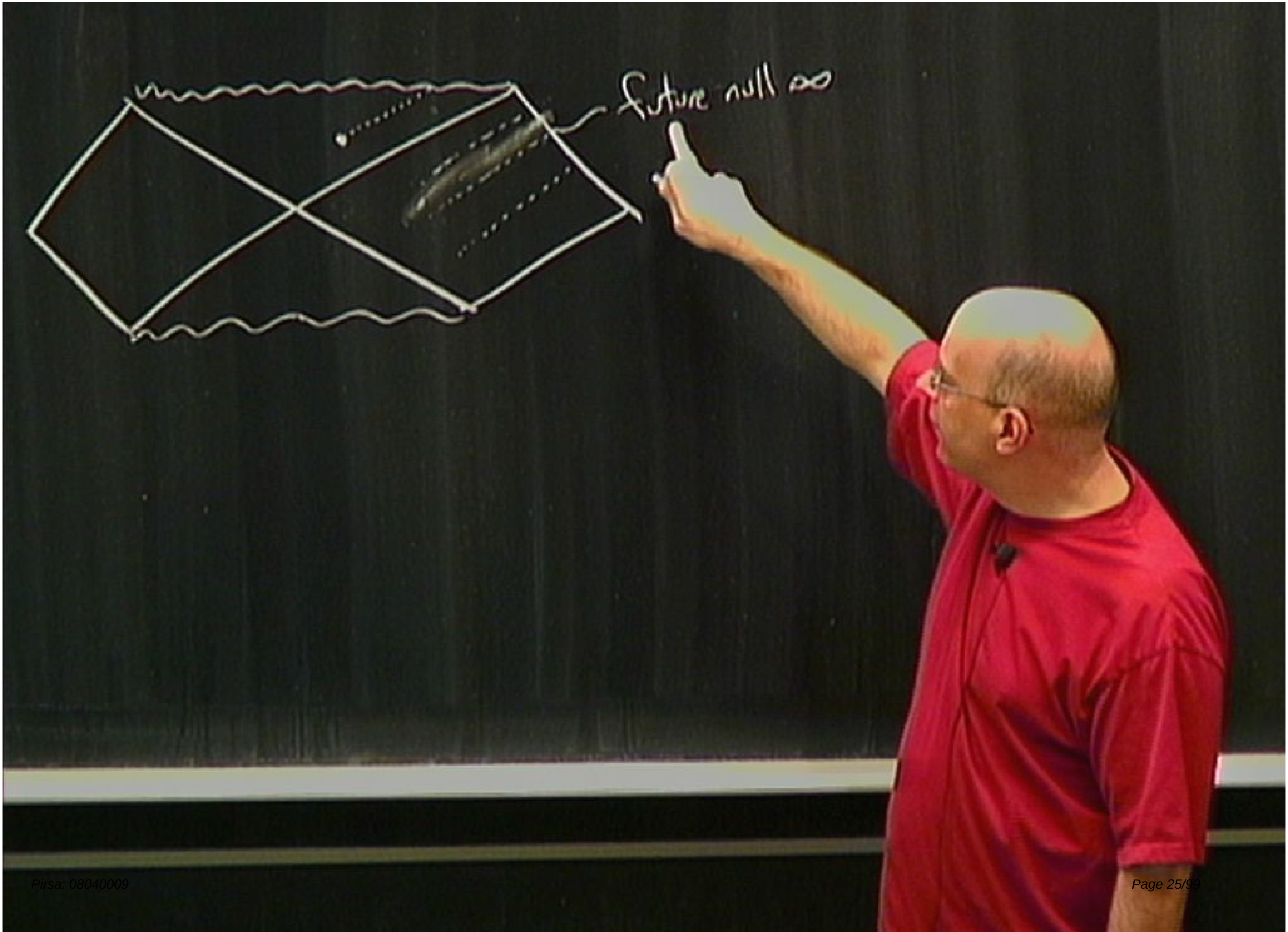









future null ∞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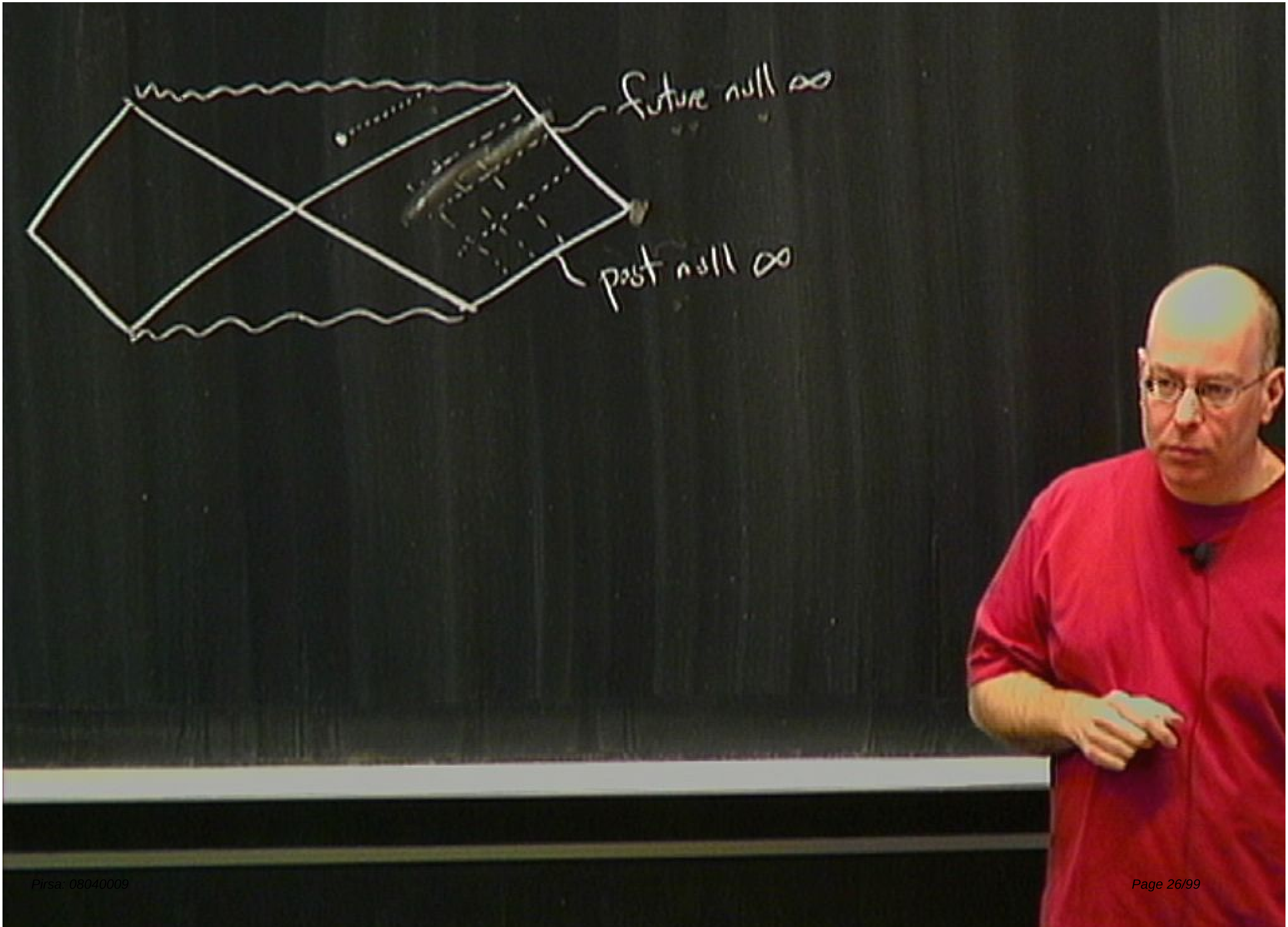
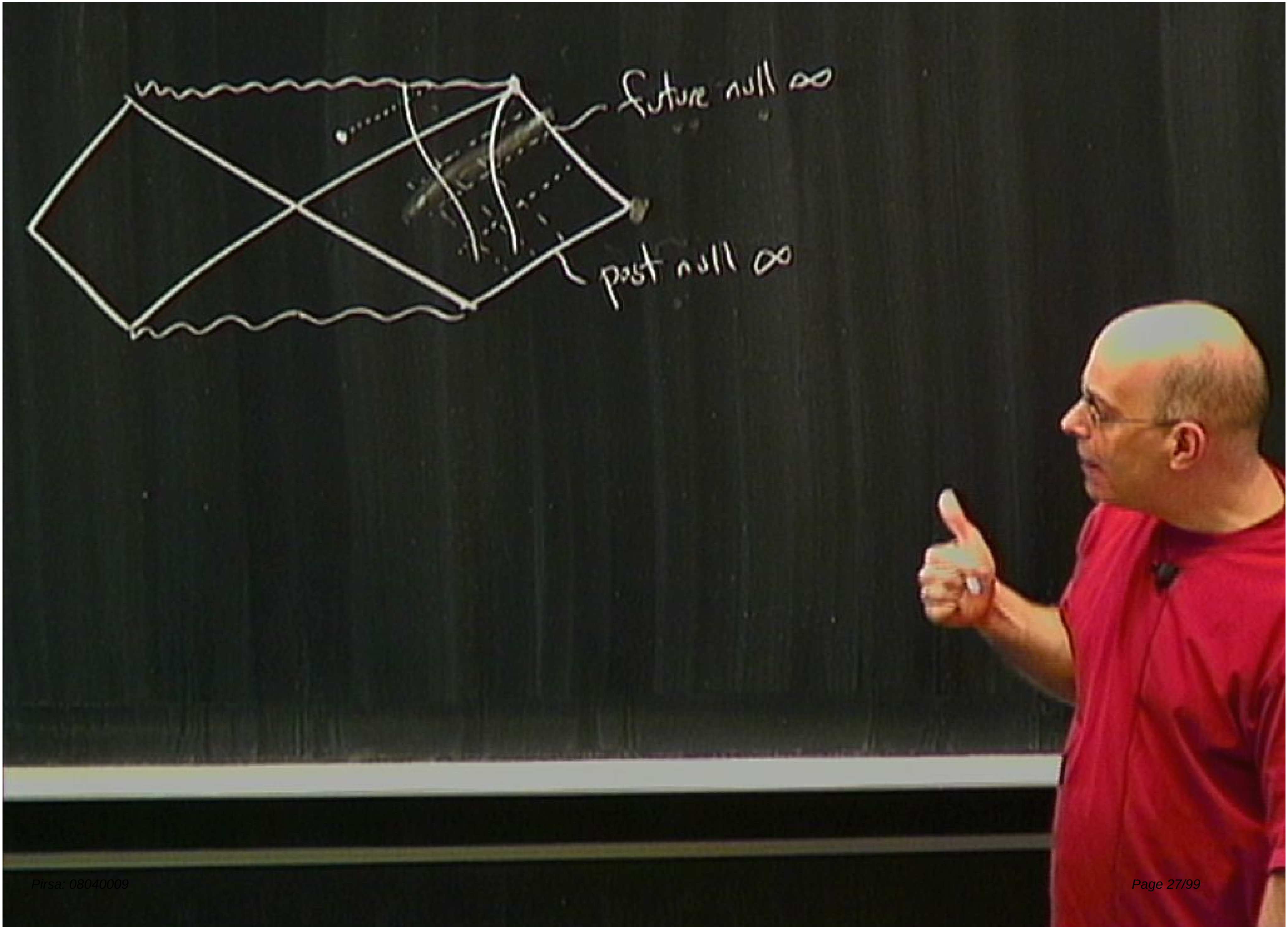
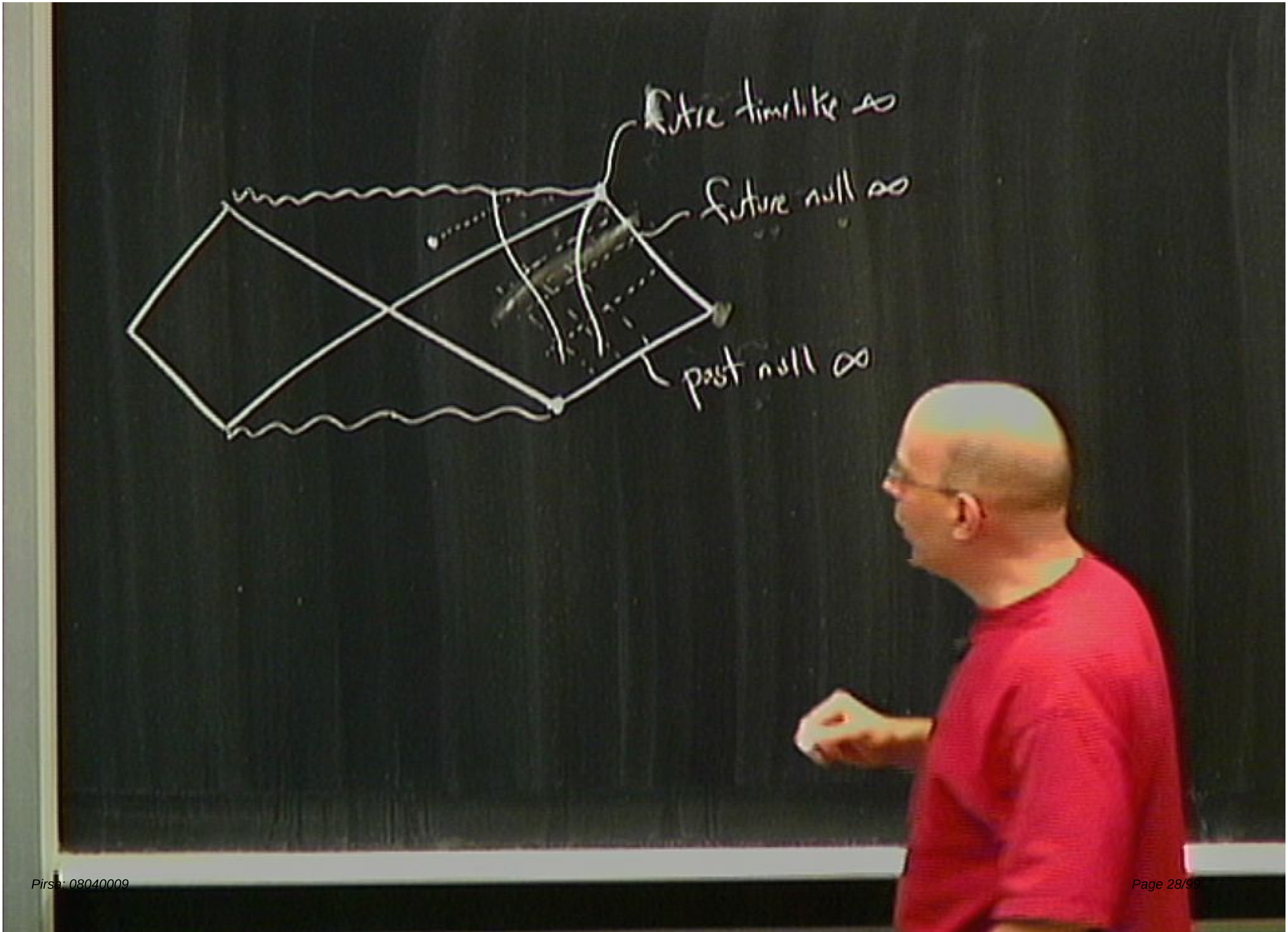
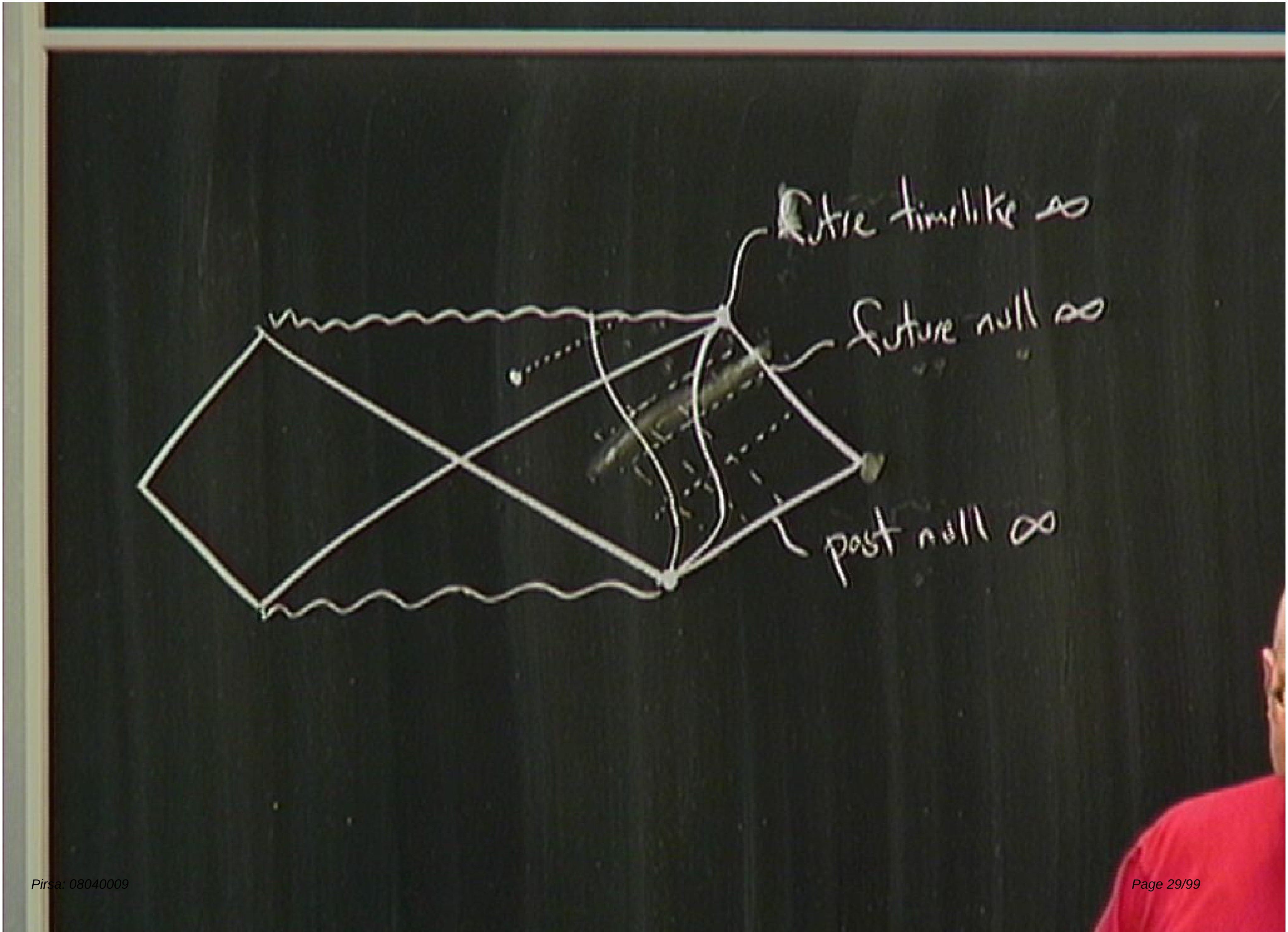
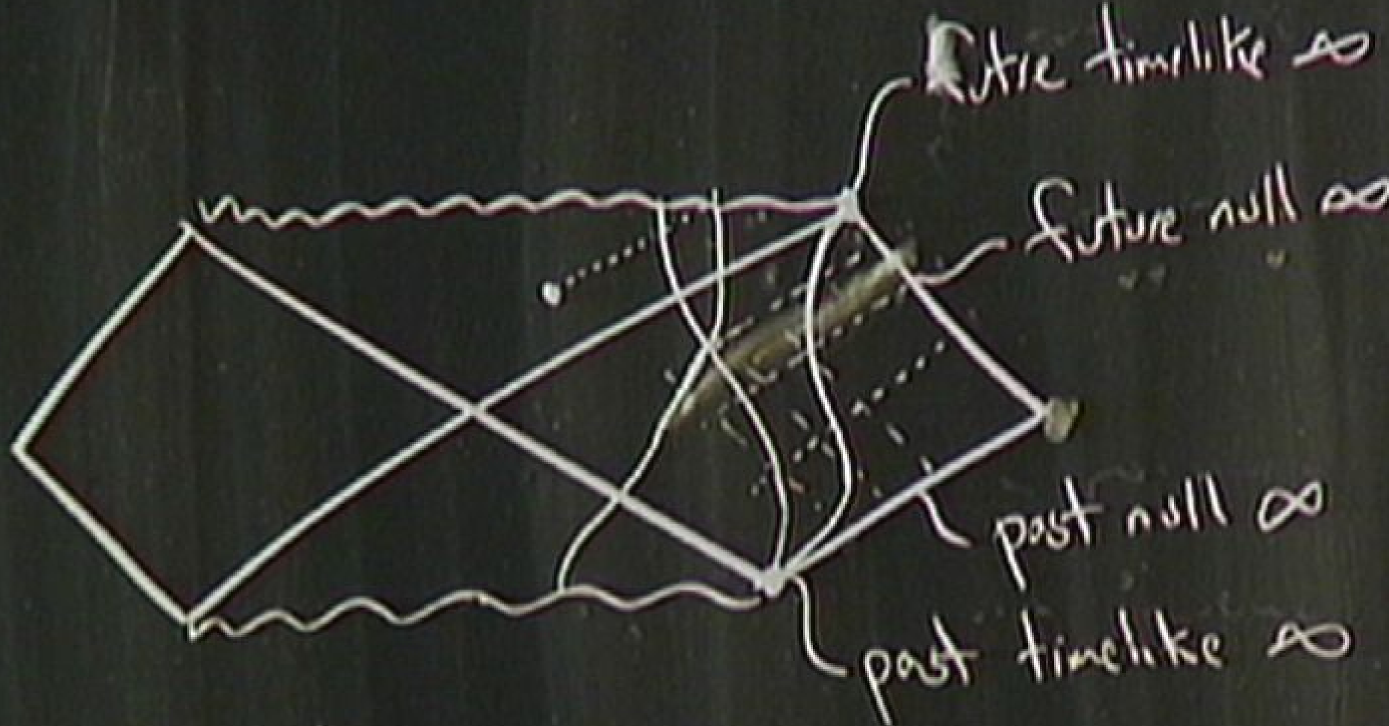
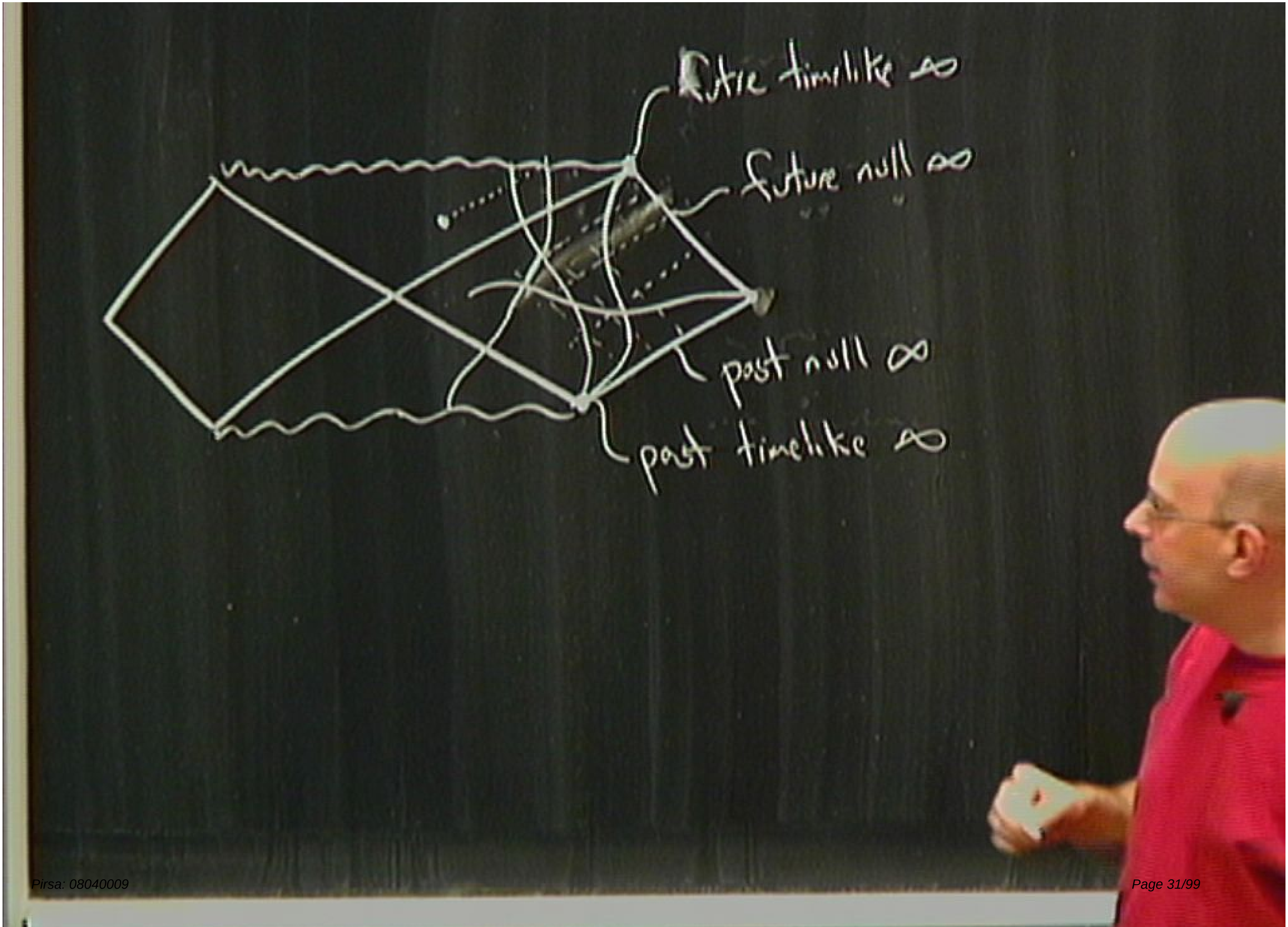

future null ∞
past null ∞

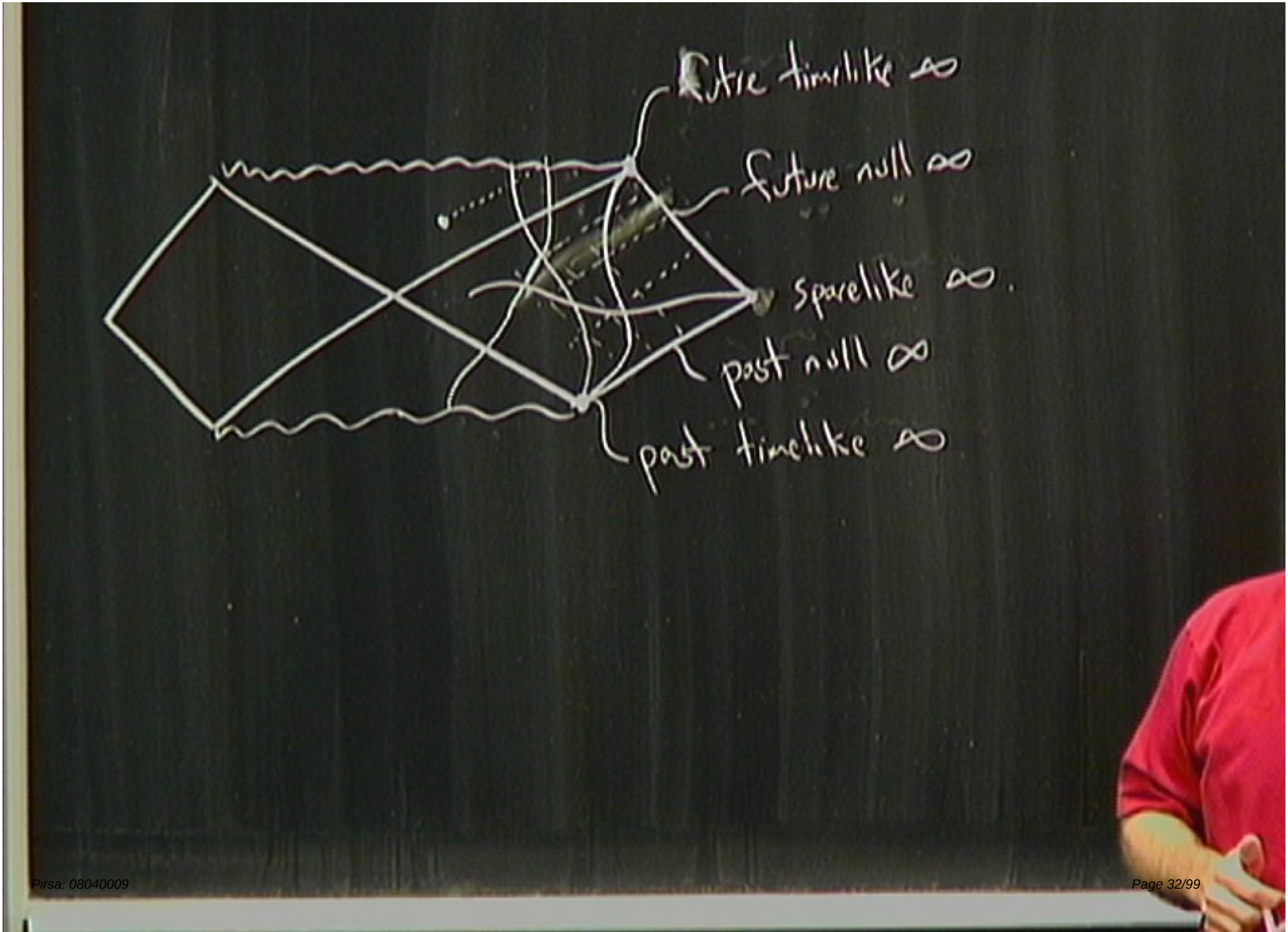

future null ∞
past null ∞

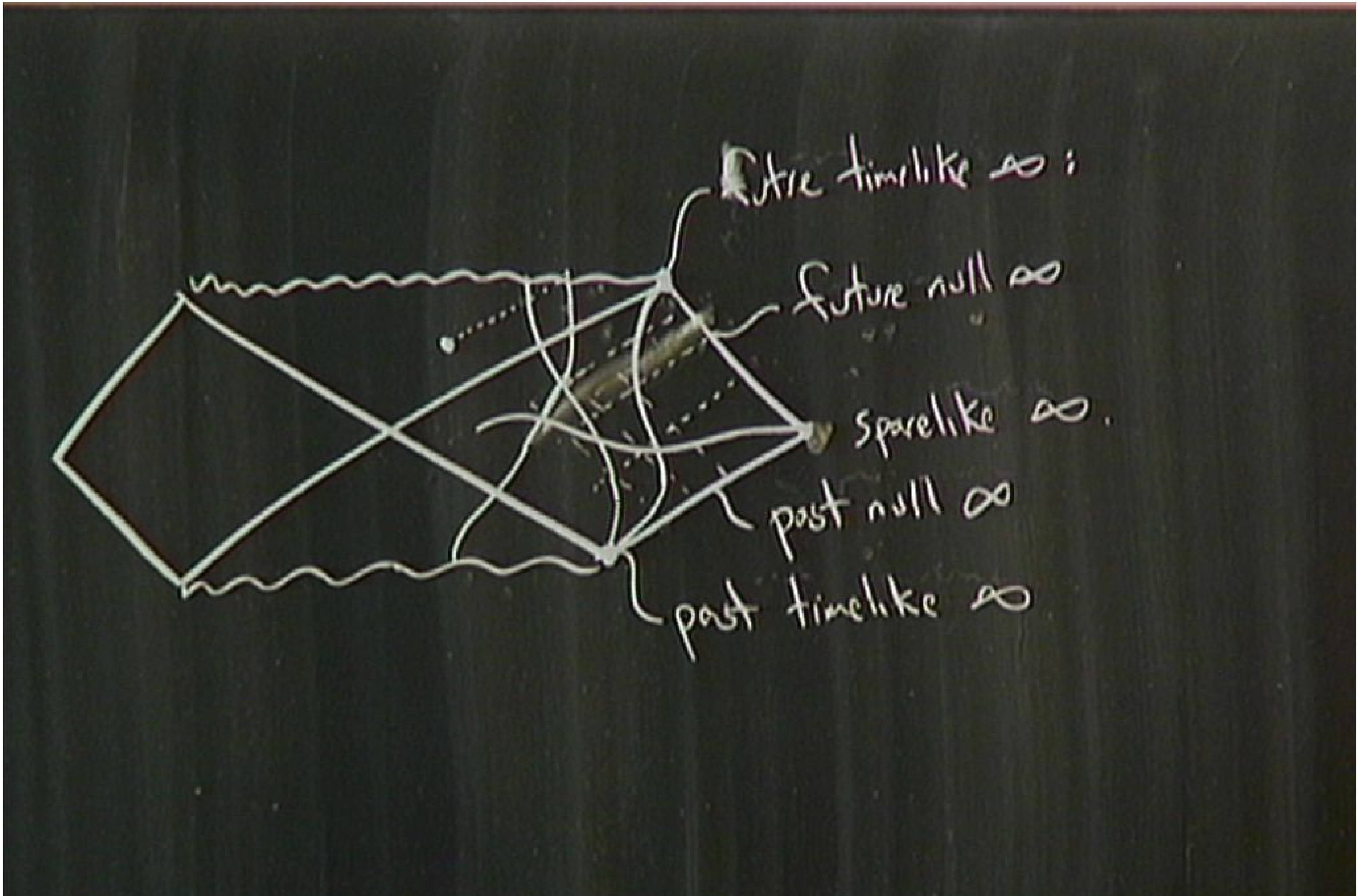

future timelike ∞
future null ∞
past null ∞

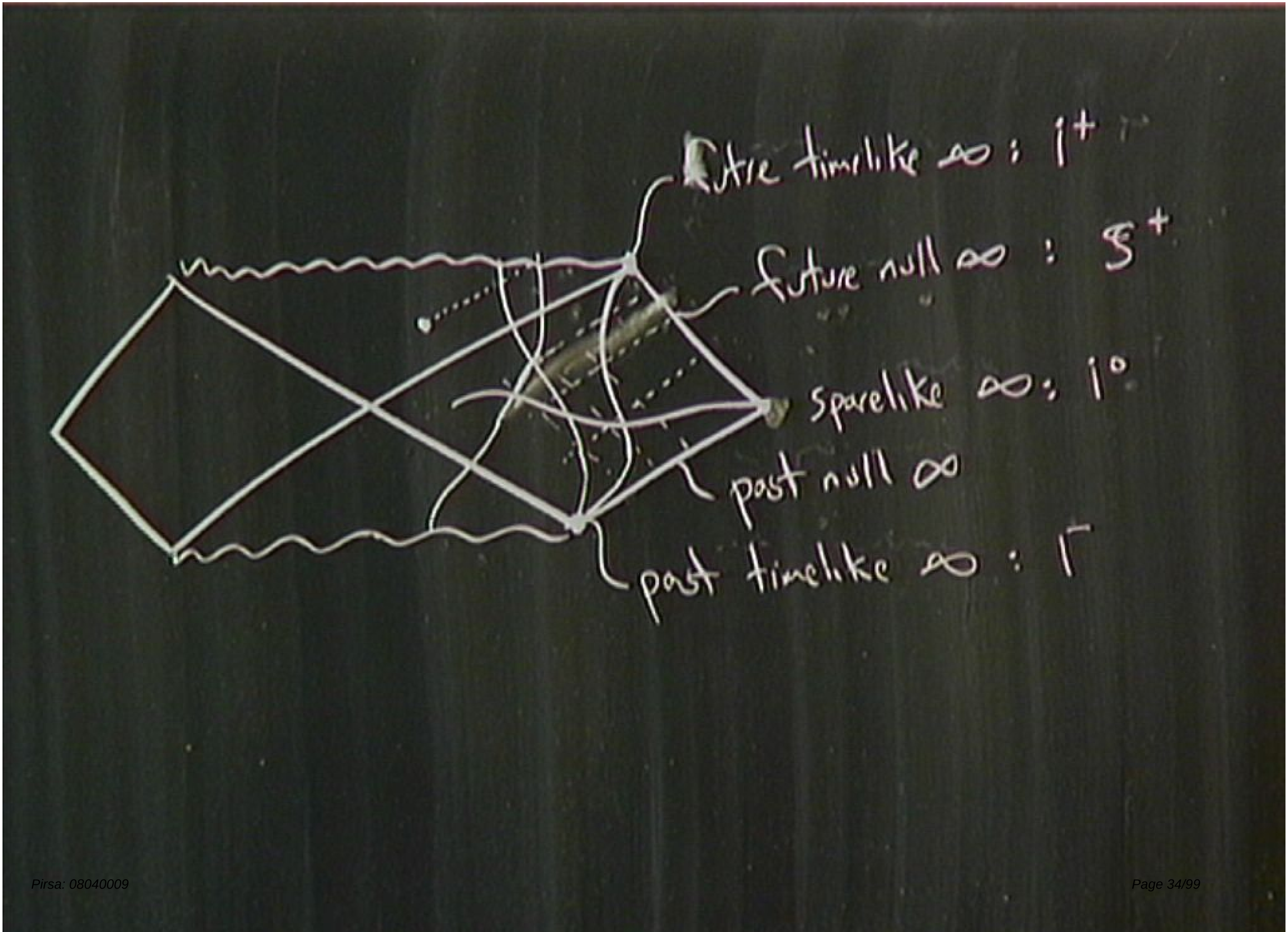

future timelike ∞
future null ∞
past null ∞

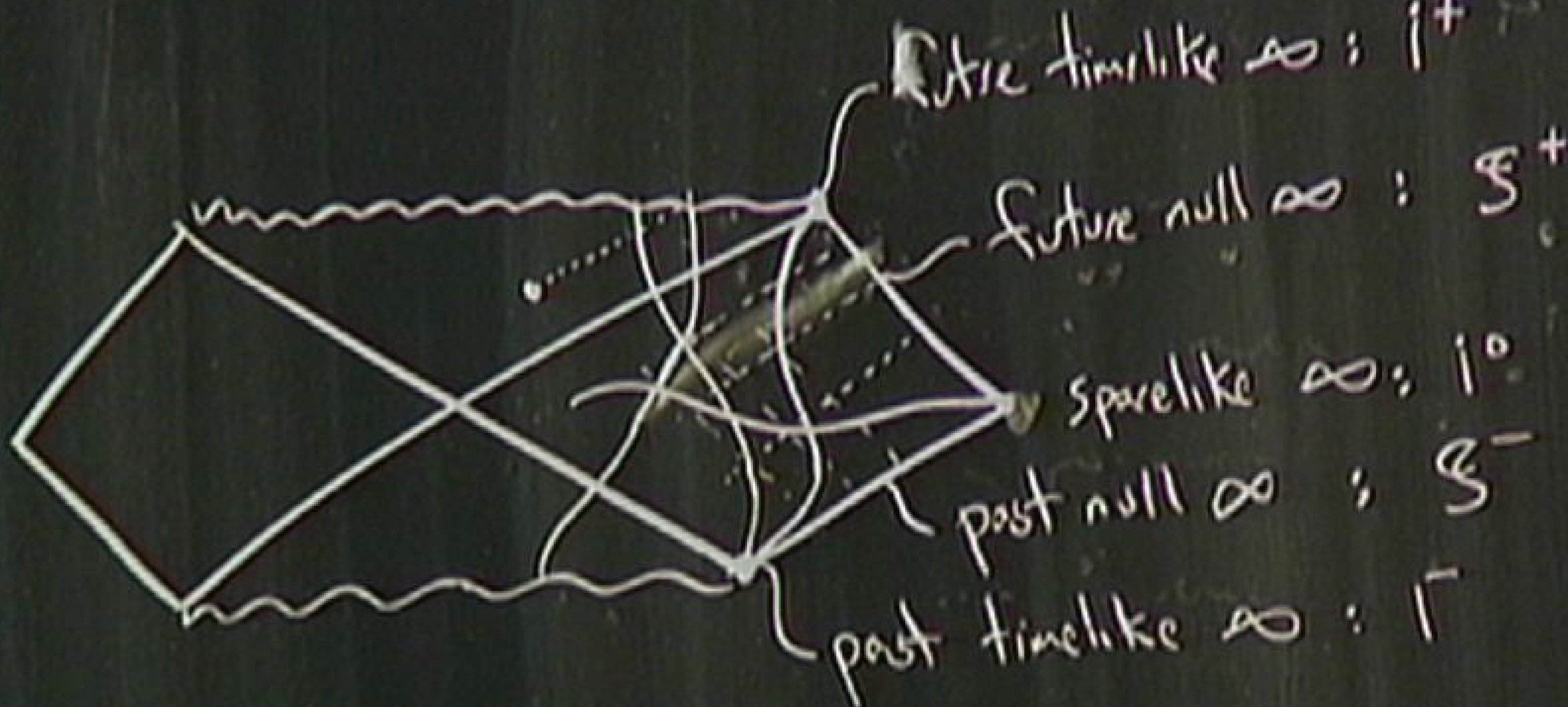

future timelike ∞
future null ∞
past null ∞
past timelike ∞


future timelike ∞
future null ∞
past null ∞
past timelike ∞


futre timelike ∞
future null ∞
sparelike ∞.
past null ∞
past timelike ∞


future timelike ∞ :
future null ∞
spacelike ∞ .
past null ∞
past timelike ∞


future timelike ∞ : i^+
future null ∞ : $\mathscr{I}^+$
spacelike ∞ : i^0
past null ∞
past timelike ∞ : i^-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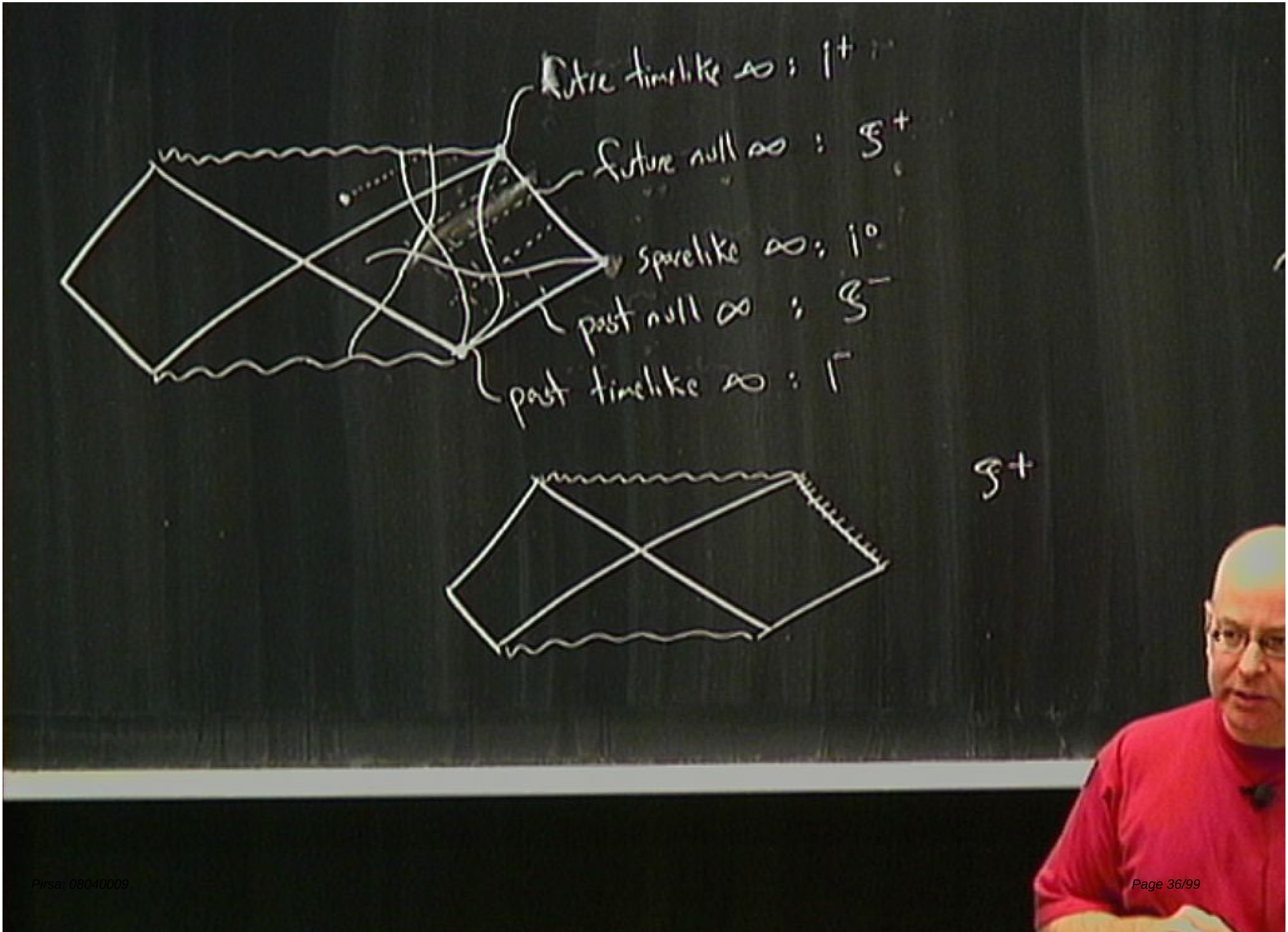
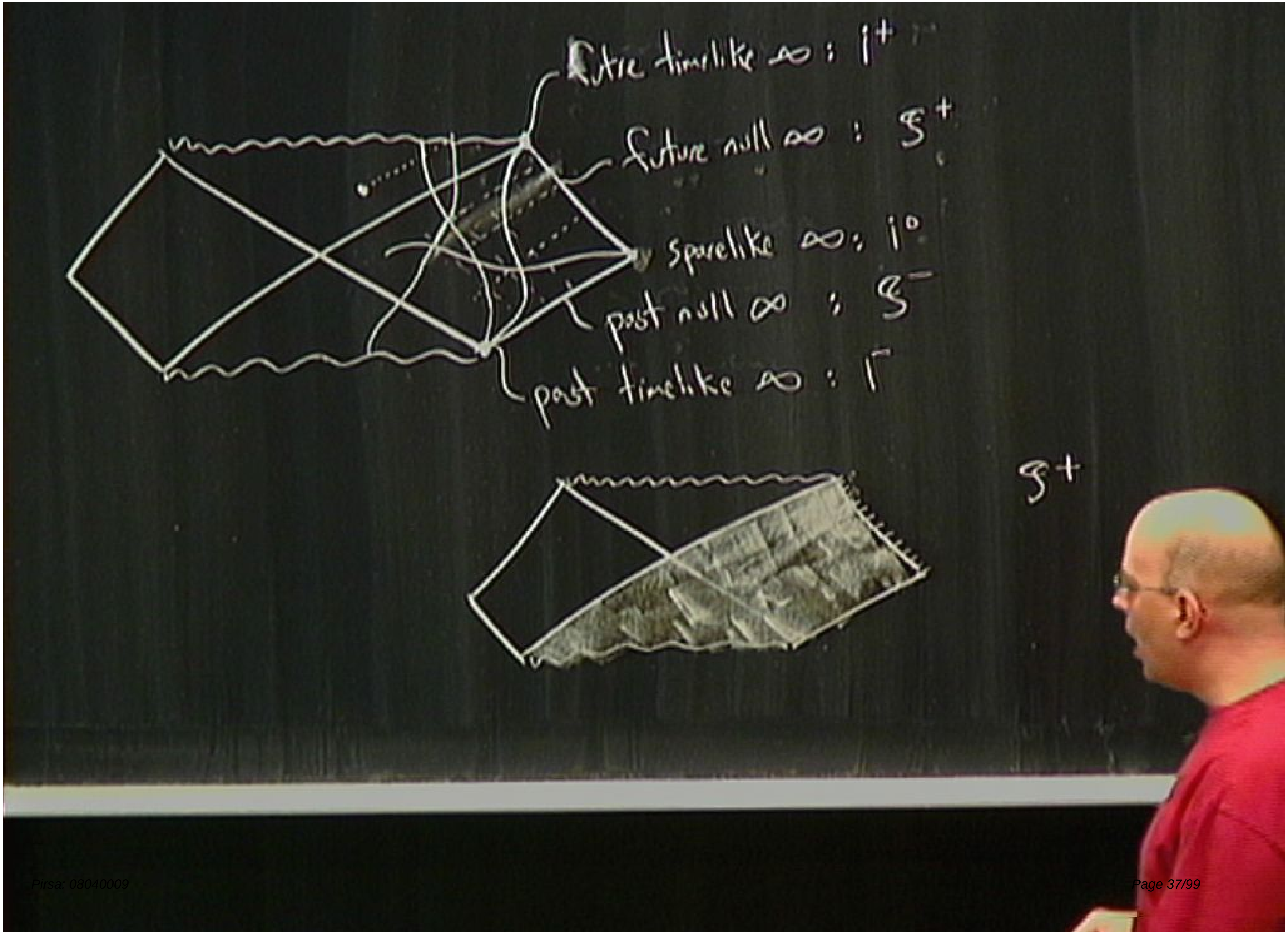
past timelike ∞ : i^-


Future timelike ∞ : i^+
future null ∞ : $\mathscr{I}^+$
spacelike ∞ : i^0
past null ∞ : $\mathscr{I}^-$
past timelike ∞ : i^-
$\mathscr{I}^+$


future timelike ∞ : i^+
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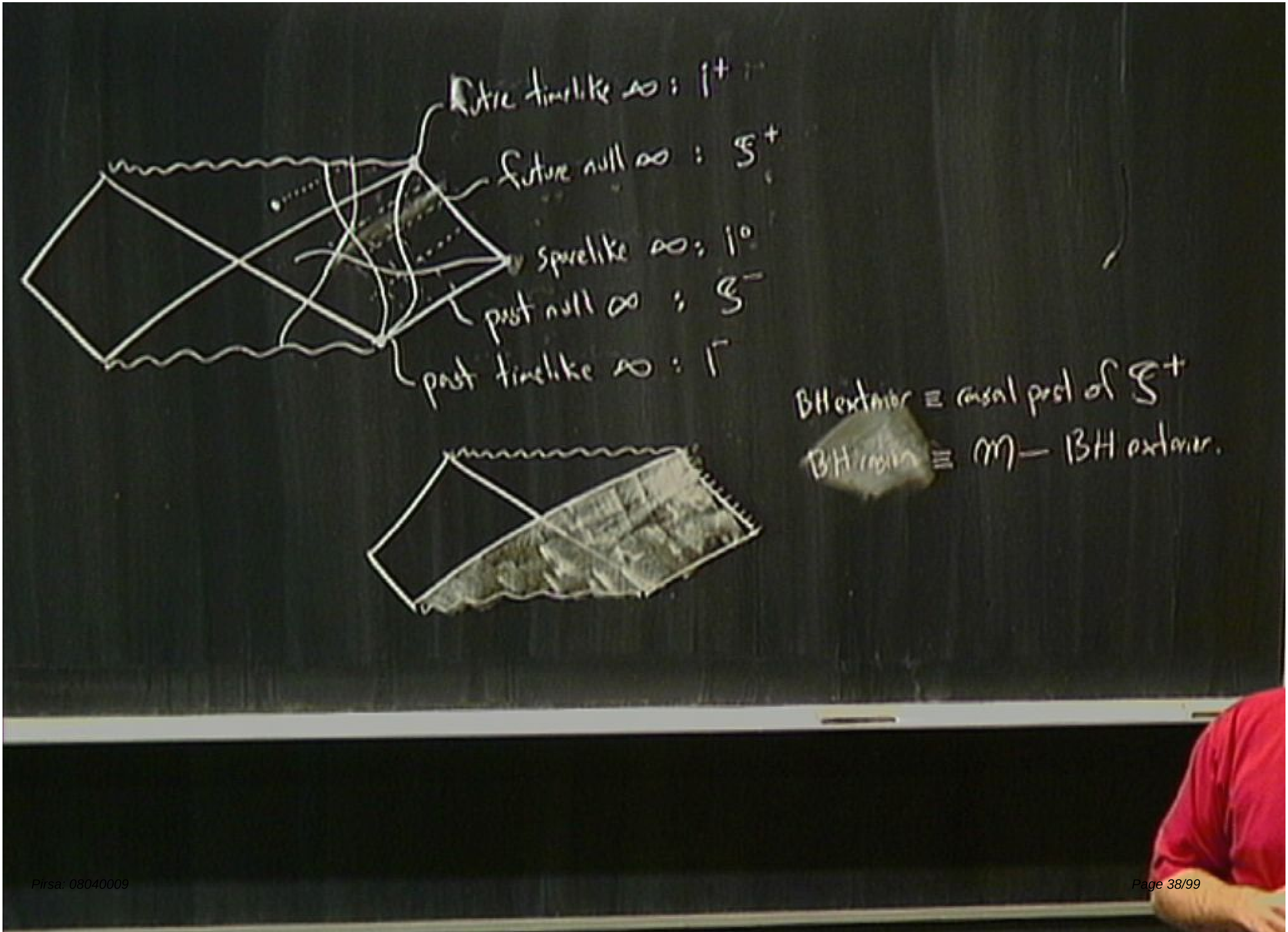
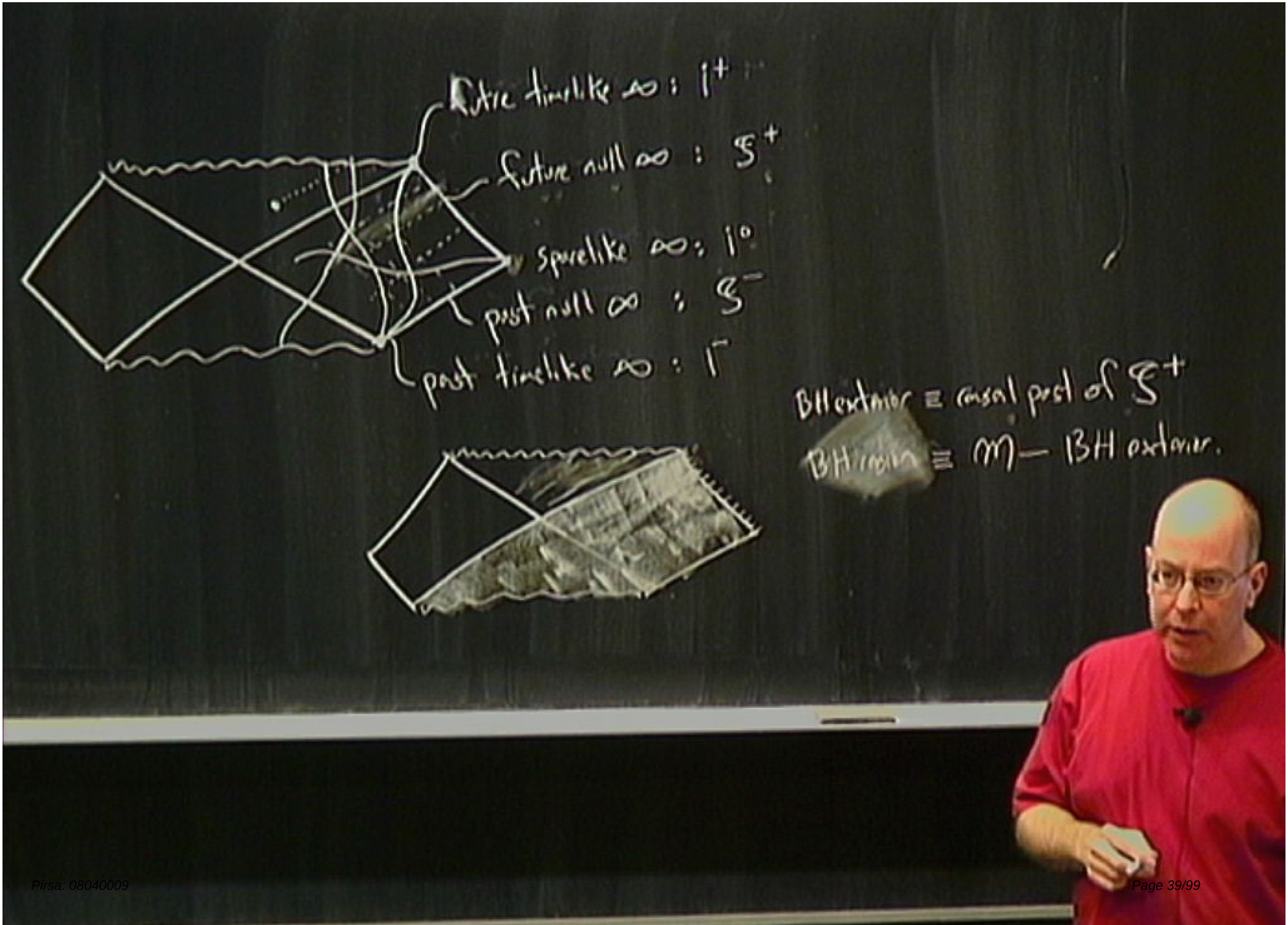
spacelike ∞ : i^0
past null ∞ : $\mathscr{I}^-$
past timelike ∞ : i^-
$\mathscr{I}^+$


Future timelike ∞ : i+
future null ∞ : ℐ+
spacelike ∞ : i0
past null ∞ : ℐ-
past timelike ∞ : i-
BH exterior ≡ causal past of ℐ+
BH interior ≡ M − BH exterior.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spacelike ∞ : i^0
past null ∞ : $\mathscr{I}^-$
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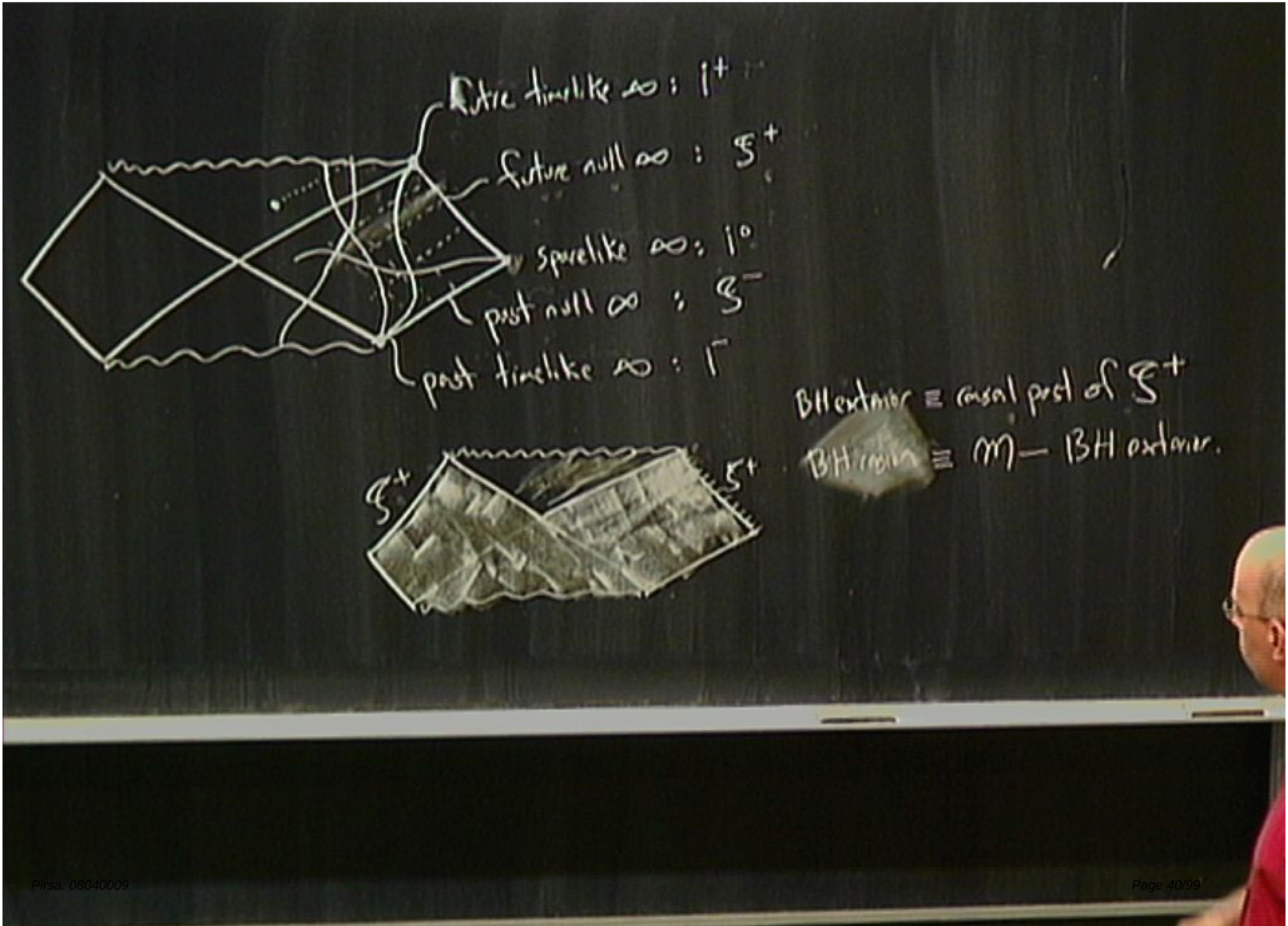
BH exterior ≡ causal past of $\mathscr{I}^+$
BH interior ≡ M − BH exterior.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future null ∞ : $\mathscr{I}^+$
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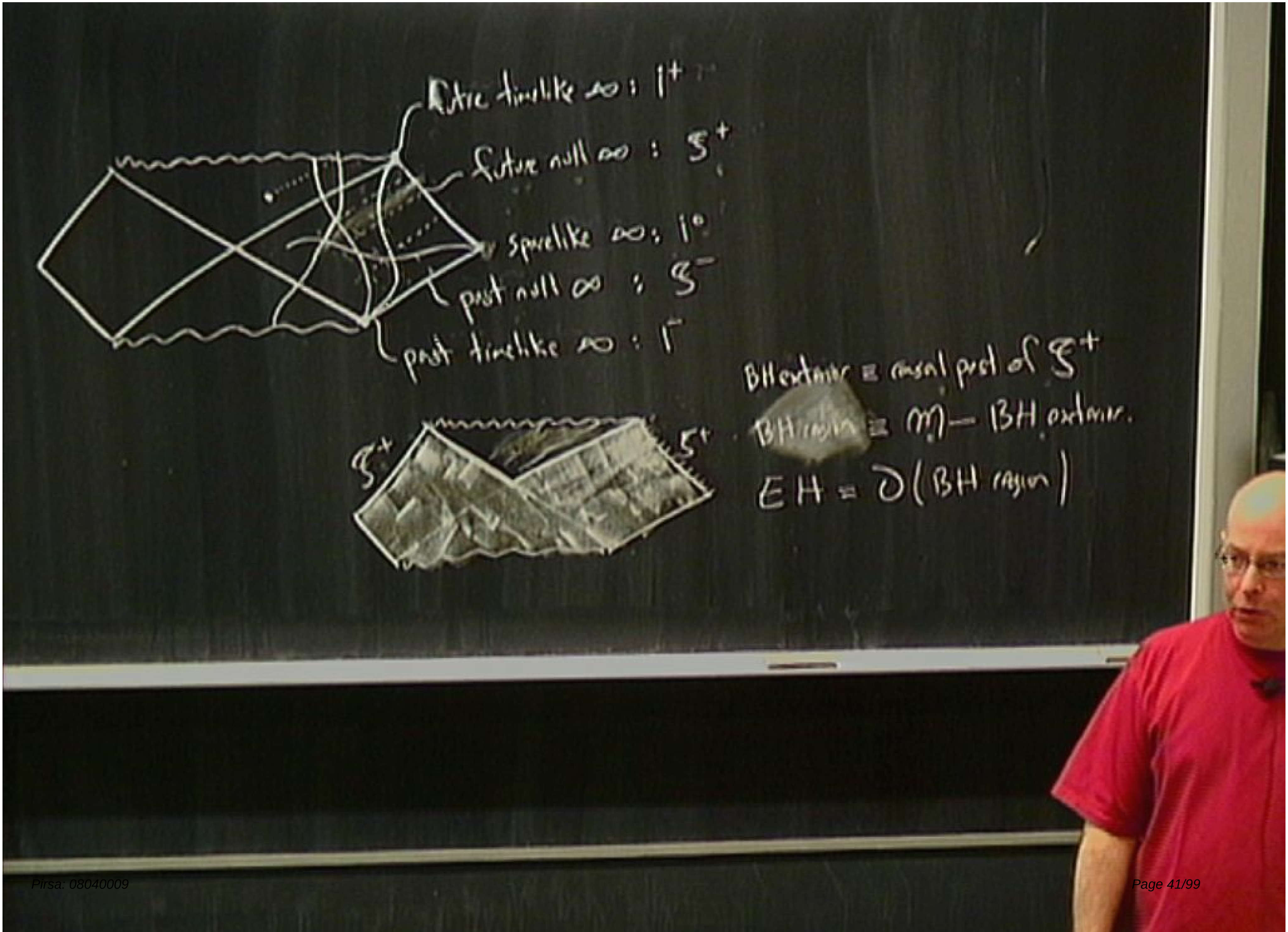
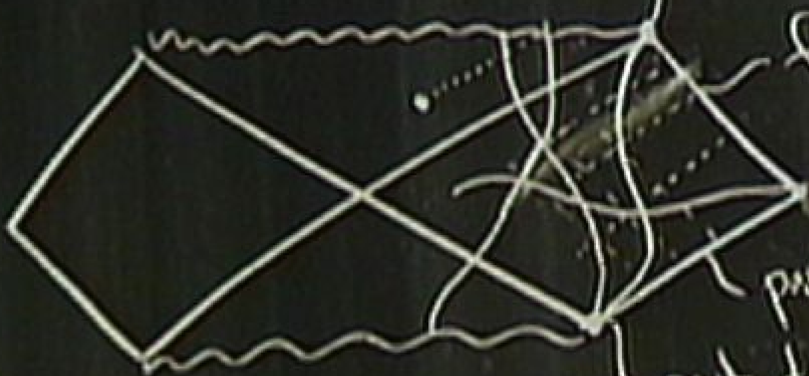
past null ∞ : $\mathscr{I}^-$
past timelike ∞ : i^-
BH exterior ≡ causal past of $\mathscr{I}^+$
BH region ≡ M − BH exterior.
$\mathscr{I}^+$
$\mathscr{I}^+$


future timelike ∞ : i+
future null ∞ : ℐ+
spacelike ∞ : i0
past null ∞ : ℐ−
past timelike ∞ : i−
BH exterior ≡ causal past of ℐ+
BH region ≡ M − BH exterior.
EH = ∂(BH region)
ℐ+
ℐ+



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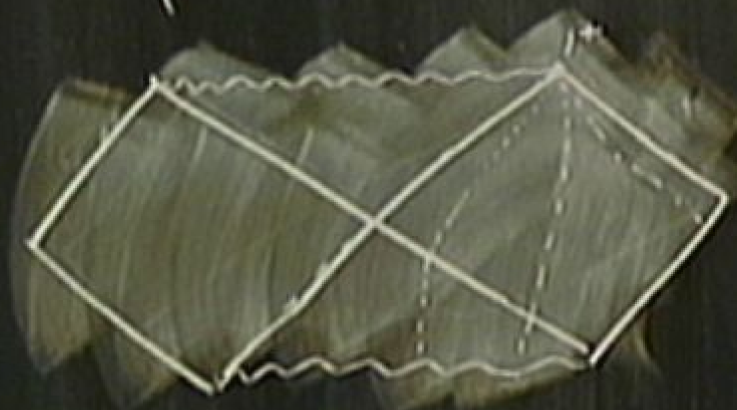
future null ∞ : $\mathscr{I}^+$

spacelike ∞ : i^0

past null ∞ : $\mathscr{I}^-$

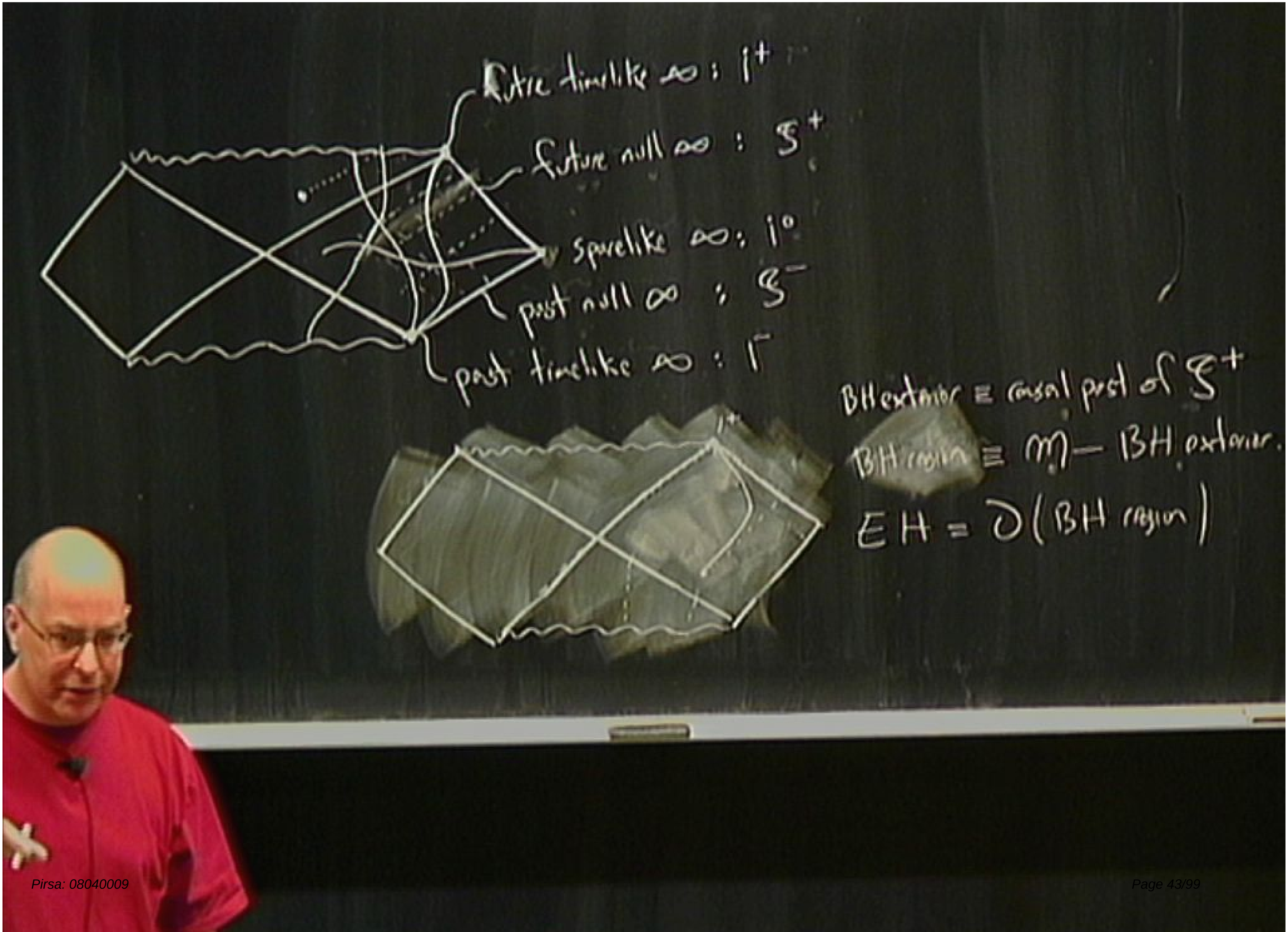
past timelike ∞ : i^-



BH exterior $\equiv$ causal past of $\mathscr{I}^+$

BH region $\equiv$ M $-$ BH exterior.

$EH = \partial(\text{BH region})$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future null ∞ : ℐ+
spacelike ∞ : i0
past null ∞ : ℐ−
past timelike ∞ : i−
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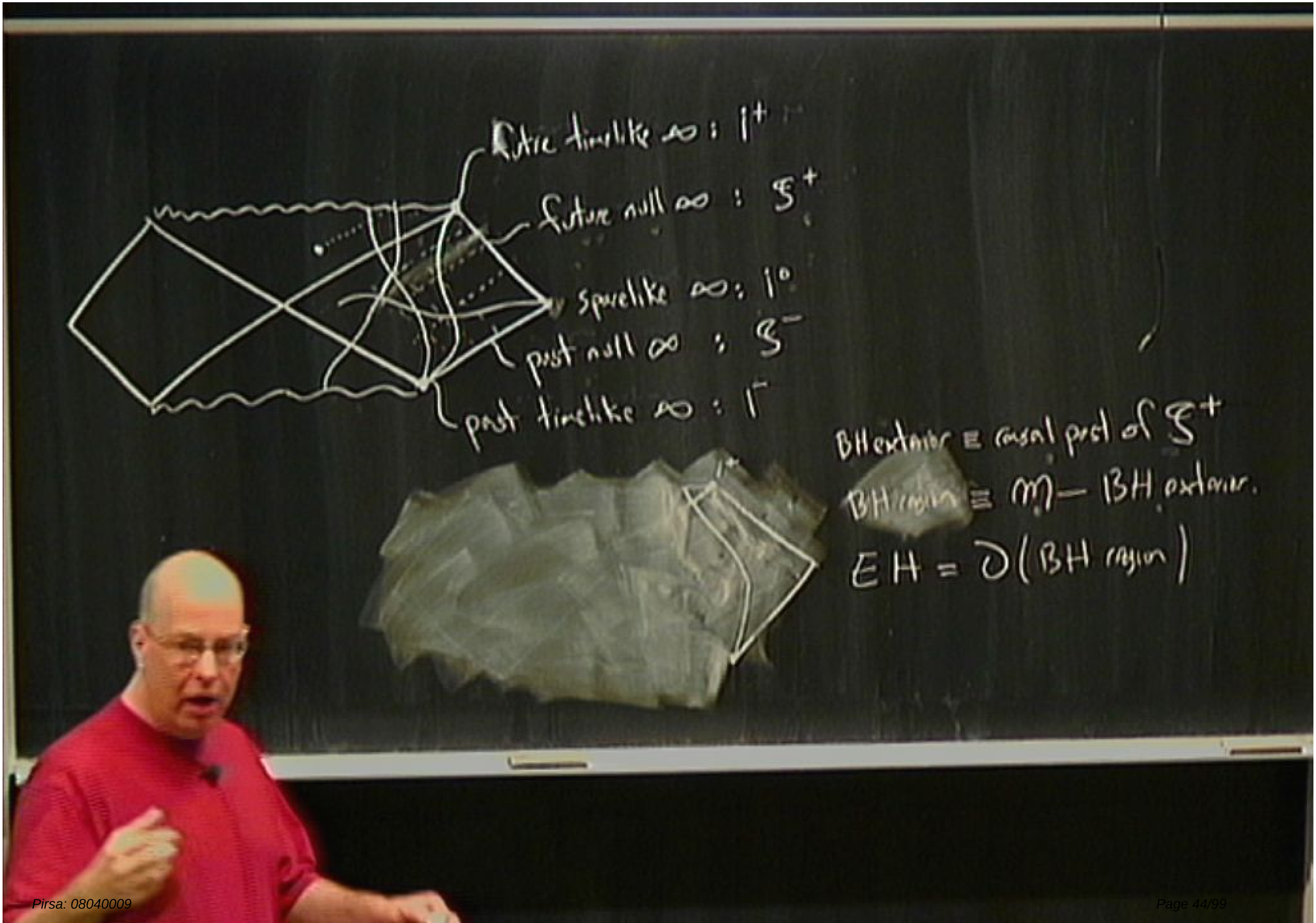
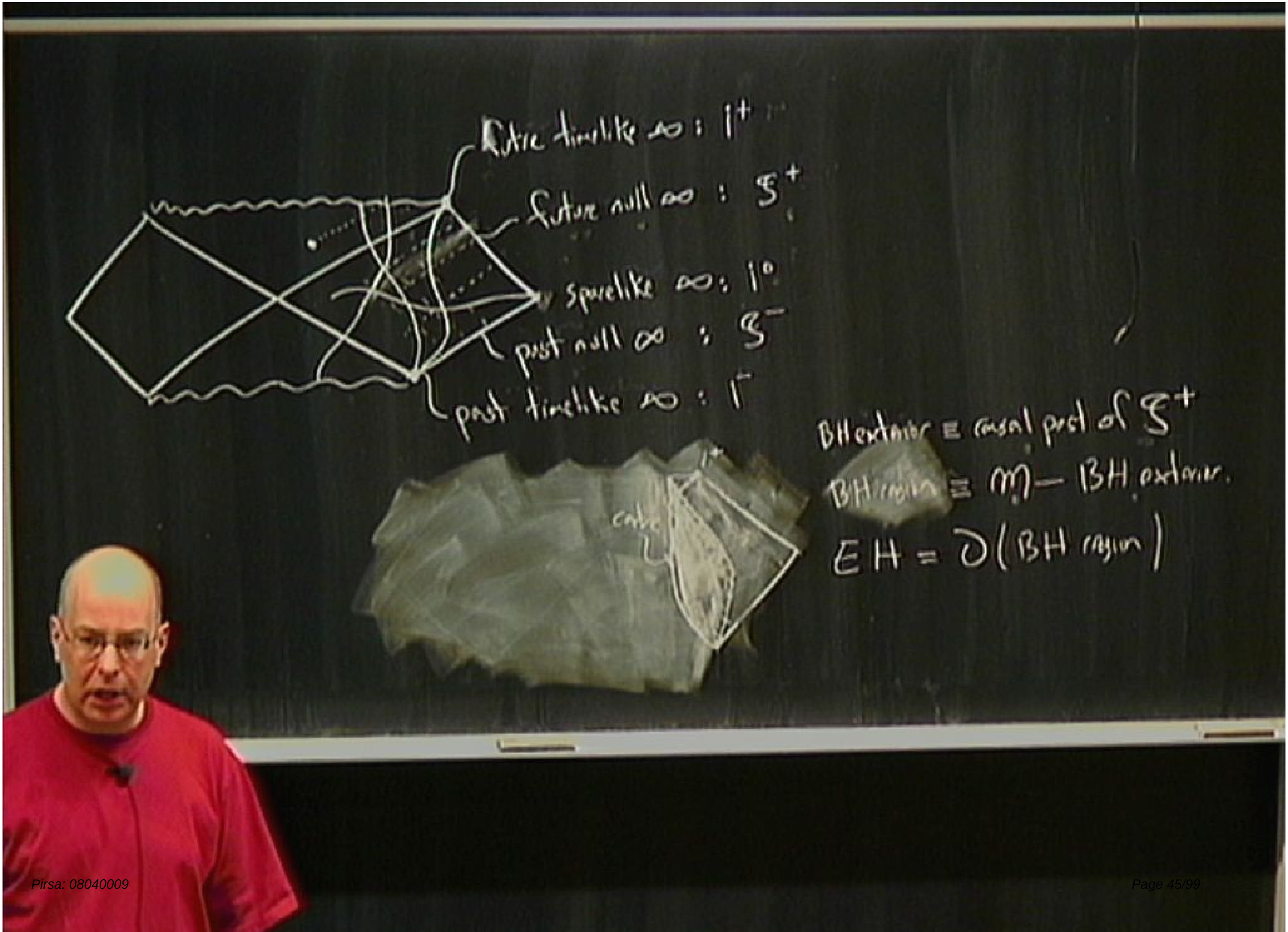
BH region ≡ M − BH exterior.
EH = ∂(BH region)



future timelike ∞ : i^+
future null ∞ : $\mathscr{I}^+$
spacelike ∞ : i^0
past null ∞ : $\mathscr{I}^-$
past timelike ∞ : i^-
BH exterior ≡ causal past of $\mathscr{I}^+$
BH region ≡ M − BH exterior.
EH = ∂(BH region)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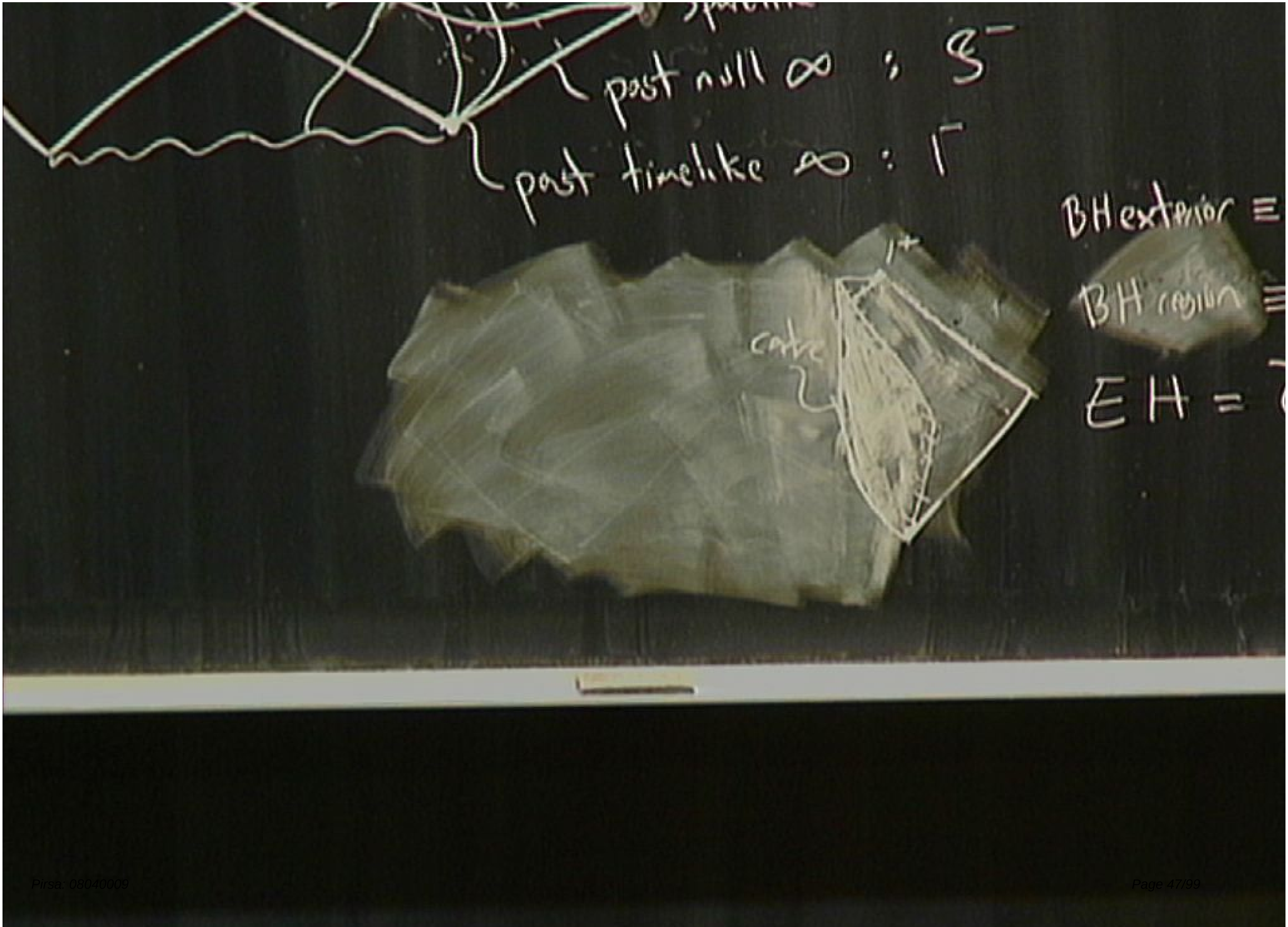
past timelike ∞ : i^-
BH exterior ≡ causal past of $\mathscr{I}^+$
BH region ≡ M − BH exterior.
EH = ∂(BH region)

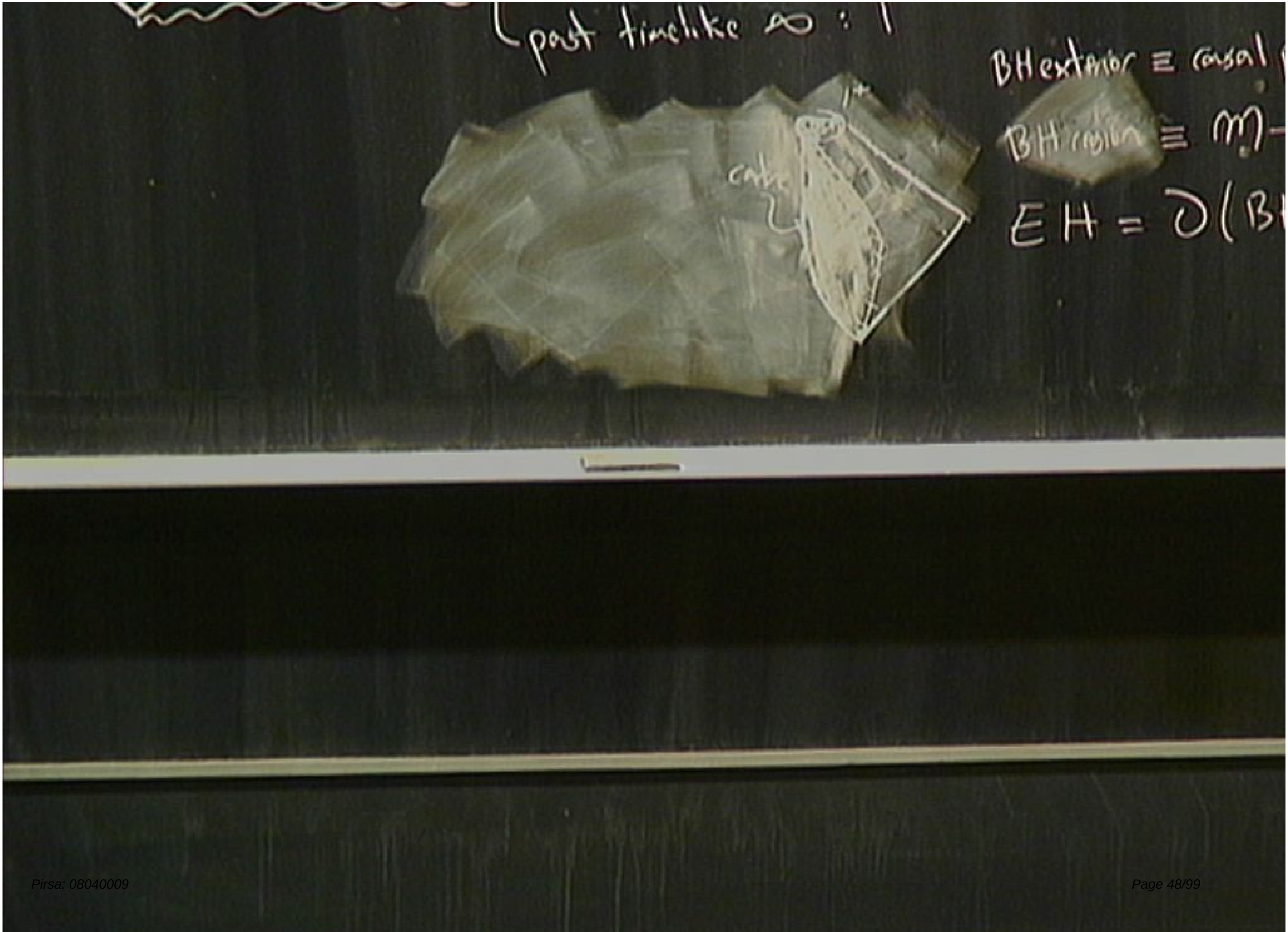
past timelike ∞ : Γ

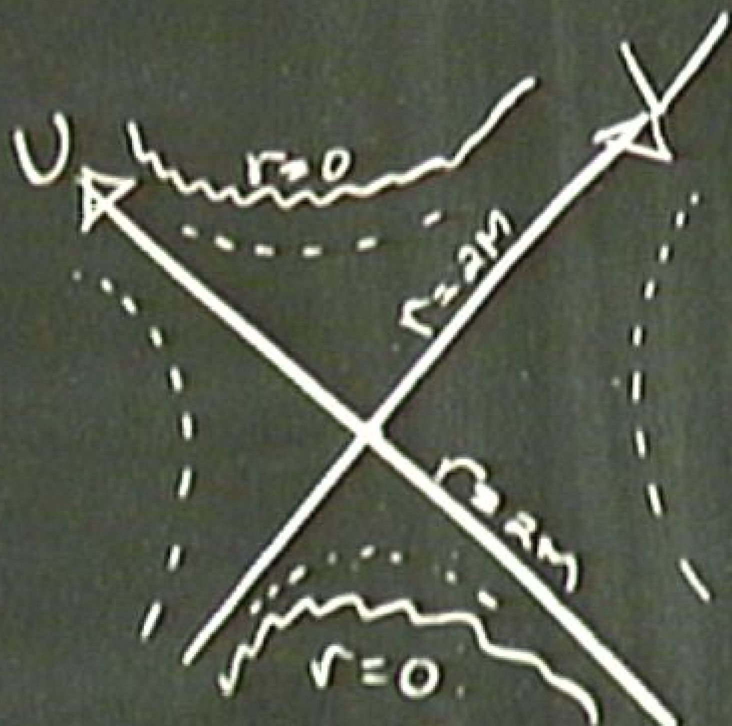
BH exterior =

BH region

EH =


past null ∞ : ℐ⁻
past timelike ∞ : i⁻
BH exterior ≡
BH region ≡
EH =
centre
i⁺


past timelike ∞ :
BHexterior ≡ causal
BH region ≡ M−
EH = ∂(B

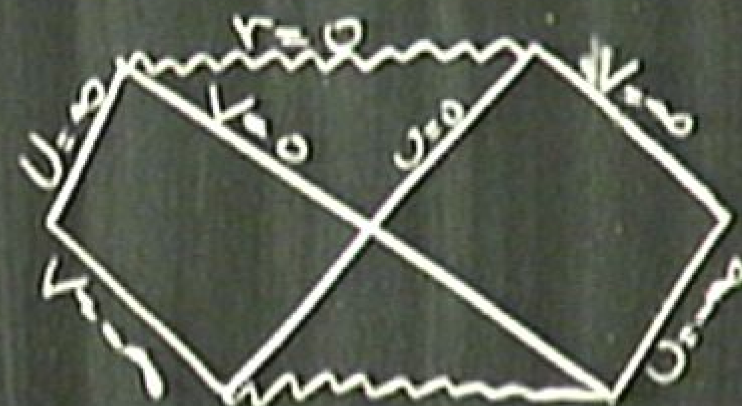


Penrose-Carter diagram:

$$\tilde{U} = \arctan U$$

$$\tilde{V} = \arctan V$$

$$-\frac{\pi}{2} < (\tilde{U}, \tilde{V}) < \frac{\pi}{2}$$



SCHWARZSCHILD

$$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2$$

$f(r)$ such that $f(r_0) = 0$

$$\boxed{\begin{array}{l} f = 1 - \frac{2m}{r} \\ r_0 = 2m \end{array}}$$

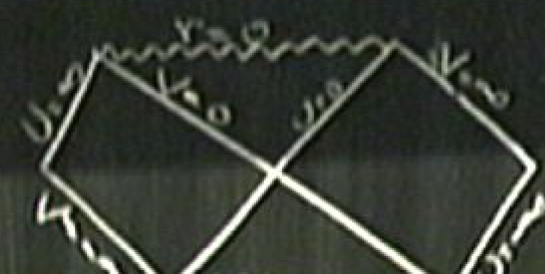
$$\left.\begin{array}{l} U = t - r^* \\ V = t + r^* \end{array}\right\} r^* = \int \frac{dr}{f(r)}$$

Kruskal: $\;\mathcal{U} = -e^{-\kappa U}$

$$\mathcal{V} = e^{\kappa V}$$

$$\left.\boxed{\begin{array}{l} \kappa = \frac{1}{2} f'(r_0) \\ \quad = \frac{m}{r_0^2} = \frac{1}{4M} \\ \quad = \text{surface gravity} \end{array}}\right\} \text{Schw.}$$

$$\mathcal{U}\mathcal{V} = -e^{\kappa(V-U)}$$
$$= -e^{2\kappa r^*}$$



Kruskal: $U = - e$

$V = e$

$$UV = - e^{k(v-u)}$$

$$= - e^{2k r^*}$$

Kruskal: $U = -e^{-ku}$

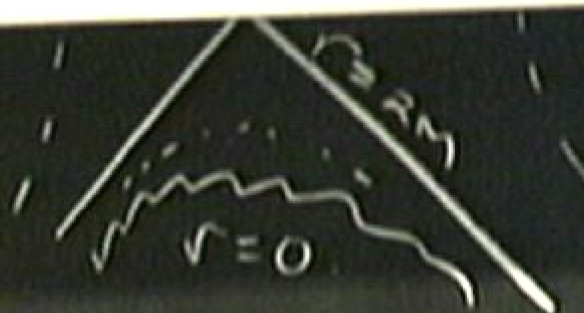
$V = e^{kv}$

$\boxed{k = \ldots}$

$UV = -e^{k(v-u)}$

$= -e^{2kr^*}$

$r(UV)$



Kruskal: $U = -e^{-\kappa u}$

$V = e^{\kappa v}$

$\kappa =$

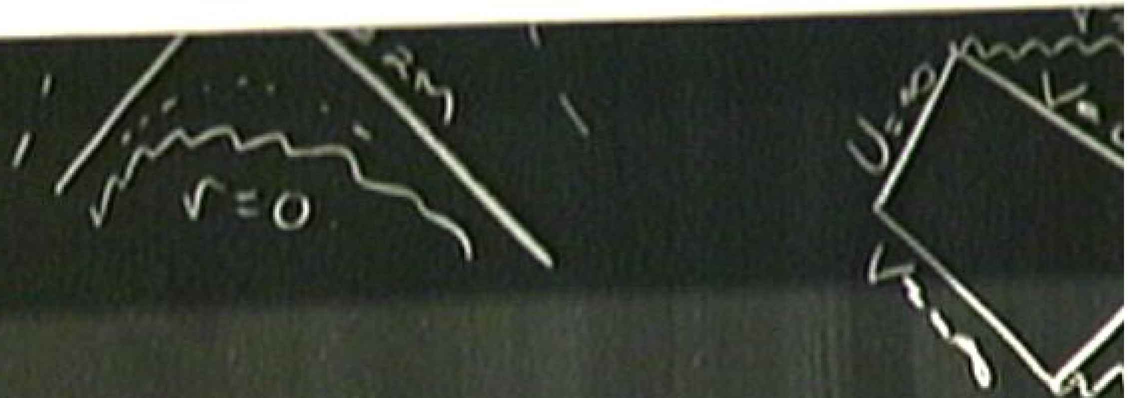
$=$

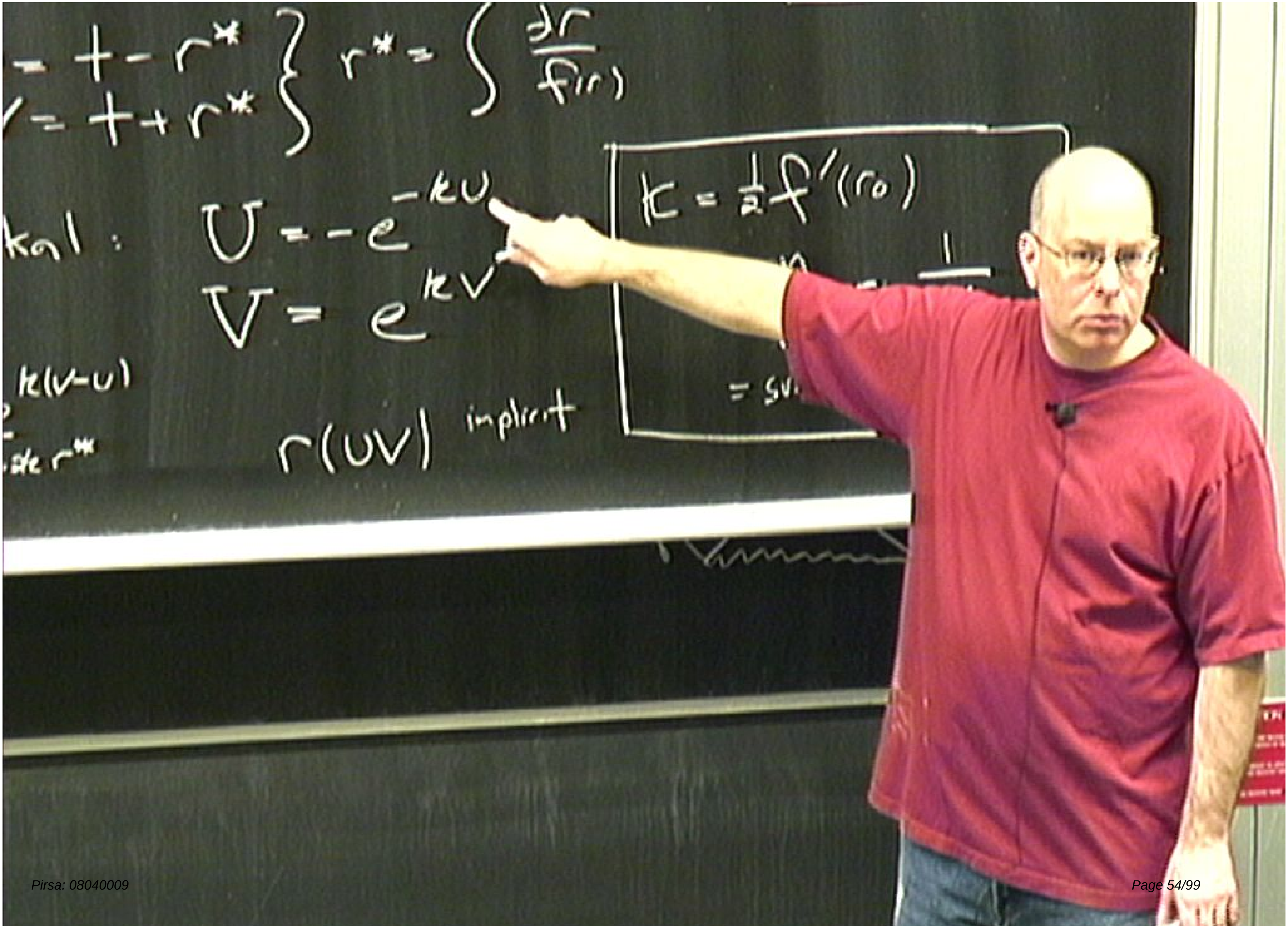
$=$

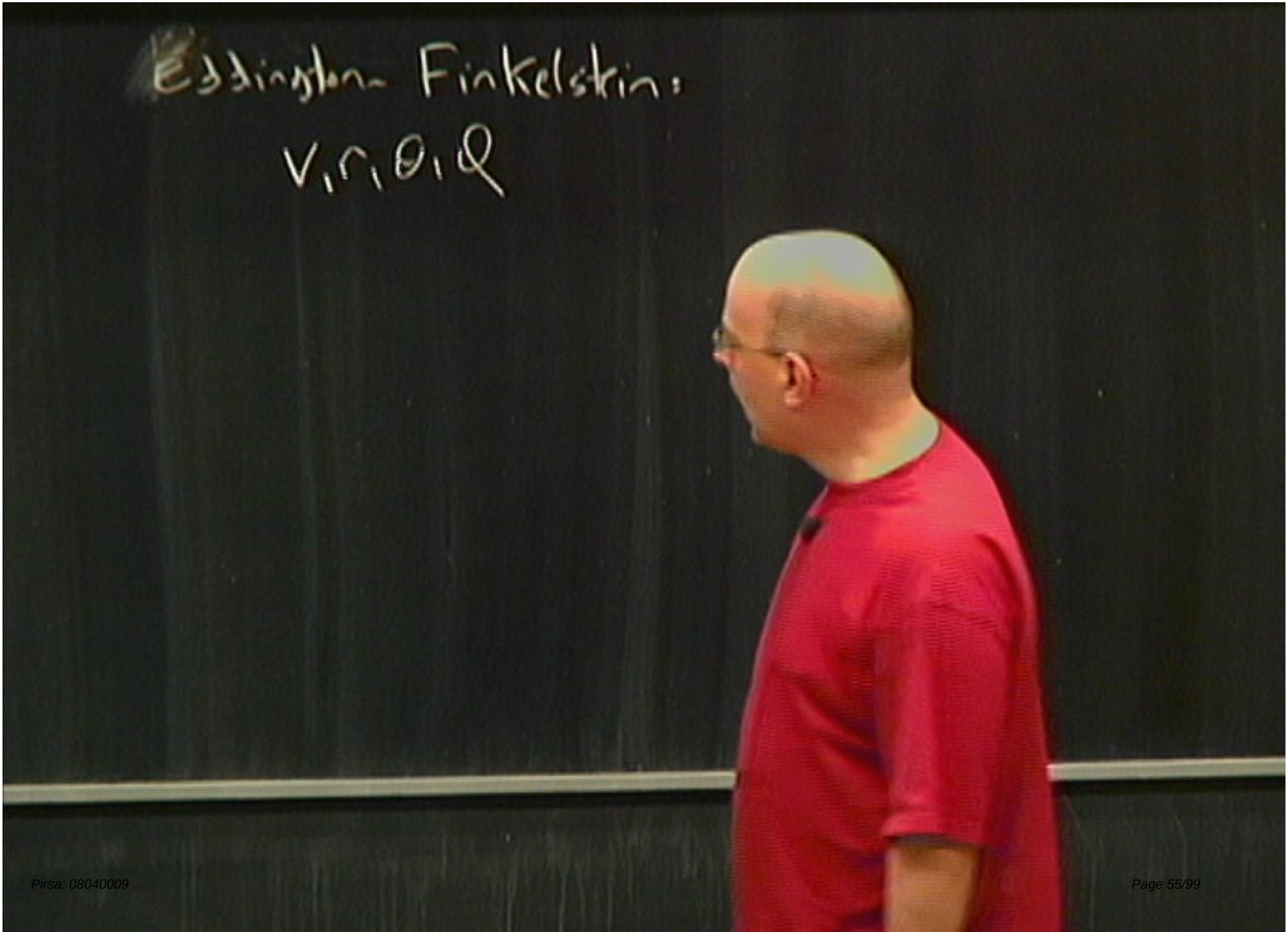
$UV = -e^{\kappa(v-u)}$

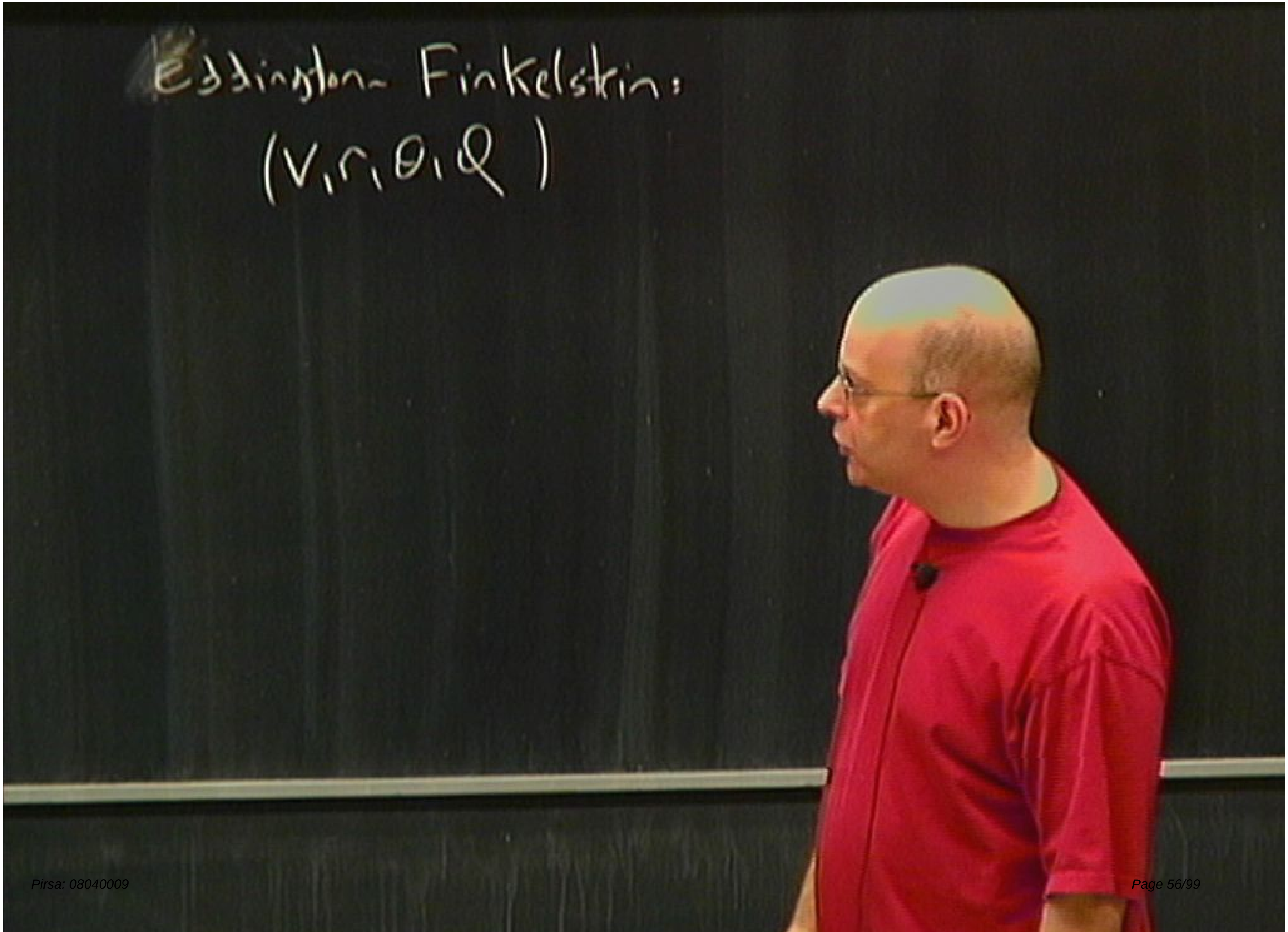
$= -e^{2\kappa r^*}$

$r(UV)$ implicit




= t - r*
V = t + r*
r* = ∫ dr/f(r)
kal :
U = - e^{-kU}
V = e^{kV}
k(v-u)
r(UV) implicit
k = ½ f'(r₀)

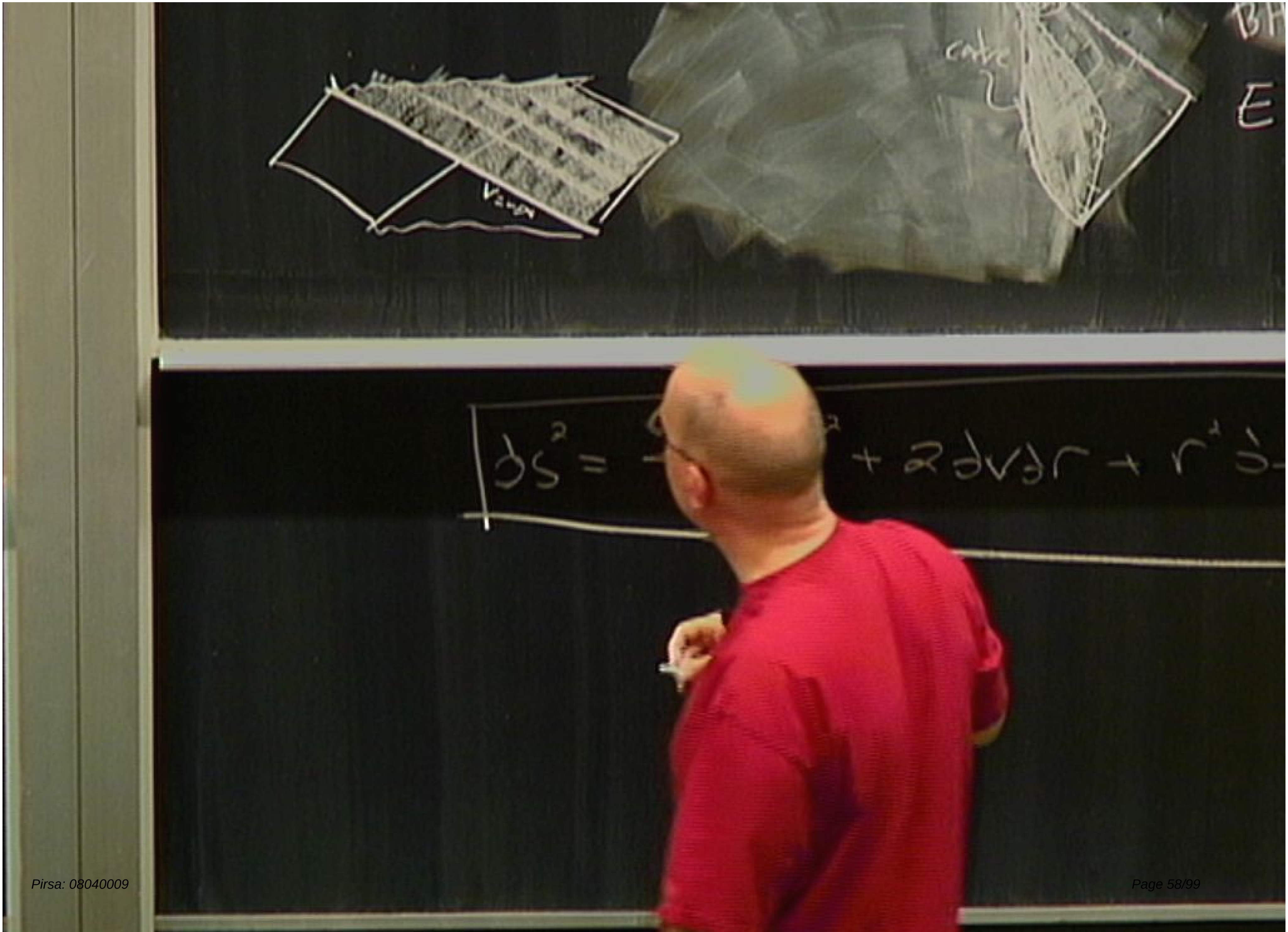

Eddington Finkelstein:
v, r, θ, φ

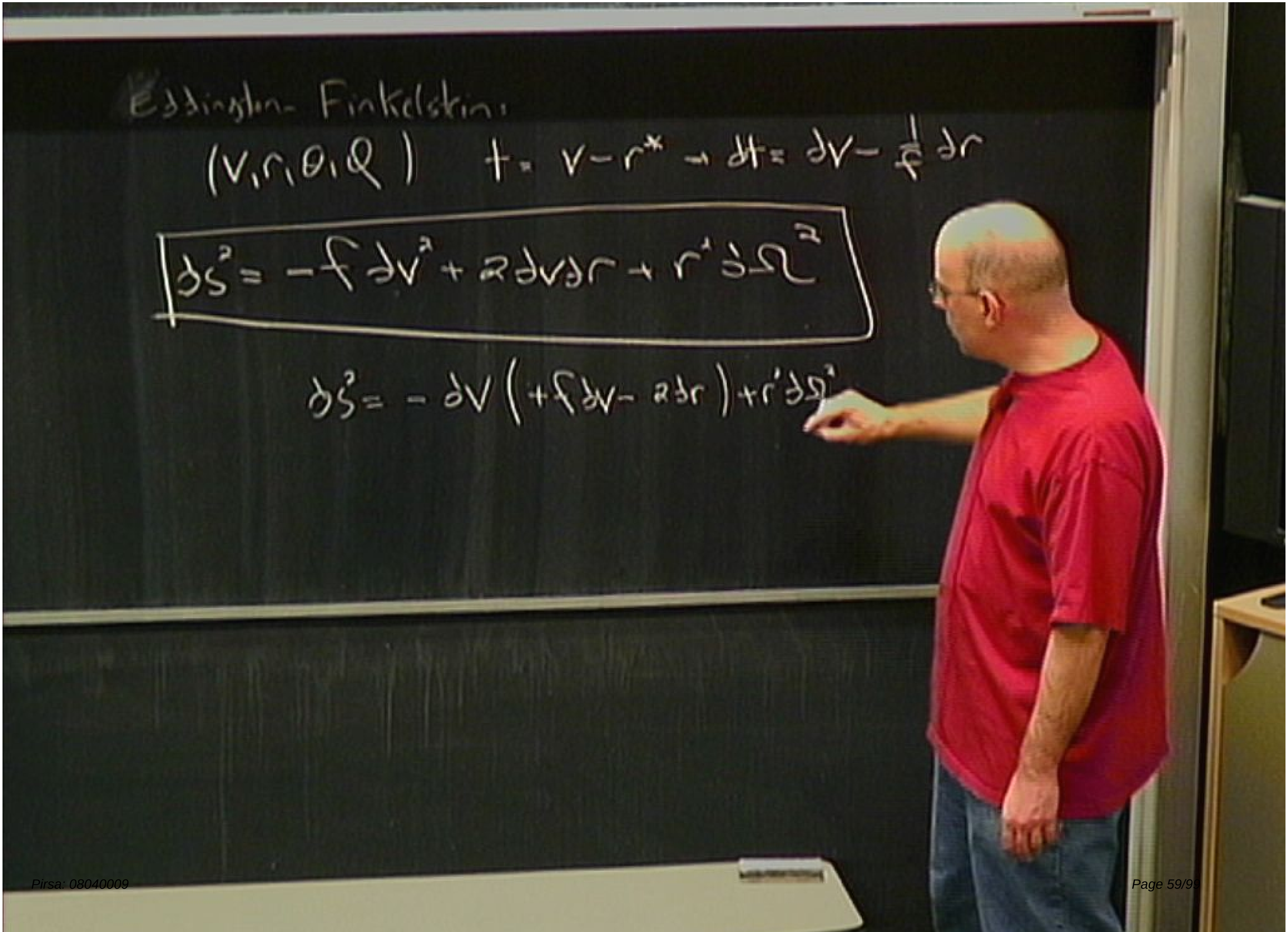

Eddington- Finkelstein:
(v, r, θ, φ)

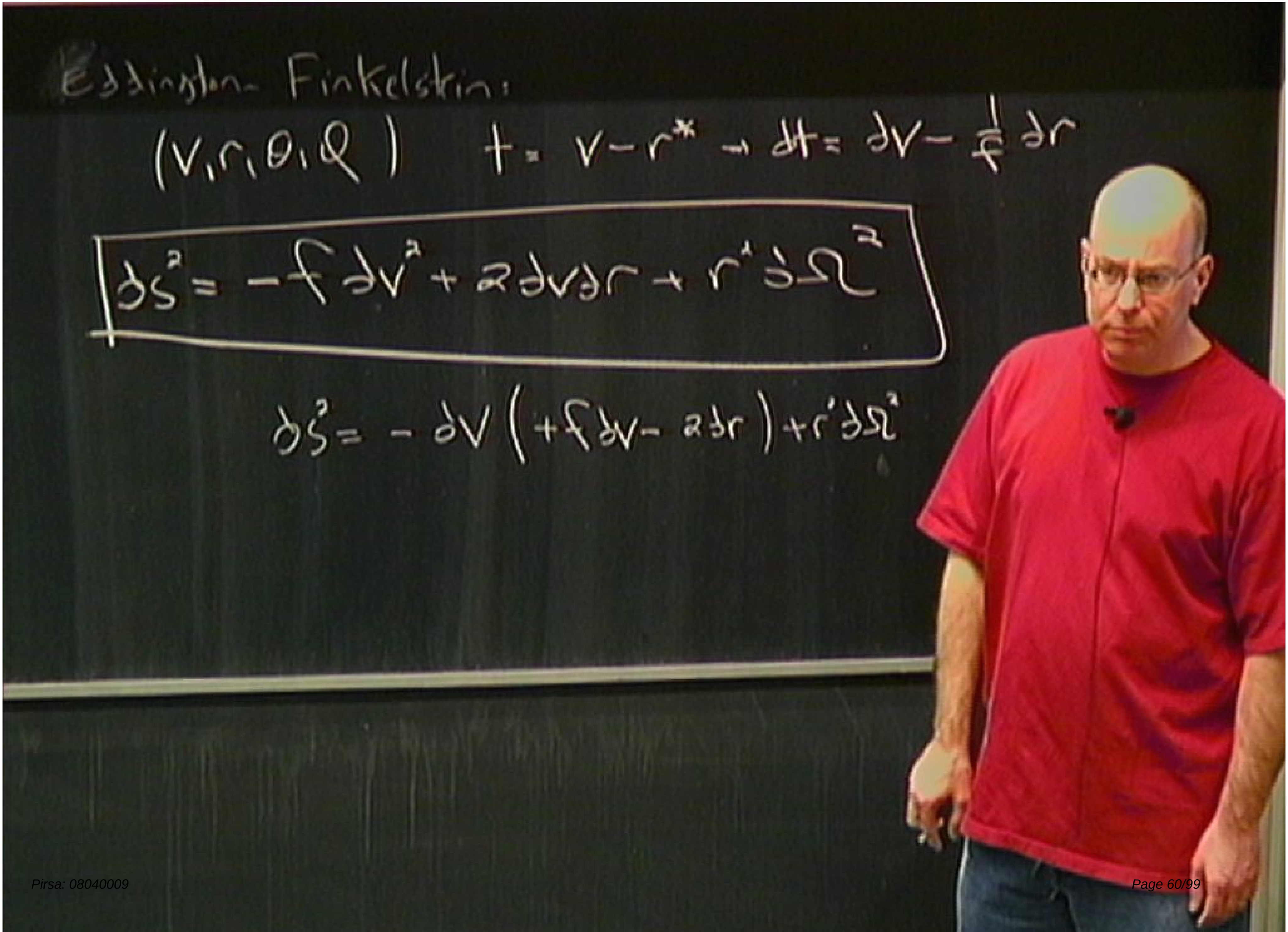
Eddington-Finkelstein:

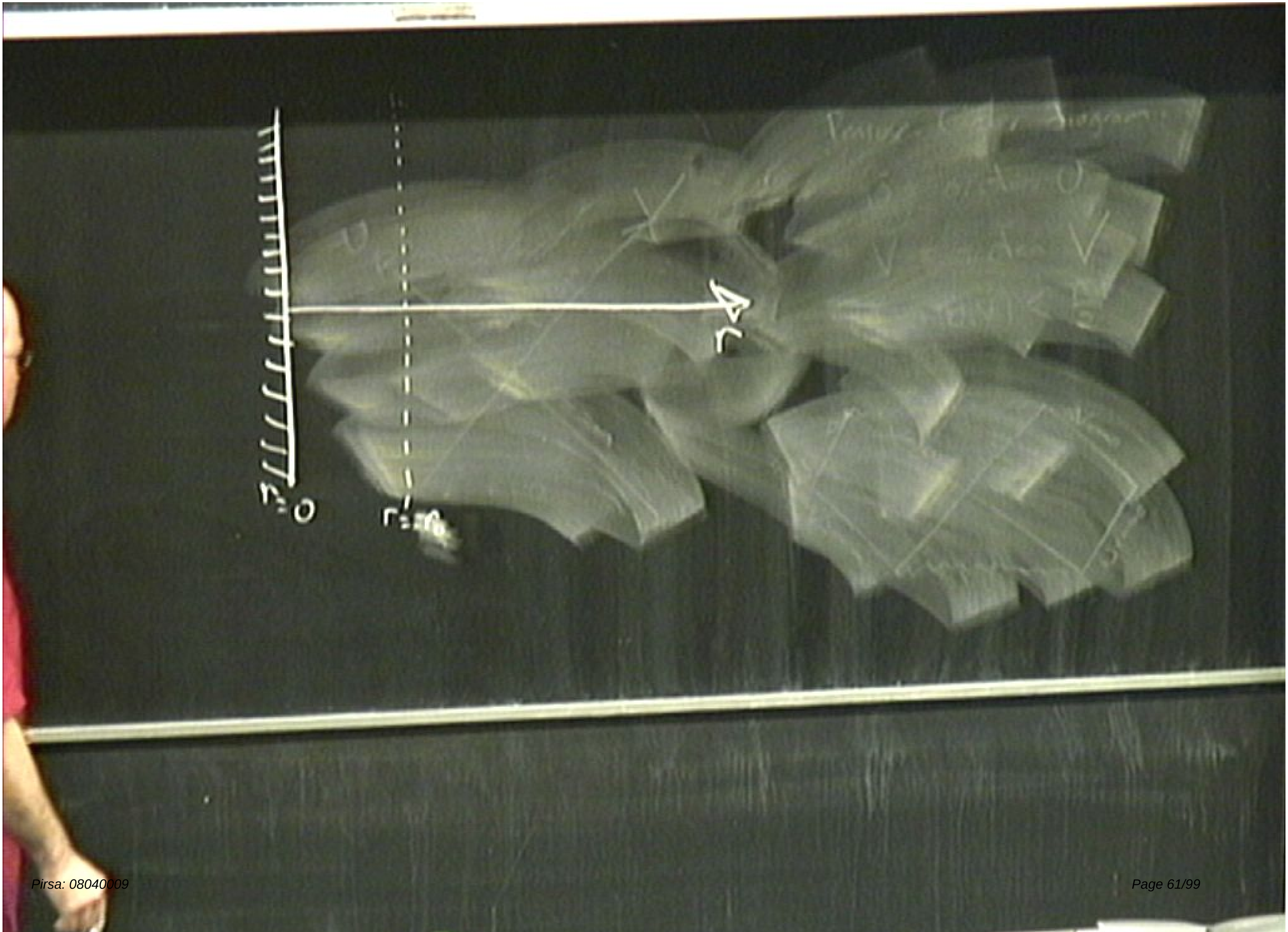
$(v, r, \theta, \varphi) \qquad t = v - r^* \rightarrow dt = dv - \frac{1}{f} dr$

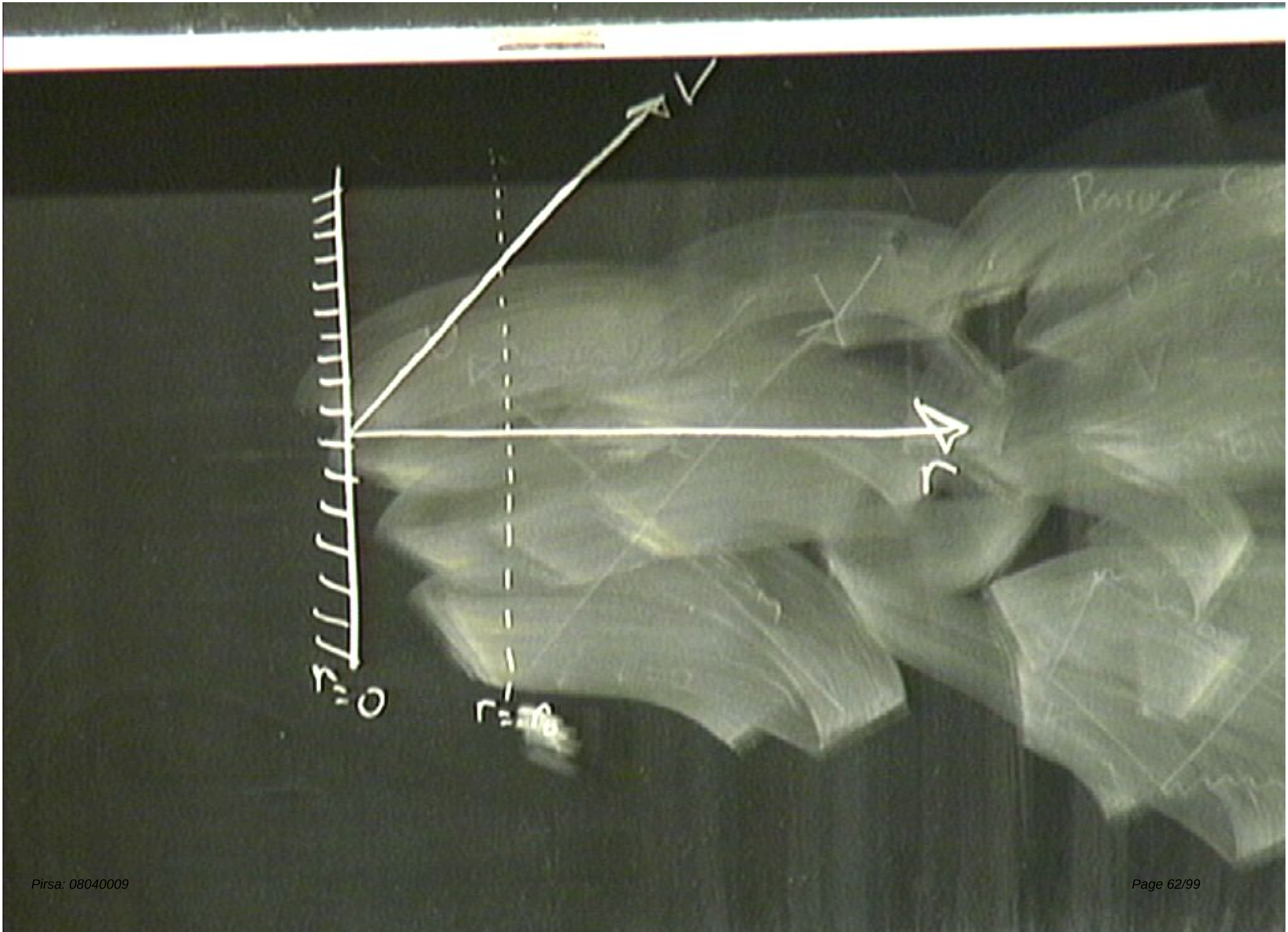
$$\boxed{ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2 d\Omega^2}$$

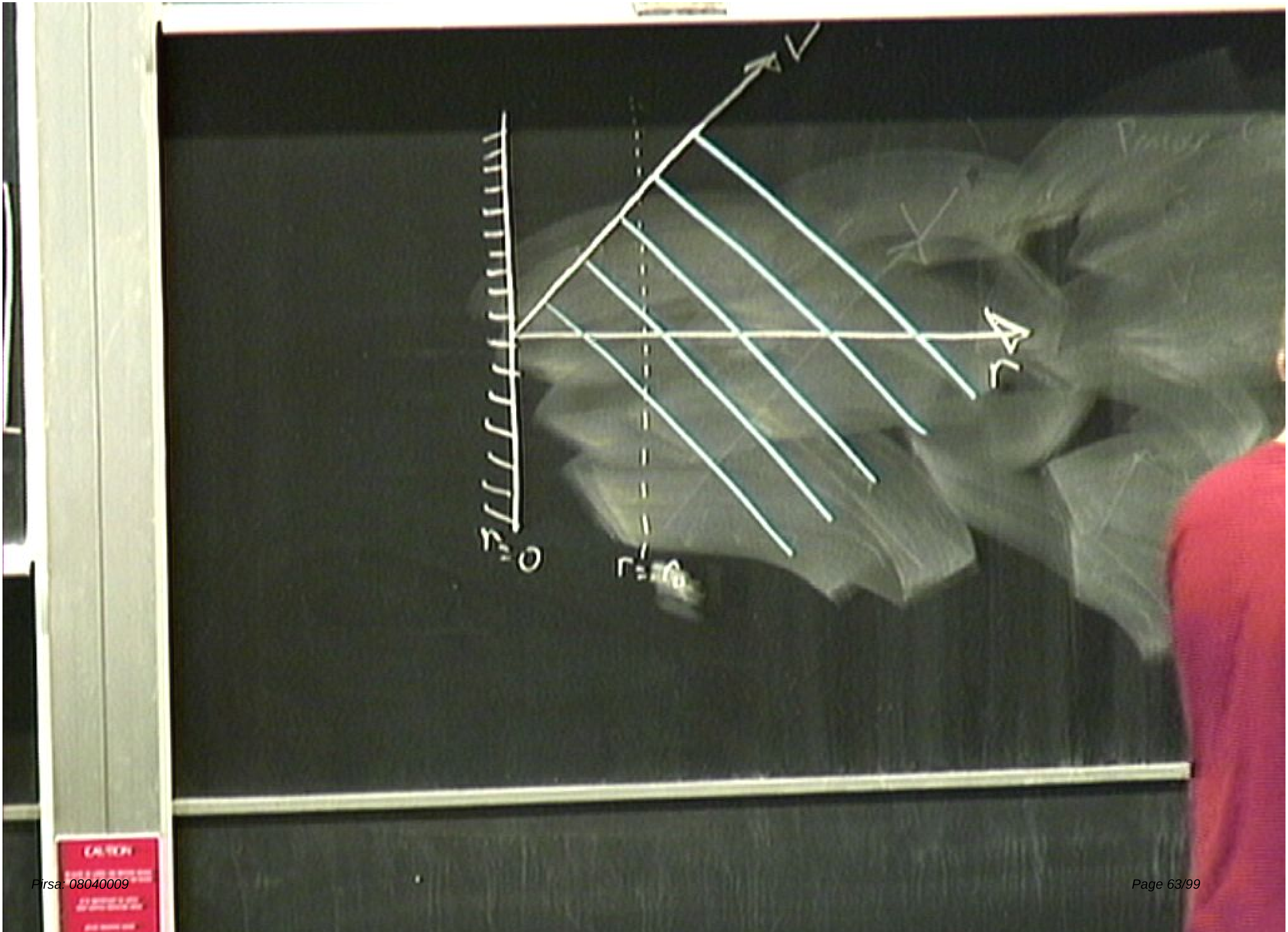

ds² =
+ 2dvdr + r²d

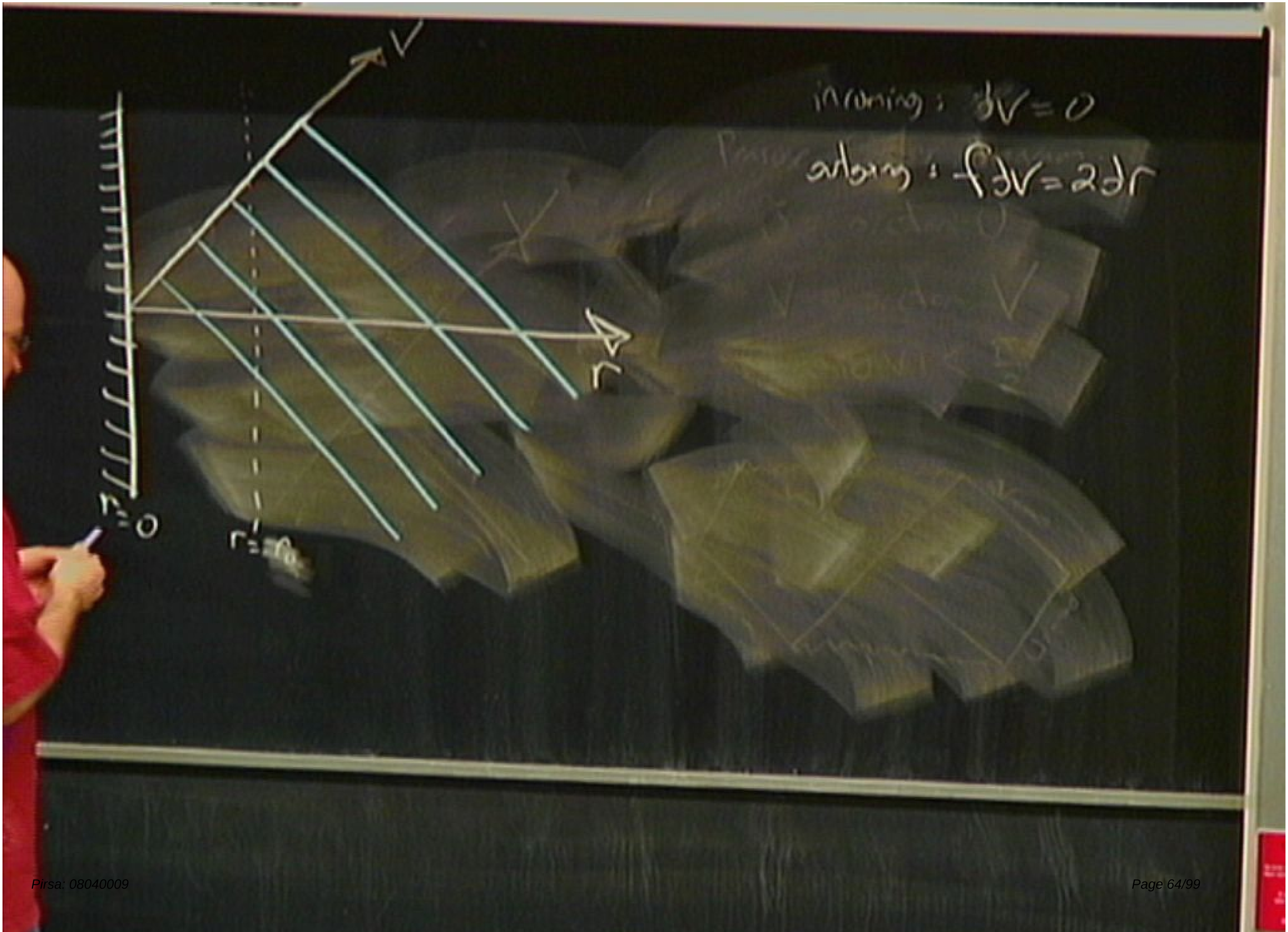

Eddington-Finkelstein:
(v, r, θ, φ) $t = v - r^* \rightarrow dt = dv - \frac{1}{f} dr$
$\boxed{ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2 d\Omega^2}$
$ds^2 = -dv\,(+f\,dv - 2\,dr) + r^2 d\Omega^2$


Eddington-Finkelstein:
(v, r, θ, φ) $t = v - r^* \rightarrow dt = dv - \frac{1}{f} dr$
$\boxed{ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2 d\Omega^2}$
$ds^2 = -dv\,(+f\,dv - 2\,dr) + r^2 d\Omega^2$

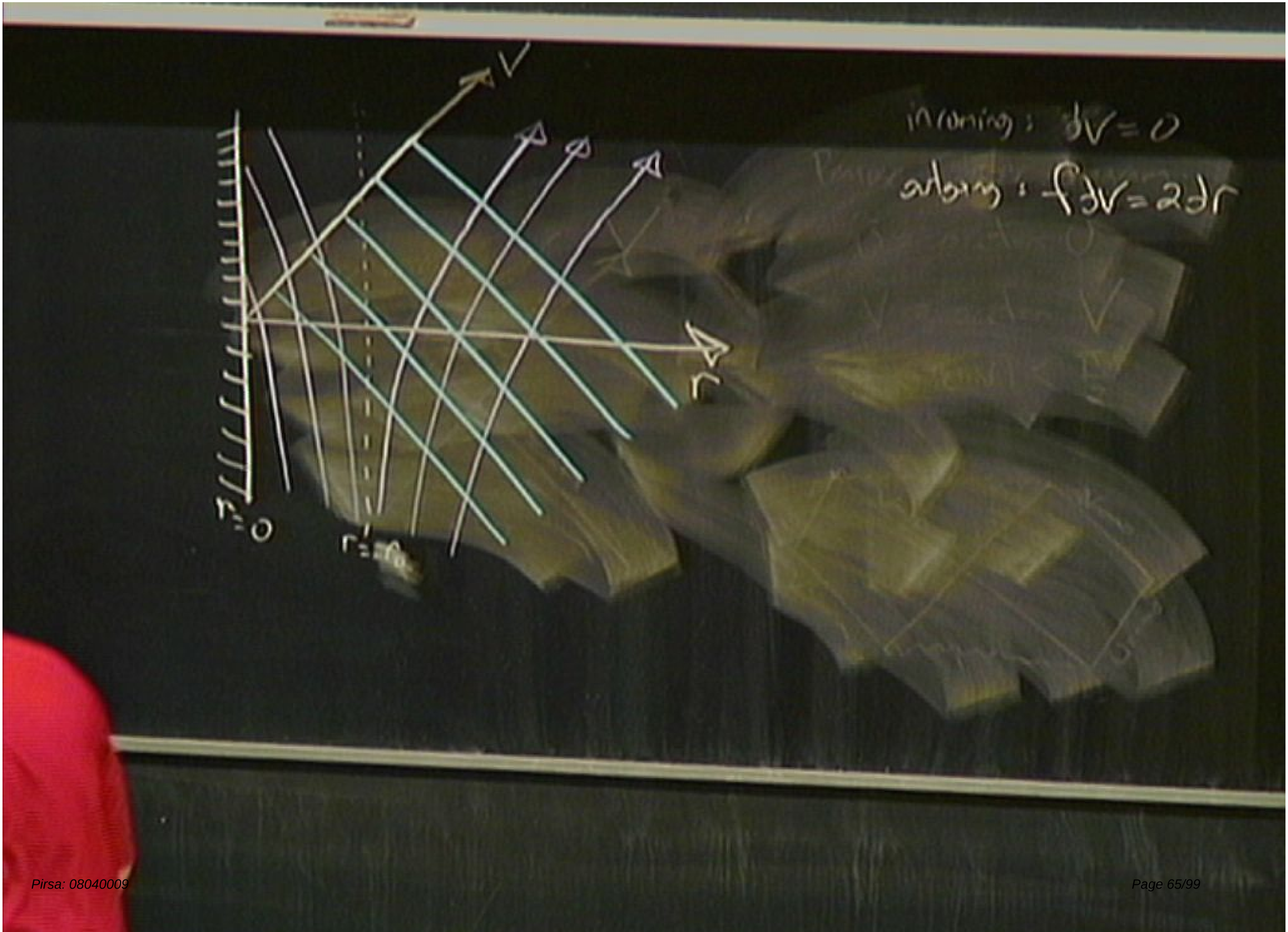

r=0
r

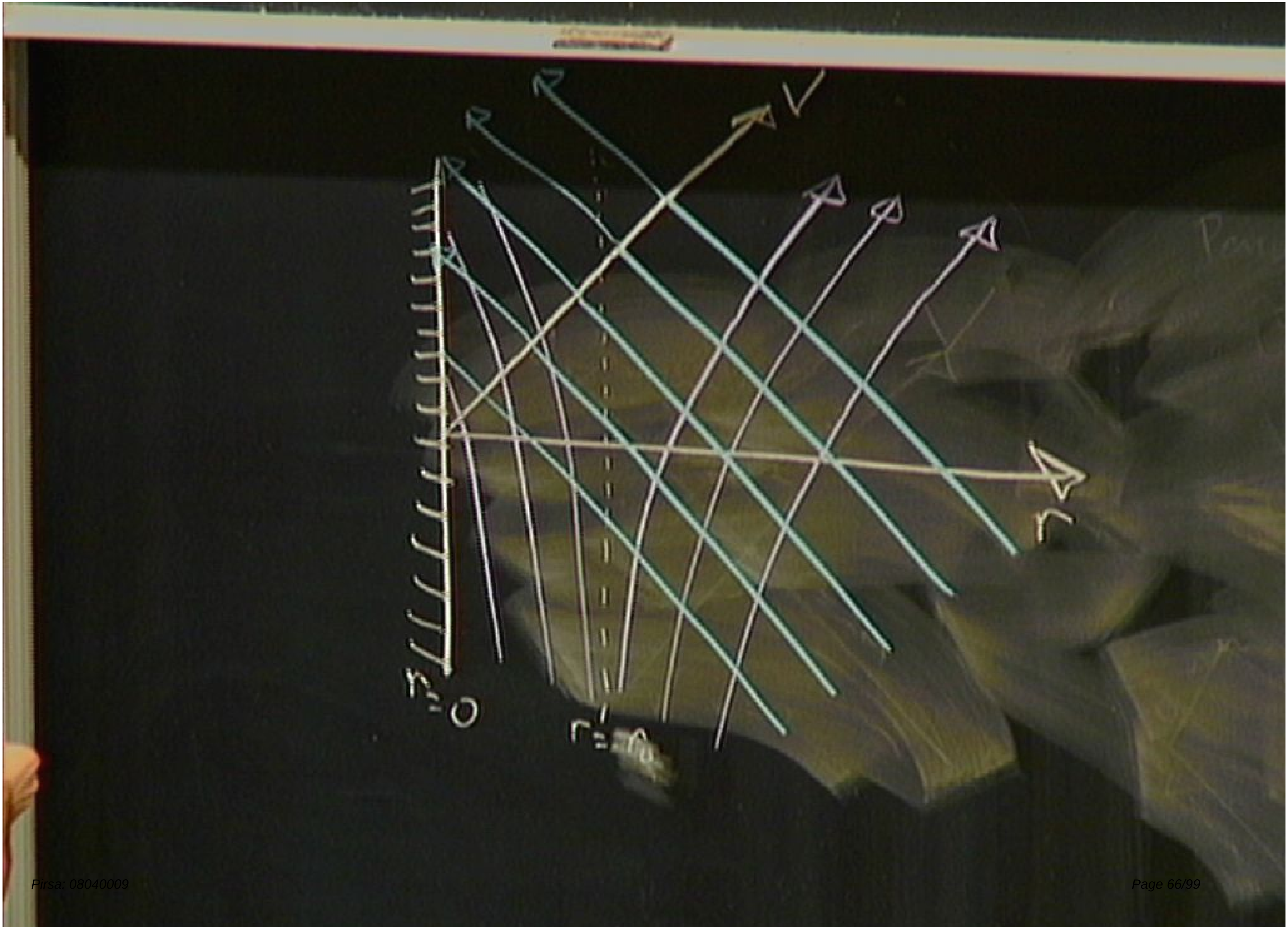

v
r
r=0

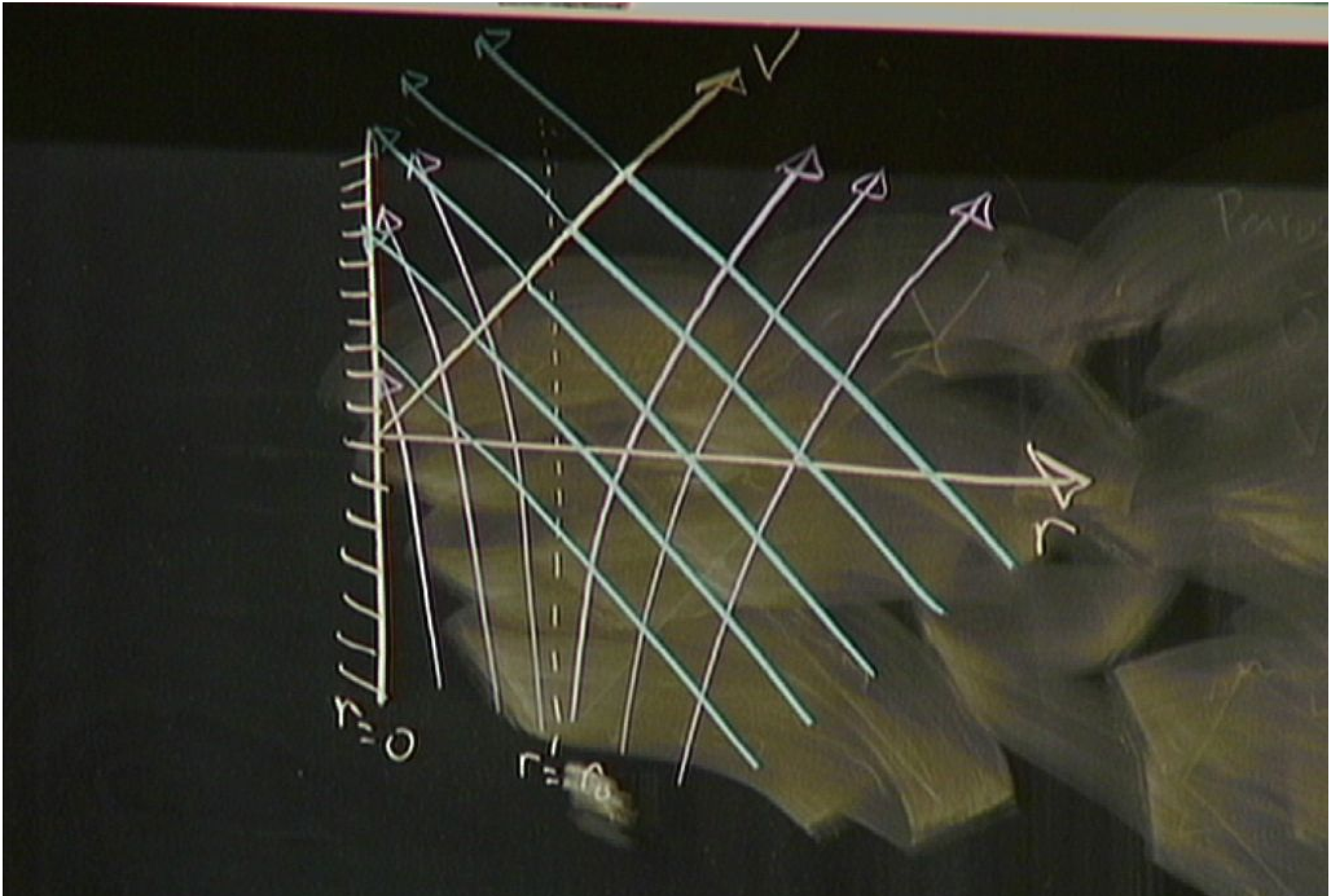

v
r
r=0

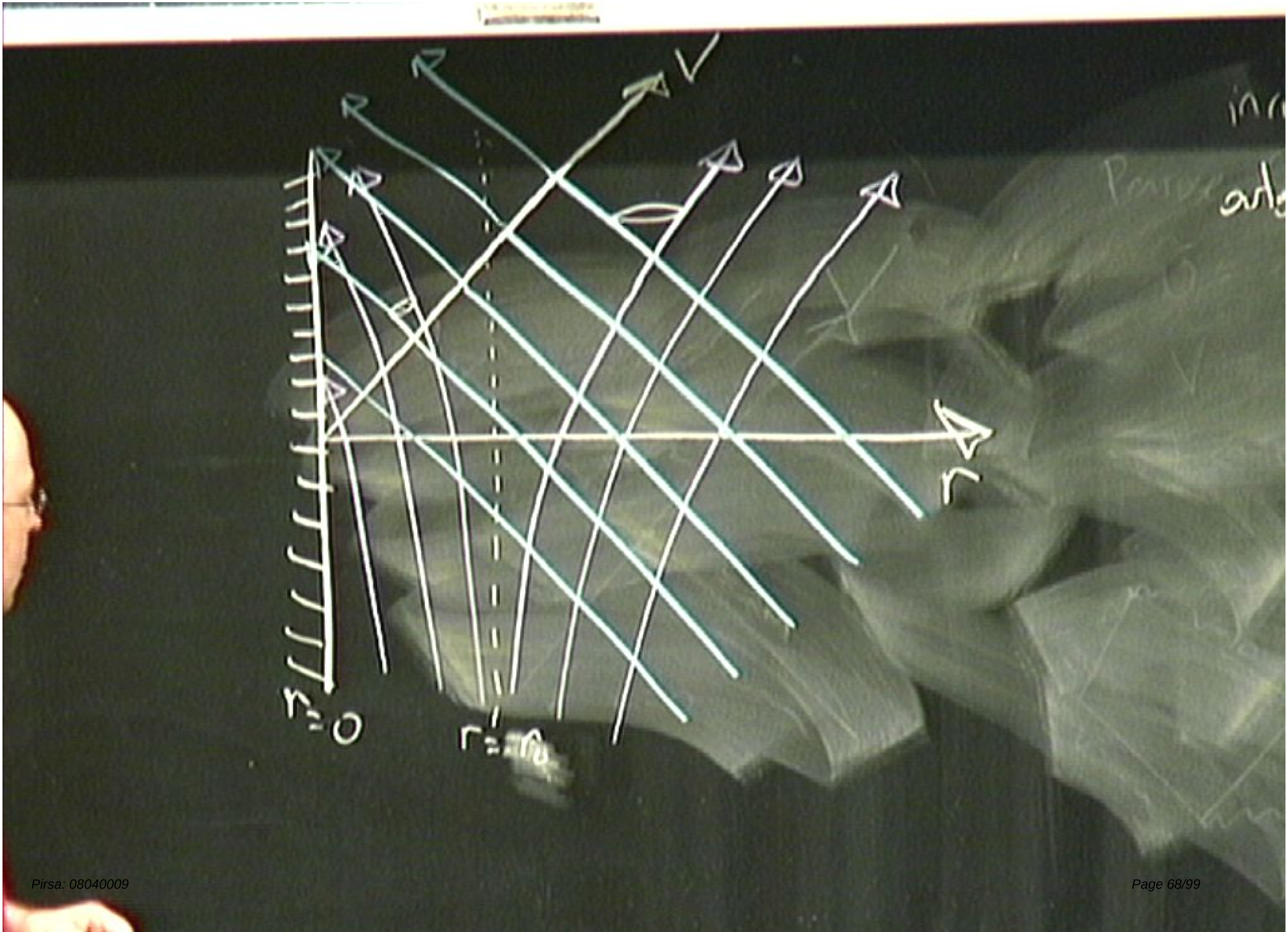


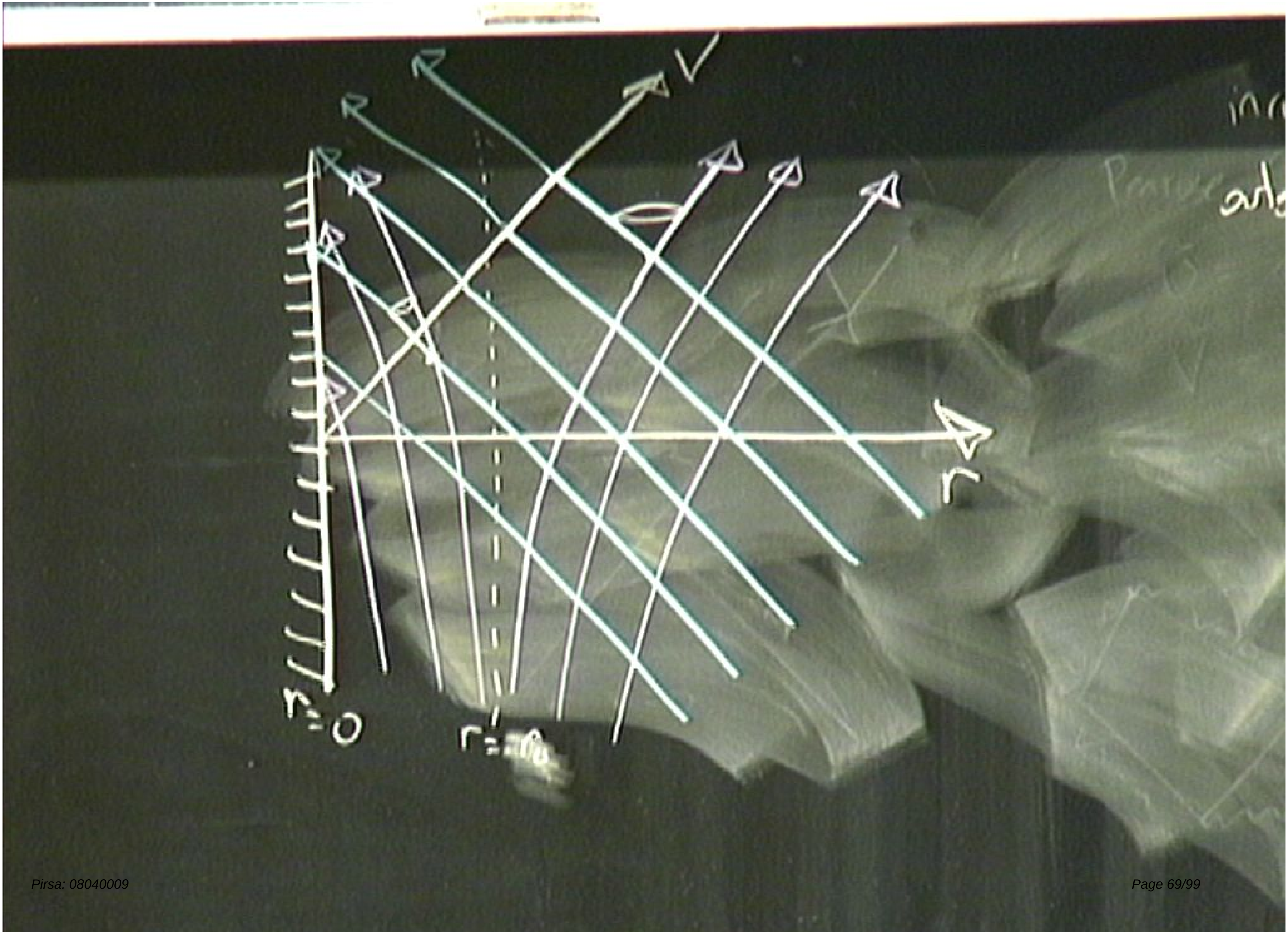
incoming : ∂v = 0
outgoing : f∂v = 2∂r
v
r
r=0
r=r0


v
incoming : ∂v = 0
outgoing : f∂v = 2∂r
r
r=0


v
r
r=0

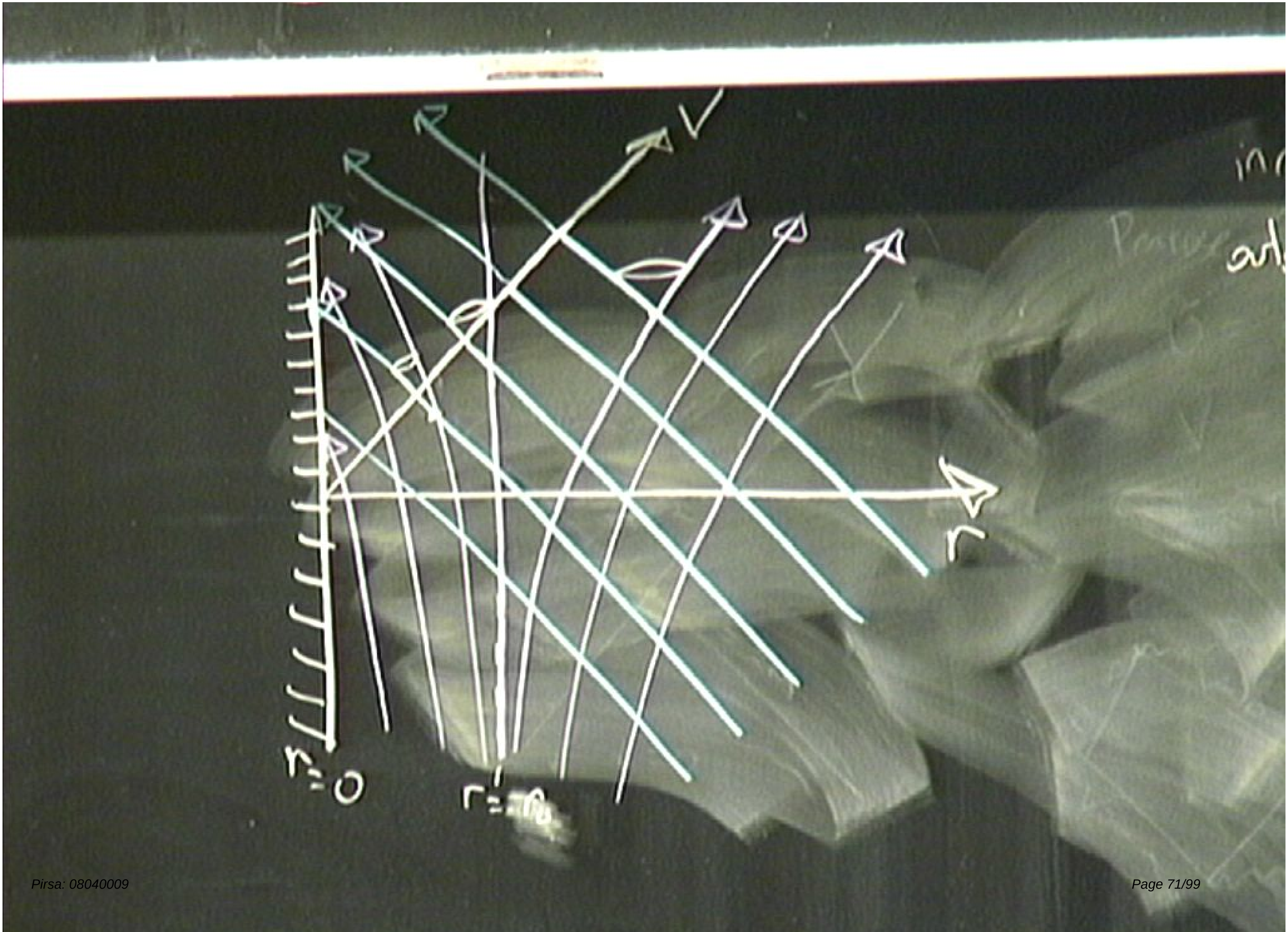

v
r
r=0

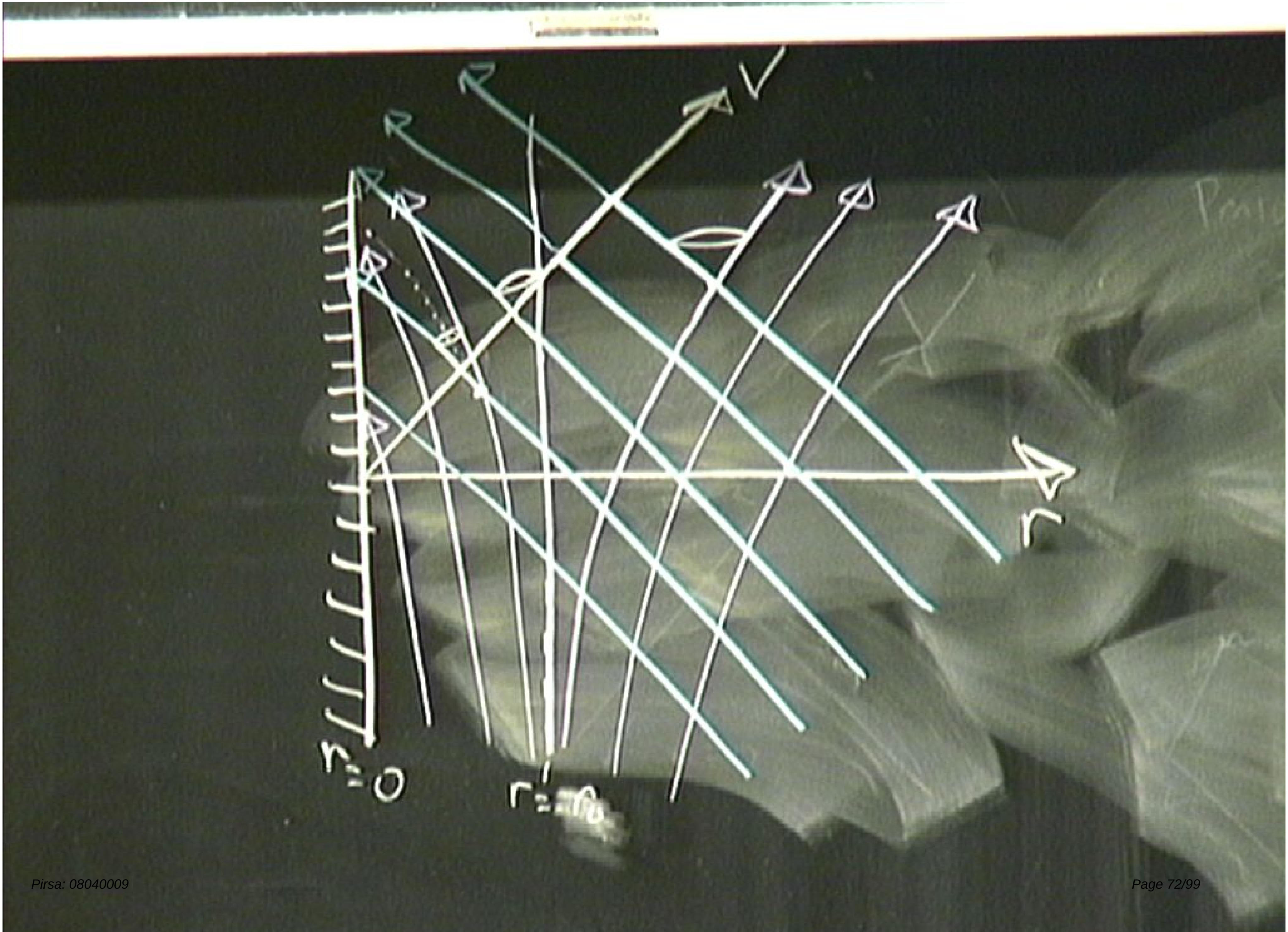

v
r
r=0


v
r
r=0

incoming: $\partial v = 0$

outgoing: $f \partial v = 2 \partial r$

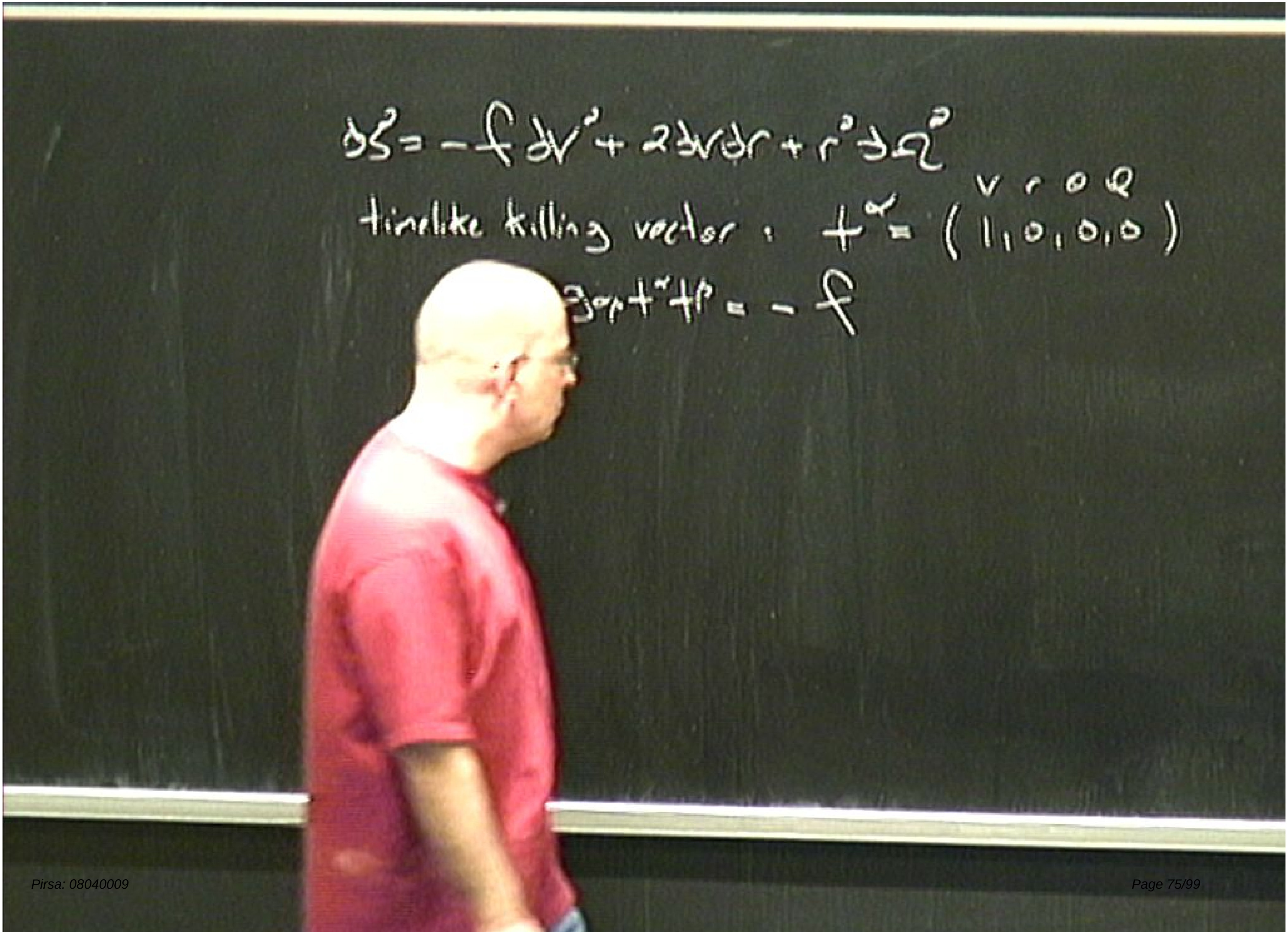

V
r
r=0
r=r₀

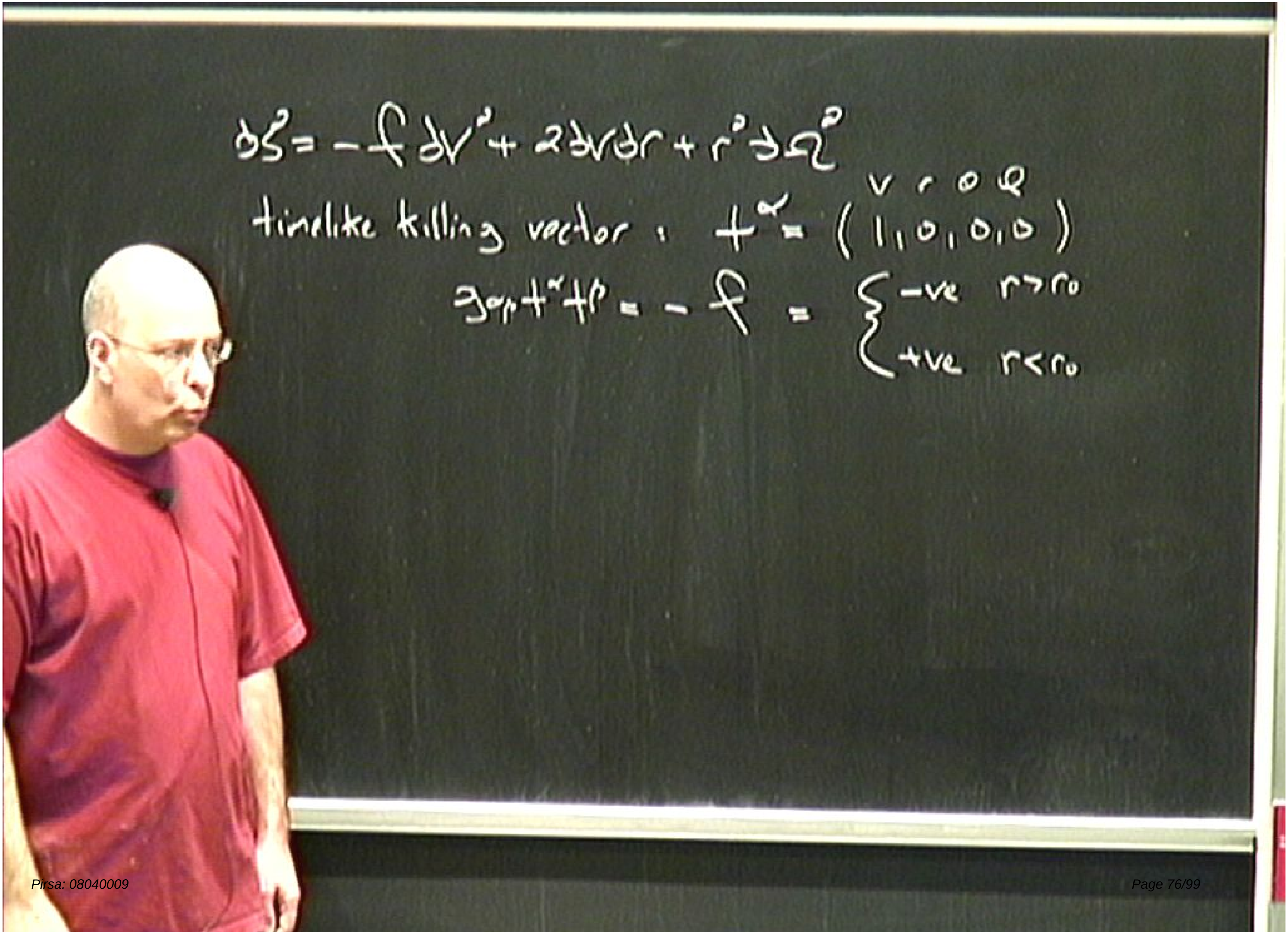

v
r
r=0

$$ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$$

$$ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$$

timelike killing vector : $t^\alpha = \begin{array}{c} \quad v \quad\; r \quad\; \theta \quad\; \phi \\ (\,1,\; 0,\; 0,\; 0\,) \end{array}$


$ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$
timelike killing vector : $t^\alpha = \begin{array}{c} \;\; v \;\; r \;\; \theta \;\; \phi \\ (1, 0, 0, 0) \end{array}$
$g_{\alpha\beta}\, t^\alpha t^\beta = -f$


$ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$
timelike killing vector : $t^\alpha = \begin{array}{c} \;\; v \;\; r \;\; \theta \;\; \phi \\ (\,1,\,0,\,0,\,0\,) \end{array}$
$g_{\alpha\beta}t^\alpha t^\beta = -f = \begin{cases} -ve & r > r_0 \\ +ve & r < r_0 \end{cases}$

$$ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2 d\Omega^2$$

timelike killing vector : $t^\alpha = \begin{array}{c} \begin{array}{cccc} v & r & \theta & \varphi \end{array} \\ (1, 0, 0, 0) \end{array}$

$$g_{\alpha\beta} t^\alpha t^\beta = -f = \begin{cases} -ve & r > r_0 \\ 0 & r = r_0 \\ +ve & r < r_0 \end{cases}$$

$$ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2 d\Omega^2$$

timelike killing vector : $t^\alpha = \begin{array}{c} \ \ v \ \ r \ \ \theta \ \ \phi \\ (1, 0, 0, 0) \end{array}$

$$g_{\alpha\beta} t^\alpha t^\beta = -f = \begin{cases} -\text{ve} & r > r_0 \\ 0 & r = r_0 \\ +\text{ve} & r < r_0 \end{cases}$$

$$a^\alpha = t^\alpha{}_{;\beta} t^\beta$$

$$ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$$

timelike killing vector : $t^\alpha = \begin{array}{c} \quad v \;\; r \;\; \theta \;\; \varphi \\ (1, 0, 0, 0) \end{array}$

$$g_{\alpha\beta}t^\alpha t^\beta = -f = \begin{cases} -\text{ve} & r > r_0 \\ 0 & r = r_0 \\ +\text{ve} & r < r_0 \end{cases}$$

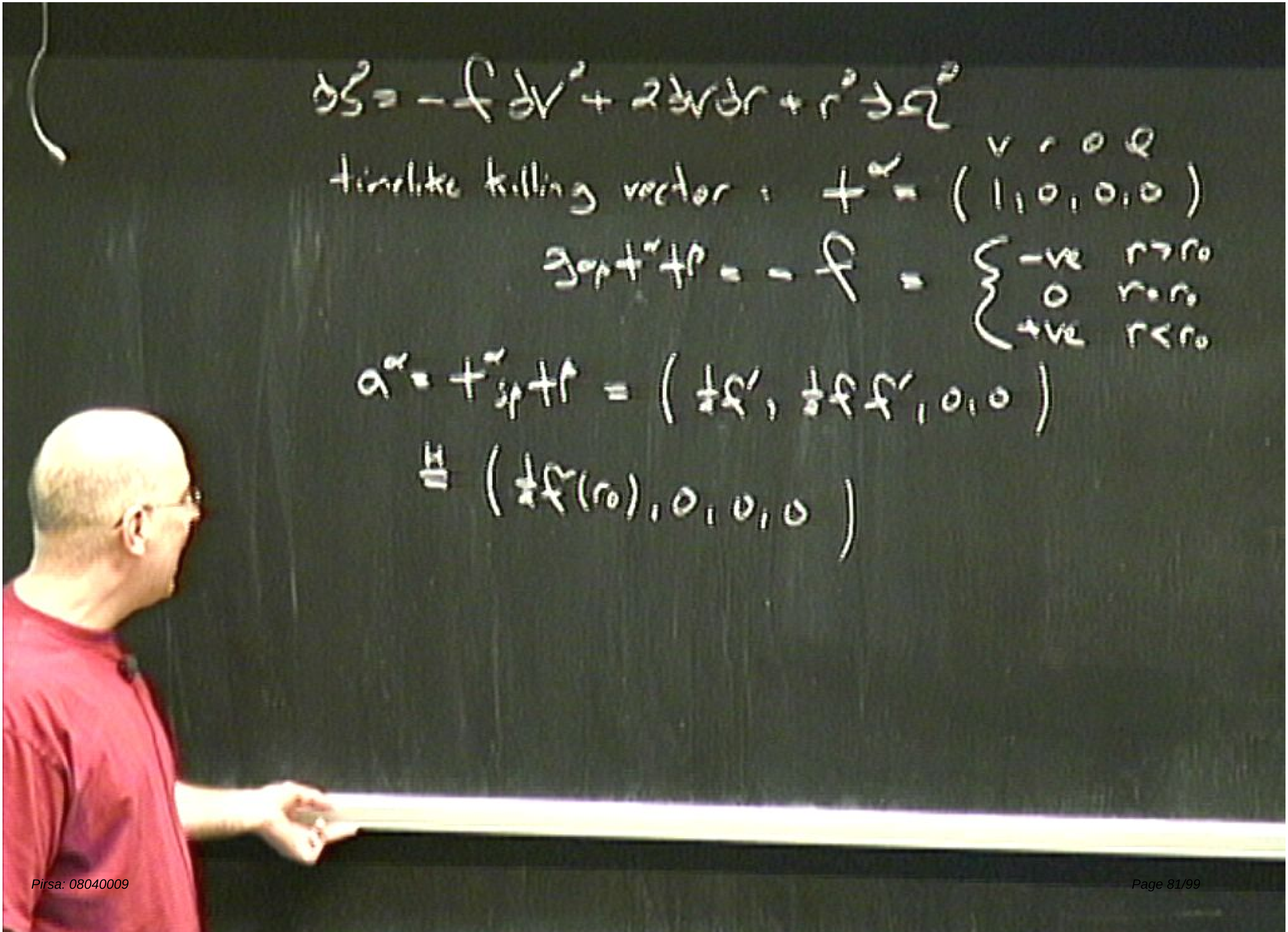
$$a^\alpha = t^\alpha{}_{;\beta}t^\beta$$

$$ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2\,d\Omega^2$$

$$\begin{array}{ll} & \quad\;\; v \;\; r \;\; \theta \;\; \varphi \\ \text{timelike killing vector : } t^\alpha = & (\,1, 0, 0, 0\,) \end{array}$$

$$g_{\alpha\beta}t^\alpha t^\beta = -f = \begin{cases} -ve & r > r_0 \\ 0 & r = r_0 \\ +ve & r < r_0 \end{cases}$$

$$a^\alpha = t^\alpha{}_{;\beta}t^\beta = \left(\tfrac{1}{2}f', \tfrac{1}{2}f f', 0, 0\right)$$


$ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2 d\Omega^2$
$v \quad r \quad \theta \quad \varphi$
timelike killing vector : $t^\alpha = (1, 0, 0, 0)$
$g_{\alpha\beta} t^\alpha t^\beta = -f = \begin{cases} -ve & r > r_0 \\ 0 & r = r_0 \\ +ve & r < r_0 \end{cases}$
$a^\alpha = t^\alpha{}_{;\beta} t^\beta = \left(\tfrac{1}{2} f', \tfrac{1}{2} f f', 0, 0 \right)$
$\overset{H}{=} \left(\tfrac{1}{2} f'(r_0), 0, 0, 0 \right)$

timelike killing vector : $t^\alpha = (1, 0, 0, 0)$

$$g_{\alpha\beta} t^\alpha t^\beta = -f = \begin{cases} -\text{ve} & r > r_0 \\ 0 & r = r_0 \\ +\text{ve} & r < r_0 \end{cases}$$

$$a^\alpha = t^\alpha{}_{;\beta} t^\beta = \left(\tfrac{1}{2} f', \tfrac{1}{2} f f', 0, 0 \right)$$

$$\overset{H}{=} \left(\tfrac{1}{2} f'(r_0), 0, 0, 0 \right)$$

$$t^\alpha{}_{;\beta} t^\beta \overset{H}{=}$$

timelike killing vector : $t^\alpha = (1, 0, 0, 0)$

$$g_{\alpha\beta} t^\alpha t^\beta = -f = \begin{cases} -\text{ve} & r > r_0 \\ 0 & r = r_0 \\ +\text{ve} & r < r_0 \end{cases}$$

$$a^\alpha = t^\alpha{}_{;\beta} t^\beta = \left(\tfrac{1}{2} f', \tfrac{1}{2} f f', 0, 0 \right)$$

$$\stackrel{H}{=} \left(\tfrac{1}{2} f'(r_0), 0, 0, 0 \right)$$

$$t^\alpha{}_{;\beta} t^\beta \stackrel{H}{=} \kappa t^\alpha$$

timelike killing vector : $t^\alpha = (1, 0, 0, 0)$

$$g_{\alpha\beta} t^\alpha t^\beta = -f = \begin{cases} -\text{ve} & r > r_0 \\ 0 & r = r_0 \\ +\text{ve} & r < r_0 \end{cases}$$

$$a^\alpha = t^\alpha{}_{;\beta} t^\beta = \left(\tfrac{1}{2} f', \tfrac{1}{2} f f', 0, 0 \right)$$

$$\overset{H}{=} \left(\tfrac{1}{2} f'(r_0), 0, 0, 0 \right)$$

$$\boxed{t^\alpha{}_{;\beta} t^\beta \overset{H}{=} \kappa t^\alpha}$$

$$g_{\alpha\beta} t^\alpha t^\beta = -f = \begin{cases} -ve & r > r_0 \\ 0 & r = r_0 \\ +ve & r < r_0 \end{cases}$$

$$a^\alpha = t^\alpha{}_{;\beta} t^\beta = \left(\tfrac{1}{2} f', \tfrac{1}{2} f f', 0, 0 \right)$$

$$\overset{H}{=} \left(\tfrac{1}{2} f'(r_0), 0, 0, 0 \right)$$

$$\boxed{t^\alpha{}_{;\beta} t^\beta \overset{H}{=} \kappa t^\alpha}$$

geodesic eqn in non-affine parametrization.

$$\boxed{\kappa = \tfrac{1}{2} f'(r_0)}$$

$$g_{\alpha\beta} t^\alpha t^\beta = -f = \begin{cases} -ve & \\ 0 & r = r_0 \\ +ve & r < r_0 \end{cases}$$

$$a^\alpha = t^\alpha{}_{;\beta} t^\beta = \left(\tfrac{1}{2} f', \tfrac{1}{2} f f', 0, 0 \right)$$

$$\stackrel{H}{=} \left(\tfrac{1}{2} f'(r_0), 0, 0, 0 \right)$$

$$\boxed{t^\alpha{}_{;\beta} t^\beta \stackrel{H}{=} \kappa t^\alpha}$$

geodesic eqn in non-affine parameterization,

$$t^\alpha = \frac{\partial x^\alpha}{\partial v}$$

$$\boxed{\kappa = \tfrac{1}{2} f'(r_0)}$$

$$ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2 d\Omega^2$$

timelike killing vector : $t^\alpha = \begin{array}{c} \begin{matrix} v & r & \theta & \varphi \end{matrix} \\ (1, 0, 0, 0) \end{array}$

$$g_{\alpha\beta} t^\alpha t^\beta = -f = \begin{cases} -ve & r > r_0 \\ 0 & r = r_0 \\ +ve & r < r_0 \end{cases}$$

Affine parameter:

$$\frac{dx^\alpha}{dv} = \exp \int k\, dv$$

$$a^\alpha = t^\alpha{}_{;\beta} t^\beta = \left(\tfrac{1}{2} f', \tfrac{1}{2} f f', 0, 0 \right)$$

$$\overset{H}{=} \left(\tfrac{1}{2} f'(r_0), 0, 0, 0 \right)$$

$$\boxed{t^\alpha{}_{;\beta} t^\beta \overset{H}{=} k\, t^\alpha}$$

$$\boxed{k = \tfrac{1}{2} f'(r_0)}$$

geodesic eqn in non-affine parametrization.

$$t^\alpha = \frac{dx^\alpha}{dv}$$

$$ds^2 = -f\,dv^2 + 2\,dv\,dr + r^2 d\Omega^2$$

$$\text{timelike killing vector}: \quad t^\alpha = \begin{array}{c} \begin{array}{cccc} v & r & \theta & \phi \end{array} \\ (1,0,0,0) \end{array}$$

$$g_{\alpha\beta} t^\alpha t^\beta = -f = \begin{cases} -ve & r > r_0 \\ 0 & r = r_0 \\ +ve & r < r_0 \end{cases}$$

Affine parameter:

$$\frac{dx^\alpha}{dv} = \exp \int k\, dv$$

$$= e^{kv}$$

$$a^\alpha = t^\alpha{}_{;\beta} t^\beta = \left(\tfrac{1}{2} f', \tfrac{1}{2} f f', 0, 0 \right)$$

$$\overset{H}{=} \left(\tfrac{1}{2} f'(r_0), 0, 0, 0 \right)$$

$$\boxed{t^\alpha{}_{;\beta} t^\beta \overset{H}{=} k\, t^\alpha}$$

$$\boxed{k = \tfrac{1}{2} f'(r_0)}$$

geodesic eqn in non-affine parametrization.

$$t^\alpha = \frac{dx^\alpha}{dv}$$

$$g_{\alpha\beta} t^\alpha t^\beta = -f = \begin{cases} -ve & r > r_0 \\ 0 & r = r_0 \\ +ve & r < r_0 \end{cases}$$

Affine parameter.

$$\frac{dx^\alpha}{dv} = \exp \int k \, dv$$

$$= e^{kv}$$

$$\lambda^* = \frac{e^{kv}}{k} = \frac{V}{k}$$

$$a^\alpha = t^\alpha{}_{;\beta} t^\beta = \left(\tfrac{1}{2} f', \tfrac{1}{2} f f', 0, 0 \right)$$

$$\stackrel{H}{=} \left(\tfrac{1}{2} f'(r_0), 0, 0, 0 \right)$$

$$\boxed{t^\alpha{}_{;\beta} t^\beta \stackrel{H}{=} k \, t^\alpha}$$

geodesic eqn in non-affine parametrization.

$$t^\alpha = \frac{dx^\alpha}{dv}$$

$$\boxed{k = \tfrac{1}{2} f'(r_0)}$$

$= \exp \int k\, \partial v$

$\overset{H}{=} \left(\frac{1}{2} f'(r_0), 0, 0, 0 \right)$

$= e^{kv}$

$= \frac{e^{kv}}{k} = \frac{V}{k}$

$$\boxed{t^{\alpha}{}_{;\beta}\, t^{\beta} \overset{H}{=} k\, t^{\alpha}}$$

$$\boxed{k = \frac{1}{2} f'(r_0)}$$

geodesic eq
parameter
$t^{\alpha} = \frac{\partial x^{\alpha}}{\partial V}$

$$= e^{kv}$$

$$\lambda^{*} = \frac{e^{kv}}{k} = \frac{V}{k}$$

$$\boxed{t^{\alpha}{}_{;\beta} t^{\beta} \stackrel{*}{=} k\, t^{\alpha}}$$

$$\boxed{k = \tfrac{1}{2} f'(r_0)}$$

geodesic eqn in non-affine parametrization,

$$t^{\alpha} = \frac{\partial x^{\alpha}}{\partial v}$$

killing's eqn: $t_{\alpha;\beta} + t_{\beta;\alpha} = 0$

$$= e^{\kappa V}$$

$$\lambda^{\alpha} = \frac{e^{\kappa V}}{\kappa} - \frac{V}{\kappa}$$

$$\boxed{t^{\alpha}{}_{;\beta}\, t^{\beta} \overset{H}{=} \kappa\, t^{\alpha}}$$

$$\boxed{\kappa = \tfrac{1}{2} f'(r_0)}$$

geodesic eqn in non-affine parametrization.

$$t^{\alpha} = \frac{\partial x^{\alpha}}{\partial v}$$

killing's eqn : $t_{\alpha;\beta} + t_{\beta;\alpha} = 0$

$$t_{\alpha;\beta}\, t^{\beta} = -t_{\beta;\alpha}\, t^{\beta} = -\tfrac{1}{2}\left(t_{\beta} t^{\beta}\right)_{;\alpha}$$

$$= e^{kv}$$

$$\lambda^\alpha = \frac{e^{kv}}{k} = \frac{V}{k}$$

$$\boxed{t^\alpha{}_{;\beta} t^\beta \overset{!!}{=} k\, t^\alpha}$$

$$\boxed{k = \tfrac{1}{2} f'(r_0)}$$

geodesic eqn in non-affine parameterization,

$$t^\alpha = \frac{\partial x^\alpha}{\partial v}$$

killing's eqn: $t_{\alpha;\beta} + t_{\beta;\alpha} = 0$

$$t_{\alpha;\beta}\, t^\beta = - t_{\beta;\alpha}\, t^\beta = -\tfrac{1}{2}\left(t_\beta t^\beta\right)_{;\alpha}$$

$$\boxed{\nabla_\alpha\left(-t_\beta t^\beta\right) = 2k\, t_\alpha}$$

$\kappa = \frac{1}{2} f'(r_0)$ $\quad t^\mu = \frac{\partial x^\mu}{\partial v}$

EH:

$\Phi = 0$

$\Phi = -t_\rho t^\rho = f$

Killing's eqn: $t_{\alpha;\beta} + t_{\beta;\alpha} = 0$

$$t_{\alpha;\beta} t^\beta = -t_{\beta;\alpha} t^\beta = -\frac{1}{2}(t_\rho t^\rho)_{;\alpha}$$

$$\boxed{\nabla_\alpha(-t_\rho t^\rho) = 2\kappa\, t_\alpha}$$

EH:

$\Phi = 0$

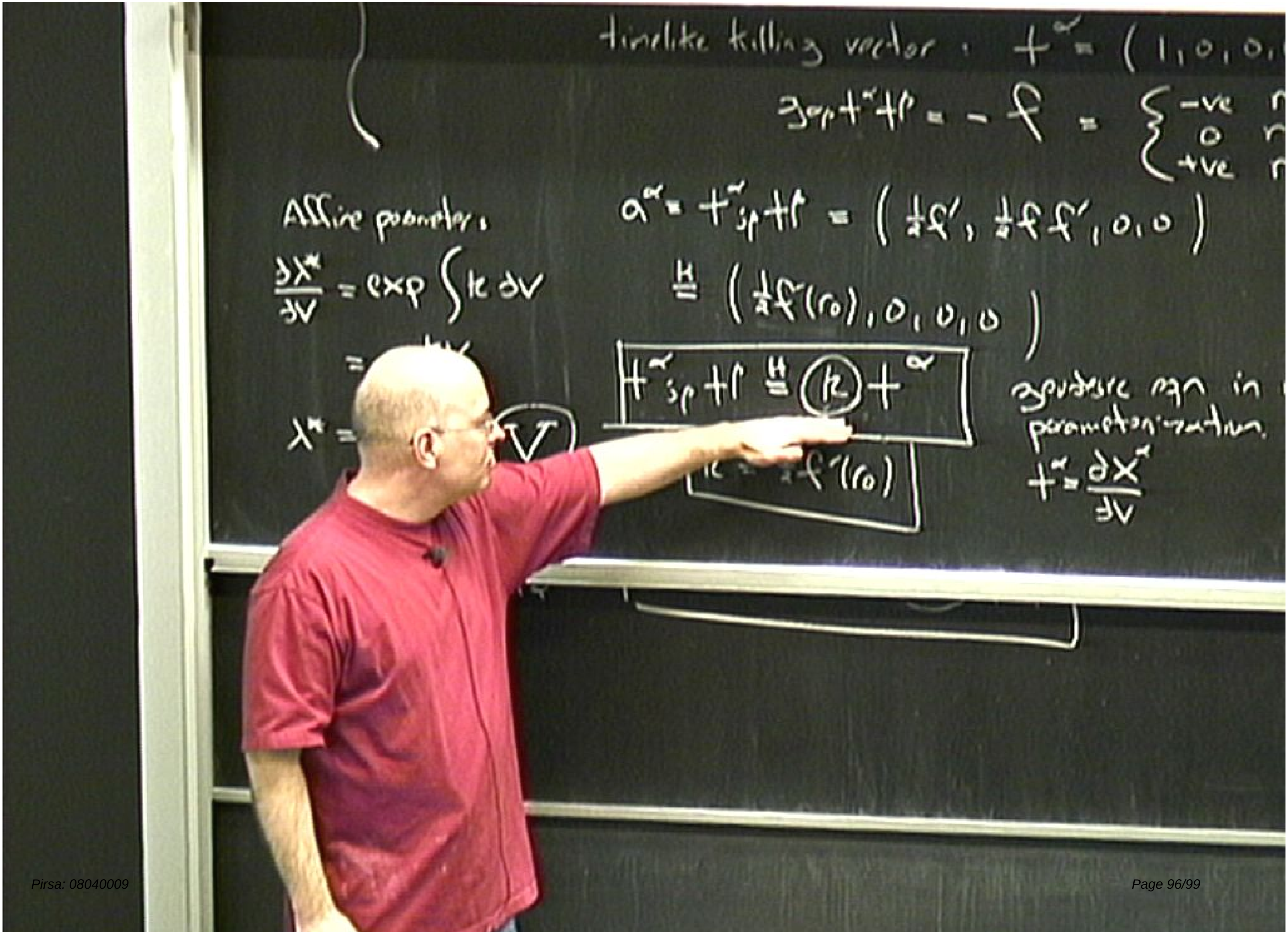
$\Phi = -t_\beta t^\beta = f$

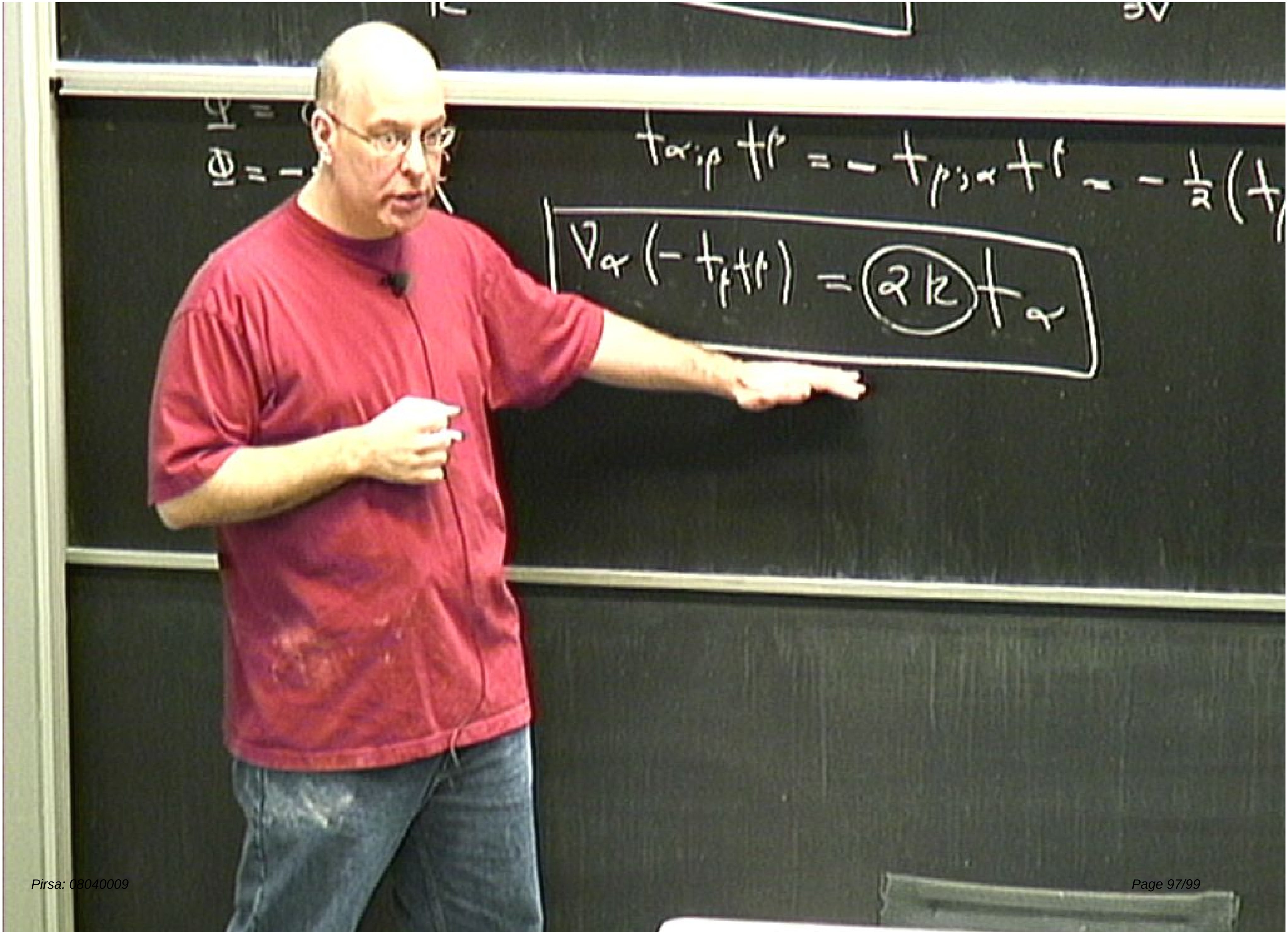
$\nabla_\alpha \Phi \propto$

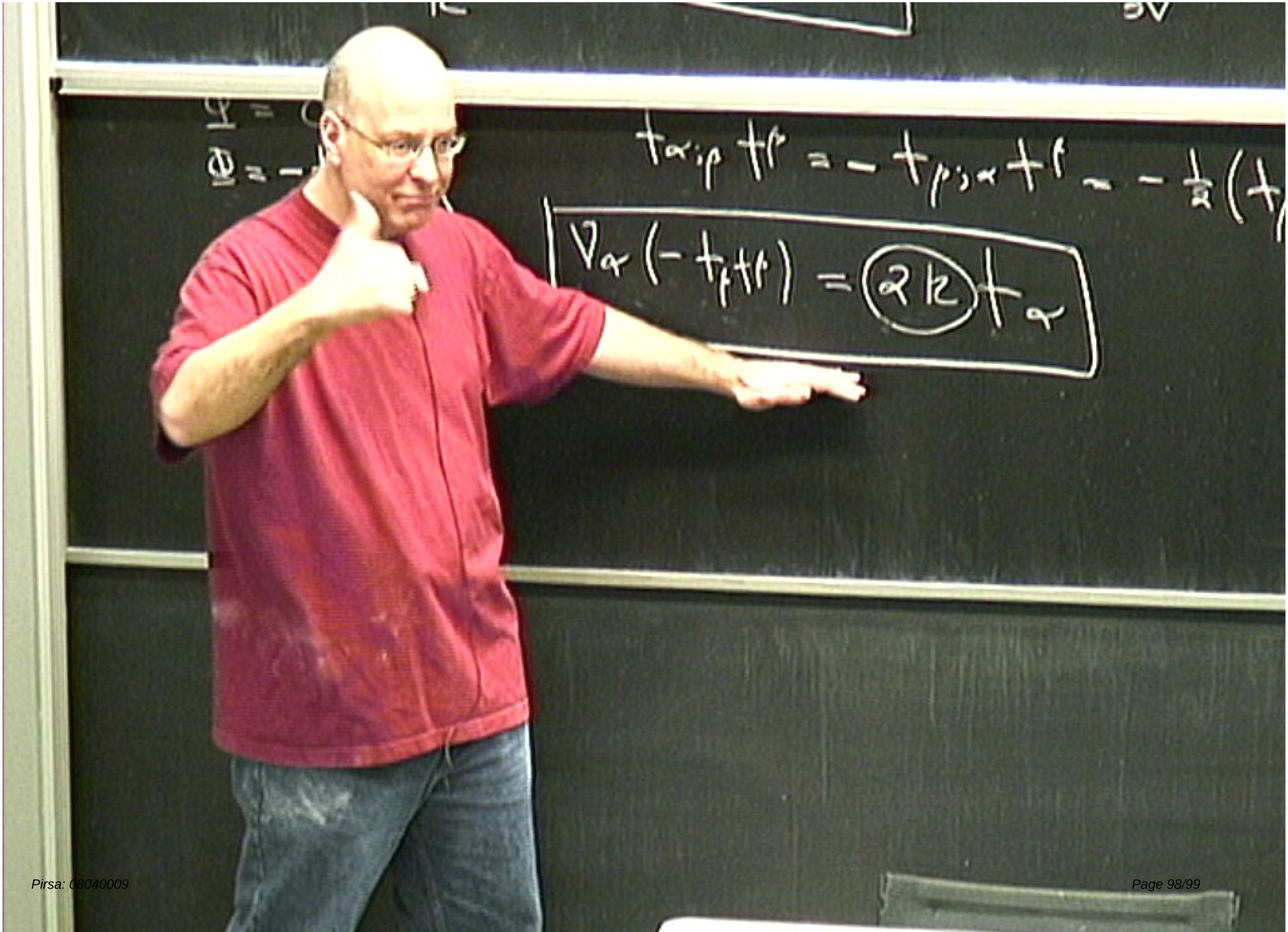
Killing's eqn: $t_{\alpha;\beta} + t_{\beta;\alpha} = 0$

$$t_{\alpha;\beta}\, t^\beta = - t_{\beta;\alpha}\, t^\beta = -\tfrac{1}{2}\left(t_\beta t^\beta\right)_{;\alpha}$$

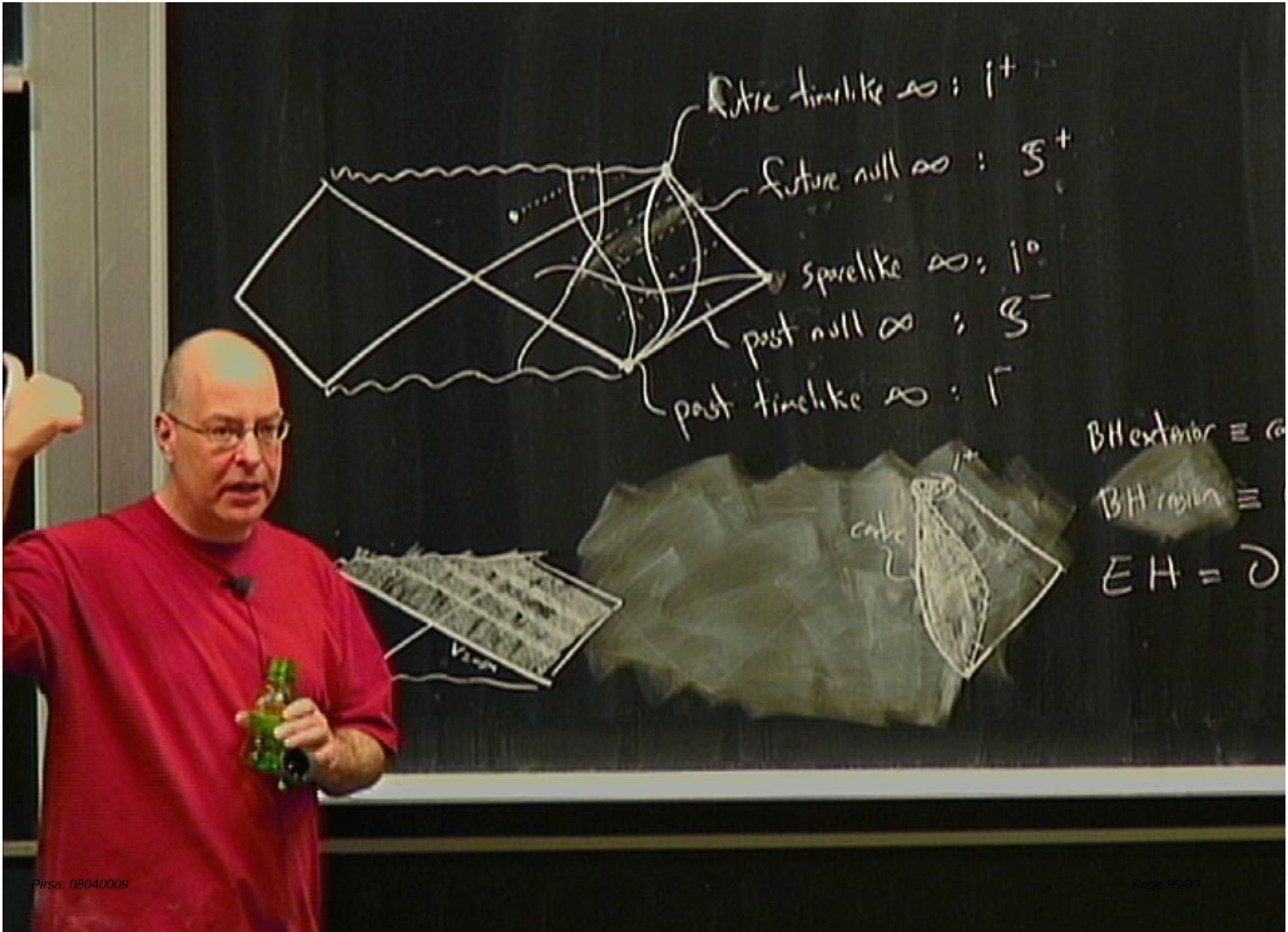
$$\boxed{\nabla_\alpha\left(-t_\beta t^\beta\right) = 2\kappa\, t_\alpha}$$


timelike killing vector : t^α = (1, 0, 0,
g_αβ t^α t^β = − f = { −ve, 0, +ve
Affine parameter:
∂x^α/∂v = exp ∫ k dv
a^α = t^α_;β t^β = (½f′, ½f f′, 0, 0)
H= (½f′(r₀), 0, 0, 0)
t^α_;β t^β H= k t^α
k = ½f′(r₀)
geodesic eqn in non-affine parametrization,
t^α = ∂x^α/∂v


$\nabla_\alpha(-t_\beta t^\beta) = 2k\, t_\alpha$



t_{α;β} t^β = − t_{β;α} t^β = − ½ (t
∇_α (− t_β t^β) = 2k t_α


future timelike ∞ : i+
future null ∞ : ℐ+
spacelike ∞: i0
past null ∞ : ℐ−
past timelike ∞ : i−
BH exterior ≡
BH region ≡
EH =