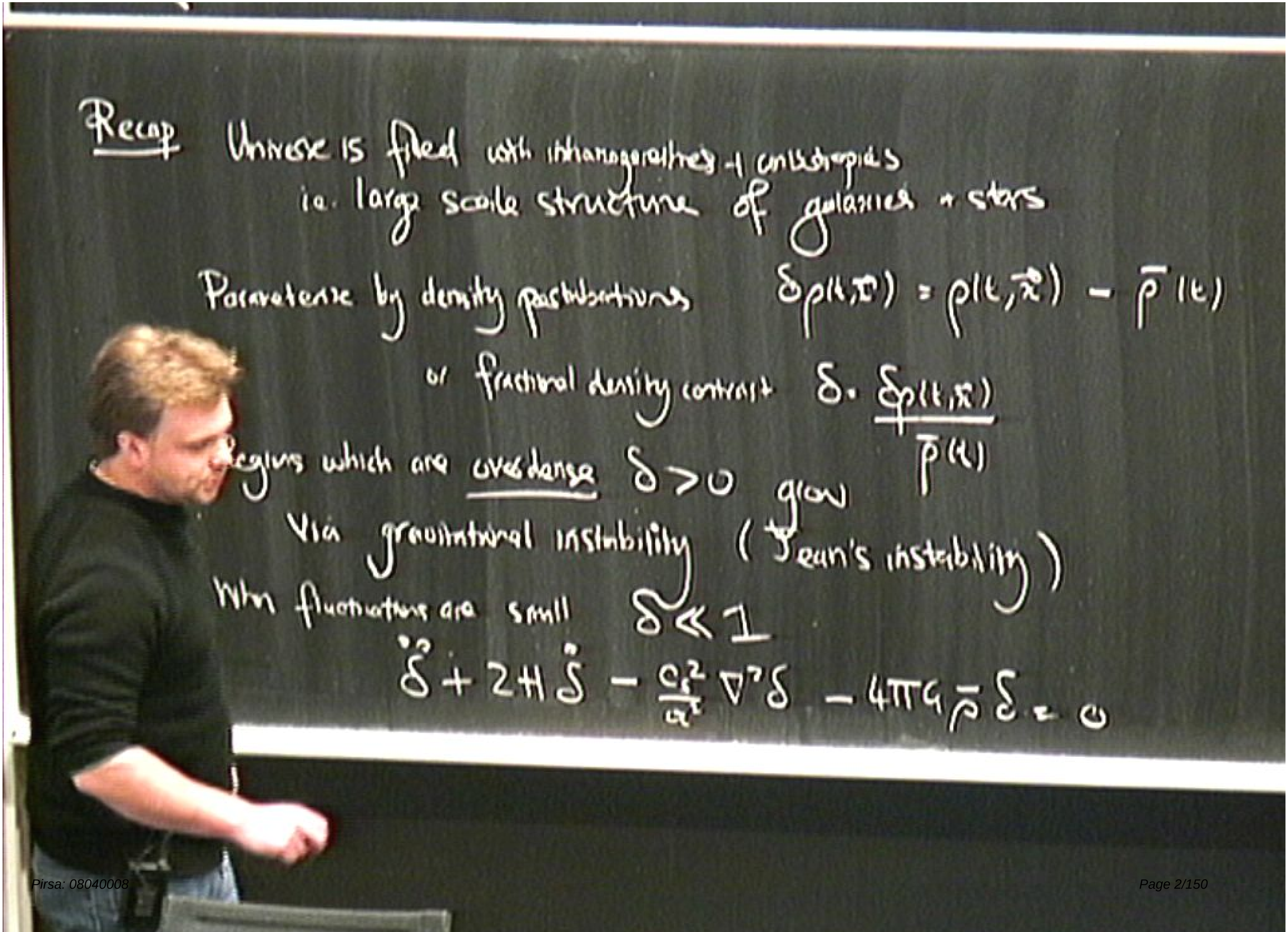
Title: Cosmology #4

Date: Apr 03, 2008 06:30 PM

URL: http://pirsa.org/08040008

Abstract: A brief history of our cosmic beginnings, Cosmic Microwave Background. How galaxies form and the existence of dark matter.


Recap Universe is filled with inhomogeneities + anisotropies
i.e. large scale structure of galaxies + stars
Parameterise by density perturbations $\delta\rho(t,\vec{x}) = \rho(t,\vec{x}) - \bar{\rho}(t)$
or fractional density contrast $\delta = \frac{\delta\rho(t,\vec{x})}{\bar{\rho}(t)}$
Regions which are overdense $\delta > 0$ grow
via gravitational instability (Jean's instability)
When fluctuations are small $\delta \ll 1$
$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta - 4\pi G\bar{\rho}\delta = 0$

Recap Universe is filled with inhomogeneities + anisotropies
i.e. large scale structure of galaxies + stars

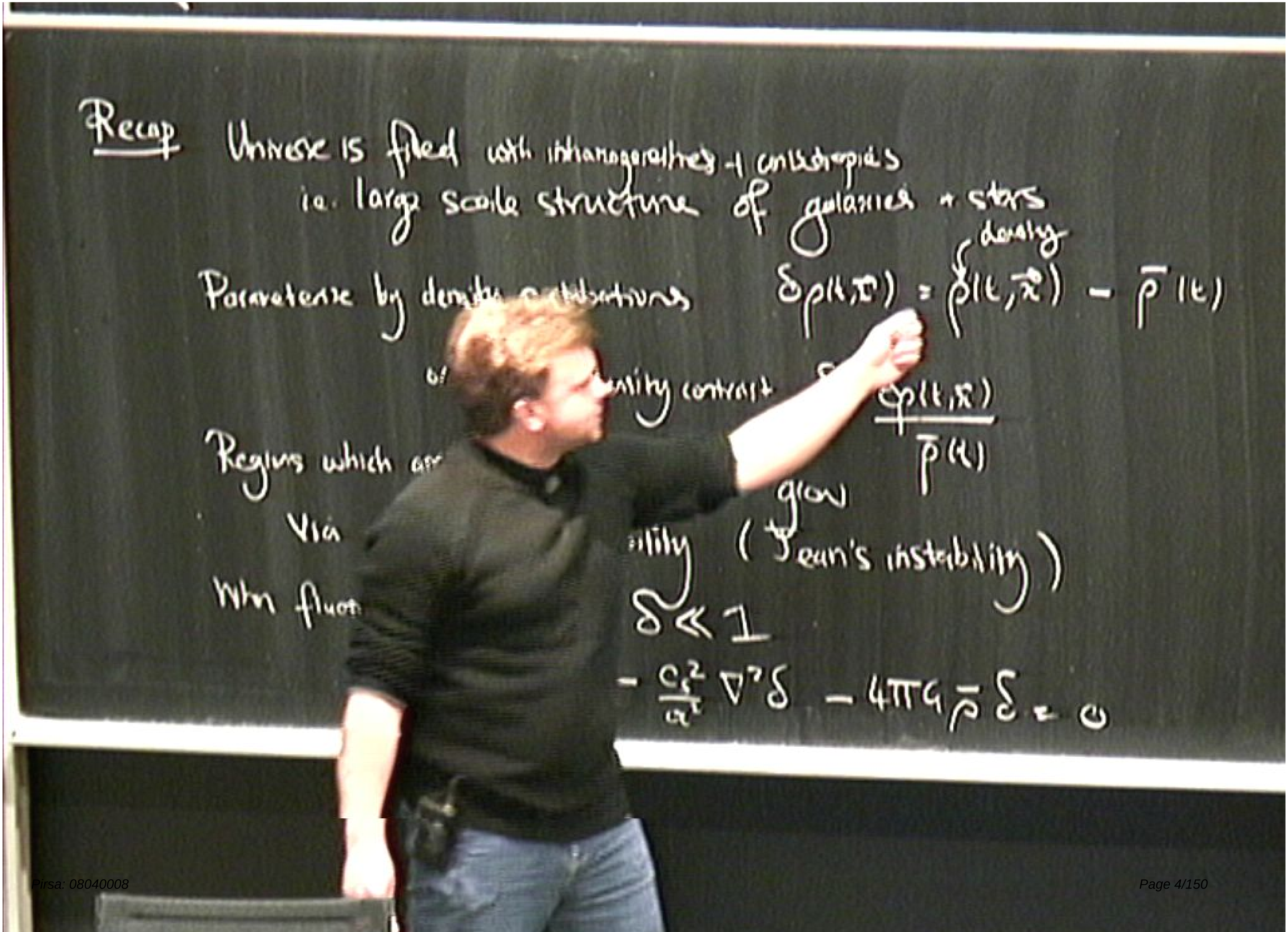
Parameterise by density perturbations $\delta\rho(t,\vec{x}) = \rho(t,\vec{x}) - \bar{\rho}(t)$

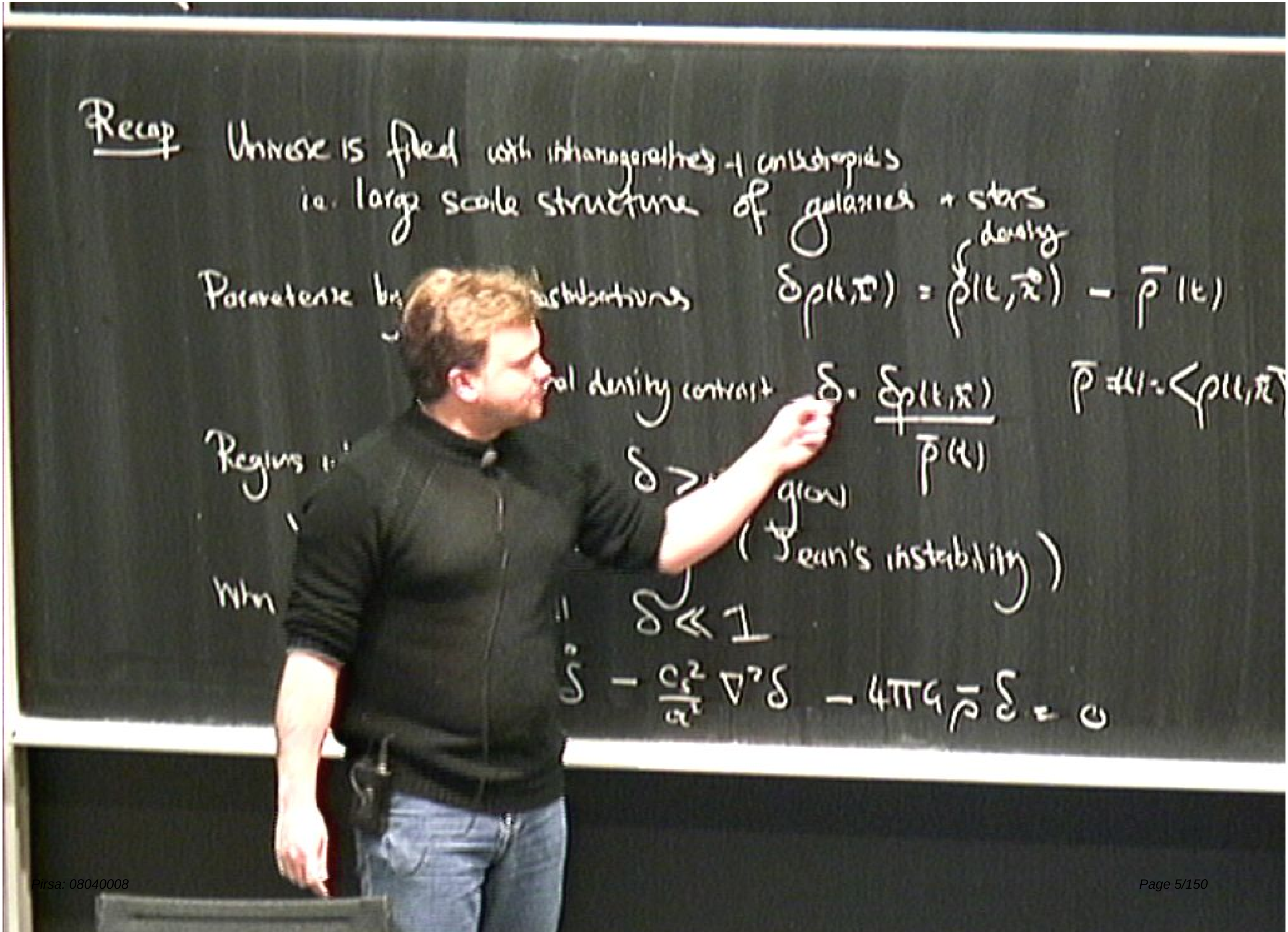
or fractional density contrast $\delta = \frac{\delta\rho(t,\vec{x})}{\bar{\rho}(t)}$

Regions which are <u>overdense</u> $\delta > 0$ grow via gravitational instability (Jean's instability)

When fluctuations are small $\delta \ll 1$

$$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta - 4\pi G\bar{\rho}\,\delta = 0$$


Recap
Universe is filled with inhomogeneities + anisotropies
i.e. large scale structure of galaxies + stars
density
Parameterise by density perturbations
δρ(t,x) = ρ(t,x) − ρ̄(t)
density contrast
δρ(t,x) / ρ̄(t)
Regions which are
grow
Via
(Jean's instability)
δ ≪ 1
− 4πG ρ̄ δ = 0


Recap
Universe is filled with inhomogeneities + anisotropies
i.e. large scale structure of galaxies + stars
density
$\delta\rho(t,\vec{x}) = \rho(t,\vec{x}) - \bar{\rho}(t)$
density contrast $\delta = \frac{\delta\rho(t,\vec{x})}{\bar{\rho}(t)}$
$\bar{\rho}(t) = \langle\rho(t,\vec{x})$
$\delta >$
grow
(Jean's instability)
$\delta \ll 1$
$\ddot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta - 4\pi G\bar{\rho}\,\delta = 0$

Recap Universe is filled with inhomogeneities + anisotropies
i.e. large scale structure of galaxies + stars

Parameterise by density perturbations $\delta\rho(t,\vec{x}) = \overbrace{\rho(t,\vec{x})}^{\text{density}} - \bar{\rho}(t)$

or fractional density contrast $\delta = \dfrac{\delta\rho(t,\vec{x})}{\bar{\rho}(t)}$ $\quad \bar{\rho}(t) = \langle \rho(t,\vec{x}) \rangle$

Regions which are <u>overdense</u> $\delta > 0$ grow

ρ

via gravitational instability (Jean's instability)

When fluctuations are small $\delta \ll 1$

$$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta - 4\pi G\bar{\rho}\,\delta = 0$$

Recap Universe is filled with inhomogeneities + anisotropies
i.e. large scale structure of galaxies + stars

Parameterise by density perturbations $\delta\rho(t,\vec{x}) = \underbrace{\rho(t,\vec{x})}_{\text{density}} - \bar{\rho}(t)$

or fractional density contrast $\delta = \dfrac{\delta\rho(t,\vec{x})}{\bar{\rho}(t)}$ $\quad \bar{\rho}(t) = \langle \rho(t,\vec{x}) \rangle$

Regions which are overdense $\delta > 0$ grow via gravitational instability (Jean's instability)

When fluctuations are small $\delta \ll 1$

$$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta - 4\pi G\bar{\rho}\,\delta = 0$$

Recap Universe is filled with inhomogeneities + anisotropies

i.e. large scale structure of galaxies + stars

Parameterise by density perturbations $\delta\rho(t,\vec{x}) = \overset{\text{density}}{\rho}(t,\vec{x}) - \bar{\rho}(t)$

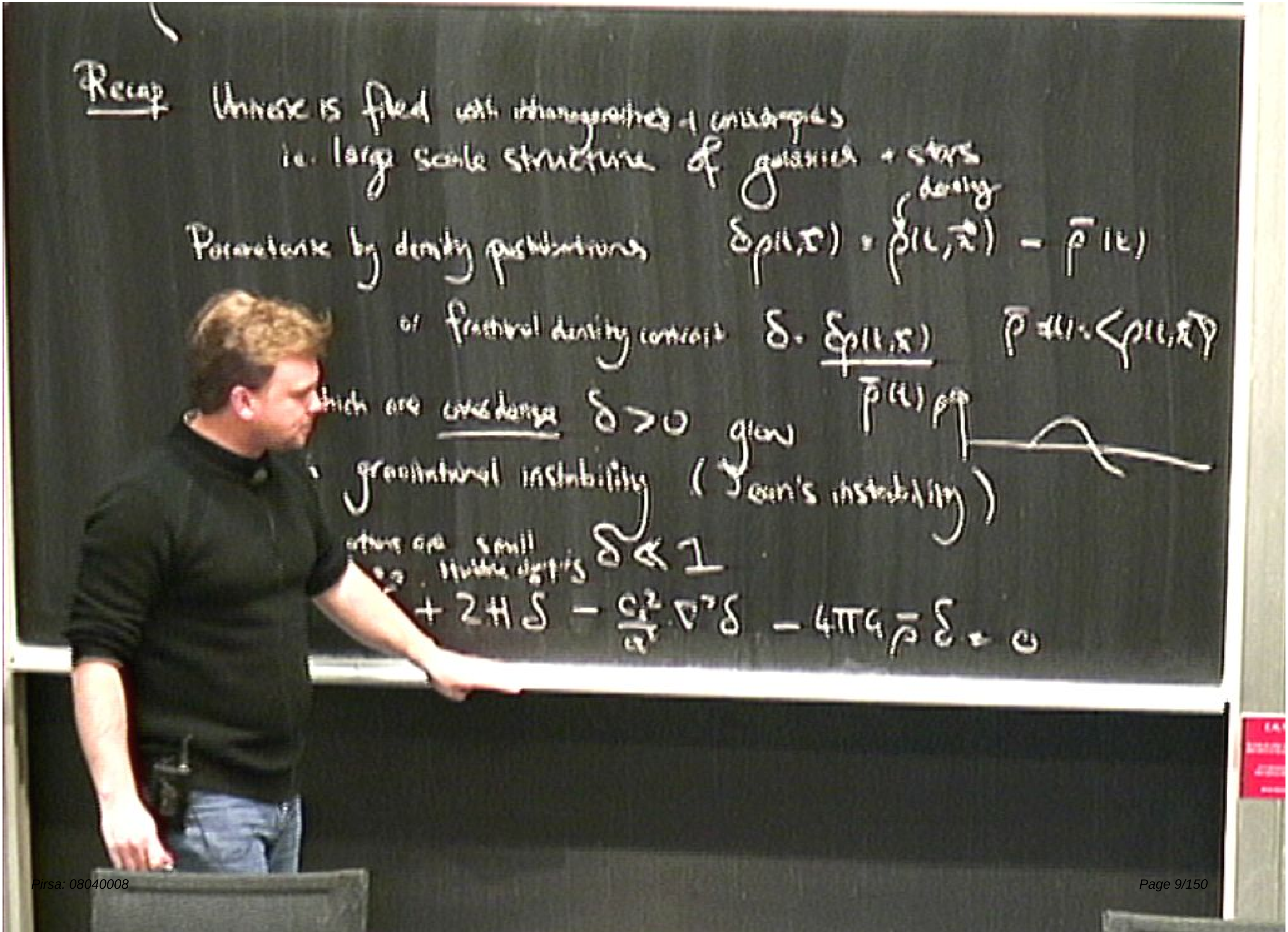
or fractional density contrast $\delta = \dfrac{\delta\rho(t,\vec{x})}{\bar{\rho}(t)}$ $\quad \bar{\rho}(t) = \langle \rho(t,\vec{x}) \rangle$

Regions which are overdense $\delta > 0$ grow

via gravitational instability (Jean's instability)

When fluctuations are small $\delta \ll 1$

$$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta - 4\pi G\bar{\rho}\,\delta = 0$$


Recap
Universe is filled with inhomogeneities + anisotropies
i.e. large scale structure of galaxies + stars
Parametrize by density perturbations $\delta\rho(t,\vec{x}) = \rho(t,\vec{x}) - \bar{\rho}(t)$
density
or fractional density contrast $\delta = \frac{\delta\rho(t,\vec{x})}{\bar{\rho}(t)}$ $\bar{\rho}(t) = \langle\rho(t,\vec{x})\rangle$
which are overdense $\delta > 0$ grow
gravitational instability (Jeans's instability)
are small
$\delta \ll 1$
$+ 2H\dot{\delta} - \frac{c_s^2}{a^2}\nabla^2\delta - 4\pi G\bar{\rho}\,\delta = 0$

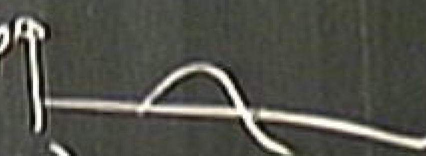
Recap Universe is filled with inhomogeneities + anisotropies

i.e. large scale structure of galaxies + stars

Parametrize by density perturbations $\delta\rho(t,\vec{x}) = \overset{\text{density}}{\rho(t,\vec{x})} - \bar{\rho}(t)$

or fractional density contrast $\delta = \dfrac{\delta\rho(t,\vec{x})}{\bar{\rho}(t)}$ $\quad \bar{\rho}(t) = \langle \rho(t,\vec{x}) \rangle$

Regions which are overdense $\delta > 0$ grow



via gravitational instability (Jean's instability)

When fluctuations are small $\delta \ll 1$

$$\ddot{\delta} + \underbrace{2H\dot{\delta}}_{\text{Hubble damping}} - \frac{c_s^2}{a^2}\nabla^2\delta \; \underbrace{-\,4\pi G\bar{\rho}\,\delta}_{\text{unstable}} = 0$$

Short wavelength perturbations are stable (pressure wins over gravity)

$\boxed{\lambda < \lambda_J}$ $\delta \sim \delta_0 e^{i\omega t - i\vec{k}\cdot\vec{x}}$ $\omega^2 = c_s^2 \vec{k}^2$

Long wavelength perts are <u>unstable</u>

(for w=0) $\delta \sim A t^{2/3} + \frac{B}{t}$

growing mode

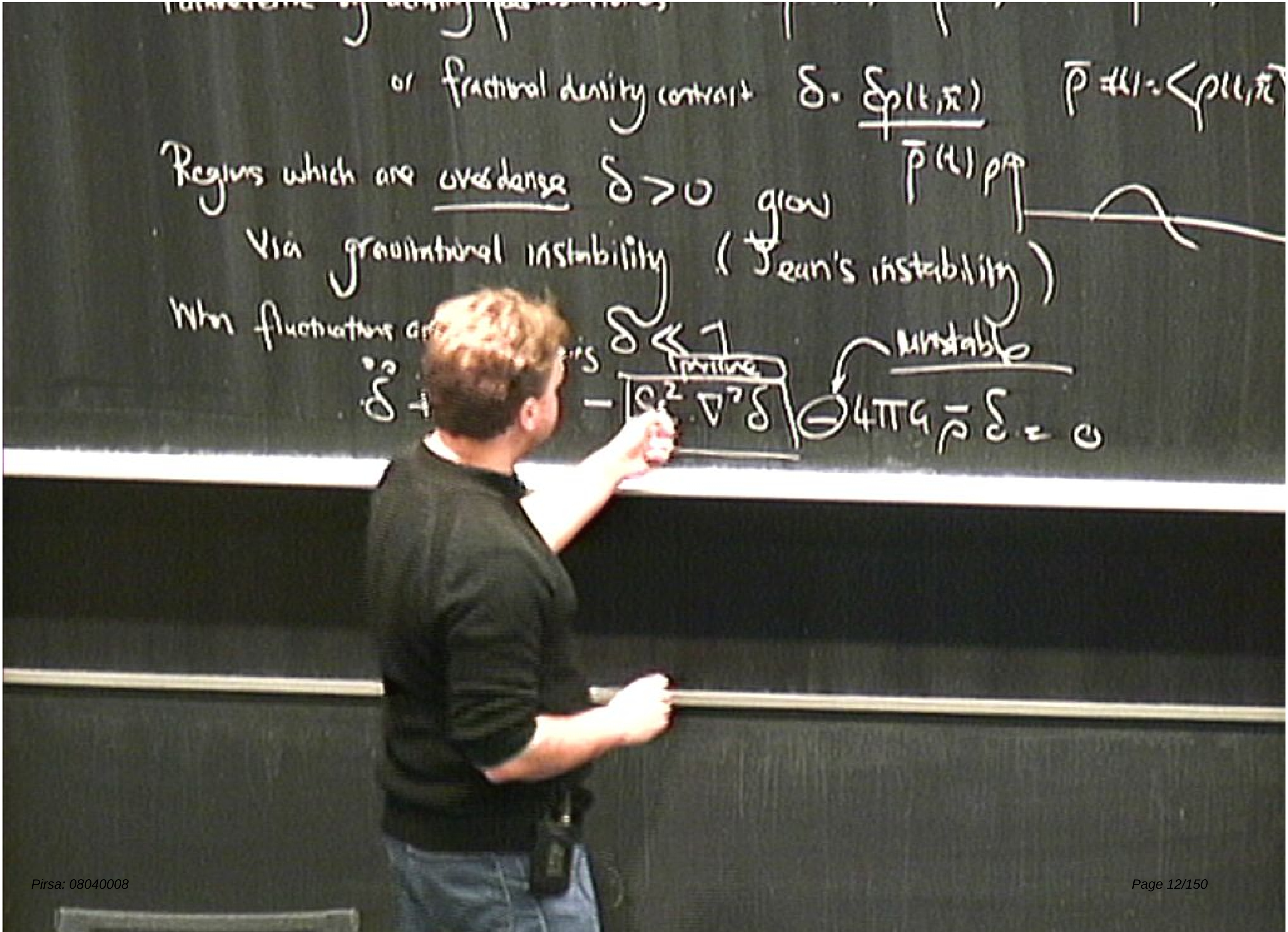
decaying mode

CMB is not perfectly isotropic

2.73K today

$\delta T(t,\vec{x}) = T(t,\vec{x}) - \bar{T}(t)$

(actually $\delta T(t,\theta,\phi) = T(t,\theta,\phi) - \bar{T}(t)$)


or fractional density contrast δ = δρ(t,x)/ρ̄(t)   ρ̄(t) = <ρ(t,x)>
Regions which are overdense δ > 0 grow
via gravitational instability (Jean's instability)
When fluctuations
δ ≪ 1
unstable
− c_s² ∇²δ − 4πG ρ̄ δ = 0

Short wavelength perturbations are stable (pressure wins over gravity)

$\boxed{\lambda < \lambda_J}$ $\quad \delta \sim \delta_0 e^{i\omega t - i\vec{k}\cdot\vec{x}} \qquad \omega^2 = c_s^2 \vec{k}^2$

Long wavelength perts are unstable $\qquad \lambda > \lambda_J$

(for w=0) $\qquad \delta \sim A t^{2/3} + \frac{B}{t}$

growing mode $\qquad$ decaying mode

CMB is not perfectly isotropic

2.73K today

$$\delta T(t,\vec{x}) = T(t,\vec{x}) - \bar{T}(t)$$

(actually $\delta T(t,\theta,\phi) = T(t,\theta,\phi) - \bar{T}(t)$)

Short wavelength perturbations are stable (pressure wins over gravity)

$\boxed{\lambda < \lambda_J}$ $\delta \sim \delta_0 e^{i\omega t - i\vec{k}\cdot\vec{x}}$ $\omega^2 = c_s^2 \vec{k}^2$

Long wavelength perts are <u>unstable</u> $\lambda > \lambda_J$

(for w=0) $\boxed{\delta \sim A t^{2/3}} + \frac{B}{t}$

growing mode. decaying mode.

CMB is not perfectly isotropic 2.73 K today

$\delta T(t,\vec{x}) = T(t,\vec{x}) - \bar{T}(t)$

(actually $\delta T(t,\theta,\phi) = T(t,\theta,\phi) - \bar{T}(t)$)

Short wavelength perturbations are stable (pressure wins over gravity)

$\boxed{\lambda < \lambda_J}$ $\delta \sim \delta_0 e^{i\omega t - i\vec{k}\cdot\vec{x}}$ $\omega^2 = c_s^2 \vec{k}^2$

Long wavelength perts are unstable $\lambda > \lambda_J$

(for w=0) $\boxed{\delta \sim A t^{2/3}} + \frac{B}{t}$

growing mode.

decaying mode.

CMB is not perfectly isotropic

2.73 K today

$\delta T(t,\vec{x}) = T(t,\vec{x}) - \bar{T}(t)$

(actually $\delta T(t,\theta,\phi) = T(t,\theta,\phi) - \bar{T}(t)$)

Short wavelength perturbations are stable (pressure wins over gravity)

$$\boxed{\lambda < \lambda_J} \quad \delta \sim \delta_0 e^{i\omega t - i\vec{k}\cdot\vec{x}} \qquad \omega^2 = c_s^2 \vec{k}^2$$

Long wavelength perts are unstable $\quad \lambda > \lambda_J$

$\delta \sim 1$ galaxy formed

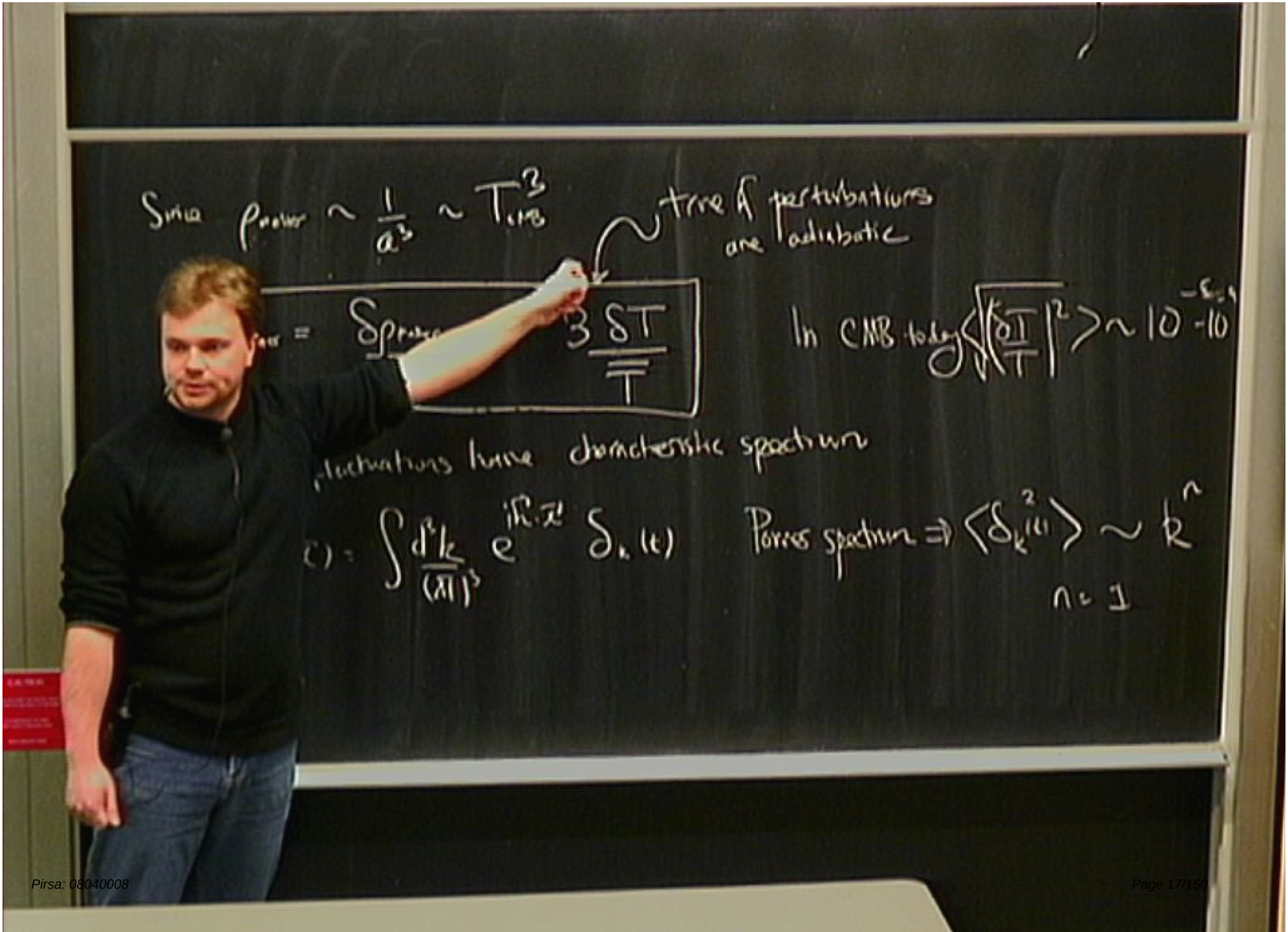
(for $k=0$) $\quad \boxed{\delta \sim A t^{2/3}} + \dfrac{B}{t}$

growing mode. $\qquad$ decaying mode.

CMB is not perfectly isotropic $\qquad$ 2.73 K today

$$\delta T(t, \vec{x}) = T(t, \vec{x}) - \bar{T}(t)$$

(actually $\delta T(t, \theta, \phi) = T(t, \theta, \phi) - \bar{T}(t)$)


true if perturbations are adiabatic
In CMB today
fluctuations have characteristic spectrum
Power spectrum
n = 1

Since $\rho_{matter} \sim \frac{1}{a^3} \sim T_{CMB}^3$ true if perturbations are adiabatic

$$\boxed{\delta_{matter} = \frac{\delta\rho_{matter}}{\bar{\rho}} = 3\frac{\delta T}{\bar{T}}}$$

In CMB today $\sqrt{\left\langle \left|\frac{\delta T}{T}\right|^2 \right\rangle} \sim 10^{-5} - 10^{-4}$

These fluctuations have characteristic spectrum

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \delta_k(t)$$

Power spectrum $\Rightarrow \langle \delta_k^2(t) \rangle \sim k^n$

$n = 1$

Since $\rho_{matter} \sim \frac{1}{a^3} \sim T_{CMB}^3$

true if perturbations are adiabatic

$$\boxed{\delta_{matter} = \frac{\delta\rho_{matter}}{\bar{\rho}} = 3\frac{\delta T}{\bar{T}}}$$

In CMB today $\sqrt{\left\langle \left|\frac{\delta T}{T}\right|^2 \right\rangle} \sim 10^{-5}$ ~ 10^{-10}

These fluctuations have characteristic spectrum

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \delta_k(t)$$

Power spectrum $\Rightarrow \langle \delta_k^2(t) \rangle \sim k^n$

$n = 1$

Since $\rho_{\text{photon}} \sim \frac{1}{a^3} \sim T_{CMB}^3$

true if perturbations are adiabatic

$$\boxed{\delta_{\text{photon}} = \frac{\delta\rho_{\text{photon}}}{\bar{\rho}} = \frac{3\,\delta T}{\bar{T}}}$$

In CMB today $\sqrt{\left\langle \left| \frac{\delta T}{T} \right|^2 \right\rangle} \sim 10^{-5} - 10^{-4}$

These fluctuations have characteristic spectrum

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, \delta_k(t)$$

Power spectrum $\Rightarrow \langle \delta_k^2(t) \rangle \sim k^n$

$n = 1$

$$\boxed{\delta_{matter} = \frac{\delta\rho_{matter}}{\bar{\rho}} = 3\frac{\delta T}{\bar{T}}}$$

In CMB today $\sqrt{\left\langle \left|\frac{\delta T}{T}\right|^2 \right\rangle} \sim 10^{-5}-10^{-6}$

These fluctuations have characteristic spectrum

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, \delta_k(t)$$

Power spectrum $\Rightarrow \langle \delta_k^2(t) \rangle \sim k^n$

$n = 1$

$t_{rec} \sim 300\,000\,\text{years}$

$$\boxed{\delta_{matter} = \frac{\delta\rho_{matter}}{\bar{\rho}} = 3\,\frac{\delta T}{\bar{T}}}$$

In CMB today $\sqrt{\left\langle \left|\frac{\delta T}{T}\right|^2 \right\rangle} \sim 10^{-5}$

These fluctuations have characteristic spectrum

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, \delta_k(t)$$

Power spectrum $\Rightarrow \langle \delta_k^2(t) \rangle \sim k^n$

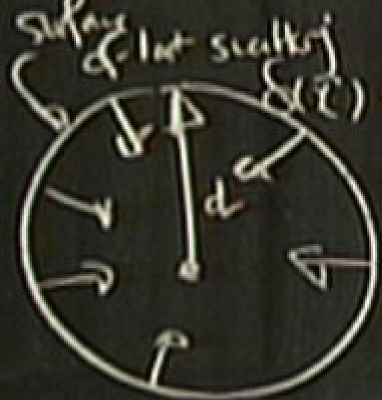
$n = 1$

$t_{re} \sim 300\,000$ years

$$\boxed{\delta_{matter} = \frac{\delta\rho_{matter}}{\bar\rho} = 3\frac{\delta T}{\bar T}}$$

In CMB today $\sqrt{\left\langle \left|\frac{\delta T}{T}\right|^2 \right\rangle} \sim 10^{-5} - 10^{-10}$

These fluctuations have characteristic spectrum



$$\delta(\vec x) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec k\cdot\vec x}\, \delta_k(t)$$

Power spectrum $\Rightarrow \langle \delta_k^2(t)\rangle \sim k^n$

$n = 1$

$t_{rec} \sim 300\,000$ years

Since $\rho_{matter} \sim \frac{1}{a^3} \sim T_{CMB}^3$

true if perturbations are adiabatic

$$\boxed{\delta_{matter} = \frac{\delta\rho_{matter}}{\bar{\rho}} = 3\frac{\delta T}{\bar{T}}}$$

In CMB today $\sqrt{\left\langle \left|\frac{\delta T}{T}\right|^2 \right\rangle} \sim 10^{-5}$

These fluctuations have characteristic spectrum

surface of last scattering

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \delta_{\vec{k}}(t)$$

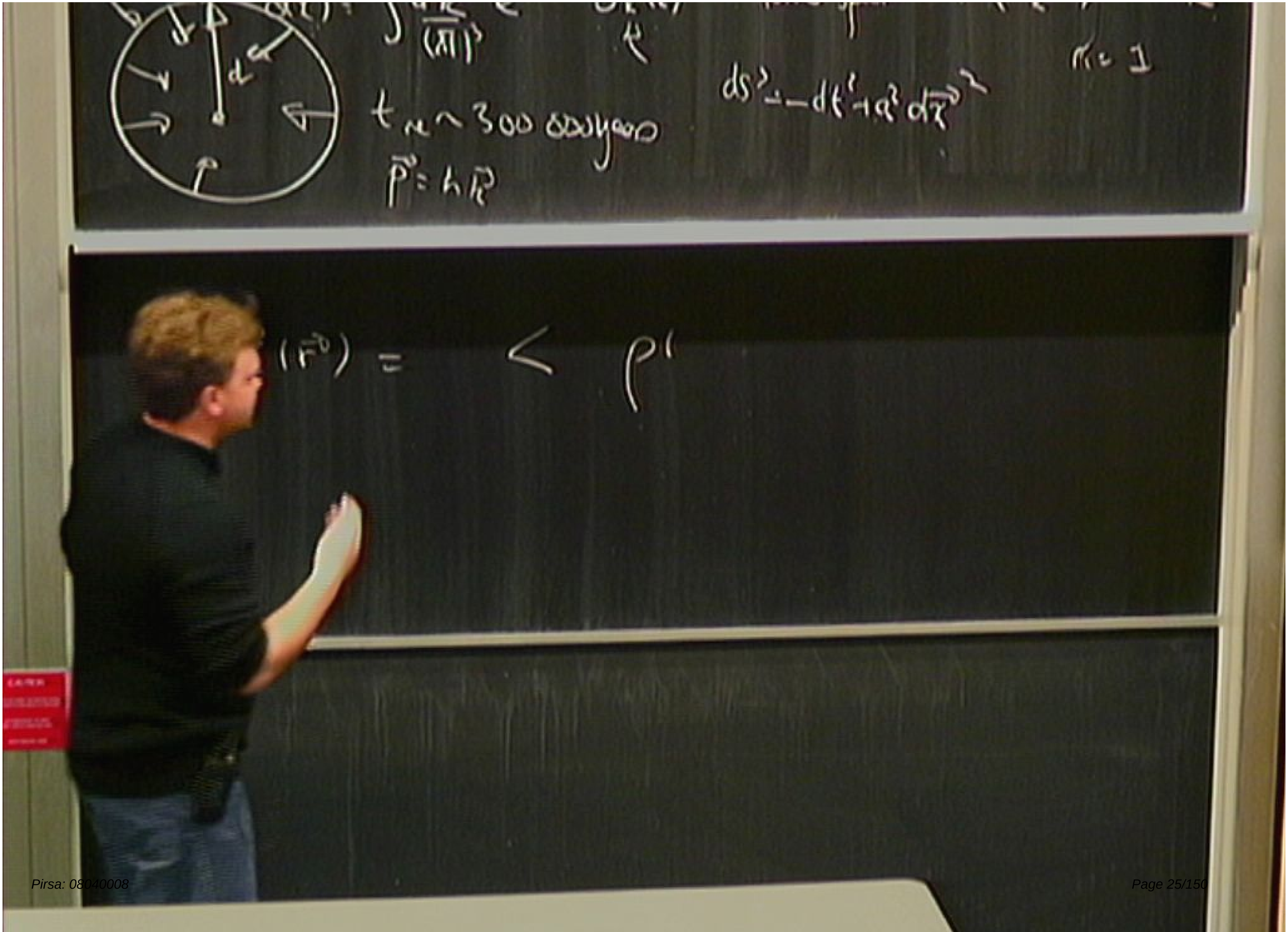
Power spectrum $\Rightarrow \langle \delta_k^2(t) \rangle \sim k^n$

$n = 1$

$ds^2 = -dt^2 + a^2 d\vec{x}^2$

$t_{re} \sim 300\,000$ years

$\vec{p} = \hbar\vec{k}$



$t_r \sim 300\,000\,\text{years}$

$\vec{p} = \hbar \vec{k}$

$ds^2 = -dt^2 + a^2 d\vec{x}^2$

$\hbar = 1$

$$\xi(\vec{r}) = \langle \rho(\vec{x} + \vec{r})\, \rho(\vec{x}) \rangle$$

$\int \frac{d^3k}{(2\pi)^3}$

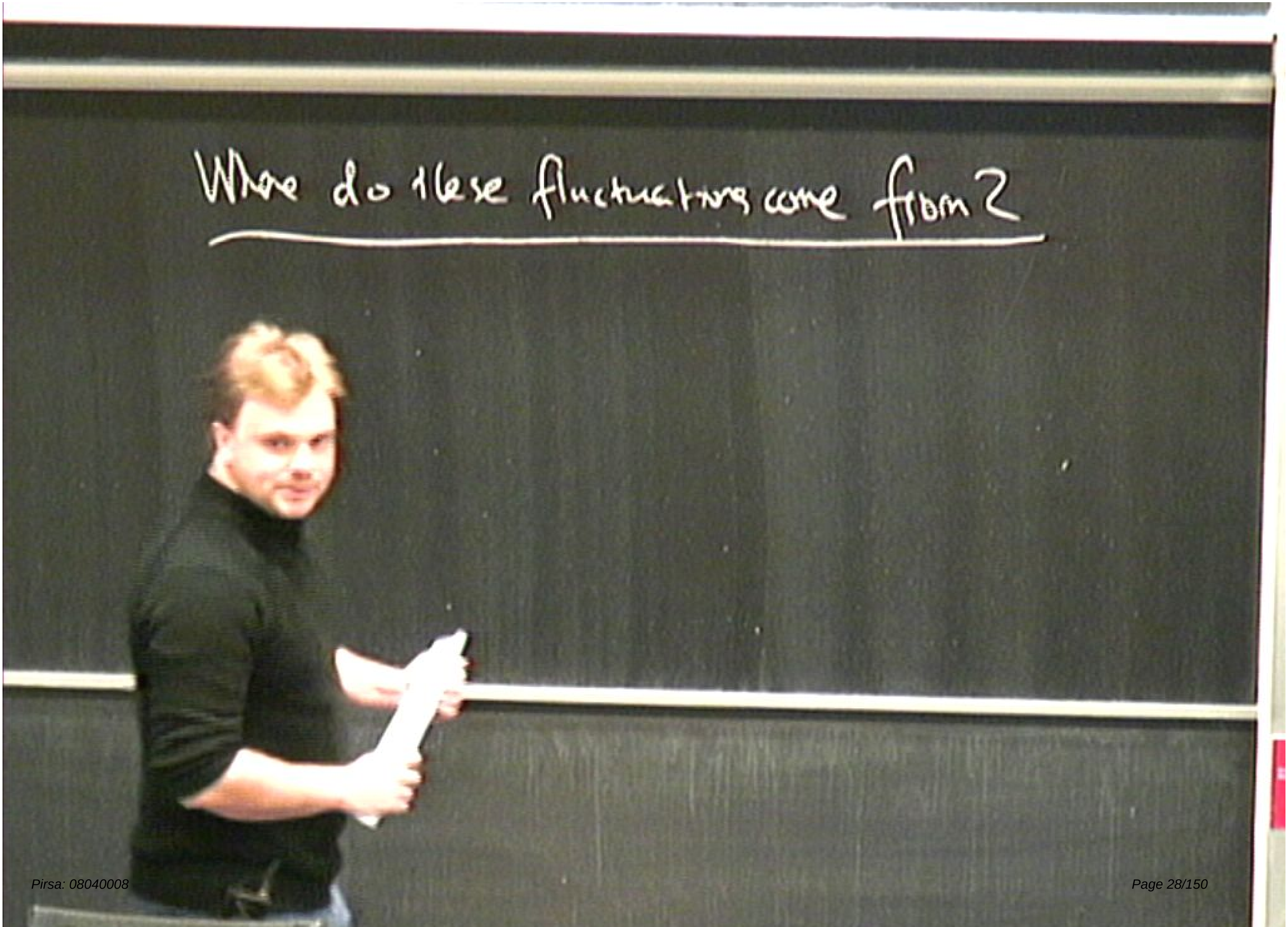
$\hbar = 1$

$t_r \sim 300\,000\ \text{years}$

$ds^2 = -dt^2 + a^2 d\vec{x}^{\,2}$

$\vec{p} = \hbar \vec{k}$

$$\xi(\vec{r}) = \langle \rho(\vec{x}+\vec{r})\,\rho(\vec{x}) \rangle$$


Where do these fluctuations come from?

Where do these fluctuations come from?

Inflation $\leftrightarrow$ why do these fluctuations have power law behavior

Where do these fluctuations come from?

Inflation $\leftrightarrow$ why do these fluctuations have power law behavior

$\leftrightarrow$ adiabatic

Where do these fluctuations come from?

Inflation $\leftrightarrow$ why do these fluctuations have power law behavior

$\leftrightarrow$ adiabatic

gaussian

x

$$p(x) = e^{-\frac{x^2}{\sigma^2}}$$

Where do these fluctuations come from?

Inflation $\leftrightarrow$ why do these fluctuations have power law behavior

x

$$p(x) = e^{-\frac{x^2}{\sigma^2}}$$

δ_k $\quad p(\delta_k) = e^{-\frac{\delta_k^2}{P(k)}}$ $\leftrightarrow$ adiabatic

gaussian

Where do these fluctuations come from?

Inflation $\leftrightarrow$ why do these fluctuations have power law behavior

x

$$p(x) = e^{-\frac{x^2}{\sigma^2}}$$

$$\delta_k \quad p(\delta_k) = e^{-\frac{\delta_k^2}{P(k)}} \leftrightarrow \frac{\text{adiabatic}}{\text{gaussian}}$$

$\bar{\rho}$ $\quad$ T

...uctuations have characteristic spectrum

$$P(k) = \xi(k)$$

$$= \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, \delta_k(t)$$

Power spectrum $\Rightarrow \langle \delta_k^2(t) \rangle \sim k$

$n = 1$

$$ds^2 = -dt^2 + a^2 d\vec{x}^2$$

$t_{re} \sim 300\,000$ years

$\vec{p} = \hbar \vec{k}$

Where do these fluctuations come from?

Inflation $\leftrightarrow$ why do these fluctuations have power law behavior

x

$p(x) = e^{-\frac{x^2}{\sigma^2}}$

δ_k $\quad p(\delta_k) = e^{-\frac{\delta_k^2}{P(k)}}$ $\Leftrightarrow$ adiabatic gaussian

$\langle \delta_{k_1} \delta_{k_2} \rangle = 0$

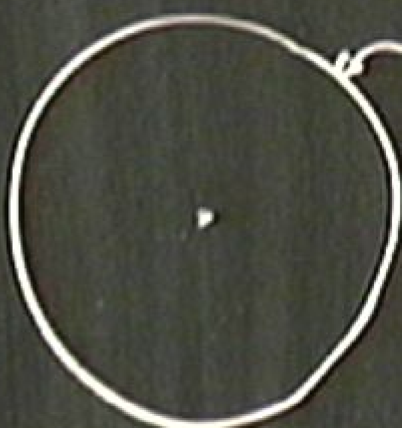
Horizon problem

Horizon problem

$$d_H = c\int_0^t dt' \, \frac{a(t_0)}{a(t')}$$

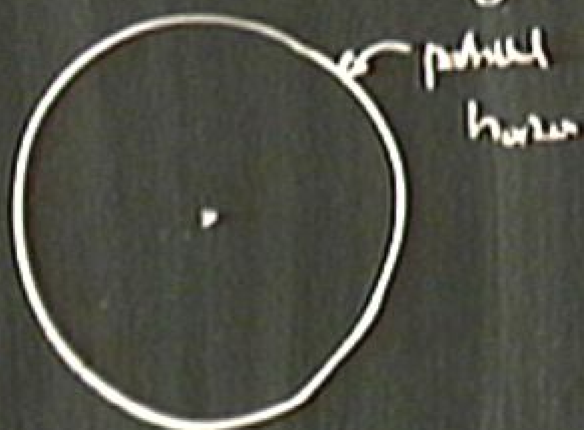
Horizon problem

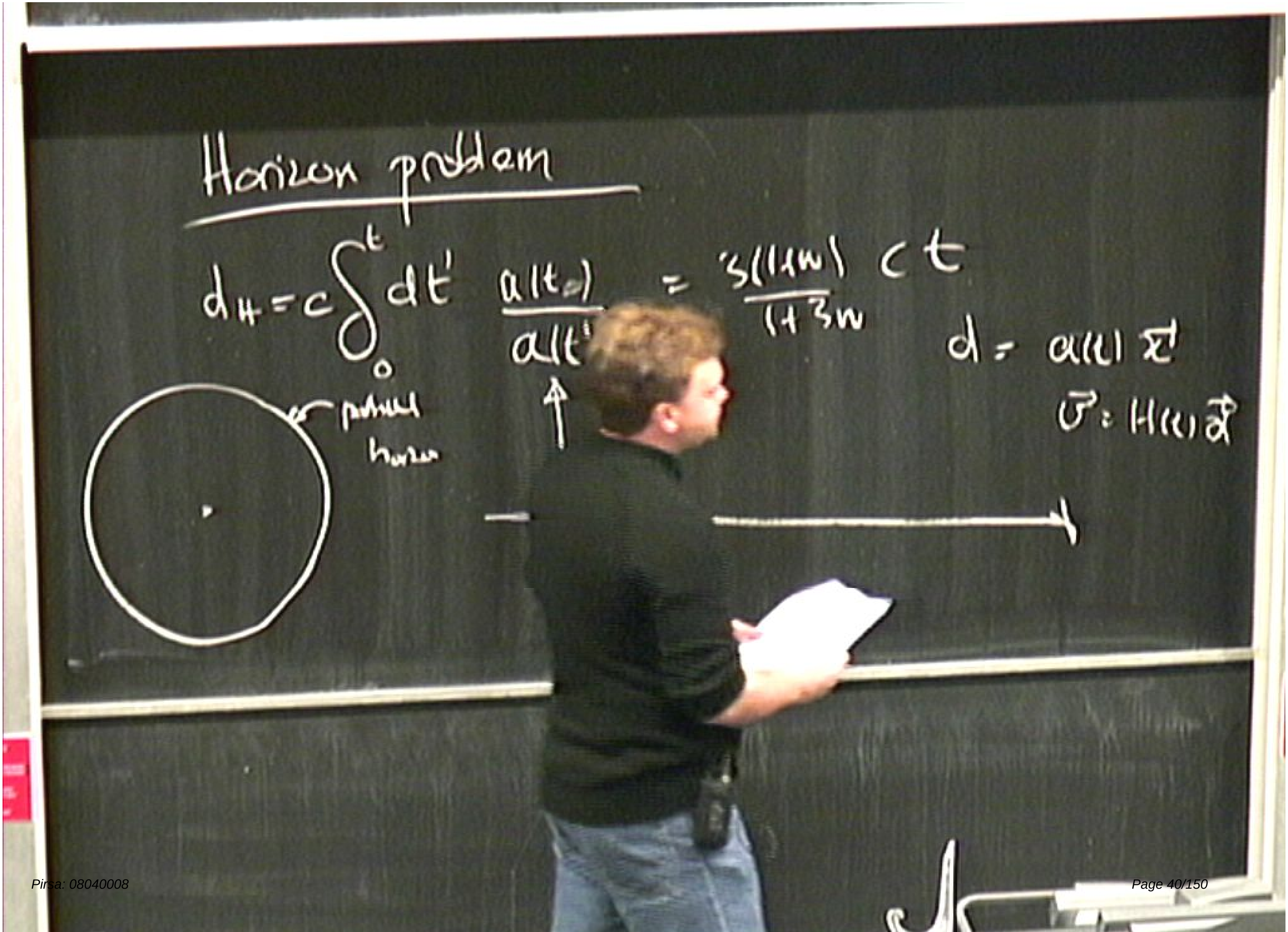
$$d_H = c\int_0^t dt' \, \frac{a(t)}{a(t')} = \frac{3(1+w)}{1+3w}\, ct$$



Horizon problem

$$d_H = c\int_0^t dt' \, \frac{a(t)}{a(t')} = \frac{3(1+w)}{1+3w}\, c\,t$$



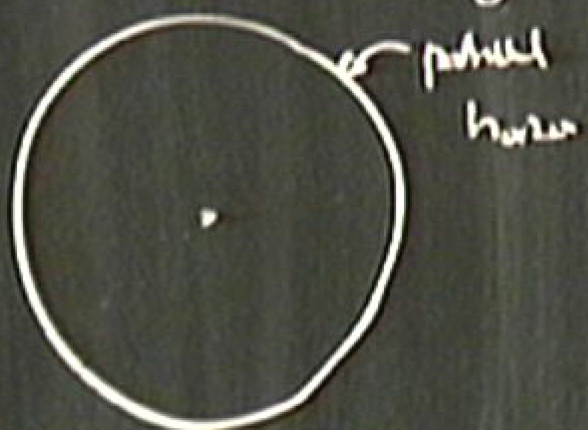

Horizon problem
d_H = c ∫_0^t dt' a(t_0)/a(t') = 3(1+w)/(1+3w) ct
d = a(t) x
v = H(t) d
particle horizon

Horizon problem

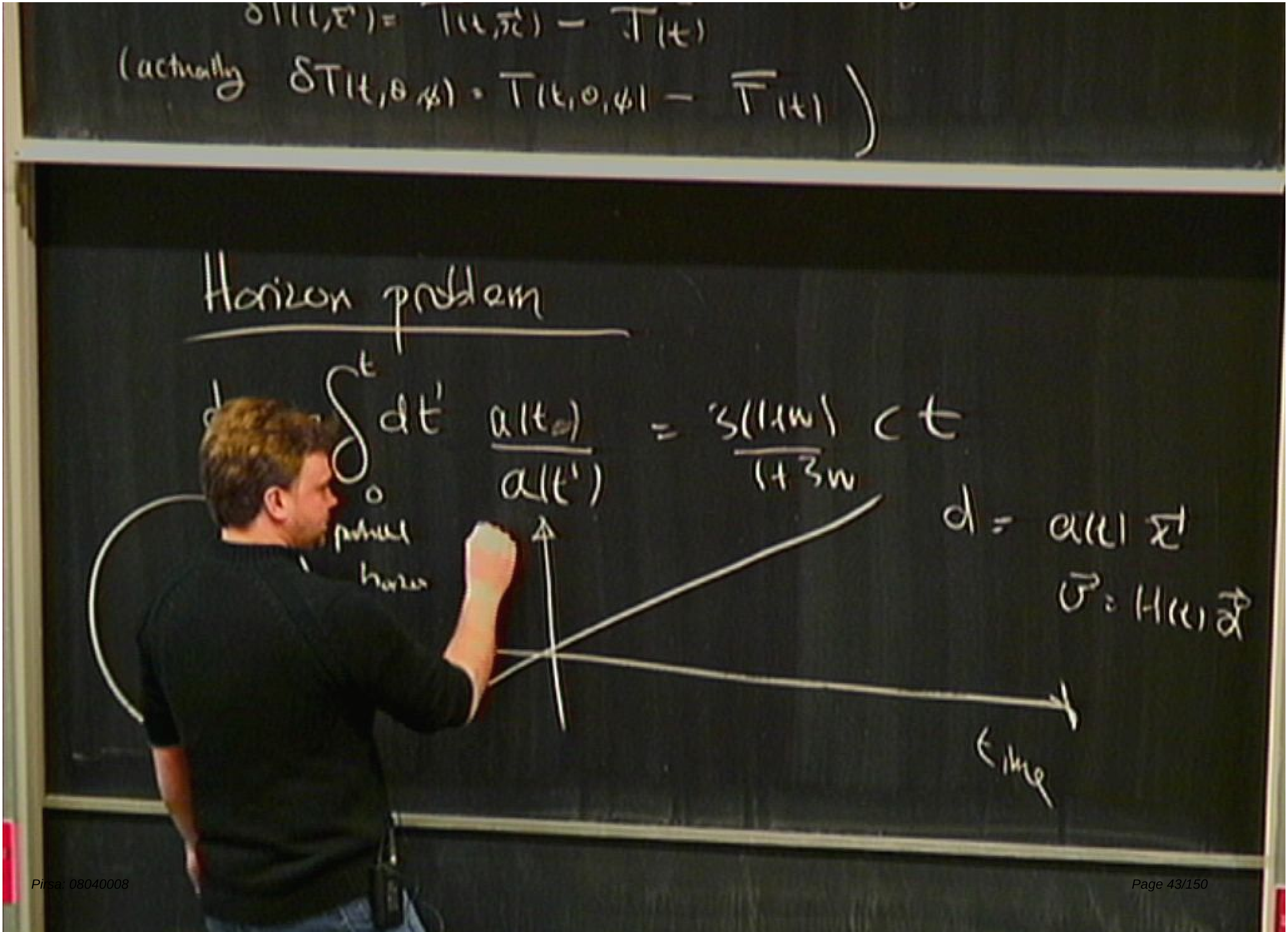
$$d_H = c\int_0^t dt' \, \frac{a(t)}{a(t')} = \frac{3(1+w)}{1+3w}\, c\,t$$

$$d = a(t)\,\vec{x}$$

$$\vec{v} = H(t)\,\vec{d}$$



$w=0 \quad a(t) \sim t^{2/3}$


$\delta T(t,\vec{x}) = T(t,\vec{x}) - \bar{T}(t)$
(actually $\delta T(t,\theta,\phi) = T(t,\theta,\phi) - \bar{T}(t)$)
Horizon problem
$d = \int_0^t dt' \frac{a(t_0)}{a(t')} = \frac{3(1+w)}{1+3w} ct$
particle horizon
$d = a(t)\vec{x}$
$\vec{v} = H(t)\vec{x}$
time

$\delta T(t,\vec{x}) = T(t,\vec{x}) - \bar{T}(t)$

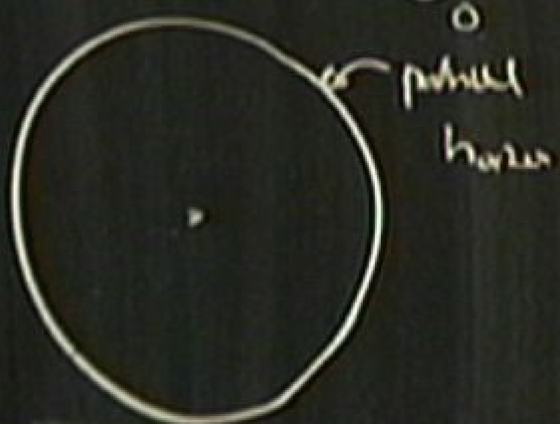
(actually $\delta T(t,\theta,\phi) = T(t,\theta,\phi) - \bar{T}(t)$)

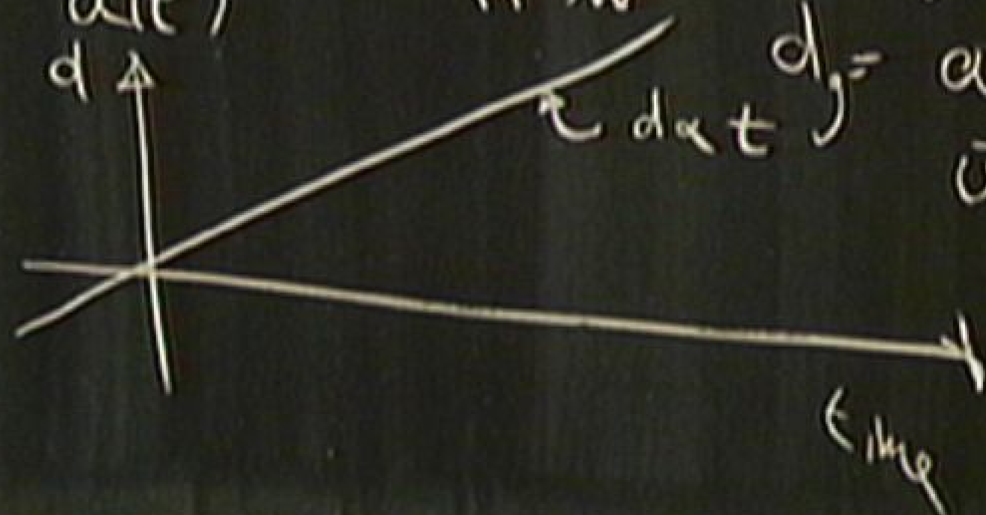
Horizon problem

$$d_H = c\int_0^t dt' \, \frac{a(t_0)}{a(t')} = \frac{3(1+w)}{1+3w}\, ct \qquad d_g \sim t^{2/3}\vec{x}$$

$$d_g = a(t)\,\vec{x}$$

$$\vec{v} = H(t)\,\vec{x}$$





$$\delta T(t,\vec{x}) = T(t,\vec{x}) - \bar{T}(t)$$

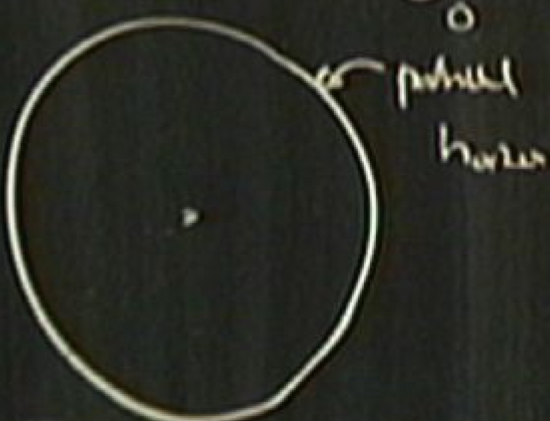
(actually $\delta T(t,\theta,\phi) = T(t,\theta,\phi) - \bar{T}(t)$)

Horizon problem

$$d_H = c\int_0^t dt' \, \frac{a(t_0)}{a(t')} = \frac{3(1+w)}{1+3w}\, ct \qquad d_g \sim t^{2/3}\vec{x}$$

$$d_g = a(t)\,\vec{x}$$

$$\vec{v} = H(t)\,\vec{x}$$



$\delta T(t,\vec{x}) = T(t,\vec{x}) - \bar{T}(t)$

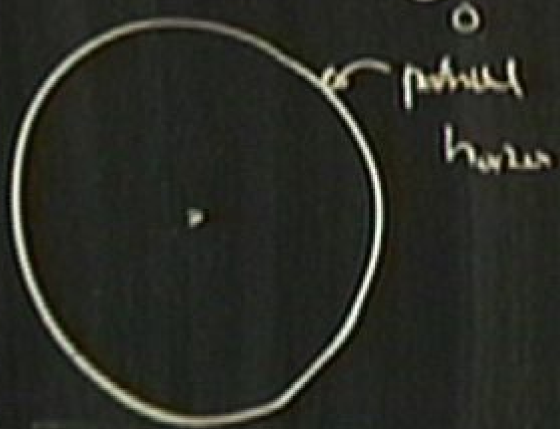
(actually $\delta T(t,\theta,\phi) = T(t,\theta,\phi) - \bar{T}(t)$)

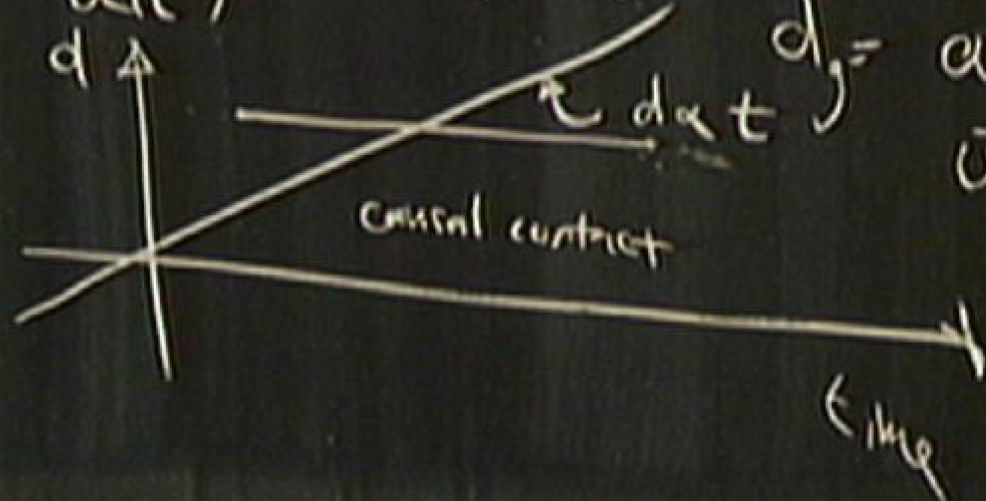
Horizon problem

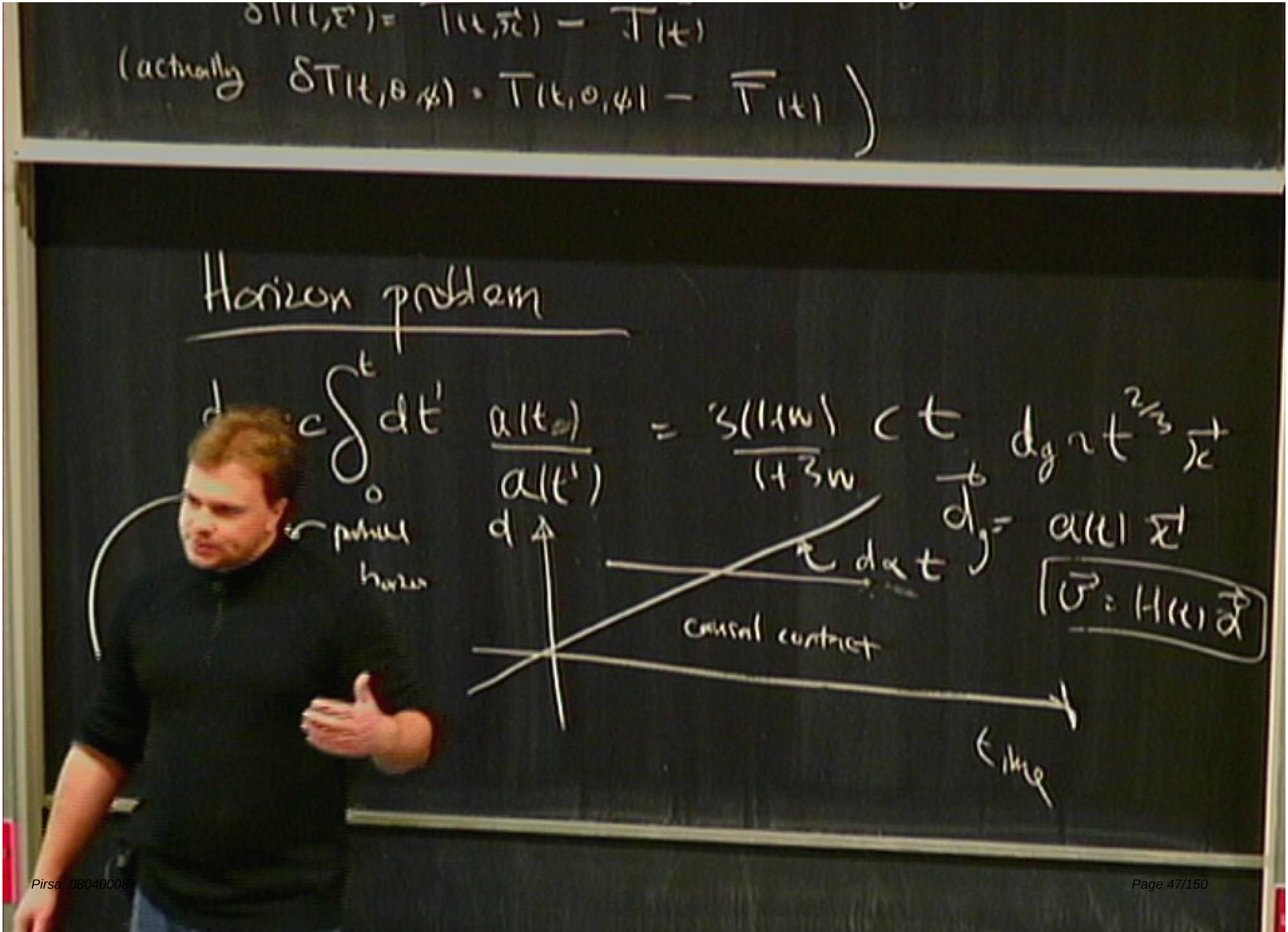
$$d_H = c\int_0^t dt' \, \frac{a(t_0)}{a(t')} = \frac{3(1+w)}{1+3w}\, c\,t \qquad d_g \sim t^{2/3}\,\vec{x}$$

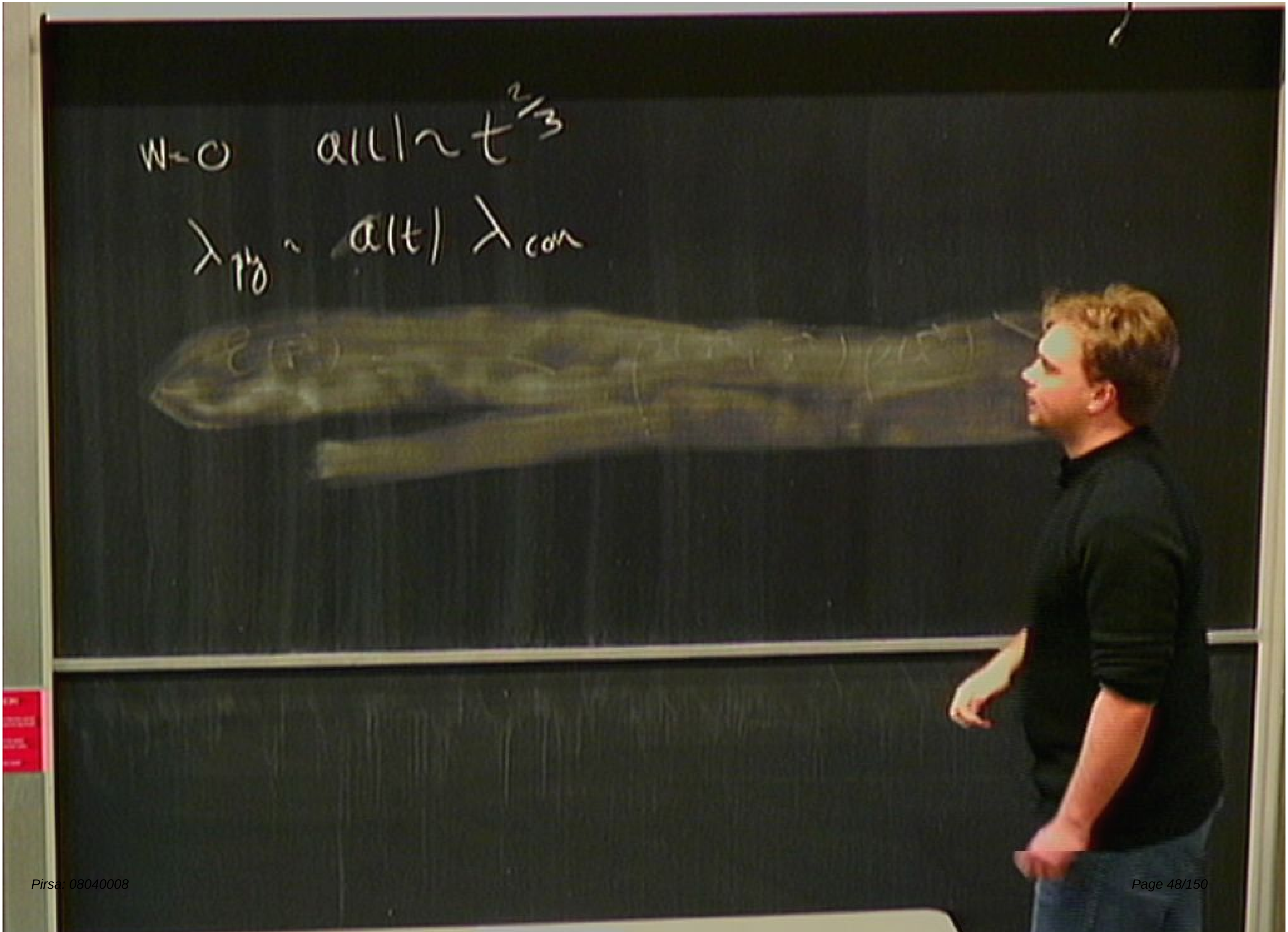
$$d_g = a(t)\,\vec{x}$$

$$\vec{v} = H(t)\,\vec{x}$$





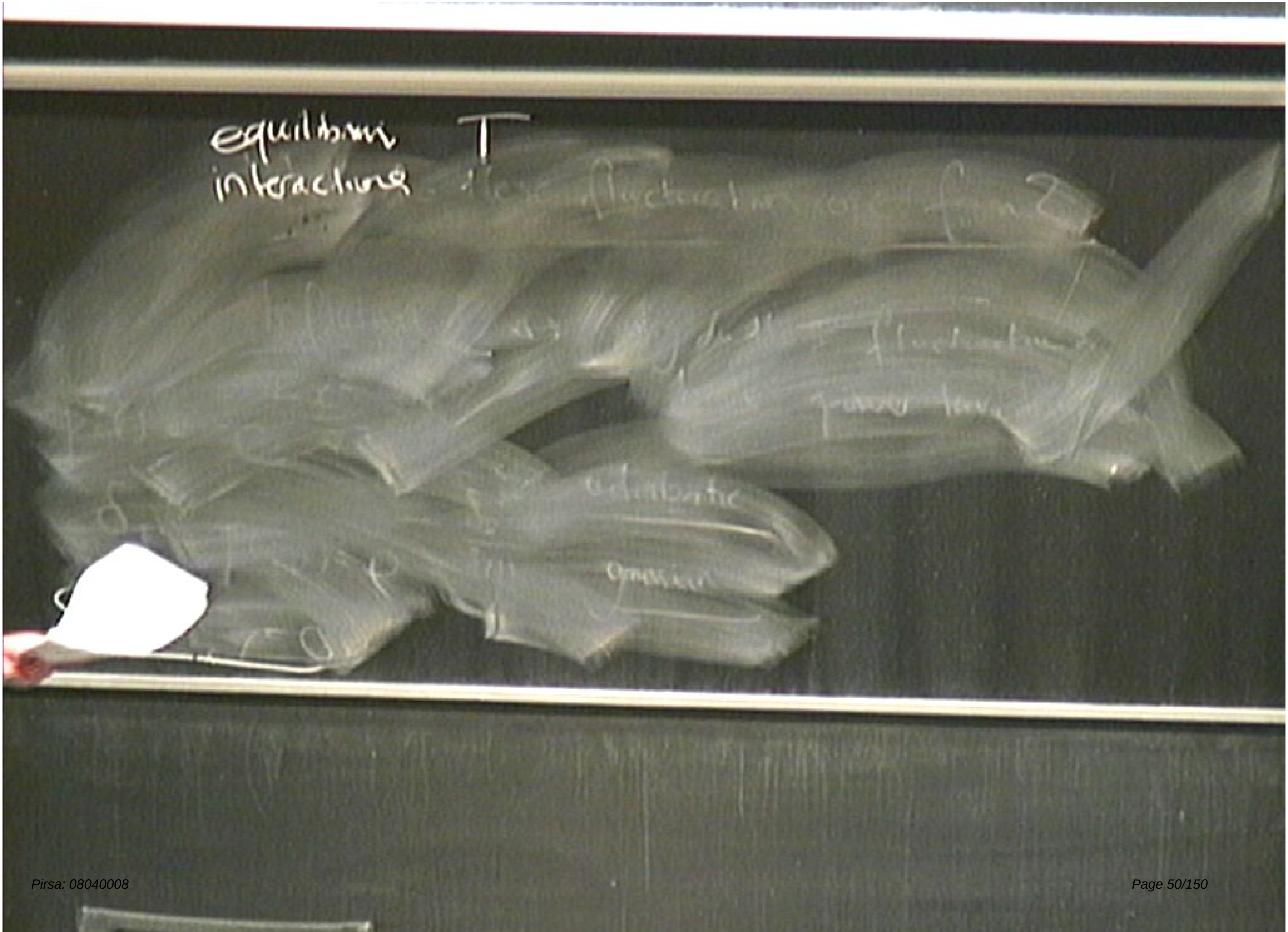

(actually δT(t,θ,φ) = T(t,θ,φ) − T̄(t))
Horizon problem
= 3(1+w) / (1+3w) c t
d_g ∼ t^{2/3} x
d_g = a(t) x
v = H(t) d
d ∝ t
causal contact
time
horizon


w=0 a(t) ~ t^2/3
λphy ~ a(t) λcon

$W = 0 \qquad a(t) \sim t^{2/3}$

$\lambda_{phy} \sim a(t)\, \lambda_{con}$

$t^{2/3}$


equilibrium
interactions
T

equilibrium T

interactions

$d \sim 10^{26}\,m \sim$ homogeneous, isotropic


equilibrium T
interactions
d ~ 10^26 m ~ homogeneous, isotropic
a(t) / a(t_0) × 10^26 m = T(t_0) / T(t) × 10^26 m

Planck length ~ quantum gravity ~ 10^{-35} m

time ~ $\frac{1}{3\times10^{8}}$ ~ 10^{-43} s

Planck length ~ quantum gravity ~ 10^{-35} m

time ~ $\frac{1}{3\times10^{8}}$ ~ 10^{-43} s

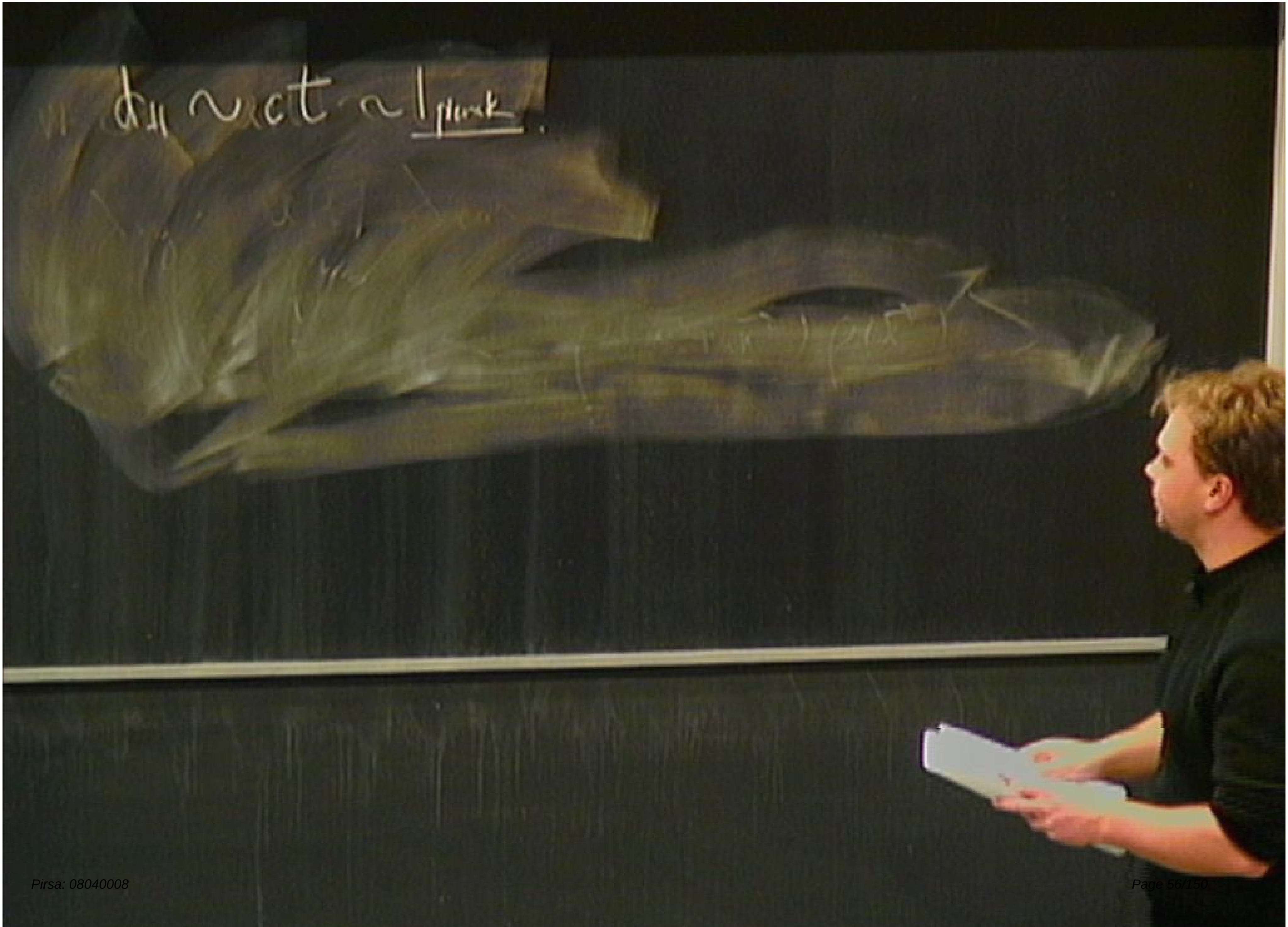
Planck temperature $T \sim 10^{32}$ K

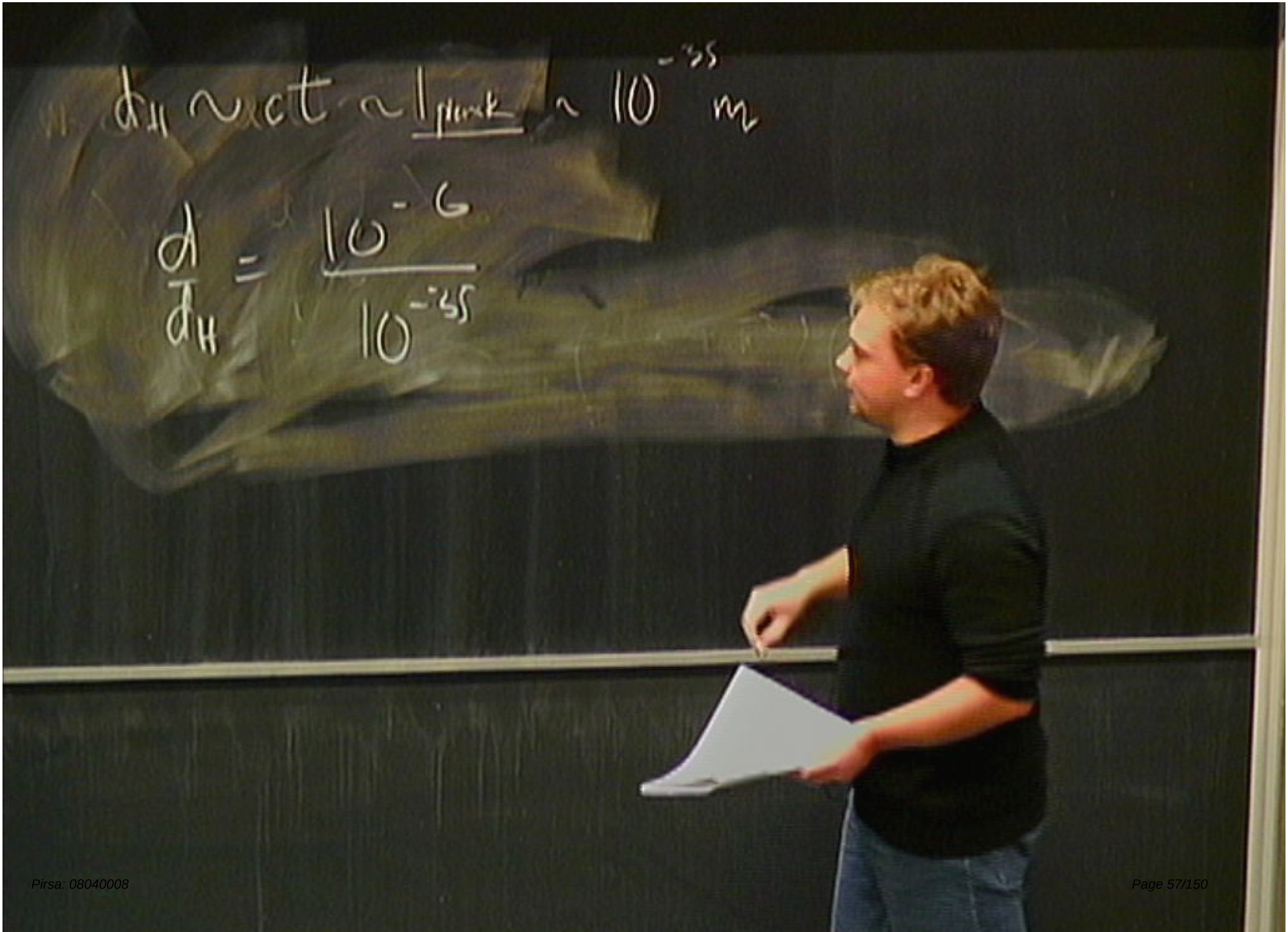
Planck length ~ quantum gravity ~ 10^{-35} m

time ~ $\frac{1}{3 \times 10^{8}}$ ~ 10^{-43} s

Planck temperature $T \sim 10^{32}$ K

$$d \sim \frac{3}{10^{32}} \times 10^{26} \sim 10^{-6}\,\text{m}$$


$d_H \sim ct \sim l_{planck}$

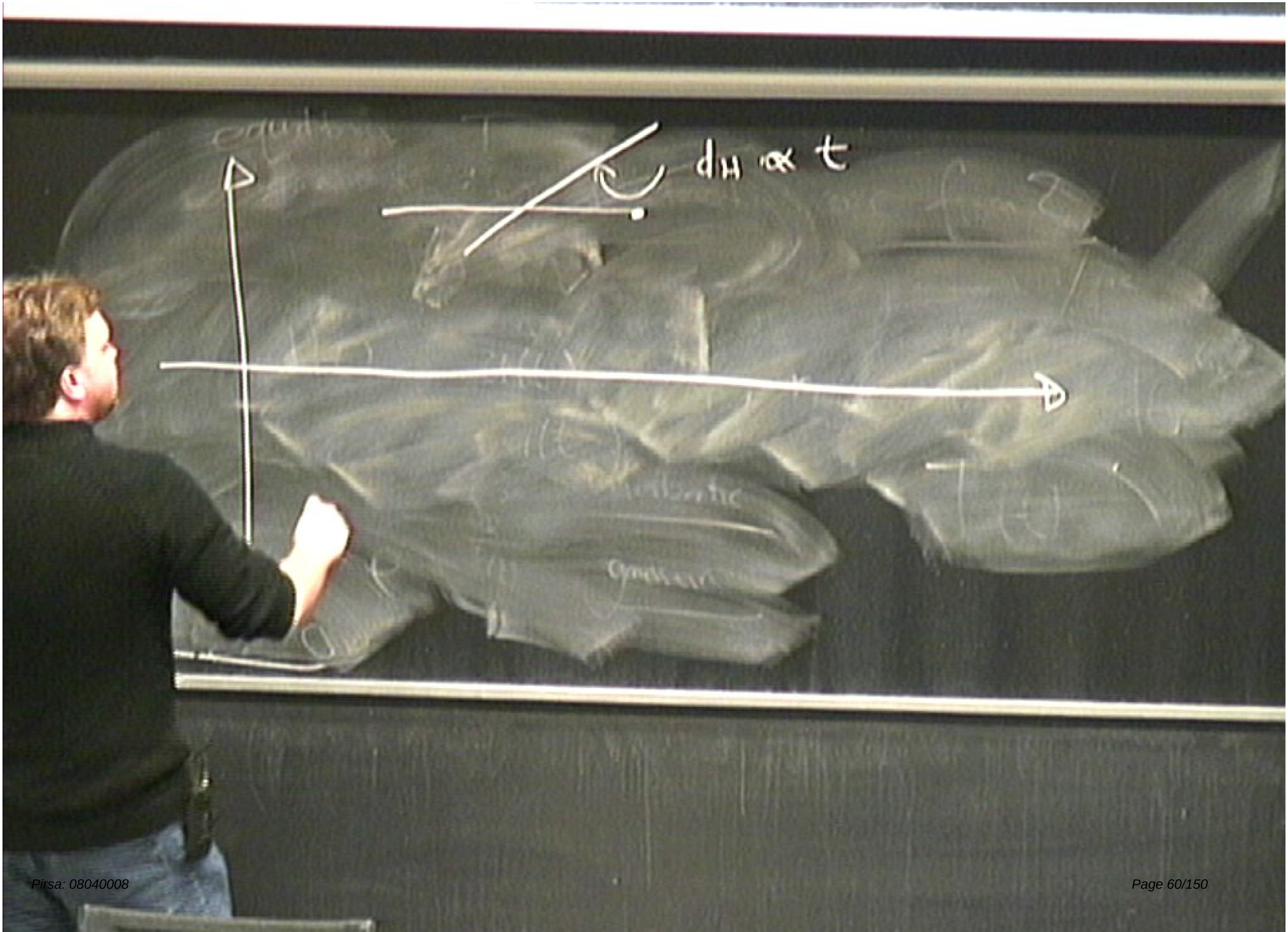

$d_H \sim ct \sim l_{planck} \sim 10^{-35}$ m
$\frac{d}{d_H} = \frac{10^{-6}}{10^{-35}}$

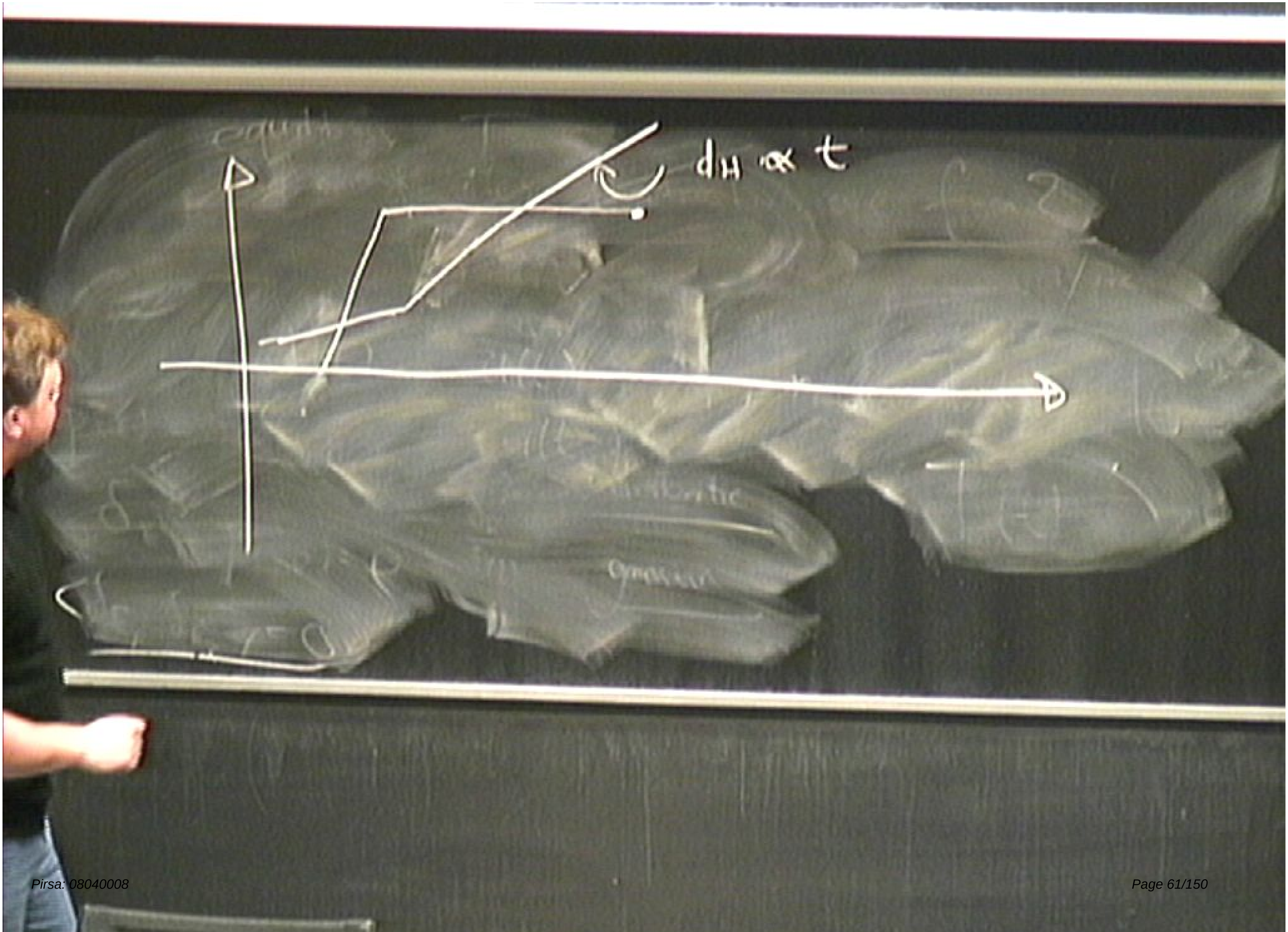
$$d_H \sim ct \sim l_{planck} \sim 10^{-35}\ m$$

$$\frac{d}{d_H} = \frac{10^{-6}}{10^{-35}} \sim 10^{29}$$

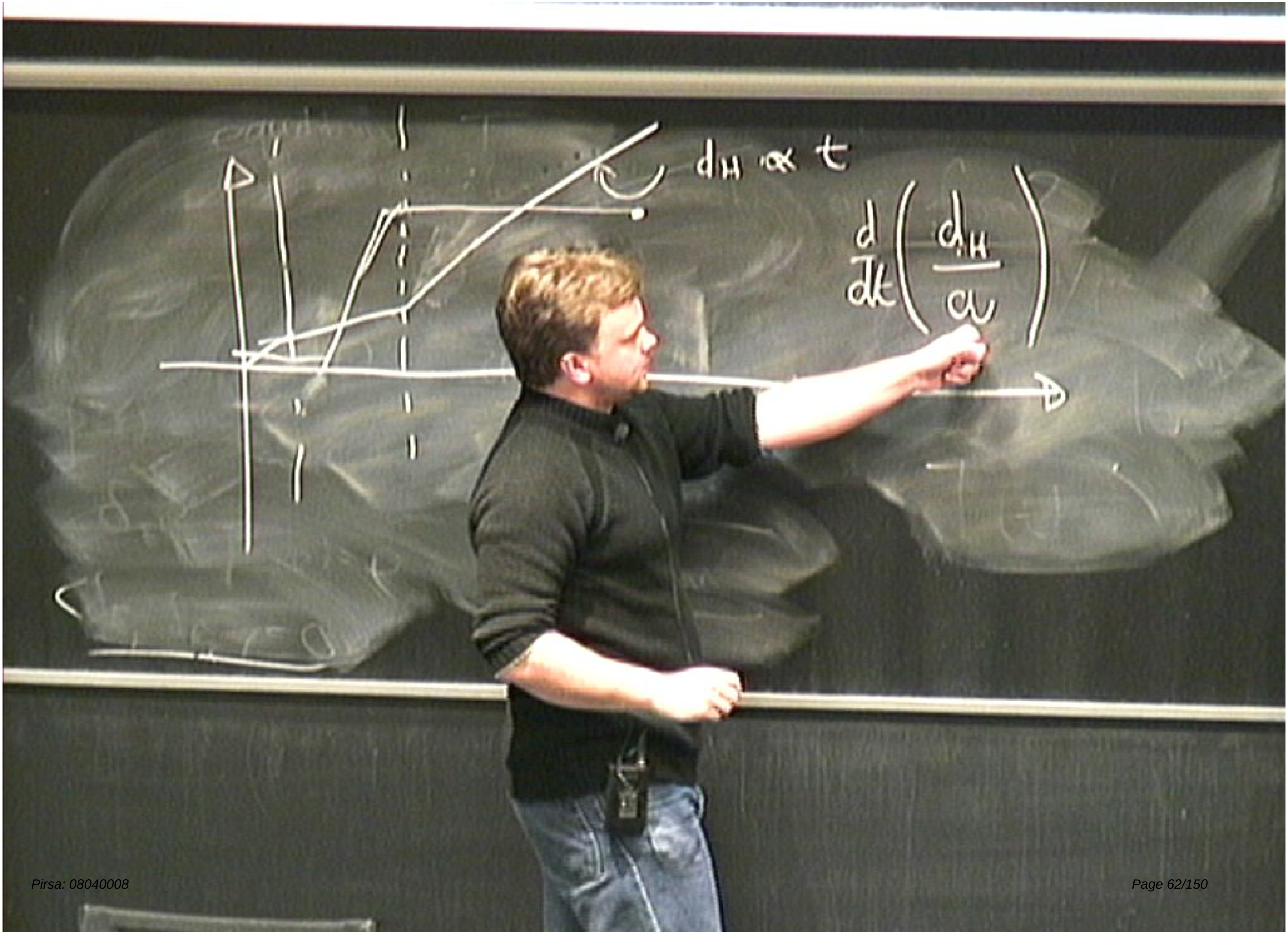
$$d_H \sim ct \sim l_{planck} \sim 10^{-35}\ m$$

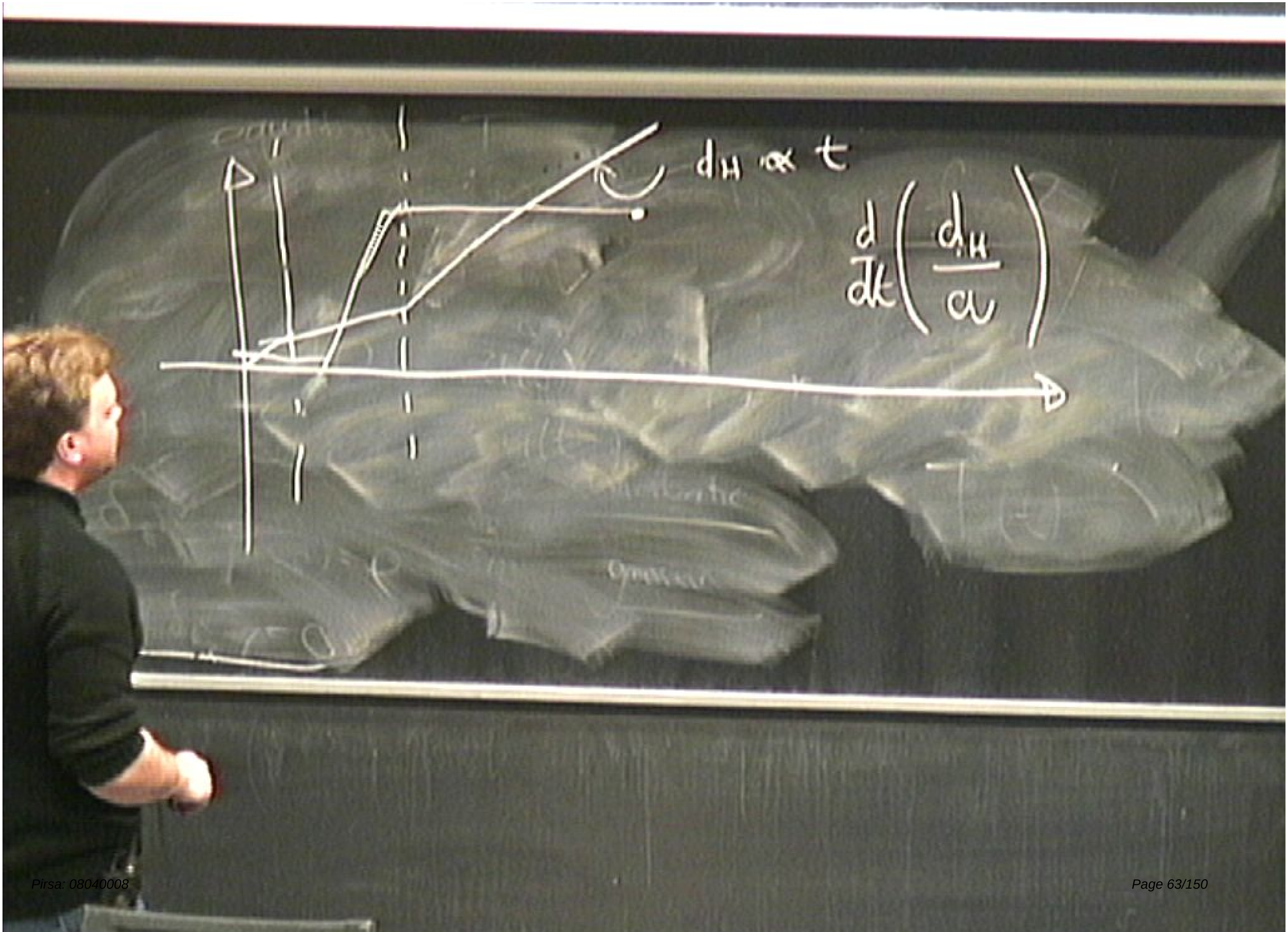
$$\frac{d}{d_H} = \frac{10^{-6}}{10^{-35}} \sim 10^{29} \sim e^{65}$$


$d_H \propto t$

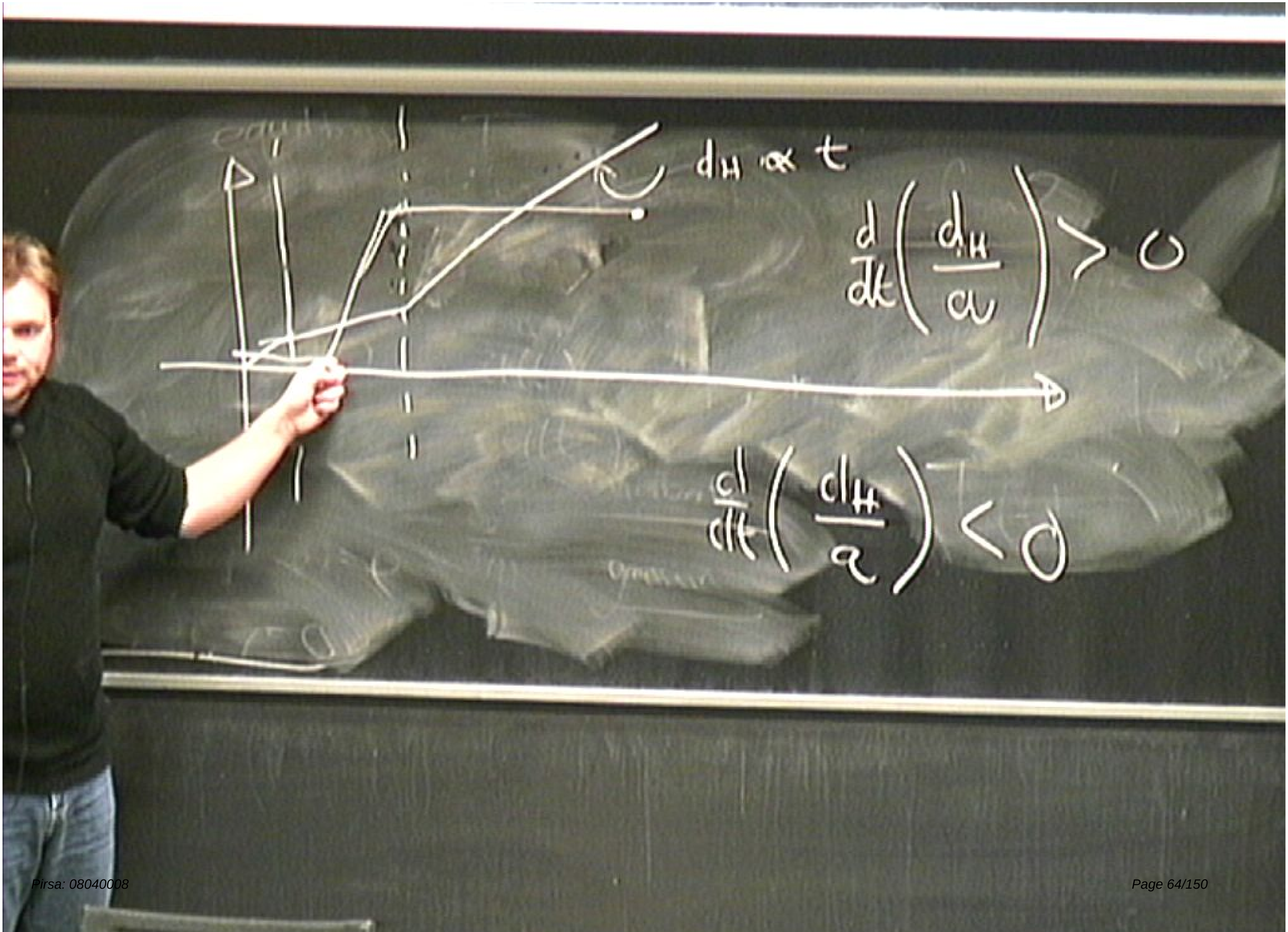


$d_H \propto t$

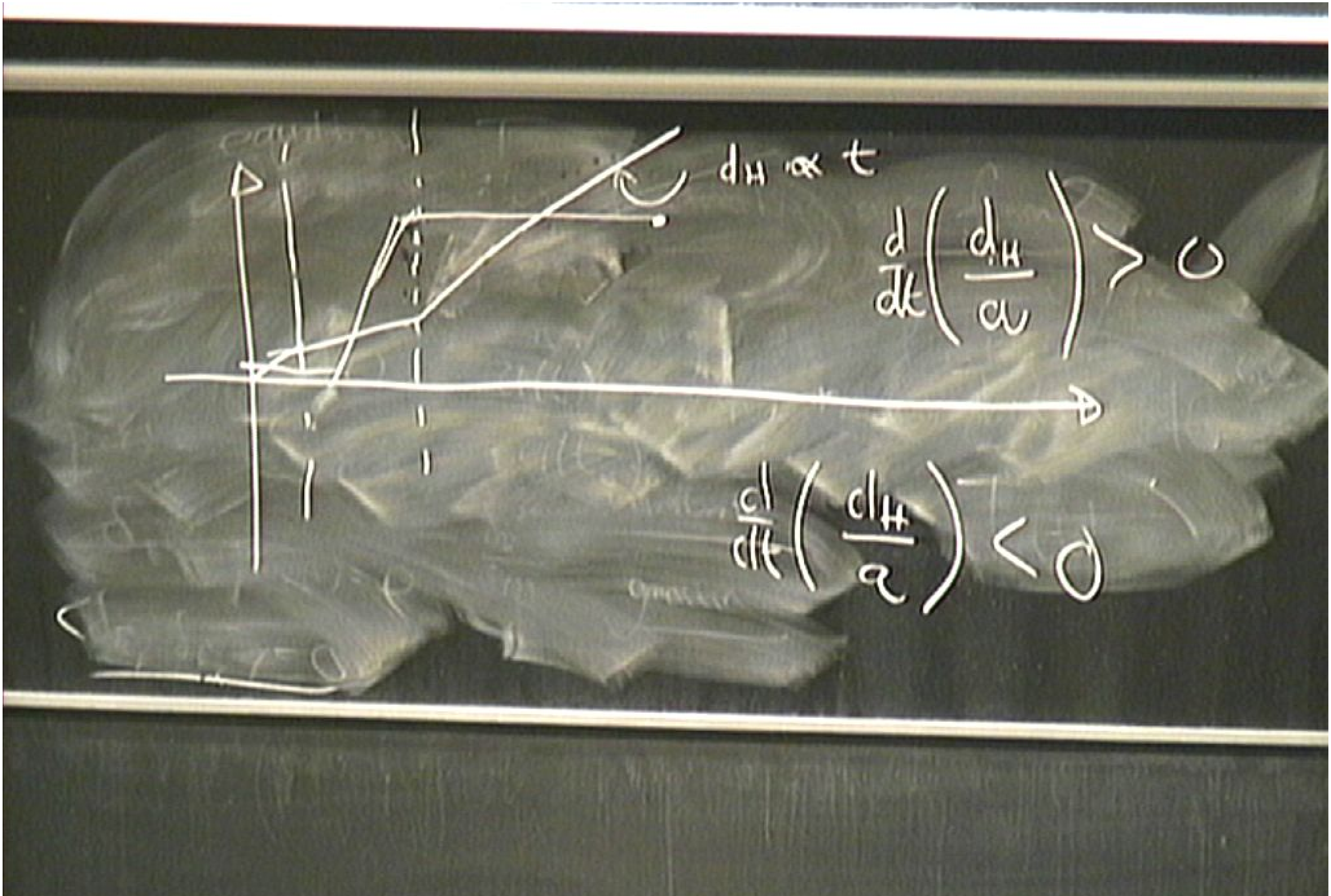

$d_H \propto t$
$\frac{d}{dt}\left(\frac{d_H}{a}\right)$



$d_H \propto t$
$\frac{d}{dt}\left(\frac{d_H}{a}\right)$



$d_H \propto t$
$\frac{d}{dt}\left(\frac{d_H}{a}\right) > 0$
$\frac{d}{dt}\left(\frac{d_H}{a}\right) < 0$



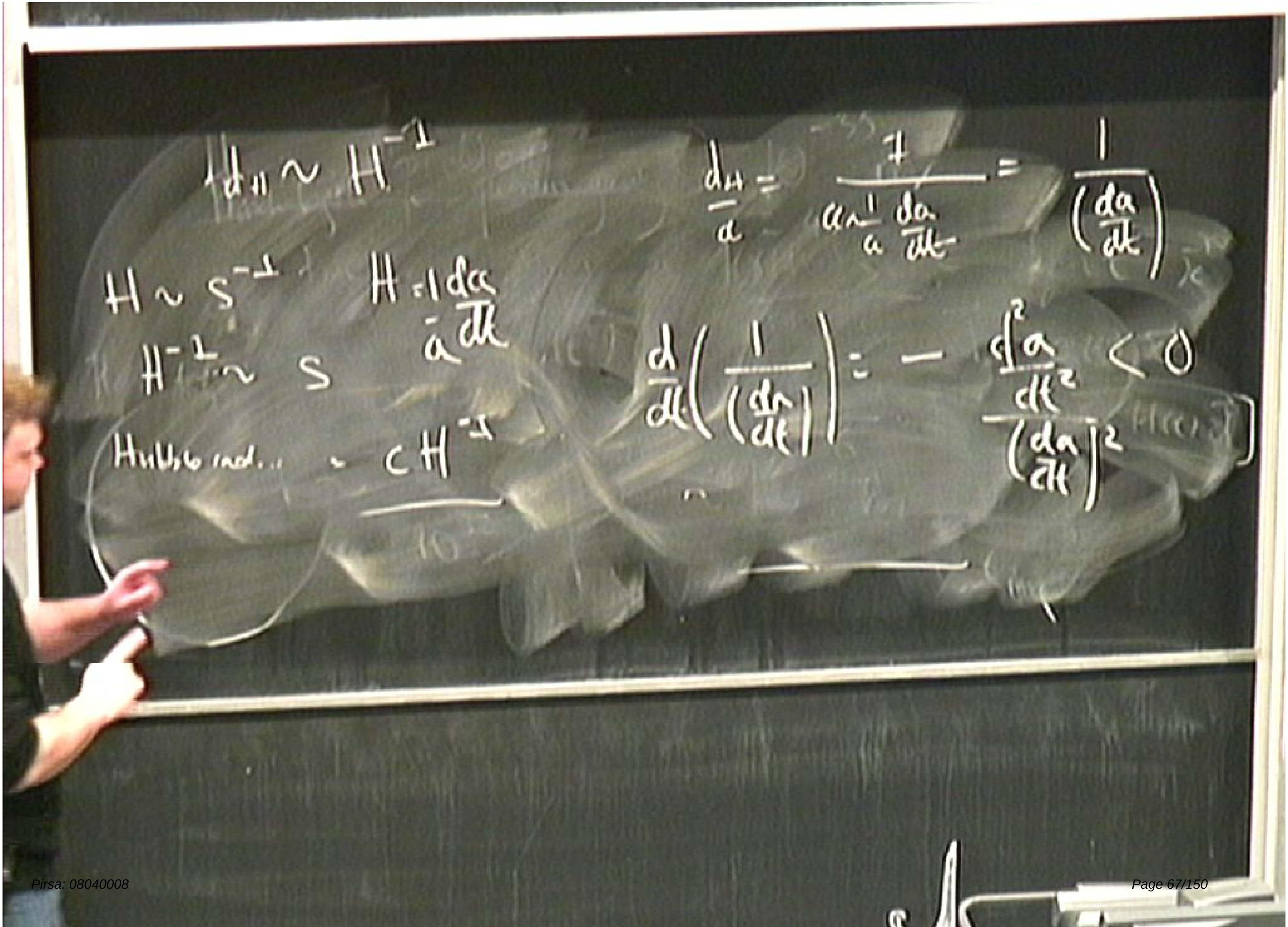
$d_H \propto t$
$\frac{d}{dt}\left(\frac{d_H}{a}\right) > 0$
$\frac{d}{dt}\left(\frac{d_H}{a}\right) < 0$

$d_H \sim H^{-1}$

$H \sim s^{-1}$ $H = \frac{1}{a}\frac{da}{dt}$

$H^{-1} \sim s$

Hubble rad... $= c H^{-1}$



$d_H \sim H^{-1}$
$H \sim S^{-1}$
$H = \frac{1}{a}\frac{da}{dt}$
$H^{-1} \sim S$
Hubble rad... $= cH^{-1}$
$\frac{d_H}{a} = \frac{1}{a \frac{1}{a}\frac{da}{dt}} = \frac{1}{\left(\frac{da}{dt}\right)}$
$\frac{d}{dt}\left(\frac{1}{\left(\frac{da}{dt}\right)}\right) = -\frac{\frac{d^2a}{dt^2}}{\left(\frac{da}{dt}\right)^2} < 0$

$d_H \sim H^{-1}$

$H \sim s^{-1}$ $\quad H = \frac{1}{a}\frac{da}{dt}$

$H^{-1} \sim s$

Hubble radius $= cH^{-1}$

$$\frac{d_H}{a} = \frac{1}{a \frac{1}{a}\frac{da}{dt}} = \frac{1}{\left(\frac{da}{dt}\right)}$$

$$\frac{d}{dt}\left(\frac{1}{\left(\frac{da}{dt}\right)}\right) = -\frac{\frac{d^2a}{dt^2}}{\left(\frac{da}{dt}\right)^2} < 0$$

$$\boxed{\frac{d^2a}{dt^2} > 0}$$ Accelerate

$d_H \sim H^{-1}$

$H \sim s^{-1}$ $\quad H = \frac{1}{a}\frac{da}{dt}$

$H^{-1} \sim s$

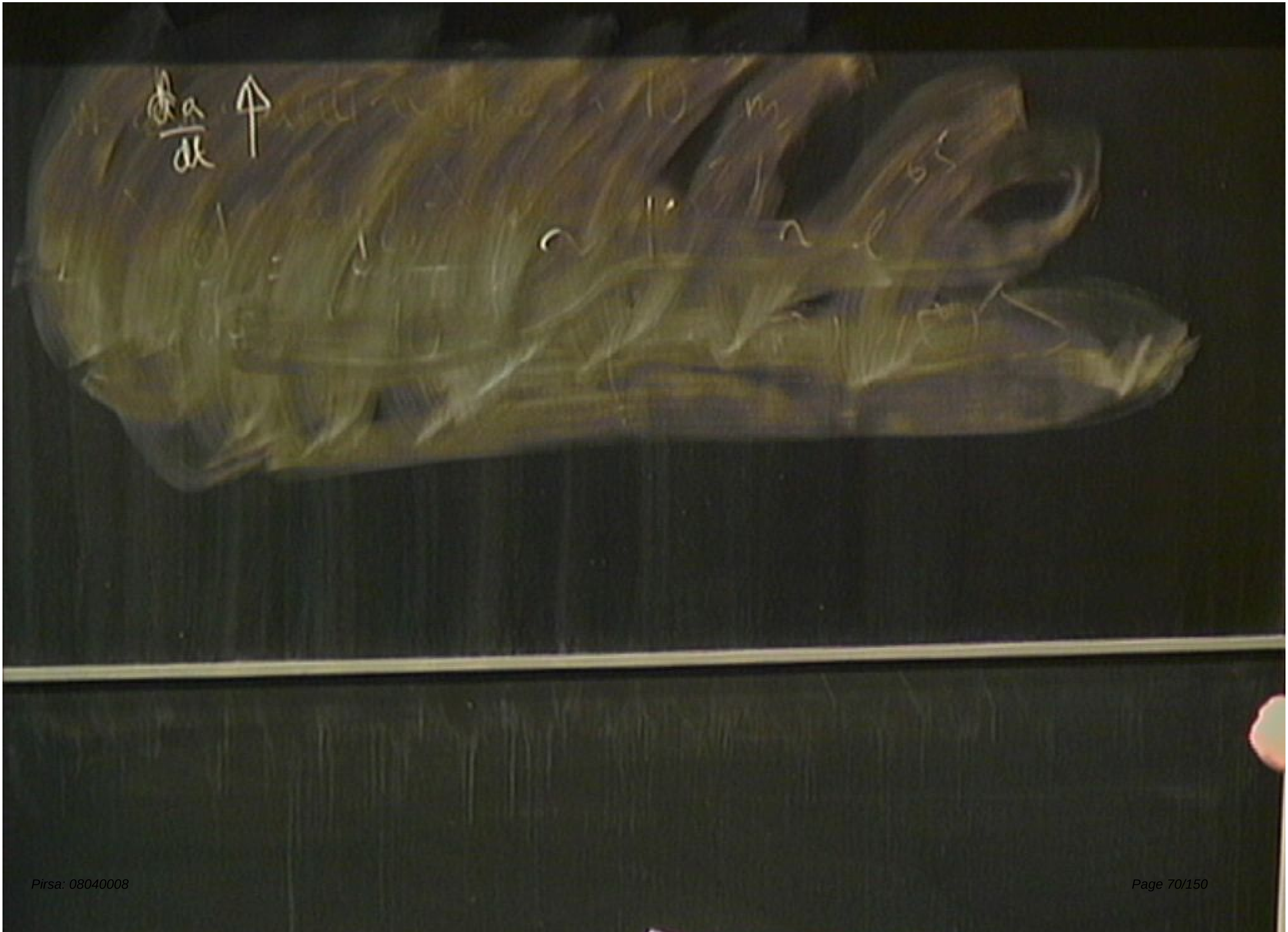
Hubble rad... $= cH^{-1}$

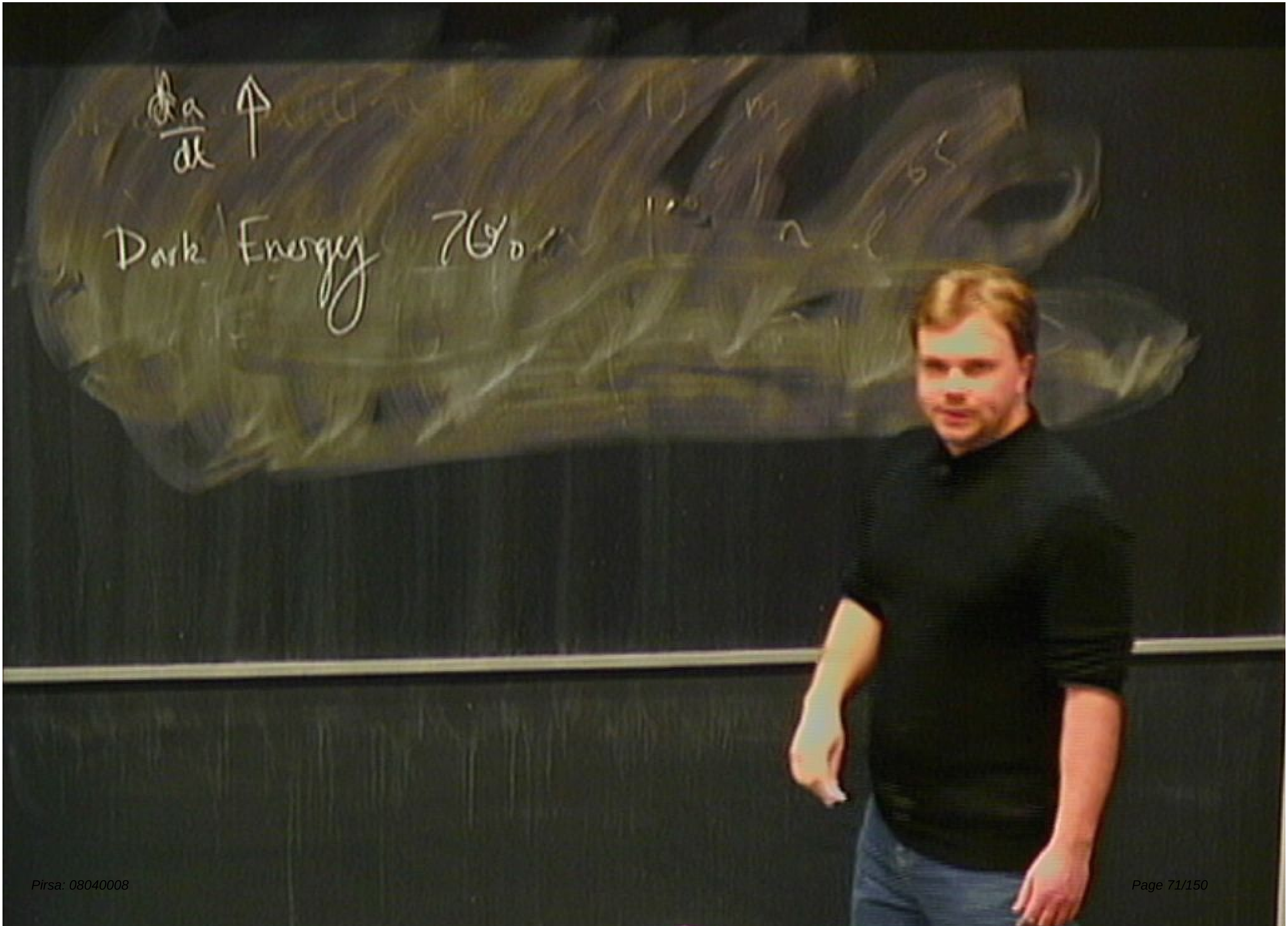
$$\frac{d_H}{a} = \frac{1}{a\,\frac{1}{a}\frac{da}{dt}} = \frac{1}{\left(\frac{da}{dt}\right)}$$

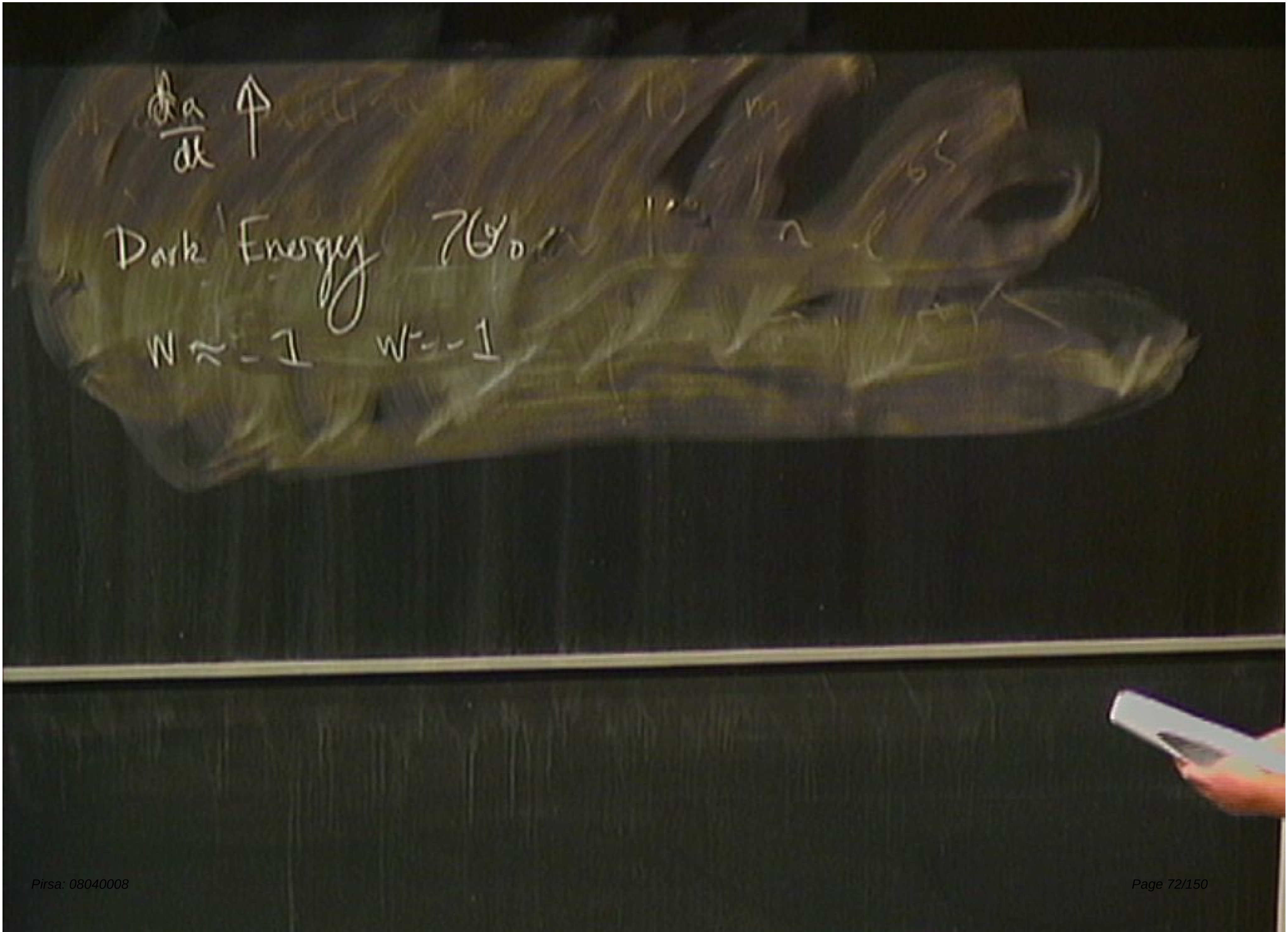
$$\frac{d}{dt}\left(\frac{1}{\left(\frac{da}{dt}\right)}\right) = -\frac{\frac{d^2a}{dt^2}}{\left(\frac{da}{dt}\right)^2} < 0$$

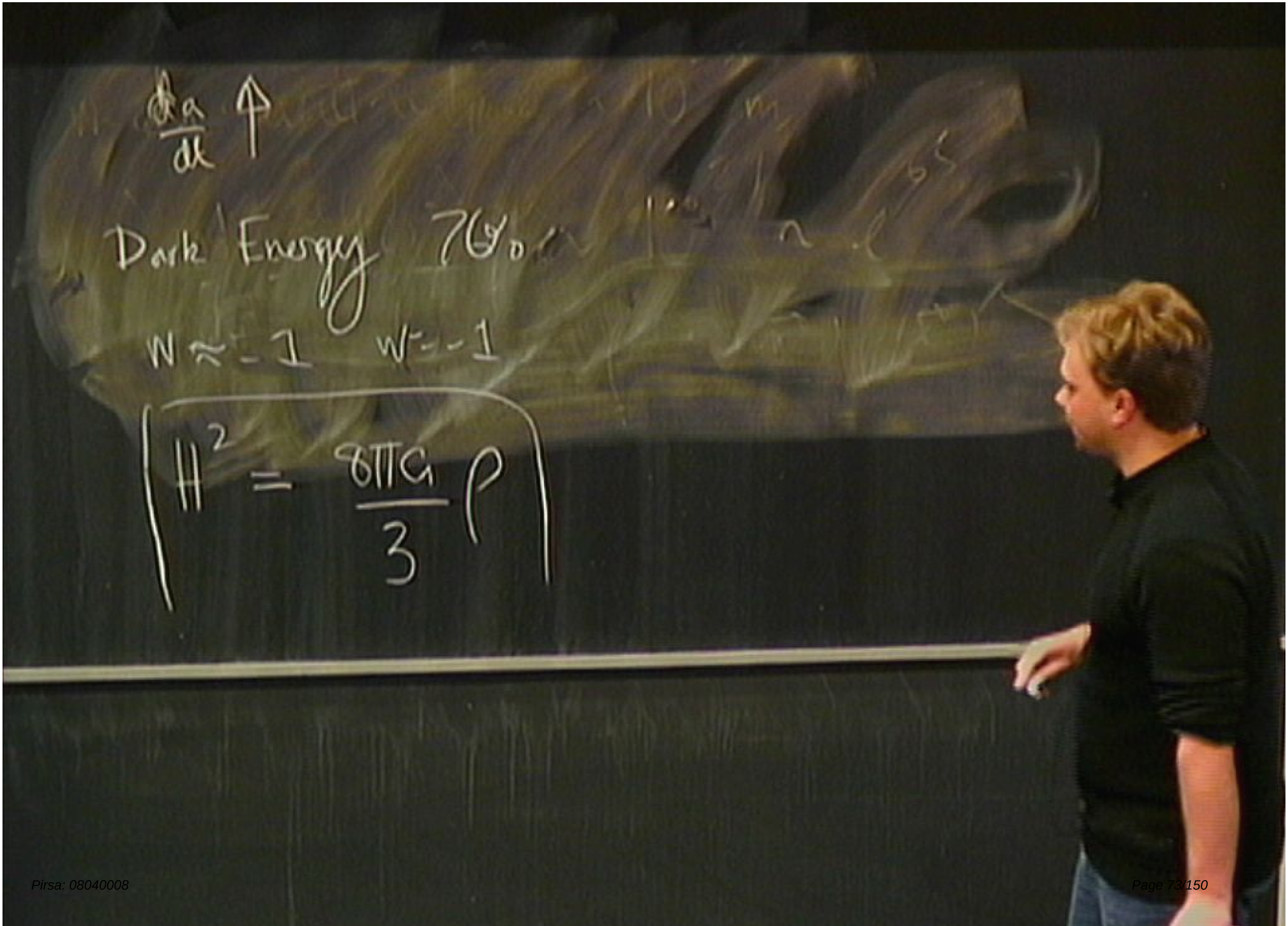
$$\boxed{\frac{d^2a}{dt^2} > 0}$$

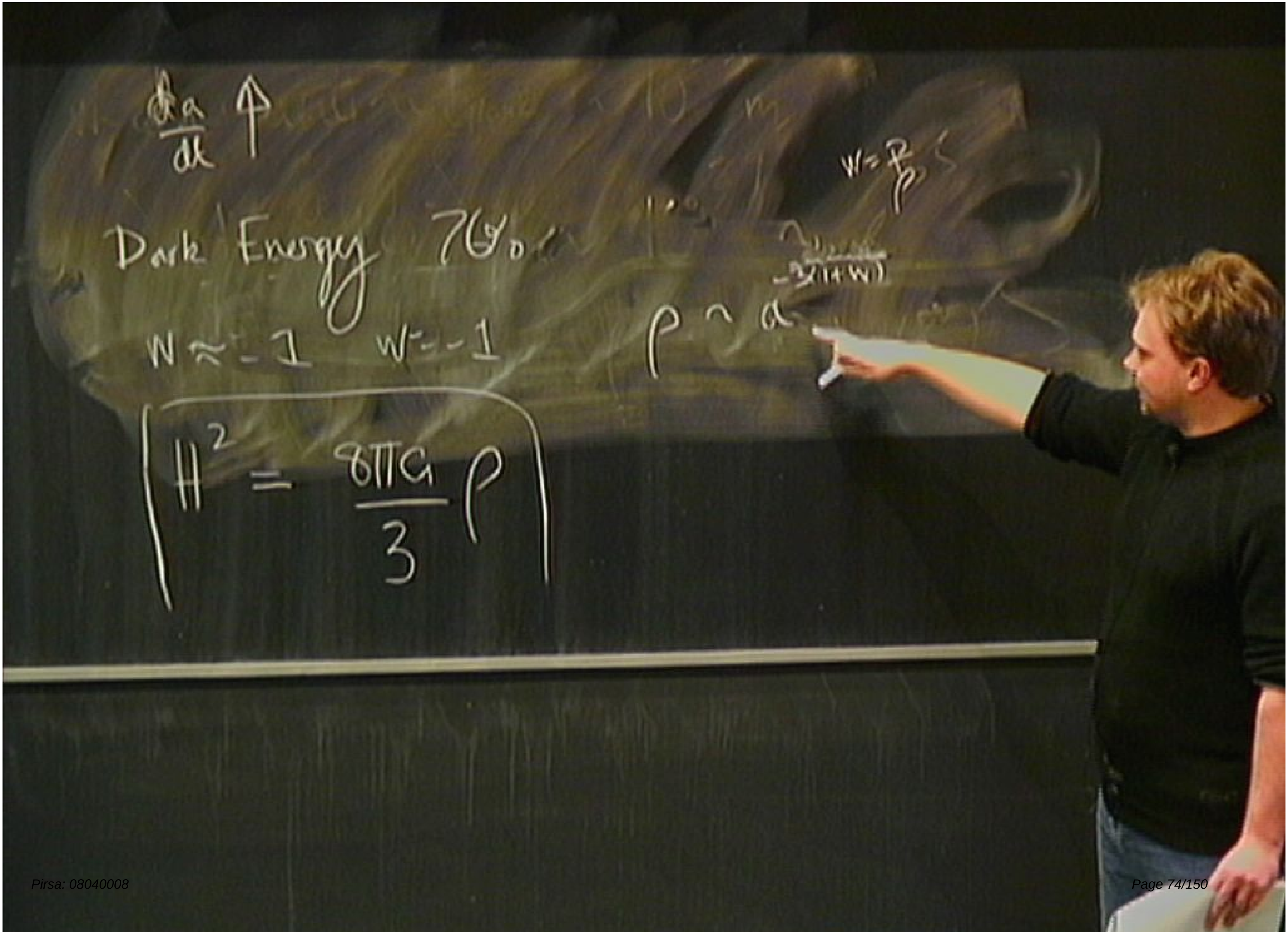
Accelerate - Inflation


$\frac{da}{dt}$ ↑


Dark Energy 76%


da/dt ↑
Dark Energy 76%
w ≈ -1 w = -1


da/dt ↑
Dark Energy 76%
w ≈ -1   w = -1
H² = 8πG/3 ρ


da/dt ↑
Dark Energy 76%
w ≈ -1
w = P/ρ
ρ ~ a^-3(1+w)
H² = 8πG/3 ρ

$\frac{da}{dt} \uparrow$

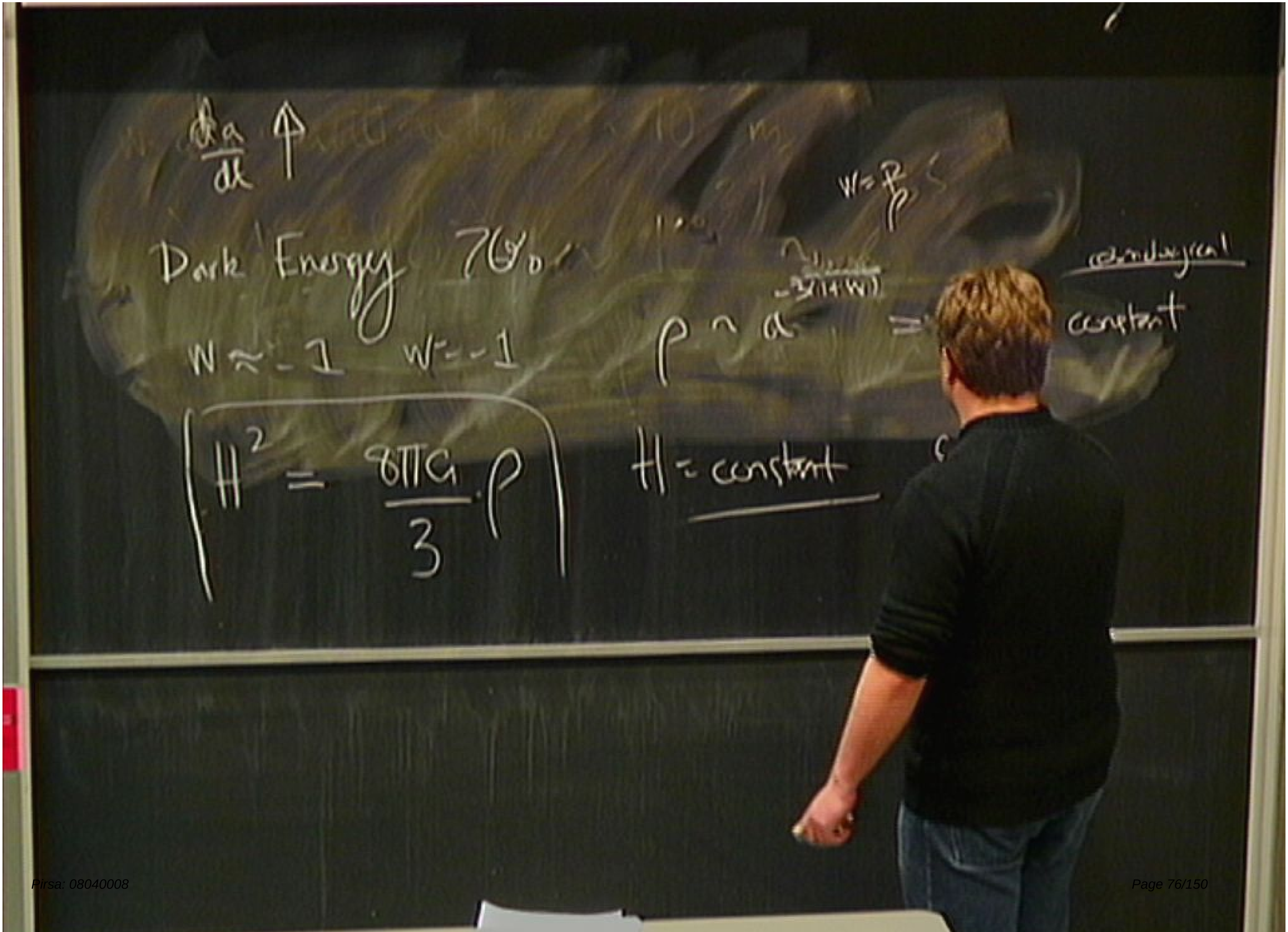
$w = \frac{p}{\rho}$

Dark Energy 70%

$w \approx -1 \quad w = -1$

$\rho \sim a^{-3(1+w)} = a^{0} = \text{constant}$ cosmological

$$\boxed{H^2 = \frac{8\pi G}{3}\rho}$$


da/dt ↑
w = p/ρ
Dark Energy 76%
cosmological
ρ ~ a^(-3(1+w))
constant
w ≈ -1   w = -1
H² = 8πG/3 ρ
H = constant

$\frac{da}{dt} \uparrow$

$w = \frac{P}{\rho}$

Dark Energy 76%

cosmological constant

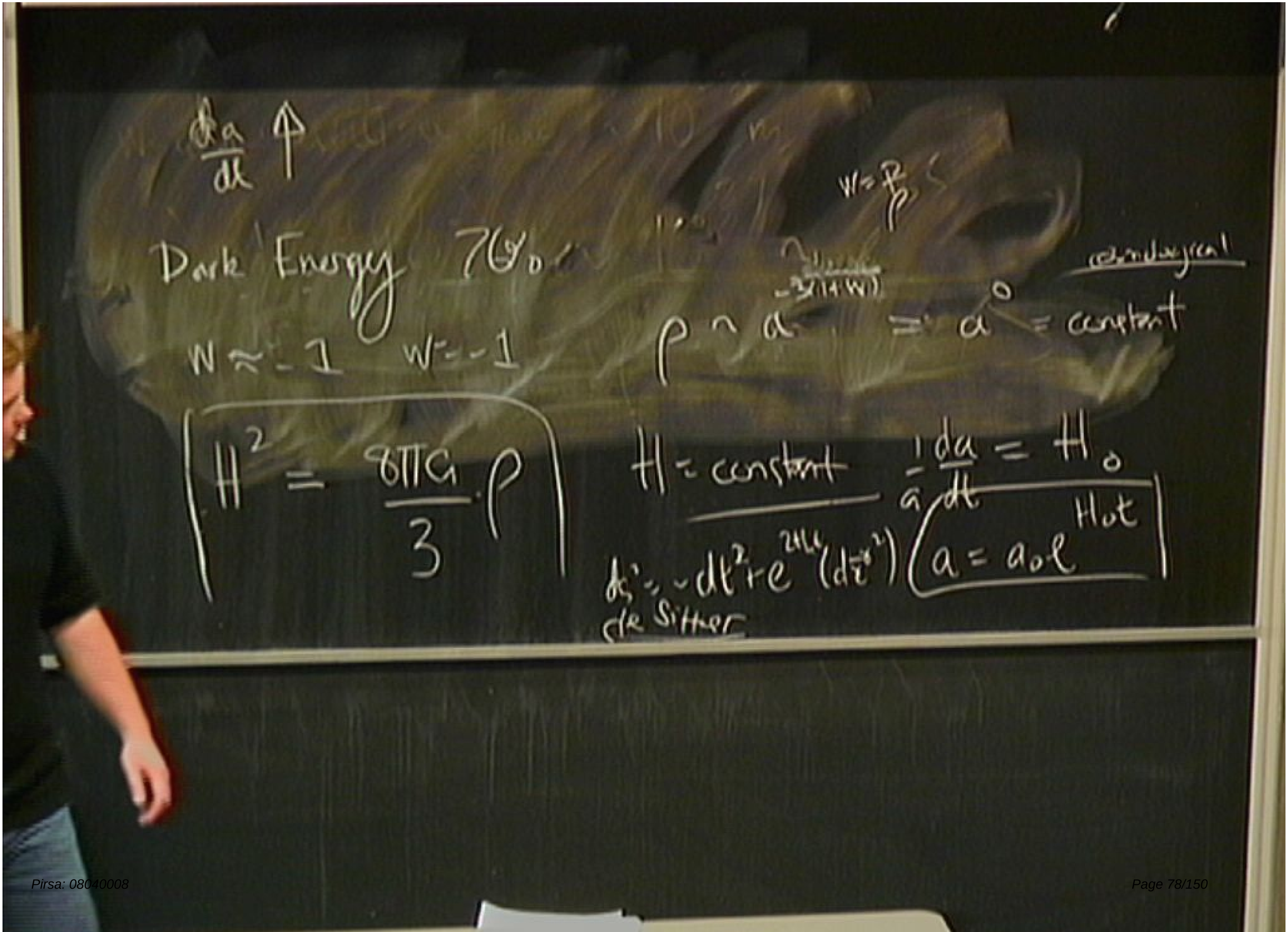
$\rho \sim a^{-3(1+w)} = a^{0} = \text{constant}$

$w \approx -1 \qquad w = -1$

$$\boxed{H^2 = \frac{8\pi G}{3}\rho}$$

$H = \text{constant} \qquad \frac{1}{a}\frac{da}{dt} = H_0$

$ds^2 = -dt^2 + e^{2Ht}(d\vec{x}^2) \qquad \boxed{a = a_0 e^{H_0 t}}$



$\frac{da}{dt} \uparrow$
Dark Energy 76%
$w = \frac{P}{\rho}$
cosmological
$\rho \sim a^{-3(1+w)} = a^{0} = \text{constant}$
$w \approx -1 \quad w = -1$
$H^2 = \frac{8\pi G}{3}\rho$
$H = \text{constant}$
$\frac{1}{a}\frac{da}{dt} = H_0$
$a = a_0 e^{H_0 t}$
$ds^2 = -dt^2 + e^{2Ht}(d\vec{x}^2)$
de Sitter

Maximally symmetric spacetime $\leftrightarrow$ Minkowski $\leftrightarrow$ $SO(4,1)$

$\frac{da}{dt} \uparrow$

$w = \frac{p}{\rho}$

Dark Energy 76%

$w \approx -1$ $w = -1$

$\rho \sim a^{-3(1+w)} \Rightarrow a^0 = \text{constant}$

cosmological constant

$$\boxed{H^2 = \frac{8\pi G}{3}\rho}$$

$H = \text{constant}$ $\frac{1}{a}\frac{da}{dt} = H_0$

$ds^2 = -dt^2 + e^{2H_0 t}(d\vec{x}^2)$ $\boxed{a = a_0 e^{H_0 t}}$

de Sitter

Maximally symmetric spacetime $\leftrightarrow$ Minkowski $\boxed{SO(4,1)}$

$\frac{da}{dt}\uparrow$

Dark Energy 76%

$w \approx -1 \qquad w = -1$

$w = \frac{p}{\rho}$

cosmological constant

$\rho \sim a^{-3(1+w)} = a^{0} = \text{constant}$

$$\boxed{H^2 = \frac{8\pi G}{3}\rho}$$

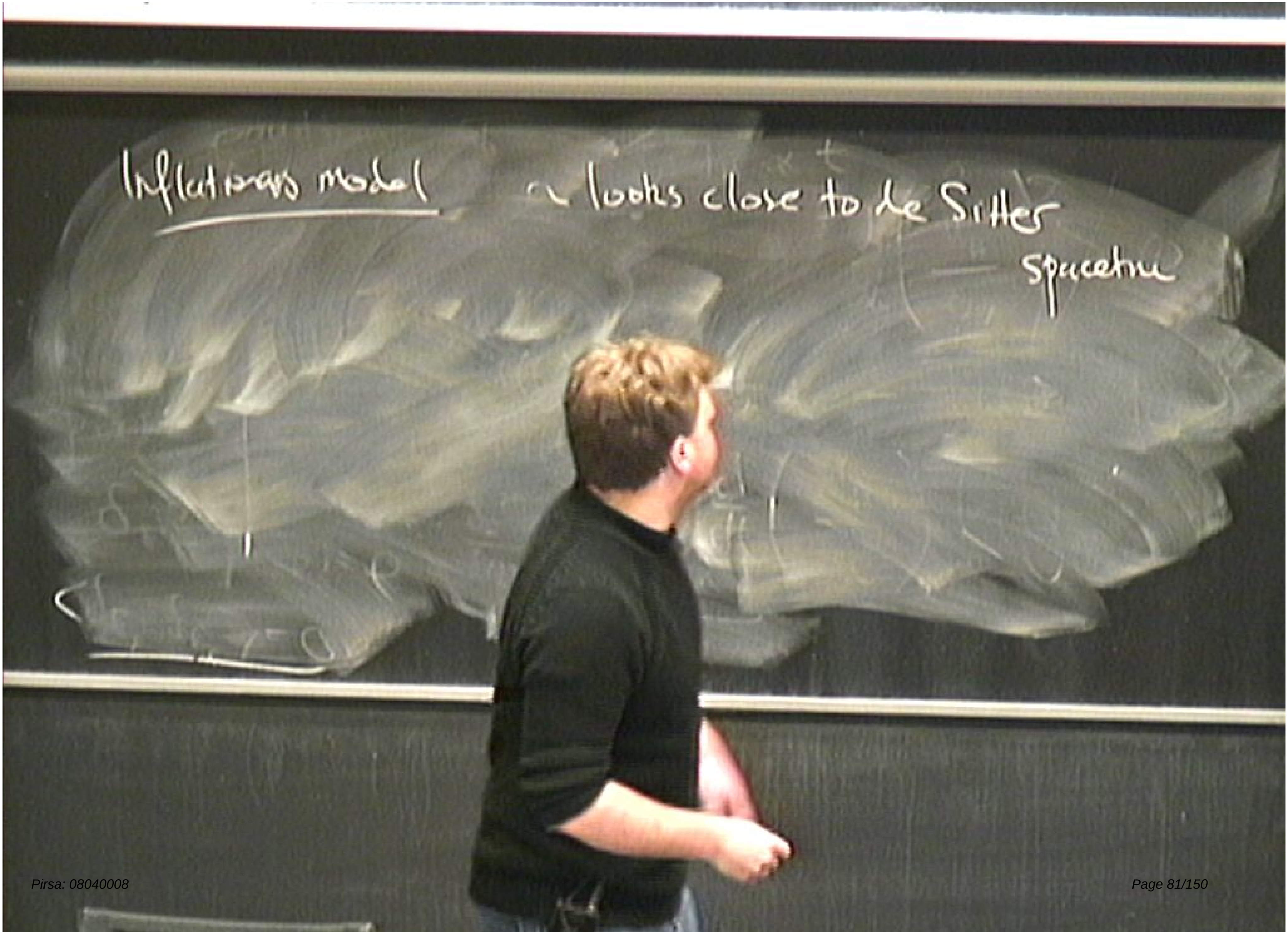
$H = \text{constant} \qquad \frac{1}{a}\frac{da}{dt} = H_0$

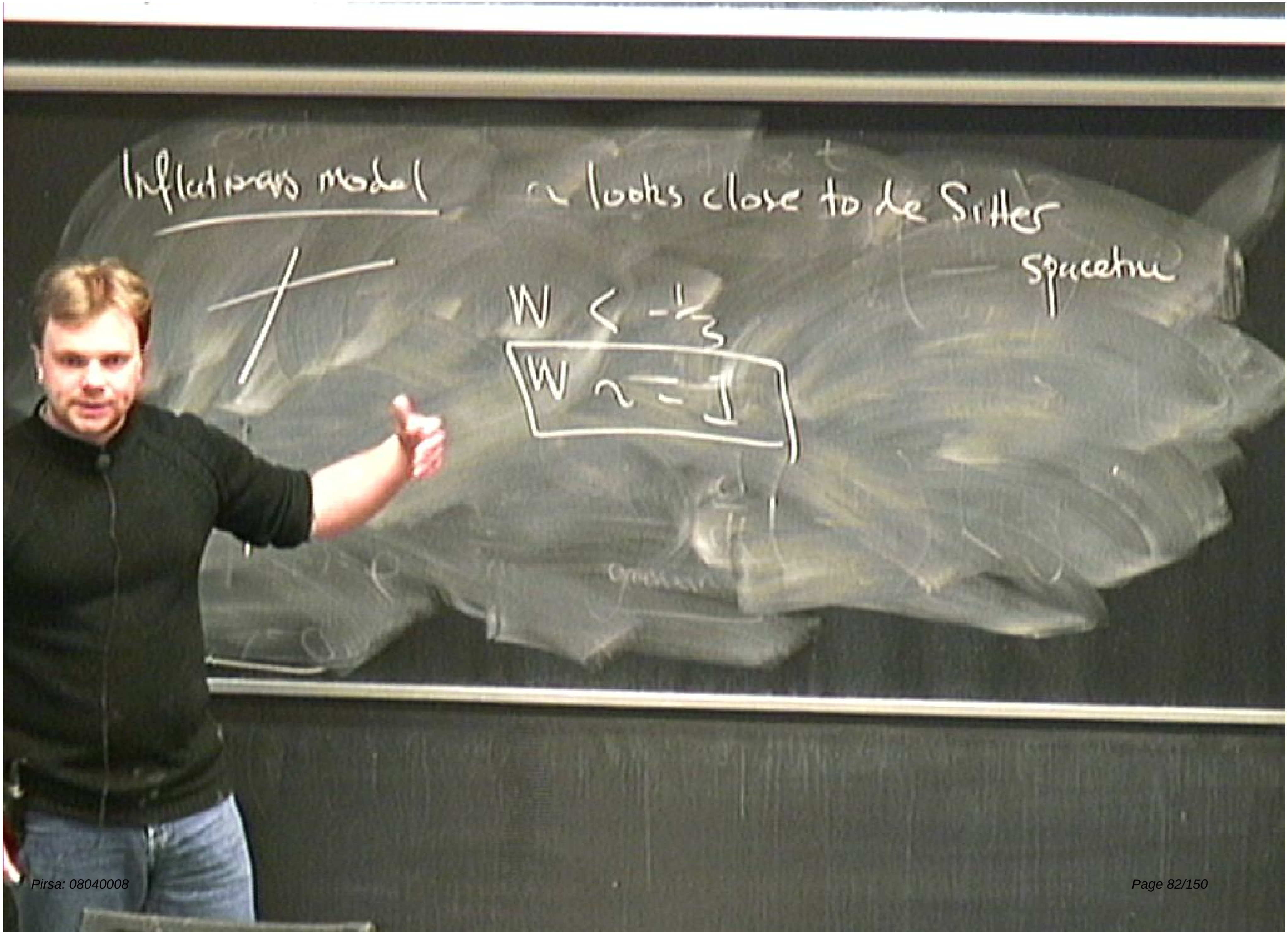
$$\boxed{a = a_0 e^{H_0 t}}$$

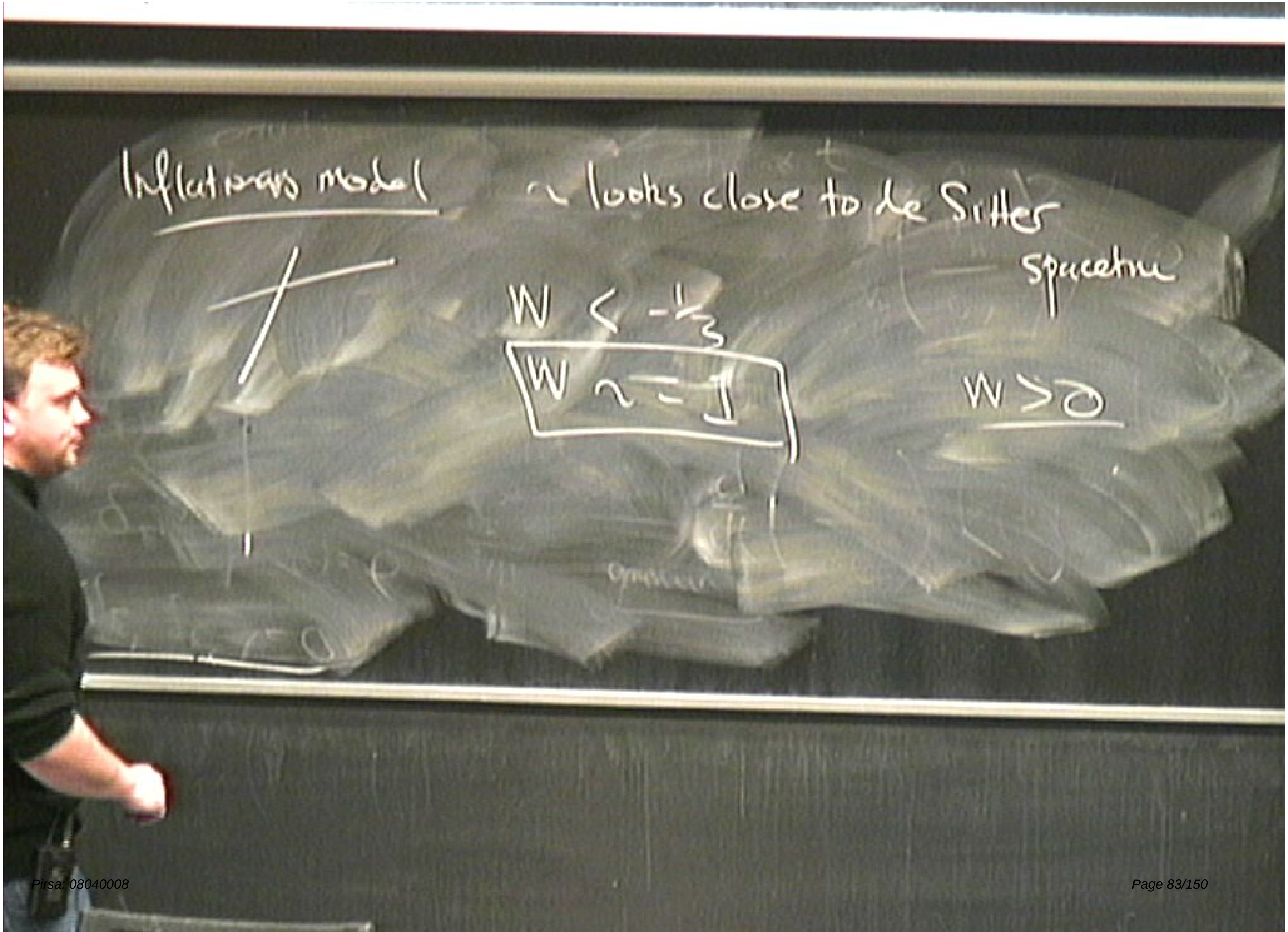
$ds^2 = -dt^2 + e^{2H_0 t}(d\vec{x}^2)$

de Sitter

$\ddot{a} = H_0^2 a_0 e^{H_0 t} > 0$


Inflationary model ~ looks close to de Sitter
spacetime


Inflationary model
~ looks close to de Sitter
spacetime
$w < -1/3$
$\boxed{w \sim -1}$


Inflationary model
~ looks close to de Sitter
spacetime
W < -1/3
W ~ -1
W > 0

Inflationary model

~ looks close to de Sitter spacetime

$W < -\frac{1}{3}$

$\boxed{W \sim -1}$

$W > 0$

$W = \frac{P}{\rho}$ $\boxed{\text{negative pressure}}$

Inflationary model

~ looks close to de Sitter spacetime

$w < -\frac{1}{3}$

$\boxed{w \sim -1}$

$w > 0$

$w = \frac{P}{\rho}$

$\boxed{\text{negative pressure}}$

Scalar field

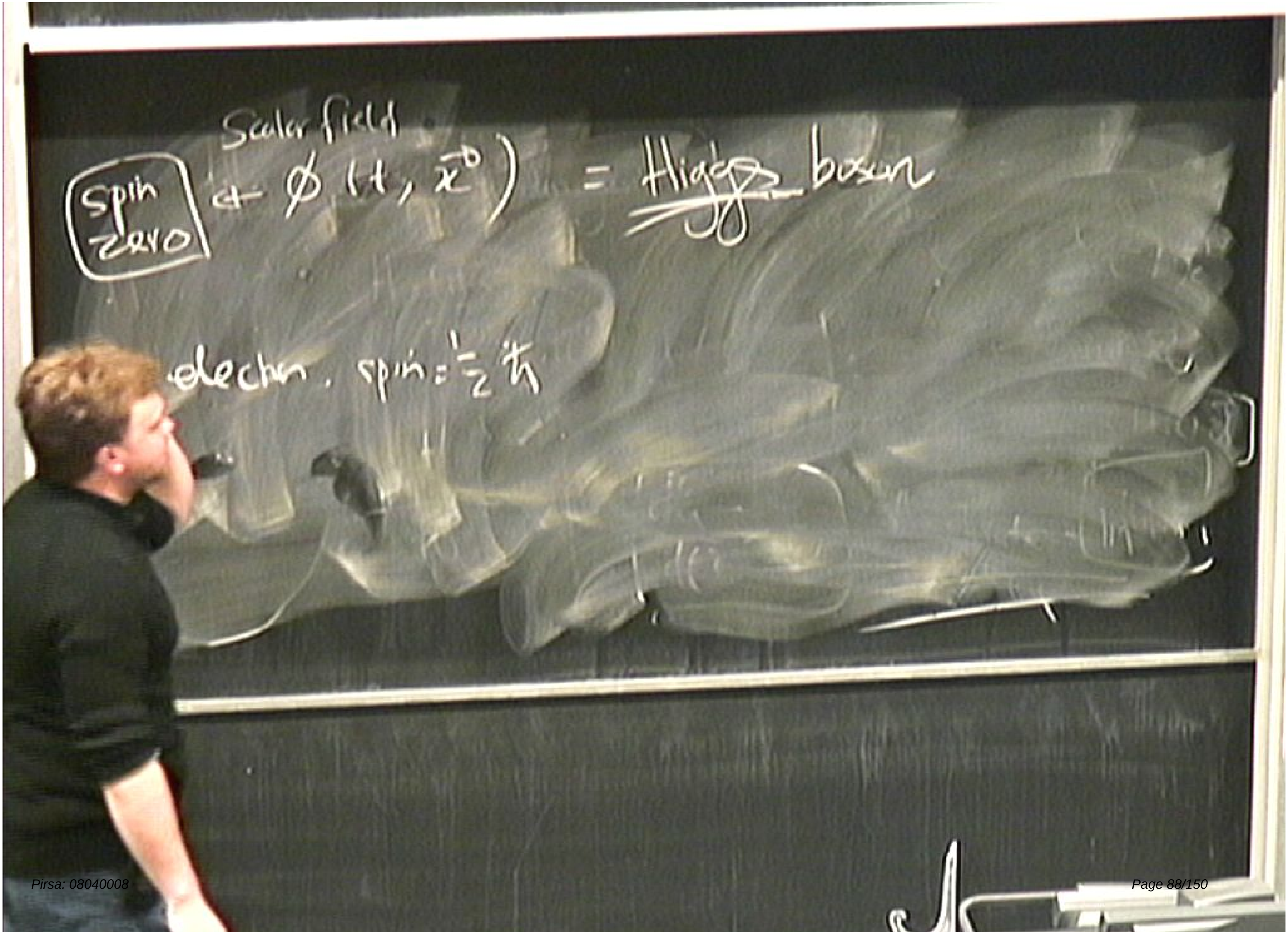
Spin zero

$\leftarrow \phi(t, \vec{x})$

Scalar field

$\boxed{\text{spin zero}}$ $\leftarrow \phi(t, \vec{x})$

electron. spin $= \frac{1}{2}\hbar$


Scalar field
spin zero
$\phi(t, \vec{x})$
= Higgs boson
electron . spin = $\frac{1}{2}\hbar$

Scalar field

spin zero $\phi(t, \vec{x})$ = Higgs boson

LHC

electron . spin $= \frac{1}{2}\hbar$

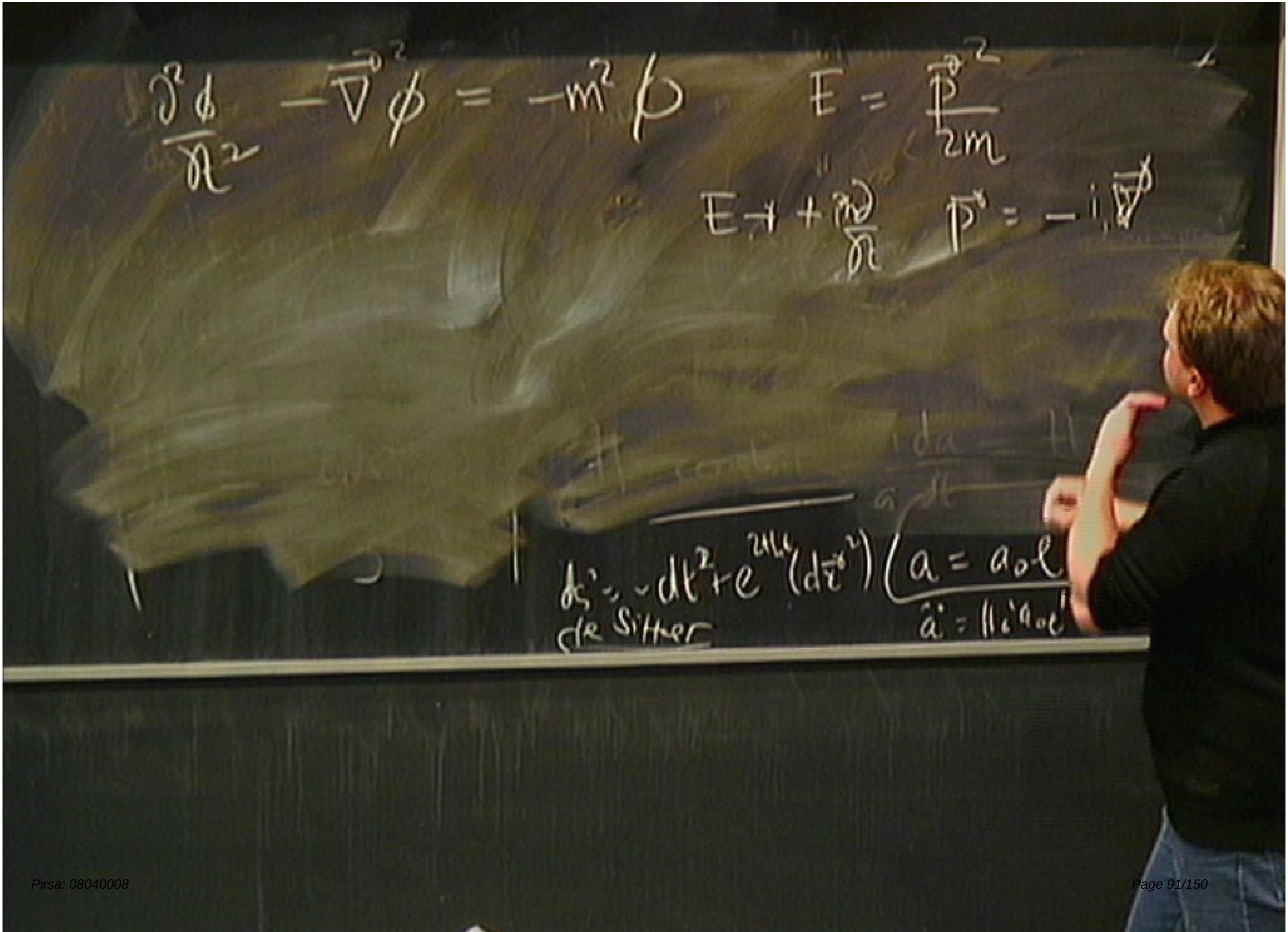
Scalar field

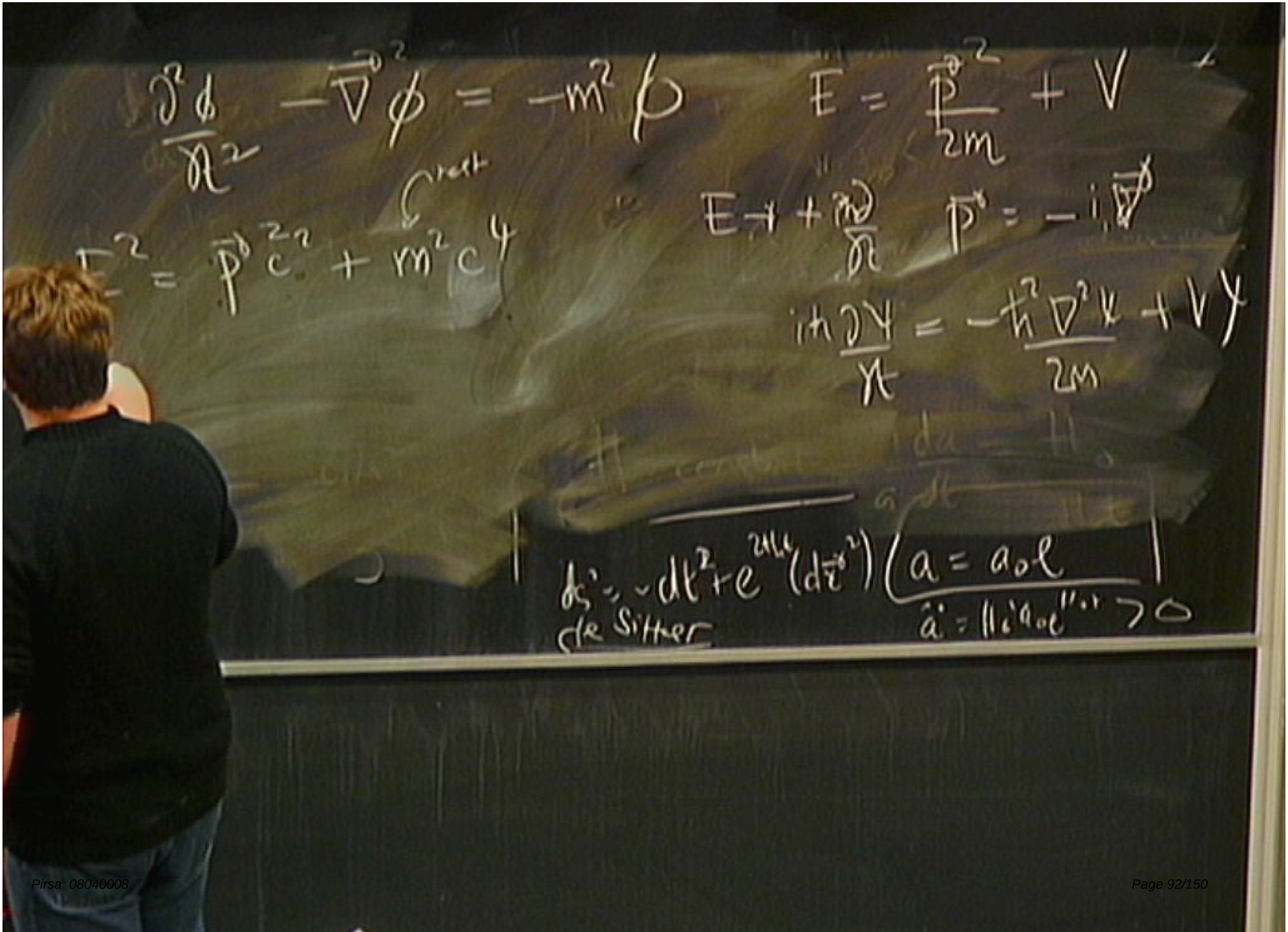
$\boxed{\text{spin zero}} \leftarrow \phi(t, \vec{x})$ = Higgs boson

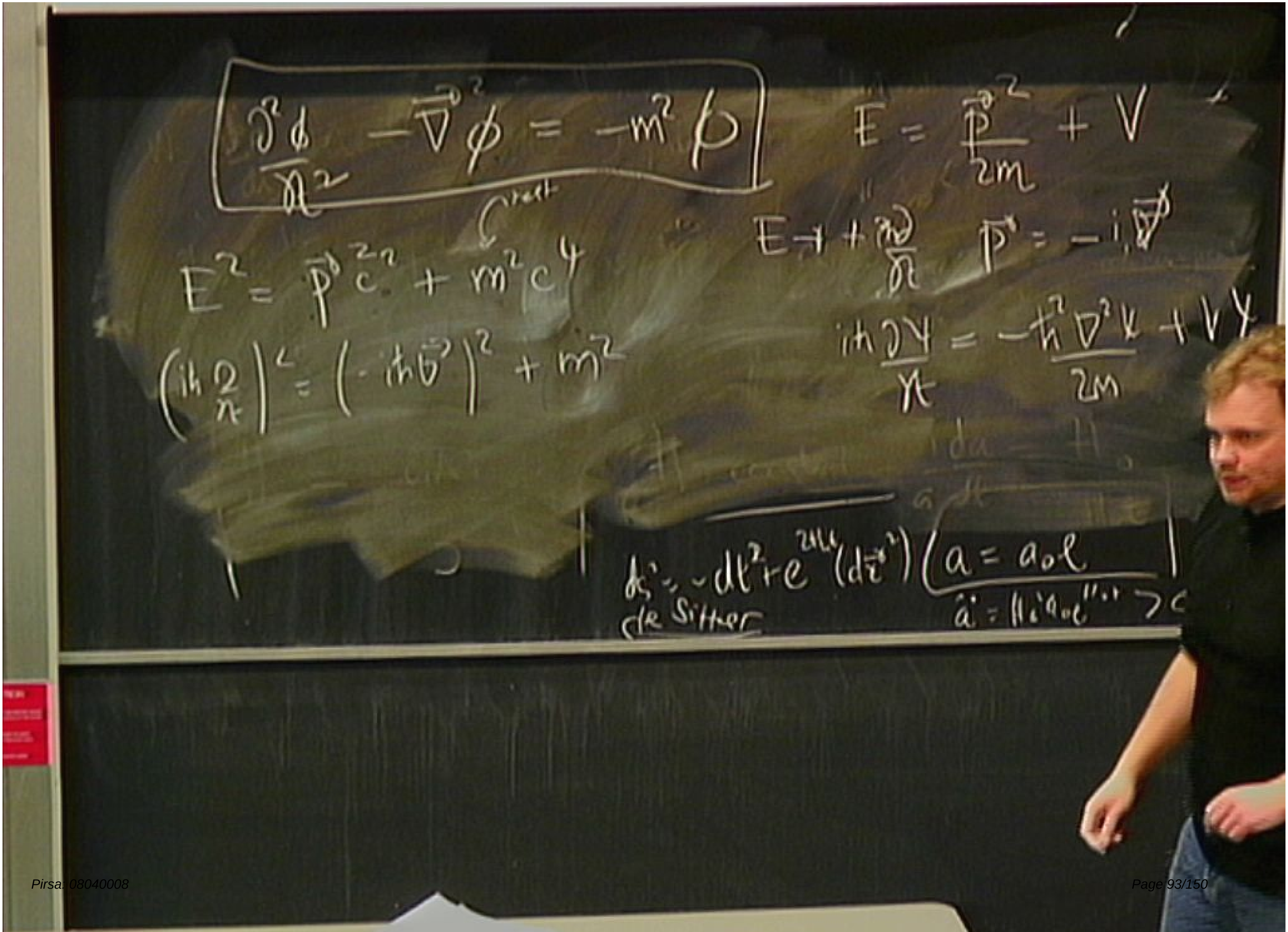
LHC

electron. spin $= \frac{1}{2}\hbar$

Wave particle duality $\longleftrightarrow$ Quantum Field theory


E = p²/2m
de Sitter


$\frac{\partial^2\phi}{\partial t^2} - \vec{\nabla}^2\phi = -m^2\phi$
$E = \frac{\vec{p}^2}{2m} + V$
rest
$E \to +\frac{i\partial}{\partial t}$ $\vec{p} = -i\vec{\nabla}$
$E^2 = \vec{p}^2c^2 + m^2c^4$
$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2\nabla^2\psi}{2m} + V\psi$
de Sitter


$\frac{\partial^2 \phi}{\partial t^2} - \vec{\nabla}^2 \phi = -m^2 \phi$
rest
$E = \frac{\vec{p}^2}{2m} + V$
$E \to i\hbar \frac{\partial}{\partial t}$ $\vec{p} = -i\hbar\vec{\nabla}$
$E^2 = \vec{p}^2 c^2 + m^2 c^4$
$\left(i\hbar \frac{\partial}{\partial t}\right)^2 = \left(-i\hbar\vec{\nabla}\right)^2 + m^2$
$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2 \nabla^2 \Psi}{2m} + V\Psi$
$ds^2 = -dt^2 + e^{2Ht}(d\vec{x}^2)$
de Sitter
$a = a_0 e^{Ht}$
$\dot{a} = H a_0 e^{Ht} > 0$

Klein-Gordon

$$\boxed{\frac{\partial^2\phi}{\partial t^2} - \vec{\nabla}^2\phi = -m^2\phi}$$

$$E = \frac{\vec{p}^{\,2}}{2m} + V$$

$$E \to +i\hbar\frac{\partial}{\partial t} \qquad \vec{p} = -i\hbar\vec{\nabla}$$

$$E^2 = \vec{p}^{\,2}c^2 + m^2c^4$$

$$\left(i\hbar\frac{\partial}{\partial t}\right)^2 = \left(-i\hbar\vec{\nabla}\right)^2 + m^2$$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2\nabla^2\psi}{2m} + V\psi$$

$$ds^2 = -dt^2 + e^{2Ht}(d\vec{x}^{\,2}) \qquad a = a_0 e^{Ht}$$

de Sitter

$$\dot{a} = H a_0 e^{Ht} > 0$$

Klein-Gordon

$$\boxed{\frac{\partial^2 \phi}{\partial t^2} - \vec{\nabla}^2 \phi = -m^2 \phi}$$

$$E = \frac{\vec{p}^2}{2m} + V$$

$$E \to i\hbar \frac{\partial}{\partial t} \qquad \vec{p} = -i\hbar\vec{\nabla}$$

treat

$$E^2 = \vec{p}^2 c^2 + m^2 c^4$$

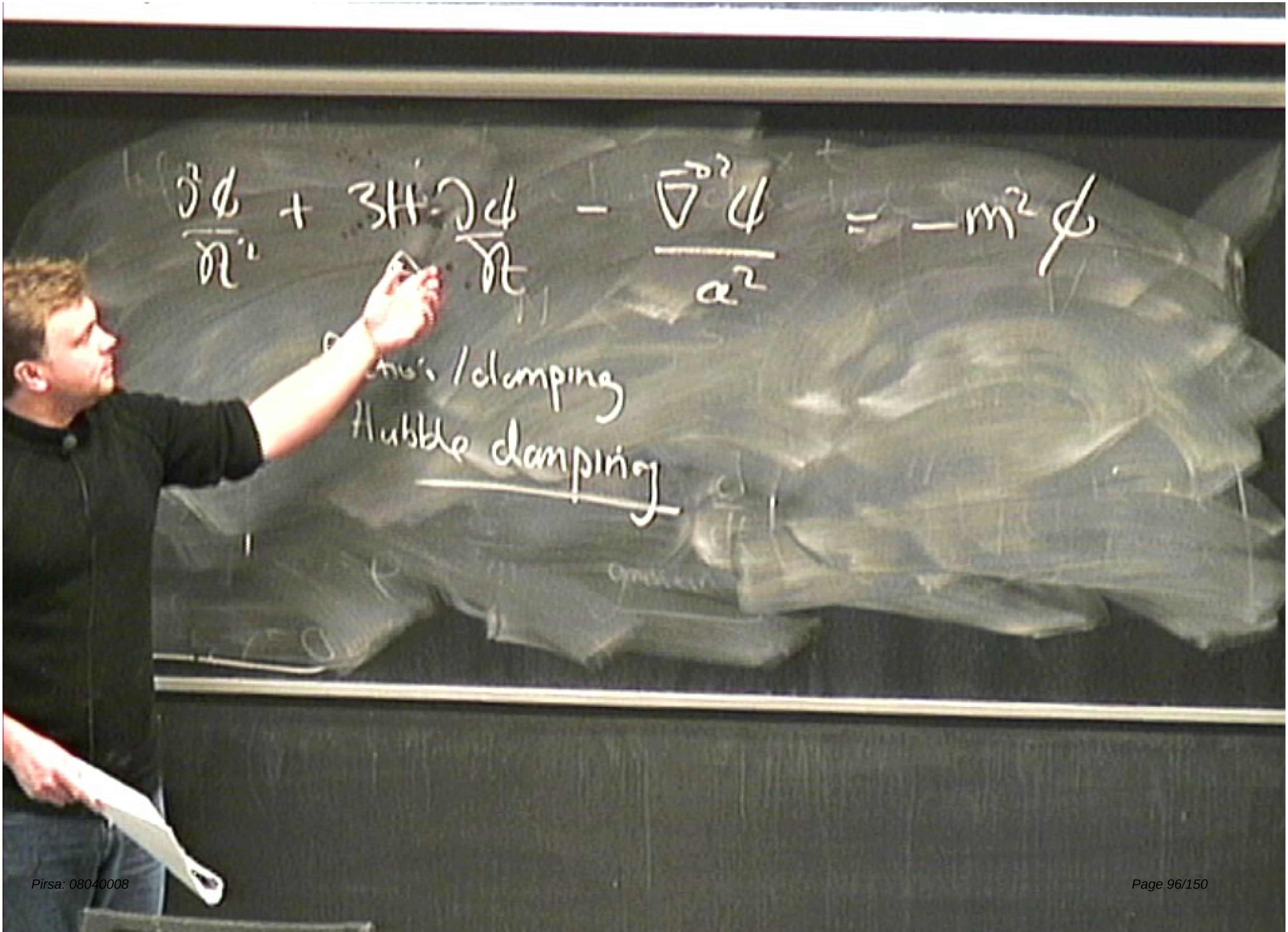
$$\left(i\hbar \frac{\partial}{\partial t}\right)^2 = \left(-i\hbar\vec{\nabla}\right)^2 + m^2$$

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2 \nabla^2 \psi}{2m} + V\psi$$

$$ds^2 = -dt^2 + e^{2Ht}(d\vec{x}^2) \qquad \boxed{a = a_0 e^{Ht}}$$

$$\dot{a} = H a_0 e^{Ht} > 0$$

de Sitter


$\frac{\partial^2 \phi}{\partial t^2} + 3H \frac{\partial \phi}{\partial t} - \frac{\vec{\nabla}^2 \phi}{a^2} = -m^2 \phi$
/damping
Hubble damping

$$\frac{\partial \phi}{\partial t^2} + 3H \frac{\partial \phi}{\partial t} - \frac{\vec{\nabla}^2 \phi}{a^2} = -m^2 \phi$$

friction/damping

Hubble damping

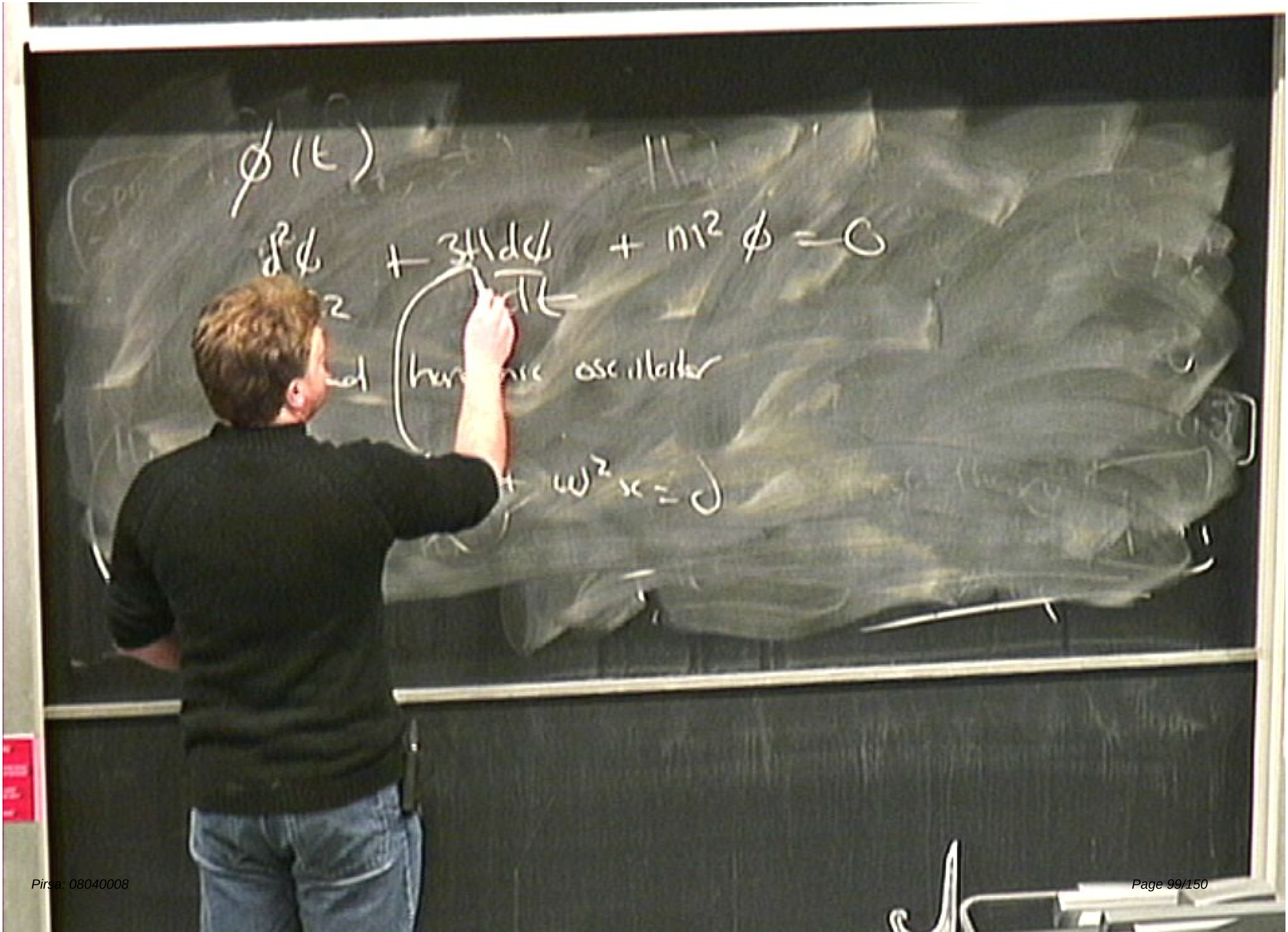
inflaton

$$\frac{\partial^2 \phi}{\partial t^2} + 3H \frac{\partial \phi}{\partial t} - \frac{\vec{\nabla}^2 \phi}{a^2} = -m^2 \phi$$

friction / damping

Hubble damping

inflaton


φ(t)
+ 3H dφ/dt + m² φ = 0
oscillator
+ ω² x = 0

$\phi(t)$

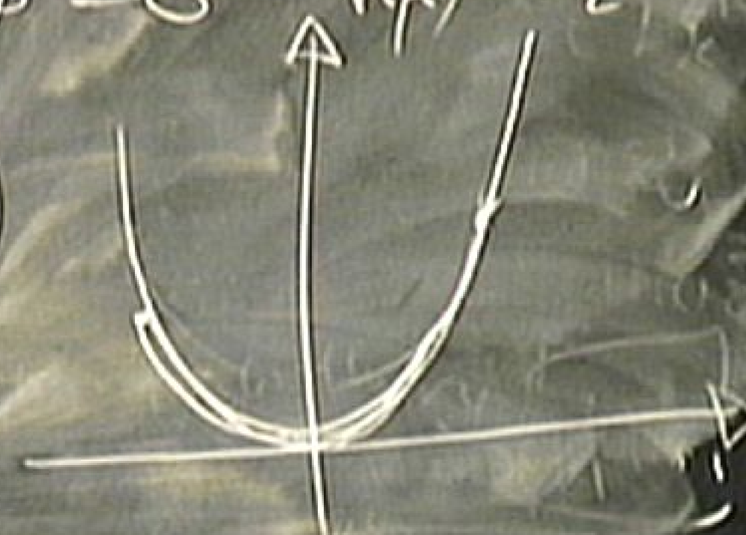
$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} + m^2\phi = 0$$

damped harmonic oscillator

$$\ddot{x} + \gamma\dot{x} + \omega^2 x = 0$$

$\phi(t)$

$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} + m^2\phi = 0 \qquad V(\phi) = \frac{1}{2}m^2\phi^2$$



damped harmonic oscillator

$$\ddot{x} + \gamma\dot{x} + \omega^2 x = 0$$

$\delta T(t,\vec{x}) = T(t,\vec{x}) - \bar{T}(t)$

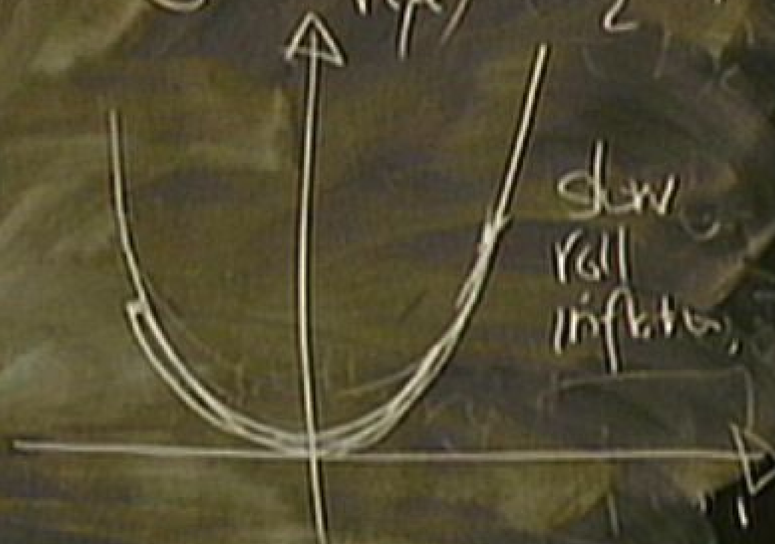
(actually $\delta T(t,\theta,\phi) = T(t,\theta,\phi) - \bar{T}(t)$)

$\phi(t)$

$$\frac{d^2\phi}{dt^2} + 3H\frac{d\phi}{dt} + m^2\phi = 0 \qquad V(\phi) = \frac{1}{2}m^2\phi^2$$

damped harmonic oscillator

$$\ddot{x} + \gamma\dot{x} + \omega^2 x = 0$$



slow roll inflation

$$\delta T(t,\vec{x}) = T(t,\vec{x}) - \bar{T}(t)$$

(actually $\delta T(t,\theta,\phi) = T(t,\theta,\phi) - \bar{T}(t)$)

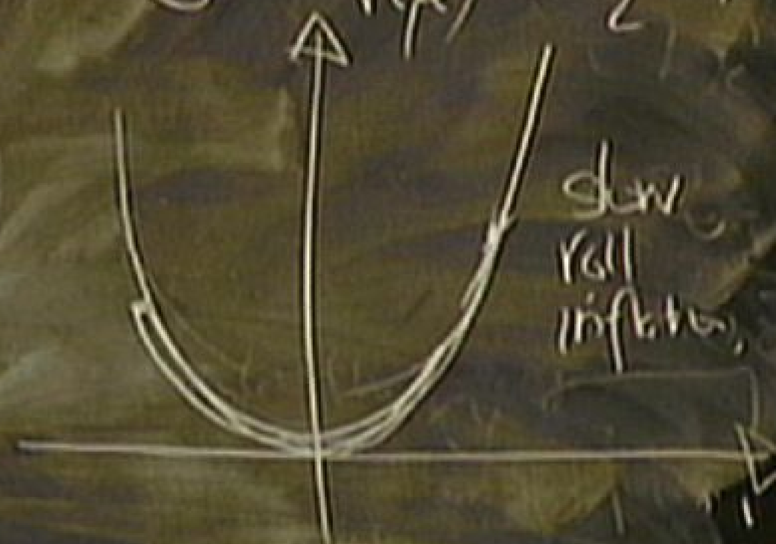
$\phi(t)$

$$\cancel{\frac{d^2\phi}{dt^2}} + 3H\frac{d\phi}{dt} + m^2\phi = 0 \qquad V(\phi) = \frac{1}{2}m^2\phi^2$$

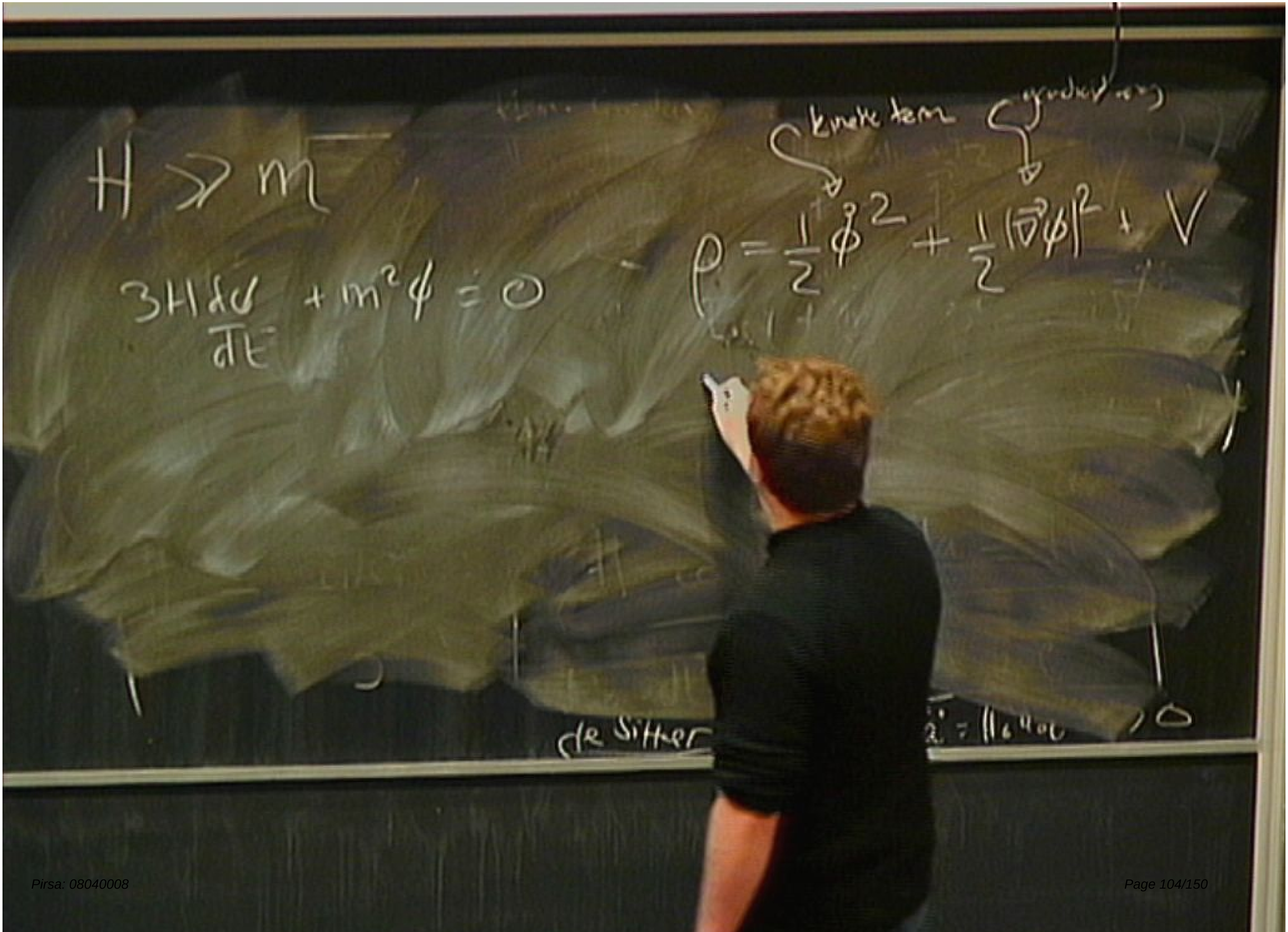
damped harmonic oscillator

$$\ddot{x} + \gamma\dot{x} + \omega^2 x = 0$$

$\gamma \gg \omega$ overdamped



slow roll inflation


H ≫ m
kinetic term
3H dφ/dt + m²φ = 0
ρ = ½ φ̇² + ½ |∇φ|² + V
de Sitter

$$H \gg m$$

$$3H\frac{d\phi}{dt} + m^2\phi = 0$$

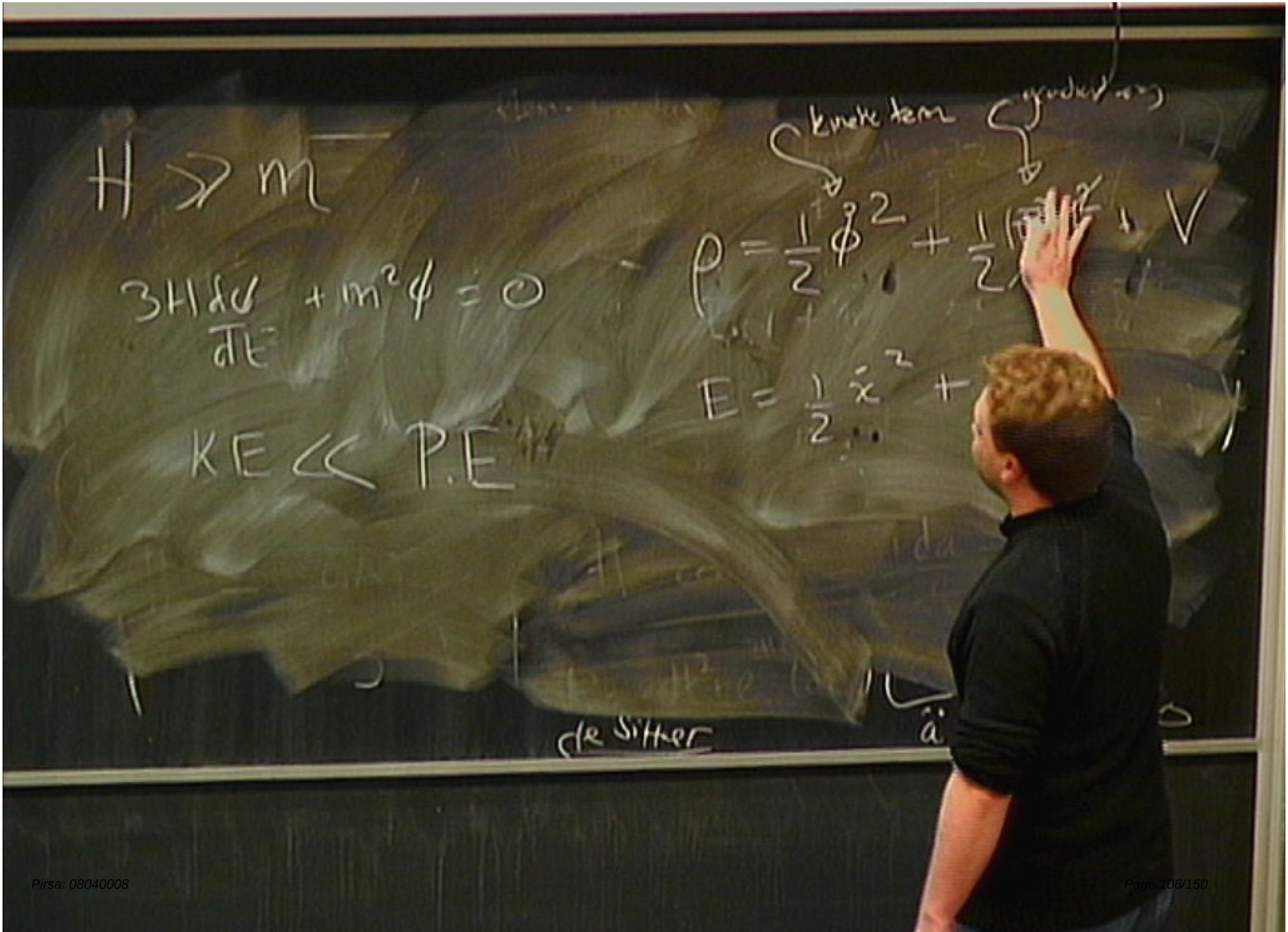
kinetic term

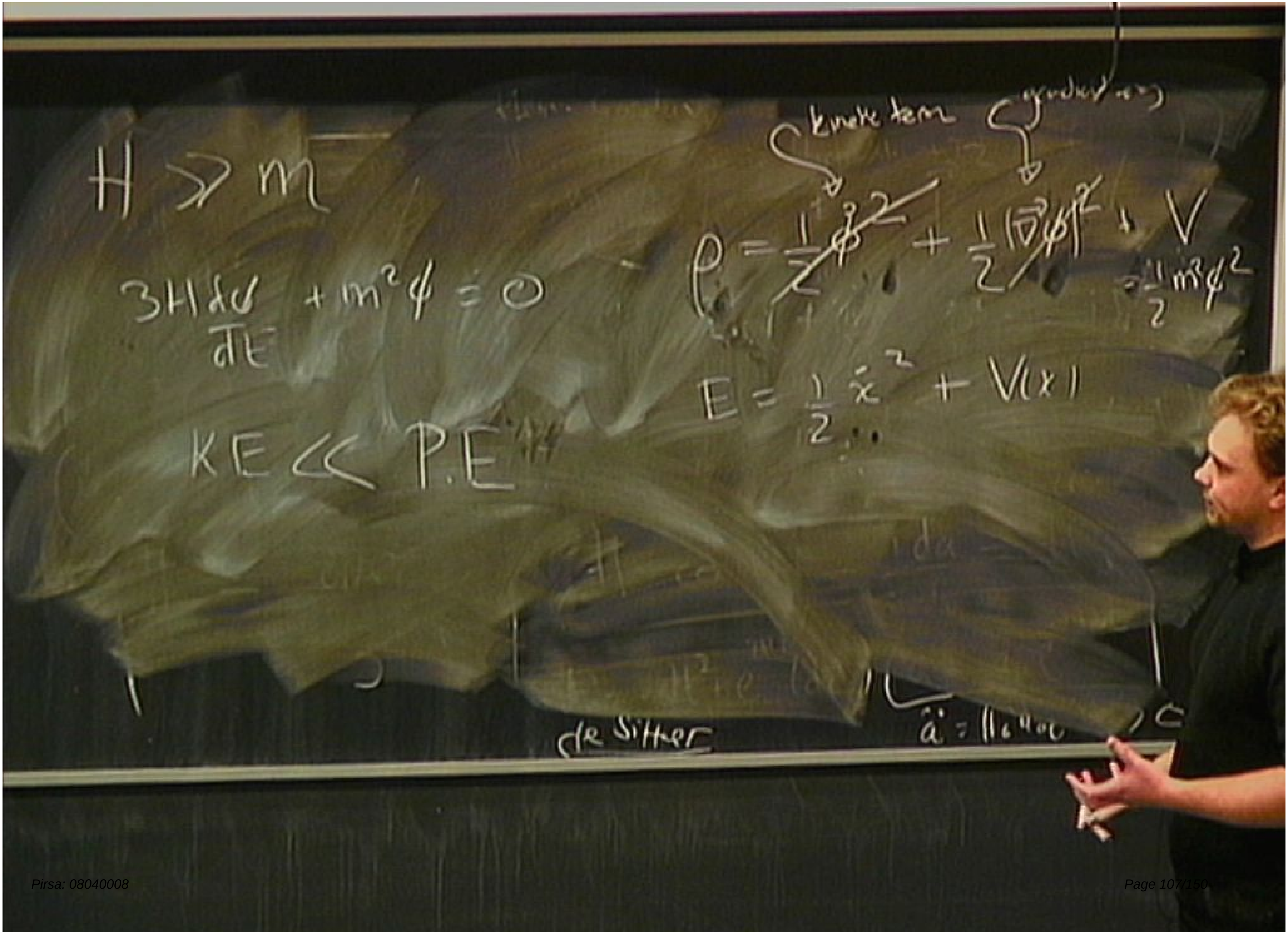
gradient energy

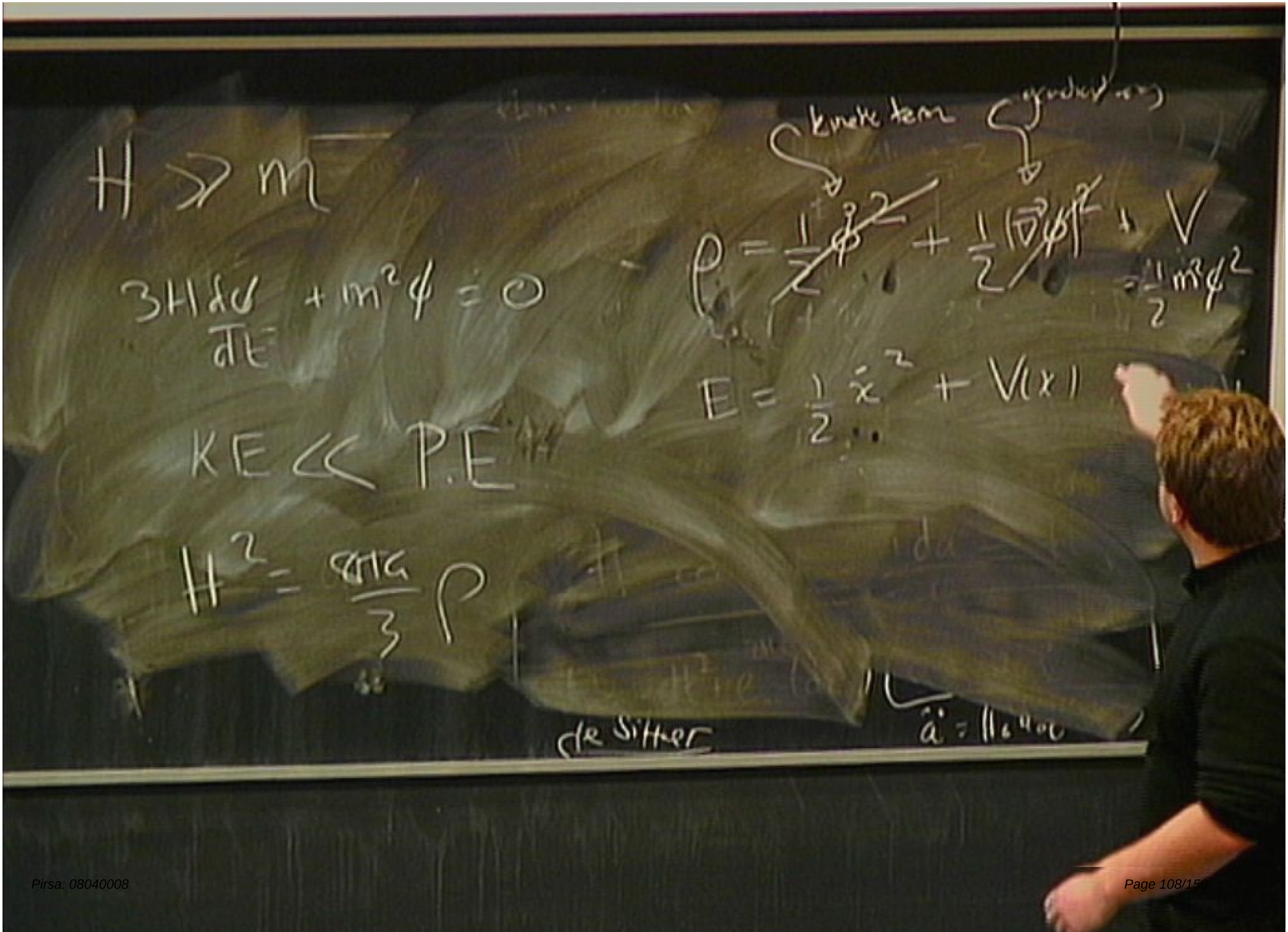
$$\rho = \frac{1}{2}\dot{\phi}^2 + \cancel{\frac{1}{2}|\vec{\nabla}\phi|^2} + V$$

$$E = \frac{1}{2}\dot{x}^2 + V(x)$$

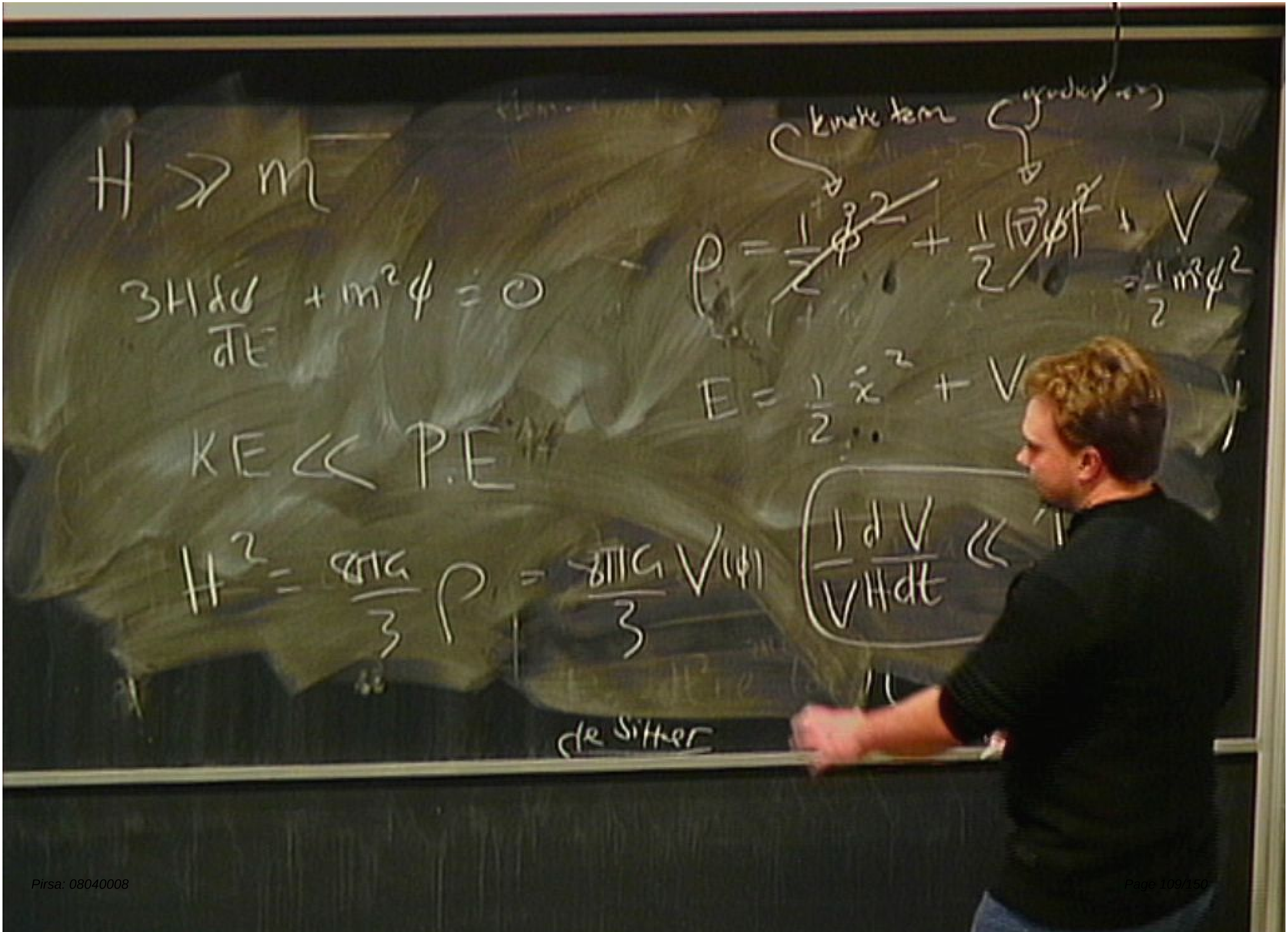
de Sitter


H >> m
KE << P.E
kinetic term
de Sitter


kinetic term
H >> m
KE << P.E
E = 1/2 ẋ² + V(x)
de Sitter



H >> m
3H dφ/dt + m²φ = 0
KE << P.E
H² = 8πG/3 ρ
kinetic term
gradient energy
ρ = ½ φ̇² + ½ |∇φ|² + V
= ½ m²φ²
E = ½ ẋ² + V(x)
de Sitter



kinetic term
gradient energy
$H \gg m$
$3H\frac{d\phi}{dt} + m^2\phi = 0$
$\rho = \cancel{\frac{1}{2}\dot{\phi}^2} + \cancel{\frac{1}{2}|\vec{\nabla}\phi|^2} + V$
$= \frac{1}{2}m^2\phi^2$
$E = \frac{1}{2}\dot{x}^2 + V$
$KE \ll P.E$
$H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}V(\phi)$
$\boxed{\frac{1}{V}\frac{dV}{H dt} \ll 1}$
de Sitter

$H \gg m$

$3H\frac{d\phi}{dt} + m^2\phi = 0$

$KE \ll P.E$

$H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}V(\phi)$

de Sitter

kinetic term

gradient energy

$\rho = \cancel{\frac{1}{2}\dot{\phi}^2} + \cancel{\frac{1}{2}|\vec{\nabla}\phi|^2} + V$

$= \frac{1}{2}m^2\phi^2$

$E = \frac{1}{2}\dot{x}^2 + V(x)$

$\boxed{\frac{1}{V}\frac{dV}{H\,dt} \ll 1}$

$\ddot{a} = \ldots > 0$

$\delta T(t,\vec{x}) = T(t,\vec{x}) - \bar{T}(t)$

(actually $\delta T(t,\theta,\phi) = T(t,\theta,\phi) - \bar{T}(t)$)

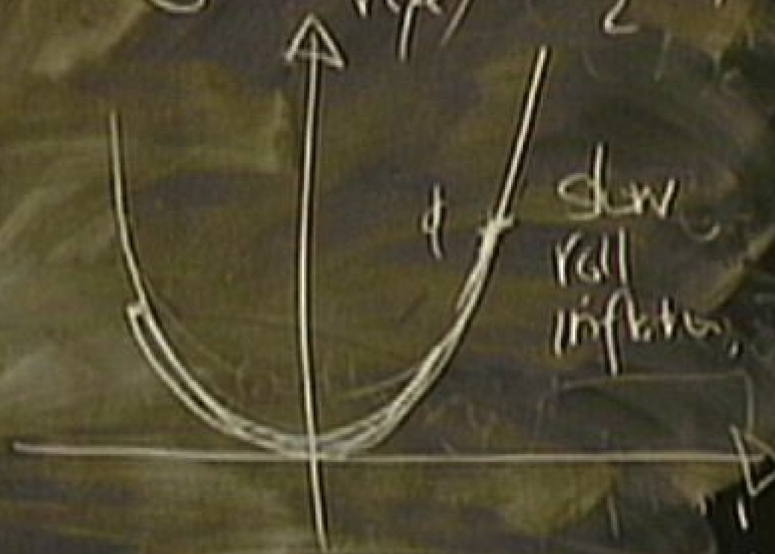
$\phi(t)$

$$\cancel{\frac{d^2\phi}{dt^2}} + 3H\frac{d\phi}{dt} + m^2\phi = 0 \qquad V(\phi) = \frac{1}{2}m^2\phi^2$$

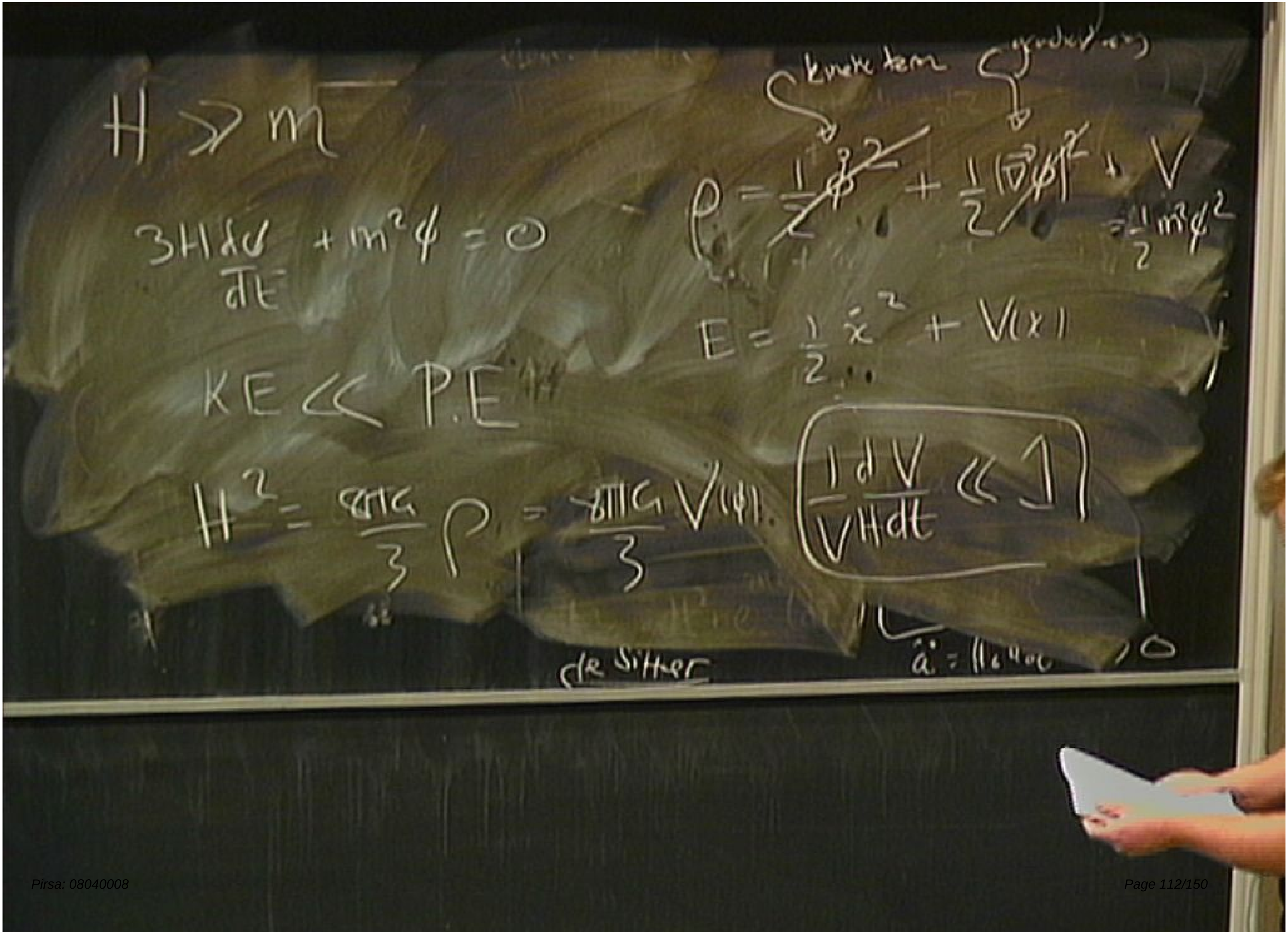
damped harmonic oscillator

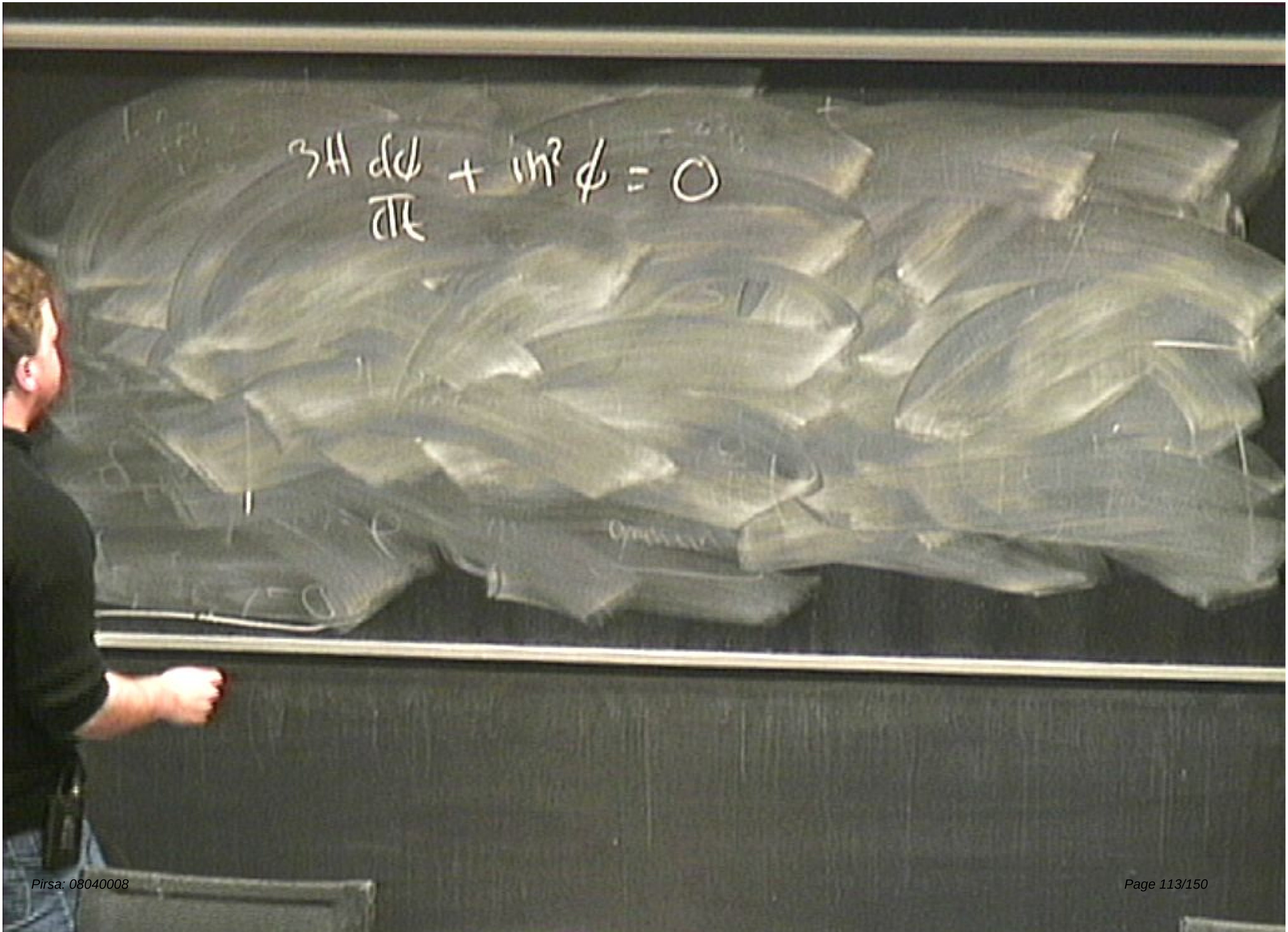
$$\ddot{x} + \gamma\dot{x} + \omega^2 x = 0$$

$\gamma \gg \omega$ overdamped



slow roll inflation


$H \gg m$
$3H\frac{d\phi}{dt} + m^2\phi = 0$
$KE \ll P.E$
$H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}V(\phi)$
kinetic term
gradient energy
$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}|\vec{\nabla}\phi|^2 + V$
$= \frac{1}{2}m^2\phi^2$
$E = \frac{1}{2}\dot{x}^2 + V(x)$
$\boxed{\frac{1}{V}\frac{dV}{Hdt} \ll 1}$
de Sitter


$3H \frac{d\phi}{dt} + m^2 \phi = 0$

$$3H \frac{d\phi}{dt} + m^2 \phi = 0$$

$$H^2 = \frac{8\pi G}{3} V(\phi) = \frac{4\pi G}{3} m^2 \phi^2$$

$$H(\phi) = \sqrt{\frac{4\pi G}{3}}\, m\phi$$

$$3H \frac{d\phi}{dt} + m^2 \phi = 0$$

$$H^2 = \frac{8\pi G}{3} V(\phi) = \frac{4\pi G}{3} m^2 \phi^2$$

$$H(\phi) = \sqrt{\frac{4\pi G}{3}}\, m \phi$$

$$3\sqrt{\frac{4\pi G}{3}}\, m\, \phi \frac{d\phi}{dt} + m^2\phi = 0$$

$$\boxed{\phi = A + B\,\frac{m}{\sqrt{12\pi G}}\,t}$$

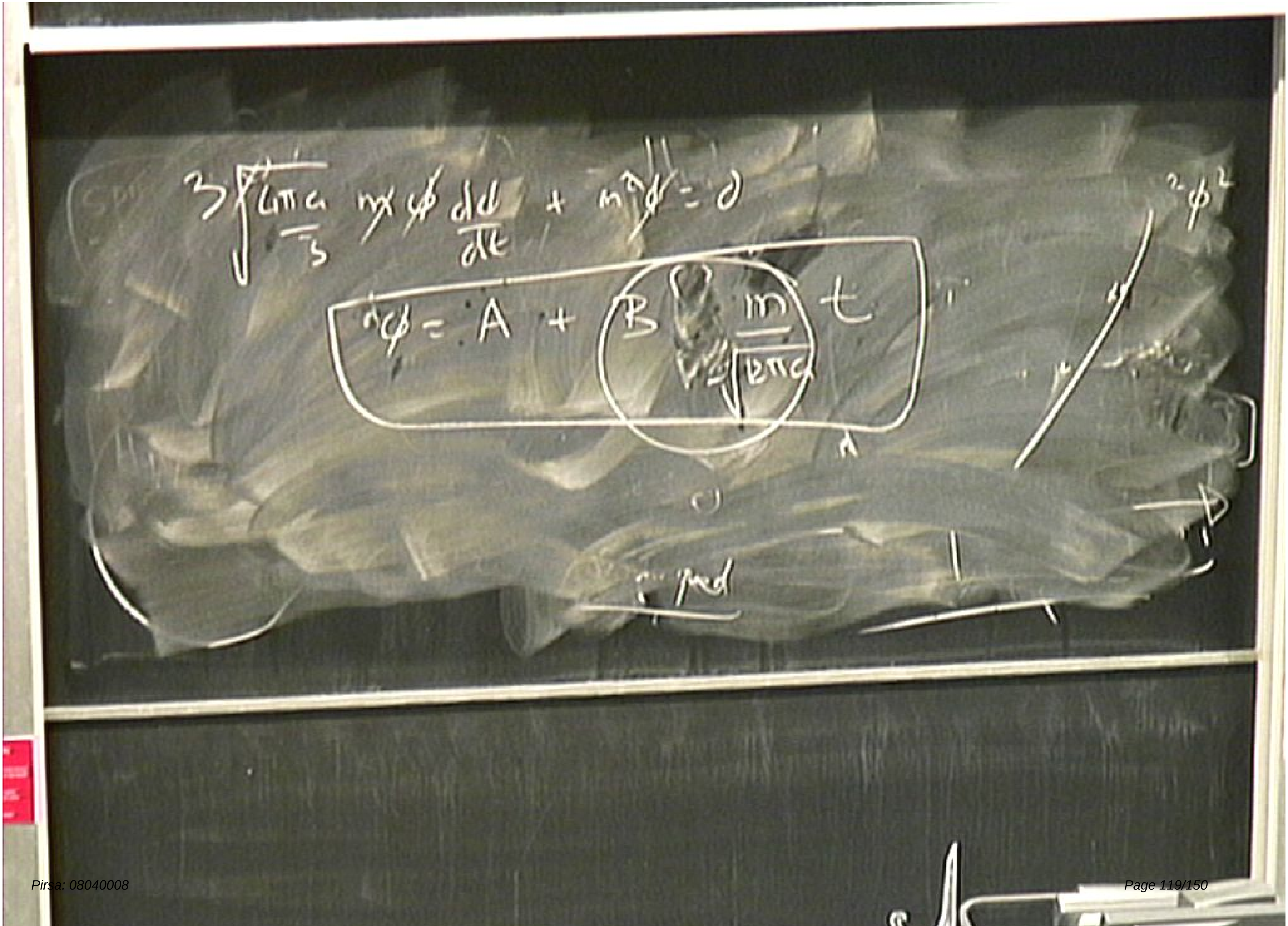
$$3\sqrt{\frac{4\pi G}{3}}\,\cancel{m}\,\cancel{\phi}\,\frac{d\phi}{dt} + m^2\cancel{\phi} = 0$$

$$\boxed{\phi = A + B\frac{m}{\sqrt{12\pi G}}\,t}$$

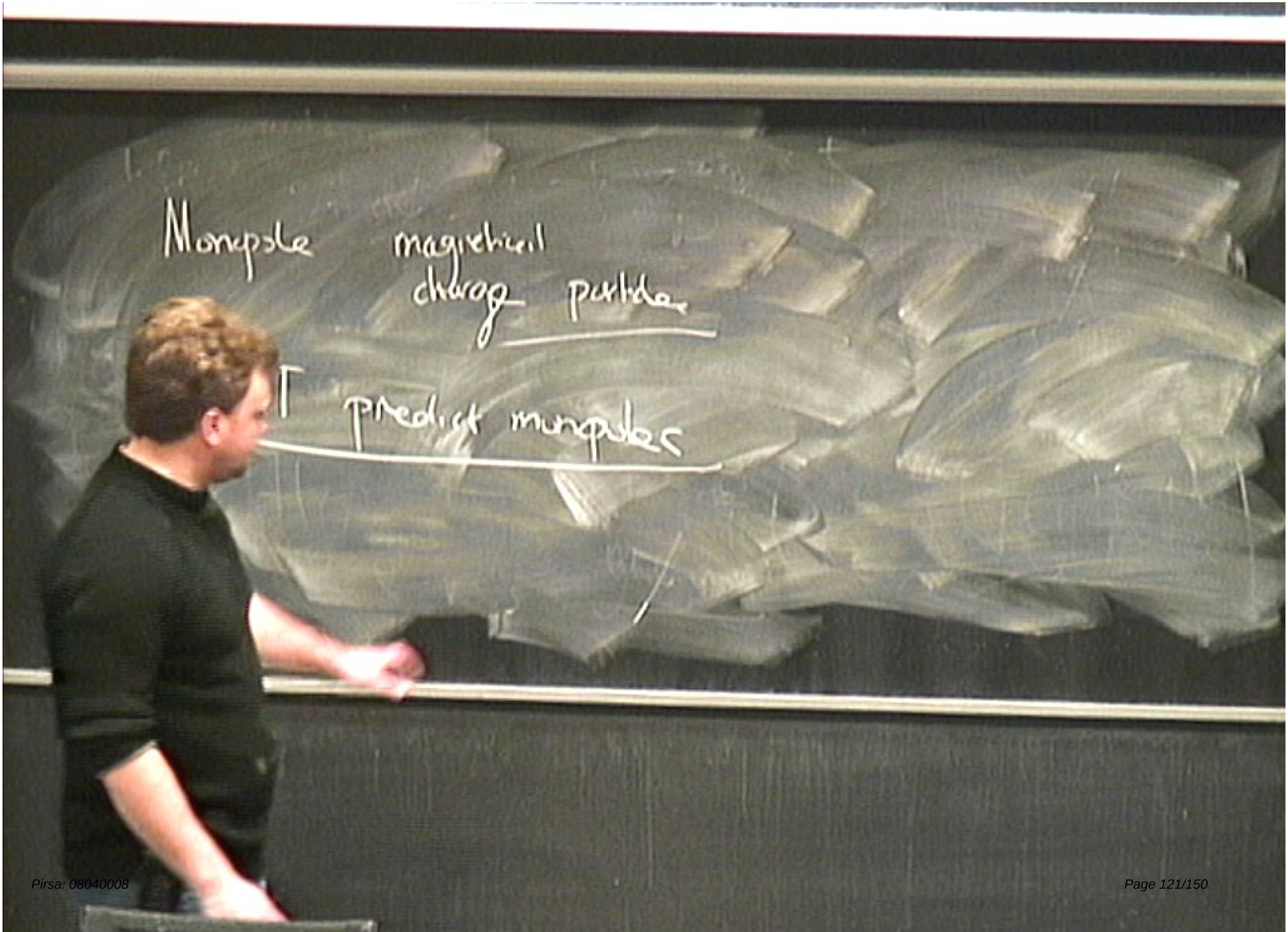
$$3\sqrt{\frac{4\pi a}{3}}\, m\, \phi \frac{d\phi}{dt} + m^2\phi = 0$$

$$\boxed{\phi = A + B\frac{m}{\sqrt{12\pi a}}\, t}$$

ϕ^2


φ = A + B m/√12πG t

Monopole magnetical
charge particle


Monopole magnetical
charge particle
T predict monopoles

Monopole magnetical
charge particle

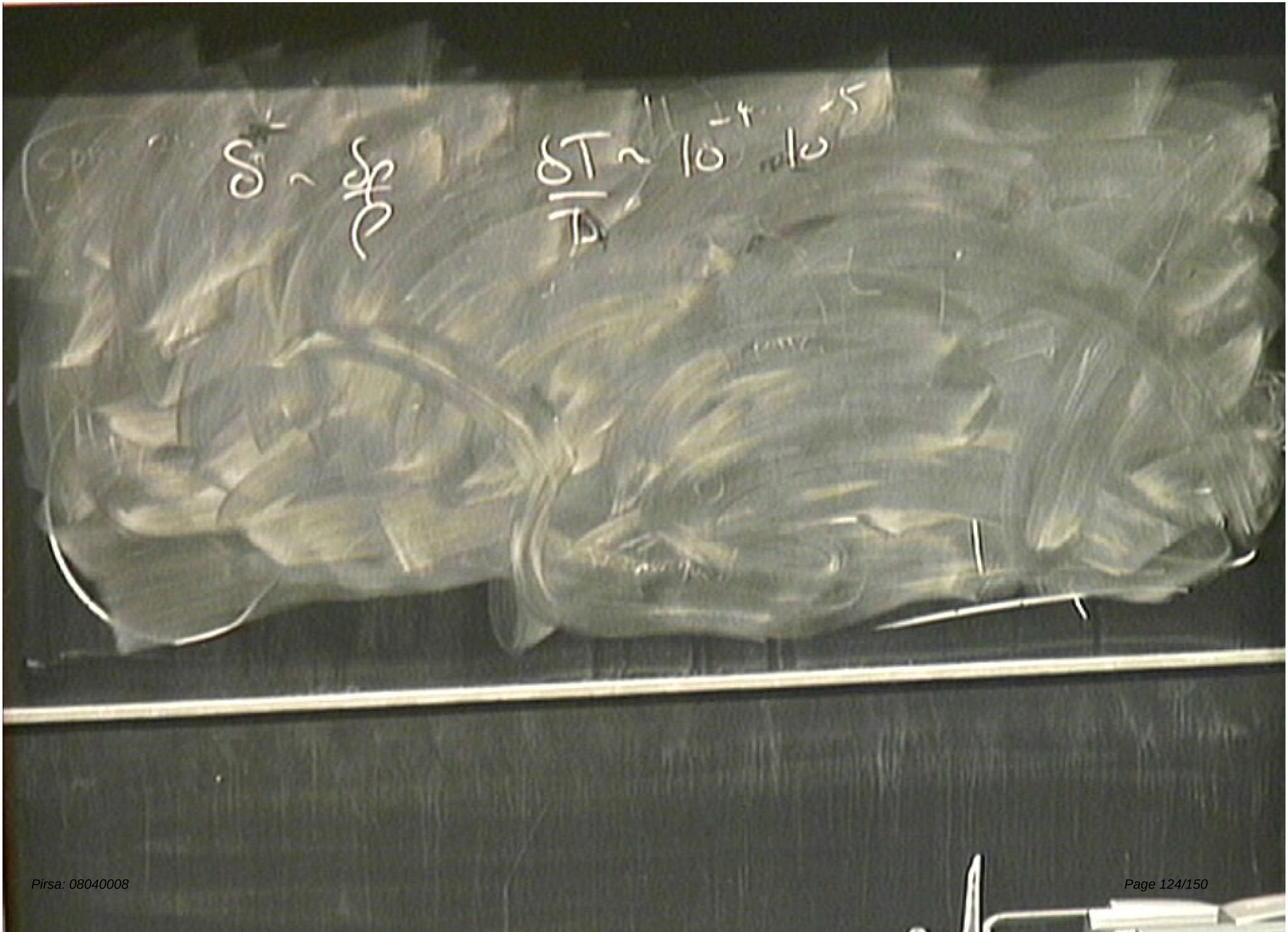
GUT predict monopoles

$\rho_{\text{monopoles}} \sim \dfrac{1}{a^3}$

Monopole magnetical charge particle

GUT predict monopoles

$\rho_{\text{monopoles}} \sim \frac{1}{a^3}$



$\delta \sim \frac{\delta\rho}{\rho}$ $\qquad$ $\frac{\delta T}{T} \sim 10^{-4} \ldots 10^{-5}$

$\phi \longleftrightarrow$ coherent state of inflaton particles

$\delta \sim \frac{\delta\rho}{\rho}$ $\qquad \frac{\delta T}{T} \sim 10^{-4} \to 10^{-5}$

$\phi \longleftrightarrow$ coherent state of inflaton particles

$\phi(t)$

Heisenberg uncertainty $\quad \Delta x \Delta p \geq \frac{1}{2}\hbar$

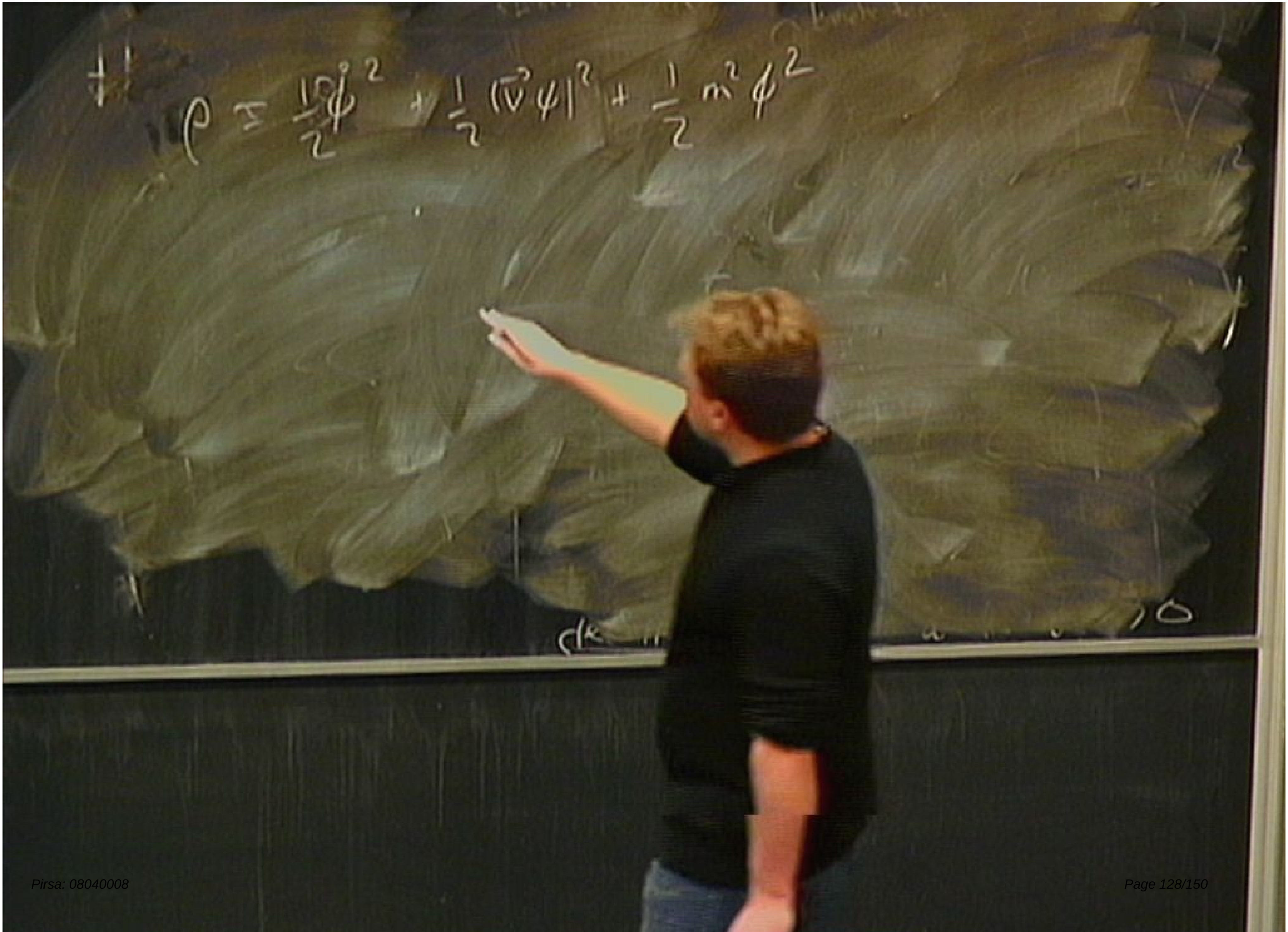
$\delta \sim \frac{\delta \rho}{\rho}$ $\qquad$ $\frac{\delta T}{T} \sim 10^{-4} \ldots 10^{-5}$

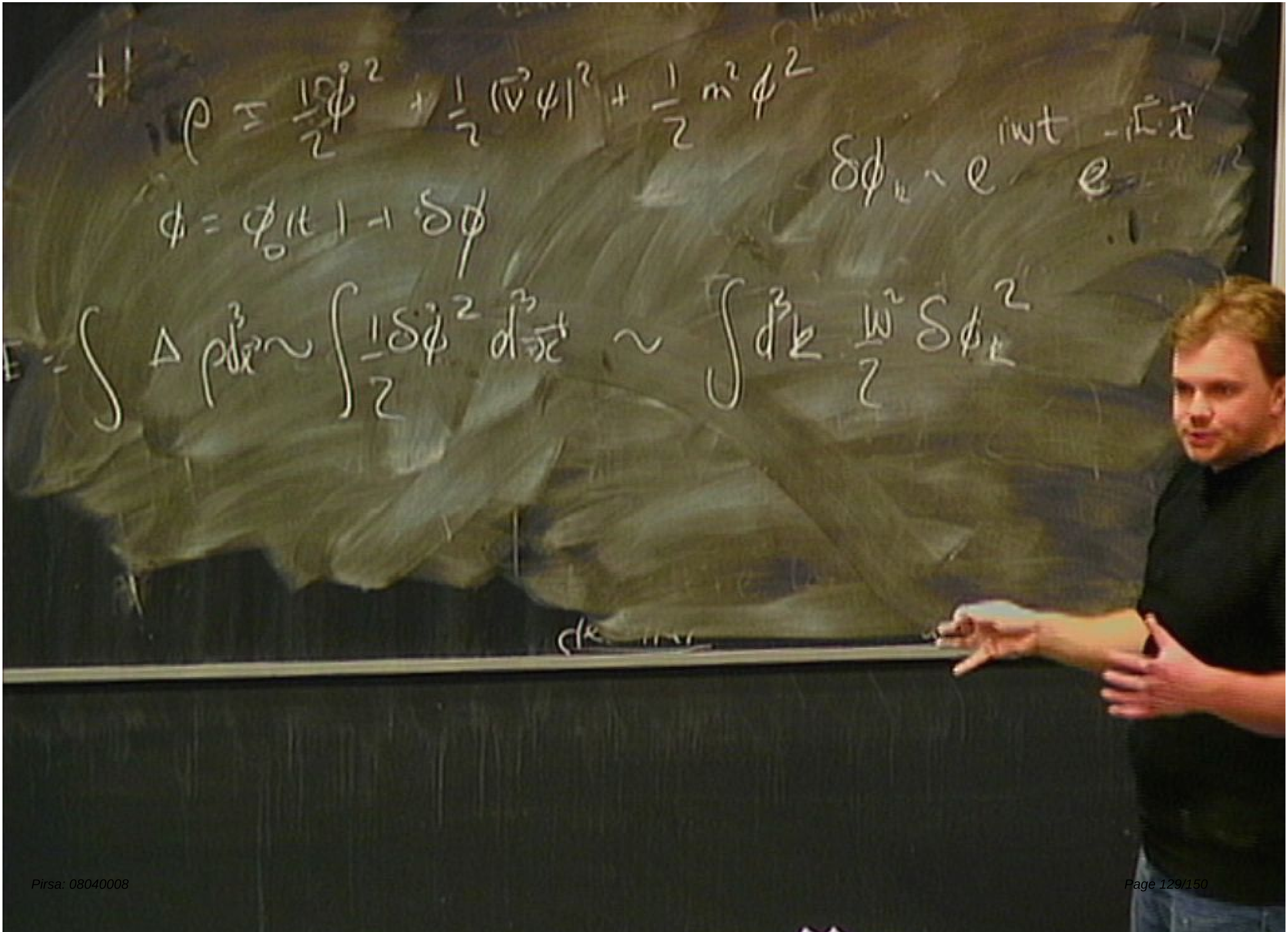
$\phi \longleftrightarrow$ coherent state of inflaton particles

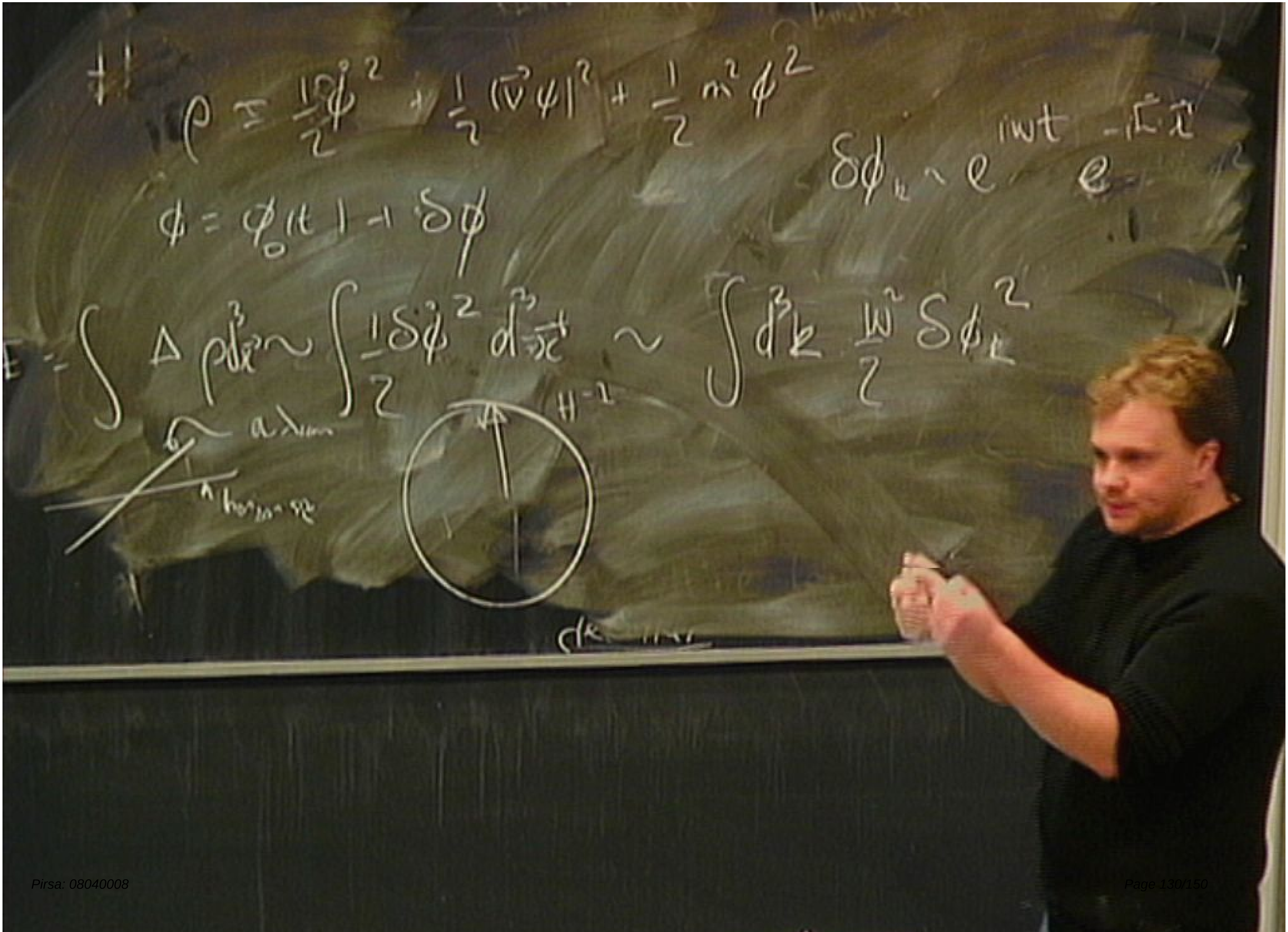
$\phi(t)$

Heisenberg $\Delta x \Delta p \geq \frac{1}{2}\hbar$

$\delta\phi(t,\vec{x})$ (Weltring)


$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}m^2\phi^2$


ρ = 1/2 φ̇² + 1/2 (∇φ)² + 1/2 m² φ²
φ = φ₀(t) + δφ
δφ_k ~ e^{iωt} e^{-ik·x}
E = ∫ Δρ d³x ~ ∫ 1/2 δφ̇² d³x ~ ∫ d³k ω²/2 δφ_k²


$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}m^2\phi^2$
$\phi = \phi_0(t) + \delta\phi$
$\delta\phi_k \sim e^{i\omega t}\, e^{-i\vec{k}\cdot\vec{x}}$
$E = \int \Delta\rho\, d^3x \sim \int \frac{1}{2}\delta\dot{\phi}^2\, d^3\vec{x} \sim \int d^3k\, \frac{\omega^2}{2}\delta\phi_k^2$
H^{-1}

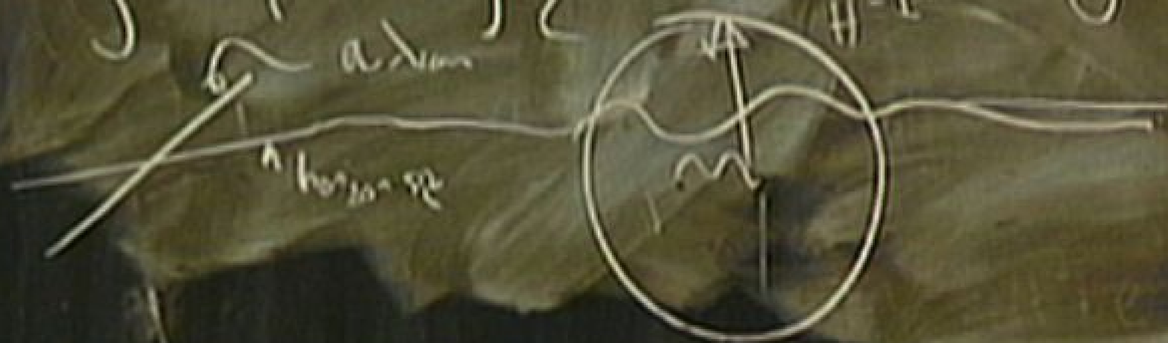
$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}m^2\phi^2$$

$$\delta\phi_k \sim e^{i\omega t}\, e^{-i\vec{k}\cdot\vec{x}}$$

$$\phi = \phi_0(t) + \delta\phi$$

$$E = \int \Delta\rho\, d^3x \sim \int \frac{1}{2}\delta\dot{\phi}^2\, d^3\vec{x} \sim \int d^3k \left(\frac{\omega^2}{2}\delta\phi_k^2\right)$$

$$E = \hbar\omega$$



$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}m^2\phi^2$$

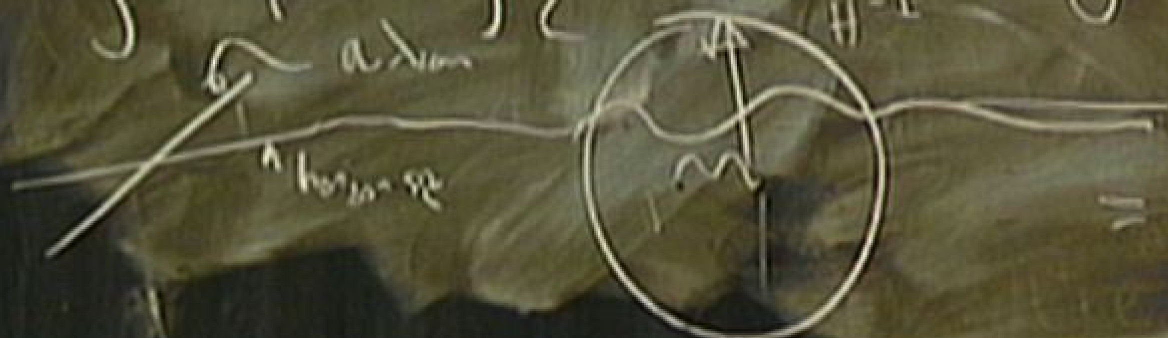
$$\delta\phi_k \sim e^{i\omega t}\, e^{-i\vec{k}\cdot\vec{x}}$$

$$\phi = \phi_0(t) + \delta\phi \qquad E^2 = p^2 \qquad \omega = |\vec{k}|$$

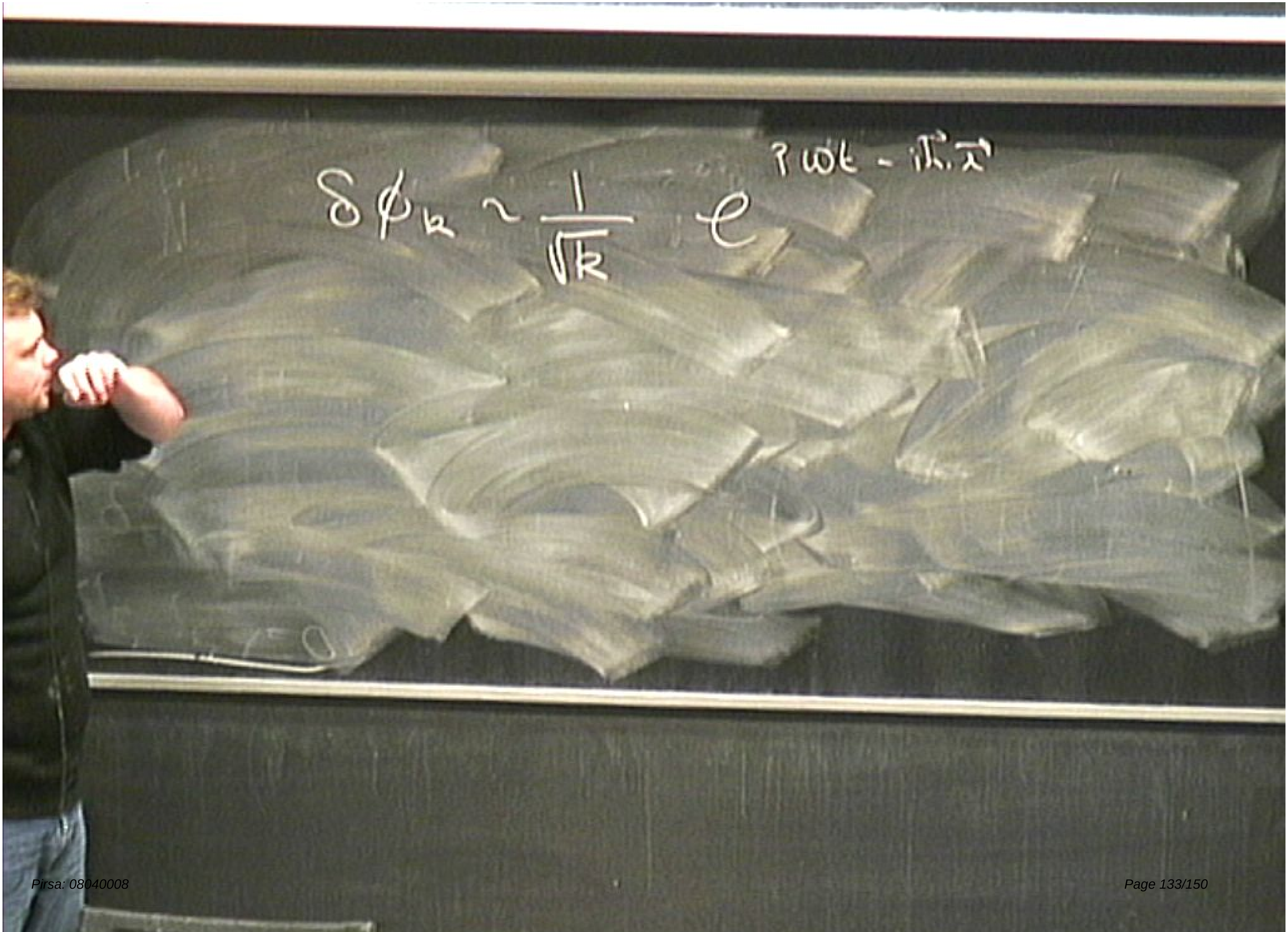
$$E = \int \Delta\rho\, d^3x \sim \int \frac{1}{2}\delta\dot{\phi}^2\, d^3\vec{x} \sim \int d^3k \left(\frac{\omega^2}{2}\delta\phi_k^2\right)$$

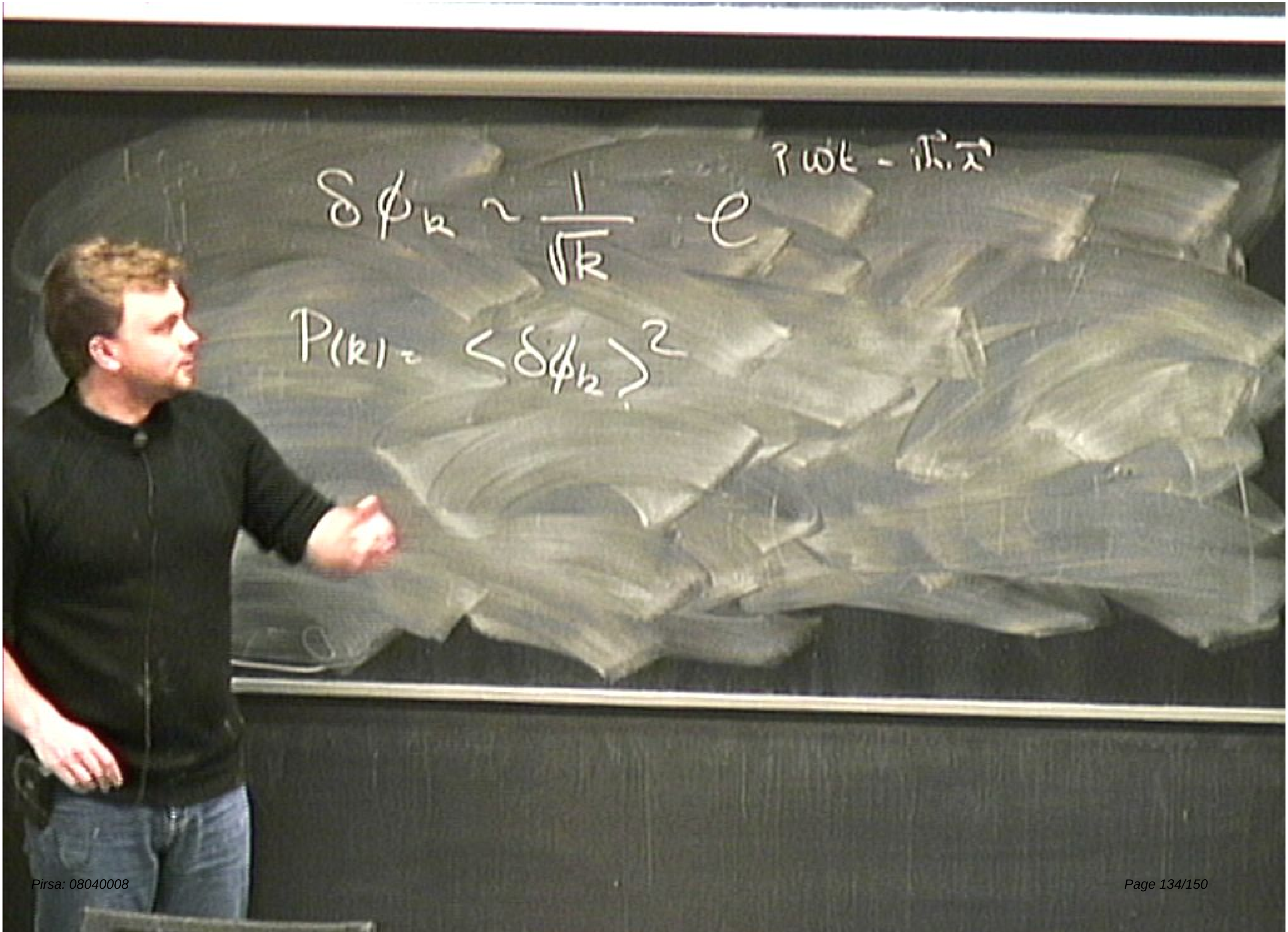
$$E = \hbar\omega$$

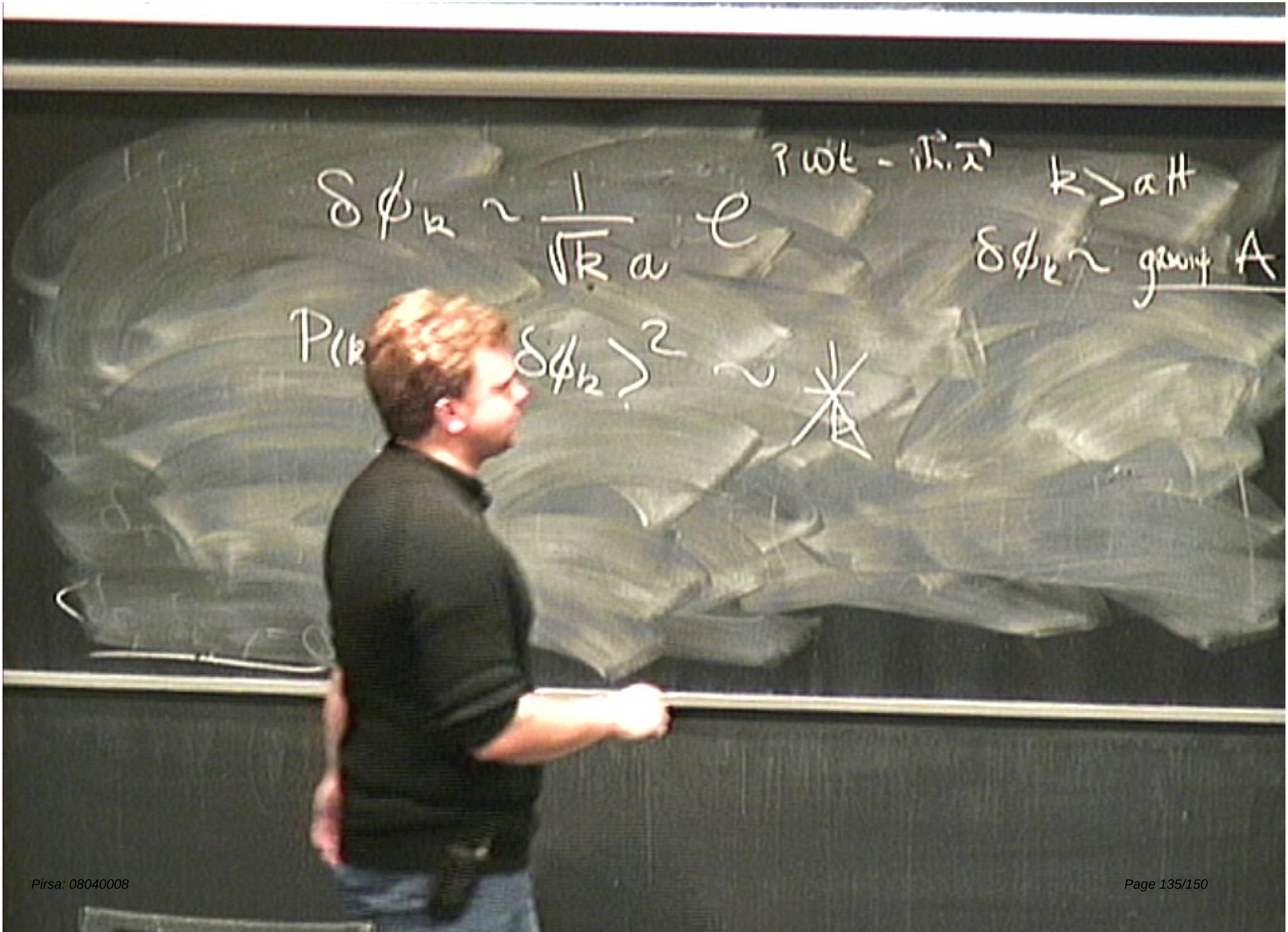
H^{-1}

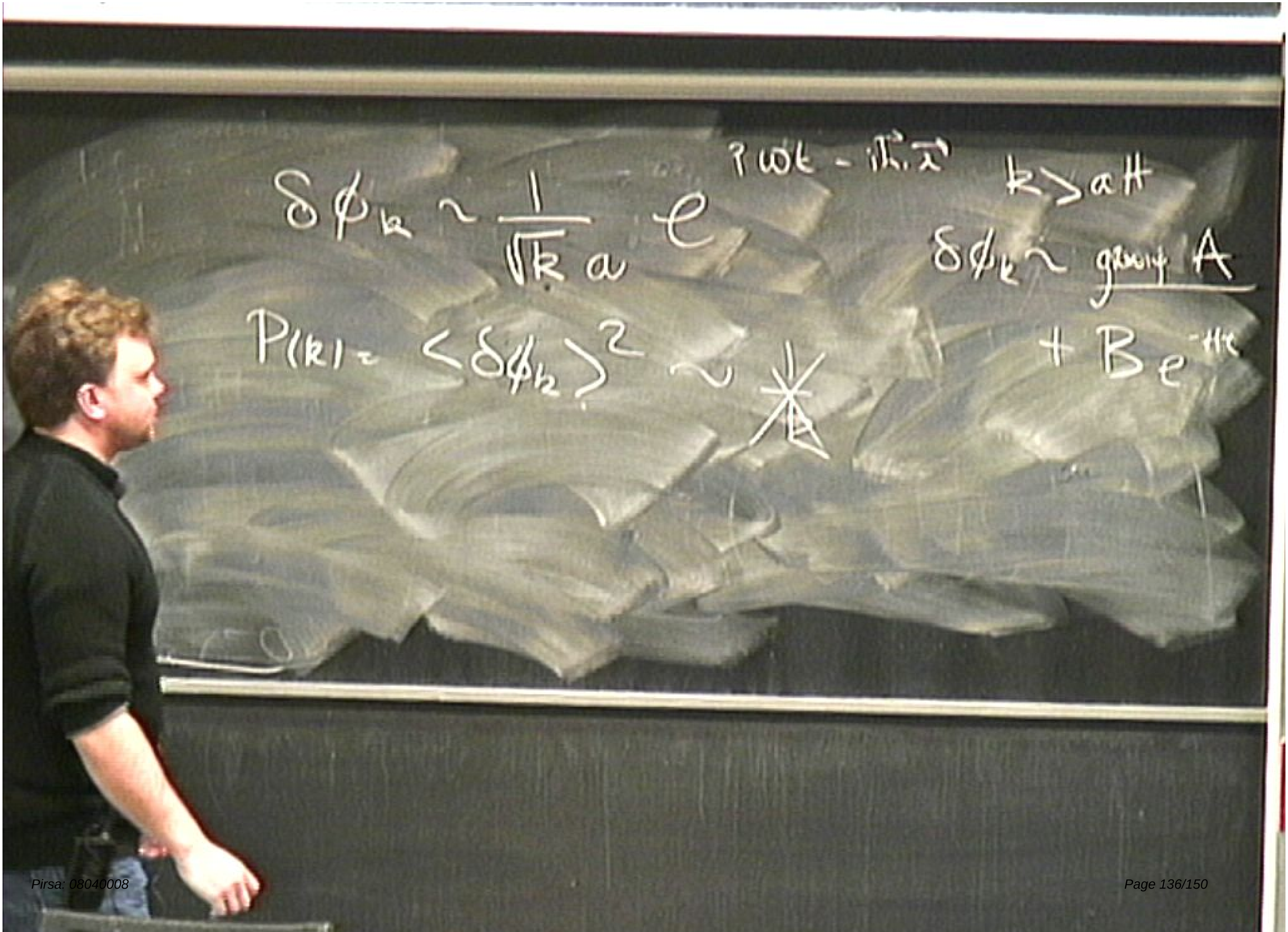


$$= \int d^3k\, \hbar\omega\, n_{\text{inter particles}}$$

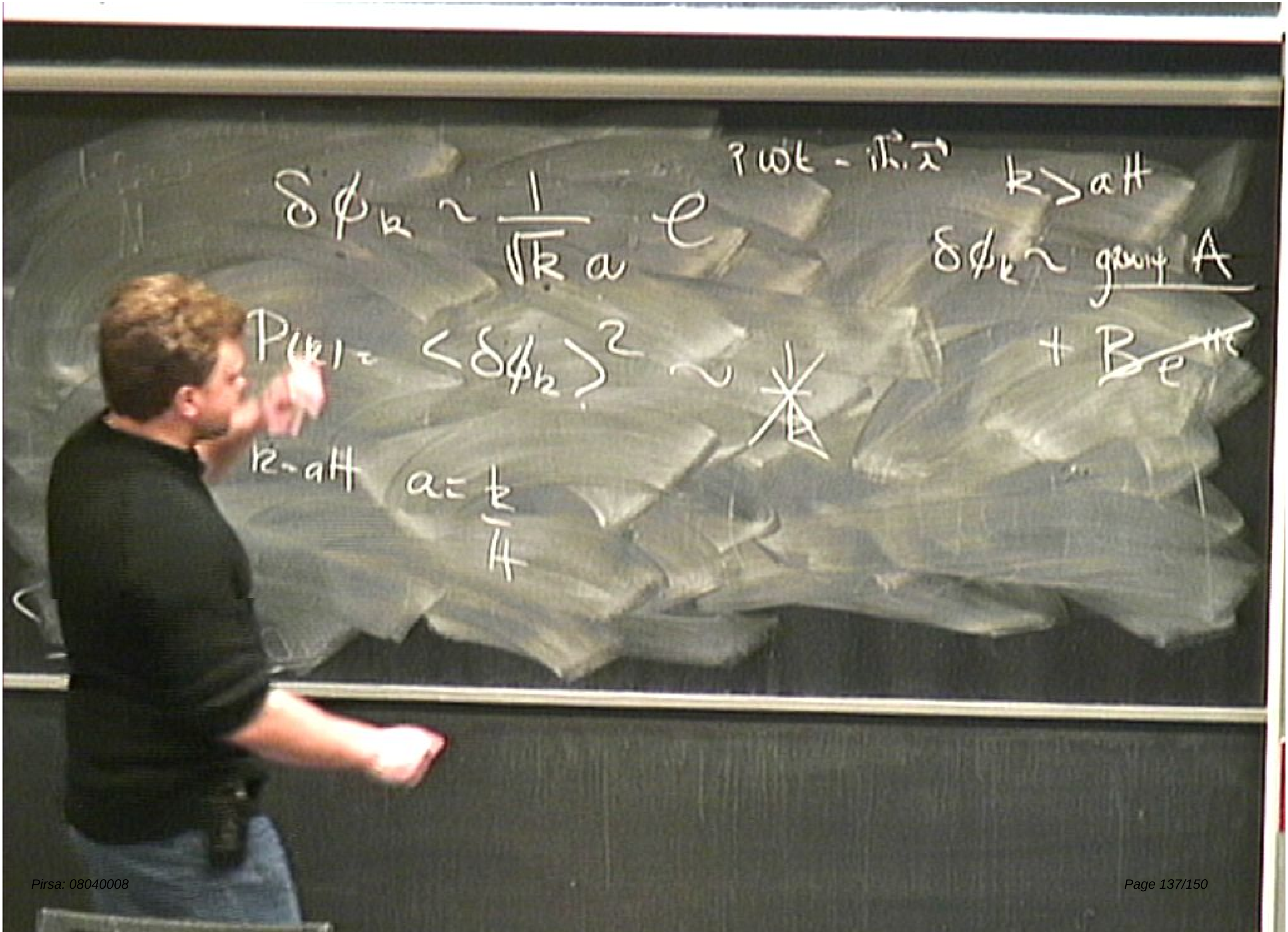

$\delta\phi_k \sim \frac{1}{\sqrt{k}} e^{i\omega t - i\vec{k}\cdot\vec{x}}$


$\delta\phi_k \sim \frac{1}{\sqrt{k}} e^{i\omega t - i\vec{k}\cdot\vec{x}}$
$P(k) = \langle \delta\phi_k \rangle^2$


δφ_k ~ 1/(√k a) e^{i ωt − i k·x}
k > aH
δφ_k ~ growing A
P(k … δφ_k)^2 ~



$\delta\phi_k \sim \frac{1}{\sqrt{k}\,a} e^{i\omega t - i\vec{k}\cdot\vec{x}}$
$k > aH$
$\delta\phi_k \sim \underline{\text{growing } A}$
$+ B e^{-Ht}$
$P(k) = \langle\delta\phi_k\rangle^2 \sim$


δφk ~ 1/(√k a) e^{iωt - ik·x}
k > aH
δφk ~ growing A + B e
P(k) = <δφk>^2 ~
k ~ aH   a = k/H

$$\delta\phi_k \sim \frac{1}{\sqrt{k}\,a}\, e^{i\omega t - i\vec{k}\cdot\vec{x}}$$

$k > aH$

$\delta\phi_k \sim$ growing A $+ \cancel{B e^{Ht}}$

$$P(k) = \langle \delta\phi_k \rangle^2 \sim \cancel{\frac{1}{k}}$$

$k \sim aH \qquad a = \frac{k}{H} \qquad \delta\phi_k \sim \frac{1}{k^{3/2}}$

$\langle \delta\phi_k \rangle^2 \sim \frac{1}{k^3}$

$$\langle \delta\phi_k \rangle^2 \sim \frac{1}{k^3}$$ scale invariant

$$\langle \delta\phi(\vec{x})\, \delta\phi(\vec{x}+\vec{r}) \rangle = \int d^3k\, \underbrace{\langle \delta\phi_k \rangle^2}_{\frac{1}{k^3}}\, e^{i\vec{k}\cdot\vec{r}}$$

$$\langle \delta\phi_k \rangle^2 \sim \frac{1}{k^3}$$

scale invariant

$$\langle \delta\phi(\vec{x})\, \delta\phi(\vec{x}+\vec{r}) \rangle = \int d^3k \, \underbrace{\langle \delta\phi_k \rangle^2}_{\frac{1}{k^3}} e^{i\vec{k}\cdot\vec{r}}$$

$$k \to \lambda k$$

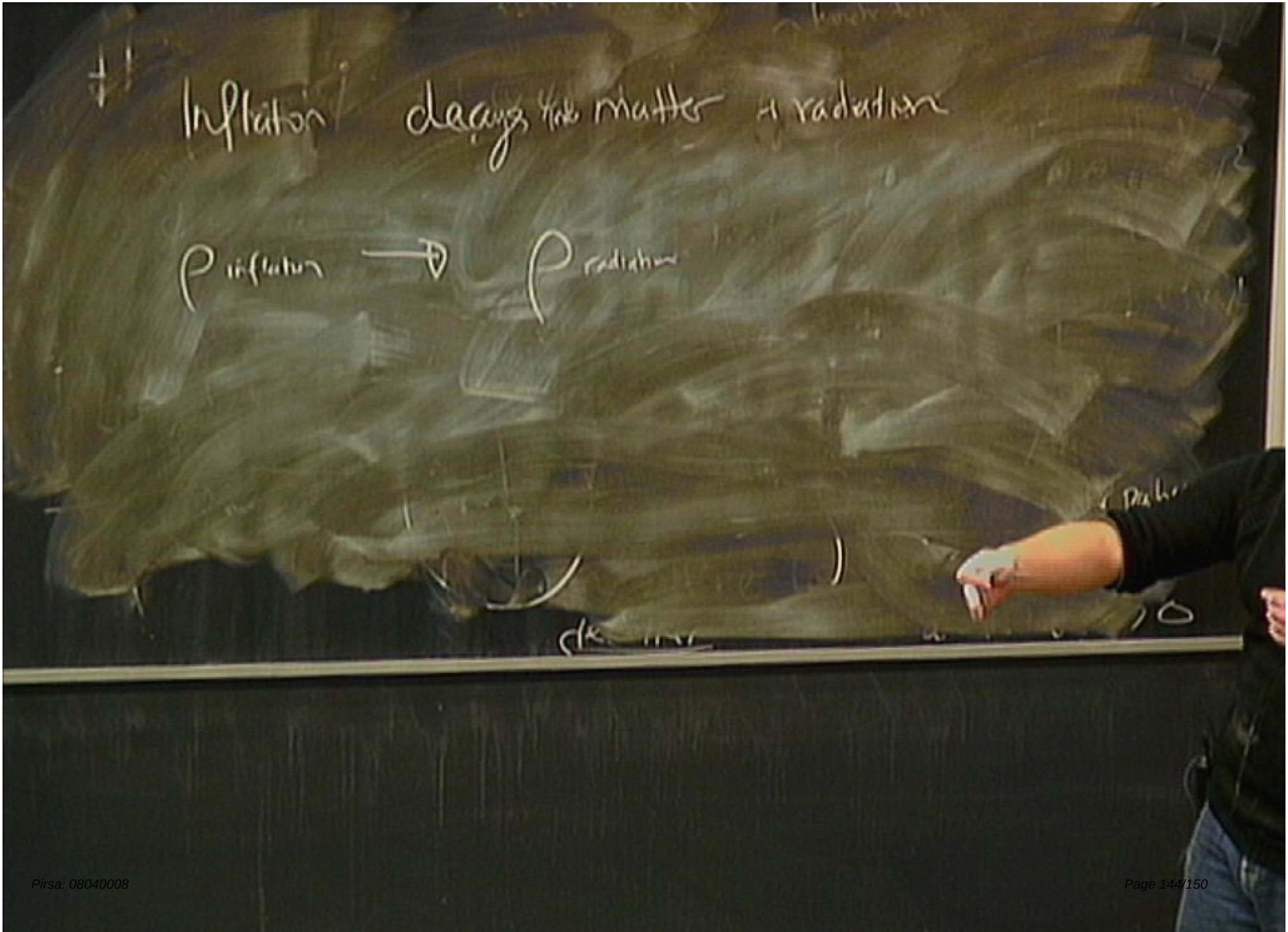
$$\langle \delta\phi_k \rangle^2 \sim \frac{1}{k^3}$$

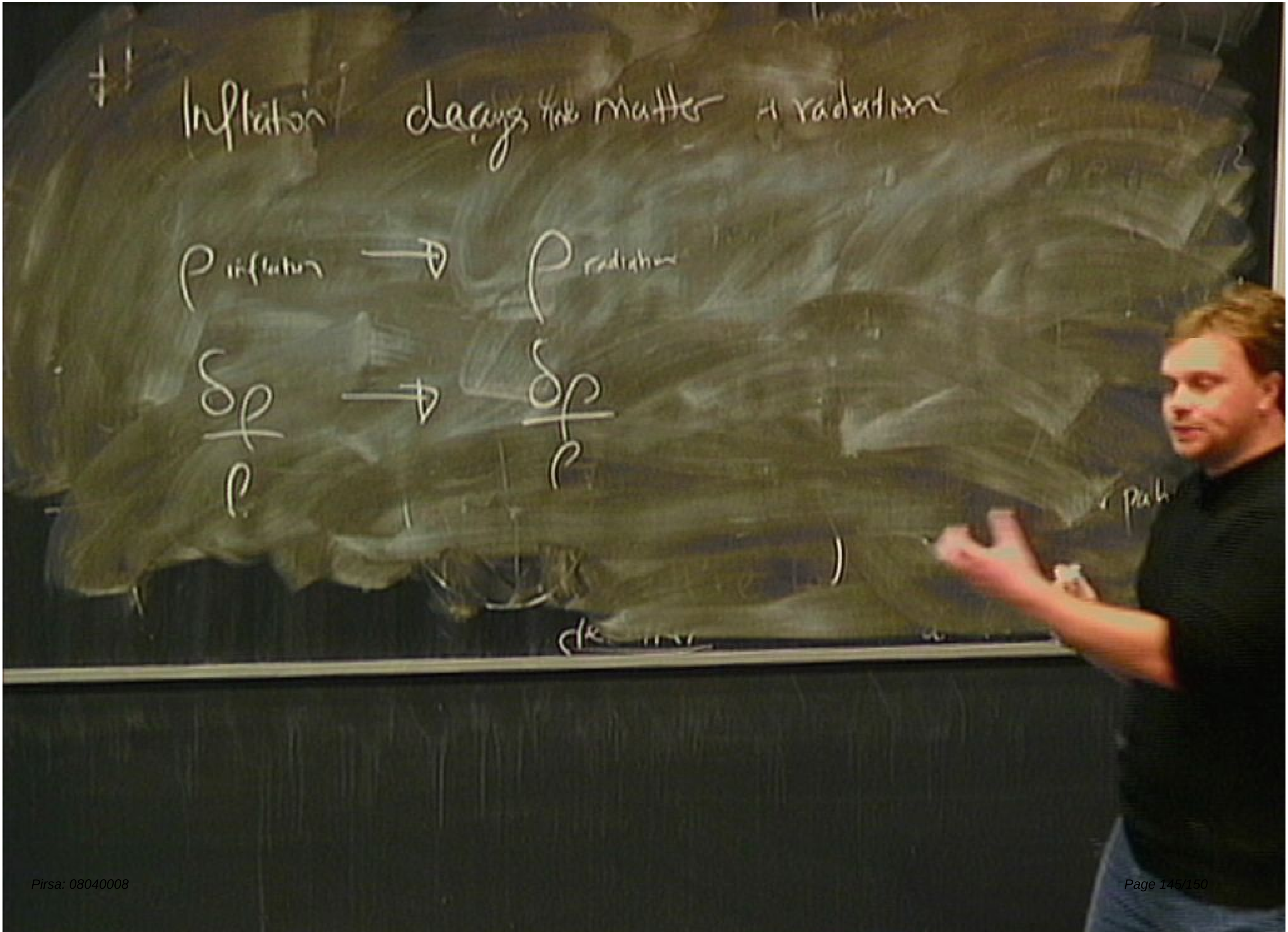
scale invariant

$$\langle \delta\phi(\vec{x})\, \delta\phi(\vec{x}+\vec{r}) \rangle = \int d^3k\, \underbrace{\langle \delta\phi_k \rangle^2}_{\frac{1}{k^3}}\, \cancel{e^{i\vec{k}\cdot\vec{r}}}$$

$$k \to \lambda k$$


Inflation decays the matter + radiation


Inflaton decays into matter + radiation
ρ inflaton → ρ radiation

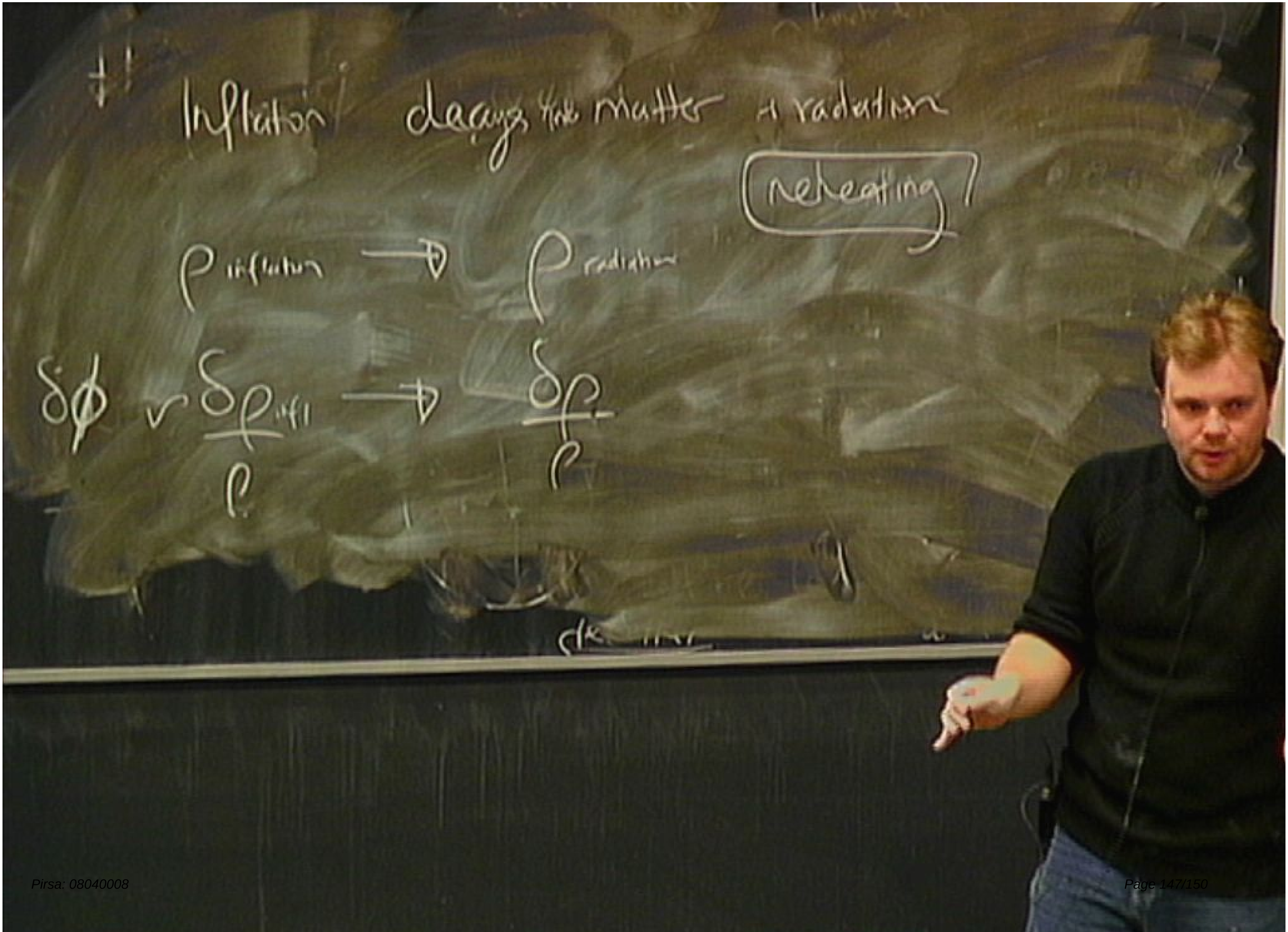

Inflaton
matter + radiation
ρ inflaton → ρ radiation
δρ/ρ → δρ/ρ

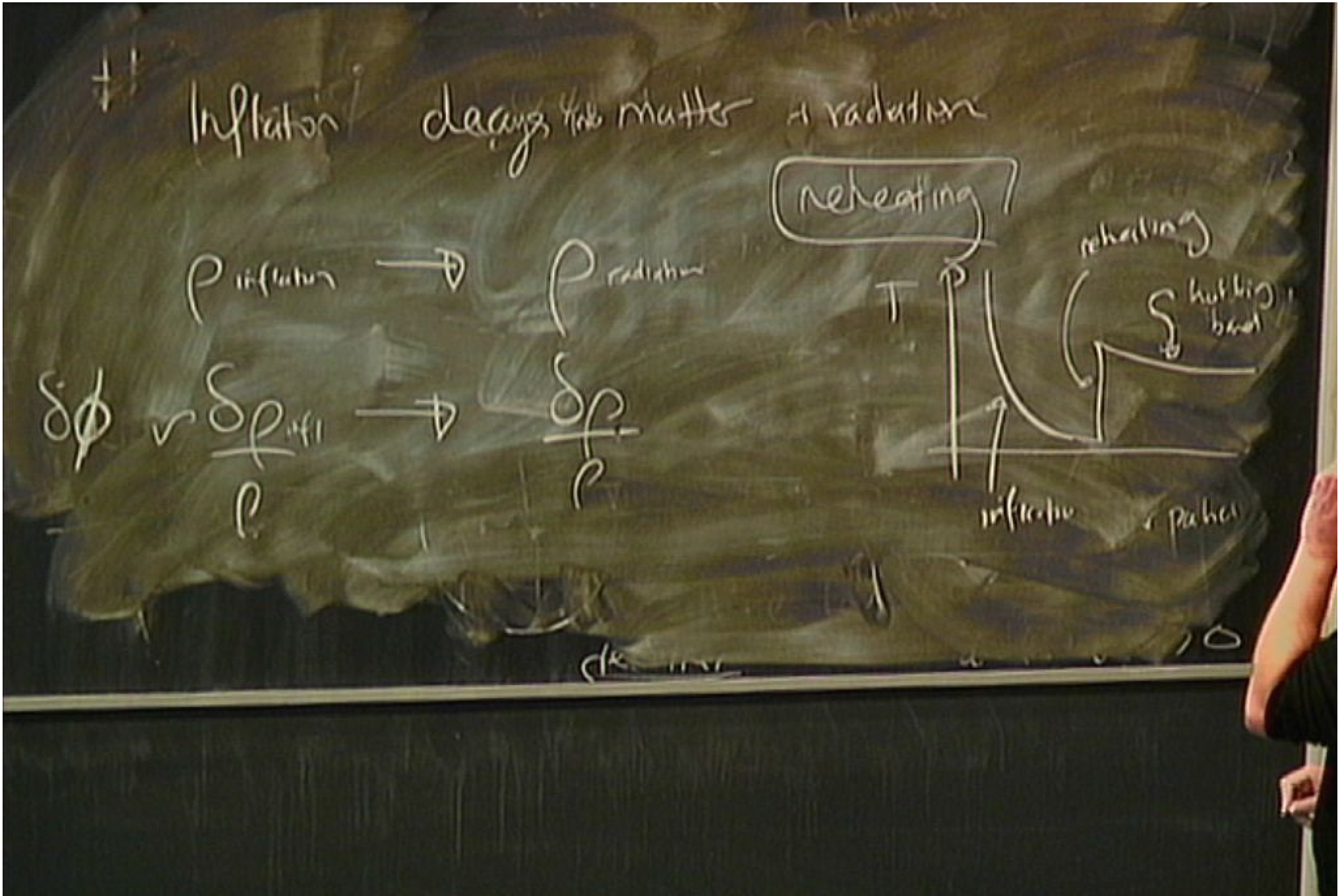
!! Inflaton decays into matter + radiation

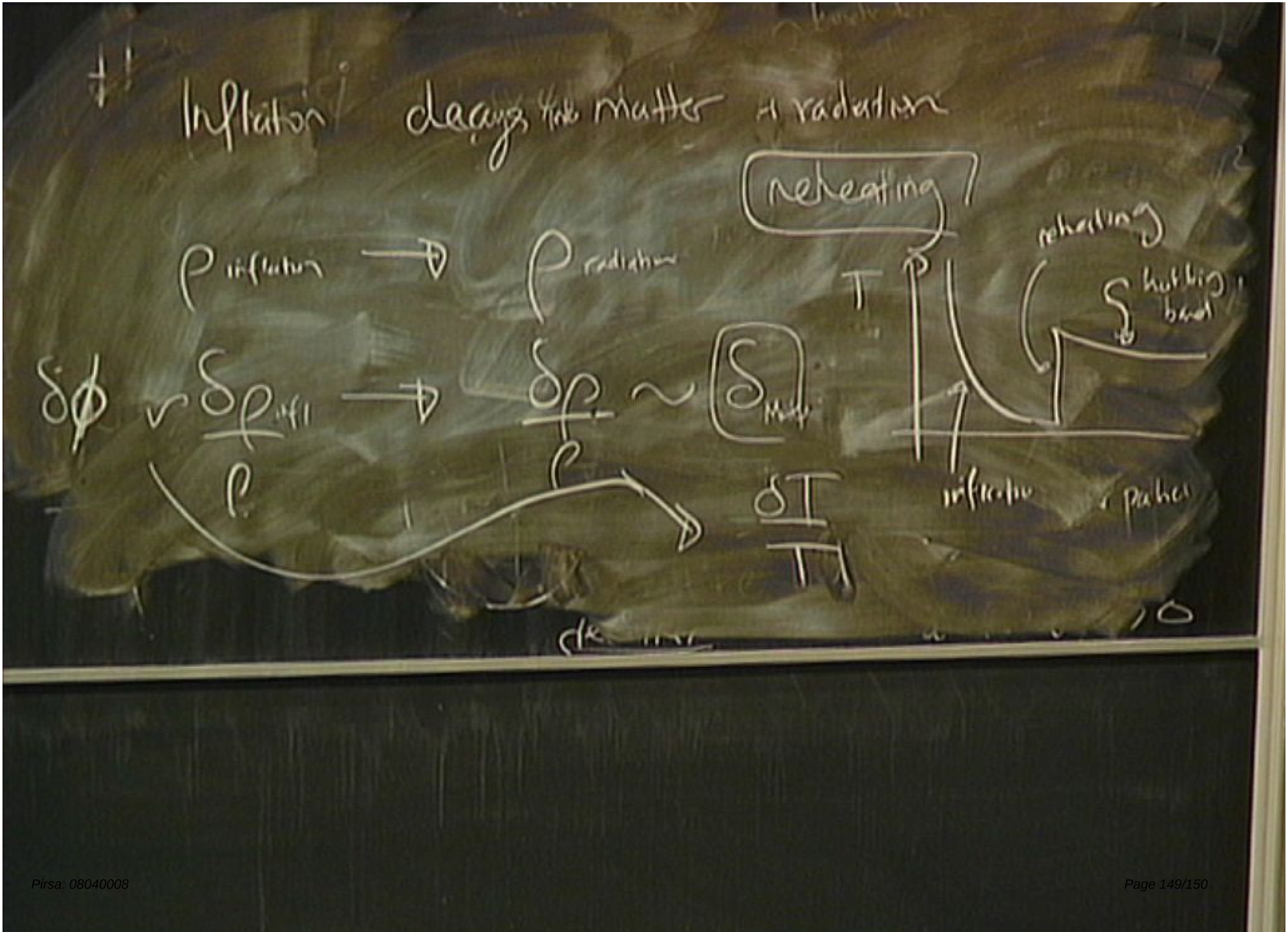
$\rho_{\text{inflaton}} \rightarrow \rho_{\text{radiation}}$

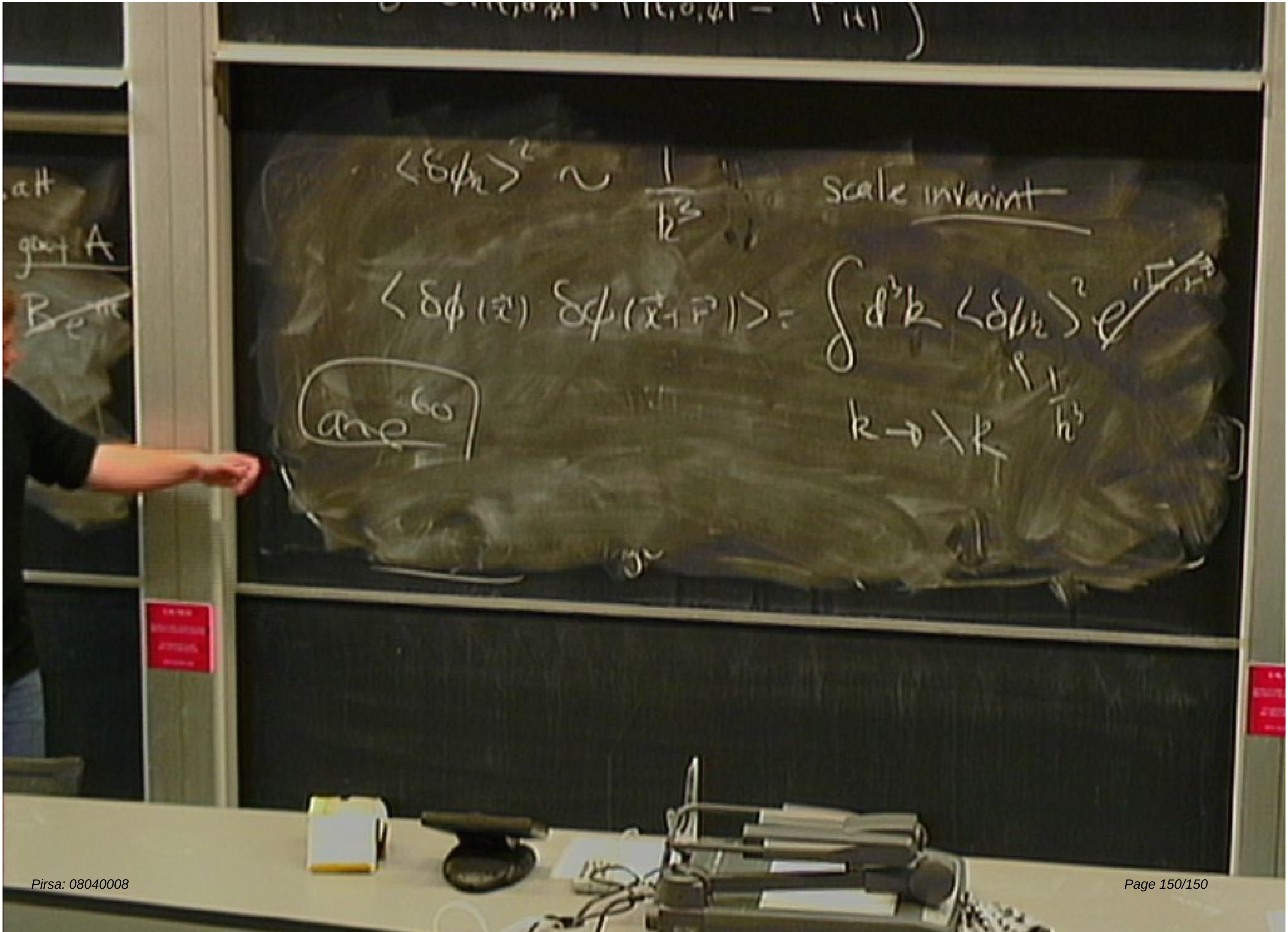
$\delta\phi \sim \frac{\delta\rho_{\text{infl}}}{\rho} \rightarrow \frac{\delta\rho}{\rho}$

$\sim$ patches


Inflaton decays into matter + radiation
reheating
ρ inflaton → ρ radiation
δφ ~ δρ infl / ρ → δρ / ρ


Inflaton decays into matter + radiation
reheating
ρ inflaton → ρ radiation
δφ ~ δρ infl / ρ → δρ / ρ
T
reheating
hot big bang
inflation


Inflaton decays into matter + radiation
reheating
inflaton


⟨δφ_k⟩² ~ 1/k³
scale invariant
⟨δφ(x) δφ(x+r)⟩ = ∫ d³k ⟨δφ_k⟩² e^{ik·r}
1/k³
k → λk
ae^60