Title: Scale dependent non-Gaussianity

Date: Mar 10, 2008 09:00 AM

URL: http://pirsa.org/08030064

Abstract:

# WHY?

- NG is a signature of interactions (self interactions and initial conditions; gravity; other fields)
- EFT (?) : must treat interactions consistently
- Small sound speed case: useful phenomenological example with an interesting string theory construction
- Observable probe of interesting physics (Mari's talk, Licia's talk)

# 3 TYPES OF QUESTIONS

- What observables can distinguish between models? (Measurable NG rules out smooth single field slow-roll!)
- What range of observables do we expect?
- What kind of guidance comes from constructions, e.g. in string theory?

(see Rachel Bean's talk)



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# PLAN

- Simple, one-parameter family of models with possibly large, scale-dependent, non-Gaussianity and a rich string theory example
- Outline:
  - Review phenomenological set-up and features
  - Discuss consistency from higher order correlations
  - Example of applying theoretical constraints

# I. PHENOMENOLOGY

# GENERAL SET-UP

- Action is a function of a single field and its first derivatives

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[M_p^2 R - 2P(X,\phi)]$$

$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- Sound speed

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

Armendariz-Picon, Damour, Mukhanov;
Garriga, Mukhanov



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# THE PARAMETERS

$$\epsilon = -\frac{\dot{H}}{H^2} \qquad \eta = \frac{\dot{\epsilon}}{H\epsilon} \qquad \kappa = \frac{\dot{c}_s}{Hc_s}$$

- Condition for inflation

$$\ddot{a} > 0 \Rightarrow \epsilon < 1$$

- Scale-dependence of the sound speed

$$c_s(k) = c_s(k_0)\left(\frac{k}{k_0}\right)^{\kappa}$$

# THE OBSERVABLES

- Scalar spectral index

$$n_s - 1 = -2\epsilon - \eta - \kappa$$

- Tensor/Scalar ratio

$$r = 16\epsilon c_s$$

- Non-Gaussianity (equilateral type)

$$f_{NL}^{eff} \propto \frac{1}{c_s^2}$$

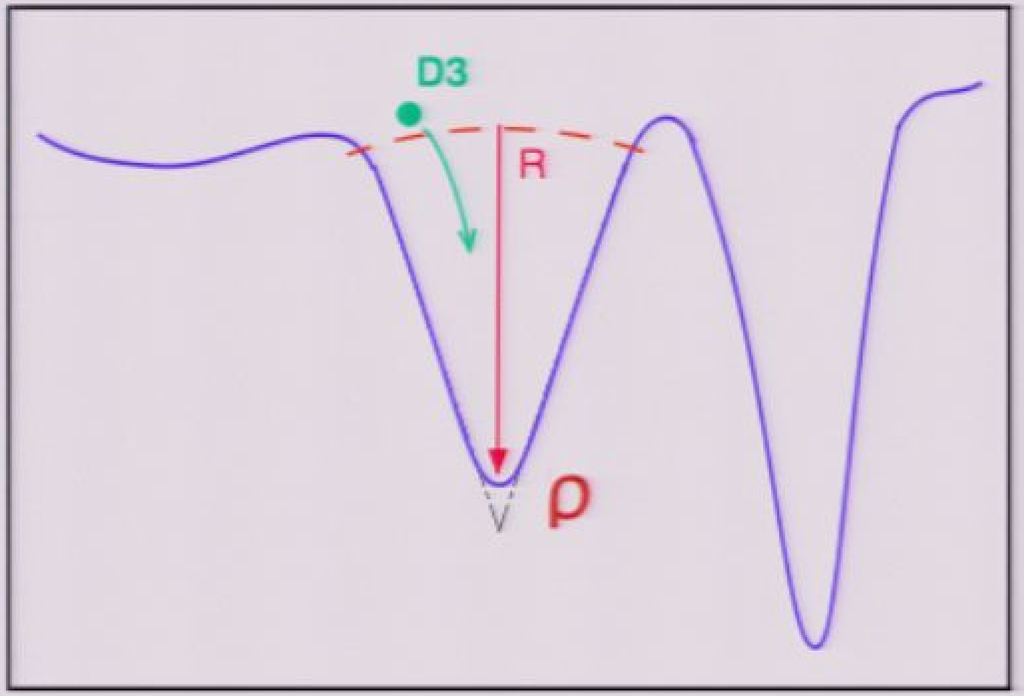
# EXAMPLE CASE: DBI

$$\gamma(\phi) = \frac{1}{\sqrt{1 - f(\phi)\dot{\phi}^2}}$$

$$\dot{\phi}^2 < f(\phi)^{-1} = Sh(\phi)^{-1}$$

$$h \approx \frac{R^4}{r^4} = \frac{R^4 T_3^2}{\phi^4}$$

$$P_{,X} = c_s^{-1} = \gamma$$



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# SUB-PLANCKIAN FIELD RANGE

- Usual chaotic inflation

$$V(\phi) = \frac{m^2\phi^2}{2}$$

$$\epsilon < 1 \Rightarrow \frac{\phi}{M_p} > \sqrt{2}$$

- With general kinetic term

$$H(\phi) = h_n\phi^n$$

$$\epsilon < 1 \Rightarrow \frac{\phi}{M_p} > \sqrt{2n^2c_s}$$



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# THE LYTH BOUND

- Beginning with

$$N_e = \int H dt$$

- Lyth wrote

$$\frac{\Delta\phi}{M_p} = N_e\sqrt{\frac{r}{8}} \quad \Rightarrow r < 0.007$$

- When sound speed decreases with scale

$$\left\{ \begin{matrix} \kappa = -0.2 & r \lesssim 0.08 \\ \kappa = -0.3 & r \lesssim 0.16 \end{matrix} \right\}$$

# II. CONSISTENCY

# CONSISTENCY

- Usual calculation of the power spectrum and NG assumes:
  - Energy in fluctuations is small compared to the background (gradient energy condition)
  - Power spectrum is calculated from term quadratic in fluctuations (interaction picture condition)

L. Leblond, S.S.



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# KEEP IN MIND...

- Violation of these bounds $\neq$ inflation ends
- Could be a different (non-perturbative/global) picture
- Another reason to keep the warped throat picture in mind...

# REMINDER: COMPUTING CORRELATIONS

- ADM formalism + gauge choice (comoving)

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$\delta\phi = 0 \qquad h_{ij} = a^2 e^{2\zeta}\delta_{ij}$$

- Finding first order for lapse functions gives the action up to terms cubic in the one remaining degree of freedom, $\zeta$
- Write action as $S = S_0 + S_2 + S_3 + \ldots$ $\quad S_0 \approx V(\phi_0)$

Maldacena; Weinberg



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# CORRELATIONS, CONT'D

- Usual computation of the power spectrum
  - Quantize

$$u(\vec{k},t) = \int d^3x \zeta(\vec{x},t) e^{-i\vec{k}\cdot\vec{x}}$$

  - Mode equation from quadratic term

$$v_k = \frac{a\sqrt{2\epsilon}}{c_s} u_k$$

$$v_k'' + \left(c_s^2 k^2 - \frac{z''}{z}\right) v_k = 0$$

  - Amplitude on horizon crossing

$$u_k \sim \frac{H}{2M_p\sqrt{\epsilon c_s k^3}}$$



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# AMPLITUDE OF FLUCTUATIONS

- Define the power spectrum and dimensionless power spectrum

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\rangle \equiv (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2)\mathcal{P}_\zeta$$
$$= (2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2)2\pi^2 P_\zeta k^{-3}$$

$$P_\zeta = A(k_0)(k/k_0)^{n_s-1}$$

- Check that in spatially flat gauge, recover the expected result

$$\langle \delta\phi^2\rangle^{1/2} = \frac{|\dot{\phi}|}{H}\langle \zeta^2\rangle^{1/2} \sim \frac{H}{2\pi\sqrt{c_s P_{,X}}}$$



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# AN APPROXIMATE APPROACH

- Bounds:
  - Gradient energy $\frac{S_2}{S_0} < 1$
  - Interaction picture $\frac{S_3}{S_2} < 1$
- Terms are evaluated at horizon crossing:

$$\zeta \to \langle \zeta^2 \rangle^{1/2} = \frac{H}{2\pi M_p \sqrt{2\epsilon c_s}}$$

- Interaction constraint is really a loop calculation, but we ignore the log term

(David Seery's talk)



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# SLOW-ROLL

- Gradient energy bound

$$\frac{S_2}{S_0} \sim \frac{\epsilon a^{-2}(\partial\zeta)^2}{H^2 M_p^2} \sim \frac{H^2}{M_p^2} < 1$$

- Interaction Bound

$$\frac{S_3}{S_2} \sim \frac{H^2 \epsilon}{M_p^2} < 1$$

(less constraining)



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# INTERLUDE, LOCAL MODEL

- Local model, defined in real space

$$\zeta = \zeta_g + \frac{3}{5} f_{NL}(\zeta_g^2 - \langle \zeta_g^2 \rangle)$$

- "Mildly non-Gaussian" means the correction to the variance is small:

$$f_{NL}^2 \langle \zeta_g^2 \rangle^2 < \langle \zeta_g^2 \rangle$$

$$f_{NL} \langle \zeta_g^2 \rangle^{1/2} < 1$$

$$f_{NL} < 10^{9/2}$$

# WITH SOUND SPEED

- Gradient Bound $$\frac{\partial}{\partial x} \sim \frac{aH}{c_s}$$

$$S_2 = M_p^2 \int dt d^3x \left( a^3 \frac{\epsilon}{c_s^2} \dot{\zeta}^2 - a\epsilon(\partial\zeta)^2 \right)$$

$$\frac{S_2}{S_0} \sim \frac{\epsilon a^{-2}(\partial\zeta)^2}{H^2 M_p^2} \sim \frac{H^2}{M_p^2 c_s^3} < 1$$

Seery, Lidsey; Chen et al; Arroja, Koyama



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# INTERACTION PICTURE CONDITION

$$\frac{S_3}{S_2} \sim \frac{\zeta}{c_s^2}(\epsilon - 2\kappa + 1 - c_s^2) < 1$$

$$\frac{H^2}{M_p^2 \epsilon} < c_s^5$$

- Rearranging terms

$$c_s^4 > \frac{H^2}{M_p^2 \epsilon c_s} \sim P_\zeta$$

(agrees with Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore)

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# WHAT ABOUT SCALE-DEPENDENCE?

- Display scale-dependence

$$P_\zeta < c_s^4$$

$$A(k_0)\left(\frac{k}{k_0}\right)^{n_s-1} < c_s^4(k_0)\left(\frac{k}{k_0}\right)^{4\kappa}$$

$$\log\left[\frac{A(k_0)}{c_s^4(k_0)}\right] < (4\kappa - n_s + 1)\log\left[\frac{k}{k_0}\right]$$

- How many e-folds before running takes the model out of the perturbative regime?

$$N_e^{max} = \frac{1}{(4\kappa - n_s + 1)}\log\left[\frac{A(k_0)}{c_s^4(k_0)}\right]$$

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$$c_s^4 > \frac{H^2}{M_p^2 \epsilon c_s} \sim P_\zeta$$

$$\frac{1}{\epsilon^2} > P_\zeta$$

SR

(agrees with Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore)

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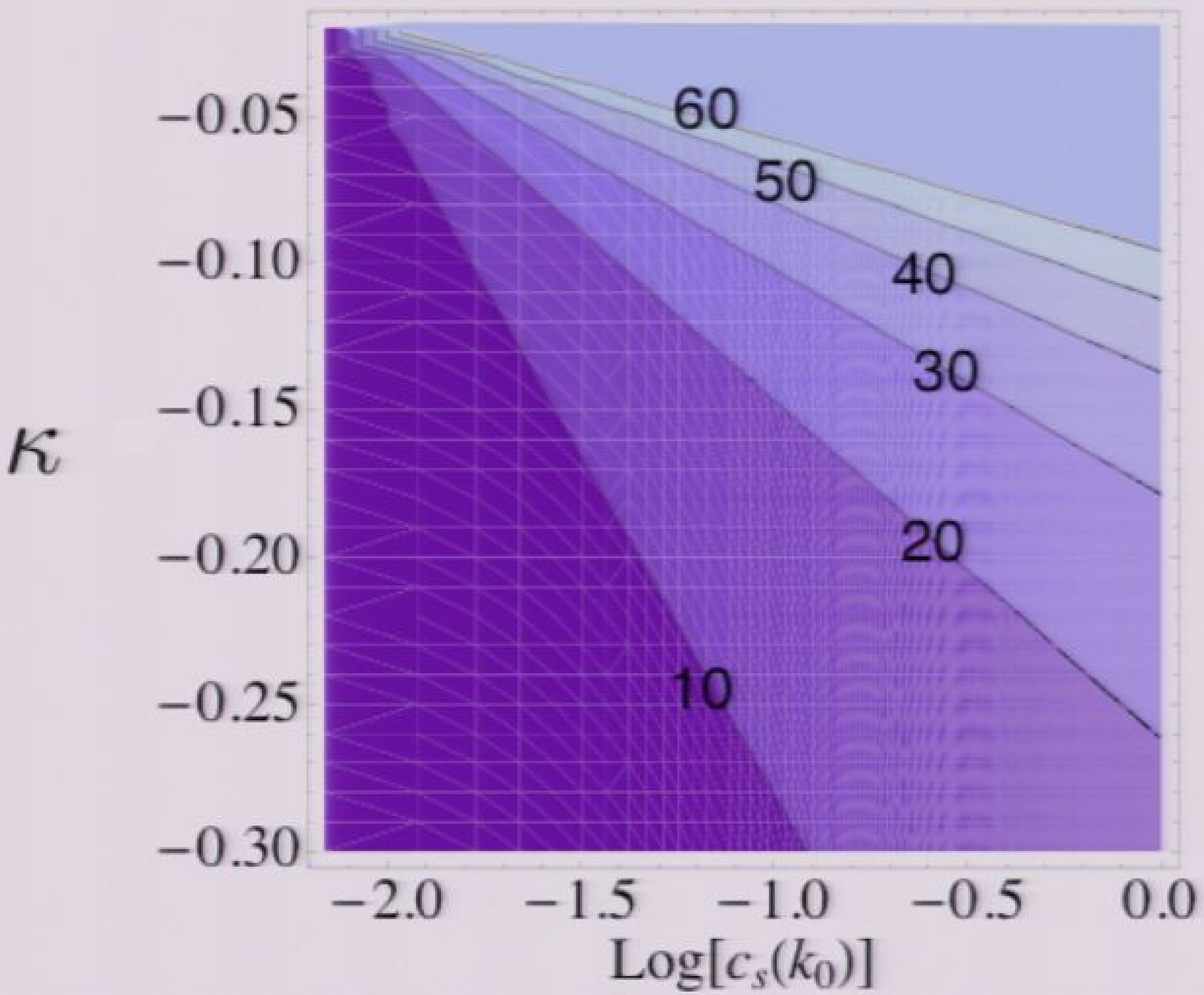




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# HIGHER ORDER TERMS

- A more general constraint $S_2 > S_n,\ n \geq 3$
- With small sound speed, terms are

$$\mathcal{L}_2 = a^3 \frac{H^4}{c_s P_{,X}} \left[ P_{,X} \left( 1 - \frac{3}{c_s^2} \right) + 2XP_{,XX} \right] + \dots$$

$$\mathcal{L}_3 = a^3 \frac{1}{2} \left( \frac{2H^4}{\dot{\phi}^2 c_s P_{,X}} \right)^{3/2} \left[ X^2 P_{,XX} \left( 1 - \frac{3}{c_s^2} \right) + \frac{2}{3} X^3 P_{,XXX} \right] + \dots$$

$$= -\mathcal{L}_2 \frac{P_\zeta^{1/2}}{2c_s^2} \left[ \frac{2\lambda c_s^2}{\Sigma} - 3(1 - c_s^2) \right]$$



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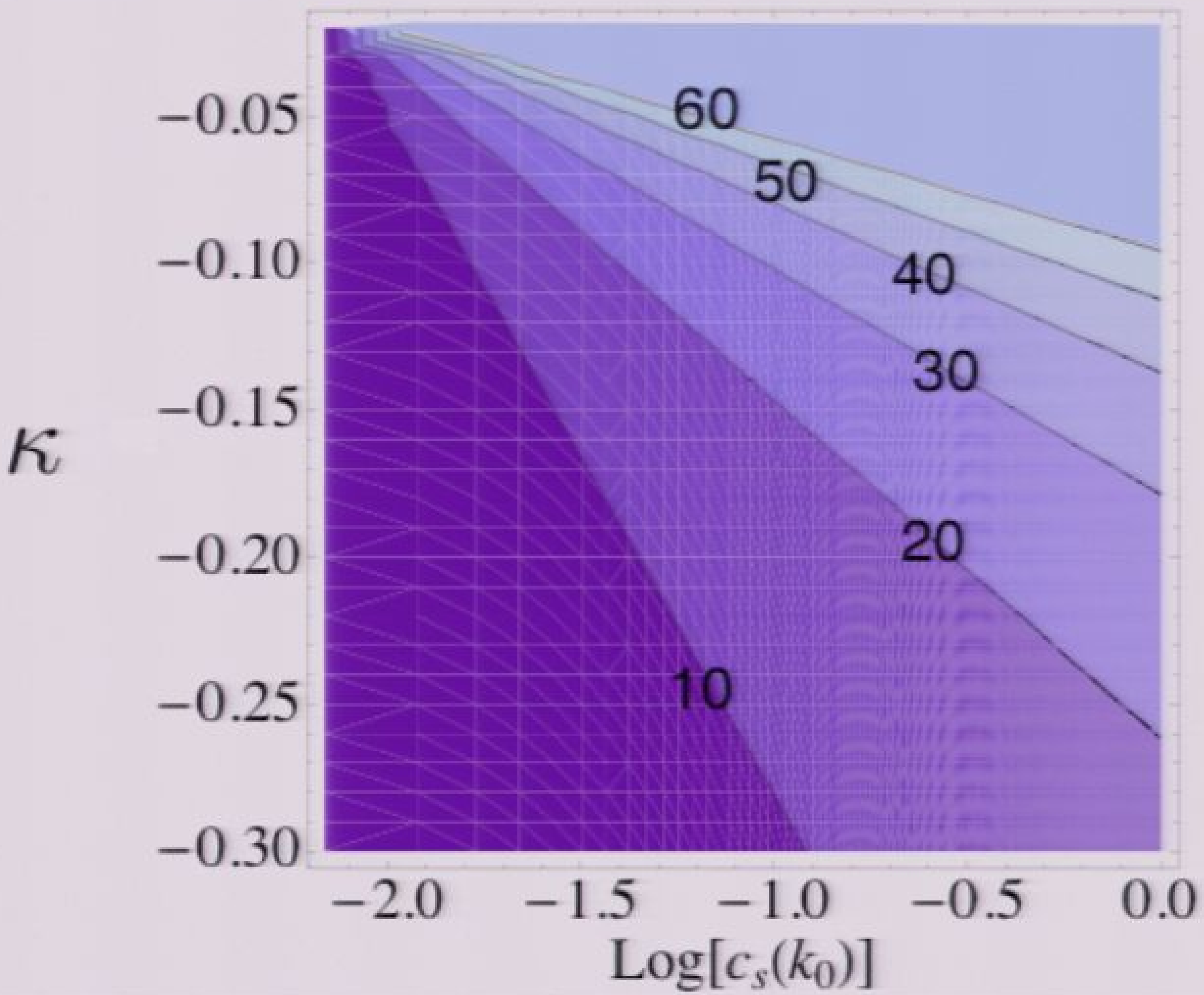




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$$= -\mathcal{L}_2 \frac{P_\zeta^{1/2}}{2c_s^2} \left[ \frac{2\lambda c_s^2}{\Sigma} - 3(1 - c_s^2) \right] \qquad \mathcal{L}_4 \sim \mathcal{L}_2 \left( \frac{P_\zeta^{1/2}}{2c_s^2} \right)^2$$

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# SCALE DEPENDENCE

- So these terms scale like

$$\hat{\mathcal{L}}_n \equiv \frac{\mathcal{L}_n}{\mathcal{L}_2} \propto L_n \left( \frac{\mathcal{P}_\zeta^{1/2}(k_0)}{c_s^2(k_0)} \right)^{(n-2)} \left( \frac{k}{k_0} \right)^{(\frac{n_s-1}{2} - 2\kappa)(n-2)}$$

$$\equiv \hat{L}_n \left( \frac{k}{k_0} \right)^{q(n-2)}$$

- Suppose $q > 0$. How can we be sure such a scale-dependent action can be truncated?

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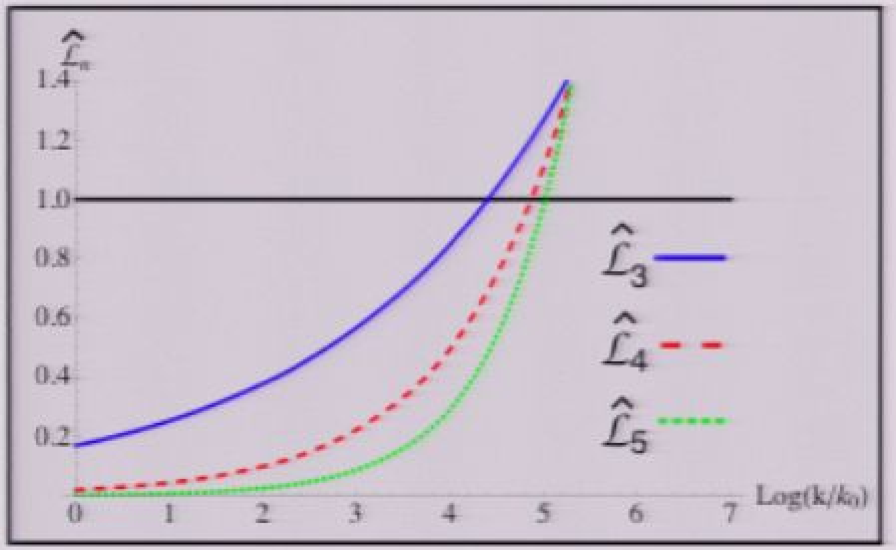


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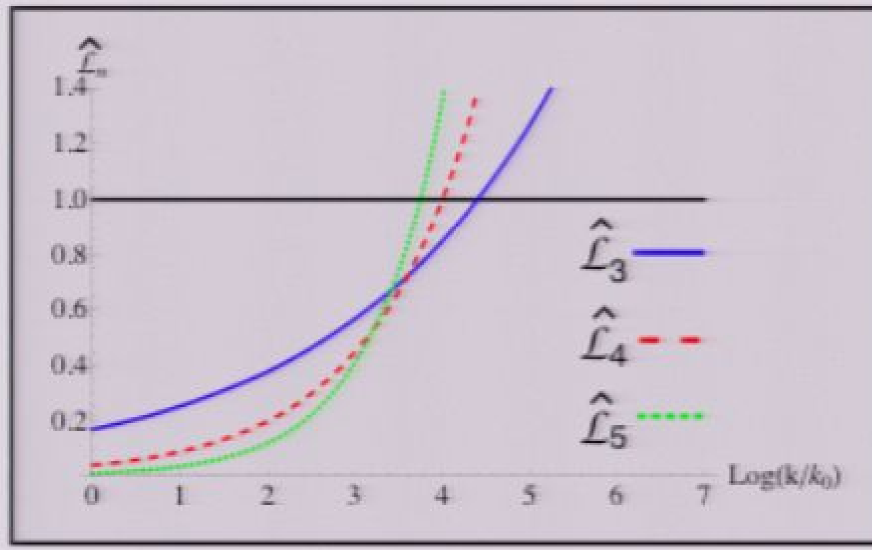


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# A POSSIBLE CONSTRAINT

- Demand that on any scale, all terms up to some order n are small (and terms are ordered)

$$\hat{\mathcal{L}}_n(k_n^*) = 1$$

$$k_n^* > k_{n-1}^*$$

- Then coefficients are constrained

$$\hat{L}_n \le (\hat{L}_{n-1})^{(n-2)/(n-3)} \Rightarrow L_n \le (L_{n-1})^{(n-2)/(n-3)}$$

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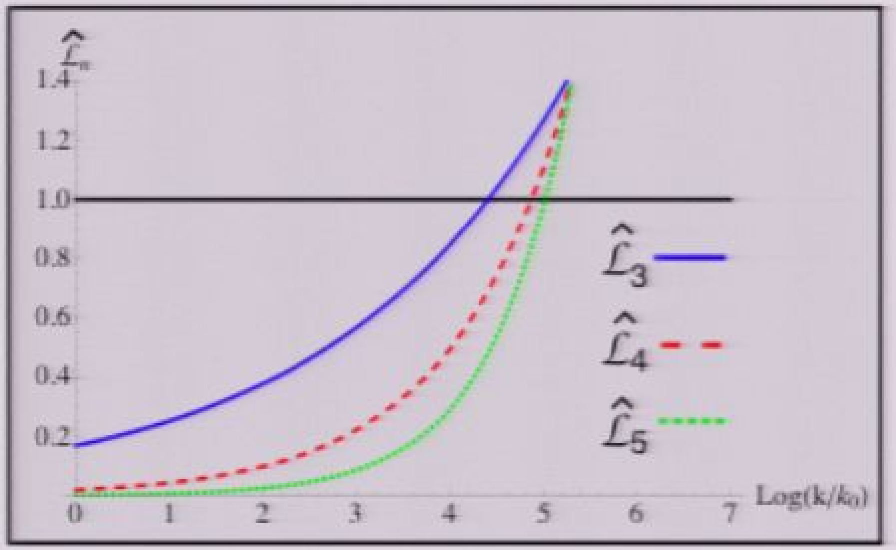


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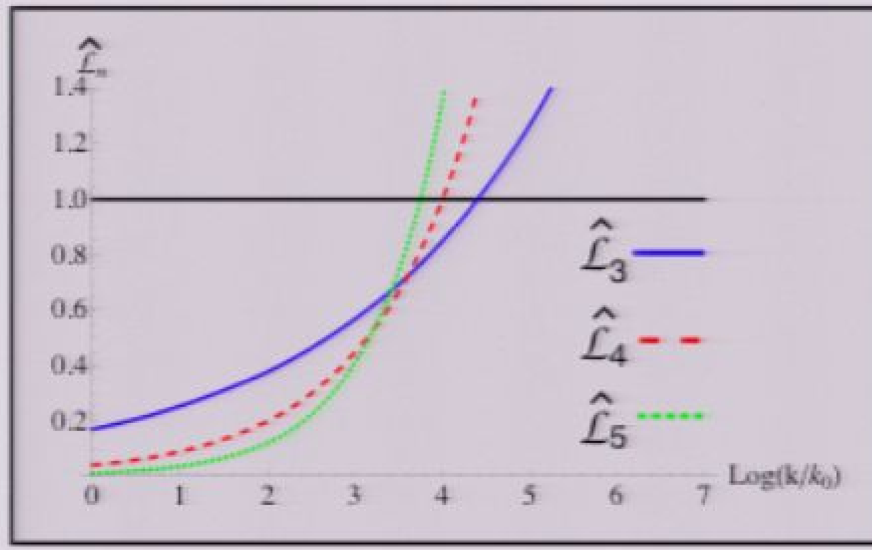


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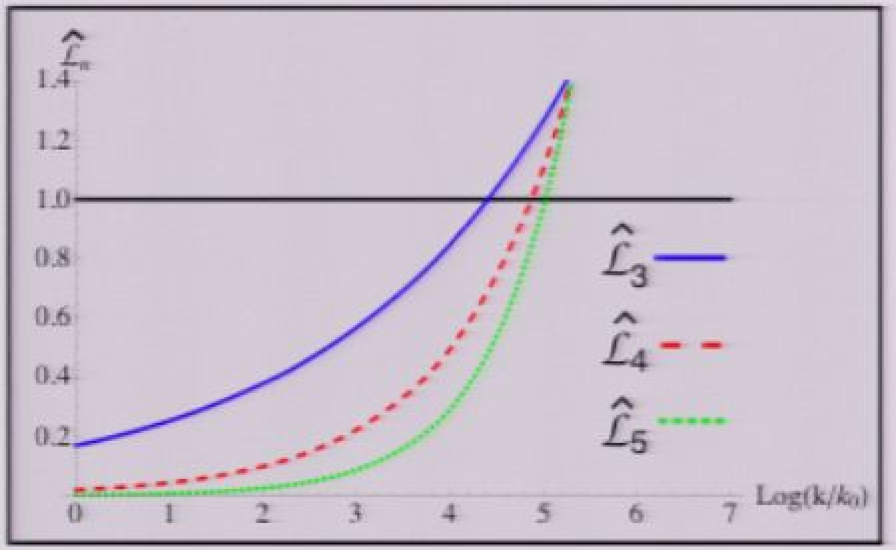


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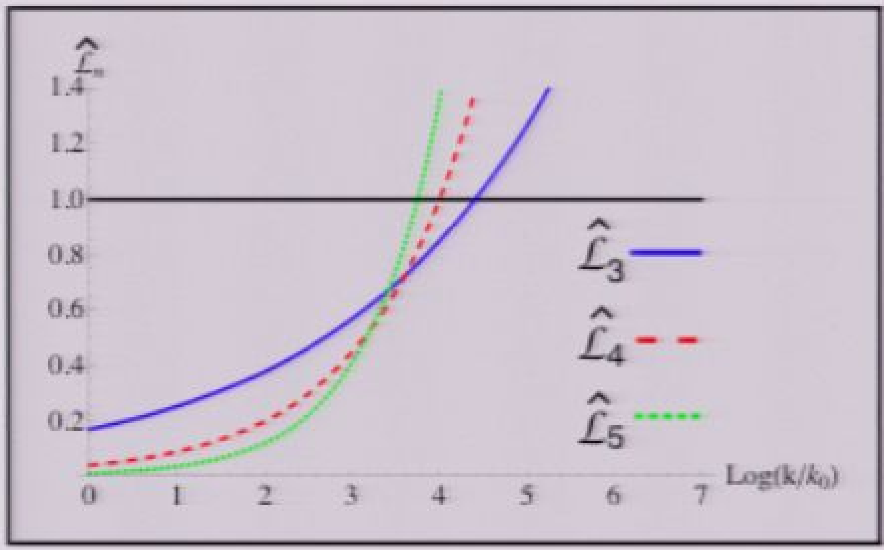


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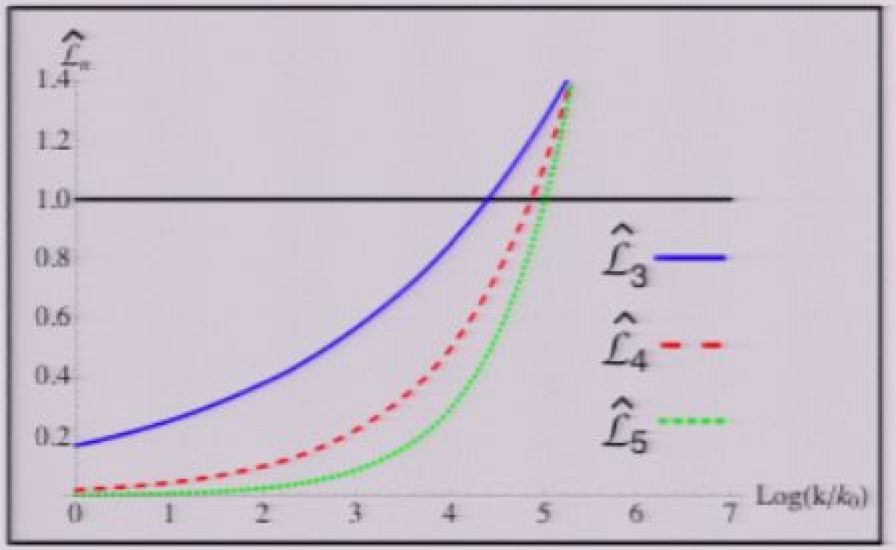


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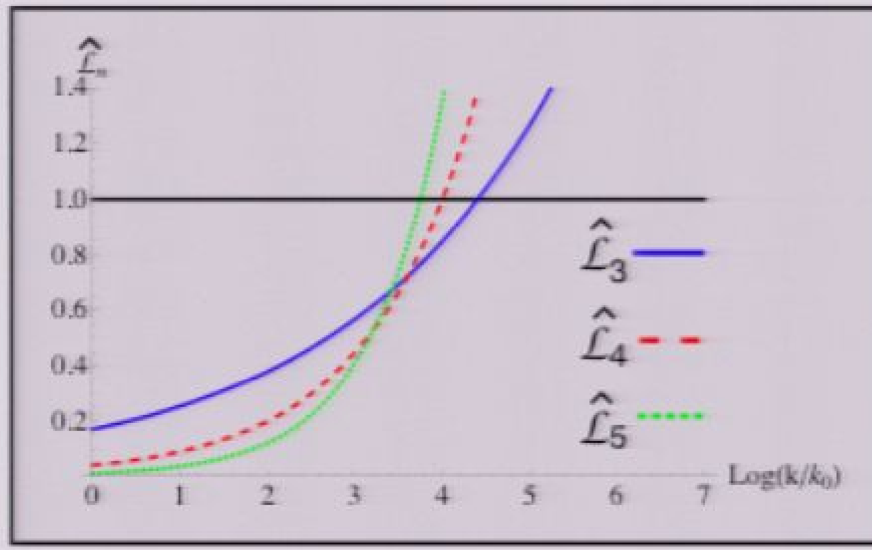


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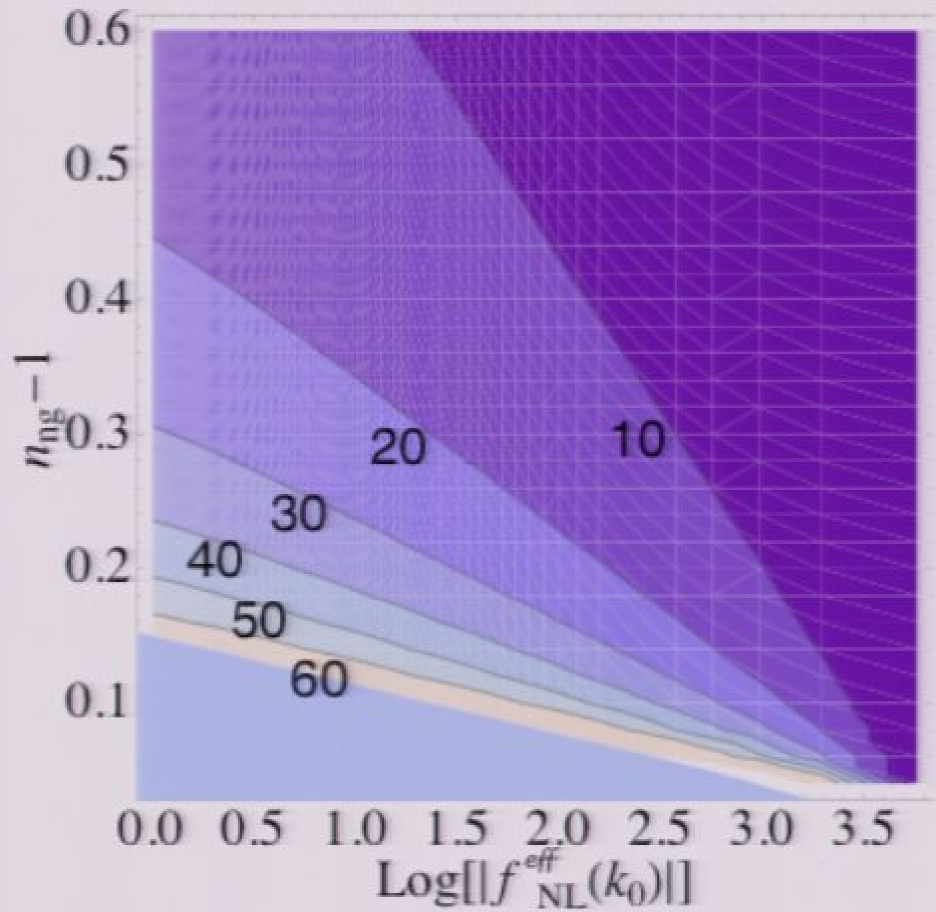
# III. APPLICATIONS: DBI AND NG IN LSS

# CONSISTENCY CONSTRAINTS: E-FOLDS

$$f_{NL}^{eff} \approx -0.32/c_s^2$$

$$f_{NL}^{eff} = f_{NL}^{eff}(k_0)\left(\frac{k}{k_0}\right)^{n_{NG}-1}$$



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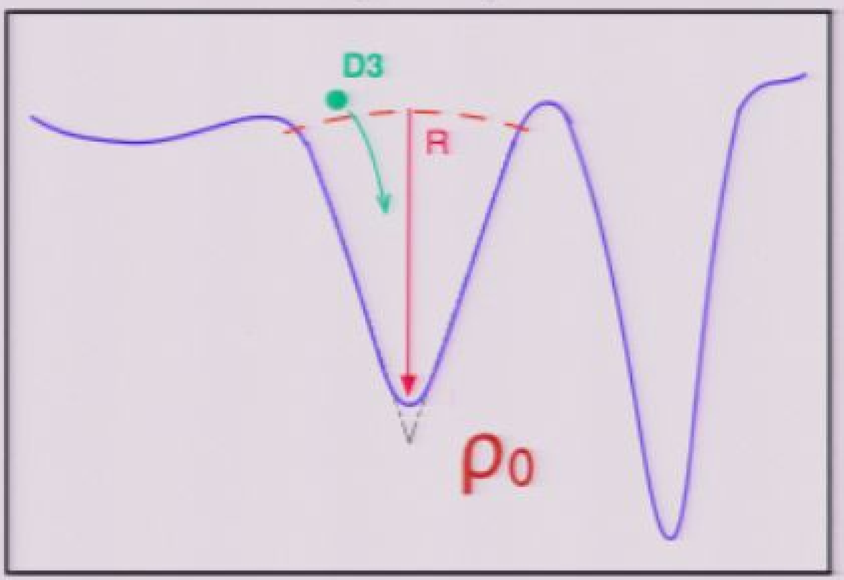
# CONSIST. CONSTRAINTS: GEOMETRY

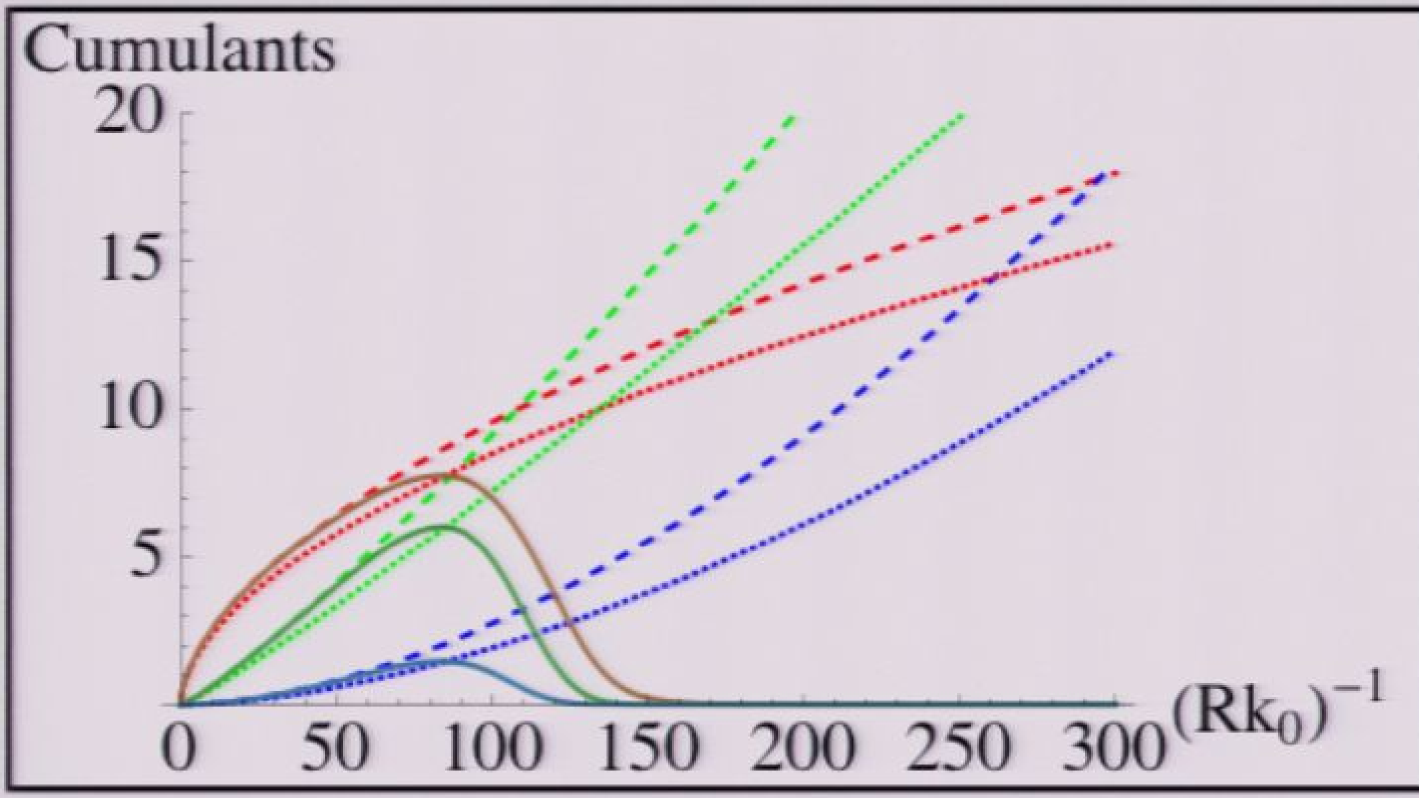
- In DBI, scale dependence of sound speed is controlled by the warp factor
- Really, the throat geometry smoothes out at the bottom anyway




$$h^{-1} \approx \frac{r^4}{R^4} \to \frac{(r^4 + r_0^4)}{R^4}$$

$$\kappa \to 0$$

# WHAT ABOUT LSS?




$$S_n \propto \left(\frac{1}{c_s^2}\right)^{n-2}$$

- - - κ constant
········ power law
—— Tanh(k/k*)



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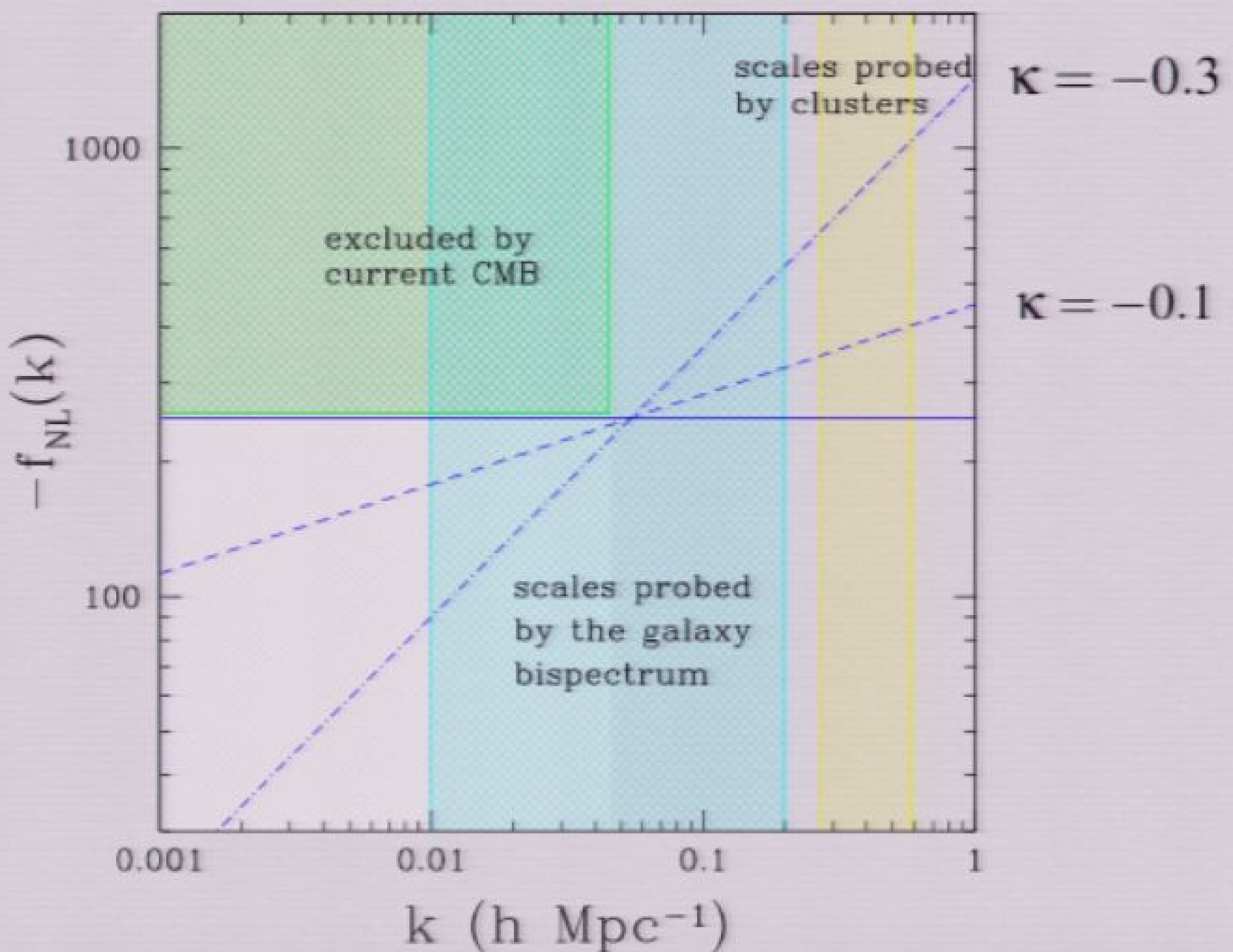
# NG ON CLUSTER SCALES?



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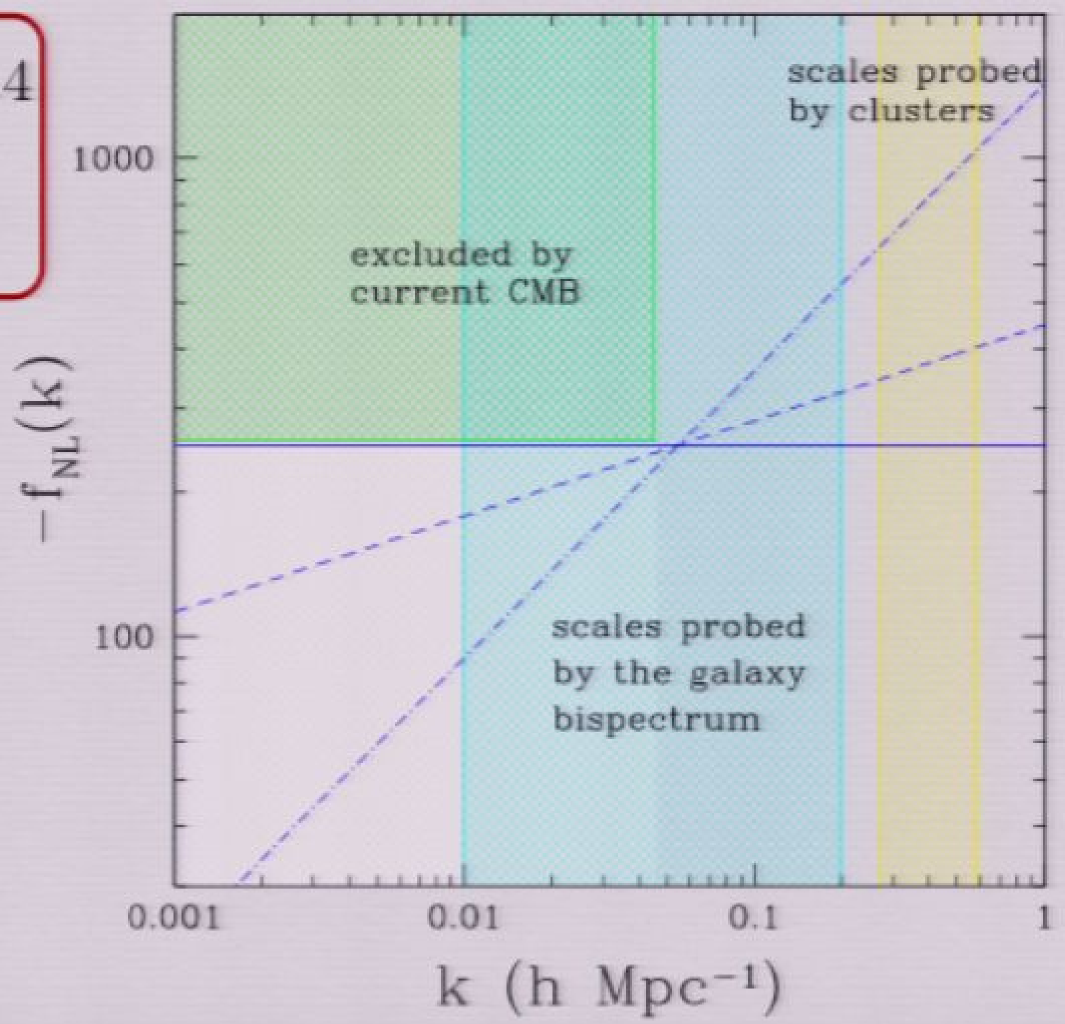
# NG ON CLUSTER SCALES?

$$Log[k/k_0] \sim 6.4$$

$$f_{NL} < 6800$$




$\kappa = -0.3$ ✓

$\kappa = -0.1$ ✓



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$$\hat{L}_n \leq (\hat{L}_{n-1})^{(n-2)/(n-3)} \Rightarrow L_n \leq (L_{n-1})^{(n-2)/(n-3)}$$

- For DBI, easy to compute largest term at any order

$$L_n = -\sum_{j=p}^{n} \frac{1}{j!} \binom{j}{n-j} (2j-3)!! \left(-\frac{3}{2}\right)^{n-j}$$

$$p = \mathrm{Int}\{n/2, (n+1)/2\}$$

- For example

$$L_3^{DBI} = 1$$
$$L_4^{DBI} = 1/2$$
$$L_5^{DBI} = -1/2$$



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# FUTURE DIRECTIONS

- Better understanding of scale-dependence
- Relate phenomenological bounds to bounds in the underlying model
- What happens outside the perturbative regime?
- What other generic consistency constraints are there?

# SUMMARY

- There are useful consistency constraints on generic inflation models (minimum sound speed, maximum NG)
- Such constraints should be considered in the 'reconstruction' programs
- Constraints can be sharpened / understood with particular realizations in, e.g., string theory
- NG, observed or not, is a fantastic tool for theorists



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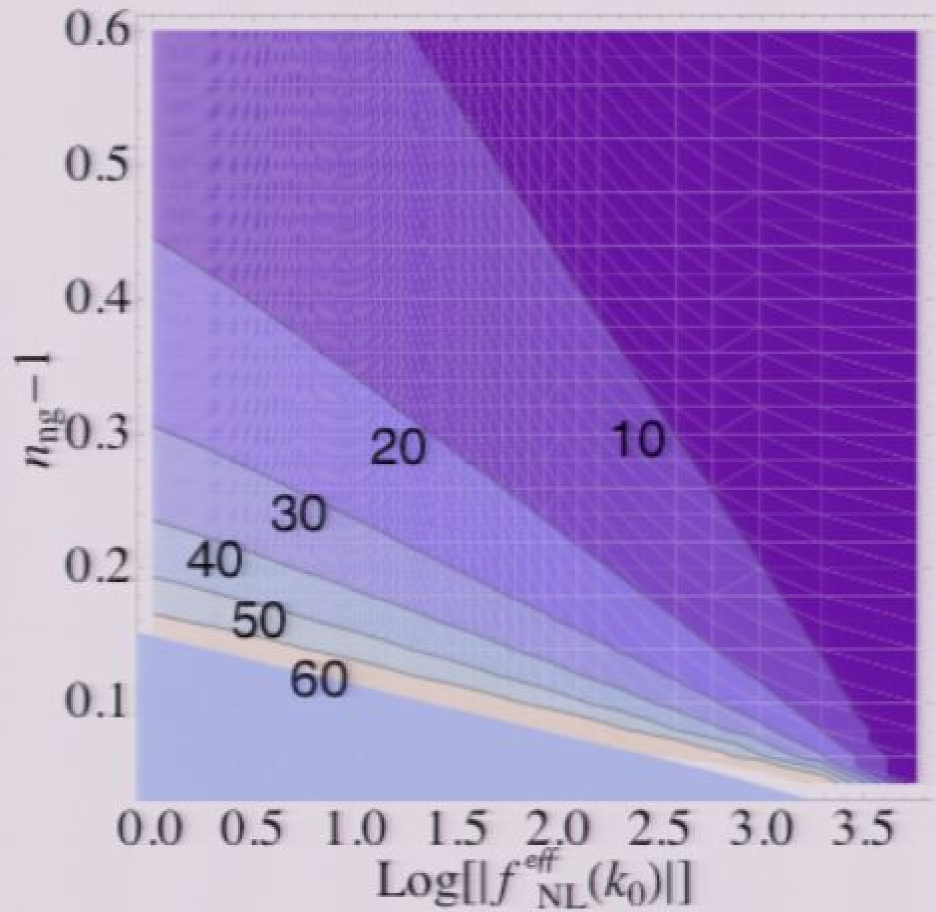




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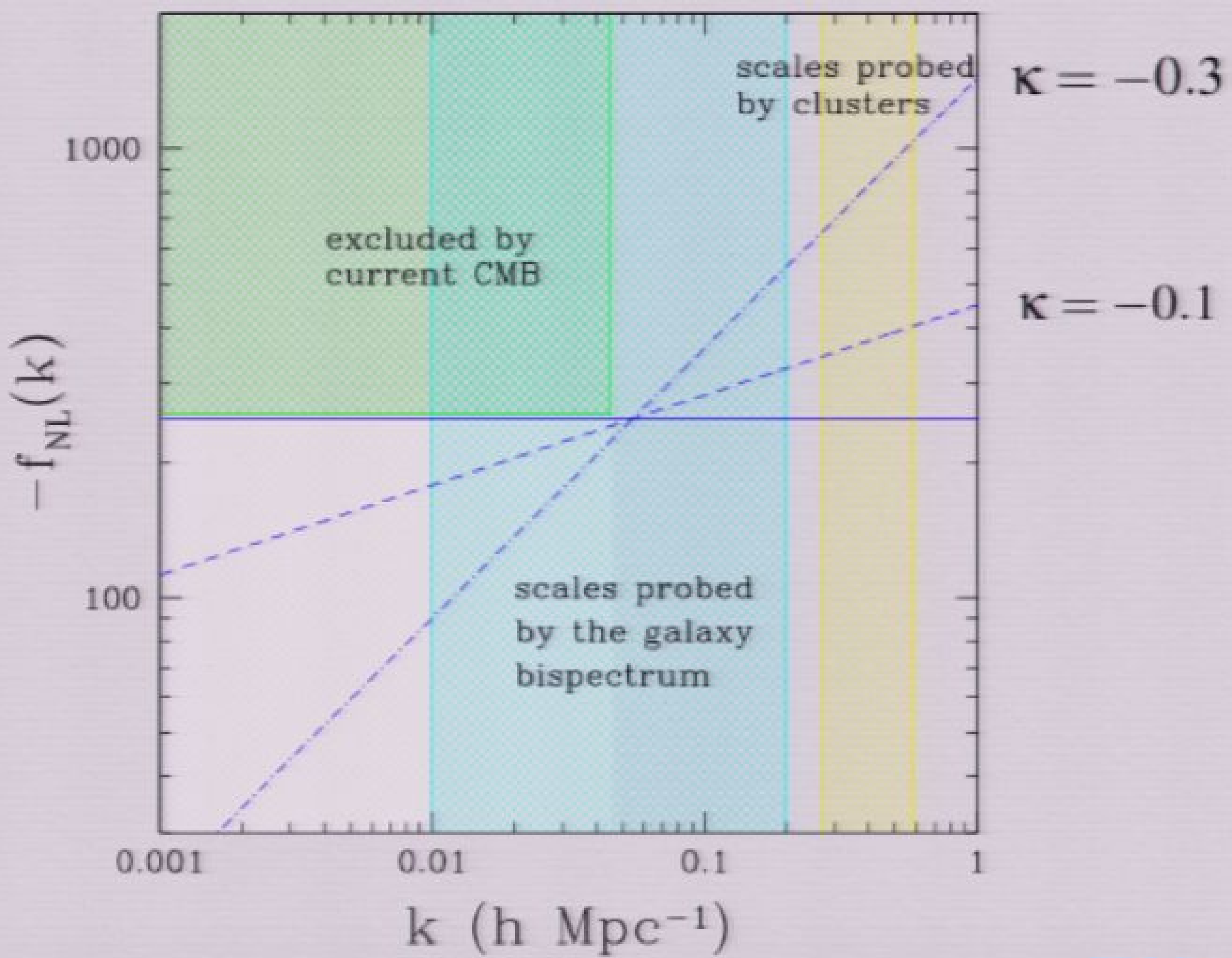
# III. APPLICATIONS: DBI AND NG IN LSS

# NG ON CLUSTER SCALES?



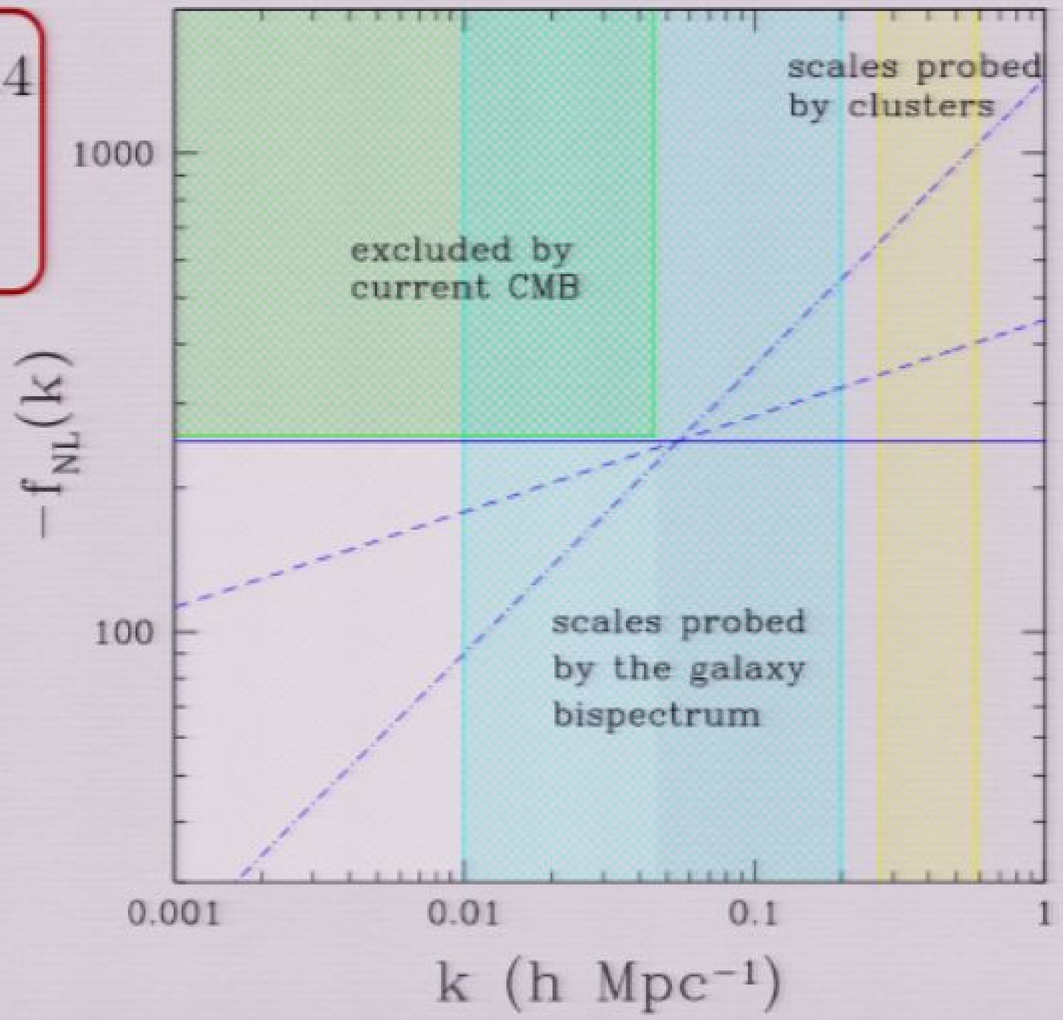



PI March 10, 2008

# NG ON CLUSTER SCALES?

$Log[k/k_0] \sim 6.4$

$f_{NL} < 6800$




$\kappa = -0.3$ ✓

$\kappa = -0.1$ ✓

No Signal

VGA-1

No Signal

VGA-1

No Signal

VGA-1

No Signal

VGA-1