Title: Nonrelativistic limit of quantum field theory

Date: Mar 18, 2008 04:00 PM

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Abstract: It is usually expected that nonrelativistic many-body Schroedinger equations emerge from some QFT models in the limit of infinite masses. For instance, from Yukawa\'s QFT, if the initial state contains 2 fermions, we expect to recover a 2-fermion nonrelativistic Schroedinger equation with 2-body Yukawa potential (in the limit of infinite fermion mass). I will give an easy (but still heuristic) derivation of this, based on the analysis of the corresponding Feynman diagrams and on the behaviour of the complete propagators for large spacetime distances. Then, I may outline another possible derivation based on the Schroedinger picture and dressed particles.

NR limit of Yukawa's QFT

- NR QM from (R)QFT in the NR limit?
- NR limit: low-energy of the initial particles

$$E = \sqrt{|\vec{p}|^2 + m^2} \rightarrow E \approx m + \frac{|\vec{p}|^2}{2m} \rightarrow \frac{|\vec{v}|}{c} \ll 1 \qquad (1)$$

- NR QM: many-body Schrödinger equation:

$$i\frac{\partial \Psi_{a_1 a_2}(t, \vec{x}_1, \vec{x}_2)}{\partial t} = (-\frac{\Delta_1}{2m} - \frac{\Delta_2}{2m} + V(\vec{x}_1 - \vec{x}_2))\Psi \qquad (2)$$

- (R)QFT: Yukawa's QFT

$$\mathcal{L}_Y = \mathcal{L}_D + \mathcal{L}_{K-G} + g\bar{\psi}\psi\phi \qquad (3)$$

Fermions of mass m. Photon of mass μ. $\mu \ll m$.

- For 2 in low-energy electrons: Eq. (2) with $V(r) = C\frac{\exp(-\mu r)}{r}$

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Schrödinger picture?

- Obtain NR SE from QFT
- Work in the SP for QFT:

$$i\frac{\partial|\Psi_t\rangle}{\partial t} = (\widehat{H}_{Y,0} + \widehat{H}_{Y,I})|\Psi_t\rangle \qquad (4)$$

- Constraint on the state preserved by the evolution. Among others, $|\Psi_t\rangle$: eigenstate of the electron number
- Electron Number? Eigenstates of $\widehat{H}_{Y,0}$ obtained by applying the operators $\widehat{c}_s^\dagger(\vec{p})$, $\widehat{d}_s^\dagger(\vec{p})$, $a^\dagger(\vec{p})$ on the vacuum $|0\rangle$.

$$N_{e^-} = \sum_s \int d^3p\, \widehat{c}_s^\dagger(\vec{p})\widehat{c}_s(\vec{p}) \qquad (5)$$

- Bare electron number! Never conserved (vacuum state example).

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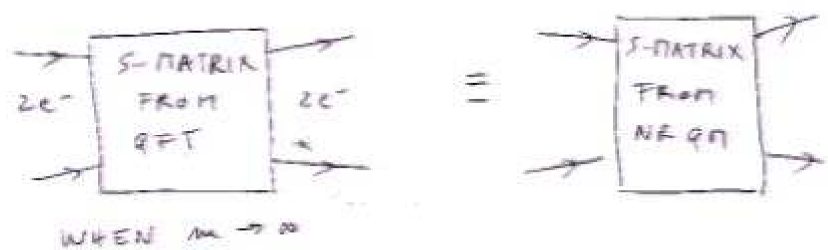
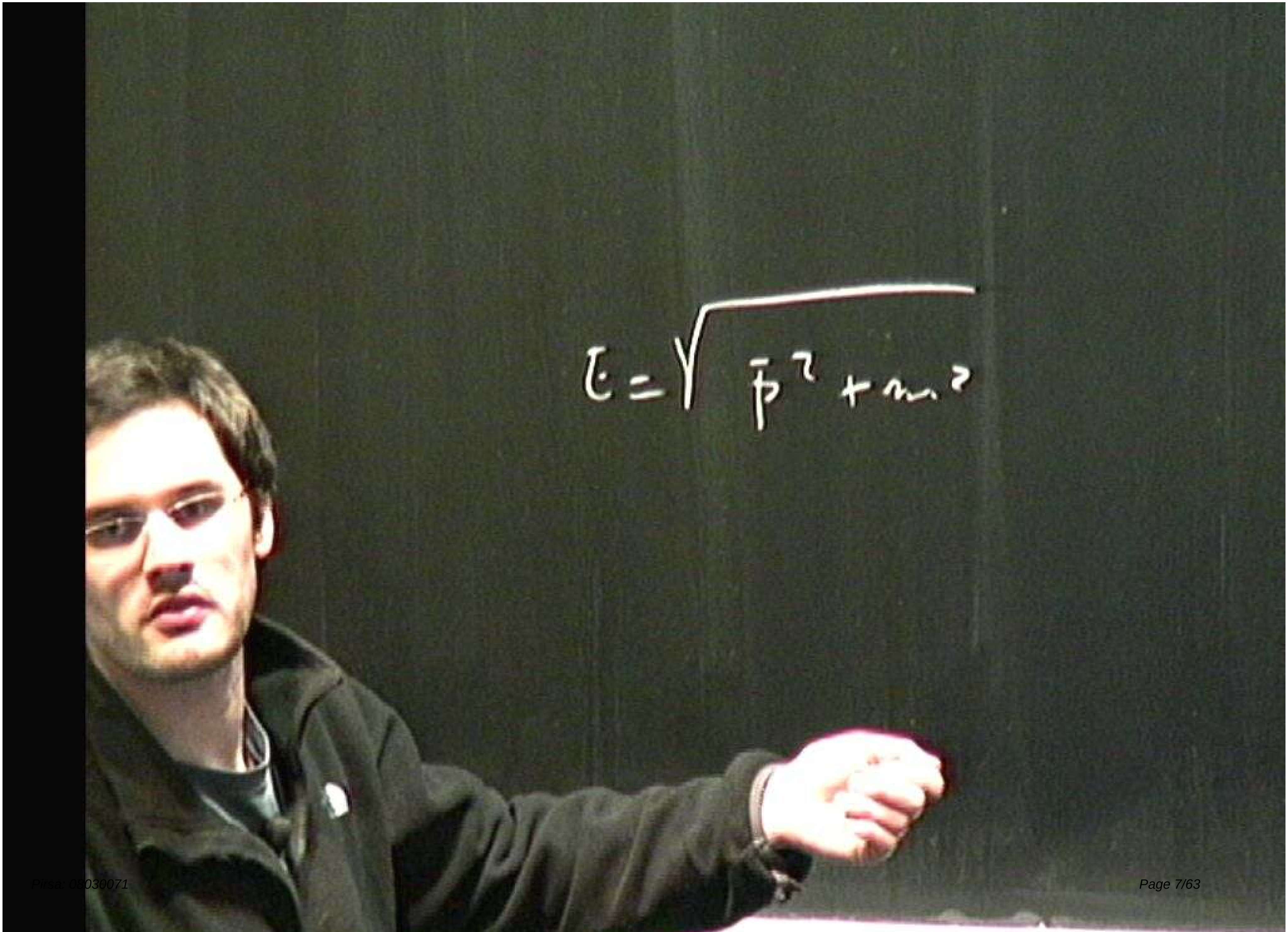
NR limit of the S-Matrix

S-Matrix, after all renormalization business, involves real parameters. S-Matrix when $m \to \infty$



- NR S-Matrix?
- Compare with the S-Matrix for $m \to \infty$
- Compute the S-Matrix? Impossible $\to$ heuristic arguments.


$E = \sqrt{\bar{p}^2 + m^2}$

$$E = \sqrt{\vec{p}^2 + m^2}$$

$$\frac{|\vec{p}|}{m} \ll 1$$

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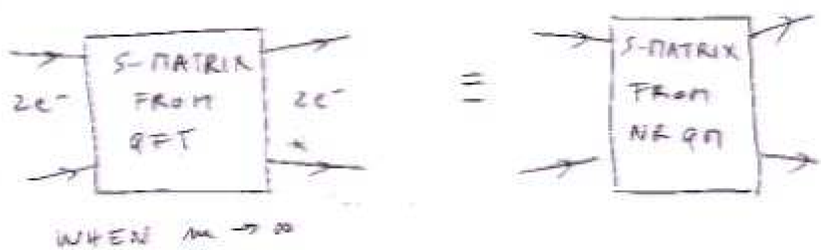

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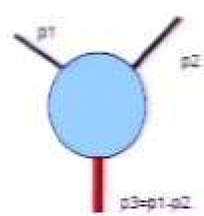
Renormalization

Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - \mu^2 \phi^2) + \bar{\psi}(i\partial\!\!\!/ - m)\psi + g\bar{\psi}\psi\phi \qquad (10)$$

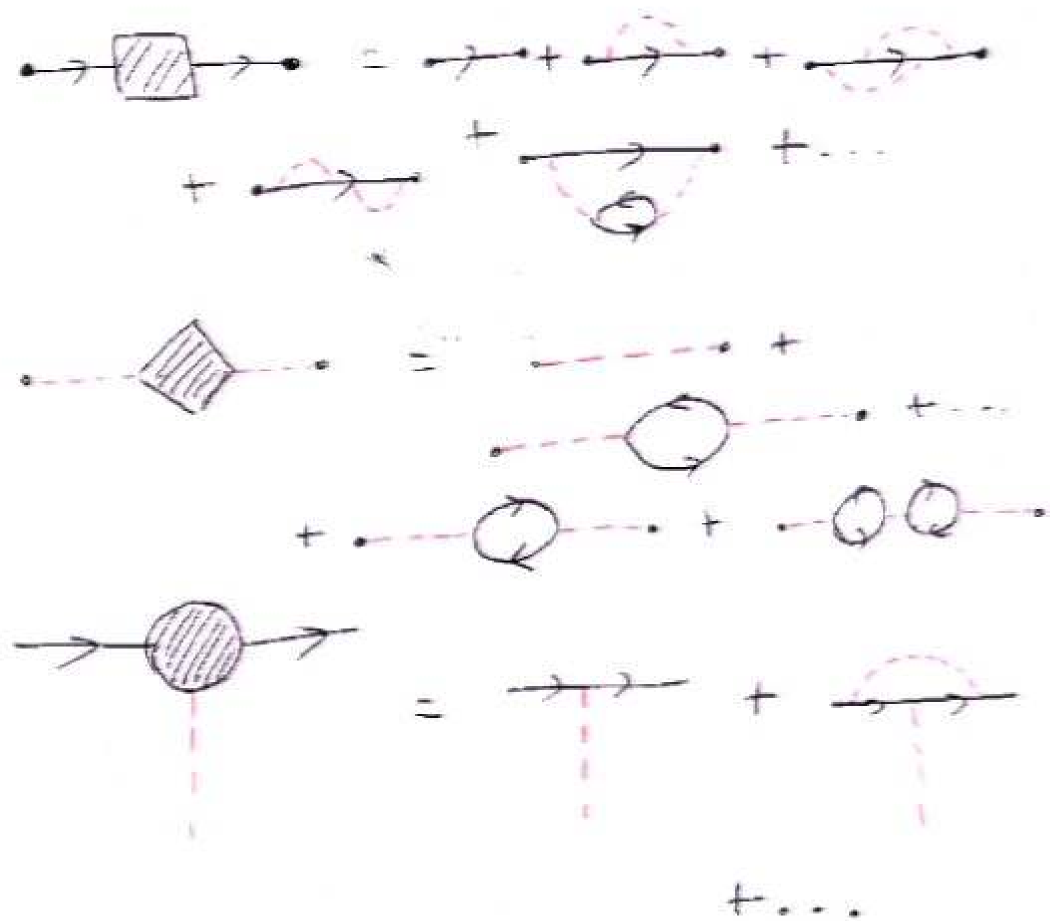
Renormalization constraints:

- Poles of the complete propagators at $p^2 = m^2$ ($p^2 = \mu^2$) for fermion (boson).
- Behaviour of the complete propagators around poles: free propagator with real mass.
- Complete vertex at some renormalization point: ig.



$\Gamma(p1, p2) =$

Renormalization



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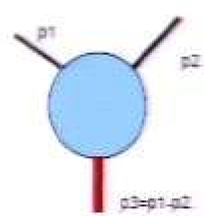

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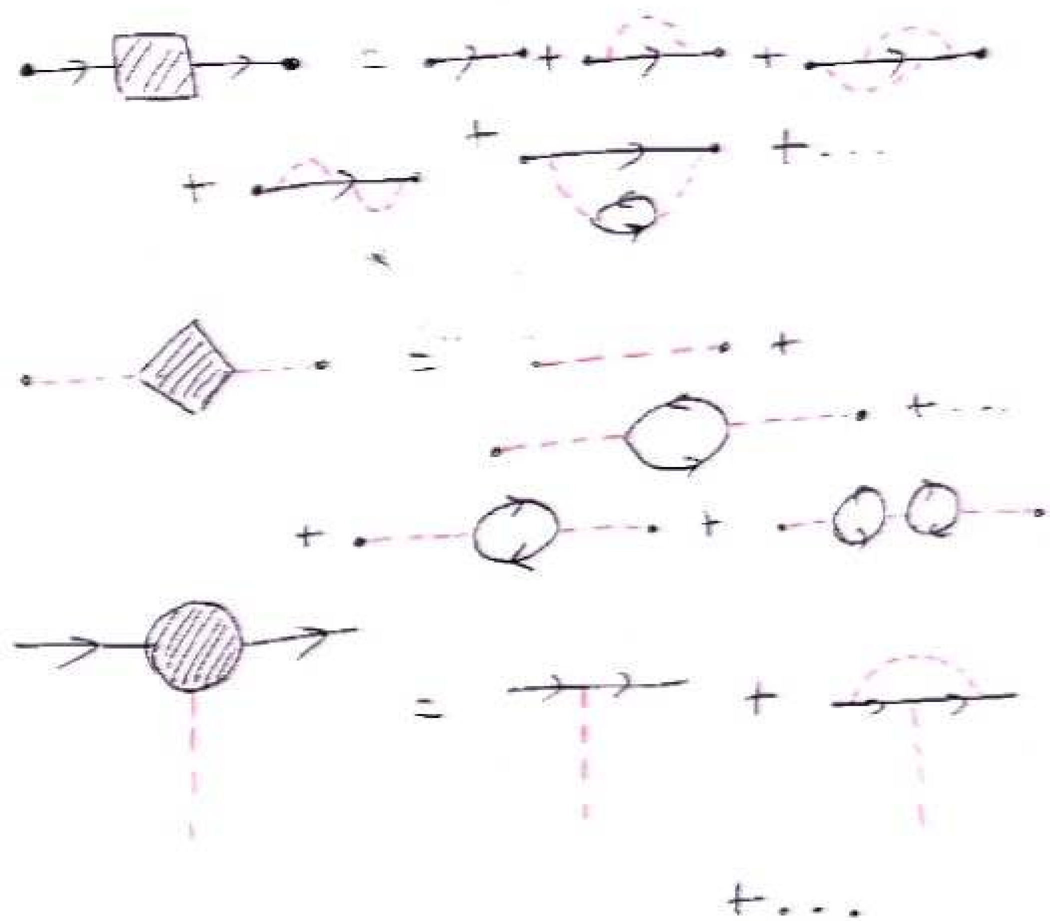

Renormalization

Bare Lagrangian:

$$\mathcal{L}_B = \frac{1}{2}(\partial_\mu \phi_B \partial^\mu \phi_B - \mu_B^2 \phi_B^2) + \bar{\psi}_B (i\partial\!\!\!/ - m_B)\psi_B + g_B \bar{\psi}_B \psi_B \phi_B \quad (11)$$

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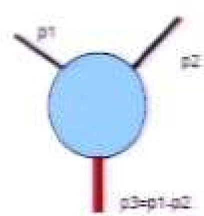

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Renormalization

5 constraints means 5 parameters → real Lagrangian:

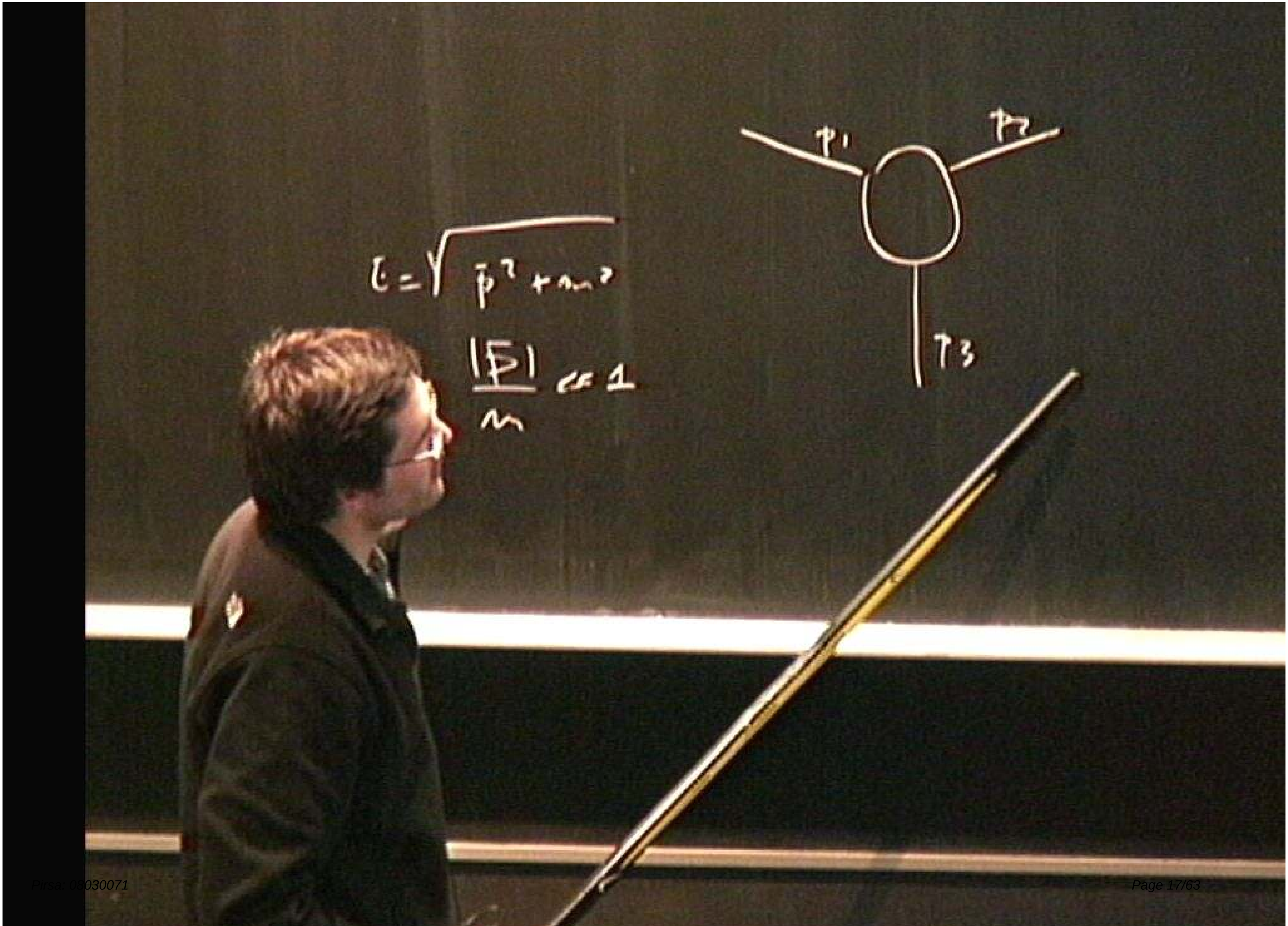
$$\mathcal{L}_R = \frac{1}{2}(Z_1 \partial_\mu \phi_R \partial^\mu \phi_R - (\mu_R^2 - \delta\mu^2)\phi_R^2) + \bar{\psi}_R(iZ_2 \not{\partial} - (m_R - \delta m))\psi_R + g_R Z_3 \bar{\psi}_R \psi_R \phi_R$$

Counter-terms:

$$\mathcal{L}_R = \frac{1}{2}(\partial_\mu \phi_R \partial^\mu \phi_R - \mu_R^2 \phi_R^2) + \bar{\psi}_R(i\not{\partial} - m_R)\psi_R + g_R \bar{\psi}_R \psi_R \phi_R + \mathcal{L}_{CT}$$

Some properties

1. Freedom to choose renormalization point for the complete vertex:
   Choose the on-shell renormalization point.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p_1
p_2
p_3

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Some properties

1. Freedom to choose renormalization point for the complete vertex:
   Choose the on-shell renormalization point.
2. Behaviour of the complete prop. for large space-time distances.
   Complete propagator $P(x' - x)$.
   When $|\Delta s^2| \gg m^{-2}$, the 4-momentum contribution comes from $(E, \vec{p})$ such that $|E^2 - |\vec{p}|^2 - m^2| \ll m^2$.
   Complete propagator $\rightarrow$ free propagator with real mass.

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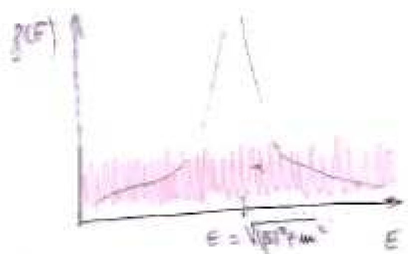
"Proof"

Complete fermion propagator for $(t' - t) \gg \lambda$ and $\Delta\vec{x} = 0$:

$$\frac{1}{(2\pi)^4} \int dE d^3p \frac{i}{\not{p} - m - \tilde{\Sigma}(\not{p}) + i\epsilon} e^{-iE(t'-t)} \qquad (12)$$



Integration over E:
Contribution from $|E^2 - |\vec{p}|^2 - m^2| \ll \lambda^{-2}$. Behaviour of the comp. prop. around the poles → free propagator with real mass.
Scale λ? Renormalization scale:

$K + V = 0$

$2m - \frac{e^2}{r} = 0$

$\Rightarrow r = \frac{e^2}{2m}$

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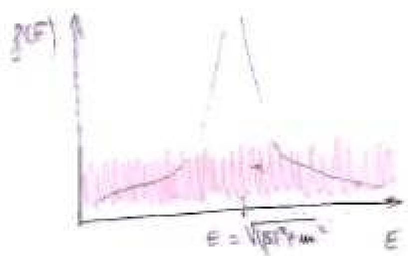

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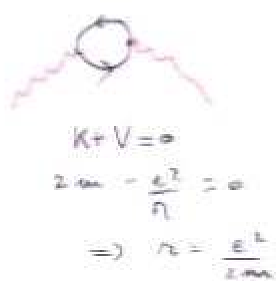

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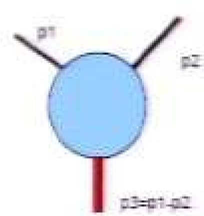

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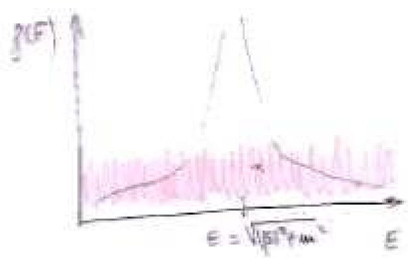

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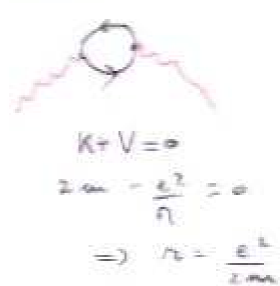

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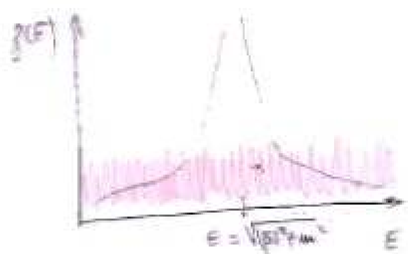

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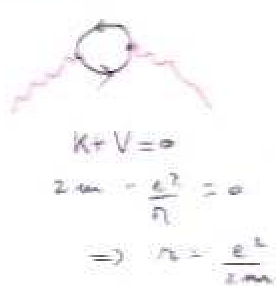

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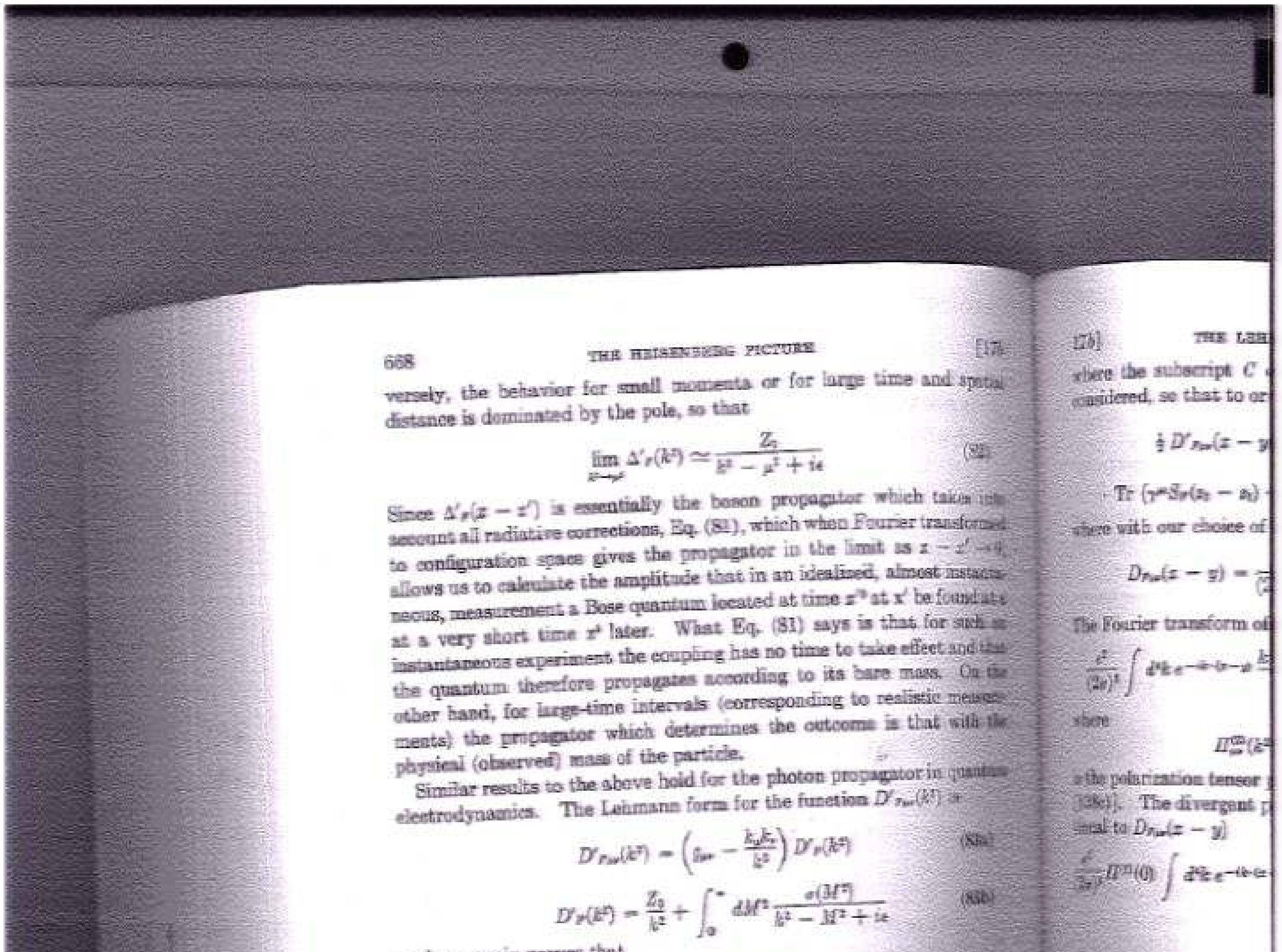
Schweber's book



versely, the behavior for small momenta or for large time and spatial distance is dominated by the pole, so that

$$\lim_{k^2 \to \mu^2} \Delta'_F(k^2) \simeq \frac{Z_2}{k^2 - \mu^2 + i\epsilon} \quad (82)$$

Since $\Delta'_F(x - x')$ is essentially the boson propagator which takes into account all radiative corrections, Eq. (81), which when Fourier transformed to configuration space gives the propagator in the limit as $x - x' \to 0$, allows us to calculate the amplitude that in an idealized, almost instantaneous, measurement a Bose quantum located at time x'^0 at $\mathbf{x}'$ be found at $\mathbf{x}$ at a very short time x^0 later. What Eq. (81) says is that for such an instantaneous experiment the coupling has no time to take effect and that the quantum therefore propagates according to its bare mass. On the other hand, for large-time intervals (corresponding to realistic measurements) the propagator which determines the outcome is that with the physical (observed) mass of the particle.

Similar results to the above hold for the photon propagator in quantum electrodynamics. The Lehmann form for the function $D'_{F\mu\nu}(k^2)$ is

$$D'_{F\mu\nu}(k^2) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) D'_F(k^2) \quad (83a)$$

$$D'_F(k^2) = \frac{Z_3}{k^2} + \int_0^\infty dM^2 \frac{\sigma(M^2)}{k^2 - M^2 + i\epsilon} \quad (83b)$$

and one again proves that

Remark and NR limit of the complete propagator

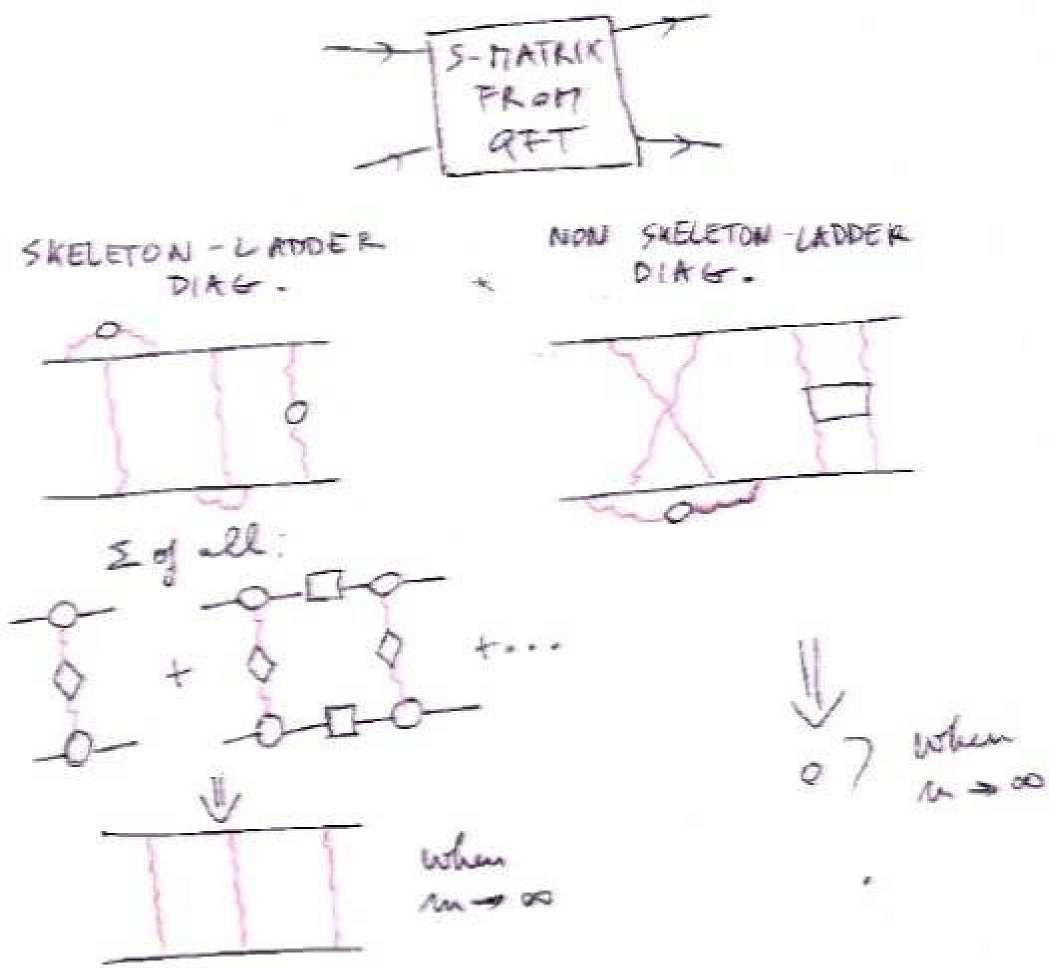
- Bare vs real rules : Small STD vs Large STD

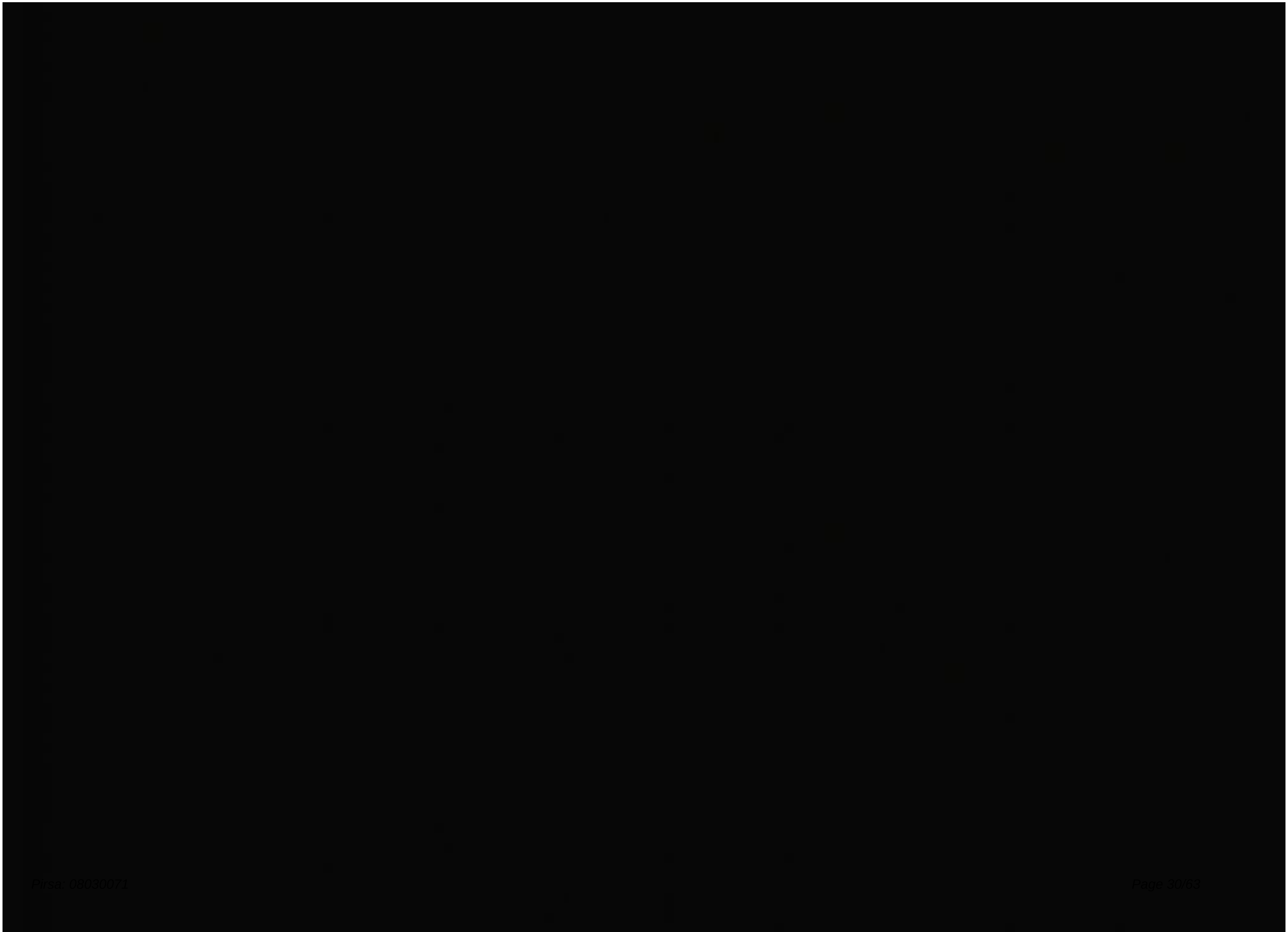
$$\frac{i}{\not{p} - m_B} VS \frac{i}{\not{p} - m_R} \tag{13}$$

Two aspects of the complete propagator.

- Limit $m \to \infty$ ($|\Delta s^2| \gg (m^{-2} \to 0)$):
  any x, x' corresponds to a LSTD.
  Complete propagator $\to$ free propagator.
  Free propagator has correct NR limit.

Skeleton-ladder VS non skeleton-ladder diagrams





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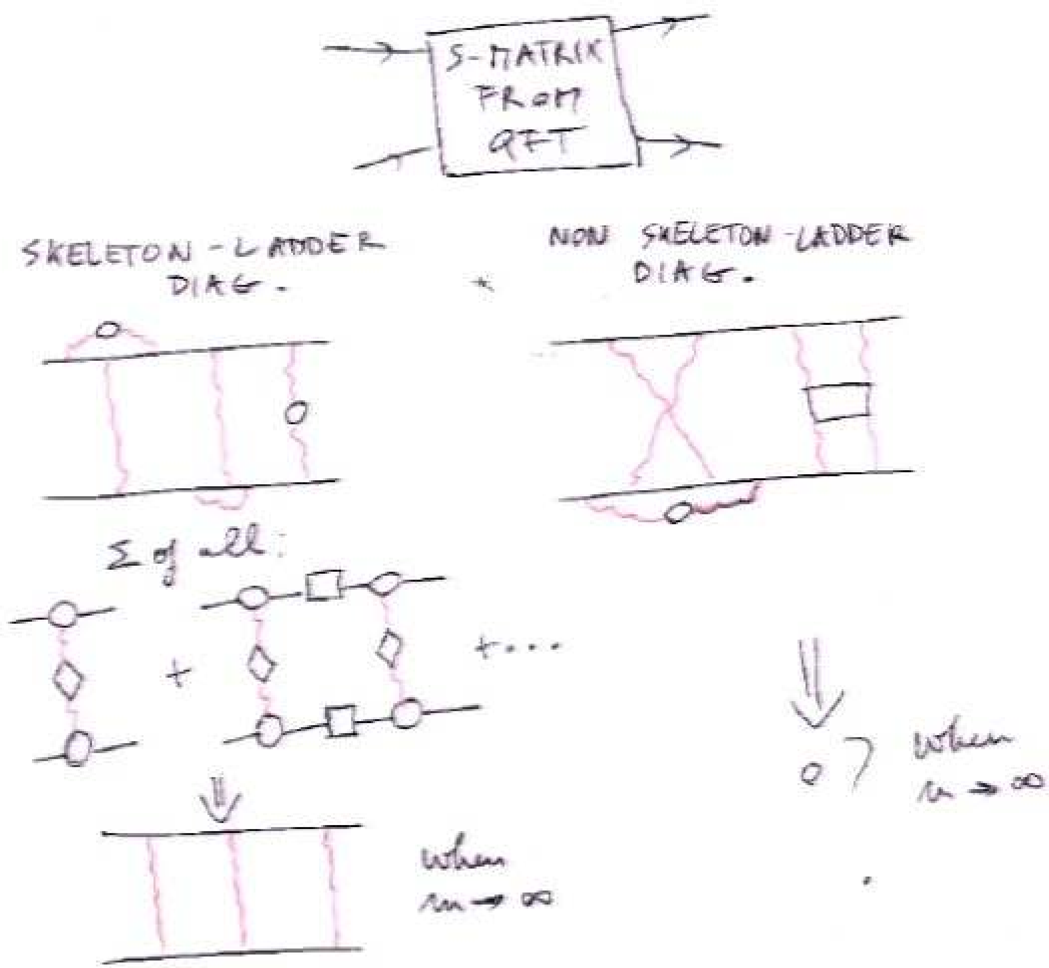

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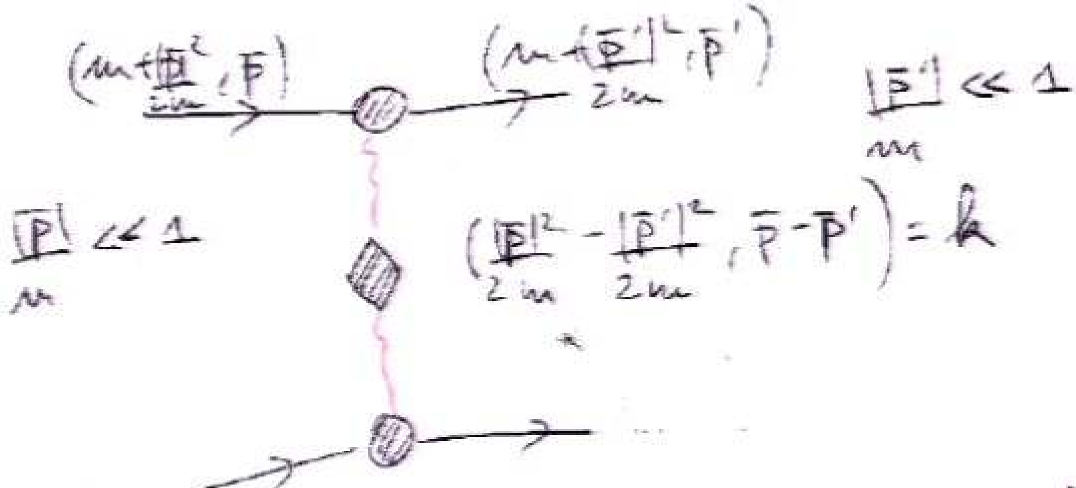
Skeleton-ladder diag. when $m \to \infty$

One-rung ladder:



1. incoming, outgoing particles on-shell
2. $|k^2 - \mu^2| \ll m^2 \Rightarrow$ ◇ $\to$ $\frac{i}{k^2 - \mu^2} \simeq \frac{-i}{|\bar{k}|^2 + \mu^2}$ INSTANTANEOUS
3. →○→ ⇒ $\xrightarrow{ig}$

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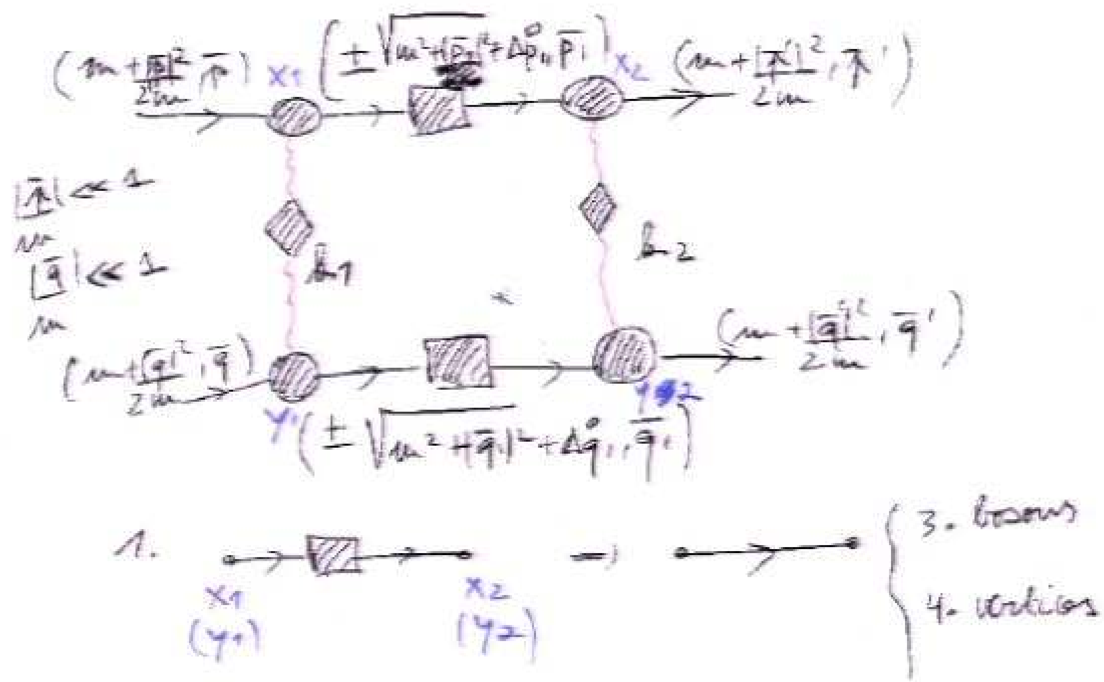
Skeleton-ladder diag. when $m \to \infty$

Two-rung ladder:



2. Compare 2 fermion propagators:

$$k_1 = \left(m + \frac{|\vec p|^2}{2m} \mp \sqrt{m^2+|\vec p_1|^2} - \Delta p_1^0, \vec p - \vec p_1\right)$$

$$2m + \frac{|\vec p|^2}{2m} + \frac{|\vec q|^2}{2m} \mp \sqrt{m^2+|\vec p_1|^2} - \Delta p_1^0 = \pm\sqrt{m^2+|\vec q_1|^2} + \Delta q_1^0$$

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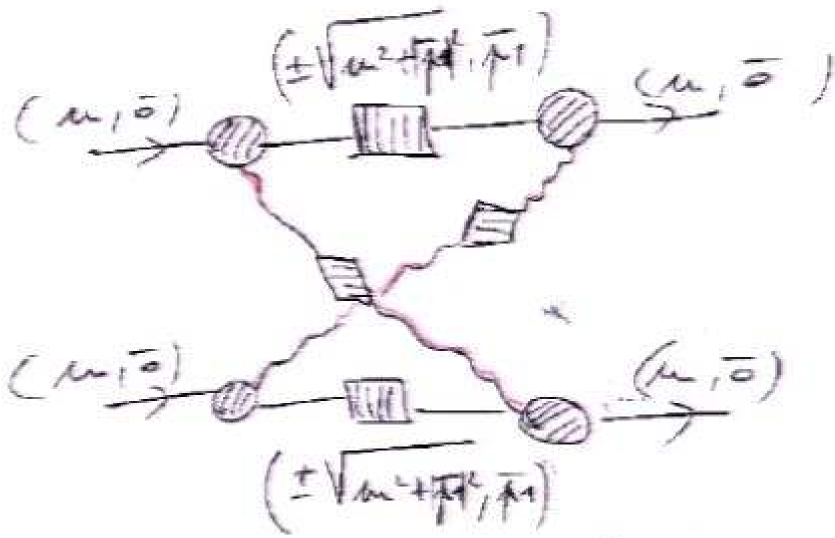
Non skeleton-ladder diagrams (order 4) when $m \to \infty$



- If $\gamma \sim$ on-shell $\Rightarrow \frac{|\vec{p}_1|}{m} \ll 1$ and $\pm$
- But there $\frac{i}{k^2 - \mu^2} \cong \frac{-i}{|\vec{k}|^2 + \mu^2}$

instantaneous interaction

$\Rightarrow$ bosons do not cross

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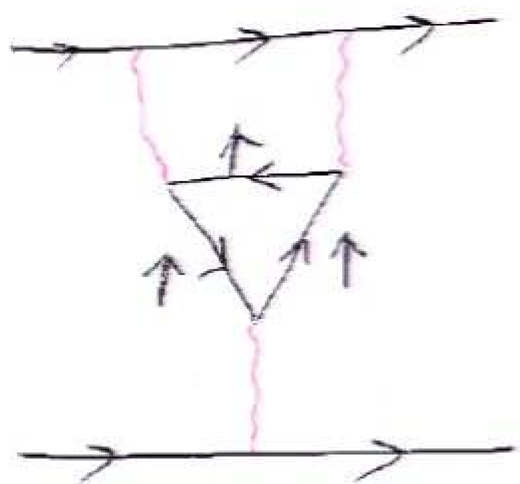
Non skeleton-ladder diagrams (order 6) when $m \to \infty$



Back to the Schrödinger picture

Problem with the Schrödinger picture comes from bare particles:

$$|0\rangle \rightarrow |0\rangle + \sum_{m,n} (c^\dagger d^\dagger)^m (a^\dagger)^n |0\rangle \tag{14}$$

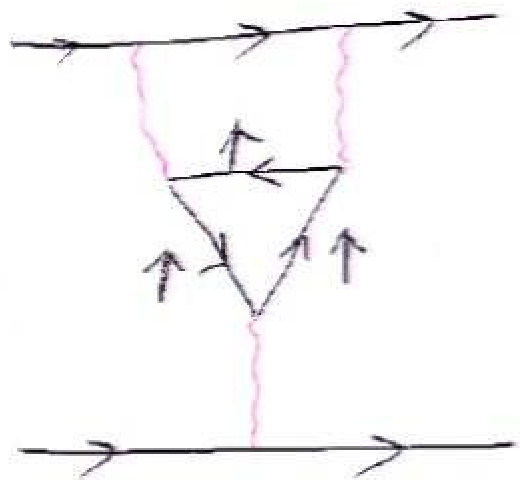
Consider instead the real vacuum:

$$H_Y |\widetilde{0}\rangle = E_{\widetilde{0}} |\widetilde{0}\rangle \tag{15}$$

The real (or dressed vacuum) is full of bare particles. Might it be empty of dressed particles?

$$\widetilde{c}_s(\vec{p})|\widetilde{0}\rangle = 0? \tag{16}$$

Non skeleton-ladder diagrams (order 6) when $m \to \infty$



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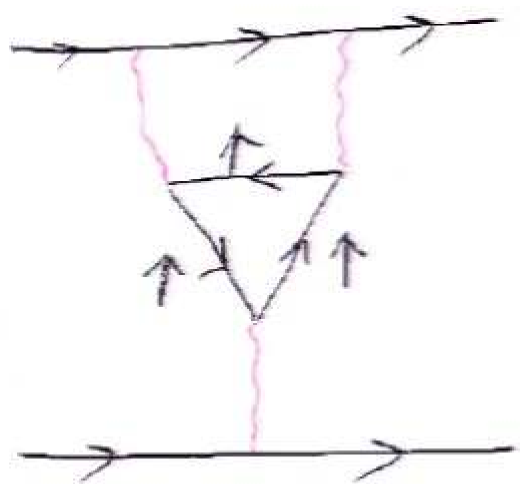
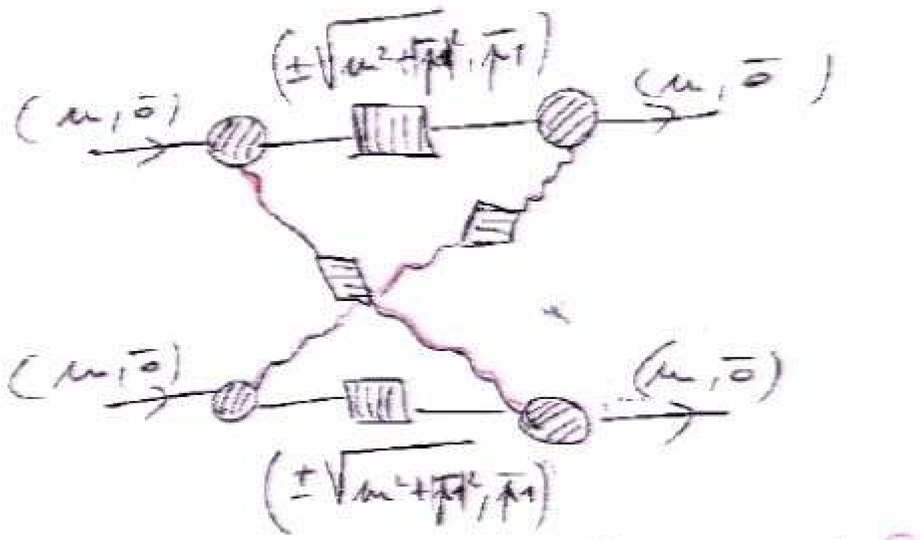


Non skeleton-ladder diagrams (order 4) when $m \to \infty$



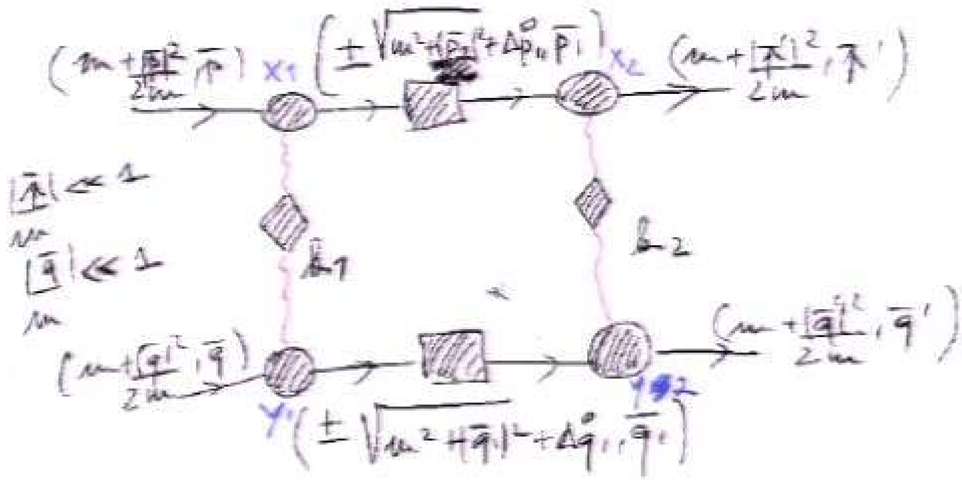
- If $\gamma \sim$ on-shell $\Rightarrow \frac{|\vec{p}|}{m} \ll 1$ and $\pm$
- But there $\frac{i}{k^2-\mu^2} \simeq \frac{-i}{|\vec{k}|^2+\mu^2}$

instantaneous interaction

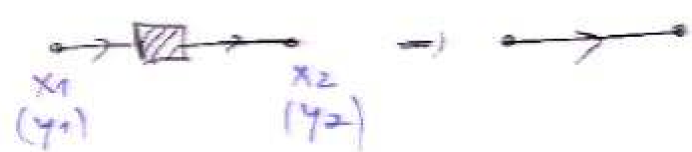
$\Rightarrow$ bosons do not cross

Skeleton-ladder diag. when $m \to \infty$

Two-rung ladder:



1.



3. bosons

4. vertices

2. Compare 2 fermion propagators:

$$k_1 = \left(m + \frac{|\vec{p}|^2}{2m} \mp \sqrt{m^2+|\vec{p}_1|^2} - \Delta p_1^0, \; \vec{p}-\vec{p}_1\right)$$

$$2m + \frac{|\vec{p}|^2}{2m} + \frac{|\vec{q}|^2}{2m} \mp \sqrt{m^2+|\vec{p}_1|^2} - \Delta p_1^0 = \pm \sqrt{m^2+|\vec{q}_1|^2} + \Delta q_1^0$$

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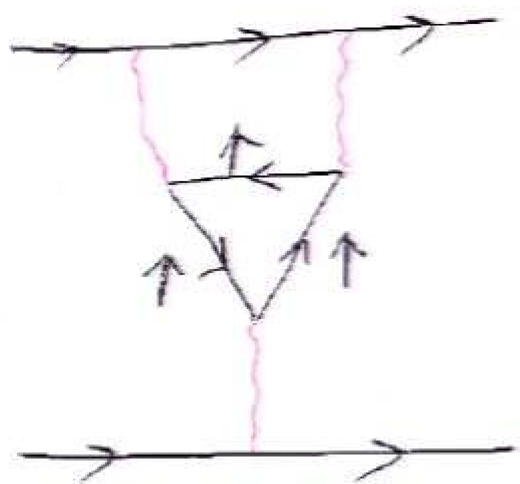
Non skeleton-ladder diagrams (order 6) when $m \to \infty$



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Back to the Schrödinger picture

Problem with the Schrödinger picture comes from bare particles:

$$|0\rangle \rightarrow |0\rangle + \sum_{m,n} (c^\dagger d^\dagger)^m (a^\dagger)^n |0\rangle \tag{14}$$

Consider instead the real vacuum:

$$H_Y |\widetilde{0}\rangle = E_{\widetilde{0}} |\widetilde{0}\rangle \tag{15}$$

The real (or dressed vacuum) is full of bare particles. Might it be empty of dressed particles?

$$\widetilde{c}_s(\vec{p}) |\widetilde{0}\rangle = 0? \tag{16}$$

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Dressed particle QFT (Greenberg-Schweber)

Requirements:

- $\widetilde{c}_s(\vec{p})|\widetilde{0}\rangle = 0$
- $H_Y \widetilde{c}_s^\dagger(\vec{p})|\widetilde{0}\rangle = (E_{\widetilde{0}} + \sqrt{|\vec{p}|^2 + m^2})\widetilde{c}_s^\dagger(\vec{p})|\widetilde{0}\rangle$
- Same canonical anti-commutation relations as the bare operators

Seems good for the NR limit. How to enforce the requirements?

- $\widetilde{c}_s(\vec{p}) = U c_s(\vec{p}) U^\dagger$ where $|\widetilde{0}\rangle = U|0\rangle$
- Hamiltonian:

$$H(a_j^\dagger, a_j) = H(U^\dagger \widetilde{a}_j^\dagger U, U^\dagger \widetilde{a}_j U) = U^\dagger H(\widetilde{a}_j^\dagger, \widetilde{a}_j) U = \widetilde{H}(\widetilde{a}_j^\dagger, \widetilde{a}_j). \tag{17}$$

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The dressed Hamiltonian

- Dressed Hamiltonian:

$$\widetilde{H}_Y = \sum_j \int d^3p \sqrt{|\vec{p}|^2 + m_j^2}\,\widetilde{a}_j^\dagger(\vec{p})\widetilde{a}_j(\vec{p}) + \ldots \text{at least 2 annihil.} \quad (18)$$

- But $\widetilde{c}_s^\dagger(\vec{p})\widetilde{c}_{s'}^\dagger(\vec{p'})|\widetilde{0}\rangle$ can't be an eigenstate of $\widetilde{H}_Y$.
- NR limit: 2 low-energy electrons remain two low-energy electrons.
- Conditions on U.

The Gell-Mann-Low theorem

- Adiabatic coupling:

$$H = H_0 + e^{-\alpha|t|} H_I \qquad \alpha \geq 0 \ . \tag{19}$$

- Evolution operator:

$$U_\alpha(t_f, t_i) = \sum_{n=0}^{n=\infty} \frac{(-i)^n}{n!} \int_{t_i}^{t_f} dt_1 \ldots \int_{t_i}^{t_f} dt_n \, e^{-\alpha|t_1|} \ldots e^{-\alpha|t_n|} T(H_I(t_1) \ldots H_I(t_n)) \ . \tag{20}$$

- Relation between the bare and dressed vacua:

$$|\tilde{0}\rangle = N \lim_{\alpha \to 0} \frac{U_\alpha(0, -\infty)|0\rangle}{\langle 0|U_\alpha(0, -\infty)|0\rangle} \ . \tag{21}$$

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The Gell-Mann-Low theorem

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Degeneracy?

- The GML T is stronger: any eigenstate of H_0 by the above transformation.
- In particular $c_s^\dagger(\vec{p})|0\rangle$ is mapped to an eigenstate of full H.
- But $c_s^\dagger(\vec{p})c_{s'}^\dagger(\vec{p'})|0\rangle$ too! and

$$U_{GML}c_s^\dagger(\vec{p})c_{s'}^\dagger(\vec{p'})|0\rangle = Uc_s^\dagger(\vec{p})U^\dagger Uc_{s'}^\dagger(\vec{p'})U^\dagger U|0\rangle = \tilde{c}_s^\dagger(\vec{p})\tilde{c}_{s'}^\dagger(\vec{p'})|\tilde{0}\rangle. \tag{22}$$

- Issue with degeneracy?

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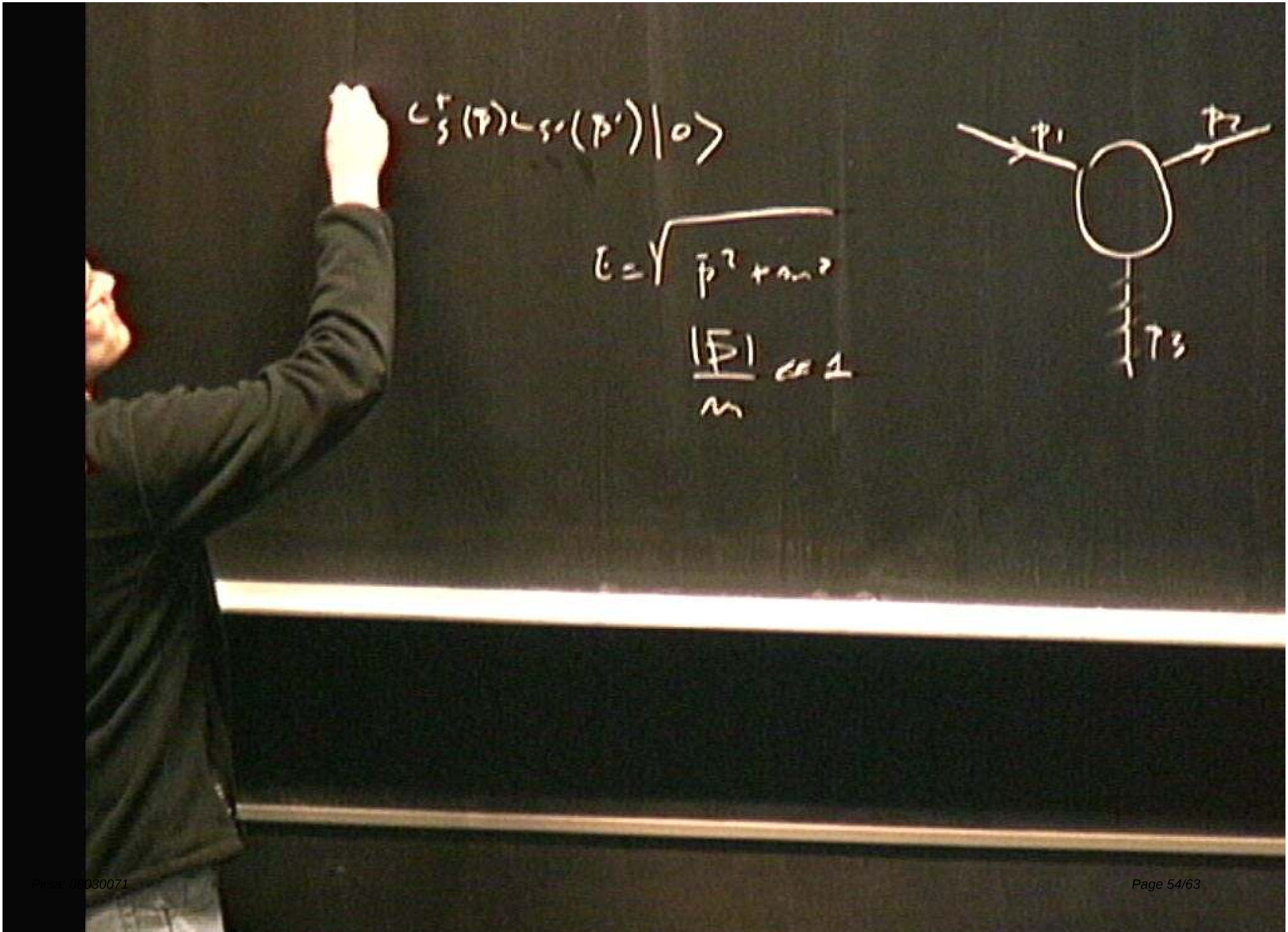
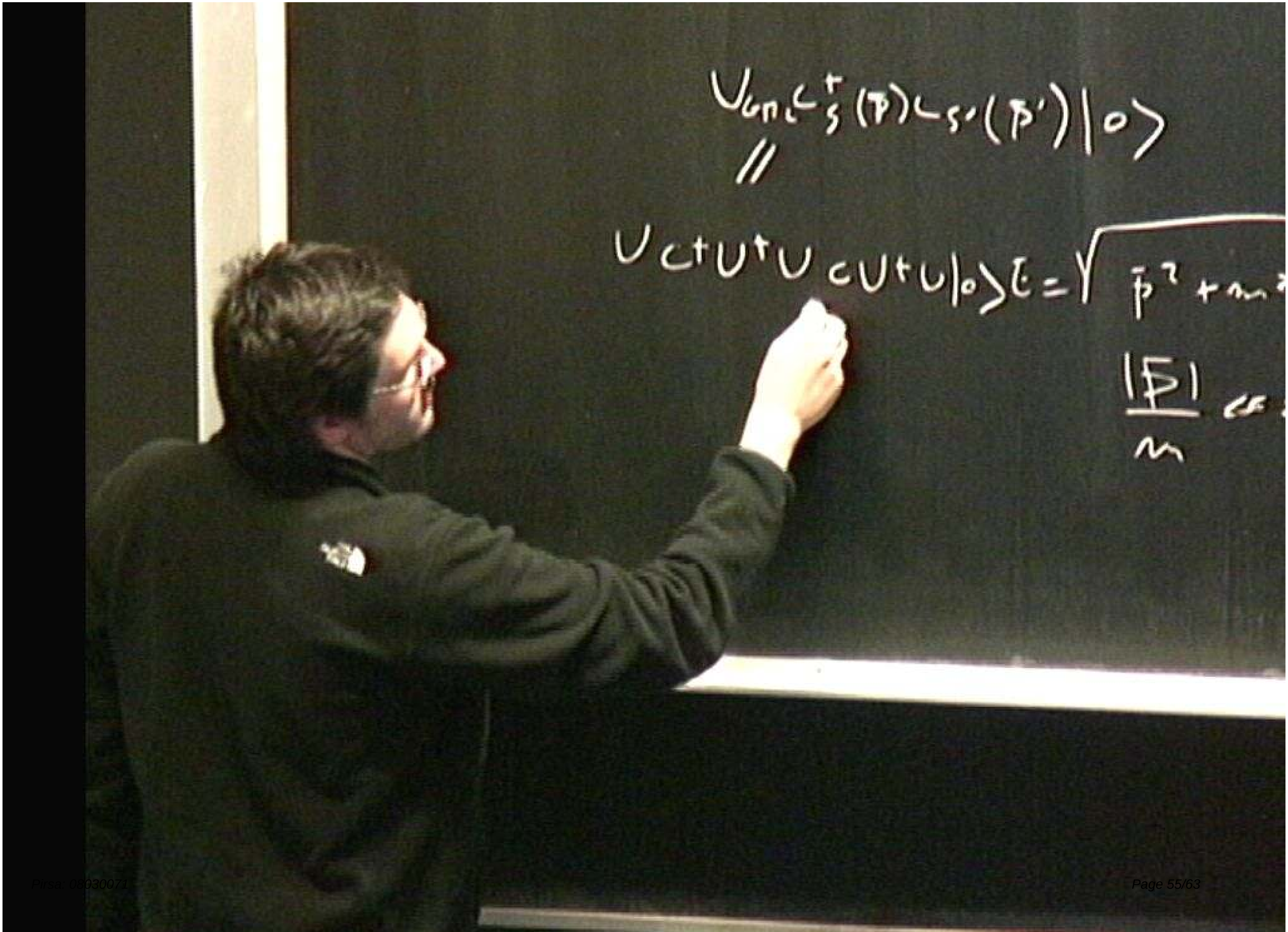
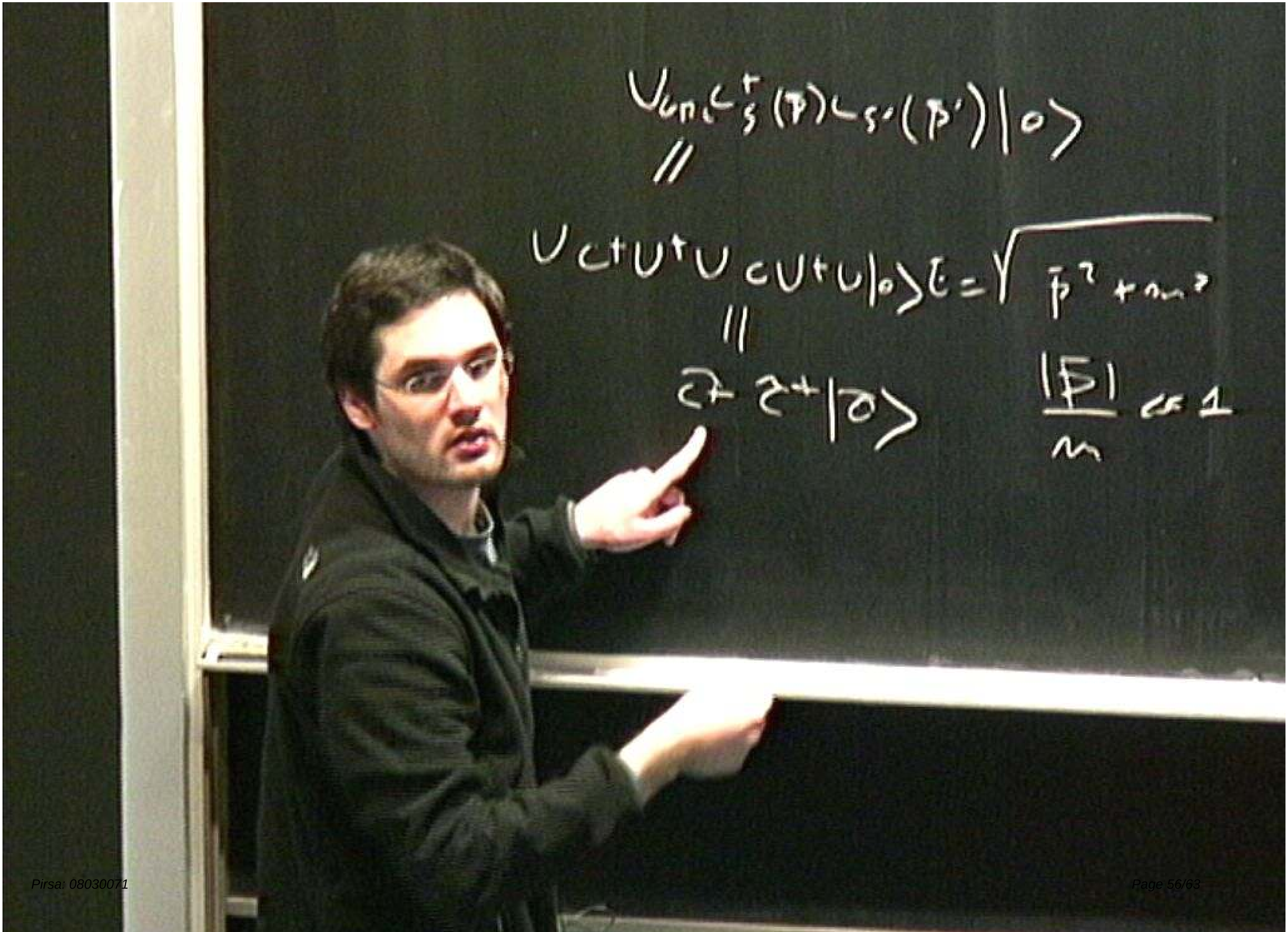
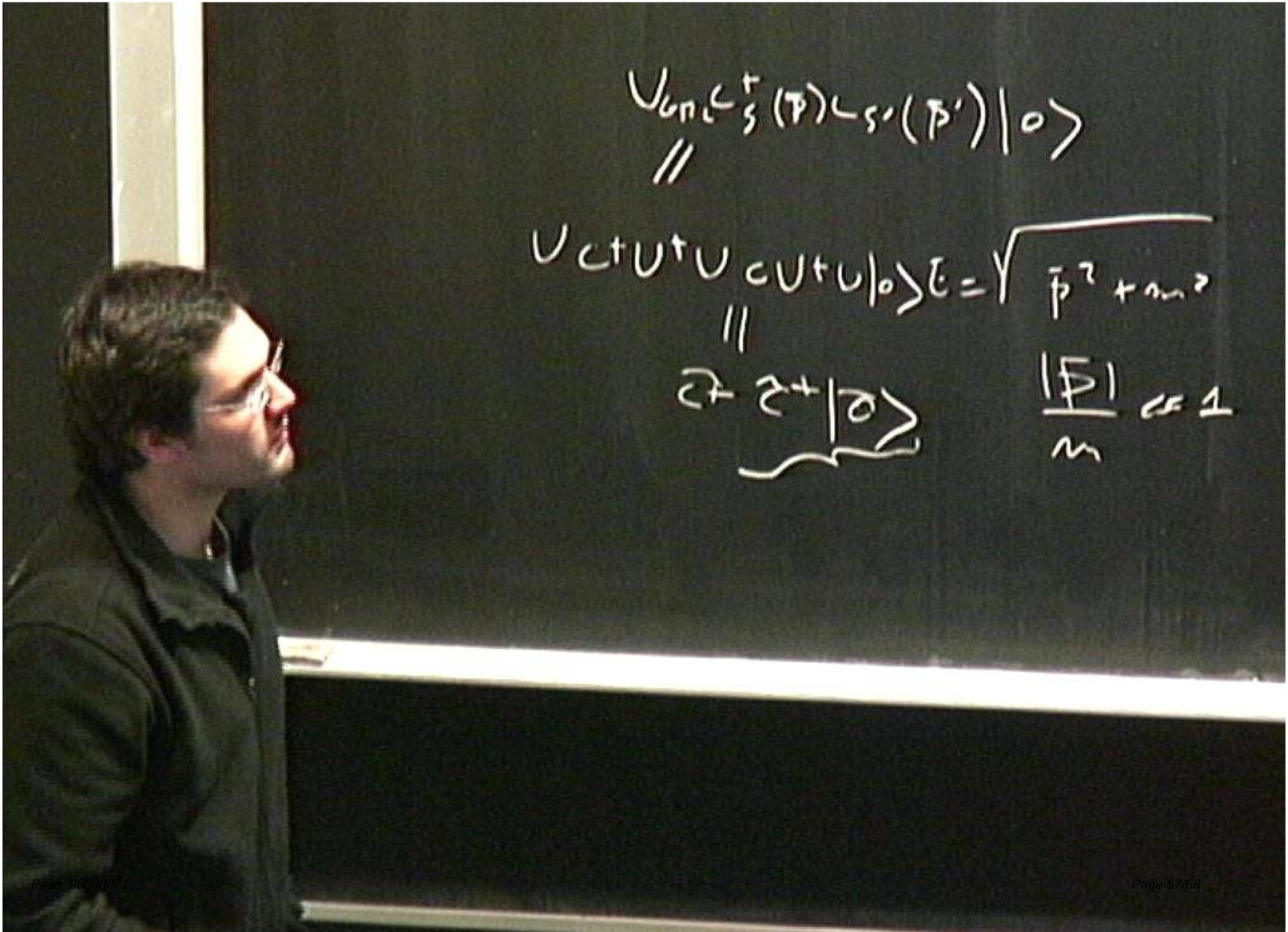
- Issue with degeneracy?


$c^{\dagger}_{s}(\vec{p})\, c_{s'}(\vec{p}')|0\rangle$
$E = \sqrt{\vec{p}^2 + m^2}$
$\frac{|\vec{p}|}{m} \ll 1$
p_1
p_2
p_3




E = √(p̄² + m²)
|p̄| / m ≪ 1



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End

1. S-Matrix when $m \to \infty$. Equivalent to S-Matrix from NR QM. Heuristic... Could it be made more rigorous?

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End

1. S-Matrix when $m \to \infty$. Equivalent to S-Matrix from NR QM. Heuristic... Could it be made more rigorous?

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End

1. S-Matrix when $m \to \infty$. Equivalent to S-Matrix from NR QM. Heuristic... Could it be made more rigorous?
2. Possible approach with dressed states. Generalization of the Gell-Mann and Low theorem for degenerate free Hamiltonian.