Title: Features in the Primordial Bispectrum

Date: Mar 10, 2008 04:00 PM

URL: http://pirsa.org/08030067

Abstract:

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Eugene A Lim (Columbia)
Xingang Chen, Richard Easther

Perimeter Institute Conference on Non-Gaussianity
March 10 2008

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Features in the Primordial Bispectrum : Beyond f_{NL}

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Outline

- "Justify your existence!"
- Describing general primordial non-Gaussianities
- $f_{NL}^{local}, f_{NL}^{equil}$ and their malcontents
- Tools for computing general primordial non-Gaussianities
- Generating non-Gaussianities : horizon-scale and subhorizon-scale

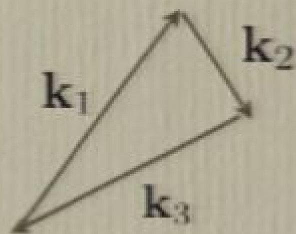
Justify your Existence!

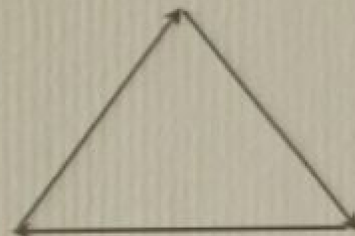
- CMB data is getting better, we are on the threshold of detecting/have already detected non-Gaussian signatures in the sky.
- Non-Gaussian signatures provide to us a way to distinguish inflationary models.
- What we need is to develop a set of analytic and numerical tools to compute *useful forms* of non-Gaussianities for every single model out of inflation.

Describing 3-pt correlation functions : Shape, Scale and Amplitude

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle$$

Shape
described by
2 angles

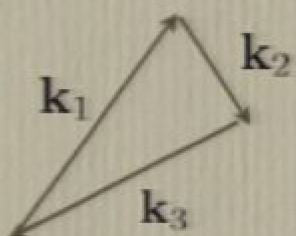




Equilateral

"Squeezed"

Scale
described by
otal momentum
$= |\mathbf{k}_1| + |\mathbf{k}_1| + |\mathbf{k}_3|$



Same *Shape*,
different *Scale*

If a 3-pt correlation is *scale-invariant*, then it is completely described by 2 angles.

Amplitude Describes the value given shape and scale $\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle \sim P_\zeta^2$

Statistical Isotropy allows us to reduce $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle \to \langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle$

Momentum Conservation $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)F(k_1, k_2, k_3)$

Scale Invariance $F(\lambda\vec{k}_1, \lambda\vec{k}_2, \lambda\vec{k}_3) = \lambda^{-6}F(\vec{k}_1, \vec{k}_2, \vec{k}_3)$

Why f_{NL}^{local} ?

- In a sense it is the most simple minded thing you can do

$$\Phi(\mathbf{x}) = \Phi_g(\mathbf{x}) + f_{NL}^{local}\Phi_g^2(\mathbf{x})$$

Salopek & Bond (90)
Komatsu & Spergel (01)

$$F^{local}(k_1,k_2,k_3) = f_{NL}^{local}P_k^2\left(\frac{1}{k_1^2k_2^2}+\frac{1}{k_1^2k_3^2}+\frac{1}{k_2^2k_3^2}\right)$$

- This form has nice properties that make it easy to work with, mostly due to the fact that the fundamental field $\Phi(\mathbf{x})$ is itself gaussian.

- Making f_{NL}^{local} type non-Gaussianities : large scale evolution of the curvature perturbation.

E.g. Multifield uncoupled inflation : $\zeta(\mathbf{x}) = \frac{d\log(a)}{d\phi}\delta\phi(\mathbf{x})_i + \frac{d^2\log(a)}{d\phi_i d\phi_j}\delta\phi(\mathbf{x})_i\delta\phi(\mathbf{x})_j + ...$

- Single field slow roll is local in the squeezed limit : physically non-G is generated around horizon crossing. Maldacena (2003), Acquaviva et al (2003)

Why f_{NL}^{equil} ?

Babich et al (2004)
Creminelli et al (2005)

- It is defined by setting the normalization of equilateral triangles to some fixed value.

$$F^{equil}(k_1,k_2,k_3) = f_{NL}^{equil} \cdot 6P_k^2\left(-\frac{1}{k_1^3k_2^3} - \frac{1}{k_1^3k_3^3} - \frac{1}{k_2^3k_3^3} - \frac{2}{k_1^2k_2^2k_3^2} + \frac{1}{k_1k_2^2k_3^2} + perm.\right)$$

- Assuming scale-invariance, this is roughly speaking orthogonal to the local shape.

- Its symmetry allows construction of simple estimators.

Beyond f_{NL}

- Breaking scale-invariance : all hell breaks lose.

$$F(\lambda\vec{k}_1, \lambda\vec{k}_2, \lambda\vec{k}_3) = \lambda^{-6} F(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

- *But why Eugene*?! Doesn't scale-invariant power spectrum guarantees scale-invariant 3-pt? No!

- More philosophically : CMB data allows us to constraint models of inflation. We can't constrain them if we do not know what they predict!

The "in-in" formalism

- The formalism we use is the Schwinger-Keldysh "in-in" formalism introduced by Maldacena (2003) into the analysis of higher point correlation functions
- Earlier works : Jordan (1986), Calzetta + Hu (1987) etc. Re-introduced to modern cosmological perturbation theory by Weinberg (2005).
- Aside : really is QFT on a dynamical background jigged to compute *correlation functions at fixed time* instead of transition amplitudes between different asymptotic states.

The Master Formula

- The correlation function

$$\langle\zeta(\mathbf{x}_1)\zeta(\mathbf{x}_2)\zeta(\mathbf{x}_3)\rangle = \langle in|\bar{T}\exp\left(i\int_{\infty}^{t} H_I(t)dt\right)\zeta^I(\mathbf{x}_1)\zeta^I(\mathbf{x}_2)\zeta^I(\mathbf{x}_3)T\exp\left(-i\int_{\infty}^{t} H_I(t)dt\right)|in\rangle$$

- For tree 3-pt correlation function

$$\langle\zeta(\tau,\mathbf{k}_1)\zeta(\tau,\mathbf{k}_2)\zeta(\tau,\mathbf{k}_3)\rangle = -i\int_{\tau_0}^{\tau} d\tau'\; a\;\langle[\zeta(\tau,\mathbf{k}_1)\zeta(\tau,\mathbf{k}_2)\zeta(\tau,\mathbf{k}_3), H_I(\tau')]\rangle$$

Interaction Hamiltonian $H_I = \int dx^3 \sum_i a^2 g_i(\epsilon,\eta,\eta')\xi_1\xi_2\xi_3$

Heisenberg "free fields"

interaction couplings : govern non-Gaussianities

- Advantage (numerical) : we just need to solve the *linear* equations of motion

- Advantage (analytical) : we can import (some of) our intuition from QFT into the analysis.

Shut up and Integrate!

- No mention of inflation : the formalism is valid for all spacetime backgrounds!
- Only early-time boundary conditions needed to be imposed : crucial advantage of "in-in".

In principle we can integrate through horizon crossing (or not), radiation domination, horizon reentry etc until it hits our eye

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(i) Single field inflation, perturbations freeze-out and all interactions vanish.
(ii) Evolution outside horizon generates non-gaussianities of the local type (spatial derivative interactions vanish) : does not distinguish between isocurvature/adiabatic types.

(iii) See also David Seery's talk!

2-Prong approach : Analytical and Numerical

- (Semi-)Analytical derivation of 3-pt
  - Factorizable form very useful for constructing bispectrum estimators $F(k_1, k_2, k_3) = f_1(k_1) f_2(k_2) f_3(k_3) \times \sin(k_1 + k_2 + k_3) \times \frac{1}{(k_1 + k_2 + k_3)^n}$

    Smith & Zaldarriaga 06
  - understanding the physics behind what's going on
- Numerical Methods
  - exact forms (for use in non-bispectrum base statistical tools like Minkowski Functionals?)
  - amazingly useful tool cross check with analytical approximations : scaling with parameters, gross features etc.
  - on principle : we should be able to do this!

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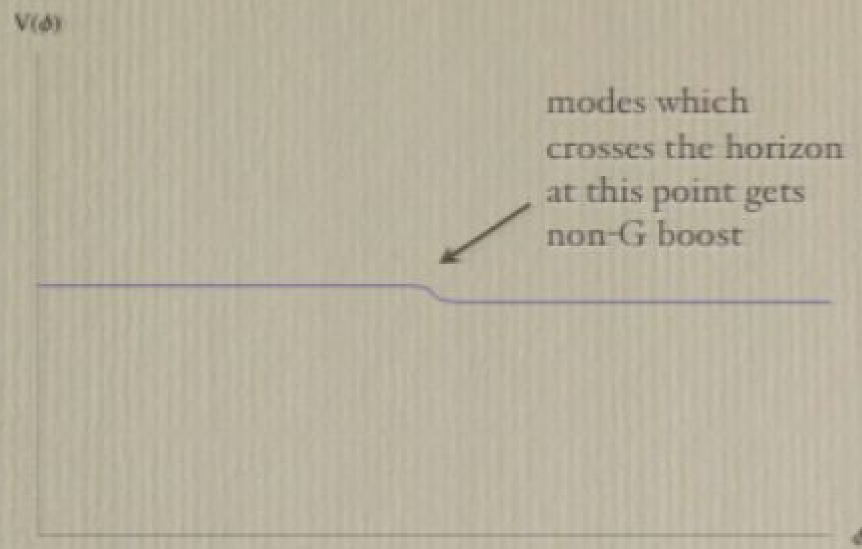
Generating Mechanisms

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = iu_1(\tau_e)u_2(\tau_e)u_3(\tau_e)\int_{\tau_{-\infty}}^{\tau_e} d\tau a^2(\epsilon^2+\epsilon\eta')u_1^*(\tau)u_2^*(\tau)u_3^*(\tau)(2\pi)^3\delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})$$

- Inflation likes to smooth things out : encoded in the small slow roll parameters.
- The forces of non-linearities : *gravitational collapse and direct interaction couplings (repulsive/attractive).*
- We classify mechanisms into two broad classes : horizon-scale (bump) and sub-horizon scale (resonance).

Horizon-scale Generation : "bump" models

Bumps will induce *temporal* breaking of slow-roll, and *all modes that crosses the horizon* at this time will get a large boost.



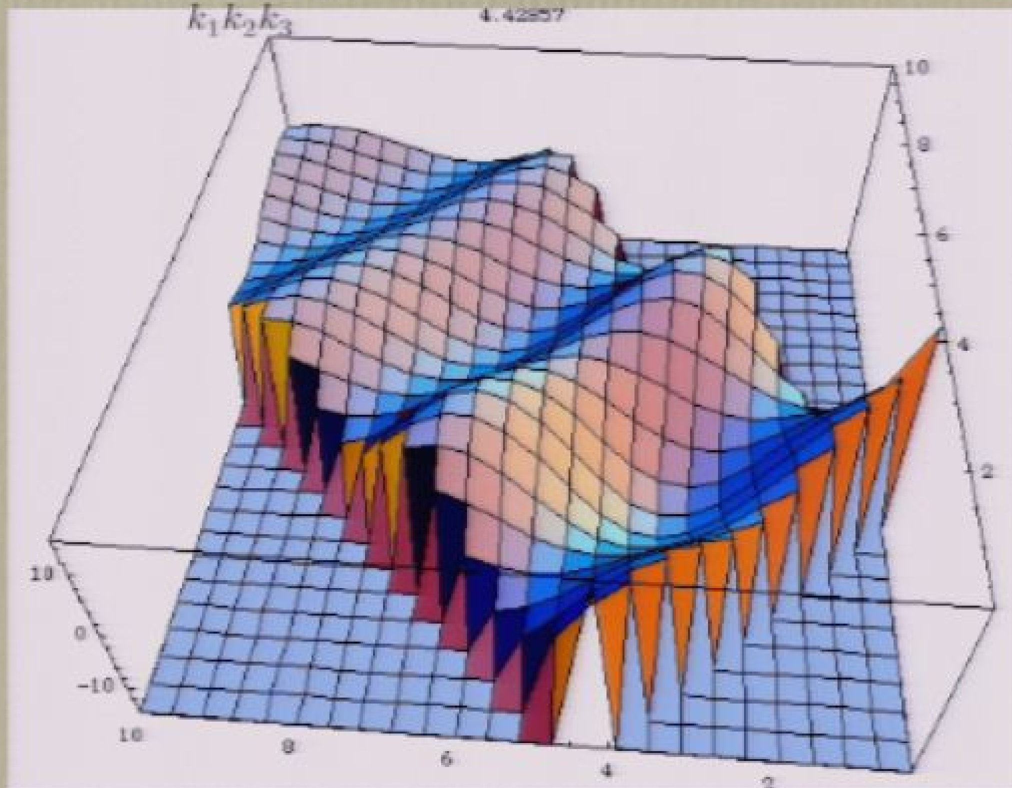
Physics : during a bump $\eta' \gg 1$, hence interaction coupling is large. Subhorizon modes do not get the boost because their oscillatory timescale << the timescale of the bump.

Only selected modes are boosted : very *scale-dependent.*

Models : $V(\phi) = \frac{1}{2}m^2\phi^2\left[1 + c\tanh\left(\frac{\phi - \phi_{step}}{d}\right)\right]$ "step" model (Adams et al 2001, Covi et al 2006, Chen et al 2006)

$V(\phi) = \frac{1}{2}m^2\phi^2\left[1 + c\cosh\left(\frac{\phi - \phi_{step}}{d}\right)\right]$ "dimple" model

"Step" Bi-spectrum in 3D!



Movie shows $\frac{G(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{k_1 k_2 k_3}$ for $0.1 < (k_1, k_2, k_3) < 10$

k_1 and k_2 are *x* and *y* axes, while k_3 is the movie time.

The maximum value for $\frac{G(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{k_1 k_2 k_3}$ is *O(1000)* times more than slow roll inflation : possibly observable with PLANCK (Chen, Smith, Easther, Komatsu, Lim, Peiris.)

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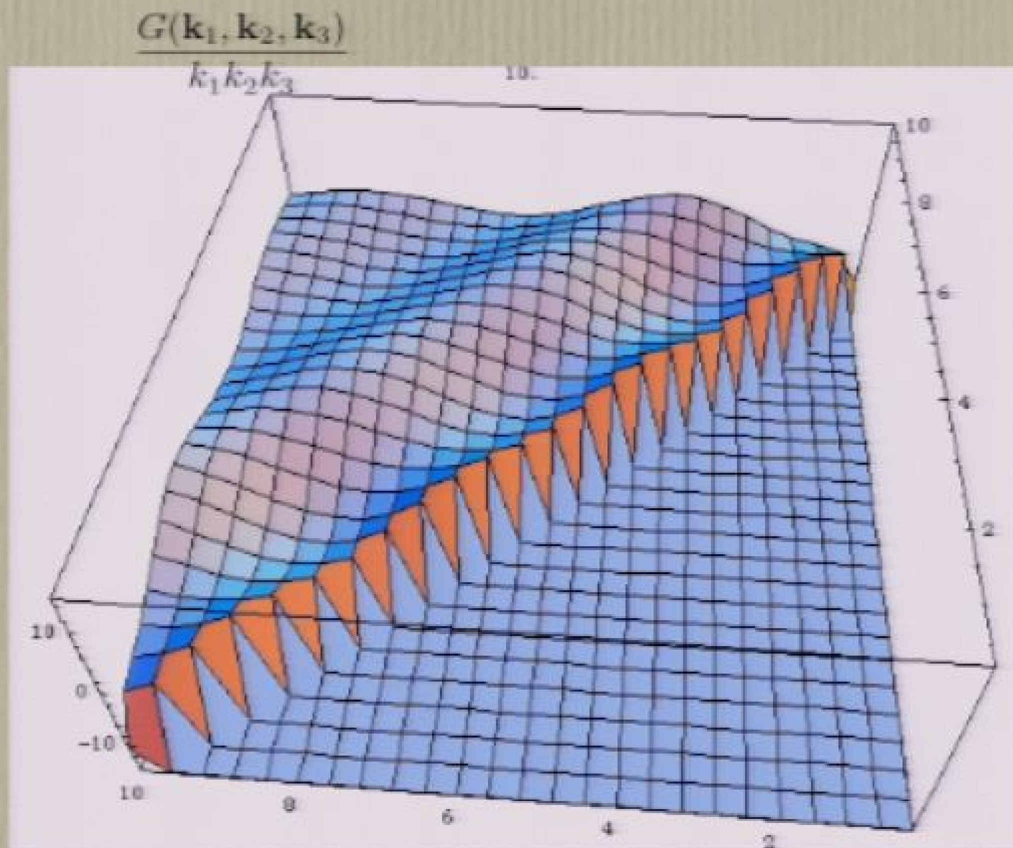

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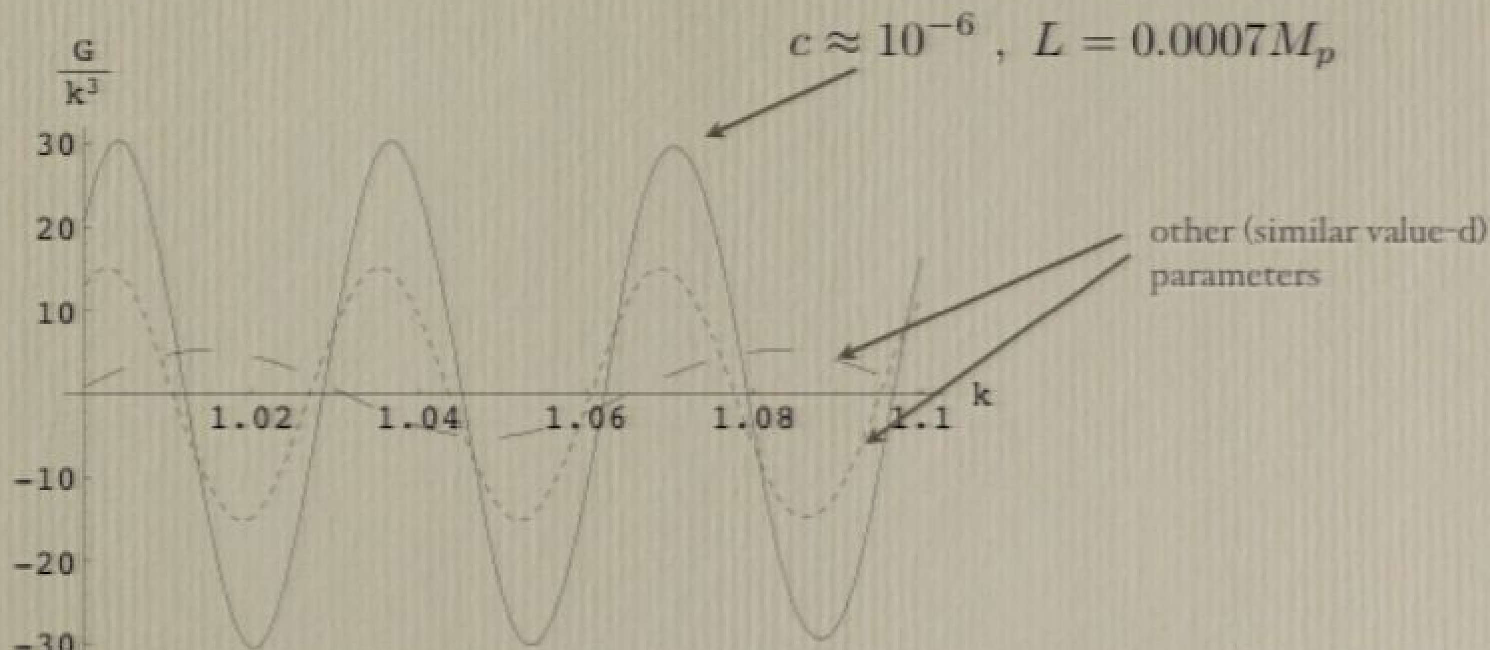
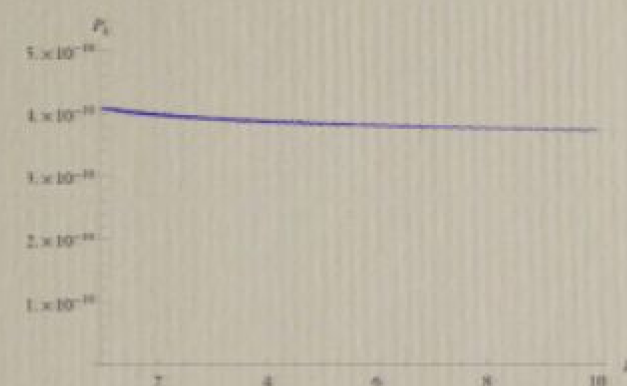
Resonance model : large non-decaying non-Gaussianities

Movie available on request!



Since c is so small : *tiny undetectable change in the power spectrum when compared to plain models!* The point is : we can only distinguish between this and plain vanilla slow roll models via their 3-pt correlation function.



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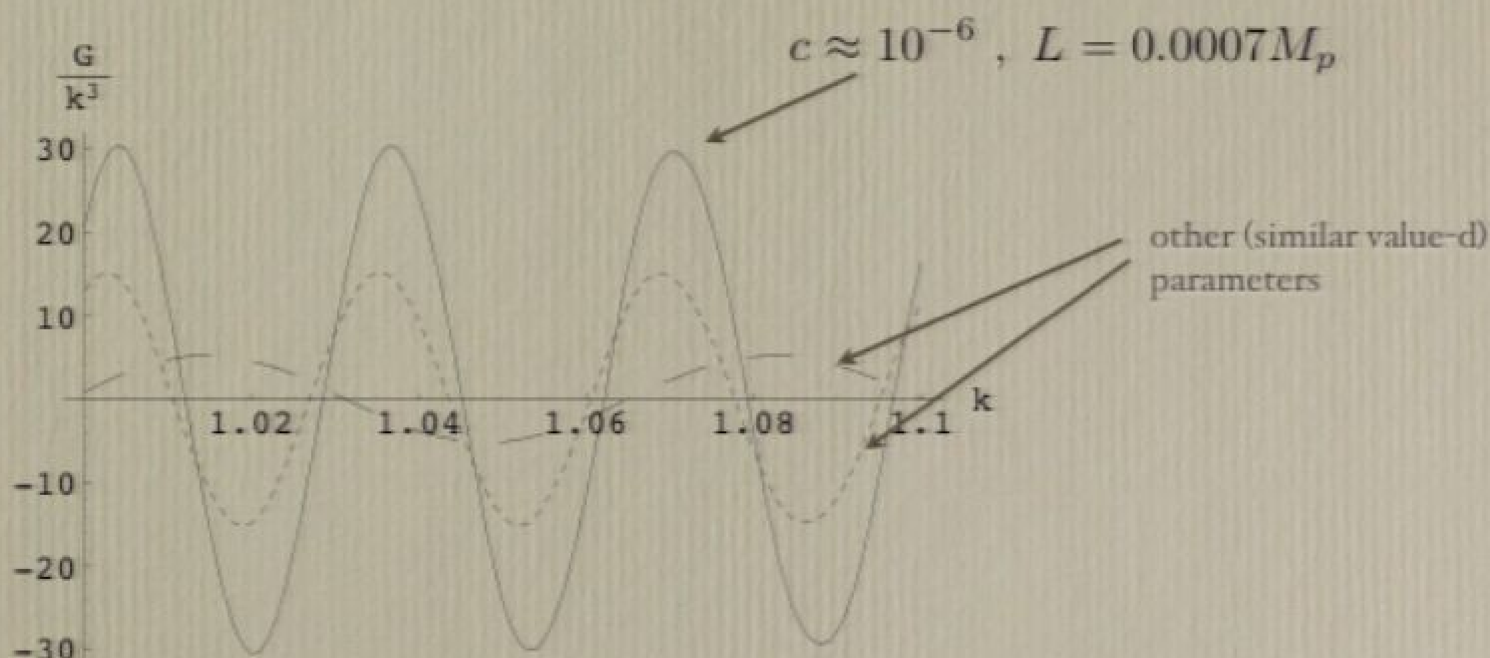

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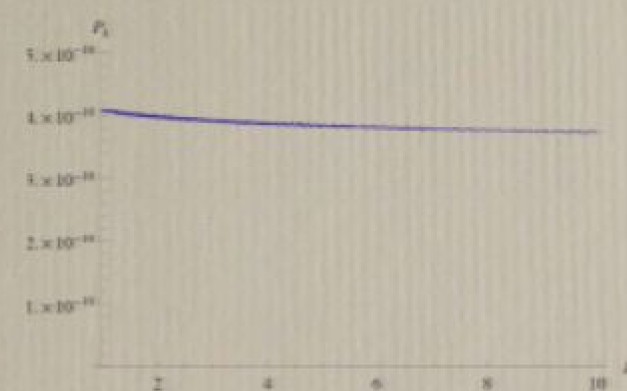
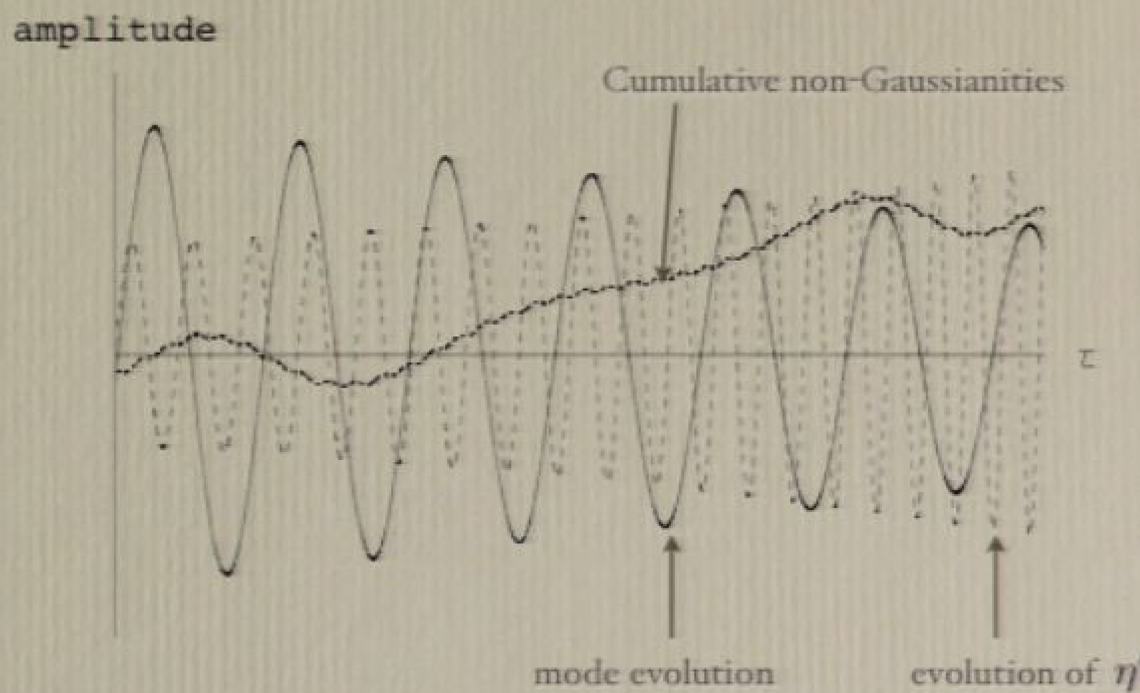


Resonance model : mechanism

Simple model :

$$V(\phi) = \frac{1}{2}m^2\phi^2\left[1 + c\sin\left(\frac{\phi}{L}\right)\right]$$



Modes resonate for a few oscillations, *well within the horizon* :

$$\omega \sim 0.1M_p \times \frac{H}{L} \approx 0.02M_p \gg H \ , \ L \approx 10^{-4}M_p$$

Since all modes redshifts, *they will all undergo resonance at least once* : large *non-decaying* non-Gaussianities are generated.

This is an example of a model with a *frequency* scale.

Resonance model : large non-decaying non-Gaussianities

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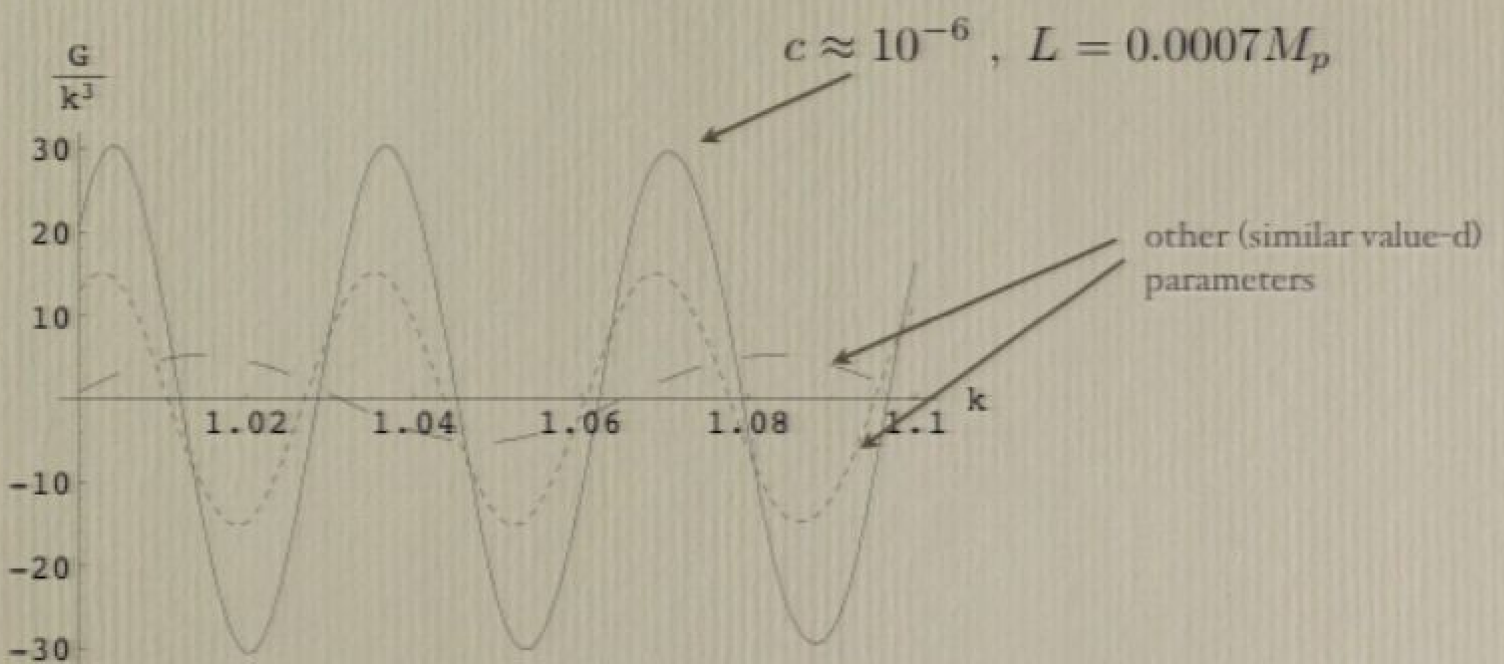

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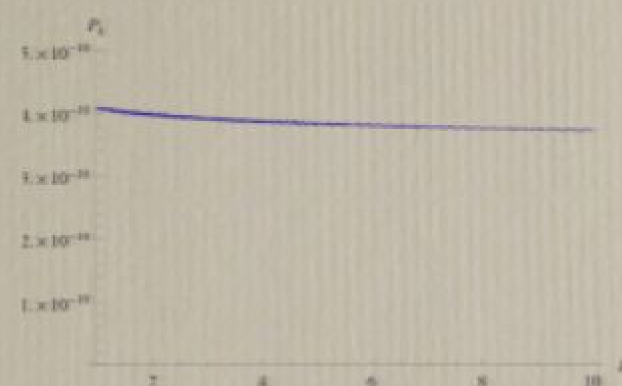

Analytical Estimate

- Frequency Scale : $\omega \sim 0.1 M_p \times \frac{H}{L} \approx 0.02 M_p \gg H\ ,\ L \approx 10^{-4} M_p$

  $m = 3 \times 10^{-6} M_p\ ,\ c = 5 \times 10^{-7}\ ,\ L = 0.0007 M_p\ ,\ \phi \approx 15 M_p$

- $\epsilon \approx 0.01$ and $\dot{\eta}_{oscl} \approx 10^{-4} \sin(\omega t)$, and we can estimate the number of oscillations to be $\mathcal{O}(10)$.

$$F^{res}(k_1, k_2, k_3) = f_{res} P_k \frac{1}{\Pi k_i^3} \sin\left(\frac{2}{\phi L} \ln K + phase\right)$$

$$f_{res} = \frac{9}{4} \frac{c}{L^{2.5} \phi^{0.5}}$$

- Factorizable once we expand around some fiducial K .

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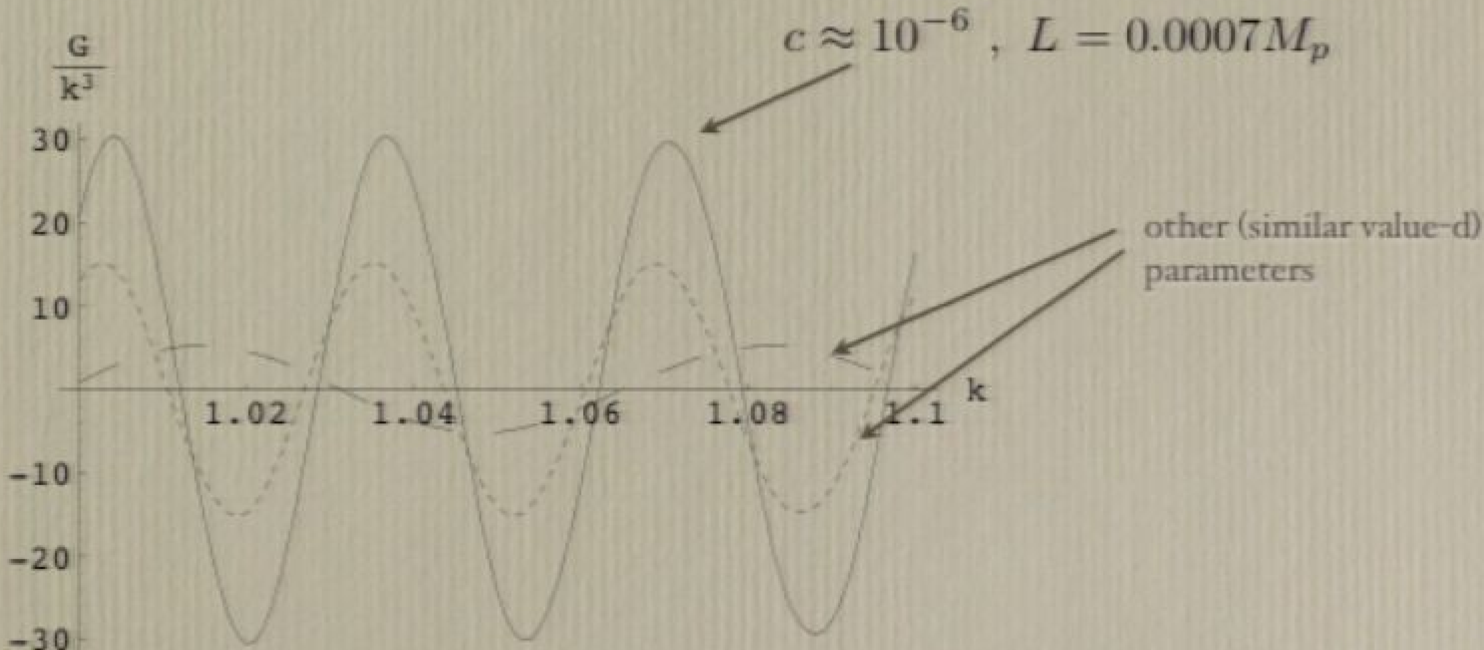

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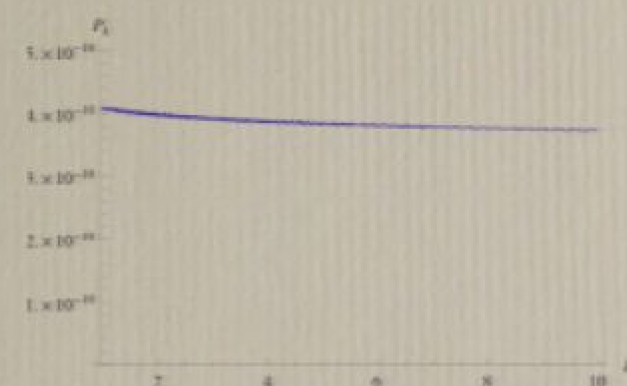

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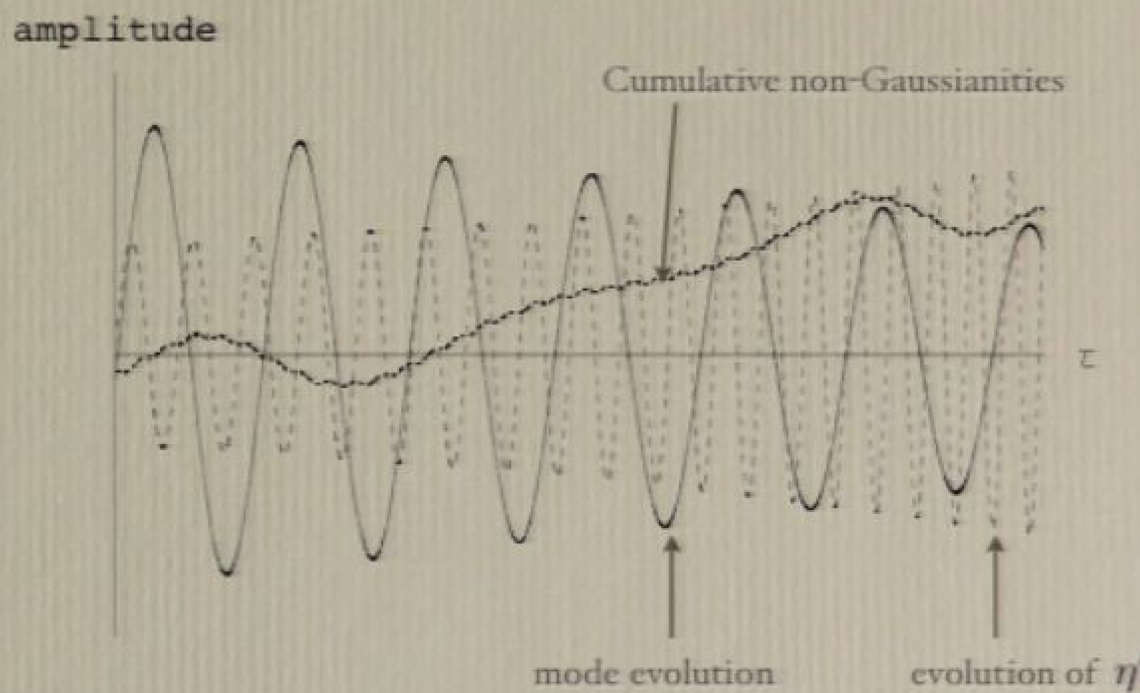

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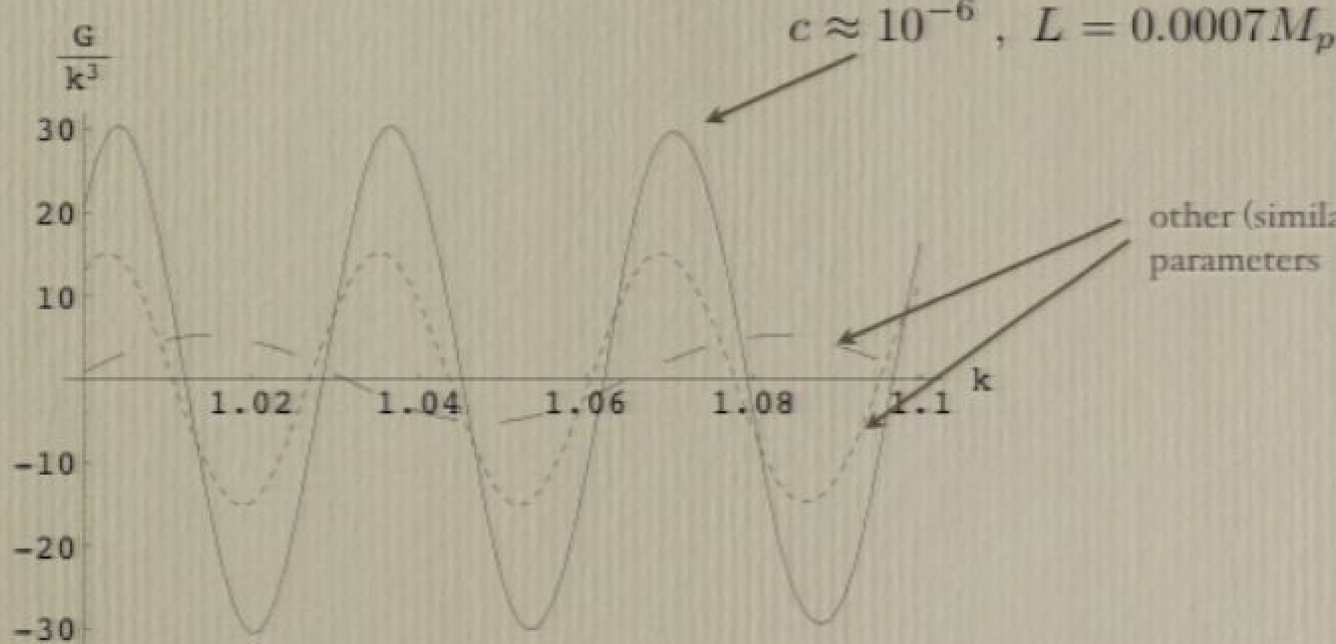

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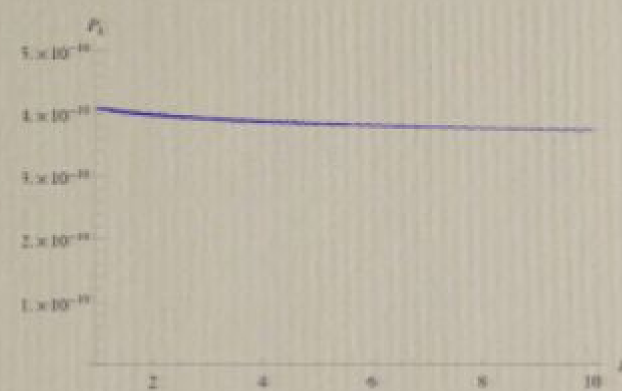

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- Factorizable once we expand around some fiducial K .

Advertisement for NONG™ code

Chen, Easther, Lim 2008

- For single field canonical kinetic models with arbitrary non-slow roll potential, arbitrary initial conditions

$$S = \int dx^4 \sqrt{g} \left[\frac{R}{2} + \frac{1}{2}(\partial\phi)^2 + V(\phi) \right]$$

- Robust "boundary regulator" : past infinities regulated in a controlled way with arbitrary accuracy.

- Being extended for general k-field models (DBI, varying sound speed etc.) Chen, Lim, Xu 2008 in painful progress

- Write your very own code : details published.

Grandiose/Philosophical Conclusions

- Experiments *and analysis tools* are getting close to pinning down non-gaussianities

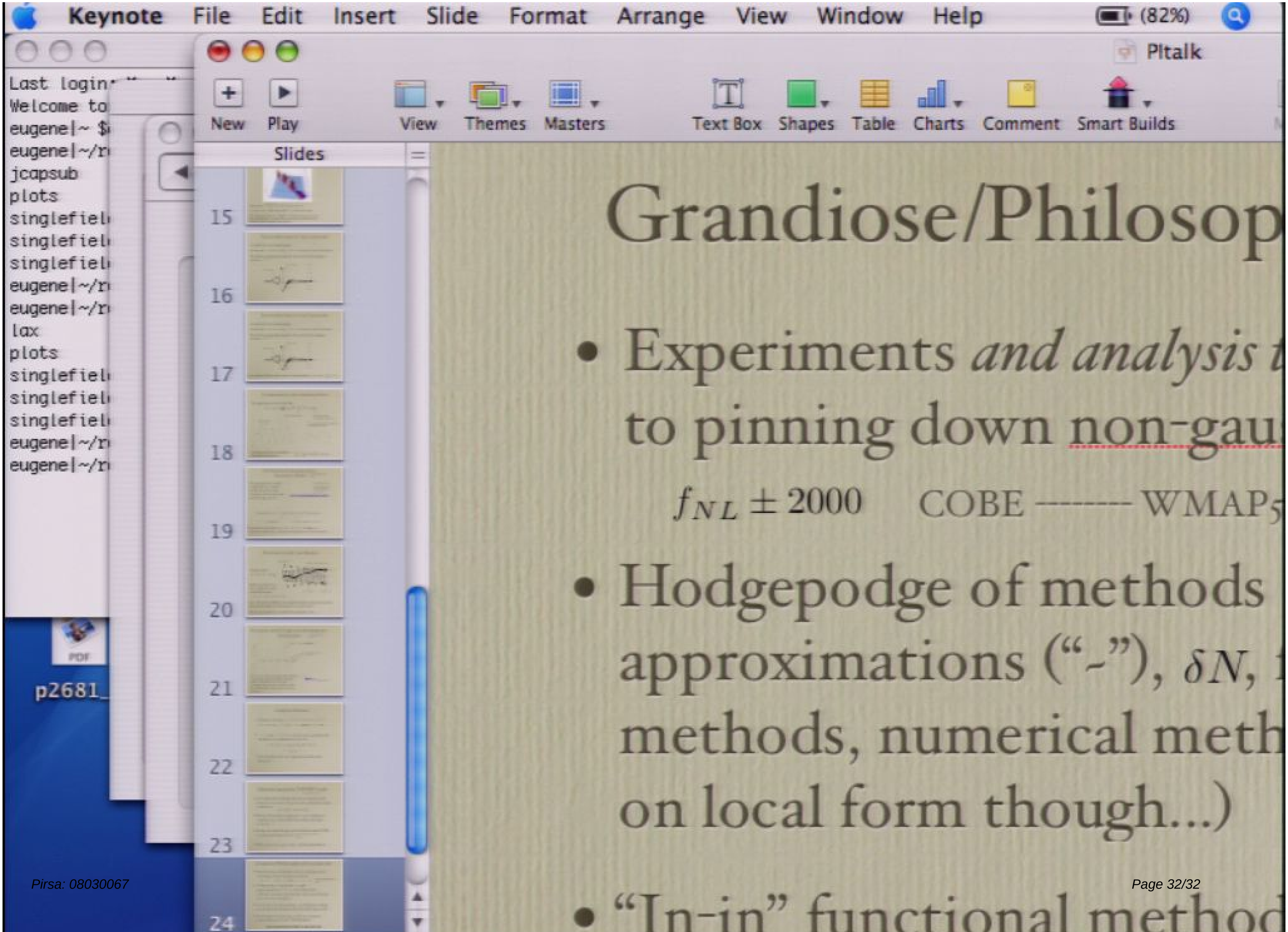
  $f_{NL} \pm 2000$ COBE ——— WMAP5 $-9 < f_{NL} < 111$ WMAP9/PLANCK/ LSS??

- Hodgepodge of methods : rough approximations ("-"), δN, field-theoretic methods, numerical methods. (Too much focus on local form though...)

- "In-in" functional methods : possibly providing us a unified and numerically friendly approach.

- Development of generic codes to compute primordial non-G ala' "NONGFast".

MUCH MORE WORK TO BE DONE!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Slides
Grandiose/Philosop
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