Title: Nonlocality and string field theory

Date: Apr 22, 2008 02:00 PM

URL: http://pirsa.org/08040005

Abstract: We introduce a formalism allowing us to localize a certain class of theories with an infinite number of derivatives (nonlocal), which include effective actions of string field theory. The number of degrees of freedom is finite and the Cauchy problem, Hamiltonian and quantization are all well-defined. As applications, the rolling tachyon of cubic string field theory and some cosmological toy models are considered.

Nonlocal systems and string field theory

Gianluca Calcagni

University of Sussex PENNSTATE

April 22th, 2008

Based on

1. G.C., *Cosmological tachyon from cubic string field theory*, JHEP 05 (2006) 012 [hep-th/0512259].
2. G.C., M. Montobbio, G. Nardelli, *Route to nonlocal cosmology*, PRD D **76**, 126001 (2007) 0705.3043 [hep-th].
3. G.C., G. Nardelli, *Tachyon solutions in boundary and cubic string field theory*, 0708.0366 [hep-th].
4. G.C., M. Montobbio, G. Nardelli, *Localization of nonlocal theories*, PLB **662**, 285 (2008) [0712.2237 [hep-th]].
5. G.C., G. Nardelli, *Nonlocal instantons and solitons*, 0802.4395 [hep-th].

Outline

1. Motivations
   - Tachyon and tachyons
   - Three questions
2. Localization
   - $1 + D$ dimension
   - Construction of solutions
3. Examples of solutions
   - $\Phi(0, x)$ nontrivial
   - $\Phi(0, x)$ distribution
4. SFT correspondences
   - RG flow along r
   - SFT correspondences

The many faces of the string tachyon

The many faces of the string tachyon

- Conformal field theory [Sen 2002].

The many faces of the string tachyon

- Conformal field theory [Sen 2002].
- Dirac–Born–Infeld (DBI) effective action [Garousi 2000; Bergshoeff et al. 2000; Kluson 2000; Gibbons, Hori, and Yi 2001].

The many faces of the string tachyon

- Conformal field theory [Sen 2002].
- Dirac–Born–Infeld (DBI) effective action [Garousi 2000; Bergshoeff et al. 2000; Kluson 2000; Gibbons, Hori, and Yi 2001].
- Boundary string field theory (BSFT) [Witten 1992,1993; Shatashvili 1993a,b].

The many faces of the string tachyon

- Conformal field theory [Sen 2002].
- Dirac–Born–Infeld (DBI) effective action [Garousi 2000; Bergshoeff et al. 2000; Klusoň 2000; Gibbons, Hori, and Yi 2001].
- Boundary string field theory (BSFT) [Witten 1992,1993; Shatashvili 1993a,b].
- Open string field theory (OSFT, CSFT) [Witten 1986a,b; Preitschopf et al. 1990; Aref'eva et al. 2002; Aref'eva et al. 1990a,b; Berkovits 1996,1999].

The many faces of the string tachyon

- Conformal field theory [Sen 2002].
- Dirac–Born–Infeld (DBI) effective action [Garousi 2000; Bergshoeff et al. 2000; Klusoň 2000; Gibbons, Hori, and Yi 2001].
- Boundary string field theory (BSFT) [Witten 1992,1993; Shatashvili 1993a,b].
- Open string field theory (OSFT, CSFT) [Witten 1986a,b; Preitschopf et al. 1990; Aref'eva et al. 2002; Aref'eva et al. 1990a,b; Berkovits 1996,1999].

For reviews see Sen (hep-th/0410103) and Ohmori (hep-th/0102085).

Bosonic OSFT tachyon

Bosonic OSFT tachyon

Cubic SFT effective action at level $(0,0)$ (metric $-+++\ldots$)

Bosonic OSFT tachyon

Cubic SFT effective action at level $(0,0)$ (metric $-+++\dots$)

$$\bar{S}_\phi = \frac{1}{g_o^2}\int d^D x\left[\frac{1}{2\alpha'}\phi(\Box+1)\phi - \frac{e^{3r_*}}{3}\left(e^{r_*\Box}\phi\right)^3 - \Lambda\right]$$

where $r_* = \ln(3^{3/2}/4) \approx 0.26$

Bosonic OSFT tachyon

Cubic SFT effective action at level $(0,0)$ (metric $-+++\dots$)

$$\bar{S}_\phi = \frac{1}{g_o^2}\int d^Dx\left[\frac{1}{2\alpha'}\phi(\Box+1)\phi - \frac{e^{3r_*}}{3}\left(e^{r_*\Box}\phi\right)^3 - \Lambda\right]$$

where $r_* = \ln(3^{3/2}/4) \approx 0.26$ and

$$e^{r_*\Box} = \sum_{\ell=0}^{+\infty}\frac{r_*^\ell}{\ell!}\Box^\ell$$

Bosonic OSFT tachyon

Cubic SFT effective action at level $(0,0)$ (metric $-+++\ldots$)

$$\bar{S}_\phi = \frac{1}{g_o^2}\int d^D x\left[\frac{1}{2\alpha'}\phi(\Box+1)\phi - \frac{e^{3r_*}}{3}\left(e^{r_*\Box}\phi\right)^3 - \Lambda\right]$$

where $r_* = \ln(3^{3/2}/4) \approx 0.26$ and

$$e^{r_*\Box} = \sum_{\ell=0}^{+\infty}\frac{r_*^\ell}{\ell!}\Box^\ell$$

Theories with an ∞ number of derivatives are called nonlocal.

Outline

1. Motivations
   - Tachyon and tachyons
   - Three questions
2. Localization
   - $1 + D$ dimension
   - Construction of solutions
3. Examples of solutions
   - $\Phi(0, x)$ nontrivial
   - $\Phi(0, x)$ distribution
4. SFT correspondences
   - RG flow along r
   - SFT correspondences

1: How to deal with nonlocal theories?

1: How to deal with nonlocal theories?

- Non-standard Cauchy problem:

1: How to deal with nonlocal theories?

- Non-standard Cauchy problem: infinite initial conditions, to know them means to find the solution if analytic in its domain [Moeller and Zwiebach 2002].

1: How to deal with nonlocal theories?

- Non-standard Cauchy problem: infinite initial conditions, to know them means to find the solution if analytic in its domain [Moeller and Zwiebach 2002].
- Infinitely many conjugate momenta, Hamiltonian and quantization unclear.

1: How to deal with nonlocal theories?

- Non-standard Cauchy problem: infinite initial conditions, to know them means to find the solution if analytic in its domain [Moeller and Zwiebach 2002].
- Infinitely many conjugate momenta, Hamiltonian and quantization unclear.
- Difficult systematic construction of solutions. Only perturbative methods available [Eliezer and Woodard 1989; Cheng et al. 2002].

1: How to deal with nonlocal theories?

- Non-standard Cauchy problem: infinite initial conditions, to know them means to find the solution if analytic in its domain [Moeller and Zwiebach 2002].
- Infinitely many conjugate momenta, Hamiltonian and quantization unclear.
- Difficult systematic construction of solutions. Only perturbative methods available [Eliezer and Woodard 1989; Cheng et al. 2002].
- Nonlocal theories at odds with the inflationary paradigm:

1: How to deal with nonlocal theories?

- Non-standard Cauchy problem: infinite initial conditions, to know them means to find the solution if analytic in its domain [Moeller and Zwiebach 2002].
- Infinitely many conjugate momenta, Hamiltonian and quantization unclear.
- Difficult systematic construction of solutions. Only perturbative methods available [Eliezer and Woodard 1989; Cheng et al. 2002].
- Nonlocal theories at odds with the inflationary paradigm: while the latter tends to erase any memory of the initial conditions, nonlocality does preserve this memory.

1: How to deal with nonlocal theories?

- Non-standard Cauchy problem: infinite initial conditions, to know them means to find the solution if analytic in its domain [Moeller and Zwiebach 2002].
- Infinitely many conjugate momenta, Hamiltonian and quantization unclear.
- Difficult systematic construction of solutions. Only perturbative methods available [Eliezer and Woodard 1989; Cheng et al. 2002].
- Nonlocal theories at odds with the inflationary paradigm: while the latter tends to erase any memory of the initial conditions, nonlocality does preserve this memory. SR approximation unclear:

1: How to deal with nonlocal theories?

- Non-standard Cauchy problem: infinite initial conditions, to know them means to find the solution if analytic in its domain [Moeller and Zwiebach 2002].
- Infinitely many conjugate momenta, Hamiltonian and quantization unclear.
- Difficult systematic construction of solutions. Only perturbative methods available [Eliezer and Woodard 1989; Cheng et al. 2002].
- Nonlocal theories at odds with the inflationary paradigm: while the latter tends to erase any memory of the initial conditions, nonlocality does preserve this memory. SR approximation unclear: The cosmological eom's are nonlinear and involve the whole infinite SR tower.

2: What are the nonperturbative solutions of CSFT?

2: What are the nonperturbative solutions of CSFT?

- Coefficients of the series oscillatory solution difficult to compute [Kiermaier et al. 2007].

2: What are the nonperturbative solutions of CSFT?

- Coefficients of the series oscillatory solution difficult to compute [Kiermaier et al. 2007].
- Undetermined convergence properties of perturbative solutions [Coletti et al. 2005; Forini et al. 2006].

3: What are the cosmological predictions of CSFT tachyon?

3: What are the cosmological predictions of CSFT tachyon?

- SFT tachyon: finite-order/perturbative solutions [Aref'eva et al. 2007].

3: What are the cosmological predictions of CSFT tachyon?

- SFT tachyon: finite-order/perturbative solutions [Aref'eva et al. 2007].
- DBI tachyon (local): extensively studied both as inflaton and dark energy field

3: What are the cosmological predictions of CSFT tachyon?

- SFT tachyon: finite-order/perturbative solutions [Aref'eva et al. 2007].
- DBI tachyon (local): extensively studied both as inflaton and dark energy field but problematic or ineffective in both cases.

3: What are the cosmological predictions of CSFT tachyon?

- SFT tachyon: finite-order/perturbative solutions [Aref'eva et al. 2007].
- DBI tachyon (local): extensively studied both as inflaton and dark energy field but problematic or ineffective in both cases.
- The equation of state of CSFT tachyon is less rigid than the DBI one.

3: What are the cosmological predictions of CSFT tachyon?

- SFT tachyon: finite-order/perturbative solutions [Aref'eva et al. 2007].
- DBI tachyon (local): extensively studied both as inflaton and dark energy field but problematic or ineffective in both cases.
- The equation of state of CSFT tachyon is less rigid than the DBI one. A comparison would open up interesting possibilities. ▸ Skip details

General nonlocal system

$$\mathcal{K}e^{-2r_*\Box}\phi(x) = U'[\phi(x)]$$

General nonlocal system

$$\boxed{\mathcal{K} e^{-2r_* \Box} \phi(x) = U'[\phi(x)]}$$

OSFT: $r_* \approx 0.26, \qquad \mathcal{K} = \Box - m^2, \qquad U \sim \phi^3, \phi^4$

General nonlocal system

$$\boxed{\mathcal{K}e^{-2r_*\Box}\phi(x) = U'[\phi(x)]}$$

OSFT: $r_* \approx 0.26, \quad \mathcal{K} = \Box - m^2, \quad U \sim \phi^3, \phi^4$

p-adic string: $r_* = (\ln p)/4, \quad \mathcal{K} = 1, \quad U \sim \phi^{p+1}$

An analytic recipe

We propose a systematic method which allows:

An analytic recipe

We propose a systematic method which allows:

1:

To address all issues of nonlocality, including problem of initial conditions and consistent quantization.

An analytic recipe

We propose a systematic method which allows:

1:

To address all issues of nonlocality, including problem of initial conditions and consistent quantization.

2:

To construct nonperturbative solutions (at least approximate, sometimes exact) of general nonlocal systems on Minkowski and curved backgrounds. Perturbative solutions are automatically recovered.

An analytic recipe

We propose a systematic method which allows:

1:

To address all issues of nonlocality, including problem of initial conditions and consistent quantization.

2:

To construct nonperturbative solutions (at least approximate, sometimes exact) of general nonlocal systems on Minkowski and curved backgrounds. Perturbative solutions are automatically recovered.

See Joukovskaya 2007 for a recent numerical method.

Outline

1. Motivations
   - Tachyon and tachyons
   - Three questions
2. Localization
   - $1 + D$ dimension
   - Construction of solutions
3. Examples of solutions
   - $\Phi(0, x)$ nontrivial
   - $\Phi(0, x)$ distribution
4. SFT correspondences
   - RG flow along r
   - SFT correspondences

A constant which is not such

A constant which is not such

- Interpret r_* as a fixed value of an auxiliary evolution variable r.

A constant which is not such

- Interpret r_* as a fixed value of an auxiliary evolution variable r.
- The scalar field is thought to live in $1 + D$ dimensions, $\phi(x) \to \Phi(r, x)$.

A constant which is not such

- Interpret r_* as a fixed value of an auxiliary evolution variable r.
- The scalar field is thought to live in $1+D$ dimensions, $\phi(x) \to \Phi(r, x)$.
- The solution $\phi_{\rm loc}(x) = \Phi(0, x)$ of the local system $(r_* = 0)$ is the "initial field configuration" of a flow along r, while $\phi(x) = \Phi(r_*, x)$.

Localized action

with M. Montobbio and G. Nardelli, PLB **662**, 285 (2008) [0712.2237 [hep-th]]

Localized action

with M. Montobbio and G. Nardelli, PLB **662**, 285 (2008) [0712.2237 [hep-th]]

$$\mathcal{S}_\Phi = \int d^D x \sqrt{-g} \int dr \left(\mathcal{L}_\Phi + \mathcal{L}_\chi \right)$$

Localized action

with M. Montobbio and G. Nardelli, PLB **662**, 285 (2008) [0712.2237 [hep-th]]

$$\mathcal{S}_\Phi = \int d^D x \sqrt{-g} \int dr \, (\mathcal{L}_\Phi + \mathcal{L}_\chi)$$

$$\mathcal{L}_\Phi = \tfrac{1}{2}\Phi(r,x)\mathcal{K}\Phi(r-2\gamma r,x) - U[\Phi(r,x)]$$

Localized action

with M. Montobbio and G. Nardelli, PLB **662**, 285 (2008) [`0712.2237 [hep-th]`]

$$\mathcal{S}_\Phi = \int d^D x \sqrt{-g} \int dr \, (\mathcal{L}_\Phi + \mathcal{L}_\chi)$$

$$\begin{aligned} \mathcal{L}_\Phi &= \tfrac{1}{2}\Phi(r,x)\mathcal{K}\Phi(r-2\gamma r,x) - U[\Phi(r,x)] \\ \mathcal{L}_\chi &= \int_0^r ds \, \chi(r',x)(\gamma\partial_{r''} - \Box)\Phi(r'',x) \end{aligned}$$

Localized action

with M. Montobbio and G. Nardelli, PLB **662**, 285 (2008) [0712.2237 [hep-th]]

$$\mathcal{S}_\Phi = \int d^D x \sqrt{-g} \int dr \left(\mathcal{L}_\Phi + \mathcal{L}_\chi \right)$$

$$\begin{aligned} \mathcal{L}_\Phi &= \tfrac{1}{2}\Phi(r,x)\mathcal{K}\Phi(r-2\gamma r,x) - U[\Phi(r,x)] \\ \mathcal{L}_\chi &= \int_0^r ds\, \chi(r',x)(\gamma\partial_{r''} - \Box)\Phi(r'',x) \end{aligned}$$

where

$$r' \equiv r - 2\gamma s\,, \qquad r'' \equiv r(1-2\gamma) + 2\gamma s$$

Equations of motion

Equations of motion

Proper e.o.m.:

$$\mathcal{K}\Phi(r_* - 2\gamma r_*, x) - U'[\Phi(r_*, x)] = 0.$$

Equations of motion

Proper e.o.m.:

$$\mathcal{K}\Phi(r_* - 2\gamma r_*, x) - U'[\Phi(r_*, x)] = 0.$$

Diffusion equation:

$$(\Box - \gamma\partial_r)\Phi(r, x) = 0.$$

Equations of motion

Proper e.o.m.:

$$\mathcal{K}\Phi(r_* - 2\gamma r_*, x) - U'[\Phi(r_*, x)] = 0.$$

Diffusion equation:

$$(\Box - \gamma\partial_r)\Phi(r, x) = 0.$$

Constraint:

$$\chi(r, x) = \mathcal{K}\Phi(r, x).$$

The diffusion equaton

The diffusion equaton

Its solution is

$$\Phi(r,x) = e^{r\Box/\gamma}\Phi(0,x)$$

The diffusion equaton

Its solution is

$$\Phi(r,x) = e^{r\Box/\gamma}\Phi(0,x)$$

$e^{q\Box}$ operators simply shift the auxiliary variable r:

$$e^{q\Box}\Phi(r,x) = e^{\gamma q\,\partial_r}\phi(r,x) = \phi(r+\gamma q,x)$$

The diffusion equaton

Its solution is

$$\Phi(r,x) = e^{r\Box/\gamma}\Phi(0,x)$$

$e^{q\Box}$ operators simply shift the auxiliary variable r:

$$e^{q\Box}\Phi(r,x) = e^{\gamma q\,\partial_r}\phi(r,x) = \phi(r+\gamma q,x)$$

The system becomes *local* in x, for any background metric.

The diffusion equaton

Its solution is

$$\Phi(r,x) = e^{r\Box/\gamma}\Phi(0,x)$$

$e^{q\Box}$ operators simply shift the auxiliary variable r:

$$e^{q\Box}\Phi(r,x) = e^{\gamma q\,\partial_r}\phi(r,x) = \phi(r+\gamma q, x)$$

The system becomes *local* in x, for any background metric.

!

Cauchy problem and Hamiltonian well-defined: Two d.o.f., four initial conditions (Φ, $\dot{\Phi}$, χ, $\dot{\chi}$)

Energy-momentum tensor ($\mathcal{K} = \Box$)

Energy-momentum tensor ($\mathcal{K} = \Box$)

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + \int_0^r ds\, \tilde{T}_{\mu\nu}(s)\,,$$

Energy-momentum tensor ($\mathcal{K} = \Box$)

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + \int_0^r ds\, \tilde{T}_{\mu\nu}(s)\,,$$

$$T^{(0)}_{\mu\nu} = \partial_\mu \Phi(r,x)\partial_\nu \Phi(r-2\gamma r,x) - g_{\mu\nu}\left[\tfrac{1}{2}\partial_\sigma \Phi(r,x)\partial^\sigma \Phi(r-2\gamma r,x) + U\right],$$

Energy-momentum tensor ($\mathcal{K} = \Box$)

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + \int_0^r ds\, \tilde{T}_{\mu\nu}(s)\,,$$

$$T^{(0)}_{\mu\nu} = \partial_\mu \Phi(r,x)\partial_\nu \Phi(r-2\gamma r,x) - g_{\mu\nu}\left[\tfrac{1}{2}\partial_\sigma \Phi(r,x)\partial^\sigma \Phi(r-2\gamma r,x) + U\right],$$

$$\tilde{T}_{\mu\nu}(s) \equiv g_{\mu\nu}(\partial_\sigma \chi \partial^\sigma \Phi + \chi \Box \Phi) - 2\partial_\mu \chi \partial_\nu \Phi.$$

Energy-momentum tensor ($\mathcal{K} = \Box$)

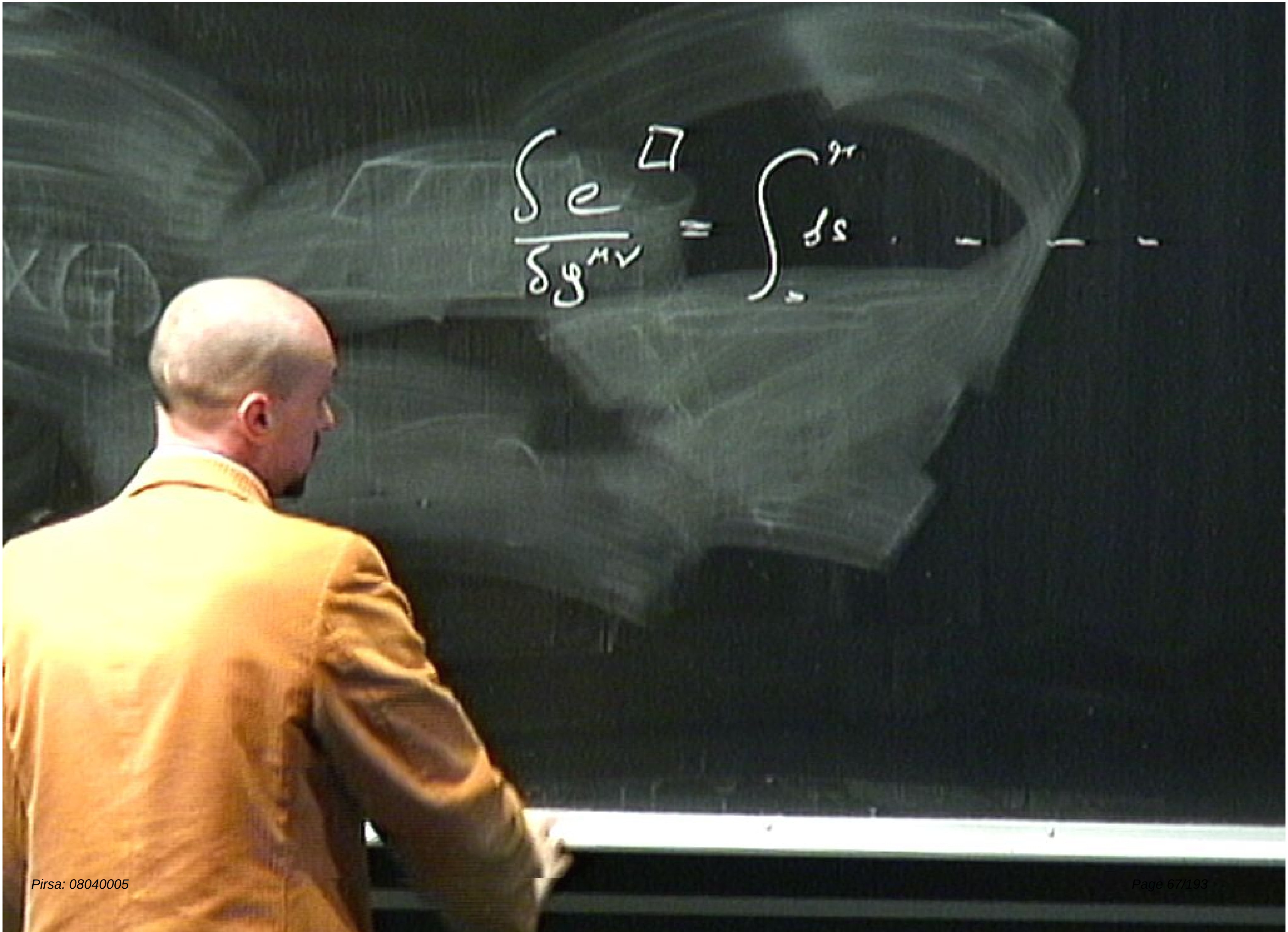
$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + \int_0^r ds\, \tilde{T}_{\mu\nu}(s)\,,$$

$$T^{(0)}_{\mu\nu} = \partial_\mu \Phi(r,x)\partial_\nu \Phi(r-2\gamma r,x) - g_{\mu\nu}\left[\tfrac{1}{2}\partial_\sigma \Phi(r,x)\partial^\sigma \Phi(r-2\gamma r,x) + U\right],$$

$$\tilde{T}_{\mu\nu}(s) \equiv g_{\mu\nu}(\partial_\sigma \chi \partial^\sigma \Phi + \chi \Box \Phi) - 2\partial_\mu \chi \partial_\nu \Phi.$$

!

Its structure exactly reproduces that of the nonlocal tensor.


δe^□ / δg^μν = ∫ ds

Energy-momentum tensor ($\mathcal{K} = \Box$)

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + \int_0^r ds\, \tilde{T}_{\mu\nu}(s)\,,$$

$$T^{(0)}_{\mu\nu} = \partial_\mu \Phi(r,x)\partial_\nu \Phi(r-2\gamma r,x) - g_{\mu\nu}\left[\tfrac{1}{2}\partial_\sigma \Phi(r,x)\partial^\sigma \Phi(r-2\gamma r,x) + U\right],$$

$$\tilde{T}_{\mu\nu}(s) \equiv g_{\mu\nu}(\partial_\sigma \chi \partial^\sigma \Phi + \chi \Box \Phi) - 2\partial_\mu \chi \partial_\nu \Phi.$$

!

Its structure exactly reproduces that of the nonlocal tensor.

Hamiltonian formalism

Conjugate momenta (simple block-diagonal metric)

Matter Lagrangian

$$L \equiv \int d^{D-1}x \int dr \sqrt{h} \left(\mathcal{L}_\Phi + \mathcal{L}_\chi \right).$$

Hamiltonian formalism

Conjugate momenta (simple block-diagonal metric)

Matter Lagrangian

$$L \equiv \int d^{D-1}x \int dr \sqrt{h} \left(\mathcal{L}_\Phi + \mathcal{L}_\chi \right).$$

Hamiltonian formalism

Conjugate momenta (simple block-diagonal metric)

Matter Lagrangian

$$L \equiv \int d^{D-1}x \int dr \sqrt{h}\,(\mathcal{L}_\Phi + \mathcal{L}_\chi).$$

$$\pi_\Phi(\bar{r},x) \equiv \frac{\delta L}{\delta \dot{\Phi}(\bar{r},x)} = \frac{\sqrt{h}}{2}\left[\dot{\Phi}(\bar{r}(1-2\gamma),x) + \frac{1}{|1-2\gamma|}\dot{\Phi}\left(\frac{\bar{r}}{1-2\gamma},x\right)\right.$$

$$\left. -\frac{1}{|\gamma|}\int dr\,\dot{\chi}(2r(1-\gamma)-\bar{r},x)\right],$$

$$\pi_\chi(\bar{r},x) \equiv \frac{\delta L}{\delta \dot{\chi}(\bar{r},x)} = -\frac{\sqrt{h}}{2|\gamma|}\int dr\,\dot{\Phi}(2r(1-\gamma)-\bar{r},x).$$

Hamiltonian formalism

Hamiltonian

Hamiltonian formalism

Hamiltonian

Nonvanishing equal-time Poisson brackets:

$$\begin{aligned} \{\Phi(r_1,x_1),\, \pi_\Phi(r_2,x_2)\}_{t_1=t_2} &= \delta(r_1-r_2)\,\delta^{(D-1)}(\mathbf{x}_1-\mathbf{x}_2)\,, \\ \{\chi(r_1,x_1),\, \pi_\chi(r_2,x_2)\}_{t_1=t_2} &= \delta(r_1-r_2)\,\delta^{(D-1)}(\mathbf{x}_1-\mathbf{x}_2)\,. \end{aligned}$$

Hamiltonian formalism
Hamiltonian

Nonvanishing equal-time Poisson brackets:

$$\begin{aligned} \{\Phi(r_1,x_1),\, \pi_\Phi(r_2,x_2)\}_{t_1=t_2} &= \delta(r_1-r_2)\,\delta^{(D-1)}(\mathbf{x}_1-\mathbf{x}_2)\,, \\ \{\chi(r_1,x_1),\, \pi_\chi(r_2,x_2)\}_{t_1=t_2} &= \delta(r_1-r_2)\,\delta^{(D-1)}(\mathbf{x}_1-\mathbf{x}_2)\,, \end{aligned}$$

Matter Hamiltonian:

$$H = \int d^{D-1}x \int dr \left[\pi_\Phi(r,x)\dot{\Phi}(r,x) + \pi_\chi(r,x)\dot{\chi}(r,x)\right] - L$$

Hamiltonian formalism
Hamiltonian

Nonvanishing equal-time Poisson brackets:

$$\{\Phi(r_1,x_1),\, \pi_\Phi(r_2,x_2)\}_{t_1=t_2} = \delta(r_1-r_2)\,\delta^{(D-1)}(\mathbf{x}_1-\mathbf{x}_2)\,,$$
$$\{\chi(r_1,x_1),\, \pi_\chi(r_2,x_2)\}_{t_1=t_2} = \delta(r_1-r_2)\,\delta^{(D-1)}(\mathbf{x}_1-\mathbf{x}_2)\,.$$

Matter Hamiltonian:

$$H = \int d^{D-1}x \int dr\left[\pi_\Phi(r,x)\dot{\Phi}(r,x) + \pi_\chi(r,x)\dot{\chi}(r,x)\right] - L$$

Nontrivial check: diffusion equation is $\dot{\pi}_\chi(r,x) = \{\pi_\chi(r,x),\, H\}$

Hamiltonian formalism
Hamiltonian

Nonvanishing equal-time Poisson brackets:

$$\begin{aligned}\{\Phi(r_1,x_1),\, \pi_\Phi(r_2,x_2)\}_{t_1=t_2} &= \delta(r_1-r_2)\,\delta^{(D-1)}(\mathbf{x}_1-\mathbf{x}_2)\,,\\ \{\chi(r_1,x_1),\, \pi_\chi(r_2,x_2)\}_{t_1=t_2} &= \delta(r_1-r_2)\,\delta^{(D-1)}(\mathbf{x}_1-\mathbf{x}_2)\,,\end{aligned}$$

Matter Hamiltonian:

$$H=\int d^{D-1}x\int dr\left[\pi_\Phi(r,x)\dot{\Phi}(r,x)+\pi_\chi(r,x)\dot{\chi}(r,x)\right]-L$$

Nontrivial check: diffusion equation is $\dot{\pi}_\chi(r,x)=\{\pi_\chi(r,x),\, H\}$ and scalar equation is $\dot{\pi}_\Phi(r,x)=\{\pi_\Phi(r,x),\, H\}$.

Energy-momentum tensor ($\mathcal{K} = \Box$)

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + \int_0^r ds\, \tilde{T}_{\mu\nu}(s)\,,$$

$$T^{(0)}_{\mu\nu} = \partial_\mu \Phi(r,x)\partial_\nu \Phi(r-2\gamma r,x) - g_{\mu\nu}\left[\tfrac{1}{2}\partial_\sigma \Phi(r,x)\partial^\sigma \Phi(r-2\gamma r,x) + U\right],$$

$$\tilde{T}_{\mu\nu}(s) \equiv g_{\mu\nu}(\partial_\sigma \chi \partial^\sigma \Phi + \chi \Box \Phi) - 2\partial_\mu \chi \partial_\nu \Phi.$$

!

Its structure exactly reproduces that of the nonlocal tensor.

The diffusion equaton

Its solution is

$$\Phi(r,x) = e^{r\Box/\gamma}\Phi(0,x)$$

$e^{q\Box}$ operators simply shift the auxiliary variable r:

$$e^{q\Box}\Phi(r,x) = e^{\gamma q\,\partial_r}\phi(r,x) = \phi(r+\gamma q,x)$$

The system becomes *local* in x, for any background metric.

!

Cauchy problem and Hamiltonian well-defined: Two d.o.f., four initial conditions (Φ, $\dot{\Phi}$, χ, $\dot{\chi}$)

Hamiltonian formalism
Hamiltonian

Nonvanishing equal-time Poisson brackets:

$$\begin{aligned} \{\Phi(r_1,x_1),\, \pi_\Phi(r_2,x_2)\}_{t_1=t_2} &= \delta(r_1-r_2)\,\delta^{(D-1)}(\mathbf{x}_1-\mathbf{x}_2)\,, \\ \{\chi(r_1,x_1),\, \pi_\chi(r_2,x_2)\}_{t_1=t_2} &= \delta(r_1-r_2)\,\delta^{(D-1)}(\mathbf{x}_1-\mathbf{x}_2)\,. \end{aligned}$$

Choice 1: $\Phi(0, x)$ nontrivial

Outline

1. Motivations
   - Tachyon and tachyons
   - Three questions
2. Localization
   - $1 + D$ dimension
   - Construction of solutions
3. Examples of solutions
   - $\Phi(0, x)$ nontrivial
   - $\Phi(0, x)$ distribution
4. SFT correspondences
   - RG flow along r
   - SFT correspondences

Choice 1: $\Phi(0, x)$ nontrivial

Choice 1: $\Phi(0, x)$ nontrivial

1. Find the eigenstates of the d'Alembertian:

Choice 1: $\Phi(0,x)$ nontrivial

1. Find the eigenstates of the d'Alembertian:

$$\Box G(\mu,x) = \mu^2 G(\mu,x)\,.$$

2. Write the local solution as an expansion in the basis of eigenstates of the $\Box$ (integral transform):

Choice 1: $\Phi(0,x)$ nontrivial

1. Find the eigenstates of the d'Alembertian:

$$\Box G(\mu,x) = \mu^2 G(\mu,x)\,.$$

2. Write the local solution as an expansion in the basis of eigenstates of the $\Box$ (integral transform):

$$\Phi(0,x) = \int d\mu\, G(\mu,x) f(\mu)\,.$$

Choice 1: $\Phi(0,x)$ nontrivial

1. Find the eigenstates of the d'Alembertian:

$$\Box G(\mu,x) = \mu^2 G(\mu,x)\,.$$

2. Write the local solution as an expansion in the basis of eigenstates of the $\Box$ (integral transform):

$$\Phi(0,x) = \int d\mu\, G(\mu,x) f(\mu)\,.$$

3. Write the nonlocal function (Gabor transform):

Choice 1: $\Phi(0,x)$ nontrivial

1. Find the eigenstates of the d'Alembertian:

$$\Box G(\mu,x) = \mu^2 G(\mu,x)\,.$$

2. Write the local solution as an expansion in the basis of eigenstates of the $\Box$ (integral transform):

$$\Phi(0,x) = \int d\mu\, G(\mu,x) f(\mu)\,.$$

3. Write the nonlocal function (Gabor transform):

$$\Phi(r,x) = \int d\mu\, e^{r\mu^2/\gamma} G(\mu,x) f(\mu)\,.$$

Choice 1: $\Phi(0,x)$ nontrivial

1. Find the eigenstates of the d'Alembertian:

$$\Box G(\mu,x) = \mu^2 G(\mu,x)\,.$$

2. Write the local solution as an expansion in the basis of eigenstates of the $\Box$ (integral transform):

$$\Phi(0,x) = \int d\mu\, G(\mu,x) f(\mu)\,.$$

3. Write the nonlocal function (Gabor transform):

$$\Phi(r,x) = \int d\mu\, e^{r\mu^2/\gamma} G(\mu,x) f(\mu)\,.$$

It satisfies the heat equation by definition

Choice 1: $\Phi(0,x)$ nontrivial

1. Find the eigenstates of the d'Alembertian:

$$\Box G(\mu,x) = \mu^2 G(\mu,x)\,.$$

2. Write the local solution as an expansion in the basis of eigenstates of the $\Box$ (integral transform):

$$\Phi(0,x) = \int d\mu\, G(\mu,x) f(\mu)\,.$$

3. Write the nonlocal function (Gabor transform):

$$\Phi(r,x) = \int d\mu\, e^{r\mu^2/\gamma} G(\mu,x) f(\mu)\,.$$

It satisfies the heat equation by definition

4. Check that $\Phi(r,x)$ is a solution of the nonlocal e.o.m.s for some γ.

Choice 2: $\Phi(0, x)$ constant almost everywhere

Choice 2: $\Phi(0, x)$ constant almost everywhere

The nonlocal function $\Phi(r, x)$ is simply the "regularization" at $r \neq 0$ of a distribution.

Outline

1. Motivations
   - Tachyon and tachyons
   - Three questions
2. Localization
   - $1 + D$ dimension
   - Construction of solutions
3. Examples of solutions
   - $\Phi(0, x)$ nontrivial
   - $\Phi(0, x)$ distribution
4. SFT correspondences
   - RG flow along r
   - SFT correspondences

Susy OSFT on Minkowski (1)

with G. Nardelli, `0708.0366 [hep-th]`

Susy OSFT on Minkowski (1)

with G. Nardelli, 0708.0366 [hep-th]

Exact susy potential $U \propto (e^{r_* \Box} \tilde{\phi}^2)^2$,

Susy OSFT on Minkowski (1)

with G. Nardelli, `0708.0366 [hep-th]`

Exact susy potential $U \propto (e^{r_* \Box} \tilde{\phi}^2)^2$, rolling solution:

$$\psi(r_*, t) = -\int_{-\infty}^{+\infty} d\sigma \, \partial_\sigma K(\sigma, r_*) \, \ln(\cosh t + \sin \sigma)$$

where $K(\sigma, r) \propto e^{-\frac{\sigma^2}{4r}}$.

$$\frac{\delta e^{\Box}}{\delta g^{\mu\nu}} = \int ds \ \ldots$$

$$\tilde{\phi} = e^{\mu_* \Box} \phi$$

Susy OSFT on Minkowski (1)

with G. Nardelli, 0708.0366 [hep-th]

Exact susy potential $U \propto (e^{r_* \Box} \tilde{\phi}^2)^2$, rolling solution:

$$\psi(r_*, t) = -\int_{-\infty}^{+\infty} d\sigma\, \partial_\sigma K(\sigma, r_*)\, \ln(\cosh t + \sin \sigma)$$

where $K(\sigma, r) \propto e^{-\frac{\sigma^2}{4r}}$.

Susy OSFT on Minkowski (1)

with G. Nardelli, 0708.0366 [hep-th]

Exact susy potential $U \propto (e^{r_* \Box} \tilde{\phi}^2)^2$, rolling solution:

$$\psi(r_*, t) = -\int_{-\infty}^{+\infty} d\sigma \, \partial_\sigma K(\sigma, r_*) \, \ln(\cosh t + \sin \sigma)$$

where $K(\sigma, r) \propto e^{-\frac{\sigma^2}{4r}}$.

Susy OSFT on Minkowski (1)

with G. Nardelli, 0708.0366 [hep-th]

Exact susy potential $U \propto (e^{r_* \Box} \tilde{\phi}^2)^2$, rolling solution:

$$\psi(r_*, t) = -\int_{-\infty}^{+\infty} d\sigma\, \partial_\sigma K(\sigma, r_*)\, \ln(\cosh t + \sin \sigma)$$

where $K(\sigma, r) \propto e^{-\frac{\sigma^2}{4r}}$.

- A series representation is also available.

Susy OSFT on Minkowski (1)

with G. Nardelli, `0708.0366 [hep-th]`

Exact susy potential $U \propto (e^{r_*\Box}\tilde{\phi}^2)^2$, rolling solution:

$$\psi(r_*, t) = -\int_{-\infty}^{+\infty} d\sigma\, \partial_\sigma K(\sigma, r_*)\, \ln(\cosh t + \sin\sigma)$$

where $K(\sigma, r) \propto e^{-\frac{\sigma^2}{4r}}$.

- A series representation is also available.
- Global solution valid at all times.

Susy OSFT on Minkowski (1)

with G. Nardelli, 0708.0366 [hep-th]

Exact susy potential $U \propto (e^{r_* \Box} \tilde{\phi}^2)^2$, rolling solution:

$$\psi(r_*, t) = -\int_{-\infty}^{+\infty} d\sigma\, \partial_\sigma K(\sigma, r_*)\, \ln(\cosh t + \sin\sigma)$$

where $K(\sigma, r) \propto e^{-\frac{\sigma^2}{4r}}$.

- A series representation is also available.
- Global solution valid at all times.
- Almost exact:

$$\Delta \equiv \left| \frac{\mathcal{K}\Phi - U'(\Phi)}{\mathcal{K}\Phi + U'(\Phi)} \right| \sim 10^{-8}$$

Susy OSFT on Minkowski (1)

with G. Nardelli, 0708.0366 [hep-th]

Exact susy potential $U \propto (e^{r_* \Box}\tilde{\phi}^2)^2$, rolling solution:

$$\psi(r_*, t) = -\int_{-\infty}^{+\infty} d\sigma \, \partial_\sigma K(\sigma, r_*) \, \ln(\cosh t + \sin \sigma)$$

where $K(\sigma, r) \propto e^{-\frac{\sigma^2}{4r}}$.

- A series representation is also available.
- Global solution valid at all times.
- Almost exact:

$$\Delta \equiv \left| \frac{\mathcal{K}\Phi - U'(\Phi)}{\mathcal{K}\Phi + U'(\Phi)} \right| \sim 10^{-8}$$

- The same result can also be found via another powerful technique (valid only on Minkowski) based on harmonic functions.

Susy OSFT on Minkowski (2)

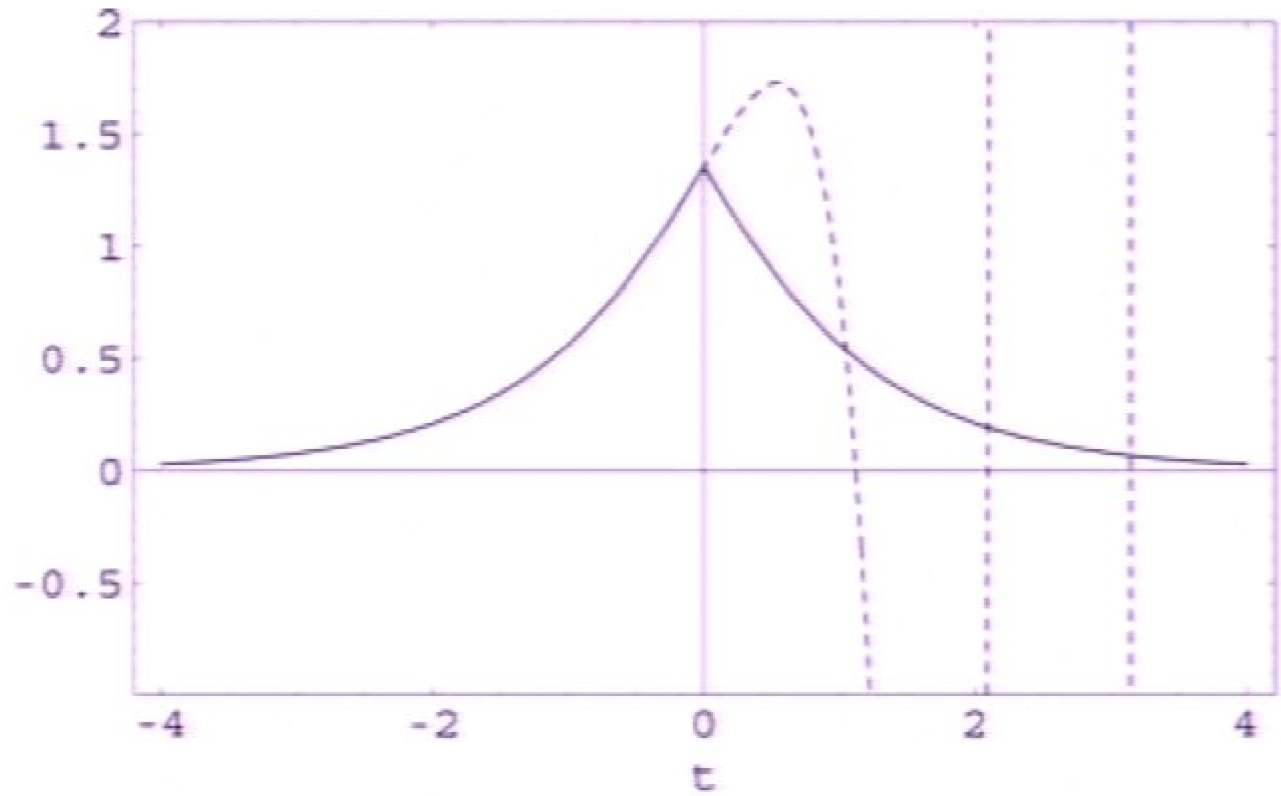


Figure: Solid: global solution. Dashed: asymptotic solution.

The analytic properties of these solutions are all under control.

Energy-momentum tensor ($\mathcal{K} = \Box$)

Equations of motion

Equations of motion

Proper e.o.m.:

$$\mathcal{K}\Phi(r_* - 2\gamma r_*, x) - U'[\Phi(r_*, x)] = 0.$$

An analytic recipe

We propose a systematic method which allows:

General nonlocal system

$$\mathcal{K} e^{-2r_* \Box} \phi(x) = U'[\phi(x)]$$

General nonlocal system

$$\mathcal{K}e^{-2r_*\Box}\phi(x) = U'[\phi(x)]$$

OSFT: $r_* \approx 0.26, \quad \mathcal{K} = \Box - m^2, \quad U \sim \phi^3, \phi^4$

p-adic string: $r_* = (\ln p)/4, \quad \mathcal{K} = 1, \quad U \sim \phi^{p+1}$

An analytic recipe

We propose a systematic method which allows:

Hamiltonian formalism

Hamiltonian

Outline

1. Motivations
   - Tachyon and tachyons
   - Three questions
2. Localization
   - $1 + D$ dimension
   - Construction of solutions
3. Examples of solutions
   - $\Phi(0, x)$ nontrivial
   - $\Phi(0, x)$ distribution
4. SFT correspondences
   - RG flow along r
   - SFT correspondences

Outline

1. Motivations
   - Tachyon and tachyons
   - Three questions
2. Localization
   - $1 + D$ dimension
   - Construction of solutions
3. Examples of solutions
   - $\Phi(0, x)$ nontrivial
   - $\Phi(0, x)$ distribution
4. SFT correspondences
   - RG flow along r
   - SFT correspondences

Susy OSFT on Minkowski (2)

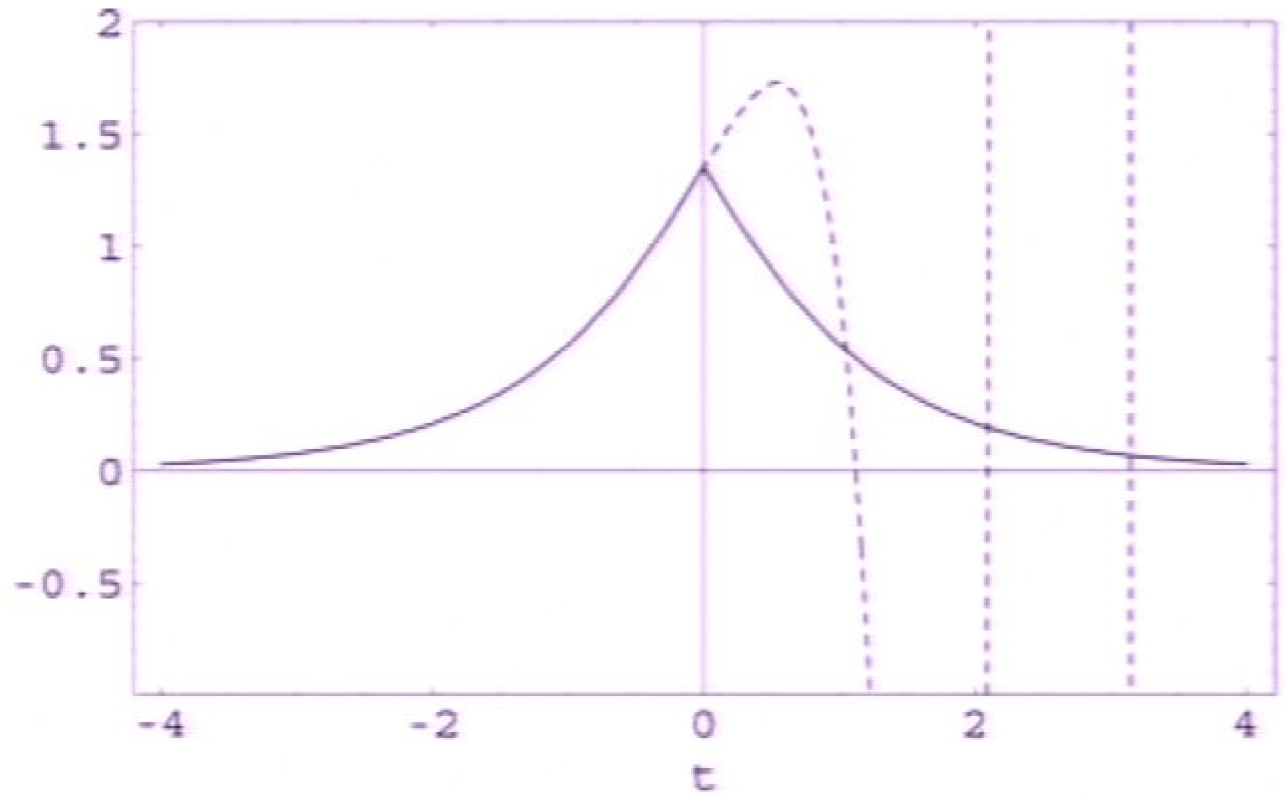
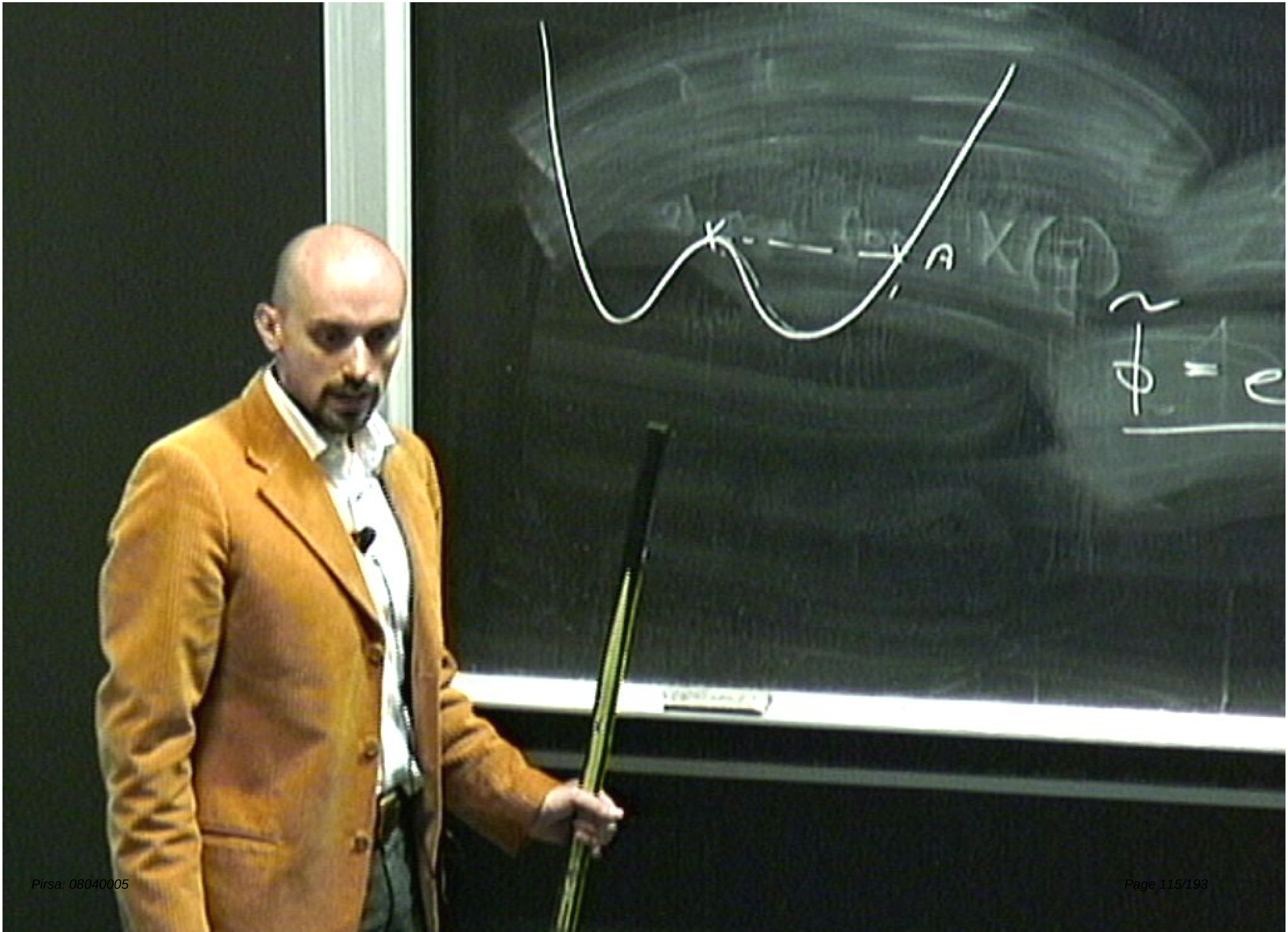
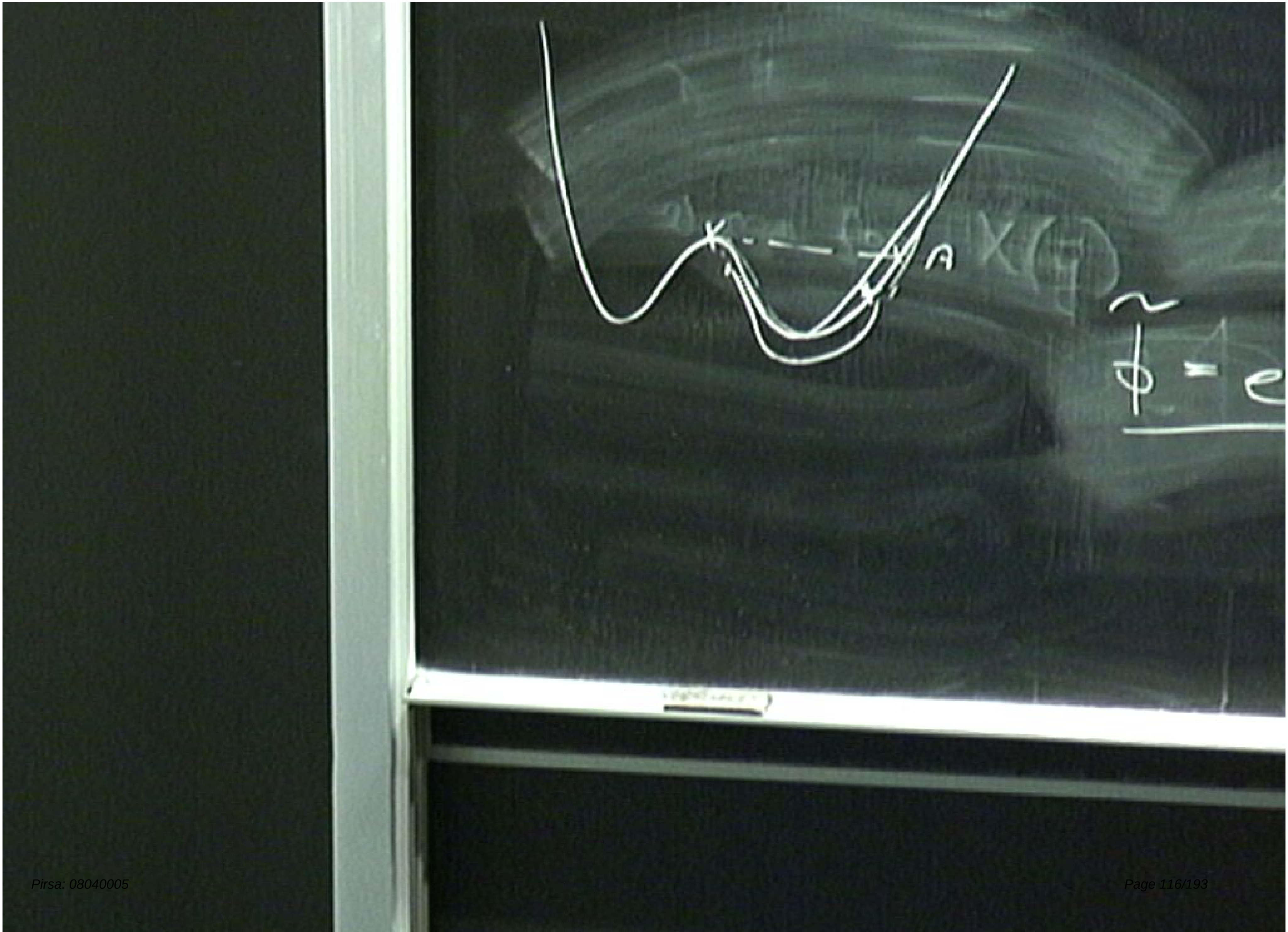


Figure: Solid: global solution. Dashed: asymptotic solution.

The analytic properties of these solutions are all under control.




A
φ̃ = e

Susy OSFT on Minkowski (2)

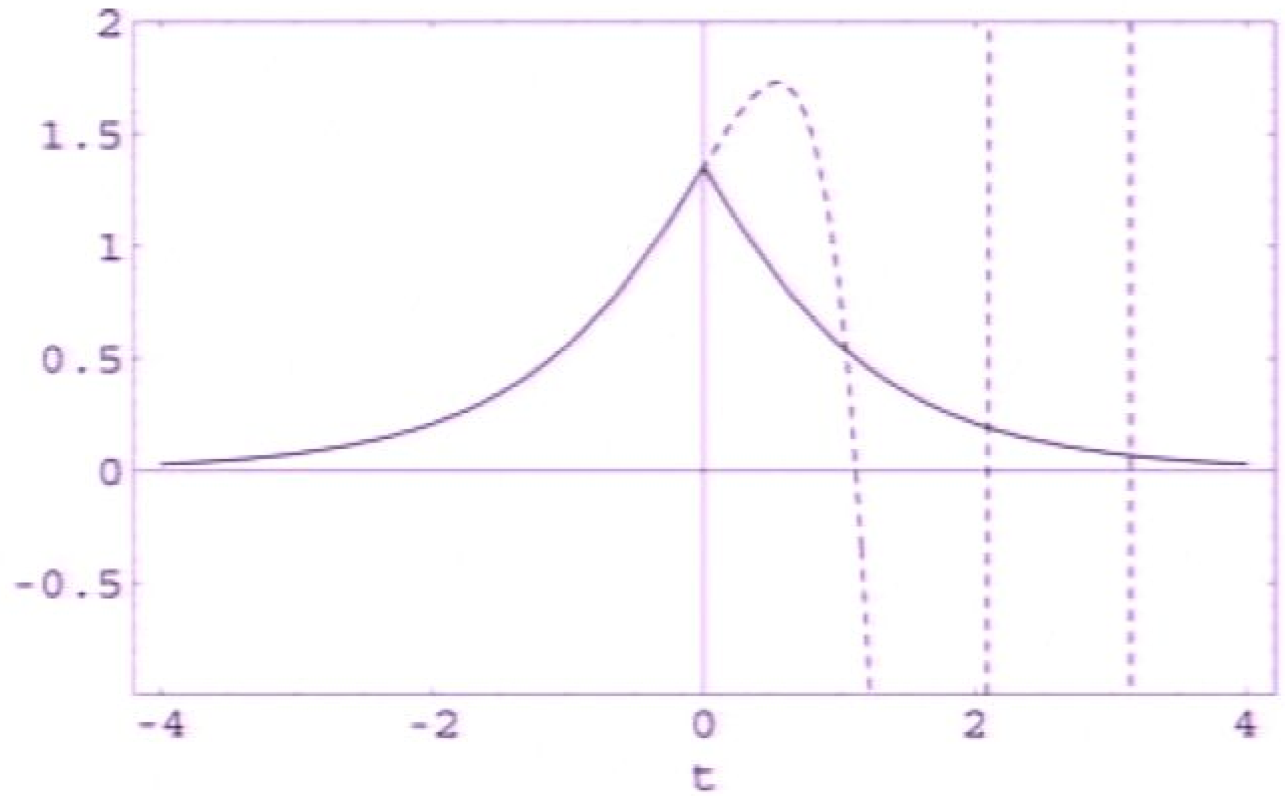


Figure: Solid: global solution. Dashed: asymptotic solution.

The analytic properties of these solutions are all under control.

Susy OSFT on Minkowski (1)

with G. Nardelli, 0708.0366 [hep-th]

Exact susy potential $U \propto (e^{r_* \Box}\tilde{\phi}^2)^2$, rolling solution:

$$\psi(r_*, t) = -\int_{-\infty}^{+\infty} d\sigma\, \partial_\sigma K(\sigma, r_*)\, \ln(\cosh t + \sin\sigma)$$

where $K(\sigma, r) \propto e^{-\frac{\sigma^2}{4r}}$.

- A series representation is also available.
- Global solution valid at all times.
- Almost exact:

$$\Delta \equiv \left|\frac{\mathcal{K}\Phi - U'(\Phi)}{\mathcal{K}\Phi + U'(\Phi)}\right| \sim 10^{-8}$$

- The same result can also be found via another powerful technique (valid only on Minkowski) based on harmonic functions.

Susy OSFT on Minkowski (2)

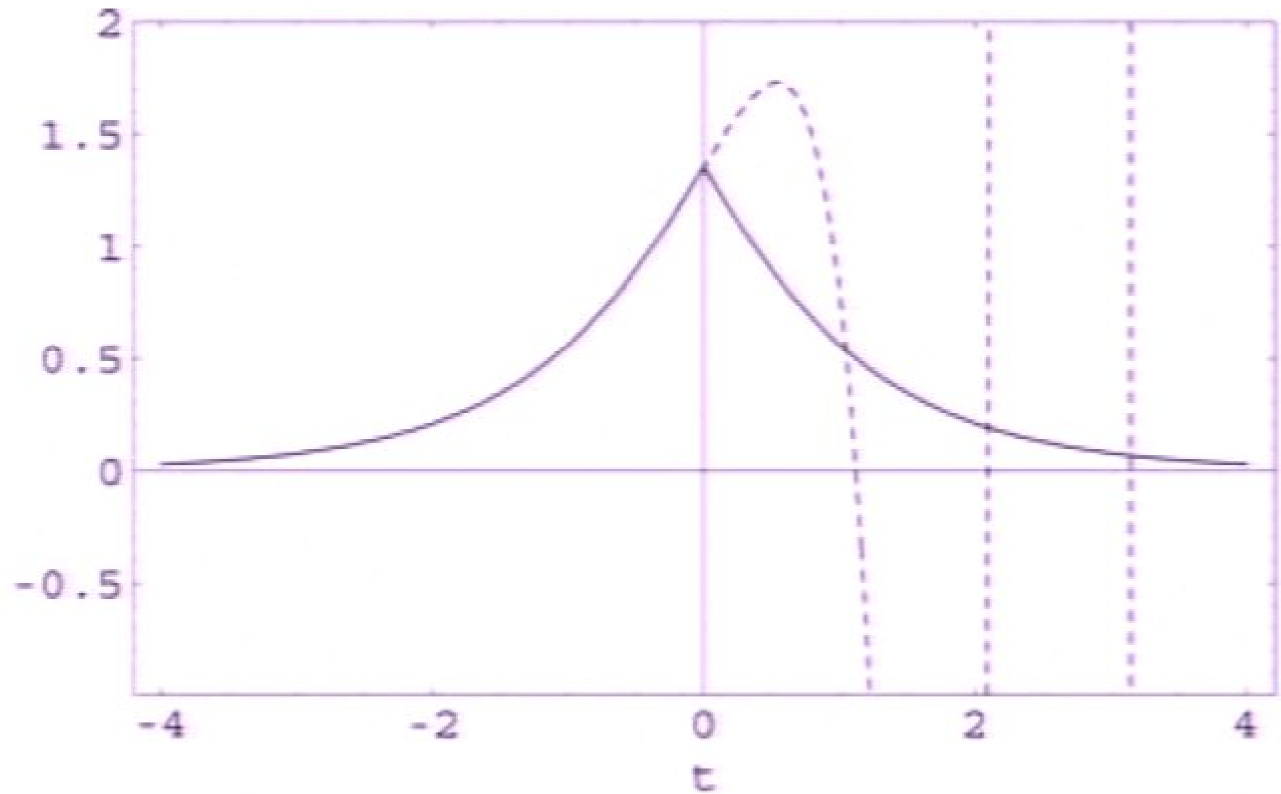
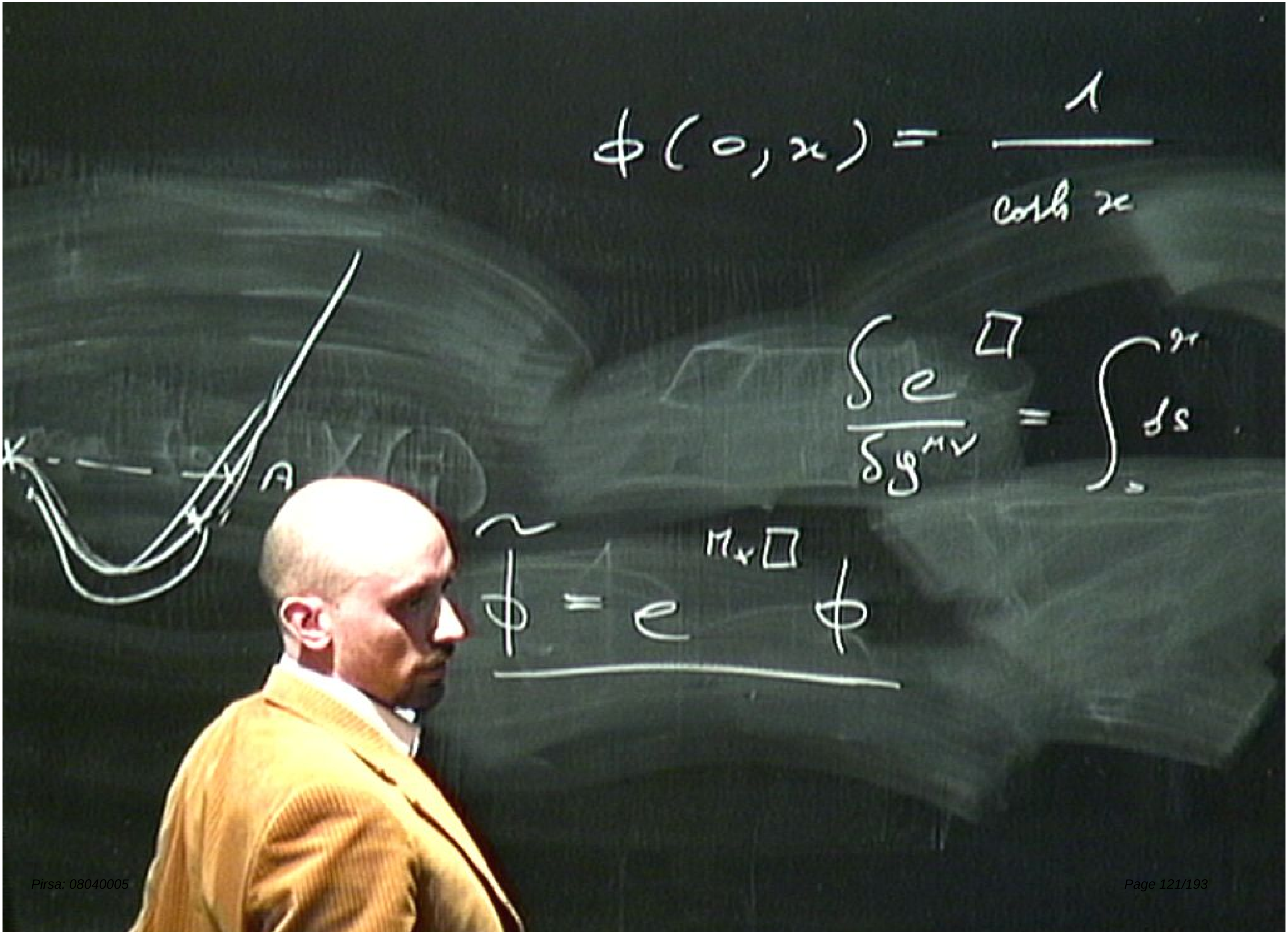


Figure: Solid: global solution. Dashed: asymptotic solution.

The analytic properties of these solutions are all under control.

Outline

1. Motivations
   - Tachyon and tachyons
   - Three questions
2. Localization
   - $1 + D$ dimension
   - Construction of solutions
3. Examples of solutions
   - $\Phi(0, x)$ nontrivial
   - $\Phi(0, x)$ distribution
4. SFT correspondences
   - RG flow along r
   - SFT correspondences


$\phi(0,x) = \frac{1}{\cosh x}$
$\frac{\delta e^{\Box}}{\delta g^{\mu\nu}} = \int_0^x ds$
$\tilde{\phi} = e^{M_* \Box} \phi$
A

p-adic soliton

p-adic soliton

Starting from $\Phi(0,x)=\delta(x)$ one gets the exact solution

$$\Phi(r_*,x)=p^{c/2(p-1)}\exp\left[-\frac{(p-1)T(x)}{2p\ln p}\right]$$

p-adic soliton

Starting from $\Phi(0,x) = \delta(x)$ one gets the exact solution

$$\Phi(r_*, x) = p^{c/2(p-1)} \exp\left[-\frac{(p-1)T(x)}{2p \ln p}\right]$$

Minkowski soliton: $T = x^2$, $c = 1$ [Brekke et al. 1988]

p-adic soliton

Starting from $\Phi(0, x) = \delta(x)$ one gets the exact solution

$$\Phi(r_*, x) = p^{c/2(p-1)} \exp\left[-\frac{(p-1)T(x)}{2p\ln p}\right]$$

Minkowski soliton: $T = x^2$, $c = 1$ [Brekke et al. 1988]
FRW lump: $T = -t^2$, $H \sim (c-1)/t$

Susy OSFT instanton

p-adic soliton

Starting from $\Phi(0,x)=\delta(x)$ one gets the exact solution

$$\Phi(r_*,x)=p^{c/2(p-1)}\exp\left[-\frac{(p-1)T(x)}{2p\ln p}\right]$$

Minkowski soliton: $T=x^2$, $c=1$ [Brekke et al. 1988]
FRW lump: $T=-t^2$, $H\sim(c-1)/t$

Susy OSFT instanton

Susy OSFT instanton

Starting from $\Phi(0, x) = \mathrm{sgn}(x)$ one gets the approximate global solution

$$\Phi(r_*, x) = \mathrm{erf}\left(\pm\sqrt{\frac{\gamma}{4r_*}}x\right) ,$$

where $\gamma \approx r_*/(1 + 2r_*)$.

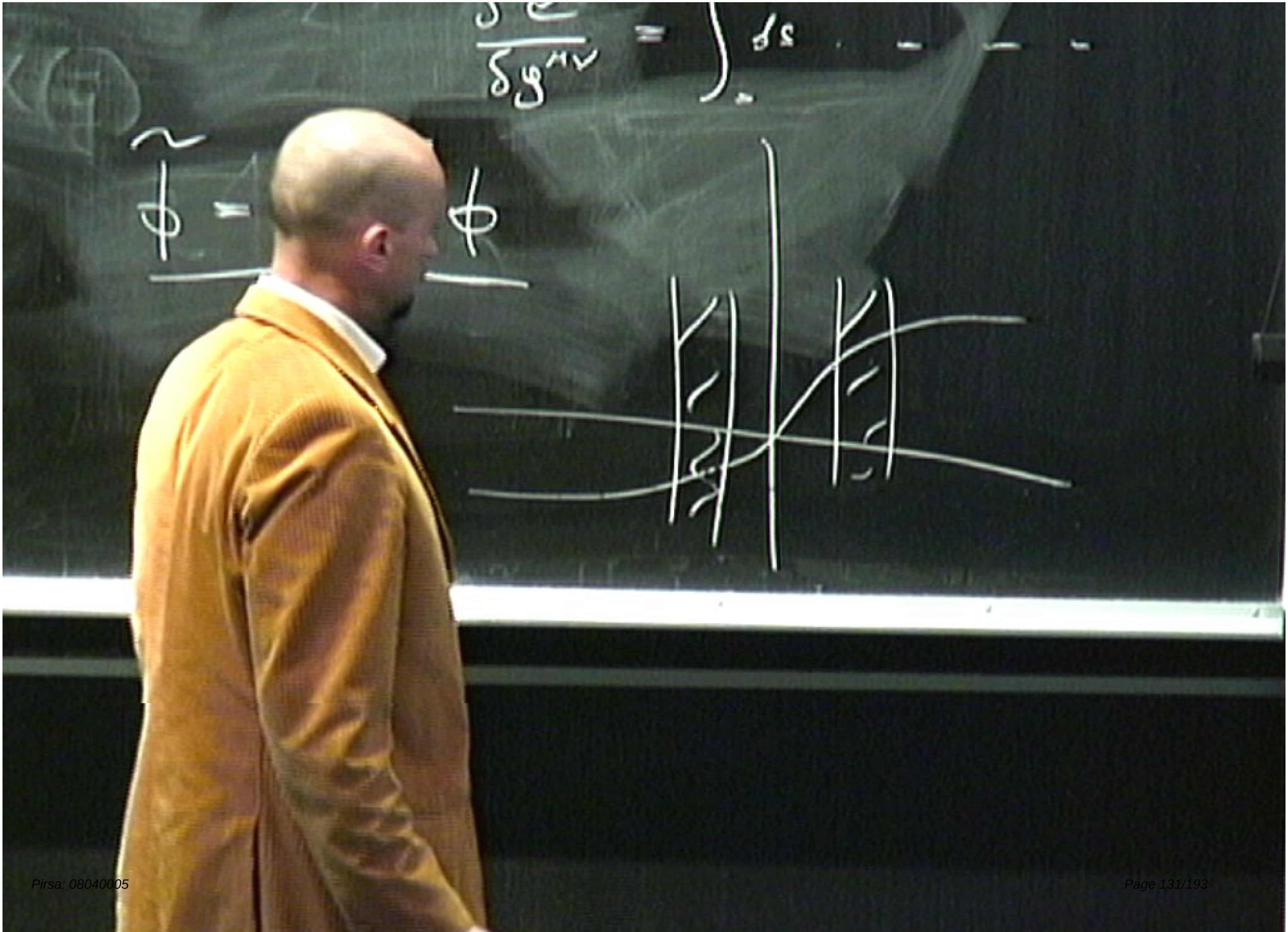
Susy OSFT instanton

Starting from $\Phi(0,x) = \mathrm{sgn}(x)$ one gets the approximate global solution

$$\Phi(r_*,x) = \mathrm{erf}\left(\pm\sqrt{\frac{\gamma}{4r_*}}x\right),$$

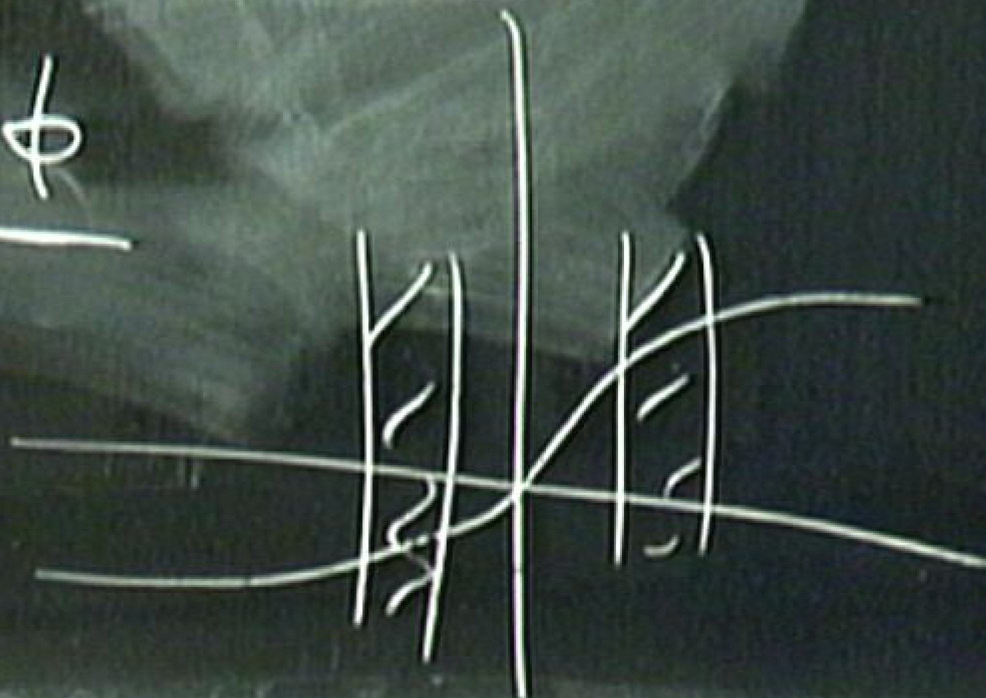
where $\gamma \approx r_*/(1+2r_*)$.

$\Delta \approx 1.5\%$ at $x \sim \pm 3$. At the origin, $\Delta < 1\%$, while $\Delta < 0.1\%$ and decays to zero for $|x| \gtrsim 5$.



$$\frac{\delta S}{\delta g^{\mu\nu}} = \int ds \ldots$$

$$\tilde{\phi} = e^{M_* \Box} \phi$$



Susy OSFT instanton

Starting from $\Phi(0, x) = \mathrm{sgn}(x)$ one gets the approximate global solution

$$\Phi(r_*, x) = \mathrm{erf}\left(\pm\sqrt{\frac{\gamma}{4r_*}}\, x\right),$$

where $\gamma \approx r_*/(1 + 2r_*)$.

$\Delta \approx 1.5\%$ at $x \sim \pm 3$. At the origin, $\Delta < 1\%$, while $\Delta < 0.1\%$ and decays to zero for $|x| \gtrsim 5$.

Outline

1. Motivations
   - Tachyon and tachyons
   - Three questions
2. Localization
   - $1 + D$ dimension
   - Construction of solutions
3. Examples of solutions
   - $\Phi(0, x)$ nontrivial
   - $\Phi(0, x)$ distribution
4. SFT correspondences
   - RG flow along r
   - SFT correspondences

Susy OSFT instanton

Starting from $\Phi(0,x) = \text{sgn}(x)$ one gets the approximate global solution

$$\Phi(r_*, x) = \text{erf}\left(\pm\sqrt{\frac{\gamma}{4r_*}}x\right),$$

where $\gamma \approx r_*/(1+2r_*)$.

$\Delta \approx 1.5\%$ at $x \sim \pm 3$. At the origin, $\Delta < 1\%$, while $\Delta < 0.1\%$ and decays to zero for $|x| \gtrsim 5$.

$$\phi(0, x) =$$

$$\left| \frac{LHS - RHS}{LHS + RHS} \right|$$

$$\frac{\int e^{\square}}{\delta g^{\mu\nu}}$$

$$\tilde{\phi} = e^{M_* \square} \phi$$

$\phi(0, x) =$

$$\left| \frac{LHS - RHS}{LHS + RHS} \right|$$

$$\frac{\int e^{\Box}}{\delta g^{\mu\nu}}$$

$$\tilde{\phi} = e^{M_* \Box} \phi$$

$\phi(0,x) =$

$$\left| \frac{LHS - RHS}{LHS + RHS} \right|$$

$$\frac{\int e^{\Box}}{\delta g^{\mu\nu}}$$

$$\tilde{\phi} = e^{M_* \Box} \phi$$

$$\left| \frac{LHS - RHS}{LHS + RHS} \right|$$

$$\tilde{\phi} = e^{M_* \Box}$$

$$\left|\frac{LHS - RHS}{LHS + RHS}\right|$$

$$\tilde{\phi} = e^{M_* \Box}$$

$$\left| \frac{LHS - RHS}{LHS + RHS} \right|$$

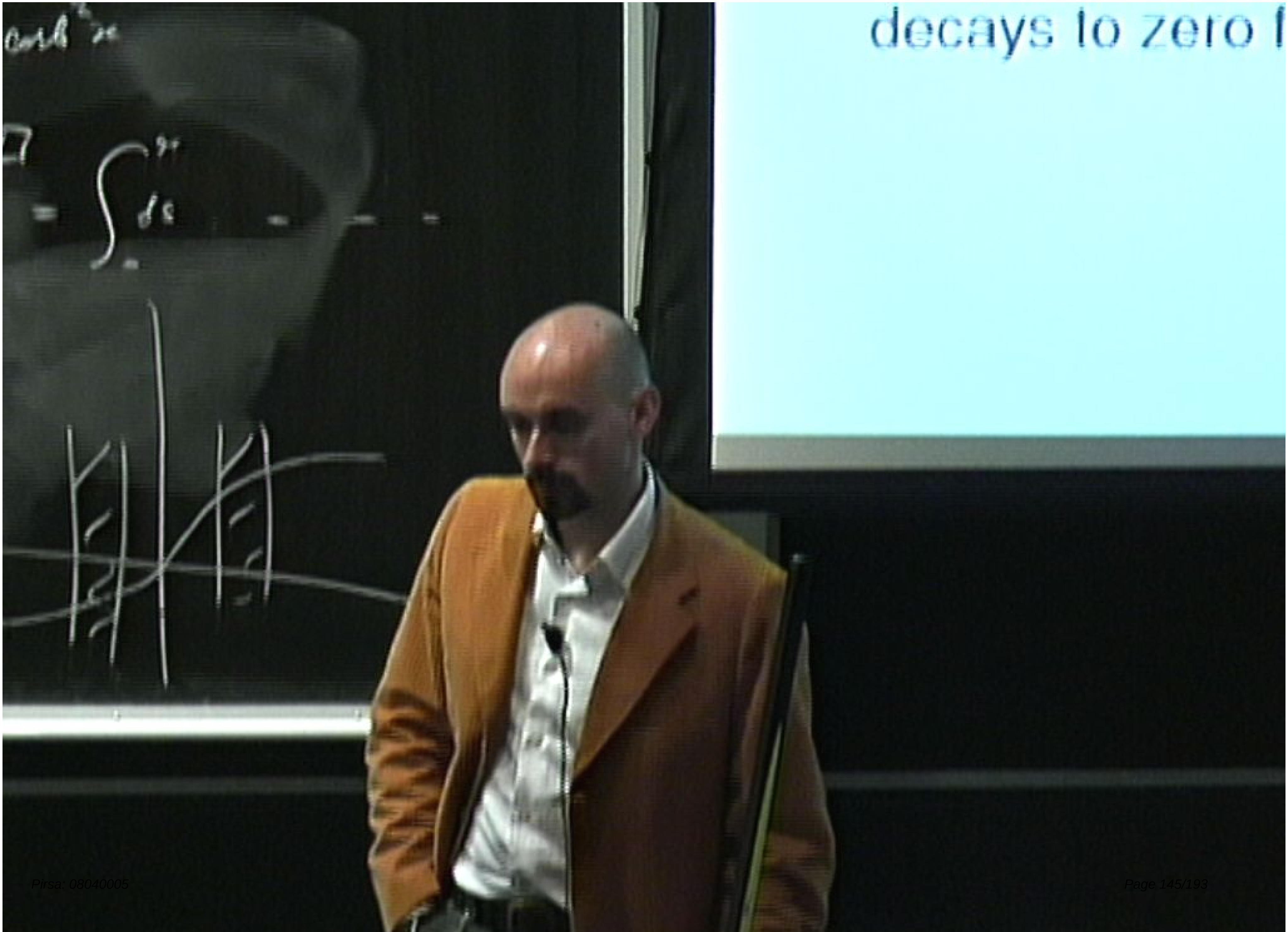
$$\tilde{\phi} = e^{M_* \Box}$$

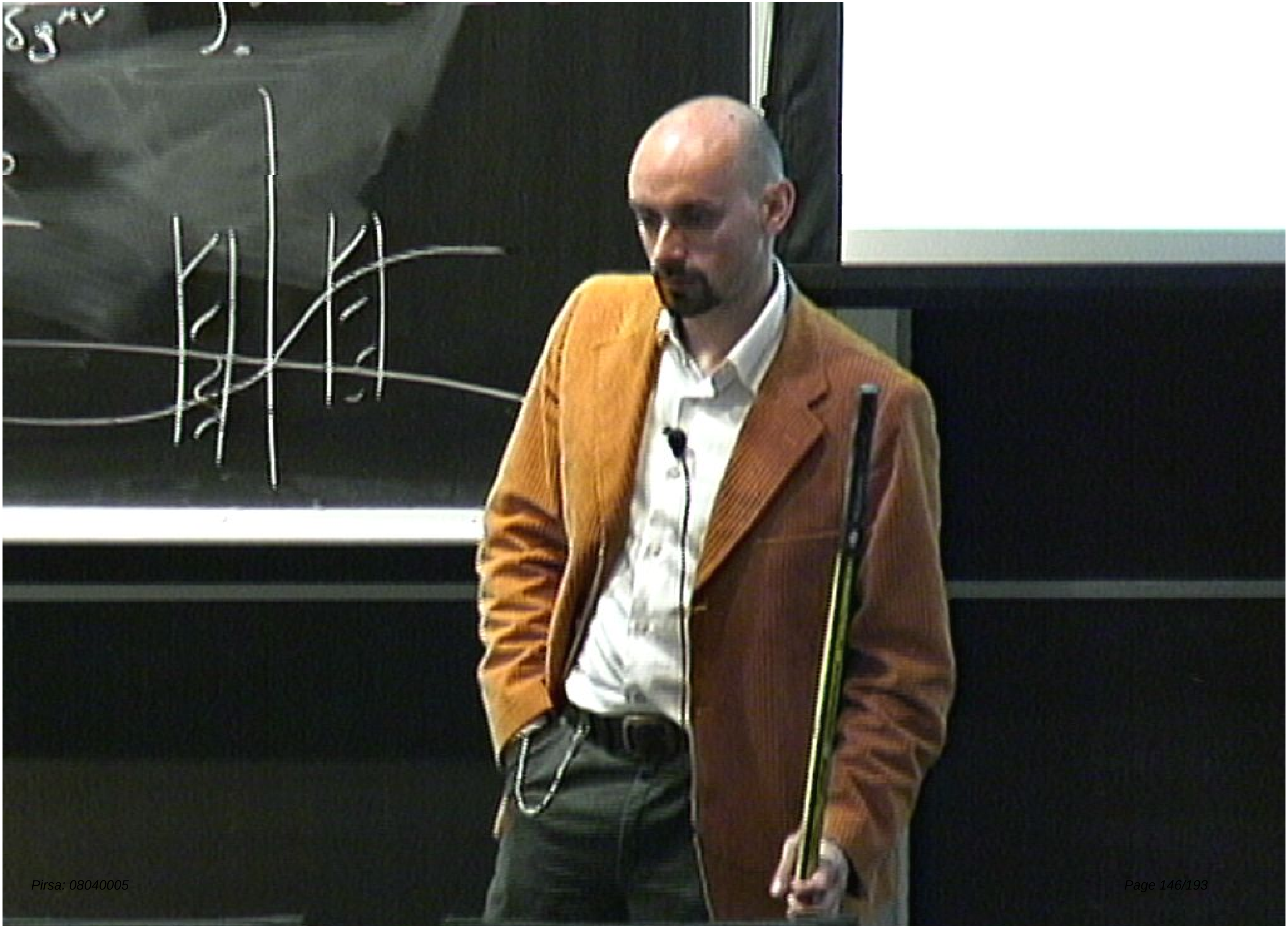
$$\left| \frac{LHS - RHS}{LHS + RHS} \right|$$

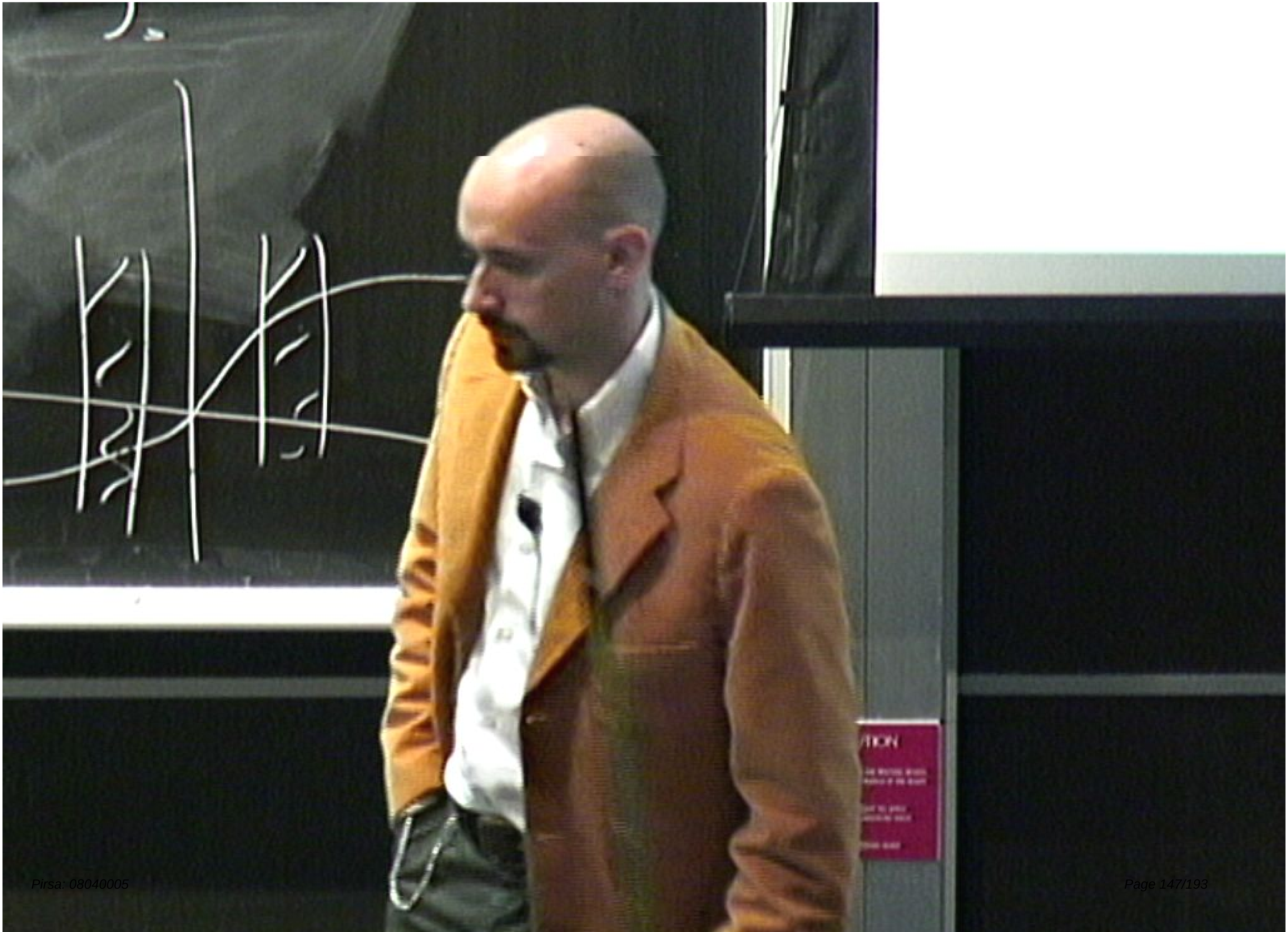
$$\tilde{\phi} = e^{M_* \Box}$$

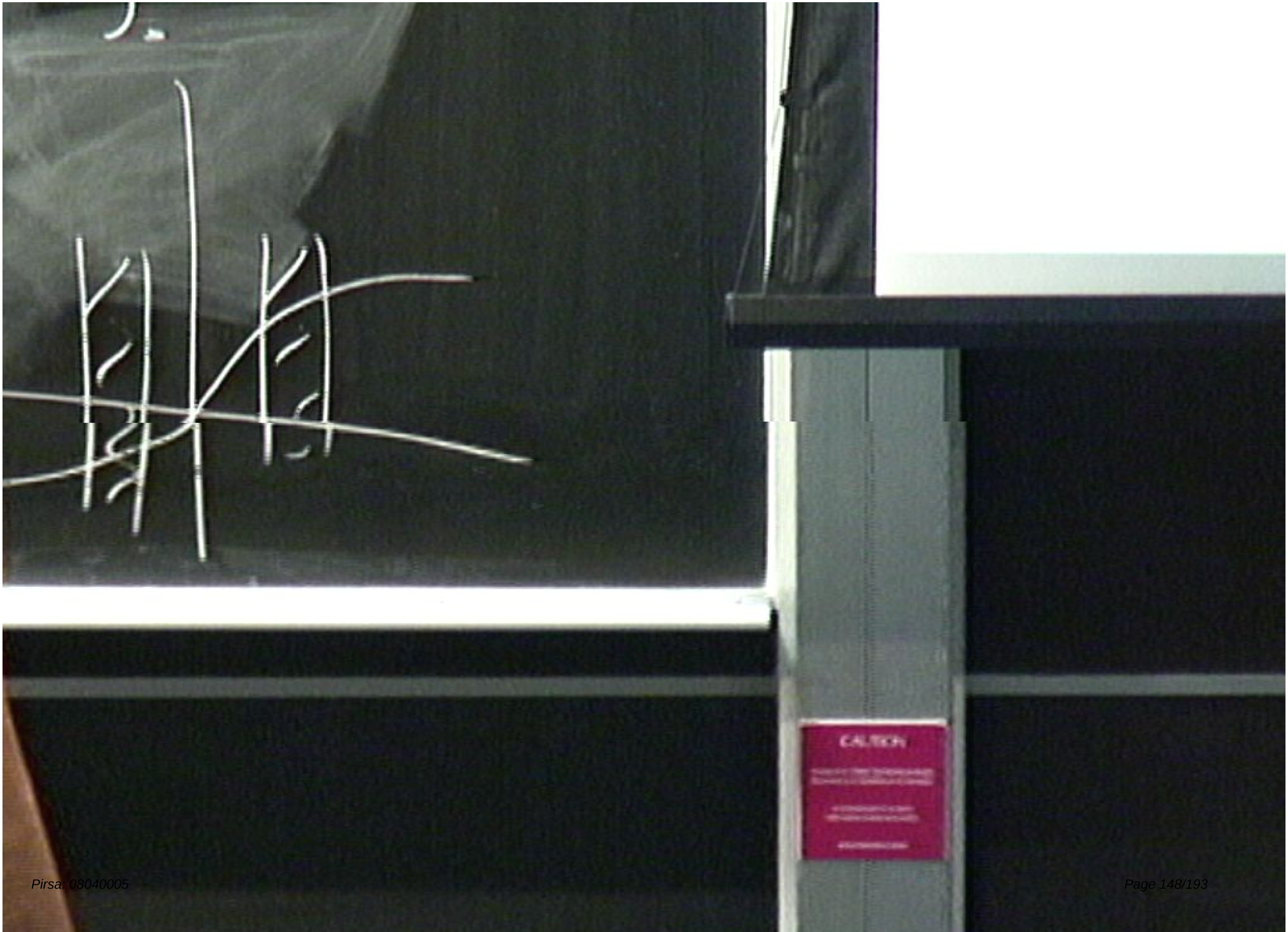
$$\left| \frac{LHS - RHS}{LHS + RHS} \right|$$

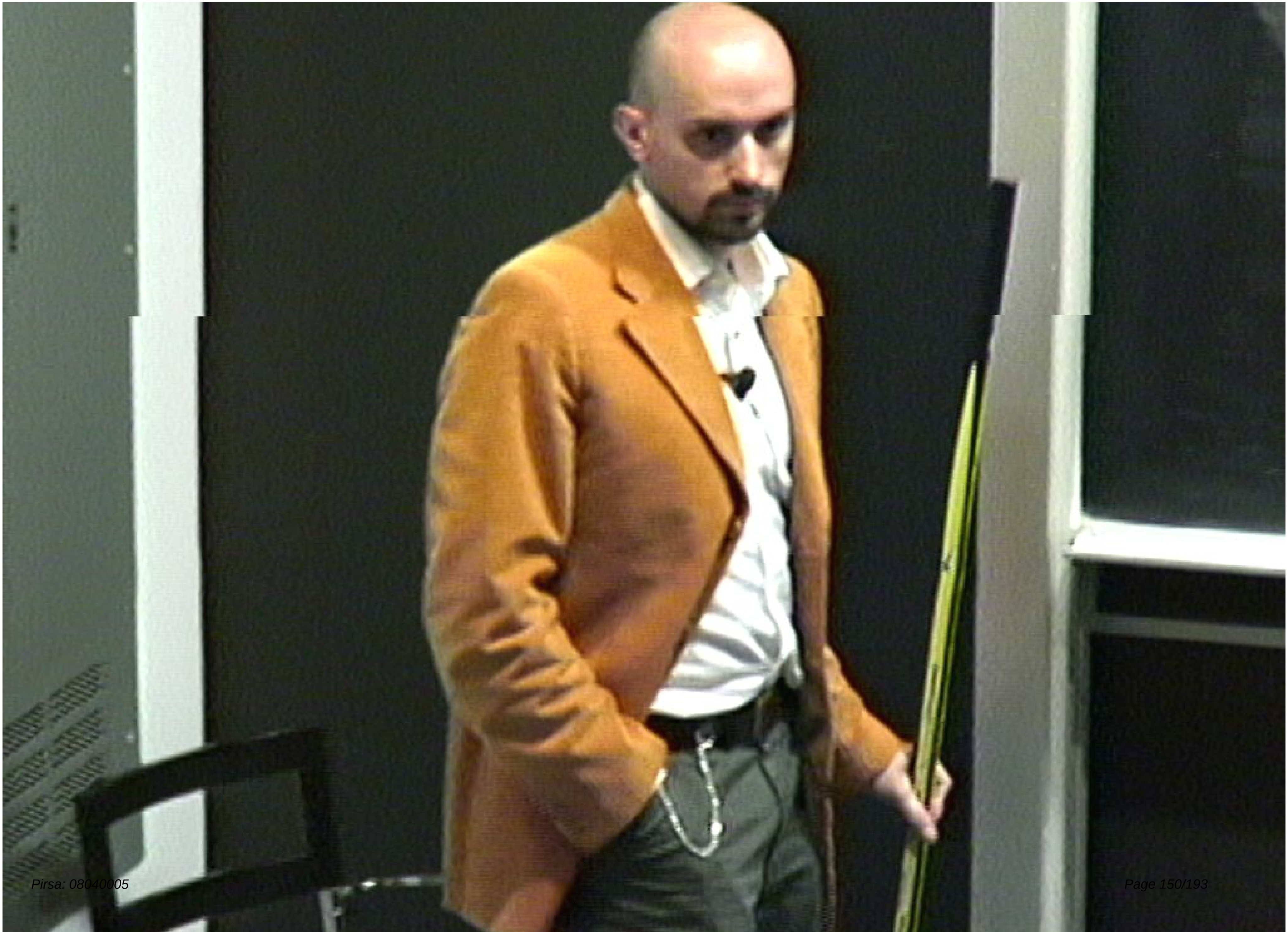
$$\tilde{\phi} = e^{M_* \Box}$$

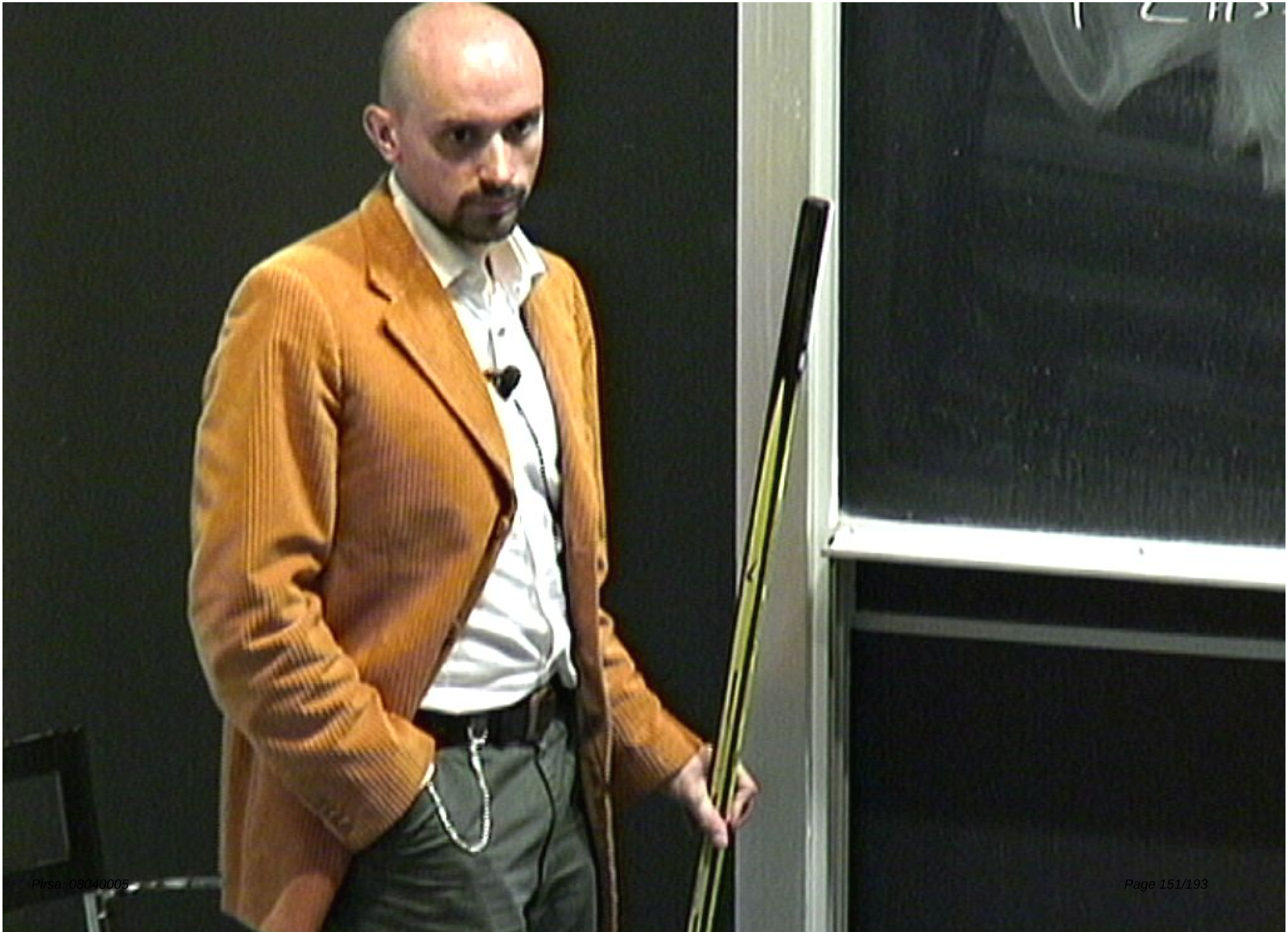

decays to zero

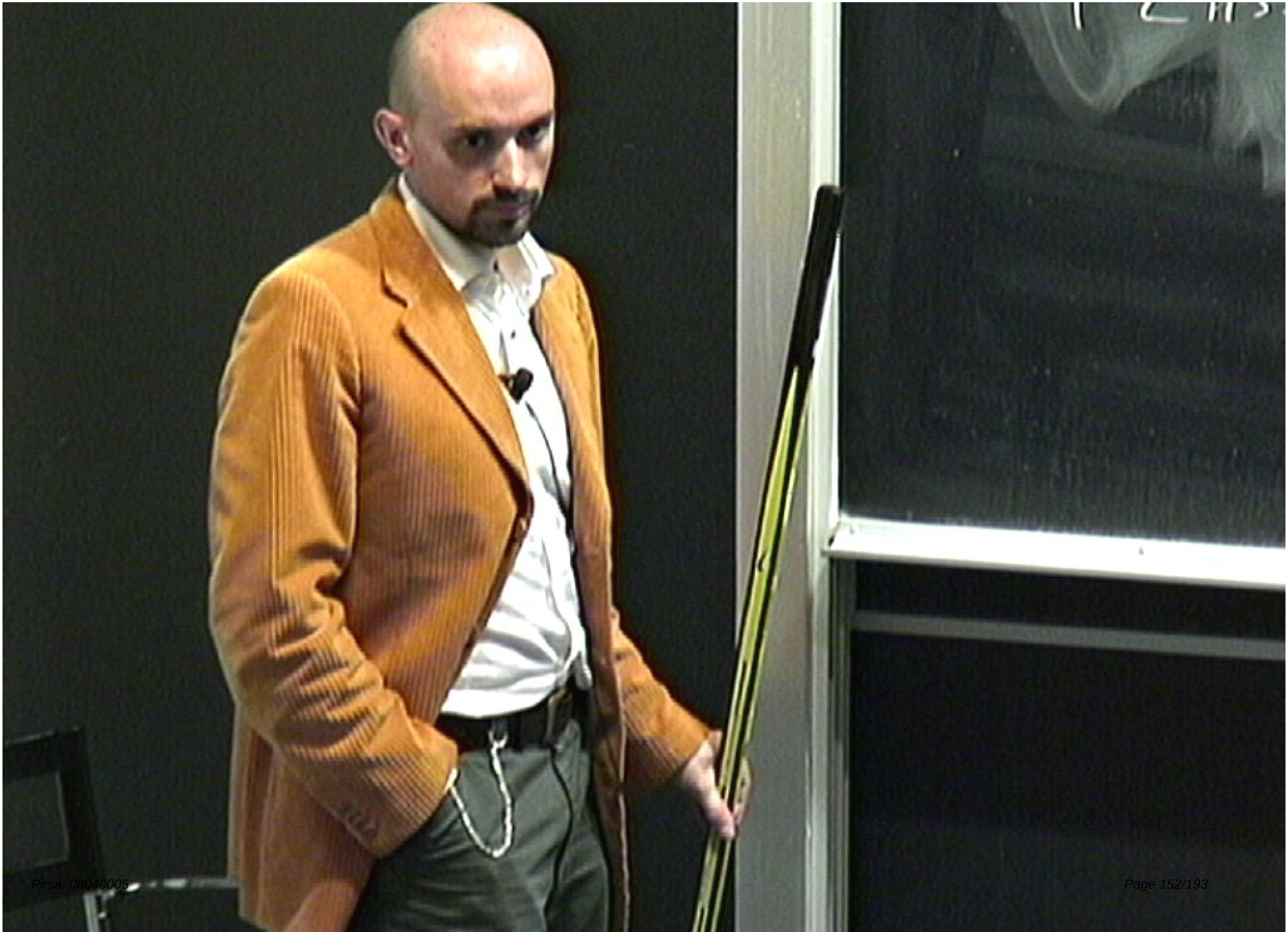












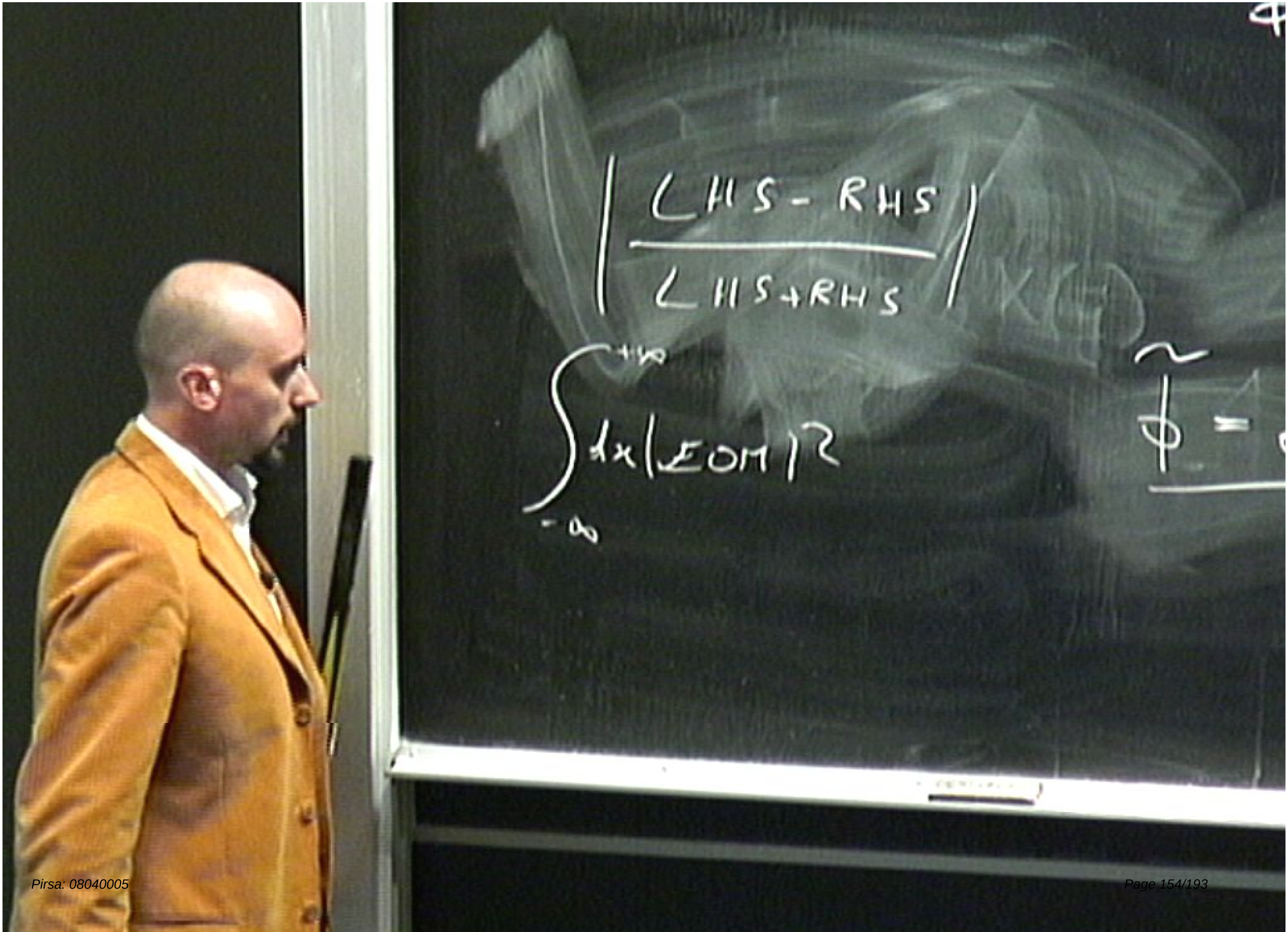
Susy OSFT instanton

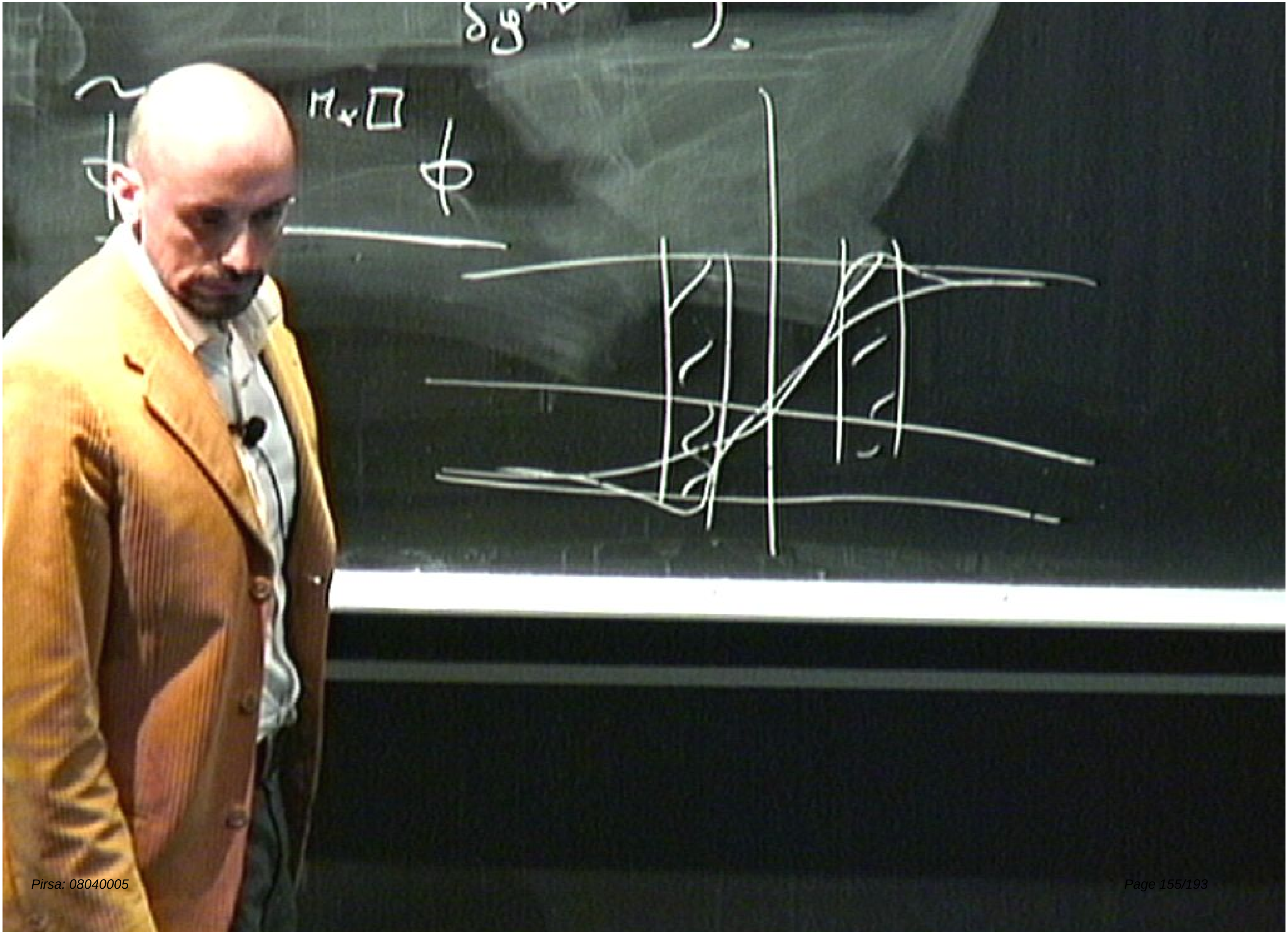
Starting from $\Phi(0,x) = \mathrm{sgn}(x)$ one gets the approximate global solution

$$\Phi(r_*, x) = \mathrm{erf}\left(\pm\sqrt{\frac{\gamma}{4r_*}}x\right),$$

where $\gamma \approx r_*/(1+2r_*)$.

$\Delta \approx 1.5\%$ at $x \sim \pm 3$. At the origin, $\Delta < 1\%$, while $\Delta < 0.1\%$ and decays to zero for $|x| \gtrsim 5$.


LHS - RHS
LHS + RHS
dx |EOM|2



Outline

1. Motivations
   - Tachyon and tachyons
   - Three questions
2. Localization
   - $1 + D$ dimension
   - Construction of solutions
3. Examples of solutions
   - $\Phi(0, x)$ nontrivial
   - $\Phi(0, x)$ distribution
4. SFT correspondences
   - RG flow along r
   - SFT correspondences

Interpretation: RG flow

Interpretation: RG flow

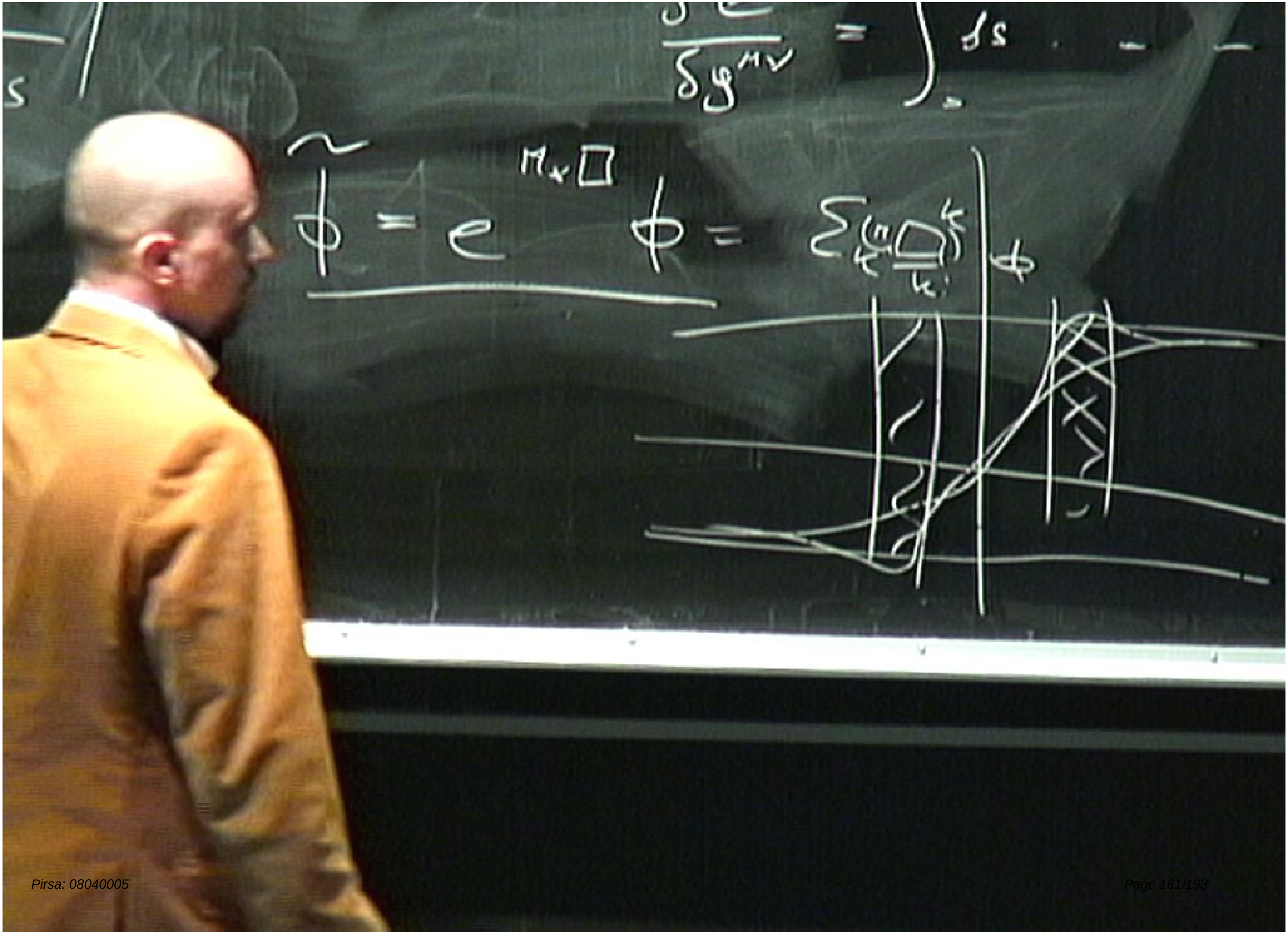
- $r = 0$ point at the IR: local regime, theory under control.

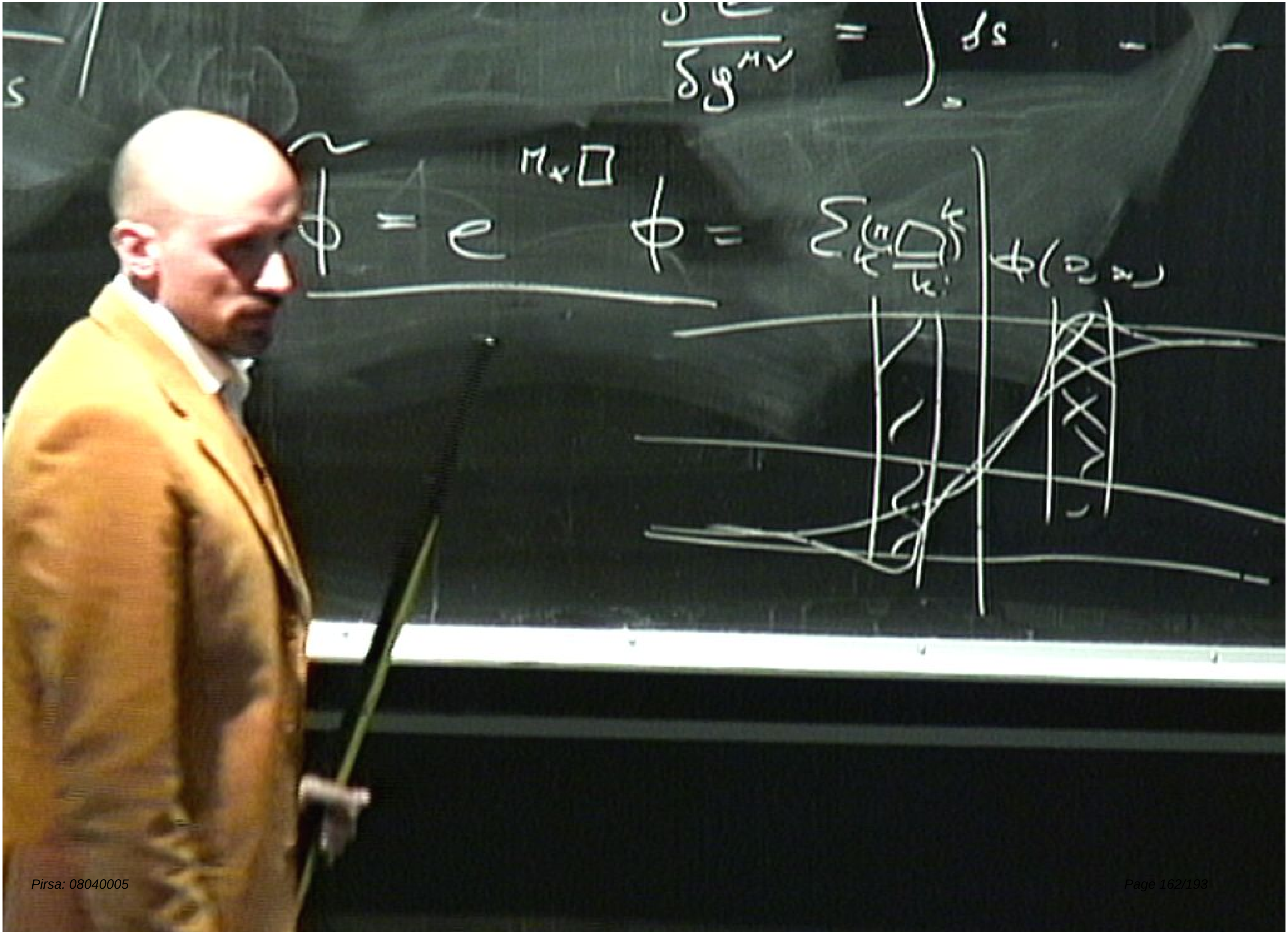
Interpretation: RG flow

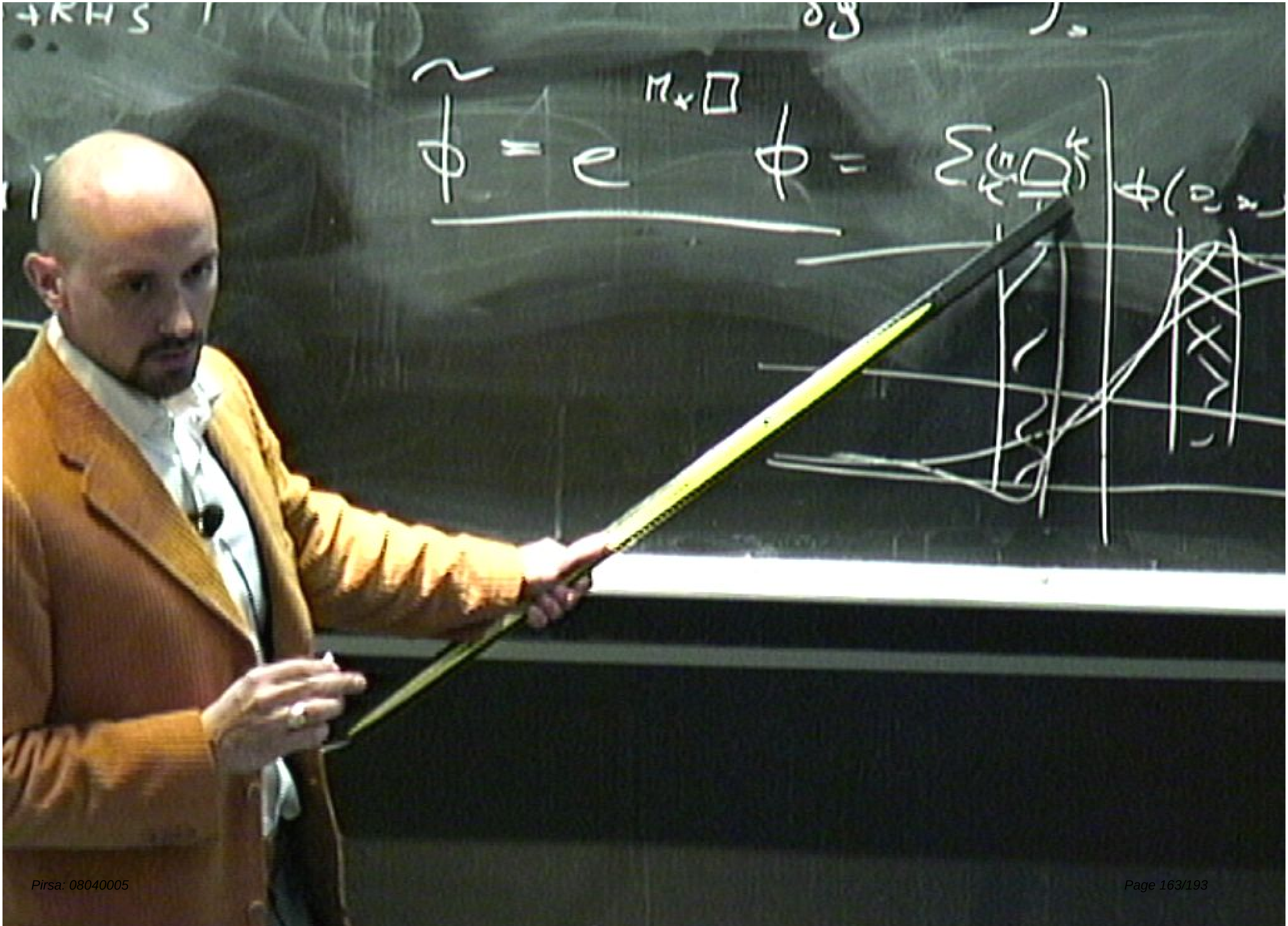
- $r = 0$ point at the IR: local regime, theory under control.
- $r = r_*$ physical UV scale at which observations take place: nonlocal effects.

Interpretation: RG flow

- $r = 0$ point at the IR: local regime, theory under control.
- $r = r_*$ physical UV scale at which observations take place: nonlocal effects.
- Truncation of $e^{\Box}$ at finite order: perturbative expansion which diverges in the UV.







Interpretation: RG flow

- $r = 0$ point at the IR: local regime, theory under control.
- $r = r_*$ physical UV scale at which observations take place: nonlocal effects.
- Truncation of $e^{\Box}$ at finite order: perturbative expansion which diverges in the UV.
- Euclidean propagator of the free field Φ:

$$G_{r_*}(k_E) = \frac{e^{-2r_* k_E^2}}{\mathcal{K}(k_E)}\,.$$

- UV cutoff $k_c \equiv (2r_*)^{-1/2}$: modes with momentum $k_E \gg k_c$ (responsible of UV perturbative divergences) are exponentially suppressed.

Interpretation: RG flow

- $r = 0$ point at the IR: local regime, theory under control.
- $r = r_*$ physical UV scale at which observations take place: nonlocal effects.
- Truncation of $e^{\Box}$ at finite order: perturbative expansion which diverges in the UV.
- Euclidean propagator of the free field Φ:

$$G_{r_*}(k_E) = \frac{e^{-2r_* k_E^2}}{\mathcal{K}(k_E)}\,.$$

- UV cutoff $k_c \equiv (2r_*)^{-1/2}$: modes with momentum $k_E \gg k_c$ (responsible of UV perturbative divergences) are exponentially suppressed.

Interpretation: RG flow

- $r = 0$ point at the IR: local regime, theory under control.
- $r = r_*$ physical UV scale at which observations take place: nonlocal effects.
- Truncation of $e^{\Box}$ at finite order: perturbative expansion which diverges in the UV.
- Euclidean propagator of the free field Φ:

$$G_{r_*}(k_E) = \frac{e^{-2r_* k_E^2}}{\mathcal{K}(k_E)}\,.$$

- UV cutoff $k_c \equiv (2r_*)^{-1/2}$: modes with momentum $k_E \gg k_c$ (responsible of UV perturbative divergences) are exponentially suppressed.

Outline

1. Motivations
   - Tachyon and tachyons
   - Three questions
2. Localization
   - $1 + D$ dimension
   - Construction of solutions
3. Examples of solutions
   - $\Phi(0, x)$ nontrivial
   - $\Phi(0, x)$ distribution
4. SFT correspondences
   - RG flow along r
   - SFT correspondences

p-adic vs. OSFT (general)

with G. Nardelli, 0802.4395 [hep-th]

p-adic vs. OSFT (general)

with G. Nardelli, 0802.4395 [hep-th]

The p-adic string can be seen as the limit of a non-propagating OSFT tachyon, where the UV cut-off $k_c \to 0$.

p-adic vs. OSFT (general)

with G. Nardelli, 0802.4395 [hep-th]

The p-adic string can be seen as the limit of a non-propagating OSFT tachyon, where the UV cut-off $k_c \to 0$.

Coordinate transformation $x^\mu \to \sqrt{r_*/r}\, x^\mu$

p-adic vs. OSFT (general)

with G. Nardelli, 0802.4395 [hep-th]

The p-adic string can be seen as the limit of a non-propagating OSFT tachyon, where the UV cut-off $k_c \to 0$.

Coordinate transformation $x^\mu \to \sqrt{r_*/r}\, x^\mu$

$$\left(\frac{r}{r_*}\Box - m^2\right) e^{-2r\Box}\Phi = \sigma\Phi^p$$

p-adic vs. OSFT (general)

with G. Nardelli, 0802.4395 [hep-th]

The p-adic string can be seen as the limit of a non-propagating OSFT tachyon, where the UV cut-off $k_c \to 0$.

Coordinate transformation $x^\mu \to \sqrt{r_*/r}\, x^\mu$

$$\left(\frac{r}{r_*}\Box - m^2\right) e^{-2r\Box}\Phi = \sigma\Phi^p$$

- $r \to r_* \approx 0.26$: OSFT tachyon equation of motion.

p-adic vs. OSFT (general)

with G. Nardelli, 0802.4395 [hep-th]

The p-adic string can be seen as the limit of a non-propagating OSFT tachyon, where the UV cut-off $k_c \to 0$.

Coordinate transformation $x^\mu \to \sqrt{r_*/r}\, x^\mu$

$$\left(\frac{r}{r_*}\Box - m^2\right) e^{-2r\Box}\Phi = \sigma\Phi^p$$

- $r \to r_* \approx 0.26$: OSFT tachyon equation of motion.
- $r_* \to \infty$: p-adic equation if the fixed point is $r = r_p$.

p-adic vs. BSFT (on lump solution)

with G. Nardelli, 0802.4395 [hep-th]

p-adic vs. BSFT (on lump solution)

with G. Nardelli, 0802.4395 [hep-th]

The effective action whose equation of motion with $\mathcal{K} = \Box$ is the diffusion equation is precisely that of BSFT up to two derivatives and with a tachyon profile $T \sim x^2$.

p-adic vs. BSFT (on lump solution)

with G. Nardelli, 0802.4395 [hep-th]

The effective action whose equation of motion with $\mathcal{K} = \Box$ is the diffusion equation is precisely that of BSFT up to two derivatives and with a tachyon profile $T \sim x^2$.

Set $\alpha = \frac{2r_*}{\gamma}$, $c = 0$, $\gamma = r_*/2$:

p-adic vs. BSFT (on lump solution)

with G. Nardelli, `0802.4395 [hep-th]`

The effective action whose equation of motion with $\mathcal{K} = \Box$ is the diffusion equation is precisely that of BSFT up to two derivatives and with a tachyon profile $T \sim x^2$.

Set $\alpha = \frac{2r_*}{\gamma}$, $c = 0$, $\gamma = r_*/2$:

$$\mathcal{L} = e^{-T}\left[\partial_\mu T \partial^\mu T + (1+T)\right] .$$

OSFT vs. BSFT (on Minkowski solutions)

with G. Nardelli, 0708.0366 [hep-th]

p-adic vs. BSFT (on lump solution)

with G. Nardelli, 0802.4395 [hep-th]

The effective action whose equation of motion with $\mathcal{K} = \Box$ is the diffusion equation is precisely that of BSFT up to two derivatives and with a tachyon profile $T \sim x^2$.

Set $\alpha = \frac{2r_*}{\gamma}$, $c = 0$, $\gamma = r_*/2$:

$$\mathcal{L} = e^{-T}\left[\partial_\mu T \partial^\mu T + (1+T)\right] .$$

OSFT vs. BSFT (on Minkowski solutions)

with G. Nardelli, 0708.0366 [hep-th]

OSFT vs. BSFT (on Minkowski solutions)

with G. Nardelli, 0708.0366 [hep-th]

- The partition function (effective action) of BSFT evaluated on the tachyon profile $T \sim e^x$ and with propagator ambiguity r_* is a kink which is the primitive of the bosonic OSFT solution.

OSFT vs. BSFT (on Minkowski solutions)

with G. Nardelli, 0708.0366 [hep-th]

- The partition function (effective action) of BSFT evaluated on the tachyon profile $T \sim e^x$ and with propagator ambiguity r_* is a kink which is the primitive of the bosonic OSFT solution.
- Another mathematical relation holds in the supersymmetric case.

Achieved results

- Developed a systematic method by which to find nonperturbative solutions of nonlocal systems.

Achieved results

- Developed a systematic method by which to find nonperturbative solutions of nonlocal systems.
- Bosonic and susy OSFT, p-adic global solutions found in Minkowski.

Achieved results

- Developed a systematic method by which to find nonperturbative solutions of nonlocal systems.
- Bosonic and susy OSFT, p-adic global solutions found in Minkowski.
- Asymptotic cosmological solutions found in toy models and p-adic string.

Achieved results

- Developed a systematic method by which to find nonperturbative solutions of nonlocal systems.
- Bosonic and susy OSFT, p-adic global solutions found in Minkowski.
- Asymptotic cosmological solutions found in toy models and p-adic string.
- Interpretation issues are all addressed.

Achieved results

- Developed a systematic method by which to find nonperturbative solutions of nonlocal systems.
- Bosonic and susy OSFT, p-adic global solutions found in Minkowski.
- Asymptotic cosmological solutions found in toy models and p-adic string.
- Interpretation issues are all addressed.

Open issues

Achieved results

- Developed a systematic method by which to find nonperturbative solutions of nonlocal systems.
- Bosonic and susy OSFT, p-adic global solutions found in Minkowski.
- Asymptotic cosmological solutions found in toy models and p-adic string.
- Interpretation issues are all addressed.

Open issues

- Search for “string” cosmological solutions still in progress.

Achieved results

- Developed a systematic method by which to find nonperturbative solutions of nonlocal systems.
- Bosonic and susy OSFT, p-adic global solutions found in Minkowski.
- Asymptotic cosmological solutions found in toy models and p-adic string.
- Interpretation issues are all addressed.

Open issues

- Search for “string” cosmological solutions still in progress.
- Implementation of nonlocality to the gravity sector.

Achieved results

- Developed a systematic method by which to find nonperturbative solutions of nonlocal systems.
- Bosonic and susy OSFT, p-adic global solutions found in Minkowski.
- Asymptotic cosmological solutions found in toy models and p-adic string.
- Interpretation issues are all addressed.

Open issues

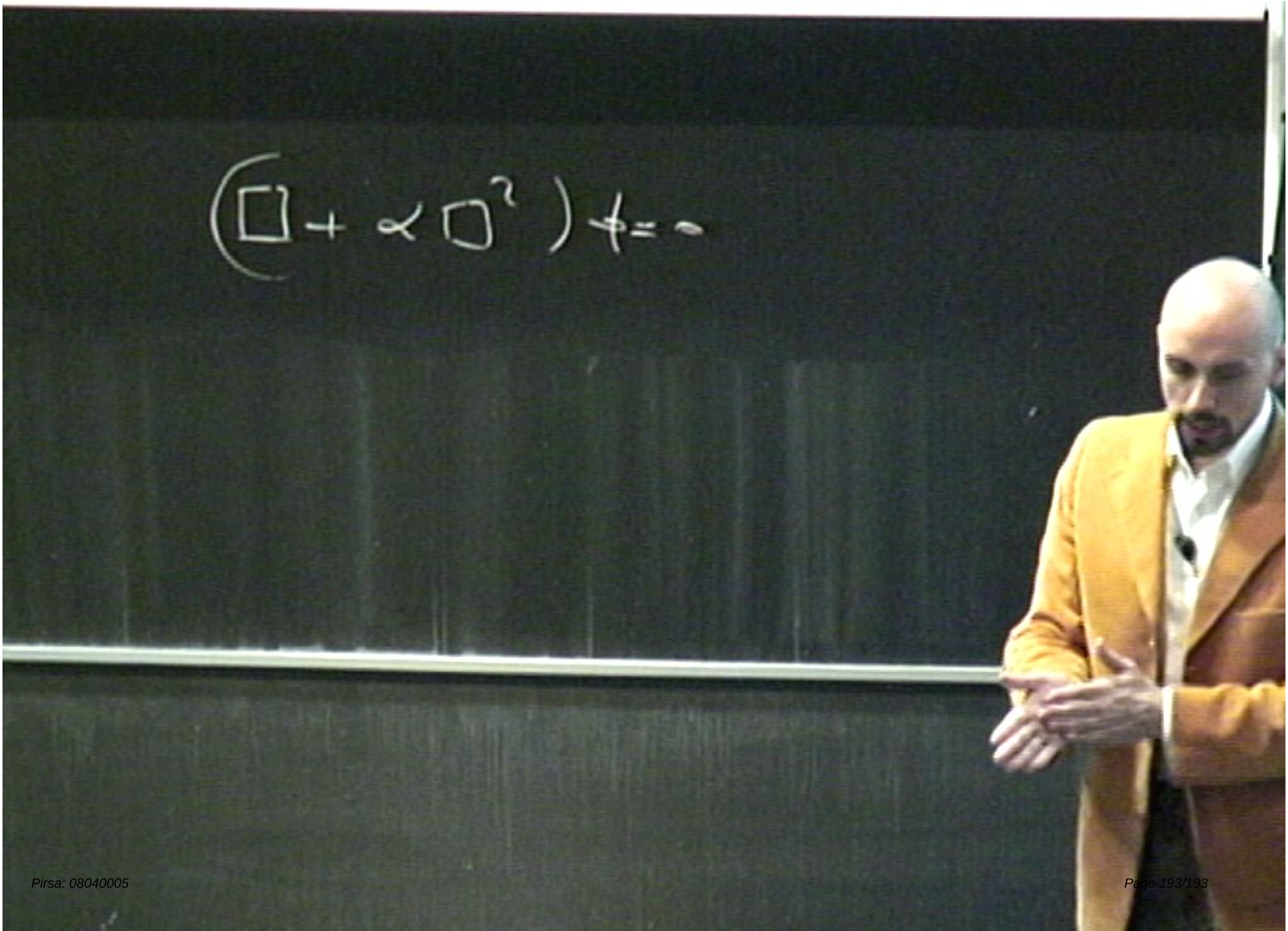
- Search for "string" cosmological solutions still in progress.
- Implementation of nonlocality to the gravity sector.
- Puzzling correspondences between OSFT, BSFT and p-adic.

Nonlocal systems and string field theory

Gianluca Calcagni

University of Sussex PENNSTATE

April 22th, 2008


(□ + α □²) φ = 0