Title: Primordial non-Gaussianity: Two "shapes" to look for

Date: Mar 10, 2008 01:00 PM

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Abstract:

Primordial non-Gaussianity: two "shapes" to look for

In "vanilla" models of inflation, the initial fluctuations are Gaussian
The 3-point correlation function function is zero:

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = 0$$

However, more exotic models can predict nonzero three-point functions
"Local" shape: e.g. curvaton model

$$\zeta(x) = \zeta_G(x) + f_{NL}^{\mathrm{local}} \zeta_G(x)^2$$

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle \sim f_{NL}^{\mathrm{local}} \left(\frac{1}{k_1^3 k_2^3} + \mathrm{symm.} \right) \delta^3 \left(\sum_i k_i \right)$$

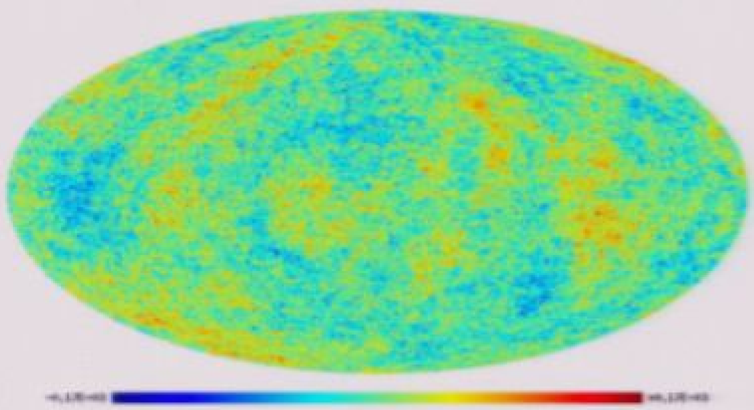
"Equilateral" shape: higher-derivative interactions, DBI inflation

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle \sim f_{NL}^{\mathrm{equil.}} \left(\prod_{i=1}^{3} \frac{k_1 + k_2 + k_3 - 2k_i}{k_i^3} \right) \delta^3 (\sum_i k_i)$$

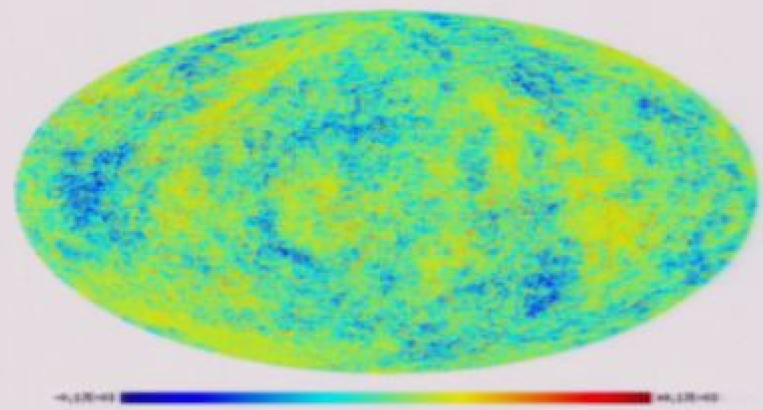
Three-point function in the observed CMB

Non-gaussian initial conditions from inflation
+ linear transfer functions = non-Gaussian CMB

$f_{NL}^{\rm local} = 0$



$f_{NL}^{\rm local} = 3000$



Michele Liguori

Optimal estimator: sum over triples (l1,l2,l3) with inverse signal-to-noise weighting

$$\langle T(l_1)T(l_2)T(l_3)\rangle \propto f_{NL}\delta^2\left(\sum_i l_i\right)$$

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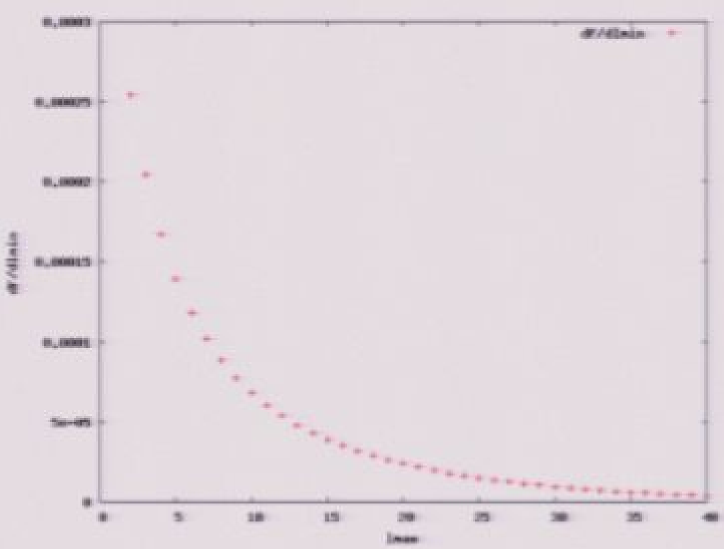
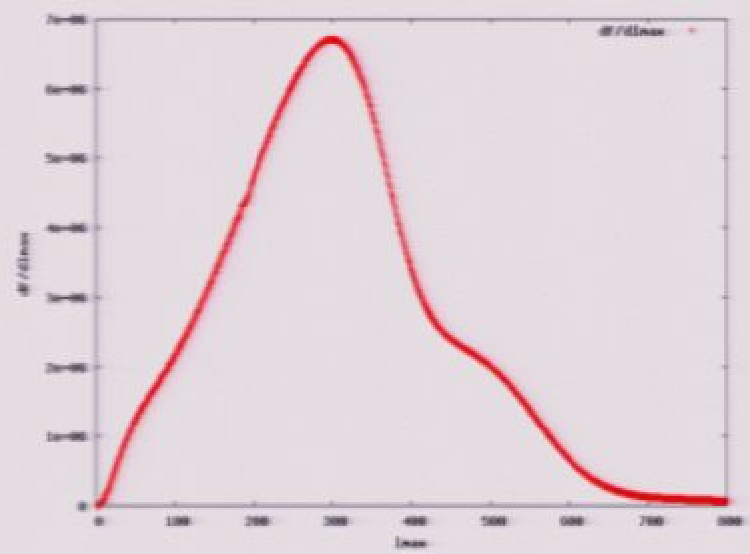
"Local fnl": signal is in squeezed triangles (l1 << l2,l3),
sign of bispectrum is always negative (in squeezed triangles)





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but a weak function of position on the sky (A -> A(n))

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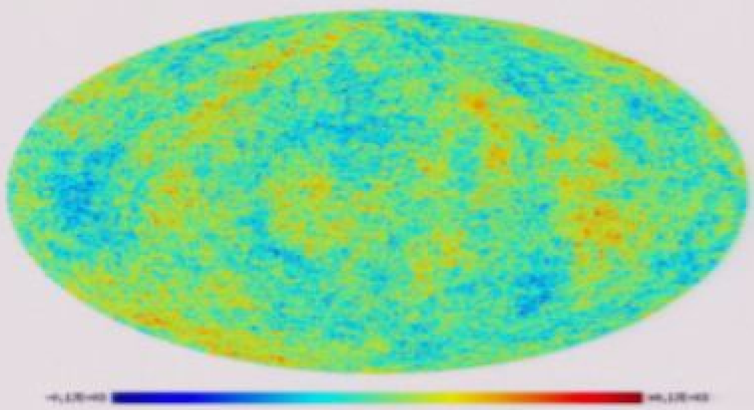

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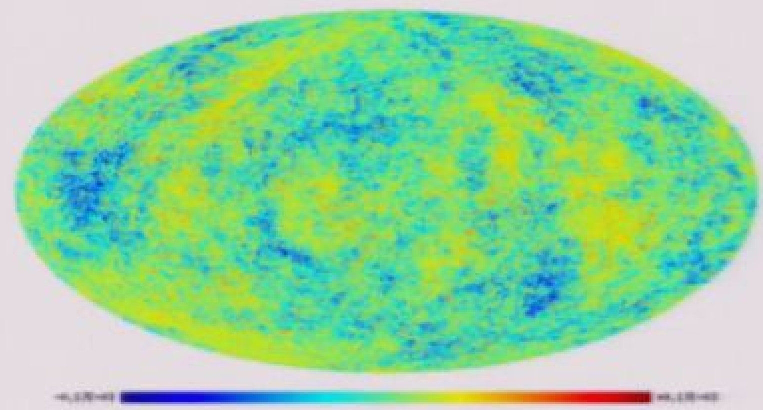

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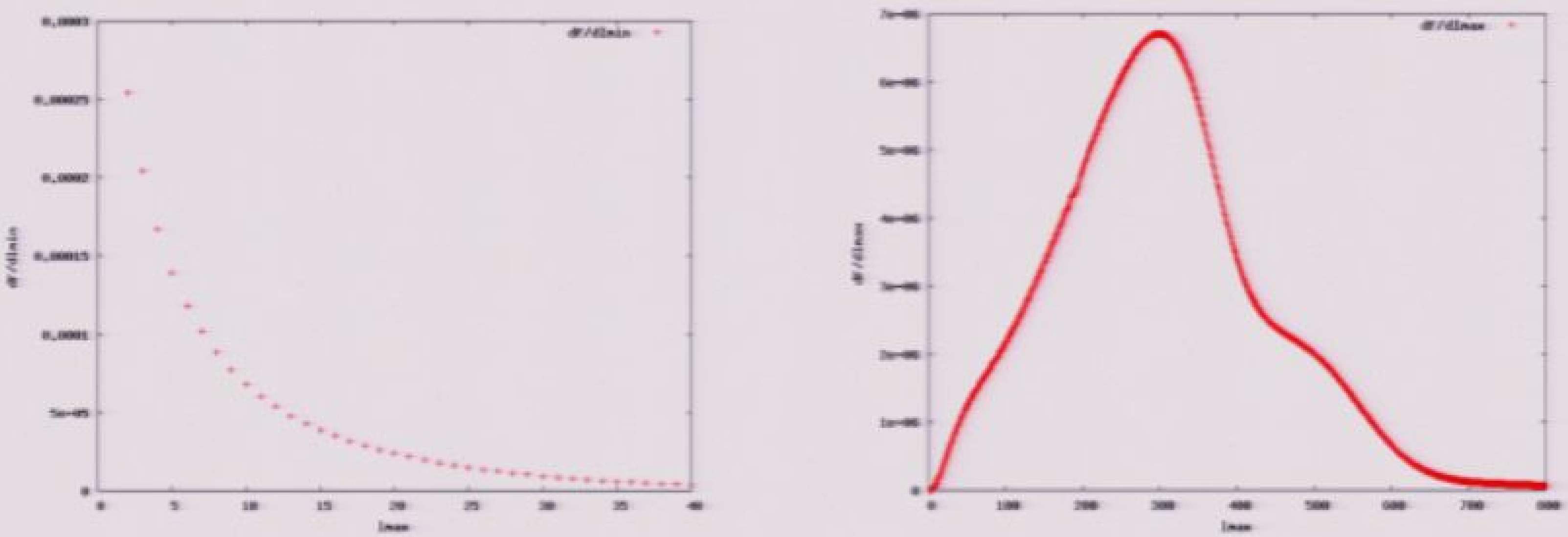

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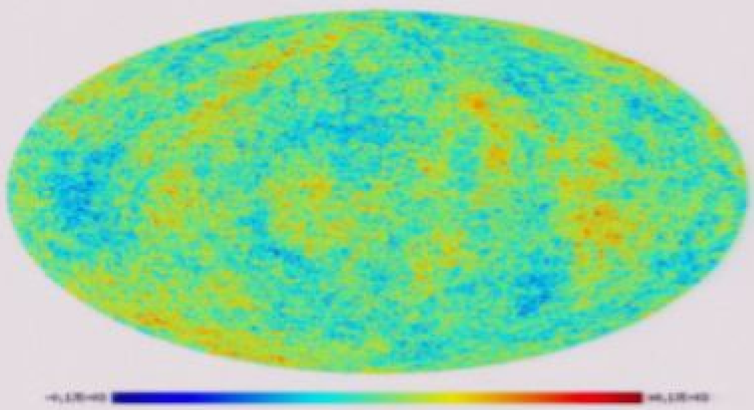

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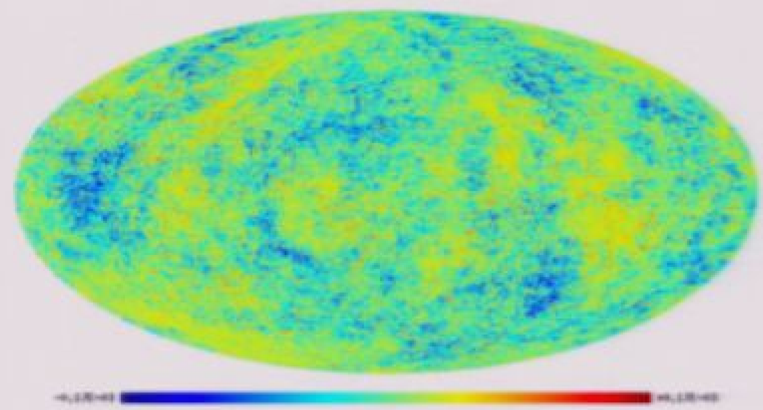

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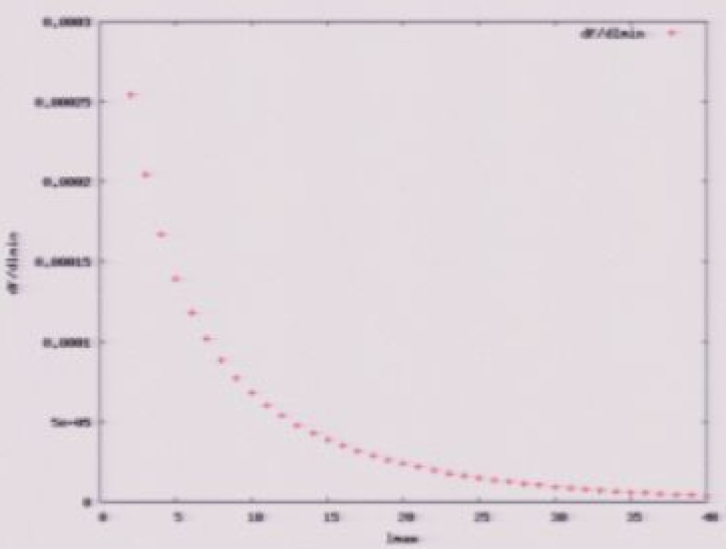

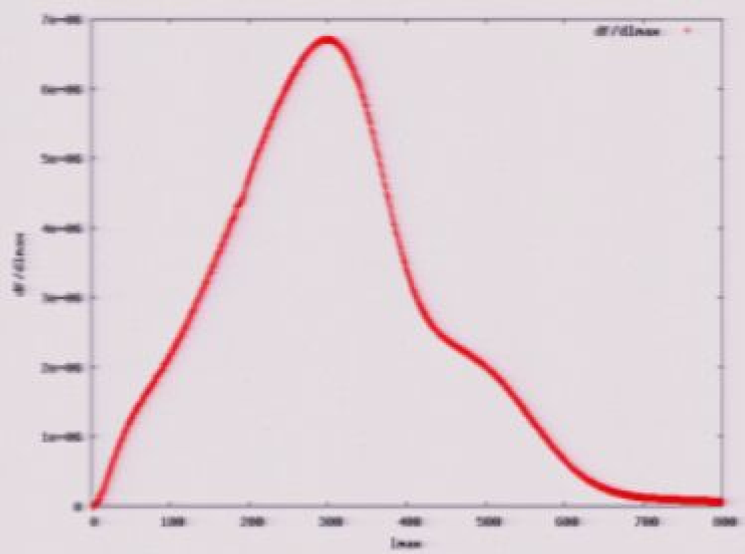

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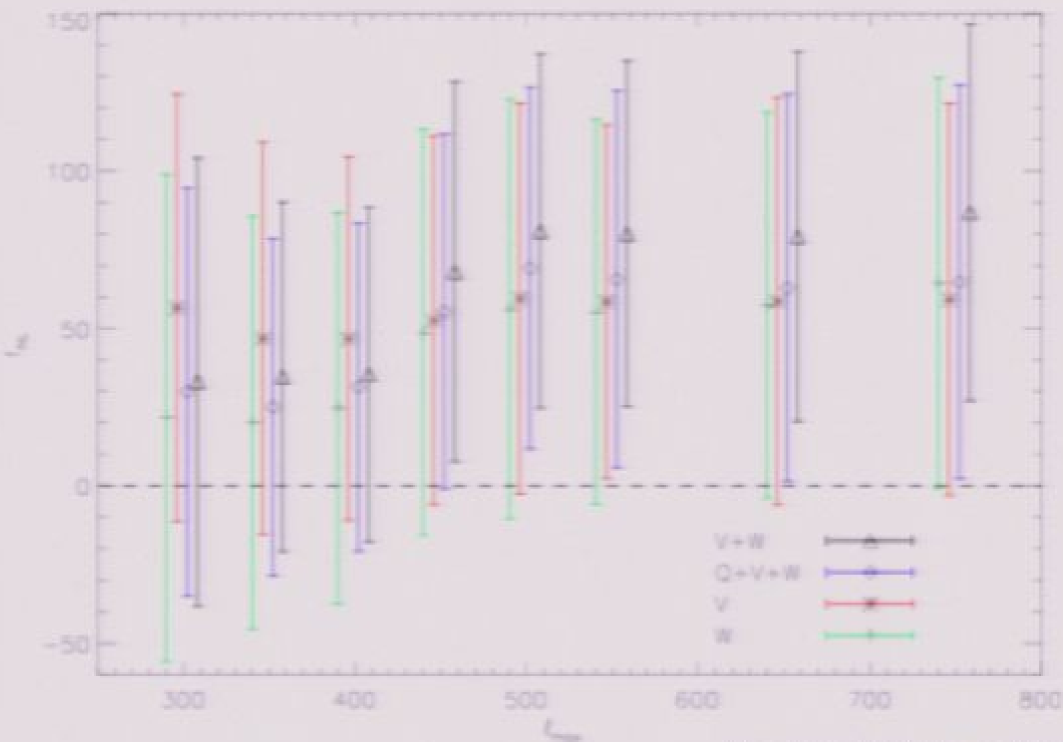
$$f_{NL}^{\rm local} = 32 \pm 34$$ Creminelli et al (WMAP3)

$$f_{NL}^{\rm local} = 87 \pm 30$$ Yadav & Wandelt (WMAP3!!)

$$f_{NL}^{\rm local} = 55 \pm 30$$ Komatsu et al (WMAP5)

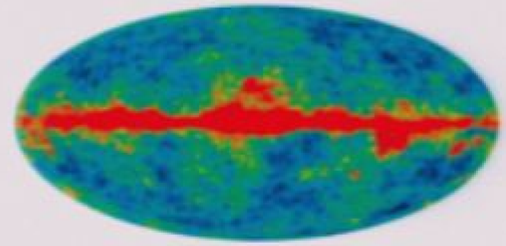
A robust detection would rule out most models of inflation! (e.g. slow-roll)
Which analysis should be believed?

Reason for the discrepancy: "step" at l=450

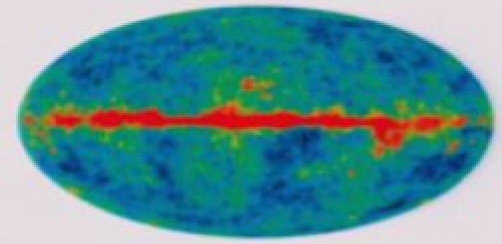


Yadav & Wandelt

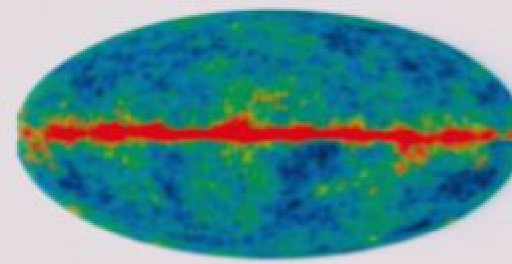
Q-band
(40 GHz)



V-band
(60 GHz)



W-band
(90 GHz)



Must be careful to avoid making a posteriori choices.... !

Use of V+W is motivated a priori

Use of lmax=750 is not

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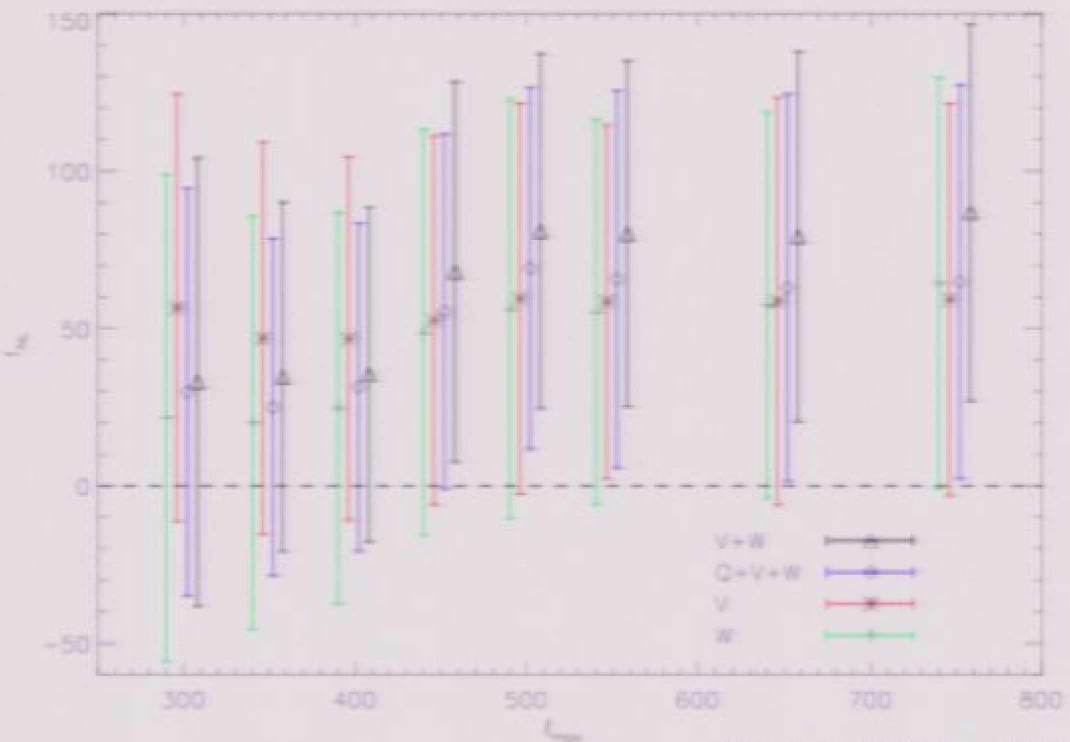

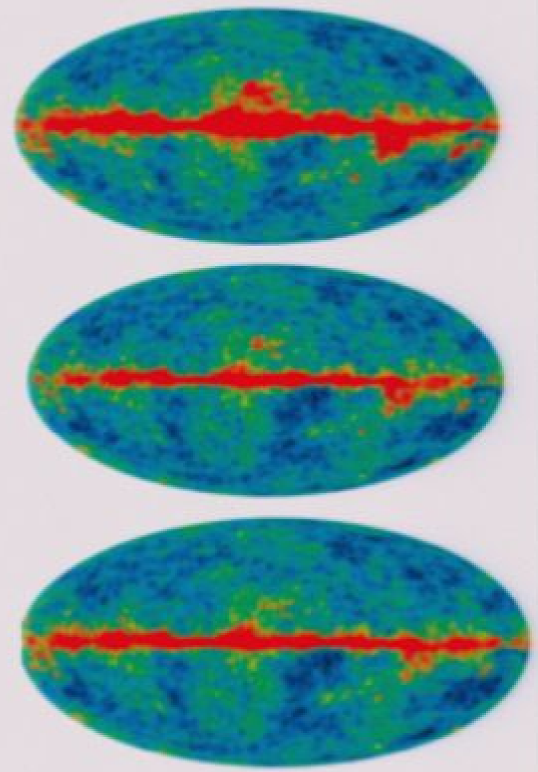

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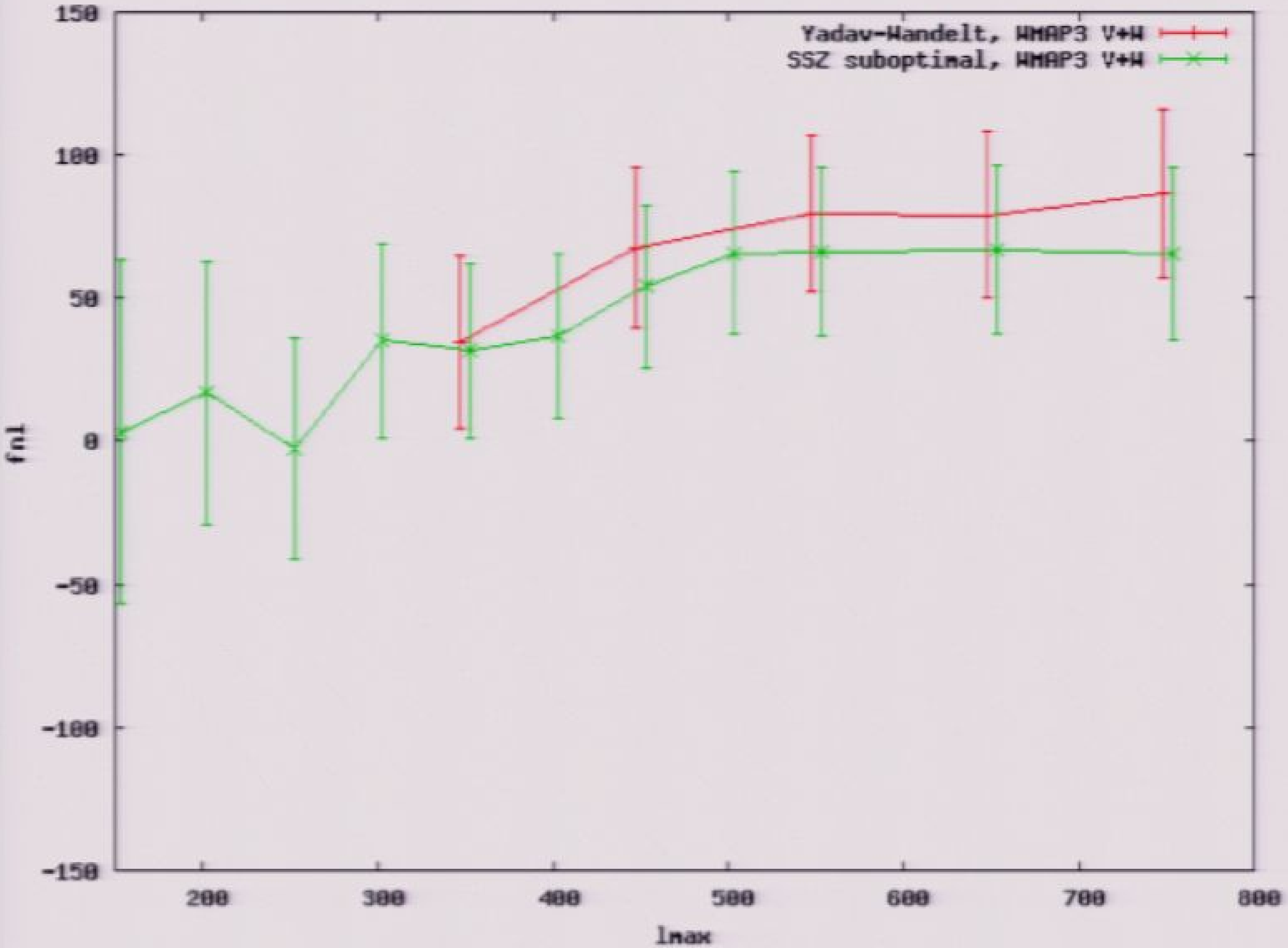

2.9 sigma

2.1 sigma

Two fnl analyses can differ because several arbitrary choices are made (pixel weighting, channel weighting, l weighting, mode subtraction): estimator is suboptimal

Systematics: general picture

Question: is the "step" at l=450 evidence of systematic contamination?

Looking for contaminants which correlate level of small-scale power to large-scale modes

Positive correlation = negative fnl

fnl = O(100) corresponds to a correlation of order 10^(-3) !

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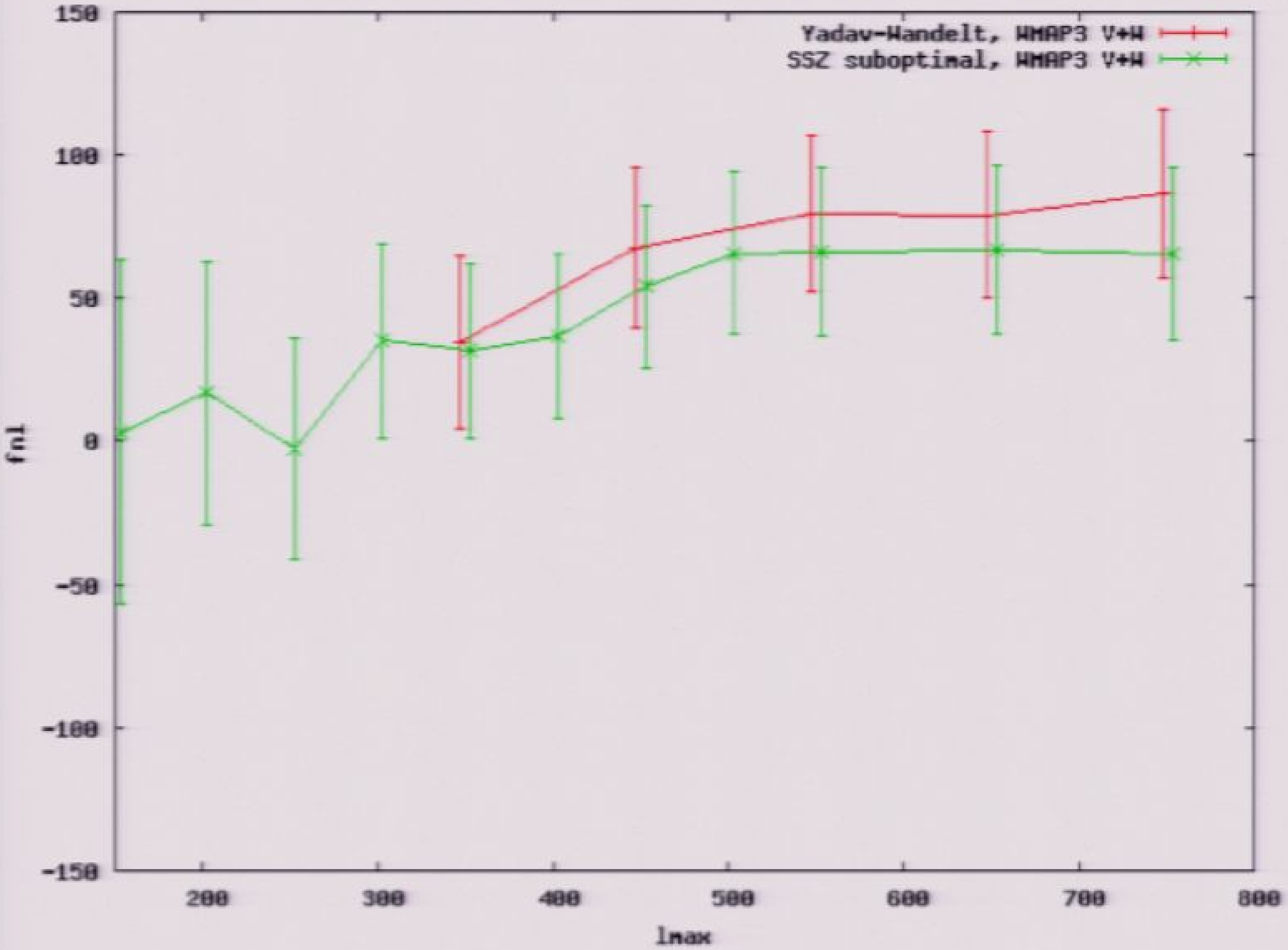

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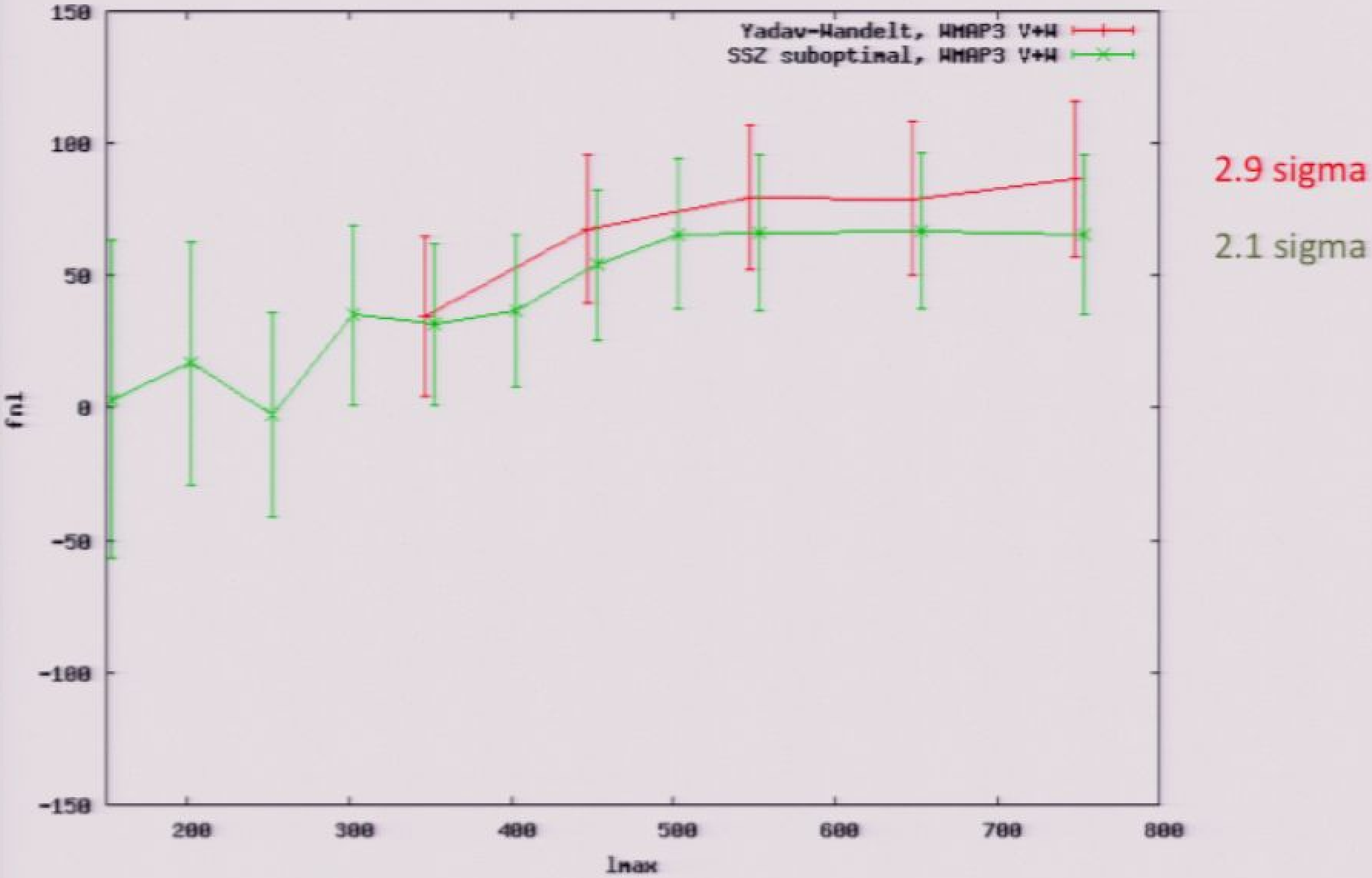

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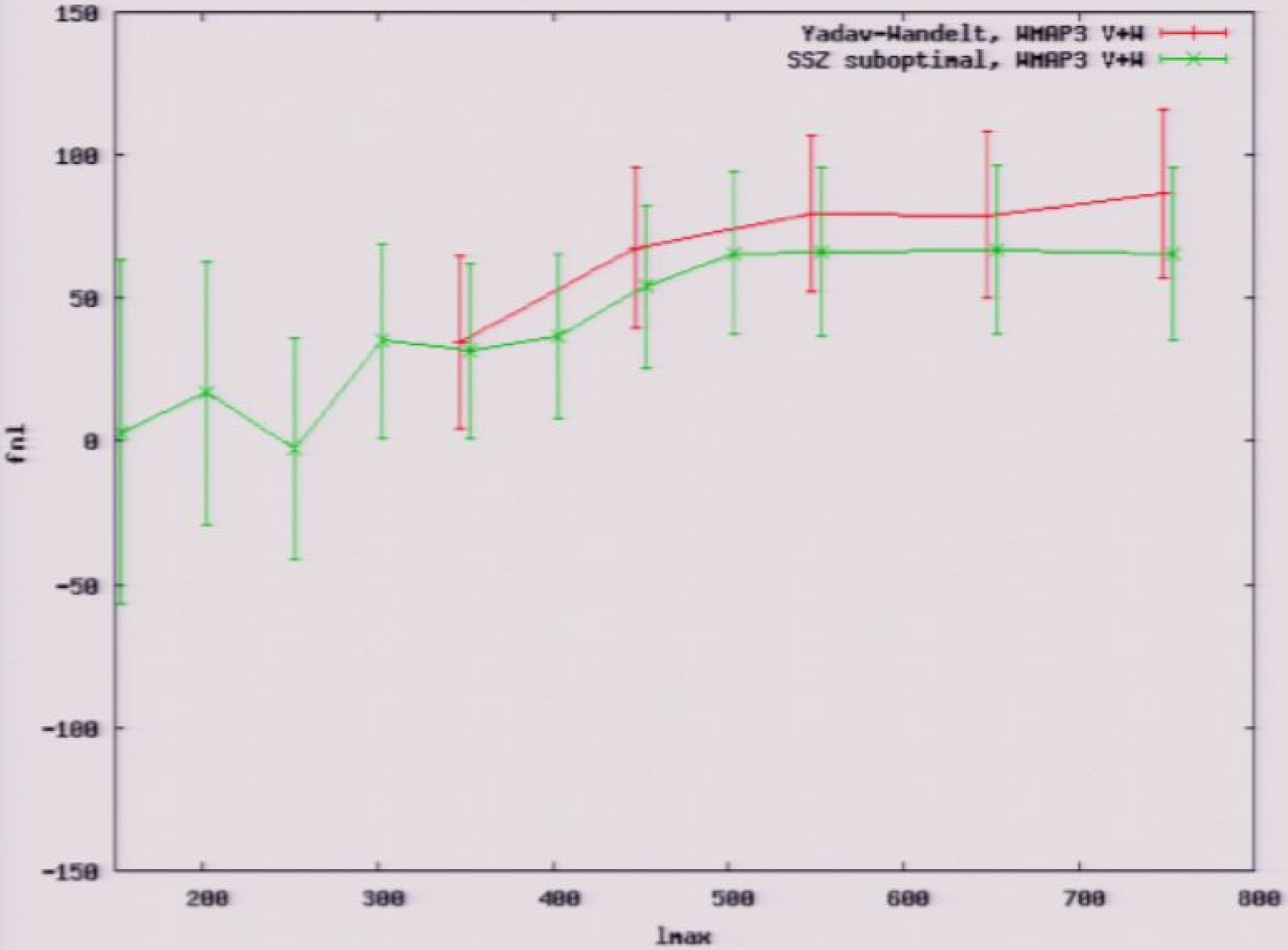

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Systematics: galactic foregrounds

Galactic foregrounds: bias to local fnl is always negative?

Heuristic argument: foregrounds are emissive,
more small-scale "blobs" in regions of high emission

Empirical evidence: compare foreground masks

ℓ_{max}	VW				Q	QVW			
	Kp12	Kp2	Kp0	Kp0+	Kp0	Kp12	Kp2	Kp0	Kp0+
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Yadav & Wandelt

From templates, foreground bias in a conservative mask seems to be small (order 1)

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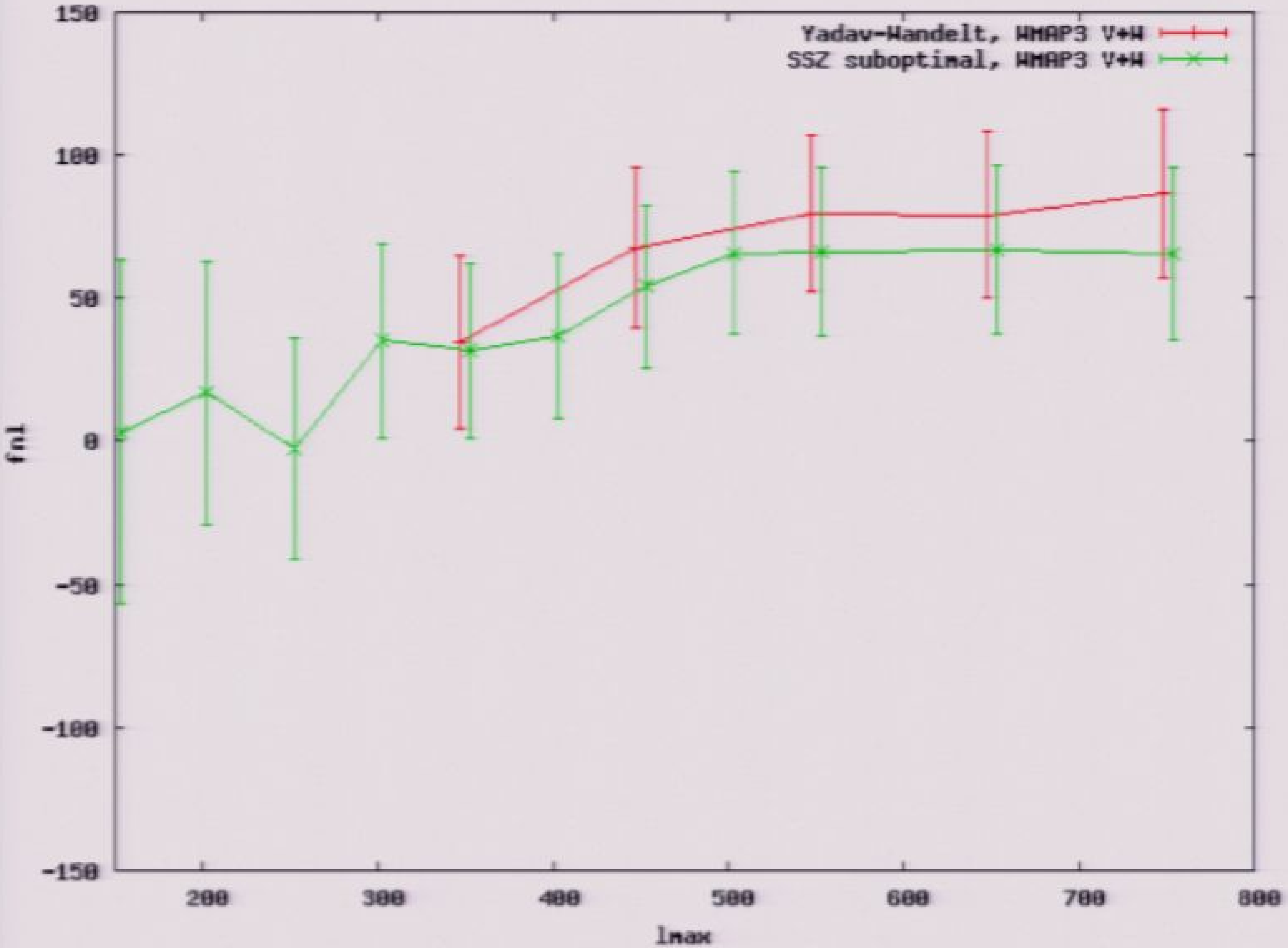

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Systematics: extragalactic contaminants

Compute (X,Y,Z) bispectrum,
where X,Y,Z = ISW, lensing, point sources, SZ, kSZ, Rees-Sciama, ...

To overlap with local fnl, need X=ISW, expect fnl bias to be positive

ISW-lensing:
Delta(fnl) ~ 5 (Smith & Zaldarriaga 2006, Serra & Cooray 2008)

ISW-(PS+SZ,PS+SZ):
Delta(fnl) < 1 (Babich & Pierpaoli, today)

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Systematics: point sources

Only get overlap with local fnl if point source density has gradients on large scales:
point source clustering
unresolved galactic sources

Expect bias to be negative

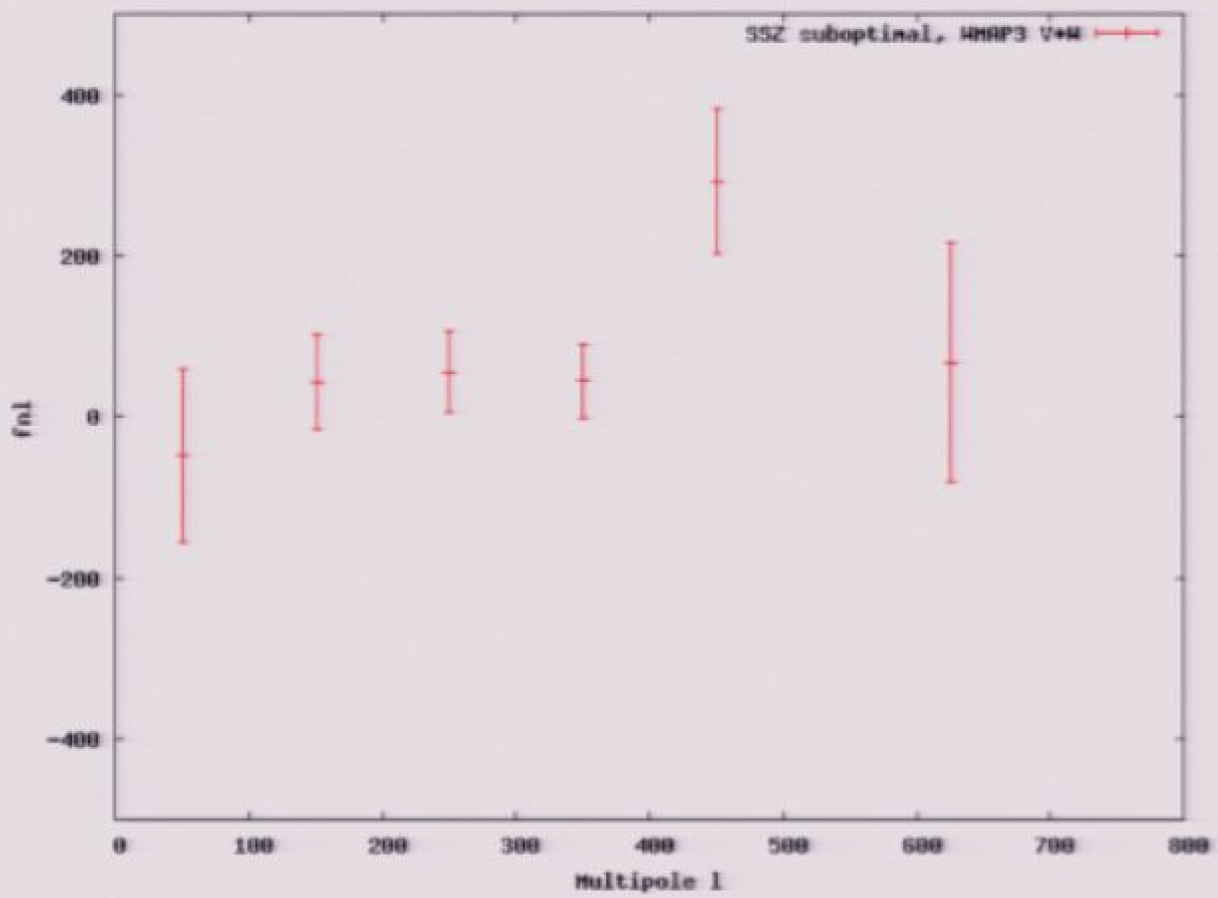
In simulations, we find that Delta(fnl) is of order 1, for realistic point source models

Possible instrumental systematics also appear to be small

Conclusion: expected level of systematic contamination is small,
this is a "pure statistics" problem.....

Statistical significance of WMAP3 "step"

Second question: is the "step" at l=450 within statistics?



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(assigns too much statistical weight to high l)

Optimal estimator: motivation

The WMAP3 "step" is 1.8-2.8 sigma in simulations, depending on endpoints chosen
With optimal estimator: such a jump would be >4 sigma

Motivation for constructing optimal estimator:

1. smaller error bar! (WMAP5 V+W: Delta(fnl)=21)
2. no arbitrary choices, two implementations should agree,
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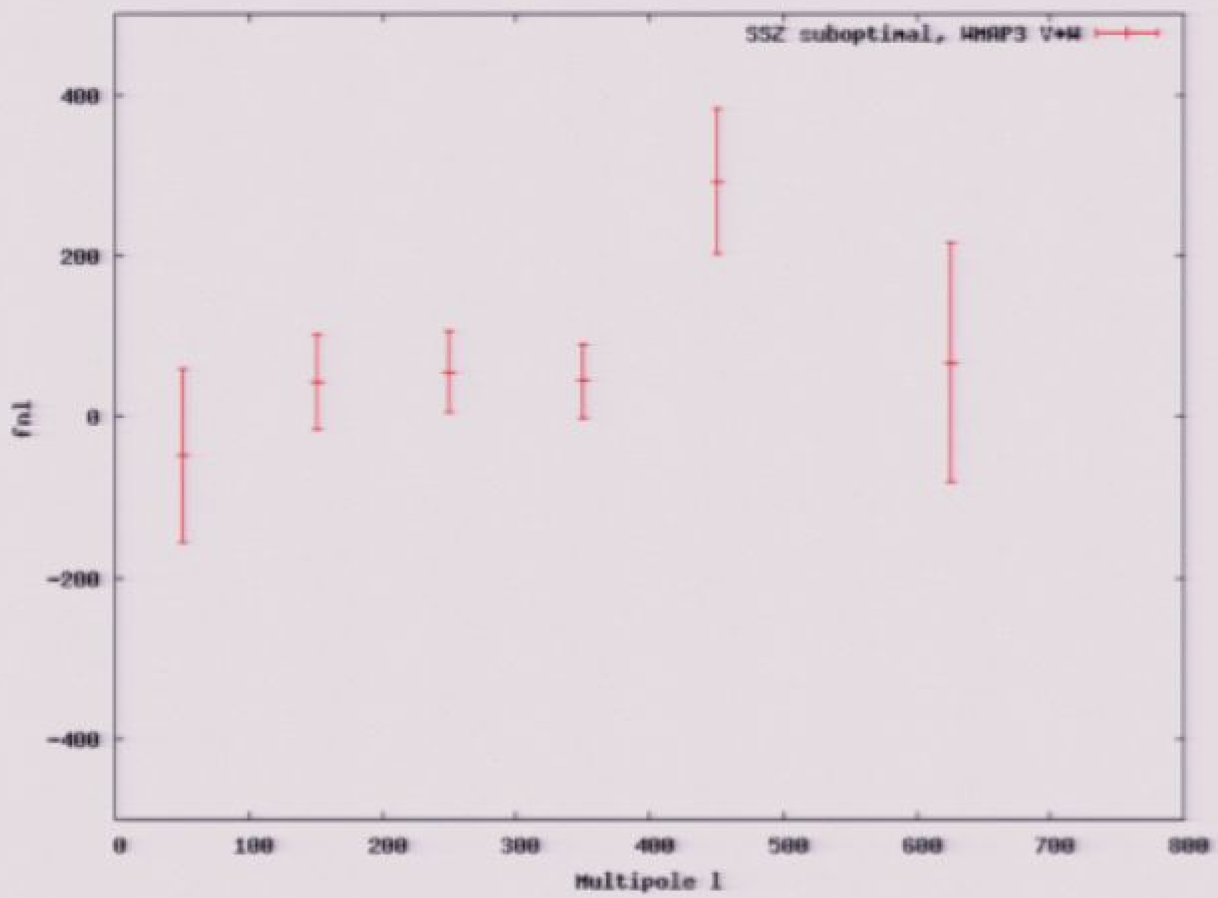

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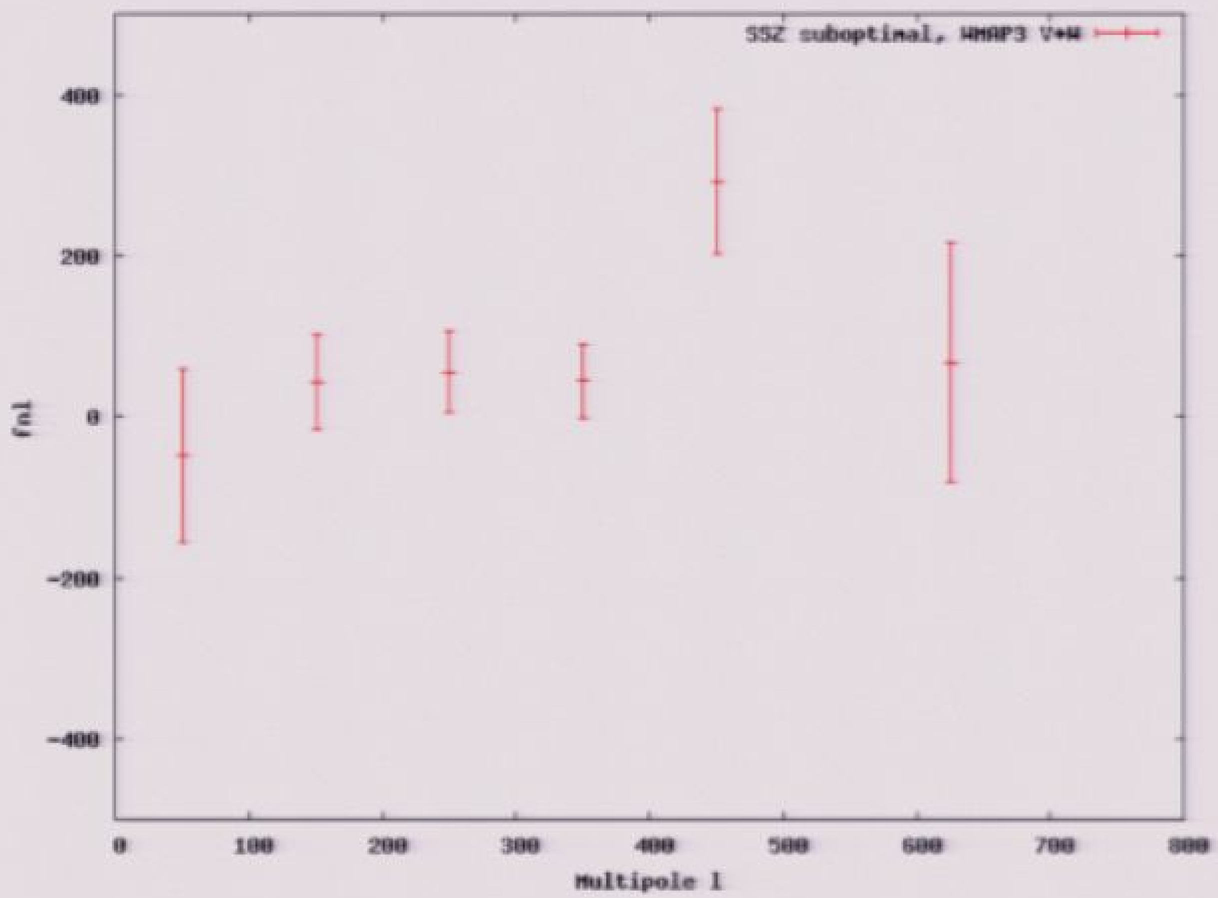

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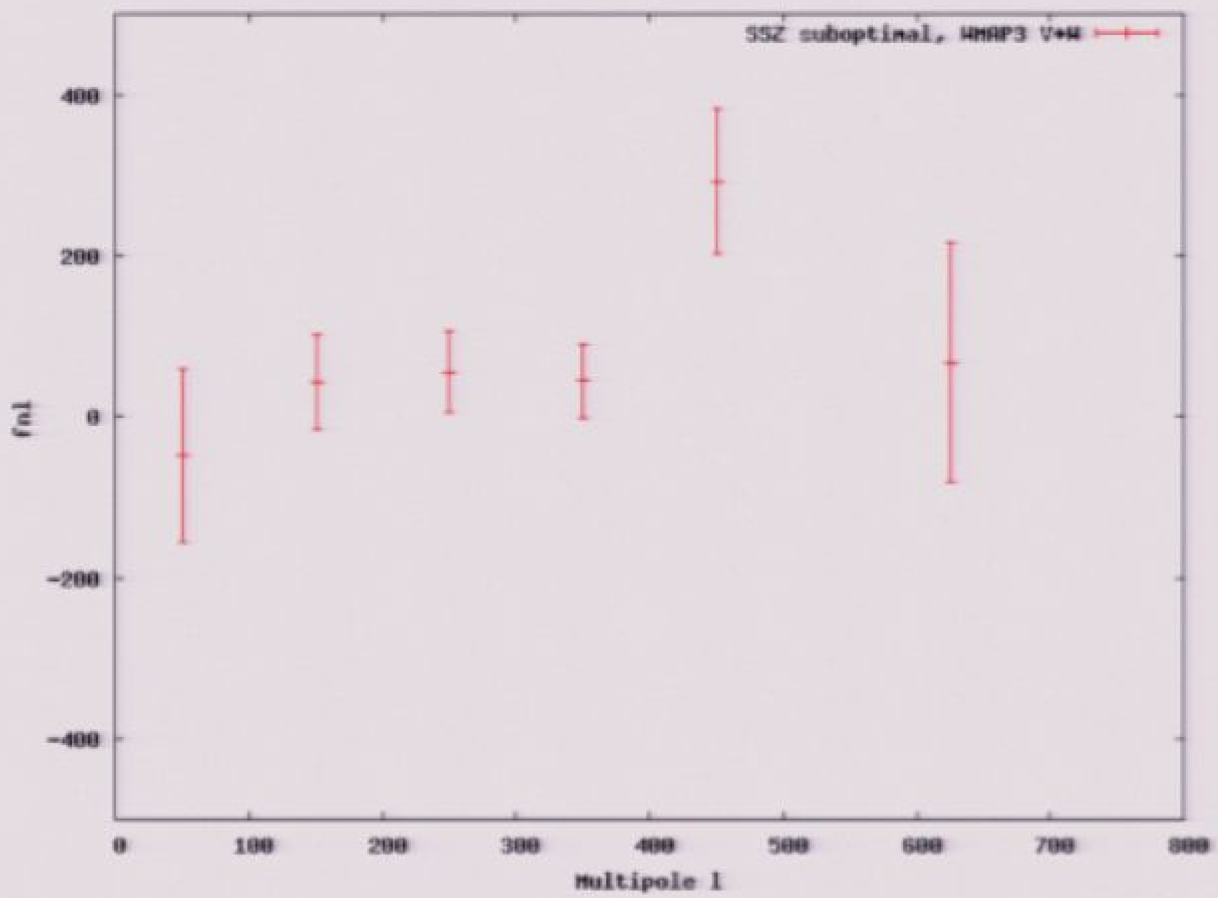

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Optimal estimator: construction

1. filter WMAP map m by inverse signal + noise: m -> (S+N)^{-1} m
   combines optimal channel weighting, pixel weighting and l weighting
2. estimate fnl from the filtered map (intuition: estimate small-scale power in degree-scale patches, correlate back to CMB)

Suboptimal estimator:

1. apply some heuristic filter intended to approximate (S+N)^{-1}
2. estimate fnl from filtered map in same way

Implementational challenge: (S+N)^{-1}

Use multigrid conjugate gradient inversion, ~20 CPU-min per (S+N)^{-1} multiplication

(Pen 2003, Smith et al 2007)

Our noise model ("N") includes:

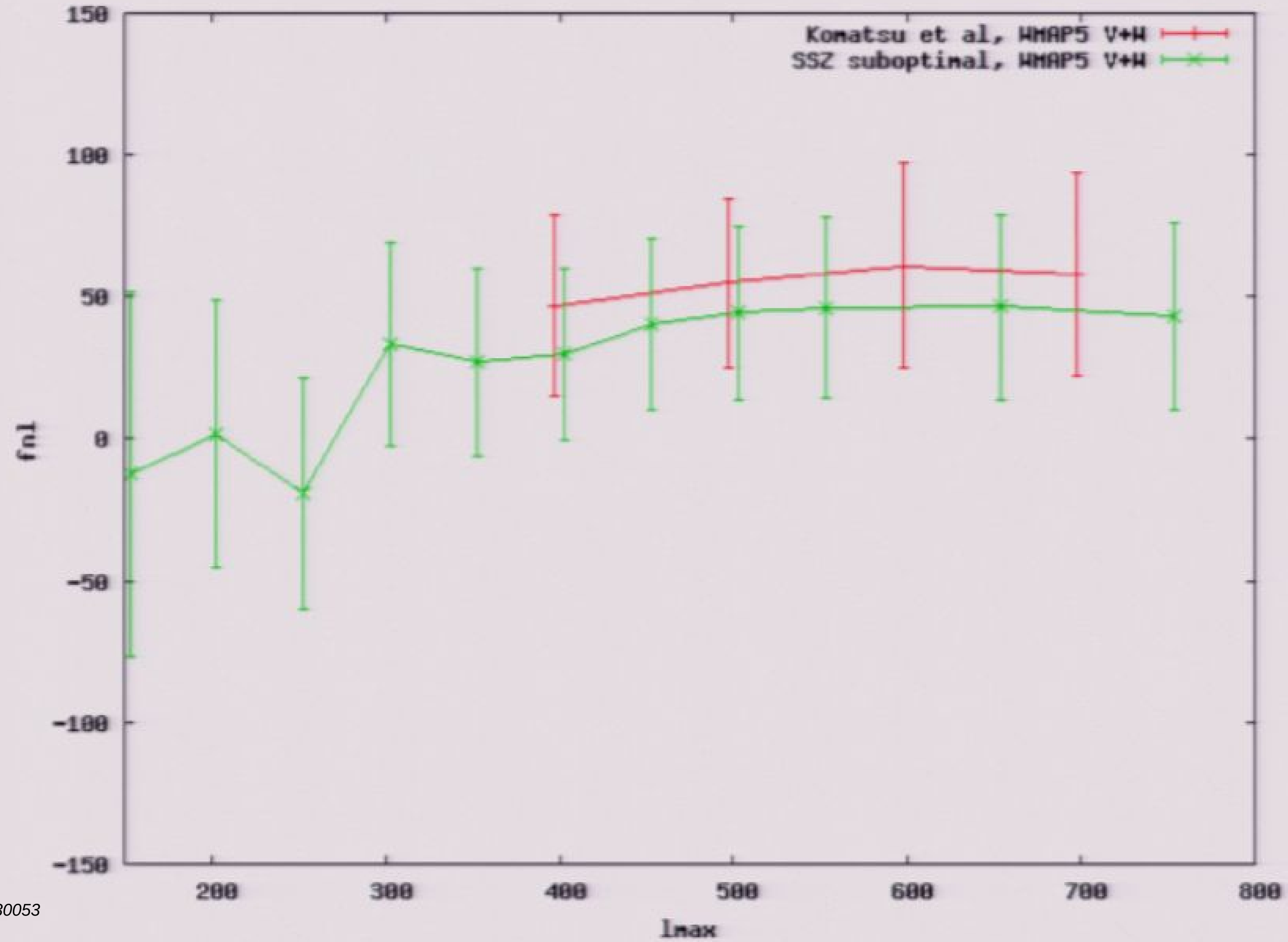
WMAP detector noise

KQ75 sky cut

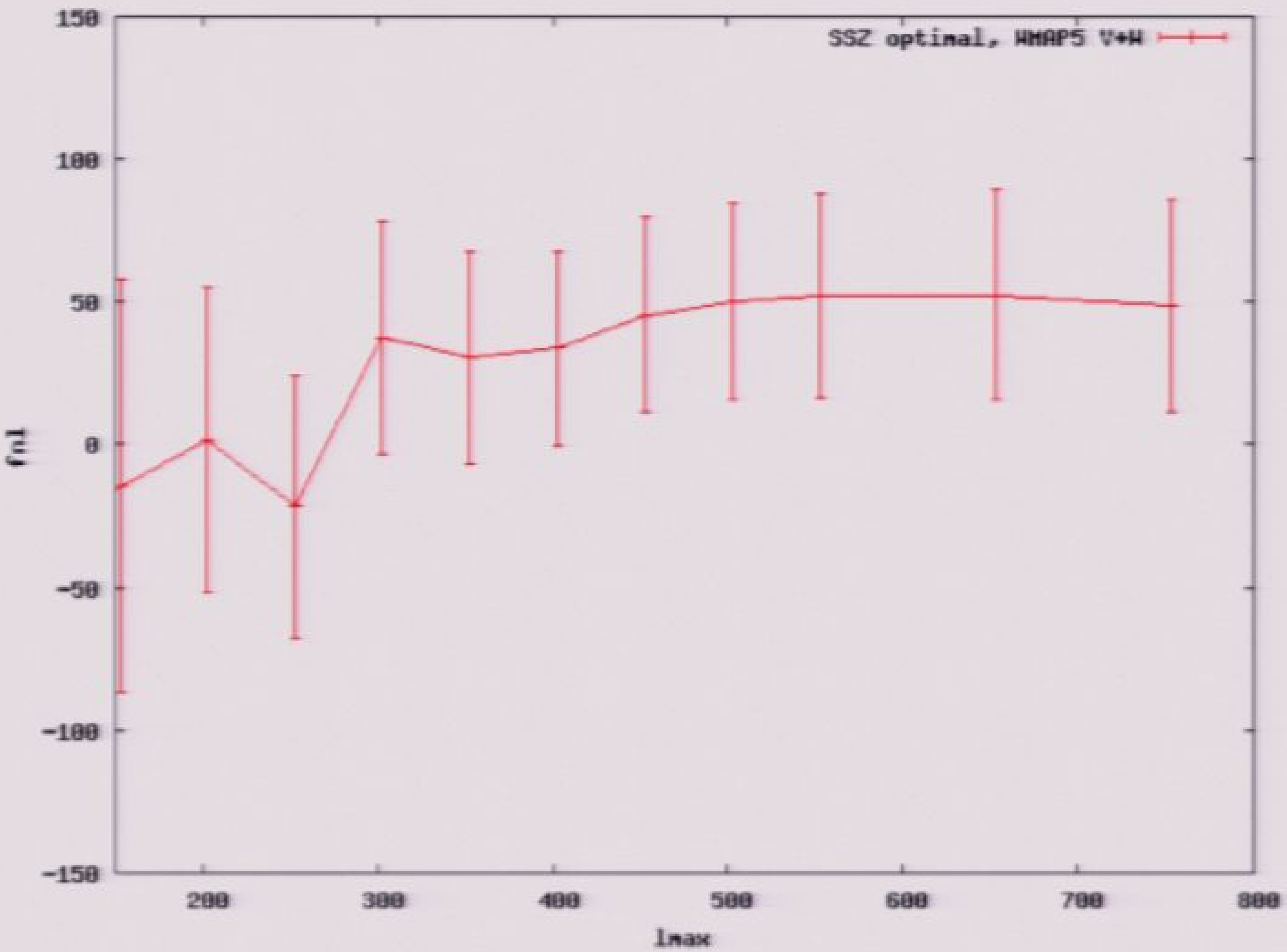
monopole/dipole marginaliation

foreground template marginalization

WMAP5: suboptimal estimator



WMAP5: optimal estimator



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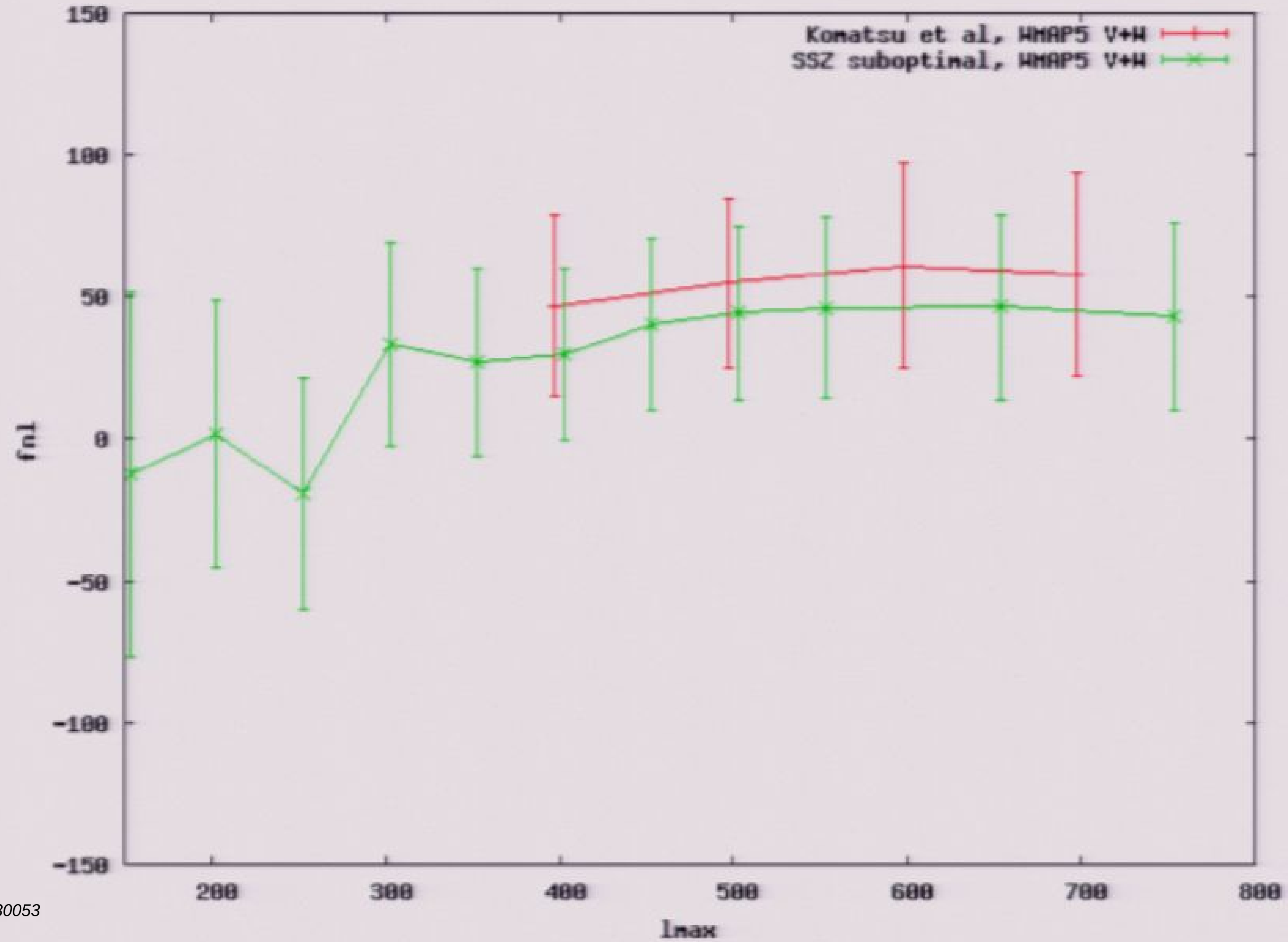

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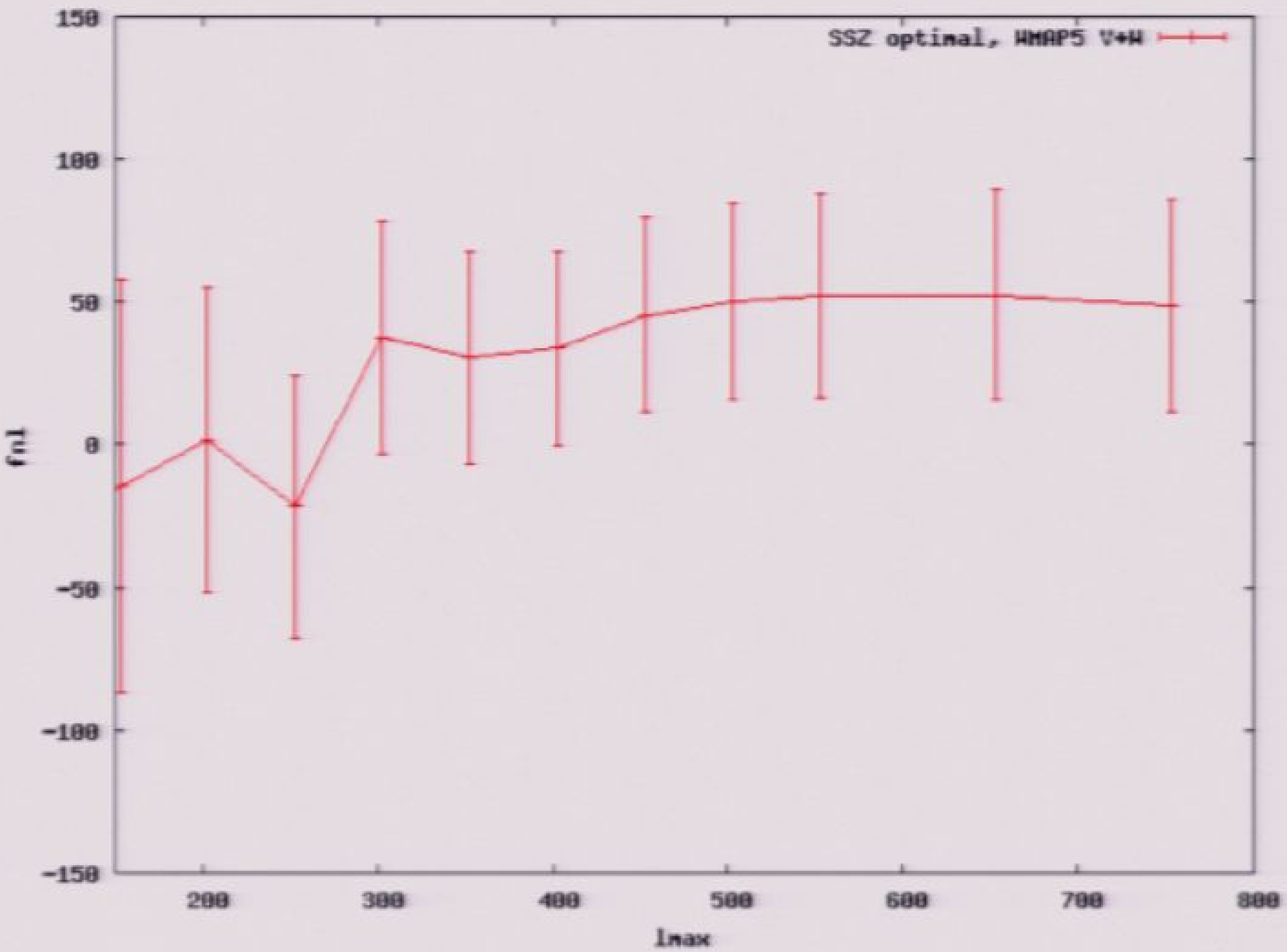

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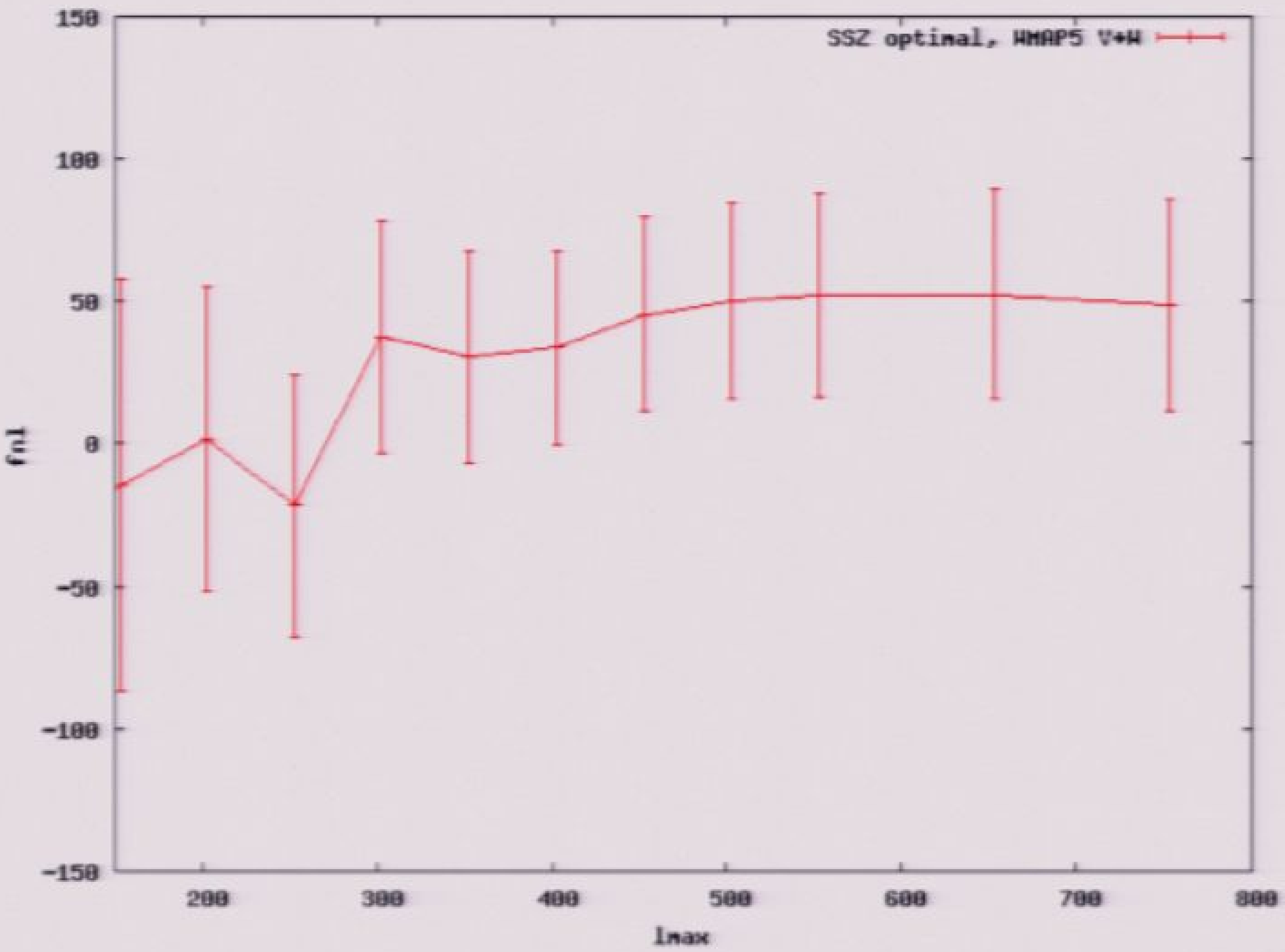

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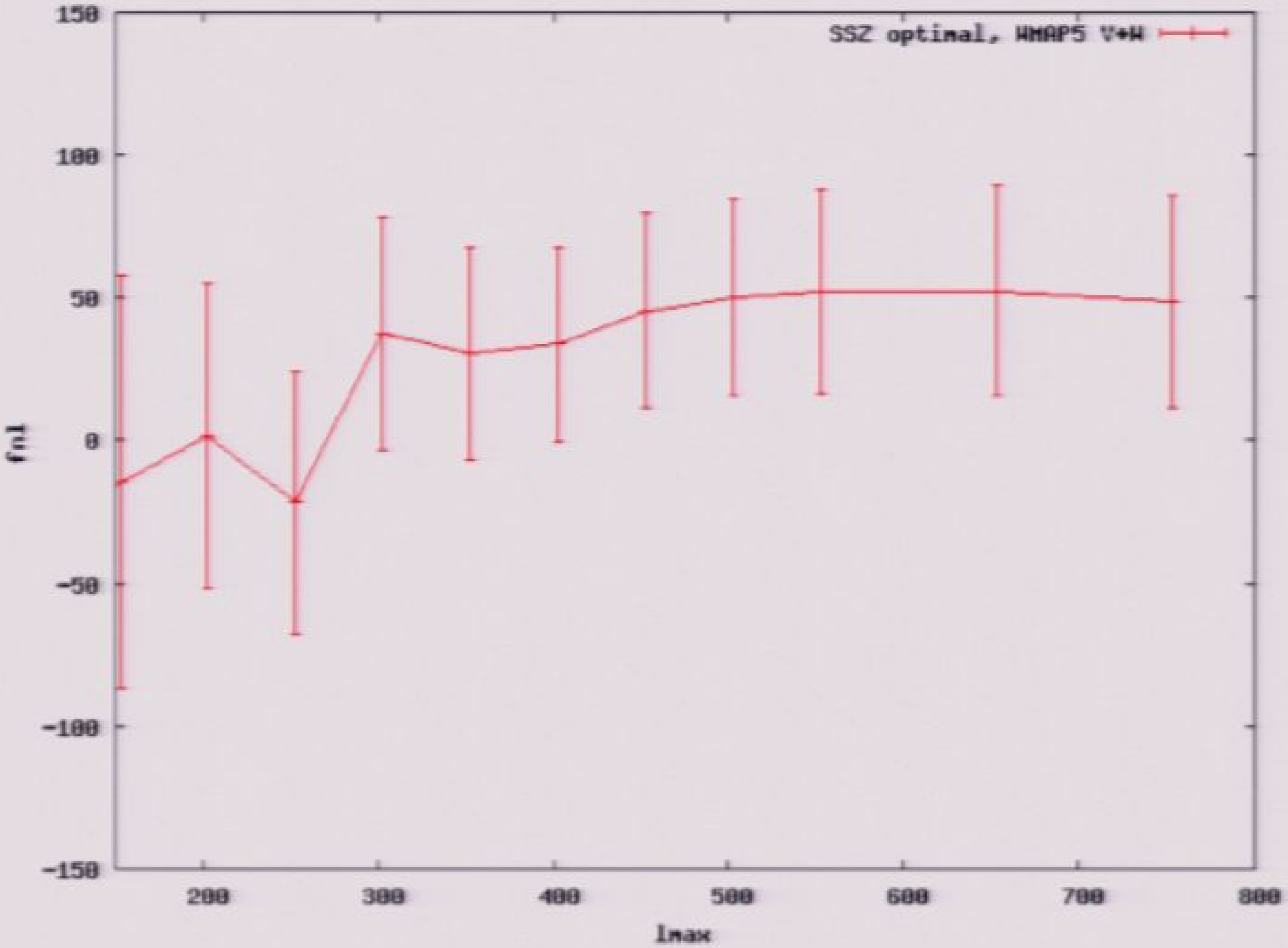

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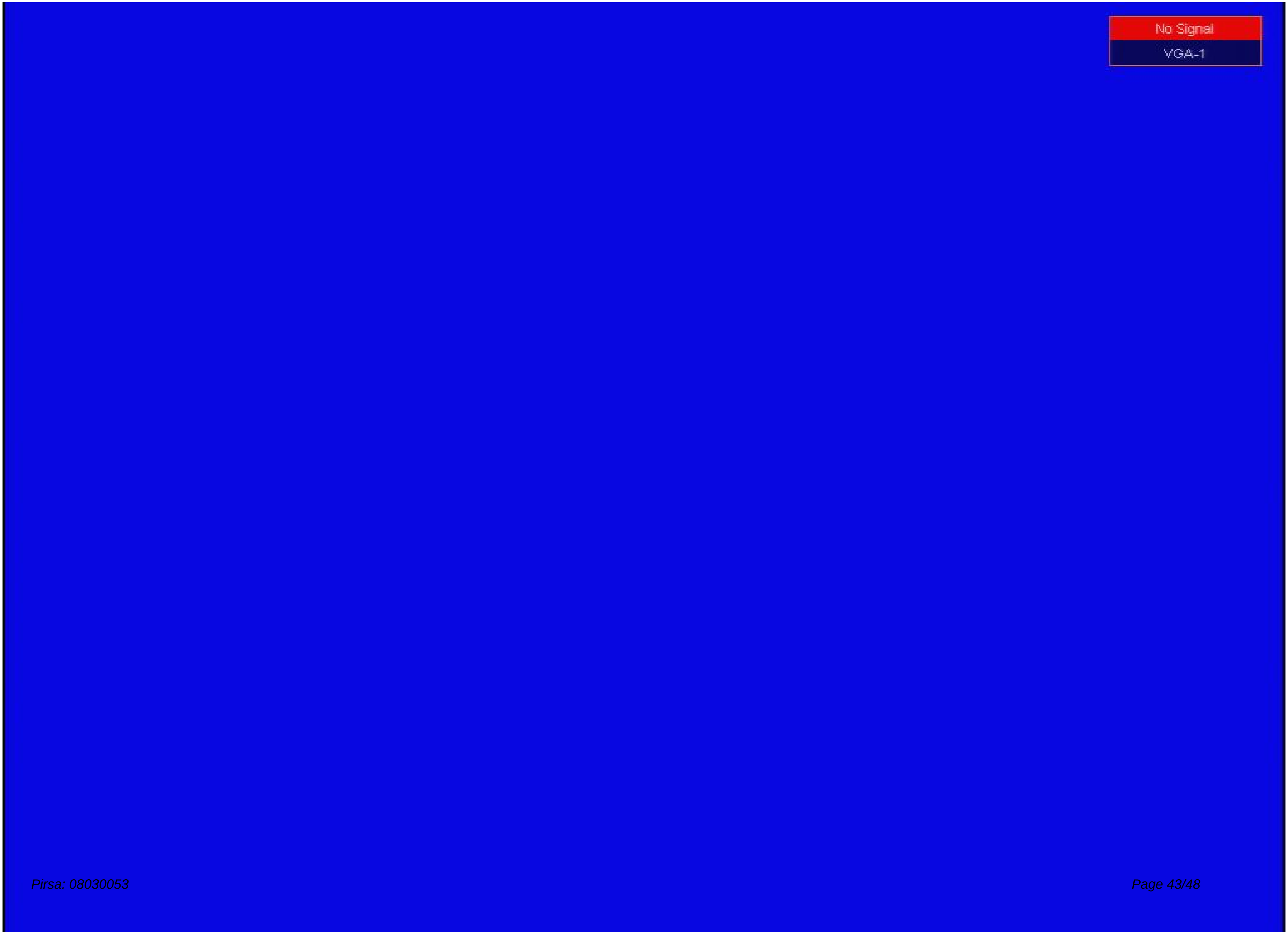
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VGA-1

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