Title: Induced gravity on intersecting brane-worlds

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Abstract: TBA

Cosmology in Intersecting Brane-World Models with Induced Gravity Terms

Gianmassimo Tasinato, UAM Madrid

Based on arXiv:0712.0385 [hep-th]
arXiv:0803.1850 [hep-th]
arXiv:09... [hep-th]

with Olindo Corradini and Kazuya Koyama

Outline

- Motivations
- The maximally symmetric case
- The FRW case
- Conclusions

Theoretical Motivations

- BW models during the last ten years offered new insights for building effective frameworks that extend Einstein GR in a (possibly) consistent way.
- They suggested new ways to test string motivated models

  ADD models proposed to consider frameworks with low foundamental scale.

  - Black Holes at colliders
  - Deviations from Newtonian inverse square law

  RS models found important connections with stringy ideas.

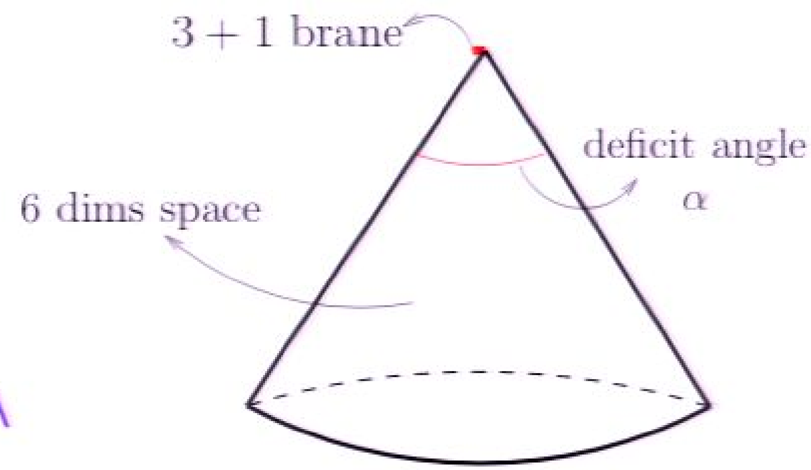
  - AdS/CFT
  - Warped Throats in CY compactifications with fluxes

- Cosmology model building has been also deeply influenced:
  - Higher codimension BWs considered to address the cc problem: $\Rightarrow$ Selftuning
  - BWs with induced gravity terms to explain present day acceleration: $\Rightarrow$ Selfacceleration

although these cannot be considered, today, completely successful attempts...

An Example: codimension two brane-world on a cone

Consider a BW sitting on the
tip of a cone



$$S = \int d^6x \sqrt{-g_6}\, M_6^4\, R_6 - \int d^4x \sqrt{-g_4}\, \lambda$$

Why it is interesting in order to address the cc problem?

- The tension of the brane does not curve our space: only modifies the extra dimensions

$$\text{Selftuning :} \qquad \lambda = 2\pi\, M_6^4\, (1-\alpha)$$

However, to get a finite 4d Planck mass, we must compactify the extra space.

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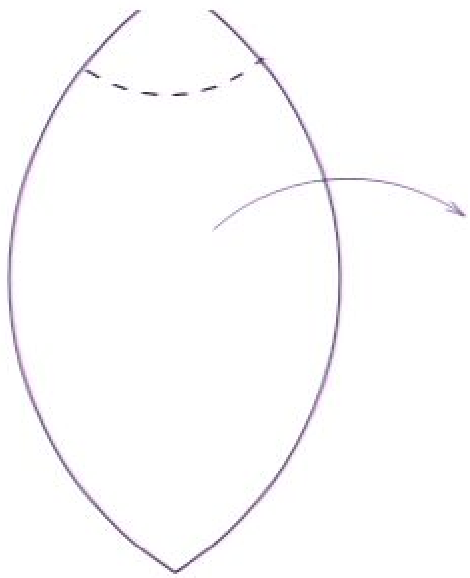

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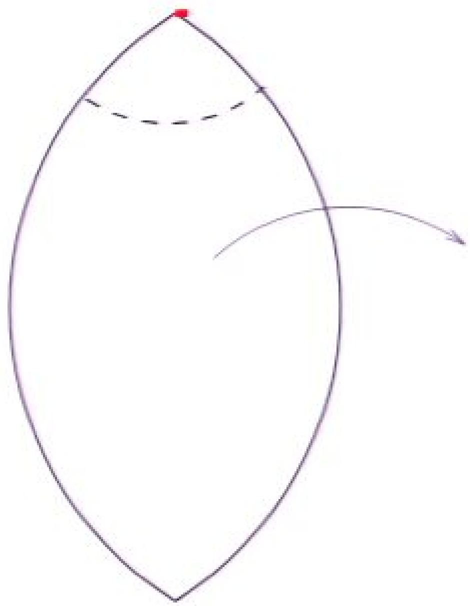

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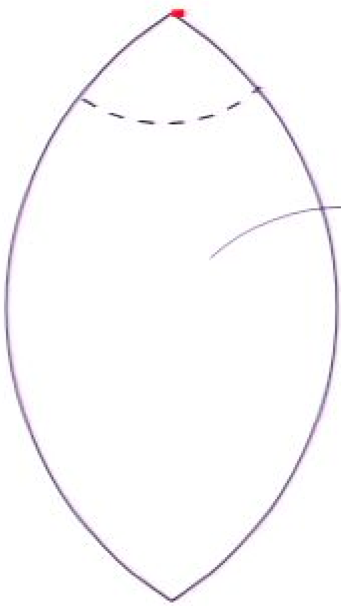

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Gauge field
and bulk cosmological constant
spontaneously compactify the space
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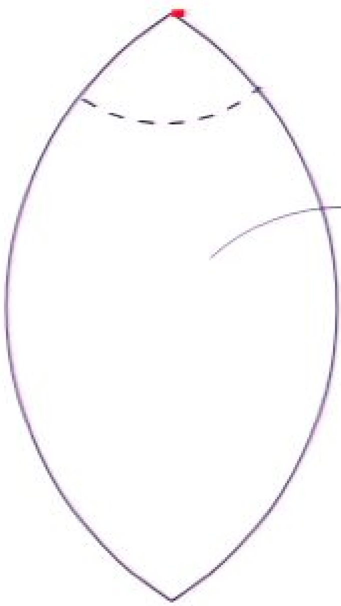

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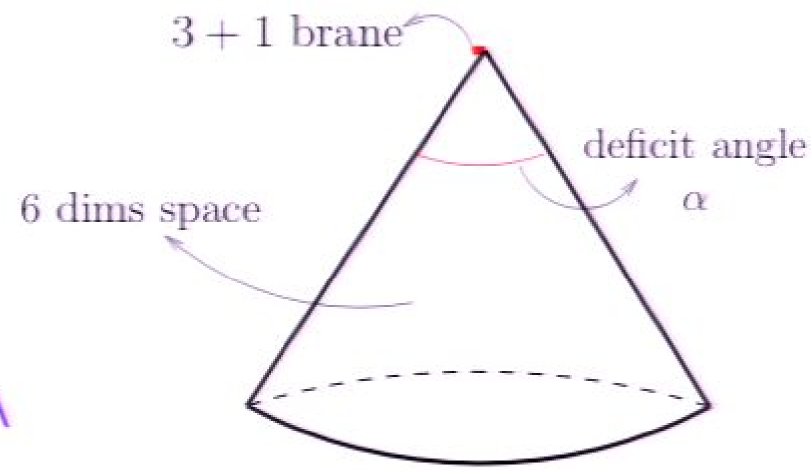

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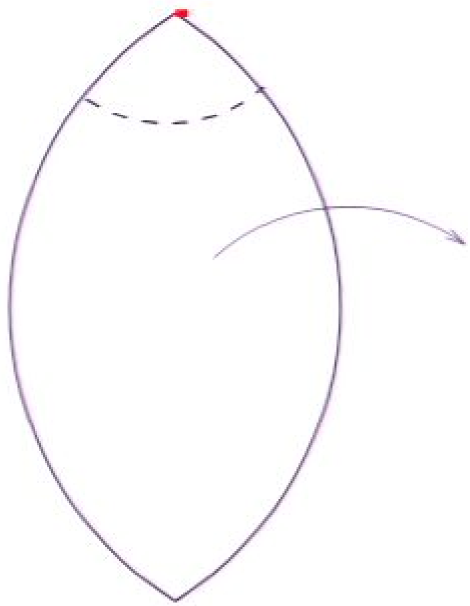

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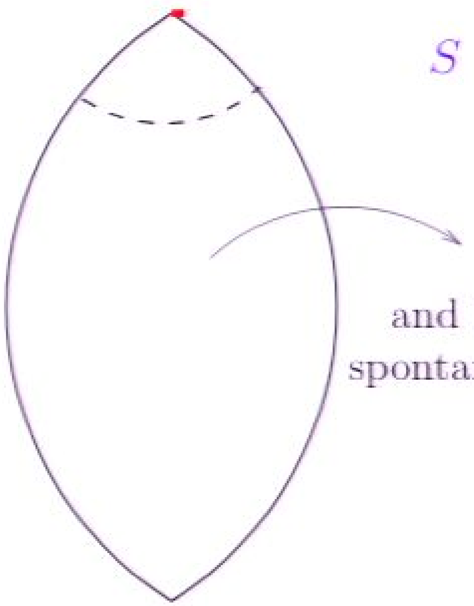

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Unfortunately, the deficit angle α results quantized: $\alpha \propto n^2$.

What about if we avoid to compactify by adding induced gravity terms on the brane?

- Does selftuning work in this case?
- Are there selfaccelerating configurations in higher codimension brane-worlds?

Selfacceleration

- Consider a tensionless $3+1$ dimensional brane on an empty bulk.
  Add **localised gravity terms** on the brane.
- Assume a FRW Ansatz for the induced brane metric.
  The equation that controls brane dynamics is

$$H = \epsilon \frac{M_5^3}{M_4^2} + \sqrt{\frac{\rho}{3M_4^2} + \frac{M_5^6}{M_4^4}}$$

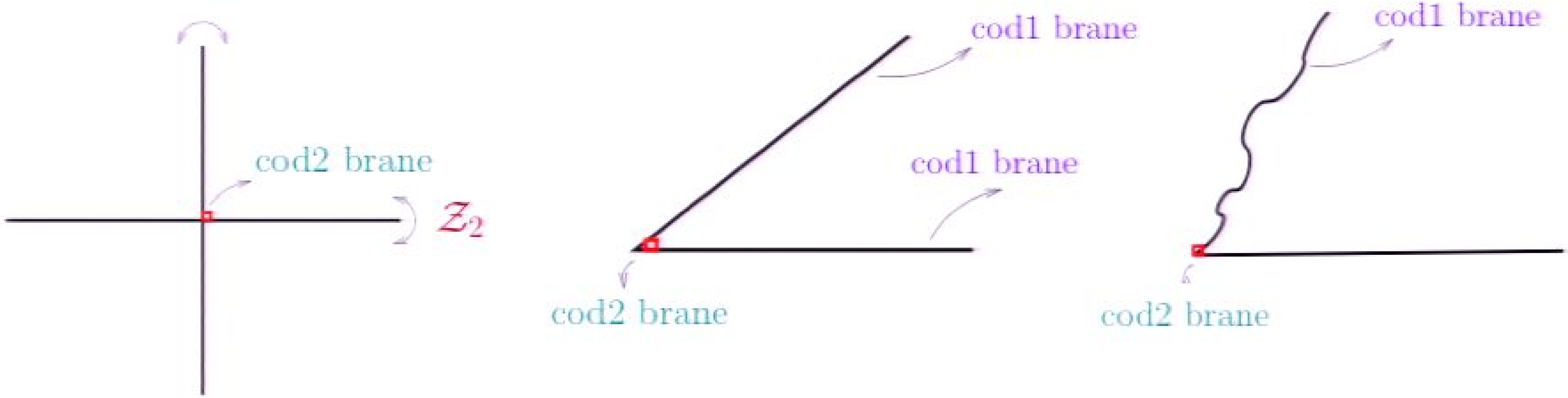
  when $\epsilon = +1$ one has selfacceleration.
  $\Rightarrow$ Even if $\rho \to 0$ one has $H > 0$.

- But this branch of solutions is **plagued by ghosts**.
  Intuitive understanding: choosing this sign of ϵ corresponds to choose the side of the brane where the induced gravity term mimics EMT that violates energy conditions.

The system under consideration: Intersecting brane-world model

The action we are interested in is

$$S = \int_{\text{bulk}} d^6x\sqrt{-g}\left(\frac{M_6^4}{2}R - \Lambda_B\right) + \sum_{i=1}^{2}\int_{\Sigma_i} d^5x\sqrt{-g_{(i)}}\left(\frac{M_{5,i}^3}{2}R_{(i)} + L_{(i)}\right)$$
$$+ \int_{\Sigma_\cap} d^4x\sqrt{-g_\cap}\left(\frac{M_4^2}{2}R_\cap + L_\cap\right),$$



Then the questions I want to consider are:

- How does matter on the brane backreact on the geometry?
- is there a regime in which acceptable $4d$ gravity is obtained?

- Can we find further selfaccelerating configurations at the intersection?

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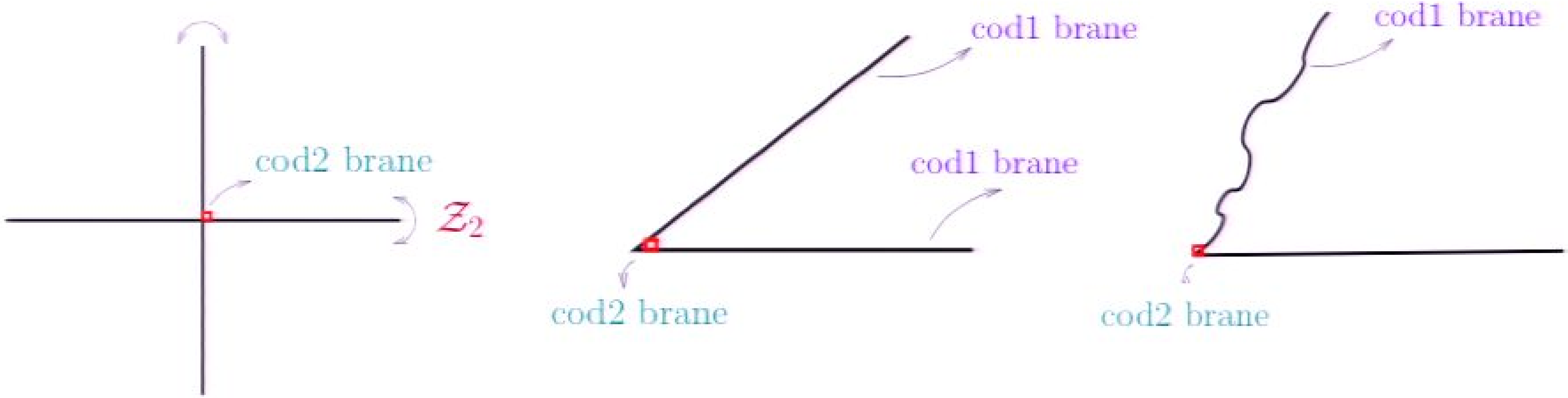

How do we **tackle** these questions?

- We determine the equations that control cosmological expansion on the brane.

  $\Rightarrow$ A FRW Ansatz is imposed on the brane geometry.

It is certainly a very simplified set-up, but a way to get interesting information.

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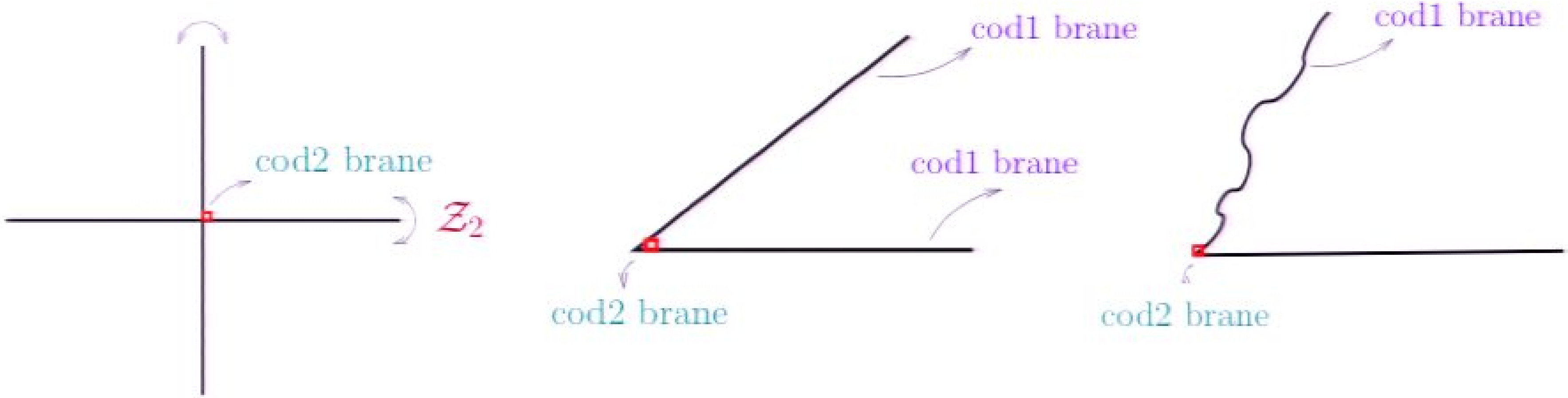
\end{aligned}
$$



Why this system and not others with similar features?

- Because cosmological equations can be easily extracted from the Israel junction conditions on the codimension one branes.

First part: Maximally symmetric configurations

Focus on the case in which brane and bulk geometries are maximally symmetric.

Which is the relation between expansion rates and brane tensions ?

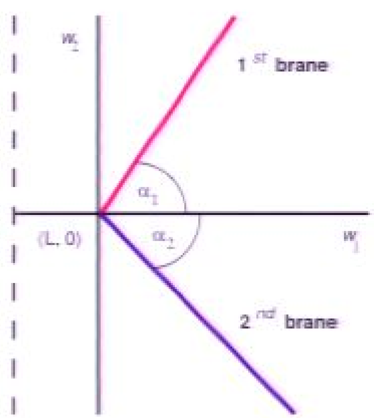
- The bulk geometry is given by

$$ds^2 = A^2(t, z^1, z^2)\left(\eta_{\mu\nu}dx^\mu dx^\nu + \delta_{kh}dz^k dz^h\right) \quad , \quad A(t, z^1, z^2) = \frac{1}{Ht + k_i z^i}$$

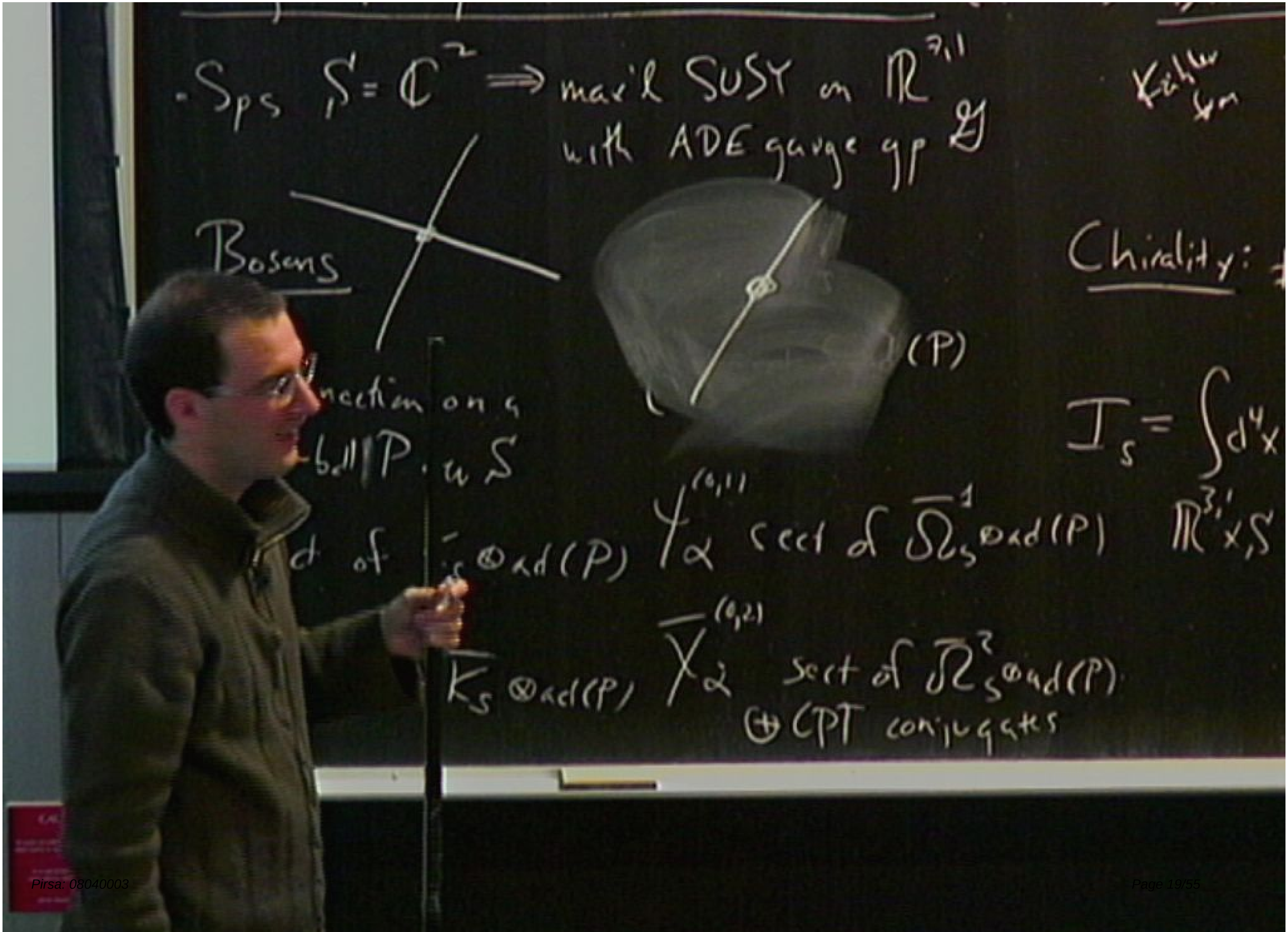
where the parameters satisfy $\quad \dfrac{\Lambda_B}{10} = H^2 - k_1^2 - k_2^2$

- The branes form generic angles, characterized by normal vectors

$$\begin{aligned}\mathbf{n}^{(1)} &= (\sin\alpha_1, -\cos\alpha_1)\\ \mathbf{n}^{(2)} &= (\sin\alpha_2, \cos\alpha_2)\end{aligned}$$



angle between the brane $\alpha = \alpha_1 + \alpha_2$


Sps S = C² ⟹ max'l SUSY on R^{7,1}
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Bosons
Chirality:
(P)
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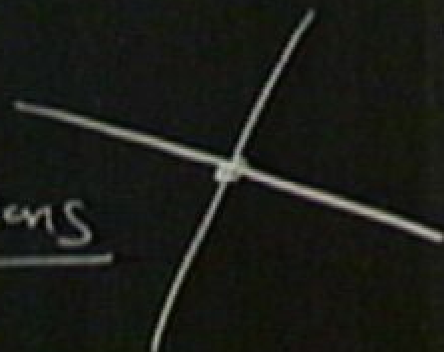
Kähler fm



(P)

Bosons

A connection on a $\mathcal{G}$-bdl P over S

Chirality:

$$I_S = \int d^4x$$

$\varphi^{(2,0)}$ sect of $K_S \otimes ad(P)$

$\psi_\alpha^{(0,1)}$ sect of $\overline{\Omega}^1_S \otimes ad(P)$

$\mathbb{R}^{3,1} \times S$

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First part: Maximally symmetric configurations

- It is convenient to define new coordinates parallel to the brane

$$\tilde{z}^k = \mathbf{n}^{(k)} \cdot \mathbf{z} \quad \text{with} \quad \mathbf{z} = (z^1, z^2)$$

Israel junction equations allow to determine brane energy momentum tensor that supports this configuration.

- Codimension one brane Σ

Characterized by velocity vectors $V^P_{(m)} \equiv \frac{\partial X^p}{\partial x^m}$ and normal n_Q.

The extrinsic curvature K_{mn} is given by

$$K_{mn} = V^P_{(m)} V^Q_{(n)} \nabla_P n_Q$$

Israel conditions require that the brane energy momentum tensor S_{mn} satisfy

$$[K_{mn}]_{br} = -\frac{S_{mn}}{M_F^{d-2}}$$

If the bulk is maximally symmetric, Codazzi equations ensure

$$\nabla^m K_{mn} = 0 \quad \Rightarrow \quad \nabla^m S_{mn} = 0$$

conservation of energy on the brane.

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Concentrate on the brane Σ_1 located at $\tilde{z}_1 = 0$ to impose the junction conditions.

- Reconsider the system in the new coordinates parallel to the branes:

  $\Rightarrow$ Impose Z_2 symmetry at the position of the second brane $\tilde{z}_2 = 0$

  – The bulk metric rewrites

$$
\begin{aligned}
ds^2 &= A^2(t, \tilde{z}^1, |\tilde{z}^2|)\left(\eta_{\mu\nu} dx^\mu dx^\nu + \tilde{\gamma}_{mn}\, d\tilde{z}^m d\tilde{z}^n\right), \\
A(t, \tilde{z}^1, |\tilde{z}^2|) &= \frac{1}{Ht + \mathcal{C}_1 \tilde{z}^1 + \mathcal{C}_2 |\tilde{z}^2|}, \\
\mathcal{C}_1 &= \frac{k_1 \cos\alpha_2 - k_2 \sin\alpha_2}{\sin\alpha}, \quad \mathcal{C}_2 = \frac{k_1 \cos\alpha_1 + k_2 \sin\alpha_1}{\sin\alpha}.
\end{aligned}
$$

with

$$
\tilde{\gamma}_{mn} = \frac{1}{\sin^2\alpha}\begin{pmatrix} 1 & \cos\alpha\, sgn(\tilde{z}^2) \\ \cos\alpha\, sgn(\tilde{z}^2) & 1 \end{pmatrix}.
$$

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  – The induced metric on Σ_1 reads

$$ds^2_{\Sigma_1} = A^2(t, 0, |\tilde{z}^2|) \left(\eta_{\mu\nu} dx^\mu dx^\nu + \frac{d\tilde{z}_2 d\tilde{z}_2}{\sin^2 \alpha} \right)$$

$\Rightarrow$

$$H_1^2 = H^2 - \sin^2 \alpha \, \mathcal{C}_2^2 \,,$$

- The extrinsic curvature is calculated straightforwardly, and

$$K_{\tilde{z}_2 \tilde{z}_2} = \tilde{\nabla}_{\tilde{z}_2} \tilde{n}_{\tilde{z}_2} = -g_{\tilde{z}_2 \tilde{z}_2} \Big(\mathcal{C}_1 - \cos \alpha \, \mathcal{C}_2 \Big) + 2A \, \delta(\tilde{z}_2) \, \frac{\cos \alpha}{\sin^2 \alpha}$$

It contains a **singular piece** that is compensated by matter at the intersection.

First part: Maximally symmetric configurations

Calling H_1, H_2, H the Hubble parameters on branes Σ_1, Σ_2, $\Sigma_\cap$
Λ_1, Λ_2, λ the tensions on branes Σ_1, Σ_2, $\Sigma_\cap$

Israel conditions provide the following system of equations

$$\begin{aligned}
\Lambda_1 &= 6M_{5,1}^3 H_1^2 - \frac{8M_6^4}{\sin\alpha}\left(\epsilon_2\sqrt{H^2 - H_2^2} - \epsilon_1 \cos\alpha\sqrt{H^2 - H_1^2}\right), \\
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\lambda &= 3M_4^2 H^2 - 6\left(\epsilon_1 M_{5,2}^3\sqrt{H^2 - H_1^2} + \epsilon_2 M_{5,1}^3\sqrt{H^2 - H_2^2}\right) + 4M_6^4\,\frac{\cos\alpha}{\sin\alpha}
\end{aligned}$$

and

$$\cos\alpha = \frac{\epsilon_1\epsilon_2\sqrt{\left(H^2 - H_1^2\right)\left(H^2 - H_2^2\right)} - \sqrt{\left(H_1^2 - \frac{\Lambda_B}{10}\right)\left(H_2^2 - \frac{\Lambda_B}{10}\right)}}{H^2 - \frac{\Lambda_B}{10}}$$

The solution depends on parameters $\epsilon_i = \pm 1$.

Selfaccelerating solutions

Are there solutions with positive Hubble parameters when the tensions vanish?

1. **Symmetric solutions**: $H_1 = H_2$ and $\epsilon_1 = \epsilon_2$

$$\begin{aligned} H_1^2 &= \frac{H^2}{2}\,(1-\cos\alpha) \\ H &= \frac{8\epsilon_1}{3\sin\alpha}\frac{M_6^4}{M_5^3}\sqrt{\frac{1+\cos\alpha}{2}} \\ 4+3\cos\alpha &= \frac{8\,M_6^4\,M_4^2\,(1+\cos\alpha)}{3\,M_5^3\,\sin\alpha} \end{aligned}$$

2. **Asymmetric solutions**: $H_1 \neq H_2$

$$\begin{aligned} \frac{1}{1-\cos\alpha} &= \frac{6M_5^3}{8M_6^4\,\sin\alpha}\left(\epsilon_1\sqrt{H^2-H_1^2}+\epsilon_2\sqrt{H^2-H_2^2}\right) \\ \sqrt{H_1^2+H_2^2} &= \frac{4M_6^4}{3M_5^3} \\ H^2 &= \frac{4M_6^4}{3M_4^2}\frac{2+\cos\alpha}{\sin\alpha} \\ \sin\alpha\,(2+\cos\alpha) &= \frac{4M_6^4M_4^2}{3M_5^6} \end{aligned}$$

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Observations

- Scales of gravity

  In both cases, one gets $H \simeq \frac{M_5^3}{M_4^2}$ like in DGP.

  For present day acceleration, $M_5 = 10^{-2}$ GeV with $M_4 = 10^{18}$ GeV.

  When $\alpha \simeq \pi/2$, then $M_6^4 \simeq \frac{M_5^6}{M_4^2} \simeq M_6 = 10^{-3}$eV

  The cross-over scales are $r_{c,5} \sim r_{c,6} \sim H^{-1}, \quad r_{c,5} = \frac{M_4^2}{M_5^3}, \quad r_{c,6} = \frac{M_4}{M_6^2}$

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Observations

- Scales of gravity

  In both cases, one gets $H \simeq \frac{M_5^3}{M_4^2}$ like in DGP.

  For present day acceleration, $M_5 = 10^{-2}$ GeV with $M_4 = 10^{18}$ GeV.

  When $\alpha \simeq \pi/2$, then $M_6^4 \simeq \frac{M_5^6}{M_4^2} \simeq M_6 = 10^{-3}$eV

  The cross-over scales are $r_{c,5} \sim r_{c,6} \sim H^{-1}, \quad r_{c,5} = \frac{M_4^2}{M_5^3}, \quad r_{c,6} = \frac{M_4}{M_6^2}$

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- Are there ghosts?

  Both these solutions require to take at least one of the ϵ to be positive.

  This strongly suggests that ghosts are present in the codimension one branes.

  – One should understand if and how they couple to the intersection.

  – With no matter on the codimension one branes, they may be **harmless**.

Selftuning configurations

Are there solutions in which the tension λ controls only the angle α?

Here it is

$$\begin{aligned}
\Lambda_1 &= 6M_{5,1}^3 H_1^2 \\
\Lambda_2 &= 6M_{5,2}^3 H_2^2 \\
H^2 &= H_1^2 = H_2^2 = \frac{\Lambda_B}{10} \\
\cos^2\alpha &= \frac{\left(\lambda - 3M_4^2 H^2\right)^2}{\left(\lambda - 3M_4^2 H^2\right)^2 + 16M_6^8}
\end{aligned}$$

- Changing λ, no other parameter besides α is modified.

  $\Rightarrow$ The value of the Hubble parameter H is independent on the value of the cc.

  No need to touch the parameters in the initial action, then no fine-tuning.

  $\Rightarrow$ The viability of this scenario is associated to the behavior of gravity on $\Sigma_\cap$.

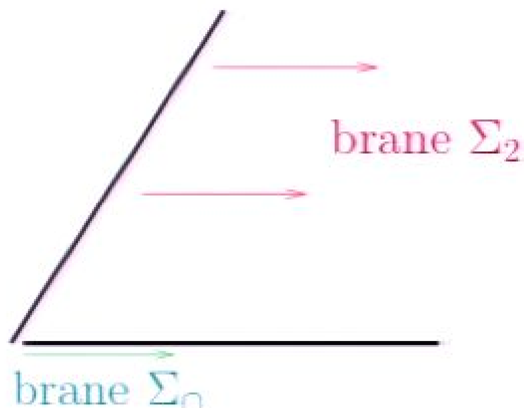
- A way to get some intuition is to study cosmological aspects of this model.

  $\Rightarrow$ These can be analyzed by a suitable generalization of these techniques.

Second part: FRW configurations

In order to study cosmology, we adopt a mirage approach.

- The branes can move through a static bulk and can deform their shape.
- This motion is interpreted as cosmological expansion from brane observers.



- If the bulk remains static, it is expected that fine-tuning relations are imposed on energy densities on Σ_2 and $\Sigma_\cap$.

Second part: FRW configurations

In order to study cosmology, we adopt a mirage approach.

We proceed again by focussing on the junction conditions.

- The moving brane Σ_2 is characterised by an embedding

$$X^M_{(2)} = (t,\, \vec{x}_3,\, \mathcal{Z}_1(t,\omega_1),\, \mathcal{Z}_2(t,\omega_1))$$

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- We pass to coordinates parallel to the brane. The normal reads

$$\tilde{n}^{(2)}_M = A\left(\frac{\dot{\mathcal{Z}}_1\mathcal{Z}_2' - \dot{\mathcal{Z}}_2\mathcal{Z}_1'}{\mathcal{N}},\, \vec{0}_3,\, 0,\, 1\right).$$

with

$$\mathcal{N} \equiv \sqrt{\mathcal{Z}_1'^2 + \mathcal{Z}_2'^2 - \left(\dot{\mathcal{Z}}_1\mathcal{Z}_2' - \dot{\mathcal{Z}}_2\mathcal{Z}_1'\right)^2}\,.$$

- Also here one component of the extrinsic curvature contains singular piece

$$K_{w_1 w_1}|_{sing} = -2A\mathcal{N}\,\frac{\mathcal{Z}_2'}{\mathcal{Z}_1'}\left(\mathcal{Z}_1'^2 + \mathcal{Z}_2'^2\right)\,\delta(w_1)$$

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- The Codazzi equation holds also in this case:

$\nabla^m S^{(2)}_{mn} = 0$ **no energy exchange with the bulk.**

But this doesn't forbid **energy exchange between $\Sigma_\cap$ and Σ_2.**

An application

- We consider a natural embedding

$$X^M = (t,\, \vec{x}_3,\, \mathcal{Z}_1(t,w_1),\, \mathcal{Z}_2(t,w_1))$$

with

$$\begin{aligned} \mathcal{Z}_1 &= w_1 \cos\alpha(t,w_1) \\ \mathcal{Z}_2 &= z_2(t,w_1) + w_1 \sin\alpha(t,w_1) \end{aligned}$$

Brane Σ_2 can move, and the angle α between the branes is time dependent.
$\Rightarrow$ Consistency conditions impose that $\dot{\alpha}(t,0) = 0$

- The energy momentum tensor on the moving Σ_2 is

$$S^a{}_b = \begin{pmatrix} -\rho_2 & 0 & 0 & 0 & 0 \\ 0 & p_2 & 0 & 0 & 0 \\ 0 & 0 & p_2 & 0 & 0 \\ 0 & 0 & 0 & p_2 & 0 \\ \chi & 0 & 0 & 0 & p_2 \end{pmatrix}, \quad \text{at } w_1 = 0$$

in order to satisfy the Israel conditions relative to this embedding.

An application

The cosmological equations that govern expansion at the intersection are, in the case $M_5^{(2)} = 0$:

$$\rho \;=\; 3M_4^2 H^2 + 6M_5^3\, k_2 \sqrt{1 + \frac{H^2}{k_2^2}} + 4M_6^5\, k_2\, \tan\alpha \left[\frac{\sqrt{H^2 + k_2^2}}{H^2 \sin^2\alpha + k_2^2}\right]$$

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– The induced gravity term at the intersection M_4 is necessary.

– When $\alpha \neq 0$, the intersection feels six d bulk effects.

$\Rightarrow$ They vanish at large H: **relativistic effect**?

– There is exchange of energy between $\Sigma_\cap$ and Σ_2:

$\Rightarrow$ An interplay between energy densities on different branes,

no good for the cc problem, because the branes get too coupled.

Can six dimensional effects alone drive acceleration?

Ad additional simplification of the previous system of equations for $\Sigma_\cap$ is obtained

- Setting $k_1 = 0$, and $k_2^2 = \beta^2 \sin^2\alpha\, M_6^2$ for β small.
- Take H^2 much bigger than $\beta^2 M_6^2$.

Then the equations reduce to

$$\dot\rho + 3H\,(\rho + p) = -4\,\epsilon\,\frac{\beta\, M_6^5}{\cos\alpha}\,\frac{\partial}{\partial t}\,\frac{1}{H}$$

with $\epsilon \equiv -\frac{\sin\alpha}{|\sin\alpha|} = \pm 1.$

$$H = \epsilon\frac{M_5^3}{M_4^2} + \sqrt{\frac{\rho}{3M_4^2} + \frac{M_5^6}{M_4^4} + \epsilon\,\frac{4\,\beta\,M_6^5}{3\,\cos\alpha\,M_4^2\,H}}$$

To achieve acceleration

- The choice $\epsilon = 1$ corresponds to the selfaccelerating DGP branch.

  It corresponds to the one already discussed.

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$$\rho = \rho_0\left(\frac{a(t)}{a_0}\right)^{-3(1+\omega)}\left(1 - \frac{4\beta M_6^5}{3\,H^3\,M_4^2\cos\alpha}\right)^{\frac{1}{3}}$$

we have acceleration when

$$\omega < \frac{2}{3\left(1 - \frac{4\beta M_6^5}{3\,H^3\,M_4^2\cos\alpha}\right)} - 1$$

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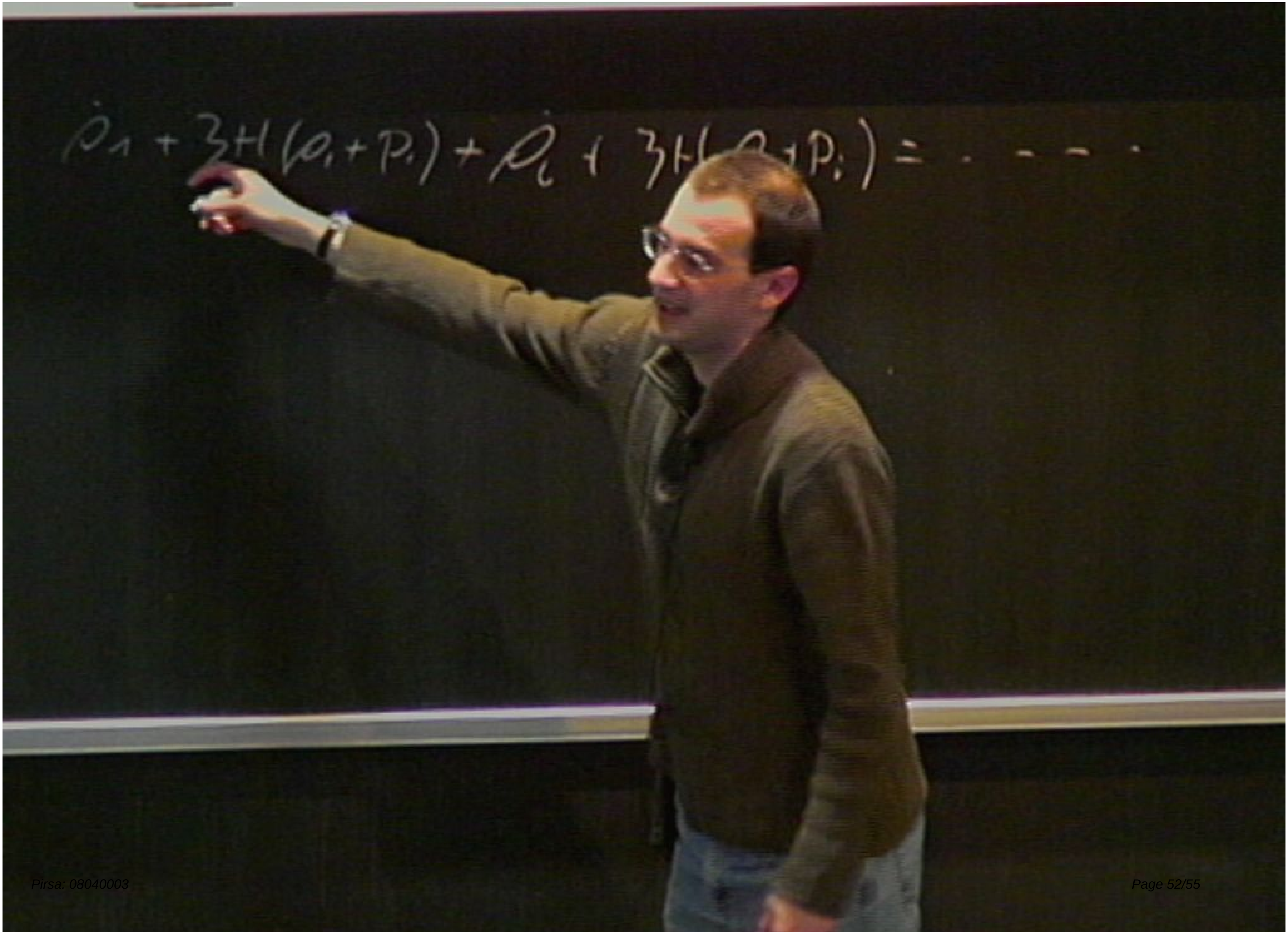
Outlook

- I presented new scenarios to address dark energy issues in the framework of higher codimension BWs with localised gravity terms.
- Study dynamics of metric fluctuations:
  - The bending mode will play a special role.
  - Identify ghosts and understand their coupling with matter at intersection.
- Explore cosmological solutions studying the **interplay** between branes with different dimensions.
  - Is the energy flow between branes compatible with the requirement of obtaining the right amount of acceleration?

$$\dot{\rho}_1 + 3H(\rho_1 + p_1) + \dot{\rho}_i + 3H(\rho_1 + p_i) = \cdot$$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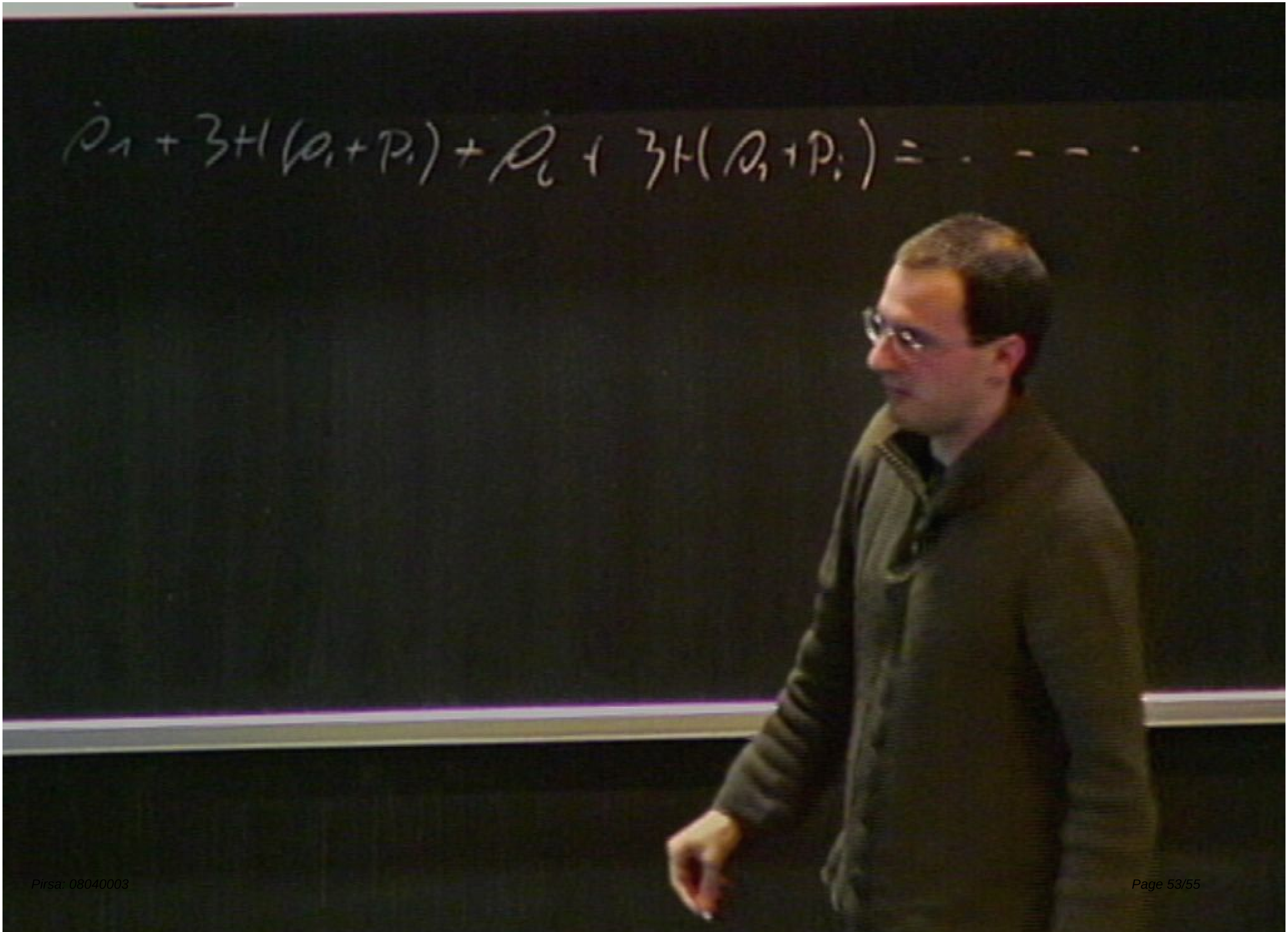
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H &= \frac{8\epsilon_1}{3\sin\alpha}\frac{M_6^4}{M_5^3}\sqrt{\frac{1+\cos\alpha}{2}} \\
4+3\cos\alpha &= \frac{8\,M_6^4\,M_4^2\,(1+\cos\alpha)}{3\,M_5^3\,\sin\alpha}
\end{aligned}
$$

2. **Asymmetric solutions**: $H_1 \neq H_2$

$$
\begin{aligned}
\frac{1}{1-\cos\alpha} &= \frac{6M_5^3}{8M_6^4\,\sin\alpha}\left(\epsilon_1\sqrt{H^2-H_1^2}+\epsilon_2\sqrt{H^2-H_2^2}\right) \\
\sqrt{H_1^2+H_2^2} &= \frac{4M_6^4}{3M_5^3} \\
H^2 &= \frac{4M_6^4}{3M_4^2}\frac{2+\cos\alpha}{\sin\alpha} \\
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