Title: Fast Scrambling

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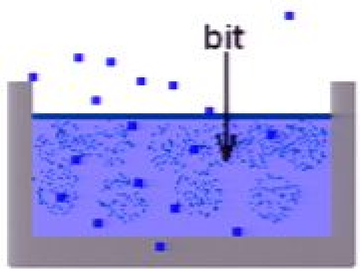
URL: http://pirsa.org/08030070

Abstract: TBA

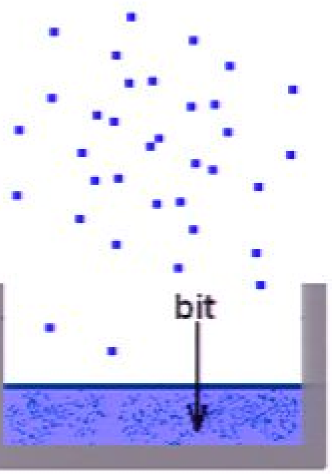
Fast Scrambling

- Patrick Hayden, John Preskill, 0708.4025

- Don Page, qc/9305007
- J. Lindesay, L. Susskind, World Sci 2005

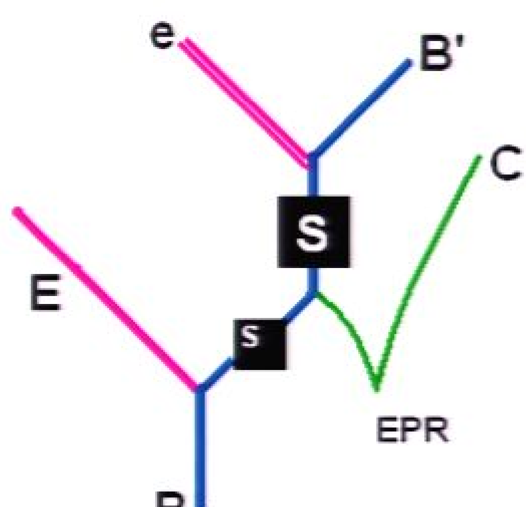


time needed to
recover bit = time to
emit 1/2 the particles



time needed to recover bit

= scrambling time t_*



S = Scrambler

How scrambled is SCRAMBLED?

Haar scrambling

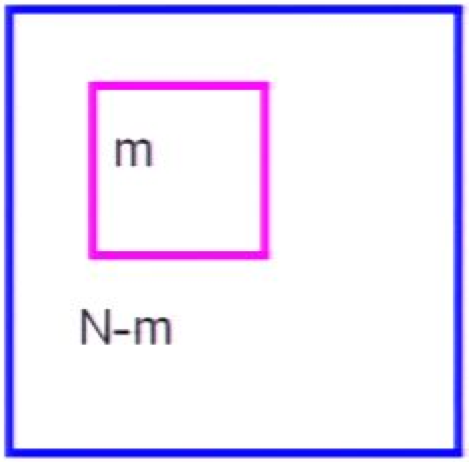
$$| R \rangle = U | \Psi_0 \rangle$$

U chosen randomly with respect to Haar measure on Hilbert space of states.

Haar scambling is very inefficient. It takes non-polyomial steps.

But it is also OVERKILL.

Page scrambling (Don Page) is good enough.



Page Scrambling means

$$S_m = \log m + \text{order } (m/N)$$

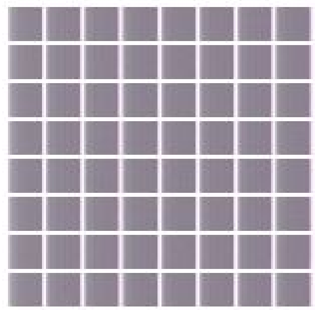
For all $m < N/2$

Haar $\longrightarrow$ Page

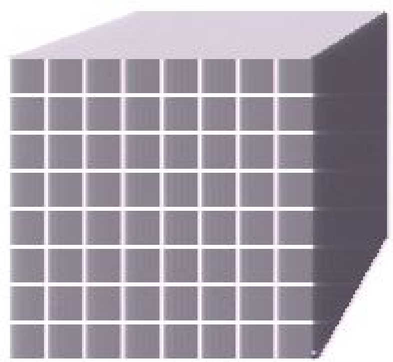
but

Page $\not\longrightarrow$ Haar

d = 1



d = 2



d =3

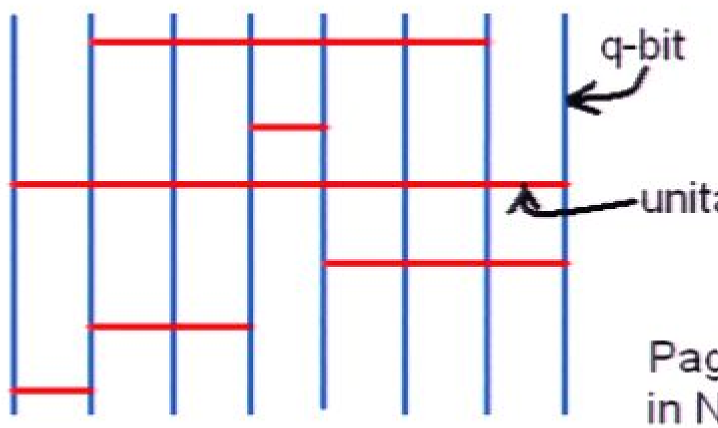
Typical scrambling time:

$$t_* = \beta N^{\frac{2}{d}}$$

$\exists$ systems that Page-scramble in time

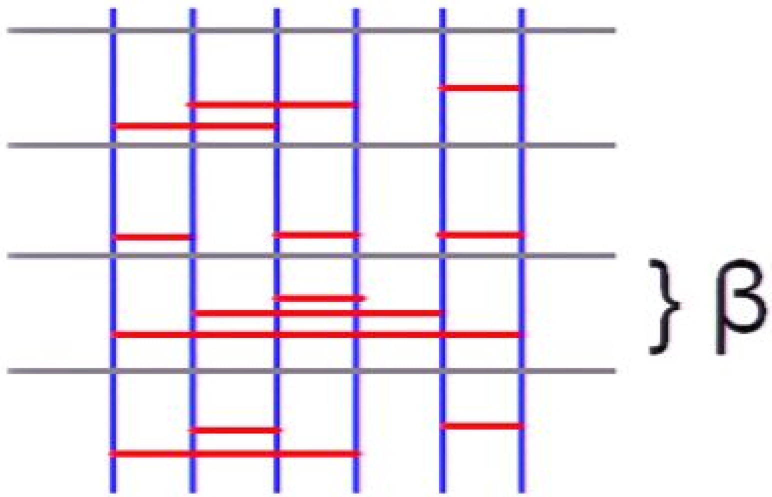
$$t_* = \log N$$

Scrambler built from 2-bit unitary operations.
Typically random but definite.



Page-scrambles
in N log N steps.

Parallel Processing



Time step = inverse temperature.

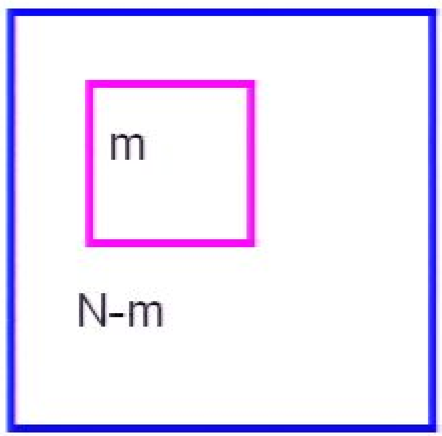
After m time-steps the 1st q-bit has directly or indirectly communicated with about 2^m others. The page-scrambling time is of order

$$t_* = \beta \log N$$

That's fast!

Hamiltonian Scramblers

$$| R \rangle = e^{iHt_*} | \Psi_0 \rangle$$



$$\rho_m = TR_{N-m} | R \rangle \langle R |$$

$$S_m = - tr \; \rho_m \log \rho_m$$

Page-thermal-scramblers satisfy

$$S_m = S_{thermal} + Order\ (m/N)$$

for $m < N/2$

Fast Hamiltonian Scramblers **?**

Let the degrees of freedom be

$$X_1, X_2, X_3, X_4, \ldots\ldots X_N$$

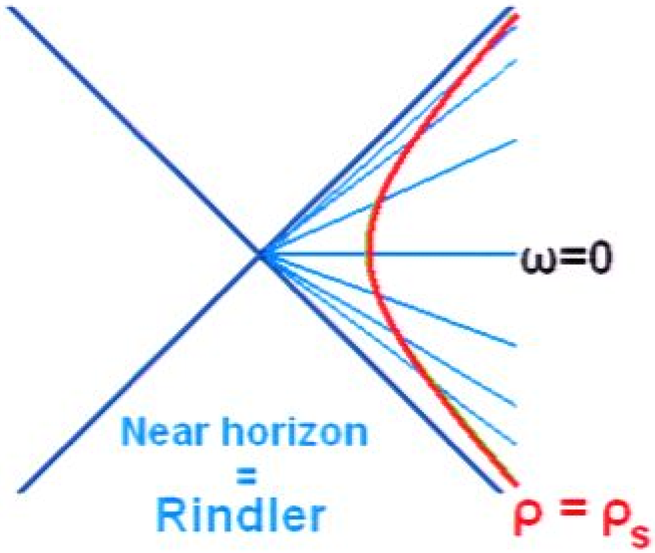
$$\mathbf{H = \sum F(X_i, X_j, X_k, X_l)}$$

Clusters of fixed size.

Can $\mathbf{t}^* = \boldsymbol{\beta} \log \mathbf{N}$ **?**

Black Holes are fast scramblers.

Hayden, Preskill
Lindesay, LS

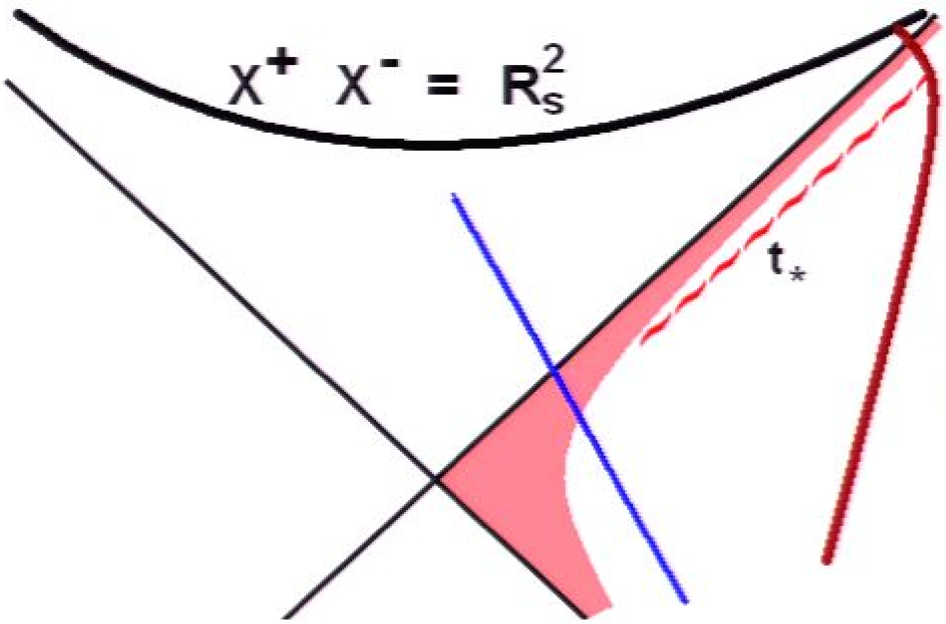


ρ_s = string scale ~ Planck scale (Stretched Horizon)

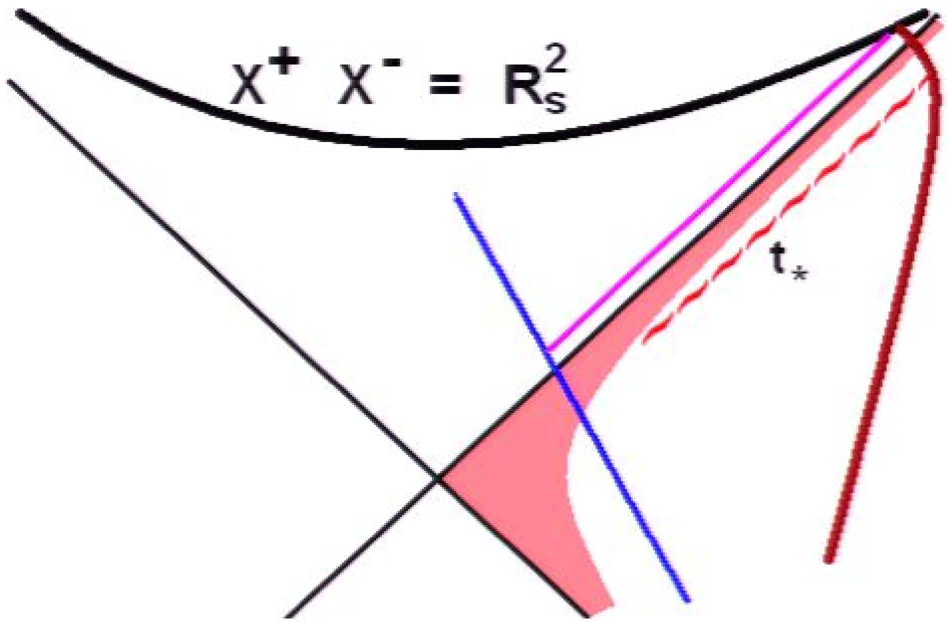
$$X^+ = \rho\, e^{\omega}$$

$$X^- = -\rho\, e^{-\omega}$$

$$\omega = t/4MG = \beta t$$



Quantum Xeroxing (cloning) behind the horizon?



$$M = \frac{R_s}{G} > \frac{1}{X^-}$$

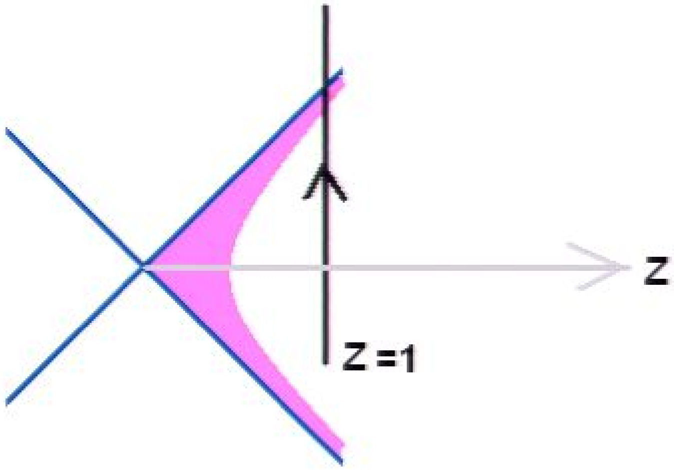
$$X^+ > \frac{R_s^3}{G}$$

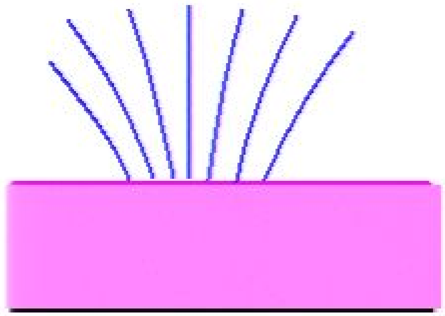
Recall $X^+ = \rho\, e^{\omega}$

Bound saturated when

$$\omega_* = \log \frac{R_s^3}{G\rho_0} \qquad t_* = \beta \log \frac{R_s^3}{G\rho_0}$$

How fast does information (charge, mass, temperature) spread over the horizon? Let's examine an electric charge falling onto a horizon.



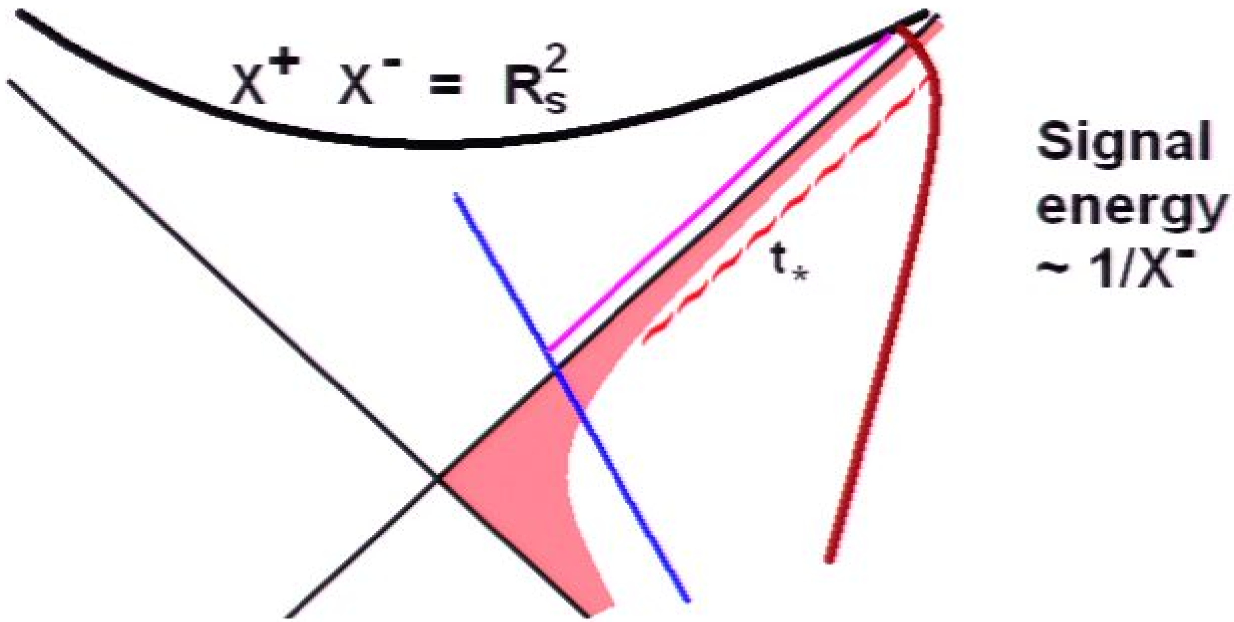


Radial electric field given by

$$E_\rho = \frac{Z-1}{[X^2 + Y^2 + (Z-1)^2]^{3/2}}$$

For large ω, on the stretched horizon this gives charge density

$$\sigma = \frac{\rho_0 \; e^{\omega}}{[X^2 + Y^2 + \rho_0^2 \, e^{2\omega}]^{3/2}}$$



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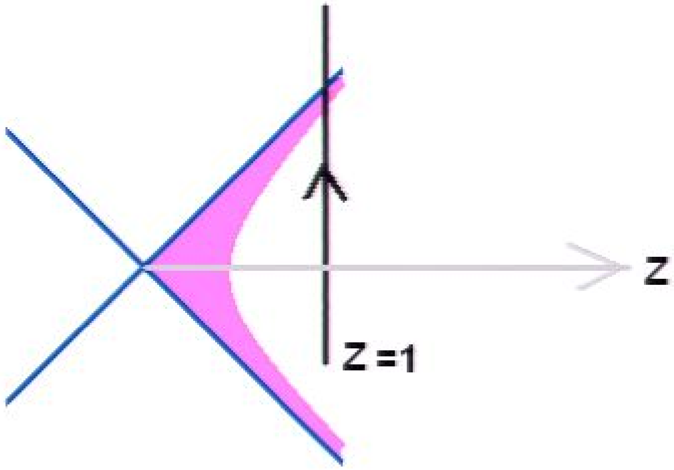

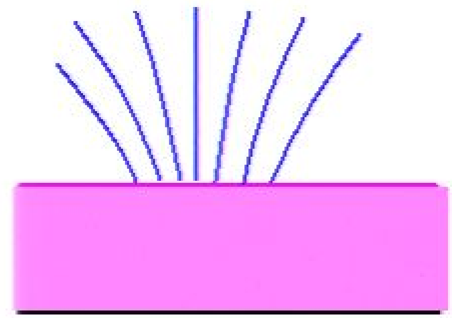

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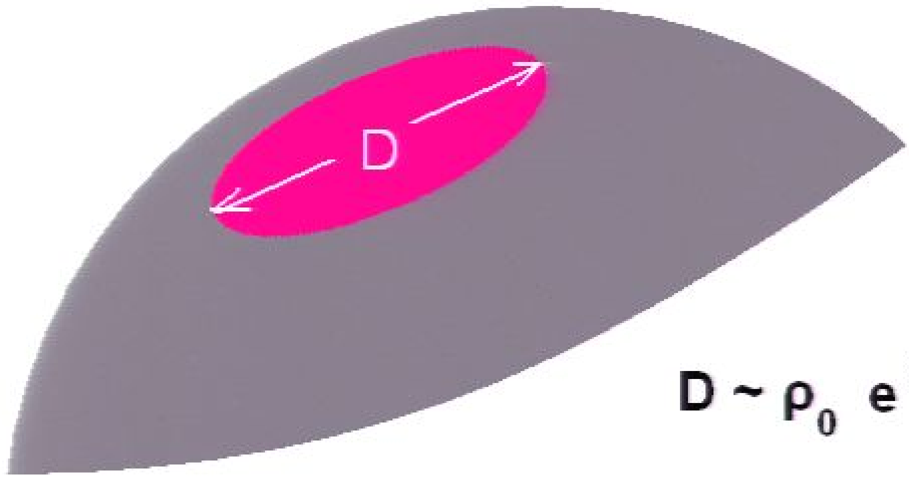


$$D \sim \rho_0 \; e^{2\omega}$$

Setting $\mathbf{D = R_s}$ gives

$$\omega_* = \log \frac{R_s}{\rho_0}$$

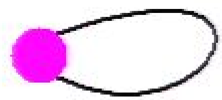
$$t_* = \beta \log \frac{R_s}{\rho_0}$$

$$t_* T = \hbar \log S$$

$$\frac{\partial \, t_* T}{\partial \log S} \geqq \hbar$$

Black holes are surely the fastest scramblers in nature. Consider a stellar mass black hole with a radius of about a kilometer and a temperature of .0000001degrees. Its scrambling time is .003 sec.

Concrete Hamiltonian? BFSS M(atrix) theory exactly describes a collection of N ten dimensional "D-particles" for large N.



X^a are quantum field operators for creating and anihilating a string attached to the DP. The string is "polarized" along the a axis where

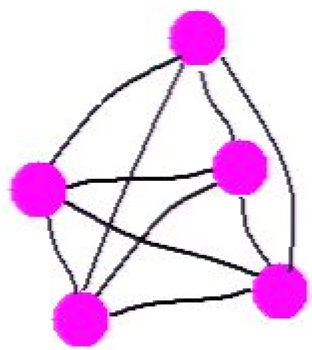
a = 1,...,9

X^a are the 9 spatial coordinates of the DP.

For slowly moving DP's

$$\mathcal{L} = \dot{X}^a \ \dot{X}^a$$

Now take an assembly of N D-particles



The "field operators" for strings connecting the m and n D-particles are (classical) matrices

$$X^a_{mn}$$

$$\mathcal{L}_{kinetic} = \dot{X}^a_{mn} \dot{X}^a_{mn} = \mathrm{Tr}\ \dot{X}^a \dot{X}^a$$

$$\mathcal{L}_{interaction} = \mathrm{Tr}\{X^a X^b X^a X^b - X^b X^a X^a X^b\}$$

Note: Every one of the $9N^2$ matrix elements is coupled to every other one.

$$X^b_{mn}\ X^a_{nl}\ X^a_{lr}\ X^b_{rn}$$

Add energy to D-particle system and it becomes a 10-dimensional non-extremal black hole with a Rindler horizon.

$$ds = f^{-1/2} \, [- (1 - R_H^7 / r^7)\, dt^2 \, \ldots\ldots]$$ Horowitz, Strominger

Using 10-D general relativity we can compute the horizon radius in terms of the temperature **T**, and the charge, **N**.

Then using the (by now) standard (Rindler) formula

$$\omega_* = \log \frac{R_H}{\rho_0}$$

one finds

$$t_* = \beta \log \frac{N}{\beta}$$

The dualities of String Theory require the M(atrix) Theory hamiltonian to be a fast scrambler.

In fact it has the correct properties:

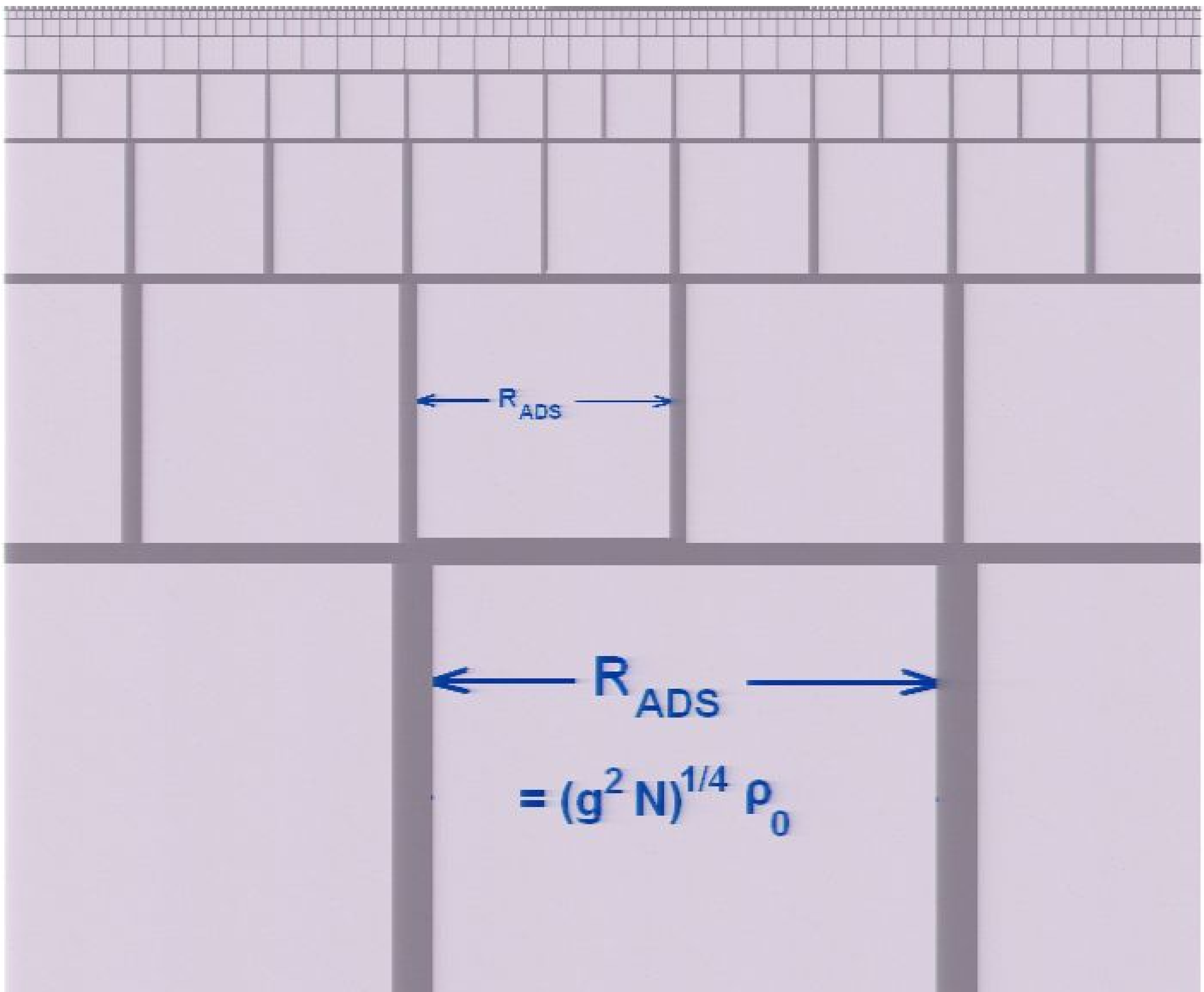
1) The individual terms in H involve at most 4 operators.

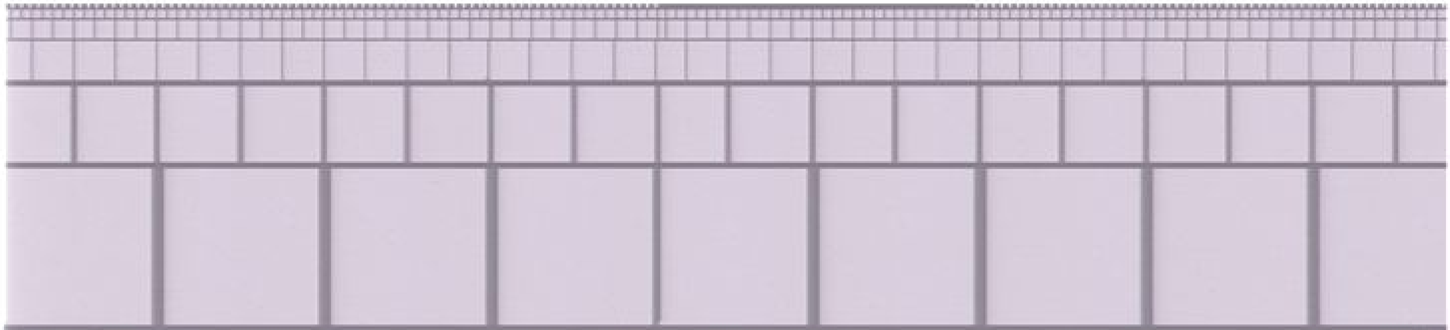
2) Every element is directly coupled to every other element.

But as usual in String Theory, no direct proof exists.

Scrambling in ADS/CFT

Each cutoff cell is described by N^2 degrees of freedom. ADS is a lattice of coupled matrix theories





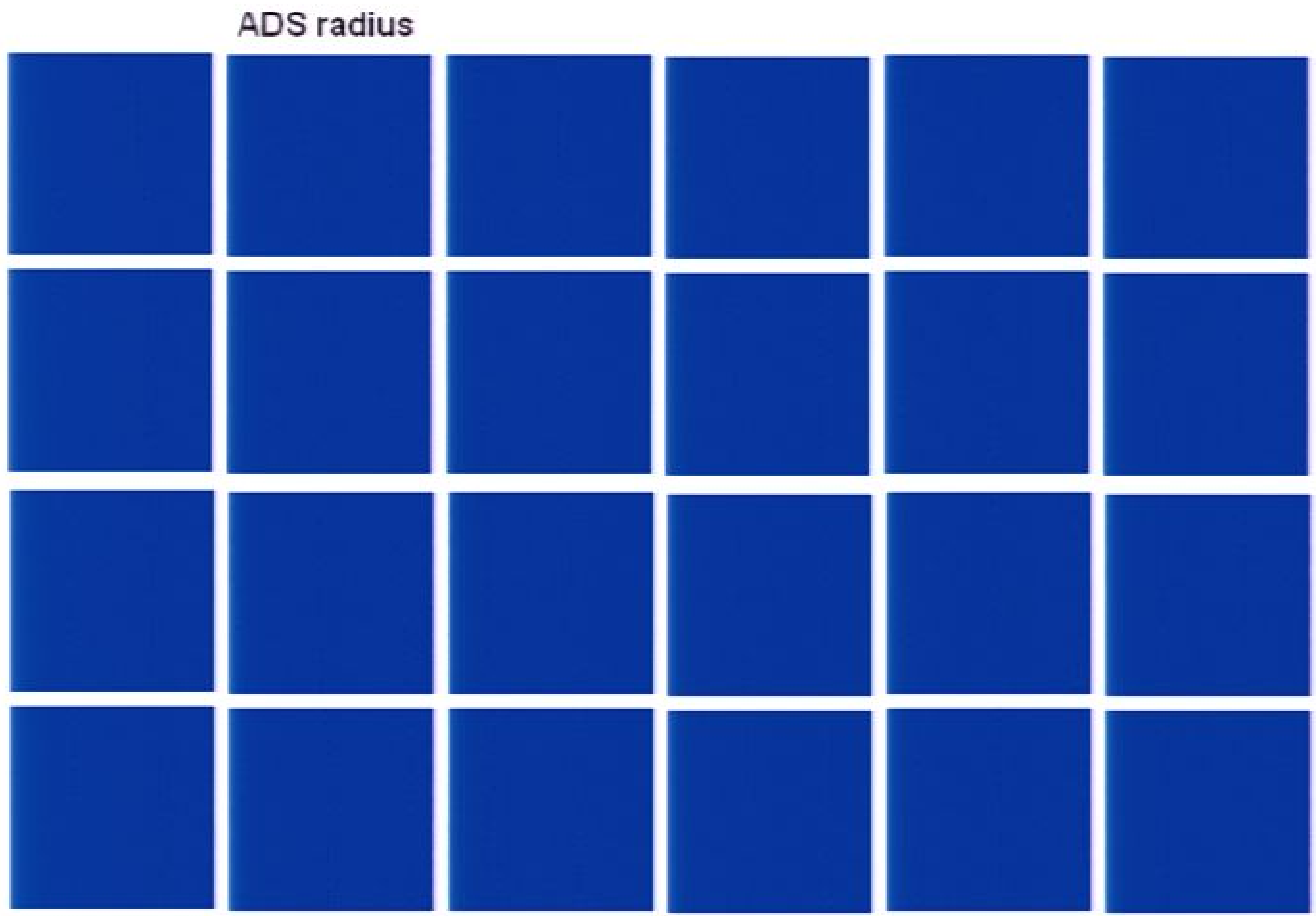
Using $t_* = \log R_{ADS}/\rho_o$ and

$R_{ADS}=(g^2N)^{1/4}$ one finds that a single

ADS volume is scrambled in time

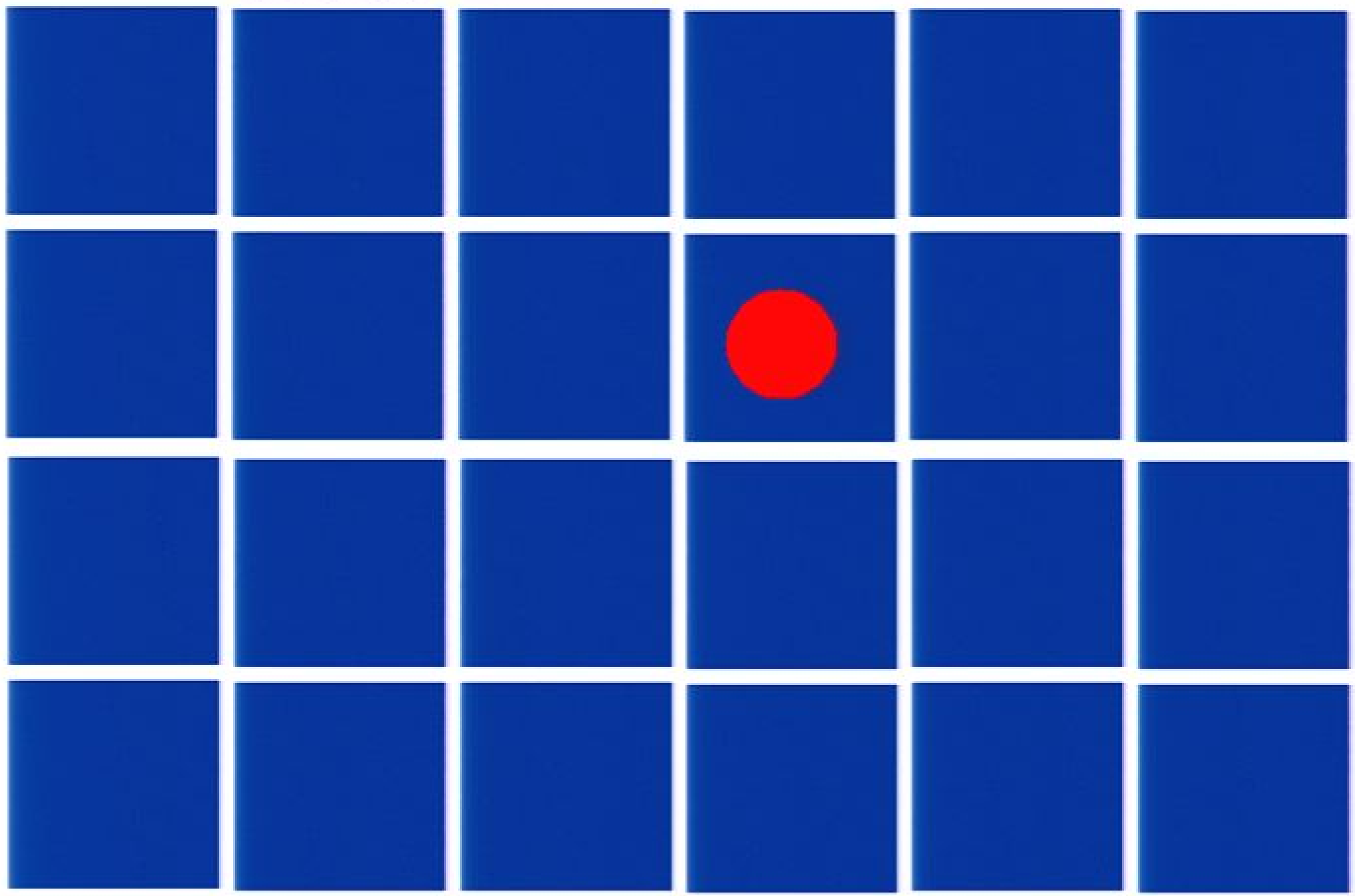
of order $t_* = \beta \log N$

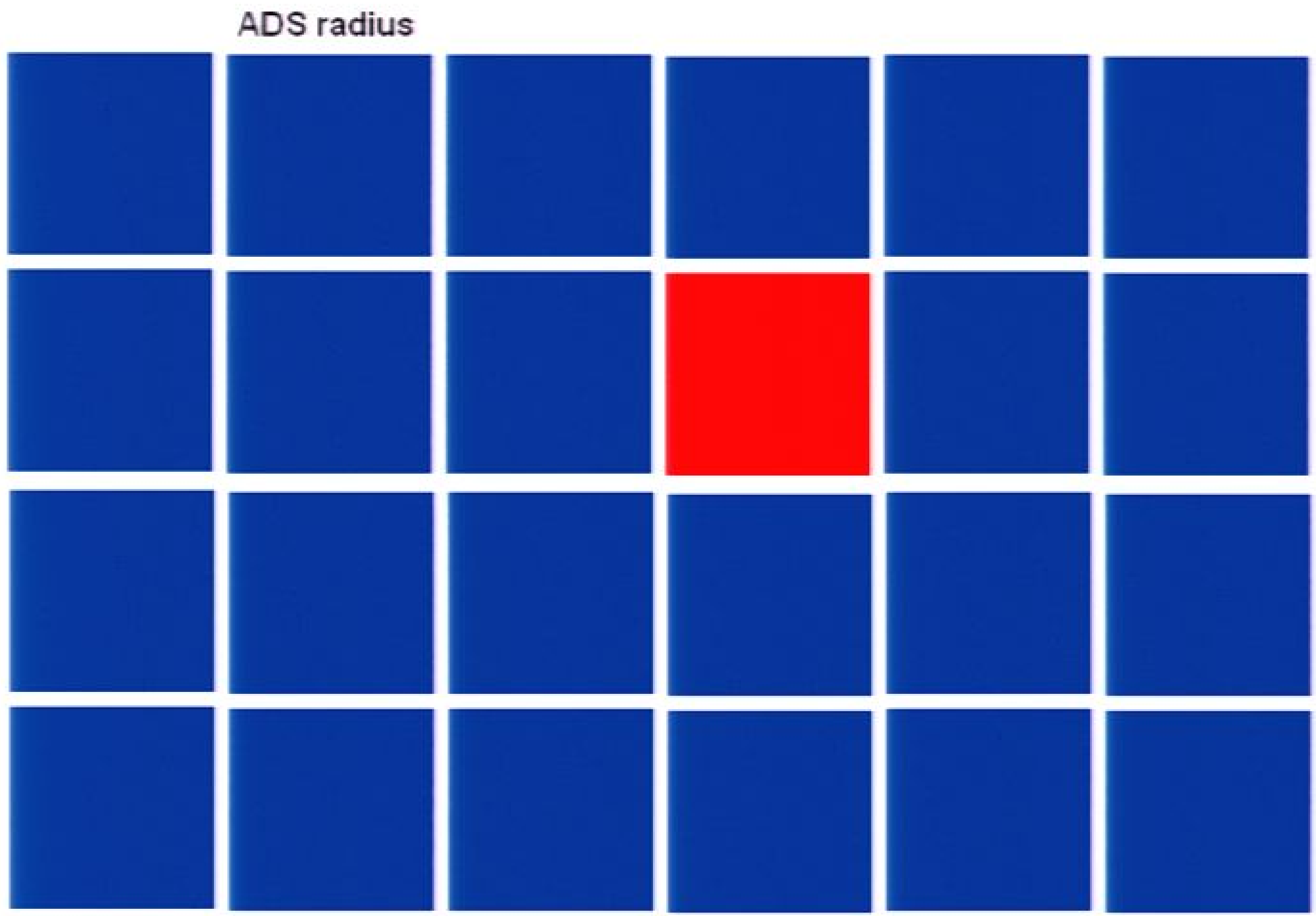
On larger scales the spreading is slow characteristic of a 3+1 dimensional system.

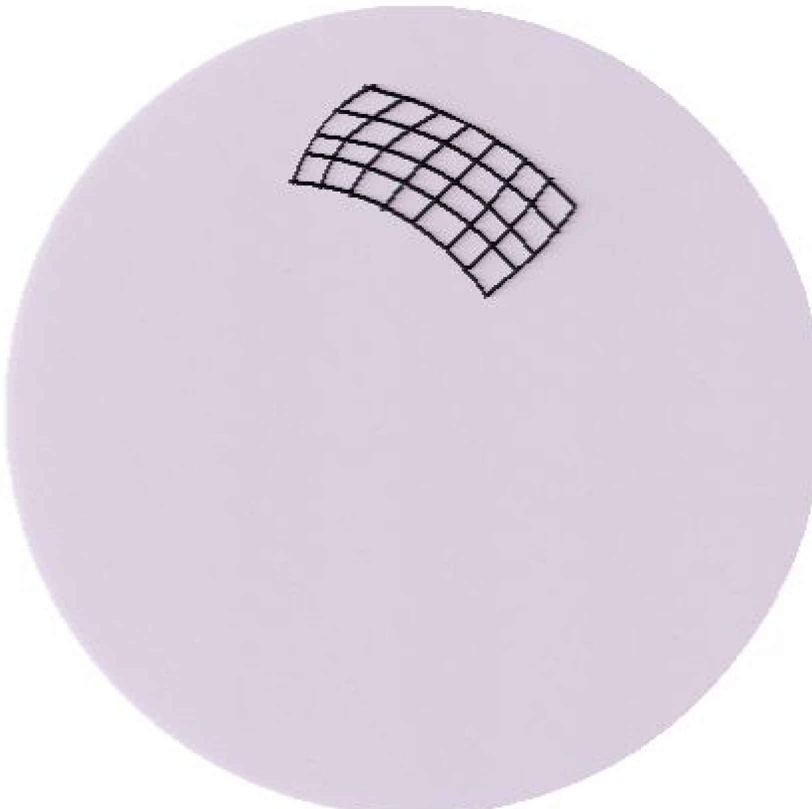


Looking down

ADS radius

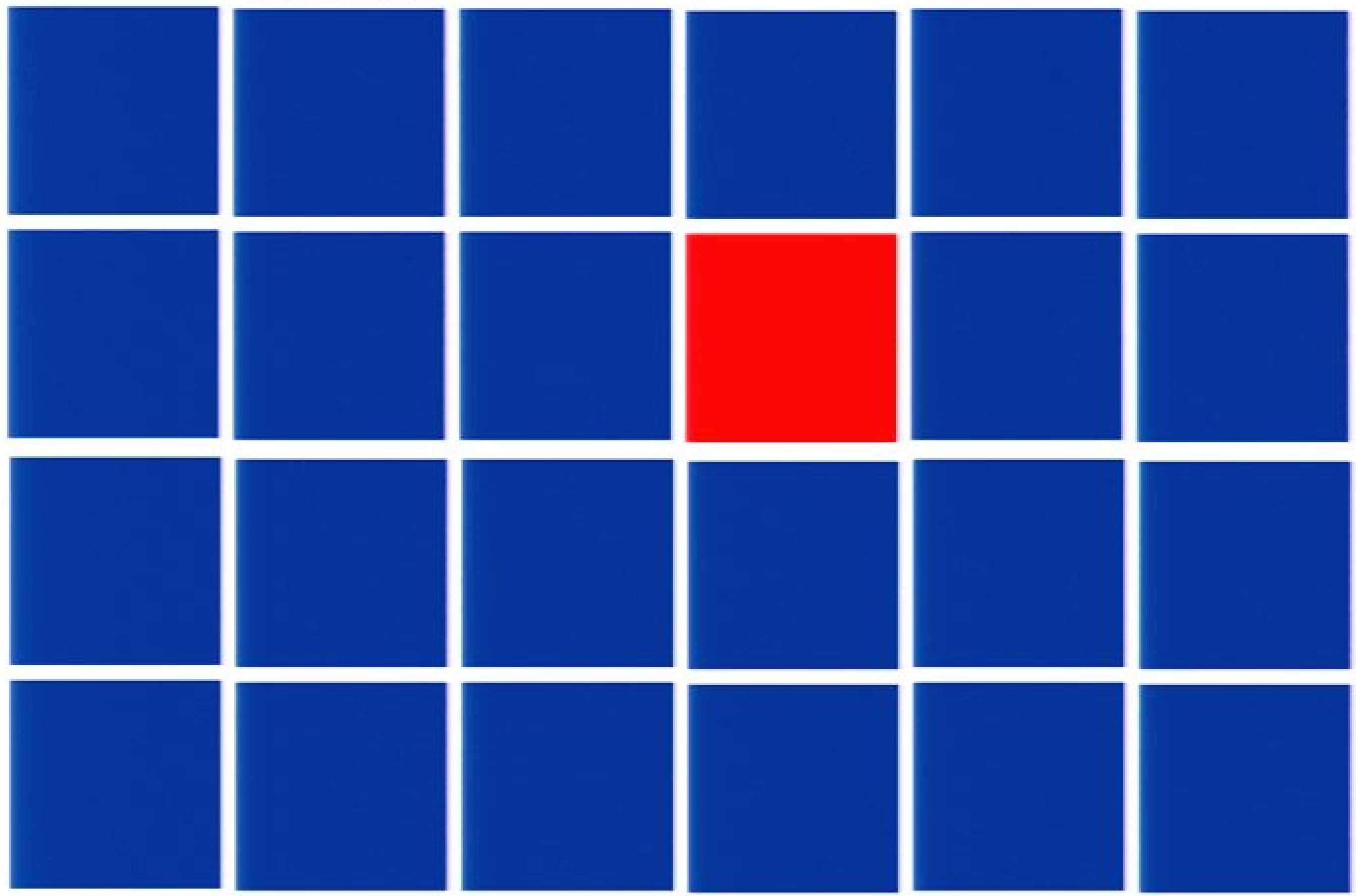



ADS radius

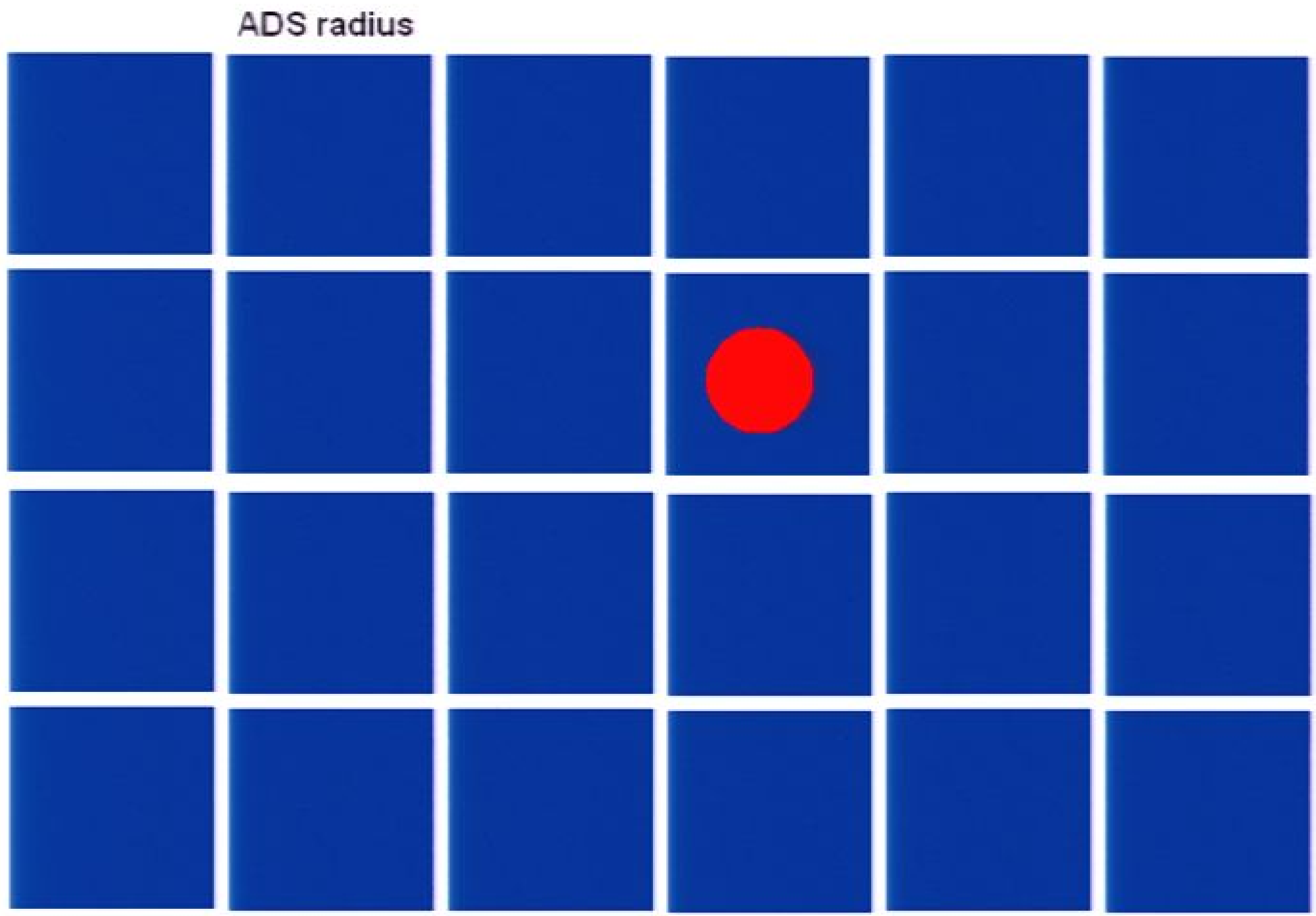


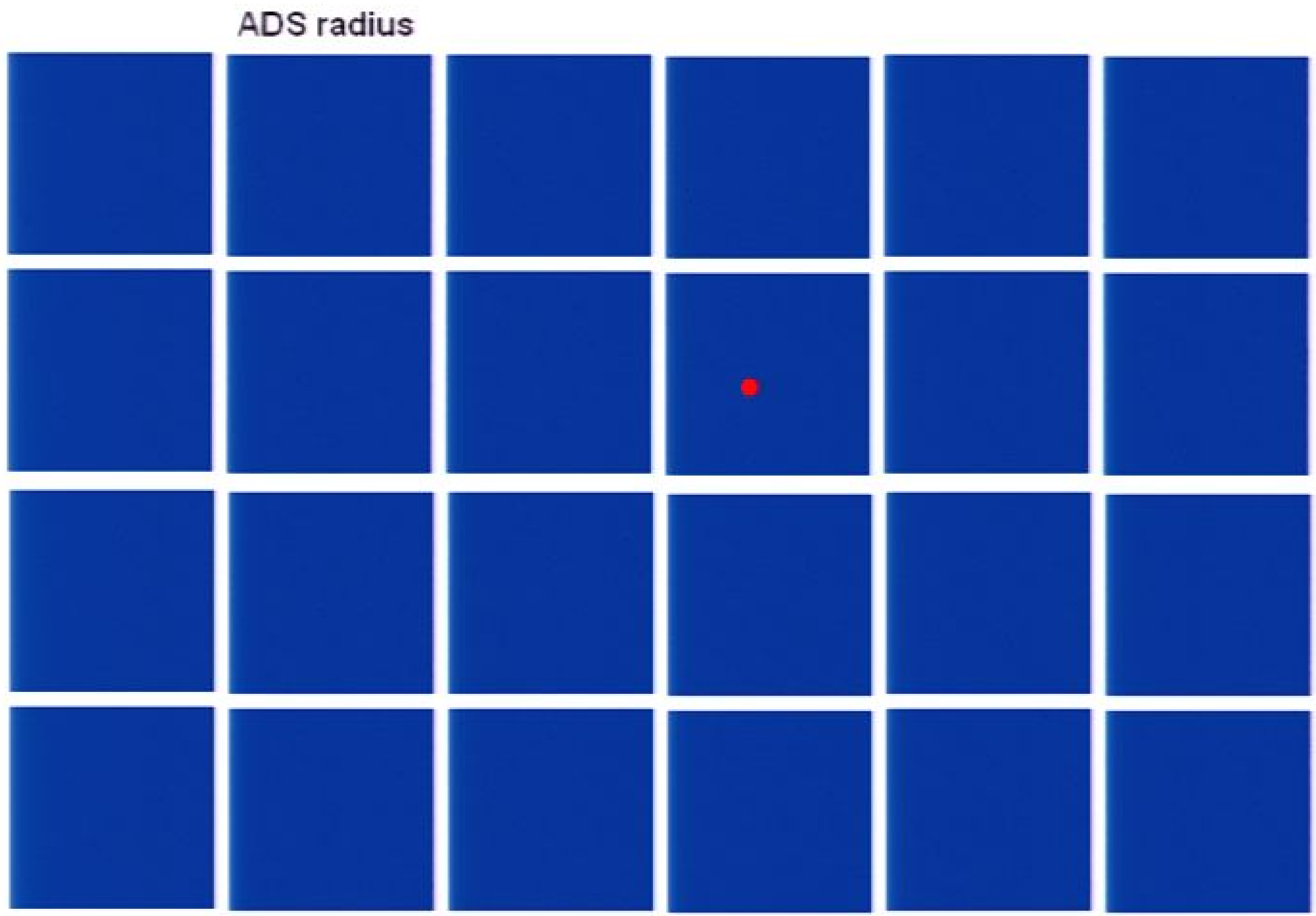
There can be no sense in which the horizon is a two dimensional collection of "pixels" interacting locally. Every pixel is coupled to every other pixel to create an ultra fast information scrambler. In a definite sense it is infinite dimensional.

ADS radius



Looking down at the horizon




ADS radius