Title: Optimal Non-Gaussian Estimators

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Abstract:

Optimal Non-Gaussian Estimators

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Motivation

- Pushing our understanding of high order statistics: Optimal, exact non-Gaussian estimators.
- CMB lensing (Hu 2002): 2+4 pt fcn. Local function (Lewis and Challinor 2006).
- f_{NL} (Komatsu, Spergel, Wandelt 2005): 3+6 pt fcn
- Simplifies because noise term (4,6 pt fcn) is Gaussian.
- What if life is more complicated? How can we quantify how much we can learn? (e.g. 21cm: Pen 2004, Zaldarriaga+Loeb 2004, Zahn and Zaldarriaga 2006, Lewis and Challinor 2007)
- Measure cosmological parameters to 10^{-8} accuracy?

Applications

- CMB is linear and clean, but only a 2-D map on the sky, with $N=l(2l+1)\sim 1,000,000$ modes.
- Limiting accuracy is $1/\mathrm{sqrt}(N) \sim 10^{-3}$.
- WMAP5 uses only CMB+BAO+SN: tiny fraction of the information in SDSS/2dF.
- 3-D structures (galaxies, lensing, 21cm, etc): many more modes, but how many are useful?
- Optimal searches with non-Gaussianity: lensing, BAO, strings.

Non-Gaussian Information

- How can we tell if a field is non-Gaussian?
- When it is non-Gaussian, how do we do optimal statistics?
- Many planned dark energy surveys are based on non-Gaussian structures – BAO, lensing, clusters. Information limited by non-Gaussianity.

Non-Gaussian Case Studies

- Non-Gaussian information saturation: Rimes and Hamilton
- Lensing: Information in the dark matter field
- Lensing of non-Gaussian sources: changes 2-pt/4-pt statistics

Information Propagation

- Measure some 2 pt statistic C(x,y)
- Find the dependence of C on your favorite parameters: $P(k,\Omega,w,w')$ or $C(\kappa,l,m)$
- Taylor expand around pivot point, and find minimum variance estimator. Fisher matrix gives errors and information.
- For Gaussian random fields and certain Baysian priors, this can be equivalent to max likelihood

Non-Gaussian Sources

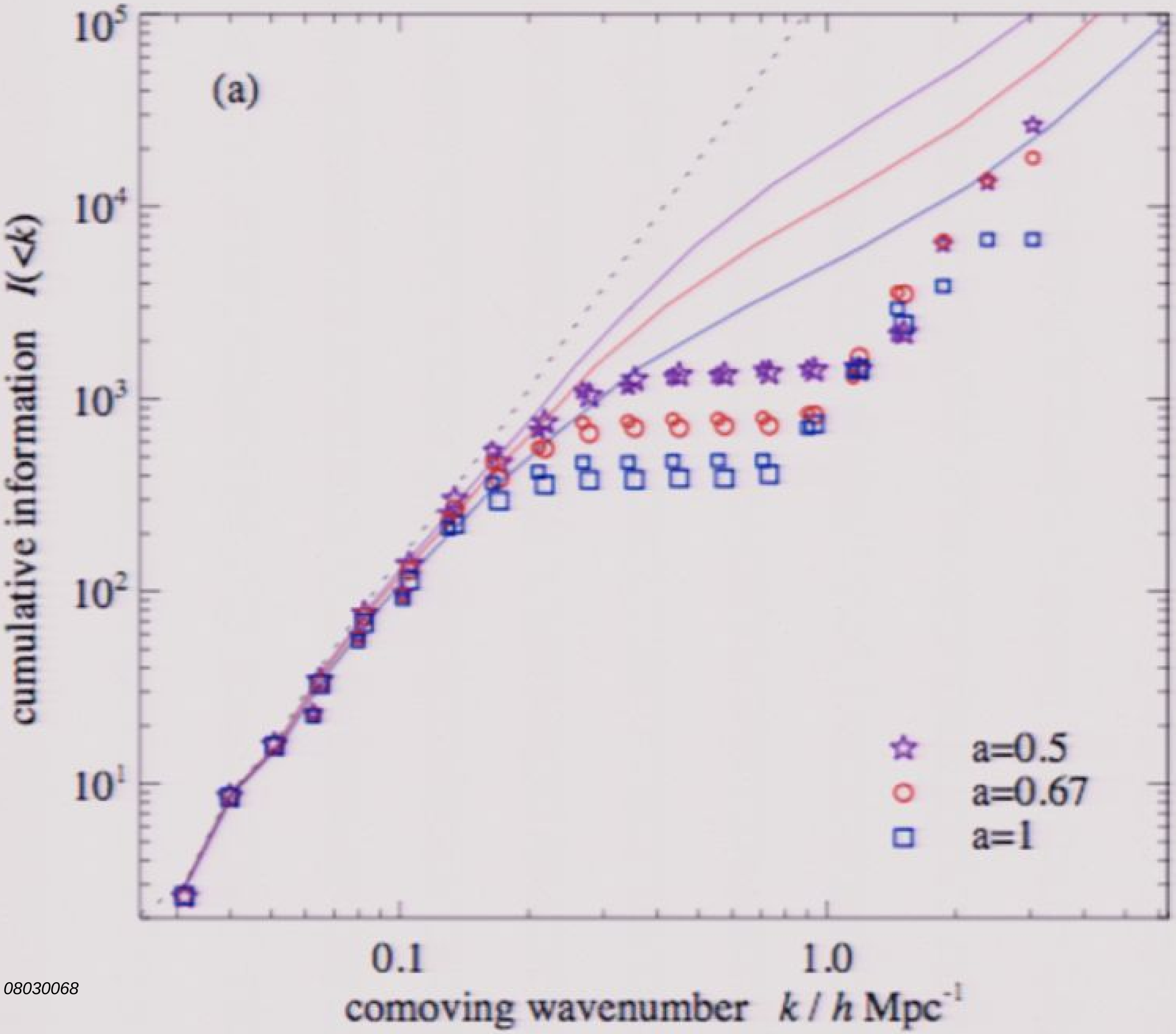
- Still have 2-pt statistics
- Minimum variance estimators are no longer derived from 2-pt+Wick.
- Need full covariance matrix of C, e.g. from simulation or data.
- May seem like daunting 4-pt function – too complicated?
- There may be more information in even higher point statistics, but that is even more daunting.

$$
\begin{aligned}
\vec{w} &\equiv \begin{pmatrix} w - w_0 \\ w' - w'_0 \\ \cdots \end{pmatrix} \\
P(k, \vec{w}) &= P(k, \vec{w}_0) + \frac{\partial P}{\partial w} \vec{w} + \vec{n} \\
\mathrm{A} \vec{w} &= \vec{y} + \vec{n} \\
\vec{w} &= \mathbf{B^{-1} A^t N^{-1}} (\vec{y} + \vec{n}) \\
\mathrm{B} &\equiv \mathbf{A^t N^{-1} A} \\
\mathbf{N} &\equiv \langle \vec{n} \vec{n}^t \rangle \sim \langle P(k) P(k') \rangle \\
\langle \vec{w} \vec{w}^t \rangle &= \mathbf{B^{-2}}
\end{aligned}
$$

- Just need to know **N**

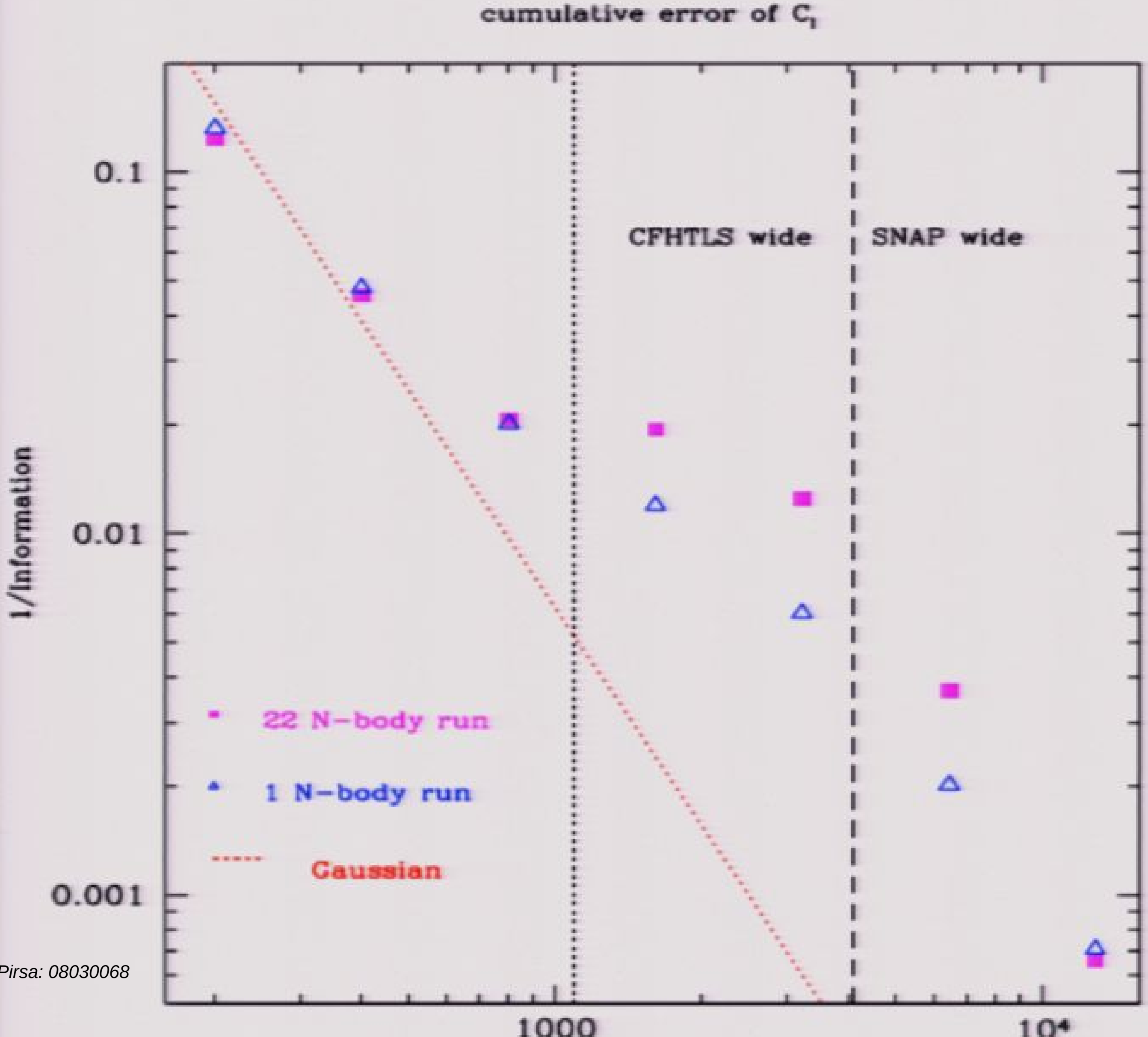
How much information?

- $F(k,k')=\langle P(k)P(k')\rangle$
- Depends on two 3-D vectors: 4-pt function with two points in one place.
- In general, $F(k,k',\cos(\theta))$: for Gaussian fields, is δ function in θ
- Legendre transform to diagonalize theta dependence -> $F(k,k', l)$: for Gaussian, independent of l
- Measure from simulations in 3-D, and propagate!
- Most observations (lensing, BAO) only need $l=0,2$



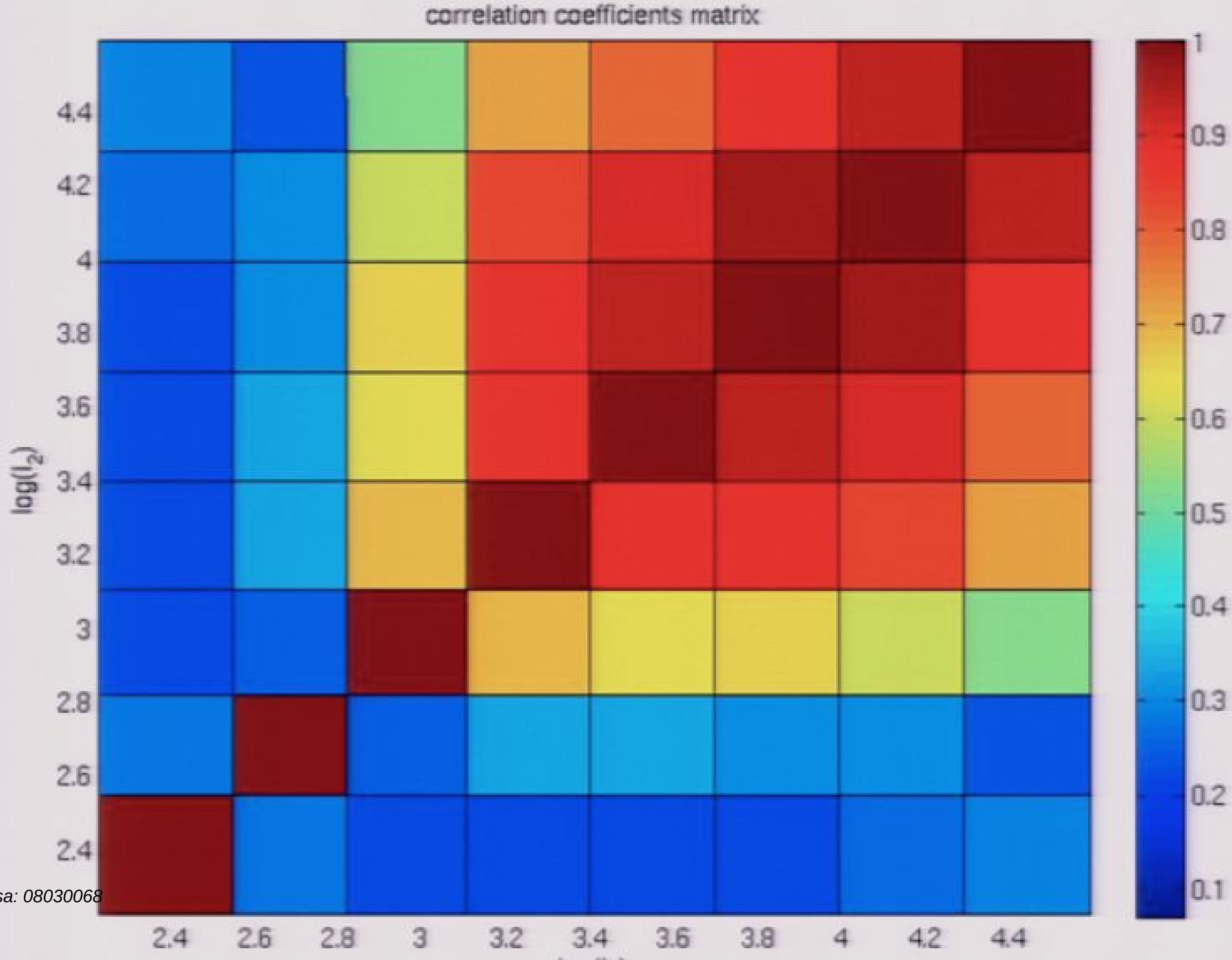
Rimes and Hamilton (2005): The cumulative Fisher information has a translinear plateau.

Intrinsic Lensing Noise

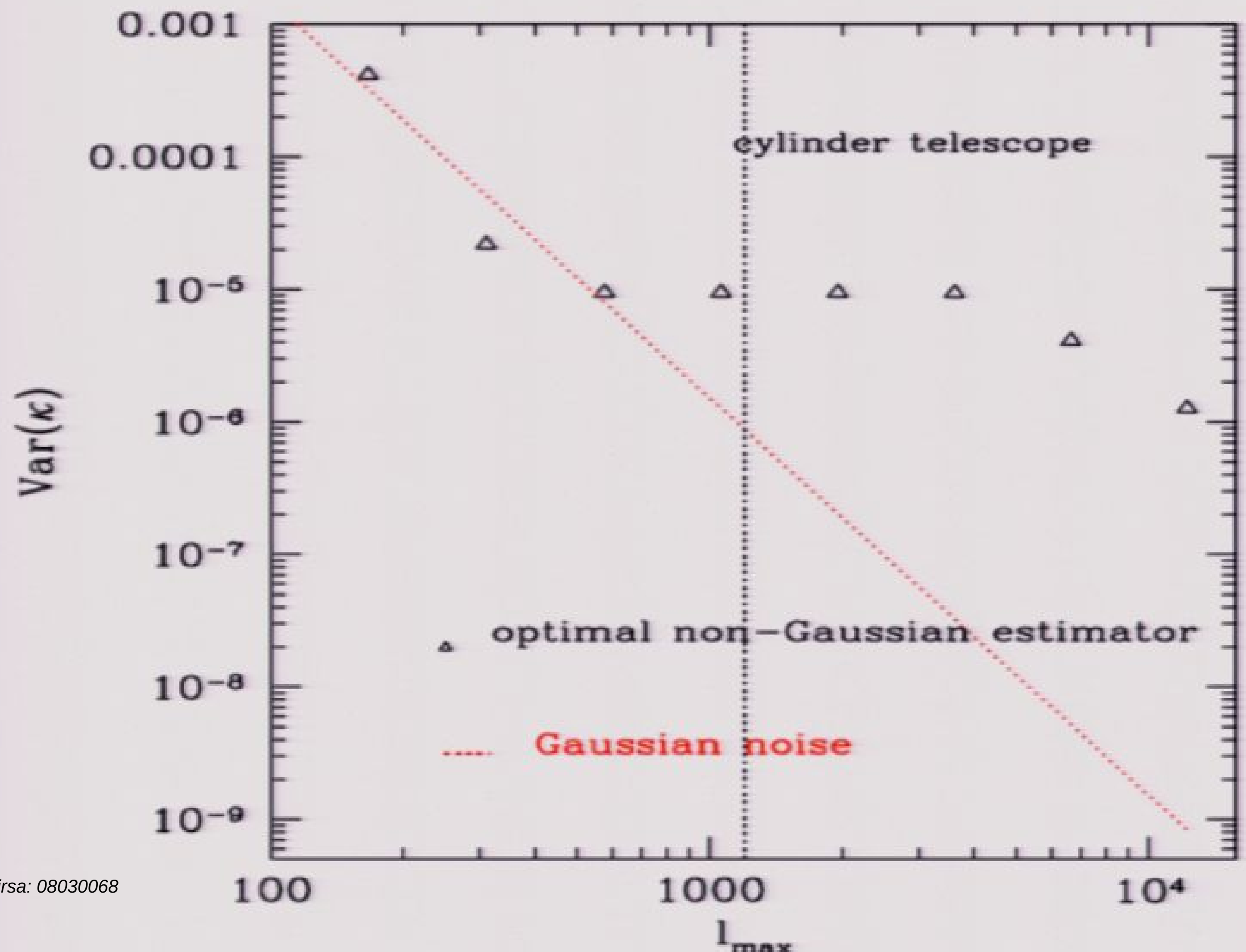


- Lu et al 2008 (in prep).
- Lensing translinear plateau at l~500.

Lensing Fisher

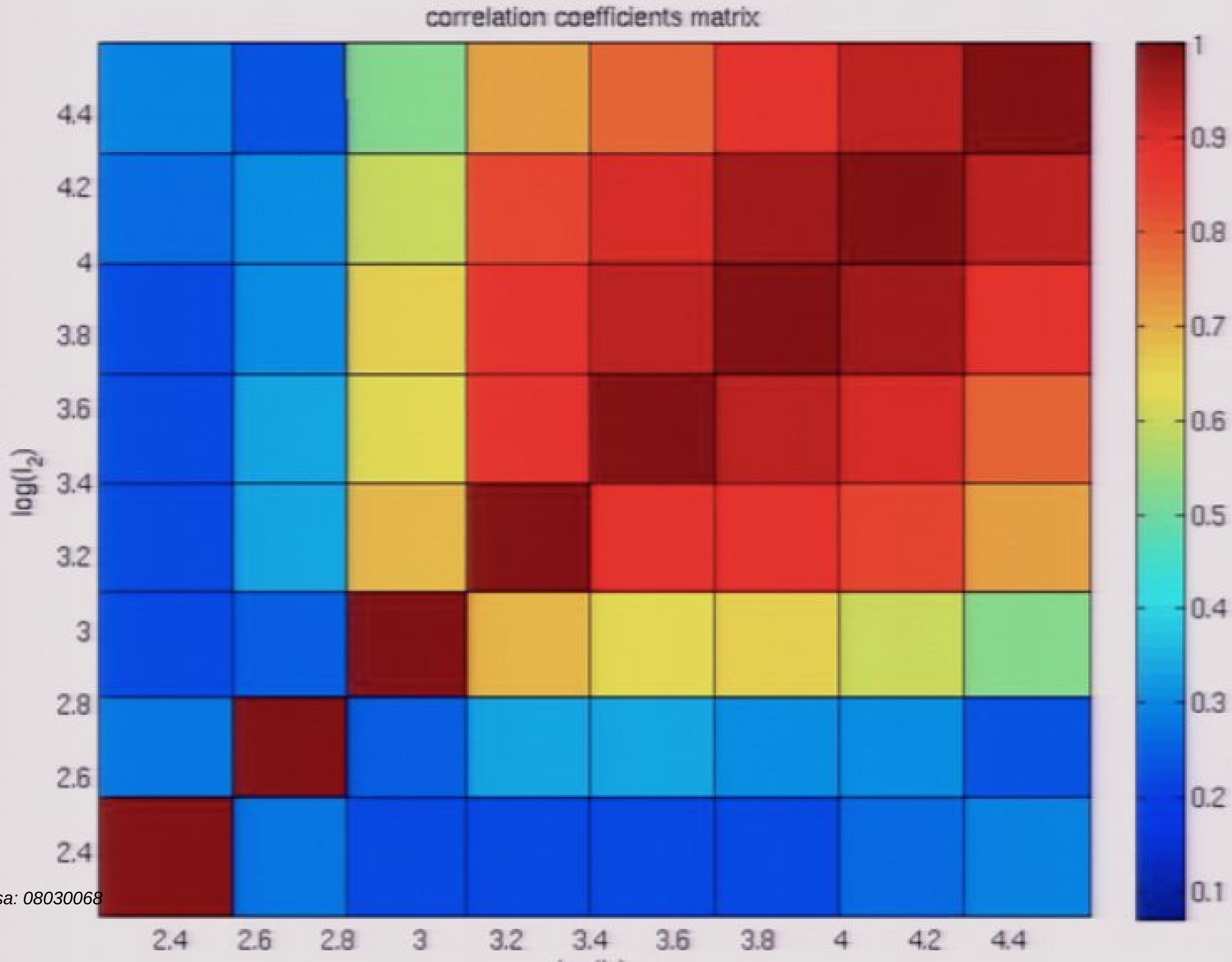


Extrinsic Lensing Noise

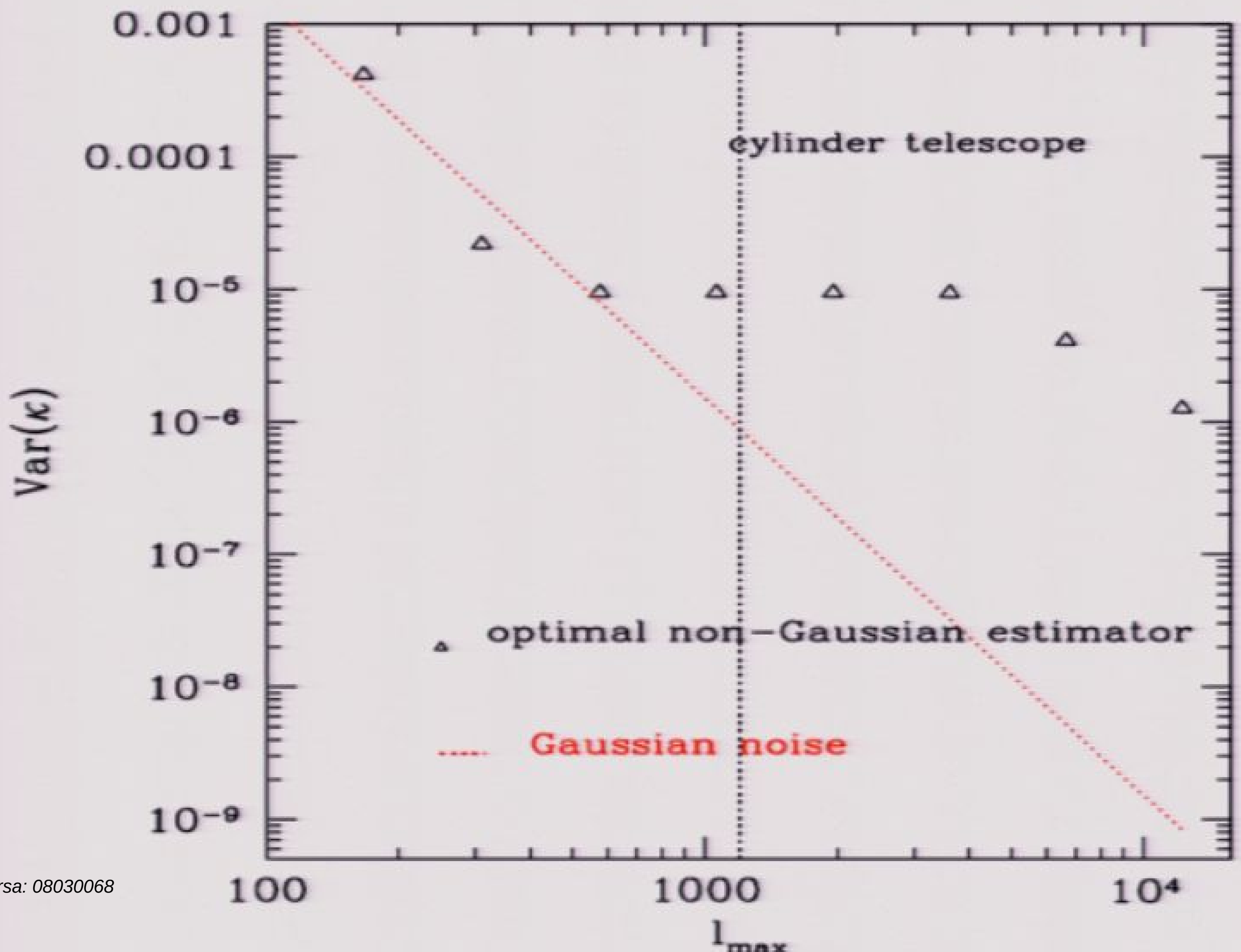


Lu et al 2008 (in prep)

Lensing Fisher



Extrinsic Lensing Noise



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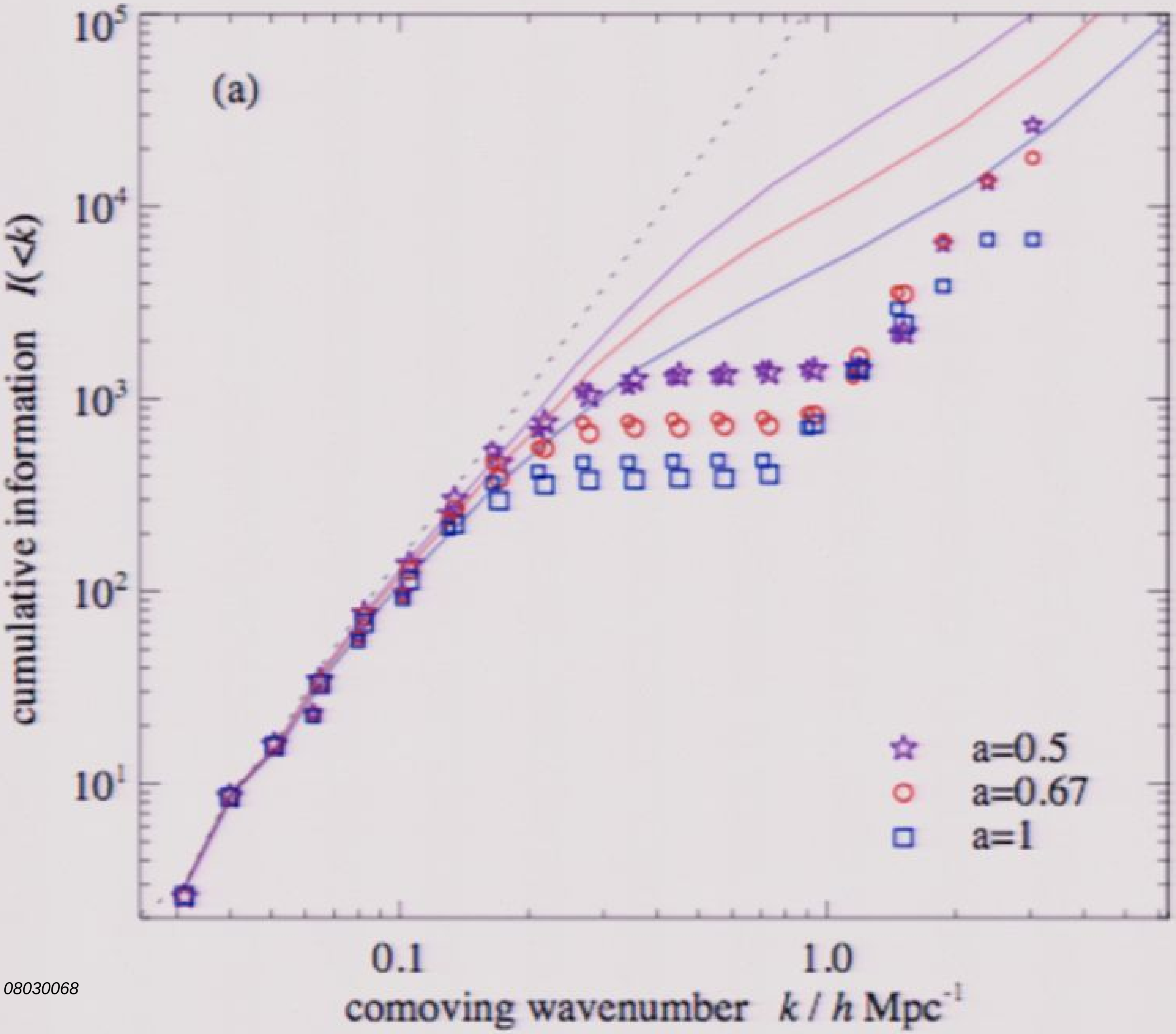
Summary

- Optimal non-Gaussian quadratic estimation solvable: 4-pt statistics. Applicable to lensing (intrinsic+extrinsic noise), BAO, etc.
- We really only need two matrices from simulations, to get all fisher propagates from there: F(k,k',l=0,2)
- Information saturation: standard 2-pt has much less information for non-Gaussian sources. Impact on lensing, BAO?
- Future 21cm surveys contain a lot of information, but less than one may think.



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