Title: Spinflation

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Abstract: TBA

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# Spinflation

Ivonne Zavala

IPPP, Durham

Based on: JHEP04(2007)026 and JCAP02(2008)010

In collaboration with: Easson, Gregory, Tasinato and Mota

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### Motivation

Inflation: very successful scenario in search of a theory.

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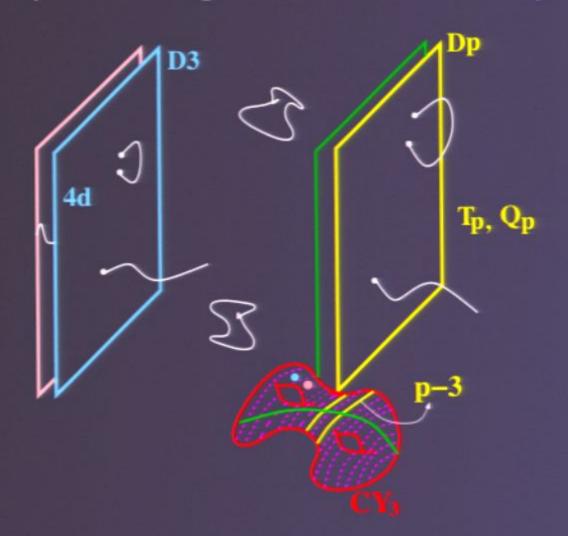
Can string theory provide a fundamental origin for the inflaton field?

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Open string inflation: brane position is the inflaton.

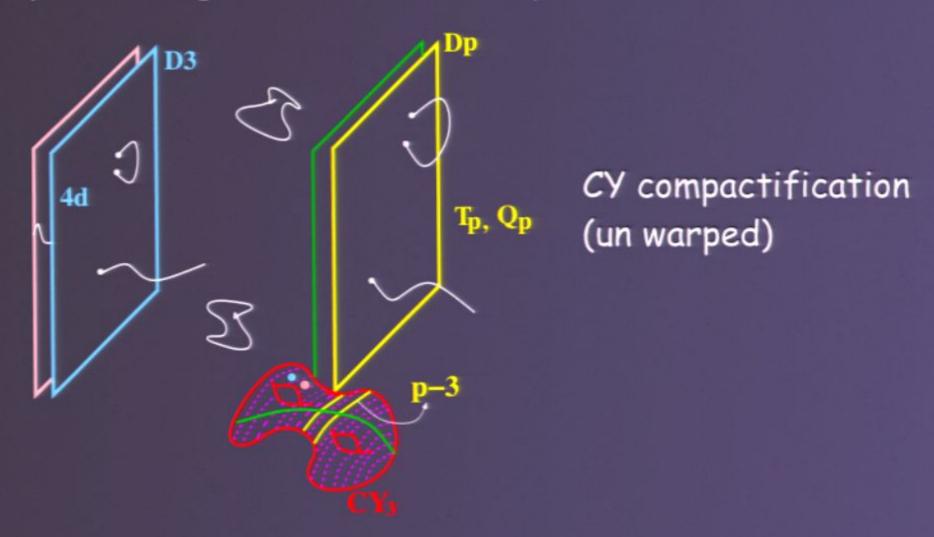
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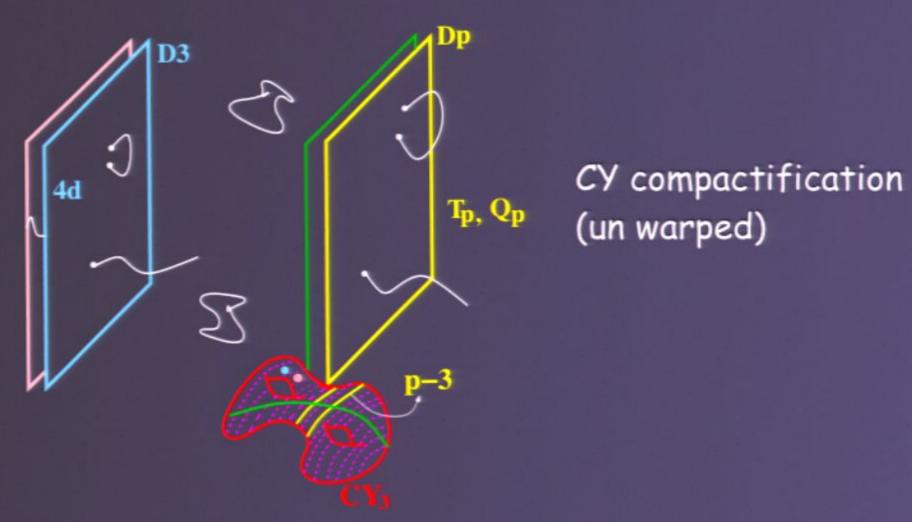
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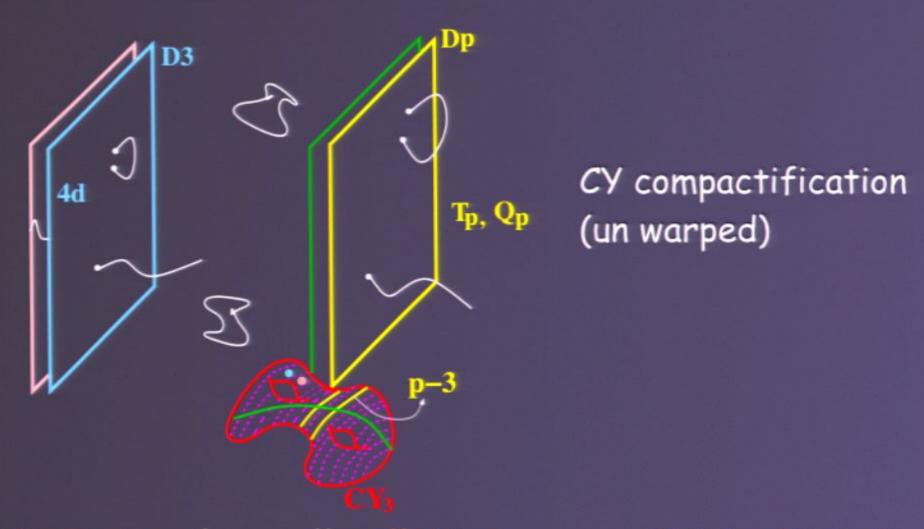


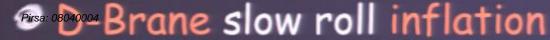
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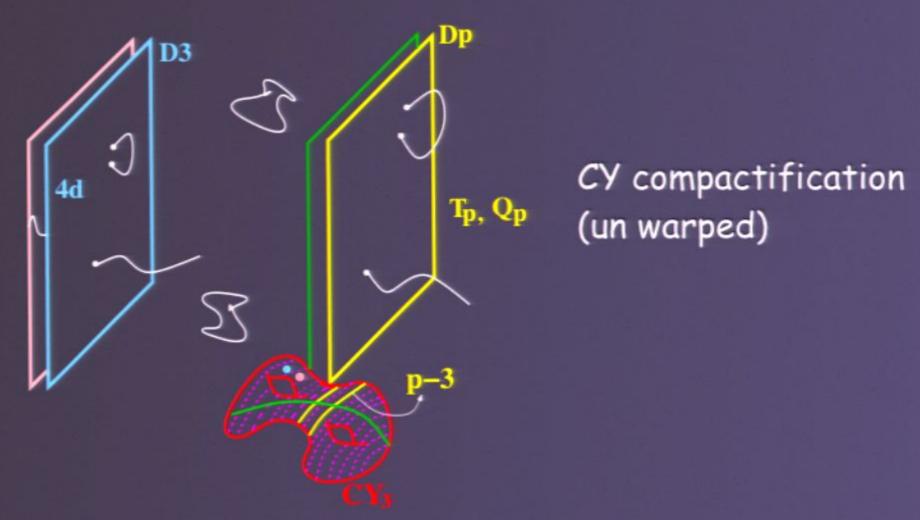
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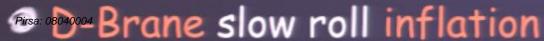




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['98 Dvali-TyPege]1/548

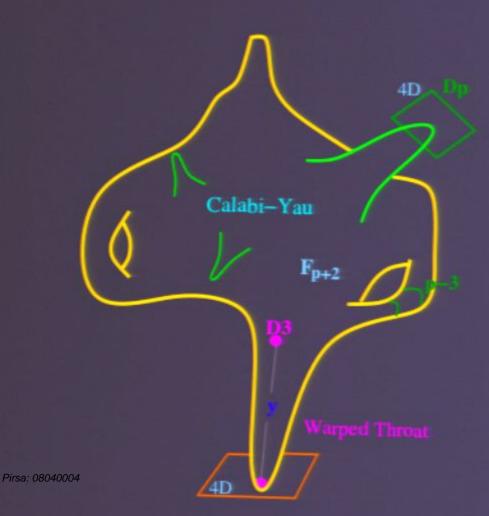
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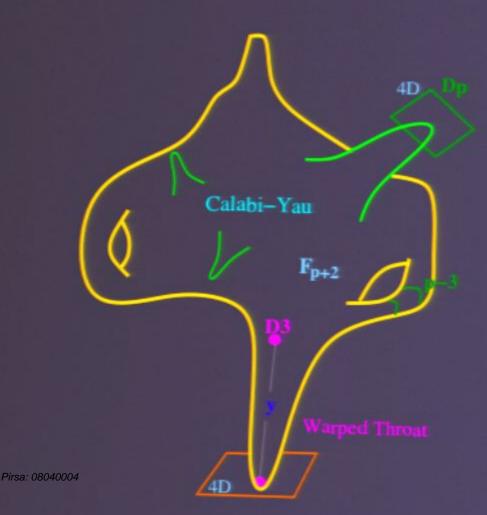
### D-Brane DBI-Inflation

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Warped CY compactification (+ fluxes)

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[Silverstein-Tong]

warped geometry

$$ds_{10}^2 = h^{-1/2}(\rho) dx_\mu dx^\mu + h^{1/2}(\rho) ds_6^2$$

$$h = \left(\frac{R}{\rho}\right)^4$$
 (warp factor)  $R^4 \propto \alpha'^2 g_s N$   
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$$v^2 = g_{\rho\rho}\dot{\rho}^2$$
  $\Longrightarrow$ 

$$hv^2 < 1$$

### Brane trajectories (q=1, branes)

Conserved energy

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Brane trajectories described by equation of motion:

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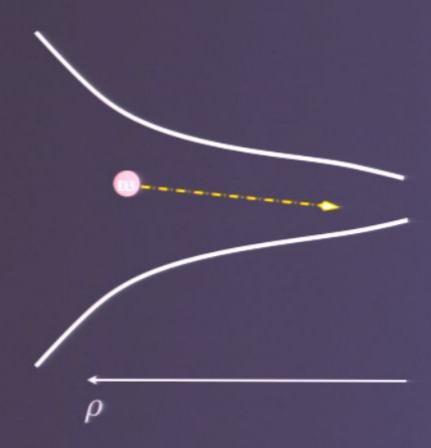
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Simple trajectories: brane moves from infinity towards

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$$\int d^4x \sqrt{-g} \left\{ \frac{M_{Pl}^2}{2} R - \left[ h^{-1} \sqrt{1 - h \, \dot{\phi}^2} - q \, h^{-1} + V(\phi) \right] \right\}$$

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# Spinflation

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Consider the more realistic flux compactification:

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 Klebanov-Strassler (GKP)

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 Effects of angular (as well as radial) motion for the brane trajectories => Mirage cosmology.

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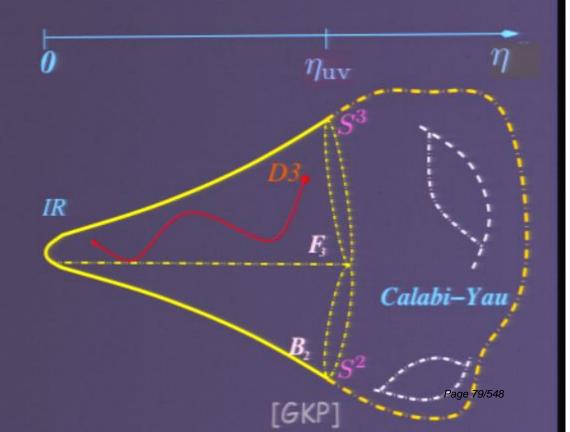
Pirsa: 08040004 Page 77/548

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Pirea: 08040004

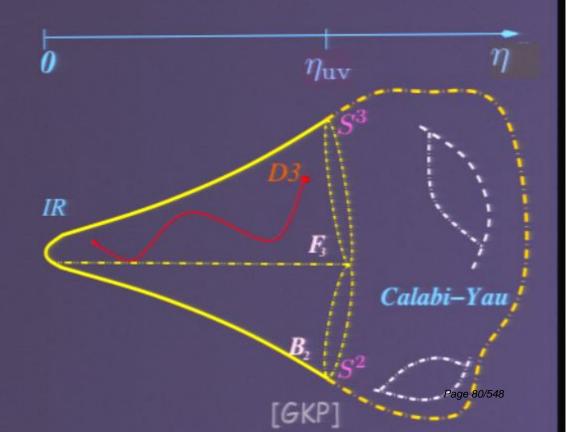
Type IIB solution with F3, H3, F5 internal fluxes

$$ds_{10}^2 = h^{-1/2}(\eta) \ dx_{\mu} dx^{\mu} + h^{1/2}(\eta) ds_6^2$$



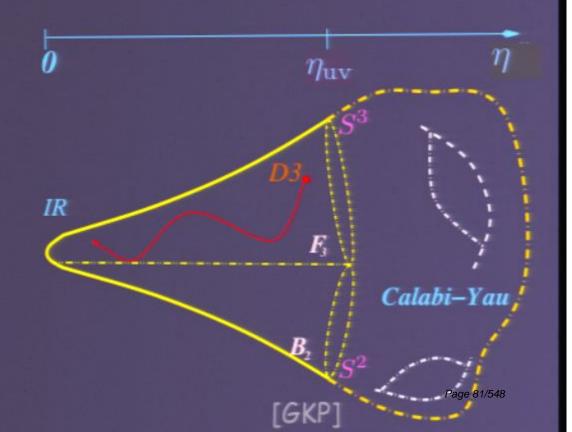
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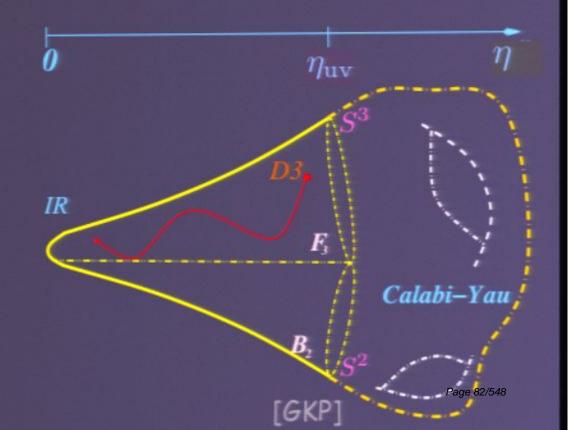
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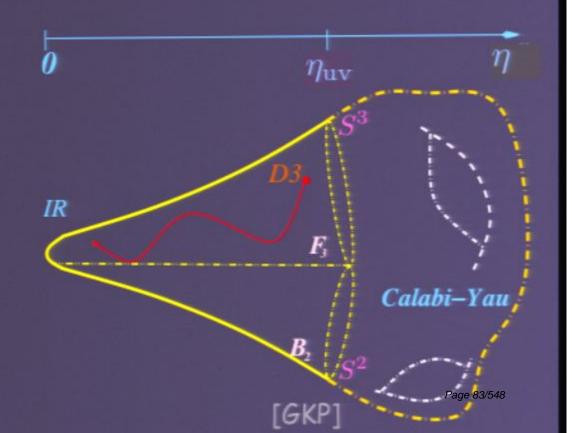
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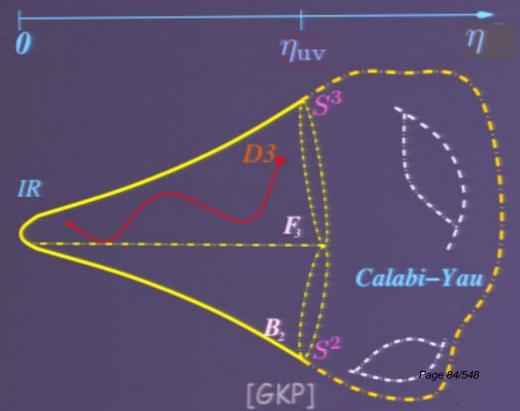
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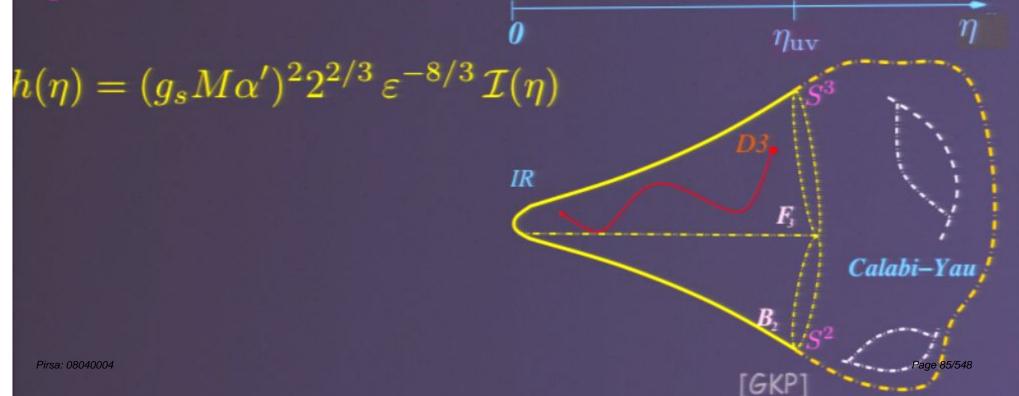
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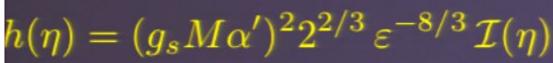
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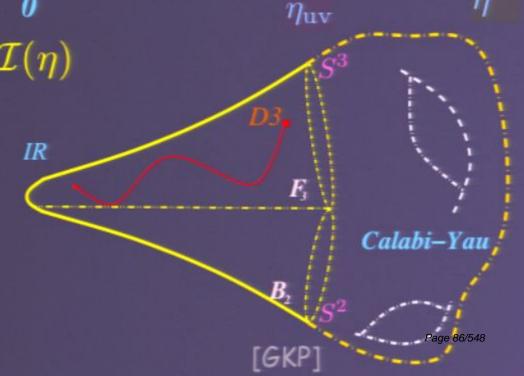
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$$M = 3 - form \ flux \ units$$



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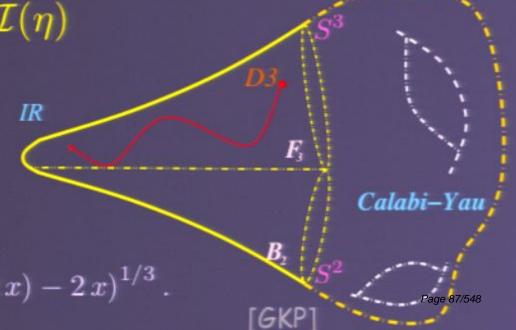
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$$h(\eta) = (g_s M \alpha')^2 2^{2/3} \varepsilon^{-8/3} \mathcal{I}(\eta)$$

$$M = 3 - form \ flux \ units$$

$$\mathcal{I}_{Pirsal.08040004}^{(\eta)} = \int_{n}^{\infty} dx \, \frac{x \, \coth x - 1}{\sinh^2 x} \, \left(\sinh (2 \, x) - 2 \, x\right)^{1/3}.$$



 $\eta_{\mathrm{uv}}$ 

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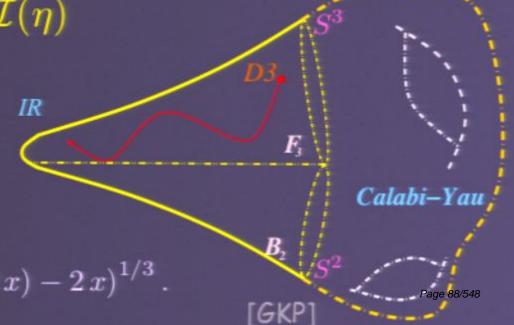
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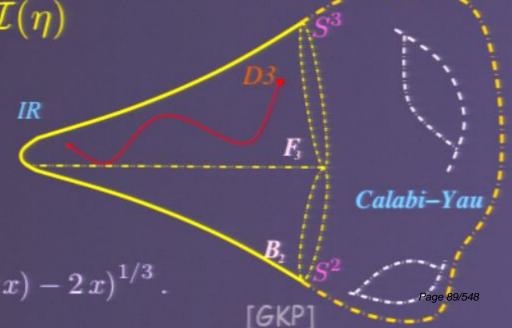
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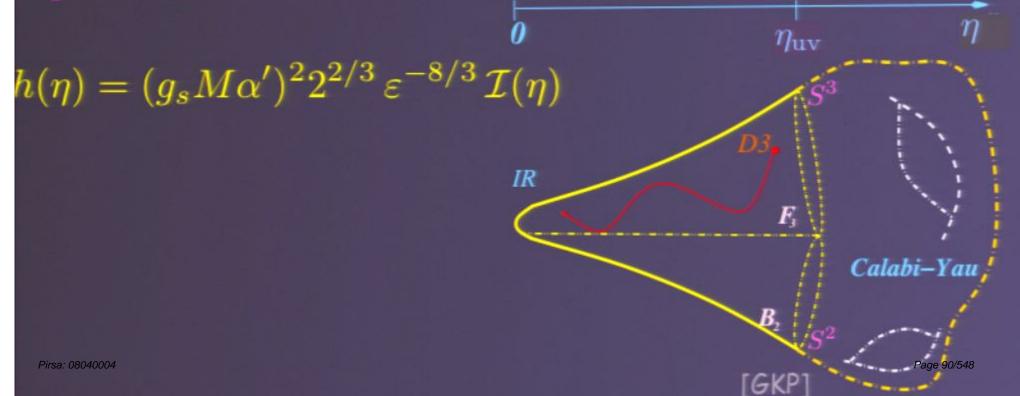


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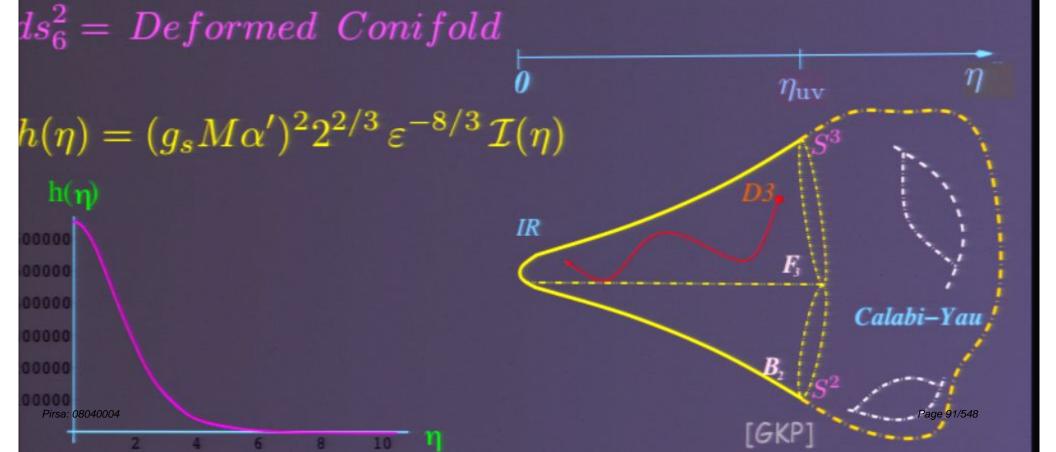
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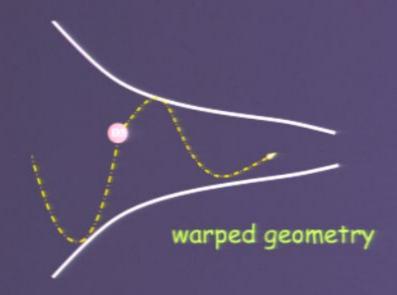
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Brane motion described by (DBI action)

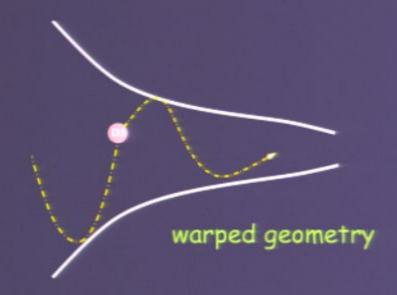
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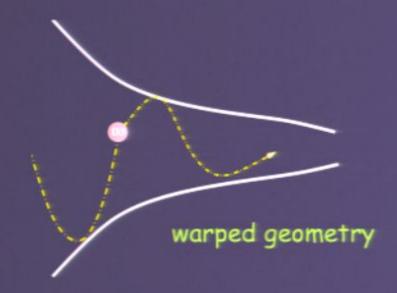
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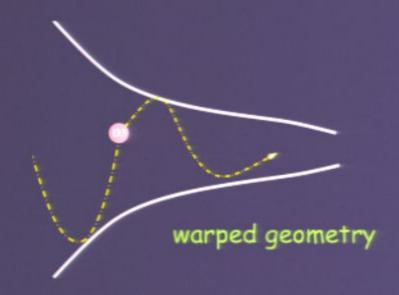
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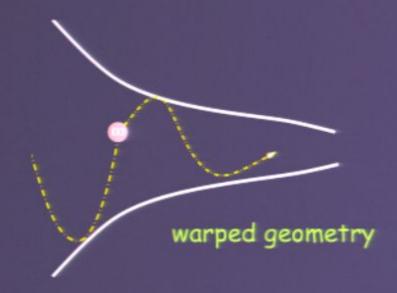
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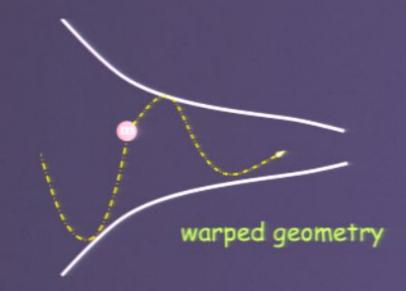


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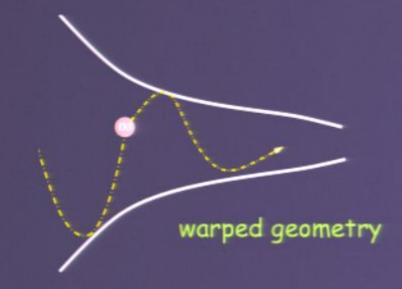
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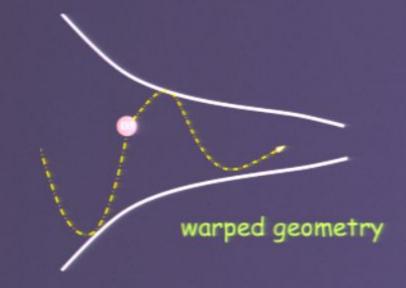
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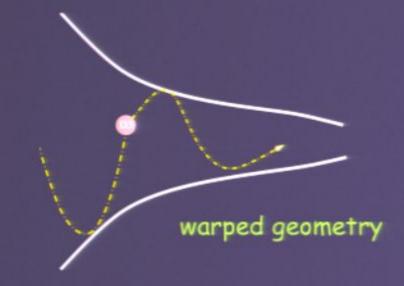


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$$\implies hv^2 < 1$$



Conserved quantities

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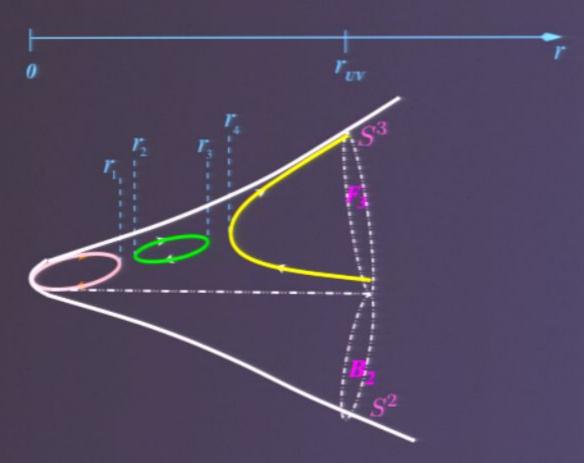
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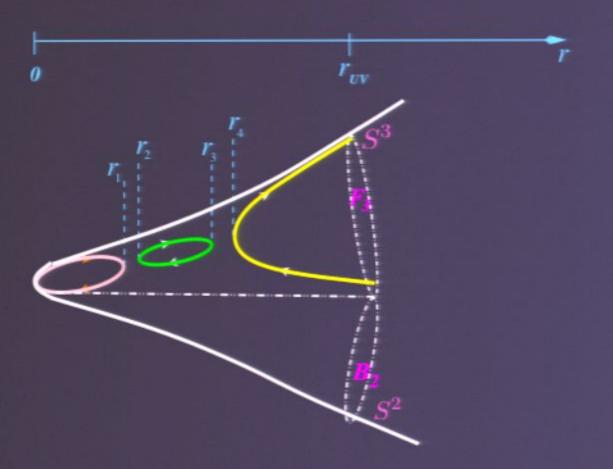
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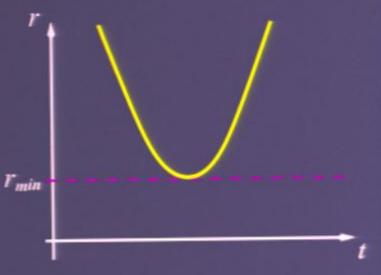
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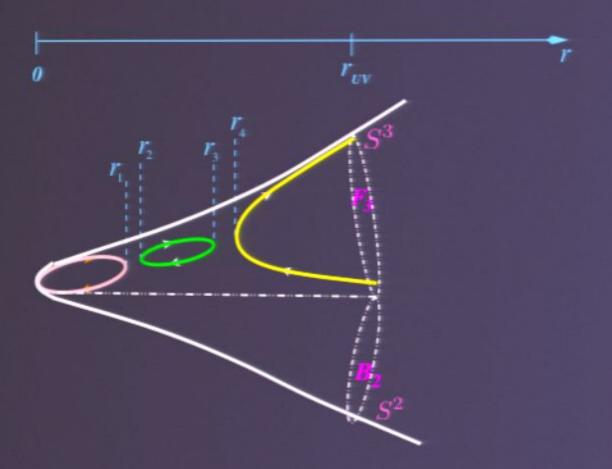
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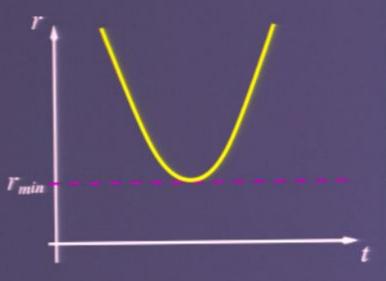
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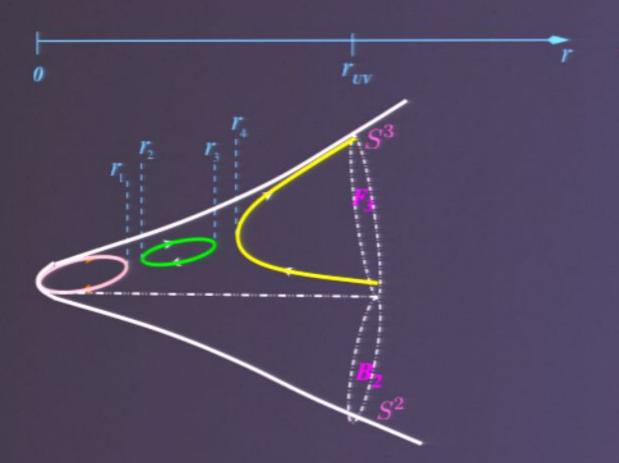


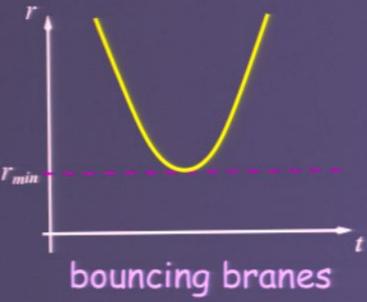


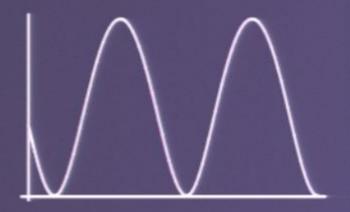


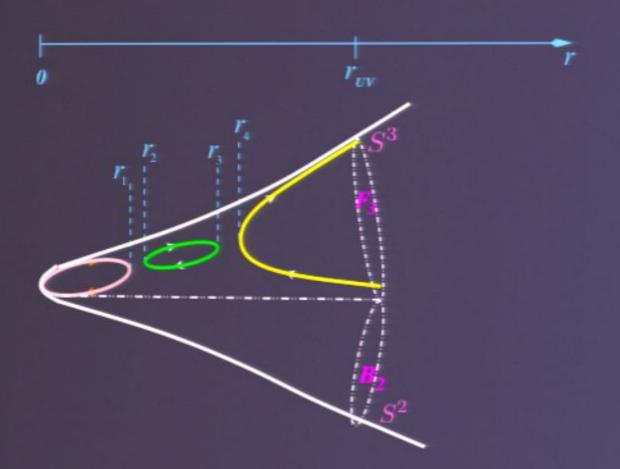


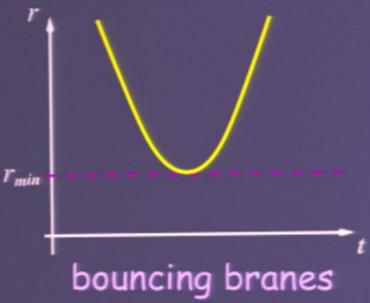






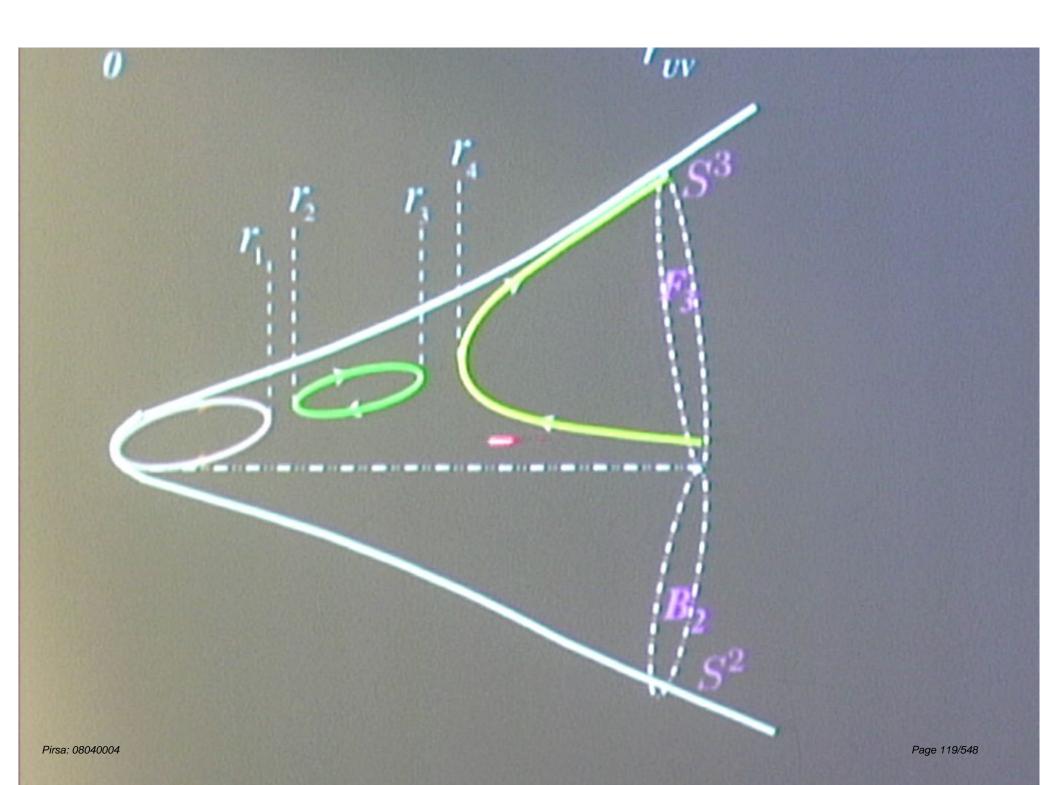


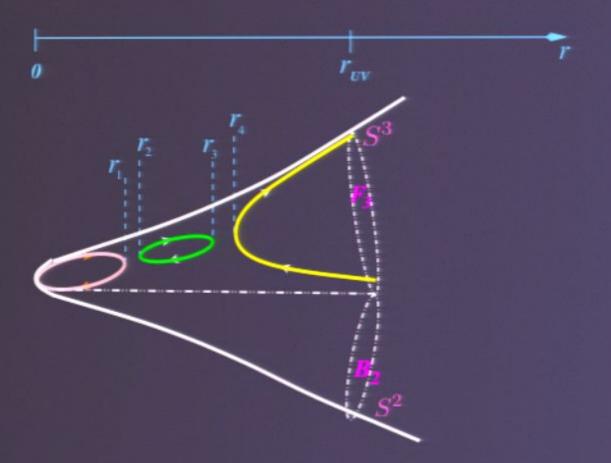


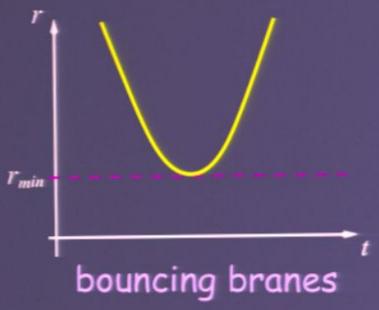




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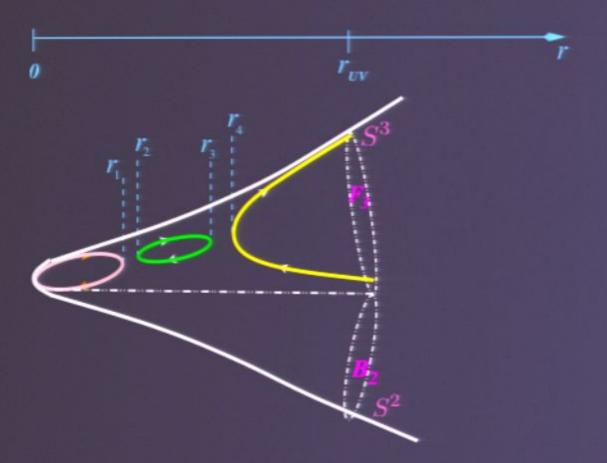


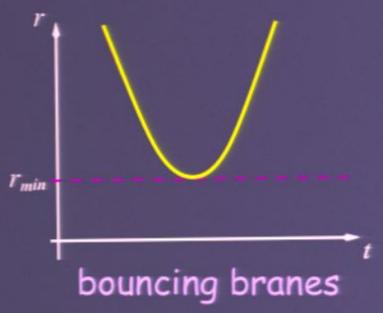






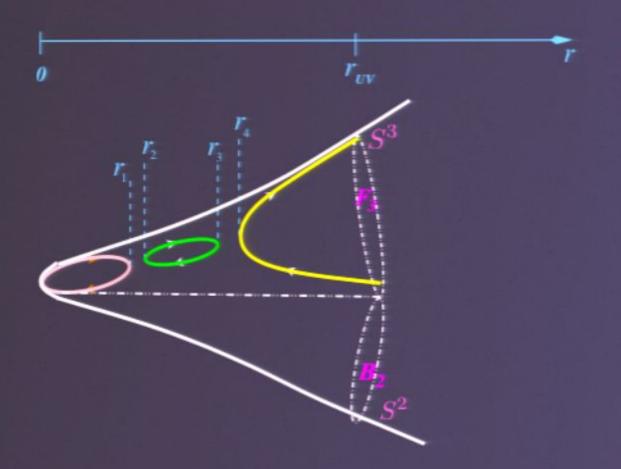
cyclic brapage 120/548

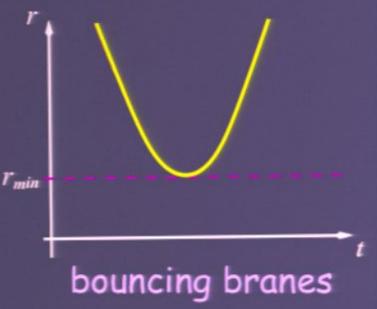






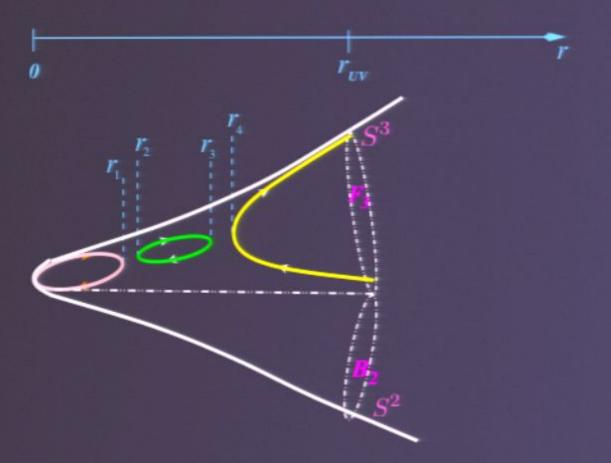
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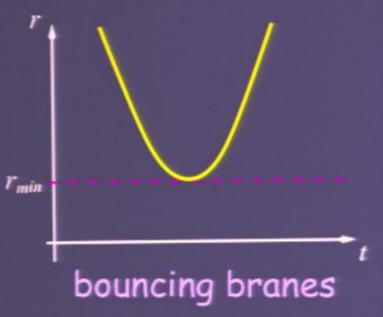






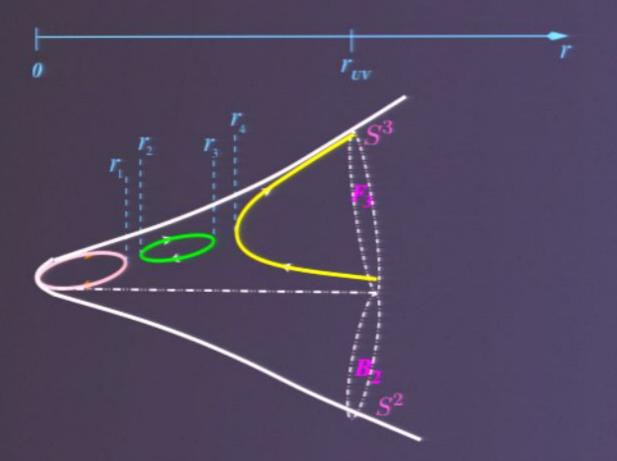
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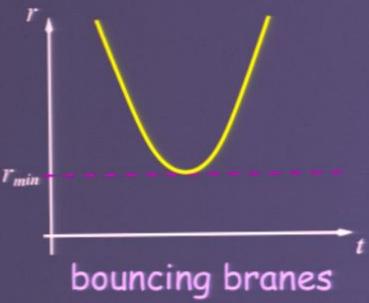






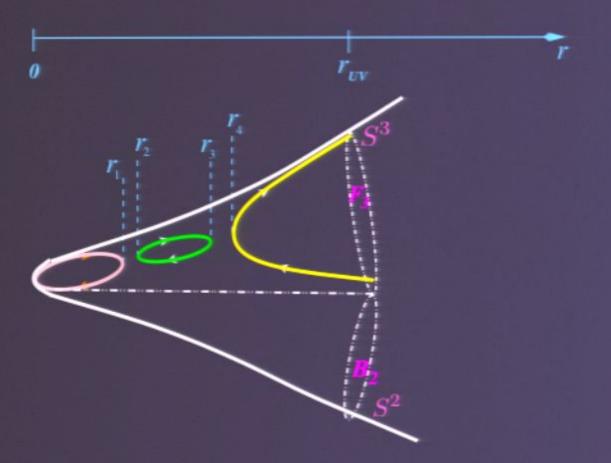
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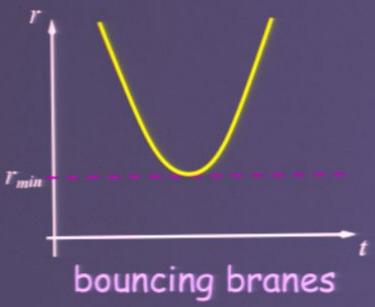






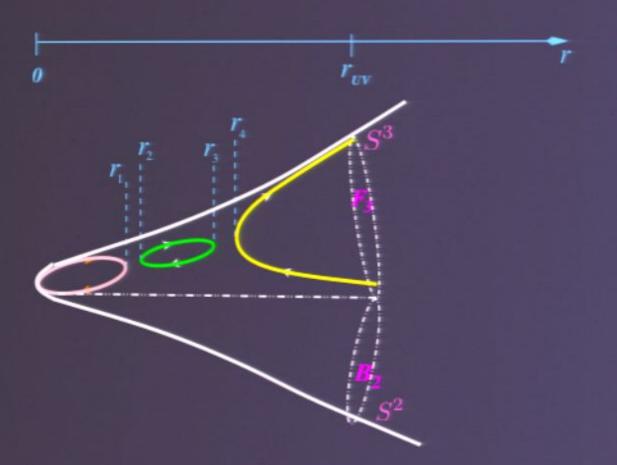
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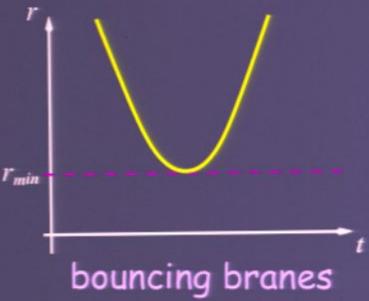






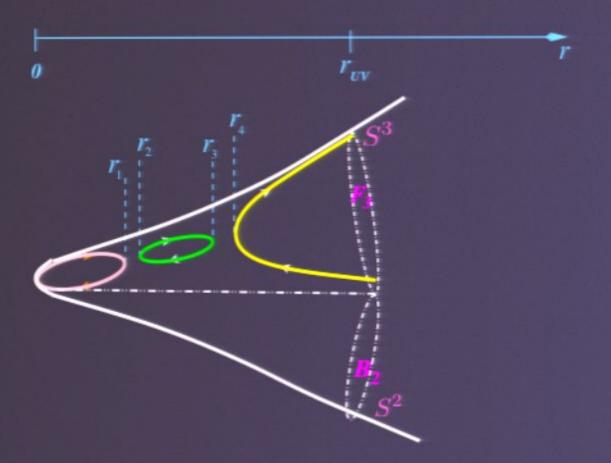
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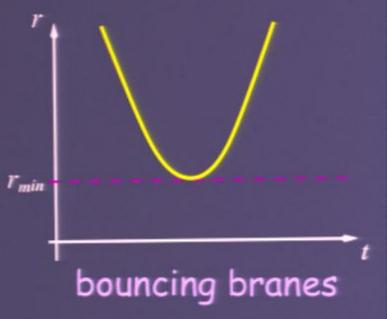






cyclic brape 126/548







cyclic bratege 127/548

$$ds_4^2 = -d\tau^2 + a^2(\tau)dx_i dx^i \,,$$

$$H_{ind}^{2} = \left(\frac{h'}{4 h^{3/4}}\right)^{2} g^{\eta \eta} \left[ E \left( h E + 2q \right) - \ell^{2}(\eta) \right]$$

 $[\ell=0$ , Kachru-McAllister '02]

where 
$$a(\tau)=h^{-1/4}(\tau)$$
 and  $H_{ind}=rac{1}{a}rac{d\,a}{d\eta}rac{d\,\eta}{d au}$  
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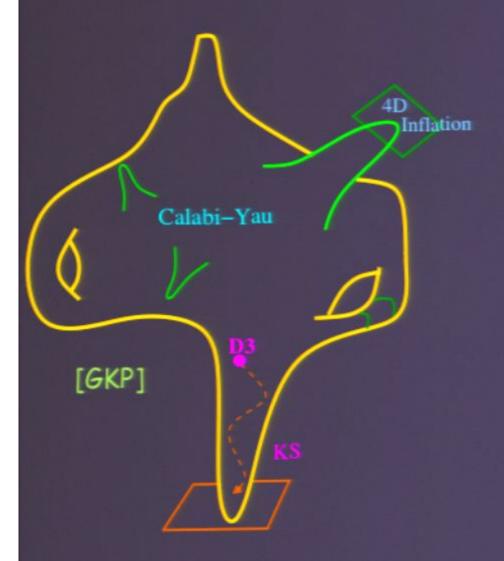
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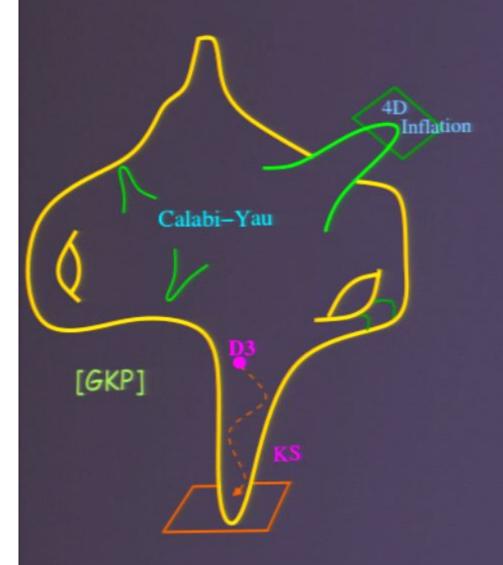
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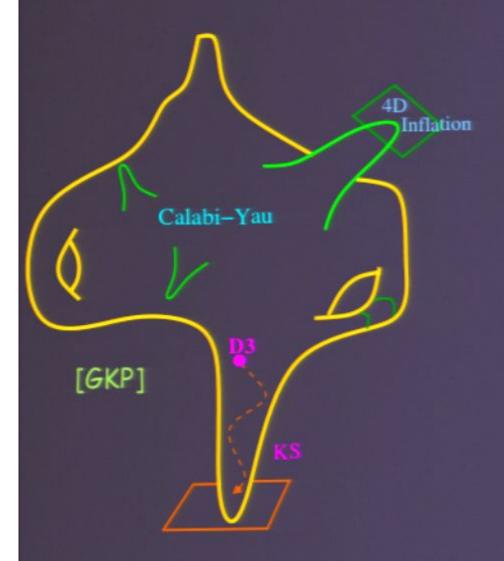
Pisa: 08040004 Mirage bouncing and cyclic universes



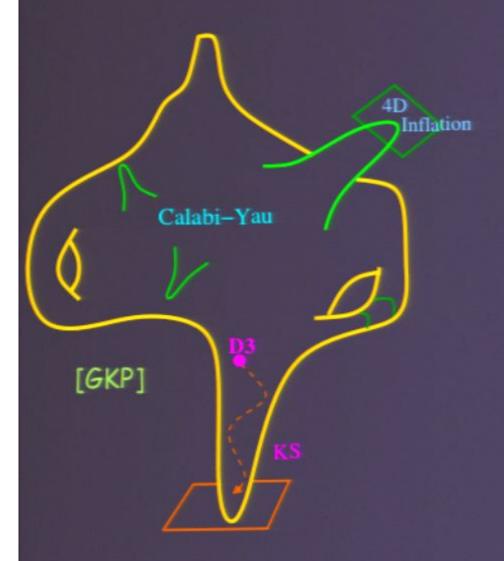
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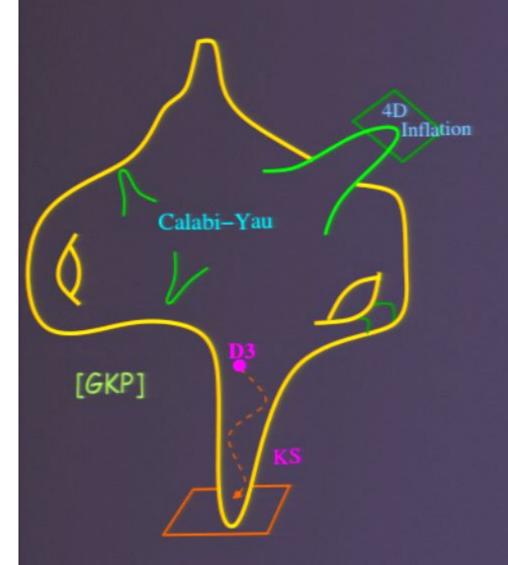
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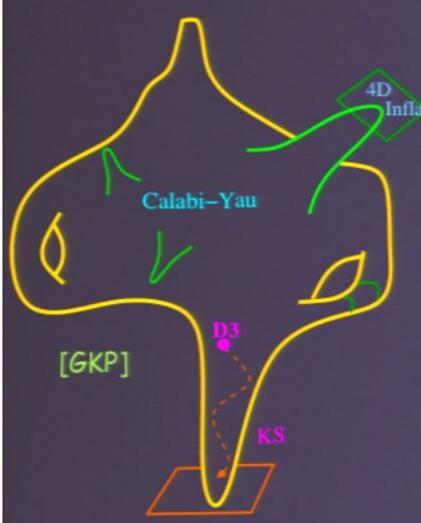
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 Couple the system to gravity to study inflationary trajectories.

[Quevedo, Gibbons, Silverstein-Tong].

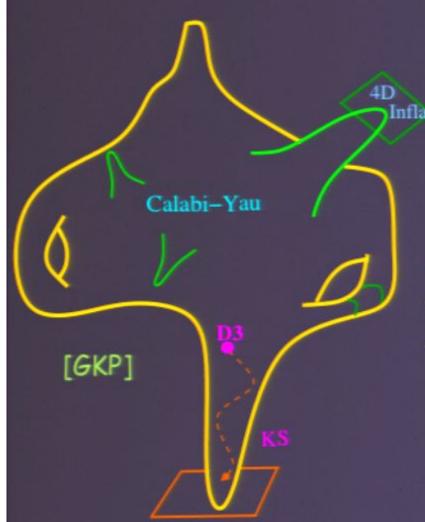
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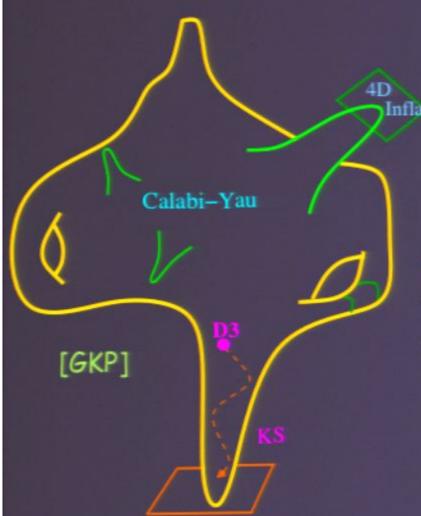
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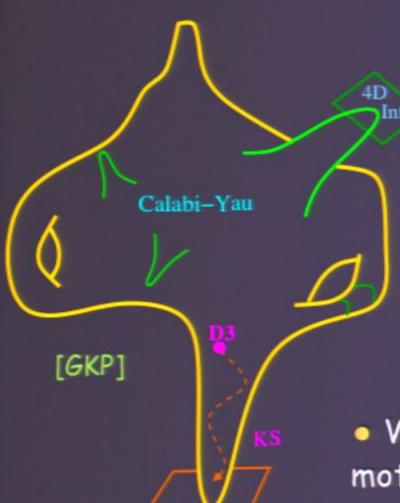
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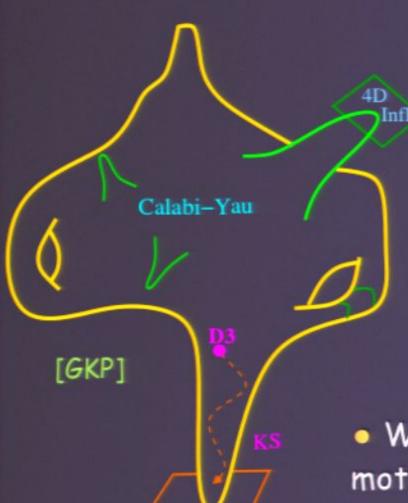
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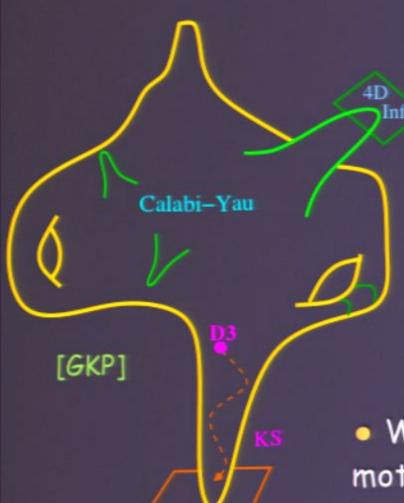


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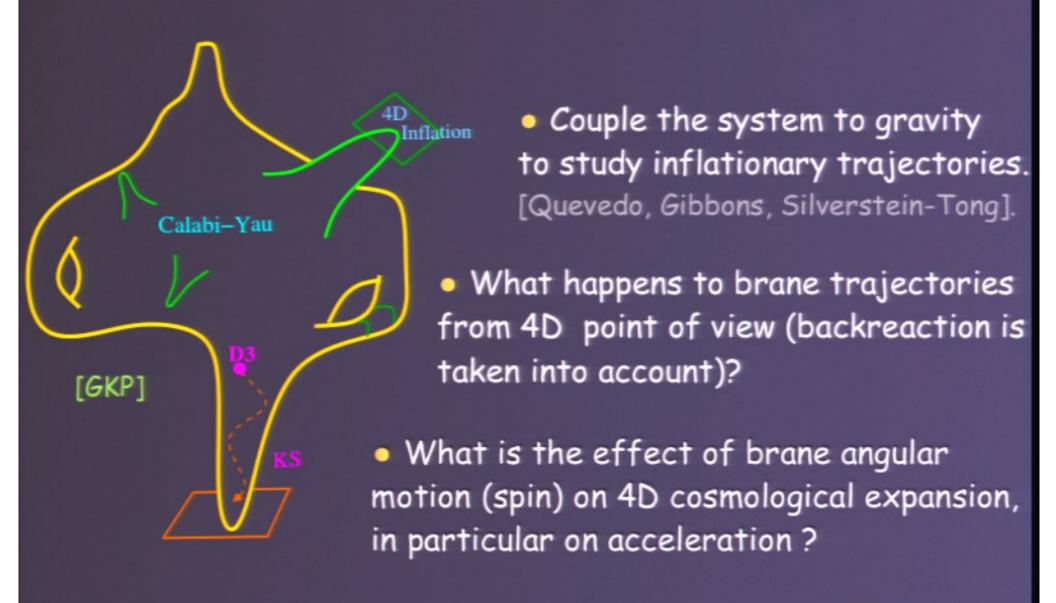
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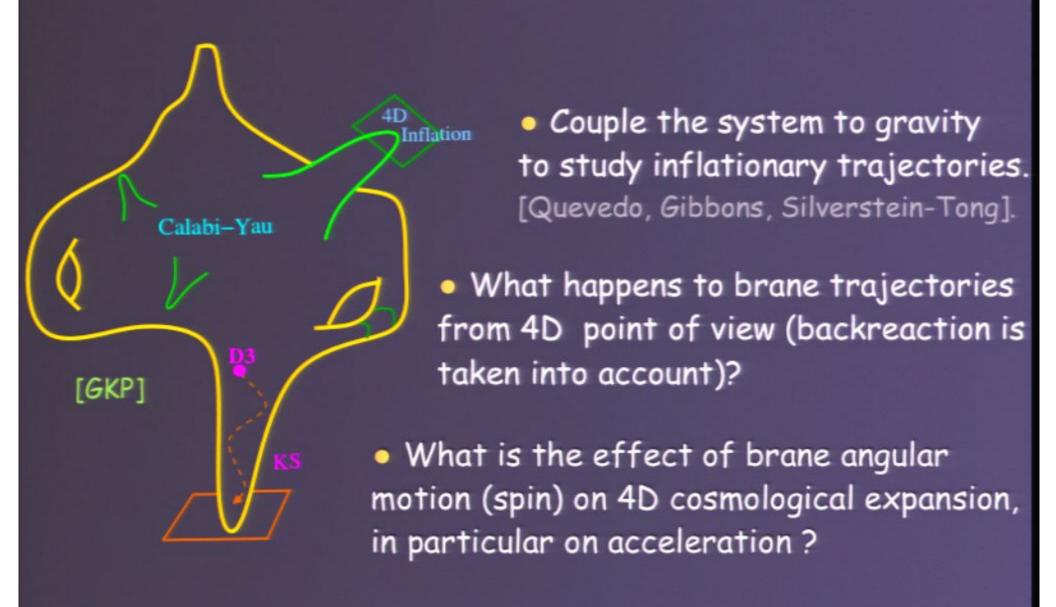
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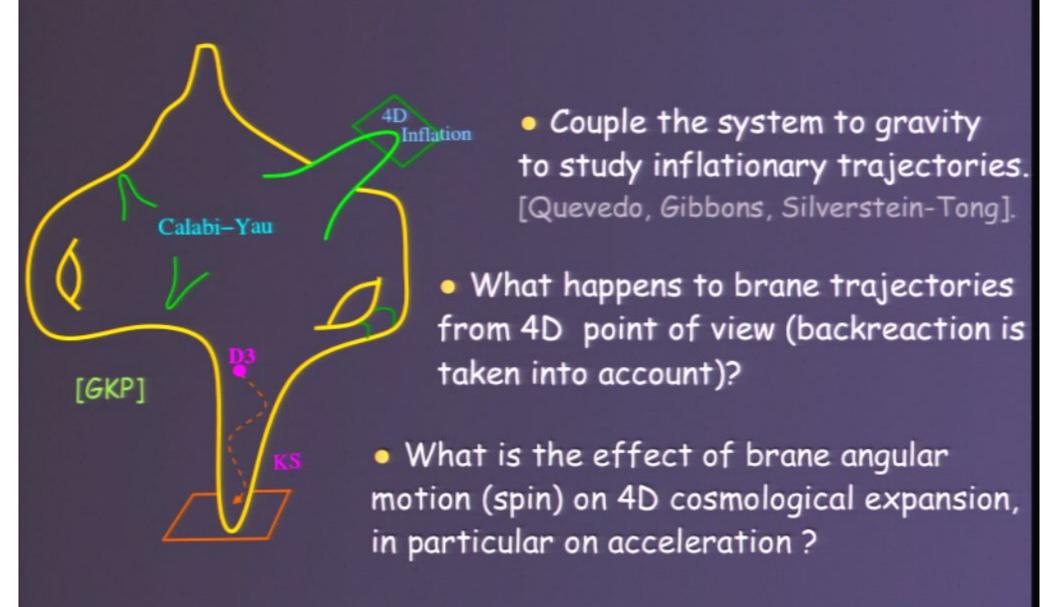
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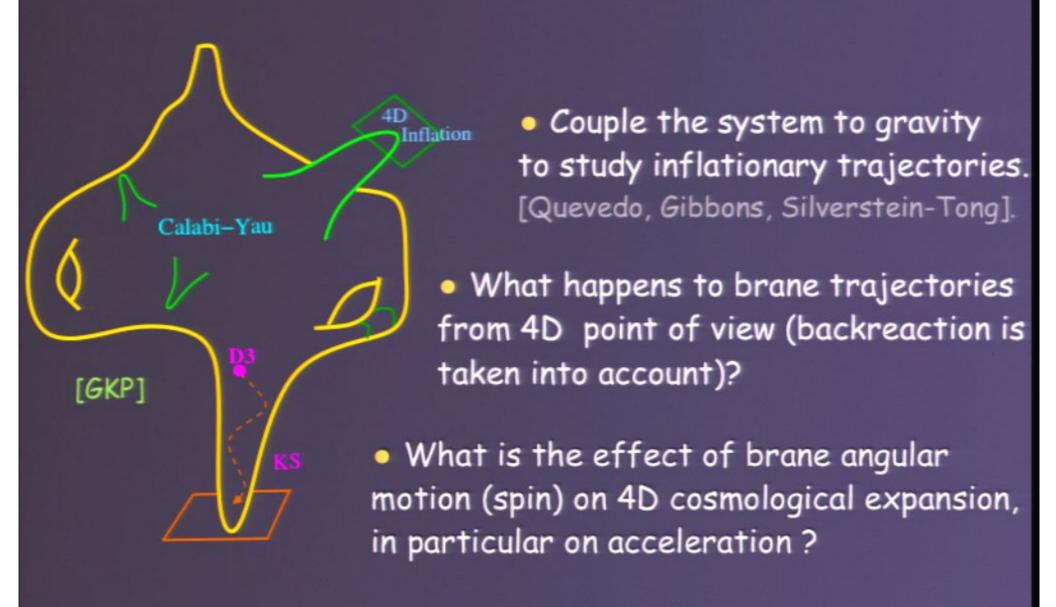
🔐 6.4001 the angular motion relax the constraints on DBI Inflation



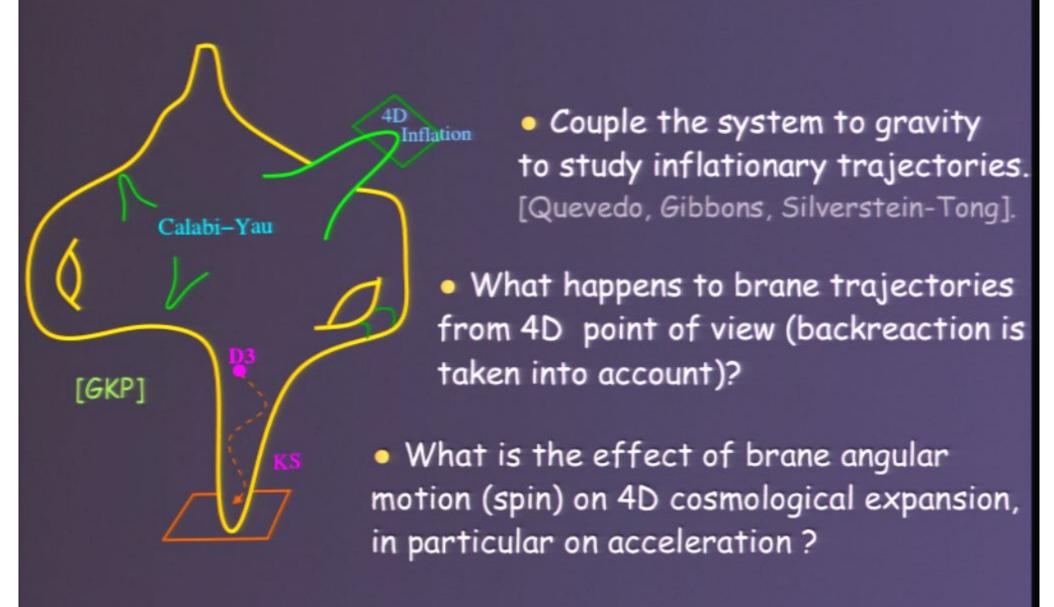
Pass Country the angular motion relax the constraints on DBI Inflation



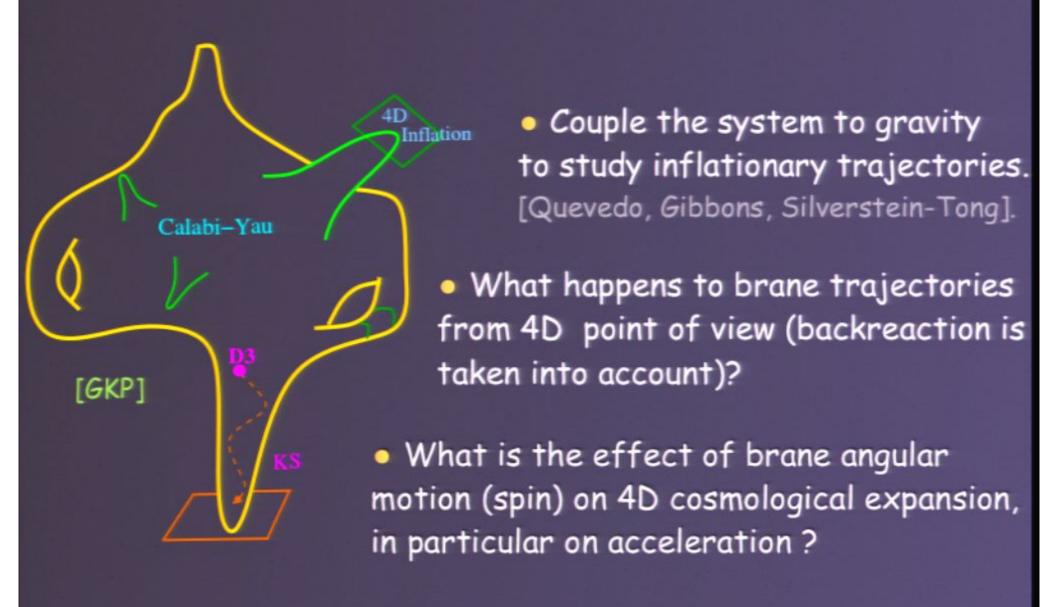
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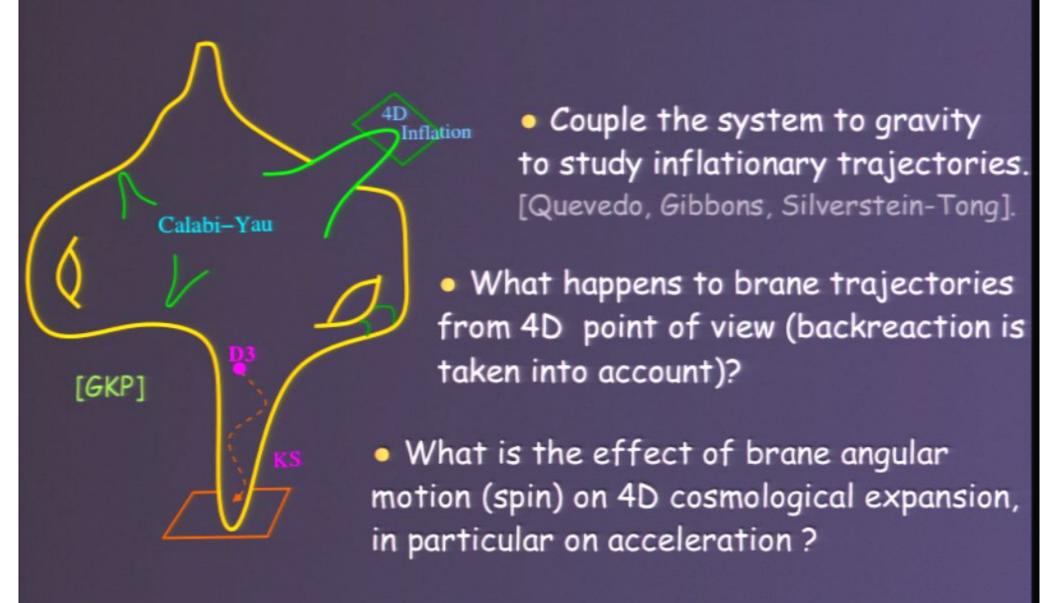
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$$M_{Pl}^2 = V_6/\kappa_{10}^2$$
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#### Four dimensional metric is of FRW form:

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$$H^2 = \frac{1}{3M_{Pl}^2}E \; ; \qquad \qquad H = \frac{\dot{a}}{a}$$

$$E = \frac{(\gamma - 1)}{h} + V;$$
  $P = \frac{(1 - \gamma^{-1})}{h} - V;$   $V = m^2 \phi^2$ 

$$\gamma = \sqrt{\frac{1 + h \ell^2(\phi)/a^6}{1 - h \dot{\phi}^2}}$$
  $l_{\theta} = a^4$ 

$$l_{\theta} = \mathbf{a}^{3} g_{\theta\theta} \,\dot{\theta} \,\gamma$$

$$\ell^2(\phi)=g^{rs}l_rl_s$$
 Page 169/540

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 Page 171/540

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Pirsa: 08040004

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Pirsa: 08040004 Page 187/548

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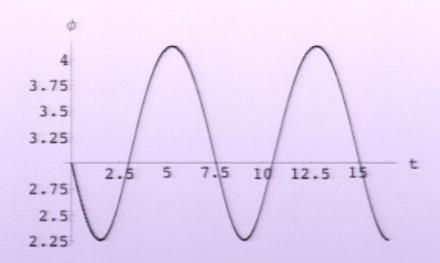
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Brane trajectory with no gravity

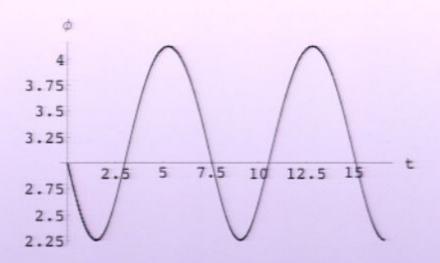
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Brane trajectory with no gravity

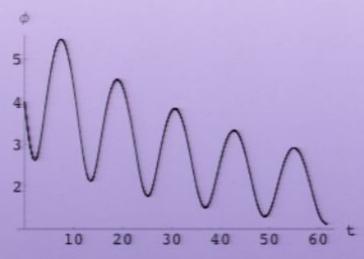


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Brane trajectory with no gravity

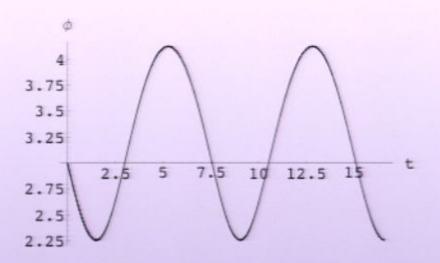


Brane position and scale factor with gravity switched on .

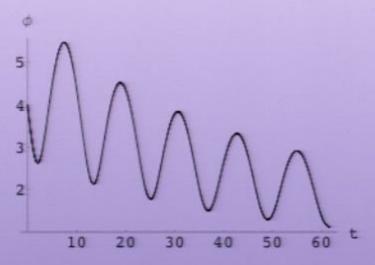


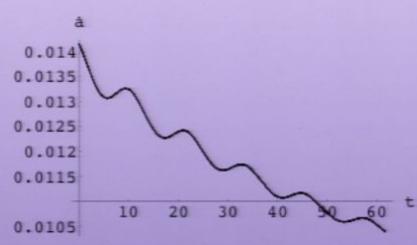
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Brane trajectory with no gravity



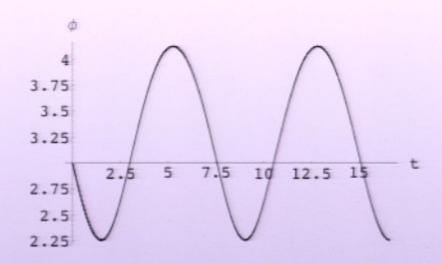
Brane position and scale factor with gravity switched on .



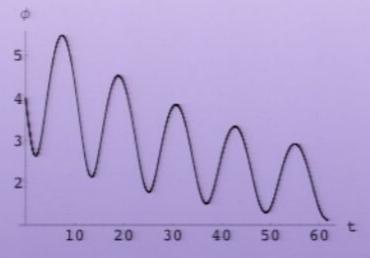


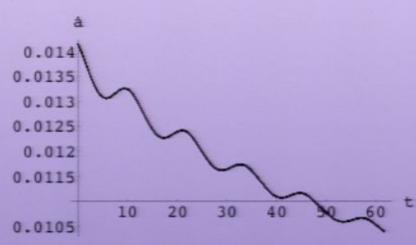
Pirsa: 08040004 Page 208/548

Brane trajectory with no gravity



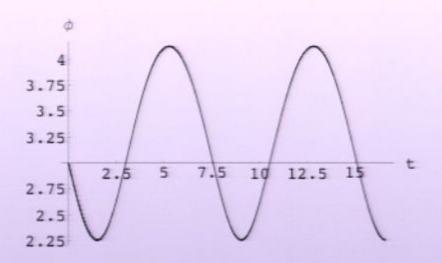
Brane position and scale factor with gravity switched on .



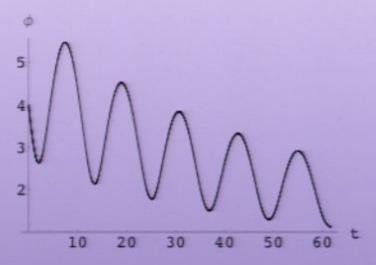


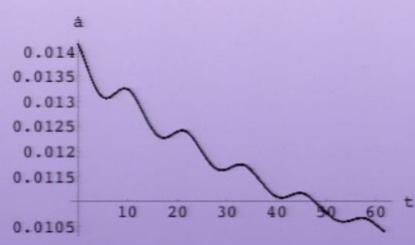
Pirsa: 08040004 Page 209/548

Brane trajectory with no gravity



Brane position and scale factor with gravity switched on .

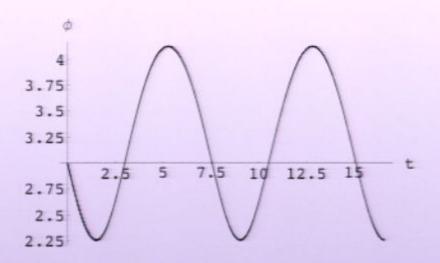




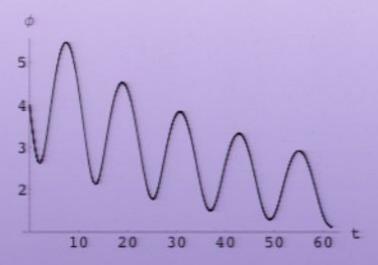
Pirsa: 08040004

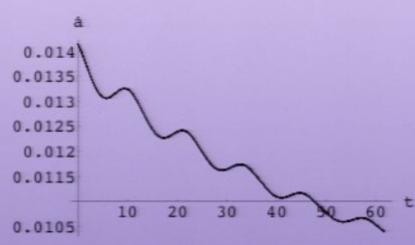
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Brane trajectory with no gravity



Brane position and scale factor with gravity switched on .

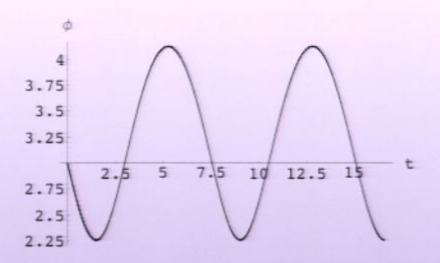




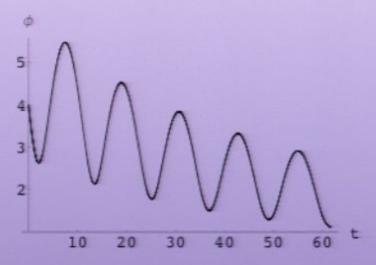
Pirsa: 08040004

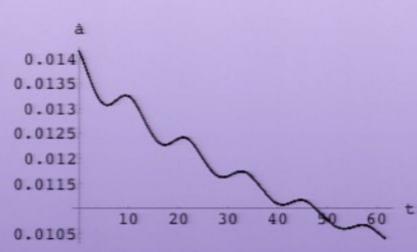
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Brane trajectory with no gravity



Brane position and scale factor with gravity switched on .

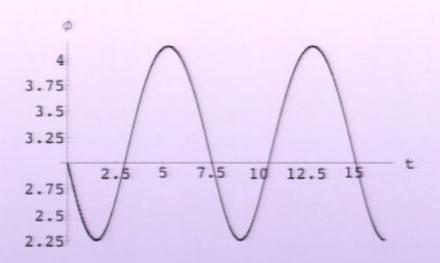




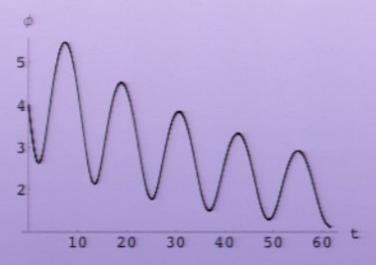
Pirsa: 08040004

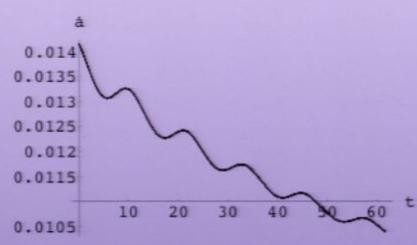
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Brane trajectory with no gravity



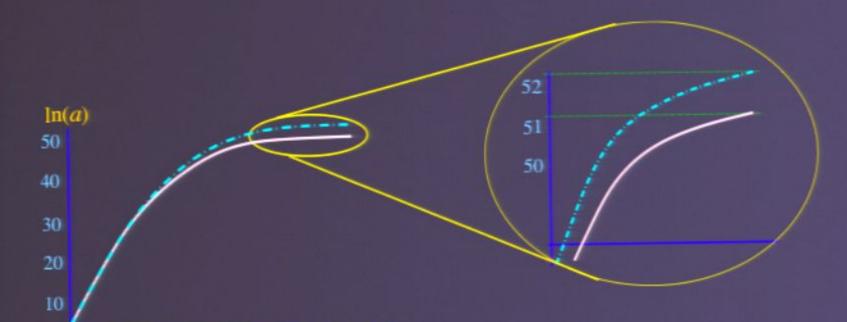
Brane position and scale factor with gravity switched on .





Pirsa: 08040004

# Accelerating Solutions: Spinflation



Inflation  $\Rightarrow mg_sM$  large

$$H^2 o egin{cases} 0 & \ell = 0 & ext{late time} \ 1/a^6 & \ell \neq 0 & ext{evolution} \end{cases}$$

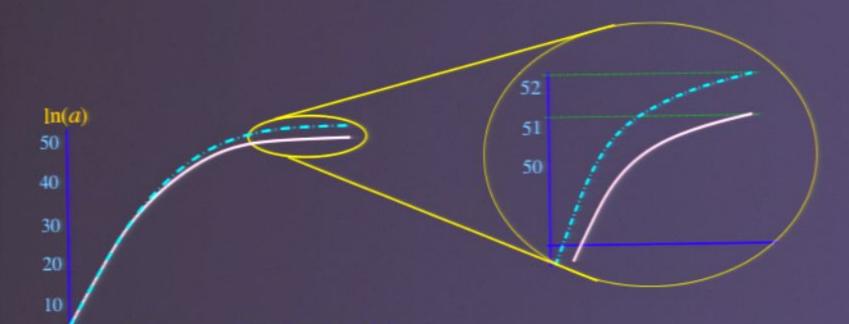
Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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# Accelerating Solutions: Spinflation



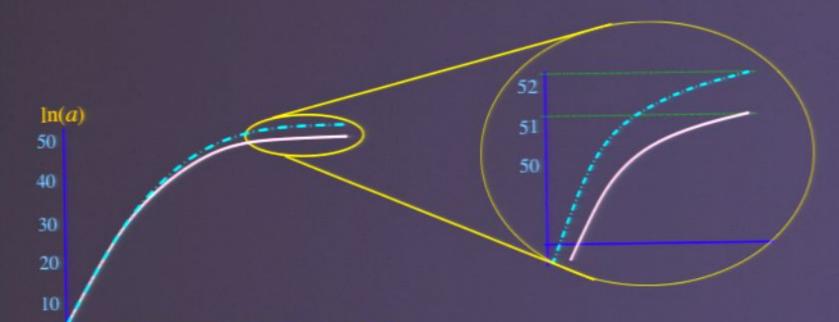
Inflation  $\Rightarrow mg_sM$  large

 $H^2 o egin{cases} 0 & \ell = 0 & \text{late time} \ 1/a^6 & \ell \neq 0 & \text{evolution} \end{cases}$ 

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

# Accelerating Solutions: Spinflation



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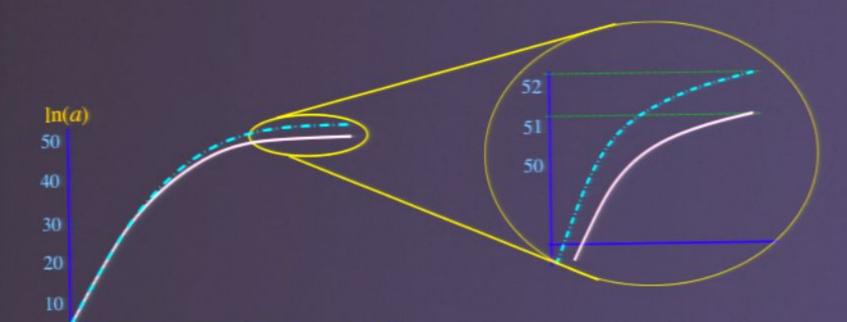
$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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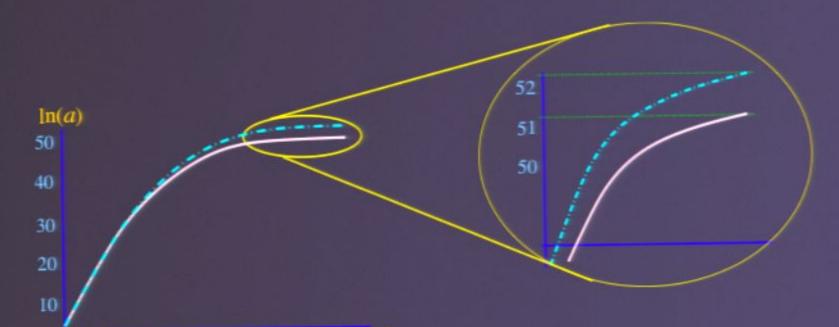
Inflation  $\Rightarrow mg_sM$  large

$$H^2 
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Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

Pirea: 08040004

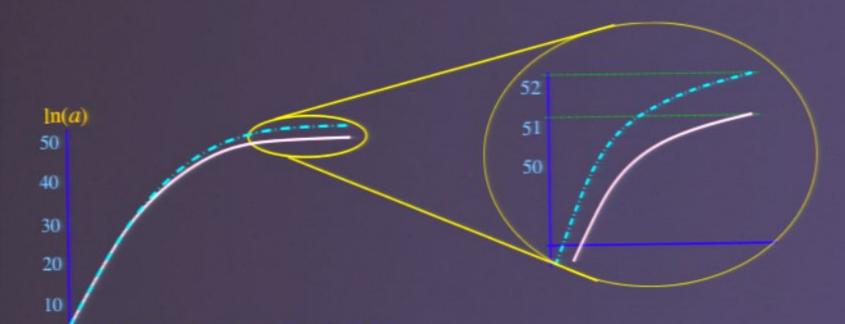


Inflation  $\Rightarrow mg_sM$  large

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Number of e-folds increases by few units with angular momentum

End of inflation ( $\gamma$  becomes => small): reheating through oscillations around tip



Inflation  $\Rightarrow mg_sM$  large

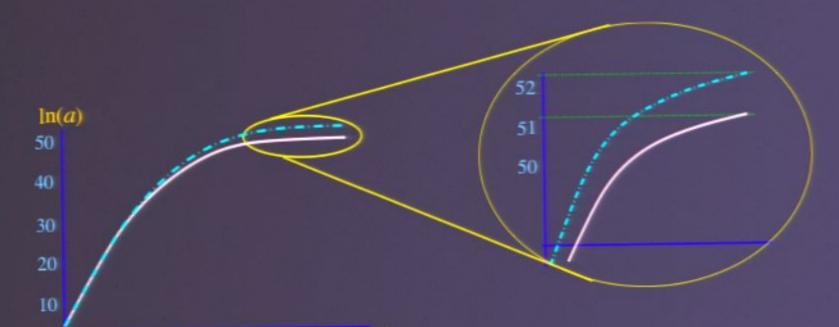
$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

Pirea: 08040004

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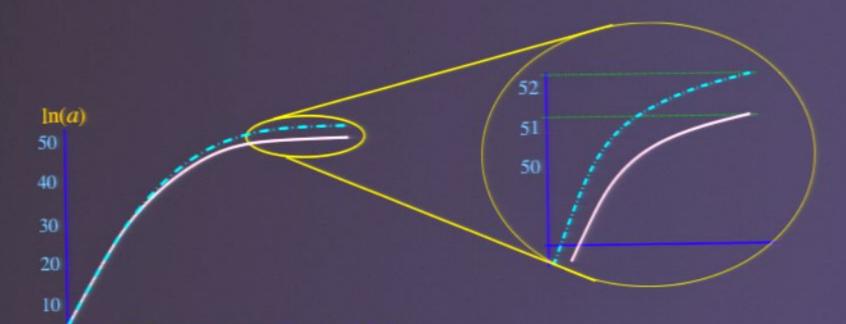
Inflation  $\Rightarrow mg_sM$  large

$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

Pirsa: 08040004



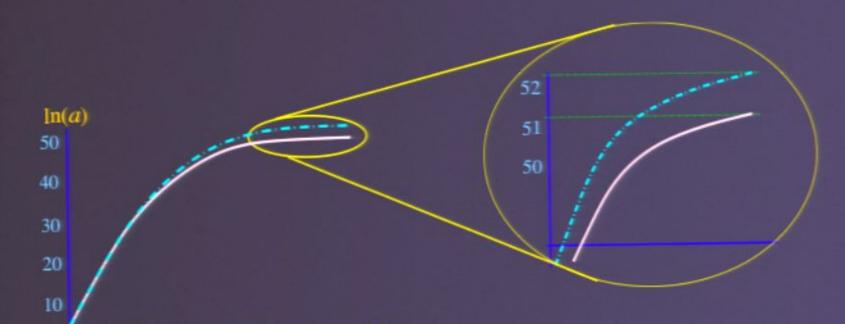
Inflation  $\Rightarrow mg_sM$  large

$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

Pirea: 08040004



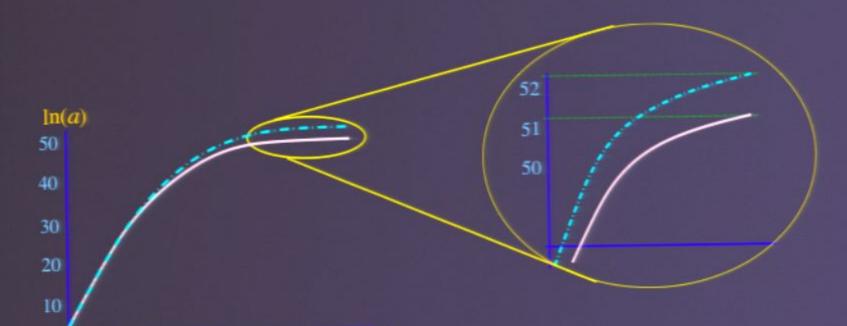
Inflation  $\Rightarrow mg_sM$  large

$$H^2 o egin{cases} 0 & \ell = 0 & \text{late time} \ 1/a^6 & \ell \neq 0 & \text{evolution} \end{cases}$$

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

Pirea: 08040004



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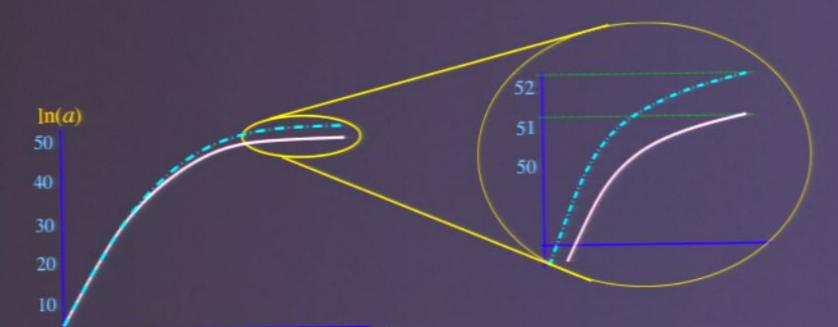
 $H^2 
ightarrow egin{cases} 0 & \ell = 0 & ext{late time} \ 1/a^6 & \ell \neq 0 & ext{evolution} \end{cases}$ 

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

Pirea: 08040004

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Inflation  $\Rightarrow mg_sM$  large

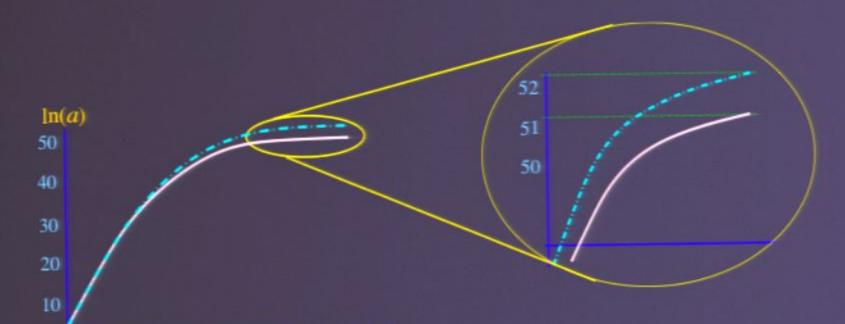
$$H^2 
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Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

Pirea: 08040004

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Inflation  $\Rightarrow mg_sM$  large

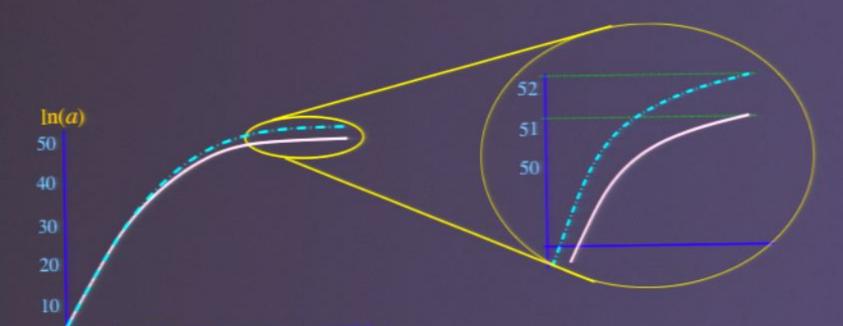
 $H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$  late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

Pirea: 08040004

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Inflation  $\Rightarrow mg_sM$  large

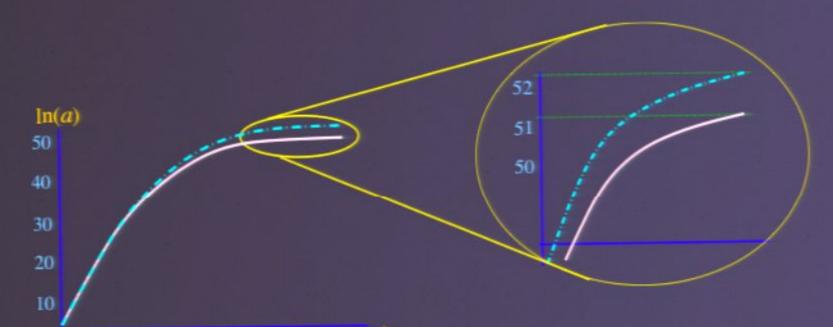
$$H^2 o egin{cases} 0 & \ell = 0 & ext{late time} \ 1/a^6 & \ell \neq 0 & ext{evolution} \end{cases}$$

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

Pirea: 08040004

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Inflation  $\Rightarrow mg_sM$  large

 $H^2 
ightarrow egin{cases} 0 & \ell = 0 & ext{late time} \ 1/a^6 & \ell \neq 0 & ext{evolution} \end{cases}$ 

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
=> small): reheating through
oscillations around tip

Pirea: 08040004

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Inflation  $\Rightarrow mg_sM$  large

$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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Inflation  $\Rightarrow mg_sM$  large

$$H^2 o egin{cases} 0 & \ell = 0 & ext{late time} \ 1/a^6 & \ell \neq 0 & ext{evolution} \end{cases}$$

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

Pirsa: 08040004

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Inflation  $\Rightarrow mg_sM$  large

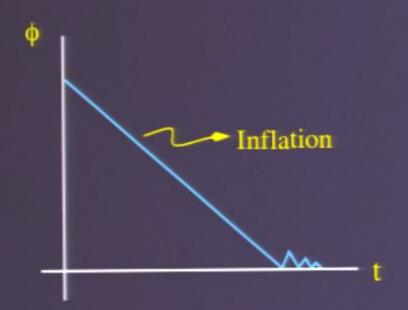
$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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Inflation  $\Rightarrow mg_sM$  large

$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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Inflation  $\Rightarrow mg_sM$  large

$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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Inflation  $\Rightarrow mg_sM$  large

$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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Inflation  $\Rightarrow mg_sM$  large

$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
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Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
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Pirsa: 08040004



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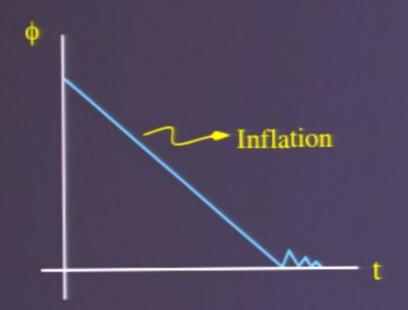
$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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Inflation  $\Rightarrow mg_sM$  large

$$H^2 o egin{cases} 0 & \ell = 0 & ext{late time} \ 1/a^6 & \ell \neq 0 & ext{evolution} \end{cases}$$

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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Inflation  $\Rightarrow mg_sM$  large

$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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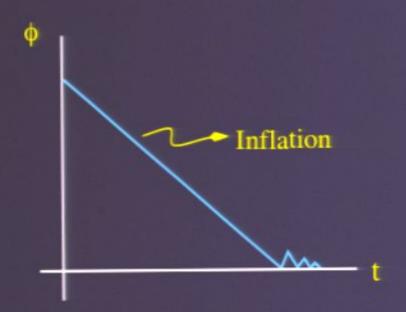
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Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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Inflation  $\Rightarrow mg_sM$  large

$$H^2 o egin{cases} 0 & \ell = 0 & ext{late time} \ 1/a^6 & \ell \neq 0 & ext{evolution} \end{cases}$$

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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Inflation  $\Rightarrow mg_sM$  large

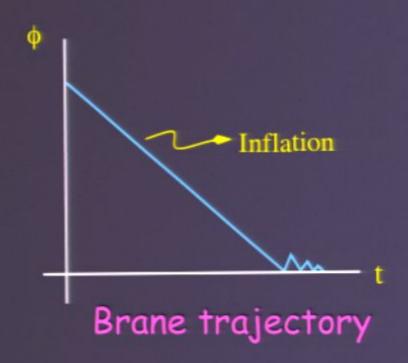
$$H^2 o \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$
 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
=> small): reheating through
oscillations around tip

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Inflation  $\Rightarrow mg_sM$  large

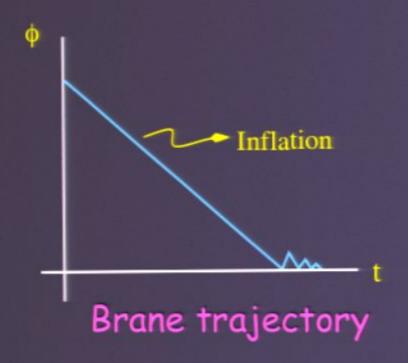
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 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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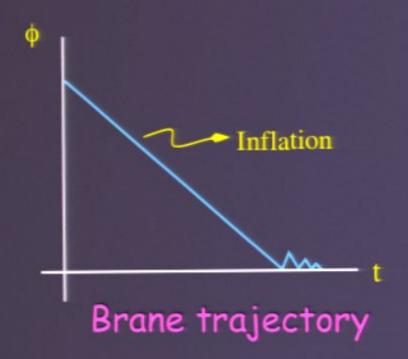
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End of inflation (γ becomes
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Pirsa: 08040004



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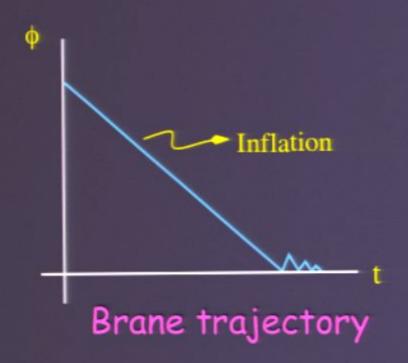
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 late time evolution

Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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Inflation  $\Rightarrow mg_sM$  large

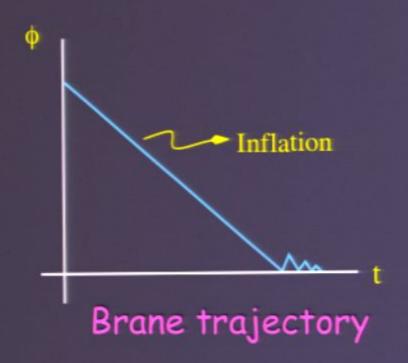
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End of inflation (γ becomes
 small): reheating through oscillations around tip

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Inflation  $\Rightarrow mg_sM$  large

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Number of e-folds increases by few units with angular momentum

End of inflation (γ becomes
 small): reheating through oscillations around tip

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- Backreaction:
  - √ acceleration of the brane has to be small in string units (validity of DBI action).
  - ✓ SUGRA approximation  $\implies g_s M \gg 1 \quad (g_s < 1)$
  - ✓ curvature of the brane as it moves at speed close to that of light: gravitational backreaction

$$\gamma - 1 \ll g_s^{-1} R^4 \ell_s^{-4} = g_s M^2$$

- UV scale:
  - √ Total 6D volume > throat volume

$$\Longrightarrow g_s M < \left(\frac{4}{\eta_{UV}}\right)^{3/2} \frac{\ell_s}{\epsilon^{2/3}} \sqrt{3} g_s \pi^2 \frac{M_{pl}}{M_s}$$

Pirsa: 084009 gular motion can help to satisfy these bounds.

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Pros 28 48 gular motion can help to satisfy these bounds.

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Pirsa: 08040004 Page 329/548

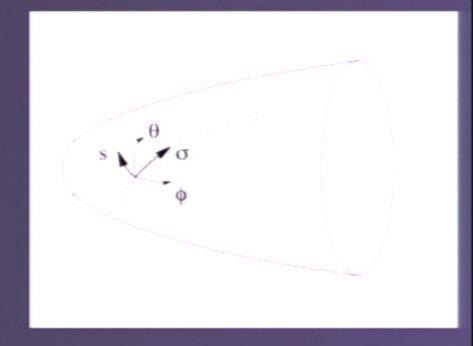
$$\cos \alpha = \frac{\dot{\phi}}{\sqrt{2X}}$$
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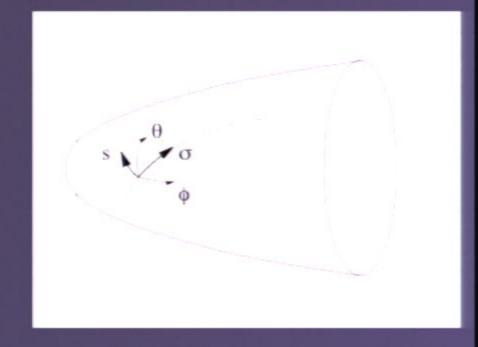
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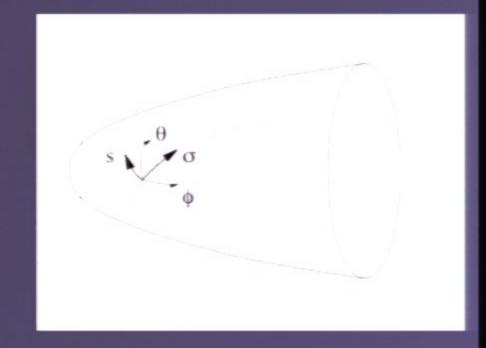
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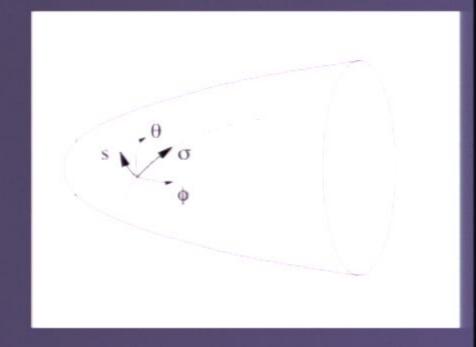
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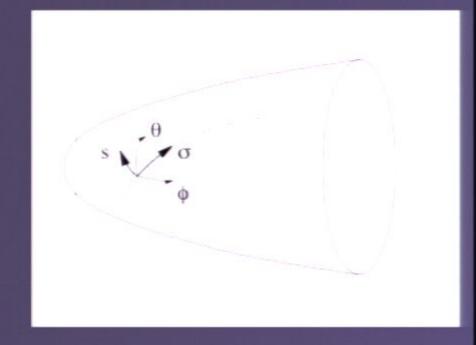
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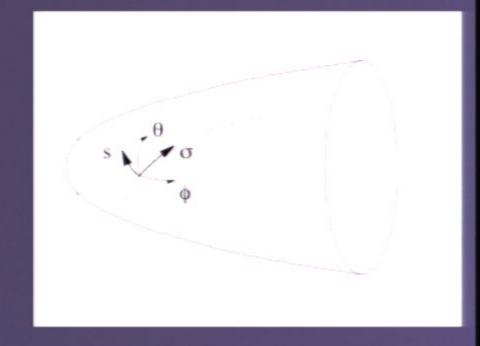
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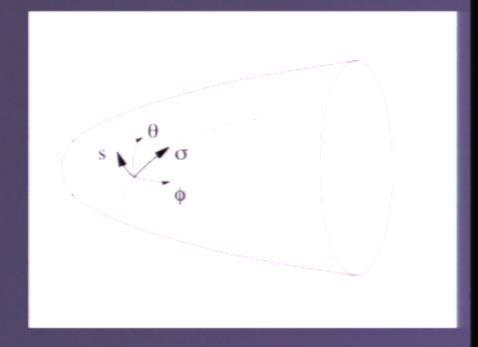
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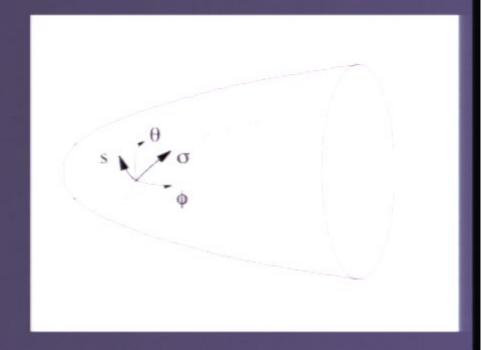
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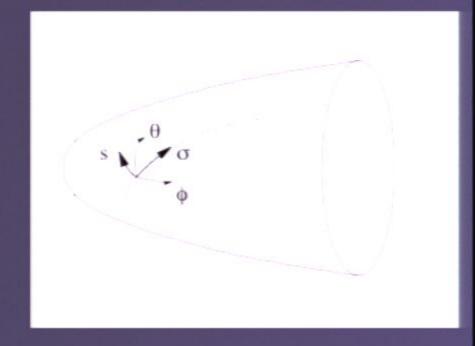
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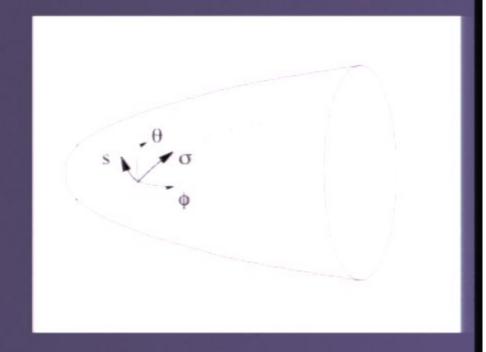
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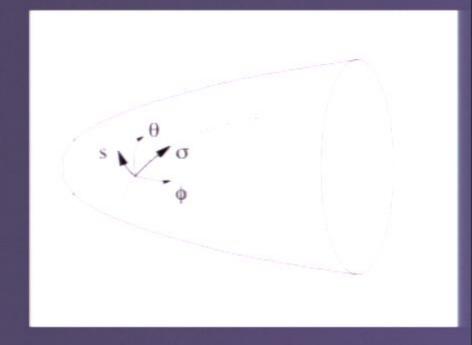
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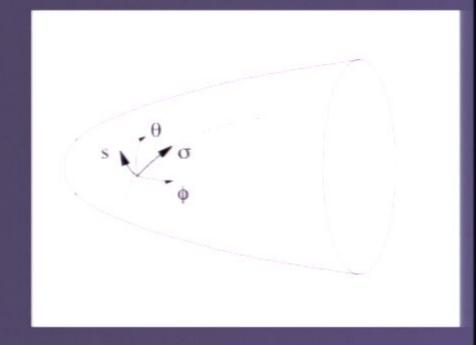
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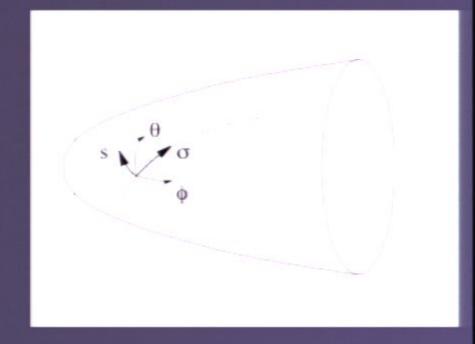
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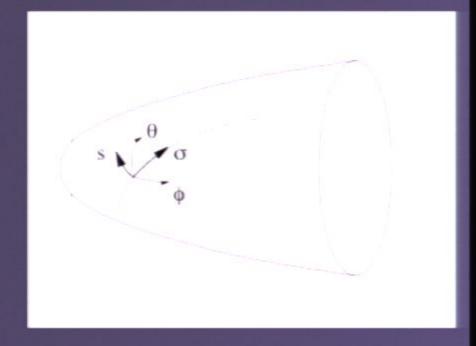
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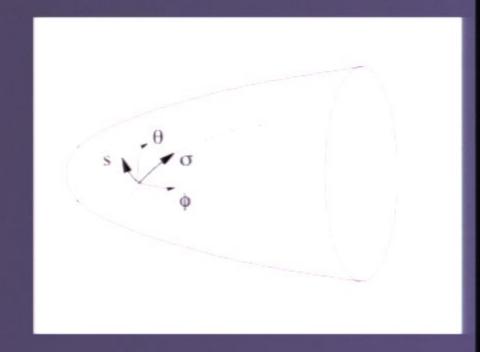
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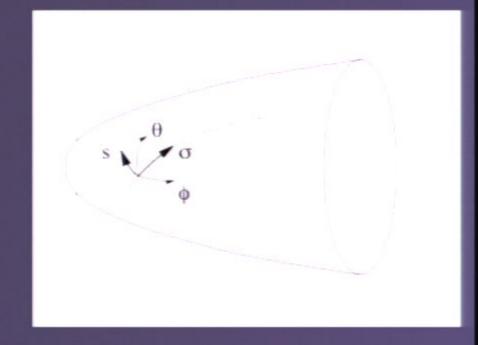
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$$\dot{\sigma}^2 = 2X \qquad \dot{s} = 0$$



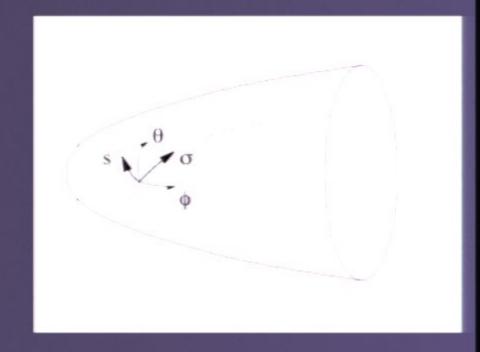
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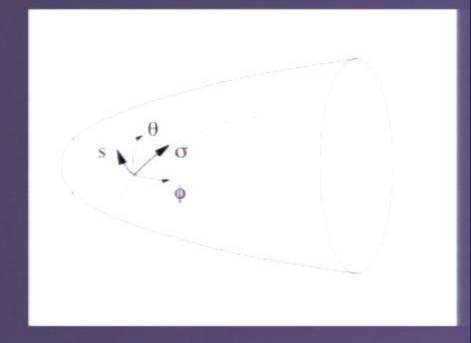
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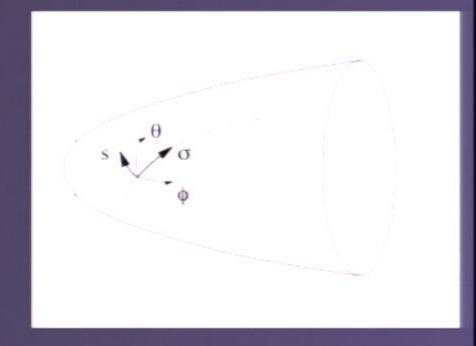
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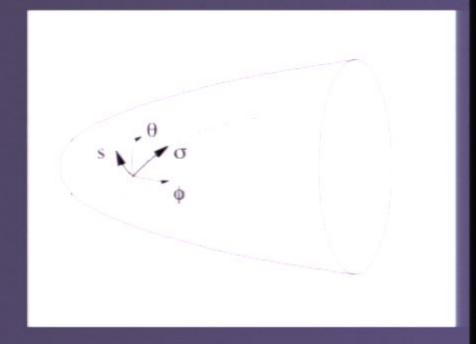
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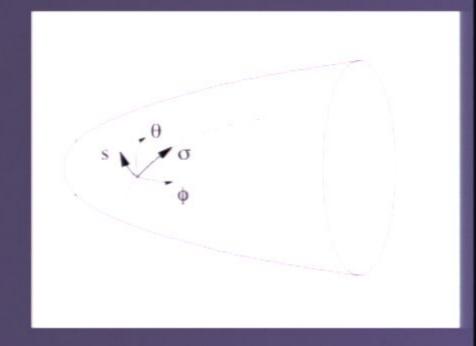
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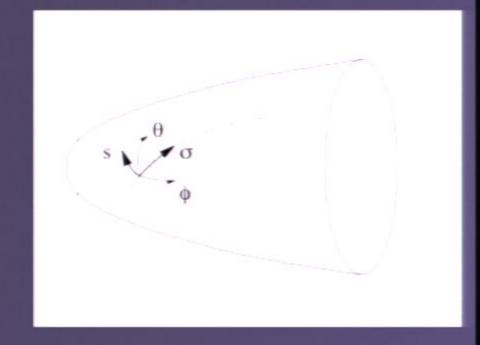
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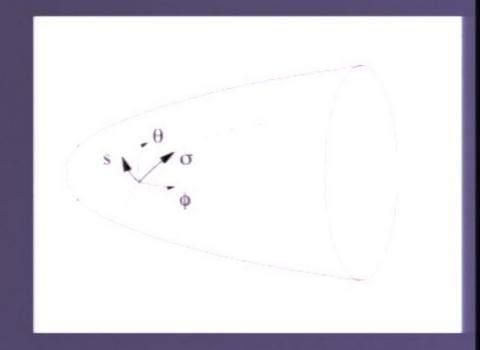
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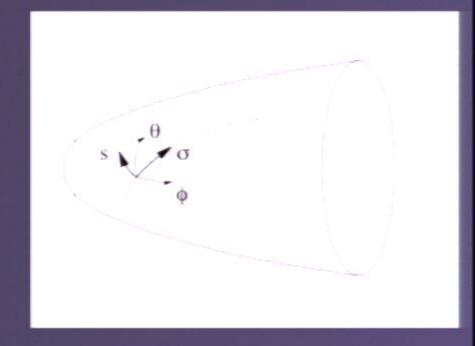
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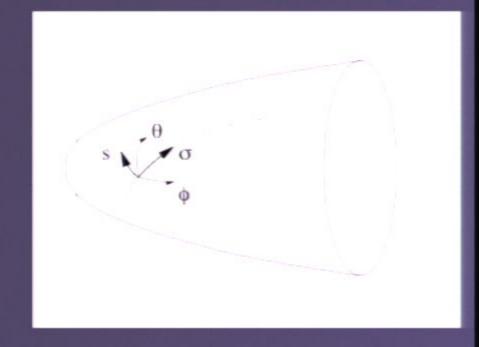
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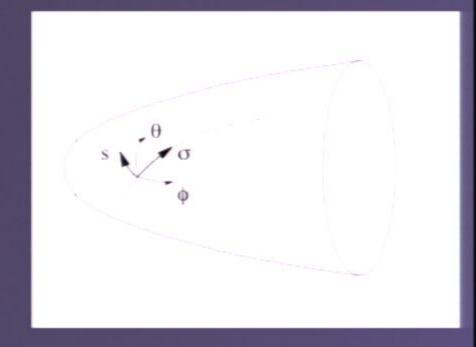
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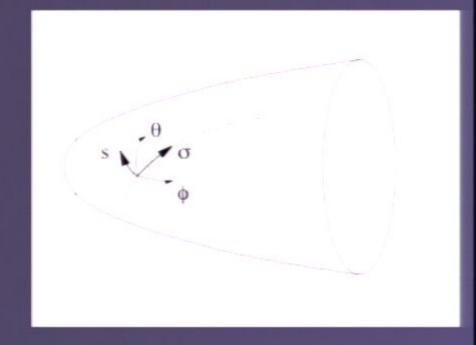
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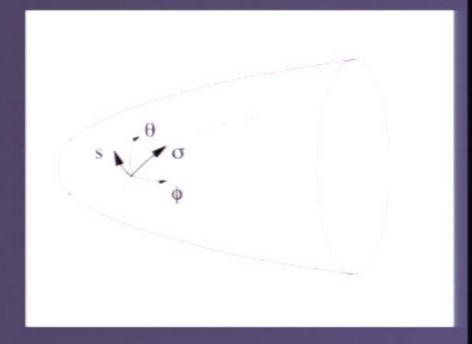
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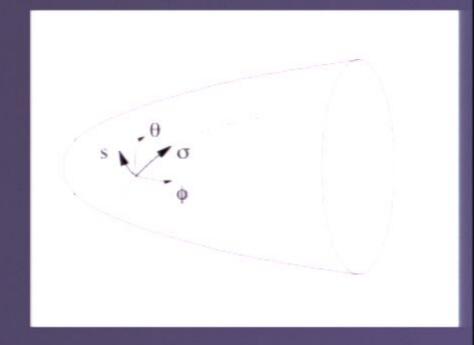
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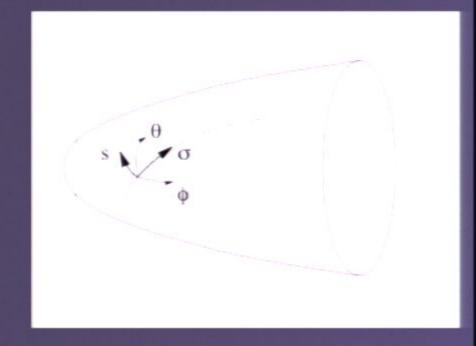
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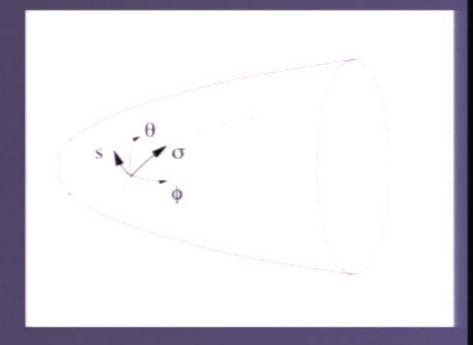
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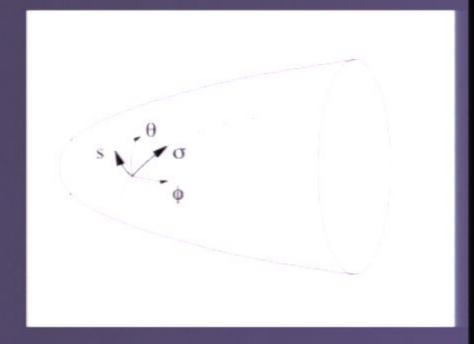
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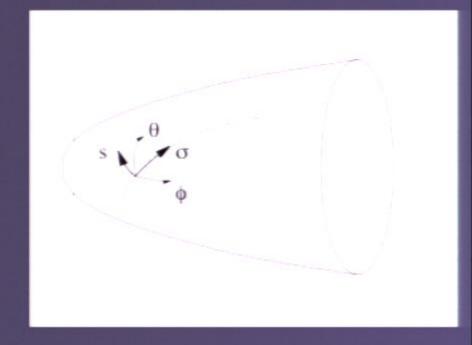
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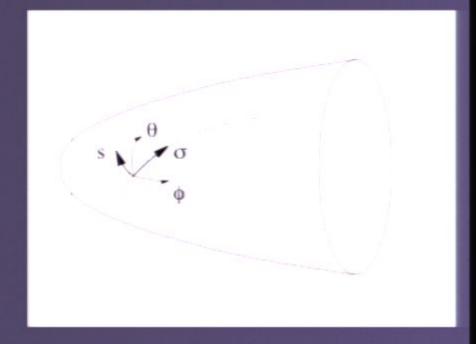
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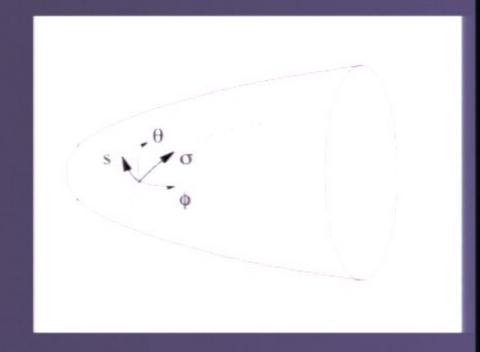
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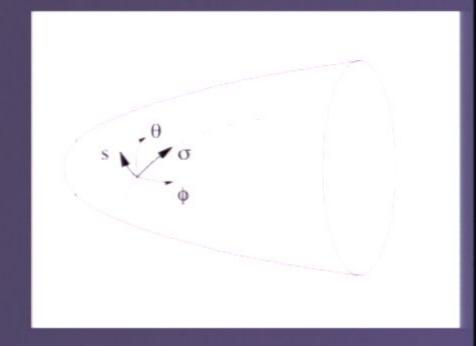
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$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

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$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

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$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\delta\dot{\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
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$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, (E+P)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f\tan \alpha} \right) \right]$$

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$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f\tan \alpha} \right) \right]$$

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where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

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where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

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$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

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$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

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where 
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where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2 c_S^2}{a^3 (E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3 (E+P)}{\dot{\sigma}H^2 c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

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where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

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$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

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Pirsa: 08040004  $c_{oldsymbol{c}}^2=\gamma^{-2}$ 

$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

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$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

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where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

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$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

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$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

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Pirsa: 08040004  $c_{f c}^2=\gamma^{-2}$ 

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

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where 
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$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

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$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

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$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

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Pirsa: 08040004  $c_{f c}^2=\gamma^{-2}$ 

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$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

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$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

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Pirsa: 08040004  $c_{oldsymbol{c}}^2=\gamma^{-2}$ 

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$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f' \dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f' \cos \alpha}{f \tan \alpha} \right) \right]$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\delta\dot{\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

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$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

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$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

Pirsa: 08040004

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Pirsa: 08040004

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#### Perturbations in Spinflation General features

- ★ Since the brane moves along different directions, various fields can contribute to the evolution of the perturbations
- ★ One can extract some general features, without explicitly solving the equations:
  - The entropy perturbation evolves independently of the curvature perturbation at large scales. Yet (at large scales  $k^2/a^2\ll 1$ ) entropy perturbation seeds curvature one.
  - ✓ Curvature and entropy perturbations evolve at different speeds. Curvature perturbations move with a speed  $c_S^2 = \gamma^{-2} \ll 1$ . Entropy perturbations move at speed of light. Different perturbations cross the horizon at different times.

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Pirsa: 08040004

- ★ D3 probe to spin in a warped flux SUGRA background (KS): DBI cosmology in multifield case.
- ★ Inflationary solutions when angular momentum is turned on give rise to Spinflation. Angular momentum sources accelerated expansion, providing a handful of e-folds at beginning of inflation.
- ★ Expanding universes with short kicks of acceleration.
- ★ DBI inflation is in tension with consistency bounds.

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  Pirsa: OBC ies of bounces could also help with this tension page 446/548

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  Pirsa: 08040004 ies of bounces could also help with this tension, Page 464/548

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#### Perturbations in Spinflation General features

- ★ Since the brane moves along different directions, various fields can contribute to the evolution of the perturbations
- ★ One can extract some general features, without explicitly solving the equations:
  - The entropy perturbation evolves independently of the curvature perturbation at large scales. Yet (at large scales  $k^2/a^2\ll 1$ ) entropy perturbation seeds curvature one.
  - ✓ Curvature and entropy perturbations evolve at different speeds. Curvature perturbations move with a speed  $c_S^2 = \gamma^{-2} \ll 1$ . Entropy perturbations move at speed of light. Different perturbations cross the horizon at different times.

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where 
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$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f\tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f' \dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f' \cos \alpha}{f \tan \alpha} \right) \right]$$

$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f' \dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f' \cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, (E+P)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f\tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

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$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f\tan \alpha} \right) \right]$$

$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\delta\dot{\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3 \left(E + P\right)}\right) \left[\frac{a^3 \tan\alpha}{Hc_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f' \dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f' \cos \alpha}{f \tan \alpha} \right) \right]$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f' \dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f' \cos \alpha}{f \tan \alpha} \right) \right]$$

$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\delta\dot{\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H \dot{\sigma}c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi \label{eq:delta_spectrum}$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$\delta\ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\delta\dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f\tan \alpha} \right) \right]$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f' \dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f' \cos \alpha}{f \tan \alpha} \right) \right]$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\delta\dot{\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f' \dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f' \cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f\tan \alpha} \right) \right]$$

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$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f\tan \alpha} \right) \right]$$

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$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f' \dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f' \cos \alpha}{f \tan \alpha} \right) \right]$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f\tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\delta\ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\delta\dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

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$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f' \dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f' \cos \alpha}{f \tan \alpha} \right) \right]$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

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$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f' \dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f' \cos \alpha}{f \tan \alpha} \right) \right]$$

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$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

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$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta\ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right)\delta\dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right)\delta s \,=\, -\, \frac{k^2}{a^2}\, \frac{\dot{\sigma}\, \tan\alpha\, H}{a\left(E + P\right)^2}\, \left(\dot{P} - c_S^2\dot{E}\right)\,\xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f' \dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f' \cos \alpha}{f \tan \alpha} \right) \right]$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]^2$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3(E+P)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f\tan \alpha} \right) \right]$$

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$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$\ddot{\delta\sigma_{\Phi}} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\dot{\delta\sigma_{\Phi}} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right)\delta\sigma_{\Phi} = -\left(\frac{H\dot{\sigma}c_S^2}{a^3(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta \ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right) \delta \dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right) \delta s \,=\, -\, \frac{k^2}{a^2} \, \frac{\dot{\sigma} \, \tan \alpha \, H}{a \, \left(E + P\right)^2} \, \left(\dot{P} - c_S^2 \dot{E}\right) \, \xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$

$$\delta \ddot{\sigma}_{\Phi} + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right) \delta \dot{\sigma}_{\Phi} + \left(U_{\sigma_{\Phi}} + c_s^2 \frac{k^2}{a^2}\right) \delta \sigma_{\Phi} = -\left(\frac{H \dot{\sigma} c_S^2}{a^3 (E + P)}\right) \left[\frac{a^3 \tan \alpha}{H c_S^2} \left(\dot{P} - c_S^2 \dot{E}\right) \frac{\delta s}{\dot{\sigma}}\right]$$

where 
$$U_{\sigma_{\Phi}} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\left(E+P\right)} \left[ \left( \frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}} \right) \frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2} \right]$$

$$\delta\ddot{s} \,+\, \left(3H + \frac{\dot{\gamma}}{\gamma}\right)\delta\dot{s} \,+\, \left(U_s + \frac{k^2}{a^2}\right)\delta s \,=\, -\, \frac{k^2}{a^2}\, \frac{\dot{\sigma}\, \tan\alpha\, H}{a\left(E + P\right)^2}\, \left(\dot{P} - c_S^2\dot{E}\right)\,\xi$$

where 
$$U_s = \tan \alpha \left[ 3H \tan \alpha \left( \frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan \alpha} + 3Hc_S^2 - \cos \alpha \frac{f'\dot{\sigma}}{f} \right) + \dot{\sigma} \left( \frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\cos \alpha}{f \tan \alpha} \right) \right]$$

$$c_S^2 = \gamma^{-2}$$