

Title: WMAP 5-year Results: Measurement of f_{NL}

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Abstract:

WMAP 5-Year Results: Measurement of f_{NL}

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“Origins and Observations of Primordial Non-Gaussianity”

Perimeter Institute, March 8, 2008

WMAP 5-Year Papers

- **Hinshaw et al.**, “*Data Processing, Sky Maps, and Basic Results*” [0803.0732](#)
- **Hill et al.**, “*Beam Maps and Window Functions*” [0803.0570](#)
- **Gold et al.**, “*Galactic Foreground Emission*” [0803.0715](#)
- **Wright et al.**, “*Source Catalogue*” [0803.0577](#)
- **Nolta et al.**, “*Angular Power Spectra*” [0803.0593](#)
- **Dunkley et al.**, “*Likelihoods and Parameters from the WMAP data*” [0803.0586](#)
- **Komatsu et al.**, “*Cosmological Interpretation*” [0803.0547](#)

WMAP 5-Year Science Team

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- E. Wollack
- J. Dunkley
- B. Gold
- E. Komatsu
- D. Larson
- M.R.olta

Special
Thanks to
WMAP
Graduates!

- C. Barnes
- R. Bean
- O. Dore
- H.V. Peiris
- L. Verde

What is f_{NL} ?

- For a pedagogical introduction to f_{NL} , see **Komatsu, astro-ph/0206039**
- In one sentence: “ f_{NL} is a **quantitative measure** of the magnitude of primordial non-Gaussianity in curvature perturbations.^{*,†}”

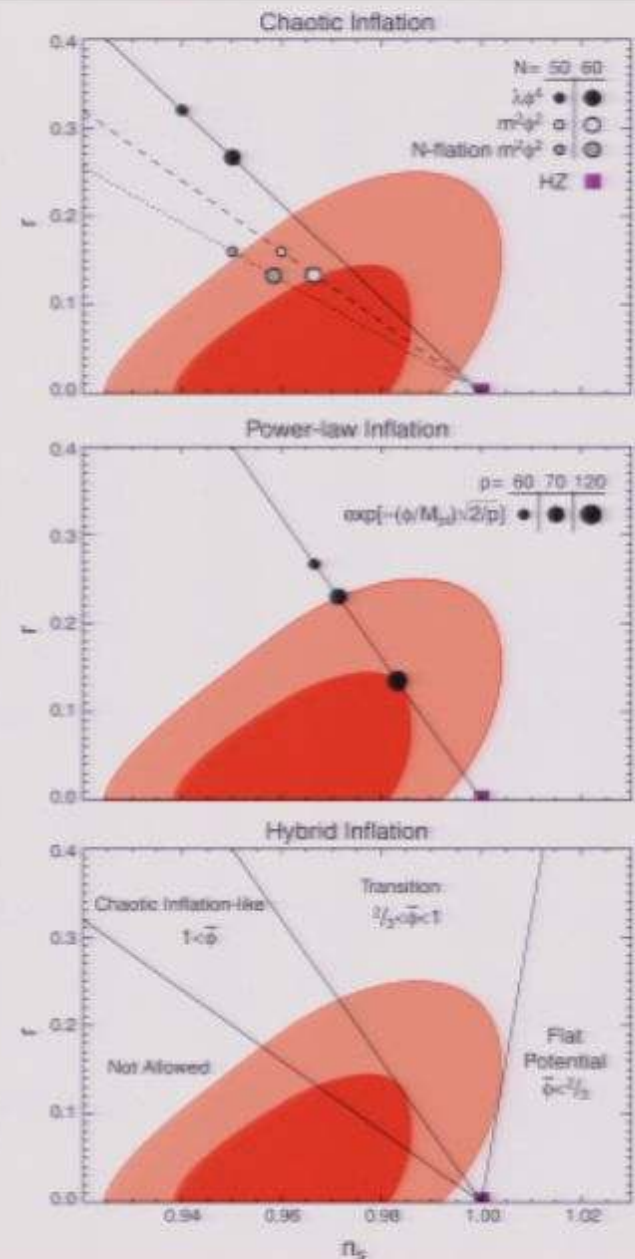
* where a positive curvature perturbation gives a negative CMB anisotropy in the Sachs-Wolfe limit

Why is Non-Gaussianity Important?

- Because a detection of f_{NL} has a best chance of ruling out the **largest** class of early universe models.
- Namely, it will rule out inflation models based upon
 - a single scalar field with
 - the canonical kinetic term that
 - rolled down a smooth scalar potential slowly, and
 - was initially in the Banch-Davies vacuum.
- **Detection of non-Gaussianity would be a major breakthrough in cosmology.**

We have r and n_s . Why Bother?

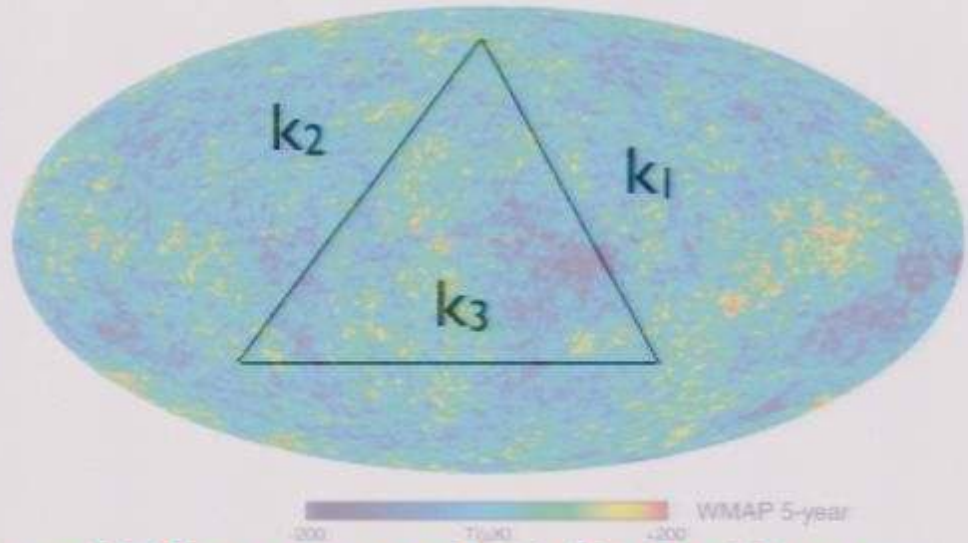
- While the current limit on the power-law index of the primordial power spectrum, n_s , and the amplitude of gravitational waves, r , have ruled out many inflation models already, many still survive (which is a good thing!)
- A convincing detection of f_{NL} would rule out most of them regardless of n_s or r .
- f_{NL} offers more ways to test various early universe models!



What if $f_{\text{NL}} \neq 0$?

- A single field, canonical kinetic term, slow-roll, and/or Bunch-Davies vacuum, must be modified.
 - Multi-field (curvaton)
 - Non-canonical kinetic term (k-inflation, DBI)
 - Temporary fast roll (features in potential; Ekpyrotic fast roll)
 - Departures from the Bunch-Davies vacuum
- It will give us a lot of clues as to what the correct early universe models should look like.

So, what is f_{NL} ?



- **f_{NL} = the amplitude of three-point function,** or also known as the “bispectrum,” $B(k_1, k_2, k_3)$, which is
 - $= \langle \Phi(k_1) \Phi(k_2) \Phi(k_3) \rangle = f_{\text{NL}} (2\pi)^2 \delta^3(k_1 + k_2 + k_3) b(k_1, k_2, k_3)$
 - where $\Phi(k)$ is the Fourier transform of the curvature perturbation, and $b(k_1, k_2, k_3)$ is a model-dependent function that defines the shape of triangles predicted by various models.

Why Bispectrum?

- The bispectrum vanishes for Gaussian random fluctuations.
- Any non-zero detection of the bispectrum indicates the presence of (some kind of) non-Gaussianity.
- A very sensitive tool for finding non-Gaussianity.

Komatsu & Spergel (2001); Babich, Creminelli & Zaldarriaga (2004)

TWO f_{NL} 's

- Depending upon the shape of triangles, one can define various f_{NL} 's:

- “Local” form



- which generates non-Gaussianity locally (i.e., at the same location) via $\Phi(\mathbf{x}) = \Phi_{\text{gaus}}(\mathbf{x}) + f_{NL}^{\text{local}} [\Phi_{\text{gaus}}(\mathbf{x})]^2$

Earlier work on the local form:

- “Equilateral” form



Salopek&Bond (1990); Gangui et al. (1994);
Verde et al. (2000); Wang&Kamionkowski (2000)

- which generates non-Gaussianity in a different way (e.g., k-inflation, DBI inflation)

Journal on f_{NL}

- Local

- $-3500 < f_{NL}^{local} < 2000$ [COBE 4yr, $l_{max}=20$] Komatsu et al. (2002)
- $-58 < f_{NL}^{local} < 134$ [WMAP 1yr, $l_{max}=265$] Komatsu et al. (2003)
- $-54 < f_{NL}^{local} < 114$ [WMAP 3yr, $l_{max}=350$] Spergel et al. (2007)
- **$-9 < f_{NL}^{local} < 111$ [WMAP 5yr, $l_{max}=500$]** Komatsu et al. (2008)

- Equilateral

- $-366 < f_{NL}^{equil} < 238$ [WMAP 1yr, $l_{max}=405$] Creminelli et al. (2006)
- $-256 < f_{NL}^{equil} < 332$ [WMAP 3yr, $l_{max}=475$] Creminelli et al. (2007)
- **$-151 < f_{NL}^{equil} < 253$ [WMAP 5yr, $l_{max}=700$]**
Komatsu et al. (2008)

Methodology

- I am not going to bother you too much with methodology...
 - Please read Appendix A of Komatsu et al., if you are interested in details.
- We use a well-established method developed over the years by: Komatsu, Spergel & Wandelt (2005); Creminelli et al. (2006); Yadav, Komatsu & Wandelt (2007)
 - There is still a room for improvement (Smith & Zaldarriaga 2006)

Data Combination

- We mainly use V band (61 GHz) and W band (94 GHz) data.
 - The results from Q band (41 GHz) are discrepant, probably due to a stronger foreground contamination
- These are *foreground-reduced maps*, delivered on the LAMBDA archive.
 - We also give the results from the raw maps.

Mask

- We have upgraded the Galaxy masks.
 - 1yr and 3yr release
 - “Kp0” mask for Gaussianity tests (76.5%)
 - “Kp2” mask for the C_l analysis (84.6%)
 - 5yr release
 - “KQ75” mask for Gaussianity tests (71.8%)
 - “KQ85” mask for the C_l analysis (81.7%)

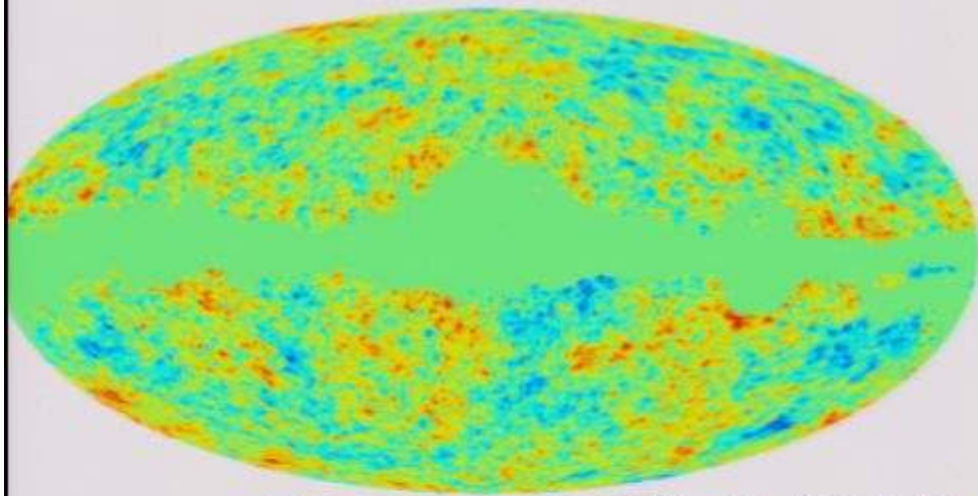
Gold et al. (2008)

- What are the KQx masks?
 - The previous KpN masks identified the bright region in the K band data, which are contaminated mostly by the synchrotron emission, and masked them.
 - “p” stands for “plus,” and N represents the brightness level above which the pixels are masked.
 - The new KQx masks identify the bright region in the K band minus the CMB map from Internal Linear Combination (the CMB picture that you always see), as well as the bright region in the Q band minus ILC.
 - Q band traces the free-free emission better than K.
 - x represents a fraction of the sky retained in K or Q.

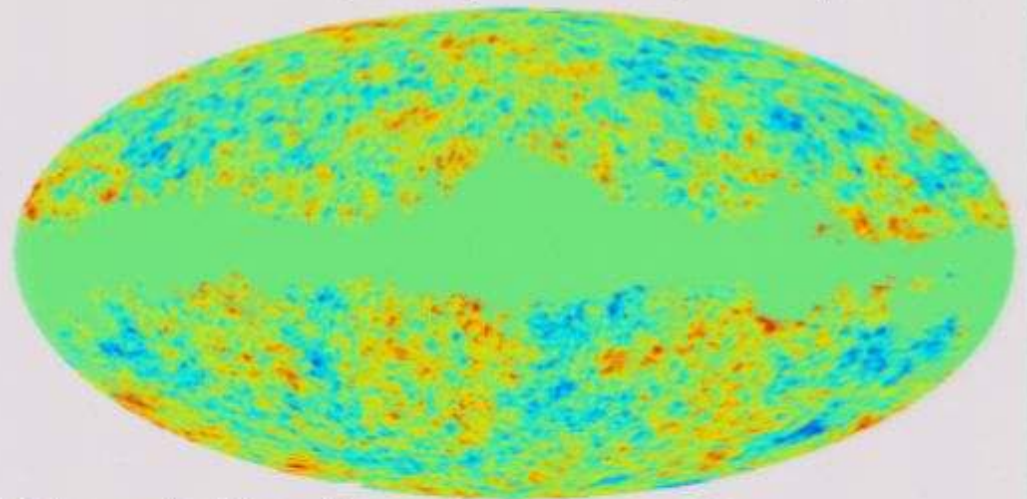
Why KQ75?

- The KQ75 mask removes the pixels that are contaminated by the free-free region better than the Kp0 mask.
- CMB was absent when the mask was defined, as the masked was defined by the K (or Q) band map minus the CMB map from ILC.
- The final mask is a combination of the K mask (which retains 75% of the sky) and the Q mask (which also retains 75%). Since Q masks the region that is not masked by K, the final KQ75 mask retains less than 75% of the sky. (It retains 71.8% of the sky for cosmology.)

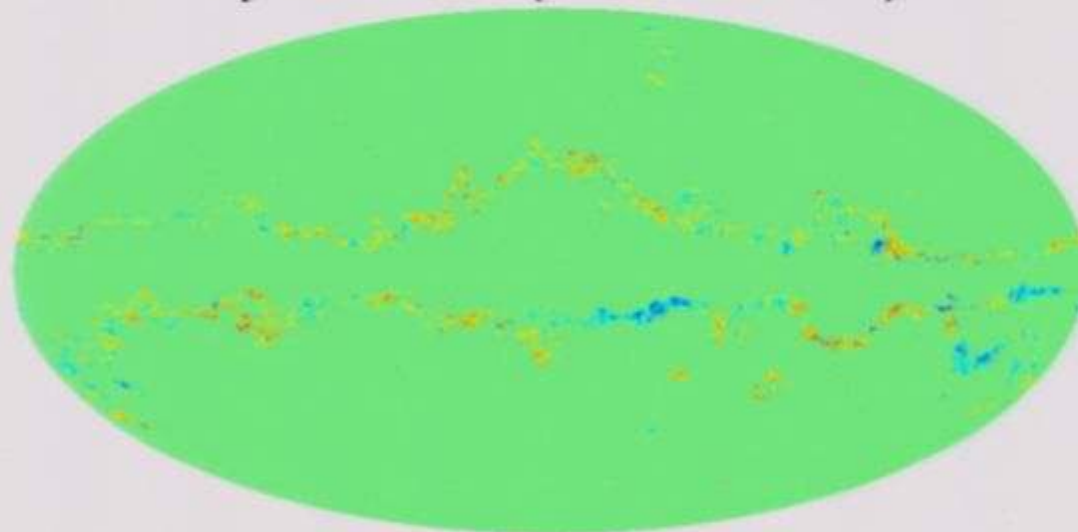
Kp0 (V band; Raw)



KQ75 (V band; Raw)

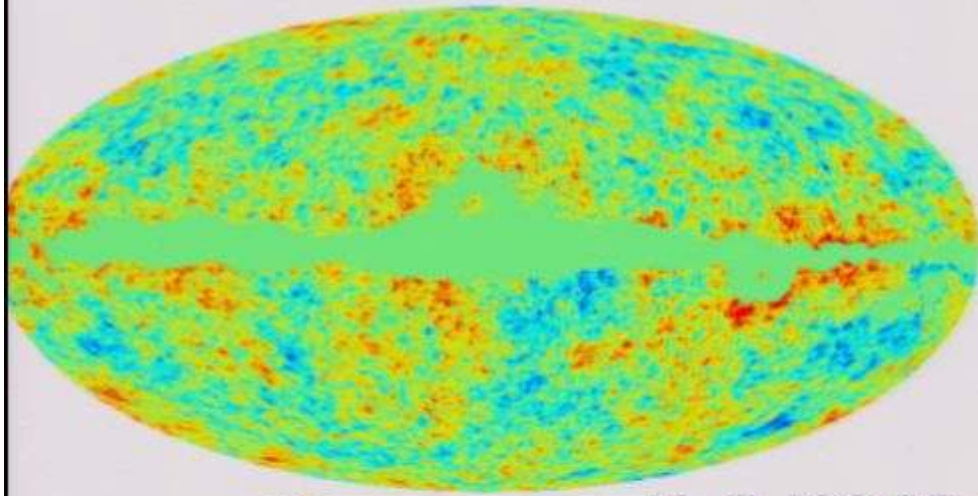


Kp0-KQ75 (V band; Raw)

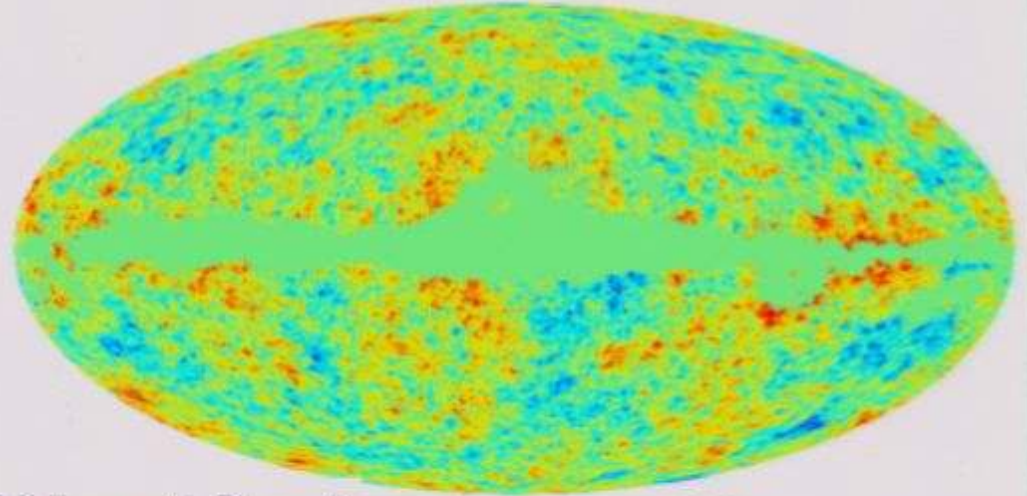


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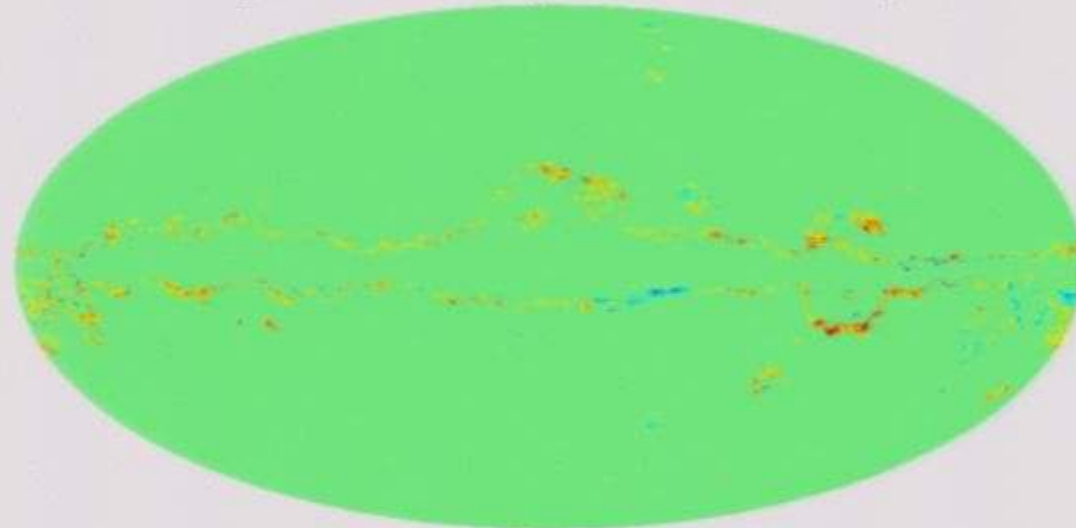
Kp2 (V band; Raw)



KQ85 (V band; Raw)

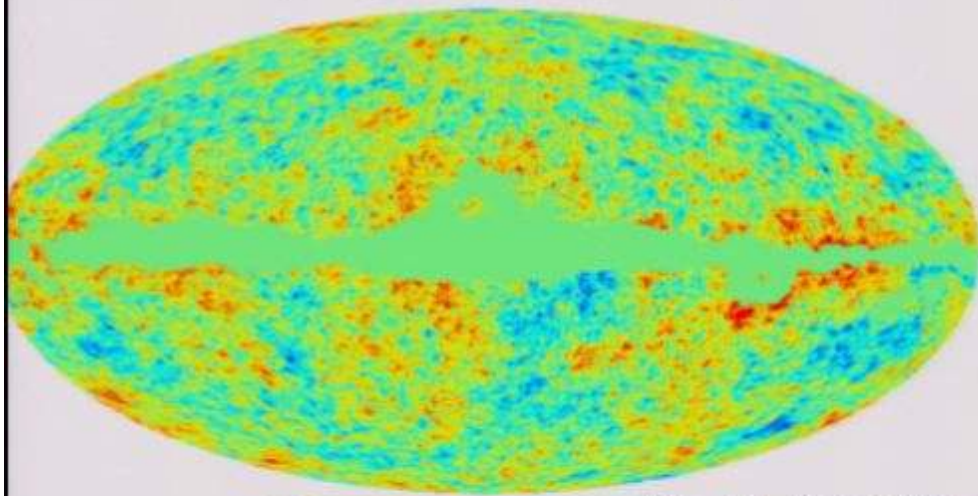


Kp2-KQ85 (V band; Raw)

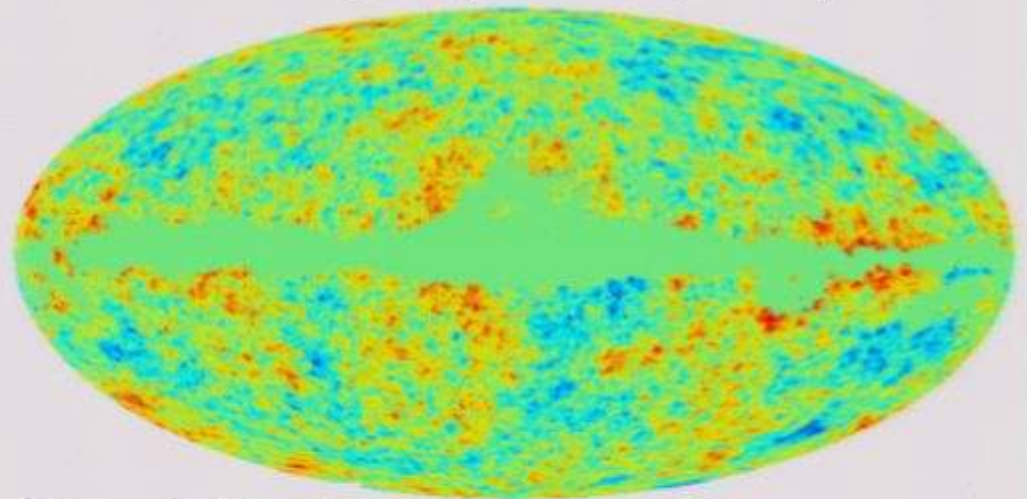


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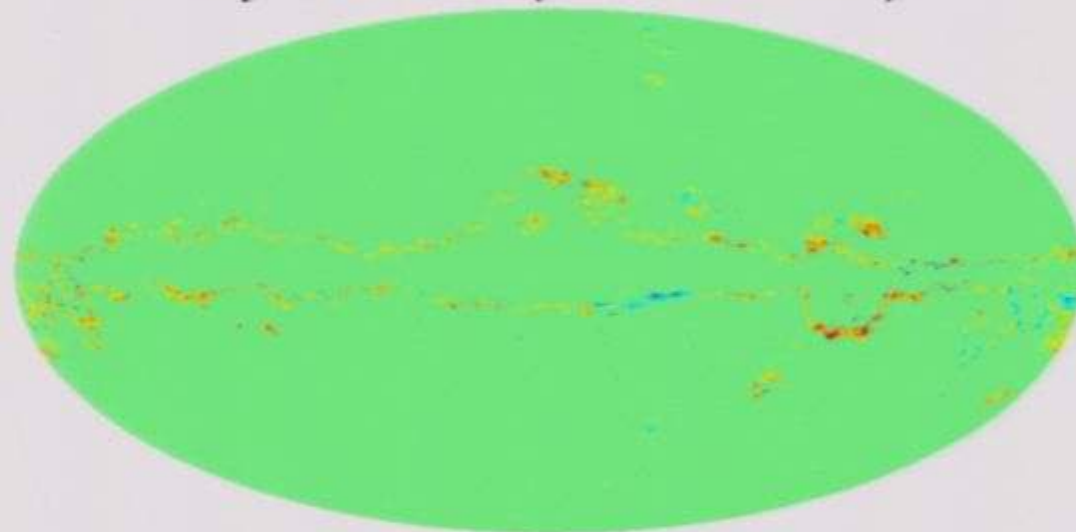
Kp2 (V band; Raw)



KQ85 (V band; Raw)

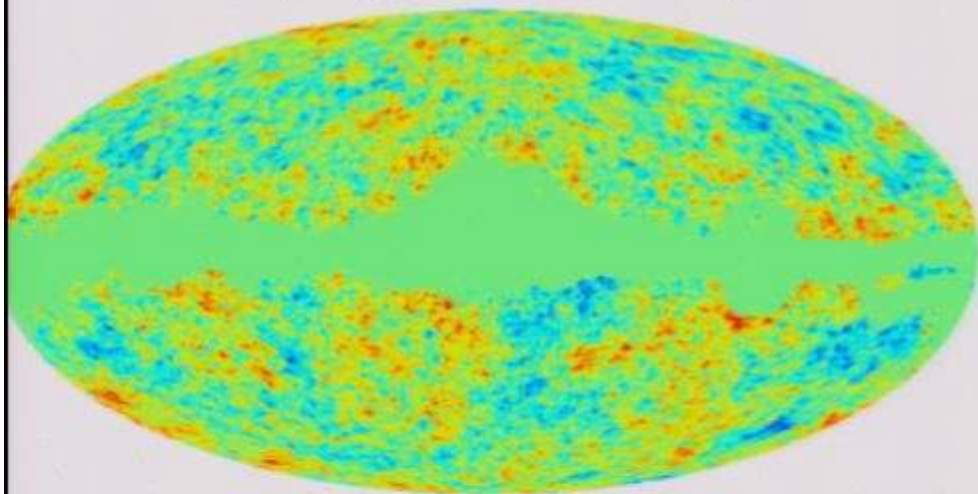


Kp2-KQ85 (V band; Raw)

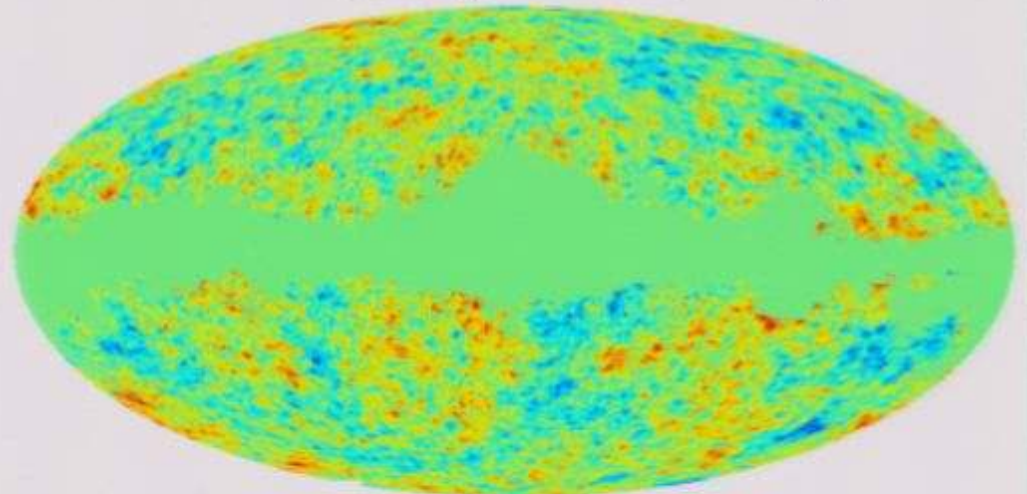


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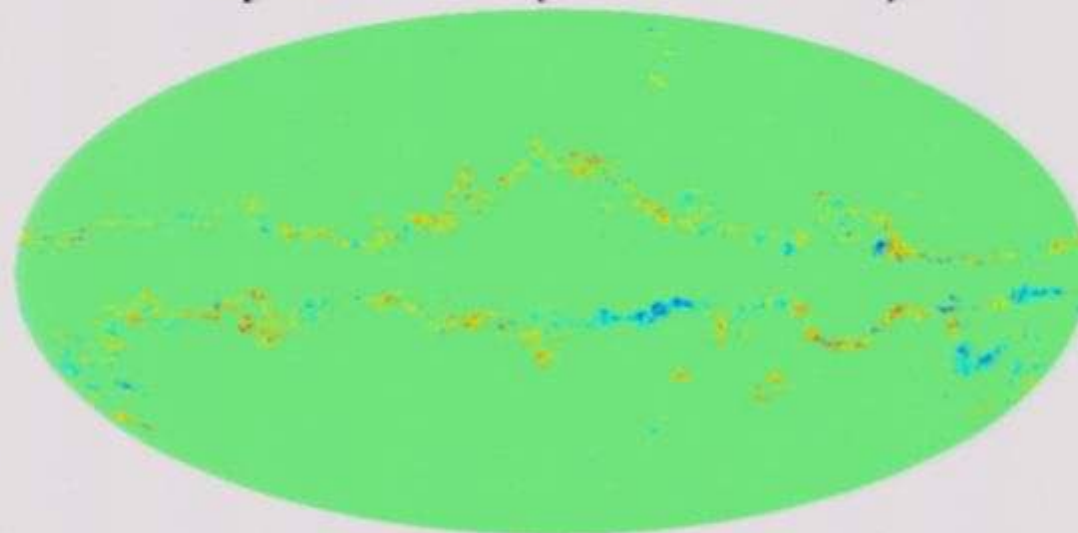
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KQ75 (V band; Raw)

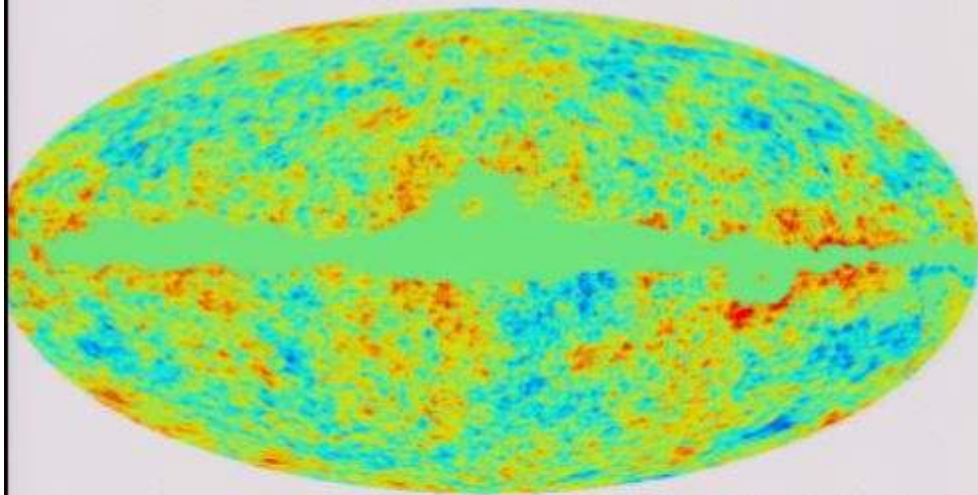


Kp0-KQ75 (V band; Raw)

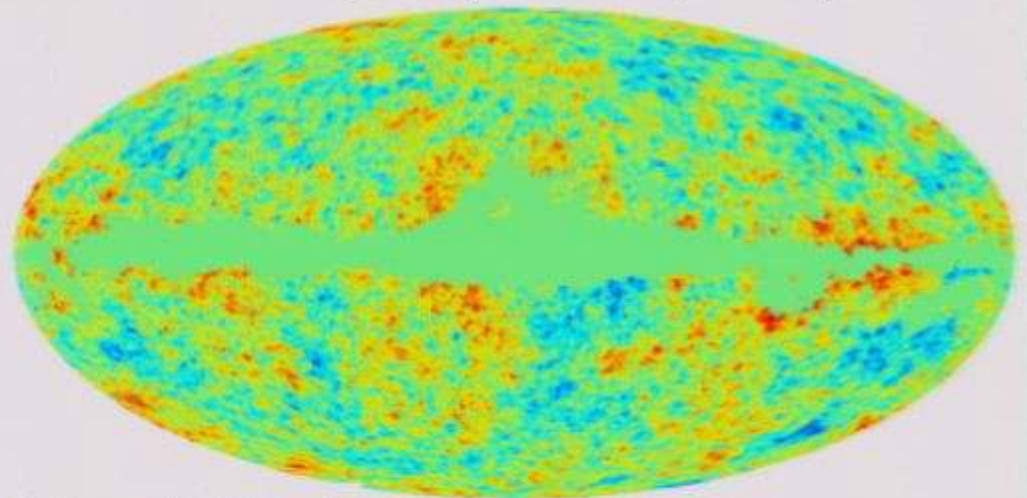


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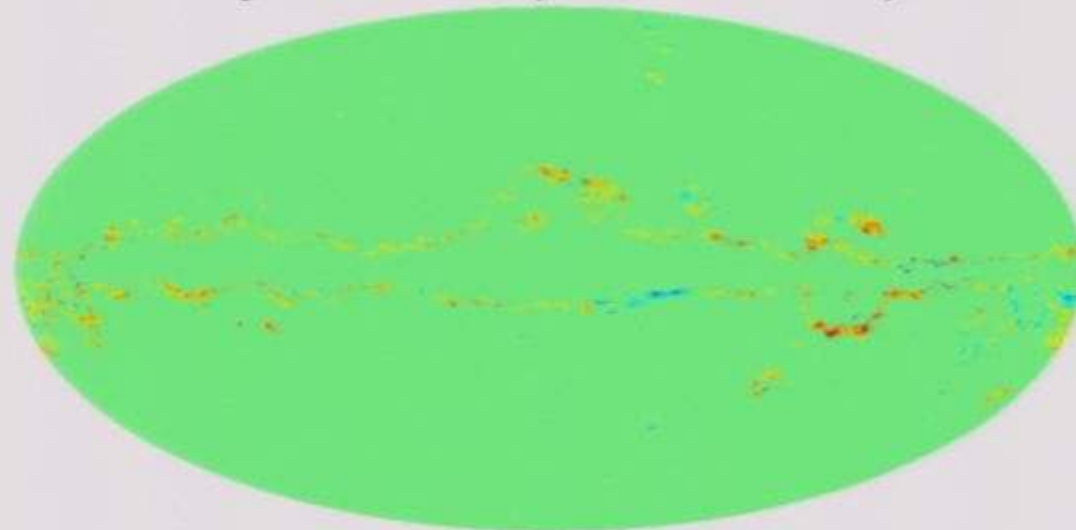
Kp2 (V band; Raw)



KQ85 (V band; Raw)

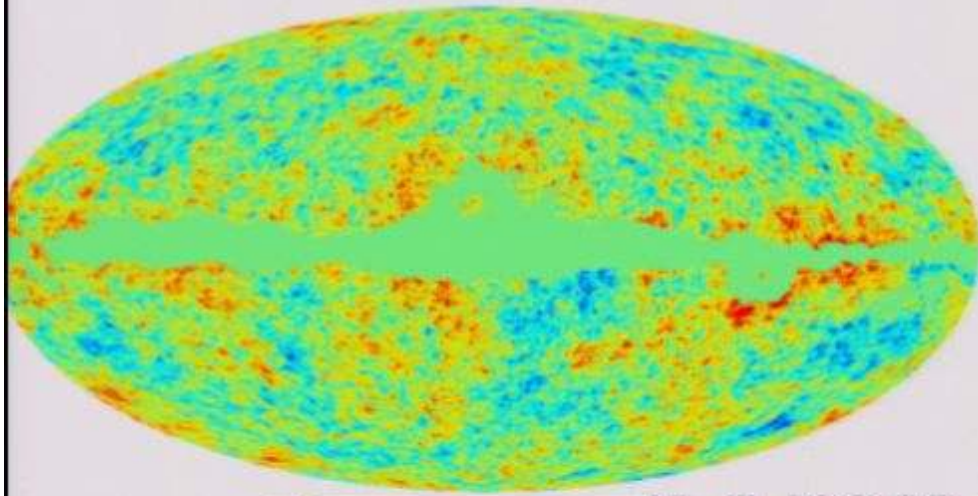


Kp2-KQ85 (V band; Raw)

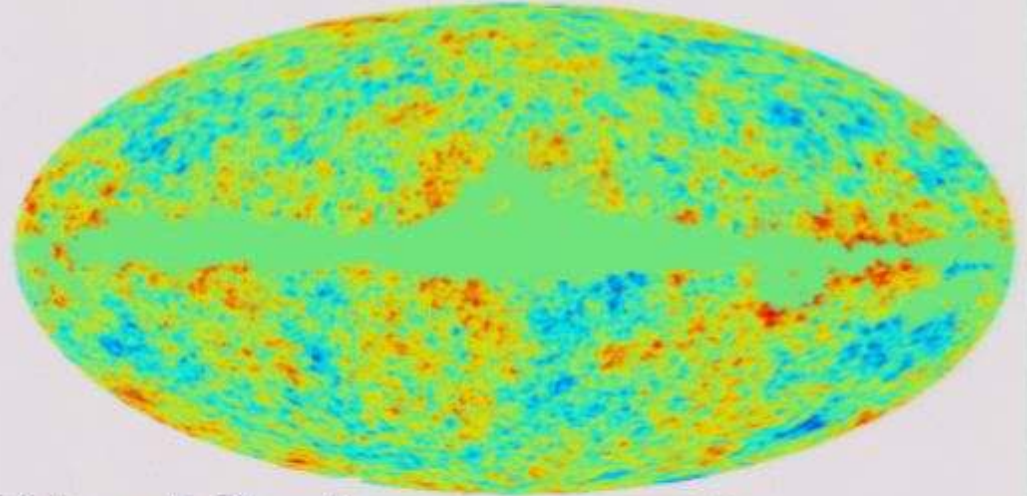


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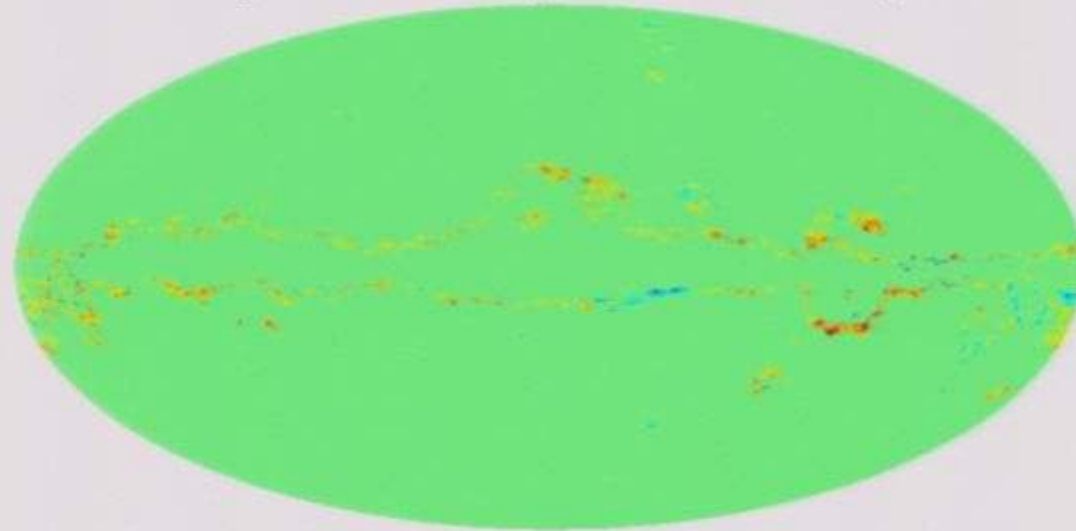
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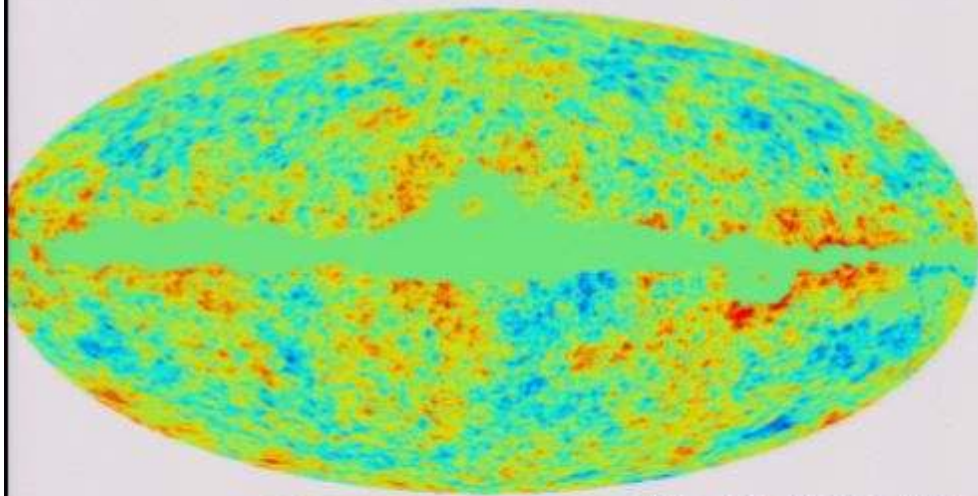


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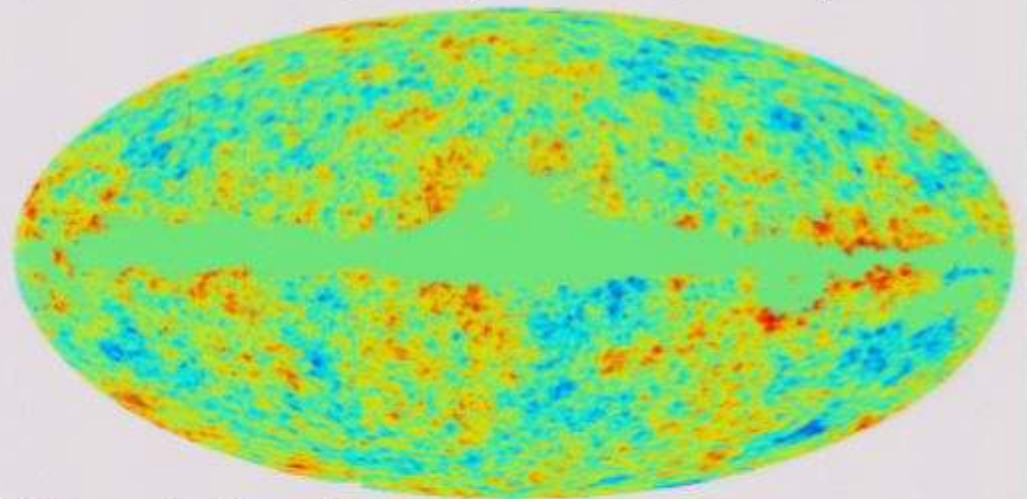


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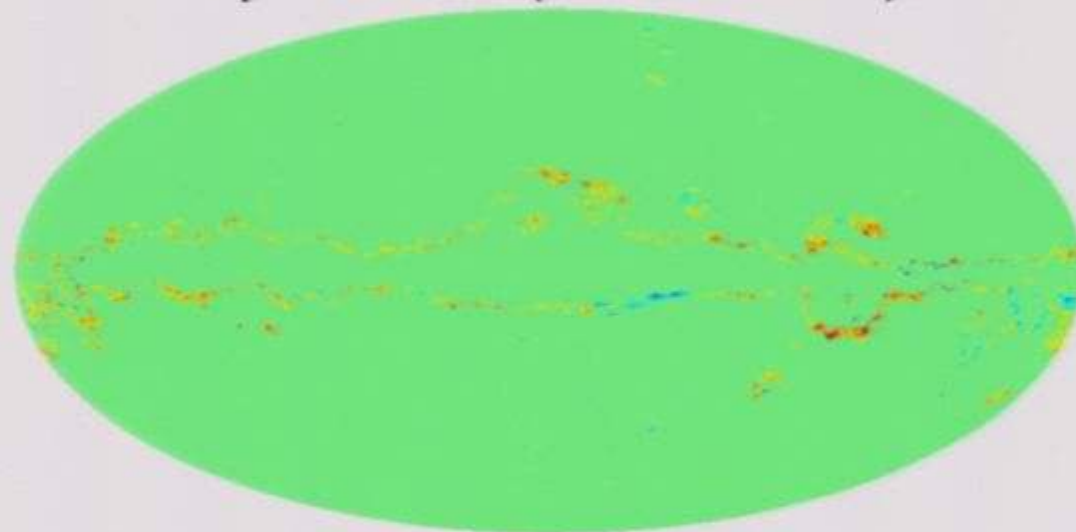
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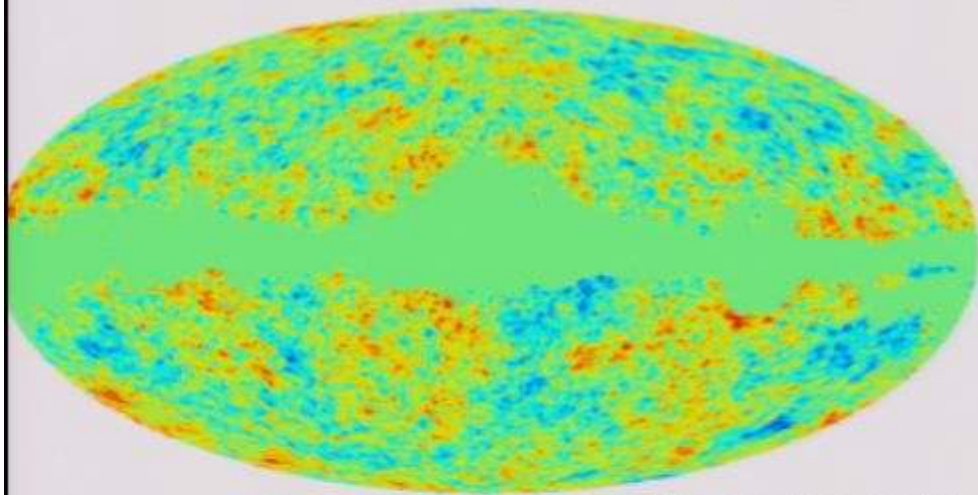


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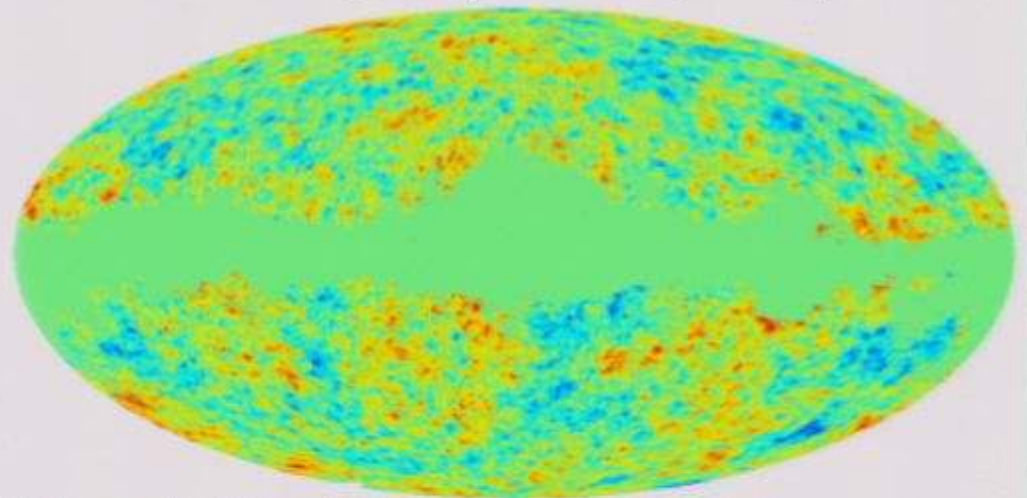



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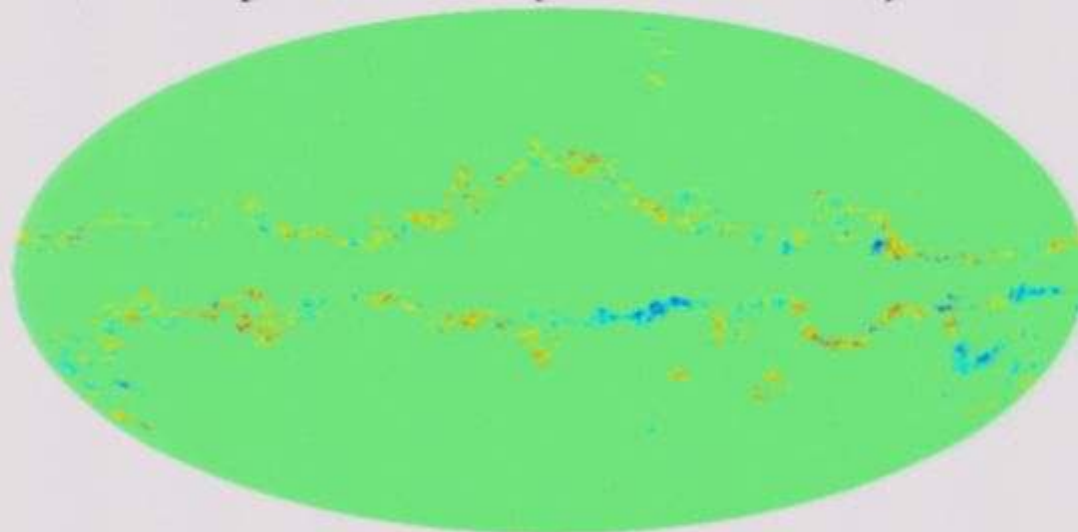
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KQ75 (V band; Raw)

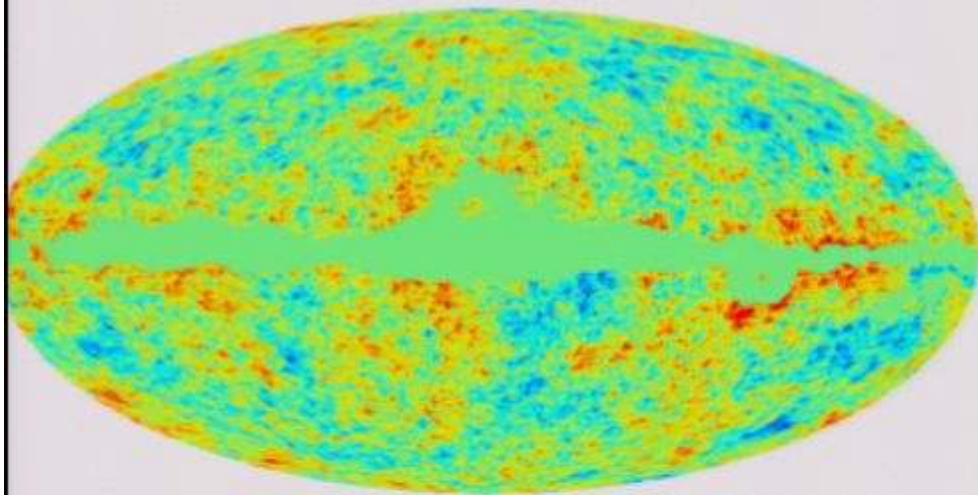


-0.30  0.30 Kp0-KQ75 (V band; Raw)

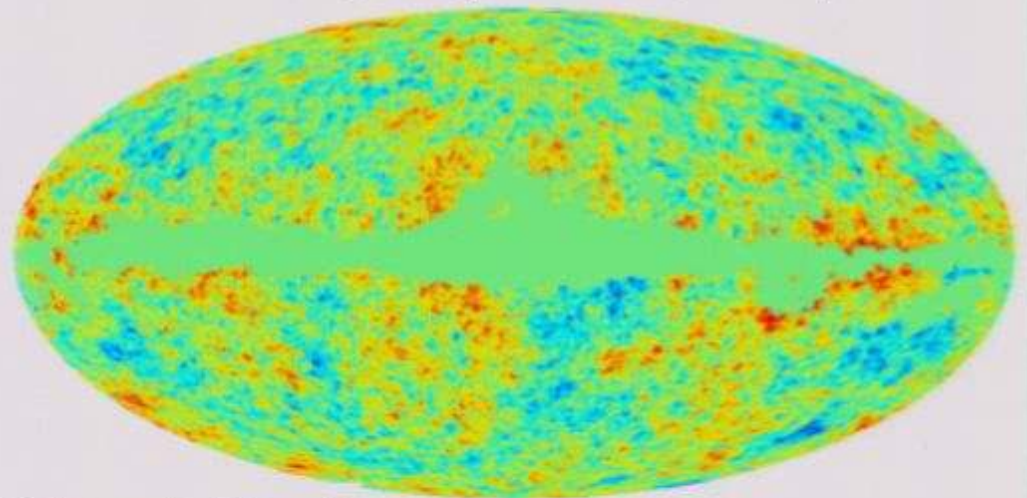



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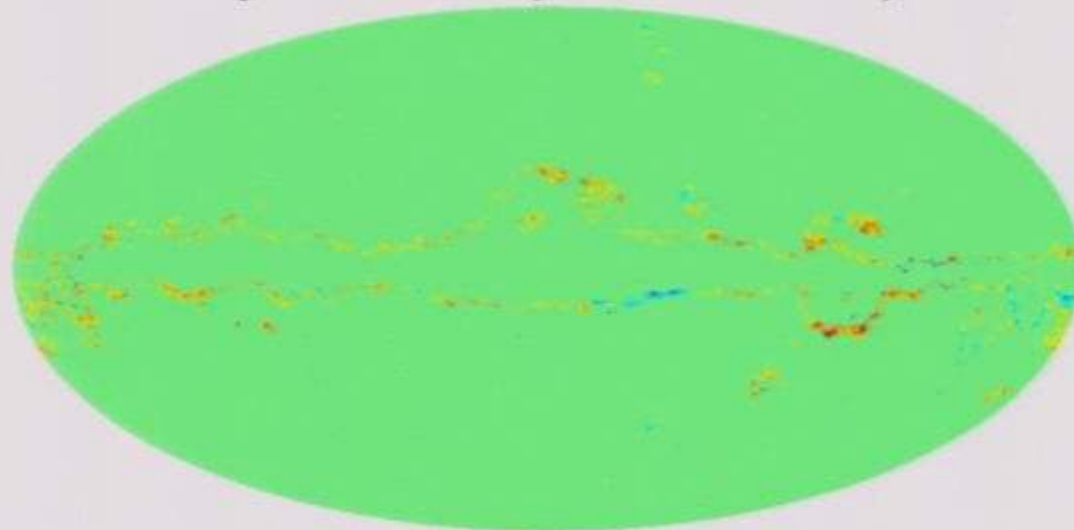
Kp2 (V band; Raw)



KQ85 (V band; Raw)



-0.30  0.30 Kp2-KQ85 (V band; Raw)



-0.30  0.30

Main Result (Local)

Band	Mask	l_{\max}	f_{NL}^{local}	$\Delta f_{NL}^{\text{local}}$	b_{src}
V+W	KQ85	400	50 ± 29	1 ± 2	0.26 ± 1.5
V+W	KQ85	500	61 ± 26	2.5 ± 1.5	0.05 ± 0.50
V+W	KQ85	600	68 ± 31	3 ± 2	0.53 ± 0.28
V+W	KQ85	700	67 ± 31	3.5 ± 2	0.34 ± 0.20
V+W	Kp0	500	61 ± 26	2.5 ± 1.5	
V+W	KQ75p1 ^a	500	53 ± 28	4 ± 2	
V+W	KQ75	400	47 ± 32	3 ± 2	-0.50 ± 1.7
V+W	KQ75	500	55 ± 30	4 ± 2	0.15 ± 0.51
V+W	KQ75	600	61 ± 36	4 ± 2	0.53 ± 0.30
V+W	KQ75	700	58 ± 36	5 ± 2	0.38 ± 0.21

- ~ 2 sigma “hint”: $f_{NL}^{\text{local}} \sim \mathbf{60 \pm 30 (68\% CL)}$
- 1.8 sigma for KQ75; 2.3 sigma for KQ85 & Kp0

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- The estimated contamination from the point sources is small, if any. (Likely overestimated by a factor of ~ 2 .)

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Null Tests

Band	Foreground	Mask	f_{NL}^{local}
Q–W	Raw	<i>KQ75</i>	-0.53 ± 0.22
V–W	Raw	<i>KQ75</i>	-0.31 ± 0.23
Q–W	Clean	<i>KQ75</i>	0.10 ± 0.22
V–W	Clean	<i>KQ75</i>	0.06 ± 0.23

- **No signal in the difference of cleaned maps.**

Frequency Dependence

Band	Foreground	Mask	f_{NL}^{local}
Q	Raw	<i>KQ75</i>	-42 ± 48
V	Raw	<i>KQ75</i>	41 ± 35
W	Raw	<i>KQ75</i>	46 ± 35
Q	Clean	<i>KQ75</i>	10 ± 48
V	Clean	<i>KQ75</i>	50 ± 35
W	Clean	<i>KQ75</i>	62 ± 35

- **Q is very sensitive to the foreground cleaning.**

V+W: Raw vs Clean ($I_{\max}=500$)

Band	Foreground	Mask	f_{NL}^{local}
V+W	Raw	<i>KQ85</i>	9 ± 26
V+W	Raw	<i>Kp0</i>	48 ± 26
V+W	Raw	<i>KQ75p1</i>	41 ± 28
V+W	Raw	<i>KQ75</i>	43 ± 30

- Clean-map results:
 - KQ85; 61 ± 26
 - Kp0; 61 ± 26
 - KQ75p1; 53 ± 28
 - KQ75; 55 ± 30

Foreground contamination is not too severe.

The Kp0 and KQ85 results may be as clean as the KQ75 results.

Our Best Estimate

- Why not using Kp0 or KQ85 results, which have a higher statistical significance?
- Given the profound implications and impact of non-zero $f_{\text{NL}}^{\text{local}}$, we have chosen a conservative limit from the KQ75 with the point source correction ($\Delta f_{\text{NL}}^{\text{local}}=4$, which is also conservative) as our best estimate.
 - The 68% limit: $f_{\text{NL}}^{\text{local}} = 51 \pm 30$ [1.7 sigma]
 - **The 95% limit: $-9 < f_{\text{NL}}^{\text{local}} < 111$**

Comparison with Y&W

- Yadav and Wandelt used the raw V+W map from the 3-year data.
 - 3yr: $f_{\text{NL}}^{\text{local}} = 68 \pm 30$ for $I_{\text{max}}=450$ & Kp0 mask
 - 3yr: $f_{\text{NL}}^{\text{local}} = 80 \pm 30$ for $I_{\text{max}}=550$ & Kp0 mask
- Our corresponding 5-year raw map estimate is
 - 5yr: $f_{\text{NL}}^{\text{local}} = 48 \pm 26$ for $I_{\text{max}}=500$ & Kp0 mask
 - C.f. clean-map estimate: $f_{\text{NL}}^{\text{local}} = 61 \pm 26$
- With more years of observations, the values have come down to a lower significance.

Main Result (Equilateral)

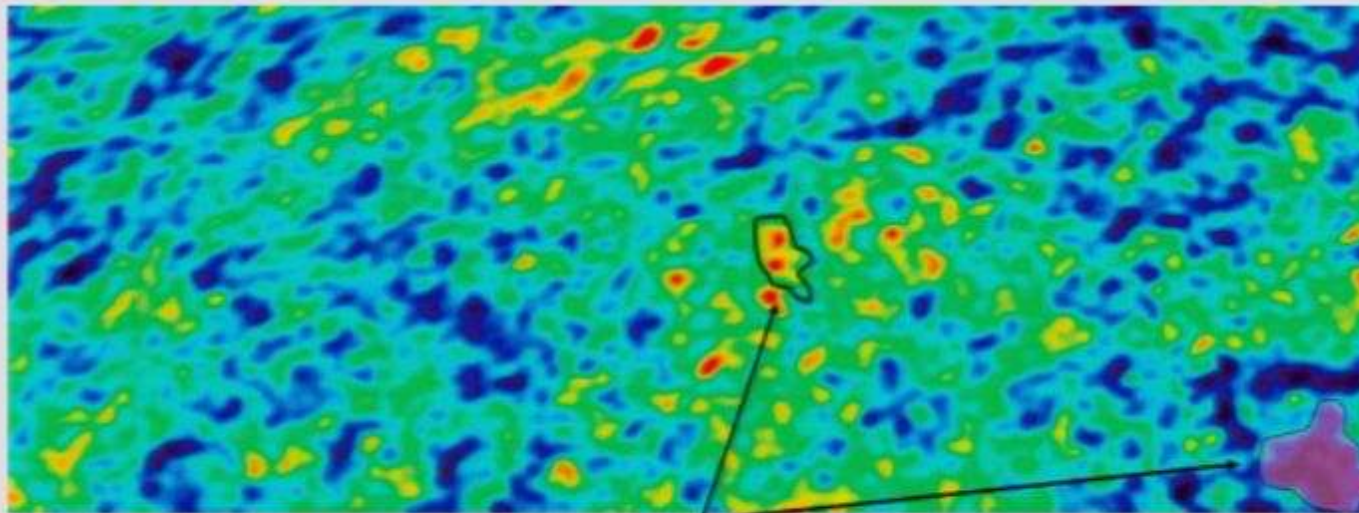
Band	Mask	l_{\max}	f_{NL}^{equil}	$\Delta f_{NL}^{\text{equil}}$
V+W	KQ75	400	77 ± 146	9 ± 7
V+W	KQ75	500	78 ± 125	14 ± 6
V+W	KQ75	600	71 ± 108	27 ± 5
V+W	KQ75	700	73 ± 101	22 ± 4

- The point-source correction is much larger for the equilateral configurations.
- Our best estimate from $l_{\max}=700$:
 - The 68% limit: $f_{NL}^{\text{equil}} = 51 \pm 101$
 - **The 95% limit: $-151 < f_{NL}^{\text{equil}} < 253$**

Forecasting 9-year Data

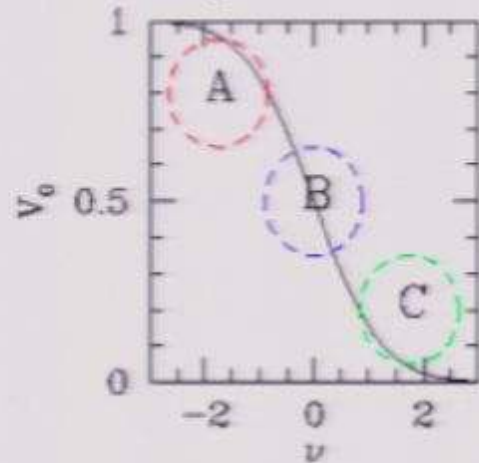
- The WMAP 5-year data do not show any evidence for the presence of $f_{\text{NL}}^{\text{equil}}$, but *do* show a (~ 2 -sigma) hint for $f_{\text{NL}}^{\text{local}}$.
- Our best estimate is probably on the conservative side, but our analysis clearly indicates that more data are required to claim a firm evidence for $f_{\text{NL}}^{\text{local}} > 0$.
- The 9-year error on $f_{\text{NL}}^{\text{local}}$ should reach $\Delta f_{\text{NL}}^{\text{local}} = 20$
 - If $f_{\text{NL}}^{\text{local}} = 50-60$, **we would see it at 2.5 to 3 sigma by 2011.**
(The WMAP 9-year survey will be complete in August 2010.)

Minkowski Functionals (MFs)

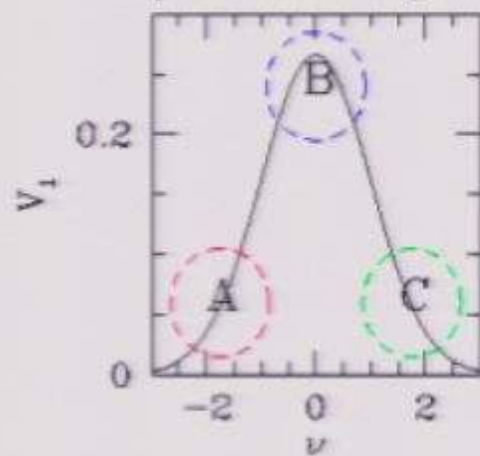


The number of hot spots minus cold spots.

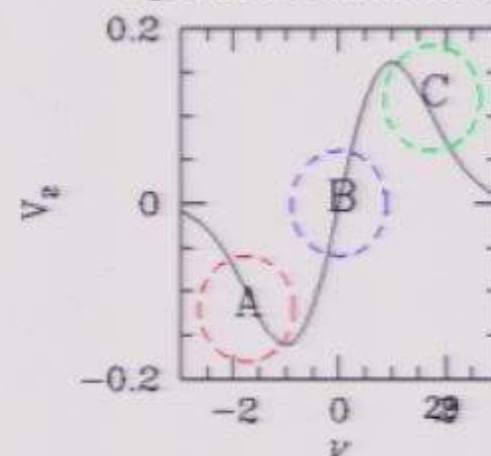
V_0 : surface area



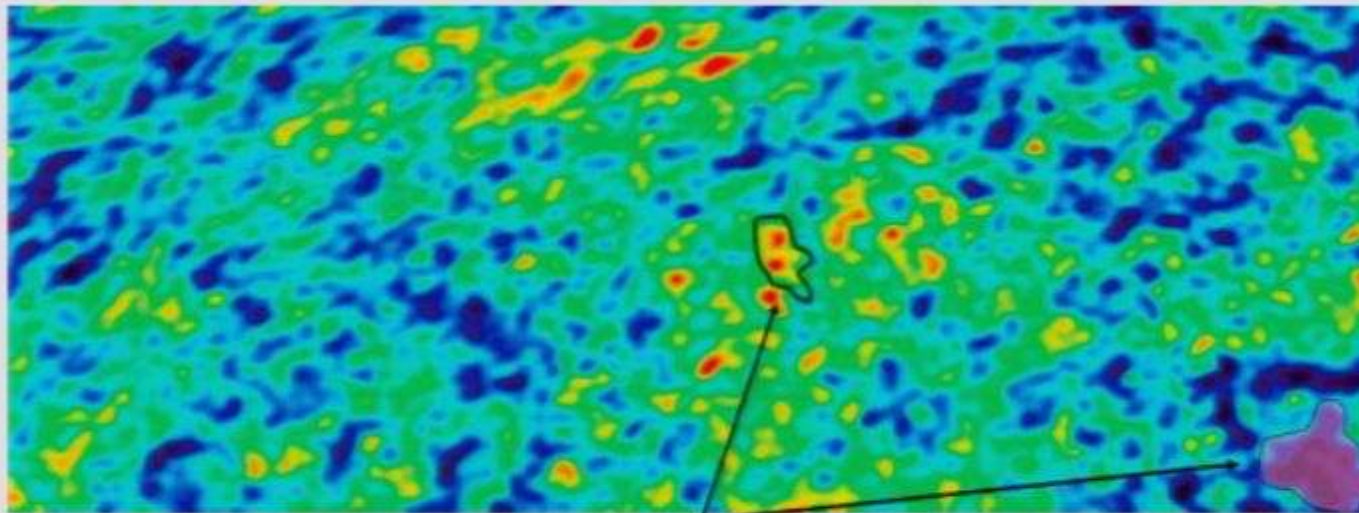
V_1 : Contour Length



V_2 : Euler Characteristic

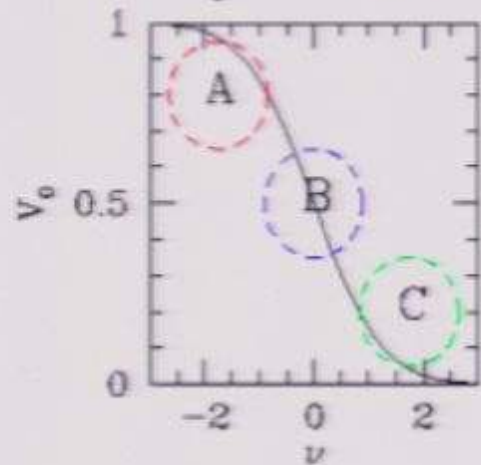


Minkowski Functionals (MFs)

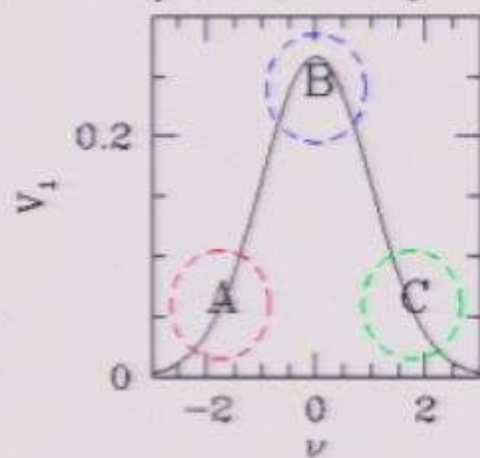


The number of hot spots minus cold spots.

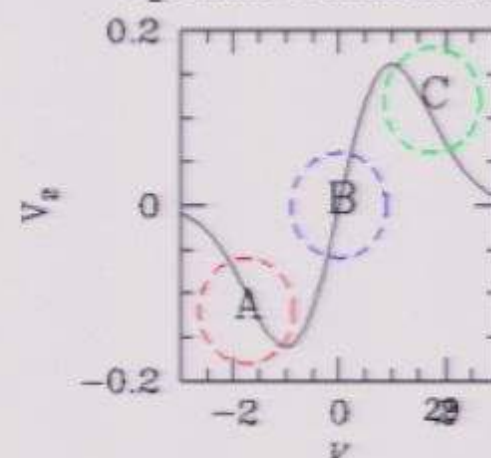
V_0 : surface area



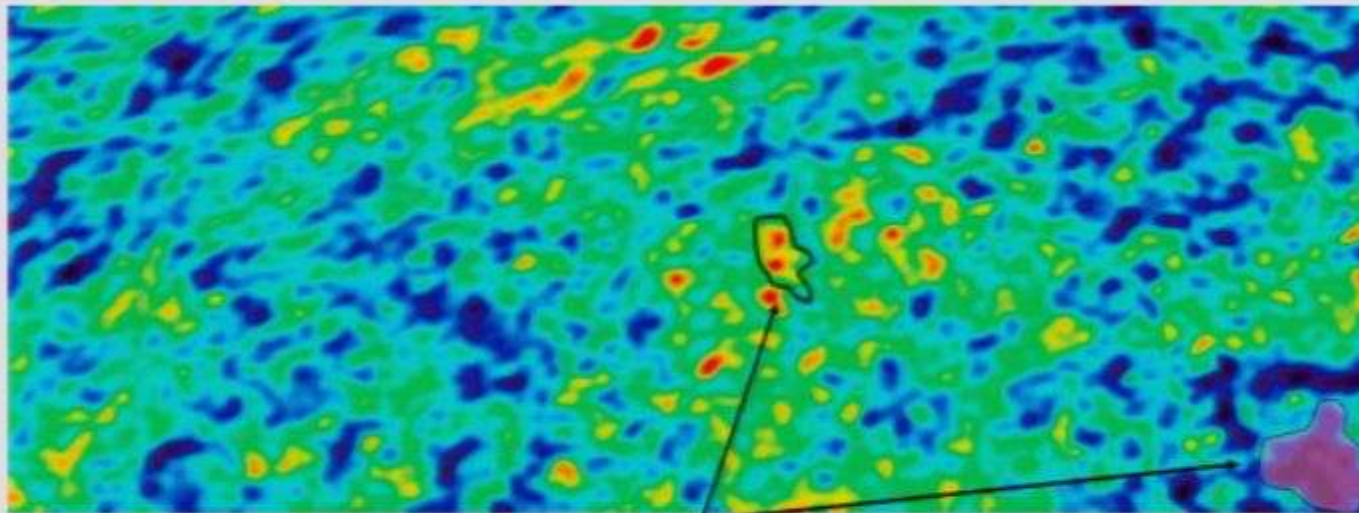
V_1 : Contour Length



V_2 : Euler Characteristic

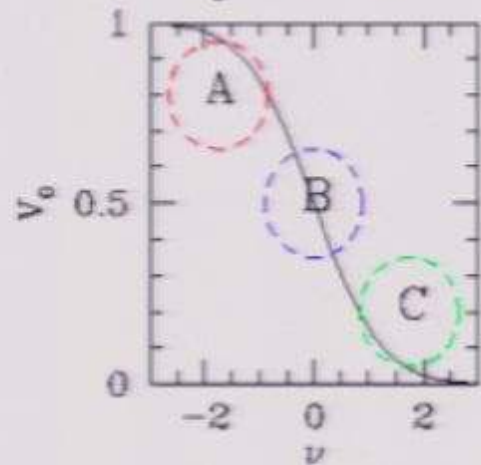


Minkowski Functionals (MFs)

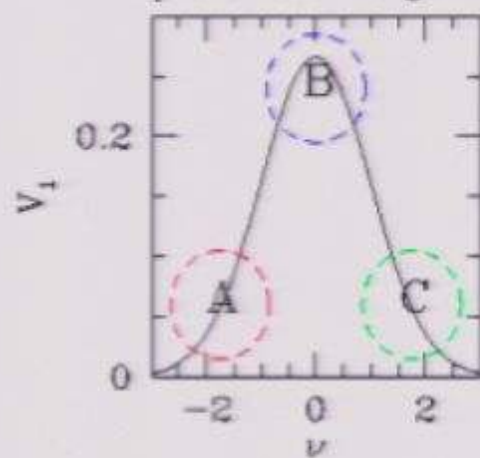


The number of hot spots minus cold spots.

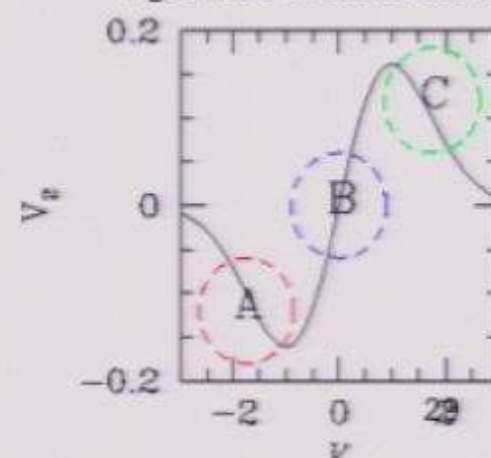
V_0 : surface area



V_1 : Contour Length



V_2 : Euler Characteristic



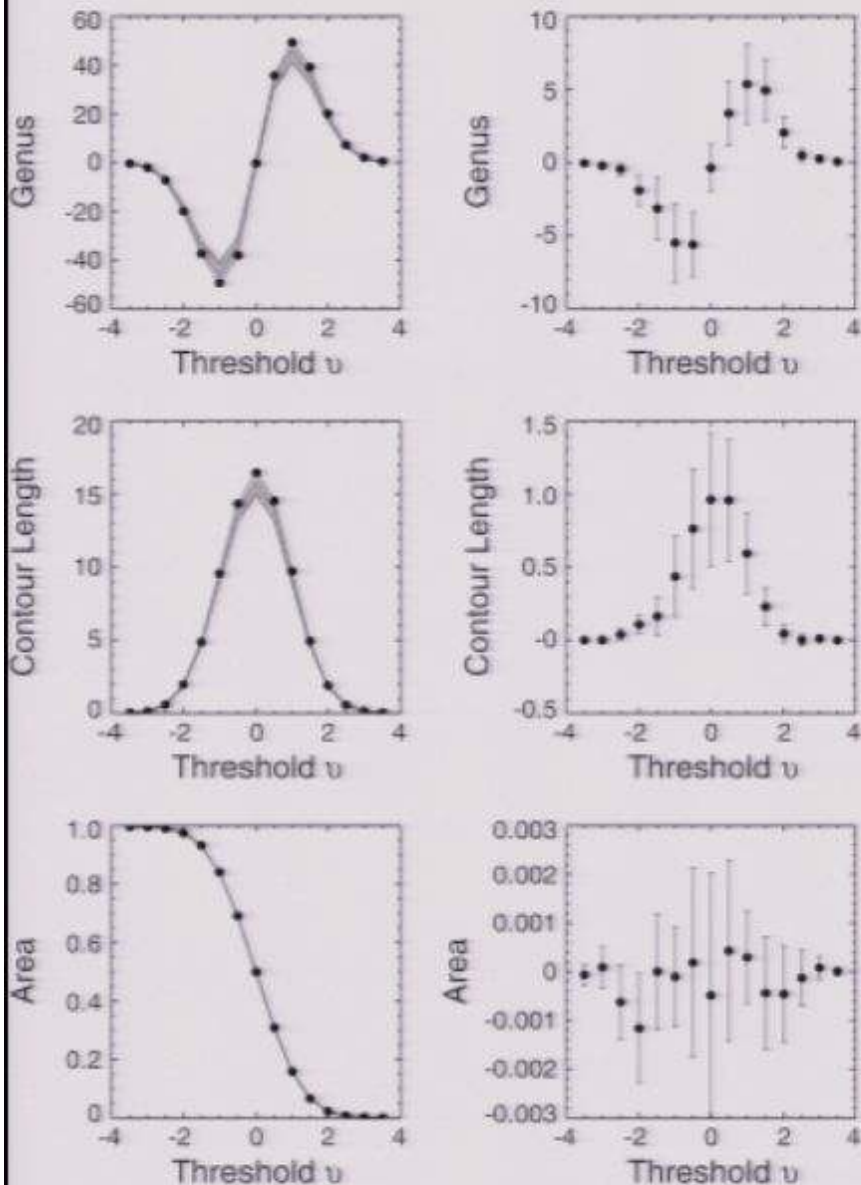
Komatsu et al. (2008)

MFs from *WMAP* 5-Year Data (V+W)

Result from a single resolution
($N_{\text{side}}=128$; 28 arcmin pixel)
[analysis done by Ai Kogut]

$$f_{\text{NL}}^{\text{local}} = -57 \pm 60 \text{ (68\% CL)}$$
$$-178 < f_{\text{NL}}^{\text{local}} < 64 \text{ (95\% CL)}$$

Cf. Hikage et al. (2008) 3-year
analysis using all the resolution:
 $f_{\text{NL}}^{\text{local}} = -22 \pm 43 \text{ (68\% CL)}$
 $-108 < f_{\text{NL}}^{\text{local}} < 64 \text{ (95\% CL)}$



“Tension?”

- **It is premature to worry about this**, but it is a little bit bothering to see that the bispectrum prefers a *positive* value, $f_{\text{NL}} \sim 60$, whereas the Minkowski functionals prefer a *negative* value, $f_{\text{NL}} \sim -60$.
- These values are derived from the same data!
- What do the Minkowski functionals actually measure?

Analytical formulae of MFs

Perturbative formulae of MFs (Matsubara 2003)

$$V_k(\mathbf{v}) = \frac{1}{(2\pi)^{k+1/2}} \frac{\omega_2}{\omega_{2-k}\omega_k} \left(\frac{\sigma_1}{\sqrt{2\sigma_0}} \right)^k e^{-\mathbf{v}^2/2} \{H_{k-1}(\mathbf{v})\} \quad \text{Gaussian term}$$

$$+ \left[\frac{1}{6} S^{(0)} H_{k+2}(\mathbf{v}) + \frac{k}{3} S^{(1)} H_k(\mathbf{v}) + \frac{k(k-1)}{6} S^{(2)} H_{k-2}(\mathbf{v}) \right] \sigma_0 + O(\sigma_0^2)$$

leading order of Non-Gaussian term

$$\sigma_j^2 = \frac{1}{4} \sum_l (2l+1) [l(l+1)] C_l W_l^2 \quad W_l: \text{smoothing kernel}$$

$$\omega_0 = 1, \omega_1 = 1, \omega_2 = \pi, \omega_3 = 4\pi/3 \quad H_k: k\text{-th Hermite polynomial}$$

$$S^{(a)}: \text{skewness parameters (a = 0, 1, 2)}$$

In weakly non-Gaussian fields ($\sigma_0 \ll 1$), the non-Gaussianity in MFs is characterized by three skewness parameters $S^{(a)}$.

3 “Skewness Parameters”

- Ordinary skewness

$$S^{(0)} \equiv \frac{\langle f^3 \rangle}{\sigma_0^4},$$

- Second derivative

$$S^{(1)} \equiv -\frac{3}{4} \frac{\langle f^2(\nabla^2 f) \rangle}{\sigma_0^2 \sigma_1^2},$$

- (First derivative)² x Second derivative

$$S^{(2)} \equiv -\frac{3d}{2(d-1)} \frac{\langle (\nabla f) \cdot (\nabla f)(\nabla^2 f) \rangle}{\sigma_1^4},$$

$$S^{(0)} = \frac{3}{2\pi\sigma_0^4} \sum_{2 \leq l_1 \leq l_2 \leq l_3} I_{l_1 l_2 l_3}^2 b_{l_1 l_2 l_3} W_{l_1} W_{l_2} W_{l_3}, \quad ($$

$S^{(0)}$: Simple average of $b_{l_1 l_2 l_3}$

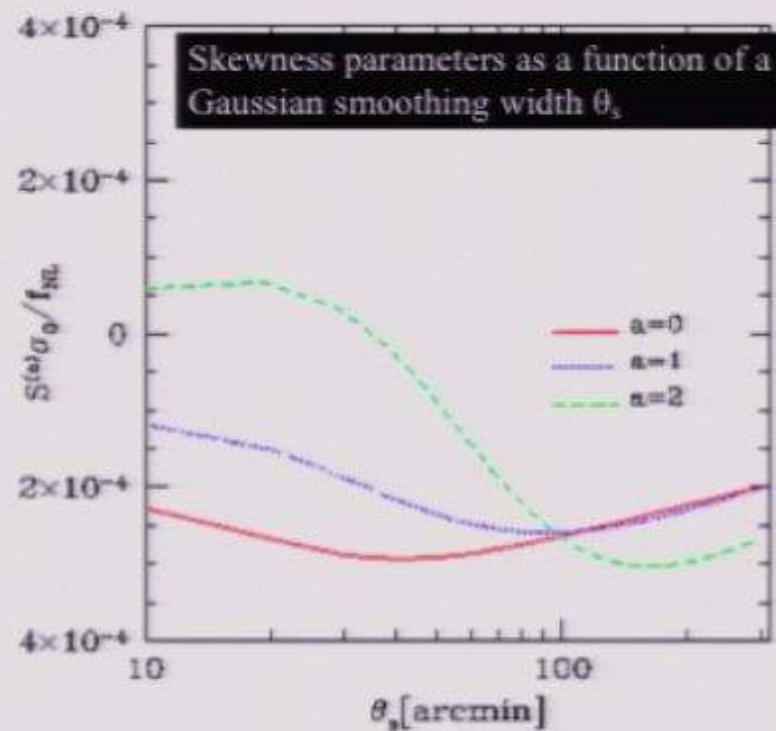
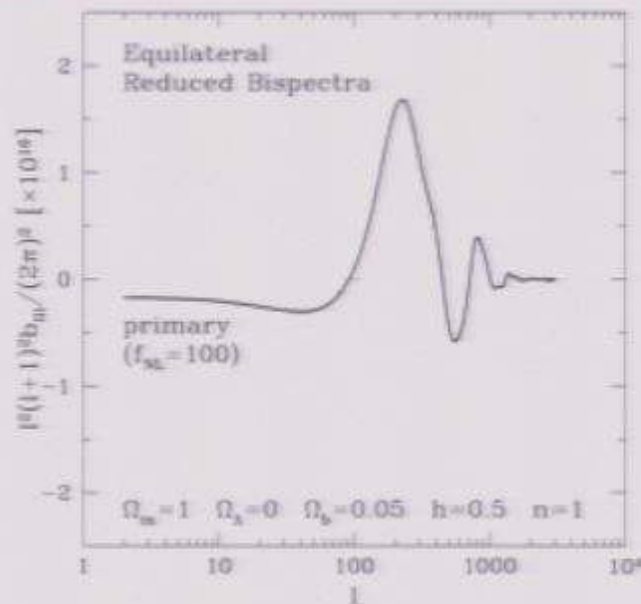
$$S^{(1)} = \frac{3}{8\pi\sigma_0^2\sigma_1^2} \sum_{2 \leq l_1 \leq l_2 \leq l_3} [l_1(l_1 + 1) + l_2(l_2 + 1) + l_3(l_3 + 1)] \\ \times I_{l_1 l_2 l_3}^2 b_{l_1 l_2 l_3} W_{l_1} W_{l_2} W_{l_3}, \quad ($$

$S^{(1)}$: l^2 weighted average

$$S^{(2)} = \frac{3}{4\pi\sigma_1^4} \sum_{2 \leq l_1 \leq l_2 \leq l_3} \{[l_1(l_1 + 1) + l_2(l_2 + 1) - l_3(l_3 + 1)] \\ \times l_3(l_3 + 1) + (\text{cyc.})\} I_{l_1 l_2 l_3}^2 b_{l_1 l_2 l_3} W_{l_1} W_{l_2} W_{l_3}, \quad ($$

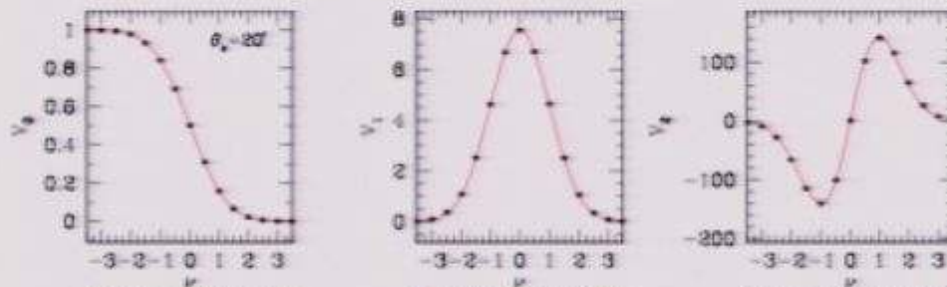
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Analytical predictions of bispectrum at $f_{\text{NL}}=100$
(Komatsu & Spergel 2001)

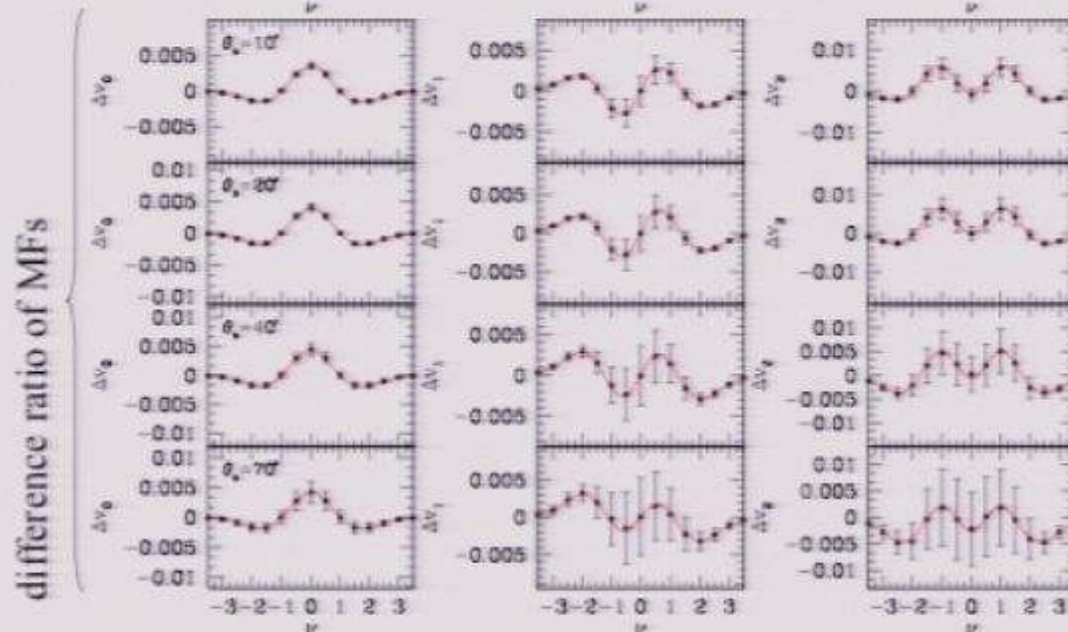


Comparison of analytical formulae with Non-Gaussian simulations

Surface area Contour Length Euler Characteristic



Comparison of MFs between analytical predictions and non-Gaussian simulations with $f_{NL}=100$ at different Gaussian smoothing scales, θ_s



Simulations are done for WMAP.

Analytical formulae agree with non-Gaussian simulations very well.

Application of the Minkowski Functionals

- The skewness parameters are the direct observables from the Minkowski functionals.
- The skewness parameters can be calculated directly from the bispectrum.
- It can be applied to *any* form of the bispectrum!
 - Statistical power is weaker than the full bispectrum, but the application can be broader than the bispectrum estimator that is tailored for a very specific form of non-Gaussianity.

$$S^{(0)} = \frac{3}{2\pi\sigma_0^4} \sum_{2 \leq l_1 \leq l_2 \leq l_3} I_{l_1 l_2 l_3}^2 b_{l_1 l_2 l_3} W_{l_1} W_{l_2} W_{l_3}, \quad ($$

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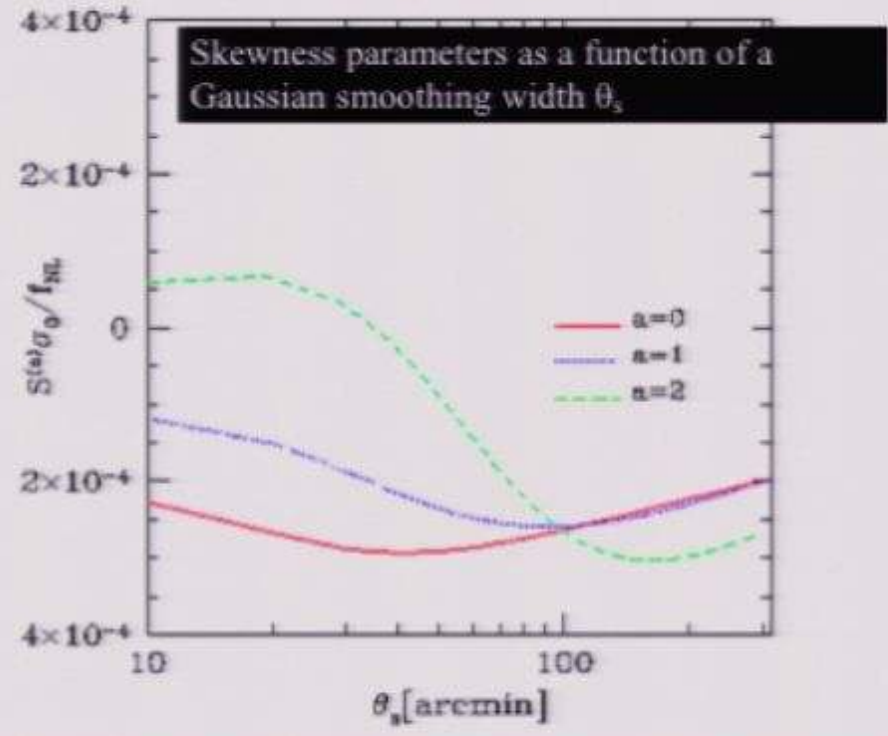
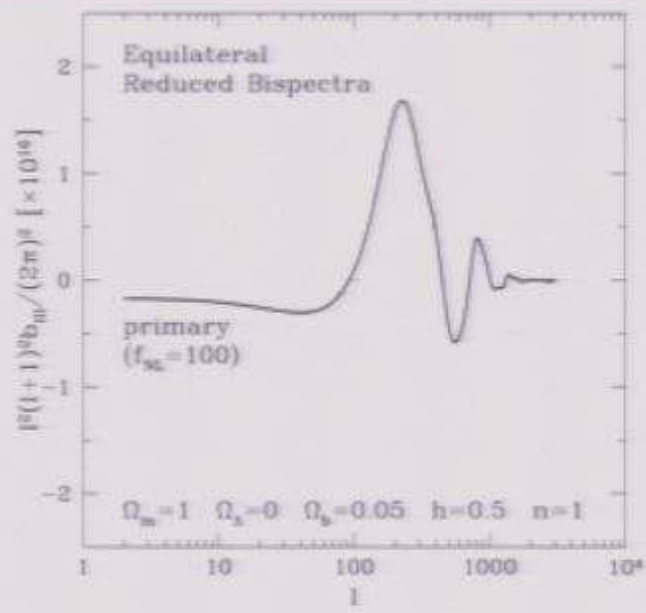
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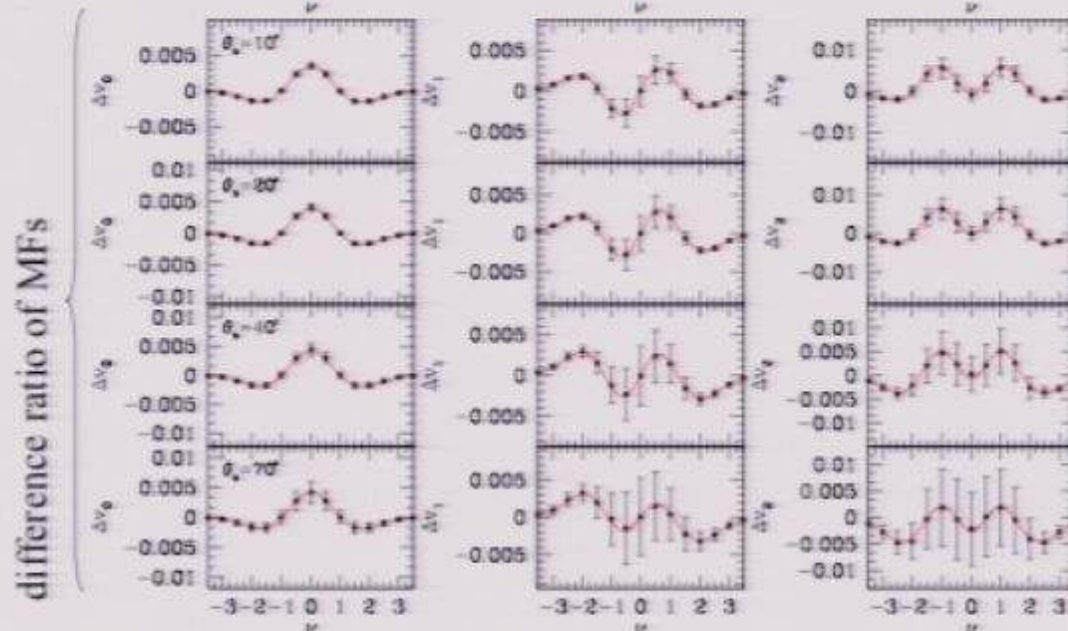
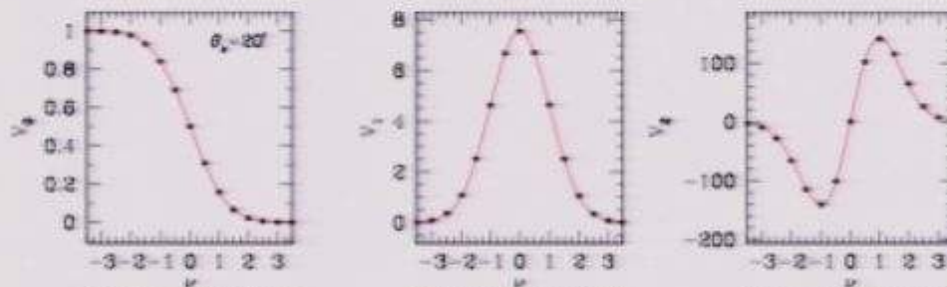
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An Opportunity?

- This apparent “tension” should be taken as an opportunity to investigate the other statistical tools, such the Minkowski functionals, wavelets, etc., in the context of primordial non-Gaussianity.
- It is plausible that various statistical tools can be written in terms of the sum of the bispectrum with various weights, in the limit of weak non-Gaussianity.
- Different tools are sensitive to different forms of non-Gaussianity - this is an advantage.

Systematics!

- Why use different statistical tools, when we know that the bispectrum gives us the maximum sensitivity?
- Systematics! Systematics!! Systematics!!!
- **I don't believe any detections, until different statistical tools give the same answer.**
 - That's why it bothers me to see that the bispectrum and the Minkowski functionals give different answers at the moment.

Summary

- The best estimates of primordial non-Gaussian parameters from the bispectrum analysis of the WMAP 5-year data are
 - $-9 < f_{\text{NL}}^{\text{local}} < 111$ (95% CL)
 - $-151 < f_{\text{NL}}^{\text{equil}} < 253$ (95% CL)
- **9-year data are required to test $f_{\text{NL}}^{\text{local}} \sim 60!$**
- The other statistical tools should be explored more.
 - E.g., estimate the skewness parameters directly from the Minkowski functionals to find the source of “tension”

Summary

best estimates of primordial non-Gaussian parameters from the bispectrum analysis of the WMAP year data are

$$-1 < f_{NL}^{local} < 111 \text{ (95\% CL)}$$

$$-51 < f_{NL}^{equil} < 253 \text{ (95\% CL)}$$

Year data are required to test $f_{NL}^{local} \sim 60!$

Other statistical tools should be explored more.

e.g., estimate the skewness parameters directly from the Minkowski functionals to find the source of "anisotropy"

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- Why not using Kp0 or KQ85 results, which have a higher statistical significance?
- Given the profound implications and impact of non-zero $f_{\text{NL}}^{\text{local}}$, we have chosen a conservative limit from the KQ75 with the point source correction ($\Delta f_{\text{NL}}^{\text{local}}=4$, which is also conservative) as our best estimate.
 - The 68% limit: $f_{\text{NL}}^{\text{local}} = 51 \pm 30$ [1.7 sigma]
 - **The 95% limit: $-9 < f_{\text{NL}}^{\text{local}} < 111$**

Main Result (Local)

Band	Mask	l_{\max}	f_{NL}^{local}	$\Delta f_{NL}^{\text{local}}$	b_{src}
V+W	KQ85	400	50 ± 29	1 ± 2	0.26 ± 1.5
V+W	KQ85	500	61 ± 26	2.5 ± 1.5	0.05 ± 0.50
V+W	KQ85	600	68 ± 31	3 ± 2	0.53 ± 0.28
V+W	KQ85	700	67 ± 31	3.5 ± 2	0.34 ± 0.20
V+W	Kp0	500	61 ± 26	2.5 ± 1.5	
V+W	KQ75p1 ^a	500	53 ± 28	4 ± 2	
V+W	KQ75	400	47 ± 32	3 ± 2	-0.50 ± 1.7
V+W	KQ75	500	55 ± 30	4 ± 2	0.15 ± 0.51
V+W	KQ75	600	61 ± 36	4 ± 2	0.53 ± 0.30
V+W	KQ75	700	58 ± 36	5 ± 2	0.38 ± 0.21

- ~ 2 sigma “hint”: $f_{NL}^{\text{local}} \sim \mathbf{60 \pm 30 (68\% CL)}$
- 1.8 sigma for KQ75; 2.3 sigma for KQ85 & Kp0

Komatsu & Spergel (2001); Babich, Creminelli & Zaldarriaga (2004)

Two f_{NL} 's

- Depending upon the shape of triangles, one can define
 - $-58 < f_{NL}^{local} < 134$ [WMAP 1yr, $l_{max}=265$] Komatsu et al. (2003)
 - $-54 < f_{NL}^{local} < 114$ [WMAP 3yr, $l_{max}=350$] Spergel et al. (2007)
 - **$-9 < f_{NL}^{local} < 111$ [WMAP 5yr, $l_{max}=500$]** Komatsu et al. (2008)
- Equilateral
 - $-366 < f_{NL}^{equil} < 238$ [WMAP 1yr, $l_{max}=405$] Creminelli et al. (2006)
 - $-256 < f_{NL}^{equil} < 332$ [WMAP 3yr, $l_{max}=475$] Creminelli et al. (2007)
 - **$-151 < f_{NL}^{equil} < 253$ [WMAP 5yr, $l_{max}=700$]** Komatsu et al. (2008)

Journal on f_{NL}

- Local

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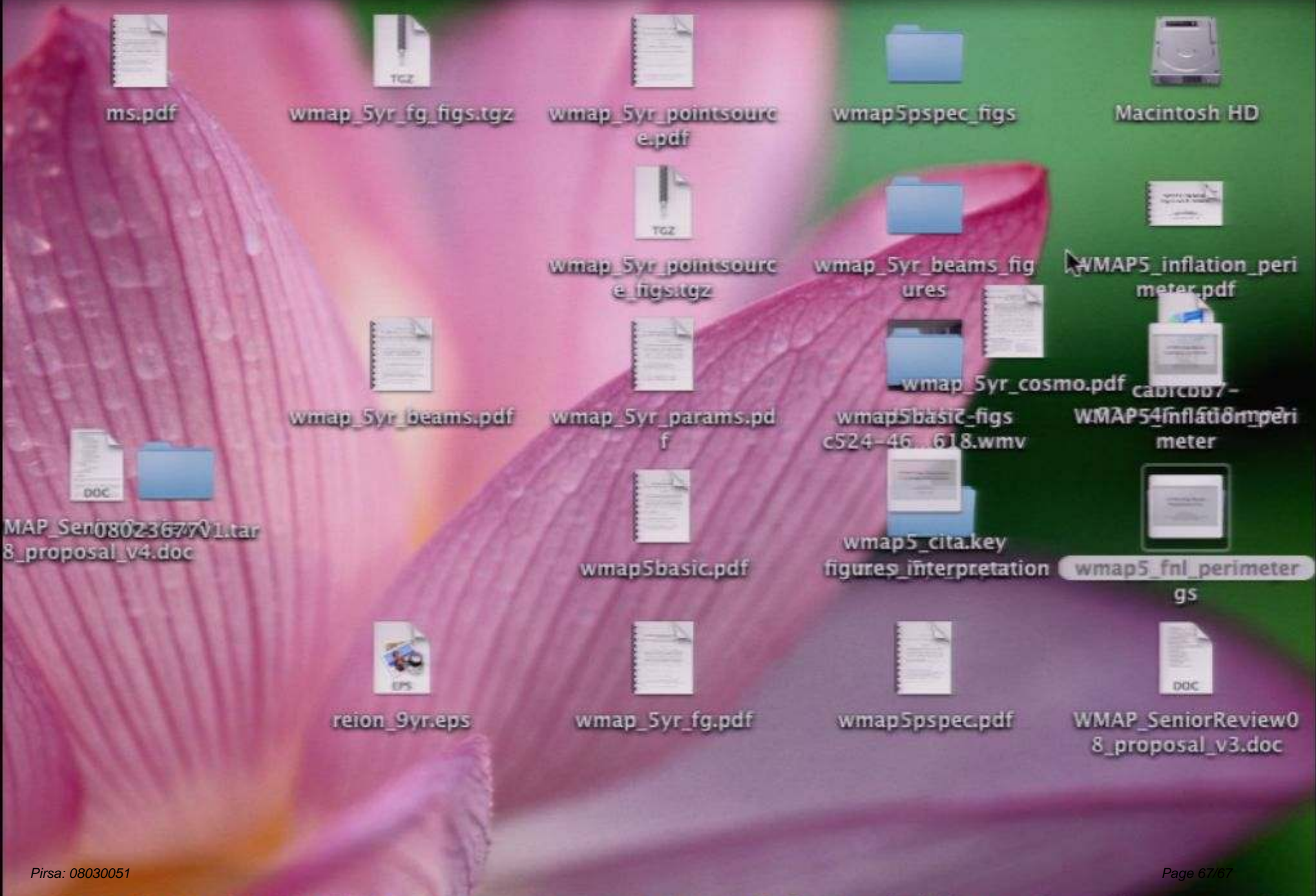
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