Title: Analytic Form of Inflationary Actions and Non-Gaussianity

Date: Mar 10, 2008 09:45 AM

URL: http://pirsa.org/08030059

Abstract:

Analytic Form of Inflationary Actions and Non-Gaussianity

Daniel Chung
(University of Wisconsin – Madison)

Collaborators:
Rachel Bean, Ghazal Geshnizjani
0801.0742

Order of Presentation

- What is the analytic form of the action consistent with likely data for minimal kinetic term single field slow-roll models?
- What is the analytic form of the action consistent with likely data for non-minimal kinetic term single field models?
- What more might we be able to say about these models?

Order of Presentation

- What is the analytic form of the action consistent with likely data for minimal kinetic term single field slow-roll models?
- What is the analytic form of the action consistent with likely c[illegible] for non-minimal kinetic term single fiel[illegible]dels?
- What more might we be able to say about these models?

Analytic Form of Inflationary Actions and Non-Gaussianity

Daniel Chung
(University of Wisconsin – Madison)

Collaborators:
Rachel Bean, Ghazal Geshnizjani
0801.0742

Order of Presentation

- What is the analytic form of the action consistent with likely data for minimal kinetic term single field slow-roll models?
- What is the analytic form of the action consistent with likely data for non-minimal kinetic term single field models?
- What more might we be able to say about these models?

Review for contrast: Minimal Kinetic Term Models

Minimal Kinetic Term Models

- Choose scalar field to avoid worrying about generating too much anisotropies.
- Choose 1 real field for minimality, simplicity, and its ability to capture the flavor more complex dynamics.

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

A model is 1D real manifold parameterized by ϕ

Solution to the homogeneous classical EOM is a parametric curve on this 1D manifold. (Most of the likely observables are controlled by this.)

- Likely observables traditionally consist of the following:
  *Amplitude of the curvature as a function of k
  *Amplitude of the tensors as a function of k

Slow-Roll Inflationary Models

- Can map $n_s(k)-1$ to the shape of the potential

Caricature:

$$n_s(k) - 1 = 2M_p^2 \frac{V''(\phi)}{V(\phi)} - 3M_p^2 \left(\frac{V'(\phi)}{V(\phi)}\right)^2 \boxed{\ll 1}$$

$$\frac{d \ln[k/k_i(T_{RH}, g_*(T_{RH}), \boxed{\phi_e})]}{d(\phi/M_p)} = -\left(\frac{1}{M_p}\frac{V(\phi)}{V'(\phi)} - \frac{M_p}{2}\frac{V'(\phi)}{V(\phi)}\right)$$

- Potential has to have small slope and curvature.
- Can map the amplitude of the gravity waves to the height of the potential

$$V = M^4$$

$$\Delta_h^2 = \frac{2V}{3\pi^2 M_p^4}$$

Not a particularly compelling argument against efforts to Measure B-modes.

- Solution to the diff eqs depend on initial conds which in turn depend on reheating.

Minimal Kinetic Term Models

- Choose scalar field to avoid worrying about generating too much anisotropies.
- Choose 1 real field for minimality, simplicity, and its ability to capture the flavor more complex dynamics.

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

A model is 1D real manifold parameterized by ϕ

Solution to the homogeneous classical EOM is a parametric curve on this 1D manifold. (Most of the likely observables are controlled by this.)

- Likely observables traditionally consist of the following:
  *Amplitude of the curvature as a function of k
  *Amplitude of the tensors as a function of k

Slow-Roll Inflationary Models

- Can map $n_s(k)-1$ to the shape of the potential

Caricature:

$$n_s(k) - 1 = 2M_p^2 \frac{V''(\phi)}{V(\phi)} - 3M_p^2 (\frac{V'(\phi)}{V(\phi)})^2 \boxed{\ll 1}$$

$$\frac{d \ln[k/k_i(T_{RH}, g_*(T_{RH}), \boxed{\phi_e})]}{d(\phi/M_p)} = -\left(\frac{1}{M_p}\frac{V(\phi)}{V'(\phi)} - \frac{M_p}{2}\frac{V'(\phi)}{V(\phi)}\right)$$

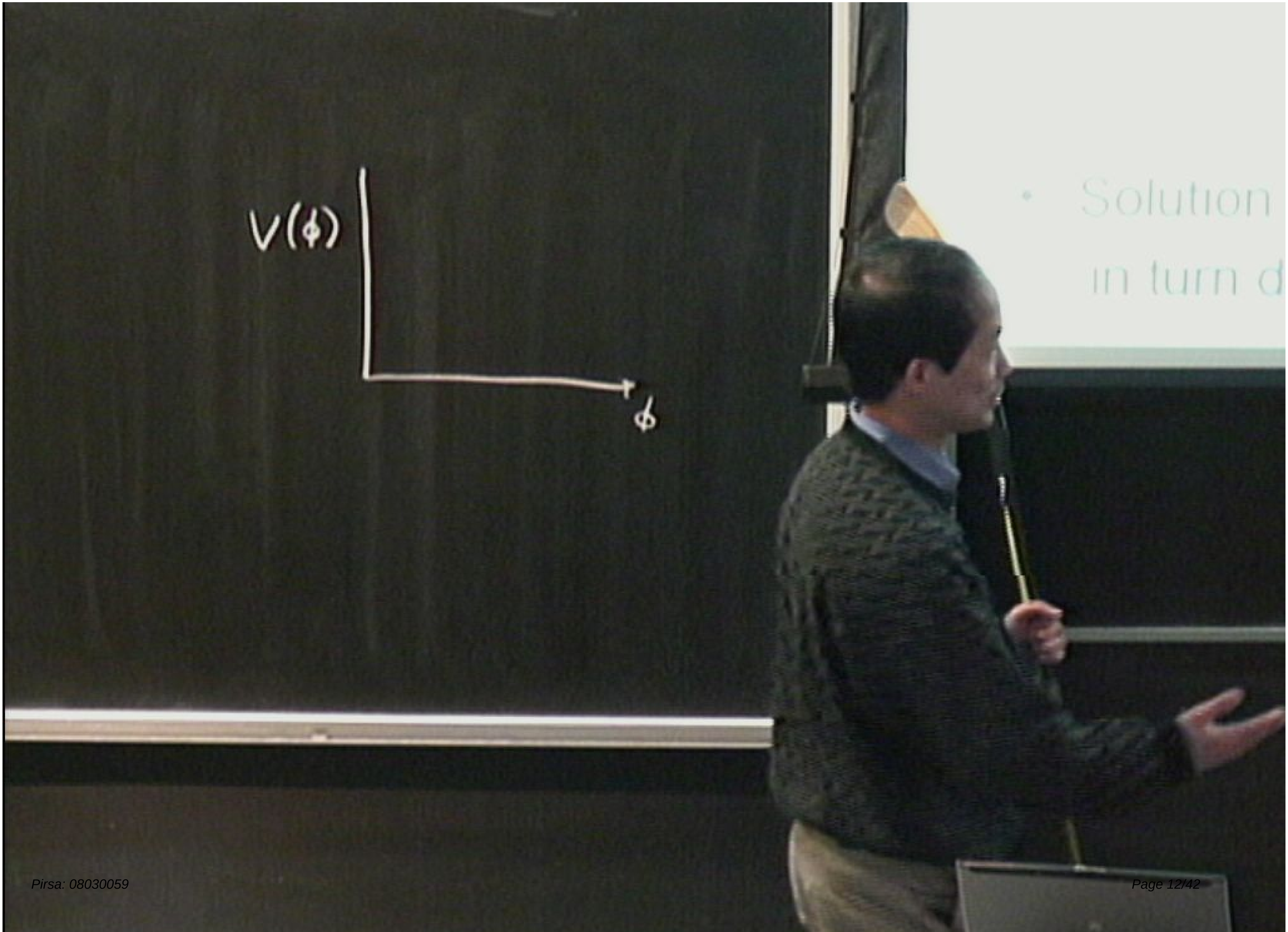
- Potential has to have small slope and curvature.
- Can map the amplitude of the gravity waves to the height of the potential

$$V = M^4$$

$$\Delta_h^2 = \frac{2V}{3\pi^2 M_p^4}$$

Not a particularly compelling argument against efforts to Measure B-modes.

- Solution to the diff eqs depend on initial conds which in turn depend on reheating.


V(ϕ)
ϕ
Solution
in turn d

Slow-Roll Inflationary Models

- Can map $n_s(k)-1$ to the shape of the potential

Caricature:

$$n_s(k) - 1 = 2M_p^2 \frac{V''(\phi)}{V(\phi)} - 3M_p^2 (\frac{V'(\phi)}{V(\phi)})^2 \boxed{\ll 1}$$

$$\frac{d \ln[k/k_i(T_{RH}, g_*(T_{RH}), \boxed{\phi_e})]}{d(\phi/M_p)} = -\left(\frac{1}{M_p}\frac{V(\phi)}{V'(\phi)} - \frac{M_p}{2}\frac{V'(\phi)}{V(\phi)}\right)$$

- Potential has to have small slope and curvature.
- Can map the amplitude of the gravity waves to the height of the potential

$V = M^4$

$$\Delta_h^2 = \frac{2V}{3\pi^2 M_p^4}$$

Not a particularly compelling argument against efforts to Measure B-modes.

- Solution to the diff eqs depend on initial conds which in turn depend on reheating.

Hopeful but Limited Potential Information

- Bottom line: Ideal measurements may yield $V(\phi(k))$ over a range of ϕ if we assume single field minimal kinetic term models
- Since slow-roll, tiny field range maps to largely varying Fourier scale range

$$\frac{d\ln[k/k_i(T_{RH}, g_*(T_{RH}), \phi_e)]}{d(\phi/M_p)} = -\left(\frac{1}{M_p}\frac{V(\phi)}{V'(\phi)} - \frac{M_p}{2}\frac{V'(\phi)}{V(\phi)}\right)$$

- When not strongly running, the potential well captured by a Taylor expansion to quartic order. (Analytic form.)
- 1 D manifold constrained by 1D manifold of data (ideal).
- Well known consistency relationship can still rule out slow roll even with limited knowledge.

Some "Well Motivated" Models Exist

Almost nobody doubts something like slow-roll inflation could have happened:

- Global U(1) neglecting natural Planck-scale violations

$$V(\phi) = V_0(1-\cos(\frac{\phi}{f})) + c\mu^4 \exp(-S_I)\cos(\frac{\phi}{f}+\Psi)$$

Typically Planckian

- D-term inflation can evade the η problem

$$U = \frac{1}{2}\Re[f_{ab}^{-1}]D_a D_b + e^{K/M_{\rm Pl}^2} g^{ij*}(D_i W)(D_j W)^* - \frac{3}{M_{\rm Pl}^2} e^{K/M_{\rm Pl}^2} W^* W$$

$$V_D = \frac{1}{2}\left(\frac{g}{2}\sum_n q_n|\phi_n|^2 + \xi\right)^2$$

- Still, most of these fall under $V(\phi) = \Lambda + b\phi + \frac{1}{2}m^2\phi^2 + \frac{1}{3}A\phi^3 + \frac{\lambda}{4}\phi^4 + \frac{\phi^{p+4}}{M^p}$
- None terribly compelling in terms of staggering beauty

More Controversial Models Exist

- Large tensor perturbations typically require large field variations (near Planckian or larger)

$$\frac{\Delta_h^2}{\Delta_S^2} = 16\epsilon \qquad \Delta N = \int_{\phi_i}^{\phi_f} \frac{d\phi}{M_p} \frac{1}{\sqrt{2\epsilon}} \sim 60 \qquad V(\phi) = \frac{m^2}{2}\phi^2 + \phi^4[\frac{\lambda}{4!} + c_2\frac{\phi^2}{M_p^2} + ...]$$

- People often quibble over whether large field variations are natural, but that is not really the issue. It is simply
  - the dynamics may not be reliably captured by a simple potential

Can no longer integrate out.

  - we have no confidence about physics above the Planck scale

What is  ?

- My two-cents: Calculability should not be taken as a fundamental requirement of physics

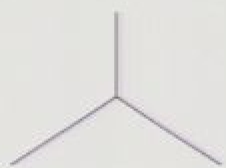
Slow-Roll Predicts Gaussianity

Slow roll parameters → flat potential → weak interaction → almost quadratic theory for $\delta\phi$ → energy density is linear in $\delta\phi$

$$\delta\rho \sim V'(\phi_0)\delta\phi$$

→ almost Gaussian field $\zeta \sim \frac{\delta\rho}{\rho + P} \propto \delta\phi$

Non-gravitational short distance contribution:



$$\mathcal{L}_{int} = \frac{1}{6}V'''(\phi_0)\delta\phi^3$$

$$\langle\zeta\zeta\zeta\rangle \propto V'''(\phi_0)\langle\delta\phi\delta\phi\rangle^3(\frac{H}{\dot{\phi}})^3\frac{1}{H} \propto (\frac{d\eta}{d\ln k} + ...)P_\zeta^2$$

This would contribute $\mathcal{O}(\epsilon^2)$ to f_{NL} .

Gravitational contribution dominates, however, as we now review.

General Slick Argument

Maldacena's slick argument for gravitationally induced NG
In the squeezed limit.

Key: Suppose a field theory is a decoupling theory.

If $\phi(z) \approx \phi_0$ then

$$\langle \phi(x)\phi(y)\phi(z)\rangle = \langle \, \langle \phi(x)\phi(y)\rangle_{\substack{\text{short distance}\\ \text{in the presence}\\ \text{of } \phi_0}} \; \phi_0 \, \rangle_{\text{long distance}}$$

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle$$

$$\vec{k}_3 = -\vec{k}_1 - \vec{k}_2 \to 0 \qquad\qquad \exp(2\zeta_G(0))|d\vec{x}|^2$$

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(-\vec{k}_1-\vec{k}_2)\rangle \sim \langle \zeta_G\zeta_G(e^{-\zeta_G(0)}|\vec{k}_1-\vec{k}_2|)\zeta_G(0)\rangle$$

$$\langle \zeta_G\zeta_G(e^{-\zeta_G(0)}k)\zeta_G(0)\rangle \sim \langle \zeta_G\zeta_G(k-\zeta_G(0)k)\zeta_G(0)\rangle$$

$$-\partial_{\ln k}\langle \zeta_G\zeta_G\rangle(k)\langle \zeta_G(0)\zeta_G(0)\rangle \propto (n_s-1)[P_\zeta(k)]^2$$

$f_{NL}^{squeeeze} \sim (n_s - 1)$ Much smaller than order unity.

Hence, if $|f_{NL}^{\text{squeeze}}| > 1$, we must violate decoupling and/or perturbative expansion and/or standard metric/gravity ansatz or

Easy to Obtain Non-Gaussian Isocurvature

In passing, one should note that Gaussianity is as much of the feature of slow roll as it is the feature of a purely the quadratic theory:

$$\langle \delta\rho_\chi \rangle \sim m^2 \langle : \chi^2 : \rangle \neq 0$$

$$\langle \delta\rho_\chi \delta\rho_\chi \delta\rho_\chi \rangle \sim m^6 \langle : \chi^2 :: \chi^2 : \rangle \langle : \chi^2 : \rangle \sim m^6 \langle \chi\chi \rangle^2 \langle : \chi^2 : \rangle \neq 0$$

even for a pure quadratic theory without gravitational constraint equation induced couplings. However, note that interaction coupling still played an important role to produce the isocurvature particles.

Hence, Such isocurvatures can also contaminate curvature perturbations:

$$\dot{\mathcal{R}} = \frac{1}{3(\rho + P)^2} \{ -\delta\rho \frac{dP}{dt} + \delta P \frac{d\rho}{dt} \}$$

General Slick Argument

Maldacena's slick argument for gravitationally induced NG
In the squeezed limit.

Key: Suppose a field theory is a decoupling theory.

If $\phi(z) \approx \phi_0$ then

$$\langle \phi(x)\phi(y)\phi(z)\rangle = \langle\ \langle \phi(x)\phi(y)\rangle_{\substack{\text{short distance}\\ \text{in the presence}\\ \text{of } \phi_0}}\ \ \phi_0\ \rangle_{\text{long distance}}$$

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle$$

$$\vec{k}_3 = -\vec{k}_1 - \vec{k}_2 \to 0 \qquad\qquad \exp(2\zeta_G(0))|d\vec{x}|^2$$

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(-\vec{k}_1-\vec{k}_2)\rangle \sim \langle \zeta_G\zeta_G(e^{-\zeta_G(0)}|\vec{k}_1-\vec{k}_2|)\zeta_G(0)\rangle$$

$$\langle \zeta_G\zeta_G(e^{-\zeta_G(0)}k)\zeta_G(0)\rangle \sim \langle \zeta_G\zeta_G(k-\zeta_G(0)k)\zeta_G(0)\rangle$$

$$-\partial_{\ln k}\langle\zeta_G\zeta_G\rangle(k)\langle\zeta_G(0)\zeta_G(0)\rangle \propto (n_s-1)[P_\zeta(k)]^2$$

$f_{NL}^{squeeze} \sim (n_s - 1)$ Much smaller than order unity.

Hence, if $|f_{NL}^{\text{squeeze}}| > 1$, we must violate decoupling and/or perturbative expansion and/or standard metric/gravity ansatz or

Easy to Obtain Non-Gaussian Isocurvature

In passing, one should note that Gaussianity is as much of the feature of slow roll as it is the feature of a purely the quadratic theory:

$$\langle \delta\rho_\chi \rangle \sim m^2 \langle : \chi^2 : \rangle \neq 0$$

$$\langle \delta\rho_\chi \delta\rho_\chi \delta\rho_\chi \rangle \sim m^6 \langle : \chi^2 :: \chi^2 : \rangle \langle : \chi^2 : \rangle \sim m^6 \langle \chi\chi \rangle^2 \langle : \chi^2 : \rangle \neq 0$$

even for a pure quadratic theory without gravitational constraint equation induced couplings. However, note that interaction coupling still played an important role to produce the isocurvature particles.

Hence, Such isocurvatures can also contaminate curvature perturbations:

$$\dot{\mathcal{R}} = \frac{1}{3(\rho + P)^2}\{-\delta\rho \frac{dP}{dt} + \delta P \frac{d\rho}{dt}\}$$

Non-minimal Kinetic Term Models

Large Interactions & Derivative Coupling

- Large derivative interactions can exist without ruining negative pressure

$$\mathcal{L}_{\text{intuition}} = f((\partial\phi)^2, \phi)(\partial\phi)^2 - m^2\phi^2$$

$$\mathcal{L}_{\text{intuition}} \sim (\partial\tilde{\phi})^2 - \frac{m^2}{Z}\tilde{\phi}^2$$

- To have a calculable EFT description, for point-like fields, one must have

$$Z \sim \mathcal{O}(1)$$

- However, DBI can in principle get around this because of the extended object nature

$$\mathcal{L} = \frac{-1}{f(\phi)}\sqrt{1-2f(\phi)X} - [V(\phi) - \frac{1}{f(\phi)}] \qquad \frac{1}{a^3}\frac{d}{dt}(a^3\gamma\dot{\phi}) + [\frac{f'(\phi)}{f(\phi)}\frac{X}{\gamma} + V'(\phi) + \frac{f'(\phi)}{f^2}(1+\frac{1}{\gamma})] = 0$$

How Well Motivated are Non-minimal Kinetic terms?

- Minimal kinetic term is a symptom of a linear wave description of a particle:

$$(\partial\phi)^2 - m^2\phi^2 \rightarrow e^{-ip\cdot x} \rightarrow p^2 = m^2$$

- Integrating out momentum shells to derive Wilsonian EFT generate higher powers $(\partial\phi)^2\frac{(\partial\phi)^{2n}}{\Lambda^{4n}}$

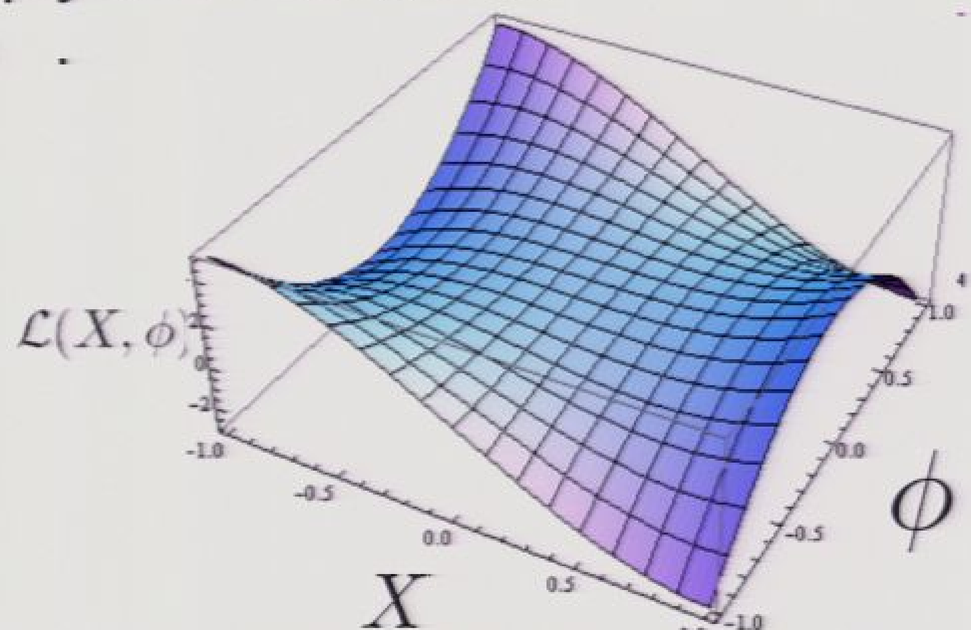
- "Non-particle" description for dynamics corresponds to non-minimal kinetic terms e.g. membrane-like object coupled to gauge fields

$$S = \mu\int d^{p+1}\sigma\sqrt{|\det(G_{\alpha\beta} + k\mathcal{F}_{\alpha\beta})|}$$

$$S = \mu\int d^{p+1}\sigma\sqrt{|\det(\eta_{\alpha\beta} + k^2\partial_\alpha\Phi^i\partial_\beta\Phi^i + kF_{\alpha\beta})}$$

What does a general single field Lagrangian of the form $\mathcal{L}(X,\phi)$ look like to be consistent with inflationary phenomenology?

- The general action is simply a 2-D manifold parameterized by (X,ϕ) .



- The question is to find the general form that satisfies the constraints from the data.
    - Ease of constructing inflationary models numerically for fitting
    - Parameterizing objects that encode data in a transparent way
    - See if there are general theoretical restrictions

Gauge Ambiguity

$$\{\phi, X_\phi\}$$

$$\phi = f(\varphi)$$

$$\{\phi, X_\phi\} \to \{\varphi, X_\varphi = X_\phi/[\partial_\varphi f]^2\}$$

$$\partial_\phi = \frac{\partial\varphi(\phi)}{\partial\phi}\partial_\varphi \quad \partial_{X_\phi} = \frac{1}{[\partial_\varphi f]^2}\partial_{X_\varphi}$$

Along a particular trajectory, $\{\phi(N_e), X_\phi(N_e)\}$

Gauge fix: adjust $f(\varphi(\phi(N_e)))$ to obtain desired $\partial_{X_\varphi}\mathcal{L}$ or $X_\varphi(N_e)$

e.g. $\partial_{X_\varphi}\mathcal{L}|_{\{\varphi(N_e), X_\varphi(N_e)\}} = \dfrac{1}{c_s(N_e)}$ or $X(N_e) = \dfrac{1}{2}$

A Canonical Transformation

- It is interesting to note that as far as the D=0+1 classical dynamics are concerned, minimal and non-minimal kinetic terms for a single scalar field can simply be a difference of field redefinition generated by a canonical transformation mixing field momentum and coordinate.
- Example:

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{4}\lambda\phi^4$$

describes the same dynamics as

$$\tilde{\mathcal{L}} = \frac{3}{4}\frac{1}{\lambda^{1/3}}[-2\tilde{\phi}\dot{\tilde{\phi}}]^{4/3} - \frac{d}{dt}\tilde{\phi}^3 - \frac{1}{2}\tilde{\phi}^4$$

$$\ddot{\phi} + \lambda\phi^3 = 0$$

$$\phi(t=0) = 0 \qquad \dot{\phi}(t=0) = A$$

$$\phi(t) = At[1 - \frac{\lambda}{20}A^2t^4 + \mathcal{O}(\lambda^2A^4t^8)]$$

$$T_{00} = \frac{1}{2}\dot{\phi}^2 + \frac{\lambda}{4}\phi^4$$

$$T_{00} \approx \frac{A^2}{2} + \mathcal{O}(t^8)$$

$$\tilde{\phi}\ddot{\tilde{\phi}} - 3(2^{-1/3})\lambda^{1/3}\tilde{\phi}^{8/3}\dot{\tilde{\phi}}^{2/3} - \dot{\tilde{\phi}}^2 = 0$$

$$\tilde{\phi}(0) = \sqrt{A} \qquad \dot{\tilde{\phi}}(0) = 0$$

$$\tilde{\phi}(t) = \sqrt{A}$$

$$\tilde{T}_{00} = \frac{1}{2}\tilde{\phi}^4 + \frac{1}{4}\frac{1}{\lambda^{1/3}}(-2\tilde{\phi}\dot{\tilde{\phi}})^{4/3}$$

$$\tilde{T}_{00} = \frac{A^2}{2}$$

Constructing Analytic Form

- Go to the $X_\varphi(N_e) = \frac{M_p^4}{2}$ gauge.
- As Rachel will explain, suppose our observables can be described up to an overall constant by

$$\epsilon(N_e) = \frac{3}{2 - \frac{\mathcal{L}(X,\phi)}{X\partial_X\mathcal{L}}} \qquad c_s^2(N_e) = \left(1 + \frac{2X\partial_X^2\mathcal{L}}{\partial_X\mathcal{L}}\right)^{-1} \qquad f_{NL}^{\text{equil}} \approx -0.28\left(1 - \frac{1}{c_s^2}\right) + 0.04\left(1 + \frac{1}{c_s^2}\right)\frac{\kappa}{2\epsilon - \eta}$$

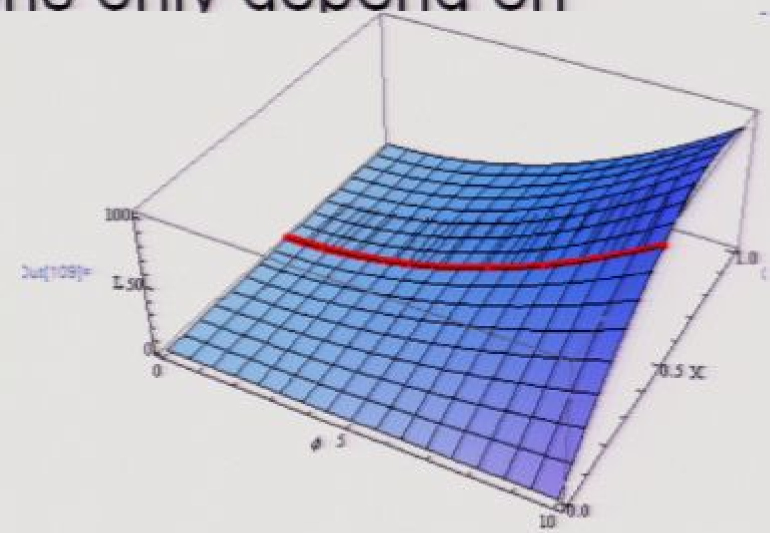
$\longrightarrow$ Boundary conditions only depend on

$$\mathcal{L}\big|_{\{\varphi(N_e), X(N_e) = \frac{M_p^4}{2}\}}$$

$$\partial_{X_\varphi}\mathcal{L}\big|_{\{\varphi(N_e), X(N_e) = \frac{M_p^4}{2}\}}$$

$$\partial_{X_\varphi}^2\mathcal{L}\big|_{\{\varphi(N_e), X(N_e) = \frac{M_p^4}{2}\}}$$

(For 3-point funct., Info. possible)

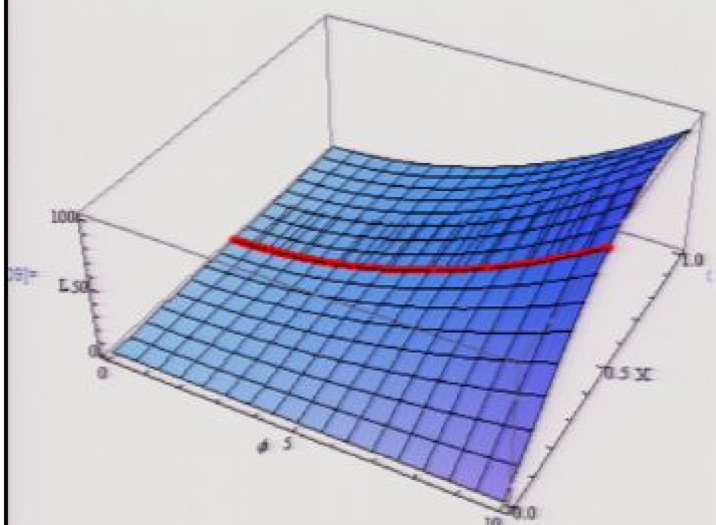


i.e. Higher derivatives with respect to N_e can be derived from $\epsilon(N_e)$ and $c_s^2(N_e)$

Analytic Form

Answer:

$$
\begin{aligned}
\mathcal{L}(\phi,X) &= q(\phi,X)+\mathcal{L}^{obs}(\phi,\frac{M_p^4}{2})-q(\phi,\frac{M_p^4}{2}) \\
&+\left[\partial_X\mathcal{L}^{obs}(\phi,\frac{M_p^4}{2})-\partial_X q(\phi,\frac{M_p^4}{2})\right](X-\frac{M_p^4}{2}) \\
&+\frac{1}{2}\left[\partial_X^2\mathcal{L}^{obs}(\phi,\frac{M_p^4}{2})-\partial_X^2 q(\phi,\frac{M_p^4}{2})\right](X-\frac{M_p^4}{2})^2
\end{aligned}
$$



You want an action consistent with data?

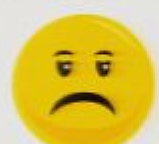
1) Extract $\mathcal{L}^{obs}|_{\{\varphi(N_e),X(N_e)=\frac{M_p^4}{2}\}}$

$\partial_{X_\varphi}\mathcal{L}^{obs}|_{\{\varphi(N_e),X(N_e)=\frac{M_p^4}{2}\}}$

$\partial^2_{X_\varphi}\mathcal{L}^{obs}|_{\{\varphi(N_e),X(N_e)=\frac{M_p^4}{2}\}}$

[See Rachel's talk to see how.]

2) Choose an "arbitrary" function $q(\phi,X)$



Simple Example

Suppose measurements give $\epsilon \sim \dfrac{1}{2N_e} \ll 1 \qquad c_s = 1 - \delta \qquad \delta \ll 1$

$$H = H_1 \exp\left(\int_1^{N_e} \frac{dN}{2N}\right) = H_1 N_e^{1/2}$$

$$\mathcal{L}\left(\frac{1}{2}, \phi\right) = H_1^2(1 - H_1^2\phi^2)$$

$$\mathcal{L}_X\left(\frac{1}{2}, \phi\right) = H_1^2$$

$$\mathcal{L}_{XX}\left(\frac{1}{2}, \phi\right) \sim 2H_1^2\delta \qquad q = 0 \longrightarrow \tilde{\mathcal{L}}_1(X, \phi) \sim H_1^2\left[-\frac{3}{4}(H_1\phi)^2 + X + \delta\, X^2\right]$$

$$q = \lambda X^3 \longrightarrow \tilde{\mathcal{L}}_2(X,\phi) = \tilde{\mathcal{L}}_1(X,\phi) + \lambda\left[X^3 - \frac{1}{8} - \frac{3}{4}(X - \frac{1}{2}) - \frac{3}{2}(X - \frac{1}{2})^2\right]$$

Note that these Lagrangians do not give the same c_s unless equation of motion is used to evaluate the background.

At this level (2 X derivatives), the two are **observationally indistinguishable**

More Explicitly

- Before putting backgd fields "on shell":
  - With $\tilde{\mathcal{L}}_1(X,\phi)$,

$$c_s = \left(1 + \frac{4X\delta}{1+2X\delta}\right)^{-1/2}$$

  - With $\tilde{\mathcal{L}}_2(X,\phi)$,

$$c_s = \left(1 + \frac{8X[\lambda(6X-3) + 2H_1^2\delta]}{3\lambda(1-2X)^2 + (4+8X\delta)H_1^2}\right)^{-1/2}$$

- After putting backgd fields "on shell", both cases give:

$$c_s = \left(1 + \frac{2\delta}{1+\delta}\right)^{-1/2}$$

Simple Example

Suppose measurements give $\epsilon \sim \frac{1}{2N_e} \ll 1 \qquad c_s = 1-\delta \qquad \delta \ll 1$

$$H = H_1 \exp\left(\int_1^{N_e} \frac{dN}{2N}\right) = H_1 N_e^{1/2}$$

$$\mathcal{L}\left(\frac{1}{2},\phi\right) = H_1^2(1 - H_1^2\phi^2)$$

$$\mathcal{L}_X\left(\frac{1}{2},\phi\right) = H_1^2$$

$$\mathcal{L}_{XX}\left(\frac{1}{2},\phi\right) \sim 2H_1^2\delta \qquad q = 0 \;\longrightarrow\; \tilde{\mathcal{L}}_1(X,\phi) \sim H_1^2\left[-\frac{3}{4}(H_1\phi)^2 + X + \delta\, X^2\right]$$

$$q = \lambda X^3 \;\longrightarrow\; \tilde{\mathcal{L}}_2(X,\phi) = \tilde{\mathcal{L}}_1(X,\phi) + \lambda\left[X^3 - \frac{1}{8} - \frac{3}{4}(X-\frac{1}{2}) - \frac{3}{2}(X-\frac{1}{2})^2\right]$$

Note that these Lagrangians do not give the same c_s unless equation of motion is used to evaluate the background.

At this level (2 X derivatives), the two are **observationally indistinguishable**

More Explicitly

- Before putting backgd fields "on shell":
  - With $\tilde{\mathcal{L}}_1(X, \phi)$,
$$c_s = \left(1 + \frac{4X\delta}{1+2X\delta}\right)^{-1/2}$$
  - With $\tilde{\mathcal{L}}_2(X, \phi)$,
$$c_s = \left(1 + \frac{8X[\lambda(6X-3) + 2H_1^2\delta]}{3\lambda(1-2X)^2 + (4+8X\delta)H_1^2}\right)^{-1/2}$$
- After putting backgd fields "on shell", both cases give:
$$c_s = \left(1 + \frac{2\delta}{1+\delta}\right)^{-1/2}$$

Simple Example

Suppose measurements give $\epsilon \sim \frac{1}{2N_e} \ll 1 \qquad c_s = 1 - \delta \qquad \delta \ll 1$

$$H = H_1 \exp\left(\int_1^{N_e} \frac{dN}{2N}\right) = H_1 N_e^{1/2}$$

$$\mathcal{L}\left(\frac{1}{2}, \phi\right) = H_1^2(1 - H_1^2\phi^2)$$

$$\mathcal{L}_X\left(\frac{1}{2}, \phi\right) = H_1^2$$

$$\mathcal{L}_{XX}\left(\frac{1}{2}, \phi\right) \sim 2H_1^2\delta \qquad q = 0 \longrightarrow \tilde{\mathcal{L}}_1(X, \phi) \sim H_1^2\left[-\frac{3}{4}(H_1\phi)^2 + X + \delta\, X^2\right]$$

$$q = \lambda X^3 \longrightarrow \tilde{\mathcal{L}}_2(X, \phi) = \tilde{\mathcal{L}}_1(X, \phi) + \lambda\left[X^3 - \frac{1}{8} - \frac{3}{4}(X - \frac{1}{2}) - \frac{3}{2}(X - \frac{1}{2})^2\right]$$

Note that these Lagrangians do not give the same c_s unless equation of motion is used to evaluate the background.

At this level (2 X derivatives), the two are **observationally indistinguishable**

More Explicitly

- Before putting backgd fields "on shell":
  - With $\tilde{\mathcal{L}}_1(X,\phi)$,

$$c_s = \left(1 + \frac{4X\delta}{1+2X\delta}\right)^{-1/2}$$

  - With $\tilde{\mathcal{L}}_2(X,\phi)$,

$$c_s = \left(1 + \frac{8X[\lambda(6X-3)+2H_1^2\delta]}{3\lambda(1-2X)^2+(4+8X\delta)H_1^2}\right)^{-1/2}$$

- After putting backgd fields "on shell", both cases give:

$$c_s = \left(1 + \frac{2\delta}{1+\delta}\right)^{-1/2}$$

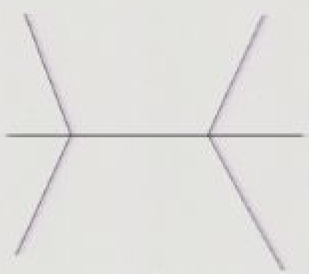
Do They Give Different Predictions for Something?

- Of course: e.g. 3 → 3 scattering in Minkowski

$$\mathcal{L}_1 = \frac{1}{2}(\partial\Phi)^2 - \frac{3}{4}H_1^2\Phi^2 + \frac{\delta}{4H_1^2 M_p^2}(\partial\Phi)^4$$

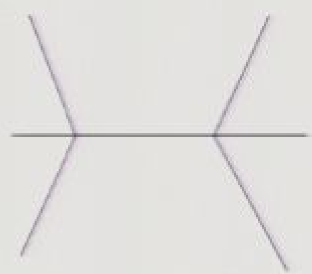
$$\mathcal{L}_2 = \frac{1}{2}(\partial\tilde{\Phi})^2 - \frac{3}{4}\frac{H_1^2}{1+\frac{3}{4}\lambda(\frac{M_p}{H_1})^2}\tilde{\Phi}^2 - \frac{\frac{\delta}{4}(\frac{H_1}{M_p})^2 - \frac{3\lambda}{8}}{[1+\frac{3}{4}\lambda(\frac{M_p}{H_1})^2]^2}(\partial\tilde{\Phi})^4 + \frac{\lambda}{8H_1^6 M_p^2}\frac{1}{[1+\frac{3}{4}\lambda(\frac{M_p}{H_1})^2]^3}(\partial\tilde{\Phi})^6$$

$\mathcal{L}_1$

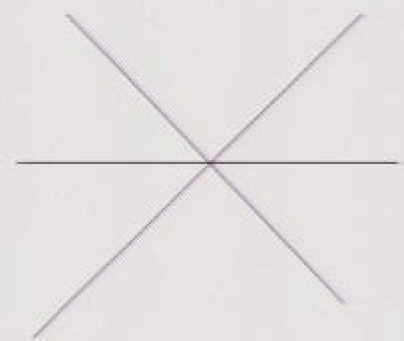


No λ dependence

$\mathcal{L}_2$



+



Effectively sensitive to 6-pt function.

Lesson: Need more observables!

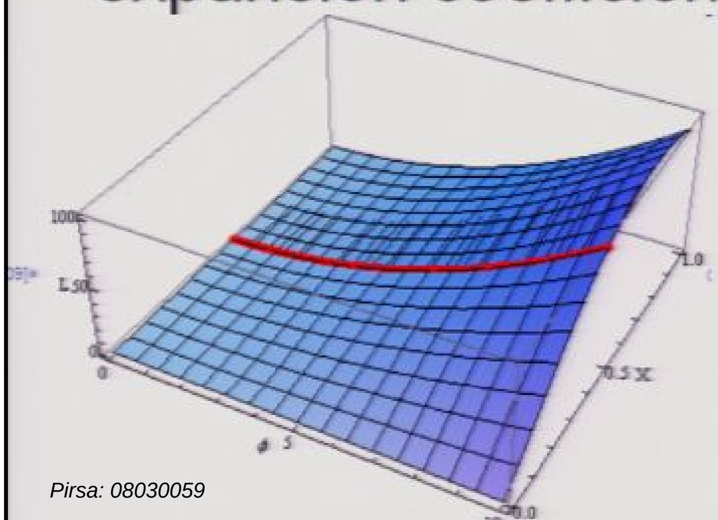
In retrospect obvious once one fixes the gauge $(e.g.\ X = \frac{M_p^2}{2})$

The determination of all

$$[\frac{\partial}{\partial X}]^n \mathcal{L}(\phi, X)|_{X=\frac{M_p^4}{2}}$$

from measurement is equivalent to specifying all Taylor expansion coefficients in the X direction.

$$c_s^2 \equiv \frac{p_X}{\rho_X} = \left(1 + 2\frac{X\mathcal{L}_{XX}}{\mathcal{L}_X}\right)^{-1}$$



Possibilities:

1) Other tree-lev terms in 3-point func.
1) Higher order correlation functions.
2) Loop corrections (probably too small)

3rd derivative from other tree level terms in 3-point Functions

In the nice papers of (Chen et al. 06) and (Seery and Lidsey 05)

$$\begin{aligned}\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle &= (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(\tilde{P}_K^\zeta)^2\frac{1}{\prod_i k_i^3} \\ &\times (\underbrace{\boxed{\mathcal{A}_\lambda+\mathcal{A}_c}}_{\text{leading}}+\mathcal{A}_o+\mathcal{A}_\epsilon+\mathcal{A}_\eta+\mathcal{A}_s)\end{aligned}$$

$$\boxed{\lambda = X^2\mathcal{L}_X + \frac{2}{3}X^3\boxed{\mathcal{L}_{XXX}}}$$

$$\begin{aligned}\mathcal{A}_\lambda &= \left(\frac{1}{c_s^2}-1-\frac{\lambda}{\Sigma}[2-(3-2\mathrm{c}_1)l]\right)_K\frac{3k_1^2k_2^2k_3^2}{2K^3} \\ \mathcal{A}_o &= \left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)_K(\epsilon F_{\lambda\epsilon}+\eta F_{\lambda\eta}+sF_{\lambda s}) \\ &+ \left(\frac{1}{c_s^2}-1\right)_K(\epsilon F_{c\epsilon}+\eta F_{c\eta}+sF_{cs}) \ ,\end{aligned}$$

Equilateral:

$$\begin{aligned}f_{NL}^\lambda &= -\frac{5}{81}\left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)+(3-2\mathrm{c}_1)\frac{l\lambda}{\Sigma} \ . \\ f_{NL}^o &= \mathcal{O}\left(\frac{\epsilon}{c_s^2},\frac{\epsilon\lambda}{\Sigma}\right)\end{aligned}$$

$$\Sigma = X\mathcal{L}_X + 2X^2\mathcal{L}_{XX}$$

Higher Order Correlation Functions

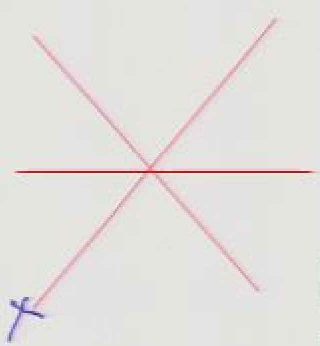
$$[\frac{\partial}{\partial X}]^n \mathcal{L}(\phi, X)|_{X=\frac{M_p^4}{2}}$$

n=2 contribution to 3-point function

$\langle\dot{\phi}\rangle \to \sqrt{2\epsilon} M_p H$

+ gravity effects

n=3 contribution to 5-point function



+ gravity effects

[Mapping under investigation,
but there should be a direct 1-1 corresp.
However, note n=3 contributes to 3-pt.]

Sound Speed and Unitarity

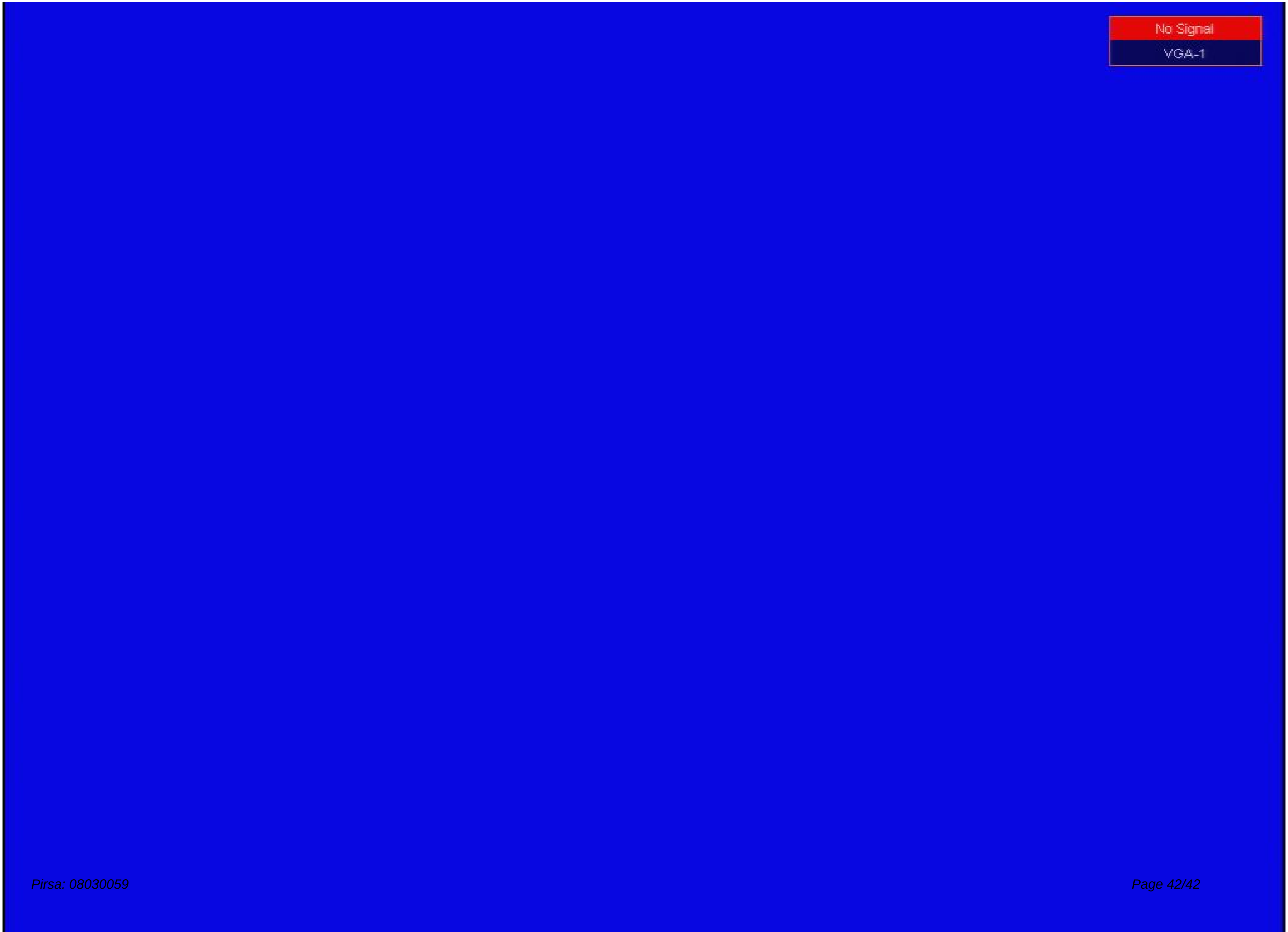
Scattering amplitudes can restrict the sign and magnitudes of the sign of the non-renormalizable kinetic terms (similar to the arguments of Adams et al.):

Im(forward scattering amplitude)=cross section

[implications for non-Gaussianity under investigation]

Summary

- One part of our work 0801.0742 examined the analytic structure of actions that can be consistent with ideal data including information coming from non-Gaussianities induced by non-minimal kinetic term interactions.
- As long as gauge transformations are non-singular, general single field actions (a 2D manifold parameterized by (ϕ, X)) can be written as Taylor expansion in the X direction which is orthogonal to the 1D manifold (parameterized by ϕ) encoding the information from the data. (A surprisingly simple result for general parameterization.)
- Unlike in the case of minimal kinetic terms, even with ideal data for 3-point functions for "all" N-efolds and fixed reheating scenarios, the set of models which are consistent with data forms an infinite set. Good news is that the infinite set organize themselves in a simple manner.
- Higher order correlation function measurements should have a one-to-one correspondence with the leading dimensions of non-renormalizable kinetic operator ambiguity.
- Rachel Bean will give (or has given) a presentation regarding the other aspect of our work.


No Signal
VGA-1