Title: Surface operators and the new holographic duality

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Abstract:

Plan

1. Review of Line and Surface Operators
   - Wilson and 't Hooft Operators
   - Gukov-Witten surface operators (of disorder type) in $\mathcal{N} = 4$ Yang-Mills theory
   - Line and Surface Operators and D-branes
2. Surface operators of order type and $D3/D7$-system
   - $D3/D7$-system
   - Construction of the field theory
   - Surface operators
3. The supergravity description
   - Probe approximation
   - Supergravity solution

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Outcome

- Holographic duality beyond field theory in flat space
- For $D3/D7$ system we have solved the problem of $D7$-brane backreaction (terms of order $\lambda \frac{M}{N}$)

Line and Surface Operators

Phase structure of gauge theories

- confining
- deconfining
- Coulomb
- Higgs
- free electric
- free magnetic

A possible way to characterize is to insert an infinitely heavy probe. For example, a heavy particle (electrically or magnetically charged) $\Longrightarrow$ The concept of **line operator**

Wilson line operator

$\mathcal{W}_{\mathcal{R}} = \mathrm{Tr}_{\mathcal{R}} e^{-q\int_L A_\mu \dot{x}^\mu ds}$, L is the worldline of a particle, $\mathcal{R}$ is the representation.

Labeled by representations

This is an example of an operator of **order type**

Constructed out of the fields in the Lagrangian

Order parameter

If L is a loop then for large L

$\mathcal{W} \sim e^{-\mathrm{perimeter}}$ Higgs phase

$\mathcal{W} \sim e^{-\mathrm{area}}$ confining phase

Charged particle can be thought as a source $\Longrightarrow dF \sim q\delta_L$

This motivates introduction of another type of line operators.

Disorder type line operators

Defined by postulating singular behavior of the gauge field (or other fields) near the line.

't Hooft operator

$\int \mathcal{D}Ae^{-S[A]}$

$F = m *_3 d\left(\frac{1}{|\overrightarrow{x}|}\right)$

F has a singularity of a magnetic monopole near $\overrightarrow{x} = 0$.

m is an element of the Cartan subalgebra of the gauge group G. Due to Dirac quantization condition m is defined up to an action of the Weyl group and can be identified with the highest weight of some representation of G^L.

Labeled by representations of $\mathbf{G^L}$

For $G = U(N)$

$G^L = U(N), \quad m = \mathrm{diag}(m_1, \ldots, m_N), \quad m_i \geq 0$

$\mathcal{T} \sim e^{-\text{perimeter}}$ confining phase

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Line operators are NOT enough to distinguish all the phases

EXAMPLE:

$\mathcal{N} = 1$ SUSY gauge theory with adjoint matter and polynomial superpotential (Cachazo, Seiberg, Witten'02)

More order parameters are needed!!

Insert a probe string-like object $\Longrightarrow$ surface operator

Currently it is not known whether surface operators are order parameters

We consider $\mathcal{N}=4, U(N)$ gauge theory. The bosonic fields are A_μ, ϕ^I

We define Gukov-Witten surface operator (Gukov, Witten'06) by postulating that A has a singularity of a cosmic string

$$A \sim d\theta, \qquad F \sim 2\pi\delta(z\bar{z})$$

near the surface

$$z=\bar{z}=0, \qquad z=re^{i\theta}$$

- We specify the group element $U \in U(N), U = Pe^{i\oint A}$– Aharonov-Bohm phase. Diagonalize $U \Longrightarrow$

$$A = \begin{pmatrix} \alpha_1 \otimes 1_{N_1} & 0 & \dots & 0 \\ 0 & \alpha_2 \otimes 1_{N_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \alpha_M \otimes 1_{N_M} \end{pmatrix} d\theta$$

- $U(N)$ is broken to the Levi group
  $L = \prod_{i=1}^{M} U(N_i), \qquad N = \sum_{i=1}^{M} N_i$
  N_i are a part of the definition of the operator.

- ϑ-angles η_i for each unbroken $U(1)$
  $\exp\left(\sum_{i=1}^{M} \eta_i \int_\Sigma \frac{F_i}{2\pi}\right)$
  (α_i, η_i) transform under S-duality

- Use the scalar fields $\phi^I, I = 4, \dots 9$.
  $\phi_{w,\bar{w}} = \phi^4 \pm i\phi^5$
  Near $z = 0$

$$\phi_w = \frac{1}{2z}\begin{pmatrix} \beta_1 + i\gamma_1 \otimes 1_{N_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \beta_M + i\gamma_M \otimes 1_{N_M} \end{pmatrix}$$

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Parameters

- The Levi group $L = \prod_{i=1}^{M} U(N_i)$
- $(\alpha_i, \eta_i; \beta_i, \gamma_i)$
  $A \sim d\theta, \quad A_z \sim \frac{1}{z}, \qquad \phi_w \sim \frac{1}{z}$

Bosonic symmetries

- Poincare symmetry along the surface $ISO(1,1)$
- Scale invariance $\Longrightarrow SO(2,2)$ conformal group in two dimensions
- $SO(4)$ rotating $\phi^6, \phi^7, \phi^8, \phi^9$
- $U(1)_{23} \times U(1)_{45} \Longrightarrow$ U(1)

Supersymmetries

Poincare supersymmetry

$$\delta\lambda = \left(\frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu} + (\nabla_\mu\phi_I)\Gamma^\mu\Gamma^I + \frac{i}{2}[\phi_I,\phi_J]\Gamma^{IJ}\right)\epsilon = 0$$

ϵ is a $D=10$ Majorana-Weyl spinor. From the $D=4$ point of view

$$\epsilon = \begin{pmatrix} \epsilon_\alpha^{\ i} \\ \bar{\epsilon}_{\dot{\alpha}i} \end{pmatrix}$$

We have $F=0, \qquad [\phi_I,\phi_J]=0, \qquad \partial_{\bar{z}}\phi_w = 0 \Longrightarrow$

$\Gamma_{zw}\epsilon = 0 \Longrightarrow \Gamma_{2345}\epsilon = -\epsilon$

$\frac{1}{2}$ of Poincare SUSY's are preserved

Similarly $\frac{1}{2}$ of superconformal SUSY's are preserved

- We have a family of half-BPS operators
- They have the same symmetries as the $D3/D3$ branes (in the near-horizon limit) intersecting along a surface

Line and Surface Operators as D-branes

It is useful to think of line and surface operators as of a system of D-branes (especially to study the supergavity duals)

- The Gukov-Witten surface operators can be represented by (the near-horizon limit of) the system

	0	1	2	3	4	5	6	7	8	9
$D3$	$\times$	$\times$	$\times$	$\times$						
$D3'$	$\times$	$\times$			$\times$	$\times$				

- The Wilson line operator in the k-symmeteric (antisymmetric) representation can be represented as the $D3/D3(D5)$ system intersecting along a line with additinal k units of the fundamental string charge

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Addition of $D7$-branes restricts $\epsilon_\alpha^{\ i}$. The restriction is

$$\tilde{\sigma}_+^{\dot{\alpha}\alpha}\epsilon_\alpha^{\ i} = 0 \qquad \text{or} \qquad \tilde{\sigma}_{\bar{z}}^{\dot{\alpha}\alpha}\epsilon_\alpha^{\ i} = 0 \qquad \Longrightarrow \qquad \epsilon_1^{\ i} = \epsilon_2^{\ i}$$

Outcome: we are dealing with a gauge theory in $D = 4$ with a $D = 2$ defect which preserves 8 Poincare supercharges with $\epsilon_1^{\ i} = \epsilon_2^{\ i}$

We are interested in $\alpha' \to 0$ limit to decouple closed strings and massive open string states

$$S = S_{\mathcal{N}=4} + \int dx^+ dx^- \ \bar{\chi}(\partial_+ + A_+ + \tilde{A}_+)\chi$$

χ is a chiral fermion in $(N, \bar{M})$ of $U(N) \times U(M)$. Under Poincare sypersymmtery

$$\delta A_\mu = -i\bar{\lambda}_{\dot{\alpha}i}\tilde{\sigma}_\mu{}^{\dot{\alpha}\alpha}\epsilon_\alpha{}^i + \text{c.c.}, \qquad \delta\chi = 0, \qquad \delta\tilde{A}_\mu = 0 \Longrightarrow$$

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Quantum-mechanically the theory is anomalous. It is convenient to split $U(N)$ into $SU(N) \times U(1)$. Denote the gauge fields by $A, \tilde{A}, a, \tilde{a}$. Consider $SU(N) \times SU(M)$ gauge transformations

$$\delta S = \frac{1}{8\pi} \int dx^+ dx^- \left[M \mathrm{Tr}_{SU(N)}(L dA) + N \mathrm{Tr}_{SU(M)}(\tilde{L} d\tilde{A}) \right]$$

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There is a difference between the Abelian and non-Abelian cases.

The anomalies are cancelled by the inflow mechanism (Green, Harvey, Moore'96). We have to take into account the Chern-Simons terms in the D-brane action

$$S_{CS}(A) = -\frac{(2\pi\alpha')^2 \tau_3}{2} \int G_1 \wedge \mathrm{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$

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In performing gauge transformations we integrate by part producing the terms like dG_1 and dG_5. In the presence of the sources

$$dG_1 = MG_{10}\tau_7\delta^2(z\bar{z}) = g_s M\delta^2(z\bar{z})$$
$$dG_5 = NG_{10}\tau_3\delta(x^4)\delta(x^5)\ldots\delta(x^9)$$

As the result,

$$\delta S_{CS}(A) + \delta S_{CS}(\tilde{A}) =$$
$$-\frac{1}{8\pi}\int dx^+dx^-\left[M\mathrm{Tr}_{SU(N)}(LdA) + N\mathrm{Tr}_{SU(M)}(\tilde{L}d\tilde{A})\right]$$

This is the mechanism to cancel the $SU(N)$ and $SU(M)$ anomalies.

The CS terms containing the $U(1)$ fields a and $\tilde{a}$ are more complicated (Itzhaki, Kutasov, Seiberg'05)

$$S_{CS}(a,\tilde{a}) = -\frac{(2\pi\alpha')^2\tau_3}{2}N\int G_1\wedge a\wedge f - \frac{(2\pi\alpha')^2\tau_7}{2}M\int G_5\wedge\tilde{a}\wedge\tilde{f}$$
$$+\frac{(2\pi\alpha')^2\tau_3}{2}N\int G_1\wedge a\wedge\tilde{f} + \frac{(2\pi\alpha')^2\tau_7}{2}M\int G_5\wedge\tilde{a}\wedge f$$

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This cancels the $U(1)$ anomalies. We have to consider the theory

$$S = S_{N=4} + S_{defect} + S_{CS}(A) + S_{CS}(\tilde{A}) + S_{CS}(a, \tilde{a})$$

Supersymmetry ???

We are taking into account G_1 which is of order $g_s M \Longrightarrow$ **the backreaction of the $D7$-branes** $\Longrightarrow$ we also have to take into account the metric and the dilaton.

$$\Longrightarrow$$

We have to consider $\mathcal{N} = 4$ Yang-Mills theory in the **$D7$-brane background !!!**

The $D7$-brane background

$$ds^2 = H_7^{-1/2}(-(dx^0)^2 + (dx^1)^2 + dx^I dx^I) + H_7^{1/2} dz d\bar{z}$$
$$e^{-\Phi} = H_7, \qquad \tau = C + ie^{-\Phi}, \qquad \partial_{\bar{z}}\tau = 0$$

This background is supersymmetric and preserves 16 supercharges

$$\epsilon = H_7^{-1/8}\epsilon_0, \qquad \gamma_{\bar{z}}\epsilon_0 = 0$$

The simplest solution with rotational $U(1)$ symmetry is given by

$$\tau = i\tau_0 + \frac{g_s M}{2\pi i}\ln z, \qquad z = re^{i\theta}$$
$$e^{-\Phi} = H_7 = \tau_0 - \frac{g_s M}{2\pi}\ln r, \quad C = \frac{g_s M}{2\pi}\theta$$

This solution has a problem: $e^{-\Phi}$ is not positive definite $\Longrightarrow$ one should view this solution as a local solution near the brane.

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Global description (Greene, Shapere, Vafa, Yau'90):

switch to Einstein frame and use $SL(2,Z)$ duality

τ is defined up to an $SL(2,Z)$ action and $Im(\tau) > 0 \Longrightarrow \tau$ takes values in the fundamental domain $\mathcal{F} = \mathcal{H}^+/SL(2,Z)$

To find a global solution we consider the one-to-one map

$j : \mathcal{F} \to \mathbf{C}$

$$j(\tau) = \frac{(\theta_2(\tau)^8+\theta_3(\tau)^8+\theta_4(\tau)^8)^3}{\eta(\tau)^{24}}$$

The $D7$-brane solution is $j(\tau(z)) = g(z)$ for any meromorphic function $g(z)$

Different choices of $g(z)$ define different $D7$-brane solutions

For a stack of M $D7$-branes at $z = 0$ we have

$$g(z) = a + \frac{b}{z^{g_s M}}$$

a sets the value of the dilaton at infinity and b is related to τ_0.

The $D7$-brane background

$$ds^2 = H_7^{-1/2}(-(dx^0)^2 + (dx^1)^2 + dx^I dx^I) + H_7^{1/2} dz d\bar{z}$$
$$e^{-\Phi} = H_7, \qquad \tau = C + ie^{-\Phi}, \qquad \partial_{\bar{z}}\tau = 0$$

This background is supersymmetric and preserves 16 supercharges

$$\epsilon = H_7^{-1/8}\epsilon_0, \qquad \gamma_{\bar{z}}\epsilon_0 = 0$$

The simplest solution with rotational $U(1)$ symmetry is given by

$$\tau = i\tau_0 + \frac{g_s M}{2\pi i}\ln z, \qquad z = re^{i\theta}$$
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$$ds^2 = -(dx^0)^2 + (dx^1)^2 + dx^I dx^I + H_7 f \bar{f} dz d\bar{z}$$

$H_7 f \bar{f}$ has to be modular invariant

Locally, one can choose a coordinate system where

$f dz = dz'$

The global modular invariant solution is

$$H_7 f \bar{f} = e^{-\Phi} \eta^2 \bar{\eta}^2 | \prod_{i=1}^{M} (z - z_i)^{-1/12} |^2$$

z_i's are the poles of $g(z)$.

Singularities:

- $z \sim z_i \Longrightarrow e^{-\Phi} \sim -\ln |z - z_i|$
- conical singularity at infinity with deficit angle $\delta = \frac{\pi M}{6}$

Summary of what we want

The theory on N $D3$-branes in the background:

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The action of the vector field A_μ

$$S_V = -\frac{T_3}{4}\int d^4x\sqrt{-g}e^{-\Phi}F_{\mu\nu}F^{\mu\nu} - \frac{T_3}{4}\int d^4x\sqrt{-g}\partial_\mu C\epsilon^{\mu\nu\rho\sigma}A_\nu F_{\rho\sigma}$$

$$T_3 = (2\pi\alpha')^2\tau_3 = \frac{1}{2\pi g_s} = \frac{1}{g^2}$$

Consider the scalars describing $D3$-brane fluctuations

Introduce vielbeins $(e^{\hat{\mu}}, e^{\hat{I}})$ and fix the static gauge

- The pullback of $(e^{\hat{I}}{}_I)$ vanishes
- The pullback of $(e^{\hat{\mu}}{}_\mu)$ = vielbein on the worldvolume

We parameterize the fluctuations by

$$\varphi^{\hat{I}} = e^{\hat{I}}_I \delta x^I$$

The index $\hat{I}$ is flat $\Longrightarrow$ $\varphi^{\hat{I}}$ transform under $SU(4) \simeq SO(6)$

while δx^I transform under diffeomorphisms

Holographic gauge theory in background fields

There is a systematic procedure of constructing the action of a **single** D-brane in an arbitrary **supersymmetric** background

- Start with the D-brane action in a curved superspace background
- Fix diffeomorphisms by choosing a static gauge
- Fix κ-supersymmetry $\Longrightarrow$ 16 fermions. Identify them with gluinos
- Since we are interested in the decoupling limit $\alpha' \to 0$ we truncate the theory at the quadratic level

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$$S_{DBI} = -\tau_3 \int d^4 x e^{-\Phi} \sqrt{-G}$$
$$G_{\mu\nu} = g_{\mu\nu} + G_{IJ}\partial_\mu \delta x^I \partial_\nu \delta x^J, \qquad G_{IJ} = H_7^{-1/2}\delta_{IJ} = e^{\Phi/2}\delta_{IJ}$$

The action of the scalars is

$$S_{Sc} = -\frac{T_3}{2}\int d^4 x\sqrt{-g}e^{-\Phi}G_{IJ}\partial_\mu \delta x^I \partial^\mu \delta x^J$$
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We have to differentiate not only $\varphi^{\hat{I}}$ but also $e^I_{\hat{I}}$

$$S_{Sc} = -\frac{T_3}{2}\int d^4 x\sqrt{-g}e^{-\Phi}(\partial_\mu \varphi^{ij}\partial^\mu \varphi_{ij} + \frac{1}{2}(\mathcal{R} + \partial^\mu \partial_\mu \Phi)\varphi^{ij}\varphi_{ij})$$

$$\varphi^{\hat{I}} = \frac{1}{2}\gamma^{\hat{I}}_{ij}\varphi^{ij}$$
$$\mathcal{R} = -\frac{3}{8}\partial^\mu \Phi \partial_\mu \Phi - \frac{1}{2}\partial^\mu \partial_\mu \Phi$$

The action of the fermions

$$S_F = T_3 \int d^4 x\sqrt{-g}e^{-\Phi}(\frac{i}{2}\bar{\lambda}_i \tilde{\sigma}^\mu D_\mu \lambda^i - \frac{i}{2}D_\mu \bar{\lambda}_i \tilde{\sigma}^\mu \lambda^i)$$
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$$- \frac{T_3}{4}\int d^4x\sqrt{-g}\partial_\mu C\bar\lambda_i\tilde\sigma^\mu\lambda^i$$

The non-Abelian extension:

- Replace $D \to \mathcal{D} = D + A$
- Replace the Chern-Simons term by its non-Abelian version $-\frac{T_3}{4}\int d^4x\sqrt{-g}\partial_\mu C\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}(A_\nu F_{\rho\sigma} - \frac{2}{3}A_\nu A_\rho A_\sigma)$
- Add the familiar non-Abelian couplings of $\mathcal{N} = 4$ SYM in flat space

$$S_{nab} = T_3\int d^4x\sqrt{-g}e^{-\Phi}\mathrm{Tr}(\bar{\lambda}_{\dot{\alpha}i}[\bar{\lambda}^{\dot{\alpha}}{}_j, \varphi^{ij}] + \lambda^{\alpha i}[\lambda_\alpha{}^j, \varphi_{ij}] - \frac{1}{2}[\varphi^{ij}, \varphi^{kl}][\varphi_{ij}, \varphi_{kl}])$$

Supersymmetry transformations are the same as in flat space except for one term in the gluino variation

$$\delta\lambda^i_\alpha = \frac{i}{2}\sigma^\mu{}_{\alpha\dot\alpha}(\partial_\mu\Phi)\varphi^{ij}\bar\epsilon^{\dot\alpha}{}_j$$

This is term is consistent with the curvature dependent "mass term".

Symmetries:

- 8 Poincare supersymmteries $\epsilon_1{}^i = \epsilon_2{}^i$
- $ISO(1,1) \times SO(6)$ bosonic symmetry
- The conformal symmetry is broken because $H_7 = H_7(z,\bar z)$

The surface operator

To obtain a surface operator, we integrate out the localized degrees of freedom χ

Rescale $\tilde{A} \to g_{D7}\tilde{A}$ and set $\alpha' \to 0$

We will eliminate $\tilde{A}$ from the defect action

$$Z = e^{iS} \int \mathcal{D}\chi \mathcal{D}\bar{\chi} e^{iS_{defect}}$$

$$S_{defect} = \int dx^+ dx^- \bar{\chi}(\partial_+ + A_+)\chi$$

The integral over χ gives the WZW model at level M

$$Z = e^{iS}\mathcal{O}_\Sigma = e^{iS} \exp\left(iM\Gamma_{WZW}(A)\right)$$

We parameterize

$$A_+ = U^{-1}\partial_+ U, \qquad A_- = V^{-1}\partial_- V$$

$$\Gamma_{WZW}(A) = -\frac{1}{8\pi}\int dx^+ dx^- \mathrm{Tr}\left(U^{-1}\partial_+ U\right)\left(U^{-1}\partial_- U\right)$$

$$-\frac{1}{24\pi}\int d^3x \epsilon^{ijk} \mathrm{Tr}\left[\left(U^{-1}\partial_i U\right)\left(U^{-1}\partial_j U\right)\left(U^{-1}\partial_k U\right)\right]$$

$$+\frac{1}{8\pi}\int dx^+ dx^- \mathrm{Tr}\left(U^{-1}\partial_+ U\right)\left(V^{-1}\partial_- V\right)$$

The last term is a local counterterm

$$\frac{1}{8\pi}\int dx^+ dx^- \mathrm{Tr}\left[\left(U^{-1}\partial_+ U\right)\left(V^{-1}\partial_- V\right)\right] = \frac{1}{8\pi}\int dx^+ dx^- \mathrm{Tr} A_+ A_-$$

$$\delta\Gamma_{WZW}(A) = \frac{1}{8\pi}\int dx^+ dx^- \mathrm{Tr}\left[L\left(\partial_+ A_- - \partial_- A_+\right)\right]$$

which is the correct anomaly of the Dirac operator

- $\mathcal{O}_\Sigma$ is an operator of **order type**
- $e^{iS} O_\Sigma$ is **gauge invariant**
- Under supersymmetry $\delta A_- \neq 0 \Longrightarrow \mathcal{O}_\Sigma$ is not supersymmetric. However, $e^{iS}\mathcal{O}_\Sigma$ is supersymmetric
- Boundary eq. of motion is $F_{+-} = 0$

Holographic duality

In the absence of $D7$-branes the theory on $D3$-branes $\Longleftrightarrow$ IIB on $AdS_5 \times S^5$

Introduce $D7$-branes and ignore the backreacktion $\Longrightarrow$ **probe** $D7$-branes in $AdS_5 \times S^5$

$$\epsilon = MG_{10}\tau_7 = g_s M = \frac{g^2}{2\pi} M$$

$g^2 M \to 0$ is the **probe approximation**

- $D7$-branes wrap $AdS_3 \times S^5$
- 8 Poincare supersymmetries and 8 superconformal supersymmetries
- $SO(2,2) \times SO(6) \times U(1)$
- If the $D7$-branes are separate we have $ISO(1,1) \times SO(6)$, 8 Poincare supersymmetries

Take the probe limit $g^2 M$ on the field theory side

$ds^2 \to \text{flat}, \qquad e^{\Phi} \to 1, \qquad G_1 \to 0$

$S = S_{\mathcal{N}=4} + \int dx^+ dx^- \, \bar{\chi}(\partial_+ + A_+)\chi$

What happens to the anomaly?

$A_\mu \to g A_\mu \Longrightarrow$

$\delta_L S = \frac{g^2 M}{8\pi} \int dx^+ dx^- \mathrm{Tr}_{U(N)}(L dA) \Longrightarrow$

subleading in the **probe approximation**

- $SO(2,2) \times SO(6) \times U(1)$
- 8 Poincare supersymmtries $\tilde{\sigma}_+^{\dot{\alpha}\alpha} \epsilon_\alpha^{\ i} = 0$
- 8 superconformal supersymmteries supersymmtries $\tilde{\sigma}_-^{\dot{\alpha}\alpha} \varepsilon_\alpha^{\ i} = 0$
- $SU(1,1|4) \times SL(2,R)$

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The backreacktion of the $D7$-branes (g^2M corrections) breaks these symmetries to $ISO(1,1)\times SO(6)$, 8 Poincare supercharges

(Super)conformal symmetry is broken by quantum ($g^2M=\lambda\frac{M}{N}$) effects

Dual supergravity solution

Start with the $D3/D7$-solution and take the near-horizon limit

$$ds^2=-H_3^{-1/2}H_7^{-1/2}dx^+dx^-+H_3^{-1/2}H_7^{1/2}dzd\bar{z}+H_3^{1/2}H_7^{-1/2}dx^Idx^I$$

$$e^{-\Phi}=H_7,\qquad \tau=C+ie^{-\Phi},\qquad \partial_{\bar{z}}\tau=0$$

$$F_{0123I}=H_7\partial_I H_3^{-1}$$

The preserved supersymmetry is

$$\epsilon=H_3^{-1/8}H_7^{-1/8}\epsilon_0,\qquad \gamma_+\epsilon_0=0,\qquad \gamma_{\bar{z}}\epsilon_0=0\Longrightarrow$$

1/4 BPS solution (8 supercharges)

Take the near-horizon limit

$$ds^2 = H_7^{-1/2}\left(ds^2_{AdS_3} + L^2 d\Omega_5\right) + \frac{\rho^2}{L^2} H_7^{1/2} dz d\bar{z}$$

$$ds^2_{AdS_3} = -\frac{\rho^2}{L^2} dx^+ dx^- + L^2 \frac{d\rho^2}{\rho^2}$$

This is the gravity dual solution!

- Go to the conformal boundary

  $$ds^2 = -H_7^{-1/2} dx^+ dx^- + H_7^{1/2} dz d\bar{z}$$

  Exactly the field theory background! $\Longrightarrow$

  holographic field theory in curved background!!

- $ISO(1,1) \times SO(6)$

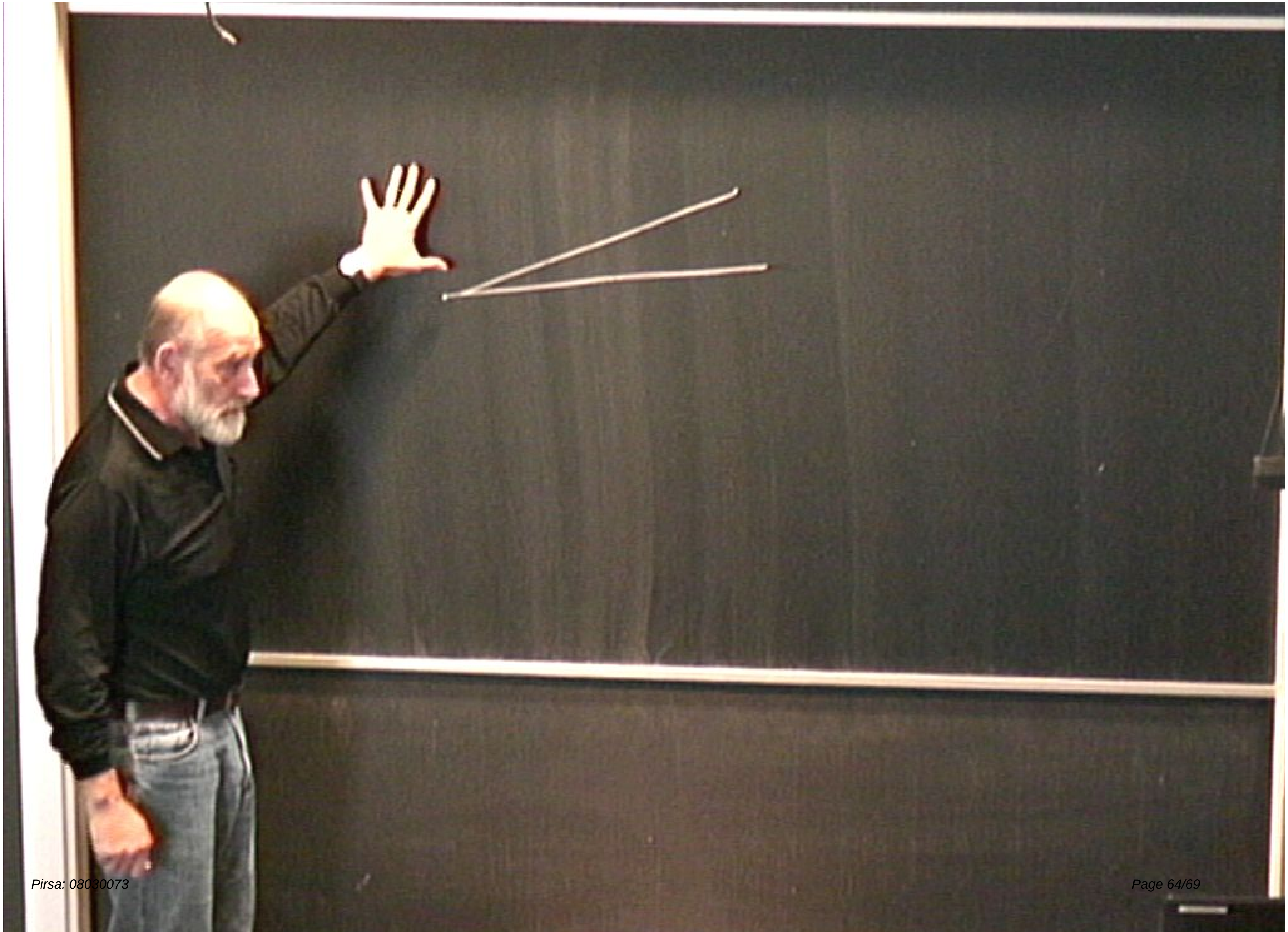
  Super(conformal) symmetry is broken by the backreaction ($g^2 M = \lambda \frac{M}{N}$) effects

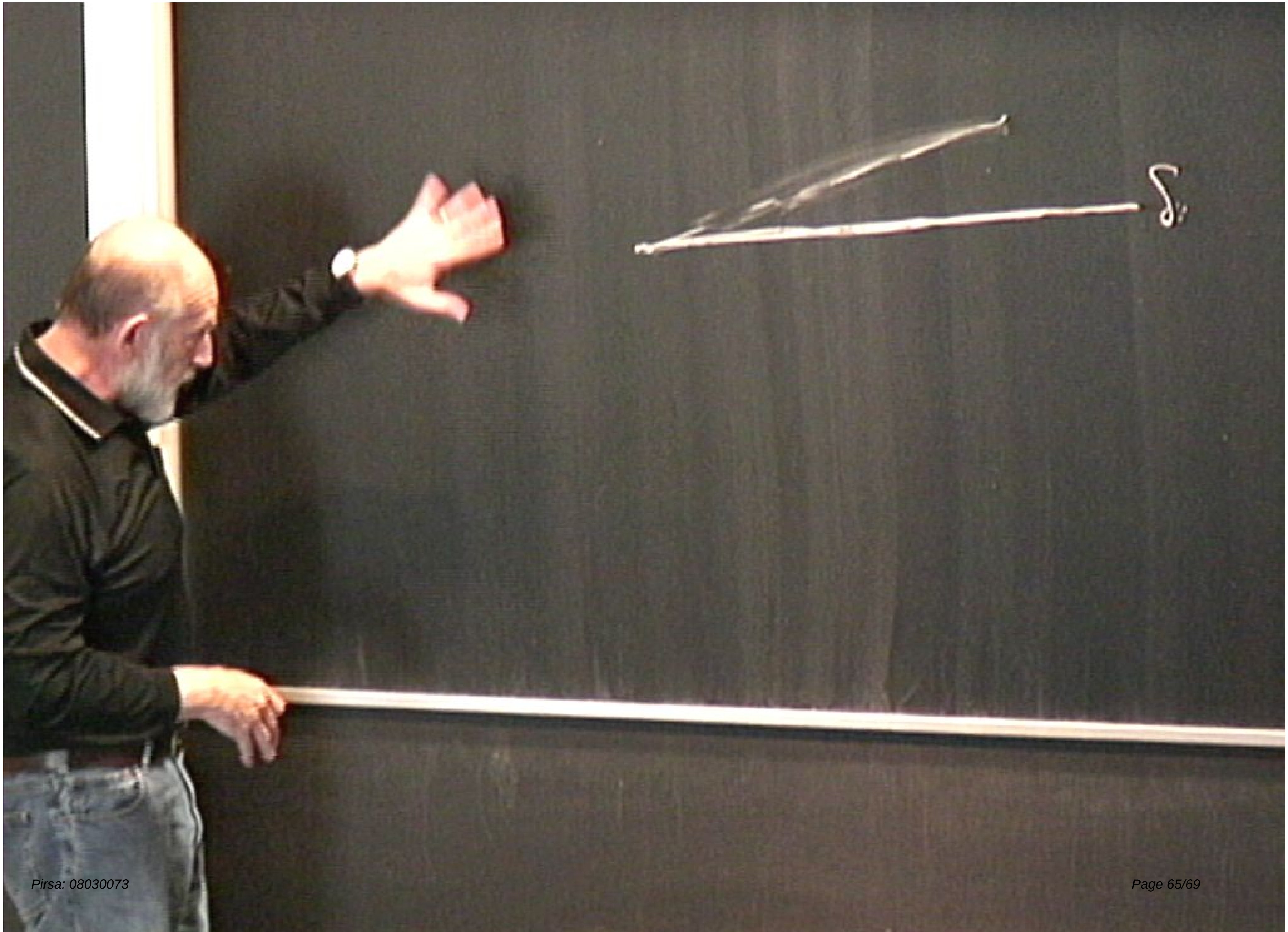
- 1/4 BPS solution (8 supercharges)

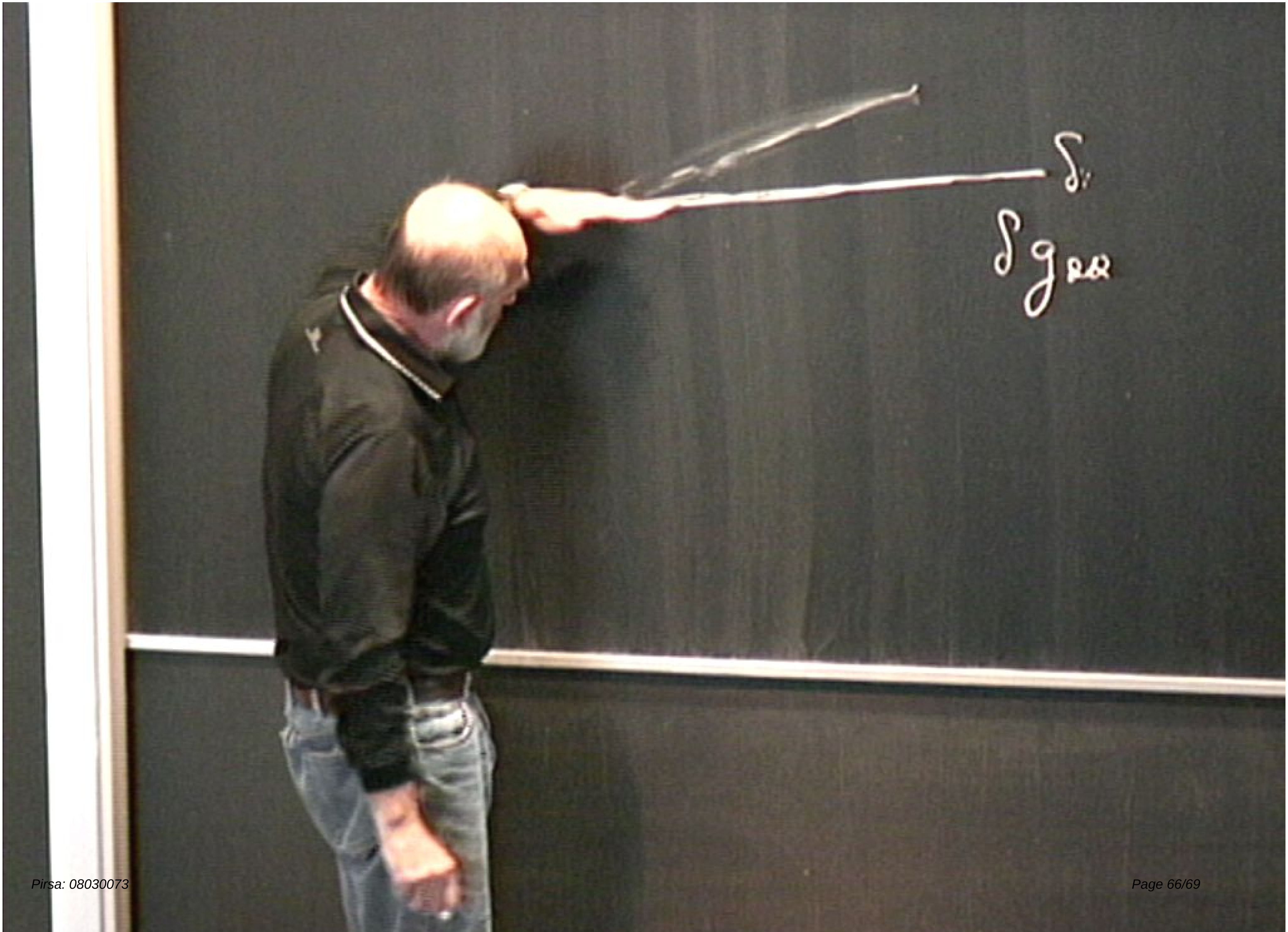
  $$\epsilon = \hat{h} H_7^{-1/2} \rho^{1/2} \epsilon_0, \qquad \gamma_+ \epsilon = 0, \qquad \gamma_{\bar{z}} \epsilon = 0$$

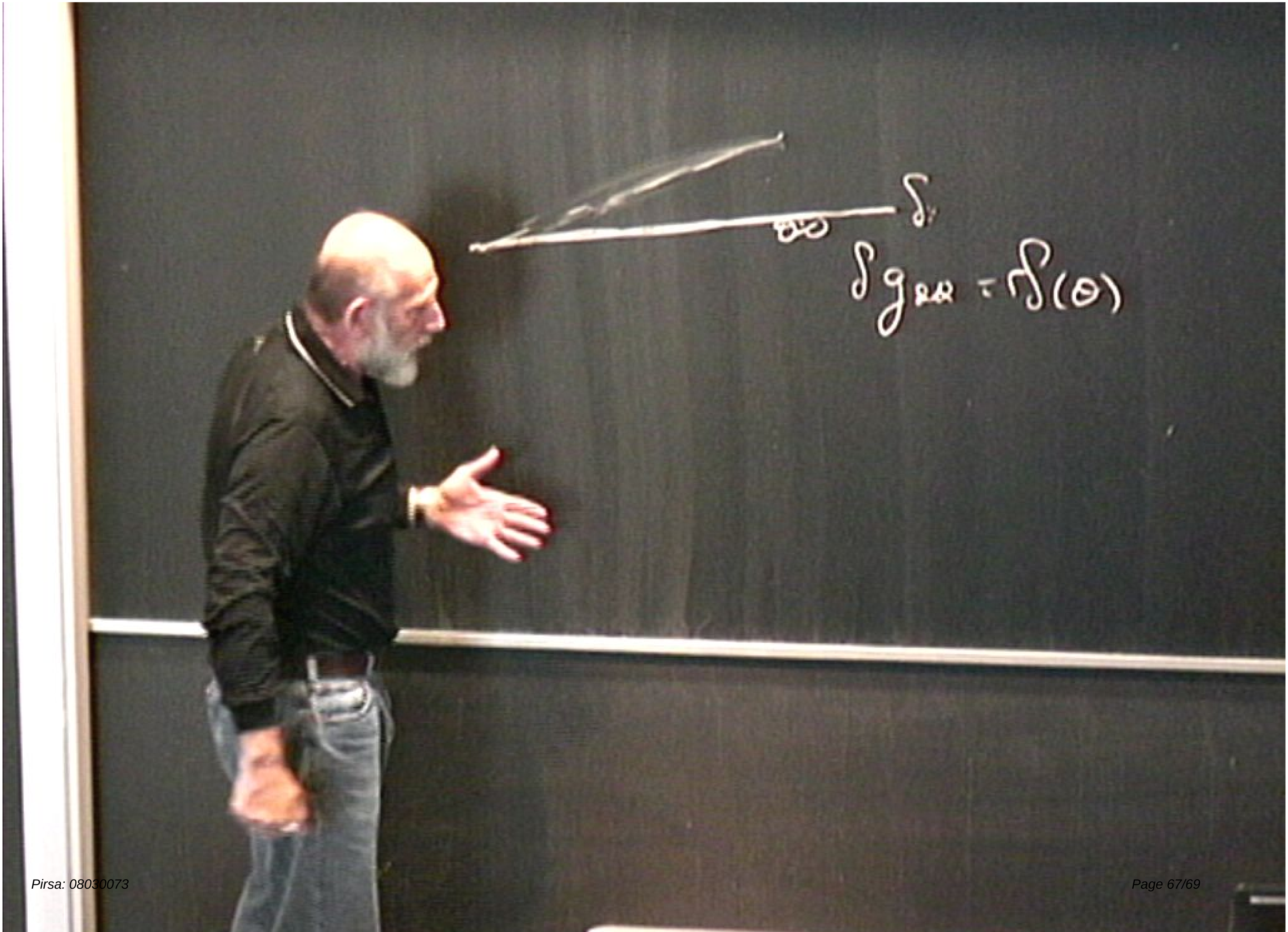
Conclusion

- Construction of 1/4 BPS WZW surface operators of order type from $D3/D7$-system
- Construction of the field theory on the $D3$-branes in the presence of $D7$-branes (the backreaction is taken into account)
- Dual gravity description
- Holographic duality beyond field theory in flat space








$\delta g_{\theta\theta} = \eta\,\delta(\theta)$