Title: Cosmology #3

Date: Apr 01, 2008 06:30 PM

URL: http://pirsa.org/08040007

Abstract: A brief history of our cosmic beginnings, Cosmic Microwave Background. How galaxies form and the existence of dark matter.

Recap

Current composition of universe $\quad w = \frac{p}{\rho}$

~ 4% visible matter (stars + galaxies) $\quad w = 0$

~ 20% dark matter (?) $\quad w = 0$

negligible radiation (mostly cosmic microwave background) $\quad w = \frac{1}{3}$

~ 76% dark energy ?? $\quad w \approx -1$

Radiation domination	$t = 10^{11}$ s Matter domination	Dark Energy domination

→ time

Cosmic Microwave Background

— highly isotropic thermal (blackbody) radiation $T_{today} = 2.73K$

$$T(t) = \frac{T(t_0)\,a(t_0)}{a(t)}$$

Cosmic Microwave Background

— highly isotropic thermal (blackbody) radiation $T_{today} = 2.75K$

$$T(t) = \frac{T(t_0)\,a(t_0)}{a(t)}$$

Particle horizon (finite size of observable universe)

$$d_H = c\int_0^{t_0} \frac{a(t_0)}{a(t)}\,dt = \frac{3(1+w)ct_0}{(1+3w)}$$

Cosmic Microwave Background

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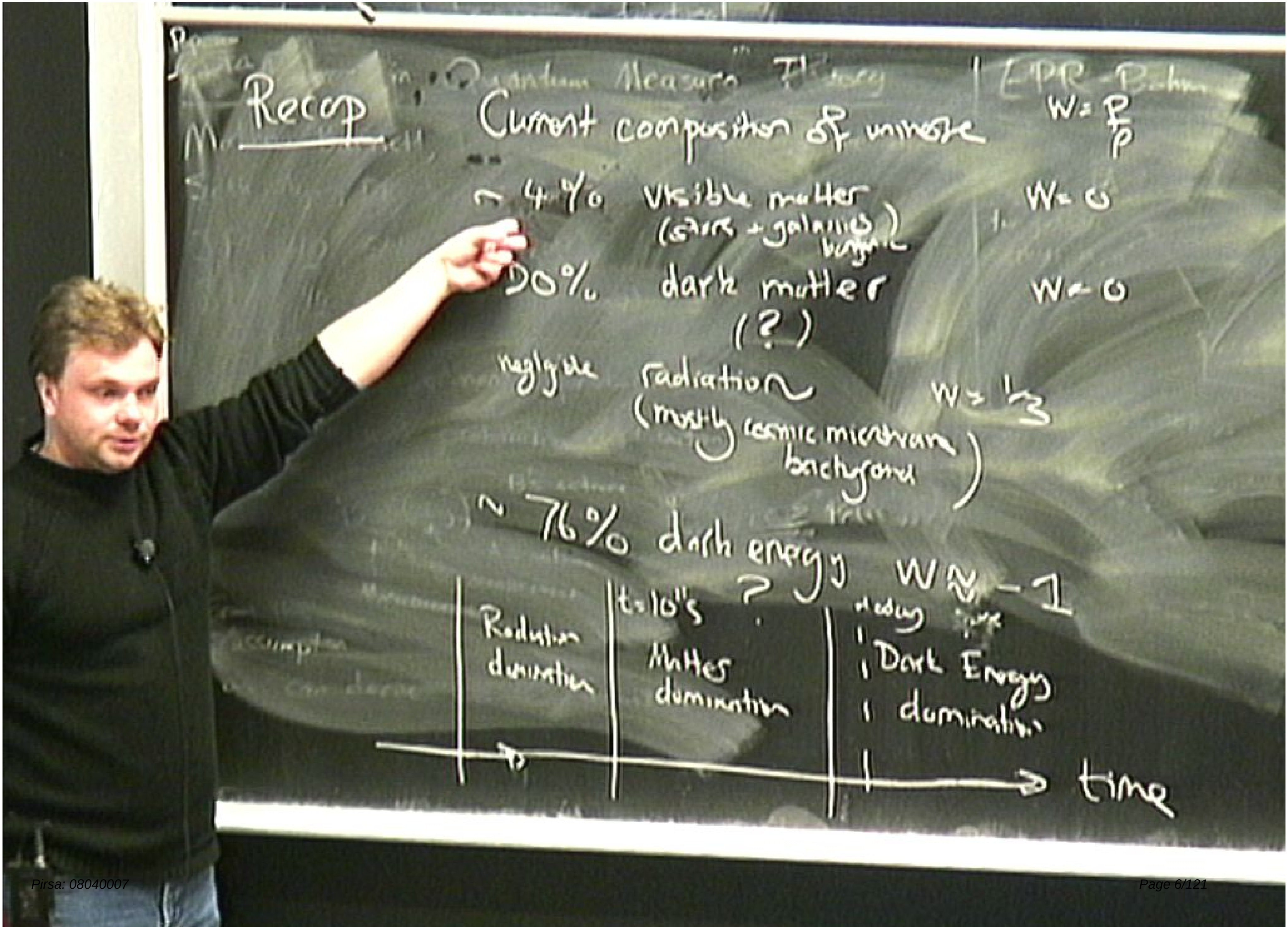
Decoupling Two "species" remain in equilibrium if the rate of interactions Γ satisfies $\Gamma \gg H$

if $\Gamma < H$ they decouple



Recap
Current composition of universe
w = p/ρ
~ 4% visible matter
(stars + galaxies)
baryonic
w = 0
20% dark matter
(?)
w ≈ 0
negligible radiation
(mostly cosmic microwave background)
w = 1/3
~ 76% dark energy
?
w ≈ -1
Radiation domination
Matter domination
Dark Energy domination
time

Recap

Current composition of universe $\quad w = \frac{p}{\rho}$

~ 4% visible matter (stars + galaxies) $\quad w = 0$

~ 20% dark matter (?) $\quad w = 0$

negligible radiation (mostly cosmic microwave background) $\quad w = \frac{1}{3}$

$$\rho = \frac{\rho_r^0}{a^4} + \frac{\rho_m^0}{a^3} + \rho_{de}$$

~ 76% dark energy ?? $\quad w \approx -1$

$\boxed{w = -1 \text{ cosmological constant}}$ $\quad \rho_{de}$ = constant

Radiation domination	t≈10's Matter domination	today Dark Energy domination

→ time


Cosmic Microwave Background
— highly isotropic thermal (blackbody) radiation
Recombination
T(t)
Particle horizon (finite size of observable universe)
$d_H = c\int_0^{t_0} \frac{a(t_0)}{a(t)}\,dt = \frac{3(1+w)ct_0}{(1+3w)}$
Two species remain in equilibrium if
satisfied Γ >> H
Γ < H they decouple

Cosmic Microwave Background

— highly isotropic thermal (blackbody) radiation $T_{today} = 2.73K$

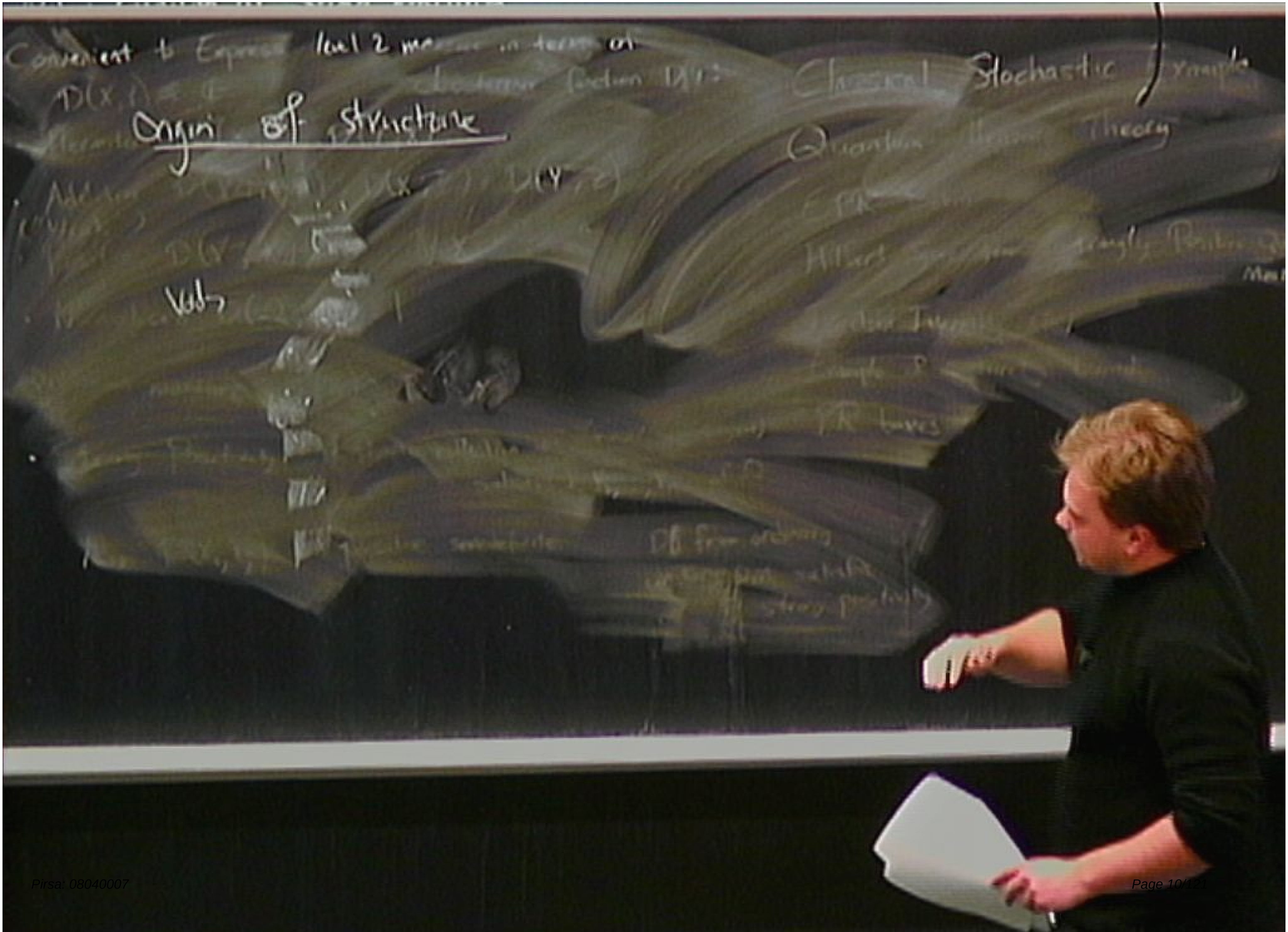
Recombination $T(t) = \frac{T(t_0)\,a(t_0)}{a(t)}$

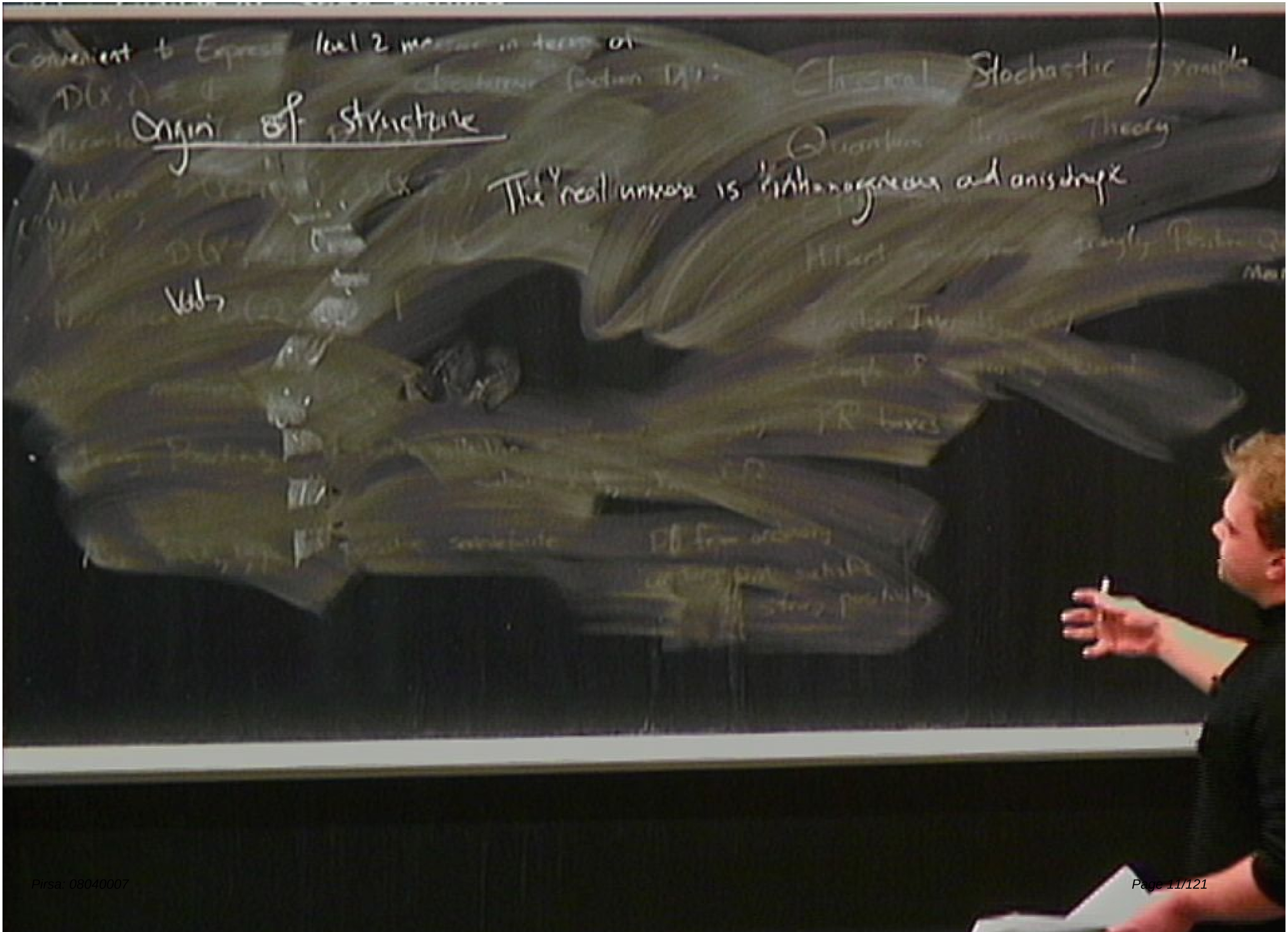
Particle horizon (finite size of observable universe)

$$d_H = c\int_0^{t_0} \frac{a(t_0)}{a(t)}\,dt = \frac{3(1+w)ct_0}{(1+3w)}$$

Decoupling Two 'species' remain in equilibrium if the rate of interactions Γ satisfies $\Gamma \gg H$

if $\Gamma < H$ they decouple


Origin of Structure
Classical Stochastic Example

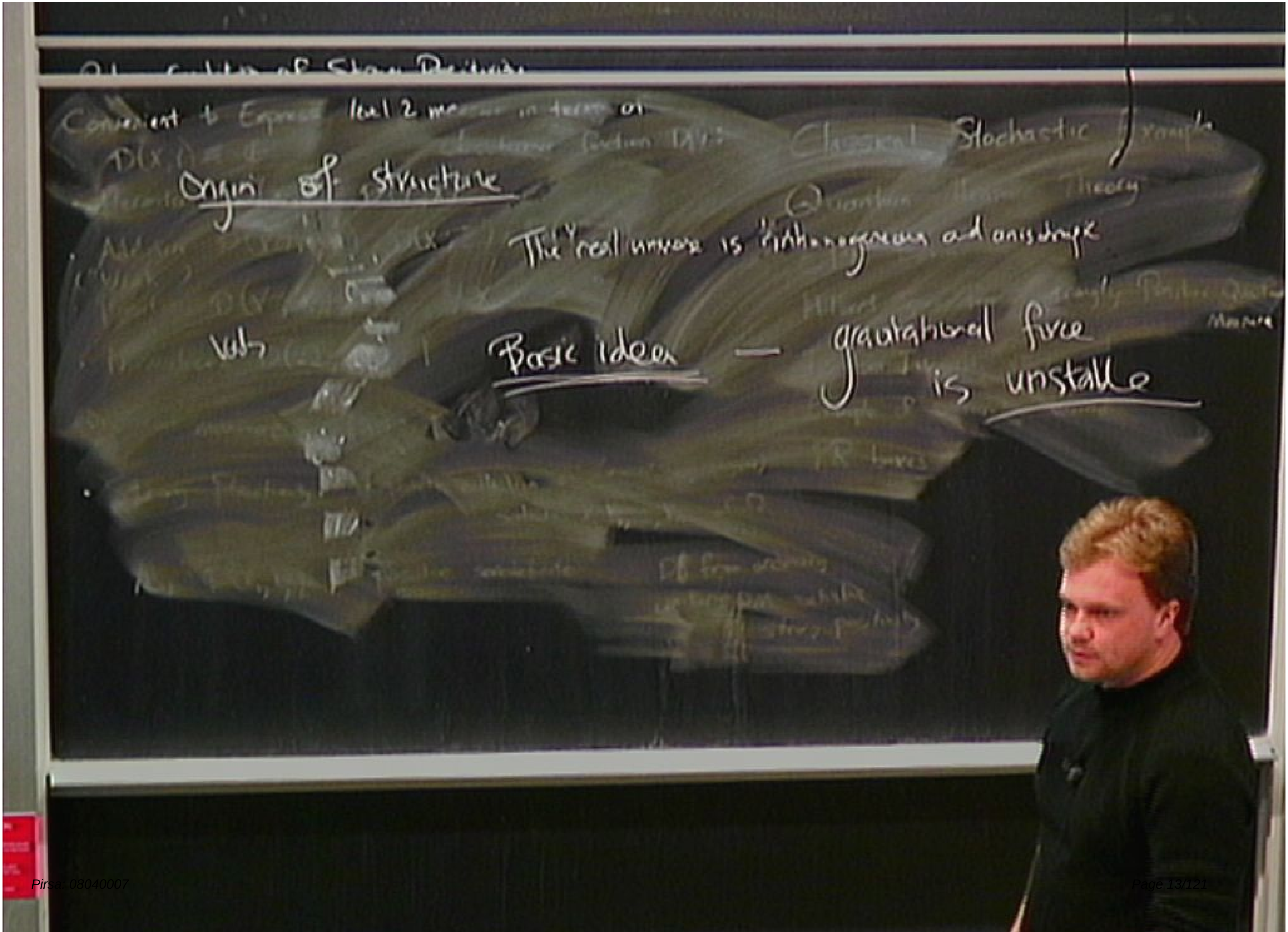

Origin of Structure
Classical Stochastic Example
The real universe is inhomogeneous and anisotropic

Classical Stochastic

Origin of structure

The real universe is inhomogeneous and anisotropic

Vals


Origin of Structure
The real universe is inhomogeneous and anisotropic
Basic idea — gravitational force is unstable
Classical Stochastic Example

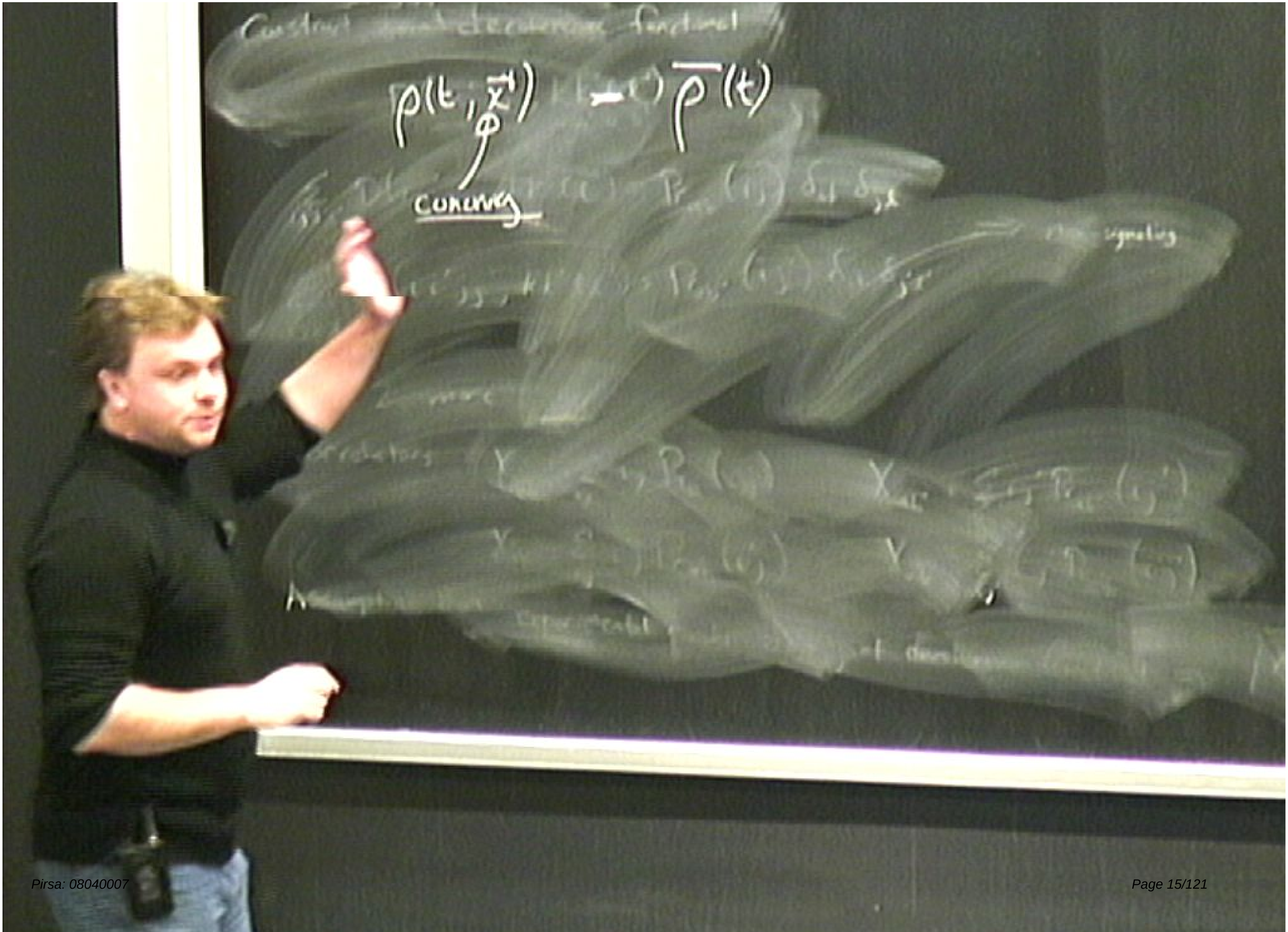
Origin of Structure

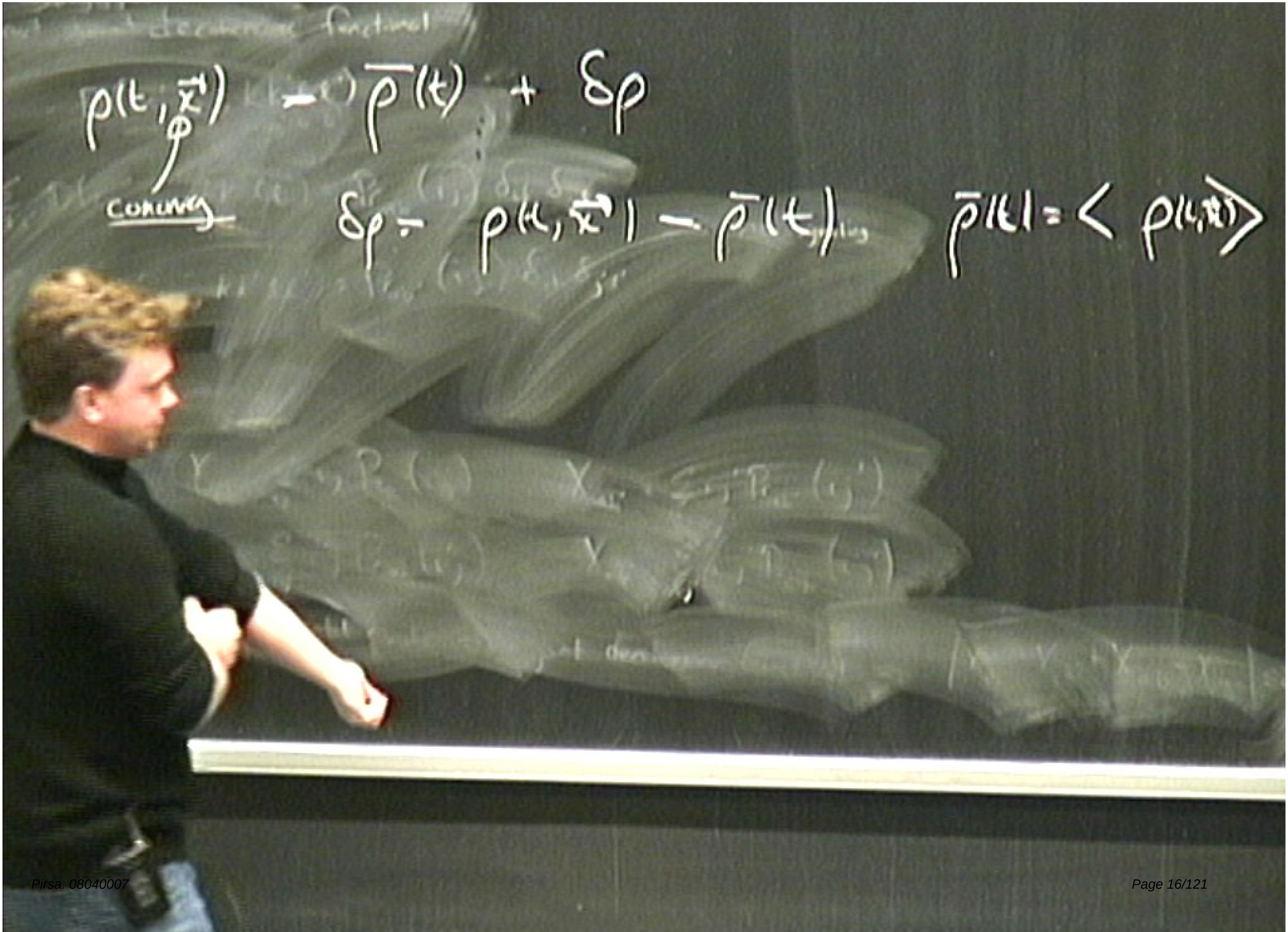
The real universe is inhomogeneous and anisotropic

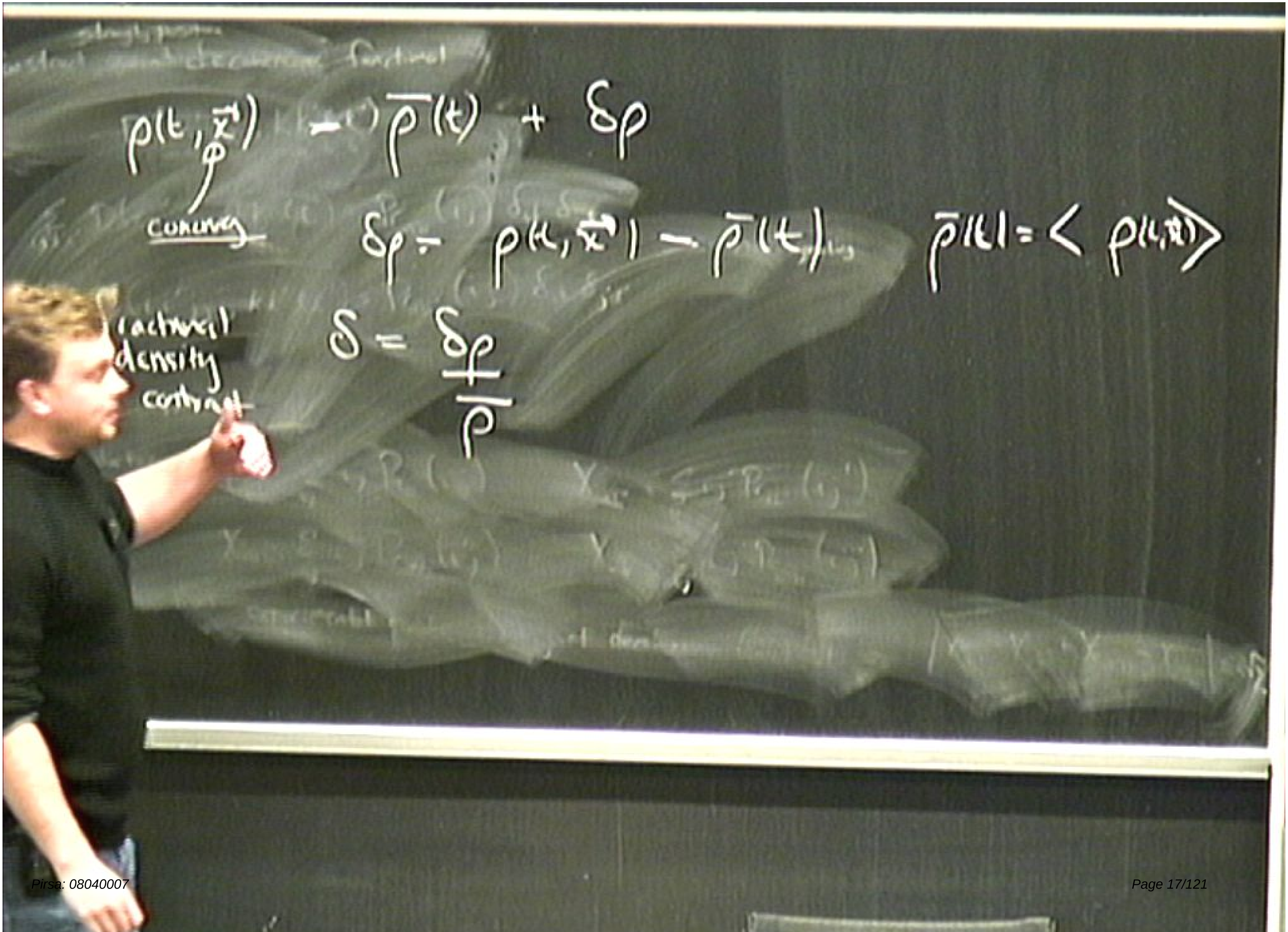
Basic idea — gravitational force is unstable

$$F_N = -\frac{m_1 m_2 G}{r^2}$$

$$\boxed{m > 0}$$


$\rho(t, \vec{x}) \to \bar{\rho}(t)$
coarsening


$\rho(t,\vec{x}) = \bar{\rho}(t) + \delta\rho$
$\delta\rho = \rho(t,\vec{x}) - \bar{\rho}(t)$
$\bar{\rho}(t) = \langle \rho(t,\vec{x}) \rangle$


ρ(t, x⃗) = ρ̄(t) + δρ
contrast
δρ = ρ(t, x⃗) − ρ̄(t)
ρ̄(t) = < ρ(t, x⃗) >
fractional density contrast
δ = δρ / ρ̄

$$\rho(t,\vec{x}) = \bar{\rho}(t) + \delta\rho$$

contrast

$$\delta\rho = \rho(t,\vec{x}) - \bar{\rho}(t) \qquad \bar{\rho}(t) = \langle \rho(t,\vec{x}) \rangle$$

fractional density contrast

$$\delta = \frac{\delta\rho}{\bar{\rho}} = \frac{\rho(t,\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\boxed{\delta = 0}$$

$$\rho(t,\vec{x}) = \bar{\rho}(t) + \delta\rho$$

comoving

$$\delta\rho = \rho(t,\vec{x}) - \bar{\rho}(t) \qquad \bar{\rho}(t) = \langle \rho(t,\vec{x}) \rangle$$

fractional density contrast

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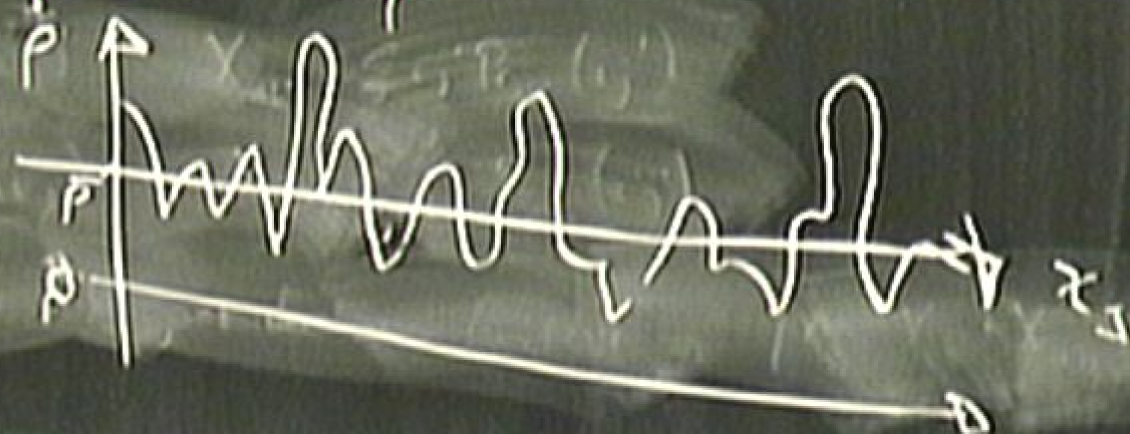


$$\rho(t,\vec{x}) = \bar{\rho}(t) + \delta\rho$$

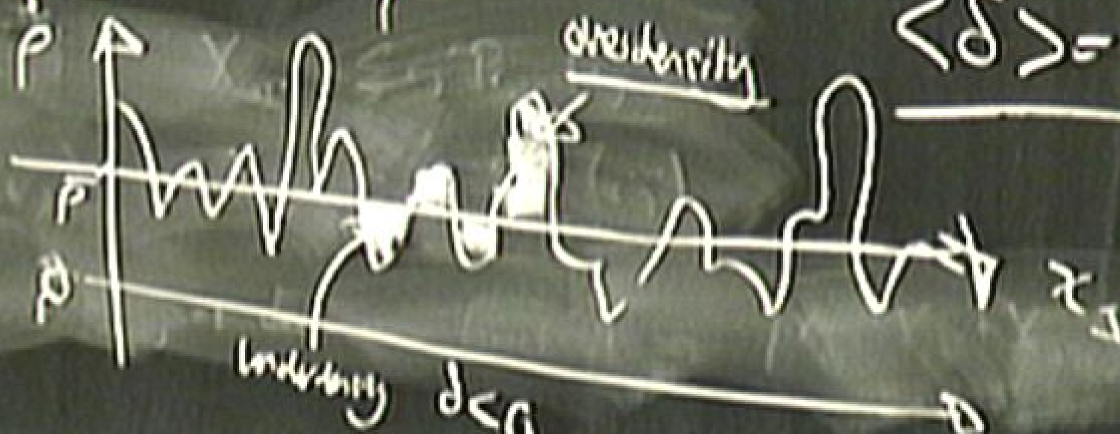
comoving

$$\delta\rho = \rho(t,\vec{x}) - \bar{\rho}(t) \qquad \bar{\rho}(t) = \langle \rho(t,\vec{x}) \rangle$$

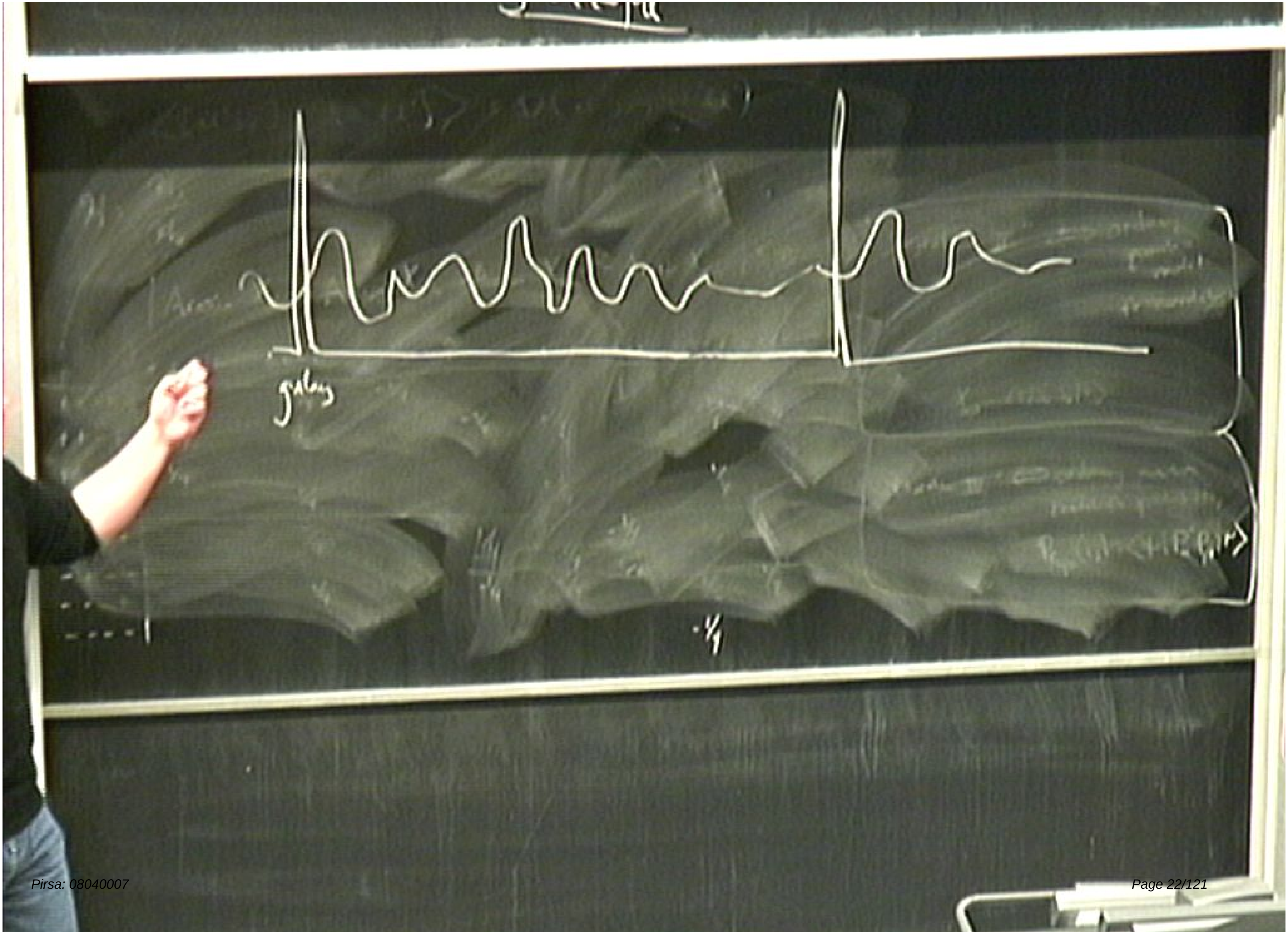
fractional density contrast

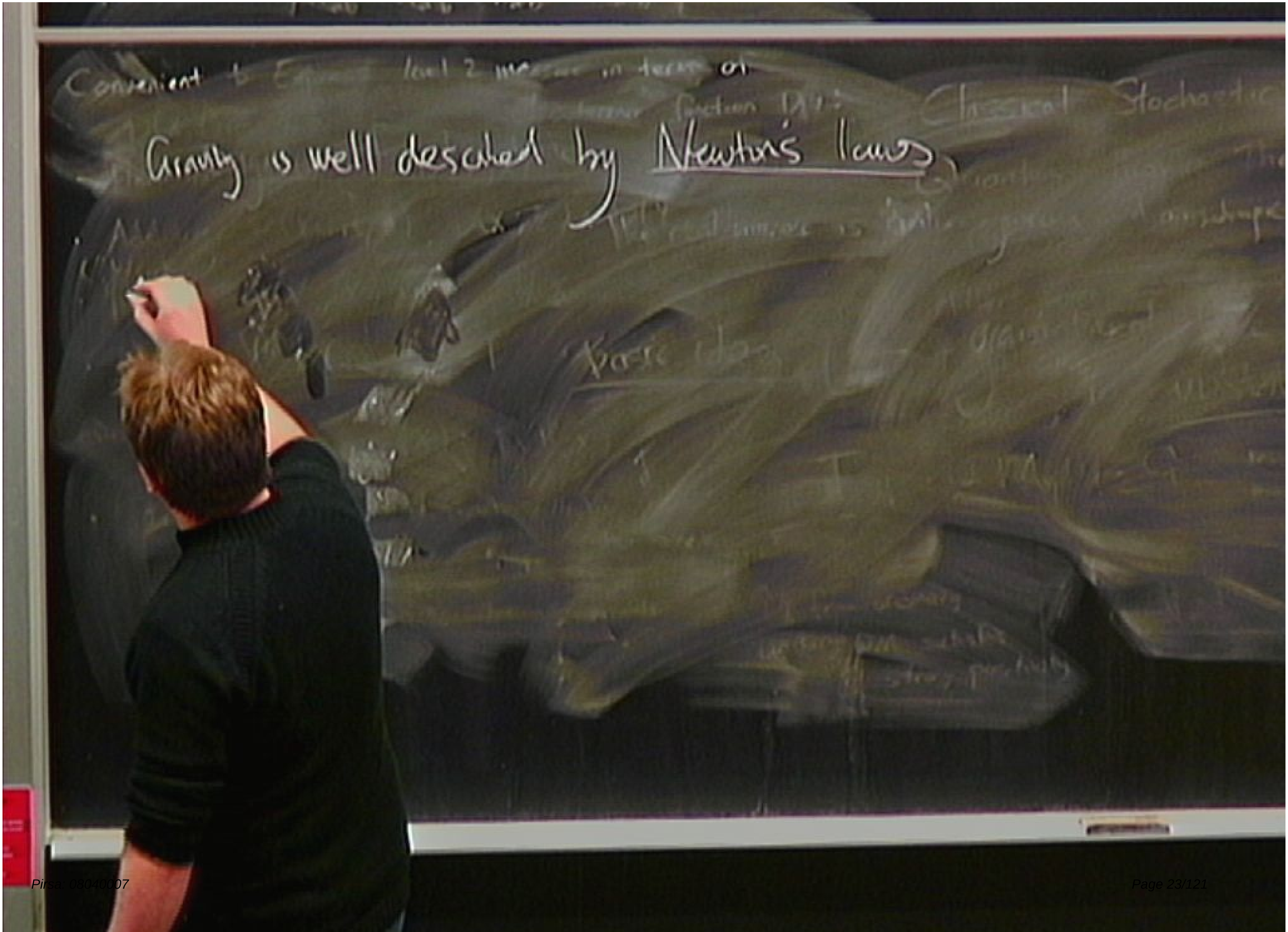
$$\delta = \frac{\delta\rho}{\bar{\rho}} = \frac{\rho(t,\vec{x}) - \bar{\rho}}{\bar{\rho}} \qquad \delta > 0$$

$$\boxed{\delta = 0} \qquad \langle \delta \rangle = 0$$




galaxy
-1/4




Gravity is well described by Newton's laws

Gravity is well described by Newton's laws

m •

$$\vec{F} = -m\vec{\nabla}\phi$$

mass density

$$\nabla^2\phi = 4\pi G\rho$$

Gravity is well described by Newton's laws

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Gravity is well described by Newton's laws

m •

$$\vec{F} = -m\vec{\nabla}\phi$$

$$\nabla^2\phi = 4\pi G\rho \quad \text{(mass density)}$$

$$\rho = M\delta^{(3)}(\vec{r})$$

$$\phi = \frac{MG}{r}$$

Gravity is well described by Newton's laws

$m \bullet \qquad \vec{F} = -m\vec{\nabla}\phi$

$$\nabla^2\phi = 4\pi G \rho \quad \text{(mass density)}$$

$$\rho = M\delta^{(3)}(\vec{r})$$

$$\phi = \frac{MG}{r}$$

Galaxies fluid ρ, p, $\vec{v}(t, \vec{x})$

Gravity is well described by Newton's laws

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Galaxies, fluid $\rho,\ P,\ \vec{v}(t,\vec{x})$

$$\frac{D\vec{v}}{Dt} = -\vec{\nabla}\phi - \frac{\vec{\nabla}P}{\rho}$$

gravitational force

Gravity is well described by Newton's laws

m • $\vec{F} = -m\vec{\nabla}\phi$ $\qquad \rho = M\delta^{(3)}(\vec{r})$

mass density

$\nabla^2\phi = 4\pi G\rho$ $\qquad \phi = \frac{MG}{r}$

Galaxies fluid ρ, P, $\vec{v}(t,\vec{x})$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v} = \frac{D\vec{v}}{Dt} = -\vec{\nabla}\phi - \frac{\vec{\nabla}P}{\rho}$$

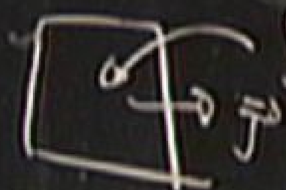
gravitational force

$\partial t \qquad \vec{\nabla}\phi \qquad \frac{\vec{\nabla}p}{\rho}$

gravitational force

Conservation law of mass

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$



$\int \rho \, dV$

$\vec{J} = \rho\vec{v}$

gravitational force

Conservation law of mass

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$$

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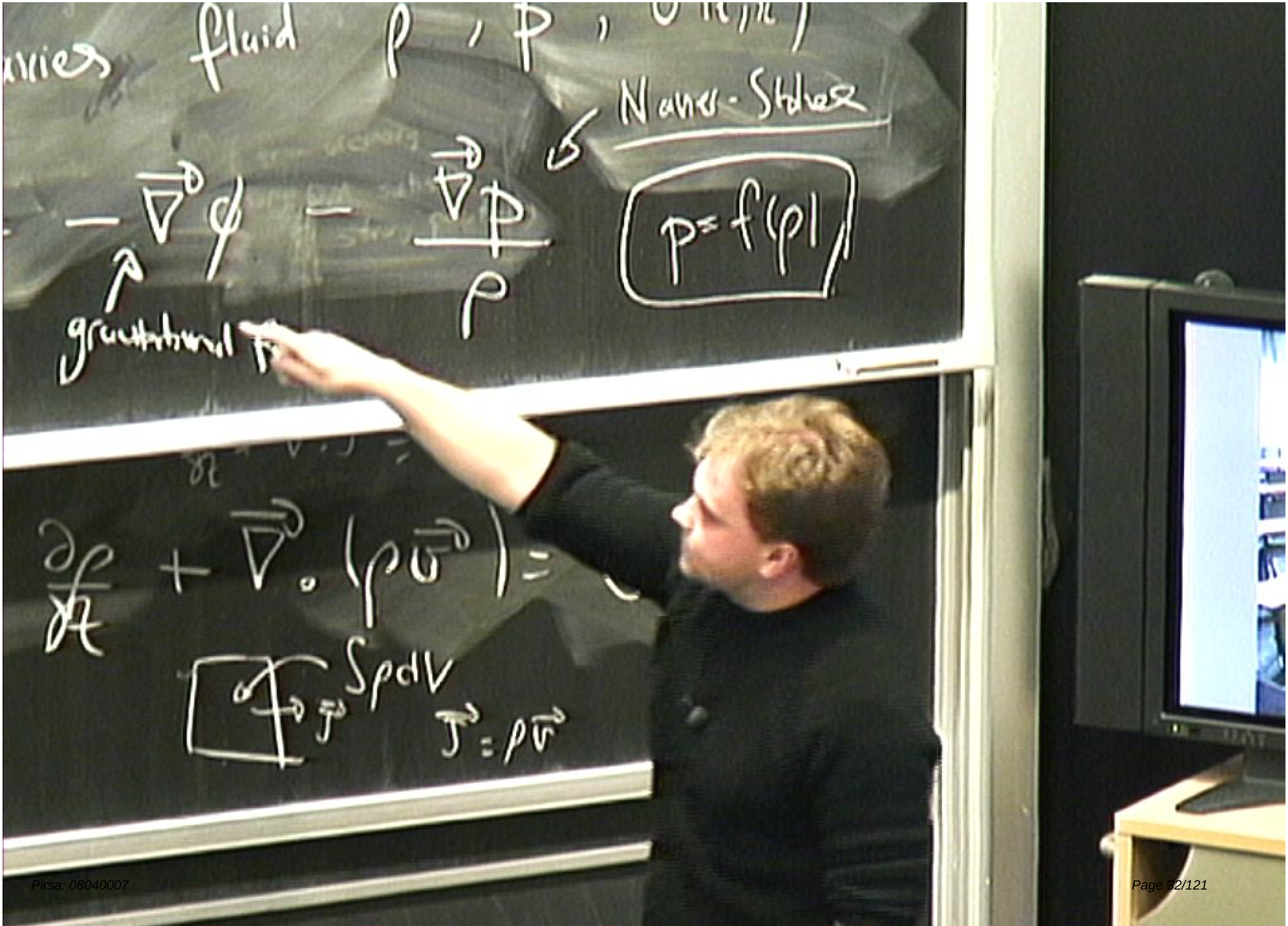
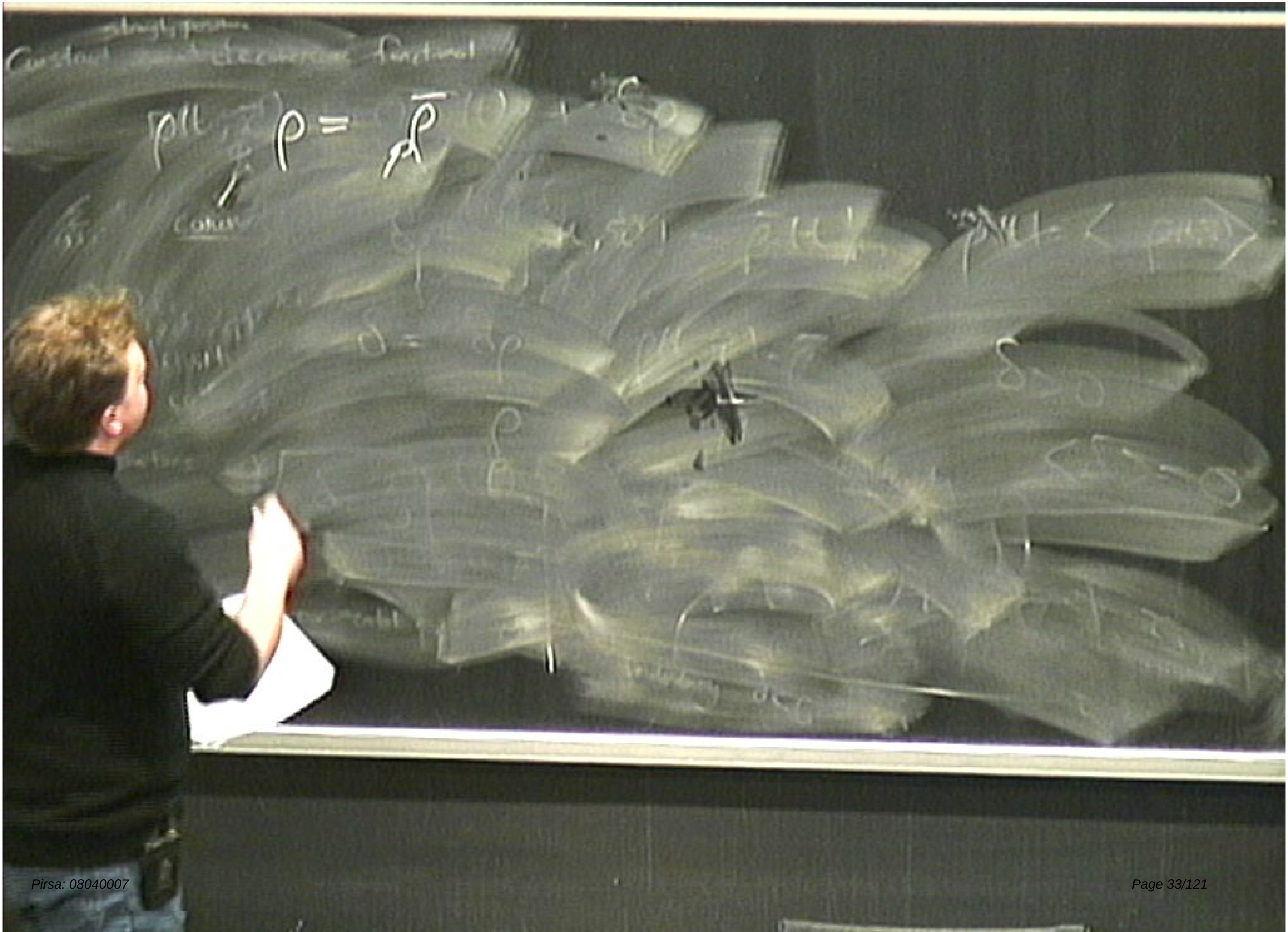
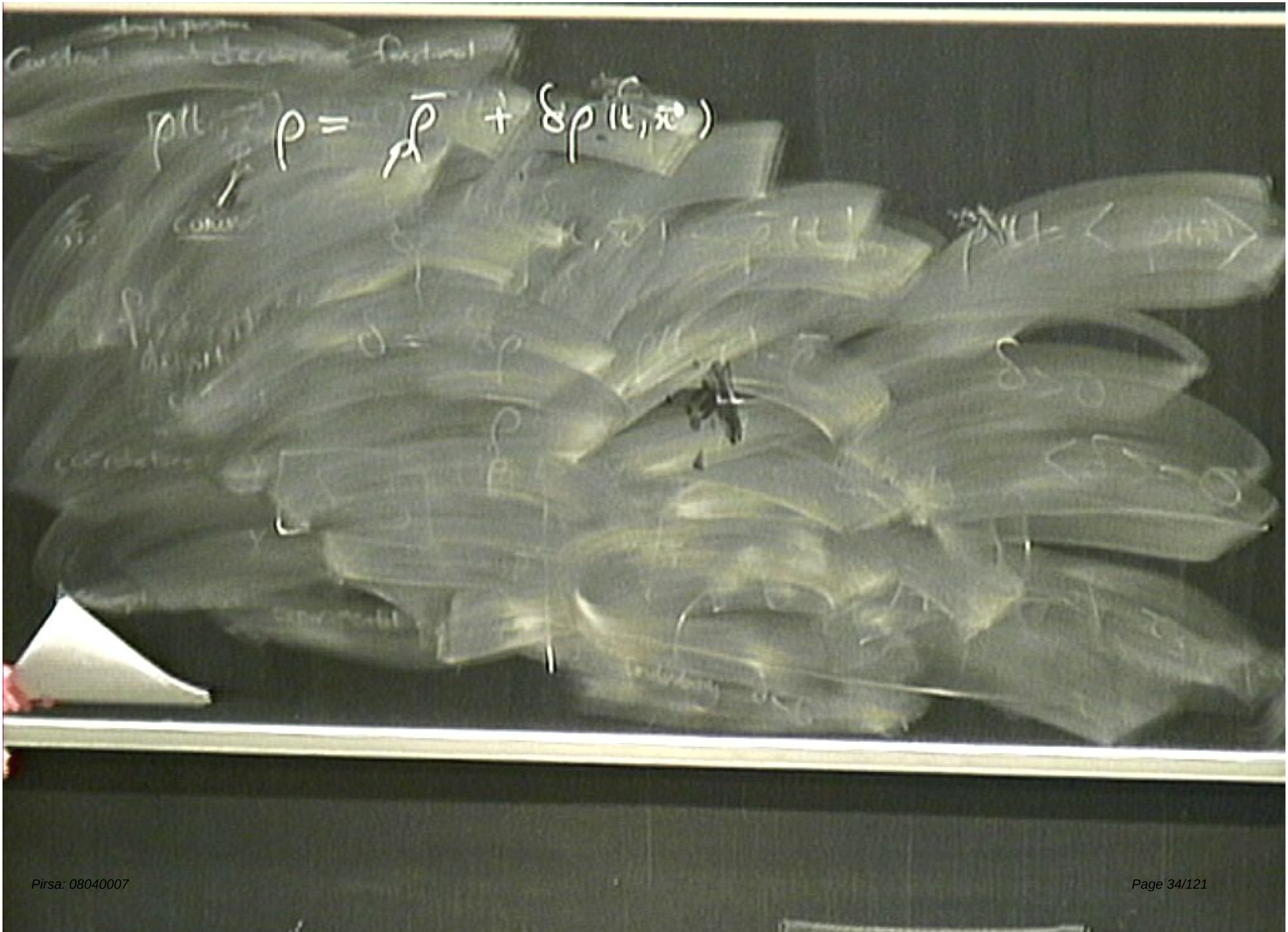
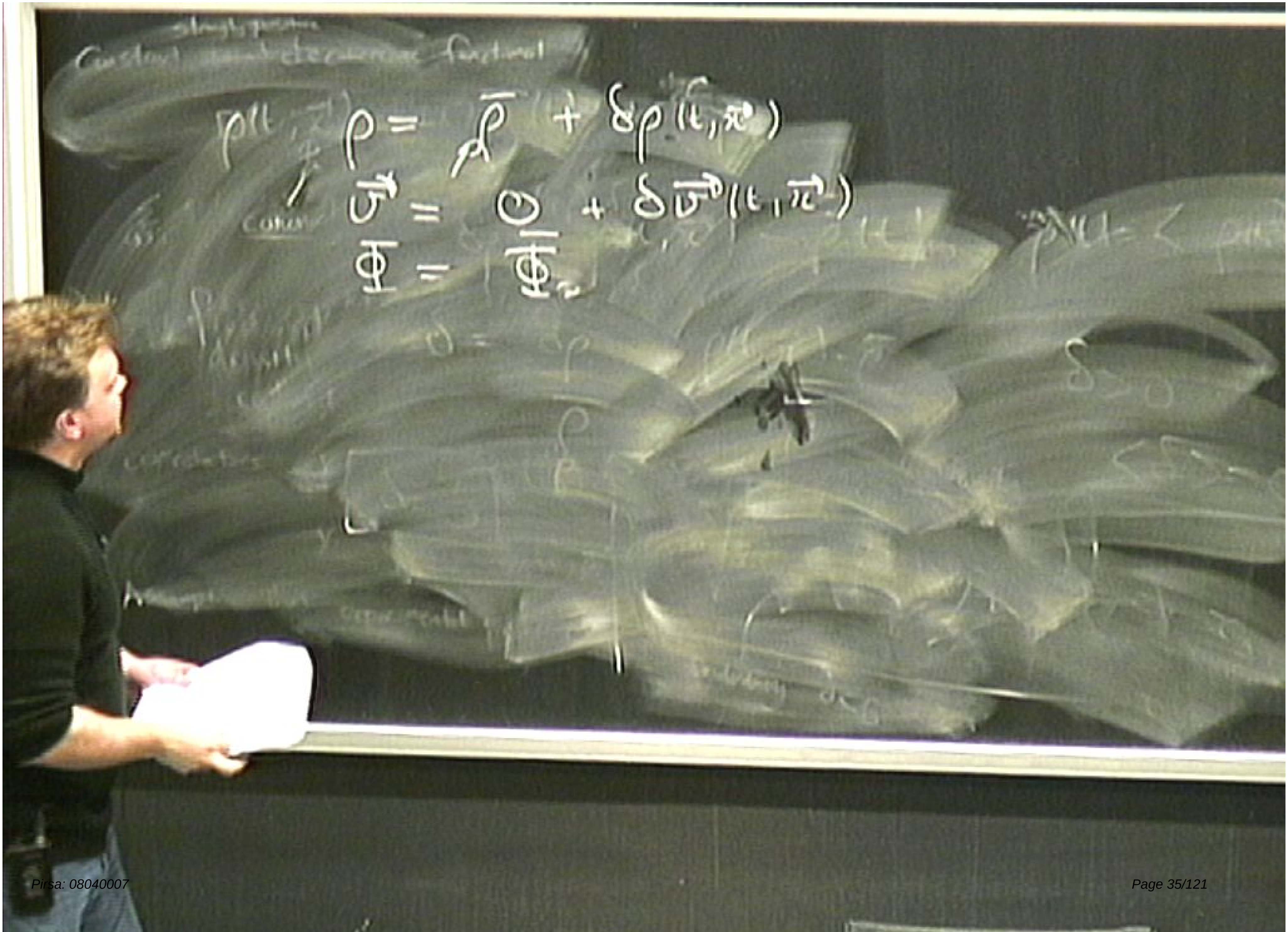
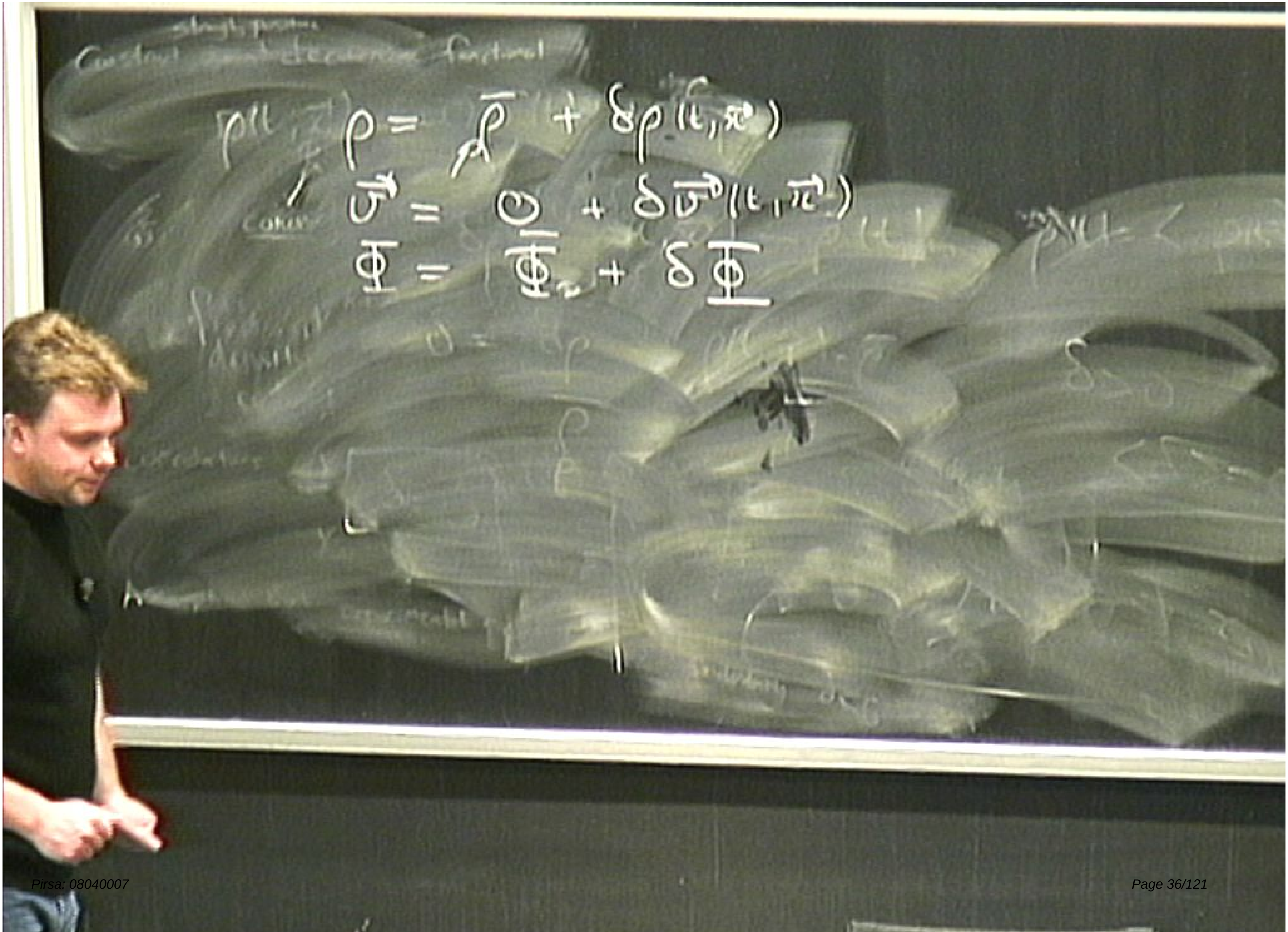

fluid
Navier-Stokes
p = f(ρ)
gravitational




$\rho = \bar{\rho} + \delta\rho(t,\vec{x})$


ρ = ρ̄ + δρ(t, x⃗)
v⃗ = 0 + δv⃗(t, x⃗)
Φ = Φ̄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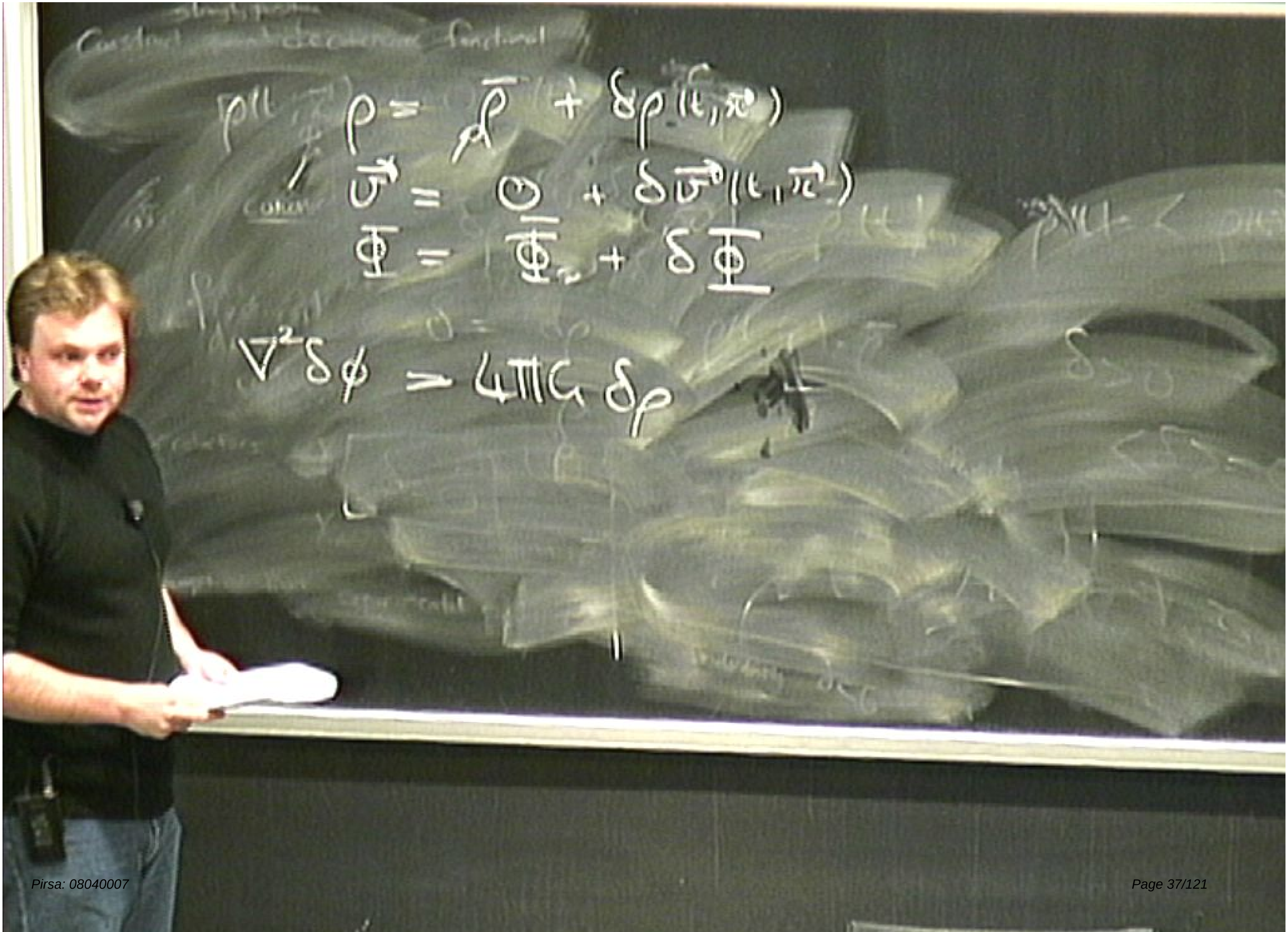
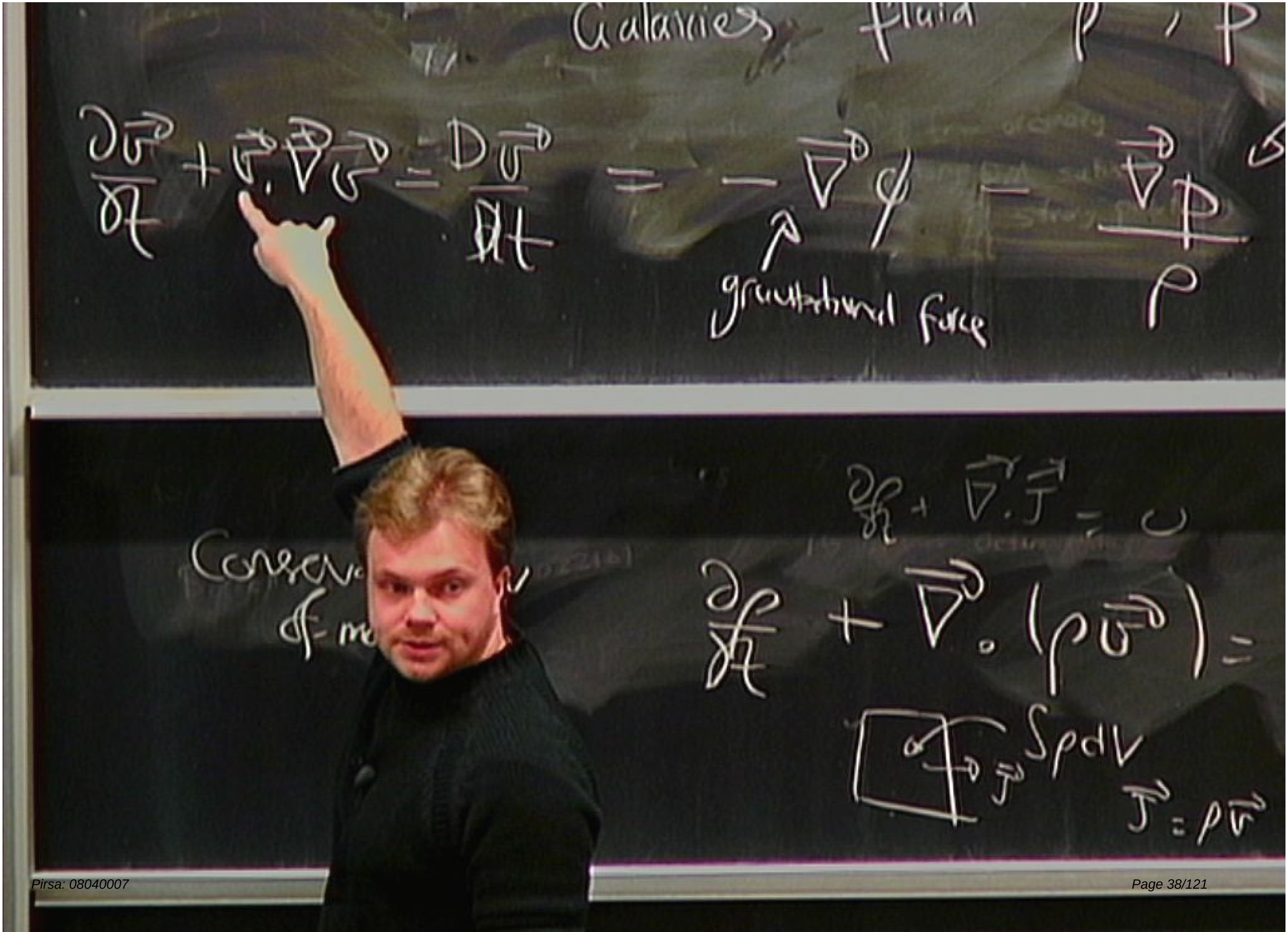

$\rho = \bar{\rho} + \delta\rho(t, \vec{x})$
$\vec{v} = 0 + \delta\vec{v}(t, \vec{x})$
$\Phi = \bar{\Phi} + \delta\Phi$
$\nabla^2 \delta\phi = 4\pi G\, \delta\rho$


Galaxies fluid ρ, P
$\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\vec{\nabla}\vec{v} = \frac{D\vec{v}}{Dt} = -\vec{\nabla}\phi - \frac{\vec{\nabla}P}{\rho}$
gravitational force
$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$
$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) =$
$\int \rho \, dV$
$\vec{J} = \rho\vec{v}$

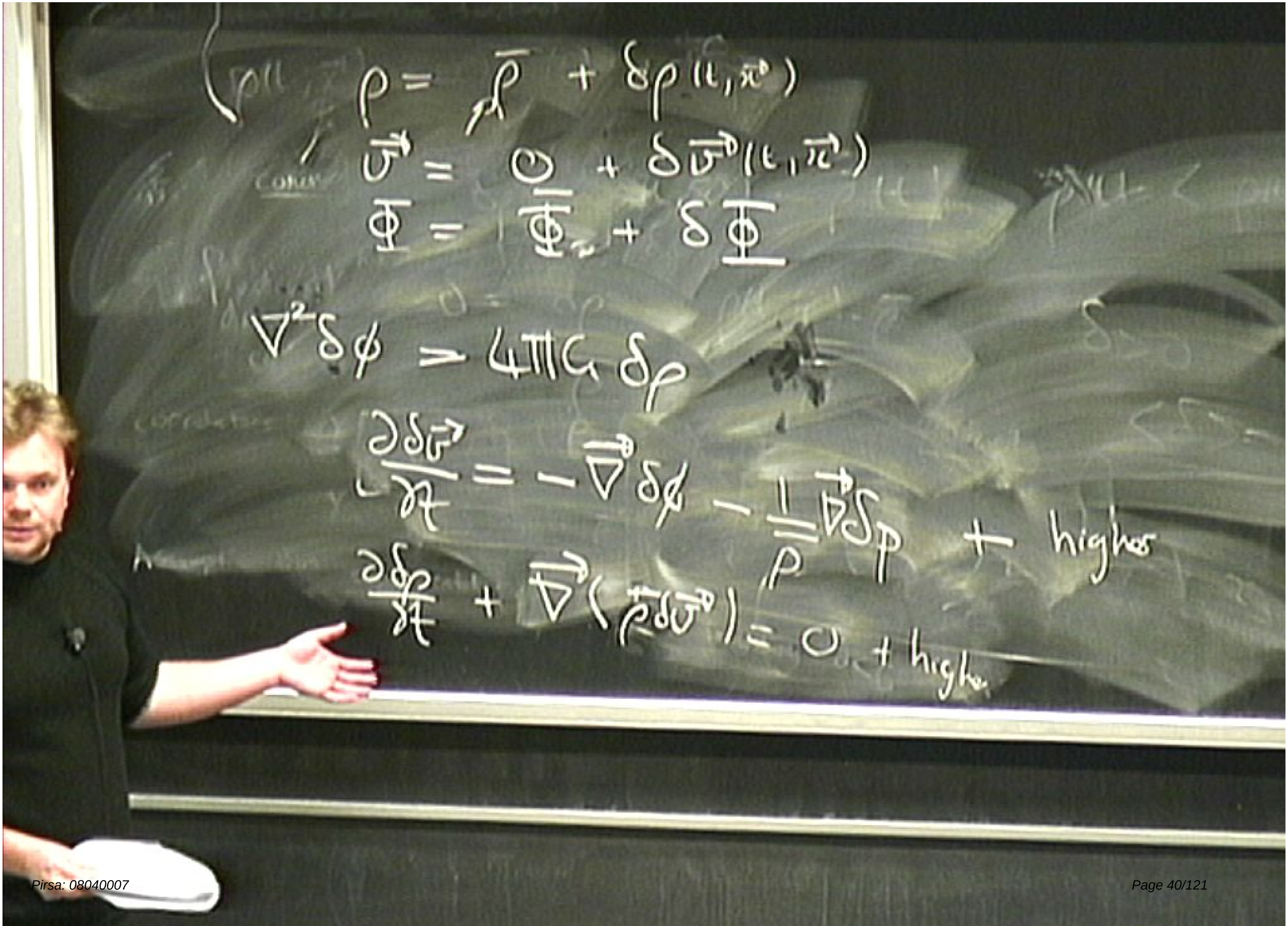
$$\rho = \bar{\rho} + \delta\rho(t,\vec{x})$$

$$\vec{v} = 0 + \delta\vec{v}(t,\vec{x})$$

$$\Phi = \bar{\Phi} + \delta\Phi$$

$$\nabla^2\delta\phi = 4\pi G\,\delta\rho$$

$$\frac{\partial\delta\vec{v}}{\partial t} = -\vec{\nabla}\delta\phi - \frac{1}{\bar{\rho}}\vec{\nabla}\delta p$$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$\nabla^2 \delta\phi = 4\pi G\, \delta\rho$
$\frac{\partial \delta\vec{v}}{\partial t} = -\vec{\nabla}\delta\phi - \frac{1}{\bar{\rho}}\vec{\nabla}\delta p + \text{higher}$
$\frac{\partial \delta\rho}{\partial t} + \vec{\nabla}(\bar{\rho}\,\delta\vec{v}) = 0 + \text{higher}$

$$\rho = \bar{\rho} + \delta\rho(t, \vec{x})$$

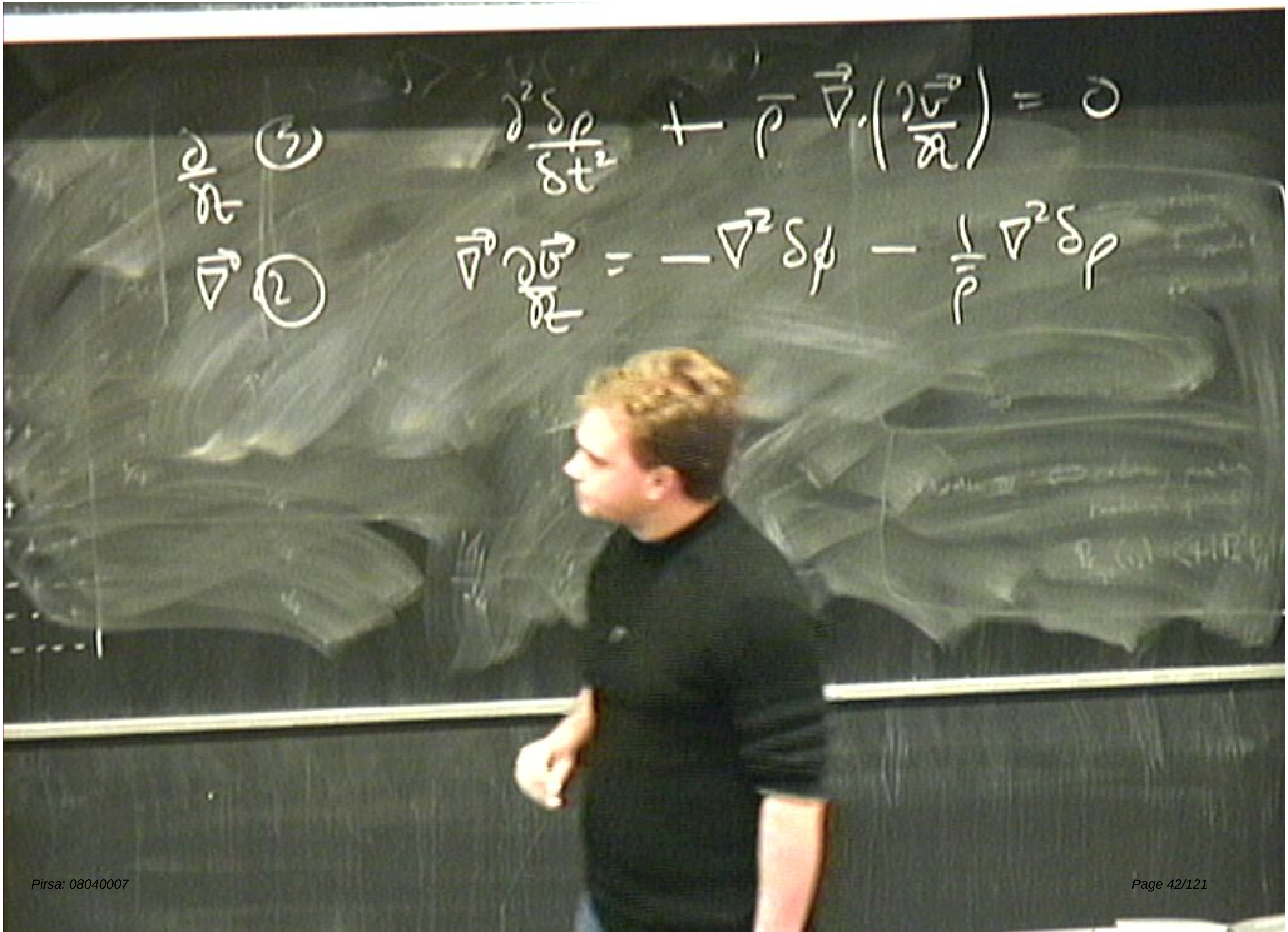
$$\vec{v} = 0 + \delta\vec{v}(t, \vec{x})$$

$$\Phi = \bar{\Phi}_0 + \delta\Phi$$

$$\nabla^2 \delta\phi = 4\pi G\, \delta\rho \qquad (1)$$

$$\frac{\partial \delta\vec{v}}{\partial t} = -\vec{\nabla}\delta\phi - \frac{1}{\bar{\rho}}\vec{\nabla}\delta p + \text{higher} \qquad (2)$$

$$\frac{\partial \delta\rho}{\partial t} + \vec{\nabla}(\bar{\rho}\,\delta\vec{v}) = 0 + \text{higher} \qquad (3)$$


$\frac{\partial}{\partial t}$ (3) $\frac{\partial^2 \delta \rho}{\partial t^2} + \bar{\rho}\, \vec{\nabla} \cdot \left(\frac{\partial \vec{v}}{\partial t} \right) = 0$
$\vec{\nabla}$ (2) $\vec{\nabla} \frac{\partial \vec{v}}{\partial t} = -\nabla^2 \delta \phi - \frac{1}{\bar{\rho}} \nabla^2 \delta p$

$\frac{\partial}{\partial t}$ (3) $\qquad \frac{\partial^2 \delta\rho}{\partial t^2} + \bar{\rho}\, \vec{\nabla}\cdot\left(\frac{\partial \vec{v}}{\partial t}\right) = 0$

$\vec{\nabla}$ (2) $\qquad \vec{\nabla}\, \frac{\partial \vec{v}}{\partial t} = -\nabla^2 \delta\phi - \frac{1}{\bar{\rho}} \nabla^2 \delta\rho$

(1) ϕ

$$\frac{1}{\bar{\rho}} \frac{\partial^2 \delta\rho}{\partial t^2} = 4\pi G\, \delta\rho - \frac{1}{\bar{\rho}} \nabla^2 \delta p \qquad \delta p = c_s^2\, \delta\rho$$

$\frac{\partial}{\partial t}$ (3) $\quad \frac{\partial^2 \delta\rho}{\partial t^2} + \bar\rho\, \vec\nabla\cdot\left(\frac{\partial \vec v}{\partial t}\right) = 0$

$\vec\nabla$ (2) $\quad \vec\nabla\, \frac{\partial \vec v}{\partial t} = -\nabla^2 \delta\phi - \frac{1}{\bar\rho}\nabla^2 \delta p$

(1) ϕ

$$\frac{1}{\bar\rho}\frac{\partial^2 \delta\rho}{\partial t^2} = 4\pi G\, \delta\rho - \frac{1}{\bar\rho}\nabla^2 \delta p$$

$\delta p = c_s^2\, \delta\rho$

$$\boxed{c_s^2 = \frac{dp}{d\rho}}$$

$\delta = \frac{\delta\rho}{\bar\rho}$

$\frac{\partial}{\partial t}$ (3) $\quad \frac{\partial^2 \delta\rho}{\partial t^2} + \bar{\rho}\, \vec{\nabla}\cdot\left(\frac{\partial \vec{v}}{\partial t}\right) = 0$

$\vec{\nabla}$ (2) $\quad \vec{\nabla}\frac{\partial \vec{v}}{\partial t} = -\underbrace{\nabla^2 \delta\phi}_{(1)\ \phi} - \frac{1}{\bar{\rho}}\nabla^2 \delta p$

$\frac{1}{\bar{\rho}}\frac{\partial^2 \delta\rho}{\partial t^2} = 4\pi G\, \delta\rho - \frac{1}{\bar{\rho}}\nabla^2 \delta p$

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$\frac{\partial}{\partial t}$ (3) $$\frac{\partial^2 \delta\rho}{\delta t^2} + \bar{\rho}\,\vec{\nabla}\cdot\left(\frac{\partial \vec{v}}{\partial t}\right) = 0$$

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(1) ρ

$$\frac{1}{\bar{\rho}}\frac{\partial^2 \delta\rho}{\partial t^2} = 4\pi G\,\delta\rho - \frac{1}{\bar{\rho}}\nabla^2 \delta p$$

$$\delta p = c_s^2\,\delta\rho$$

$$\boxed{c_s^2 = \frac{dp}{d\rho}}$$

$$\delta = \frac{\delta\rho}{\rho}$$

$$\left(\frac{\partial}{\partial t}\,③\right) \qquad \frac{\partial^2 \delta\rho}{\partial t^2} + \bar{\rho}\,\vec{\nabla}\cdot\left(\frac{\partial \vec{v}}{\partial t}\right) = 0$$

$$-\left(\vec{\nabla}\cdot ②\right) \qquad \vec{\nabla}\cdot\frac{\partial \vec{v}}{\partial t} = -\nabla^2 \delta\phi - \frac{1}{\bar{\rho}}\nabla^2 \delta p$$

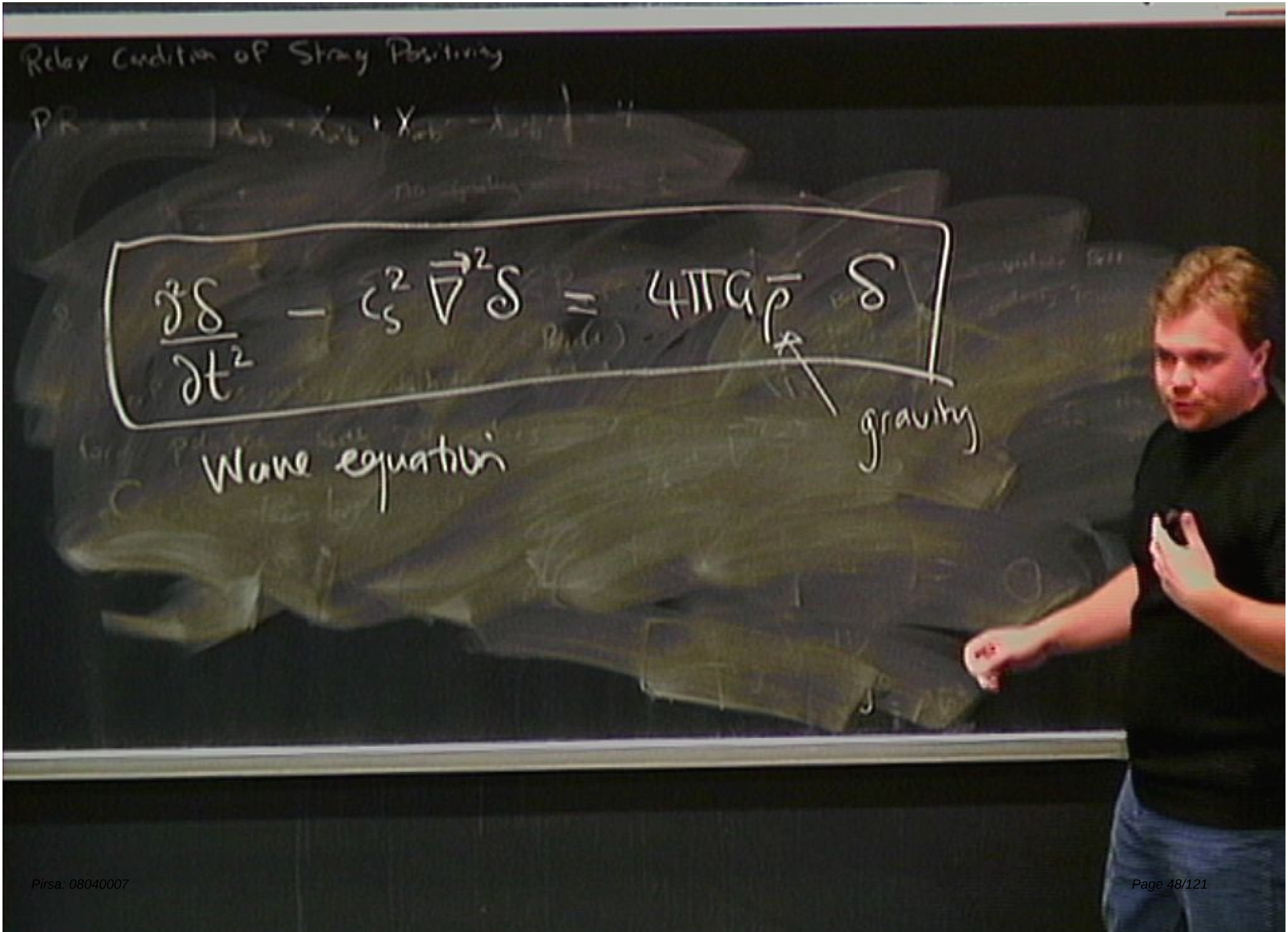
$$+\ ① \qquad ①\ \rho$$

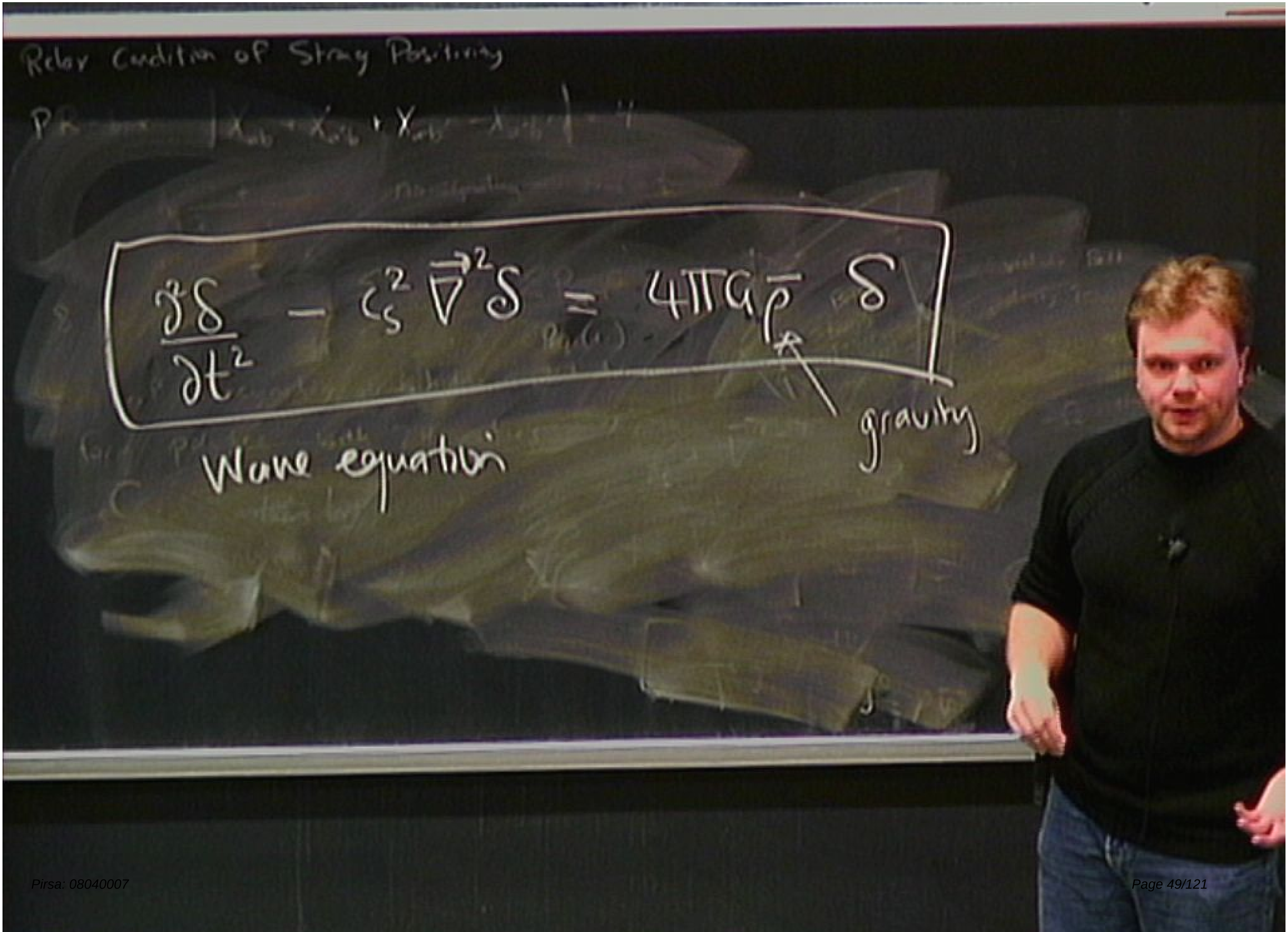
$$\frac{1}{\bar{\rho}}\frac{\partial^2 \delta\rho}{\partial t^2} = 4\pi G\,\delta\rho - \frac{1}{\bar{\rho}}\nabla^2 \delta p$$

$$\delta p = c_s^2\,\delta\rho$$

$$\boxed{c_s^2 = \frac{dp}{d\rho}}$$

$$\delta = \frac{\delta\rho}{\bar{\rho}}$$


Relax Condition of Strong Positivity
$$\boxed{\frac{\partial^2\delta}{\partial t^2} - c_s^2\vec{\nabla}^2\delta = 4\pi G\bar{\rho}\,\delta}$$
Wave equation
gravity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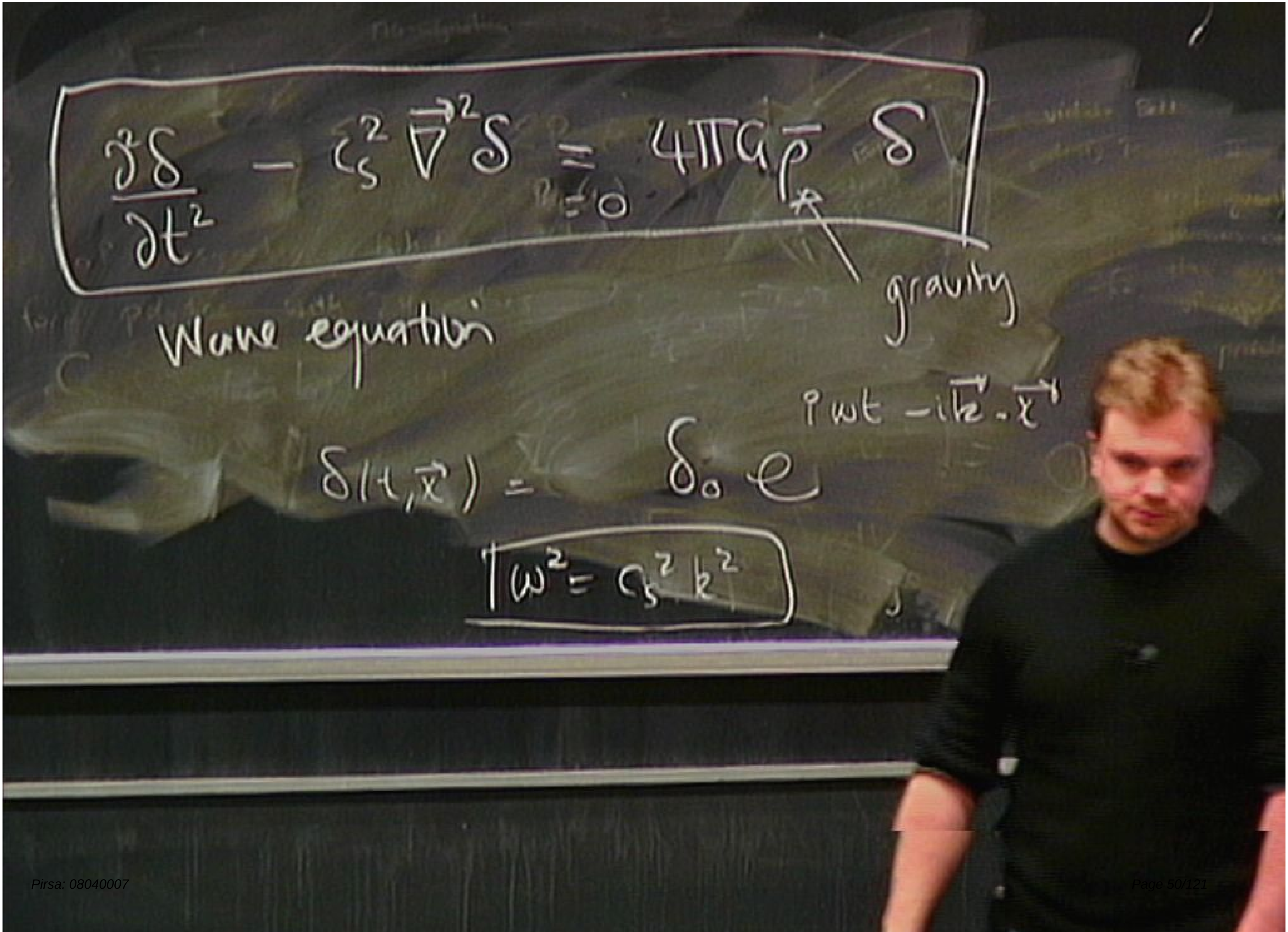
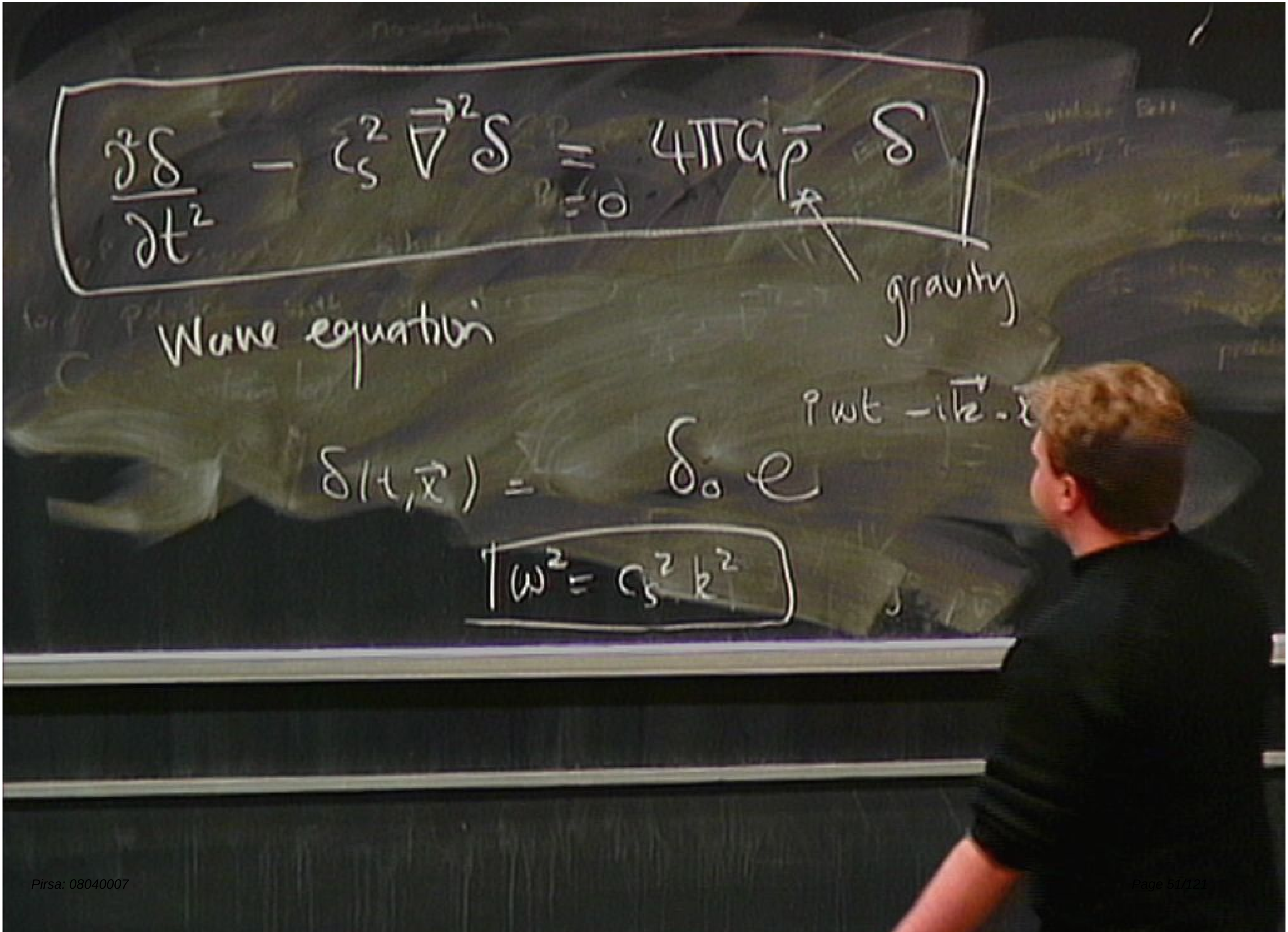

$\boxed{\frac{\partial^2\delta}{\partial t^2} - c_s^2 \vec{\nabla}^2 \delta = 4\pi G \bar{\rho}\, \delta}$
gravity
Wave equation
$\delta(t,\vec{x}) = \delta_0 e^{i\omega t - i\vec{k}\cdot\vec{x}}$
$\boxed{\omega^2 = c_s^2 k^2}$


$\frac{\partial^2\delta}{\partial t^2} - c_s^2 \vec{\nabla}^2 \delta = 4\pi G \bar{\rho}\, \delta$
$P=0$
gravity
Wave equation
$\delta(t,\vec{x}) = \delta_0 e^{i\omega t - i\vec{k}\cdot\vec{x}}$
$\omega^2 = c_s^2 k^2$

$$\boxed{\frac{\partial^2 \delta}{\partial t^2} - c_s^2 \vec{\nabla}^2 \delta = 4\pi G \bar{\rho}\, \delta}$$

gravity

Wave equation

$$\delta(t,\vec{x}) = \delta_0\, e^{i\omega t - i\vec{k}\cdot\vec{x}}$$

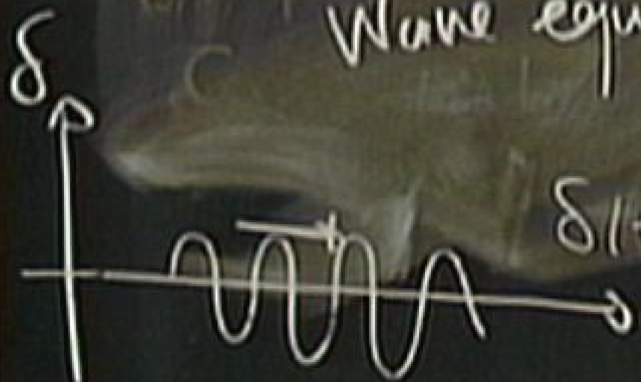
$$\boxed{\omega^2 = c_s^2 k^2}$$

Sound waves in the fluid

$$\boxed{\frac{\partial^2 \delta}{\partial t^2} - c_s^2 \vec{\nabla}^2 \delta \underset{P=0}{=} 4\pi G \bar{\rho}\, \delta}$$

gravity

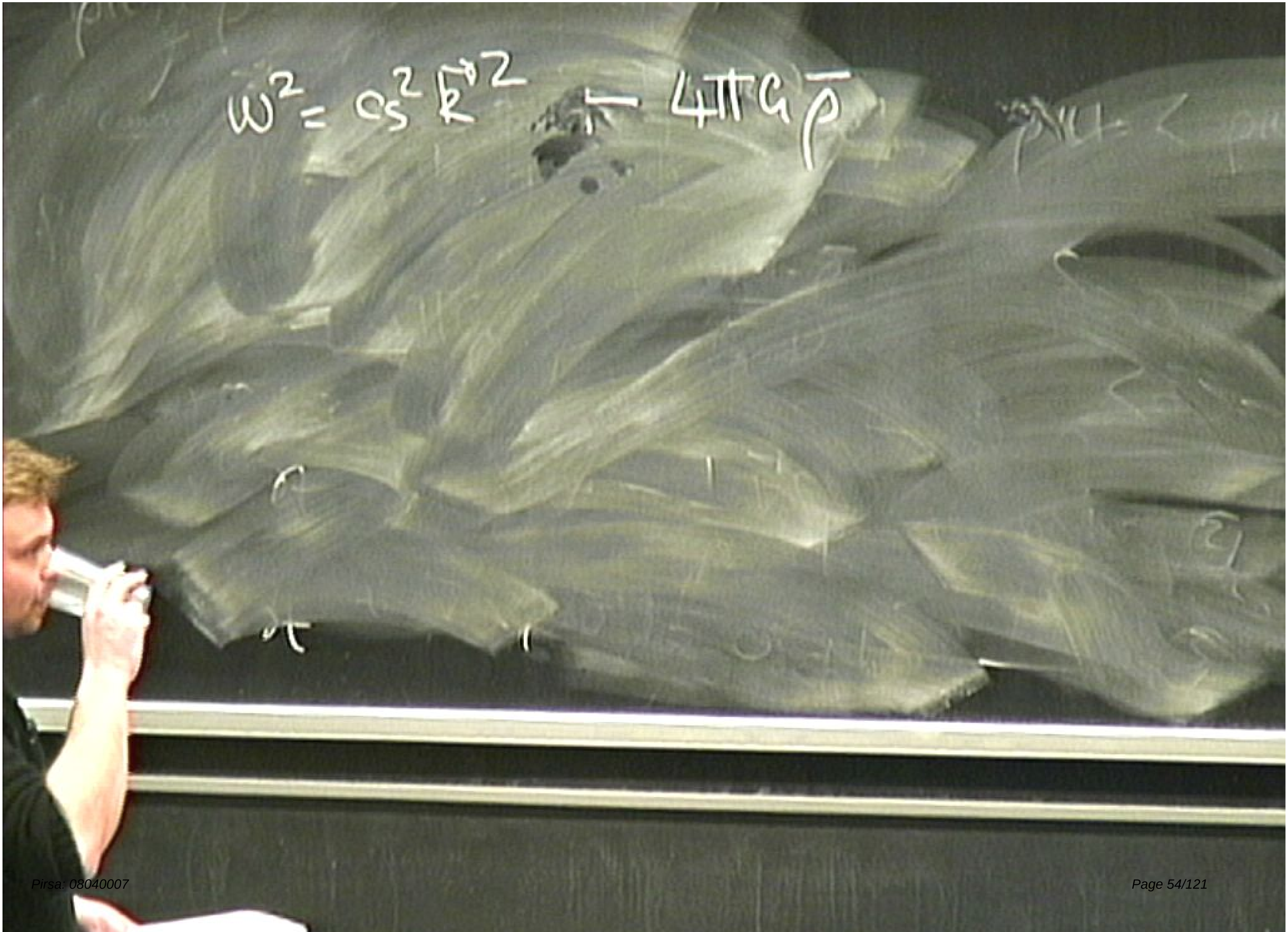
Wave equation

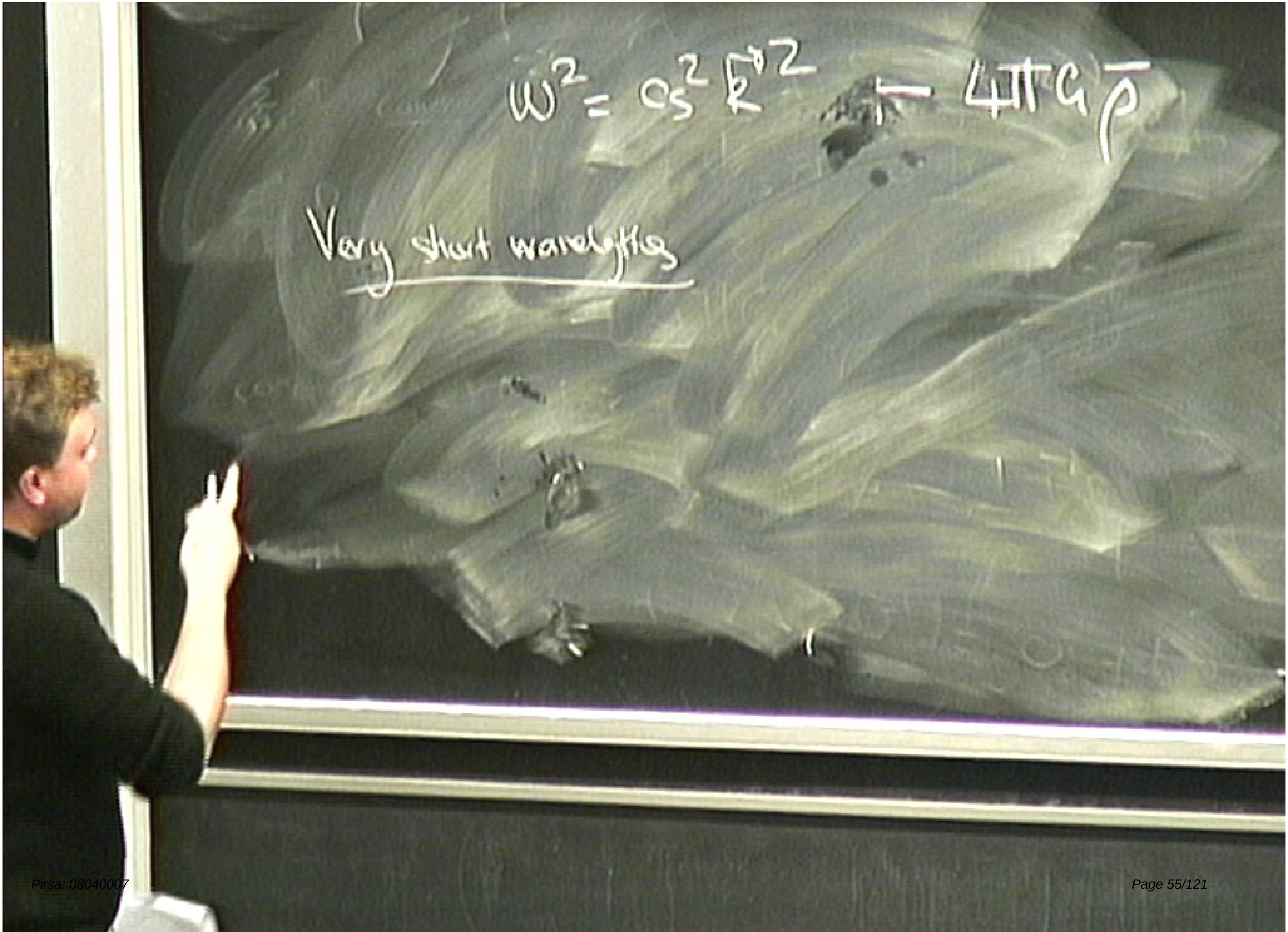


$$\delta(t, \vec{x}) = \delta_0\, e^{i\omega t - i\vec{k}\cdot\vec{x}}$$

(Sound waves in the fluid)

$$\boxed{\omega^2 = c_s^2 k^2}$$


$\omega^2 = c_s^2 \vec{k}^2 - 4\pi G \bar{\rho}$


$\omega^2 = c_s^2 \vec{k}^2 - 4\pi G \bar{\rho}$
Very short wavelengths

$$\omega^2 = c_s^2 \vec{k}^2 - 4\pi G \bar{\rho}$$

Very short wavelengths $\lambda = \frac{2\pi}{|\vec{k}|}$ $\omega \approx c_s |\vec{k}|$

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Pressure force of fluid much stronger gravity at short distances

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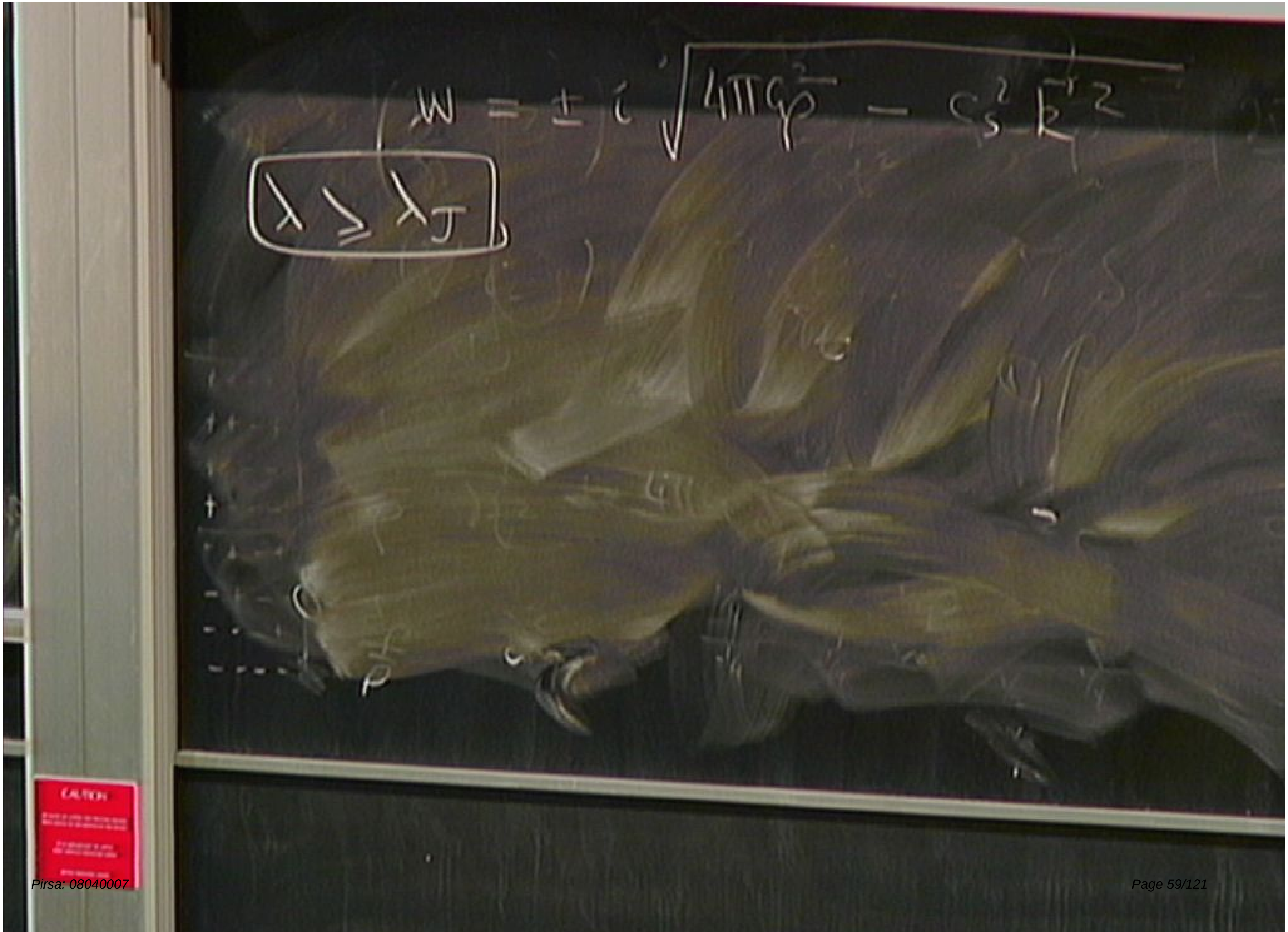
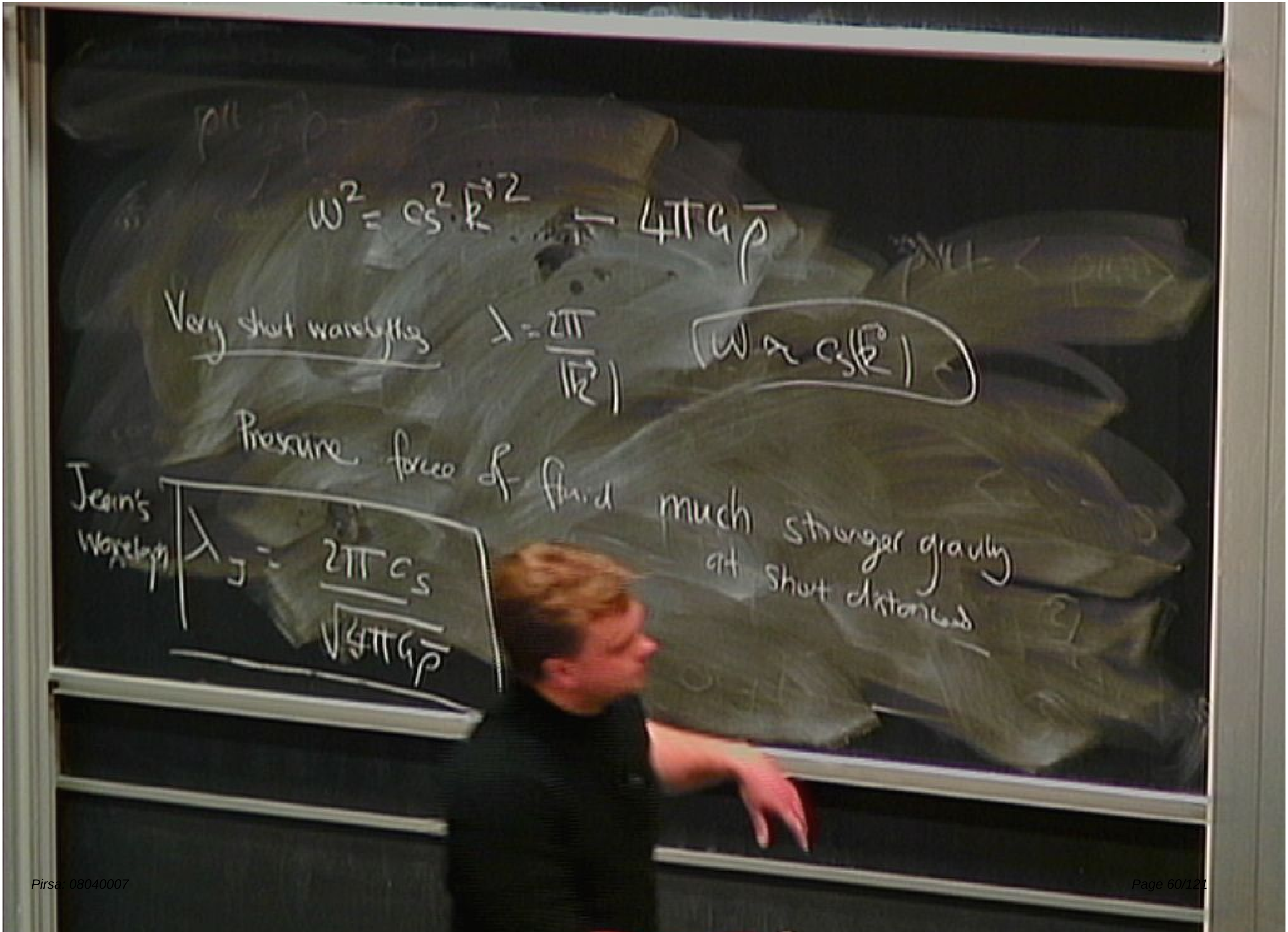
Pressure force of fluid much stronger gravity at short distances

Jean's wavelength $\boxed{\lambda_J = \frac{2\pi c_s}{\sqrt{4\pi G \bar{\rho}}}}$ $\omega = 0$


$\omega = \pm i \sqrt{4\pi G \rho - c_s^2 k^2}$
$\lambda \geq \lambda_J$
CAUTION



$\omega^2 = c_s^2 \vec{k}^2 - 4\pi G \bar{\rho}$
Very short wavelengths $\lambda = \frac{2\pi}{|\vec{k}|}$
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Jean's wavelength
$\lambda_J = \frac{2\pi c_s}{\sqrt{4\pi G\bar{\rho}}}$

$$\omega = \pm i\sqrt{4\pi G\bar{\rho} - c_s^2 \vec{k}^2}$$

$$\boxed{\lambda \geq \lambda_J}$$

$$e^{+i\omega t}\, e^{-i\vec{k}\cdot\vec{x}}$$

$$= e^{\pm\left(\sqrt{4\pi G\bar{\rho} - c_s^2 \vec{k}^2}\right)t - i\vec{k}\cdot\vec{x}}$$

c

c

ρ

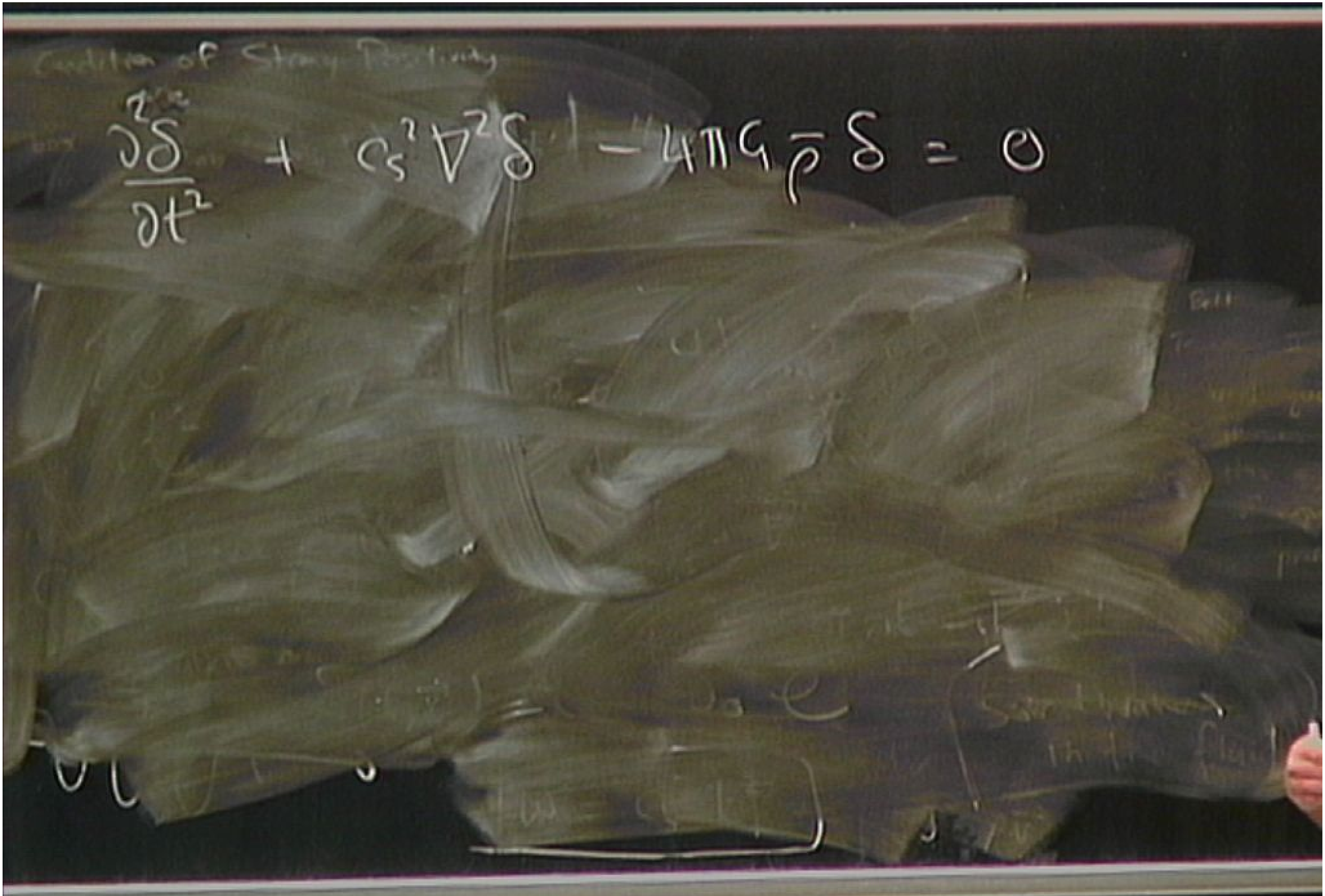
$$\omega = \pm i\sqrt{4\pi G\bar{\rho} - c_s^2 \vec{k}^2}$$

$$\boxed{\lambda \geq \lambda_J}$$

$$e^{+i\omega t}\, e^{-i\vec{k}\cdot\vec{x}}$$

$$\delta = \delta_0\, e^{\pm\left(\sqrt{4\pi G\bar{\rho} - c_s^2 \vec{k}^2}\right)t - i\vec{k}\cdot\vec{x}}$$

Jean's instability


$\frac{\partial^2 \delta}{\partial t^2} + c_s^2 \nabla^2 \delta - 4\pi G \bar{\rho} \delta = 0$

$$\frac{\partial^2\delta}{\partial t^2} + c_s^2\nabla^2\delta - 4\pi G\bar{\rho}\,\delta = 0$$

$$+ 2H\frac{\partial\delta}{\partial t}$$

friction / damping

Hubble damping

$$\ddot{x} + \gamma\dot{x} + \omega^2 x$$

friction/damping

Condition of Strong Positivity

$$\frac{\partial^2 \delta}{\partial t^2} + c_s^2 \nabla^2 \delta - 4\pi G \bar{\rho} \delta = 0$$

$$+ 2H \frac{\partial \delta}{\partial t}$$

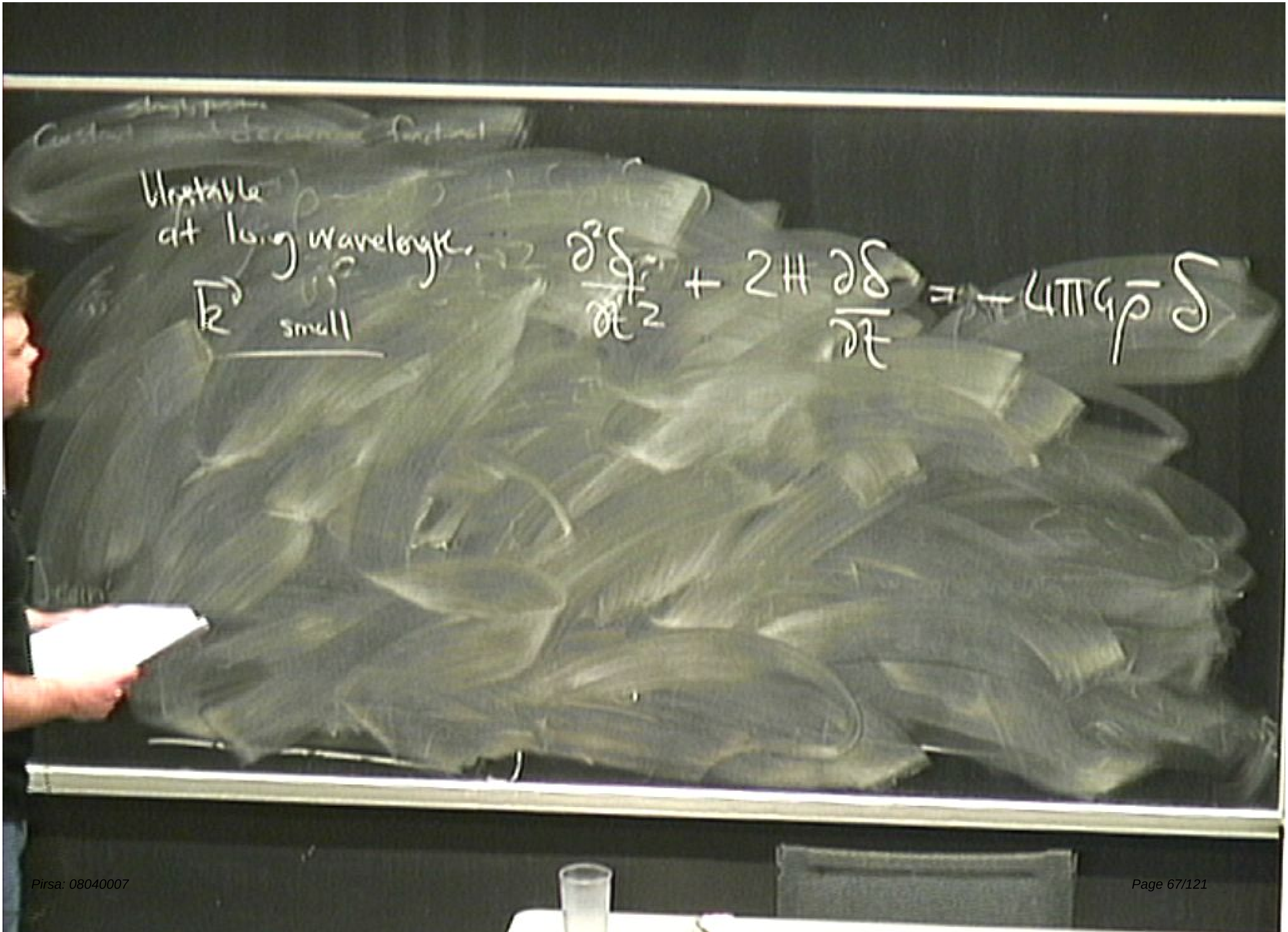
friction / damping

Hubble damping

$$\ddot{x} + \gamma \dot{x} + \omega^2 x$$

friction/damping


Unstable
at long wavelength
$\vec{k}$ small
$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} = -4\pi G \bar{\rho} \delta$


Unstable
at long wavelengths,
$\vec{k}$ small
$\frac{\partial^2\delta}{\partial t^2} + 2H\frac{\partial\delta}{\partial t} = -4\pi G\bar{\rho}\,\delta$

Unstable
at long wavelength
$\vec{k}$ small

$$\frac{\partial^2 \delta}{\partial t^2} + 2H \frac{\partial \delta}{\partial t} = 4\pi G \bar{\rho}\, \delta$$

Friedmann
equation

Unstable at long wavelength

$\vec{k}$ small

$$\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} = -4\pi G\bar{\rho}\,\delta$$

Friedmann equation $\quad H^2 = \frac{8\pi G\bar{\rho}}{3}$

$$\boxed{\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} + \frac{3}{2}H^2\delta = 0}$$

Unstable
at long wavelengths
$\vec{k}$ small

$$\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} = 4\pi G\bar{\rho}\,\delta$$

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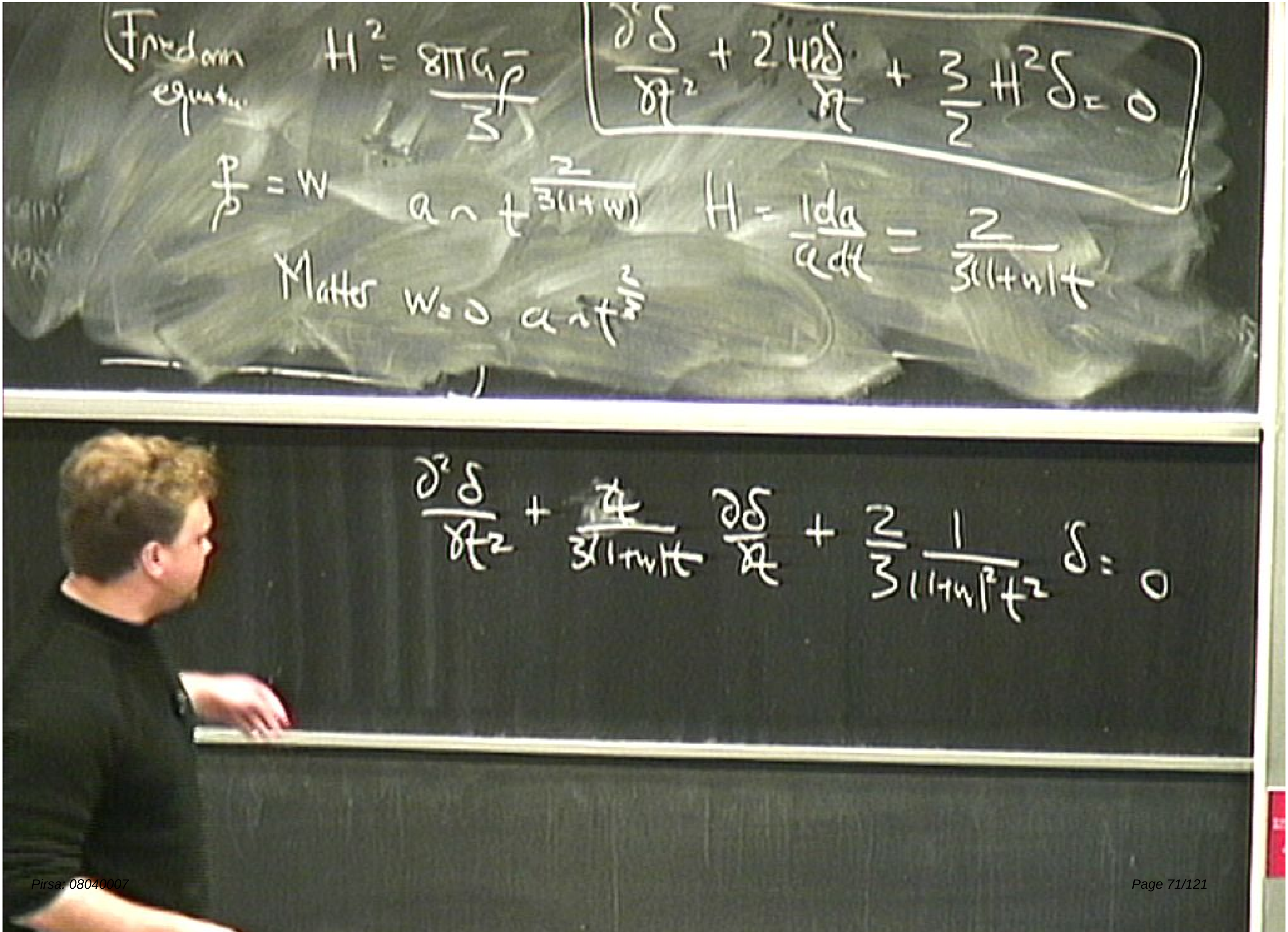
$$\boxed{\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} + \frac{3}{2}H^2\delta = 0}$$

$\frac{p}{\rho} = w \qquad a \sim t^{\frac{2}{3(1+w)}} \qquad H = \frac{1}{a}\frac{da}{dt} = \frac{2}{3(1+w)t}$

Matter $w = 0 \quad a \sim t^{\frac{2}{3}}$


Friedmann equation $H^2 = \frac{8\pi G \bar{\rho}}{3}$
$\boxed{\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} + \frac{3}{2}H^2\delta = 0}$
$\frac{p}{\rho} = w$ $a \sim t^{\frac{2}{3(1+w)}}$ $H = \frac{1}{a}\frac{da}{dt} = \frac{2}{3(1+w)t}$
Matter $w=0$ $a \sim t^{\frac{2}{3}}$
$\frac{\partial^2 \delta}{\partial t^2} + \frac{4}{3(1+w)t}\frac{\partial \delta}{\partial t} + \frac{2}{3}\frac{1}{(1+w)^2 t^2}\delta = 0$

equat...

$$H^2 = \frac{8\pi G}{3}\rho \qquad \boxed{\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} + \frac{3}{2}H^2\delta = 0}$$

$$\frac{p}{\rho} = w \qquad a \sim t^{\frac{2}{3(1+w)}} \qquad H = \frac{1}{a}\frac{da}{dt} = \frac{2}{3(1+w)t}$$

Matter $w = 0$ $a \sim t^{\frac{2}{3}}$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{4}{3(1+w)t}\frac{\partial \delta}{\partial t} + \frac{2}{3}\frac{1}{(1+w)^2 t^2}\delta = 0$$

$$\delta \sim A\, t \quad + B\, t$$

Friedmann equation $H^2 = \frac{8\pi G}{3}\bar{\rho}$

$$\boxed{\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} + \frac{3}{2}H^2\delta = 0}$$

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$\underline{w=0}$

$$\delta \sim A\, t^{2/3} + B\, t^{-1}$$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{4}{3(1+w)t}\frac{\partial \delta}{\partial t} + \frac{2}{3}\frac{1}{(1+w)^2 t^2}\,\delta = 0$$

$w = 0$

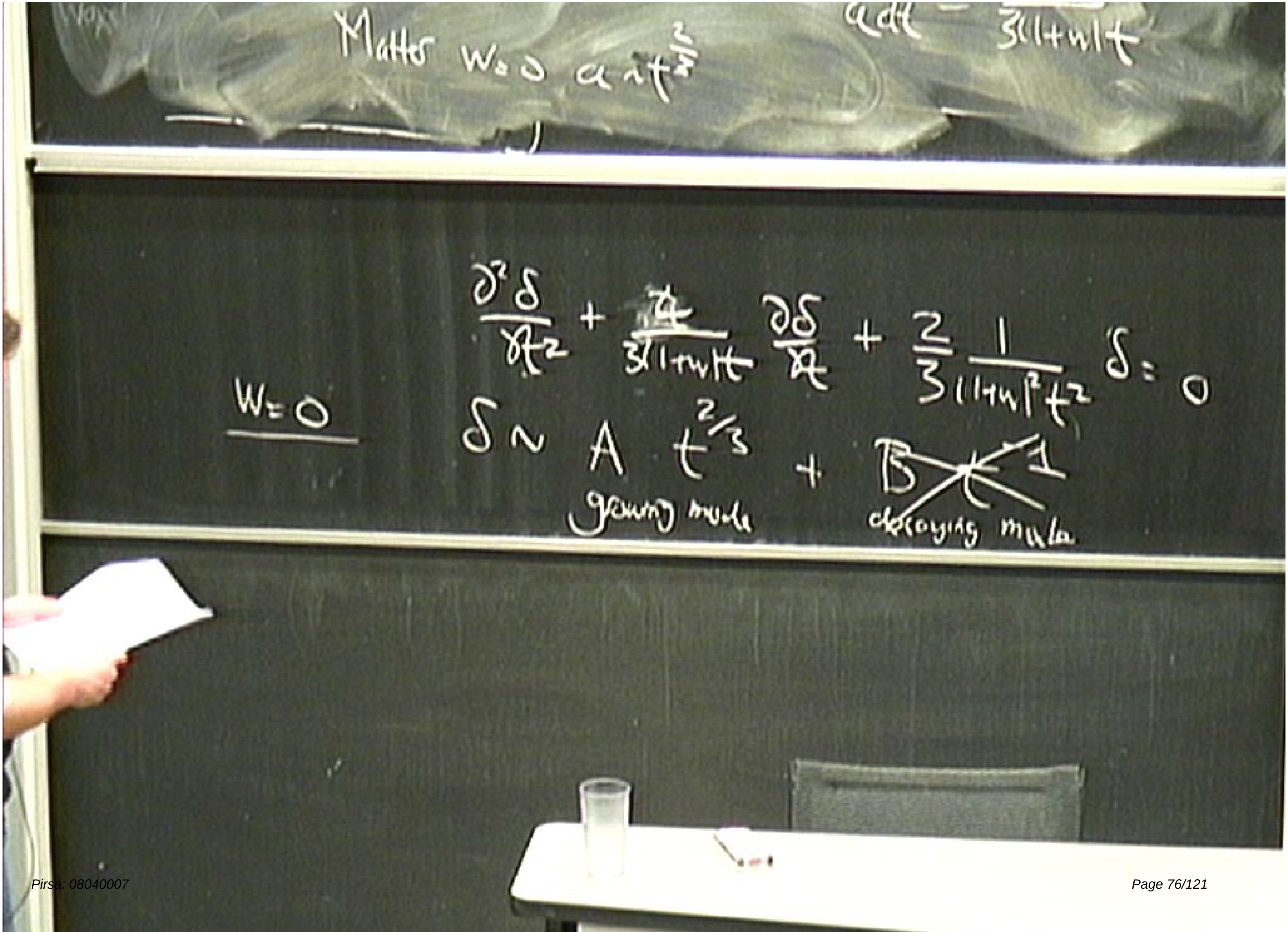
$$\delta \sim A\, t^{2/3} + B\, t^{-1}$$

Matter $w=0$ $a \propto t^{\frac{2}{3}}$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{4}{3(1+w)t}\frac{\partial \delta}{\partial t} + \frac{2}{3}\frac{1}{(1+w)^2 t^2}\,\delta = 0$$

$\underline{w=0}$

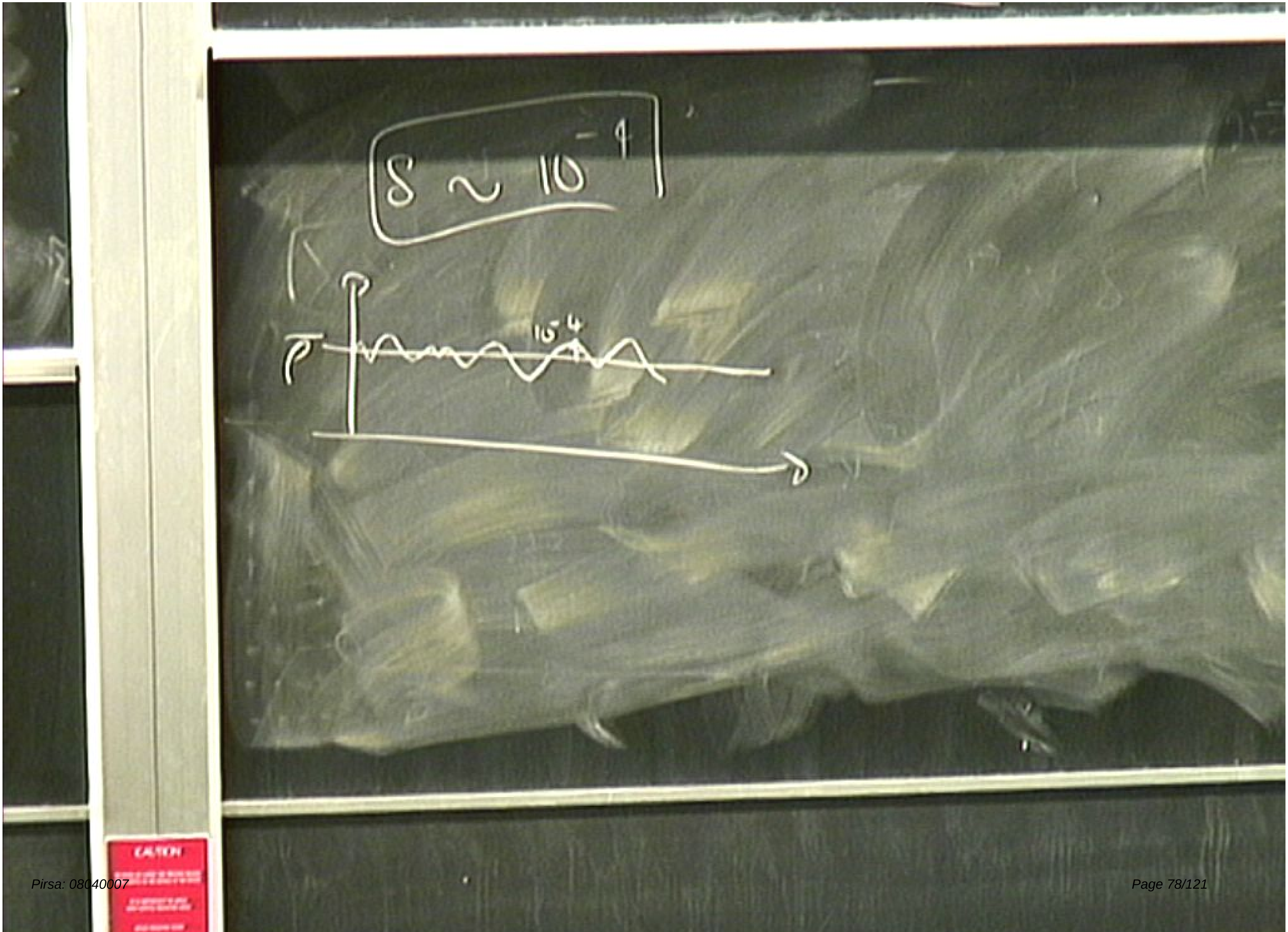
$$\delta \sim \underbrace{A\, t^{2/3}}_{\text{growing mode}} + \underbrace{B\, t^{-1}}_{\text{decaying mode}}$$

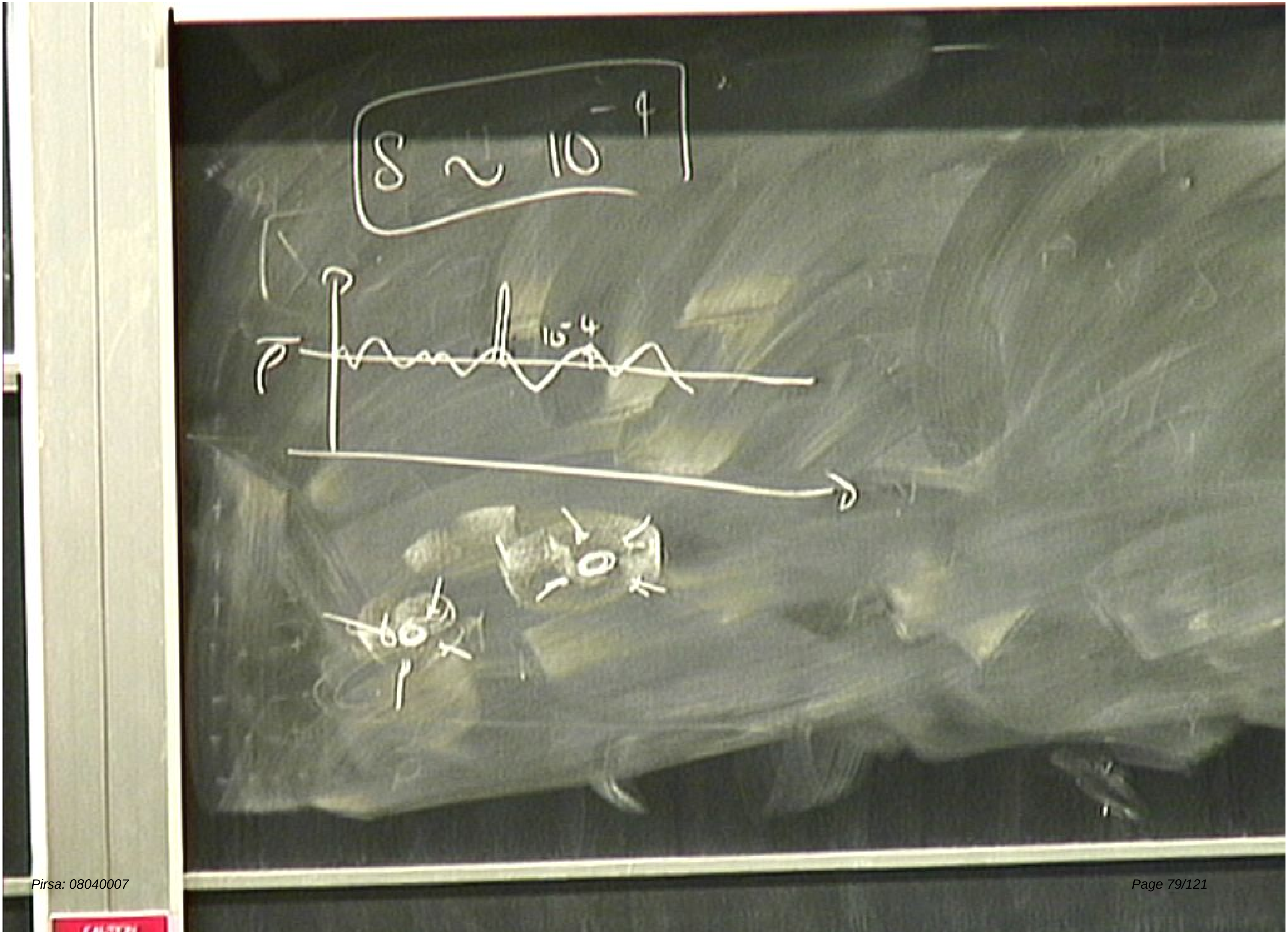

Matter $w=0$ $a \sim t^{\frac{2}{3}}$
$3(1+w)t$
$\frac{\partial^2\delta}{\partial t^2} + \frac{4}{3(1+w)t}\frac{\partial\delta}{\partial t} + \frac{2}{3}\frac{1}{(1+w)^2t^2}\delta = 0$
$w=0$
$\delta \sim A\, t^{2/3} + \cancel{B\, t^{-1}}$
growing mode
decaying mode

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{4}{3(1+w)t}\frac{\partial \delta}{\partial t} + \frac{2}{3}\frac{1}{(1+w)^2 t^2}\,\delta = 0$$

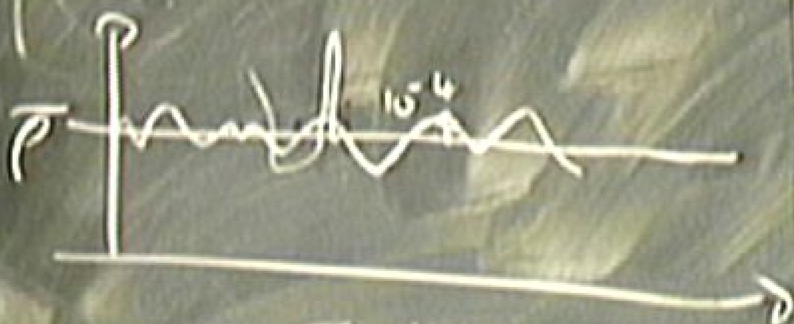
$\underline{w = 0}$

$$\delta \sim \underbrace{A\, t^{2/3}}_{\text{growing mode}} + \underbrace{\cancel{B\, t^{-1}}}_{\text{decaying mode}}$$


$\delta \sim 10^{-4}$
ρ
$\bar{\rho}$
10^{-4}
CAUTION


δ ~ 10^-4
10^-4
ρ̄
ρ

$\boxed{\delta \sim 10^{-4}}$

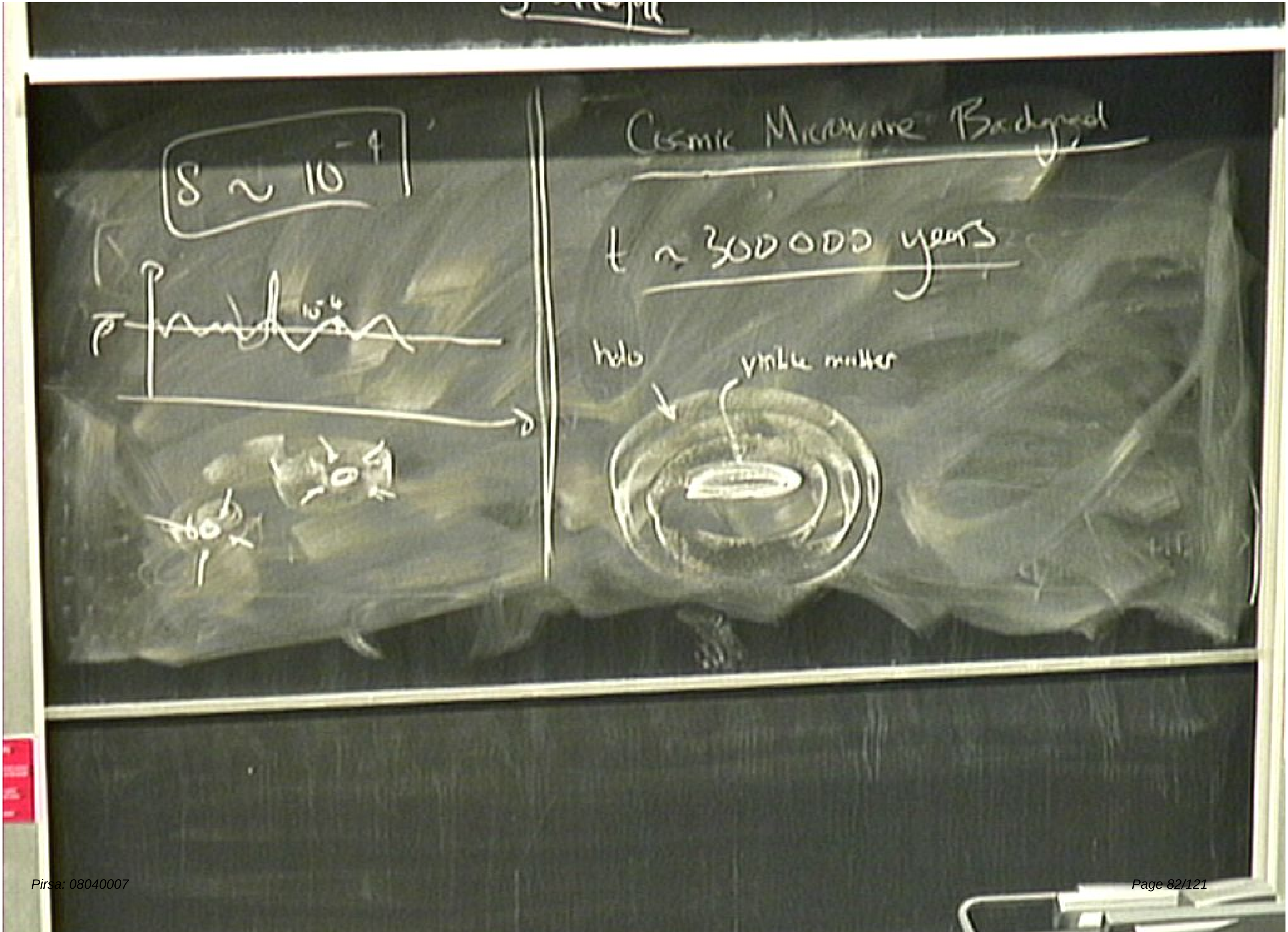


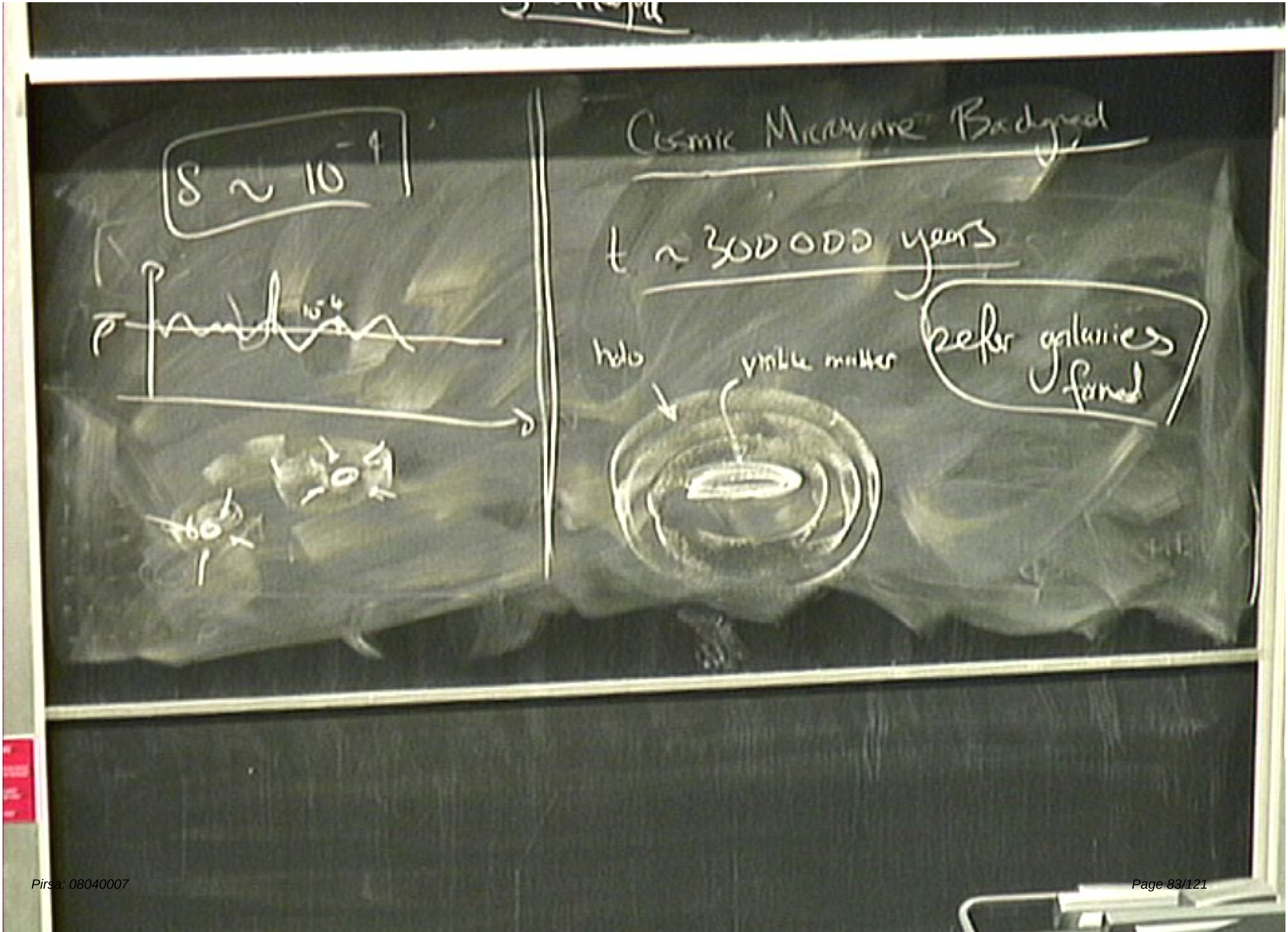
Cosmic Microwave Background

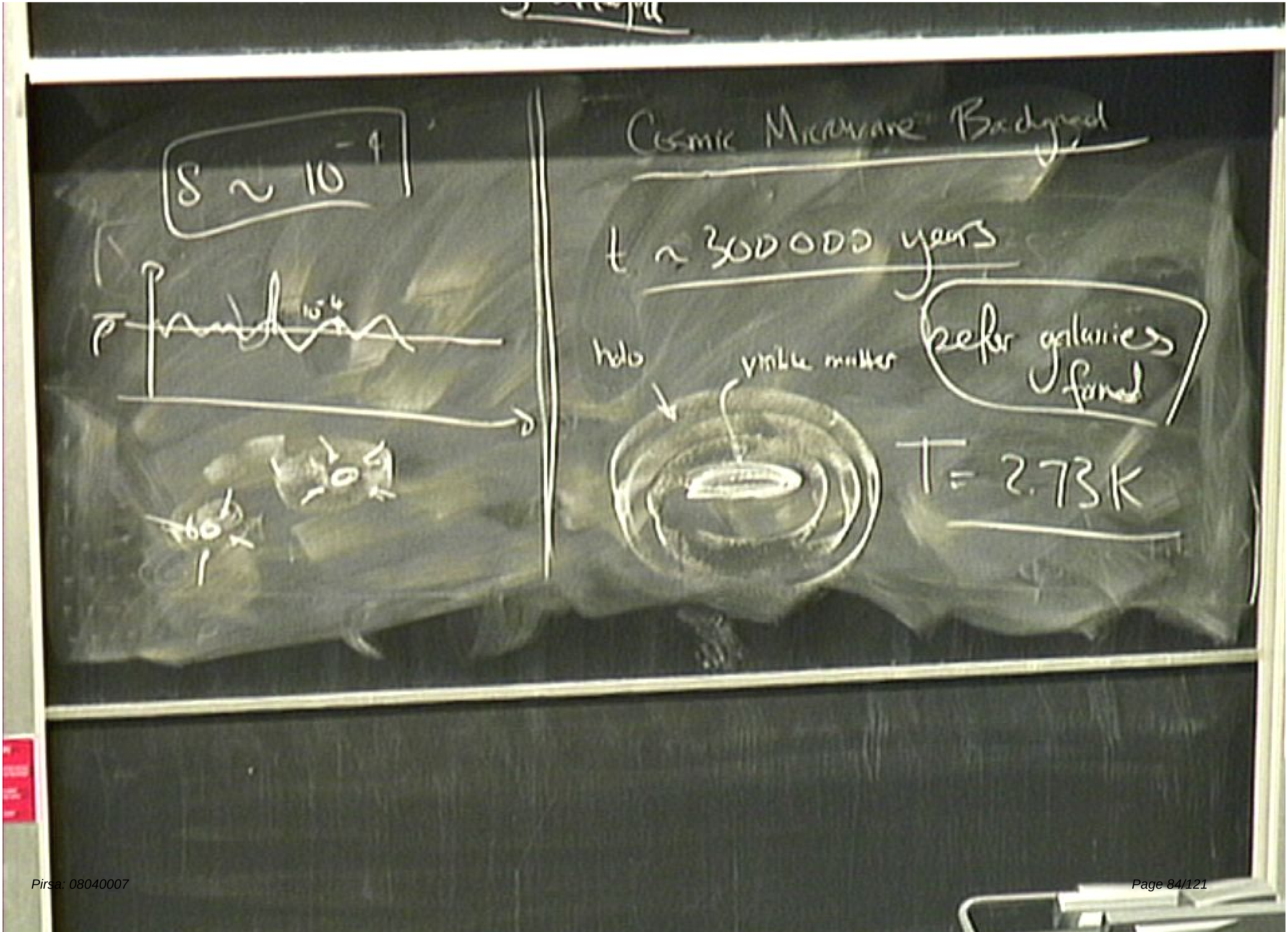
$$\boxed{\delta \sim 10^{-4}}$$

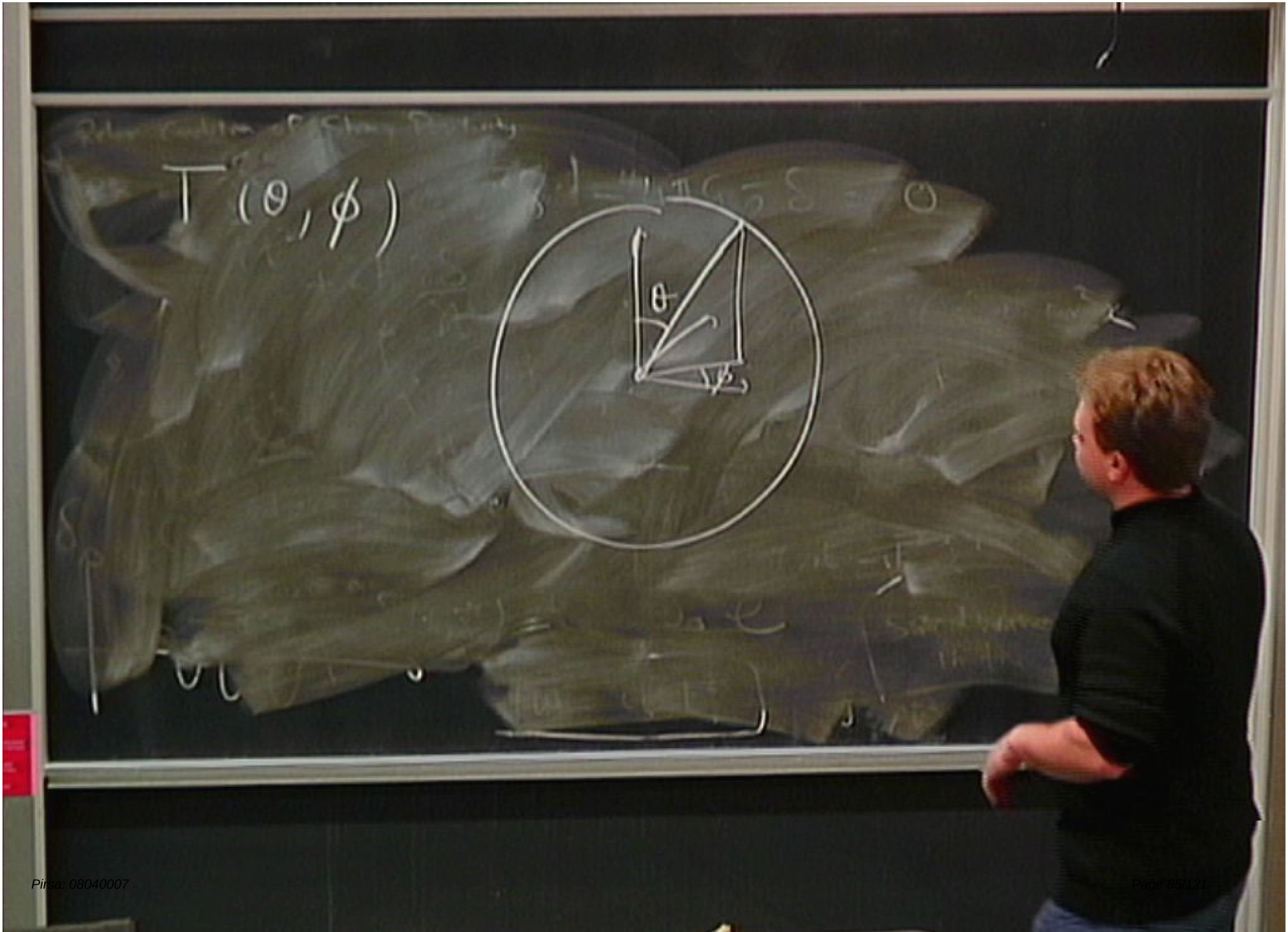
Cosmic Microwave Background

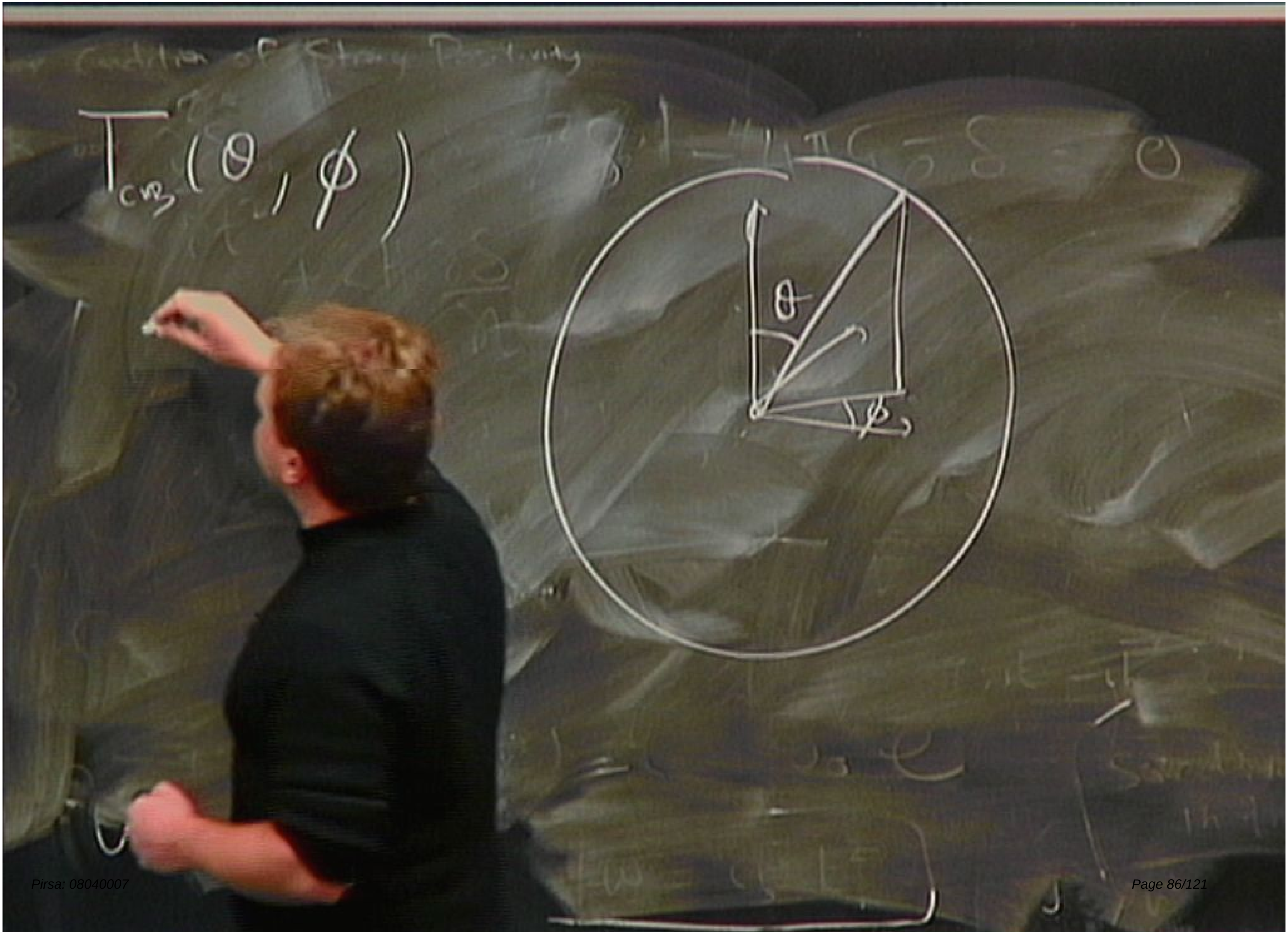
$t \approx 300000$ years


Cosmic Microwave Background
δ ~ 10⁻⁴
t ≈ 300000 years
halo
visible matter


$\delta \sim 10^{-4}$
ρ
10^{-4}
Cosmic Microwave Background
$t \approx 300000$ years
before galaxies formed
halo
visible matter

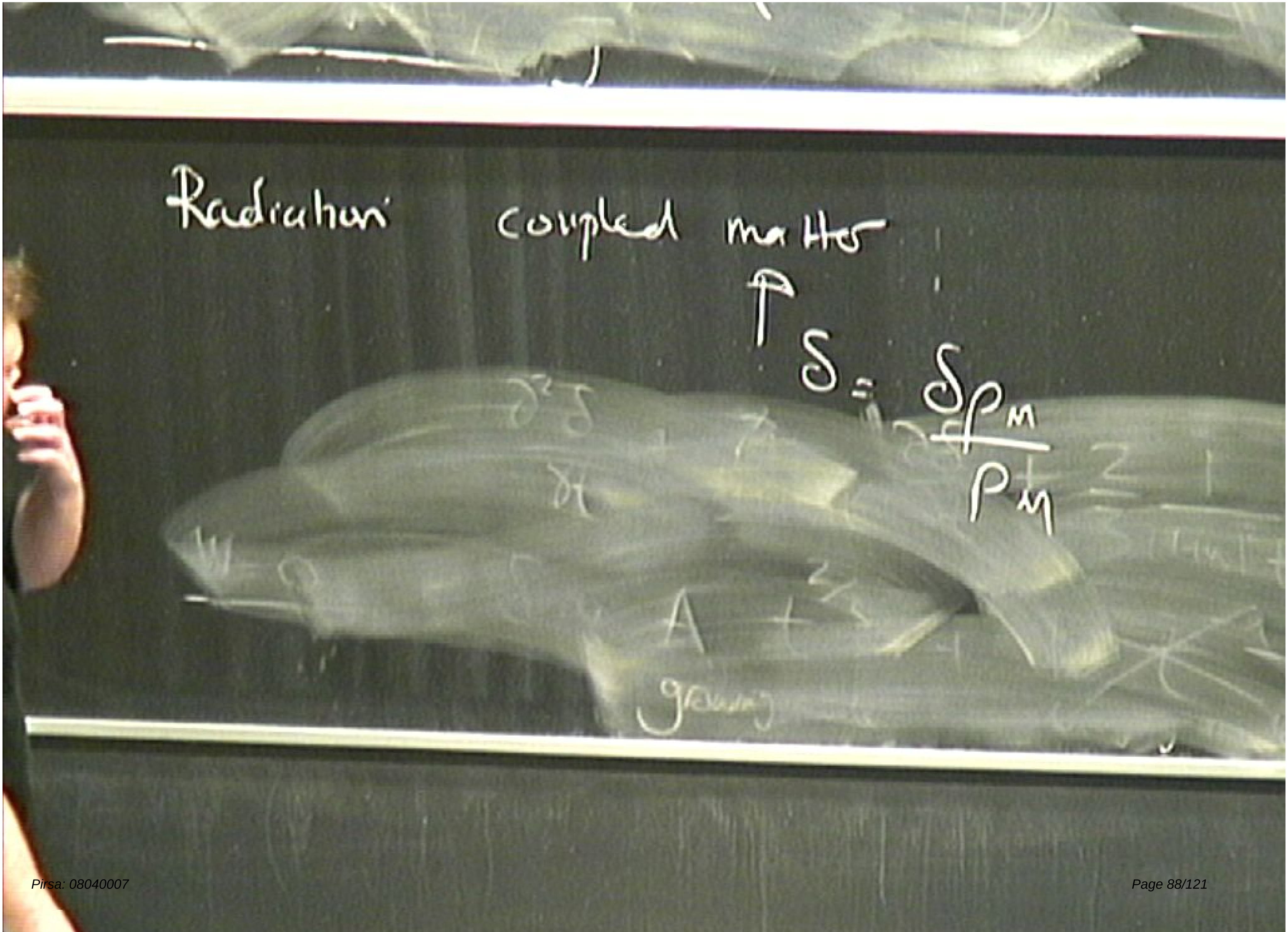

$\delta \sim 10^{-4}$
Cosmic Microwave Background
$t \approx 300000$ years
halo
visible matter
before galaxies formed
$T = 2.73K$


T (θ, φ)
θ
φ


Condition of Strong Positivity
$T_{cmb}(\theta, \phi)$
θ
ϕ

$$T_{cmb}(\theta, \phi) = \bar{T} + \delta T(\theta,\phi)$$

$$\frac{\delta T(\theta,\phi)}{\bar{T}} \sim 10^{-4}$$


Radiation coupled matter
$\delta = \frac{\delta\rho_M}{\rho_M}$
growing

Radiation   coupled matter

$$\frac{\delta T}{T} \qquad \delta = \frac{\delta \rho_M}{\bar{\rho}_M}$$

growing

Radiation coupled matter

$\frac{\delta T}{T}$

$\delta = \frac{\delta \rho_M}{\bar{\rho}_M}$

$\rho_M \sim \frac{1}{a^3}$

Radiation coupled matter

$$\frac{\delta T}{T} \qquad \delta = \frac{\delta \rho_M}{\bar{\rho}_M}$$

$$\rho_M \sim \frac{1}{a^3} \sim T^3$$

growing

Radiation coupled matter

$$\frac{\delta T}{T} \qquad T \sim \frac{1}{a} \qquad \delta = \frac{\delta \rho_M}{\bar{\rho}_M}$$

$$\rho_M \sim \frac{1}{a^3} \sim T^3$$

growing

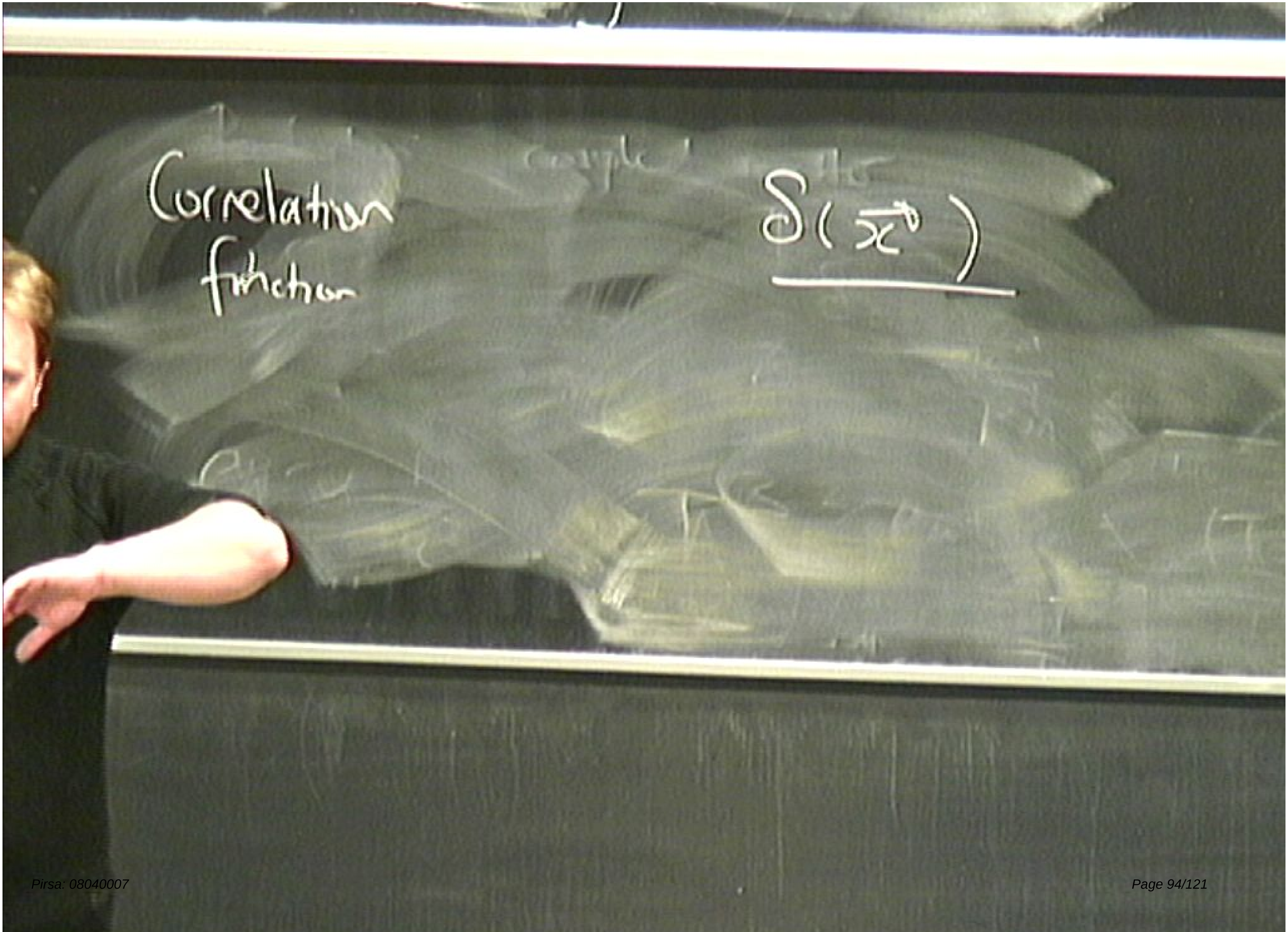
Radiation coupled matter

$$\frac{\delta T}{T} \qquad T \sim \frac{1}{a} \qquad \delta = \frac{\delta \rho_M}{\rho_M}$$

$$\rho_M \sim \frac{1}{a^3} \sim T^3 \qquad \rho_M + \delta\rho_M \propto (T + \delta T)^3$$

$$\sim T^3 + 3\,\delta T\, T^2$$

going $\frac{\delta\rho_M}{\rho_M} = 3\frac{\delta T}{T}$


Correlation function
$\delta(\vec{x})$

Correlation function

$\delta(\vec{x})$

$\bar{\delta} = \int \delta \, d^3x$

$= 0$

Correlation function

$\delta(\vec{x})$

$\bar{\delta} = \int \delta \, d^3x$

Variance

$= 0$

$\langle x^2 \rangle$ distribution $\left\langle \delta(\vec{x}) \, \delta(\vec{x}') \right\rangle$

Correlation function

$\delta(\vec{x})$

$\bar{\delta} = \int \delta \, d^3x = 0$

Variance distribution

$\langle x^2 \rangle$

$\langle \delta(\vec{x}) \, \delta(\vec{x}) \rangle$

$\langle \delta(\vec{x}) \, \delta(\vec{x} + \vec{r}) \rangle$

Correlation function

$\delta(\vec{x})$

$\bar{\delta} = \int \delta \, d^3x = 0$

Variance distribution

$\langle x^2 \rangle$

$\langle \delta(\vec{x}) \, \delta(\vec{x}) \rangle$

$\xi(\vec{r}) = \langle \delta(\vec{x}) \, \delta(\vec{x} + \vec{r}) \rangle$

Correlation function

$$\delta(\vec{x})$$

$$\bar{\delta} = \int \delta \, d^3x = 0$$

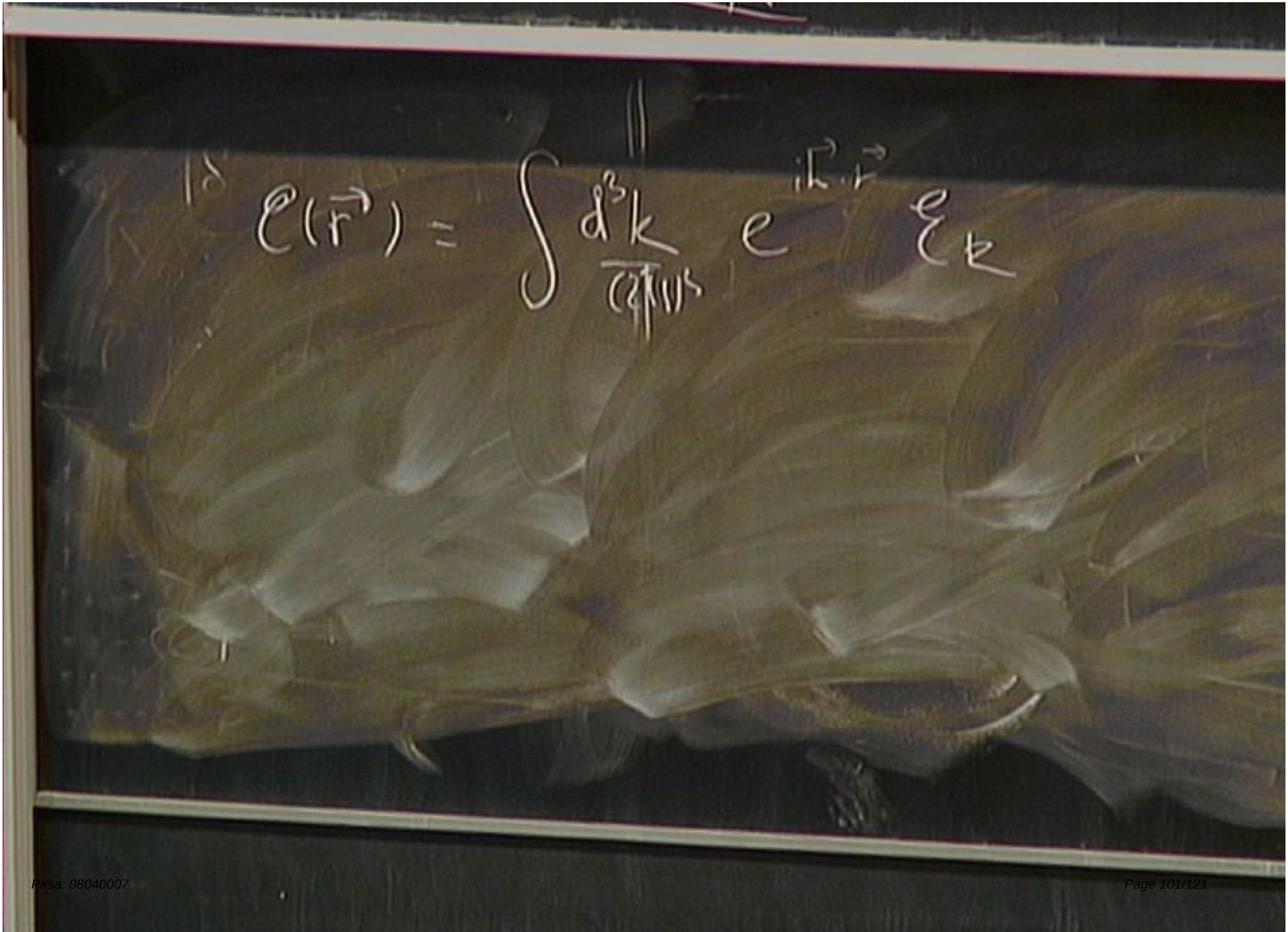
Variance distribution

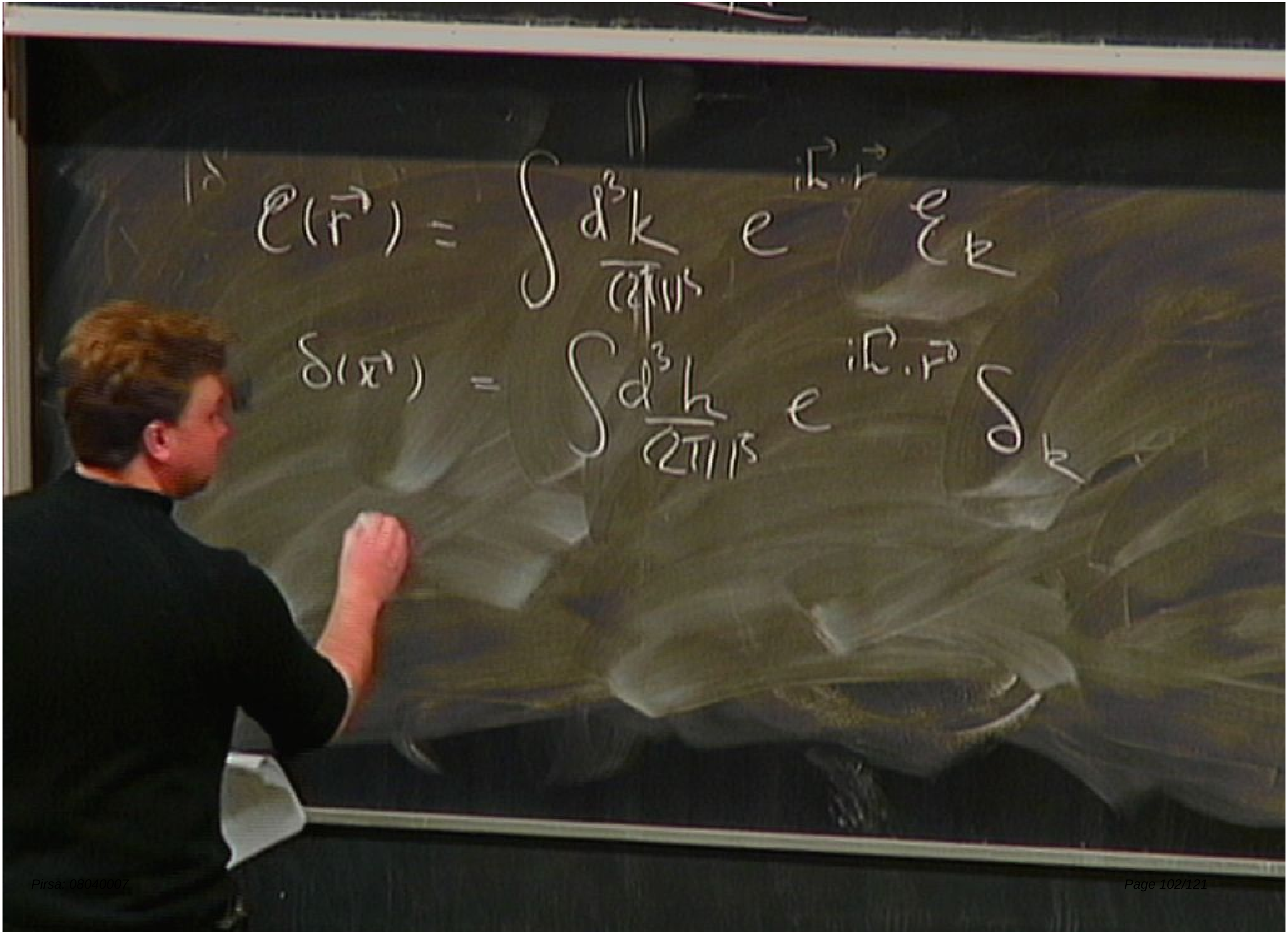
$$\langle x^2 \rangle$$

$$\langle \delta(\vec{x}) \, \delta(\vec{x}) \rangle$$

$$\boxed{\xi(\vec{r}) = \langle \delta(\vec{x}) \, \delta(\vec{x}+\vec{r}) \rangle}$$

$$\rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \rho_k$$


$\rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \rho_k$


$\rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \rho_k$
$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \delta_k$

$$\rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{r}}\, \varepsilon_k$$

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{r}}\, \delta_k$$

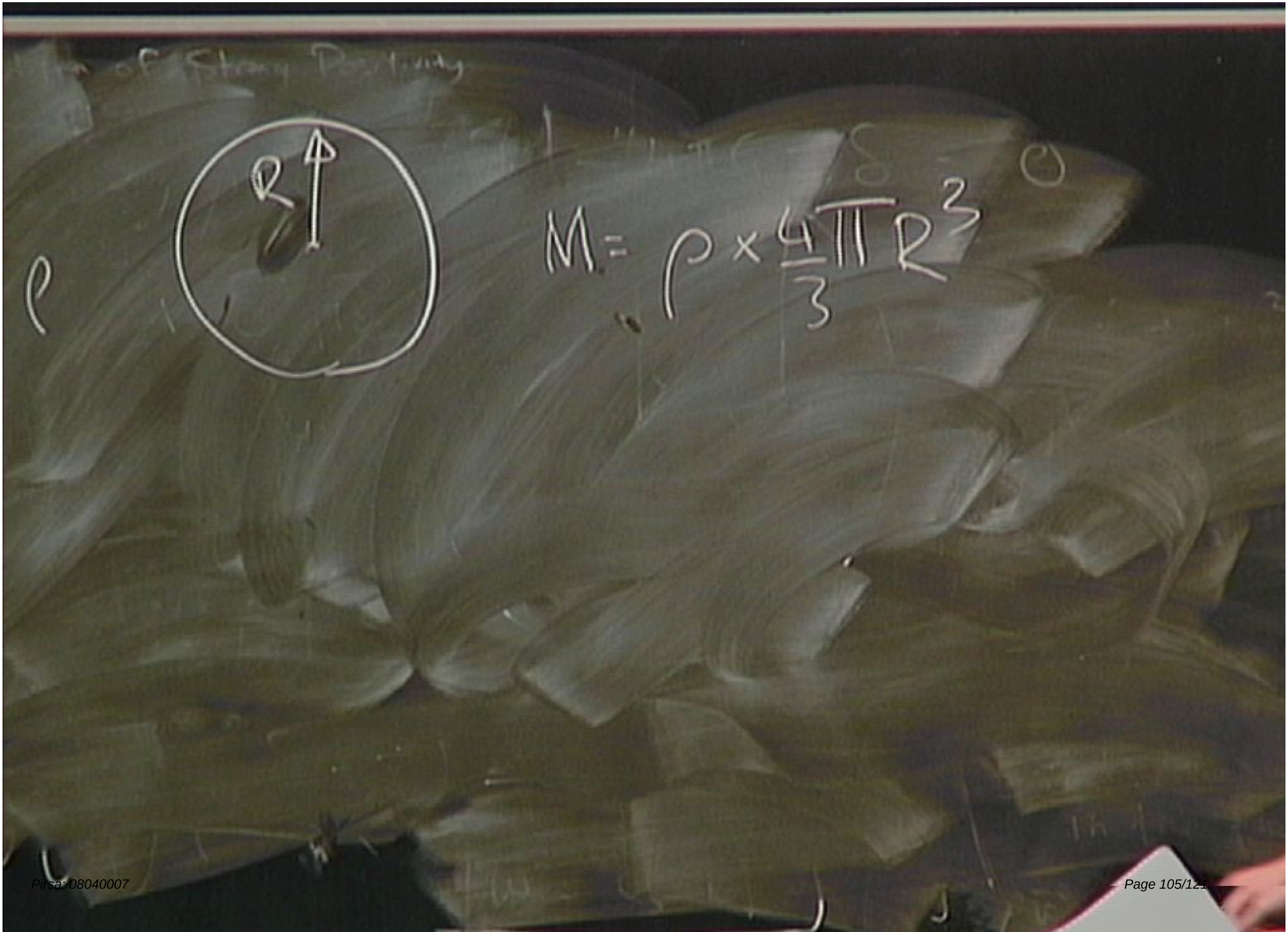
$$\boxed{\varepsilon_k = \langle \delta_k^2 \rangle}$$

$$\rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \, \varepsilon_k$$

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \, \delta_k$$

$$\boxed{\varepsilon_k = \langle \delta_k^2 \rangle}$$

Power spectrum of density fluctuation.


of Strong Positivity
R
ρ
$M = \rho \times \frac{4}{3}\pi R^3$

ρ

$$M = \rho \times \frac{4}{3}\pi R^3$$

$$\left(\frac{\delta M}{\bar{M}}\right)_h \qquad \delta M = M - \bar{M}$$

ρ

$$M = \rho \times \frac{4}{3}\pi R^3$$

$$\sigma(R) = \left(\frac{\delta M}{\bar{M}}\right)_h \qquad \delta M = M - \bar{M}$$

ρ

R

$$M = \rho \times \frac{4}{3}\pi R^3$$

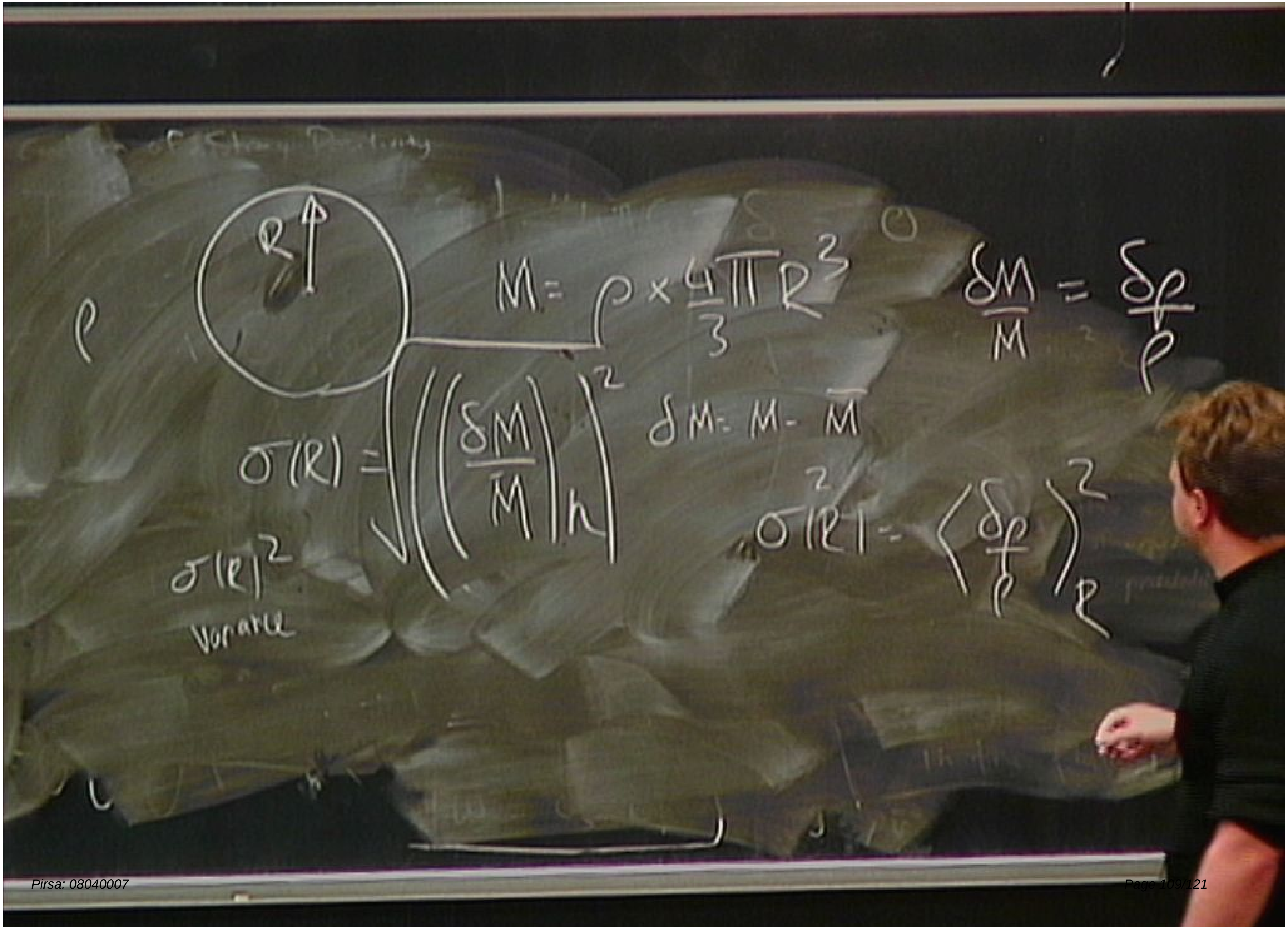
$$\frac{\delta M}{\bar{M}} = \frac{\delta \rho}{\rho}$$

$$\sigma(R) = \sqrt{\left\langle \left(\frac{\delta M}{\bar{M}} \right)_h^2 \right\rangle}$$

$$\delta M = M - \bar{M}$$

$\sigma(R)^2$

Varance


ρ
R
M = ρ × 4/3 π R^3
δM/M = δρ/ρ
σ(R) = √⟨(δM/M)²⟩_R
δM = M − M̄
σ²(R) = ⟨(δρ/ρ)²⟩_R
σ(R)²
Variance

ρ

$$M = \rho \times \frac{4}{3}\pi R^3$$

$$\frac{\delta M}{M} = \frac{\delta \rho}{\rho}$$

$$\sigma(R) = \sqrt{\left\langle \left(\frac{\delta M}{M}\right)^2 \right\rangle_R}$$

$$\delta M = M - \overline{M}$$

$$\sigma(R)^2 = \left\langle \left(\frac{\delta \rho}{\rho}\right)^2 \right\rangle_R$$

$\sigma(R)^2$

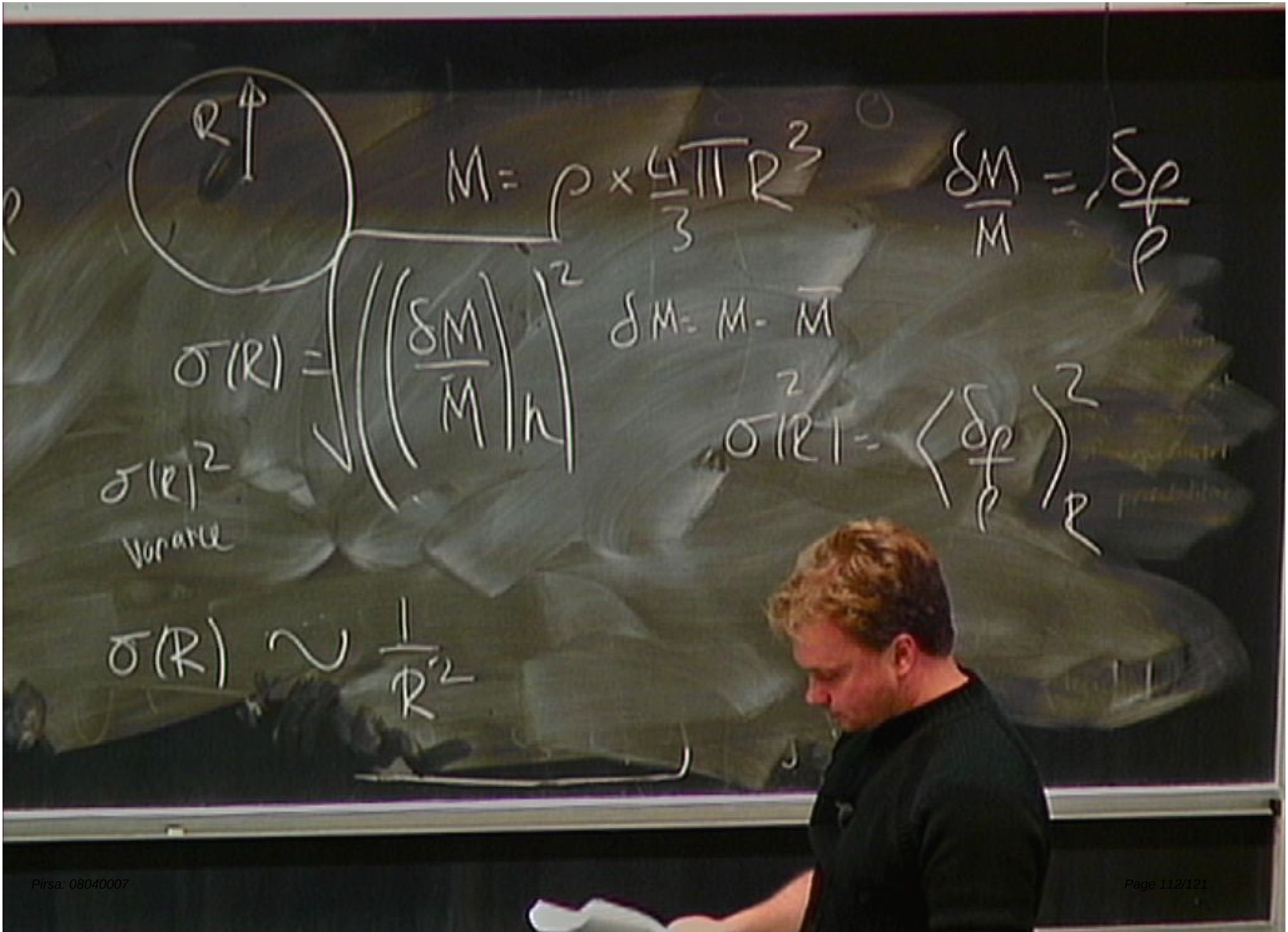
variance

$$\xi(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \mathcal{E}_k$$

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \delta_k$$

$$\boxed{\mathcal{E}_k = \langle \delta_k^2 \rangle} \qquad \sigma(R)^2 = \mathcal{E}_{k=\frac{1}{R}}$$

Power spectrum of density fluctuation.


R
$M = \rho \times \frac{4}{3}\pi R^3$
$\frac{\delta M}{M} = \frac{\delta\rho}{\rho}$
$\sigma(R) = \sqrt{\left\langle\left(\frac{\delta M}{\bar{M}}\right)^2\right\rangle_R}$
$\delta M = M - \bar{M}$
$\sigma(R)^2 = \left\langle\frac{\delta\rho}{\rho}\right\rangle_R^2$
$\sigma(R)^2$
variance
$\sigma(R) \sim \frac{1}{R^2}$

$$\rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \rho_k$$

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \delta_k$$

$$\boxed{P_k = \langle \delta_k^2 \rangle} \qquad \sigma(R)^2 = P_{k=\frac{1}{R}} \times k^3$$

$$P_{k} = \frac{1}{k} \frac{1}{R^3}$$

Power spectrum of density fluctuation

$$M = \rho \times \frac{4}{3}\pi R^3 \qquad \frac{\delta M}{M} = \frac{\delta \rho}{\rho}$$

$$\sigma(R) = \sqrt{\left(\left(\frac{\delta M}{\bar{M}}\right)_h\right)^2} \qquad \delta M = M - \bar{M}$$

$$\sigma(R)^2$$

variance

$$\sigma(R)^2 = \left\langle \frac{\delta\rho}{\rho} \right\rangle^2_R$$

$$\sigma^2(R) \sim \frac{1}{R^4} = e_{k = \frac{1}{R}} \times \frac{1}{R^3}$$

$$\sigma(R) \sim \frac{1}{R^2}$$

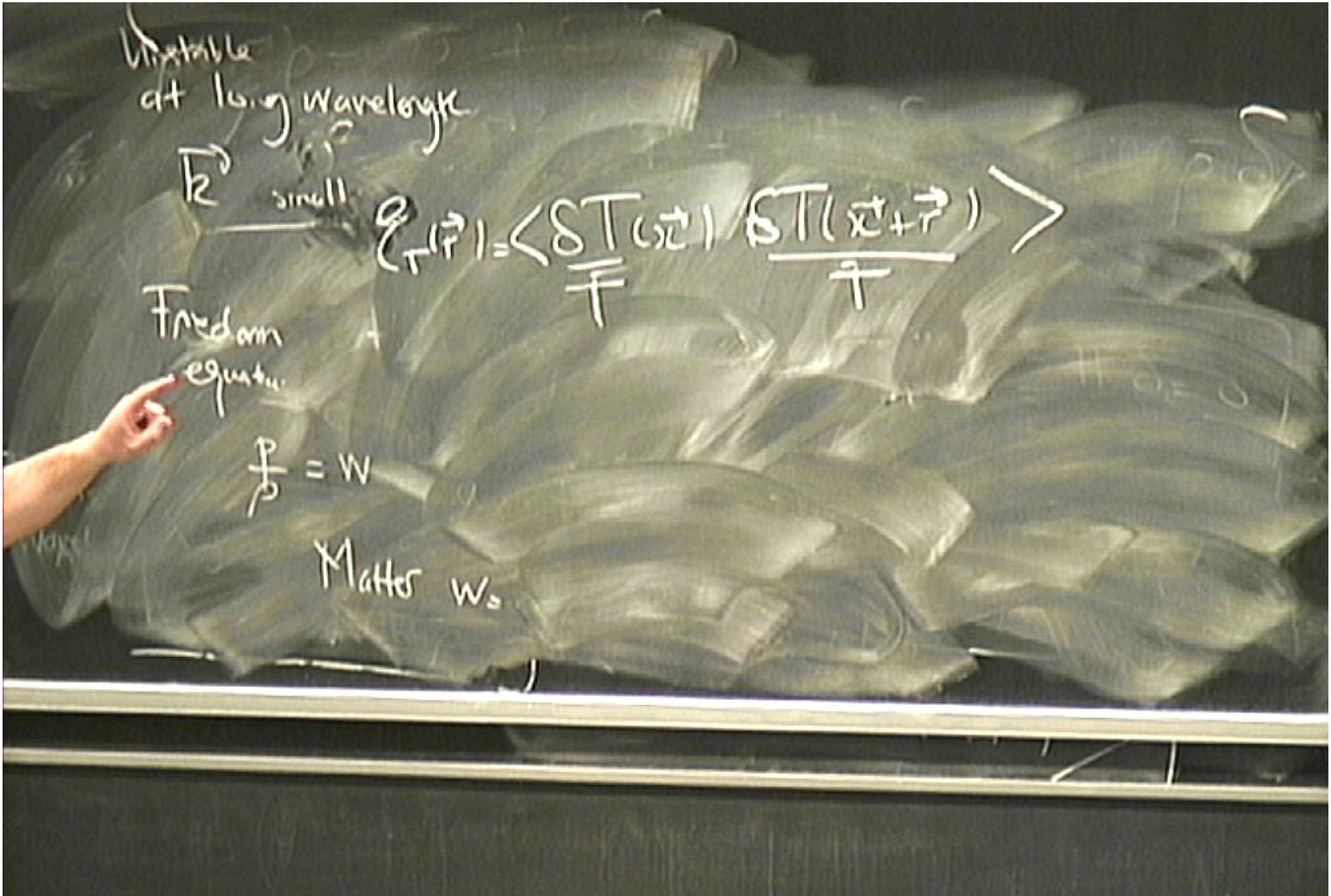
$$\boxed{e_k = \frac{1}{dk} \cdot k}$$

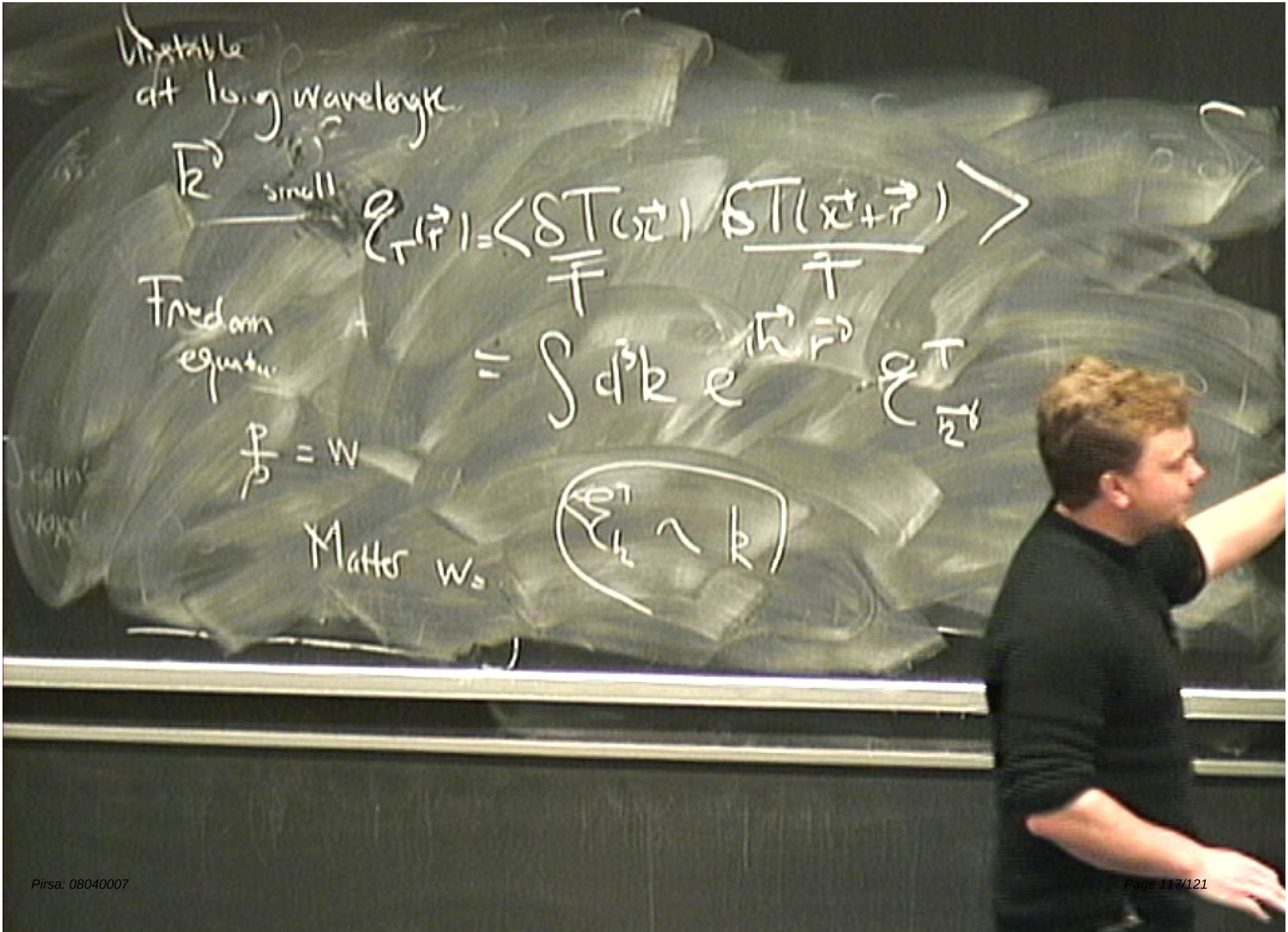
$$\rho(\vec{r}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{r}}\, \rho_k \sim \boxed{\rho_k k}$$

$$\delta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{r}}\, \delta_k$$

$$\boxed{\rho_k = \langle \delta_k^2 \rangle} \qquad \sigma(R)^2 = \rho_{k=\frac{1}{R}} \times k^3 \sim \rho_{k=\frac{1}{R}} \frac{1}{R^3}$$

Power spectrum of density fluctuation


Unstable
at long wavelength
k small
ξ_T(r) = < δT(x)/T δT(x+r)/T >
Friedman equation
p/ρ = w
Matter w=


Unstable
at long wavelength
small
Friedman
equation
Matter w=

Unstable
at long wavelength

$\vec{k}$ small

$$\xi_T(\vec{r}) = \left\langle \frac{\delta T(\vec{x})}{\bar{T}} \frac{\delta T(\vec{x}+\vec{r})}{\bar{T}} \right\rangle$$

Friedman equation

$$= \int d^3k \, e^{i\vec{k}\cdot\vec{r}} \, \xi^T_{\vec{k}}$$

$$\frac{p}{\rho} = w$$

Matter w=

$$\xi^T_k \sim k$$

Unstable at long wavelength

$\vec{k}$ small

$$\xi_T(\vec{r}) = \left\langle \frac{\delta T(\vec{x})}{T} \frac{\delta T(\vec{x}+\vec{r})}{T} \right\rangle$$

$$= \int d^3k \, e^{i\vec{k}\cdot\vec{r}} \, \xi^T_{\vec{k}}$$

Friedmann equation

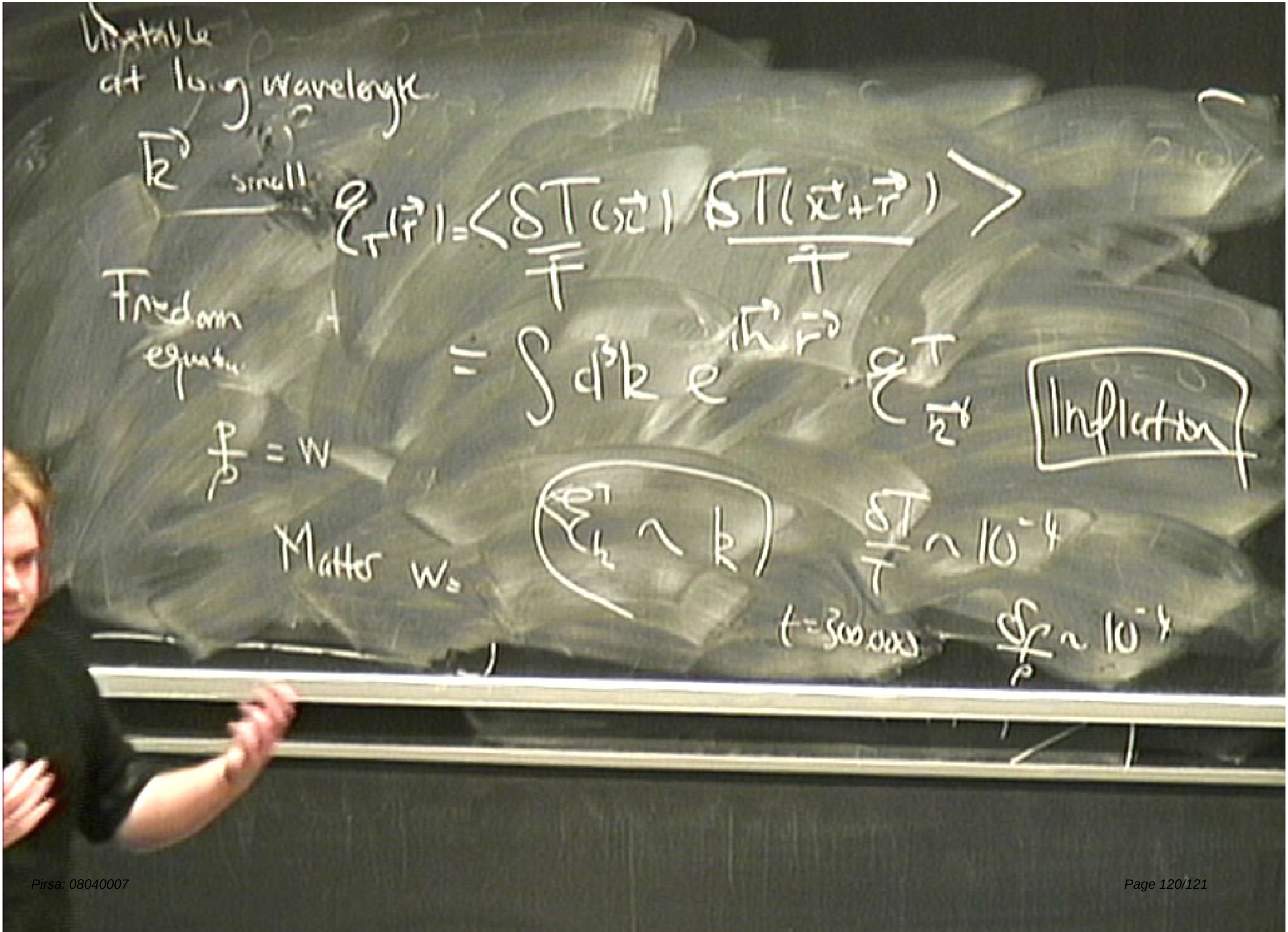
$\frac{p}{\rho} = w$

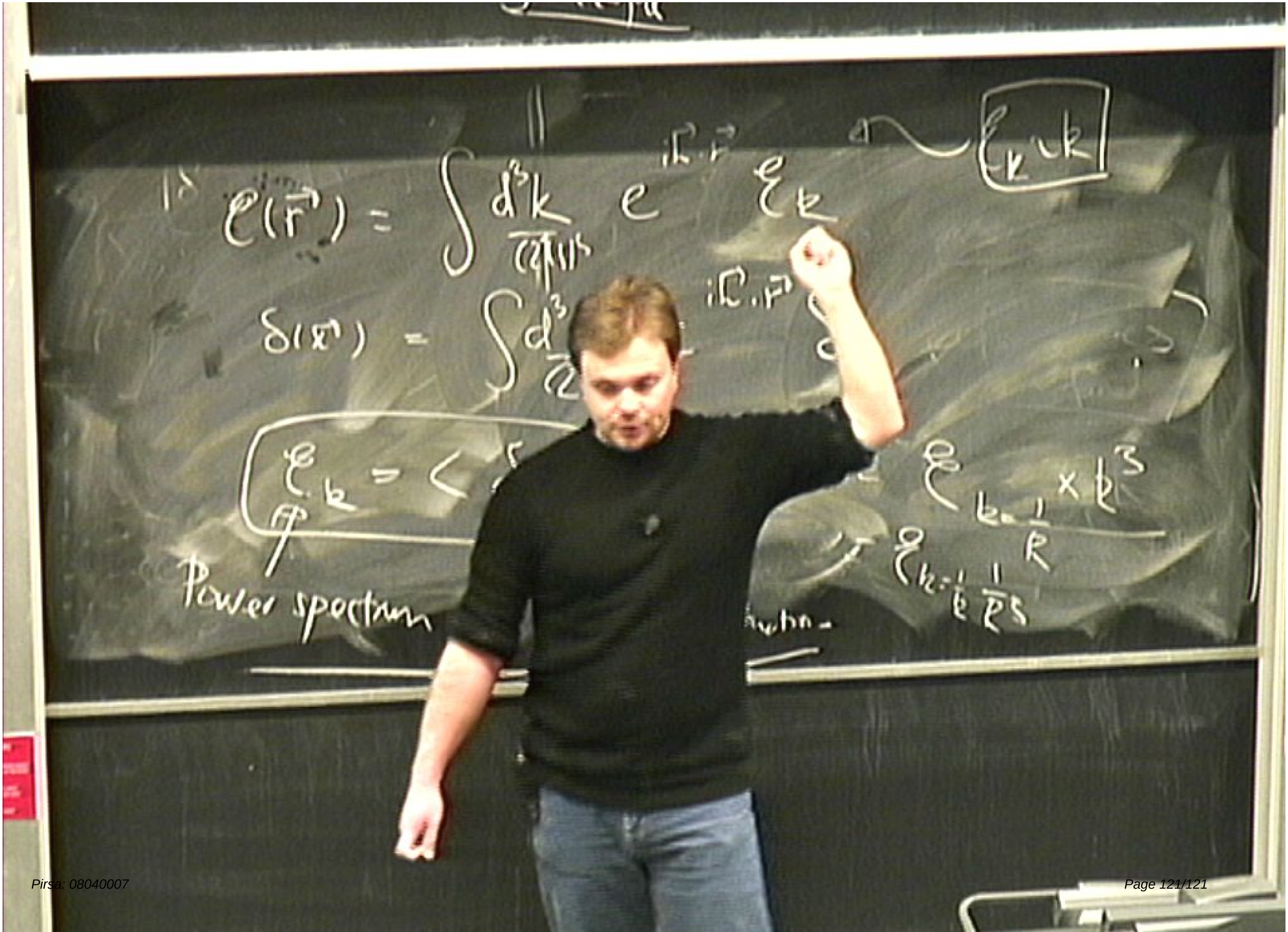
Matter $w=0$

$$\xi^T_k \sim k$$

$\frac{\delta T}{T} \sim 10^{-4}$

$t = 300{,}000$ $\quad \frac{\delta\rho}{\rho} \sim 10^{-4}$


Unstable
at long wavelength
$\vec{k}$ small
$\xi_T(\vec{r}) = \left\langle \frac{\delta T(\vec{x})}{T} \frac{\delta T(\vec{x}+\vec{r})}{T} \right\rangle$
$= \int d^3k \, e^{i\vec{k}\cdot\vec{r}} \, \xi^T_{\vec{k}}$
Friedmann equation
$\frac{p}{\rho} = w$
Inflation
Matter w=
$\xi^T_k \sim k$
$\frac{\delta T}{T} \sim 10^{-4}$
t=300,000
$\frac{\delta\rho}{\rho} \sim 10^{-4}$


Power spectrum