

Title: Cosmology #3

Date: Apr 01, 2008 06:30 PM

URL: <http://pirsa.org/08040007>

Abstract: A brief history of our cosmic beginnings, Cosmic Microwave Background. How galaxies form and the existence of dark matter.

Recap

Quantum Measure Theory | EPR-Bohm  
Current composition of universe

~ 4% visible matter (stars + galaxies)

~ 20% dark matter (?)

negligible radiation (mostly cosmic microwave background)

~ 76% dark energy

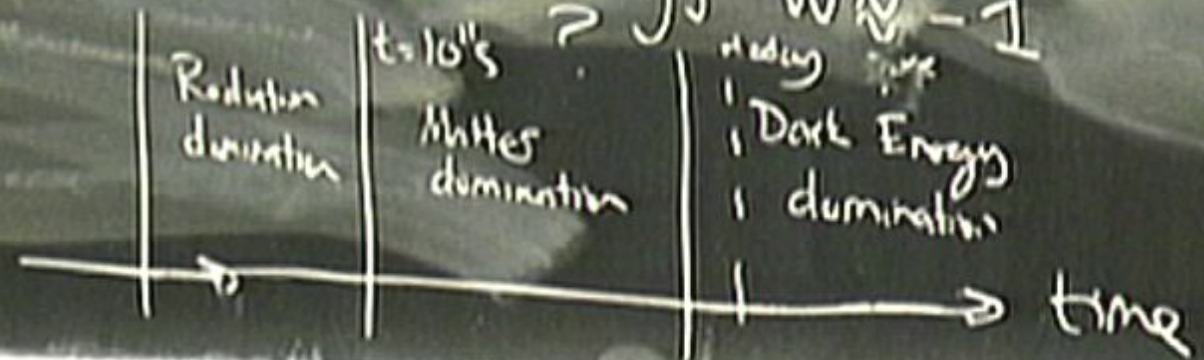
$$w = \frac{p}{\rho}$$

$$w = 0$$

$$w = 0$$

$$w = \frac{1}{3}$$

$$w = -1$$





# Cosmic Microwave Background

— highly isotropic thermal (blackbody) radiation:  $T_{\text{today}} = 2.73\text{K}$

$$T(t_1) = T(t_0) \frac{a(t_0)}{a(t_1)}$$

$$\phi(A) = \begin{cases} 1 & \text{if } A \\ 0 & \text{otherwise} \end{cases}$$

$$\phi(A \cup B) = \phi(A) + \phi(B)$$
$$\phi(A \cap B) = \phi(A) \phi(B)$$



# Cosmic Microwave Background

— highly isotropic thermal (blackbody) radiation  $T_{\text{today}} = 2.73\text{K}$

$$T(t_1) = T(t_0) \frac{a(t_0)}{a(t_1)}$$

Particle horizon (finite size of observable universe)

$$d_H = c \int_0^{t_0} \frac{a(t_0)}{a(t)} dt = \frac{3(1+w)ct_0}{(1+3w)}$$



## Cosmic Microwave Background

— highly isotropic thermal (blackbody) radiation  $T_{\text{rad}} = 2.75\text{K}$

$$T(t) = T(t_0) \frac{a(t_0)}{a(t)}$$

Particle horizon (finite size of observable universe)

$$d_H = \int_0^{t_0} \frac{a(t_0)}{a(t)} dt = \frac{3(1+w)ct_0}{(1+3w)}$$

Decoupling

Two species remain in equilibrium if the rate of interactions  $\Gamma$

satisfies  $\Gamma \gg H$

if  $\Gamma < H$  they decouple



Recap

Quantum Measure Theory

EPR-Bohm

Current composition of universe

$W = \frac{P}{P}$

~ 4% visible matter  
(stars + galaxies + baryons)

$W = 0$

20% dark matter  
(?)

$W = 0$

negligible radiation  
(mostly cosmic microwave background)

$W = \frac{1}{3}$

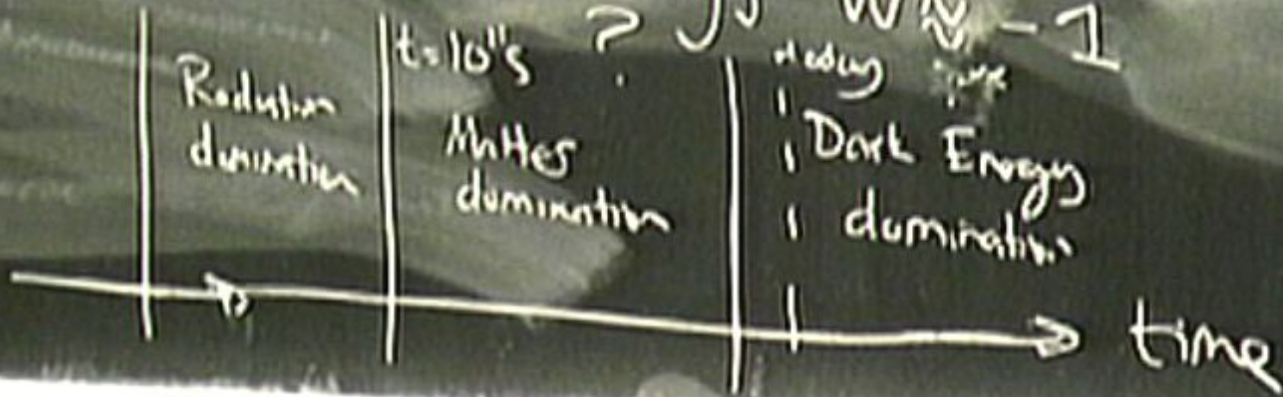
~ 76% dark energy

$W = -1$

Radiation domination

to 10's  
Matter domination

Dark Energy domination





Recap

Current composition of universe

~ 4% visible matter (stars + galaxies)

~ 20% dark matter (?)

negligible radiation (mostly cosmic microwave background)

~ 76% dark energy

EPR - Bohm  
 $w = \frac{p}{\rho}$

$w = 0$

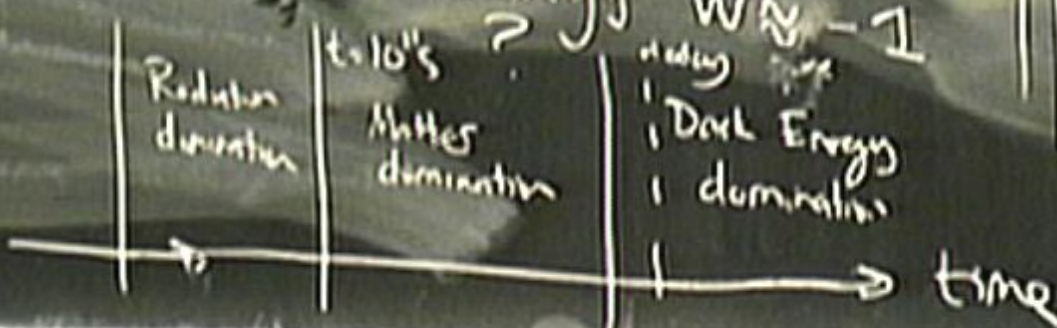
$w = 0$

$w = \frac{1}{3}$

$$\rho = \frac{\rho_r}{a^4} + \frac{\rho_m}{a^3} + \rho_{de}$$

$w = -1$   
Cosmological constant

$\rho = \text{constant}$





Cosmic Microwave Background

— highly isotropic thermal (blackbody) radiation

Recombination

Particle horizon (finite size of observable universe)

$$d_H = c \int_0^{t_0} \frac{a(t_0)}{a(t)} dt = \frac{3(1+w)ct_0}{(1+3w)}$$

Two species remain in equilibrium if

Satisfies  $T \gg H$

$T < H$  they decouple



# Cosmic Microwave Background

— highly isotropic thermal (blackbody) radiation  $T_{avg} = 2.75K$

## Recombination

$$T(t_1) = T(t_0) \frac{a(t_0)}{a(t_1)}$$

Particle horizon (finite size of observable universe)



$$d_H = \int_0^{t_0} \frac{a(t_0)}{a(t)} dt = \frac{3(1+w)ct_0}{(1+3w)}$$

expanding

Two species remain in equilibrium if the rate of interactions  $\Gamma$

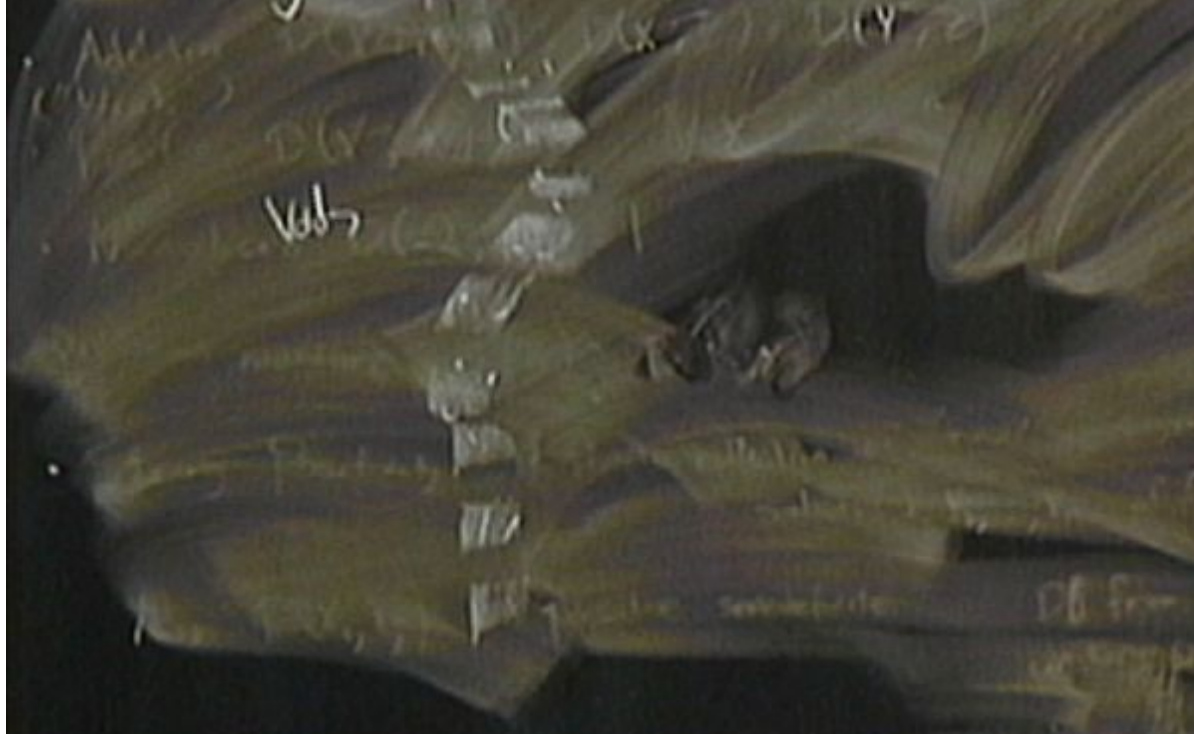
satisfies  $\Gamma \gg H$

if  $\Gamma < H$  they decouple



Convenient to Express level 2 measure in terms of  
D(x) = E

# Origin of Structure



Classical Stochastic Example

Quantum Heisenberg Theory

EPR

Hilbert space

Example 2

PR laws

DB from ordering

strong probability





Convenient to Express level 2 measure in terms of characteristic function  $\mathbb{P}(X)$

# Origin of Structure

The real world is **inhomogeneous and anisotropic**

Classical Stochastic Example

Quantum Theory

Hilbert space, Positive Q

Number Theory

Group of transformations

PR boxes

From ordinary  
continuous state  
strong locality

Web





Classical Stochastic

Quantum Theory

Hilbert

Example of

FR boxes

strongly

partially

strongly

partially

strongly

partially

strongly

partially

The real universe is inhomogeneous and anisotropic

# Origin of structure

Vols



At center of Star Physics

Convenient to Express level 2 model in terms of  
D(x) = ...

Classical Stochastic Example

Origin of structure

Quantum Theory

The real universe is inhomogeneous and anisotropic

Volts

Basic idea

gravitational force  
is unstable





Origin of Slow Degrees

convenient to Express level 2 motion in terms of stochastic function  $D(x)$

# Origin of structure

The real universe is inhomogeneous and anisotropic

Classical Stochastic Example

Quantum Gravity Theory

## Basic idea

gravitational force is unstable

$$F_N = - \frac{m_1 m_2 G}{r^2}$$

$m_1 \rightarrow \leftarrow m_2$

$m > 0$



Construct a state-dependent function

$$\rho(t, \vec{x}) \rightarrow \bar{\rho}(t)$$

CURVES

$\rho_{11}(t), \rho_{12}(t), \rho_{21}(t), \rho_{22}(t)$

$\rho_{11}(t) = \rho_{11}(0) e^{-\lambda_{11} t}$

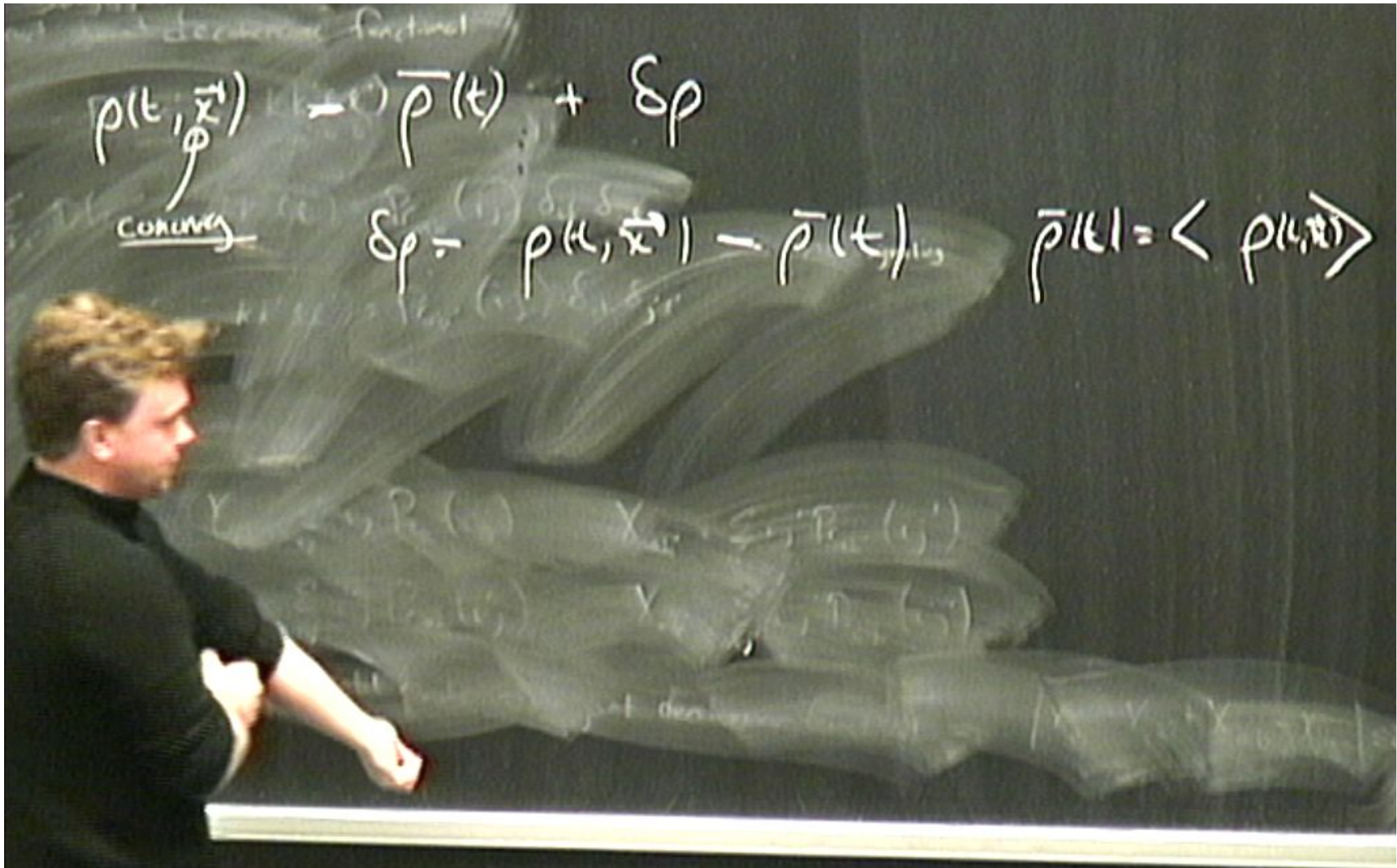
$\rho_{12}(t) = \rho_{12}(0) e^{-\lambda_{12} t}$

$\rho_{21}(t) = \rho_{21}(0) e^{-\lambda_{21} t}$

$\rho_{22}(t) = \rho_{22}(0) e^{-\lambda_{22} t}$

Experimental





$$\rho(t, \vec{x}) \approx \bar{\rho}(t) + \delta\rho$$

CONVEX

$$\delta\rho = \rho(t, \vec{x}) - \bar{\rho}(t)$$

$$\bar{\rho}(t) = \langle \rho(t, \vec{x}) \rangle$$



$$\rho(t, \vec{x}) = \bar{\rho}(t) + \delta\rho$$

CONVEX

$$\delta\rho = \rho(t, \vec{x}) - \bar{\rho}(t)$$

$$\bar{\rho}(t) = \langle \rho(t, \vec{x}) \rangle$$

$$\delta = \frac{\delta\rho}{\rho}$$

fractional  
density  
contrast



slightly positive  
constant - constant - constant - fractal

$$\rho(t, \vec{x}) = \bar{\rho}(t) + \delta\rho$$

CONVEX

$$\delta\rho = \rho(t, \vec{x}) - \bar{\rho}(t)$$

$$\bar{\rho}(t) = \langle \rho(t, \vec{x}) \rangle$$

fractal  
density  
contrast

$$\delta = \frac{\delta\rho}{\rho} = \frac{\rho(t, \vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\delta = 0$$



$$\rho(t, \vec{x}) = \bar{\rho}(t) + \delta\rho$$

coherence

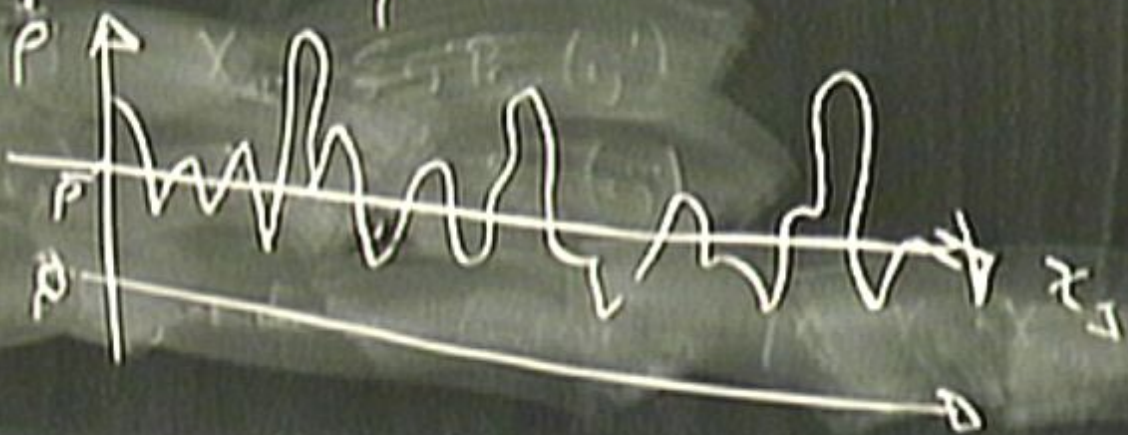
$$\delta\rho = \rho(t, \vec{x}) - \bar{\rho}(t)$$

$$\bar{\rho}(t) = \langle \rho(t, \vec{x}) \rangle$$

fractal  
density  
contrast

$$\delta = \frac{\delta\rho}{\bar{\rho}} = \frac{\rho(t, \vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\delta = 0$$





$$\rho(t, \vec{x}) = \bar{\rho}(t) + \delta\rho$$

fluctuation

$$\delta\rho = \rho(t, \vec{x}) - \bar{\rho}(t)$$

$$\bar{\rho}(t) = \langle \rho(t, \vec{x}) \rangle$$

fractional density contrast

$$\delta = \frac{\delta\rho}{\bar{\rho}} = \frac{\rho(t, \vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\delta > 0$$

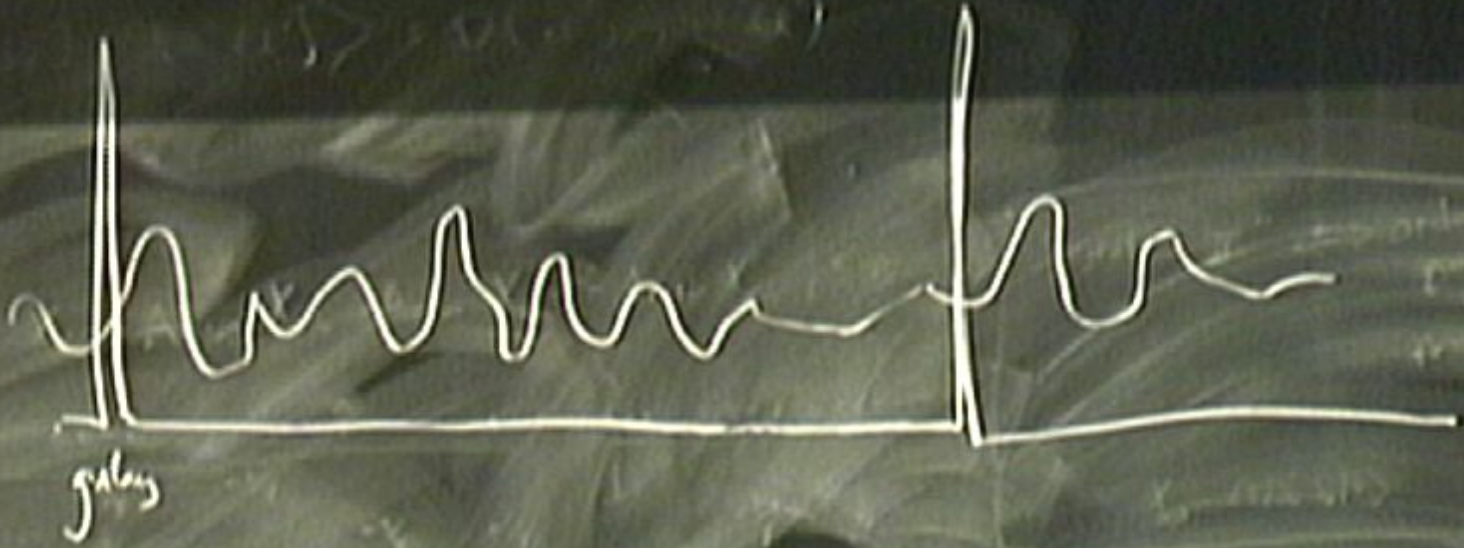
$$\langle \delta \rangle = 0$$

$$\delta = 0$$









Convenient to Express level 2 measure in terms of

Gravity is well described by Newton's laws

Classical Stochastic





Convenient to express level 2 measure in terms of

Gravity is well described by Newton's laws

$$\vec{F} = -m \vec{\nabla} \phi$$
$$\nabla^2 \phi = 4\pi G \rho$$

← mass density



Convenient to Express level 2 measure in terms of

Gravity is well described by Newton's laws

$$m \cdot \vec{F} = -m \vec{\nabla} \phi$$
$$\nabla^2 \phi = 4\pi G \rho$$

$\leftarrow$  mass density



Convenient to express level 2 measure in terms of

Gravity is well described by Newton's laws

$$m \cdot \vec{F} = -m \vec{\nabla} \phi$$

$$\nabla^2 \phi = 4\pi G \rho$$

← mass density

$$\rho = M \delta^{(3)}(\vec{r})$$

$$\phi = \frac{MG}{r}$$

Convenient to Express local 2 measure in terms of

Gravity is well described by Newton's laws

$$m \cdot \vec{F} = -m \vec{\nabla} \phi$$

$$\nabla^2 \phi = 4\pi G \rho$$

← mass density

$$\rho = M S^{(3)}(\vec{r})$$

$$\phi = \frac{MG}{r}$$

---

Galaxies fluid  $\rho, p, \vec{u}, \vec{\pi}$



Gravity is well described by Newton's laws

$$\vec{F} = -m \vec{\nabla} \phi$$

$$\nabla^2 \phi = 4\pi G \rho$$

← mass density

$$\rho = M \delta^{(3)}(\vec{r})$$

$$\phi = \frac{MG}{r}$$

Galaxies fluid  $\rho, p, \vec{v}(\vec{r}, t)$

$$\frac{D\vec{v}}{Dt} = -\vec{\nabla} \phi - \frac{\vec{\nabla} p}{\rho}$$

↑  
gravitational force

Convenient to Express local 2 measure in terms of

Gravity is well described by Newton's laws,

$m \cdot \vec{F} = -m \vec{\nabla} \phi$

$\nabla^2 \phi = 4\pi G \rho$  ← mass density

$\rho = M \delta^{(3)}(\vec{r})$   
 $\phi = \frac{MG}{r}$

Galaxies fluid  $\rho, p, \vec{v}(\vec{r}, t)$

$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = \frac{D \vec{v}}{Dt} = - \vec{\nabla} \phi - \frac{\vec{\nabla} p}{\rho}$

↑  
gravitational force



$\frac{\partial}{\partial t}$

$\uparrow$   
gravitational force

$\rho$

Conservation law  
of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

$\int_{\mathcal{V}} \rho dV$

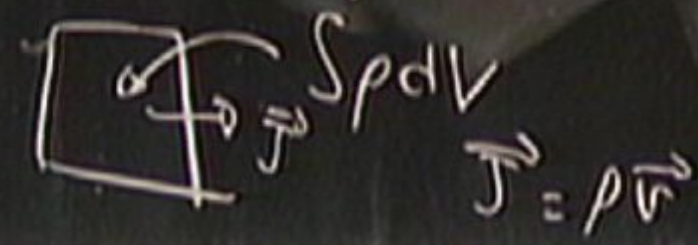
$\vec{J} = \rho \vec{v}$

Continuity equation

Conservation law  
of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$





carries fluid  $\rho, p, \vec{u}(\rho, p)$   
 Navier-Stokes  
 $-\nabla^2 \phi$   
 $\frac{\nabla^2 p}{\rho}$   
 $p = f(|\rho|)$   
 gravitation  $\vec{F}$

$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$   
 $\int_V \rho dV$   
 $\vec{J} = \rho \vec{u}$









single-point  
Constant number of degrees of freedom

$$\rho = \bar{\rho} + \delta\rho(t, \vec{x})$$

order

first order

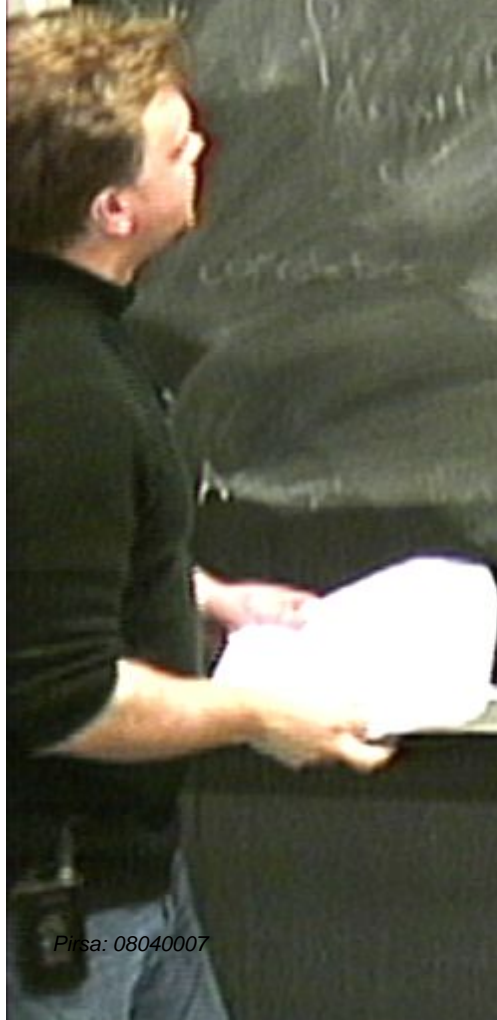
$$\delta = \frac{\delta\rho}{\rho}$$

$$\delta \rho < \rho$$

$$\delta \rho > 0$$

$$\delta \rho < 0$$





slightly more  
constant and decrease factorial

$$\rho(t, \pi) = \bar{\rho} + \delta \rho(t, \pi)$$
$$\vec{v}(t, \pi) = \vec{v}_0 + \delta \vec{v}(t, \pi)$$
$$H(t, \pi) = H_0 + \delta H(t, \pi)$$

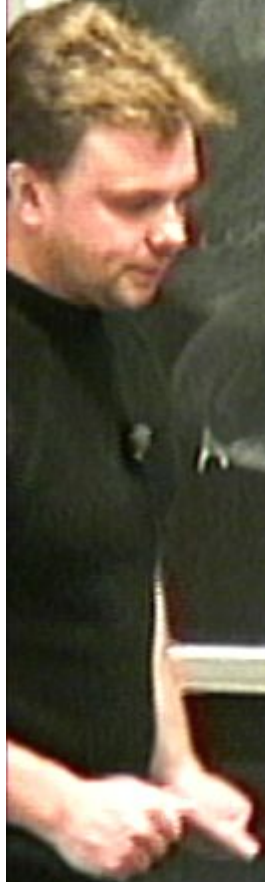


single point  
constant and discrete functional

$$\rho = \bar{\rho} + \delta\rho(t, \vec{x})$$

$$\vec{v} = \vec{v}_0 + \delta\vec{v}(t, \vec{x})$$

$$\Phi = \bar{\Phi} + \delta\Phi$$







Construct perturbation function

$$\rho = \bar{\rho} + \delta\rho(t, \vec{r})$$
$$\vec{H} = \vec{0} + \delta\vec{H}(t, \vec{r})$$
$$\Phi = \bar{\Phi} + \delta\Phi$$

$$\nabla^2 \delta\Phi = 4\pi G \delta\rho$$



Galaxies fluid  $\rho, P$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = \frac{D \vec{v}}{Dt}$$

$$= -\nabla \phi$$

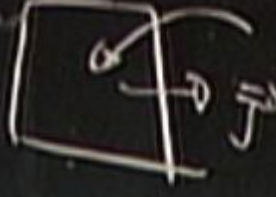
↑  
gravitational force

$$-\frac{\nabla P}{\rho}$$

Conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$



$\int_V \rho dV$   
 $\vec{J} = \rho \vec{v}$



slightly positive  
Constant...  
fundamental

$$\rho = \bar{\rho} + \delta\rho(t, \vec{r})$$

$$\vec{E} = 0 + \delta\vec{E}(t, \vec{r})$$

$$\phi = \bar{\phi} + \delta\phi$$

$$\nabla^2 \delta\phi = 4\pi C \delta\rho$$

$$\frac{\partial \delta\vec{E}}{\partial t} = -\nabla \delta\phi - \frac{1}{\epsilon_0} \nabla \delta\rho$$



$$\rho = \bar{\rho} + \delta\rho(\mathbf{r}, \mathbf{r}') \quad \text{with } \rho \ll \bar{\rho}$$

$$\vec{v} = \vec{0} + \delta\vec{v}(\mathbf{r}, \mathbf{r}')$$

$$\Phi = \bar{\Phi} + \delta\Phi$$

$$\nabla^2 \delta\phi = 4\pi G \delta\rho$$

$$\frac{\partial \delta\vec{v}}{\partial t} = -\nabla \delta\phi - \frac{1}{\rho} \nabla \delta\rho + \text{higher}$$

$$\frac{\partial \delta\rho}{\partial t} + \nabla \cdot (\rho \delta\vec{v}) = 0 + \text{higher}$$





$$\rho = \bar{\rho} + \delta\rho(t, \vec{x})$$

$$\vec{u} = \vec{0} + \delta\vec{u}(t, \vec{x})$$

$$\Phi = \bar{\Phi} + \delta\Phi$$

$$\nabla^2 \delta\phi = 4\pi G \delta\rho \quad \textcircled{1}$$

$$\frac{\partial \delta\vec{u}}{\partial t} = -\vec{\nabla} \delta\phi - \frac{1}{\rho} \vec{\nabla} \delta p + \text{higher} \quad \textcircled{2}$$

$$\frac{\partial \delta\vec{u}}{\partial t} + \vec{\nabla}(\rho \delta\vec{u}) = 0 + \text{higher} \quad \textcircled{3}$$



⑤  $\frac{\partial \rho}{\partial t}$

②  $\nabla$

$$\frac{\partial^2 \rho}{\partial t^2} + \rho \nabla \cdot \left( \frac{\partial \vec{v}}{\partial x} \right) = 0$$

$$\nabla \cdot \frac{\partial \vec{v}}{\partial t} = -\nabla^2 \rho - \frac{1}{\rho} \nabla^2 \rho$$





③

$$\frac{\partial^2 \delta p}{\partial t^2} + \bar{\rho} \vec{\nabla} \cdot \left( \frac{\partial \vec{u}}{\partial x} \right) = 0$$

②

$$\vec{\nabla} \cdot \vec{\sigma} = -\nabla^2 \delta p - \frac{1}{\rho} \nabla^2 \delta p$$

$$\frac{1}{\rho} \frac{\partial^2 \delta p}{\partial t^2} \rightarrow 4\pi G \delta p - \frac{1}{\rho} \nabla^2 \delta p$$

$$\delta p = c_s^2 \delta \rho$$



(1)  $\frac{\partial \vec{v}}{\partial t}$

(2)  $\nabla \phi$

$$\frac{\partial^2 \delta \rho}{\partial t^2} + \bar{\rho} \nabla \cdot \left( \frac{\partial \vec{v}}{\partial t} \right) = 0$$

$$\nabla \cdot \frac{\partial \vec{v}}{\partial t} = -\nabla^2 \delta \phi - \frac{1}{\bar{\rho}} \nabla^2 \delta \rho$$

$$\frac{1}{\bar{\rho}} \frac{\partial^2 \delta \rho}{\partial t^2} \rightarrow 4\pi G \delta \rho - \frac{1}{\bar{\rho}} \nabla^2 \delta \rho$$

$$\delta p = c_s^2 \delta \rho$$
$$\boxed{c_s^2 = \frac{dp}{d\rho}}$$

$\delta = \delta \rho$



③

②

$$\frac{\partial^2 \delta p}{\partial t^2} + \bar{\rho} \vec{\nabla} \cdot \left( \frac{\partial \vec{u}}{\partial x} \right) = 0$$

$$\vec{\nabla} \cdot \vec{u} = -\frac{\nabla^2 \delta p}{\rho} - \frac{1}{\rho} \nabla^2 \delta p$$

$$\frac{1}{\rho} \nabla^2 \delta p \rightarrow 4\pi G \delta p - \frac{1}{\rho} \nabla^2 \delta p$$

$$\delta p = c_s^2 \delta \rho$$

$$c_s^2 = \frac{dp}{d\rho}$$

$\delta = \delta \rho$



(3)

(2)

$$\frac{\partial^2 \delta p}{\partial t^2} + \bar{\rho} \vec{\nabla} \cdot \left( \frac{\partial \vec{u}}{\partial x} \right) = 0$$

$$\vec{\nabla} \cdot \vec{\sigma} = -\nabla^2 \delta p - \frac{1}{\bar{\rho}} \nabla^2 \delta p$$

$$\frac{1}{\rho} \nabla^2 \delta p \rightarrow 4\pi G \delta p - \frac{1}{\rho} \nabla^2 \delta p$$

$$\delta p = c_s^2 \delta \rho$$

$$c_s^2 = \frac{dp}{d\rho}$$

$\delta = \delta \rho$



$$\left( \frac{\partial \phi}{\partial t} \right) \quad (3)$$

$$+ \left( \nabla \cdot \vec{v} \right) \quad (2)$$

$$\frac{1}{\rho} \nabla^2 \delta \rho$$

$$\rightarrow 4\pi G \delta \rho$$

$$- \frac{1}{\rho} \nabla^2 \delta \rho$$

$$\delta p = c_s^2 \delta \rho$$

$$c_s^2 = \frac{dp}{d\rho}$$

$$\frac{\partial^2 \delta \rho}{\partial t^2} + \bar{\rho} \nabla \cdot \left( \frac{\partial \vec{v}}{\partial t} \right) = 0$$

$$\nabla \cdot \vec{v} = -\frac{\nabla^2 \delta \rho}{\rho} - \frac{1}{\rho} \nabla^2 \delta \rho$$



Relax Condition of String Positioning

PR  $\rightarrow (X_{ab} - X_{ab} + X_{ab} - X_{ab}) = 0$

No signaling

$$\frac{\partial^2 \delta}{\partial t^2} - c_s^2 \nabla^2 \delta = 4\pi G \bar{\rho} \delta$$

Wave equation

gravity



Relax Condition of String Positioning

$$P_{ab} = \frac{1}{2\pi\alpha'} (\dot{X}_a \dot{X}_b + X_a \ddot{X}_b - \dot{X}_b \ddot{X}_a)$$

$$\frac{\partial^2 \delta}{\partial t^2} - c_s^2 \nabla^2 \delta = 4\pi G \bar{\rho} \delta$$

Wave equation

gravity



$$\frac{\partial^2 \delta}{\partial t^2} - c_s^2 \nabla^2 \delta = 4\pi G \bar{\rho} \delta$$

Wave equation

gravity

$$\delta(t, \vec{x}) = \delta_0 e^{i\omega t - i\vec{k} \cdot \vec{x}}$$

$$\omega^2 = c_s^2 k^2$$



$$\frac{\partial^2 \delta}{\partial t^2} - c_s^2 \nabla^2 \delta = 4\pi G \bar{\rho} \delta$$

Wave equation

gravity

$$\delta(t, \vec{x}) = \delta_0 e^{i\omega t - i\vec{k} \cdot \vec{x}}$$

$$\omega^2 = c_s^2 k^2$$



$$\frac{\partial^2 \delta}{\partial t^2} - c_s^2 \nabla^2 \delta = 4\pi G \bar{\rho} \delta$$

Wave equation

gravity

$$\delta(t, \vec{x}) = \delta_0 e^{i\omega t - i\vec{k} \cdot \vec{x}}$$

$$\omega^2 = c_s^2 k^2$$

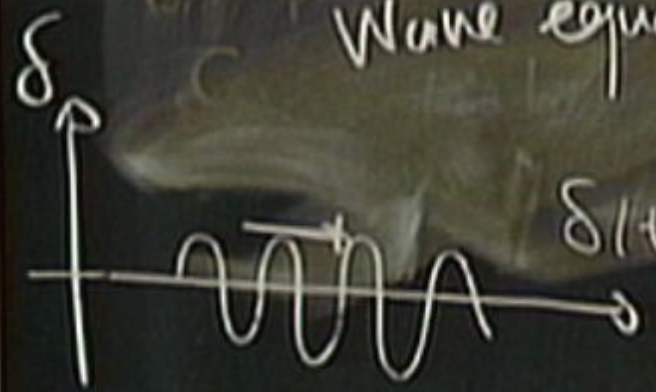
Sound waves  
in the fluid



$$\frac{\partial^2 \delta}{\partial t^2} - c_s^2 \nabla^2 \delta = 4\pi G \bar{\rho} \delta$$

Wave equation

gravity



$$\delta(t, \vec{x}) = \delta_0 e^{i(\omega t - \vec{k} \cdot \vec{x})}$$

$i(\omega t - \vec{k} \cdot \vec{x})$

$$\omega^2 = c_s^2 k^2$$

Sound waves  
in the fluid



$$\omega^2 = c_s^2 k^2 - 4\pi G \bar{\rho}$$



$$\omega^2 = c_s^2 k^2 - 4\pi G \bar{\rho}$$

Very short wavelengths



$$\omega^2 = c_s^2 k^2 = 4\pi G \bar{\rho}$$

Very short wavelengths

$$\lambda = \frac{2\pi}{|k|}$$

$$\omega \approx c_s |k|$$



$$\omega^2 = c_s^2 k^2 - 4\pi G \bar{\rho}$$

Very short wavelengths

$$\lambda = \frac{2\pi}{|k|}$$

$$(\omega \approx c_s |k|)$$

Pressure force of fluid much stronger gravitly  
at short distances



$$\omega^2 = c_s^2 k^2 - 4\pi G \bar{\rho}$$

Very short wavelengths

$$\lambda = \frac{2\pi}{|k|}$$

$$(\omega \approx c_s k)$$

Pressure force of fluid much stronger gravly at short distances

Jeein's  
Wavelength

$$\lambda_J = \frac{2\pi c_s}{\sqrt{4\pi G \bar{\rho}}}$$

$$\omega = 0$$



$$W = \pm i \sqrt{4\pi q^2 - c_s^2 k^2}$$

$$\lambda \geq \lambda_J$$

CAUTION  
Do not touch the surface  
Do not touch the surface  
Do not touch the surface



$$\omega^2 = c_s^2 k^2 \rightarrow 4\pi G \bar{\rho}$$

Very short wavelengths

$$\lambda = \frac{2\pi}{|k|}$$

$$\omega \approx c_s |k|$$

Pressure force of fluid

much stronger gravly  
at short distances

Jean's  
Wavelength

$$\lambda_J = \frac{2\pi c_s}{\sqrt{4\pi G \bar{\rho}}}$$



$$\omega = \pm c \sqrt{4\pi\epsilon_0 \rho^2 - c_s^2 k^2}$$

$$\lambda \geq \lambda_J$$

$$e^{i\omega t - i\vec{k} \cdot \vec{x}}$$

$$e^{i\omega t - i\vec{k} \cdot \vec{x}}$$

$$= e^{\pm \left( \sqrt{4\pi\epsilon_0 \rho^2 - c_s^2 k^2} \right) t - i\vec{k} \cdot \vec{x}}$$



$$\omega = \pm i \sqrt{4\pi G \rho - c_s^2 k^2}$$

$$\lambda \geq \lambda_J$$

$$e^{+i\omega t - i\vec{k} \cdot \vec{x}}$$

$$\delta = \sum_{\vec{k}} e^{\pm \left( \sqrt{4\pi G \rho - c_s^2 k^2} \right) t - i\vec{k} \cdot \vec{x}}$$

Jean's instability



Condition of Strong Positivity

$$\frac{\partial^2 \mathcal{S}}{\partial t^2} + c_s^2 \nabla^2 \mathcal{S} - 4\pi G \bar{\rho} \mathcal{S} = 0$$



Condition of Strong Resonance

$$\frac{\partial^2 \delta}{\partial t^2} + c_s^2 \nabla^2 \delta - 4\pi G \bar{\rho} \delta = 0$$

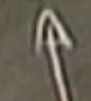
$$+ 2H \frac{\partial \delta}{\partial t}$$



friction / damping

Hubble damping

$$\ddot{x} + \gamma \dot{x} + \omega^2 x$$



friction / damping



Equation of String Perturbation

$$\frac{\partial^2 \delta}{\partial t^2} + c_s^2 \nabla^2 \delta - 4\pi G \bar{\rho} \delta = 0$$

$$+ 2H \frac{\partial \delta}{\partial t}$$

friction / damping  
Hubble damping

$$\ddot{x} + \gamma \dot{x} + \omega^2 x$$

friction / damping



slightly  
Construct and determine function

Variable  
at long wavelength

$k \rightarrow$  small

$$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} = -4\pi G \bar{\rho} \delta$$

beam  
wave



slightly positive  
constant amplitude function

Variable  
at long wavelength.

$k$  small

$$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} = -4\pi G \bar{\rho} \delta$$



slightly positive  
constant conductance factor

Unstable  
at long wavelength.

$k \rightarrow$  small

$$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} = -4\pi G \bar{\rho} \delta$$

Friedman  
equation



slightly positive  
constant and decreasing function

Unstable  
at long wavelength.

$k \rightarrow$  small

$$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} = -4\pi G \bar{\rho} \delta$$

Friedman  
equation

$$H^2 = \frac{8\pi G \bar{\rho}}{3}$$

$$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} + \frac{3}{2} H^2 \delta = 0$$



Unstable  
at long wavelength,

$k \rightarrow$   
small

$$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} = -4\pi G \bar{\rho} \delta$$

Friedman  
equation

$$H^2 = \frac{8\pi G \bar{\rho}}{3}$$

$$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} + \frac{3}{2} H^2 \delta = 0$$

$$\frac{p}{\rho} = w$$

$$a \sim t^{\frac{2}{3(1+w)}}$$

$$H = \frac{1}{a} \frac{da}{dt} = \frac{2}{3(1+w)t}$$

Matter  $w=0$   $a \sim t^{2/3}$



Friedmann  
equation

$$H^2 = \frac{8\pi G \rho}{3}$$

$$\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} + \frac{3}{2}H^2\delta = 0$$

$$\frac{\rho}{\rho} = w$$

$$a \sim t^{\frac{2}{3(1+w)}}$$

$$H = \frac{1}{a} \frac{da}{dt} = \frac{2}{3(1+w)t}$$

Matter  $w=0$   $a \sim t^{2/3}$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{4}{3(1+w)t} \frac{\partial \delta}{\partial t} + \frac{2}{3} \frac{1}{(1+w)^2 t^2} \delta = 0$$



equation

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2}{3(1+w)t} \frac{\partial \delta}{\partial t} + \frac{2}{3(1+w)^2 t^2} \delta = 0$$

$\frac{p}{\rho} = w$       $a \sim t^{\frac{2}{3(1+w)}}$       $H = \frac{1}{a} \frac{da}{dt} = \frac{2}{3(1+w)t}$

Matter  $w=0$   $a \sim t^{2/3}$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2}{3(1+w)t} \frac{\partial \delta}{\partial t} + \frac{2}{3(1+w)^2 t^2} \delta = 0$$

$$\delta \sim A t + B t^2$$



Friedman  
equation

$$H^2 = \frac{8\pi G}{3} \rho$$

$$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} + \frac{3}{2} H^2 \delta = 0$$

$$\frac{p}{\rho} = w$$

$$a \sim t^{\frac{2}{3(1+w)}}$$

$$H = \frac{1}{a} \frac{da}{dt} = \frac{2}{3(1+w)t}$$

Matter  $w=0$   $a \sim t^{2/3}$

$w=0$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{4}{3(1+w)t} \frac{\partial \delta}{\partial t} + \frac{2}{3} \frac{1}{(1+w)^2 t^2} \delta = 0$$
$$\delta \sim A t^{2/3} + B t^{-1}$$



$\omega = 0$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{4}{3(1+\omega t)} \frac{\partial \delta}{\partial t} + \frac{2}{3} \frac{1}{(1+\omega)^2 t^2} \delta = 0$$
$$\delta \approx A t^{2/3} + B t^{-1}$$



Matter  $w=0$  a  $\dot{a}^2$

$a \dot{a}^2 = 3(1+w)a^2 \dot{a}^2$

$w=0$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2}{3(1+w)a} \frac{\partial \delta}{\partial t} + \frac{2}{3} \frac{1}{(1+w)^2 a^2} \delta = 0$$

$$\delta \sim A t^{2/3} + B t^{-1}$$

growing mode                      decaying mode



Matter  $w=0$  a  $t^{2/3}$

add  $3(1+w)t$

$w=0$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2}{3(1+w)t} \frac{\partial \delta}{\partial t} + \frac{2}{3} \frac{1}{(1+w)^2 t^2} \delta = 0$$

$\delta \sim A t^{2/3}$   
growing mode

~~$\delta \sim B t^{-1}$~~   
~~decaying mode~~



$$\underline{W=0}$$

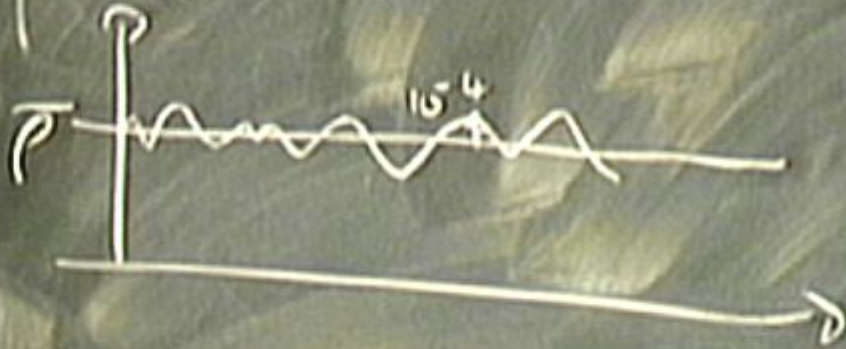
$$\frac{\partial^2 \delta}{\partial t^2} + \frac{4}{3(1+wt)} \frac{\partial \delta}{\partial t} + \frac{2}{3} \frac{1}{(1+wt)^2 t^2} \delta = 0$$

$\delta \sim A t^{2/3}$  +  ~~$B t^{-1}$~~

growing mode      decaying mode

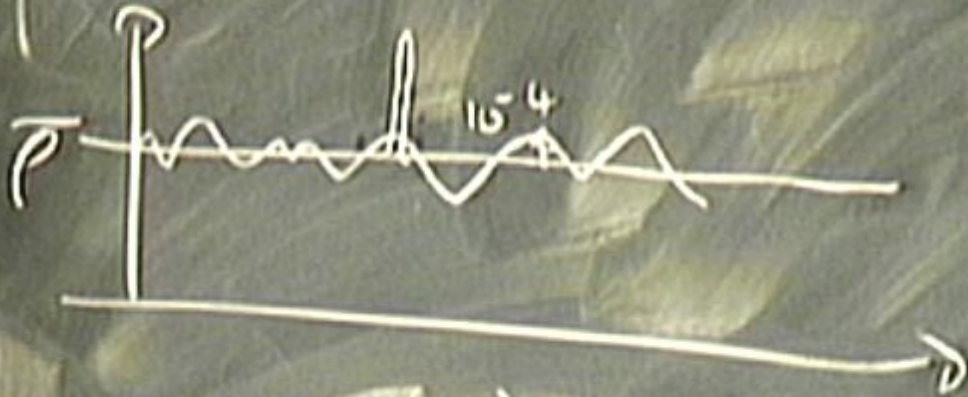


$$\delta \sim 10^{-4}$$





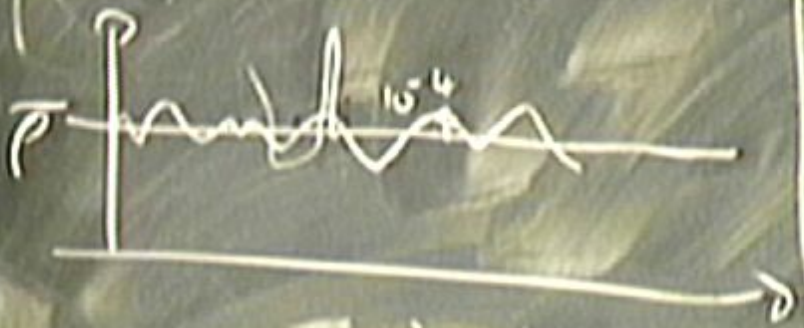
$$S \sim 10^{-4}$$





# Cosmic Microwave Background

$$\delta \sim 10^{-4}$$





$$S \sim 10^{-4}$$

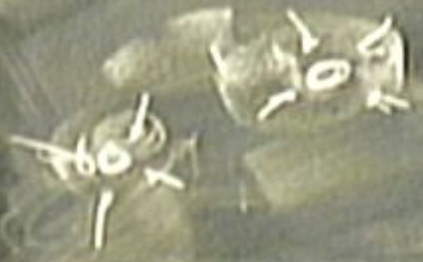


# Cosmic Microwave Background

$$t \sim 300000 \text{ years}$$



$$\sigma \sim 10^{-4}$$



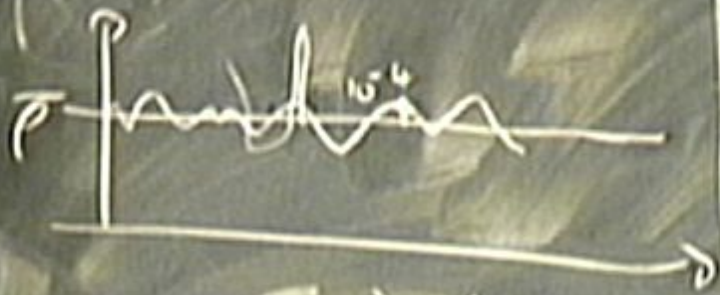
# Cosmic Microwave Background

$$t \sim 300000 \text{ years}$$





$$\rho \sim 10^{-4}$$



# Cosmic Microwave Background

$$t \sim 300000 \text{ years}$$

halo  
visible matter



before galaxies formed



$$\delta \sim 10^{-4}$$



# Cosmic Microwave Background

$$t \sim 300000 \text{ years}$$

hole  
visible matter



prefer galaxies  
found

$$T = 2.73 \text{ K}$$



Relation between  $\theta$  and  $\phi$

$$T(\theta, \phi)$$





Condition of Strong Positivity

$$T_{CW_3}(\theta, \phi)$$





Relax Condition of Strong Positivity

$$T_{CW_3}(\theta, \phi)$$

$$= T + \delta T(\theta, \phi)$$

$$\delta T(\theta, \phi) \sim 10^{-4}$$

$T$





Radiation

coupled matter

$\rho$

$S_{\text{in}}$

$S_{\text{out}}$

$P_M$

growing



Radiation

coupled matter

$$\frac{\delta T}{T}$$

$\rho$

$S =$

$$\frac{S_p}{P_{13}}$$

growing



Radiation

coupled matter

$$\frac{\delta T}{T}$$

$\rho$

$\delta_{\rho}$

$$\frac{\delta \rho}{\rho}$$

$$\rho_M \sim \frac{1}{a^3}$$

growing



Radiation

coupled matter

$$\frac{\delta T}{T}$$

$\rho$

$\delta_{\rho}$

$$\frac{\delta p_M}{p_M}$$

$$\rho_M \frac{1}{a^3}$$

$\rho$

$$T_A \rho$$

growing mode



Radiation

coupled matter

$$\frac{\delta T}{T}$$

$\rho$

$\delta \rho$

$$\frac{\delta p}{p}$$

$$\rho_M \sim$$

$$\frac{1}{a^3}$$

$\sim$

$$T^3$$

growing



Radiation coupled matter

$$\frac{\delta T}{T}$$

↑

$$\delta = \frac{\delta p_M}{p_M}$$

$$T \sim \frac{1}{a}$$

$$p_M \sim \frac{1}{a^3}$$

$$T^3$$

$$p_M + \delta p_M$$

$$(T + \delta T)^3$$

growing

$$\frac{\delta p_M}{p_M} = 3 \frac{\delta T}{T}$$

$$\sim T^3 + 3 \delta T T^2$$



Correlation  
function

couple

$$\underline{S(\vec{x})}$$



Correlation  
function

example

$$\underline{\delta(\vec{x})}$$

$$\bar{\delta} = \int \delta d^3x$$

$$= 0$$



Correlation  
function

$$\underline{\delta(\vec{x})}$$

$$\bar{\delta} = \int \delta d^3x$$

Variance  
distribution

$$\langle x^2 \rangle$$

$$\langle \delta(\vec{x}) \delta(\vec{x}') \rangle$$

$$= 0$$



Correlation  
function

$$\underline{\delta(\vec{x})}$$

$$\bar{\delta} = \int \delta d^3x$$

Variance  
distribution

$$\langle \delta(\vec{x}) \delta(\vec{x}) \rangle$$

$$\langle \delta(\vec{x}) \delta(\vec{x} + \vec{r}) \rangle$$

$$= 0$$



Correlation  
function

example!

$$\underline{\delta(\vec{x})}$$

$$\bar{\delta} = \int \delta \, d^3x$$

Variance  
distribution

$$\langle \delta(\vec{x}) \delta(\vec{x}) \rangle = 0$$

$$g(\vec{r}) = \langle \delta(\vec{x}) \delta(\vec{x} + \vec{r}) \rangle$$



Correlation  
function

$$\underline{\delta(\vec{x})}$$

$$\bar{\delta} = \int \delta d^3x$$

Variance  
distribution  
 $\langle x^2 \rangle$

$$\langle \delta(\vec{x}) \delta(\vec{x}') \rangle = 0$$

$$G(\vec{r}) = \langle \delta(\vec{x}) \delta(\vec{x} + \vec{r}) \rangle$$



$$\epsilon(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \epsilon_k$$



$$\psi \quad \rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \rho_{\vec{k}}$$



$$\rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \rho_{\vec{k}}$$

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \delta_{\vec{k}}$$



$$\rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \xi_{\vec{k}}$$

$$\delta(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \delta_{\vec{k}}$$

$$\xi_{\vec{k}} = \langle \delta_{\vec{k}}^2 \rangle$$



$$\rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \rho_{\vec{k}}$$

$$\delta(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \delta_{\vec{k}}$$

$$\rho_{\vec{k}} = \langle \delta_{\vec{k}}^2 \rangle$$

Power spectrum of density fluctuation.





$$M = \rho \times \frac{4}{3} \pi R^3$$





$$M = \rho \times \frac{4}{3} \pi R^3$$

$$\left( \frac{\delta M}{\bar{M}} \right)_h$$

$$\delta M = M - \bar{M}$$



Condition of Strong Positivity



$$M = \rho \times \frac{4\pi R^3}{3}$$

$$\sigma(R) = \left( \frac{\delta M}{\bar{M}} \right) h$$

$$\delta M = M - \bar{M}$$



Chapter 10: Error Propagation



$$M = \rho \times \frac{4}{3} \pi R^3$$

$$\frac{\delta M}{M} = \frac{\delta \rho}{\rho}$$

$$\sigma(R) = \sqrt{\left( \left( \frac{\delta M}{M} \right)^2 \right) \times R^2}$$

$$\delta M = M \cdot \frac{\delta M}{M}$$

$\sigma(R)^2$   
variance



of P. Slony. Probability



$$M = \rho \times \frac{4\pi R^3}{3}$$

$$\frac{\delta M}{M} = \frac{\delta \rho}{\rho}$$

$$\sigma(R) = \sqrt{\left(\left(\frac{\delta M}{M}\right)^2\right)_R}$$

$$\delta M = M - \bar{M}$$

$$\sigma(R) = \sqrt{\left(\frac{\delta \rho}{\rho}\right)^2_R}$$

$\sigma(R)^2$   
variance





$$M = \rho \times \frac{4\pi R^3}{3}$$

$$\frac{\delta M}{M} = \frac{\delta \rho}{\rho}$$

$$\sigma(R) = \sqrt{\left( \left( \frac{\delta M}{M} \right)^2 \right)}$$

$$\delta M = M - \bar{M}$$

$$\sigma(R)^2 = \left\langle \left( \frac{\delta \rho}{\rho} \right)^2 \right\rangle_R$$

$\sigma(R)^2$   
variance



$$\rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \xi_{\vec{k}}$$

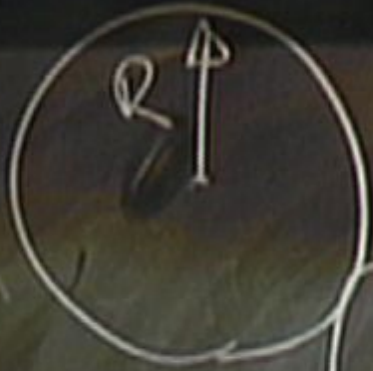
$$\delta(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \delta_{\vec{k}}$$

$$\xi_{\vec{k}} = \langle \delta_{\vec{k}}^2 \rangle$$

$$\sigma(R)^2 = \int_{k=1/R} \dots$$

Power spectrum of density fluctuation.





$$M = \rho \times \frac{4\pi R^3}{3}$$

$$\frac{\delta M}{M} = \frac{\delta \rho}{\rho}$$

$$\sigma(R) = \sqrt{\left(\left(\frac{\delta M}{M}\right) \cdot R\right)^2}$$

$$\delta M = M - \bar{M}$$

$$\sigma(R) = \left\langle \left(\frac{\delta \rho}{\rho}\right)^2 \right\rangle R$$

$\sigma(R)^2$   
variance

$$\sigma(R) \sim \frac{1}{R^2}$$



$$\rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \xi_{\vec{k}}$$

$$\delta(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \delta_{\vec{k}}$$

$$\xi_{\vec{k}} = \langle \delta_{\vec{k}}^2 \rangle$$

$$\sigma(R)^2 = \int_{k=\frac{1}{R}}^{\infty} k^3 dk = \frac{1}{2} \times \frac{1}{R^2}$$

Power spectrum of density fluctuations.





$$M = \rho \times \frac{4\pi R^3}{3}$$

$$\frac{\delta M}{M} = \frac{\delta \rho}{\rho}$$

$$\sigma(R) = \sqrt{\left(\frac{\delta M}{M}\right)^2}$$

$$\delta M = M - \bar{M}$$

$\sigma(R)^2$   
variance

$$\sigma(R) = \left\langle \frac{\delta \rho}{\rho} \right\rangle^2$$

$$\sigma^2(R) \sim \frac{1}{R^4} = e_k \cdot \frac{1}{R} \times \frac{1}{R^3}$$

$$\sigma(R) \sim \frac{1}{R^2}$$

$$e_k = \frac{1}{R} = k$$



$$1) \rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \rho_{\vec{k}}$$

$$\rho_{\vec{k}} = \rho_{-\vec{k}}$$

$$\delta(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \delta_{\vec{k}}$$

$$\rho_{\vec{k}} = \langle \delta_{\vec{k}}^2 \rangle$$

$$\sigma(R)^2 = \int_{k=1/R}^{\infty} k^3 \rho_{\vec{k}} = \int_{k=1/R}^{\infty} \frac{1}{R^3} \rho_{\vec{k}}$$

Power spectrum of density fluctuation.



limitable  
at long wavelength

$$\vec{k} \xrightarrow{\text{small}} \langle \frac{\delta T(\vec{x})}{T} \frac{\delta T(\vec{x} + \vec{r})}{T} \rangle$$

Freedom  
equation

$$\frac{p}{\rho} = w$$

Matter  $w_s$



limitable  
at long wavelength

$\vec{k}$  small

$$e_{T(\vec{r})} = \left\langle \frac{\delta T(\vec{r})}{F} \middle| \frac{\delta T(\vec{r} + \vec{r}')}{F} \right\rangle$$

Freedom  
equation

$$= \int d^3k e^{i\vec{k} \cdot \vec{r}} e_{T(\vec{r})}$$

$$\frac{p}{\beta} = W$$

Matter  $w_s$

$$\left( \frac{w_s}{k} \sim k \right)$$





limitable  
at long wavelength

$\vec{k}$  small

$$\rho_{T(\vec{r})} = \left\langle \frac{\delta T(\omega)}{F} \middle| \frac{\delta T(x+\vec{r})}{F} \right\rangle$$

Freedom  
equation

$$= \int d^3k e^{i\vec{k}\cdot\vec{r}} \rho_{\vec{k}}^T$$

$$\frac{p}{\rho} = w$$

Matter  $w_0$

$$\left( \frac{\rho}{\rho_0} \sim k \right)$$



Unstable  
at long wavelength

$\vec{k}$

small

$$e_{T(\vec{r})} = \left\langle \frac{\delta T(\vec{x})}{T} \frac{\delta T(\vec{x} + \vec{r})}{T} \right\rangle$$

Fredom  
equation

$$= \int d^3k e^{i\vec{k} \cdot \vec{r}} \frac{\partial^2}{\partial t^2}$$

$$\frac{p}{\rho} = w$$

Matter  $w_s$

$$\left( \frac{\delta T}{T} \sim k \right)$$

$$\frac{\delta T}{T} \sim 10^{-4}$$

$$t = 300,000$$

$$\frac{p}{\rho} \sim 10^{-4}$$



Unstable  
at long wavelength

$\vec{k}$  small

$$e_{T(\vec{r})} = \left\langle \frac{\delta T(\vec{x}, t)}{\bar{T}} \frac{\delta T(\vec{x} + \vec{r}, t)}{\bar{T}} \right\rangle$$

Friedman  
equation

$$\frac{\dot{\rho}}{\rho} = -w$$

Matter  $w = 0$

$$= \int d^3k e^{i\vec{k} \cdot \vec{r}} \frac{\delta T}{\bar{T}}$$

$$\frac{\delta T}{\bar{T}} \sim \frac{1}{k}$$

Inflation

$$\frac{\delta T}{\bar{T}} \sim k$$

$$\frac{\delta T}{\bar{T}} \sim 10^{-4}$$

$$t = 300,000$$

$$\frac{\delta T}{\bar{T}} \sim 10^{-4}$$



$$\psi(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \tilde{\psi}(\vec{k})$$

$$\delta(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}}$$

$$\tilde{\psi}(\vec{k}) = \langle \psi | \vec{k} \rangle$$

Power spectrum

$$\tilde{\psi}(\vec{k}) \propto \frac{1}{k^3}$$

$$\tilde{\psi}(k) \propto \frac{1}{k^2}$$

