Title: The Black Hole Information Paradox, Past and Future

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Abstract: Hawking\'s black hole information paradox is one of the great thought experiments in physics. It points to a breakdown of some central principle of physics, though which one breaks down is still in dispute. It has led to the discovery of ideas that seem to be key to unifying quantum mechanics and gravity, namely the holographic principle and gauge/gravity duality. I review this subject, and discuss ongoing work and future directions.

The Black Hole Information Paradox, Past and Future

Joseph Polchinski
Kavli Institute for Theoretical Physics
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PI, April 9, 2008

Planck's challenge:

Planck's constant $\hbar$, the speed of light c, and Newton's constant G define a natural system of units:

Planck length, $l_P = \sqrt{\hbar G / c^3} = 1.6 \times 10^{-33}\,\mathrm{cm}$

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"These necessarily retain their meaning for all times and for all civilizations, even extraterrestrial and non-human ones, and can therefore be designated as *natural units*."

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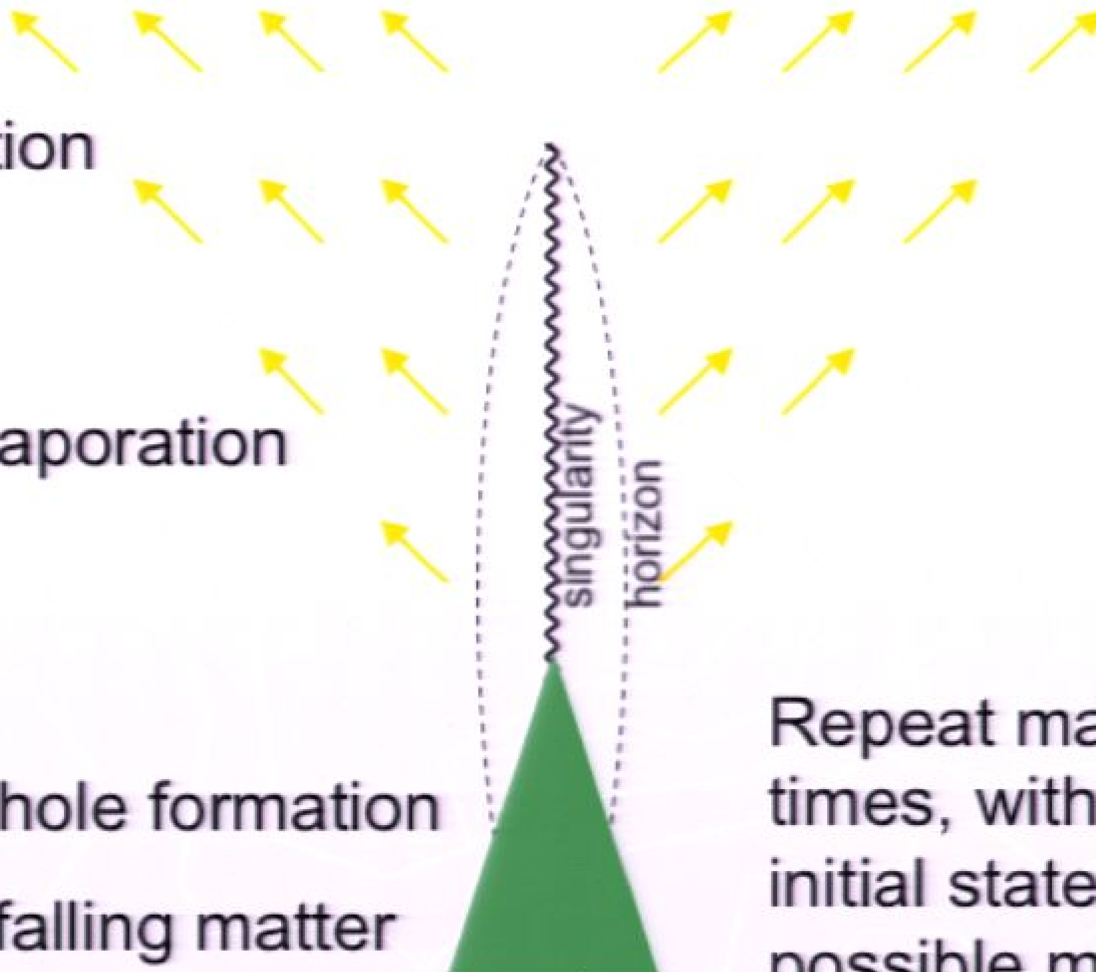
Hawking's thought experiment (1976):

4. Final state:
Hawking radiation

3. Black hole evaporation

2. Black hole formation

1. Initial state: infalling matter



Repeat many times, with same initial state and all possible measurements on the final state.

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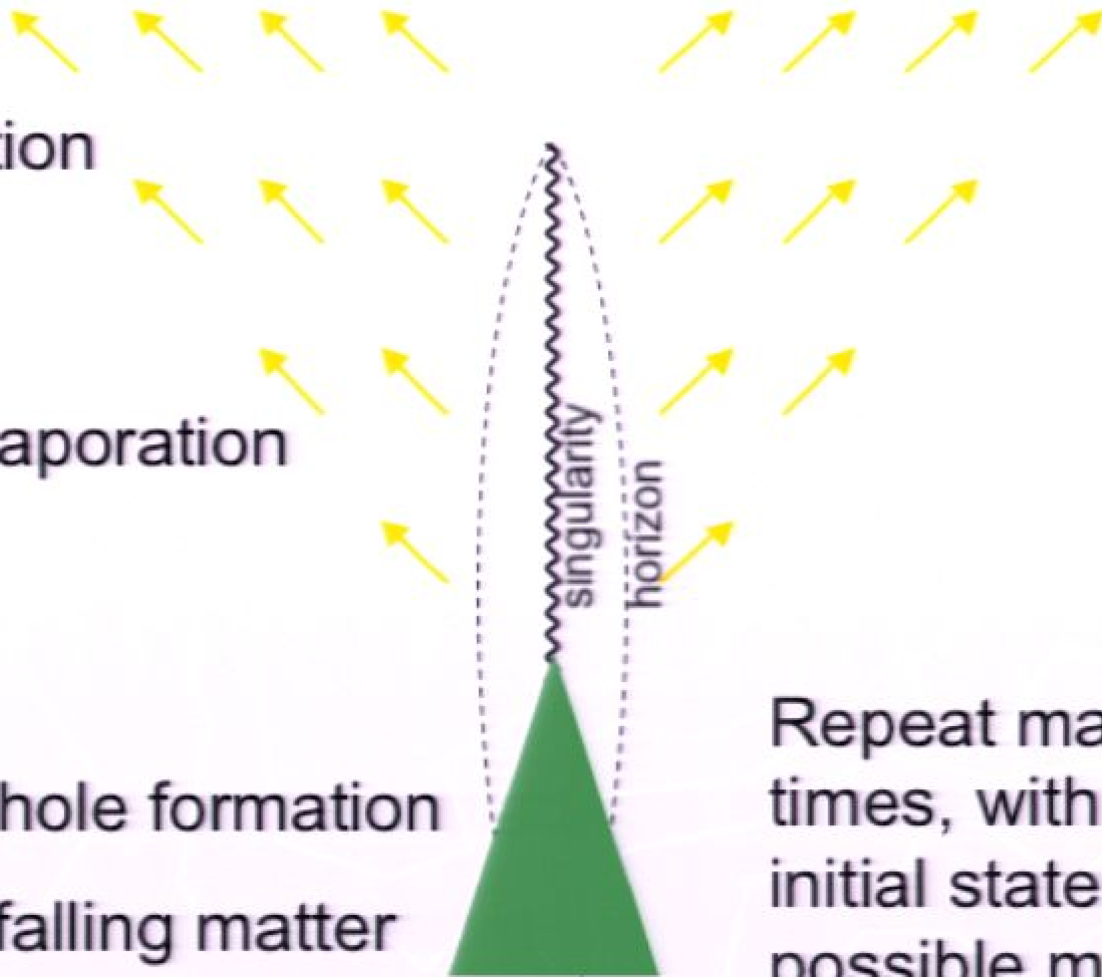
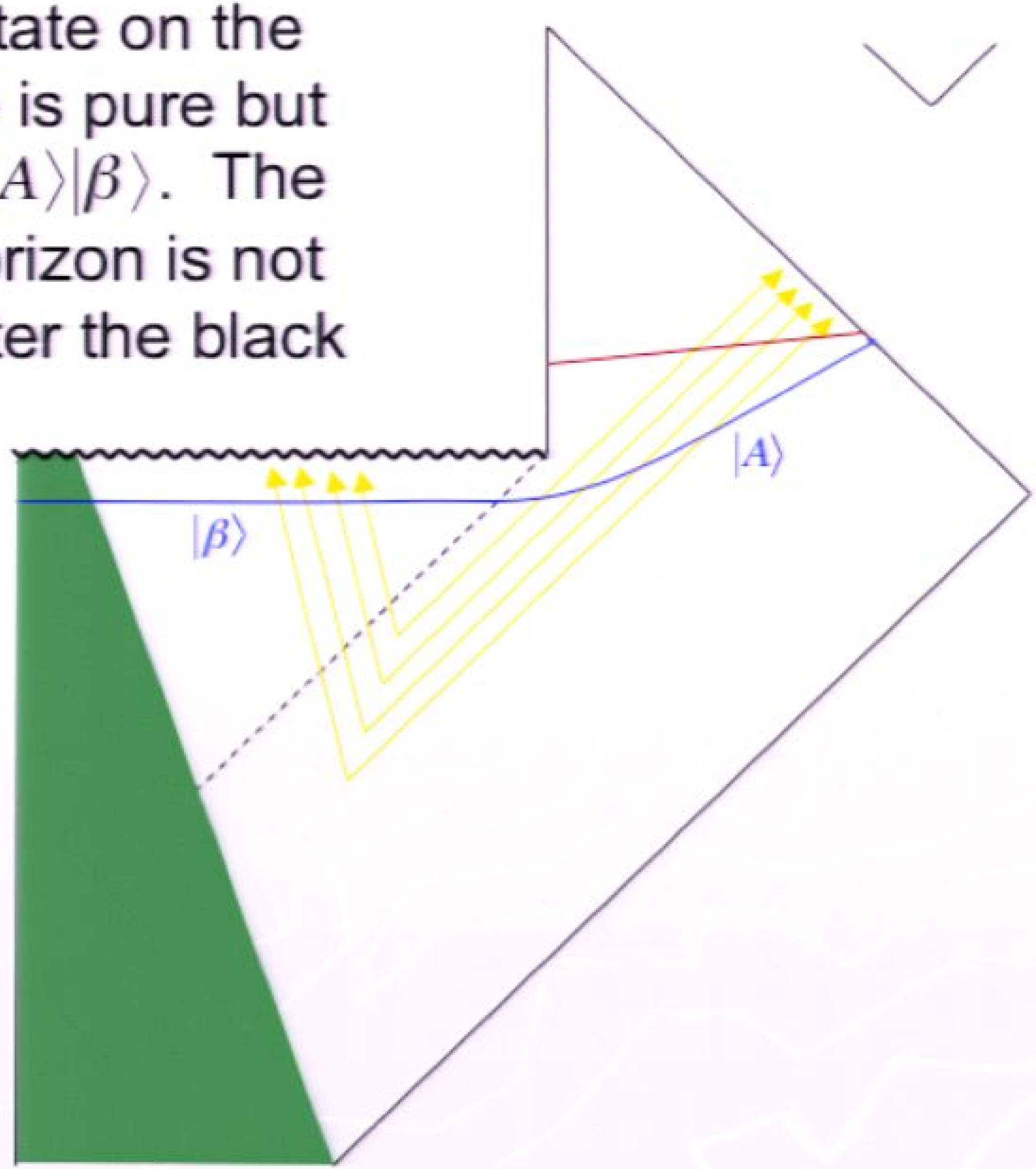


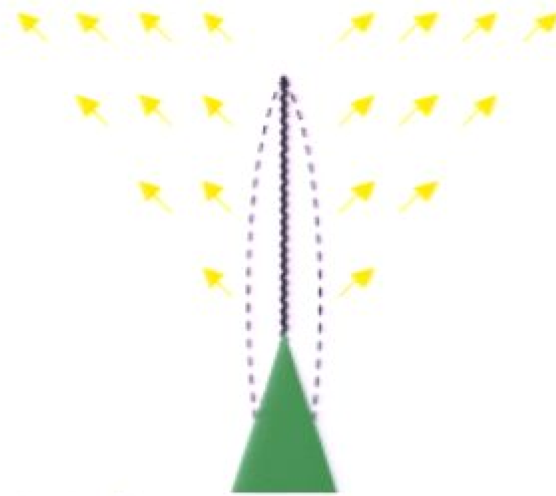
Repeat many times, with same initial state and all possible measurements on the final state.

The problem: the state on the indicated time slice is pure but entangled, $\sum_{A,\beta} S_{A\beta i}|A\rangle|\beta\rangle$. The state behind the horizon is not observable, and after the black hole evaporates this information is lost.

Thus the system after evaporation can only be described by the density matrix $\rho_{AC} = \sum_{\beta} S_{A\beta i} \overline{S}_{C\beta i}$



The final system cannot be described by a pure state $|\psi_{\mathrm{f}}\rangle$ but only by a mixed state (density matrix).



$$\rho_{\mathrm{i}} = |\psi_{\mathrm{i}}\rangle\langle\psi_{\mathrm{i}}| \quad \rightarrow \quad \rho_{\mathrm{f}} = \sum_{A,B} S_{AB\mathrm{ii}} |A\rangle\langle B| \neq |\psi_{\mathrm{f}}\rangle\langle\psi_{\mathrm{f}}|$$

That is, the expectation value of an observable O is

$\sum_{A,B} S_{AB\mathrm{ii}} \langle B|O|A\rangle$, and not the usual $\langle\psi_{\mathrm{f}}|O|\psi_{\mathrm{f}}\rangle$

(Phase) information has been lost.

"Not only does God play dice, but he sometimes throws them where they cannot be seen."

This thought experiment implies a breakdown of the ordinary rules of quantum mechanics, and this should be happening everywhere, all the time, via virtual black holes.

In addition to this *paradox* there is a related *puzzle*: black holes satisfy thermodynamic laws, with a Bekenstein-Hawking entropy $S = A/4l_P^2$. What is the underlying *statistical mechanics?* What *states* does this entropy count.

This talk will focus on the paradox.

Possible outcomes to black hole evaporation:

1. The state of the Hawking radiation is actually pure. The information (about what went into the black hole) is encoded in the Hawking radiation.

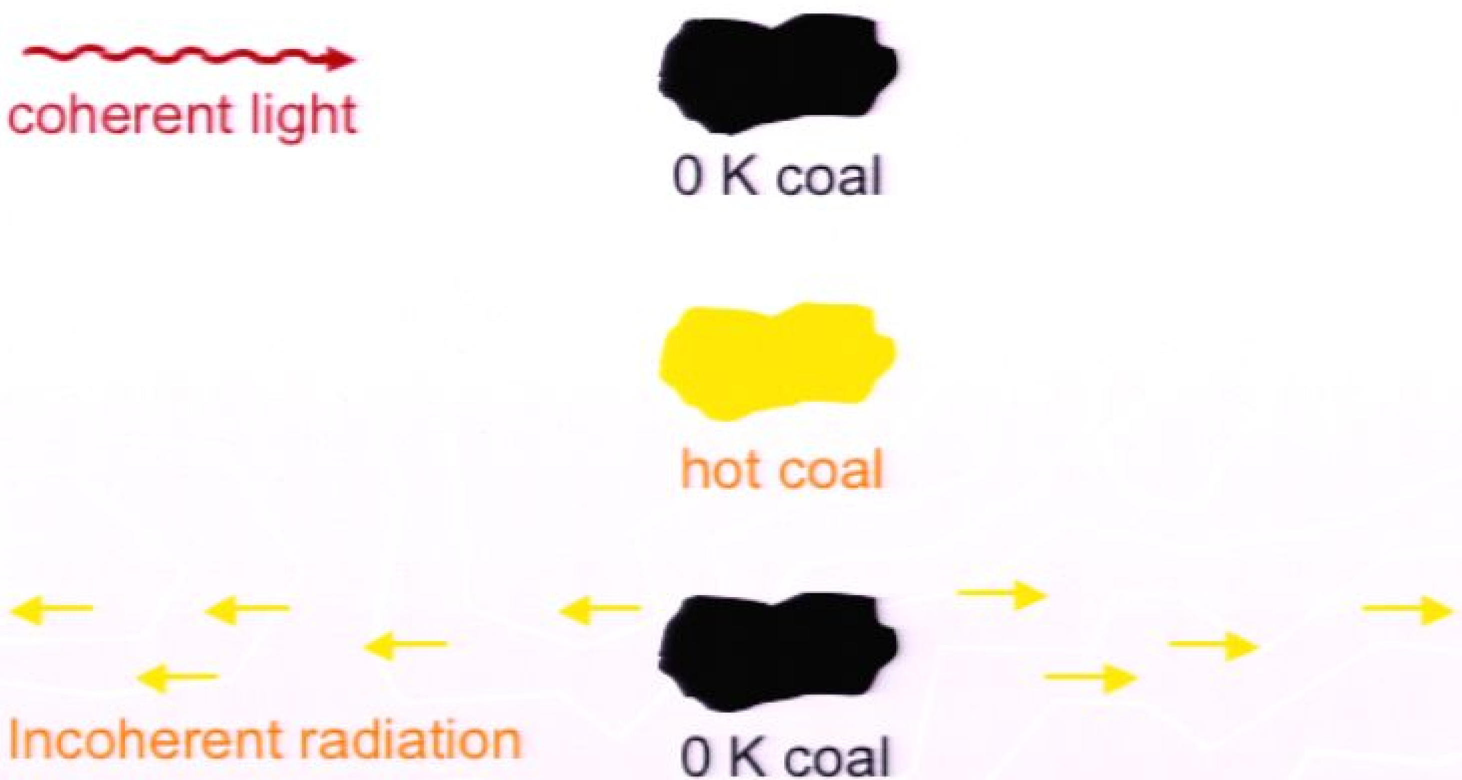
2. The state is indeed mixed. Information is lost.

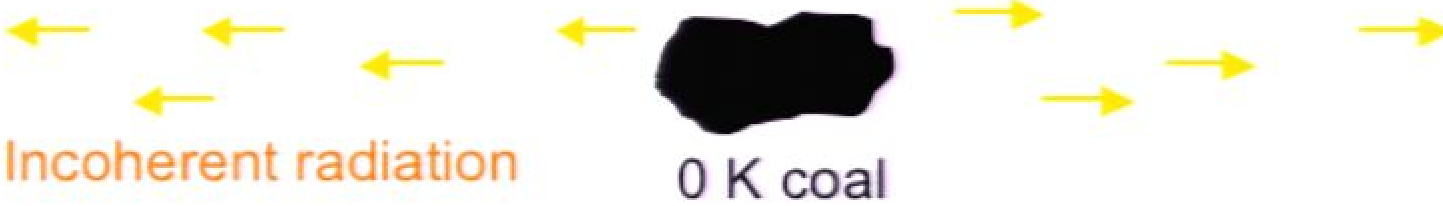
3. The evaporation does not proceed to completion, but terminates in a stable remnant with a very large number of internal states.

4. A remnant which (slowly) decays, reemitting the information.

1. The information gets out with the Hawking radiation

Consider an analogous thought experiment:





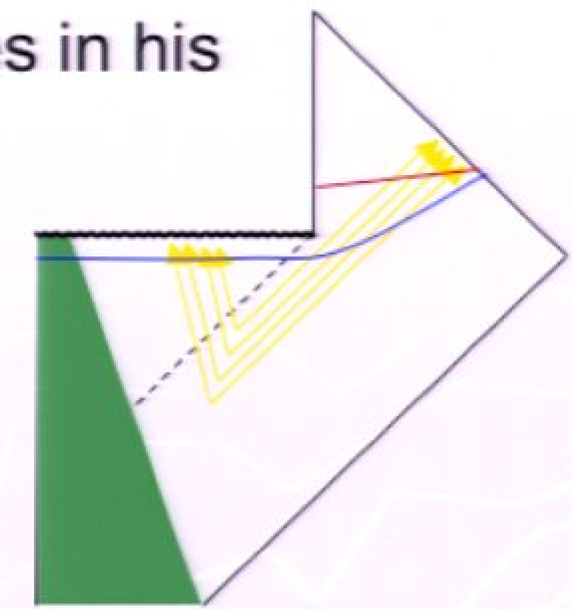
The final state is similar to the black hole case, but here the state is pure: there are definite phase correlations between the different states of the radiation.

Did Hawking miss such subtle phases in his calculation?

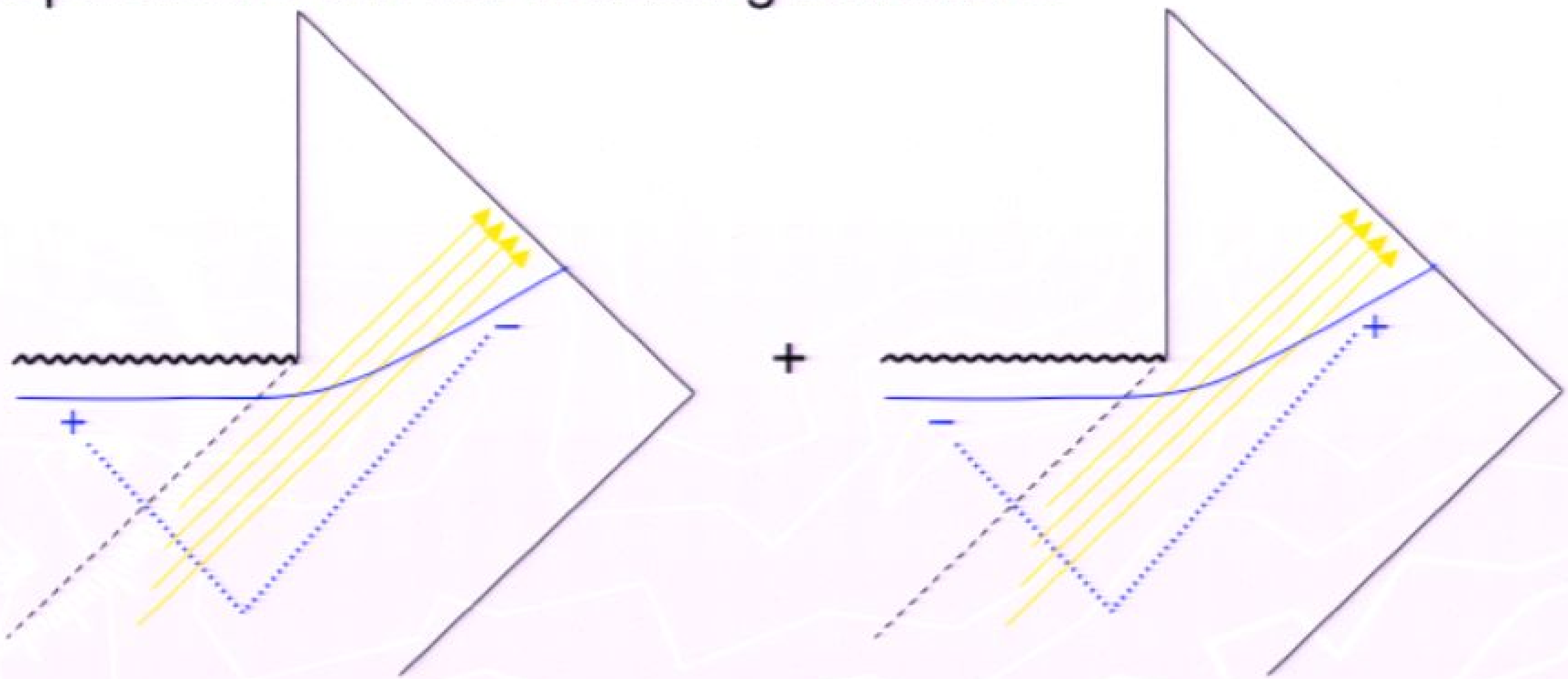
It is easy to confuse oneself:

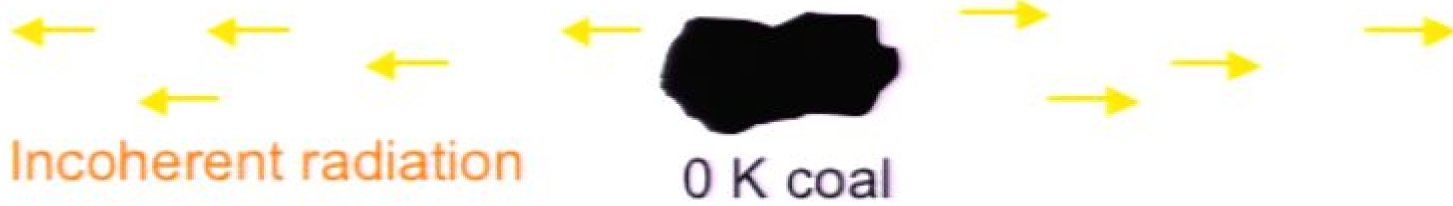
• Coordinate singularity → transplanckian energies

• Time-reversed evolution unstable.



Quantum no-Xerox principle (Wooters, Zurek, Dieks; Susskind): take some entangled bit-pairs, and throw half of each pair into the black hole. These pass through the horizon essentially unchanged (the equivalence principle) and reach the interior, which is spacelike separated from the Hawking radiation.





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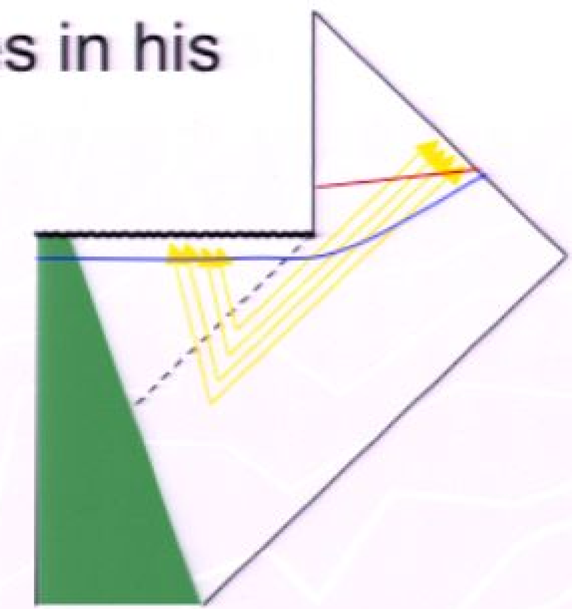

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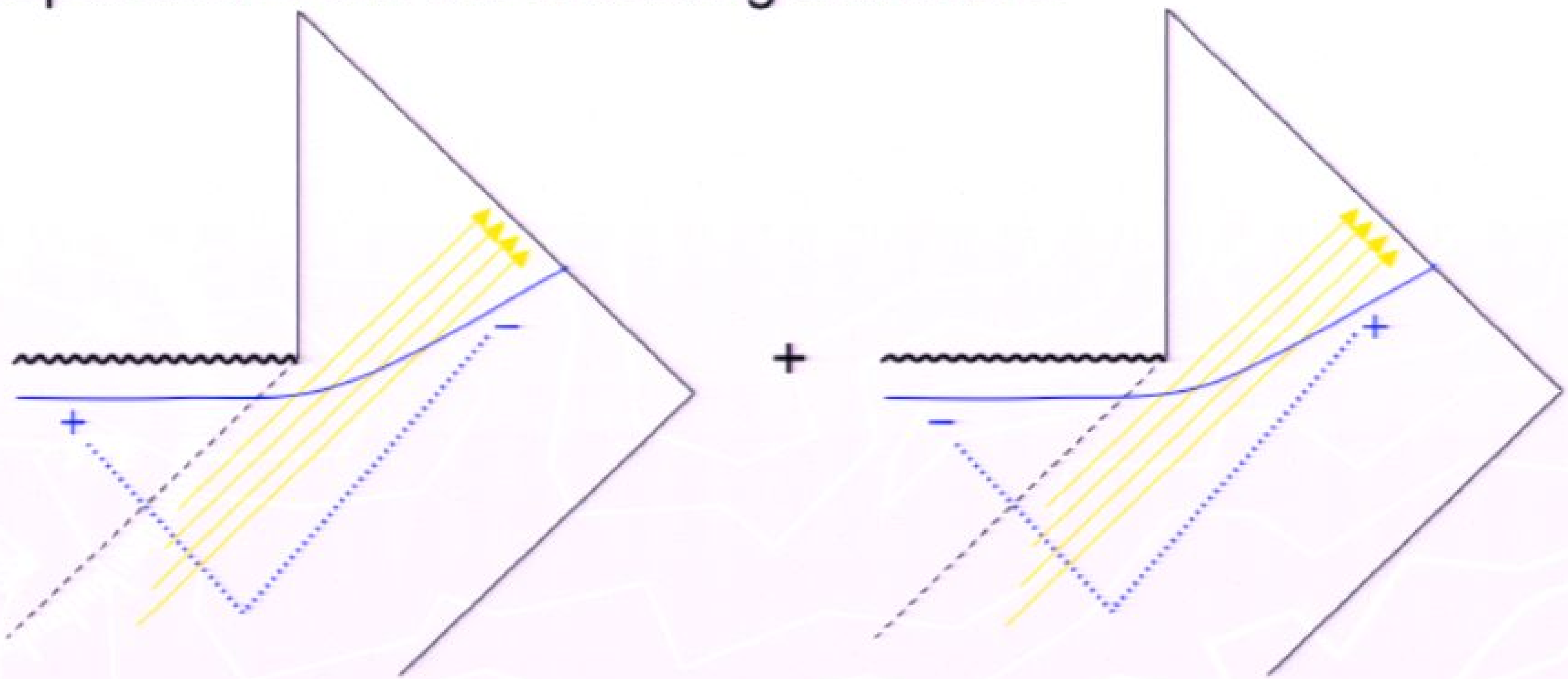
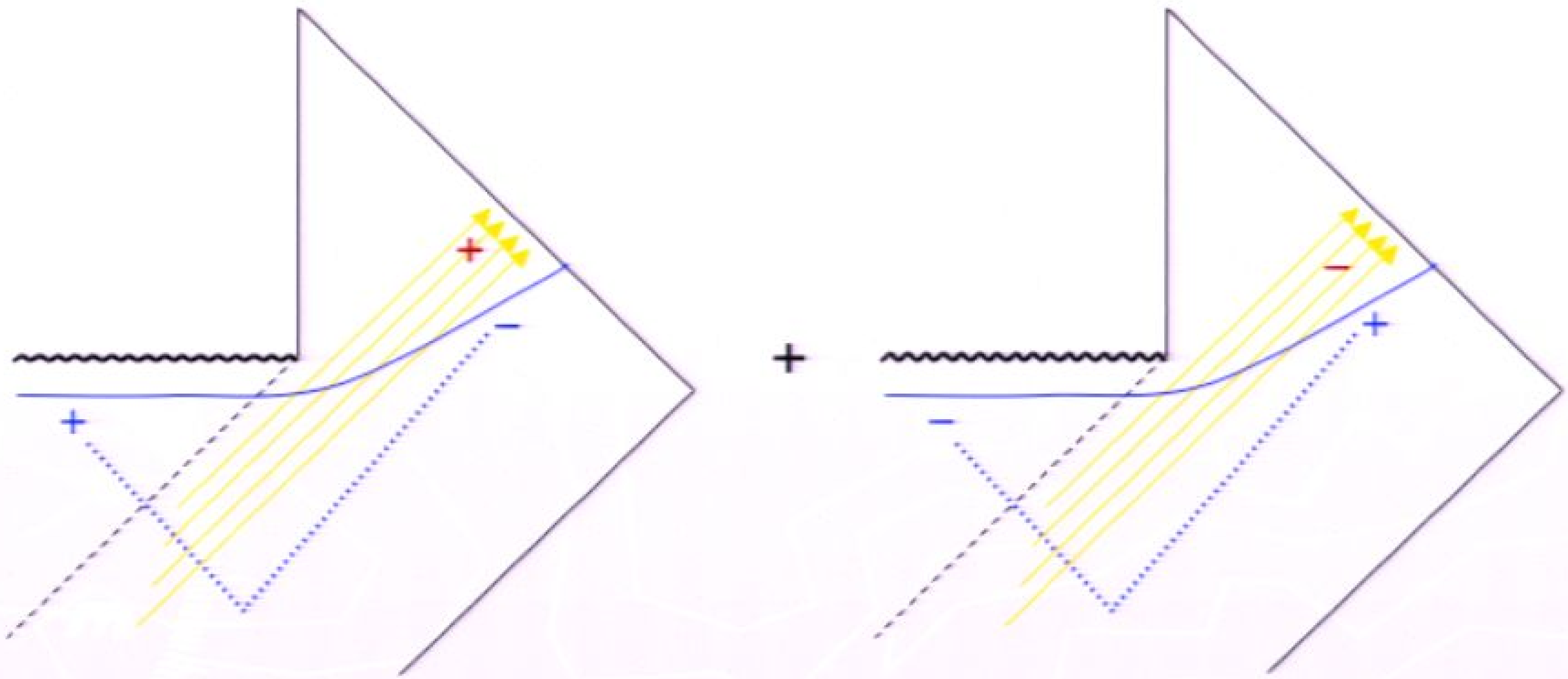


But could there also be a second copy of the inside bit encoded in the Hawking radiation?:



There is no quantum xerox machine that can take any state $|\psi\rangle$ in a single Hilbert space into a state $|\psi\rangle|\psi\rangle$ in a pair of Hilbert spaces:

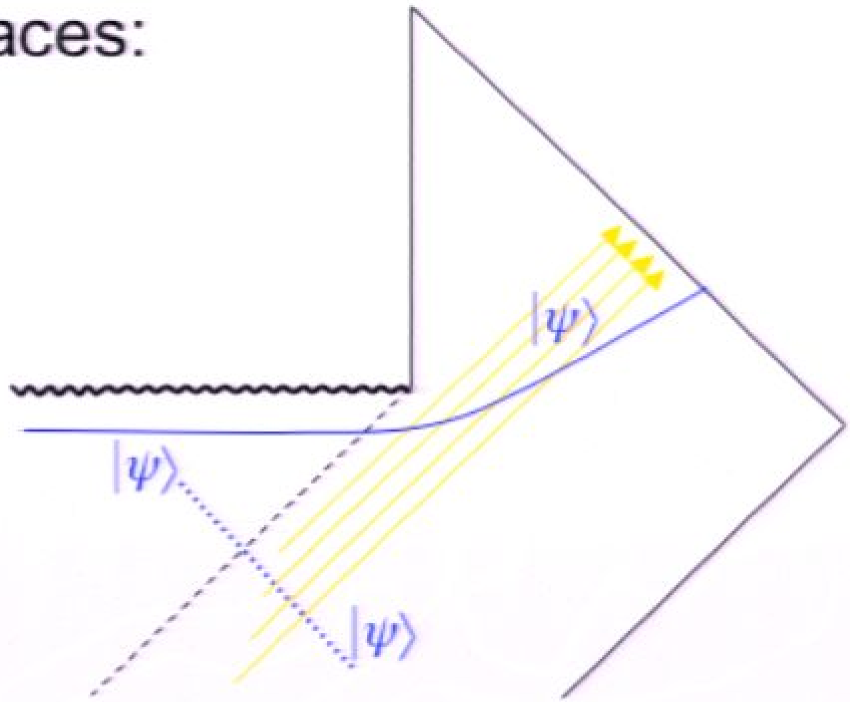
If $|\Uparrow\rangle \rightarrow |\Uparrow\rangle|\Uparrow\rangle$

and $|\Downarrow\rangle \rightarrow |\Downarrow\rangle|\Downarrow\rangle$,

then $|\Leftarrow\rangle = (|\Uparrow\rangle + |\Downarrow\rangle)/\sqrt{2}$

$\rightarrow (|\Uparrow\rangle|\Uparrow\rangle + |\Downarrow\rangle|\Downarrow\rangle)/\sqrt{2}$

$\neq |\Leftarrow\rangle|\Leftarrow\rangle$.



Quantum information thrown into the black hole cannot also be in the Hawking radiation, it is lost. A black hole is not like a lump of coal because it has a horizon.

2. Information is lost

It is problematic whether there is a consistent theoretical framework. For example, this seems to imply energy non-conservation (Banks, Peskin, Susskind, 1984). Argument: a local time-evolution law that turns pure into mixed states can be written as evolution with a spacetime-dependent Hamiltonian density (Ellis, Hagelin, Nanopoulos, Srednicki, 1984).

$$\mathcal{H}_{\text{eff}} = \sum_\alpha g_\alpha(t,x)\, O_\alpha(t,x)$$

Here, $O_\alpha(t,x)$ are some set of local operators, and $g_\alpha(t,x)$ is averaged *incoherently*. That is,

`spacetime is dirty.'

Even if the *average* distribution of dirt is translation-invariant, any given distribution is not, and so momentum and energy conservation are lost.

Possible loopholes:

- Locality --- should be OK for small or virtual black holes.
- Unobservability (Unruh & Wald, 1995). Instantaneous change in Schwarzschild mass?

3. The information is stored in a stable remnant.

The Hawking calculation breaks down when the black hole mass and curvature approach the Planck scale. Perhaps the evaporation ceases, and we have a `remnant' which has a large number of internal states (representing the states of the infallen Hawking particles), so that

remnant x Hawking radiation

is in a pure state.

Problem: since the initial black hole could have been arbitrarily large, the necessary number of internal states is unbounded. Divergent effects from virtual remnants?

4. The information is stored in a long-lived remnant, which slowly decays and releases the information.

There are of order M^2/M_P^2 Hawking quanta of energy M_P^2/M, where $M >> M_P$ is the initial black hole mass.* If the final decay of the black hole involves a few Planck-scale quanta, there are not enough degrees of freedom to restore purity. However, if the remnant decays via a large number of very low energy quanta (energy of order M_P^3/M^2).

Problem: still an unbounded number of light states, giving divergent virtual effects.

* $\hbar = c = 1$.

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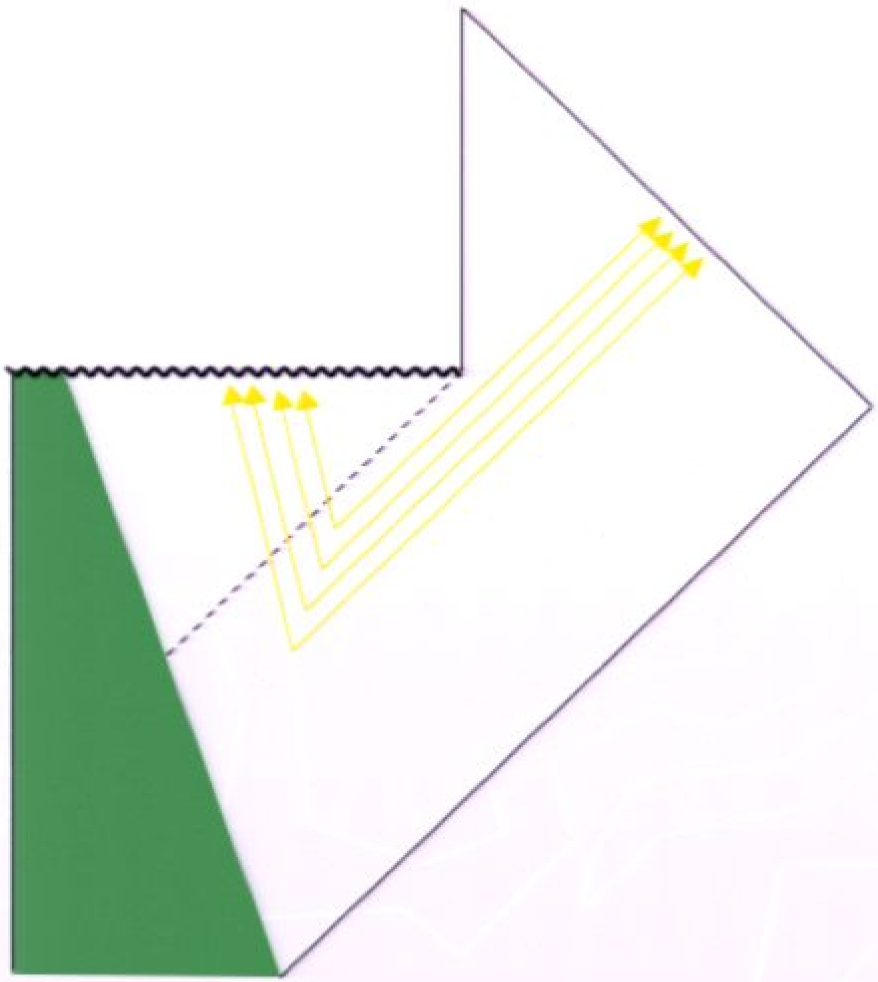
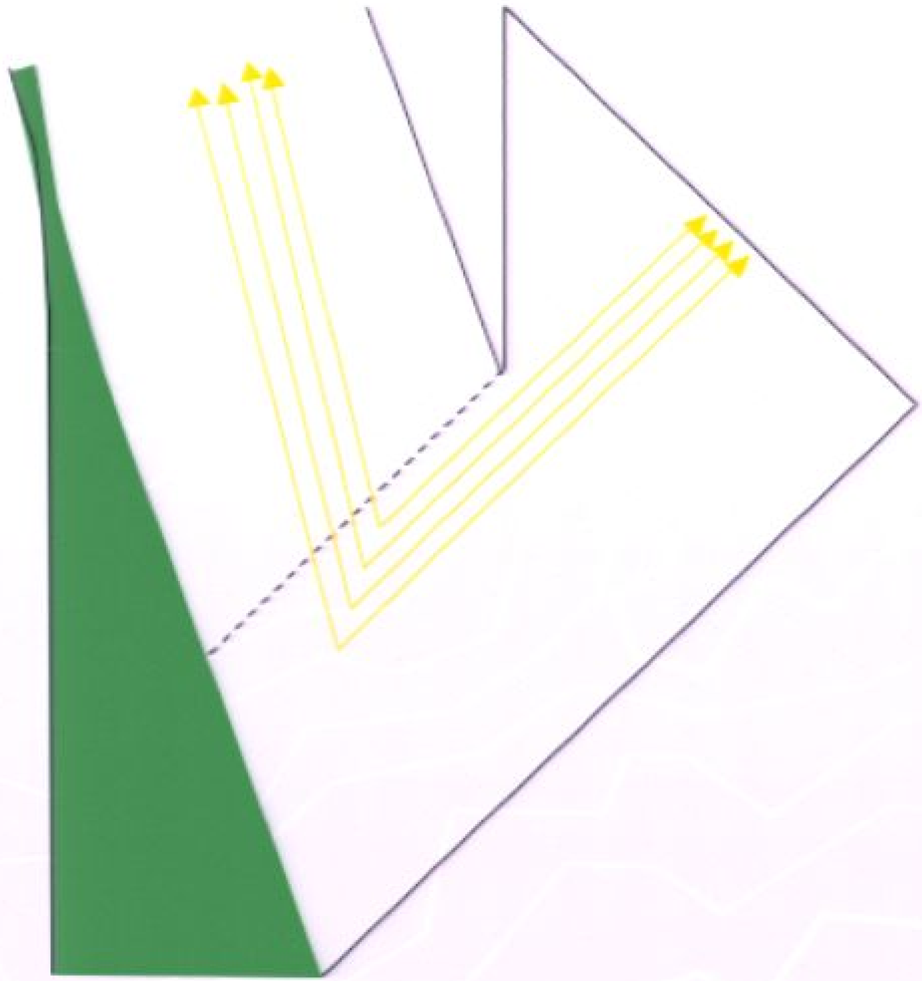
What about the singularity?

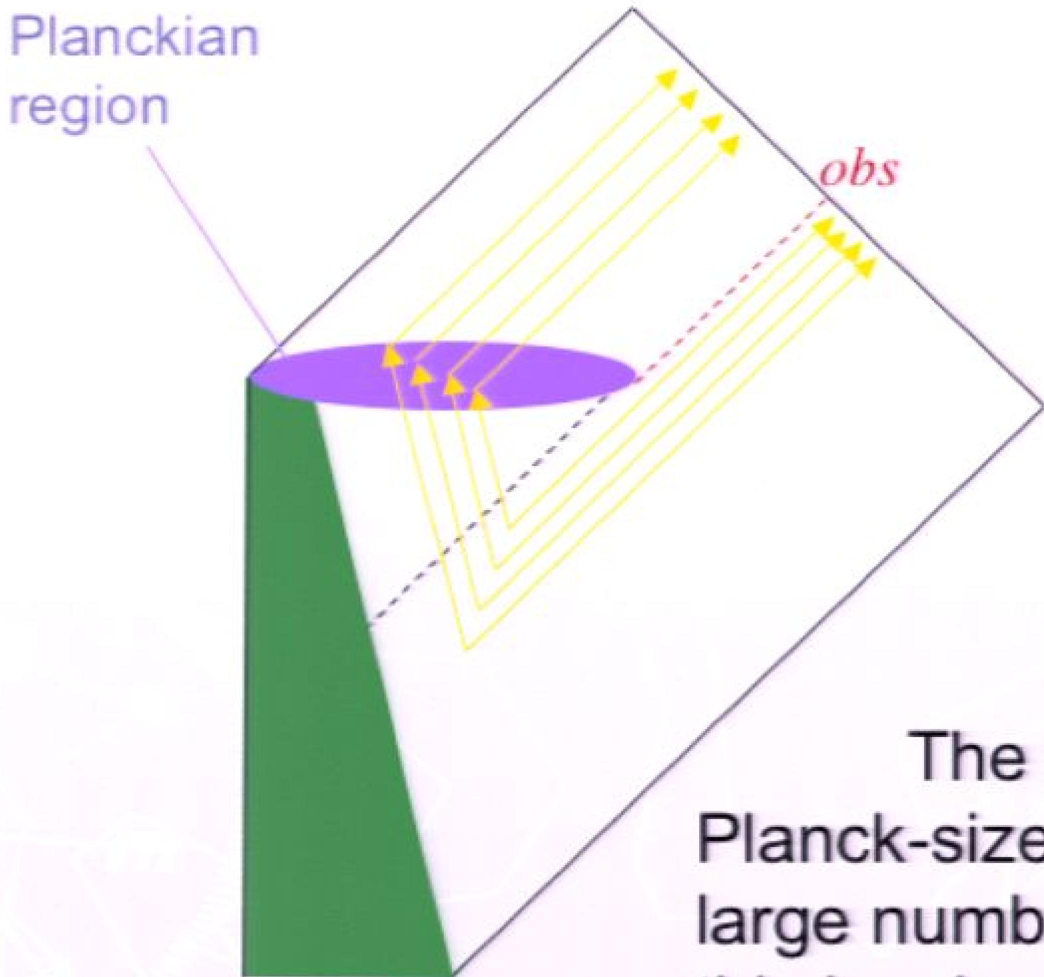


One possibility: spacetime continues past the singularity, forming a disconnected `baby universe.'



Looks like information loss, but consideration of the quantum state of the baby universe apparently leads to long-lived remnants (JP & Strominger, Strominger 1994). Long story, α-parameters, …

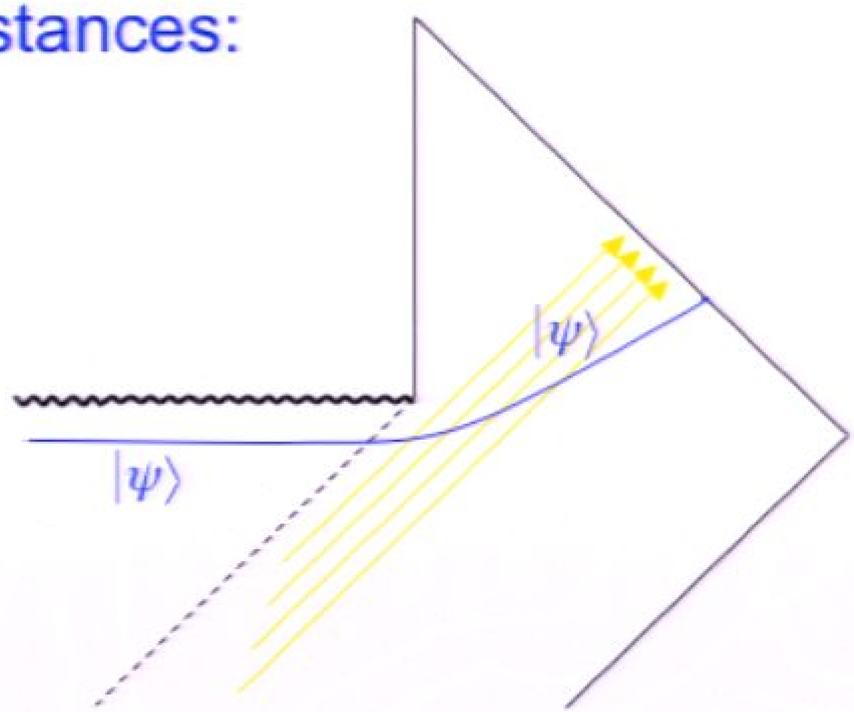
Another possibility (e.g. Ashtekar & Bojowald, 1995):



Apparently the information is emitted --- how?

The observer *obs* sees a Planck-sized object with a very large number of internal states: this is a long-lived remnant.

Lesson: In order for the information to be in the Hawking radiation, it must be transmitted over large spacelike distances:



Black hole complementarity (Susskind, 1993): these are the same state as seen by two different observers --- radically nonlocal…

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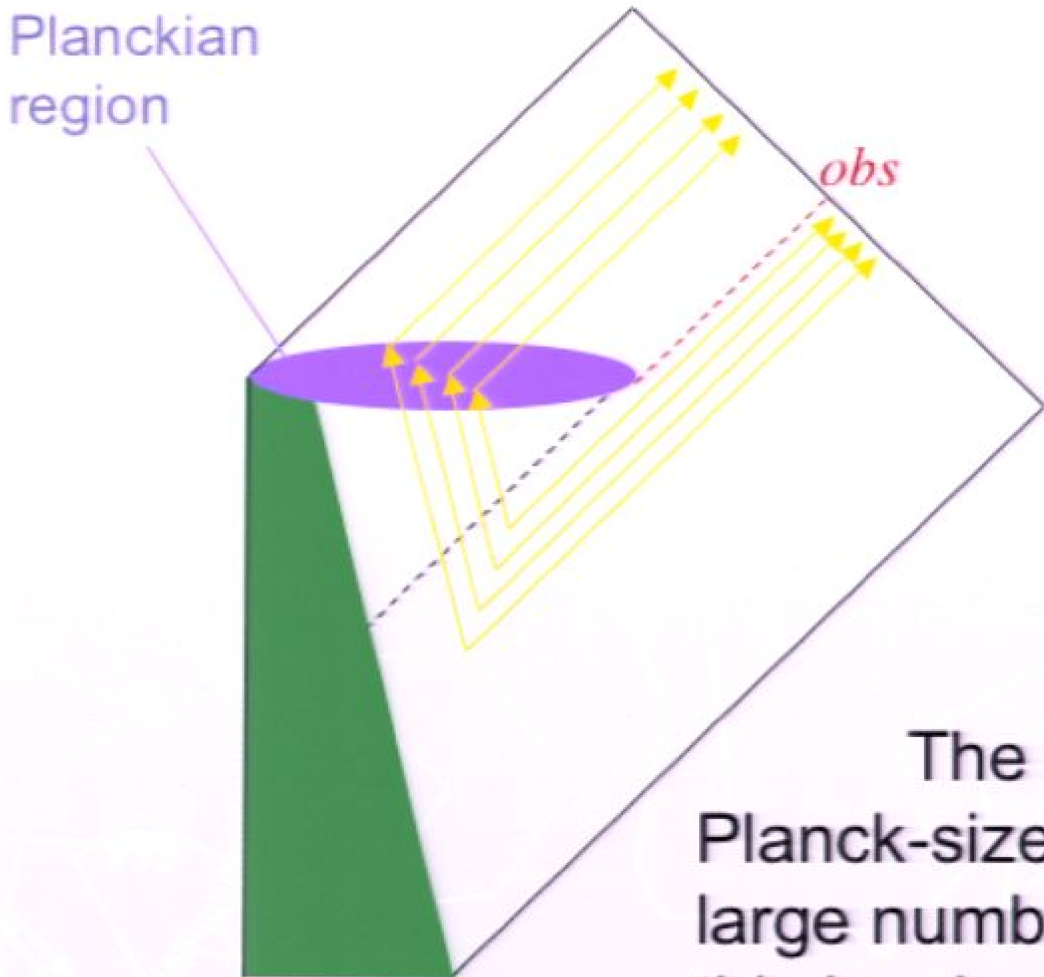

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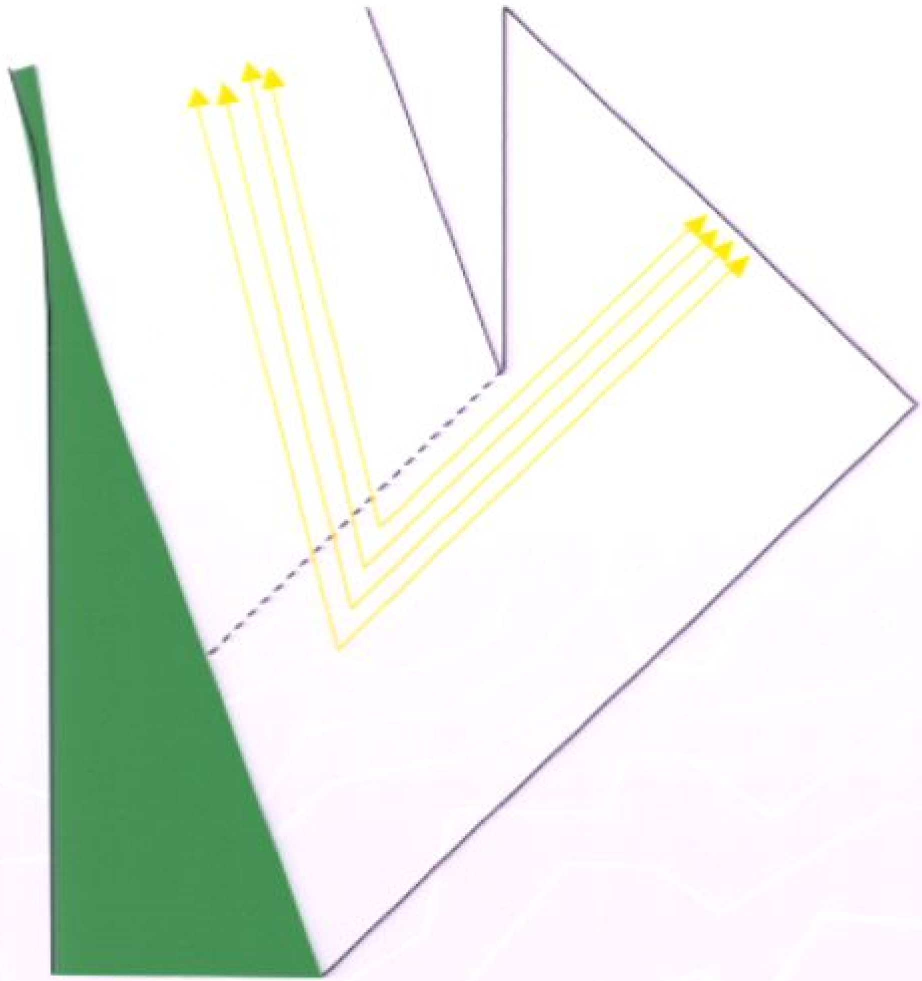

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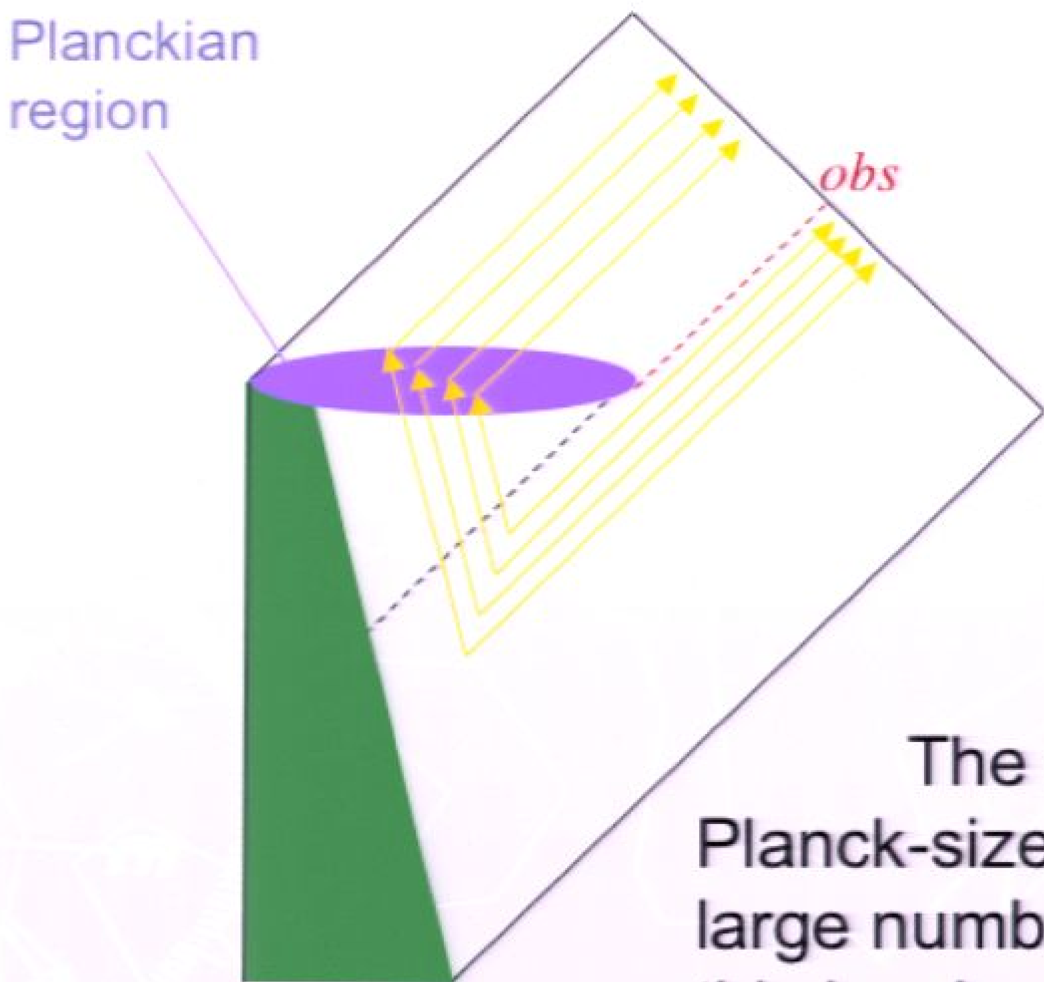

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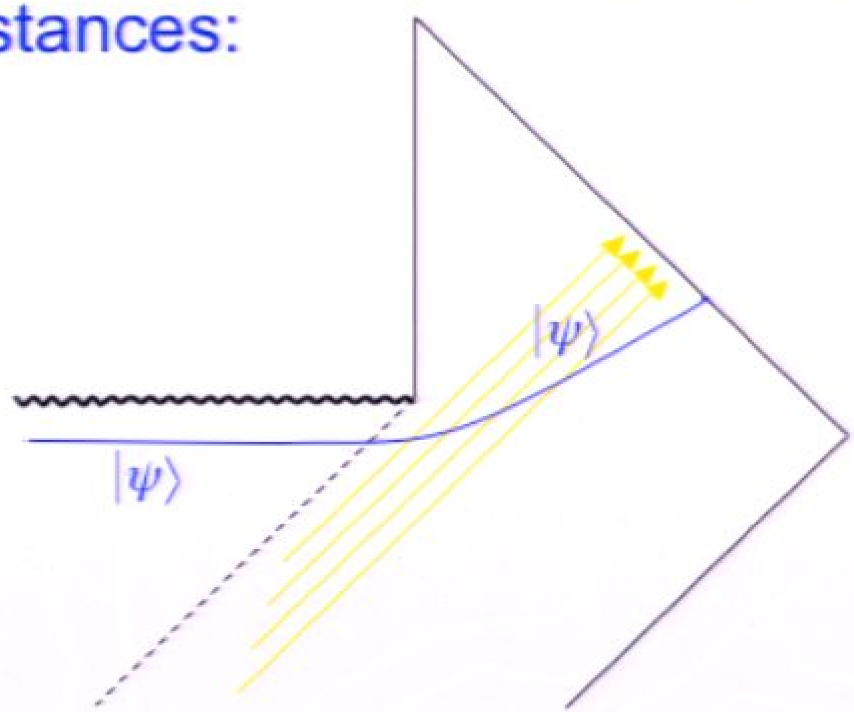
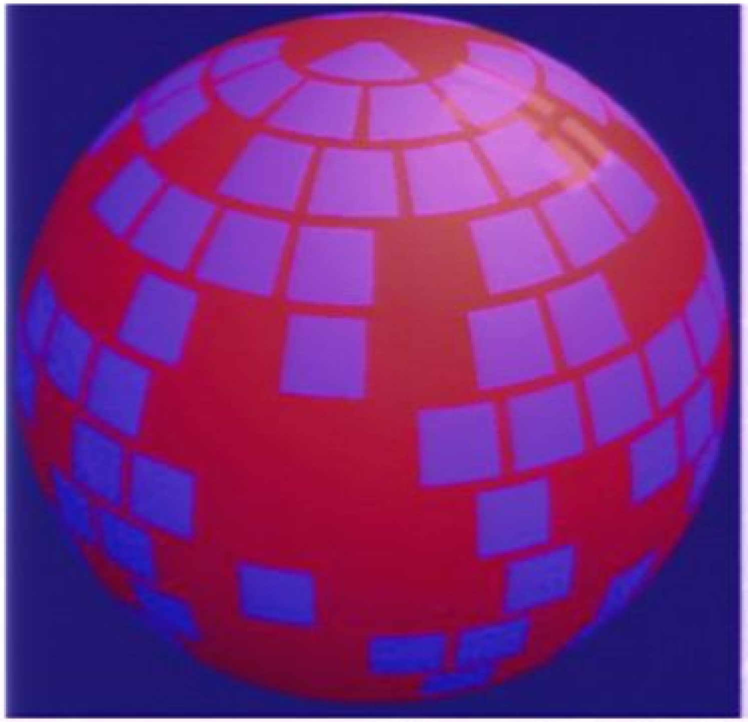


Black hole complementarity (Susskind, 1993): these are the same state as seen by two different observers --- radically nonlocal…

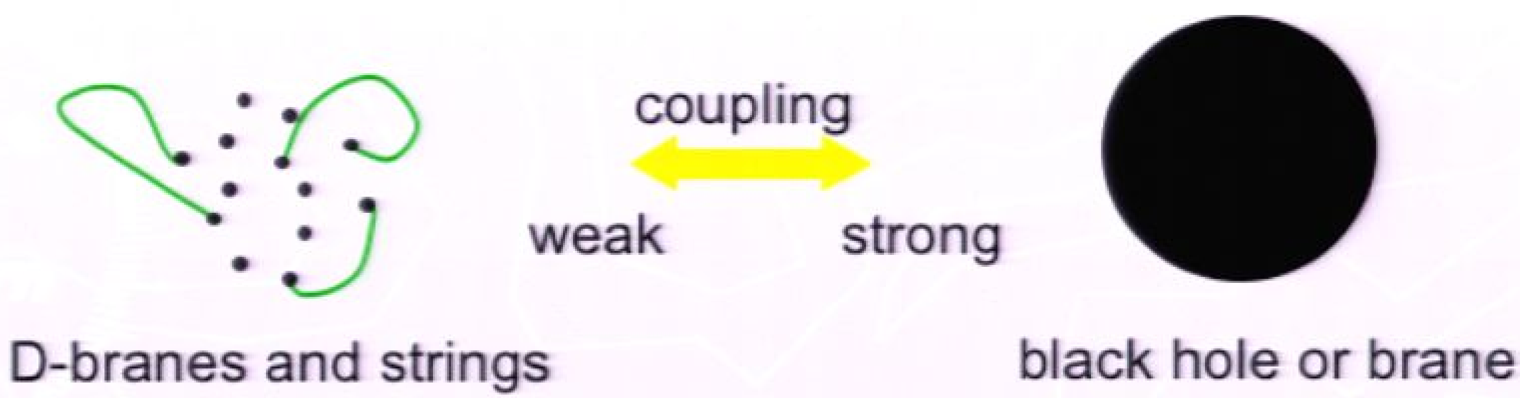
Another hint of radical nonlocality: the Bekenstein-Hawking entropy $S = A/4l_P^2$ suggests the *holographic principle* ('t Hooft, Susskind, 1993), that quantum gravity in any space can be formulated in terms of degrees of freedom living on the *boundary* of the space.



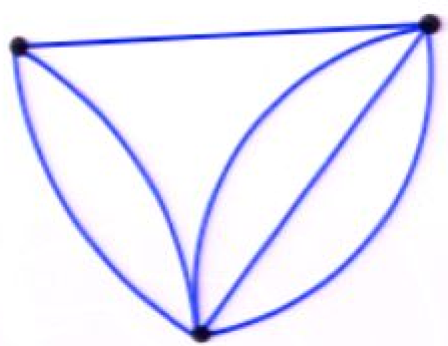
From G. 't Hooft

Black hole entropy counting:

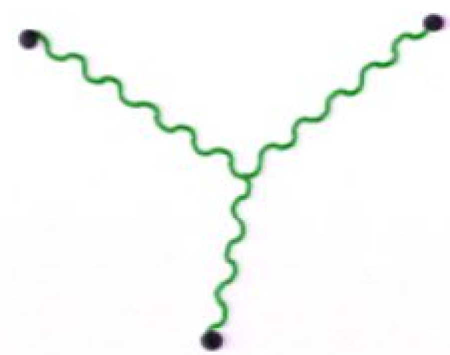
Strominger and Vafa (1996) argued that by turning down the coupling one could adiabatically turn some supersymmetric black holes into weakly coupled systems whose states can be explicitly counted, giving a statistical interpretation to the Bekenstein-Hawking entropy.



Motivated by the information paradox, various groups studied dynamical properties of this system (scattering amplitudes, decays) and found surprising agreements between very different calculations:

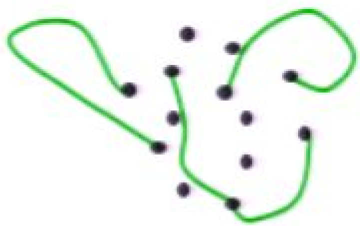


Field theory loop graph



Gravitational tree amplitude in black hole background

Maldacena (1997) explained this in terms of a new duality:



coupling

weak strong

D-branes and strings | black hole or brane

low energy limit | low energy limit

$\mathcal{N} = 4$ gauge theory ⟷ coupling (weak ⟷ strong) ⟷ IIB superstring with $AdS_5 \times S^5$ b.c.

This is another duality; whereas previously known weak-strong duality are

quantum field theory ↔ quantum field theory

string theory ↔ string theory

this is

quantum field theory ↔ string theory

That is, these are the same theory, in different variables. Like most such dualities this is not proven, but the evidence is very strong, and growing.

The deepest thing we know about the theoretical structures underlying nature, and a direct desendant of the information paradox.

With a duality, information flows in both directions:

gauge theory ⇒ gravity/string theory

gauge theory ⇐ gravity/string theory

Lessons:

⇒ Algorithmic construction of quantum gravity (in an AdS box).

⇒ We find out where the information goes.

⇐ Solution to certain strongly coupled gauge theories.

⇐ A tool for understanding heavy ion collisions.

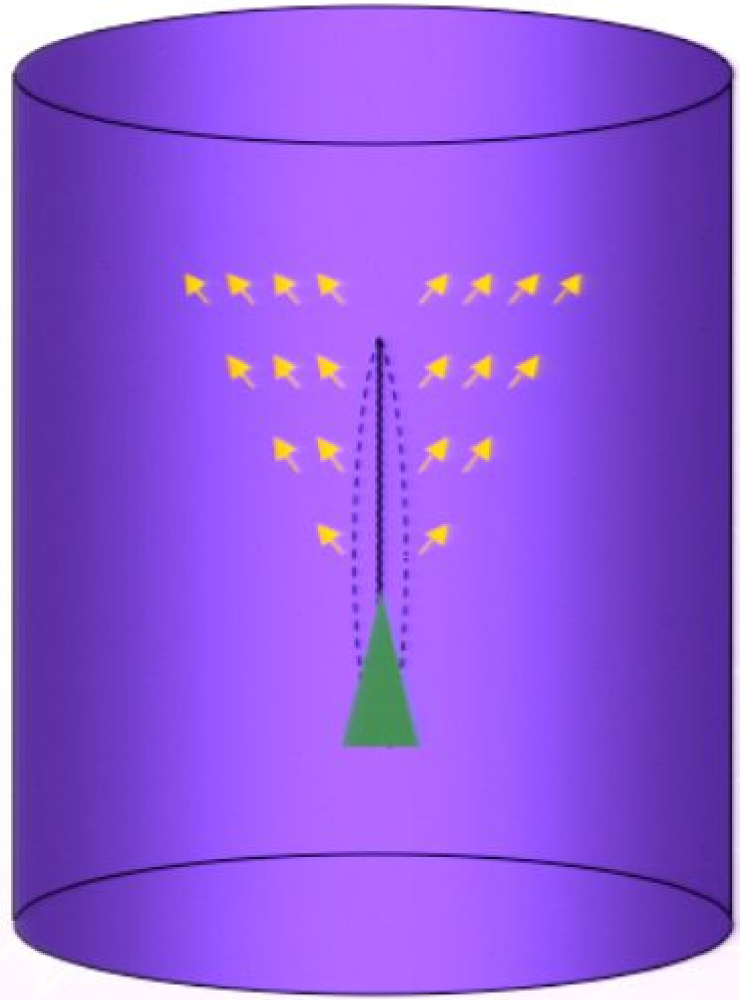
This duality provides an *algorithmic* nonperturbative constuction of string theory with AdS boundary conditions: we could simulate the dual gauge theory on a (large enough) computer. (cf. Ken Wilson and QFT).

Almost background independent:
the asymptotic geometry is fixed, but in the interior the geometry and even topology can fluctuate freely, and pass through nongeometric Planckian states.

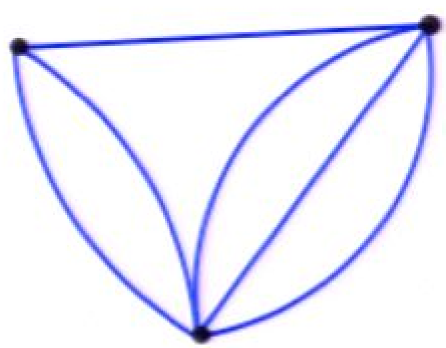
The interior spacetime, and its coordinate group are *emergent*. The coordinate invariance acts trivially on the gauge theory variables.

We can repeat our thought experiment in an AdS box. The dual description is in an ordinary coherent system: information is preserved (option 1). A black hole is dual to a plasma of hot glue, not so different from a lump of coal.

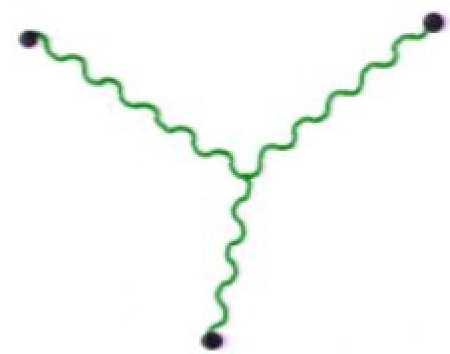
The gauge theory variables are indeed strongly nonlocal, and holographic (the gauge theory lives on the boundary).



AdS/CFT duality allows us to calculate analytically in certain strongly coupled gauge theories: we can do the calculations that we would like to do for QCD (hadron spectra, scattering amplitudes).



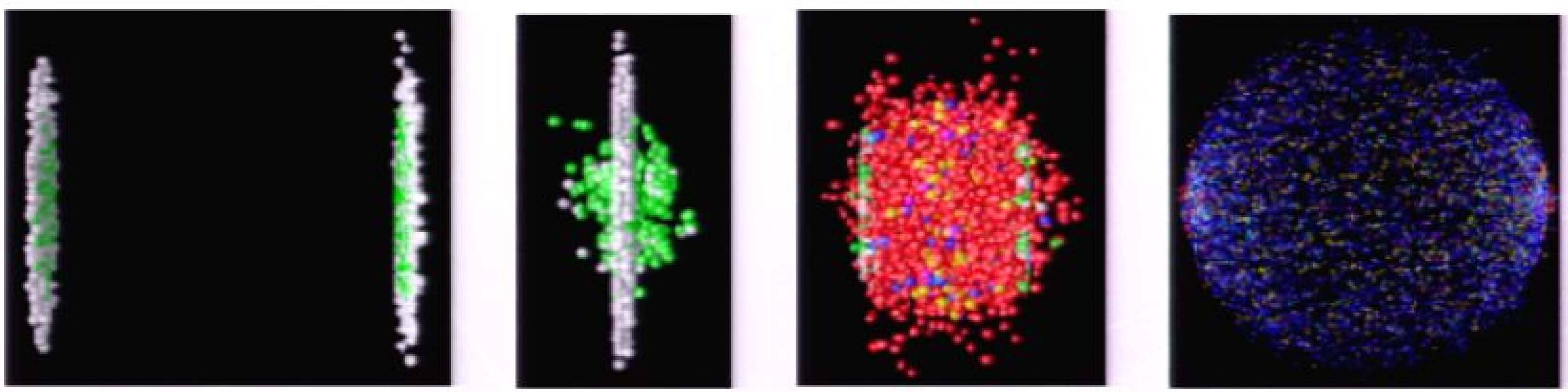
Field theory loop graph (one of *many*).



Gravitational tree amplitude in black hole background

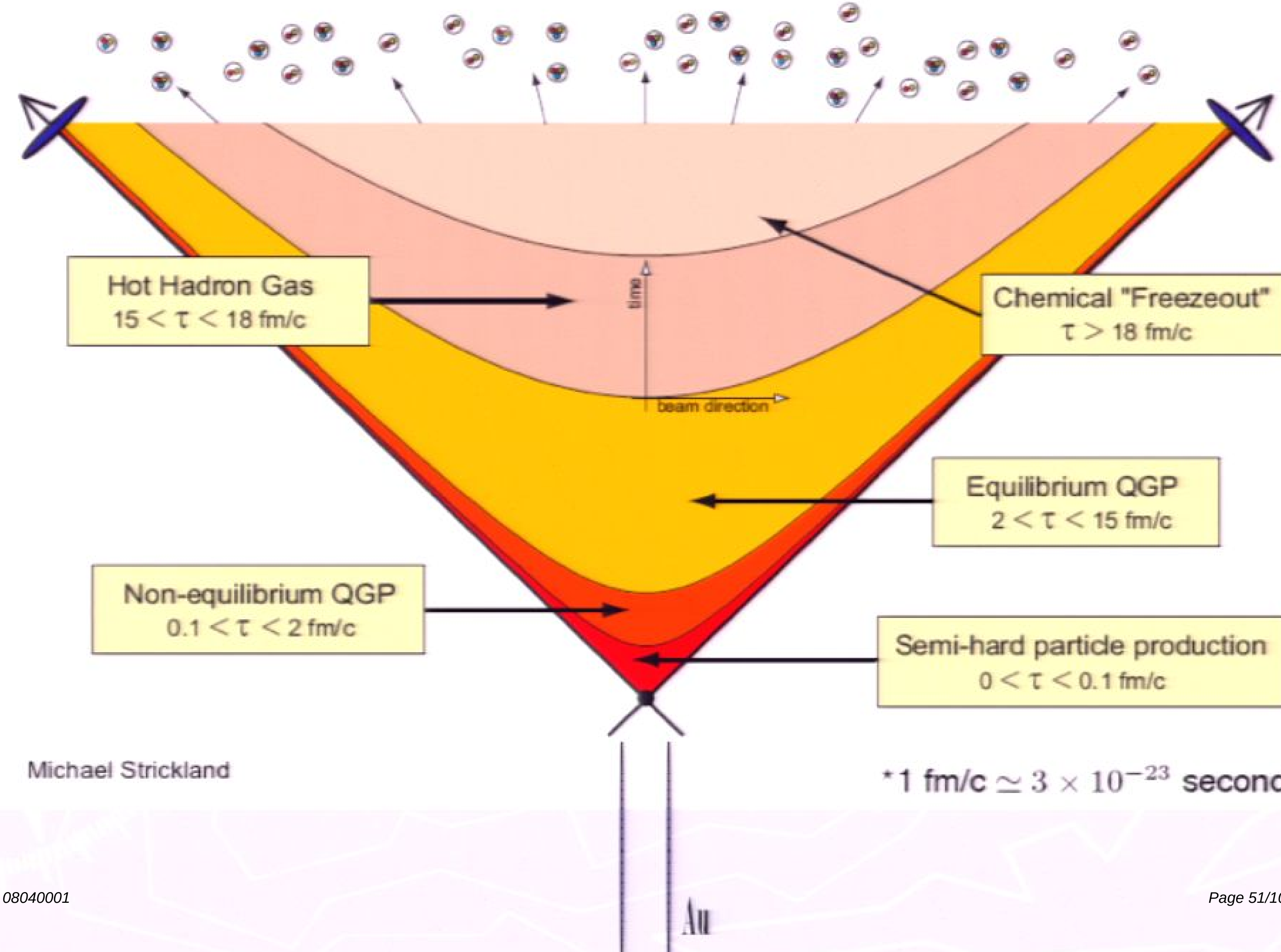
The original duality was for a gauge theory with a large amount of symmetry (super- and conformal), but one can progress towards systems of lower symmetry. Still can't do real QCD, but can do some approximate calculations.

At RHIC (Brookhaven) and in the future at the LHC, heavy nuclei are collided with the goal of making a quark-gluon plasma:



PHENIX

Heavy-ion collision timescales and "epochs" @ LHC



Michael Strickland

*1 fm/c $\simeq 3 \times 10^{-23}$ seconds

Insights from black hole dual:

- Shear viscosity
- Entropy density
- Is the plasma weakly or strongly coupled? Black hole ~ `harmonic oscillator' for strongly coupled nonequilibrium CFT.
- ...
- Recent: improved understanding of relativistic hydrodynamics.
- There may also be condensed matter applications (Subir et al.)

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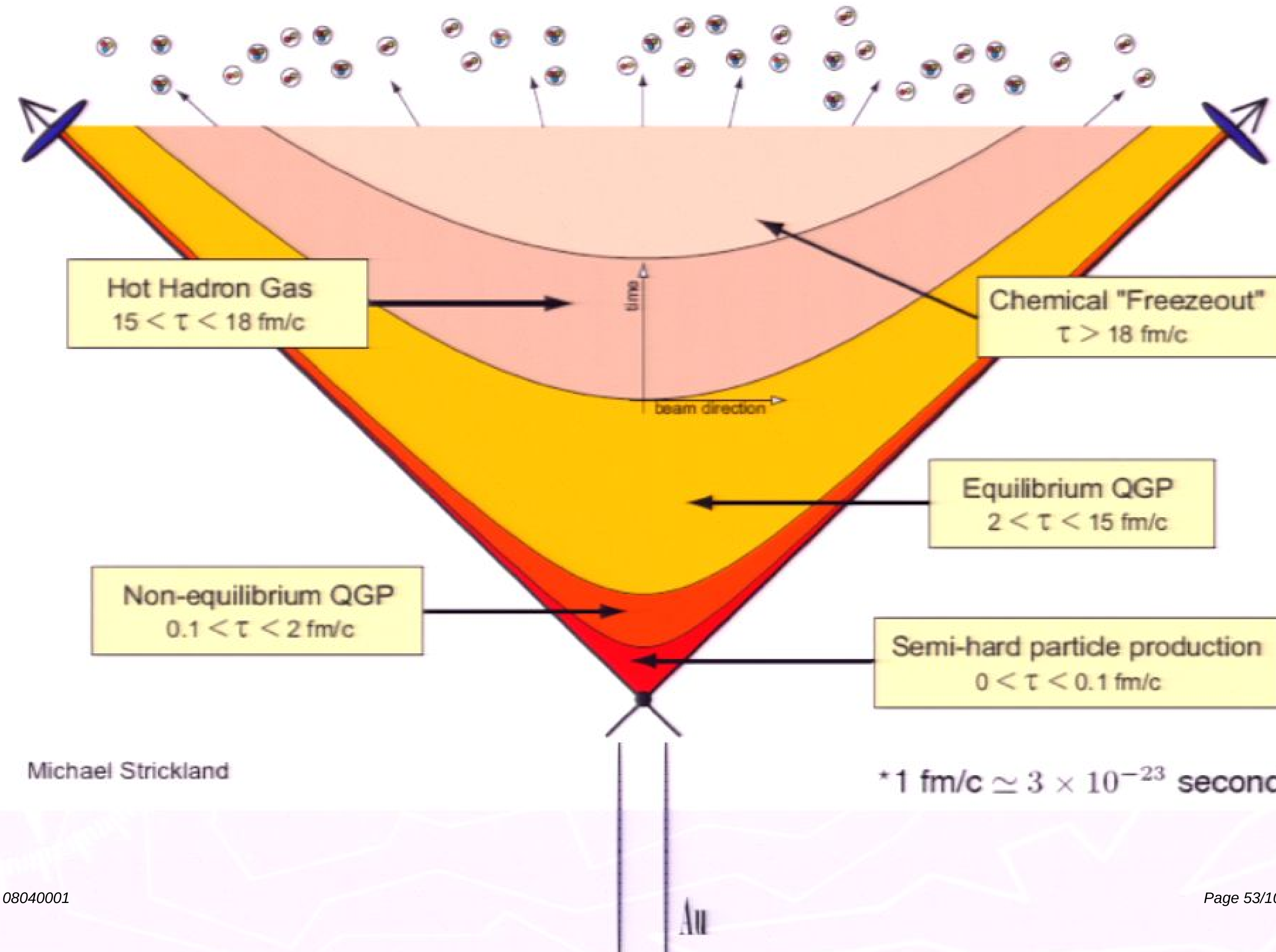

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Yes and no: duality erases distinctions that we make with our classical experience - between particle and wave, and black hole and plasma - because a single quantum theory has many classical limits.

The future…

The problems of quantum gravity:

- Spacetime foam (nonrenormalizability).
- The cosmological constant.
- Black hole entropy.
- Black hole information.
- Cosmological singularities (the Big Bang); initial conditions.
- The interpretation of quantum mechanics, applied to the whole universe.

The last two are especially important for understanding which of the many vacua of string theory we are in.

How to proceed? E.g., how to extend AdS/CFT to cosmological situations? Is there more to learn from Hawking's paradox?

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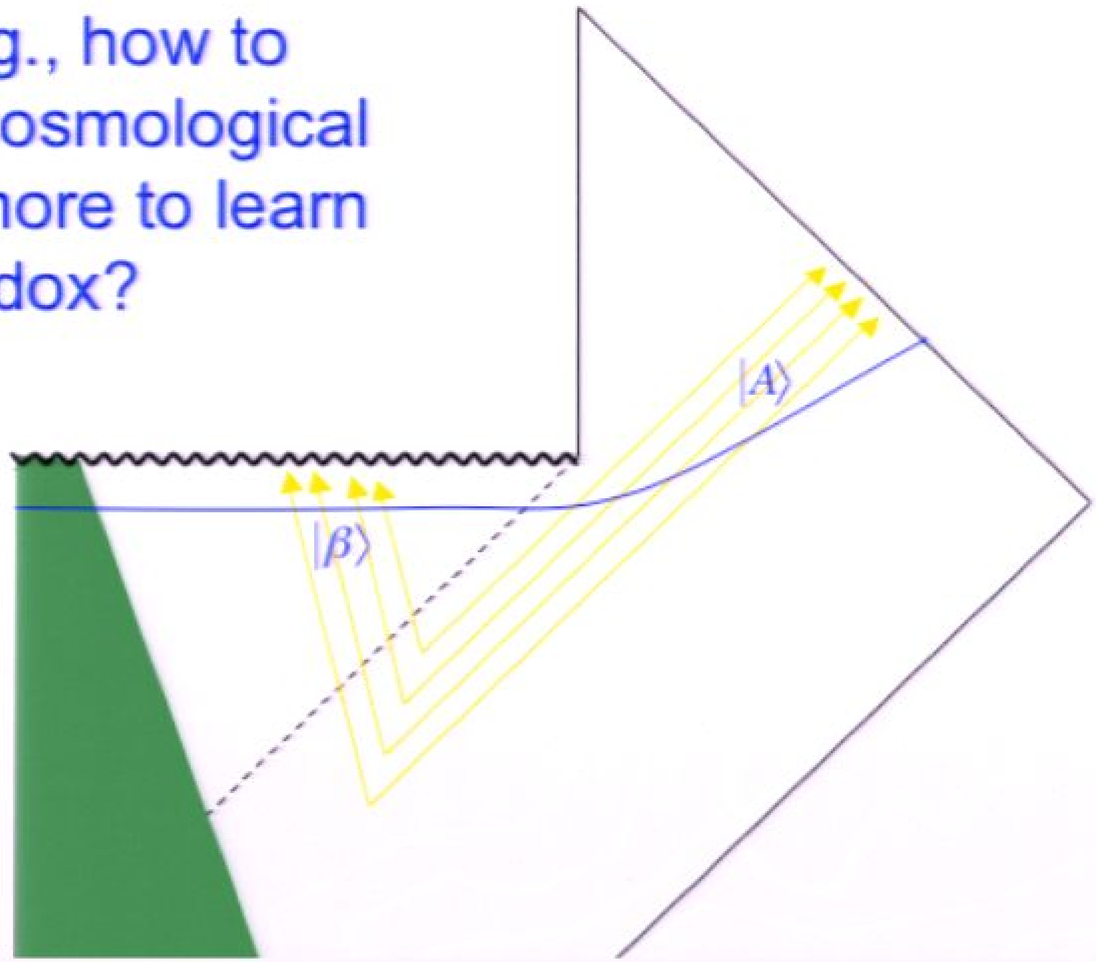

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How exactly does locality emerge, and how does it break down?

If the black hole has an S-matrix, how do we calculate it?

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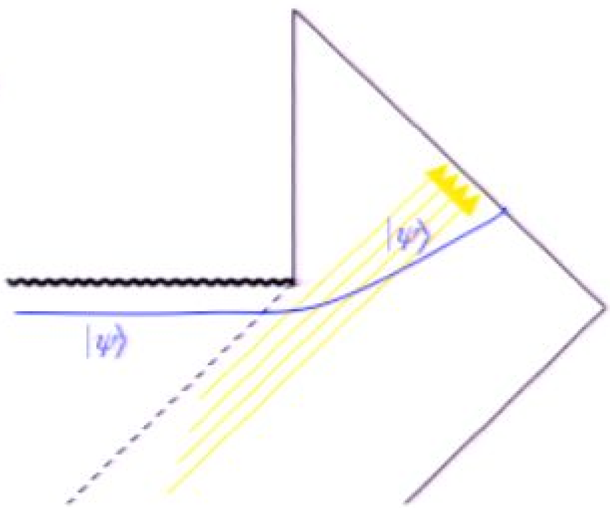

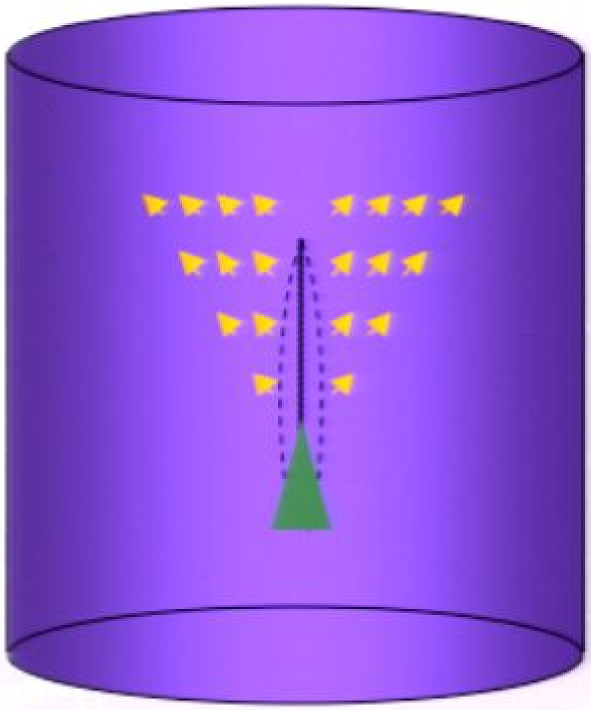

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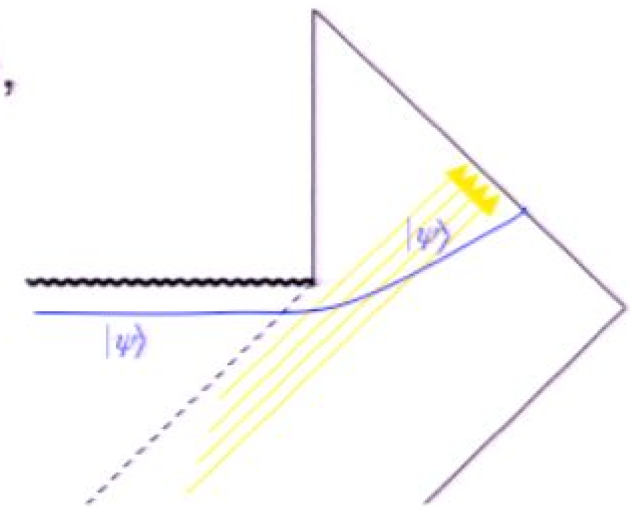

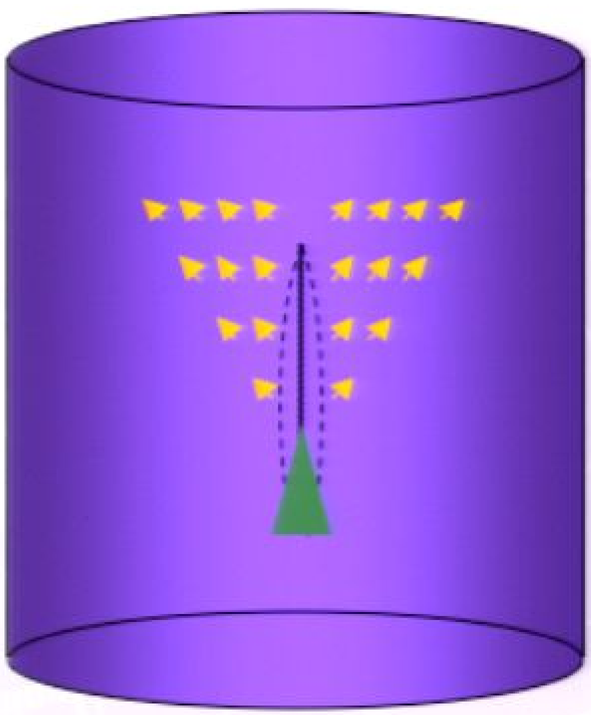

initial bulk state → initial CFT state
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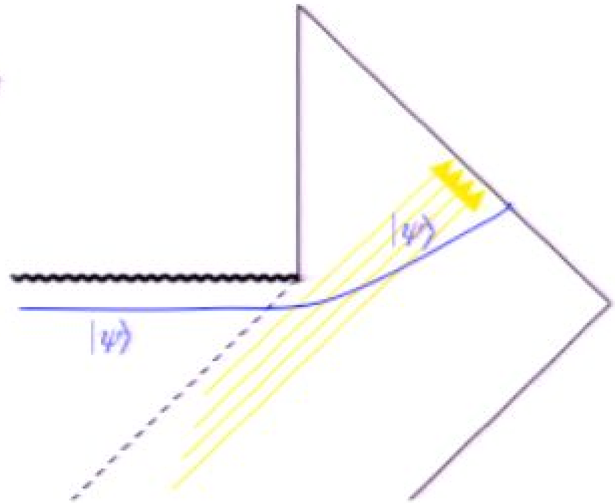

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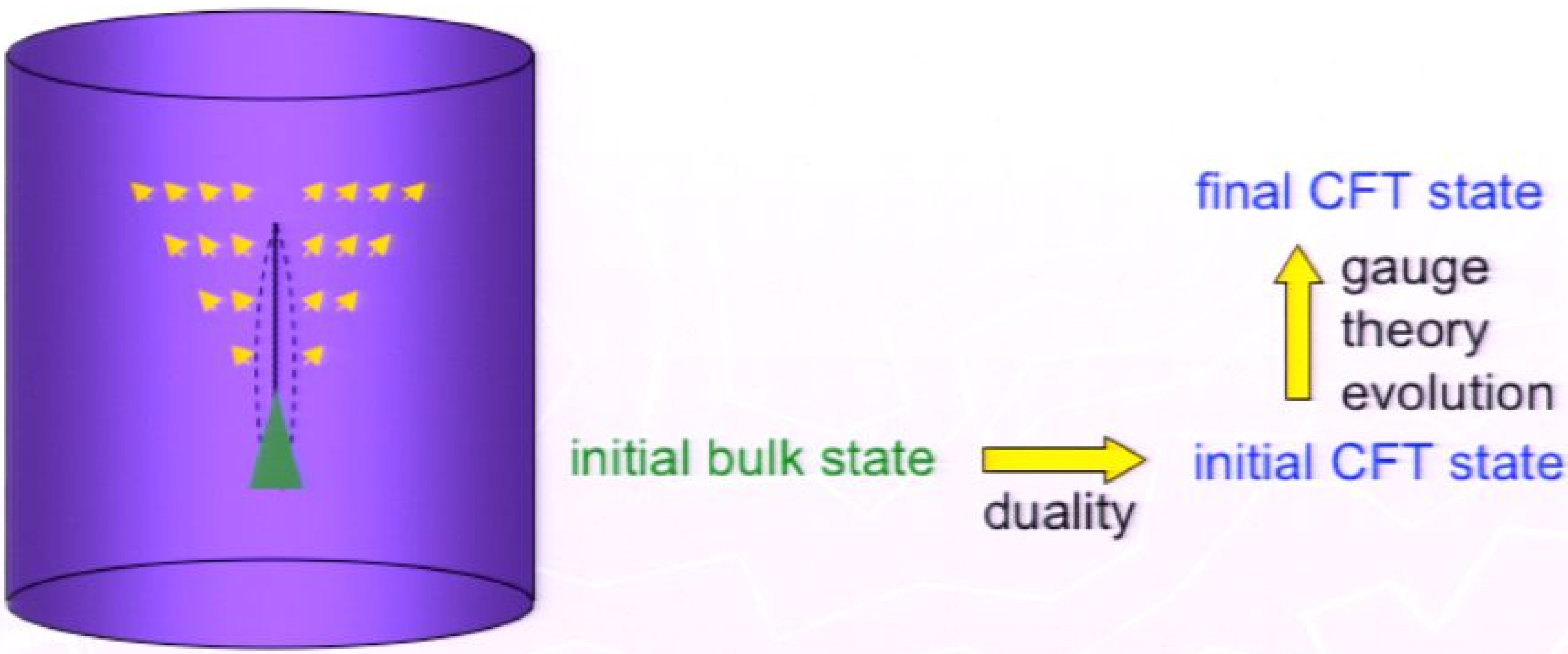

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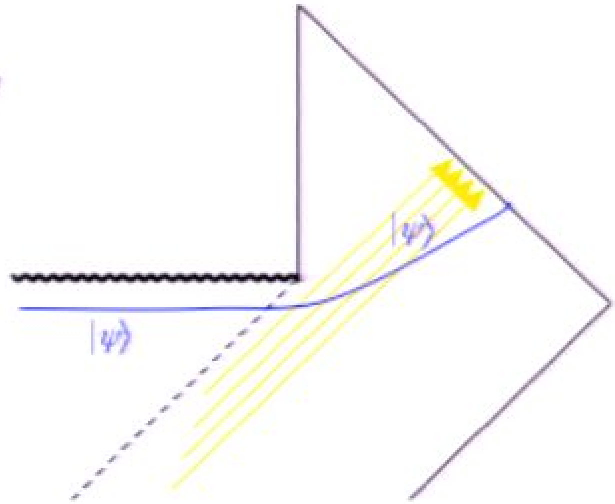

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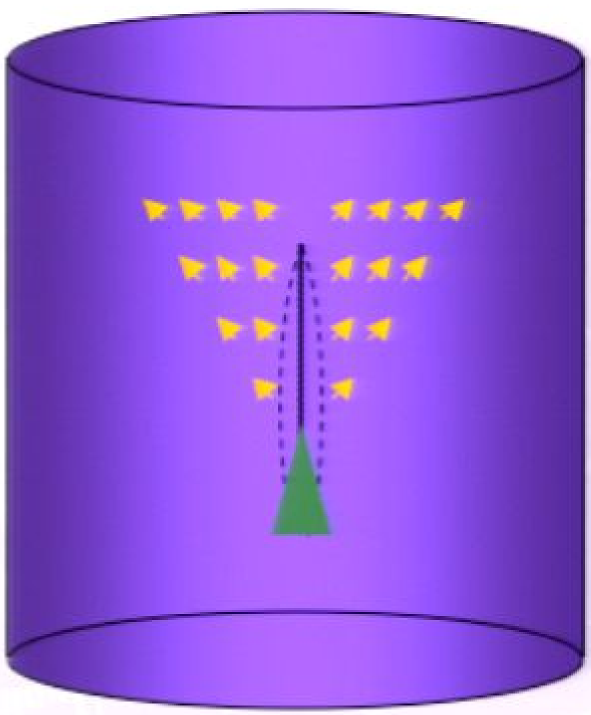

initial bulk state → duality → initial CFT state

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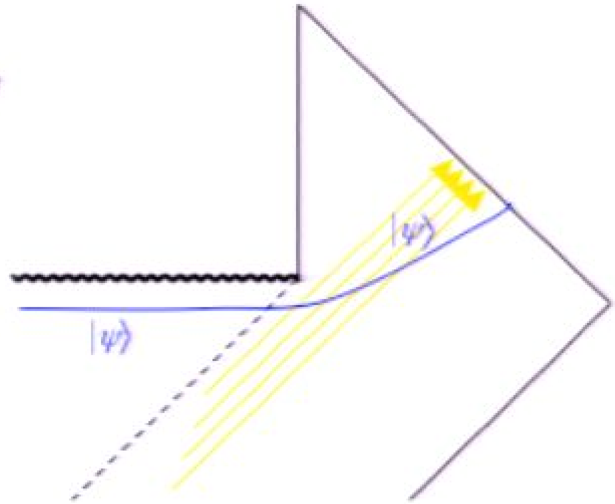

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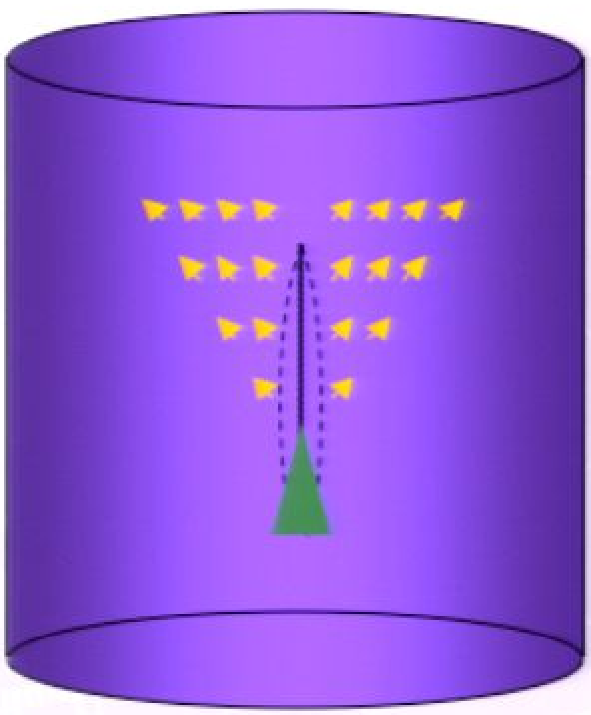

final bulk state ⟸ duality ⟸ final CFT state

gauge theory evolution ⇑

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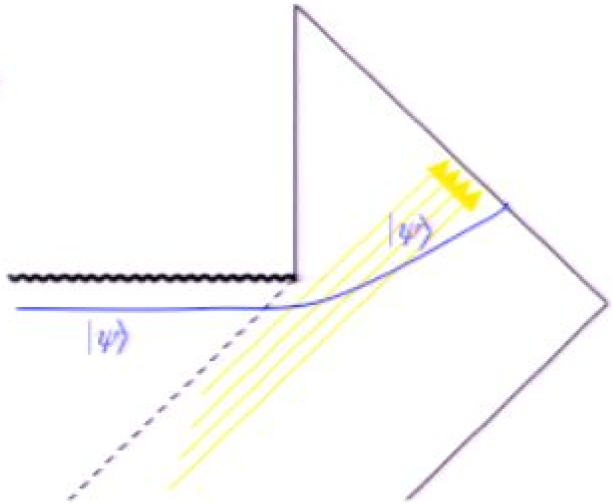

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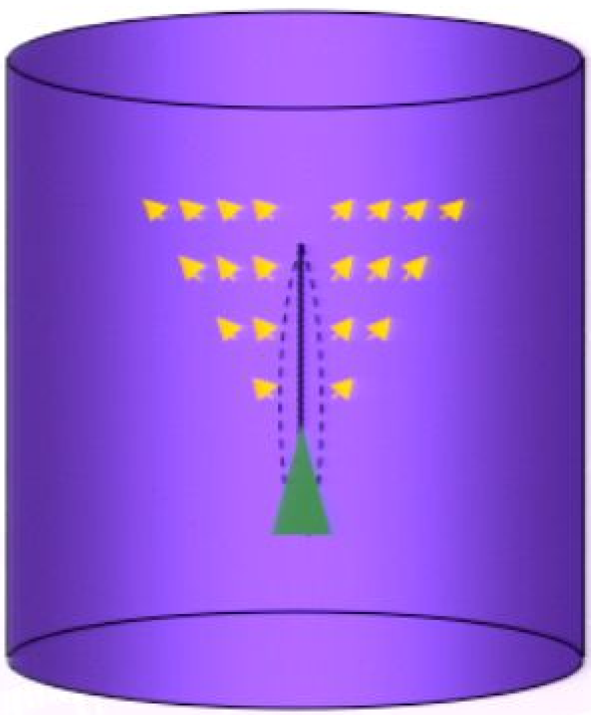

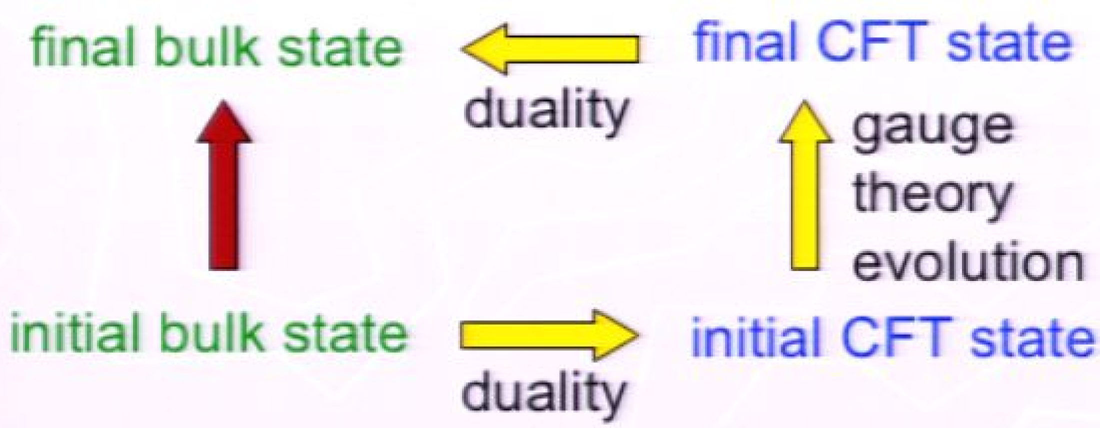

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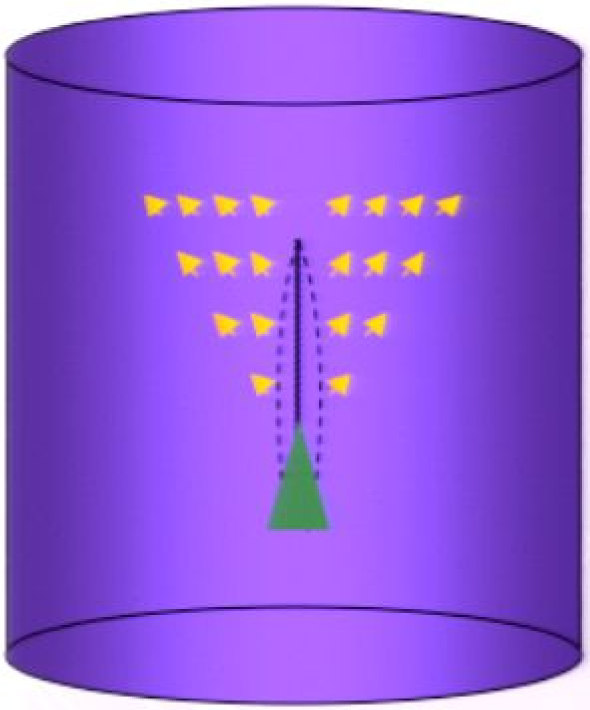

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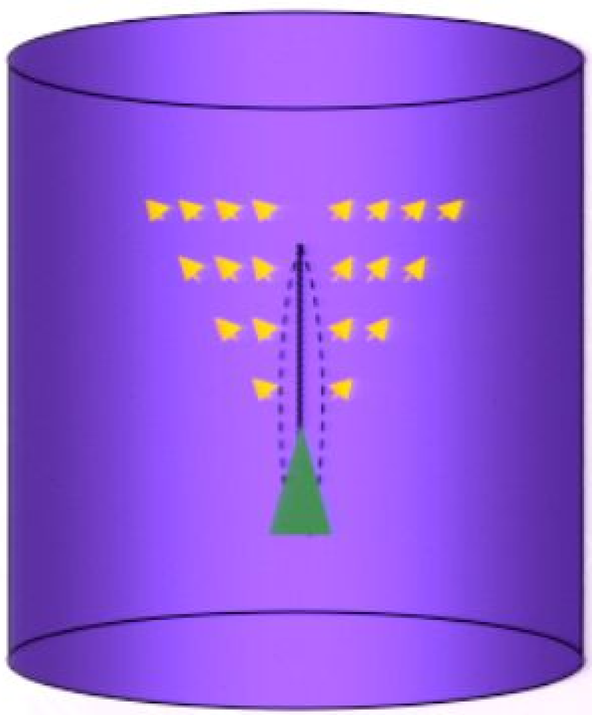

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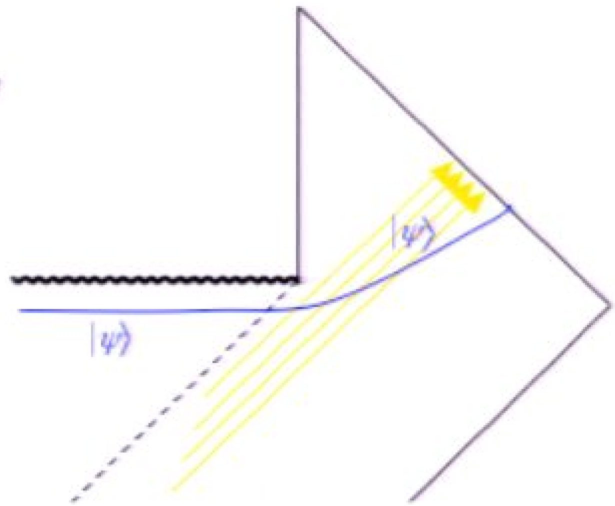

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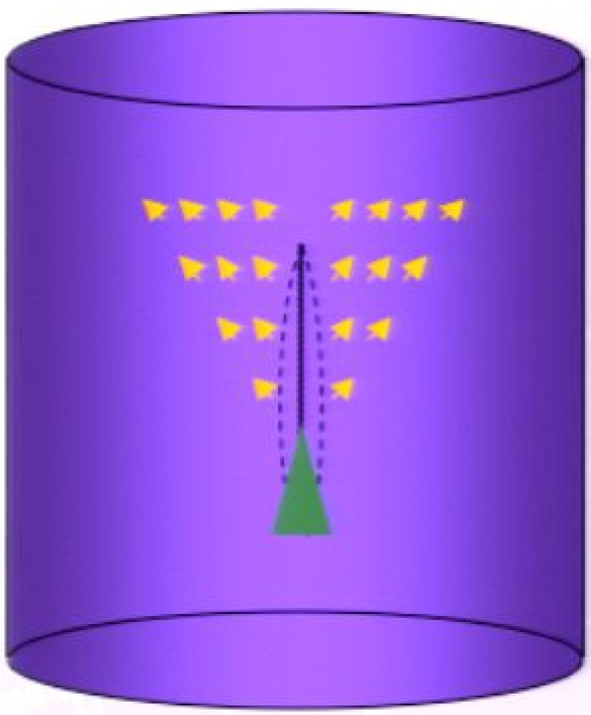

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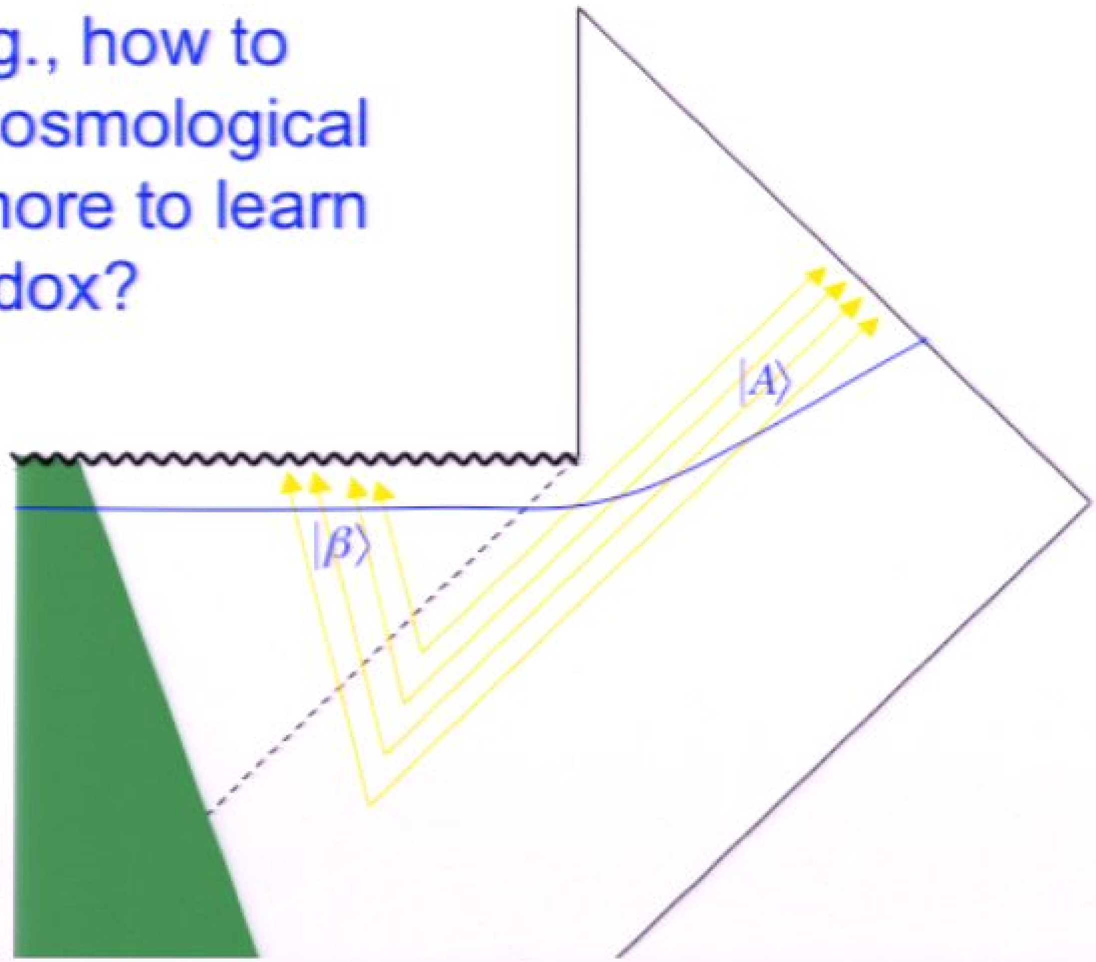

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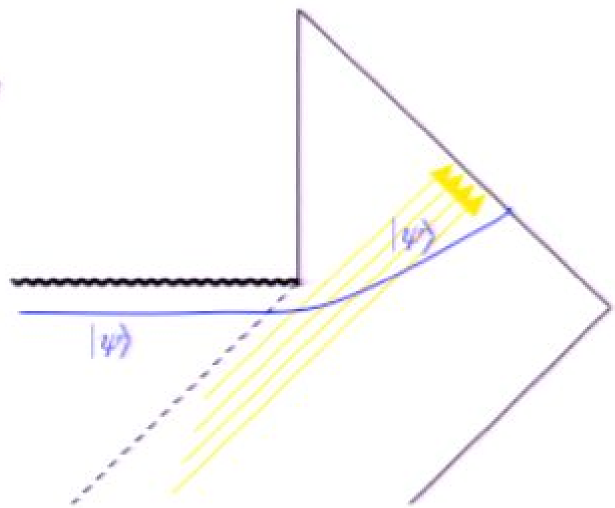

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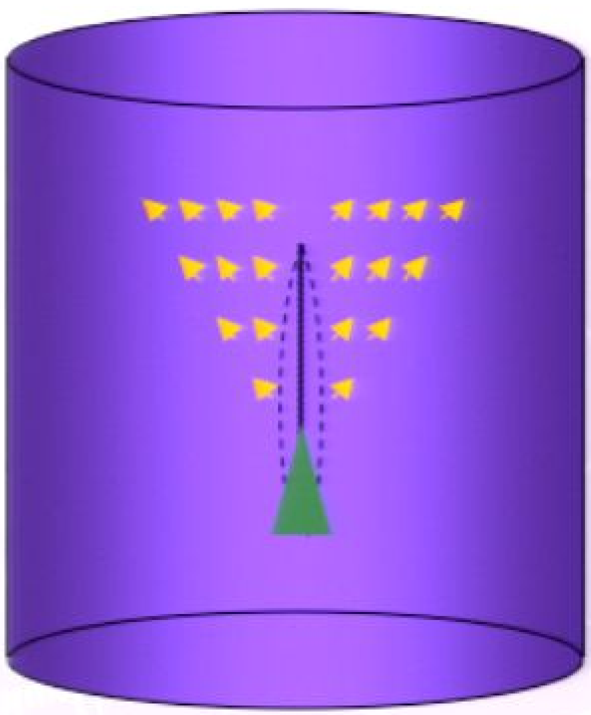

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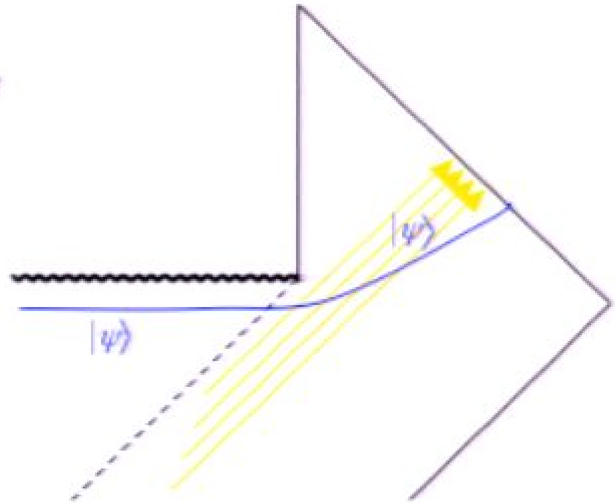

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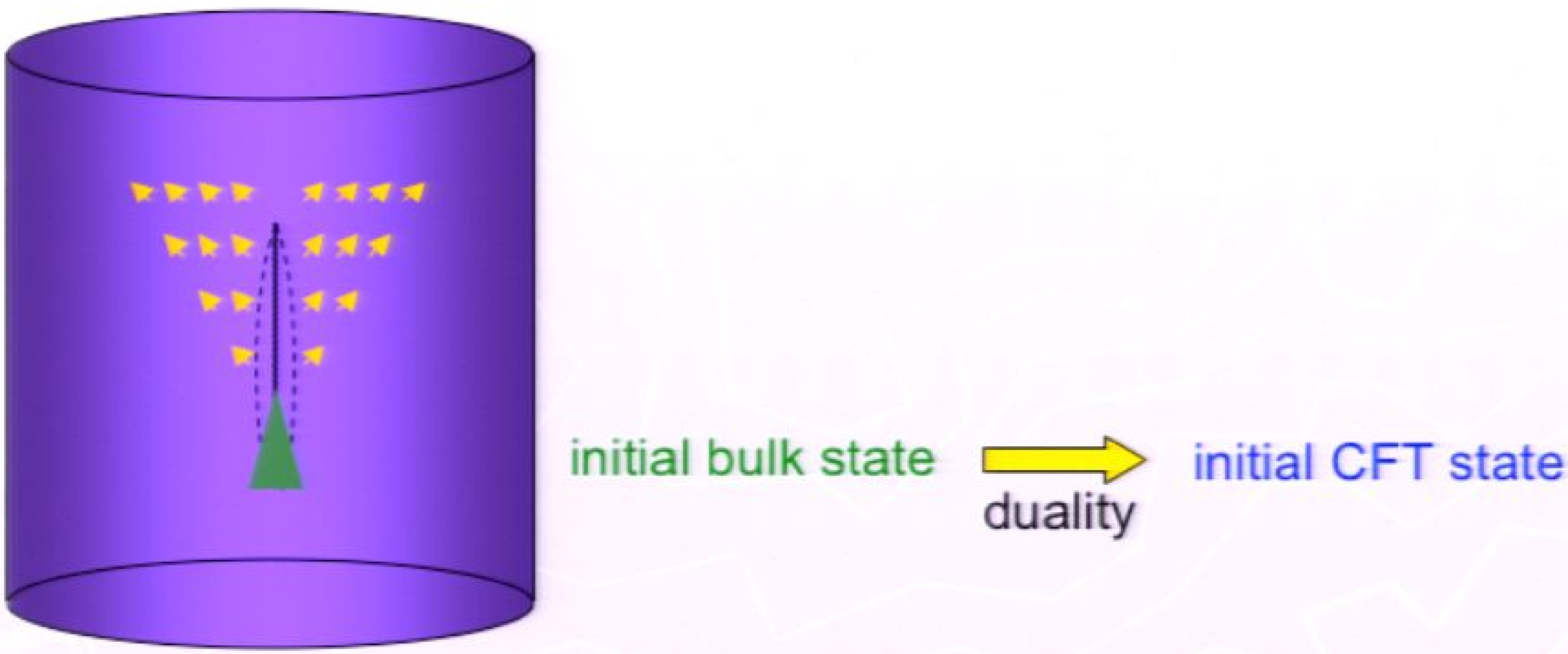

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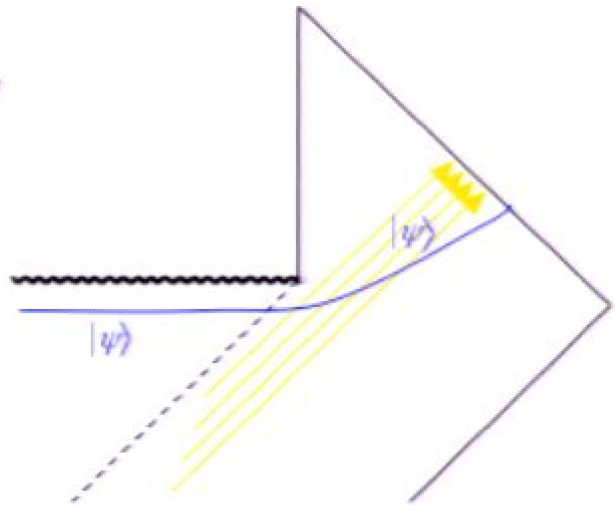

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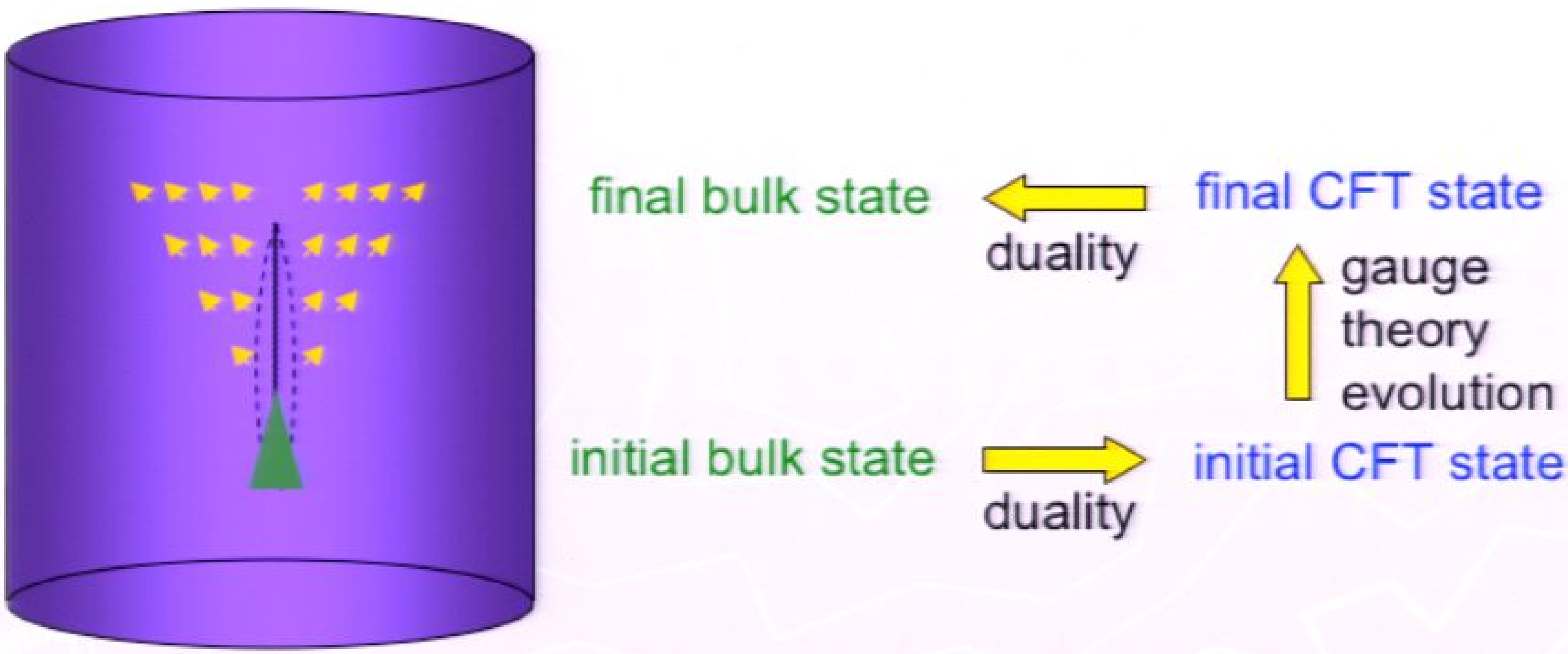

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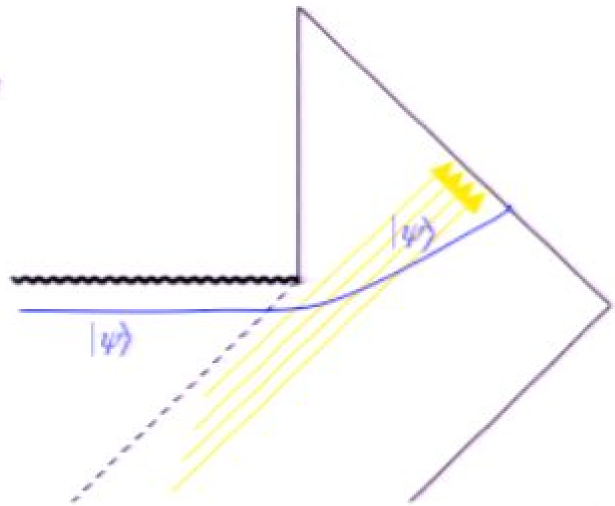

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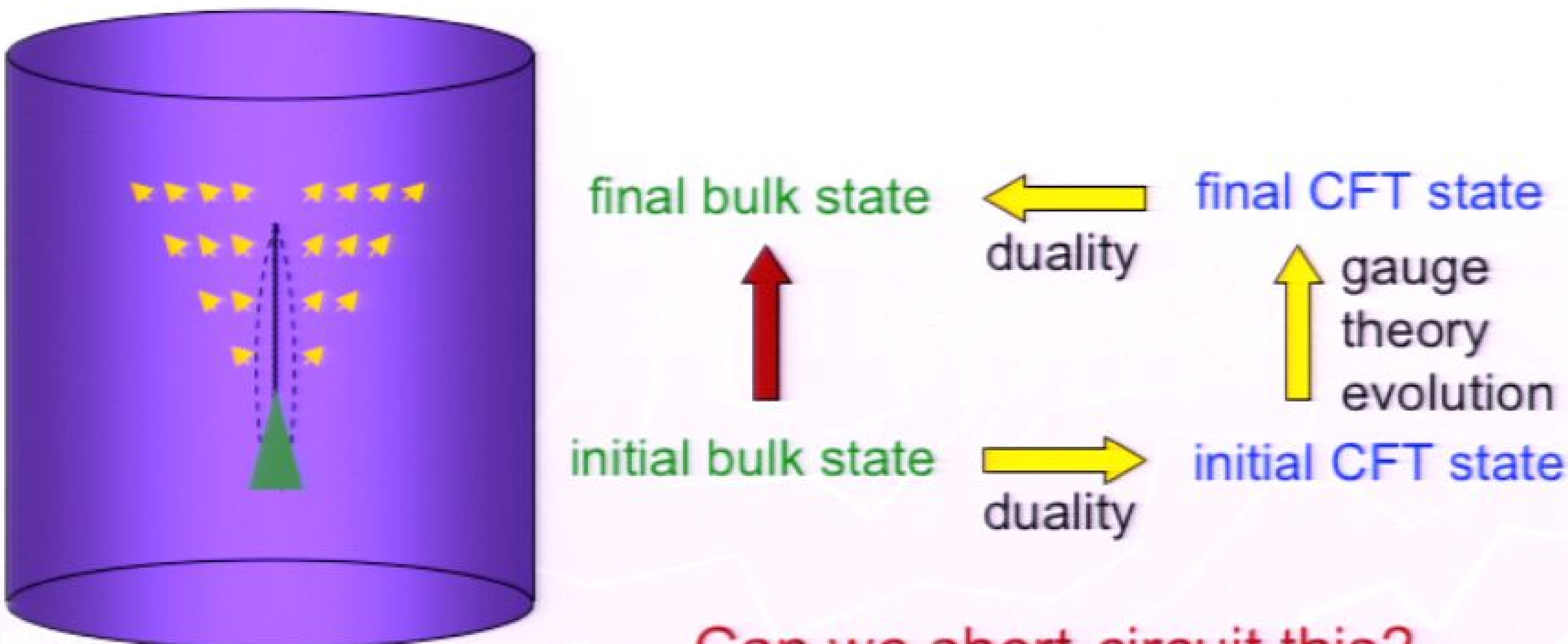

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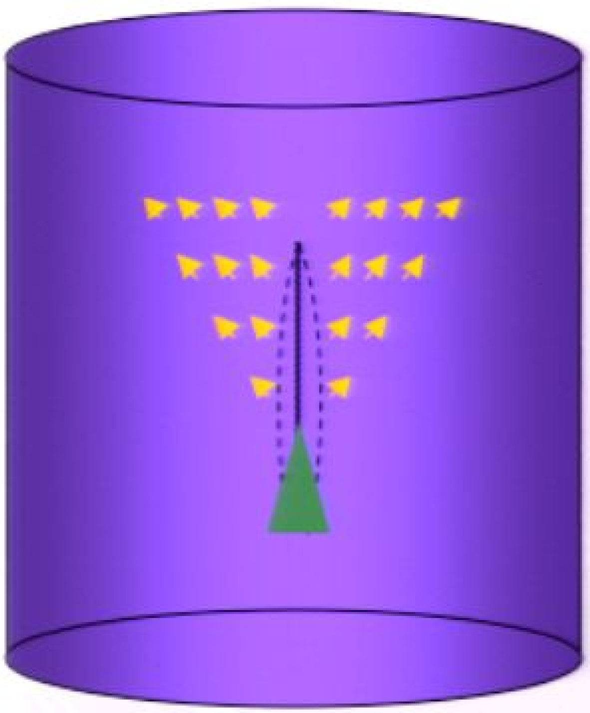

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A simple toy model (Iizuka & JP, following Festuccia & Liu):

The AdS/CFT gauge theory is an infinite number of coupled modes, which are $N \times N$ matrices in color space. One can look for reduced models which retain key features of the paradox. Example:

N^2+N harmonic oscillators X_{ij}, Y_i:

$$V = m^2 X_{ij} X_{ji} + M^2 Y_i^\dagger Y_i + g Y_i^\dagger X_{ij} Y_j$$

Think of X_{ij} as a heat bath coupled to Y_i. But also, X_{ij} ~ black hole, Y_i ~ probe.

This model captures Maldacena's version of the information problem: AdS/CFT is gravity in a box, and like any finite quantum system should have a discrete spectrum (i.e. Bekenstein-Hawking entropy is finite. Quantization of gravity as an effective field theory gives a continuous spectrum in the presence of a horizon.

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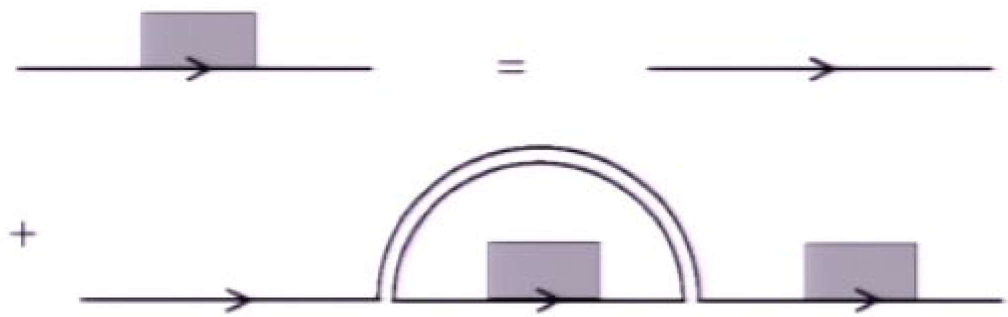
Graphical structure the same as 't Hooft's 2-D QCD:



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Close contour, get

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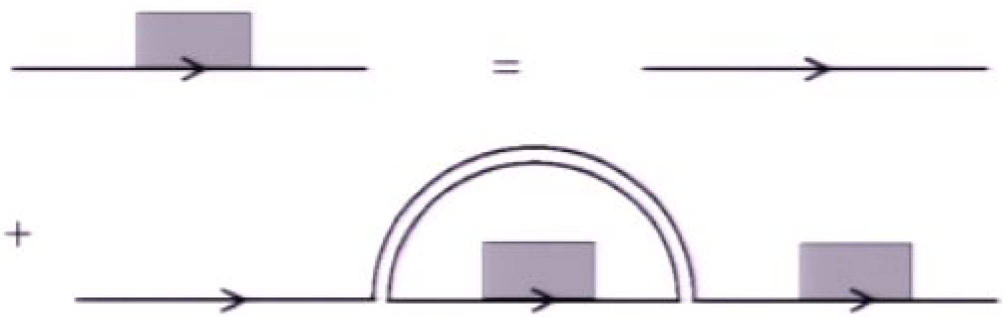

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At finite temperature we don't need full Schwinger-Keldysh, because ensemble is free. Get same Schwinger-Dyson equation with thermal propagator

$$\tilde{K}_0(T,\omega) = \frac{i}{1-e^{-m/T}}\left(\frac{1}{\omega^2 - m^2 + i\epsilon} - \frac{e^{-m/T}}{\omega^2 - m^2 - i\epsilon}\right)$$

Slightly more complicated result:

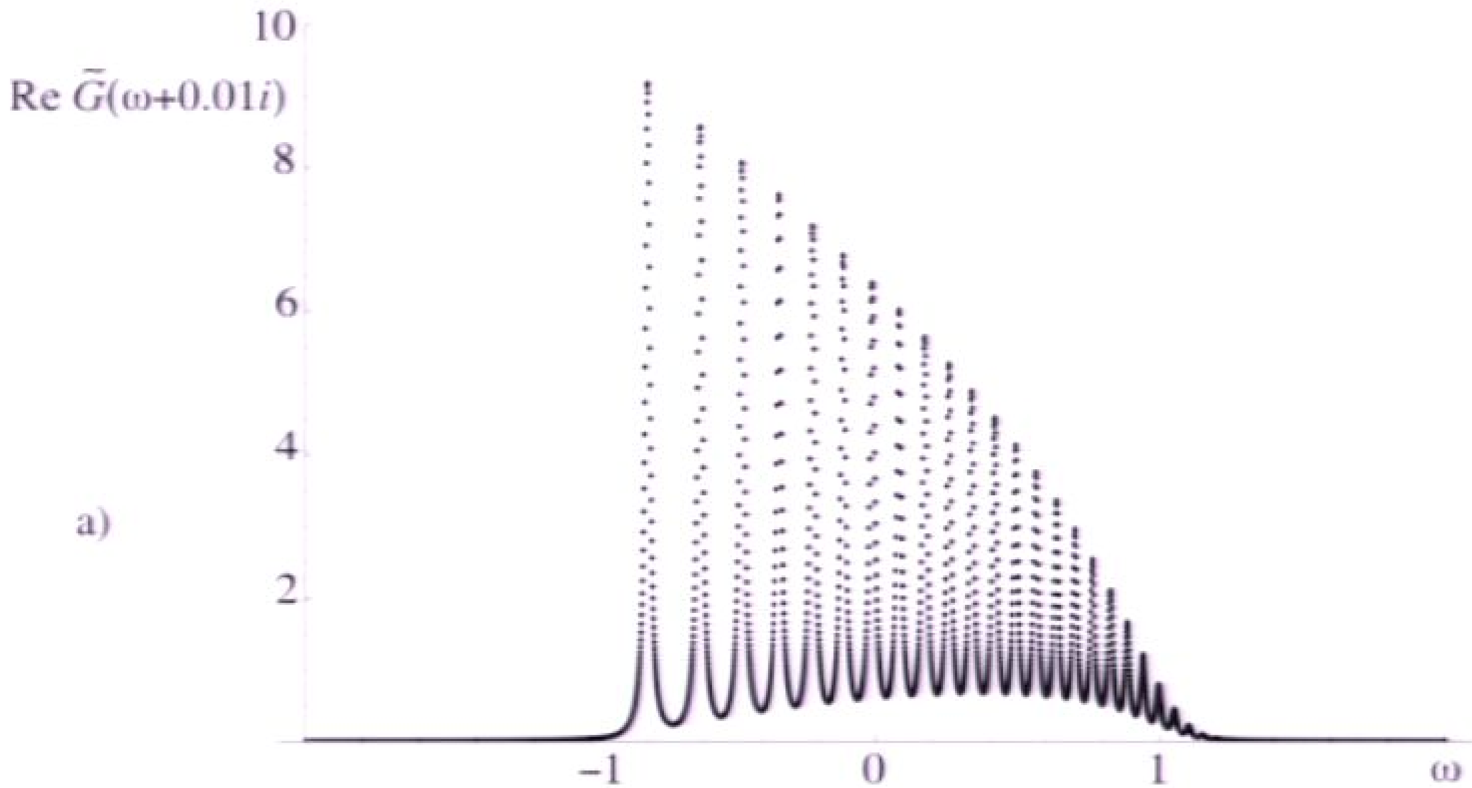
$$\tilde{G}(T,\omega) = \frac{i}{\omega}\left\{1 - \frac{\nu^2}{4(1-e^{-m/T})}\tilde{G}(T,\omega)\left[\tilde{G}(T,\omega - m) + e^{-m/T}\tilde{G}(T,\omega + m)\right]\right\}$$

$T = 0$: $\tilde{G}(\omega) = \frac{i}{\omega}\left(1 - \frac{\lambda}{2m}\tilde{G}(\omega)\tilde{G}(\omega - m)\right)$

One strategy: recurse from $\tilde{G}(T,\omega) \sim \frac{i}{\omega}$ at large $|\omega|$.

This works if recursion is stable, which is the case only for $T = 0$.

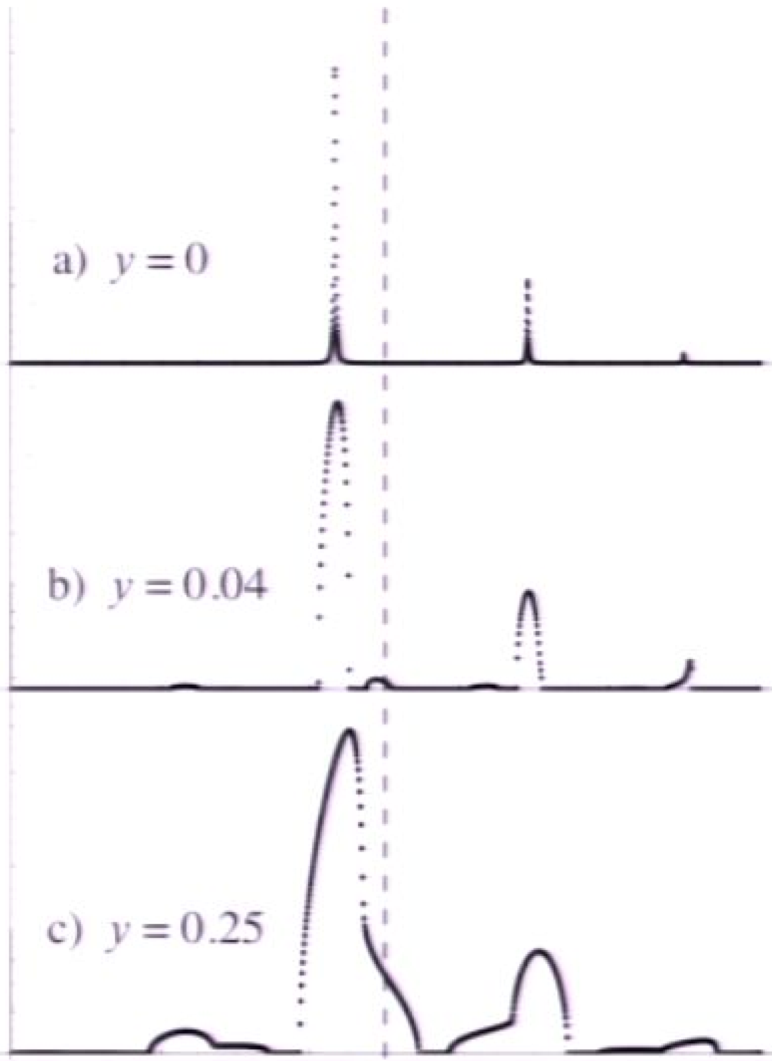
Zero temperature - poles on real axis:

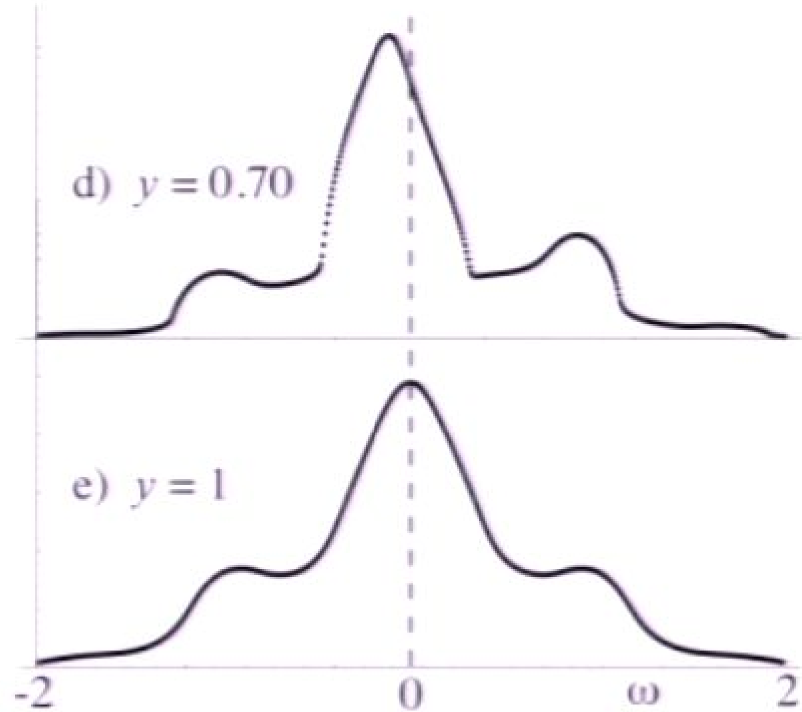


Finite temperature - poles widen into cuts, which then merge:

$$y = e^{-m/T}$$

coupling smaller than previous slide





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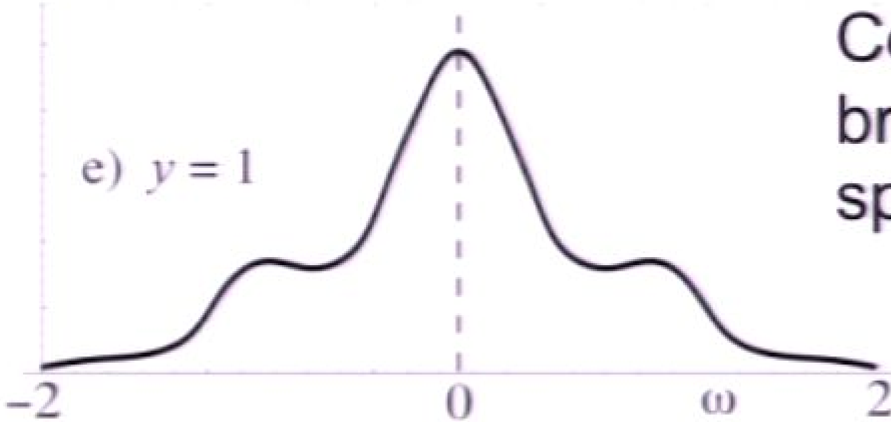
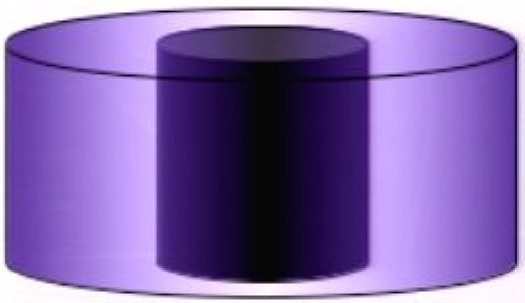


Continuous spectral weight breaks up into poles with spacing of order $\exp\{-O(N^2)\}$.

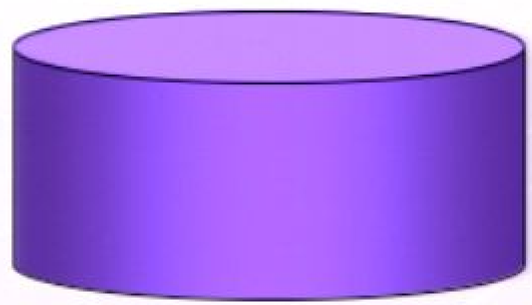
How do we see this in the $1/N$ expansion?

Conjecture of Maldacena (Hawking): additional Euclidean saddle, weight $\exp\{-O(N^2)\}$.

AdS black hole



thermal AdS



Problem (Barbon and Rabinovici): $\exp\{-O(N^2)\}$ do not have necessary secular growth.

Another conjecture…

Q: This model is so simple, what can remain?

A: The *stringy exclusion principle.* For $N \times N$ matrices, $\mathrm{Tr}(X^k)$ is not independent for $k > N$. This implies that the string Hilbert space is smaller than the naïve Fock space.

Conjecture: this is the same reduction as required by that required by black hole complementarity.

In conclusion…

Don Page (1993) showed that if the information is emitted with the Hawking radiation, there is essentially none in the first half of the radiation, and then it gradually* begins to emerge.

This is a good metaphor for the information problem itself. Hawking found the problem in 1976, and for a while much effort went in and nothing came out. The first information emerged in 1993 (complementarity, holography), and more since, but I don't think that the problem has decayed completely…

*recent interesting paper by Hayden & Preskill: information return time can be as short as $M \ln M$ (versus M^3 lifetime of black hole).

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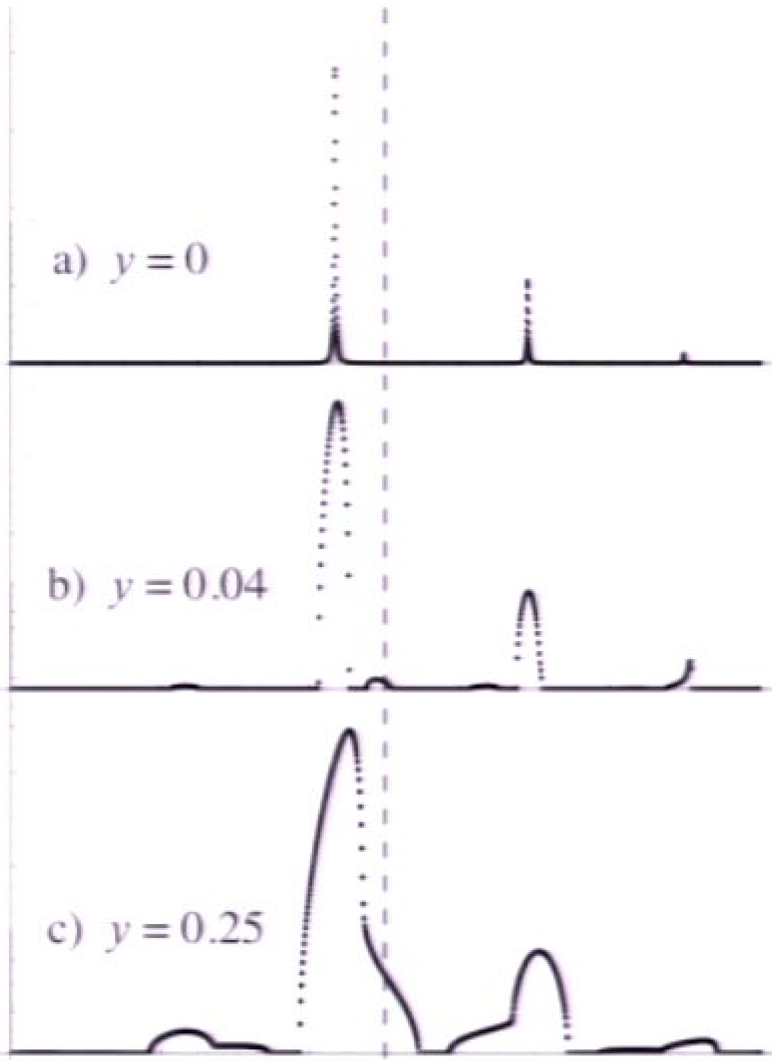

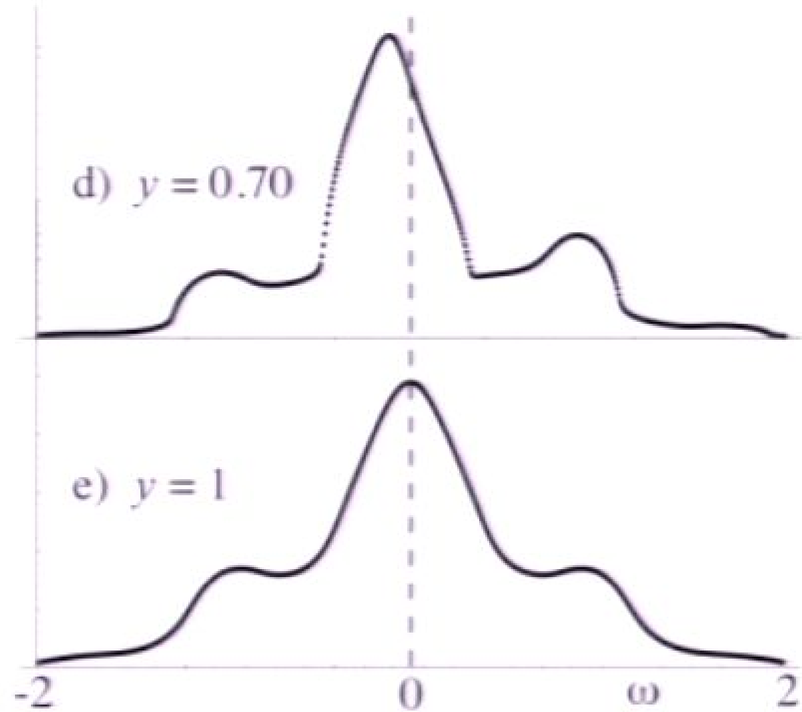

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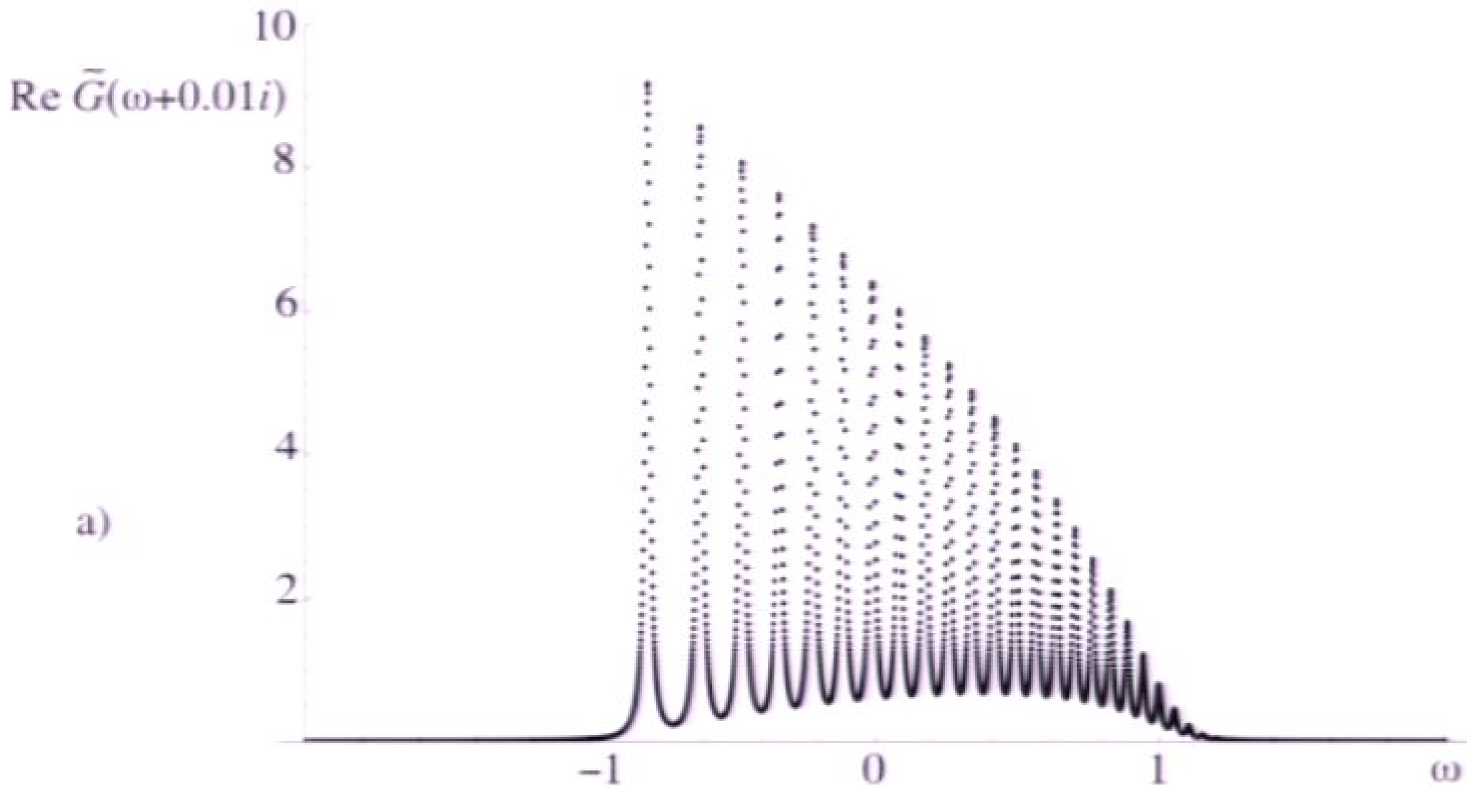

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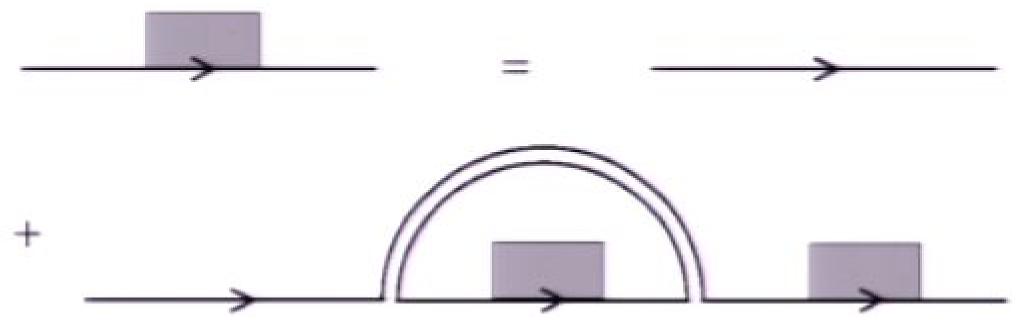

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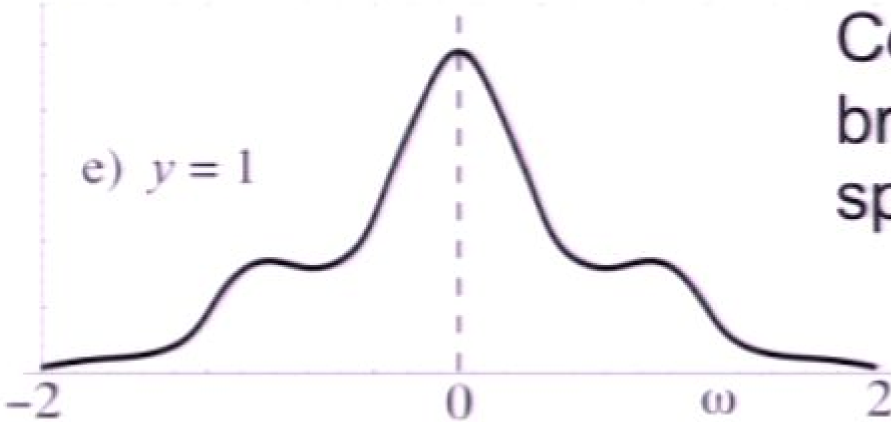

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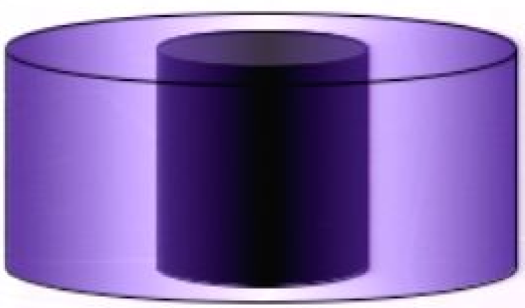

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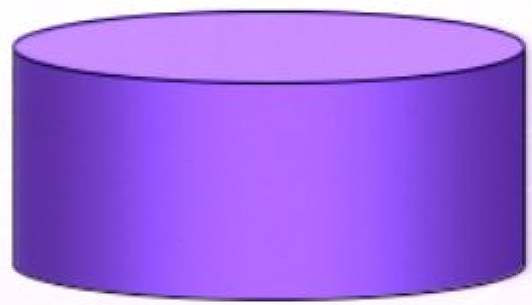

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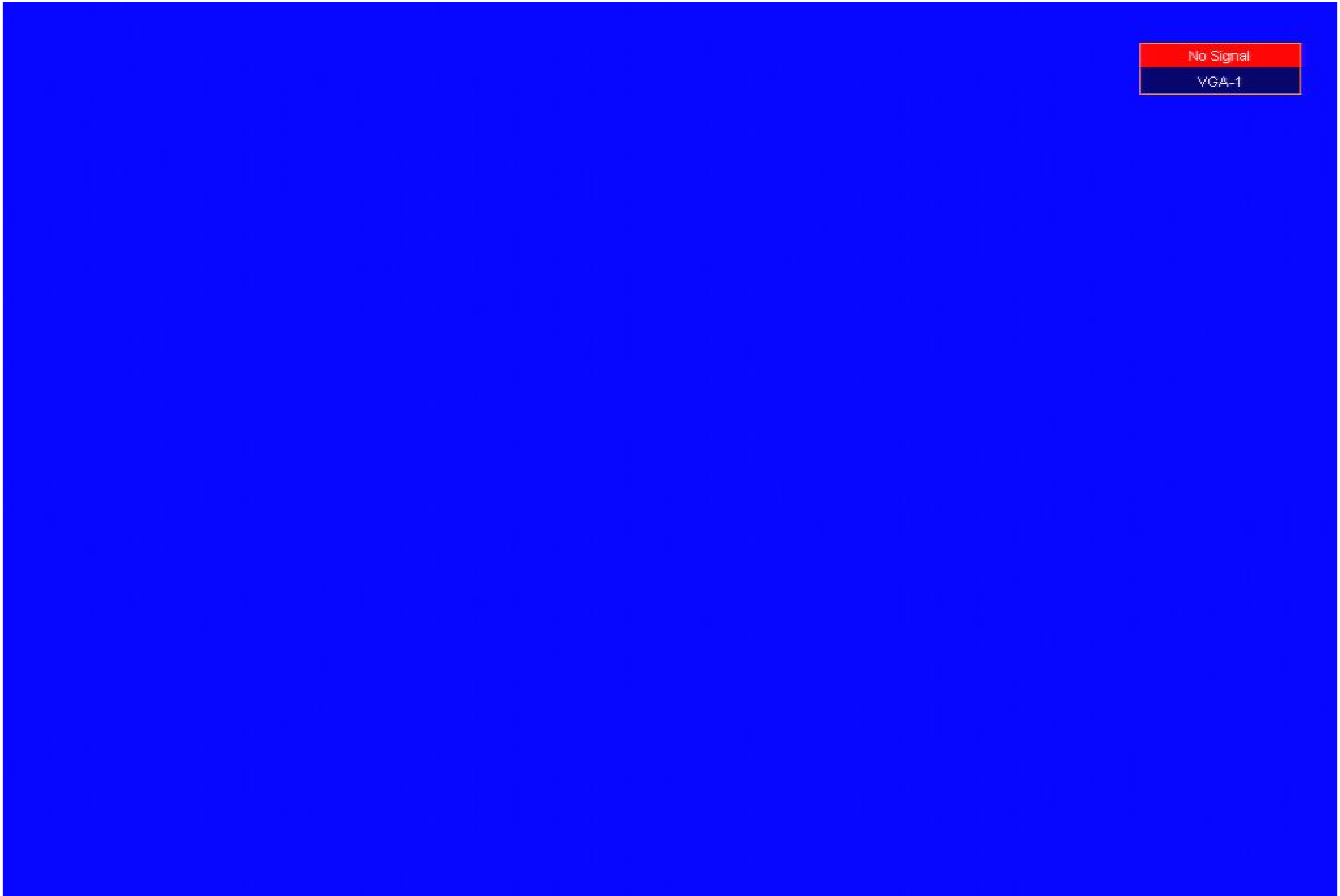
Finally, and of greatest interest to astronomy, if it is only anthropic constraints that keep the effective cosmological constant within empirical limits, then this constant should be rather large, large enough to show up before long in astronomical observations.

From Weinberg (1989).

No Signal

VGA-1

No Signal
VGA-1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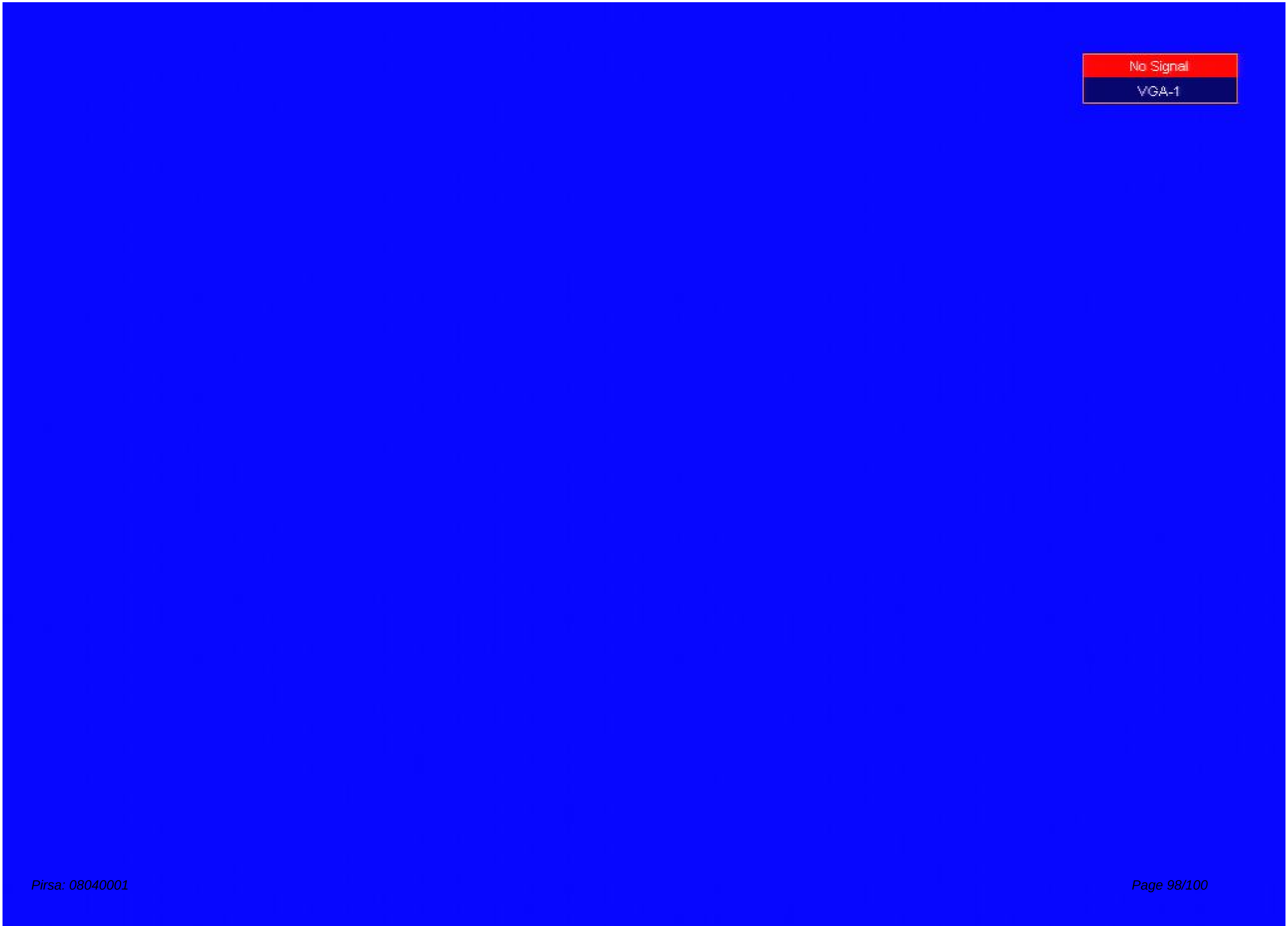
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