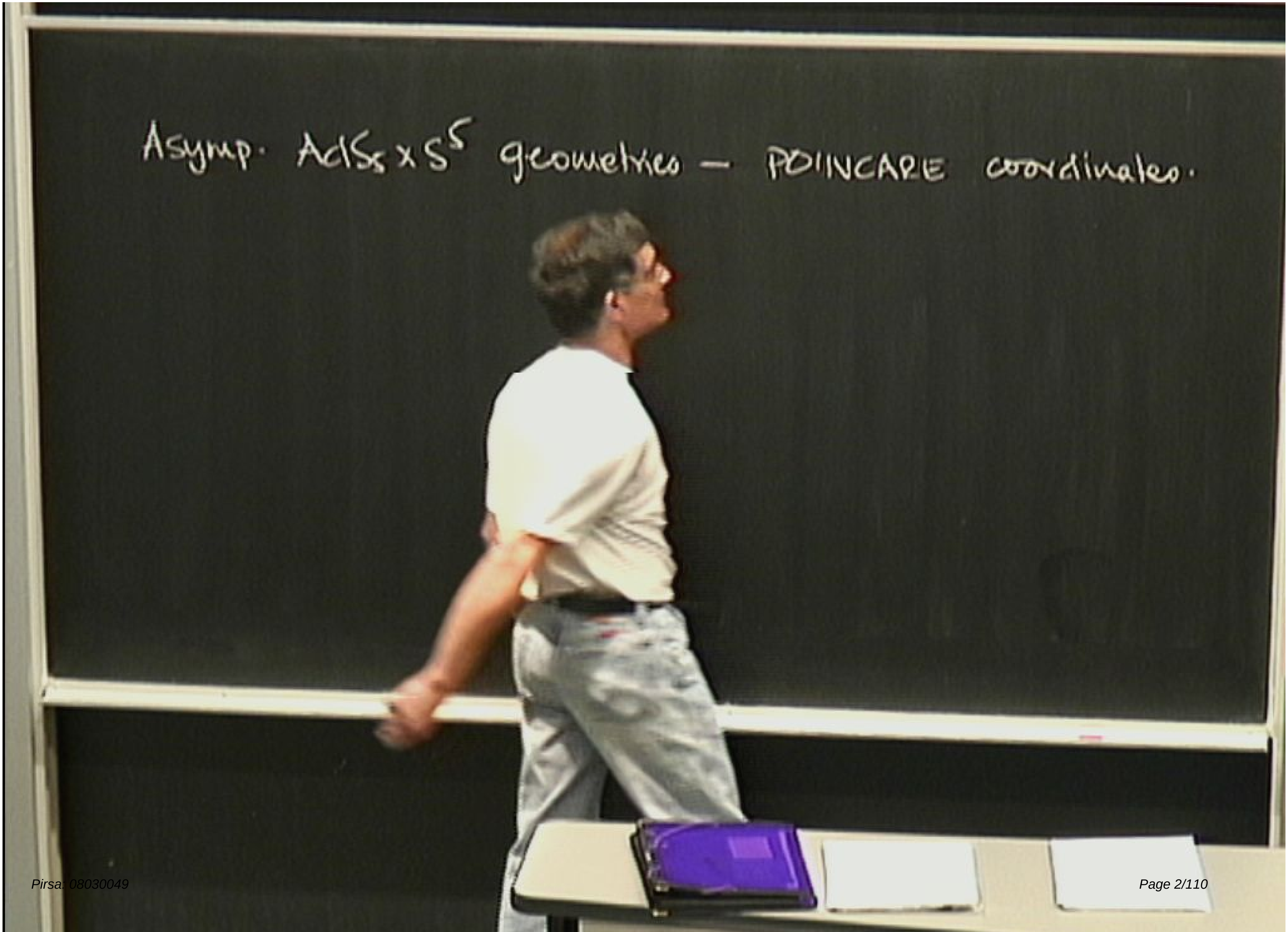
Title: Gauge Theory Duals of Cosmological Singularities

Date: Mar 08, 2008 09:00 AM

URL: http://pirsa.org/08030049

Abstract:


Asymp. $AdS_5 \times S^5$ geometries – POINCARE coordinates.

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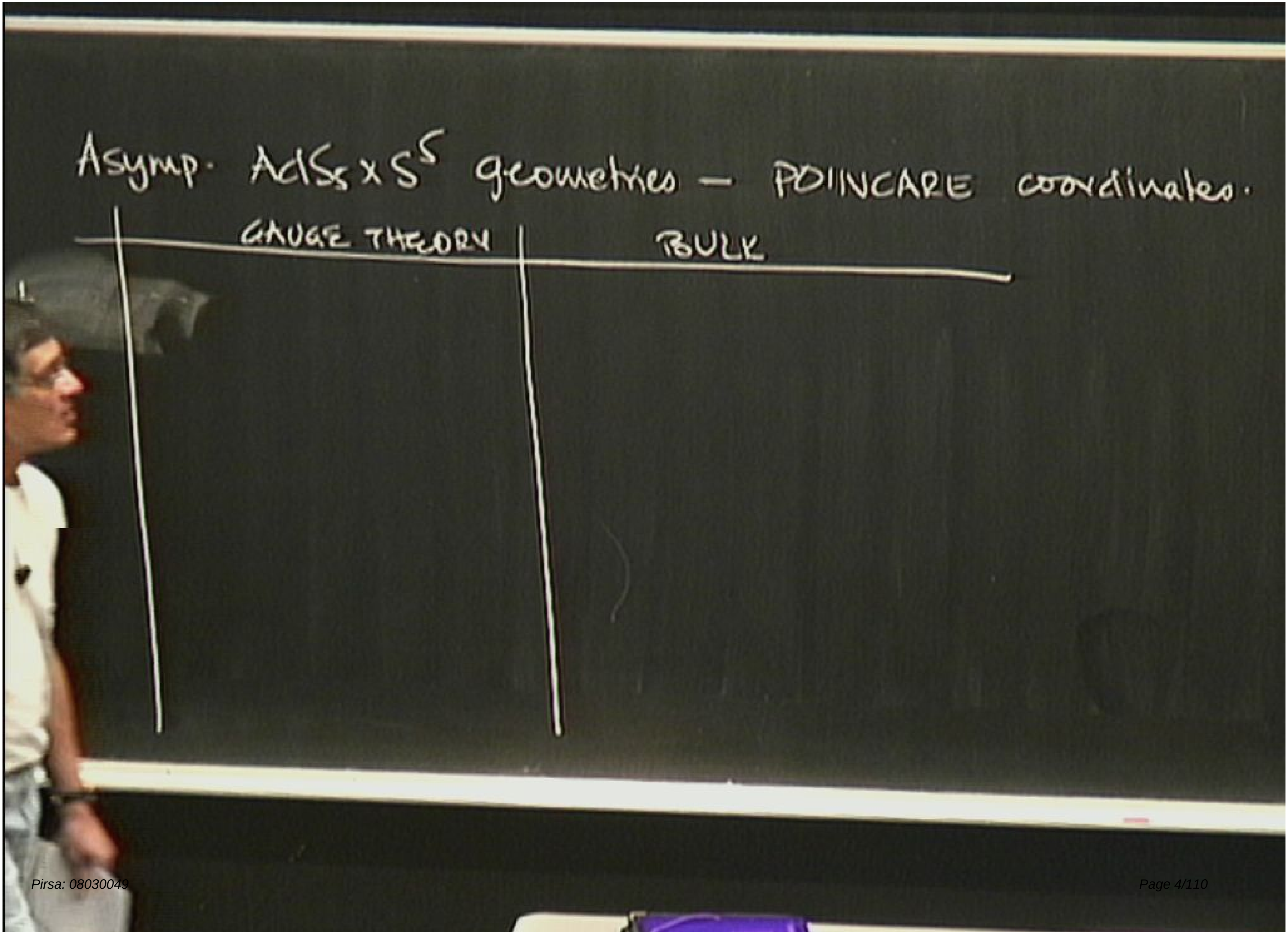
$t = -\infty$

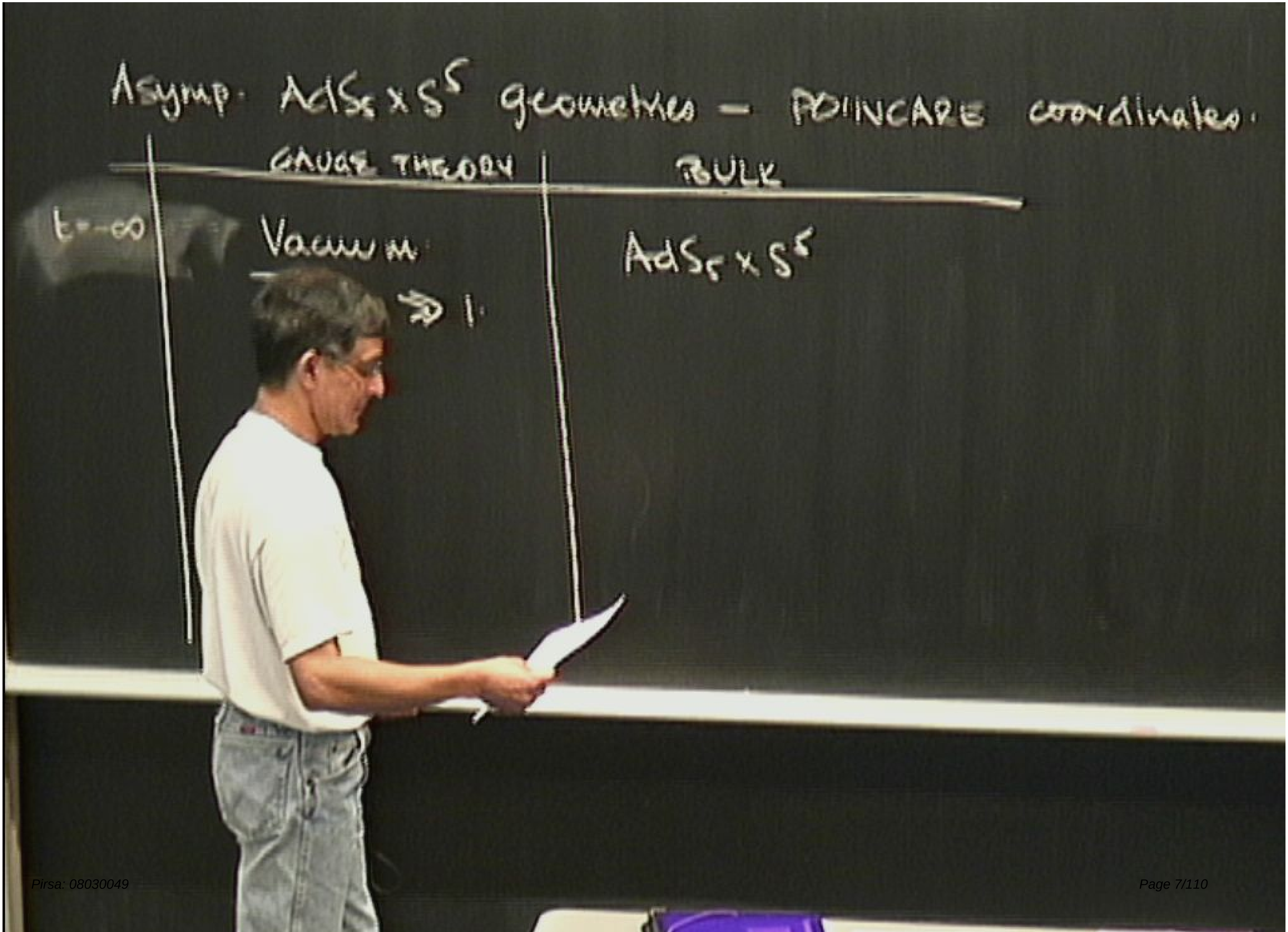

Asymp. $AdS_5 \times S^5$ geometries – POINCARE coordinates.
GAUGE THEORY
BULK

Asymp. $AdS_5 \times S^5$ geometries — POINCARE coordinates.

	GAUGE THEORY	BULK
$t=-\infty$	Vacuum $\overline{g_{YM}}$	

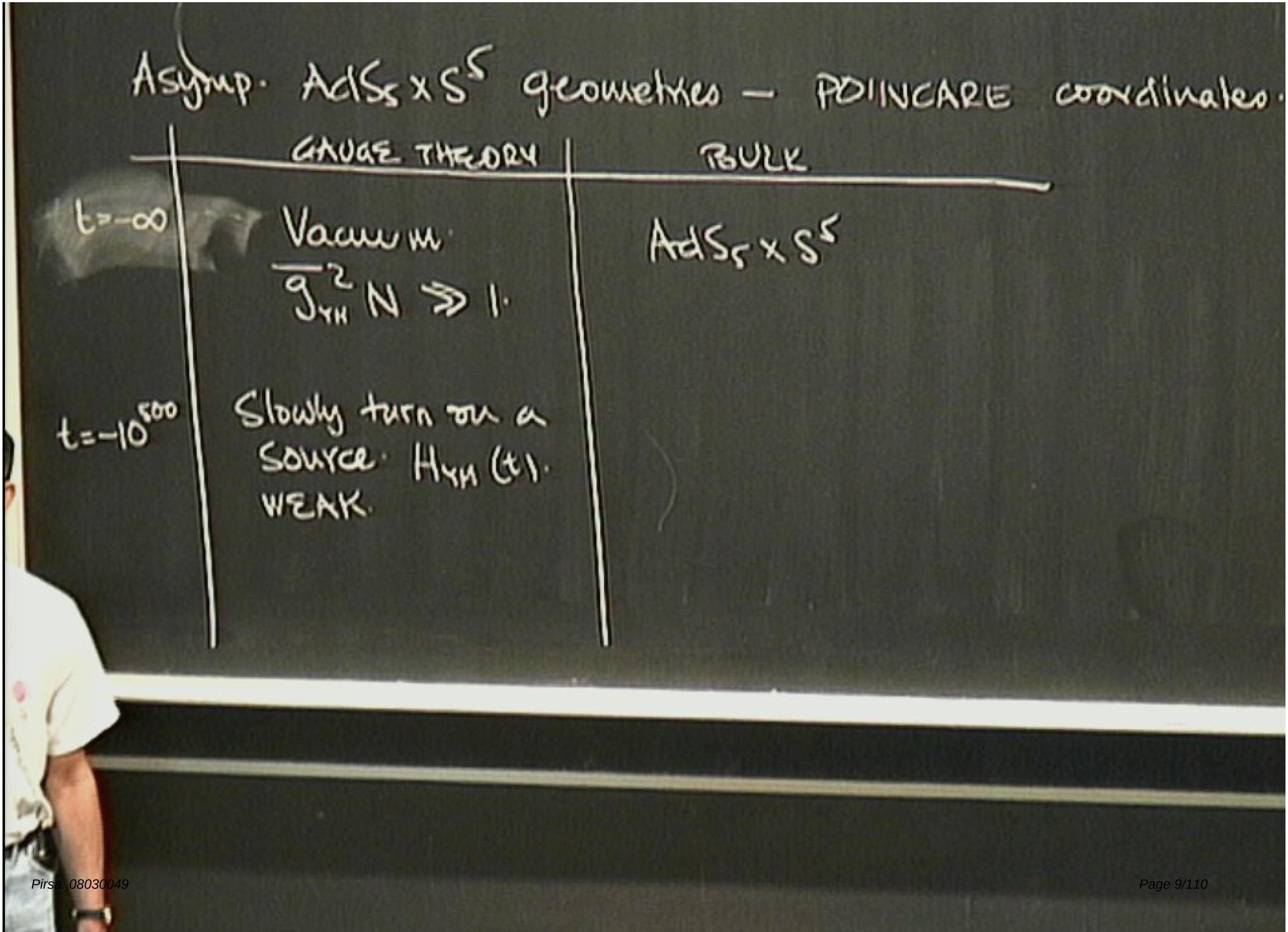
Asymp. $AdS_5 \times S^5$ geometries — POINCARE coordinates.

	GAUGE THEORY	BULK
$t=-\infty$	Vacuum. $\overline{g_{YM}^2 N} \gg 1.$	


Asymp. AdS5 x S5 geometries — POINCARE coordinates.
GAUGE THEORY
BULK
t=-∞
Vacuum
⇒ 1
AdS5 x S5

Asymp. $AdS_5 \times S^5$ geometries — POINCARE coordinates.

	GAUGE THEORY	BULK
$t=-\infty$	Vacuum. $\overline{g_{YM}^2} N \gg 1$.	$AdS_5 \times S^5$
$t=-10^{500}$	Slowly turn on a source.	


Asymp. $AdS_5 \times S^5$ geometries — POINCARE coordinates.
GAUGE THEORY
BULK
$t=-\infty$
Vacuum.
$g_{YM}^2 N \gg 1$.
$AdS_5 \times S^5$
$t=-10^{500}$
Slowly turn on a source. $H_{YM}(t)$.
WEAK.

Asymp. $AdS_5 \times S^5$ geometries – POINCARE coordinates.

	GAUGE THEORY	BULK
$t=-\infty$	Vacuum $g_{YM}^2 N \gg 1$.	$AdS_5 \times S^5$
$t=-10^{500}$	Slowly turn on a source $H_{YM}(t)$. WEAK e.g. $g_{YM}(t)$.	

Asymp. $AdS_5 \times S^5$ geometries — POINCARE coordinates.

	GAUGE THEORY	BULK
$t=-\infty$	Vacuum. $\bar{g}_{YM}^2 N \gg 1$.	$AdS_5 \times S^5$
$t=-10^{500}$	Slowly turn on a source. $H_{YM}(t)$. WEAK. e.g. $g_{YM}(t)$. $\left(\frac{1}{g_{YM}^2(t)} - \frac{1}{\bar{g}_{YM}^2}\right) \mathrm{Tr}\left(F^2 + (D\Phi^i)^2 + \ldots\right)$	

Asymp. $AdS_5 \times S^5$ geometries — POINCARE coordinates.

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$t=-\infty$	Vacuum. $\overline{g}_{YM}^2 N \gg 1$.	$AdS_5 \times S^5$
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Asymp. $AdS_5 \times S^5$ geometries — POINCARE coordinates

	GAUGE THEORY	BULK
$t=-\infty$	Vacuum. $\overline{g_{YM}^2 N} \gg 1$.	$AdS_5 \times S^5$
$t=-10^{500}$	Slowly turn on a source. $H_{YM}(t)$. WEAK. e.g. $g_{YM}(t)$. $\sim \left(\frac{1}{g_{YM}^2(t)} - \frac{1}{g_{YM}^2}\right) \mathrm{Tr}\left(F^2 + (D\Phi^i)^2 + \dots\right)$	NON-NORMALIZABLE Deformation of $AdS_5 \times S^5$

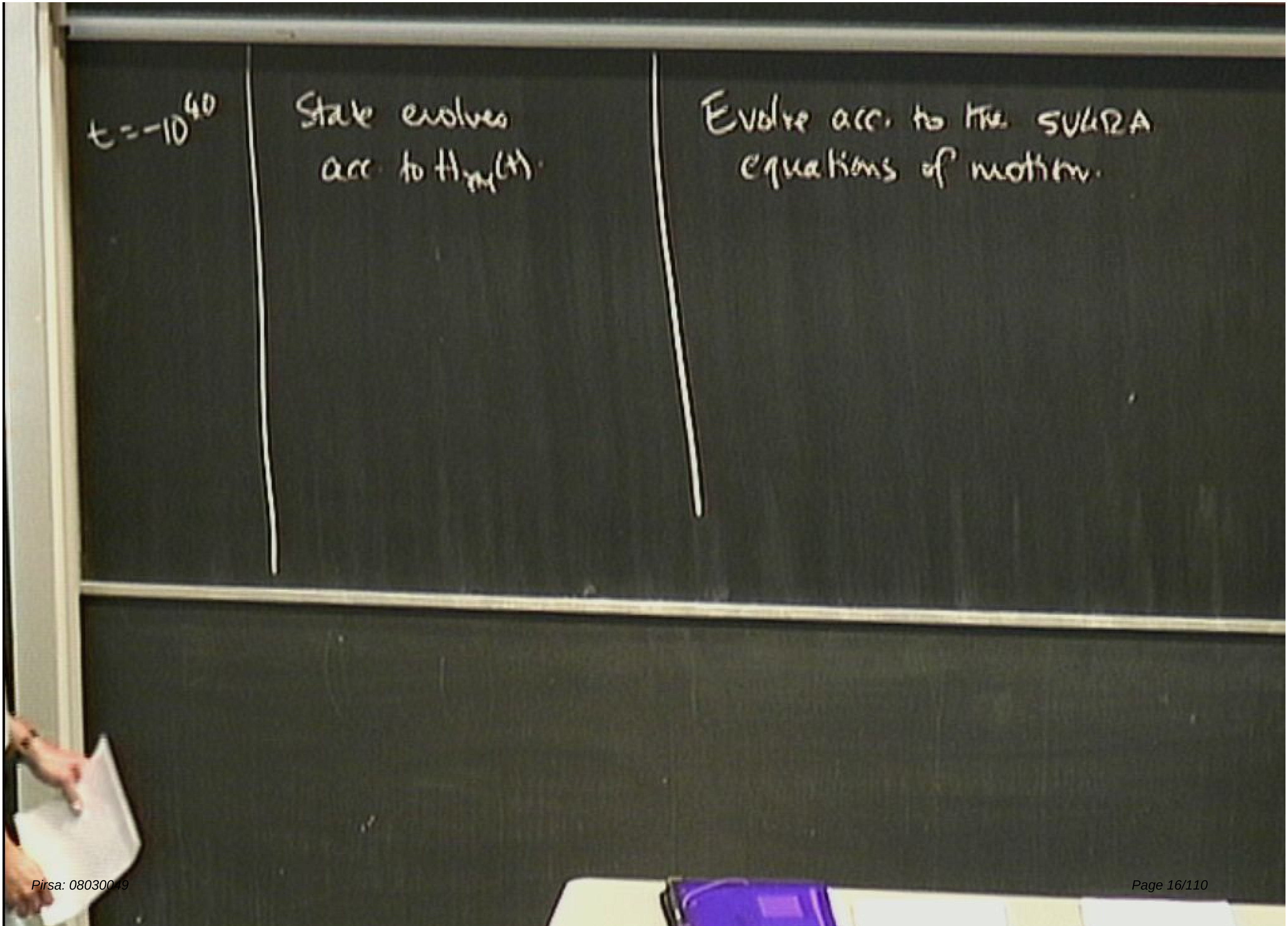
Asymp. $AdS_5 \times S^5$ geometries — POINCARE coordinates.

	GAUGE THEORY	BULK
$t=-\infty$	Vacuum. $\overline{g_{YM}^2 N} \gg 1$.	$AdS_5 \times S^5$
$t=-10^{500}$	Slowly turn on a source. $H_{YM}(t)$. WEAK. e.g. $g_{YM}(t)$. $\sim\left(\frac{1}{g_{YM}^2(t)}-\frac{1}{g_{YM}^2}\right)\mathrm{Tr}\left(F^2+(D\Phi^i)^2+\ldots\right)$	NON-NORMALIZABLE Deformation of $AdS_5 \times S^5$. Turning on a nontrivial dilaton $\phi(t)$.

$t=-10^{40}$

$t = -10^{40}$

State evolves
acc. to $H_{SM}(t)$.


t = -10^40
State evolves
acc. to H_my(t).
Evolve acc. to the SUGRA
equations of motion.

$t = -10^{40}$

State evolves acc. to $H_{YM}(t)$.

Evolve acc. to the SUGRA equations of motion.

→ Solve SUGRA eqns. of motion.

$t = 0$

$t = -10^{40}$

$t = 0$

State evolves acc. to $H_{YM}(t)$.

$g_{YM}(t) \ll 1$.

Evolve acc. to the SUGRA equations of motion.

→ Solve SUGRA equs. of motion.

$t = -10^{40}$

$t = 0$

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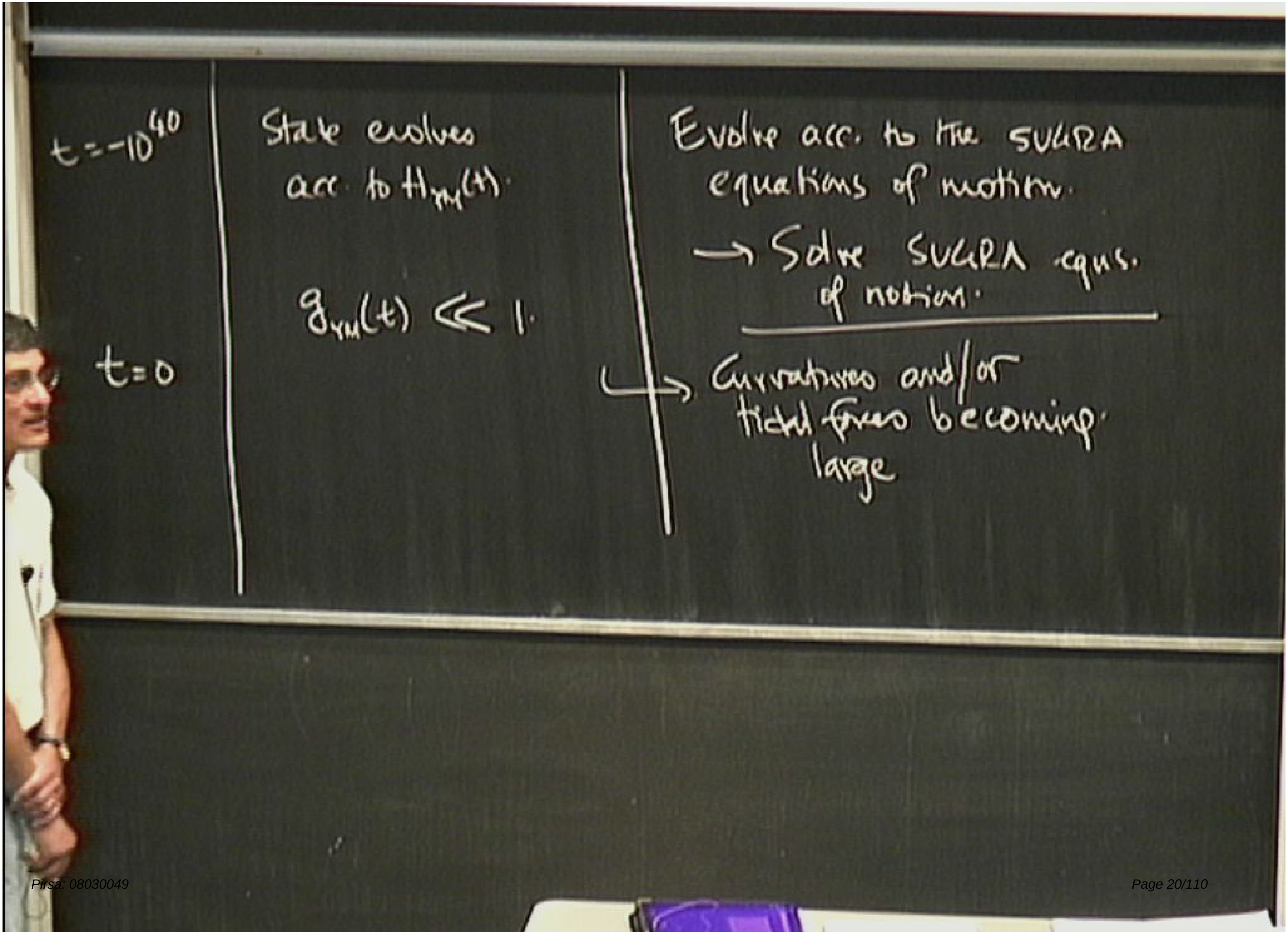
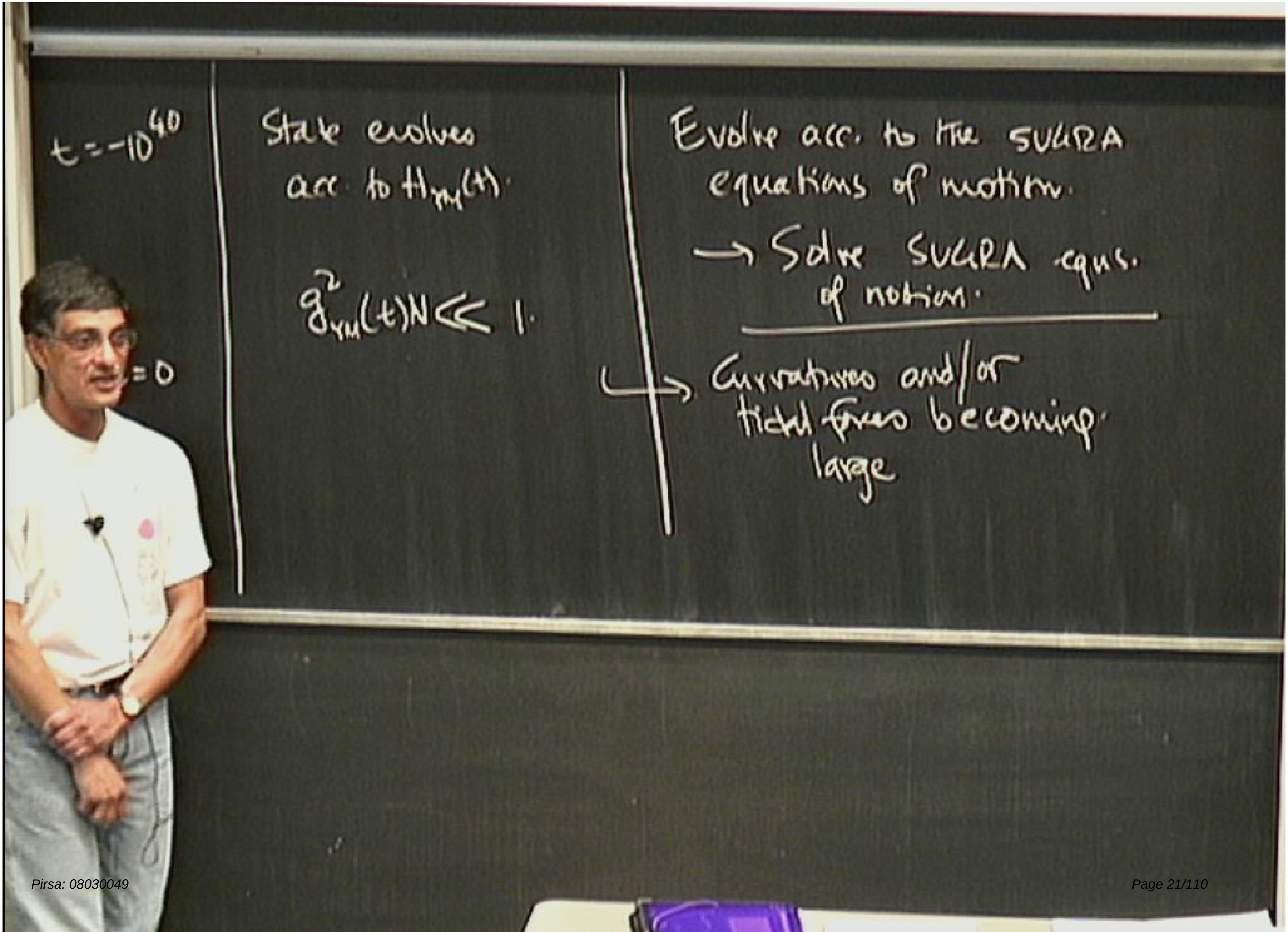
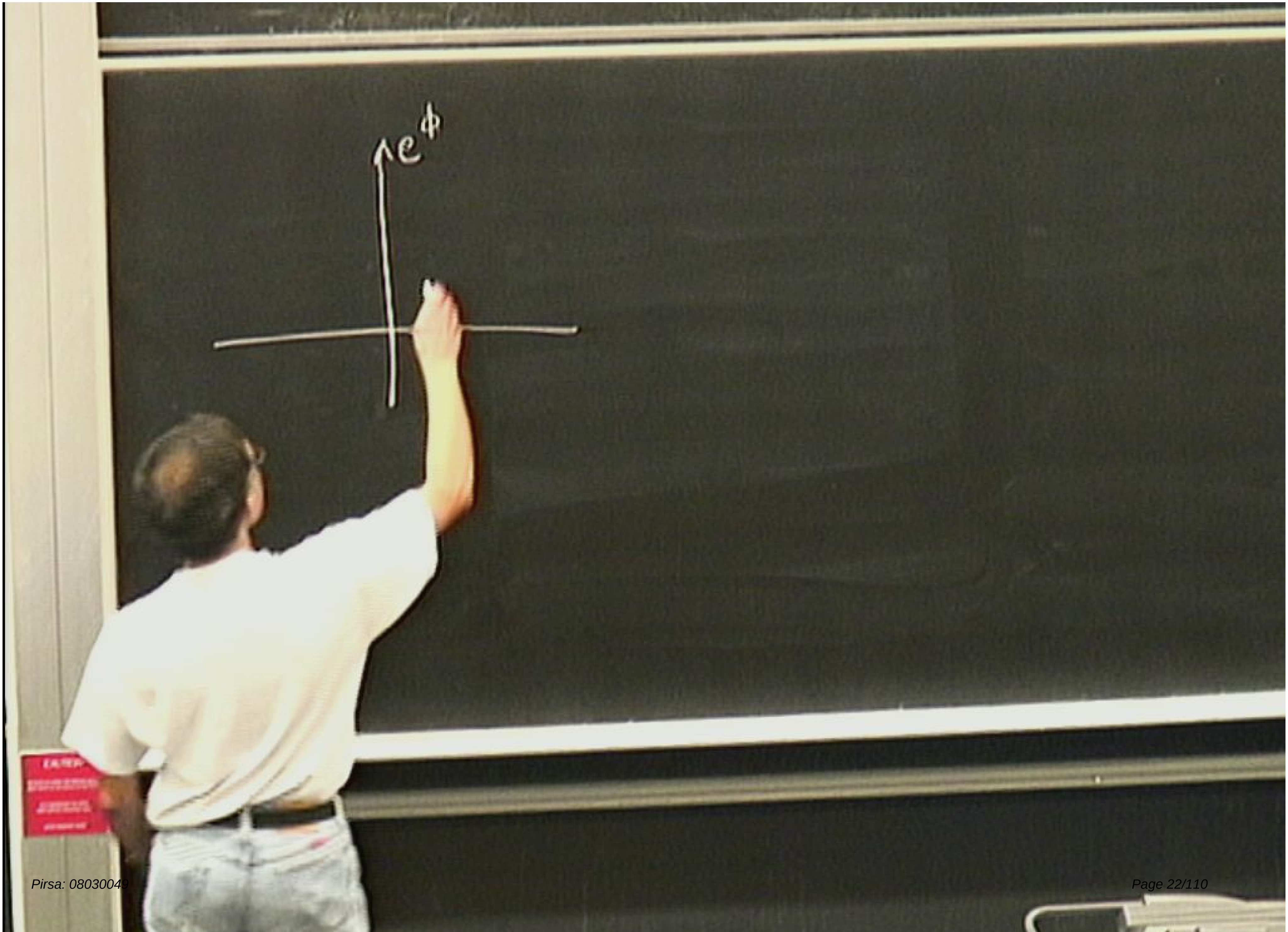
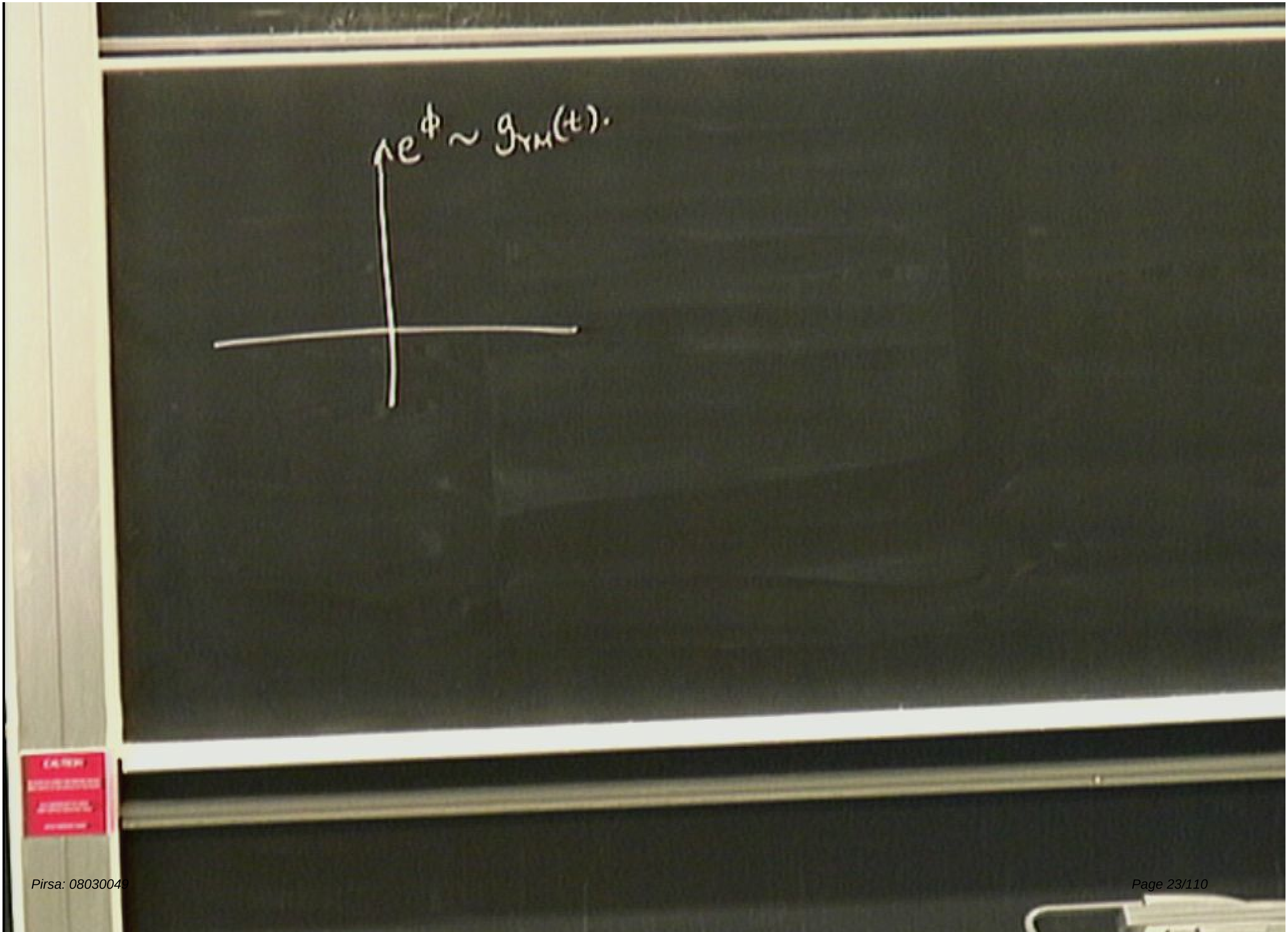
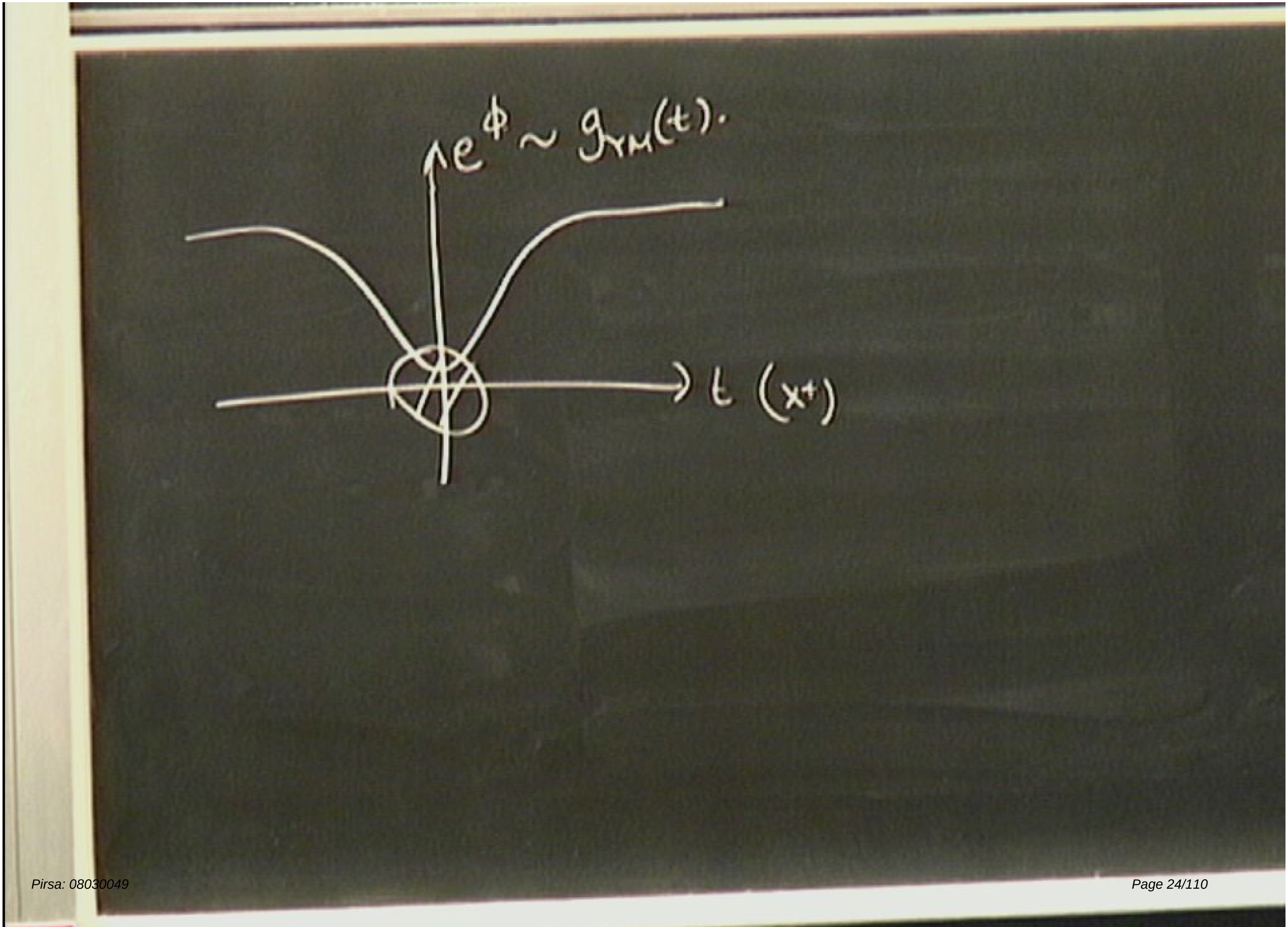

t = -10^40
State evolves acc. to H_YM(t).
g_YM(t) << 1.
t=0
Evolve acc. to the SUGRA equations of motion.
→ Solve SUGRA eqns. of motion.
↳ Curvatures and/or tidal forces becoming large

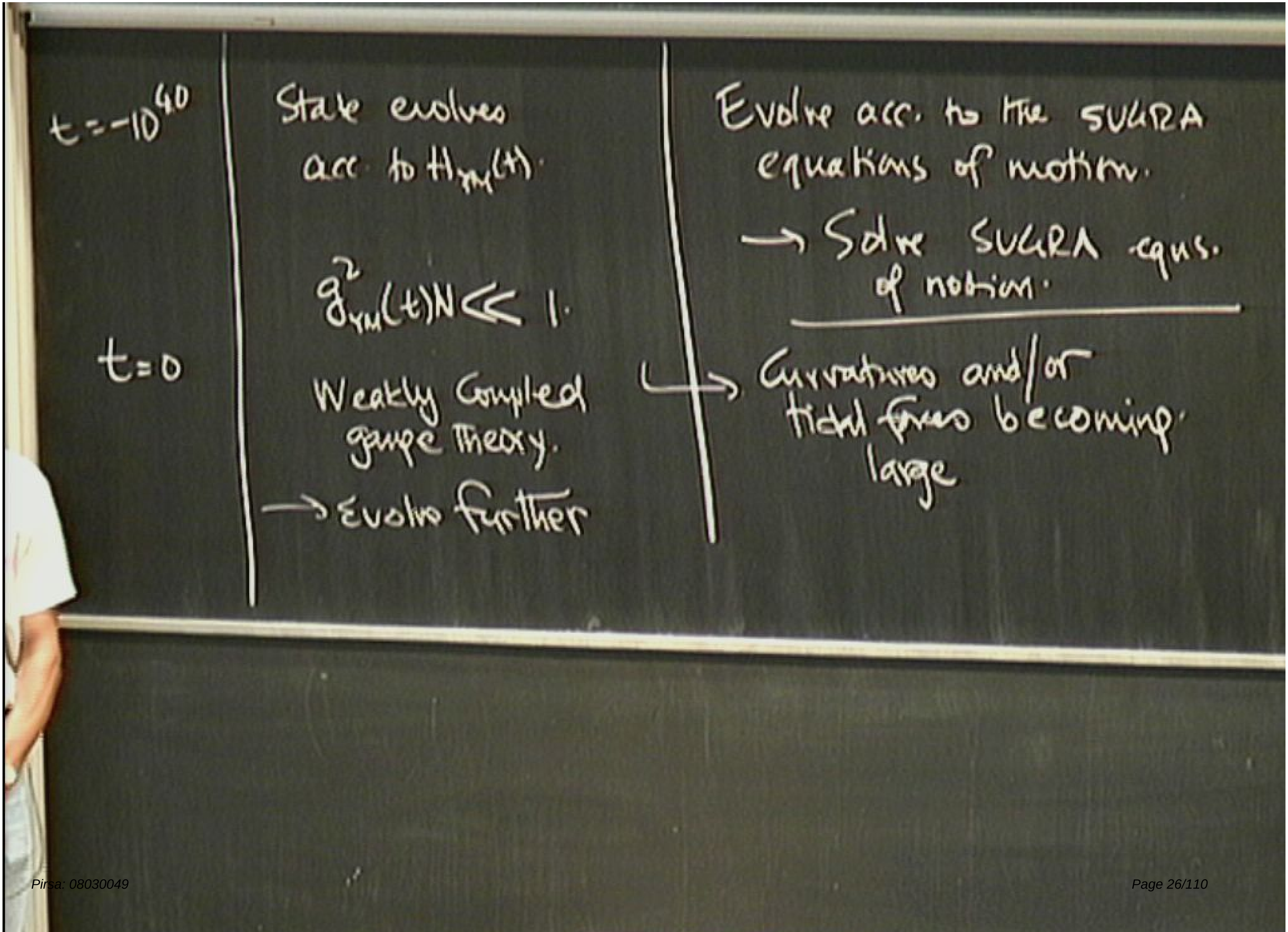

t = -10^40
= 0
State evolves
acc to H_YM(t).
g^2_YM(t)N << 1.
Evolve acc. to the SUGRA
equations of motion.
→ Solve SUGRA eqns.
of motion.
→ Curvatures and/or
tidal forces becoming
large


e^{ϕ}

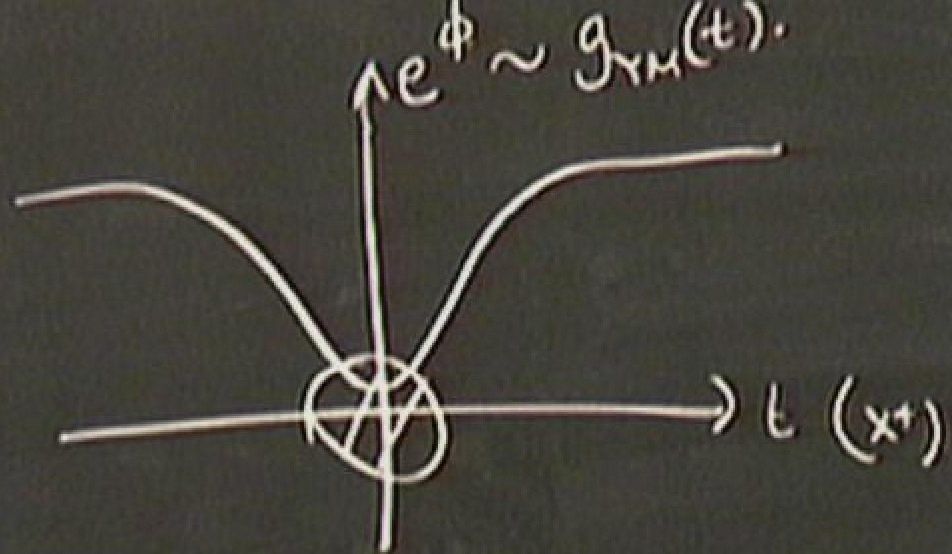

$e^{\phi} \sim g_{YM}(t).$


$e^{\phi} \sim g_{YM}(t)$.
t (x^{+})

$t = -10^{40}$	State evolves acc. to $H_{YM}(t)$.	Evolve acc. to the SUGRA equations of motion.
	$g^2_{YM}(t) N \ll 1.$	→ Solve SUGRA eqns. of motion.
$t = 0$	Weakly Coupled gauge theory.	↳ Curvatures and/or tidal forces becoming large


$t=-10^{40}$
State evolves acc. to $H_{YM}(t)$.
$g^2_{YM}(t)N \ll 1$.
$t=0$
Weakly Coupled gauge Theory.
→ Evolve further
Evolve acc. to the SUGRA equations of motion.
→ Solve SUGRA eqns. of motion.
→ Curvatures and/or tidal forces becoming large

$t = -10^{40}$	State evolves acc. to $H_{YM}(t)$.	Evolve acc. to the SUGRA equations of motion.
		→ Solve SUGRA equs. of motion.
	$g^2_{YM}(t) N \ll 1$.	
$t = 0$	Weakly Coupled gauge theory.	→ Curvatures and/or tidal forces becoming large
	→ Evolve further	NO SPACE-TIME.



Find sugra solutions of this type.

$t = -10^{500}$

Slowly turn on a source. $H_{YM}(t)$. WEAK.

e.g. $g_{YM}(t)$.

$$\left(\frac{1}{g_{YM}^2(t)} - \frac{1}{g_{YM}^2}\right) \mathrm{Tr}\left(F^2 + (D\Phi^i)^2 + \ldots\right)$$

Def

Turni

$t = -10^{40}$

State evolves acc. to $H_{YM}(t)$.

E

NULL SINGULARITIES.

NULL SINGULARITIES.

$$ds^2 = \frac{1}{w^2}\left[dw^2 + \left(-2dy^+dy^- + d\vec{y}_2^{\,2}\right) + \frac{1}{4}w^4(\partial_+\phi)^2(dy^+)^2\right] + d\Omega_5^2$$

NULL SINGULARITIES.

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$\vec{y}$ ↑ 2

$\Phi(y^+) \rightarrow$ any function.

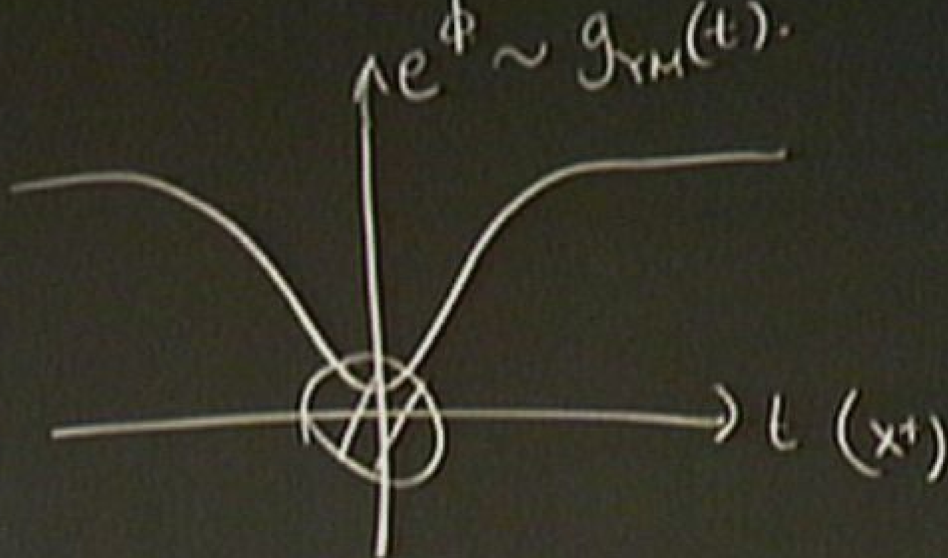
$F_5 =$

NULL SINGULARITIES.

$$ds^2 = \frac{1}{w^2}\left[dw^2 + \left(-2dy^+dy^- + d\vec{y}_2^{\,2} + \frac{1}{4}w^2(\partial_i\partial_j\Phi)(dy^+)^2\right) + d\Omega_5^2\right]$$

$\Phi(y^+) \rightarrow$ any function.

$$F_5 = \omega_5 + {}^*\omega_5$$



Find sugra solutions of this type.

NULL SINGULARITIES.

$$ds^2 = \frac{1}{w^2}\left[dw^2 + \left(-2dy^+dy^- + d\vec{y}_2^{\,2}\right) + \frac{1}{4}w^2(\partial_+\Phi)^2(dy^+)^2\right] + d\Omega_5^2$$

$\Phi(y^+) \to$ any function.

$F_5 = \omega_5 + {}^*\omega_5$ $\quad (\partial_+\Phi) \to \infty$

$e^\Phi \ll 1$ at $y^+ = 0$.

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$(\alpha) \sim (\partial_+\Phi)^2$

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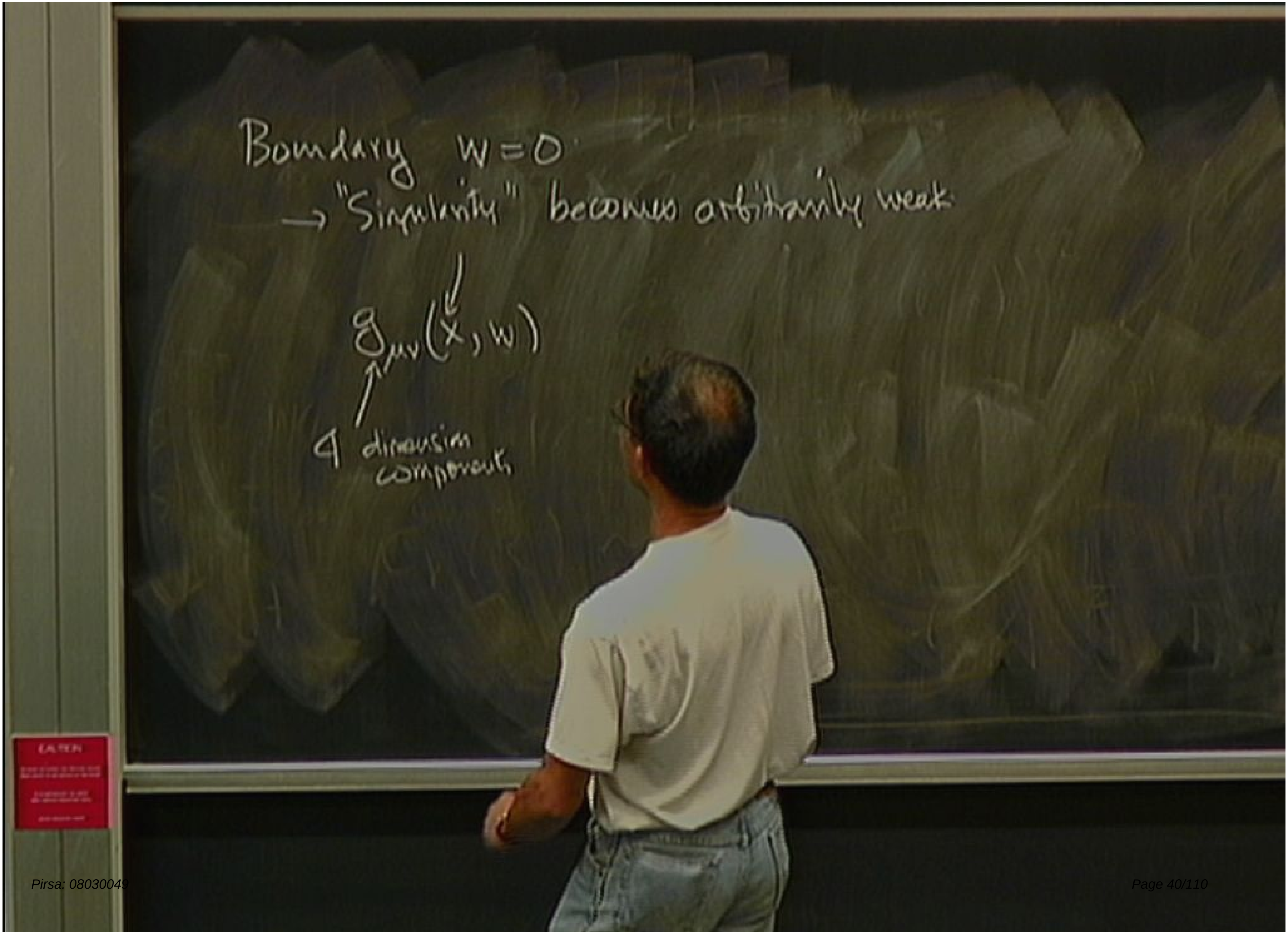
$(\partial_+\Phi) = \partial_+(e^{\Phi})\, e^{-\Phi}$

$|a| \sim (\partial_+\Phi)^2$

$a^\mu_\sigma = R^\mu{}_{\nu\lambda\sigma}\, \xi^\nu \xi^\lambda X^\nu$

$\xi \to$ Tangent $\quad X \to$ Separation



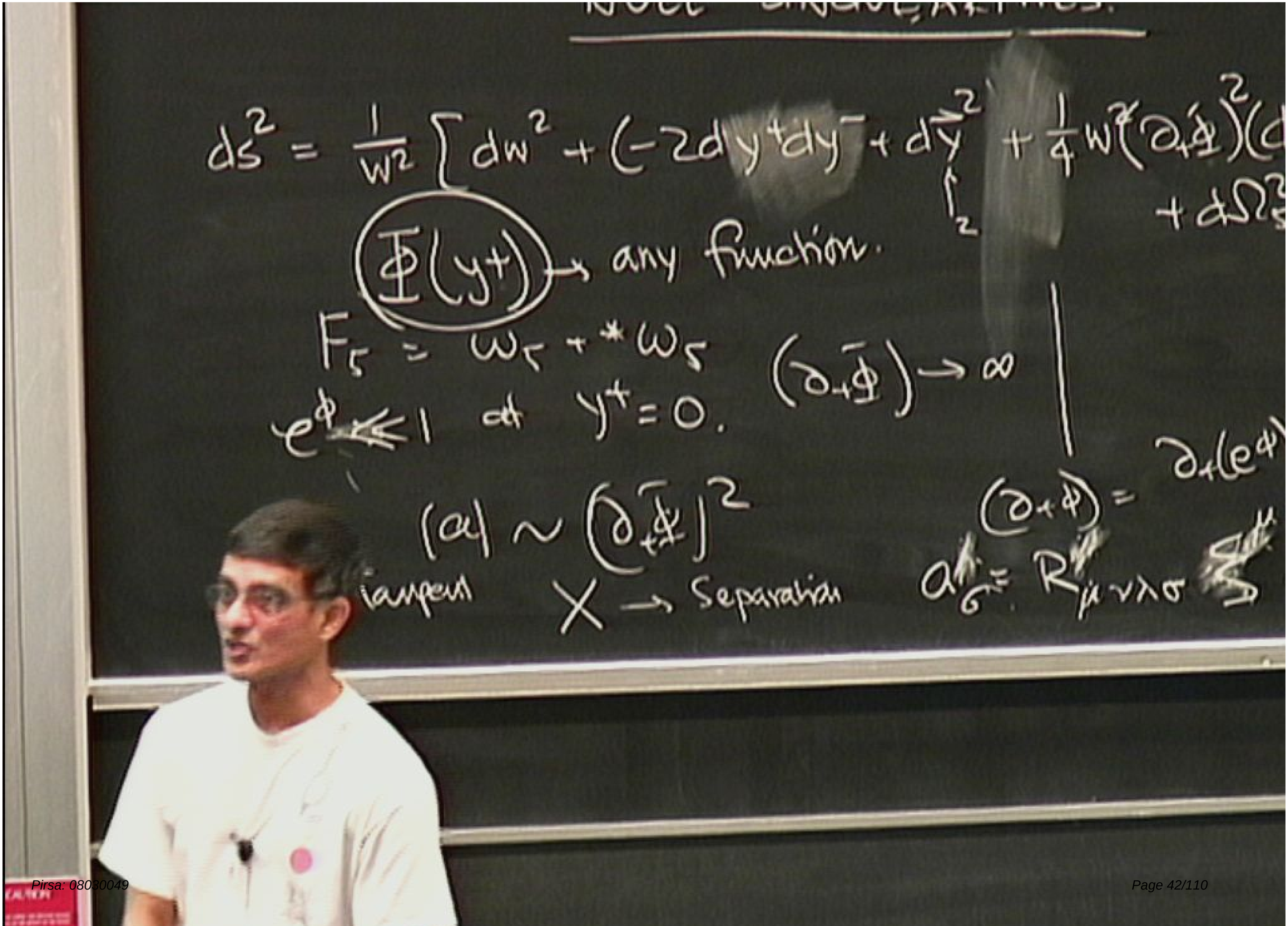
Boundary w = 0
→ "Singularity" becomes arbitrarily weak
$g_{\mu\nu}(x, w)$
4 dimension components

Boundary $w=0$

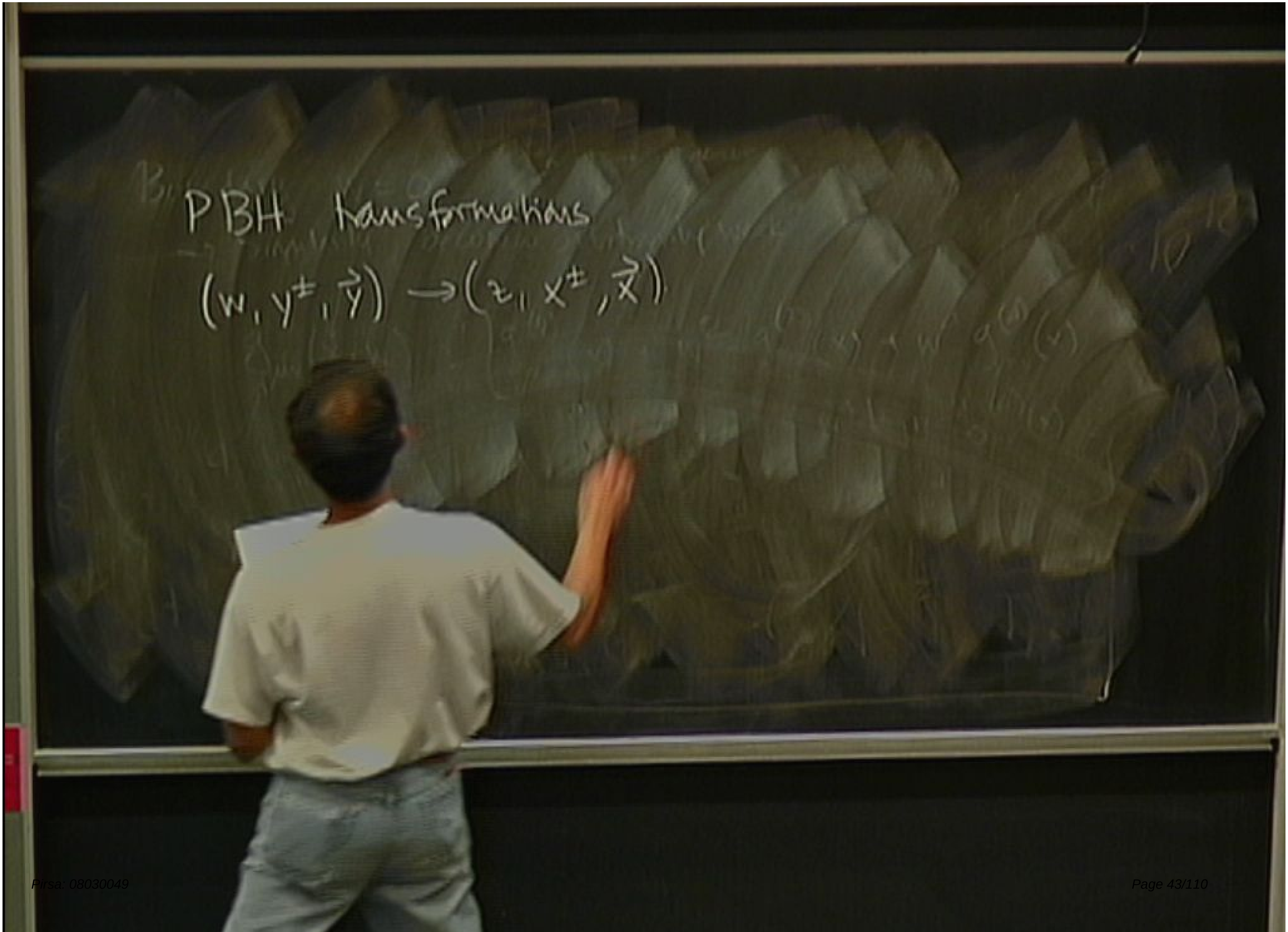
→ "Singularity" becomes arbitrarily weak

$$g_{\mu\nu}(x,w) = \frac{1}{w^2}\Big[g^{(0)}_{\mu\nu}(x) + w^2\, g^{(2)}_{\mu\nu}(x) + w^4\, g^{(4)}_{\mu\nu}(x) + w^4 \log w\, h^{(4)}_{\mu\nu}(x) + \cdots\Big]$$

4 dimension components



any function.
F5 = ω5 + *ω5
at y+ = 0.
Separation


PBH transformations
$(w, y^{\pm}, \vec{y}) \rightarrow (z, x^{\pm}, \vec{x})$

PBH transformations

$$(w, y^\pm, \vec{y}) \to (z, x^\pm, \vec{x})$$

$$z = w\, e^{f(y^+)/2}$$

$$x^- = y^- - \frac{1}{4} w^2 (\partial_+ f)$$

$$x^+ = y^+$$

$$\vec{x} = \vec{y}$$

$$\frac{1}{2}(f')^2 - f'' = \frac{1}{2}(\phi')^2$$

PBH transformations

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$$x^+ = y^+$$

$$\vec{x} = \vec{y}$$

$$\frac{1}{2}(f')^2 - f'' = \frac{1}{2}(\dot{f}')^2$$

$$ds^2 = \frac{1}{z^2}\left[dz^2 + e^{f(x^+)}\left(-2dx^+dx^- + d\vec{x}^2\right)\right]$$

PBH transformations

$$(w, y^{\pm}, \vec{y}) \longrightarrow (z, x^{\pm}, \vec{x})$$

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New boundary $z=0$ $\longrightarrow$ Conformally flat

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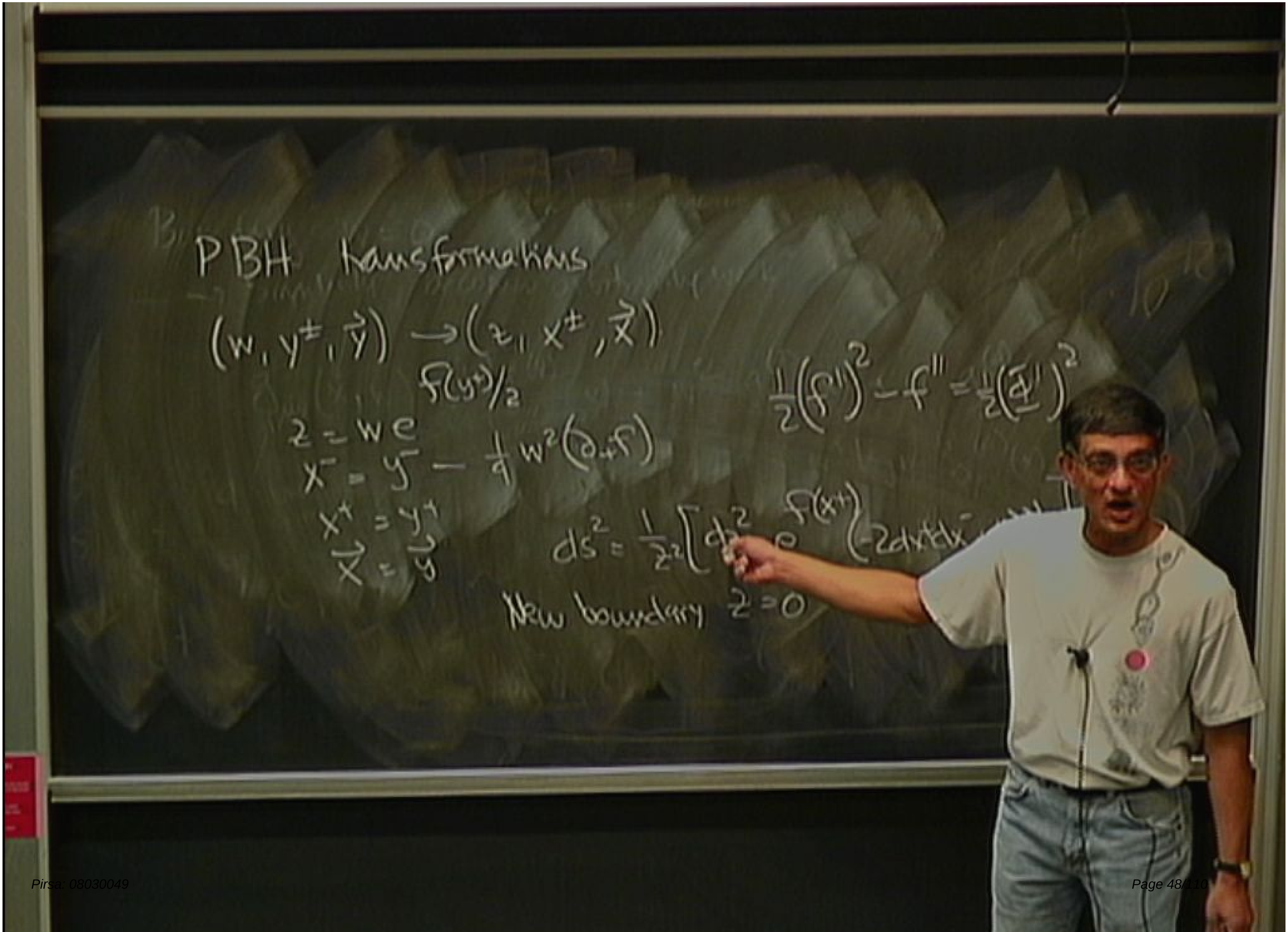
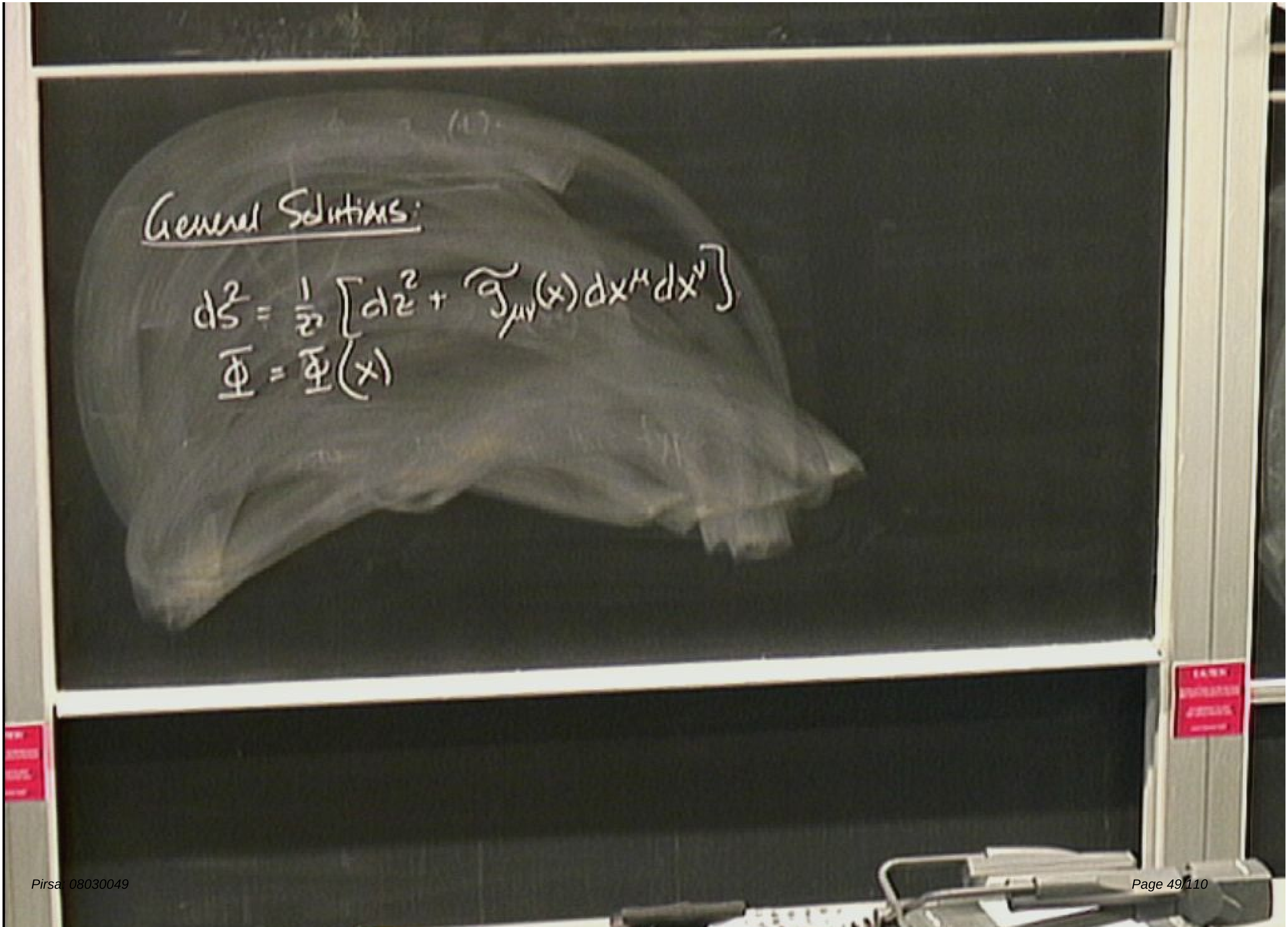
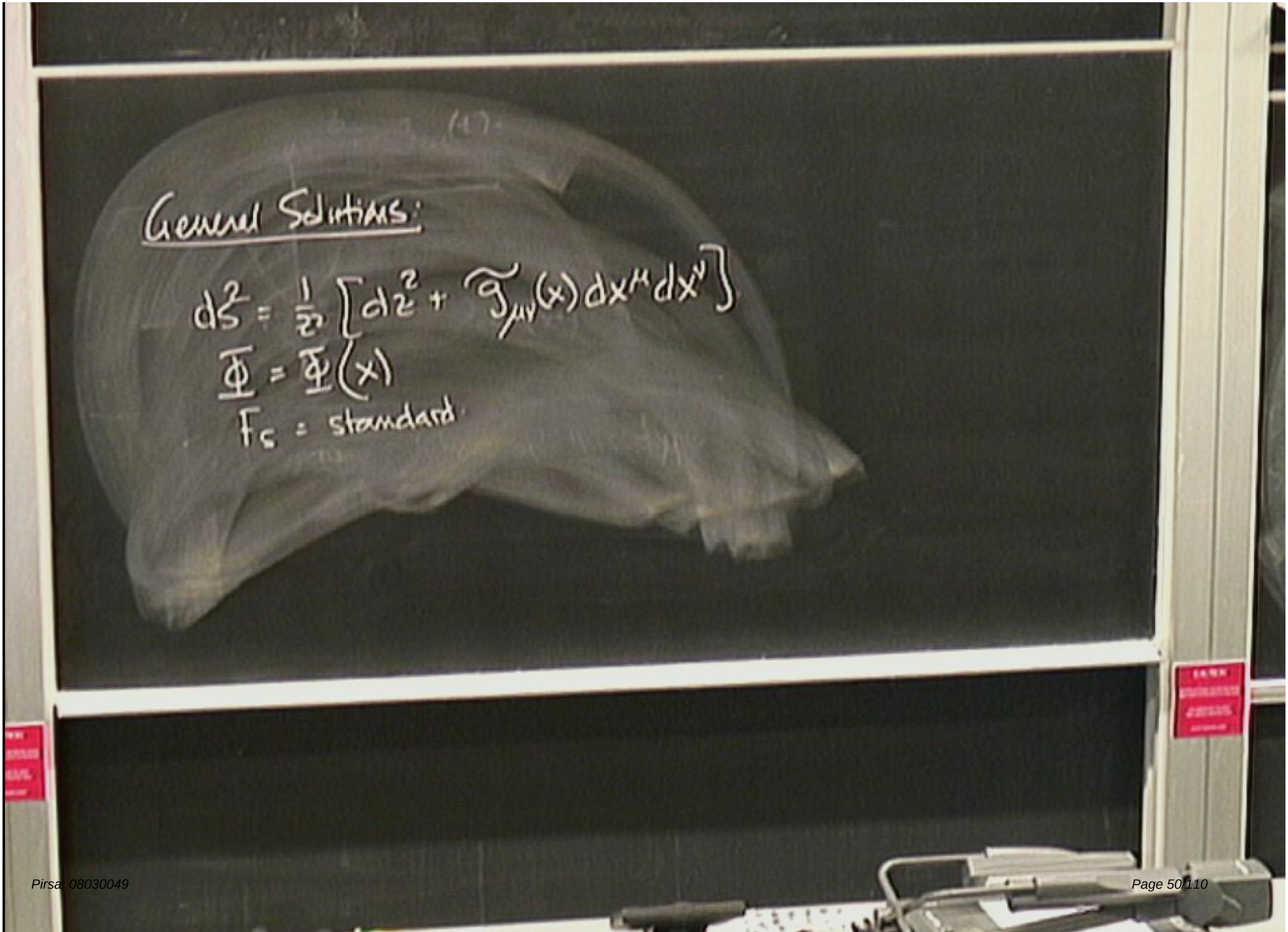

PBH transformations
(w, y±, y⃗) → (z, x±, x⃗)
z = w e^{f(y+)/2}
x⁻ = y⁻ − ¼ w²(∂₊f)
x⁺ = y⁺
x⃗ = y⃗
ds² = 1/z² [dz² + e^{f(x+)} (−2dx⁺dx⁻
New boundary z = 0


General Solutions:
$ds^2 = \frac{1}{z^2}\left[dz^2 + \tilde{g}_{\mu\nu}(x)\,dx^\mu dx^\nu\right]$
$\Phi = \Phi(x)$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$ds^2 = \frac{1}{z^2}\left[dz^2 + \tilde{g}_{\mu\nu}(x)\,dx^\mu dx^\nu\right]$
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F_5 = standard

General Solutions:

$$ds^2 = \frac{1}{z^2}\left[dz^2 + \tilde{g}_{\mu\nu}(x)\,dx^\mu dx^\nu\right] + d\Omega_5^2$$

$$\Phi = \Phi(x)$$

F_5 = standard

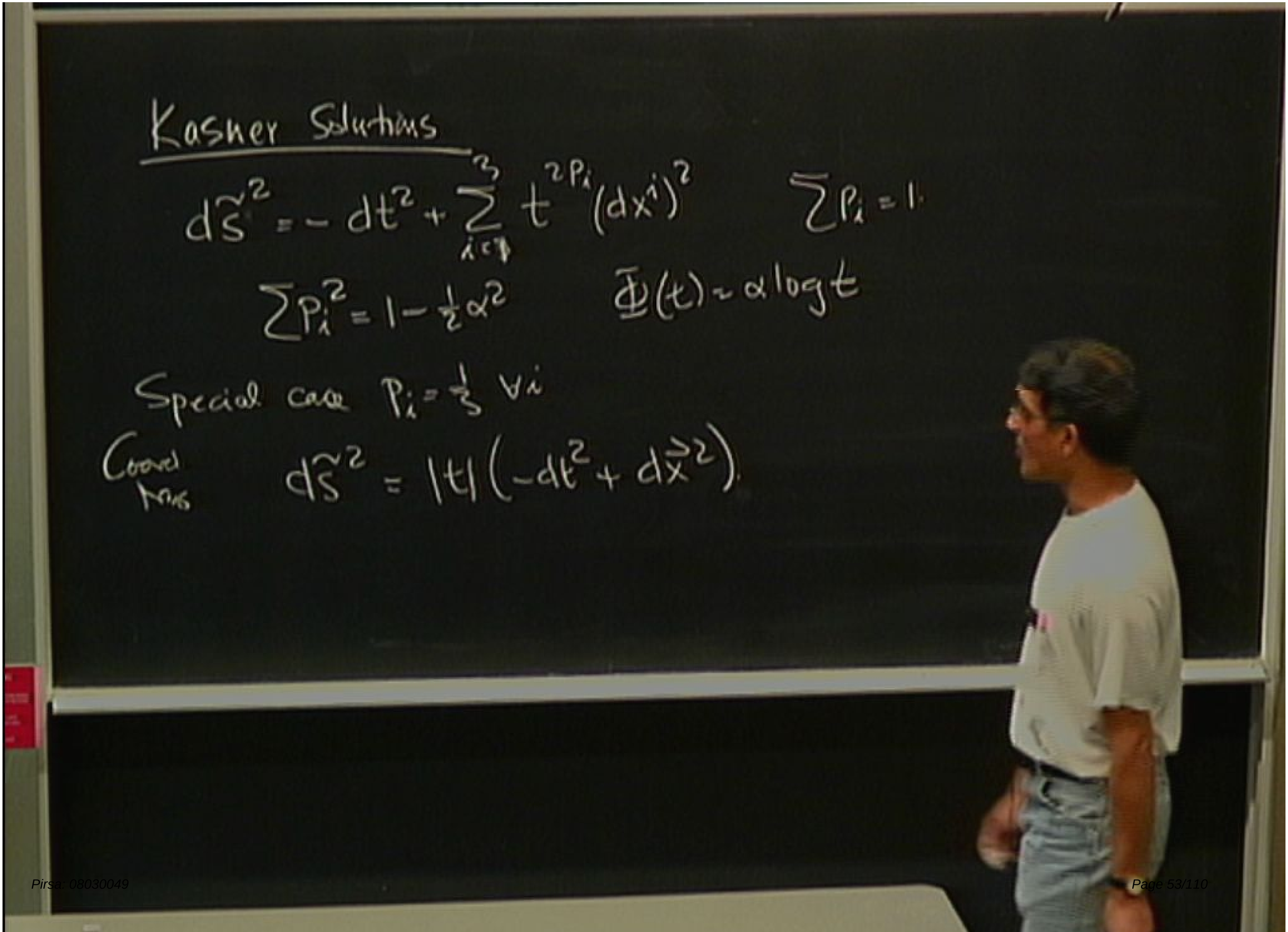
$$\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\,\partial_\nu\Phi$$

$$\tilde{\nabla}^2\Phi = 0$$

Kasner Solutions

$$d\tilde{s}^2 = -dt^2 + \sum_{i=1}^{3} t^{2p_i} (dx^i)^2 \qquad \sum p_i = 1$$

$$\sum p_i^2 = 1 - \tfrac{1}{2}\alpha^2 \qquad \Phi(t) = \alpha \log t$$


Kasner Solutions
$d\tilde{s}^2 = -dt^2 + \sum_{i=1}^{3} t^{2p_i}(dx^i)^2$
$\sum p_i = 1$
$\sum p_i^2 = 1 - \frac{1}{2}\alpha^2$
$\Phi(t) = \alpha \log t$
Special case $p_i = \frac{1}{3}$ $\forall i$
$d\tilde{s}^2 = |t|(-dt^2 + d\vec{x}^2)$

Kasner Solutions

$$d\tilde{s}^2 = -dt^2 + \sum_{i=1}^{3} t^{2p_i}(dx^i)^2 \qquad \sum p_i = 1$$

$$\sum p_i^2 = 1 - \tfrac{1}{2}\alpha^2 \qquad \Phi(t) = \alpha \log t$$

Special case $p_i = \frac{1}{3}\ \forall i$

Conf. trns

$$d\tilde{s}^2 = |t|\left(-dt^2 + d\vec{x}^2\right) \leftarrow$$

e^{ϕ} is NOT BOUNDED.

$$ds^2 = \frac{1}{\rho^2}\left[d\rho^2 - \frac{(16T^2 - 5\rho^2)^2}{256T^4} dT^2 \right.$$

$$+$$

$$ds^2 = \frac{1}{\rho^2}\left[d\rho^2 - \frac{(16T^2 - 5\rho^2)^2}{256T^4}\,dT^2 + \frac{(16T^2-\rho^2)^{4/3}(16T^2+5\rho^2)^{7/6}}{256T^4}\,d\vec{x}^{\,2}\right]$$

$$ds^2 = \frac{1}{\rho^2}\left[d\rho^2 - \frac{(16T^2 - 5\rho^2)^2}{256T^4}\, dT^2 + \frac{(16T^2-\rho^2)^{4/3}(16T^2+5\rho^2)^{7/6}}{256T^4}\, d\vec{x}^{\,2} \right]$$

$$e^{\Phi} = T^{\sqrt{3}} \left(\frac{16T^2+5\rho^2}{16T^2-\rho^2} \right).$$

Boundary at $\rho = 0$ is flat.

FRW Solutions:

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$$ds^2 = \frac{1}{z^2}\left[dz^2 + F(\eta,R)\left[-d\eta^2 + dR^2 + R^2 d\Omega_2^2\right]\right]$$

$$\Phi = \Phi(\eta, R).$$

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$$ds^2 = \frac{1}{z^2}\left[dz^2 + F(\eta, R)\left[-d\eta^2 + dR^2 + R^2 d\Omega_2^2\right]\right]$$

$$\Phi = \Phi(\eta, R).$$

Functions of $(\eta^2 - R^2)$

$$\eta = \tau \cosh \xi$$

$$R = \tau \sinh \xi.$$

$$\boxed{F(\tau) = \left|1 - \frac{1}{\tau^4}\right|} \qquad \boxed{e^{\Phi(\tau)} = \left(\frac{\tau^2 - 1}{\tau^2 + 1}\right)^{\sqrt{3}}}$$

FRW Solutions:

$$ds^2 = \frac{1}{z^2}\left[dz^2 + F(\eta,R)\left[-d\eta^2 + dR^2 + R^2 d\Omega_2^2\right]\right]$$

$\Phi = \Phi(\eta,R)$.

Functions of $(\eta^2 - R^2)$

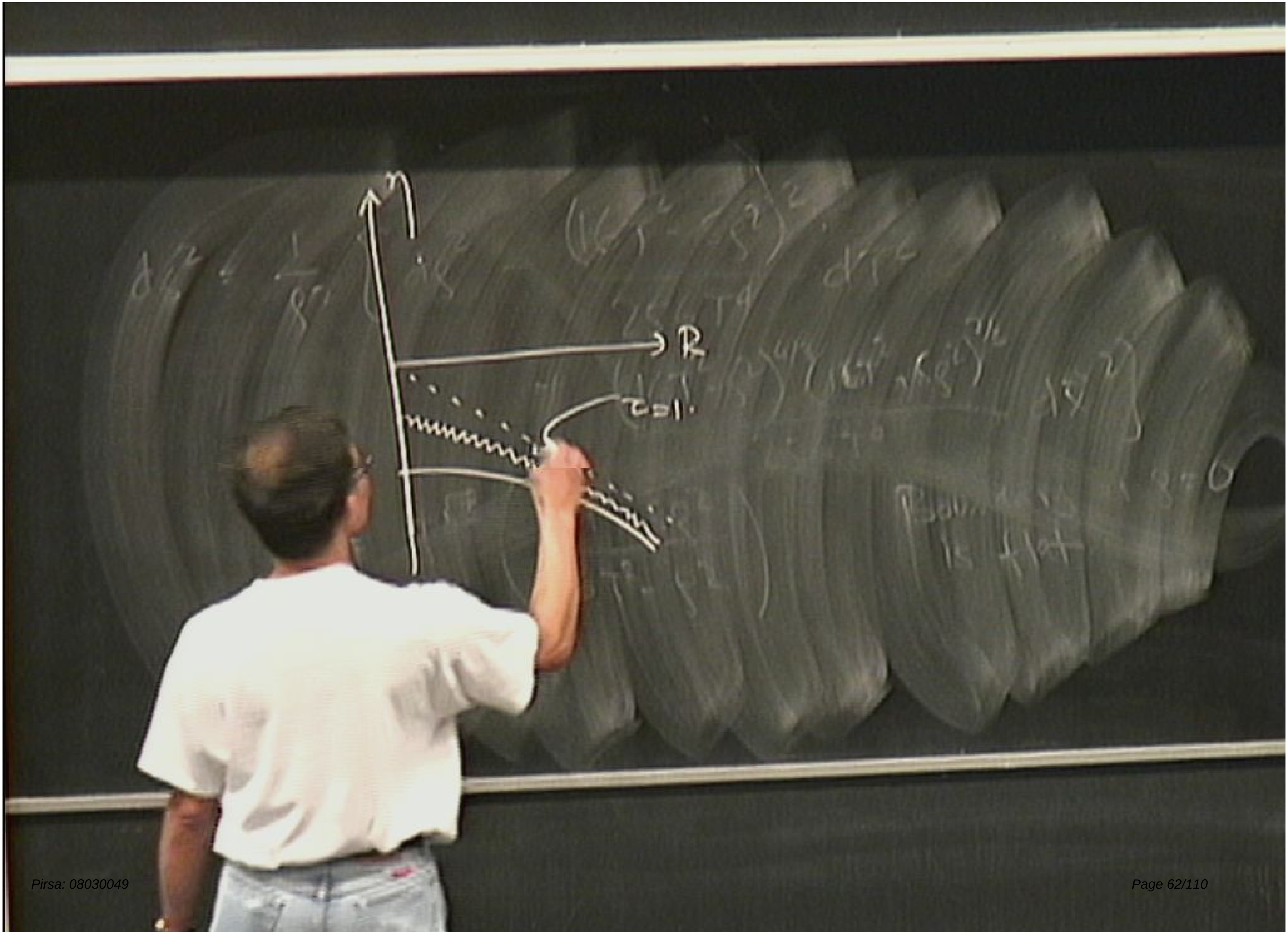
$\eta = \tau \cosh \xi$

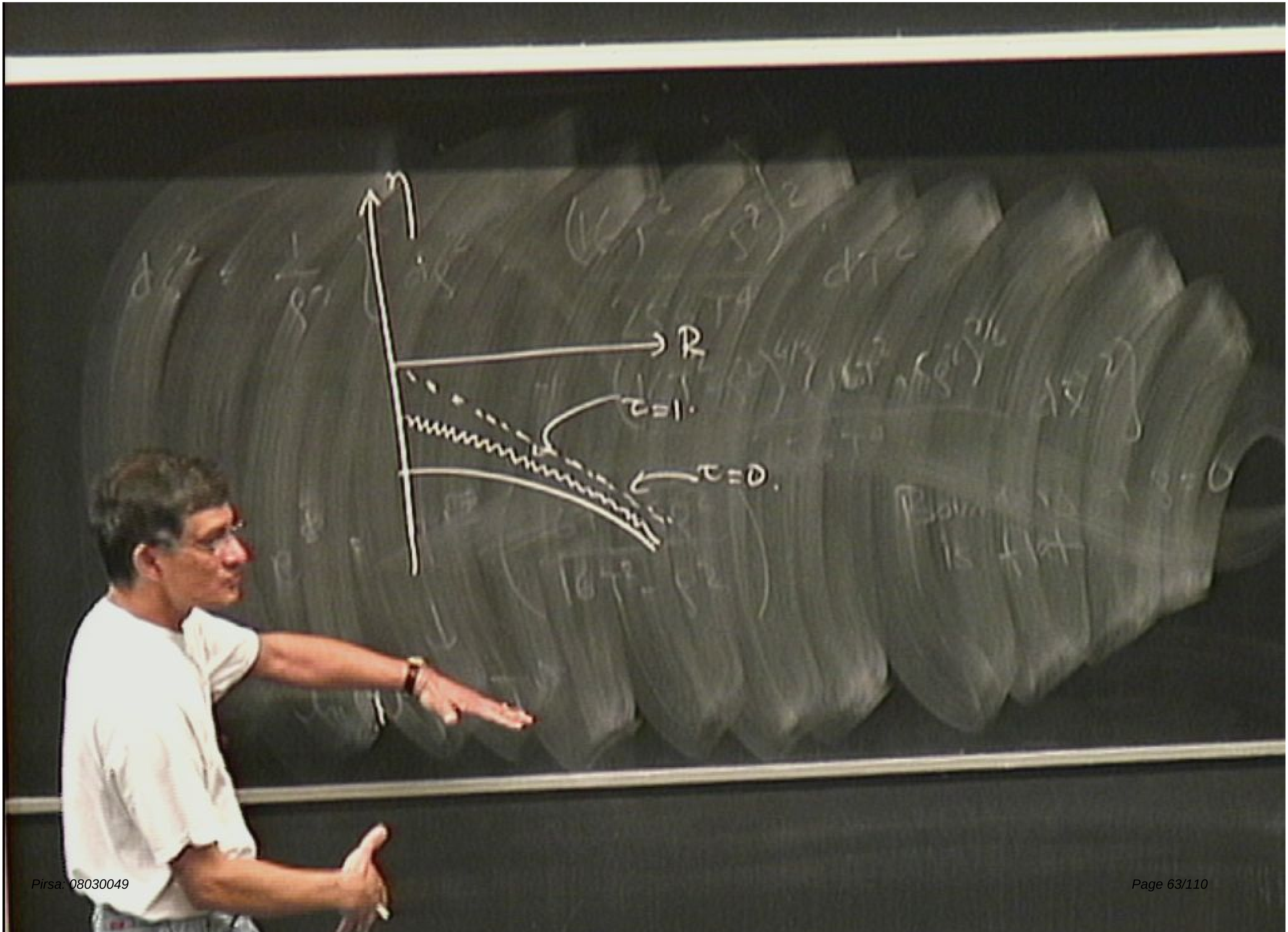
$R = \tau \sinh \xi$.

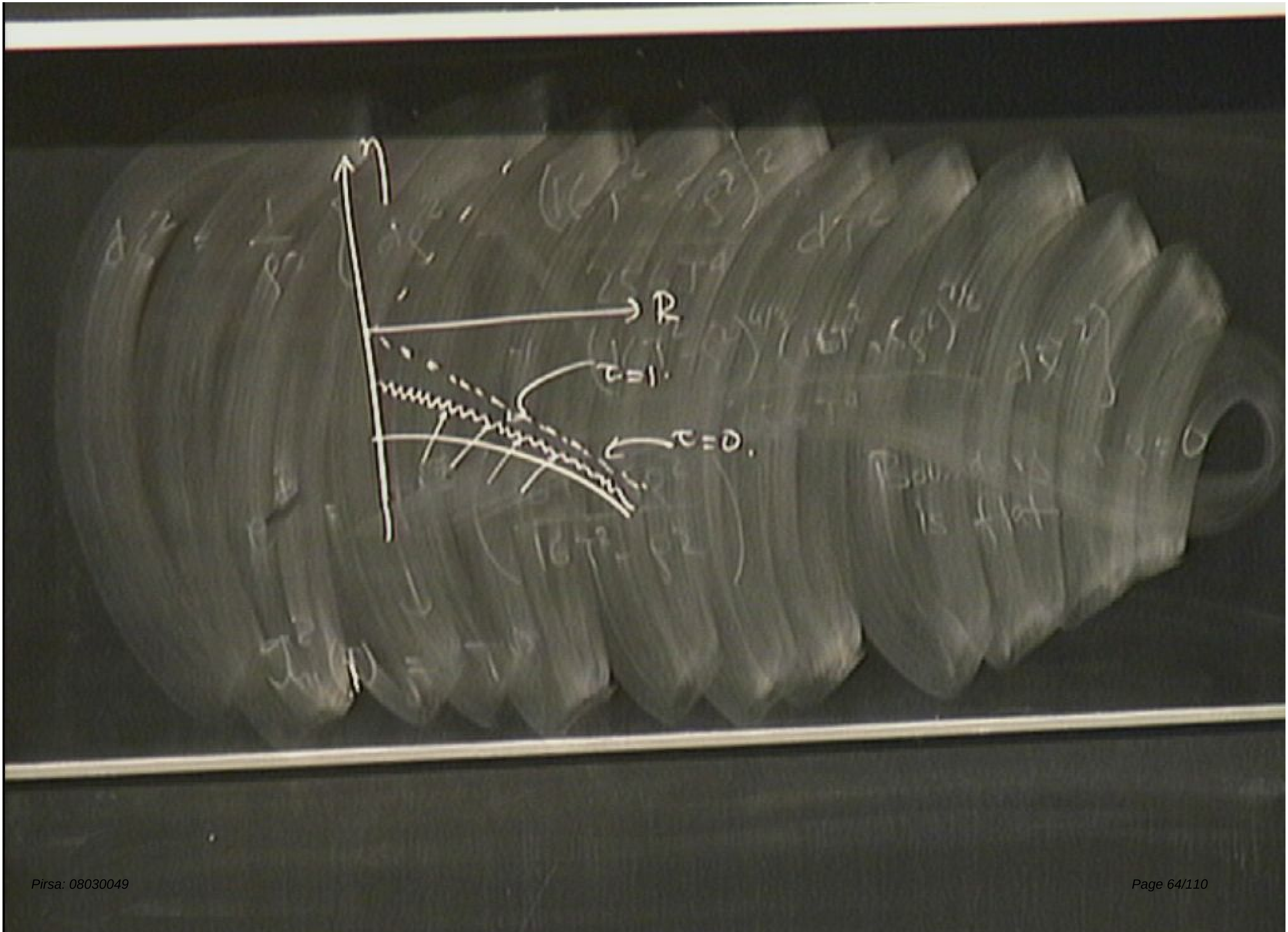
$\tau = 1$

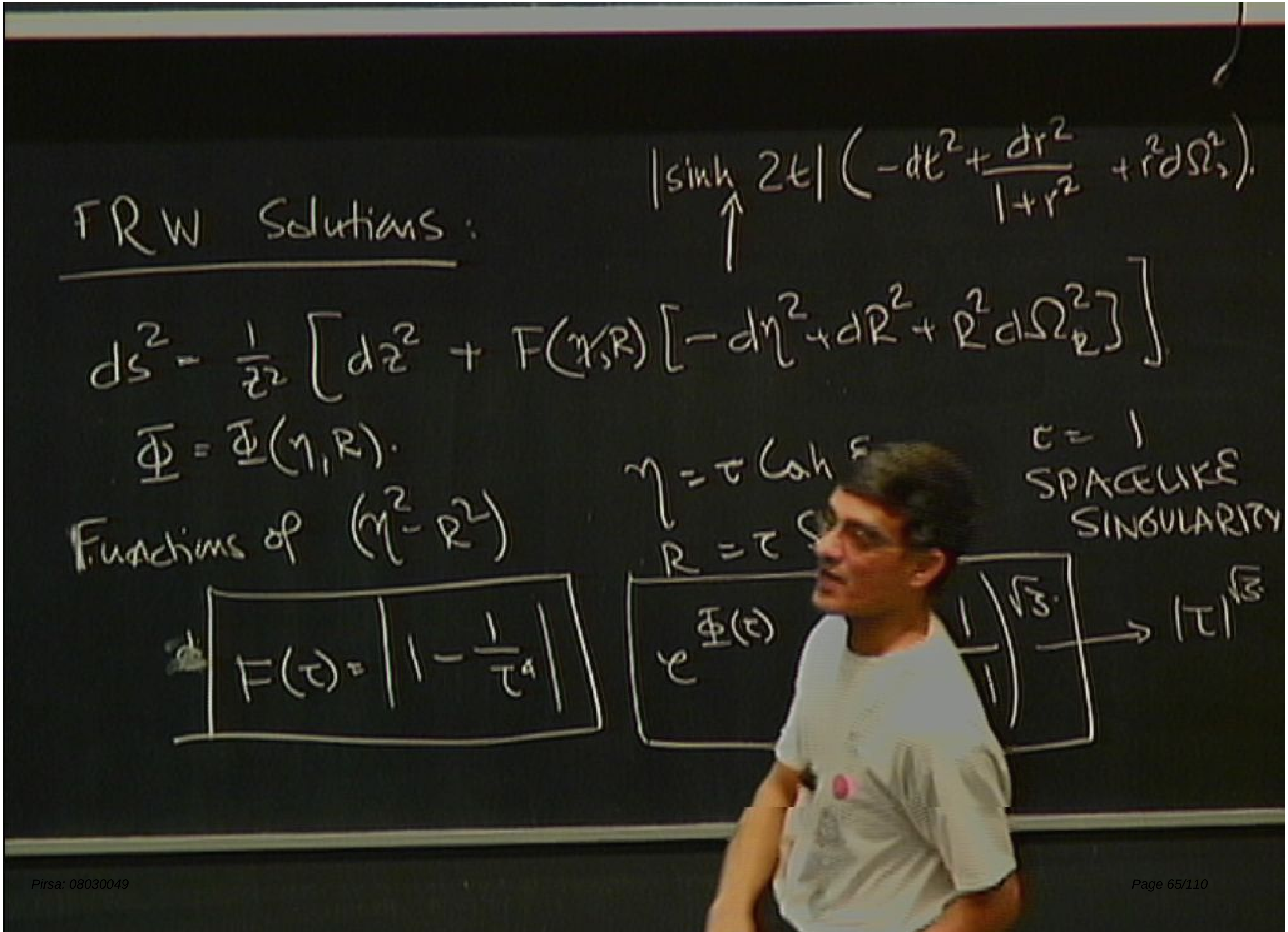
SPACELIKE SINGULARITY

$$\boxed{F(\tau) = \left|1 - \frac{1}{\tau^4}\right|} \qquad \boxed{e^{\Phi(\tau)} = \left(\frac{\tau^2 - 1}{\tau^2 + 1}\right)^{\sqrt{3}}}$$


R
τ=1


R
τ=1.
τ=0.


R
τ=1.
τ=0.



|sinh 2t| (-dt^2 + dr^2/(1+r^2) + r^2 dΩ_2^2).
FRW Solutions:
ds^2 = 1/z^2 [dz^2 + F(η,R) [-dη^2 + dR^2 + R^2 dΩ_2^2]]
Φ = Φ(η, R).
Functions of (η^2 - R^2)
η = τ Cosh
R = τ
τ = 1
SPACELIKE
SINGULARITY
F(τ) = |1 - 1/τ^4|
e^Φ(τ)
√3.
→ |τ|^√3

$$|\sinh 2t|\left(-dt^2+\frac{dr^2}{1+r^2}+r^2 d\Omega_3^2\right).$$

FRW Solutions:

$$ds^2 = \frac{1}{z^2}\left[dz^2 + F(\eta, R)\left[-d\eta^2 + dR^2 + R^2 d\Omega_2^2\right]\right]$$

$$\Phi = \Phi(\eta, R).$$

Functions of $(\eta^2 - R^2)$

$$\eta = \tau \cosh \xi$$

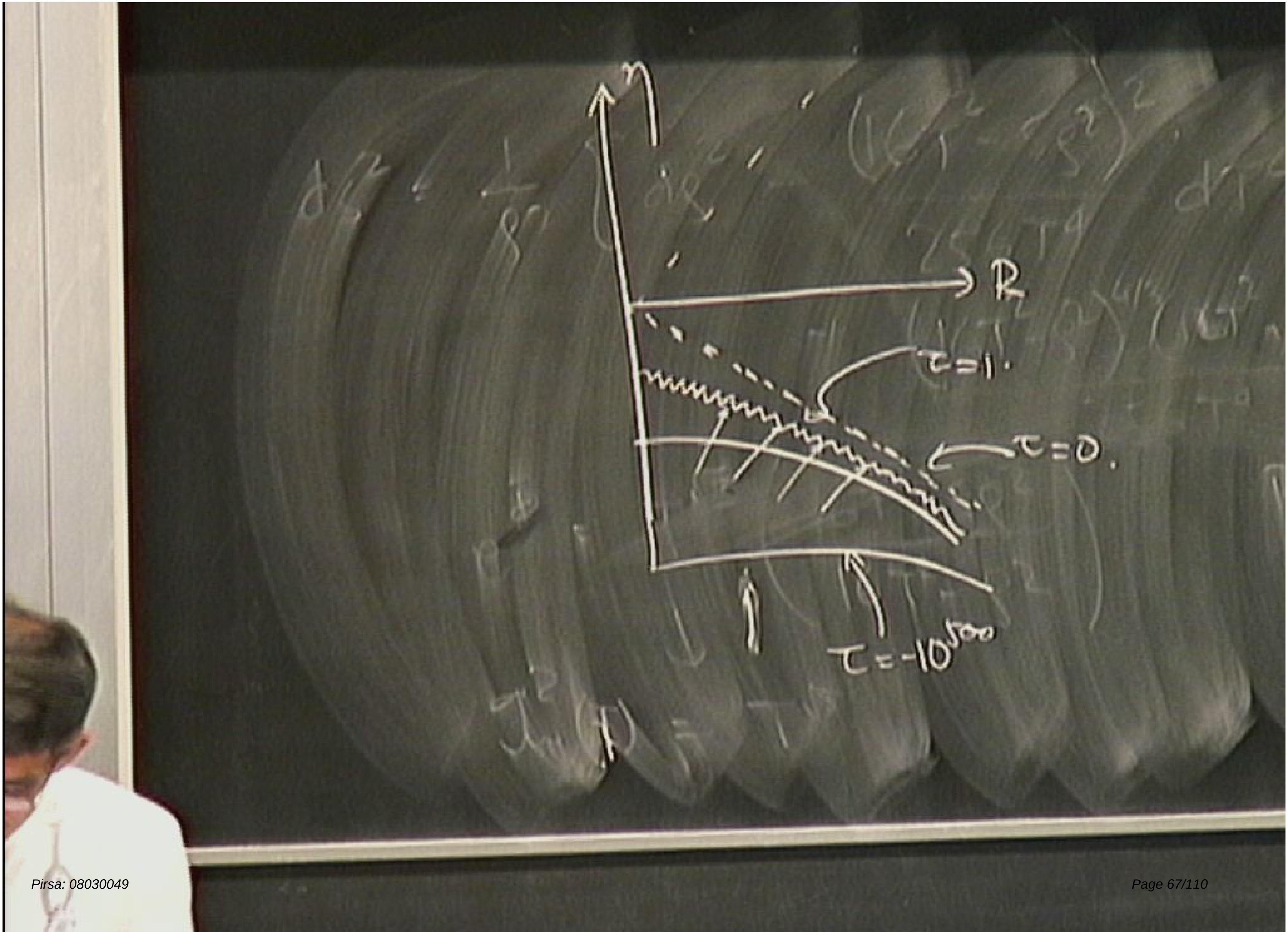
$$R = \tau \sinh \xi.$$

$\tau = 1$

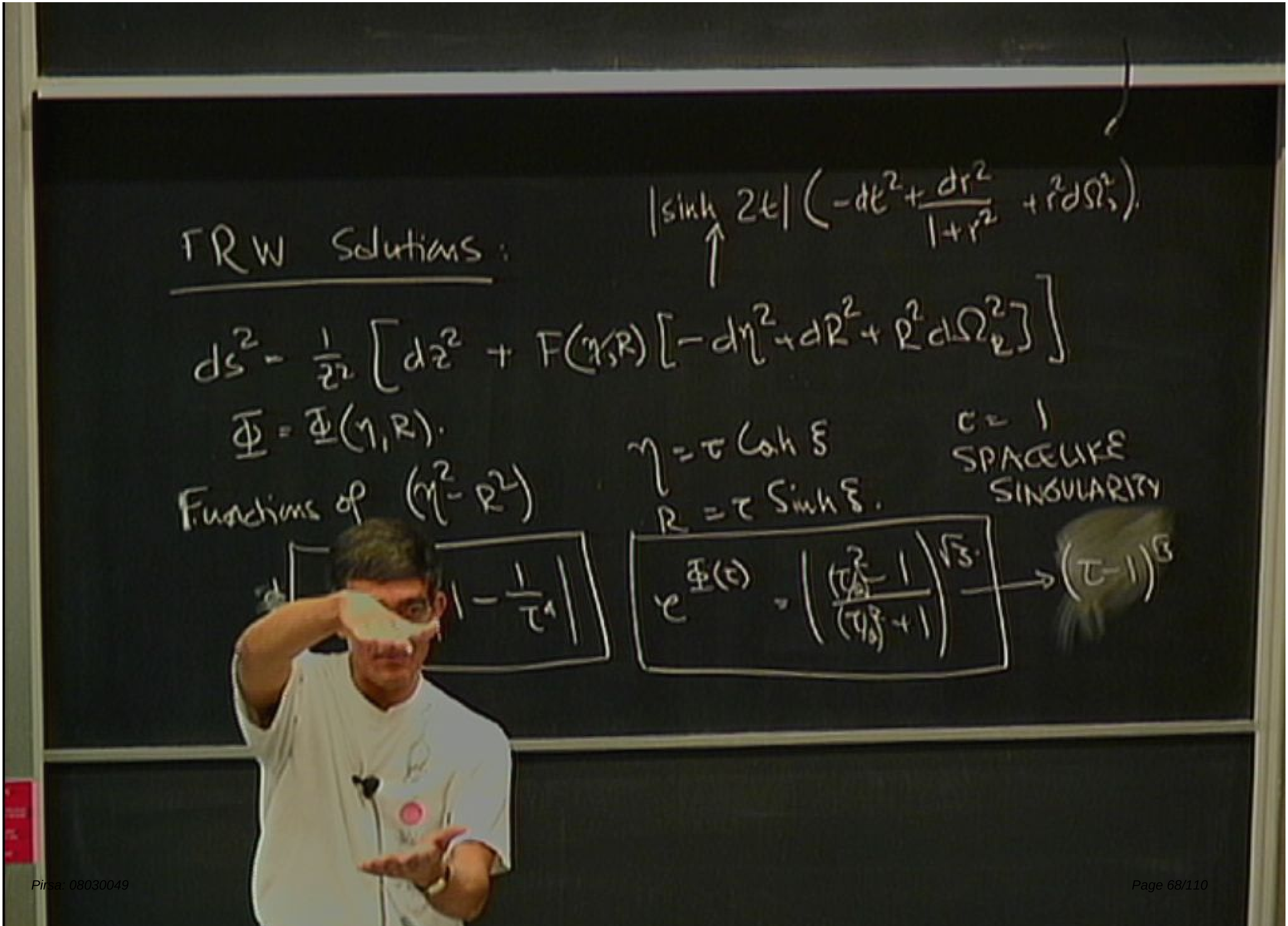
SPACELIKE SINGULARITY

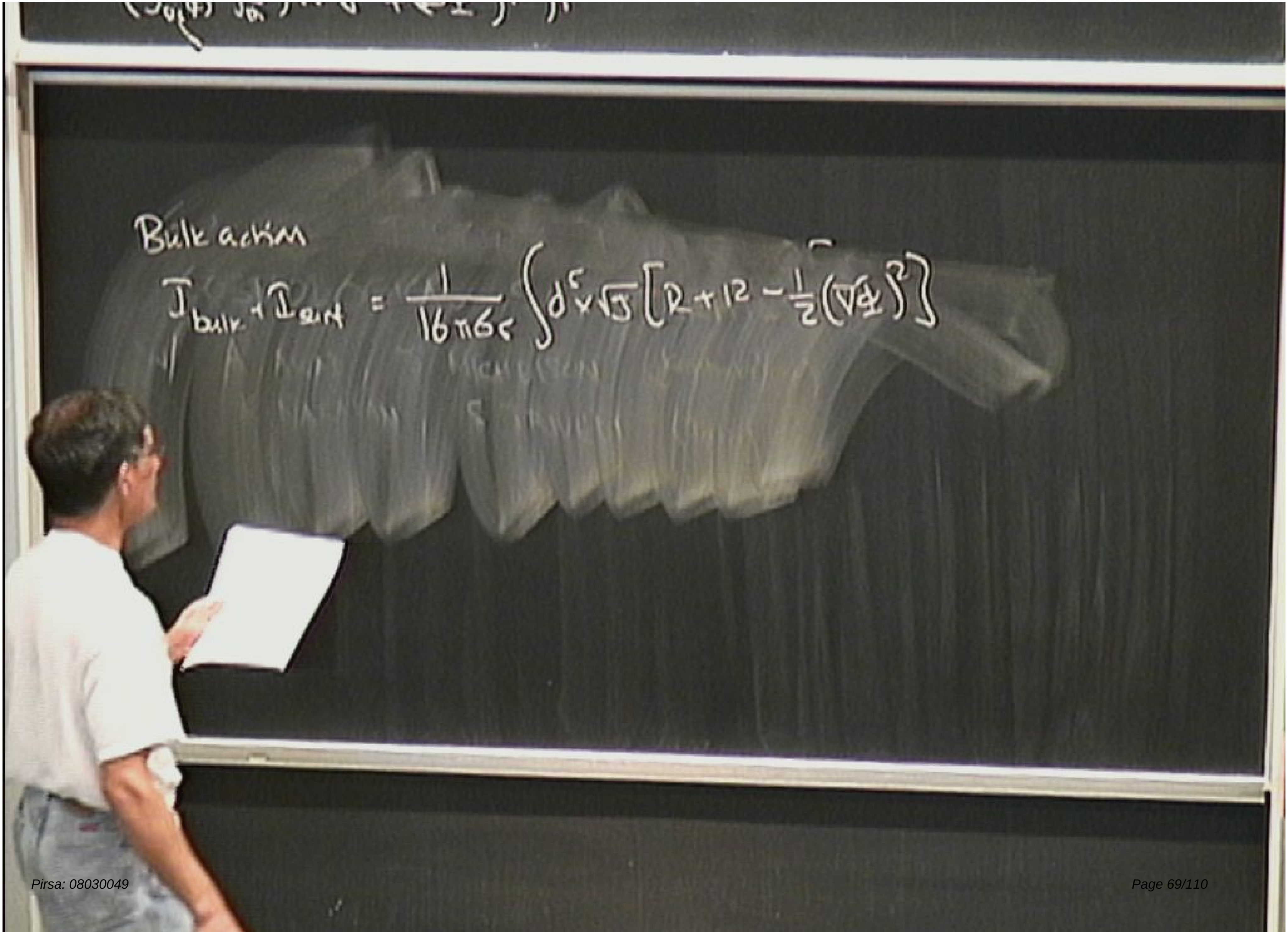
$$F(\tau) = \left|1 - \frac{1}{\tau^4}\right|$$

$$e^{\Phi(\tau)} = \left(\frac{\tau^2 - 1}{\tau^2 + 1}\right)^{\sqrt{3}} \to (\tau - 1)^{\sqrt{3}}$$



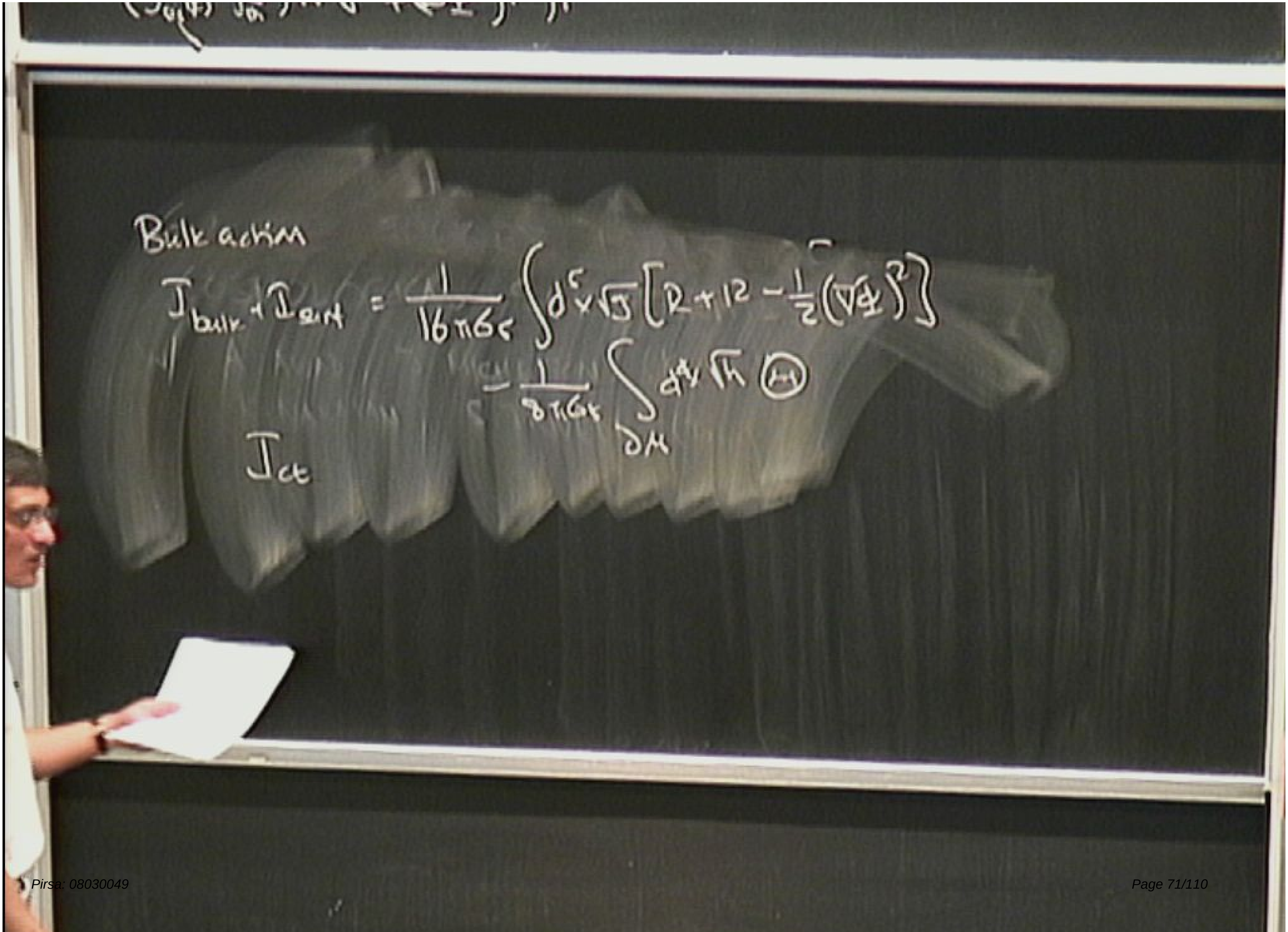
R
τ=1.
τ=0.
τ=-10^500


|sinh 2t| (-dt² + dr²/(1+r²) + r²dΩ₃²).
FRW Solutions:
ds² = 1/z² [dz² + F(η,R) [-dη² + dR² + R²dΩ₂²]]
Φ = Φ(η,R).
Functions of (η² - R²)
η = τ Cosh ξ
R = τ Sinh ξ.
c = 1
SPACELIKE
SINGULARITY
1 - 1/τ⁴
e^Φ(τ) = (((τ/τ₀)² - 1) / ((τ/τ₀)² + 1))^√3 → (τ-1)^√3


Bulk action
$I_{bulk} + I_{sint} = \frac{1}{16\pi G_5}\int d^5x\sqrt{g}\left[R + 12 - \frac{1}{2}(\nabla\Phi)^2\right]$

Bulk action

$$I_{bulk} + I_{surf} = \frac{1}{16\pi G_5}\int d^5x \sqrt{g}\left[R + 12 - \frac{1}{2}(\nabla\Phi)^2\right] - \frac{1}{8\pi G_5}\int_{\partial M} d^4x \sqrt{h}\,\Theta$$


Bulk action
I_bulk + I_surf = 1/(16πG_5) ∫ d^5x √g [R + 12 − 1/2 (∇Φ)^2]
− 1/(8πG_5) ∫_∂M d^4x √h Θ
I_ct

Bulk action

$$I_{bulk} + I_{surf} = \frac{1}{16\pi G_5}\int d^5x\sqrt{g}\left[R + 12 - \frac{1}{2}(\nabla\Phi)^2\right] - \frac{1}{8\pi G_5}\int_{\partial M} d^4x\sqrt{h}\,\Theta$$

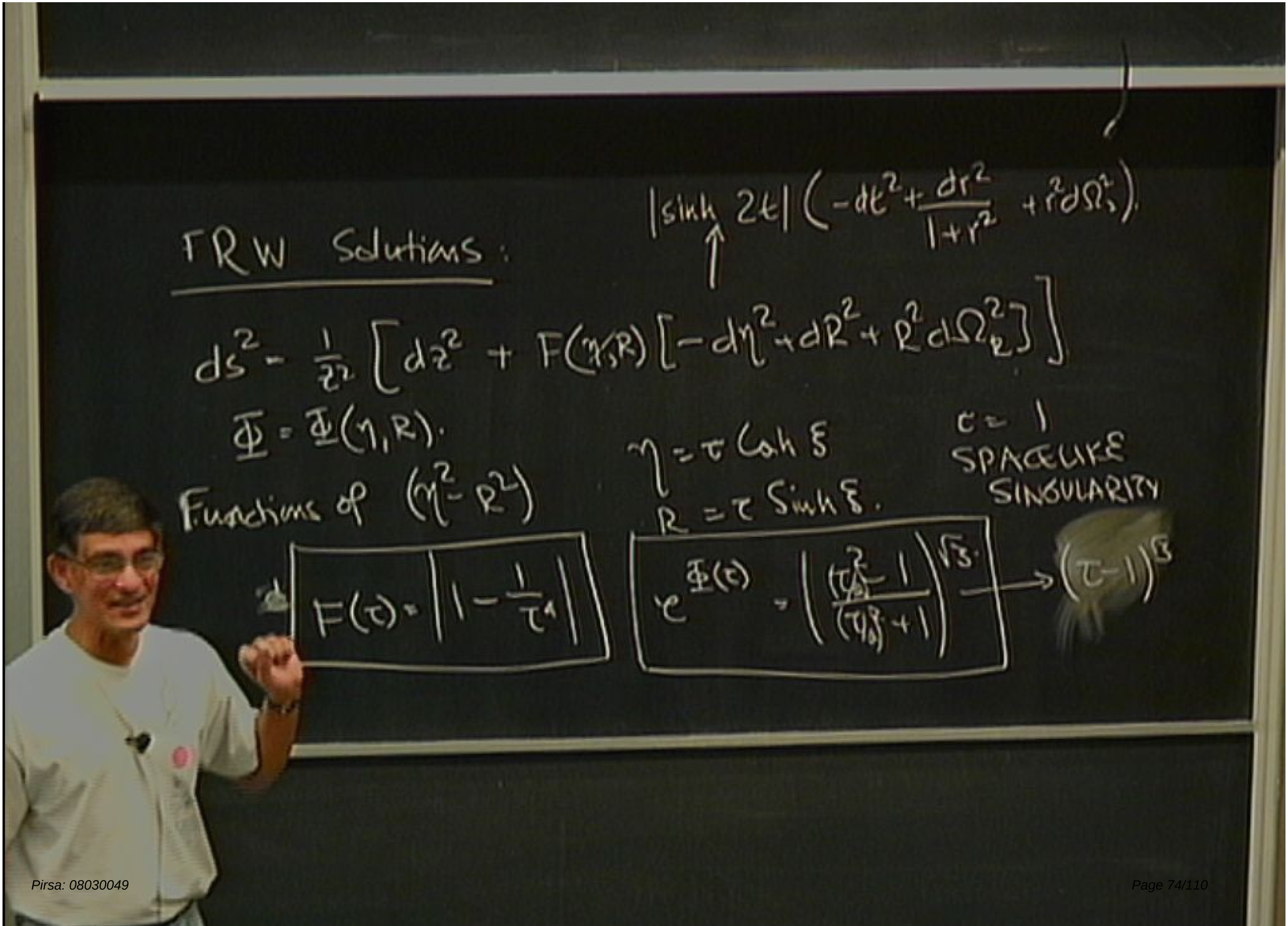
$$I_{ct} = -\frac{1}{8\pi G_5}\int d^4x\sqrt{h}\left(3 + \frac{R(h)}{4} - \frac{1}{8}(\nabla\Phi)^2 - \log \epsilon_0\, a_{(4)}\right]$$

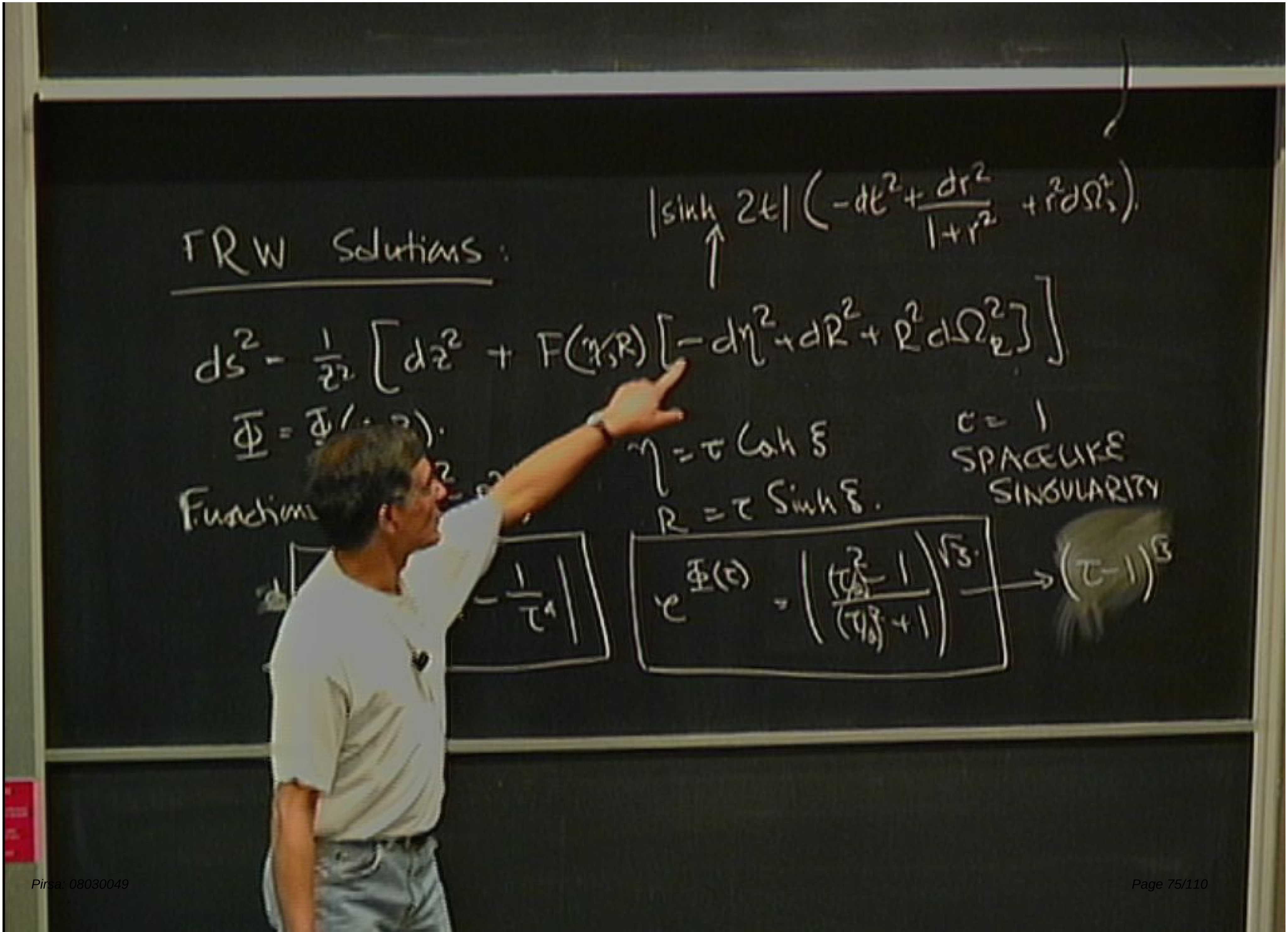
Bulk action

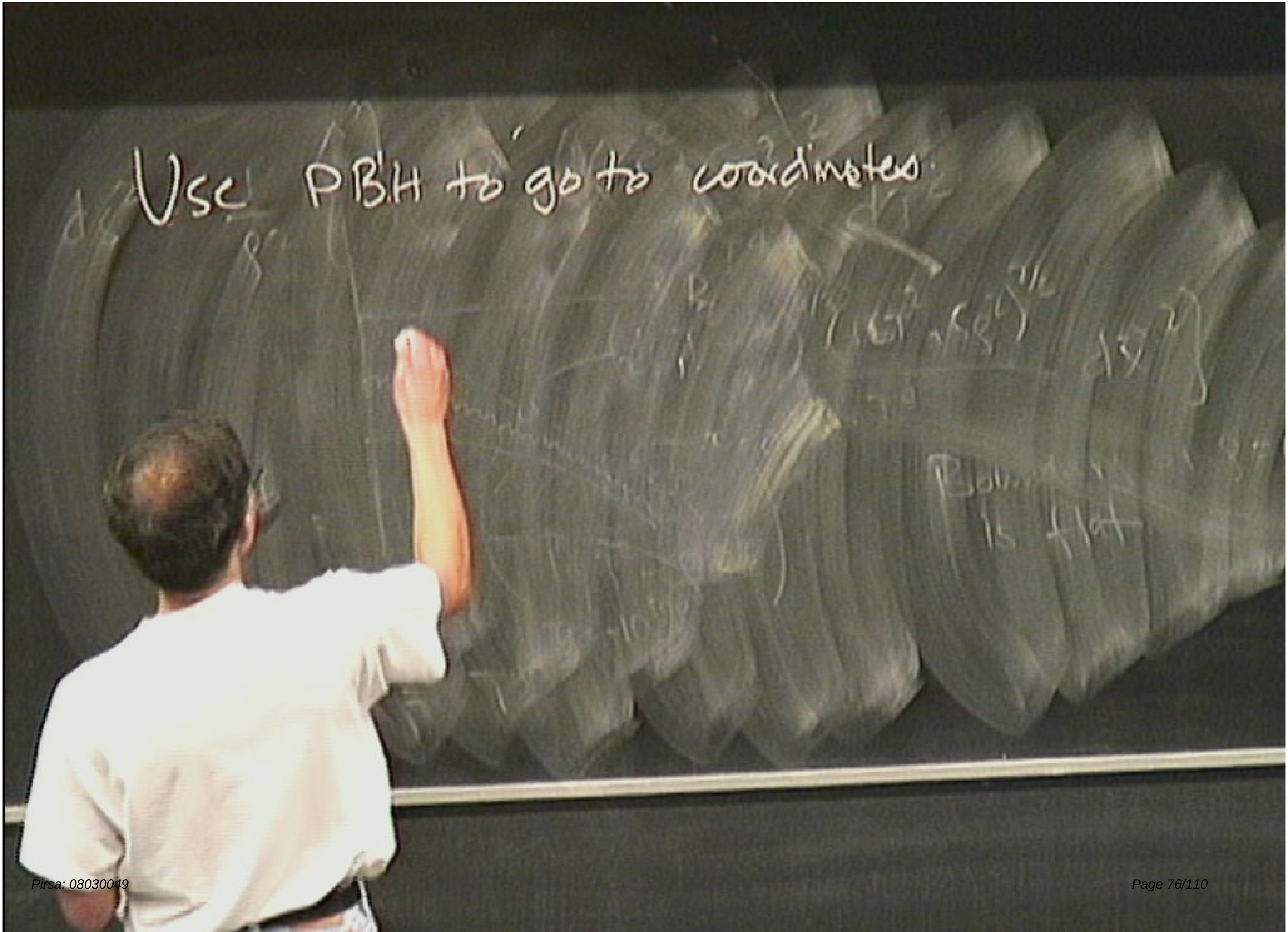
$$I_{bulk} + I_{surf} = \frac{1}{16\pi G_5}\int d^5x\sqrt{g}\left[R + 12 - \frac{1}{2}(\nabla\Phi)^2\right]$$

$$- \frac{1}{8\pi G_5}\int_{\partial M} d^4x\sqrt{h}\,\Theta$$

$$I_{ct} = -\frac{1}{8\pi G_5}\int d^4x\sqrt{h}\left(3 + \frac{R(h)}{4} - \frac{1}{8}(\nabla\Phi)^2 - \log\rho_0\, a_{(4)}\right]$$


FRW Solutions:
|sinh 2t| (-dt² + dr²/(1+r²) + r² dΩ₃²).
ds² = 1/z² [dz² + F(η,R) [-dη² + dR² + R² dΩ₂²]]
Φ = Φ(η,R).
Functions of (η² - R²)
η = τ cosh ξ
R = τ sinh ξ.
τ = 1
SPACELIKE
SINGULARITY
F(τ) = |1 - 1/τ⁴|
e^Φ(τ) = (((τ/τ₀)² - 1)/((τ/τ₀)² + 1))^√3 → (τ-1)^√3


FRW Solutions:
|sinh 2t| (−dt² + dr²/(1+r²) + r²dΩ²)
ds² = 1/z² [dz² + F(η,R) [−dη² + dR² + R²dΩ²]]
η = τ Cosh ξ
R = τ Sinh ξ.
τ = 1
SPACELIKE
SINGULARITY
(τ−1)


Use PBH to go to coordinates

Use PBH to go to coordinates

$$(z,\tau) \to (\rho, T)$$

$$ds^2 = \frac{1}{\rho^2}\Big[d\rho^2\left(1+O(\rho^4)\right) - \left(1+O(\rho^2)\right) dT^2 + \left(1+O(\rho^2)\right)\left(\tau^2 d\xi^2 + \tau^2 \sinh^2\xi \, d\Omega_2^2\right)\Big]$$

Bulk action

$$I_{bulk} + I_{surf} = \frac{1}{16\pi G_5}\int d^5x \sqrt{g}\left[R + 12 - \frac{1}{2}(\nabla\Phi)^2\right]$$

$$- \frac{1}{8\pi G_5}\int_{\partial M} d^4x \sqrt{h}\, \Theta$$

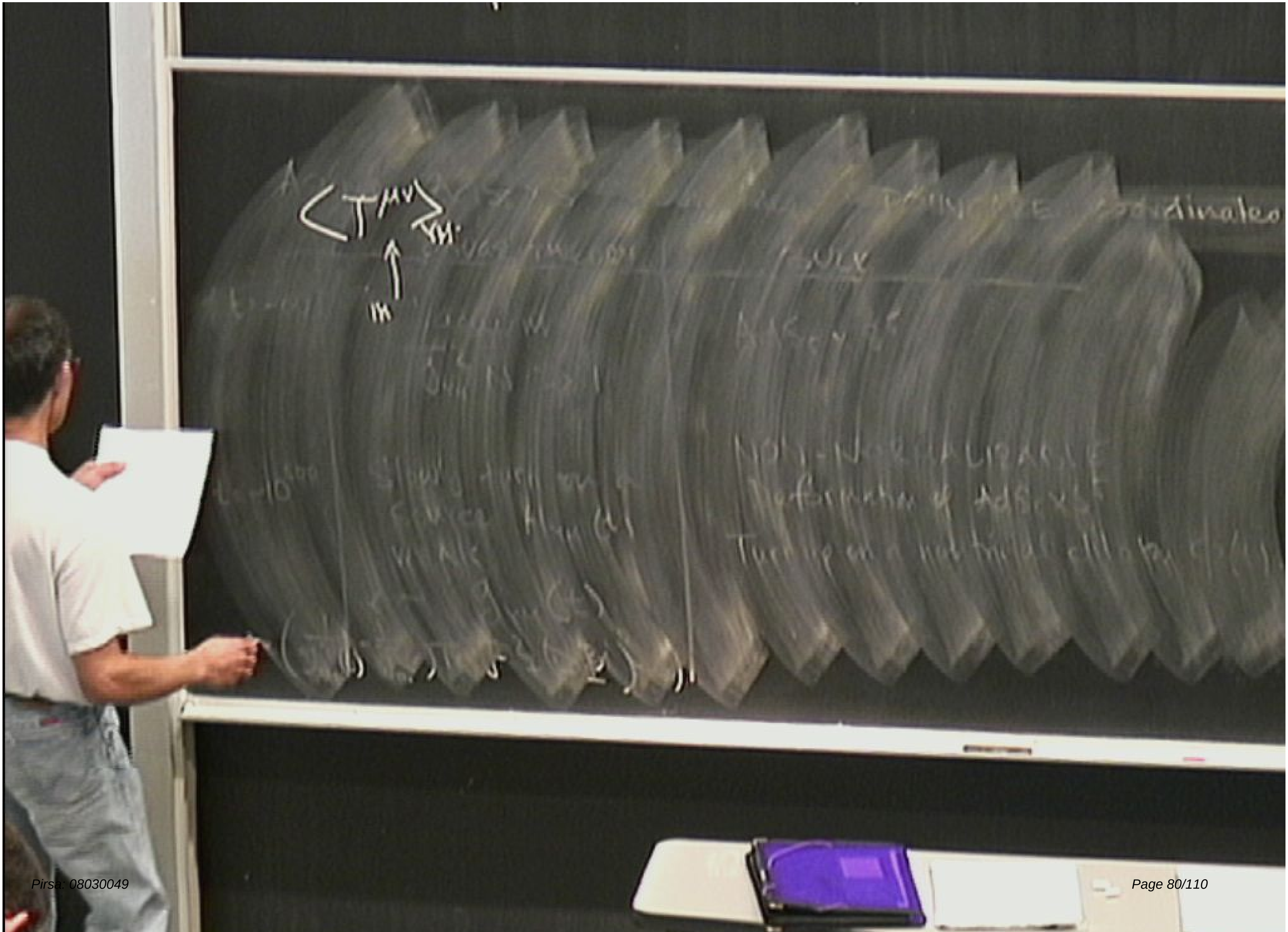
$$I_{ct} = -\frac{1}{8\pi G_5}\int d^4x \sqrt{h}\left(3 + \frac{R(h)}{4} - \frac{1}{8}(\nabla\Phi)^2 - \log \epsilon_0\, a_{(4)}\right]$$

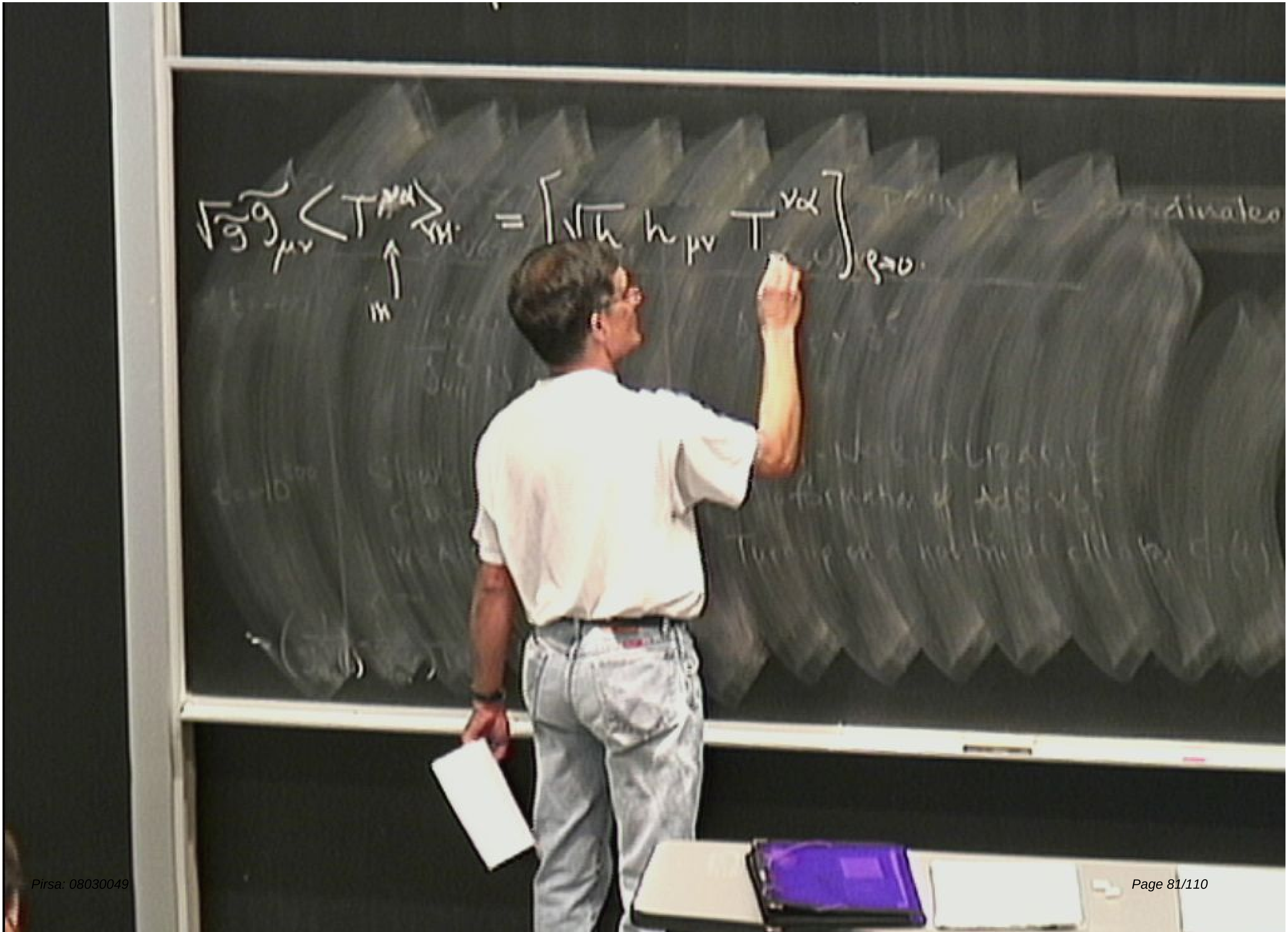
Bulk action

$$I_{bulk} + I_{surf} = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{g}\left[R + 12 - \frac{1}{2}(\nabla\Phi)^2\right] - \frac{1}{8\pi G_5}\int_{\partial M} d^4x\,\sqrt{h}\,\Theta$$

$$I_{ct} = -\frac{1}{8\pi G_5}\int d^4x\,\sqrt{h}\left(3 + \frac{R(h)}{4} - \frac{1}{8}(\nabla\Phi)^2 - \log S_0\, a_{(4)}\right]$$

$$T^{\mu\nu} = \frac{2}{\sqrt{h}}\frac{\delta I}{\delta h_{\mu\nu}} \qquad I = I_{bulk} + I_{surf} + I_{ct}$$




$\sqrt{g}\, g_{\mu\nu} \langle T^{\nu\alpha} \rangle_{VH} = \left[\sqrt{h}\, h_{\mu\nu}\, T^{\nu\alpha} \right]_{\rho\to 0}$

$$I_{ct} = -\frac{1}{8\pi G_5}\int_{\partial M} d^4x \sqrt{h}\left(3 + \frac{R(h)}{4} - \frac{1}{8}(\nabla\Phi)^2 - \log\rho_0\, a_{(4)} \ldots\right)$$

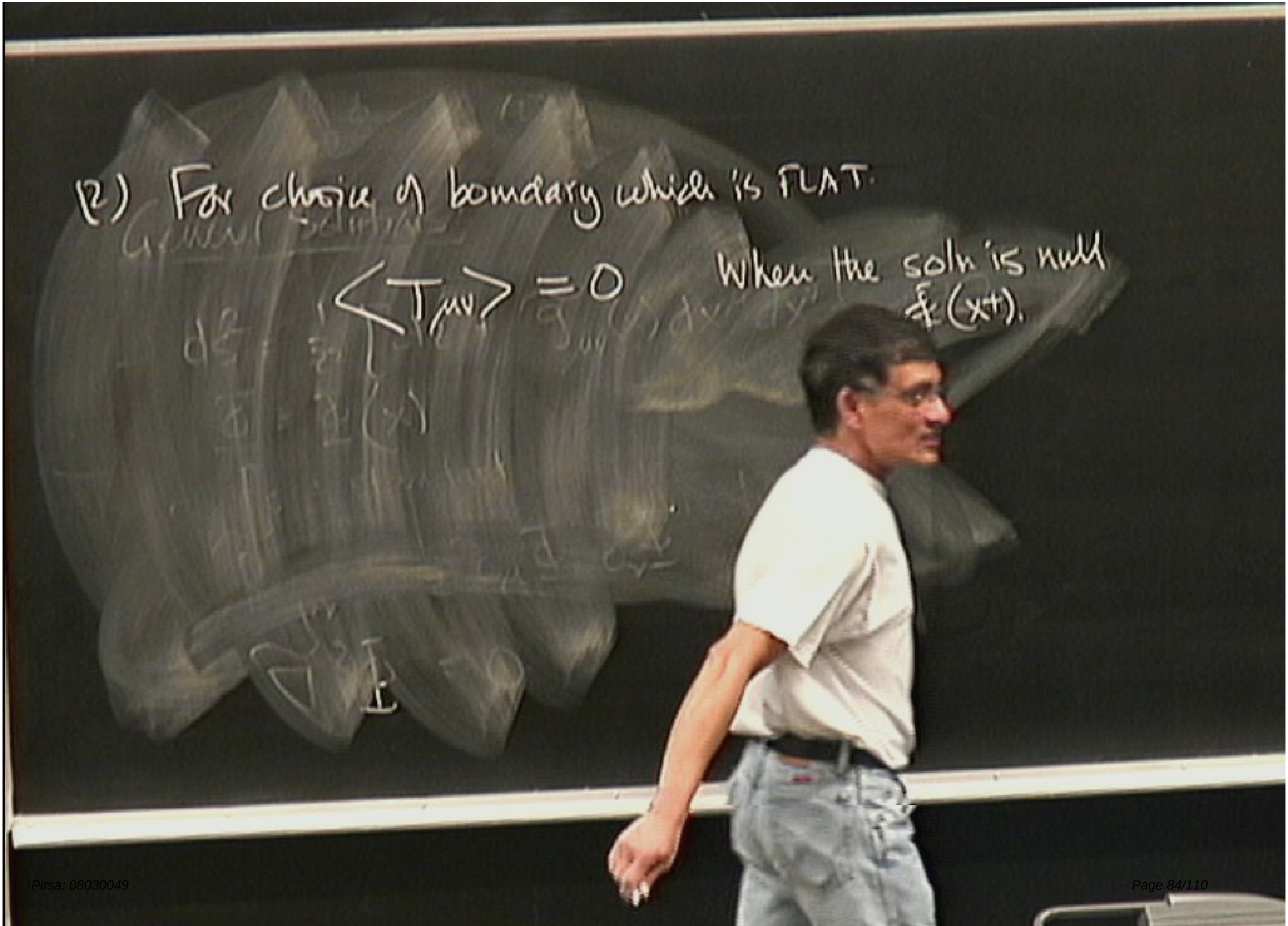
$$T^{\mu\nu}_{BY} = \frac{2}{\sqrt{h}}\frac{\delta I}{\delta h_{\mu\nu}} \qquad I = I_{bulk} + I_{surf} + I_{ct}$$

$$\sqrt{g}\, g_{\mu\nu}\langle T^{\mu\alpha}\rangle_{YM} = \left[\sqrt{h}\, h_{\mu\nu}\, T^{\nu\alpha}_{BY}\right]_{\rho\to 0}$$

$$\sqrt{g}\, g_{\mu\nu} \langle T^{\mu\alpha} \rangle_{YM} = \left[\sqrt{h}\, h_{\mu\nu}\, T^{\nu\alpha}_{BY} \right]_{\rho \to 0}$$

(1) Choice of boundary which is conformally flat

$$\langle T^{\mu\nu} \rangle_{YM} = 0 \quad \text{both for } \Phi(x,t),\ \Phi(t)$$


(2) For choice of boundary which is FLAT:
⟨T_μν⟩ = 0
When the soln is null
f(x+).

(2) For choice of boundary which is FLAT:

$$\langle T_{\mu\nu} \rangle = 0$$

When the soln is null $f(x^+)$.

(2) For choice of boundary which is FLAT.

$$\langle T_{\mu\nu} \rangle = 0$$

When the soln is null & (x+).

For time dep solns.

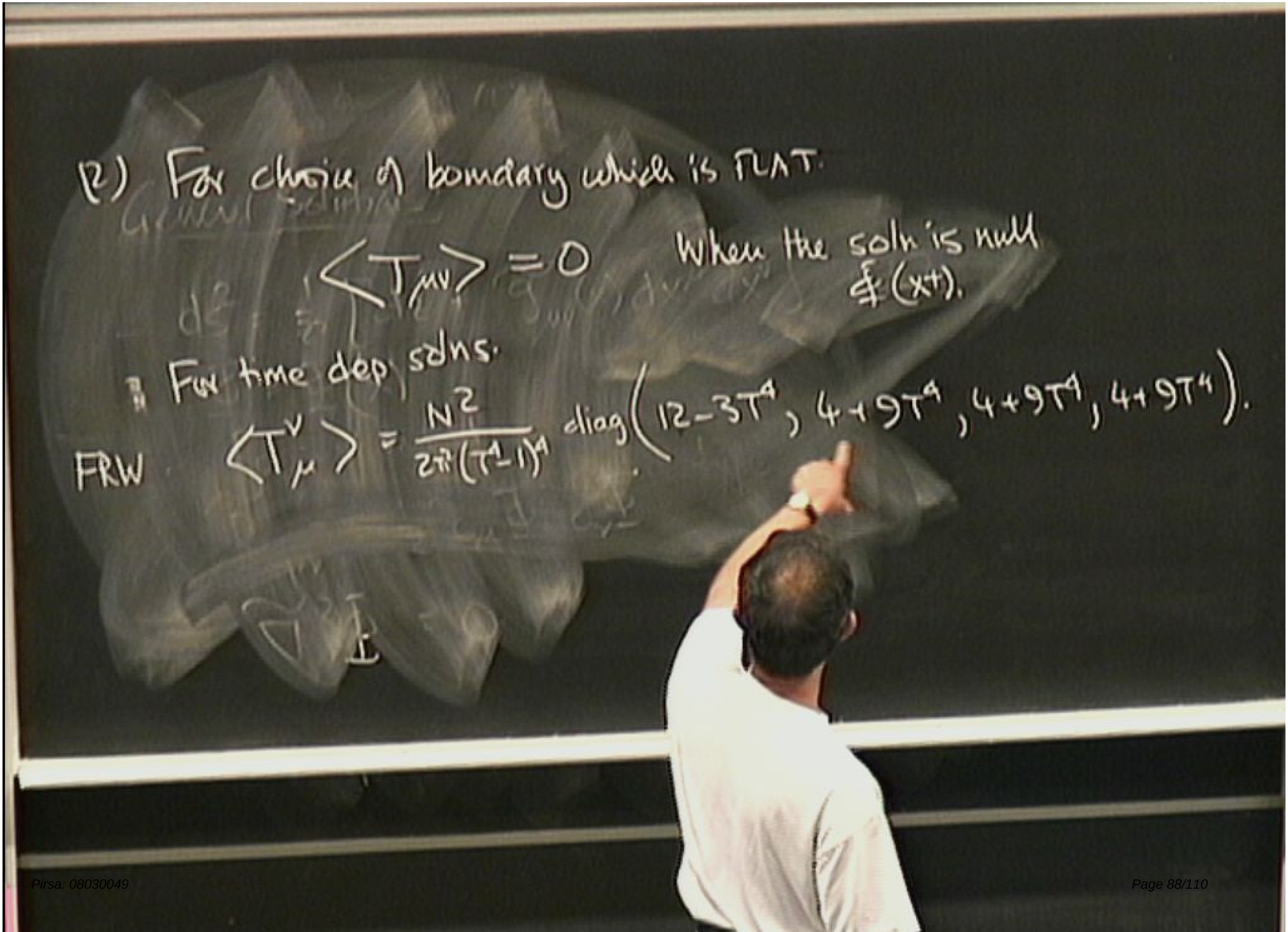
(2) For choice of boundary which is FLAT:

$$\langle T_{\mu\nu} \rangle = 0$$

When the soln is null $f(x^+)$.

For time dep solns:

FRW $\langle T^{\nu}_{\mu} \rangle = \frac{N^2}{2\pi^2(\tau^4-1)^4} \mathrm{diag}(12-3\tau^4, 4+9\tau^4, 4+9\tau^4, 4+9\tau^4$


(2) For choice of boundary which is FLAT.
$\langle T_{\mu\nu} \rangle = 0$
when the soln is null & f(x+).
For time dep solns.
FRW
$\langle T^{\nu}_{\mu} \rangle = \frac{N^2}{2\pi^2 (T^4-1)^4} \, \mathrm{diag}(12-3T^4, 4+9T^4, 4+9T^4, 4+9T^4)$.

(2) For choice of boundary which is FLAT.

$$\langle T_{\mu\nu} \rangle = 0$$ When the soln is null f(x+).

For time dep solns.

FRW $$\langle T^{\nu}_{\mu} \rangle = \frac{N^2}{2\pi^2 (T^4-1)^4} \, \mathrm{diag}\left(12-3T^4, 4+9T^4, 4+9T^4, 4+9T^4\right).$$

$T \to -\infty$ early times

$T \to 1$ singularity

$T \to 0$

(2) For choice of boundary which is FLAT.

$$\langle T_{\mu\nu} \rangle = 0$$ when the soln is null ∉ (x+).

For time dep. solns.

FRW $$\langle T^{\nu}_{\mu} \rangle = \frac{N^2}{2\pi^2 (T^2-1)^4} \, \mathrm{diag}\left(12-3T^4,\ 4+9T^4,\ 4+9T^4,\ 4+9T^4\right).$$

$T \to -\infty$ early times

$T \to 1$ singulary

$T \to 0$

$\mu, \nu = 0, 1, 2, 3$

$i, j = 1 \ldots 6$

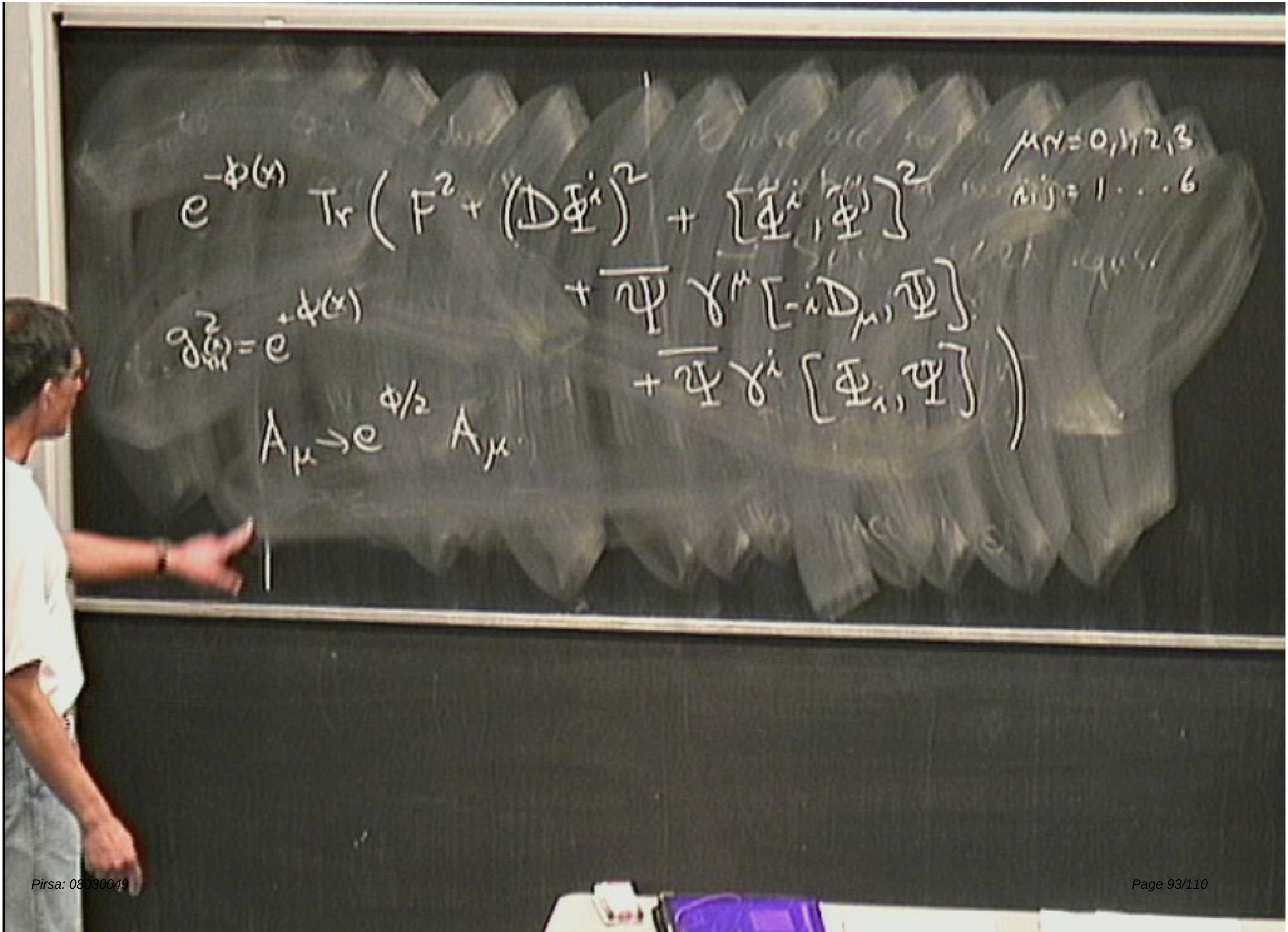
$$e^{-\phi(x)} \, \mathrm{Tr}\Big(F^2 + (D\Phi^i)^2 + [\Phi^i, \Phi^j]^2 + \bar{\Psi}\gamma^\mu [-iD_\mu, \Psi] + \bar{\Psi}\gamma^i [\Phi_i, \Psi] \Big)$$

$$e^{-\phi(x)}\, \mathrm{Tr}\Big(F^2 + (D\Phi^i)^2 + [\Phi^i, \Phi^j]^2 + \bar{\Psi}\gamma^\mu[-iD_\mu, \Psi] + \bar{\Psi}\gamma^i[\Phi_i, \Psi] \Big)$$

$\mu, \nu = 0, 1, 2, 3$

$i, j = 1 \ldots 6$

$g^2(x) = e^{+\phi(x)}$


$e^{-\phi(x)} \, \mathrm{Tr}\Big(F^2 + (D\Phi^i)^2 + [\Phi^i, \Phi^j]^2$
$+ \bar{\Psi} \gamma^\mu [-iD_\mu, \Psi]$
$+ \bar{\Psi} \gamma^i [\Phi_i, \Psi] \Big)$
$\mu, \nu = 0, 1, 2, 3$
$i, j = 1 \ldots 6$
$g^2_{YM}(x) = e^{+\phi(x)}$
$A_\mu \to e^{\phi/2} A_\mu$

$$e^{-\phi(x)}\,\mathrm{Tr}\Big(F^2 + (D\Phi^i)^2 + [\Phi^i,\Phi^j]^2 + \bar{\Psi}\gamma^\mu[-iD_\mu,\Psi] + \bar{\Psi}\gamma^i[\Phi_i,\Psi]\Big)$$

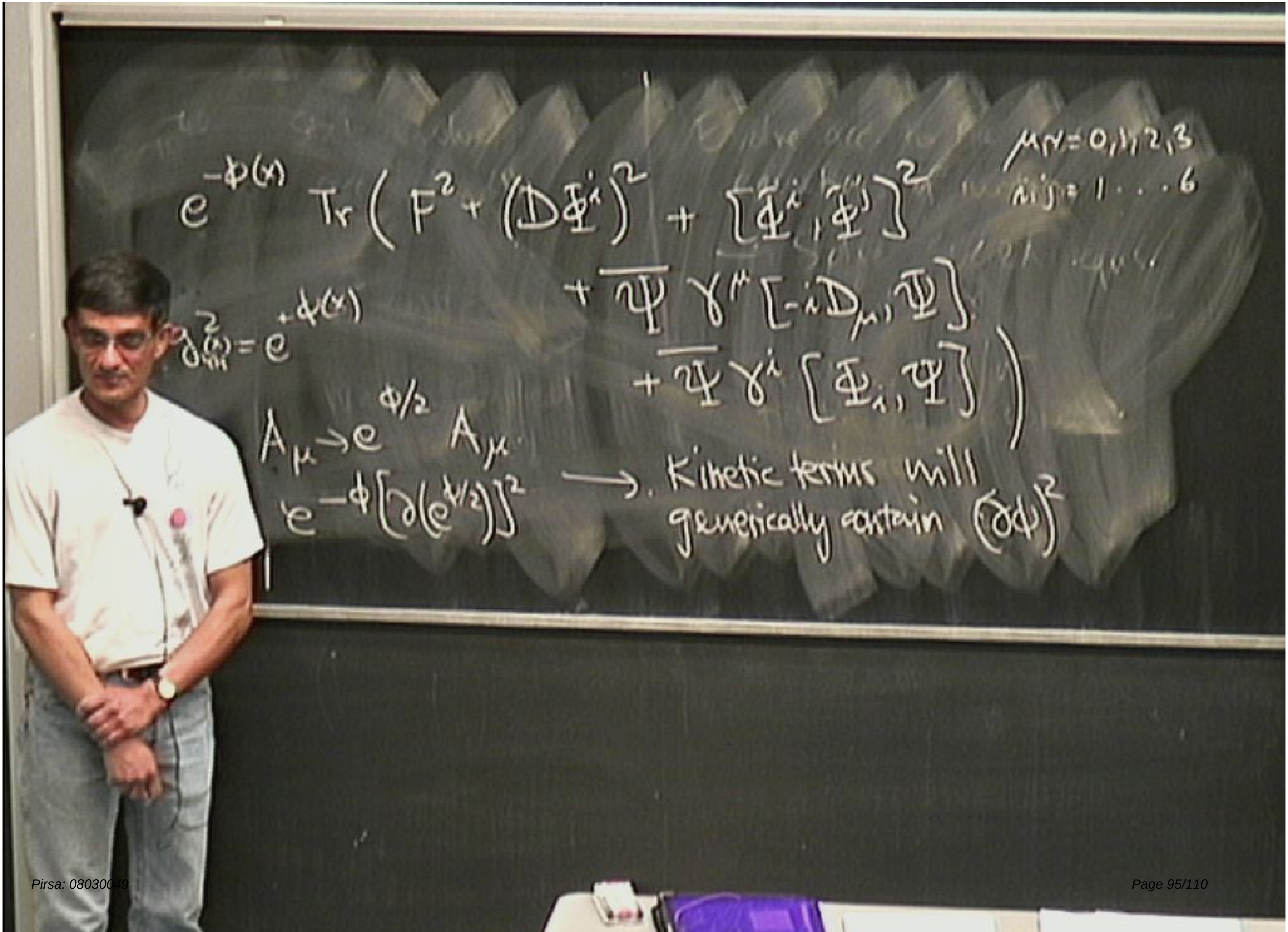
$\mu,\nu = 0,1,2,3$

$i,j = 1\ldots 6$

$$g^2_{YM}(x) = e^{+\phi(x)}$$

$$A_\mu \to e^{\phi/2} A_\mu$$

$$e^{-\phi}[\partial(e^{\phi/2})]^2$$


μ,ν = 0,1,2,3
i,j = 1 . . . 6
e^{-φ(x)} Tr(F^2 + (DΦ^i)^2 + [Φ^i, Φ^j]^2
+ Ψ̄ γ^μ [-iD_μ, Ψ]
+ Ψ̄ γ^i [Φ_i, Ψ])
g^2(x) = e^{+φ(x)}
A_μ → e^{φ/2} A_μ
e^{-φ}[∂(e^{φ/2})]^2 → Kinetic terms will
generically contain (∂φ)^2

$\mu, \nu = 0, 1, 2, 3$

$i, j = 1 \ldots 6$

$$e^{-\phi(x)} \, \mathrm{Tr}\Big(F^2 + (D\Phi^i)^2 + [\Phi^i, \Phi^j]^2 + \bar{\Psi} \gamma^\mu [-iD_\mu, \Psi] + \bar{\Psi} \gamma^i [\Phi_i, \Psi] \Big)$$

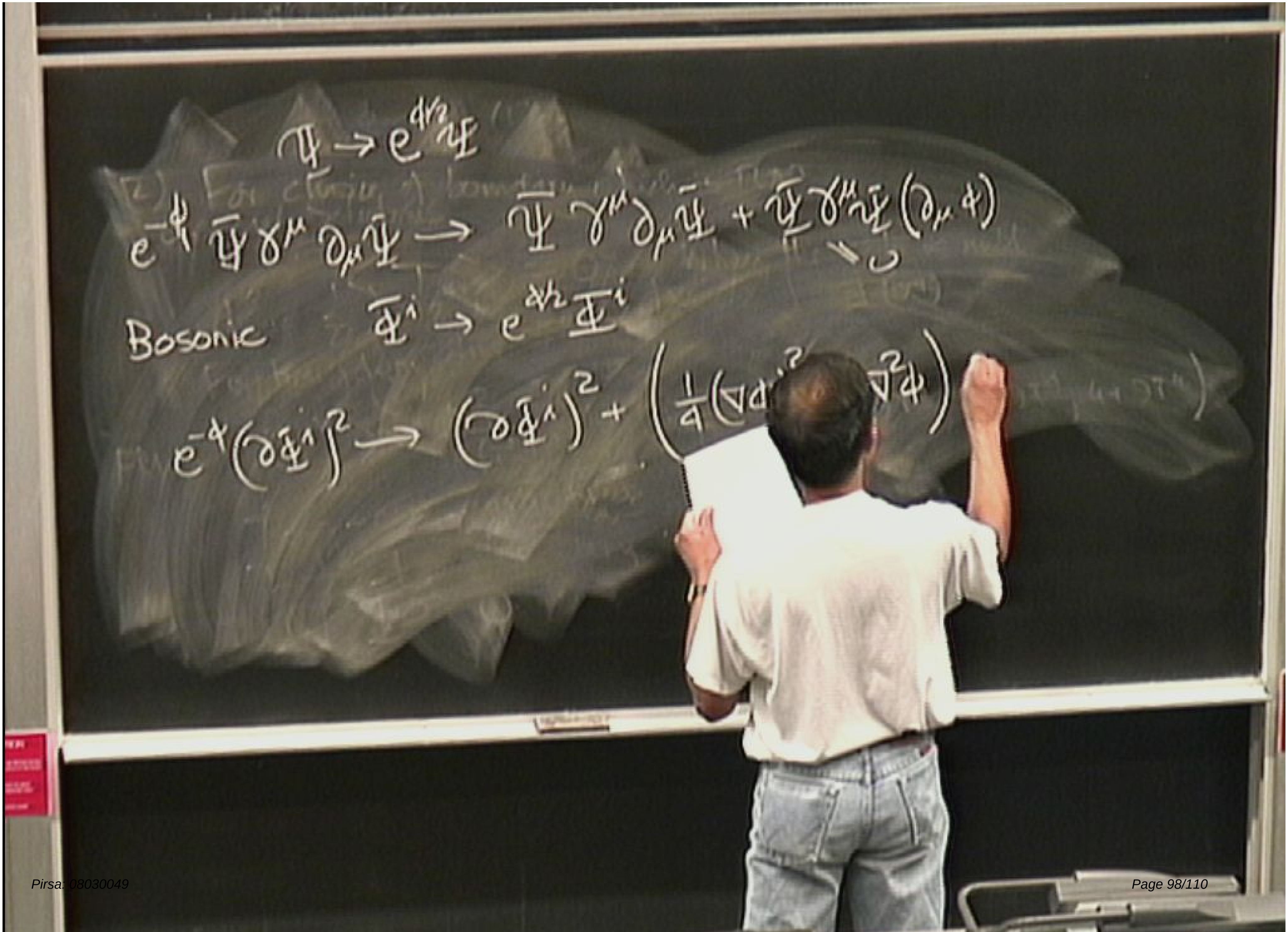
$$g^2_{YM}(x) = e^{+\phi(x)}$$

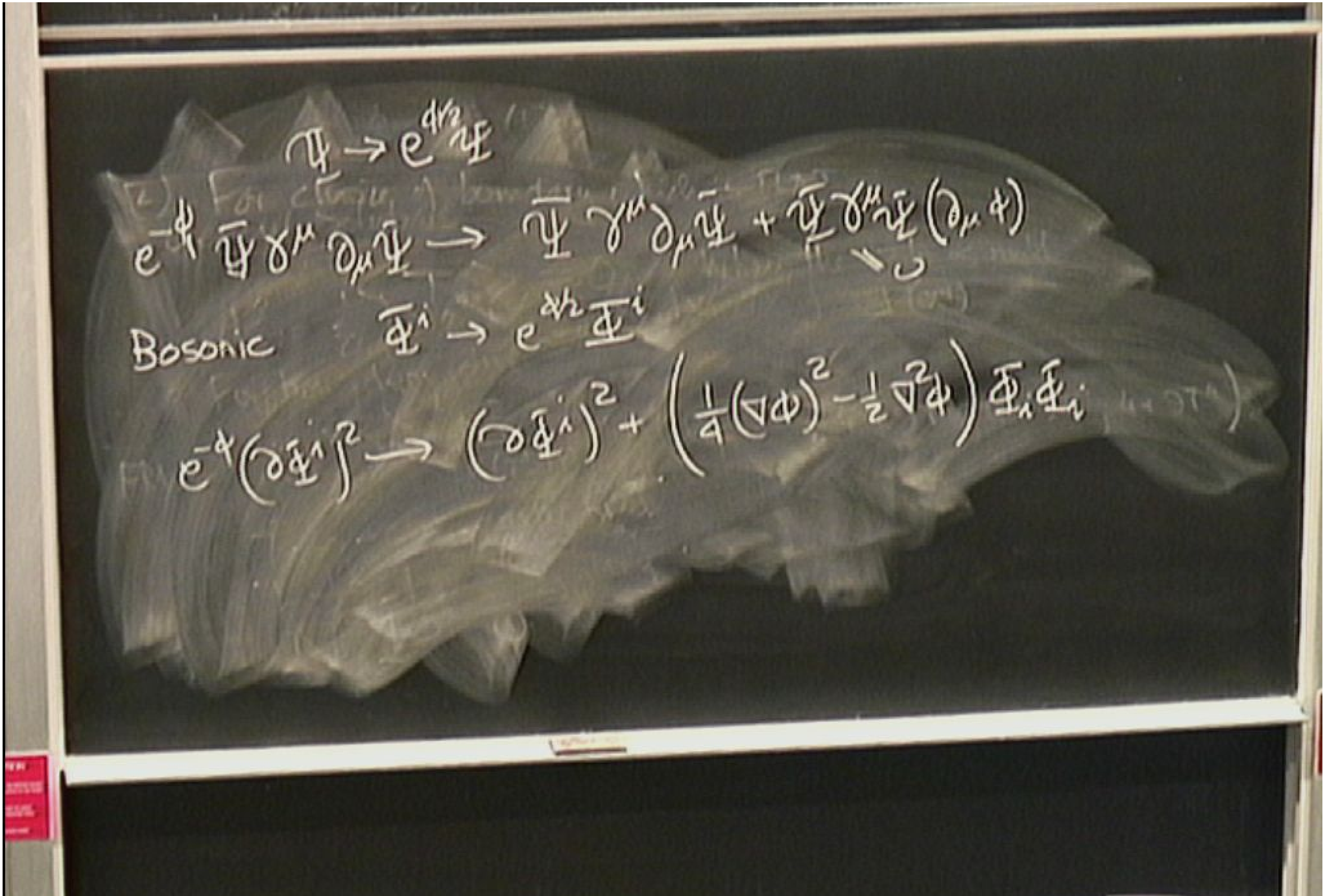
$$A_\mu \to e^{\phi/2} A_\mu$$

$e^{-\phi}[\partial(e^{\phi/2})]^2 \longrightarrow$ Kinetic terms will generically contain $(\partial\phi)^2$

$$\Psi \to e^{\phi/2}\Psi$$

$$e^{-\phi}\,\bar{\Psi}\gamma^\mu\partial_\mu\Psi \longrightarrow \bar{\Psi}\gamma^\mu\partial_\mu\Psi + \bar{\Psi}\gamma^\mu\Psi(\partial_\mu\phi)$$


$\Psi \to e^{\phi/2}\Psi$
$e^{-\phi}\bar{\Psi}\gamma^\mu\partial_\mu\Psi \longrightarrow \bar{\Psi}\gamma^\mu\partial_\mu\Psi + \bar{\Psi}\gamma^\mu\Psi(\partial_\mu\phi)$
Bosonic $\Phi^i \to e^{\phi/2}\Phi^i$
$e^{-\phi}(\partial\Phi^i)^2 \longrightarrow (\partial\Phi^i)^2 + \left(\frac{1}{4}(\nabla\phi)^2 \ldots \nabla^2\phi\right)$


$\Psi \to e^{\phi/2}\Psi$
$e^{-\phi}\,\bar\Psi\gamma^\mu\partial_\mu\Psi \longrightarrow \bar\Psi\gamma^\mu\partial_\mu\Psi + \bar\Psi\gamma^\mu\Psi(\partial_\mu\phi)$
Bosonic
$\Phi^i \to e^{\phi/2}\Phi^i$
$e^{-\phi}(\partial\Phi^i)^2 \longrightarrow (\partial\Phi^i)^2 + \left(\frac{1}{4}(\nabla\phi)^2 - \frac{1}{2}\nabla^2\phi\right)\Phi_i\Phi_i$

$$\Psi \to e^{\phi/2}\Psi$$

$$e^{-\phi}\,\bar\Psi\gamma^\mu\partial_\mu\Psi \to \bar\Psi\gamma^\mu\partial_\mu\Psi + \bar\Psi\gamma^\mu\Psi(\partial_\mu\phi)$$

$$\searrow 0$$

Bosonic $\quad \Phi^i \to e^{\phi/2}\Phi^i$

$$e^{-\phi}(\partial\Phi^i)^2 \to (\partial\Phi^i)^2 + \left(\frac{1}{4}(\nabla\phi)^2 - \frac{1}{2}\nabla^2\phi\right)\Phi_i\Phi_i$$

NULL SINGULARITIES

$$\Psi \to e^{\phi/2}\Psi$$

$$e^{-\phi}\,\bar{\Psi}\gamma^\mu\partial_\mu\Psi \longrightarrow \bar{\Psi}\gamma^\mu\partial_\mu\Psi + \bar{\Psi}\gamma^\mu\Psi(\partial_\mu\phi)$$

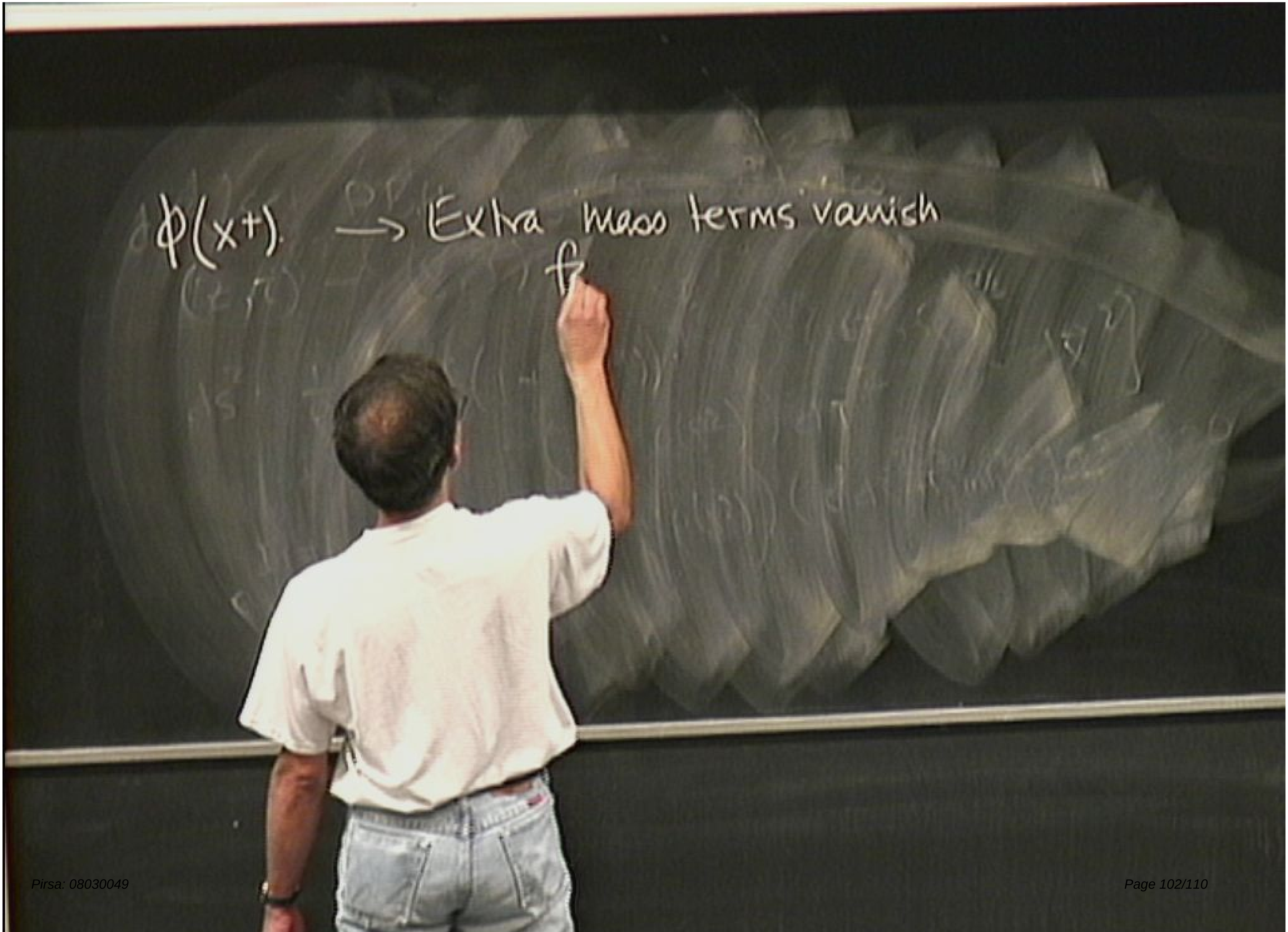
$$= 0$$

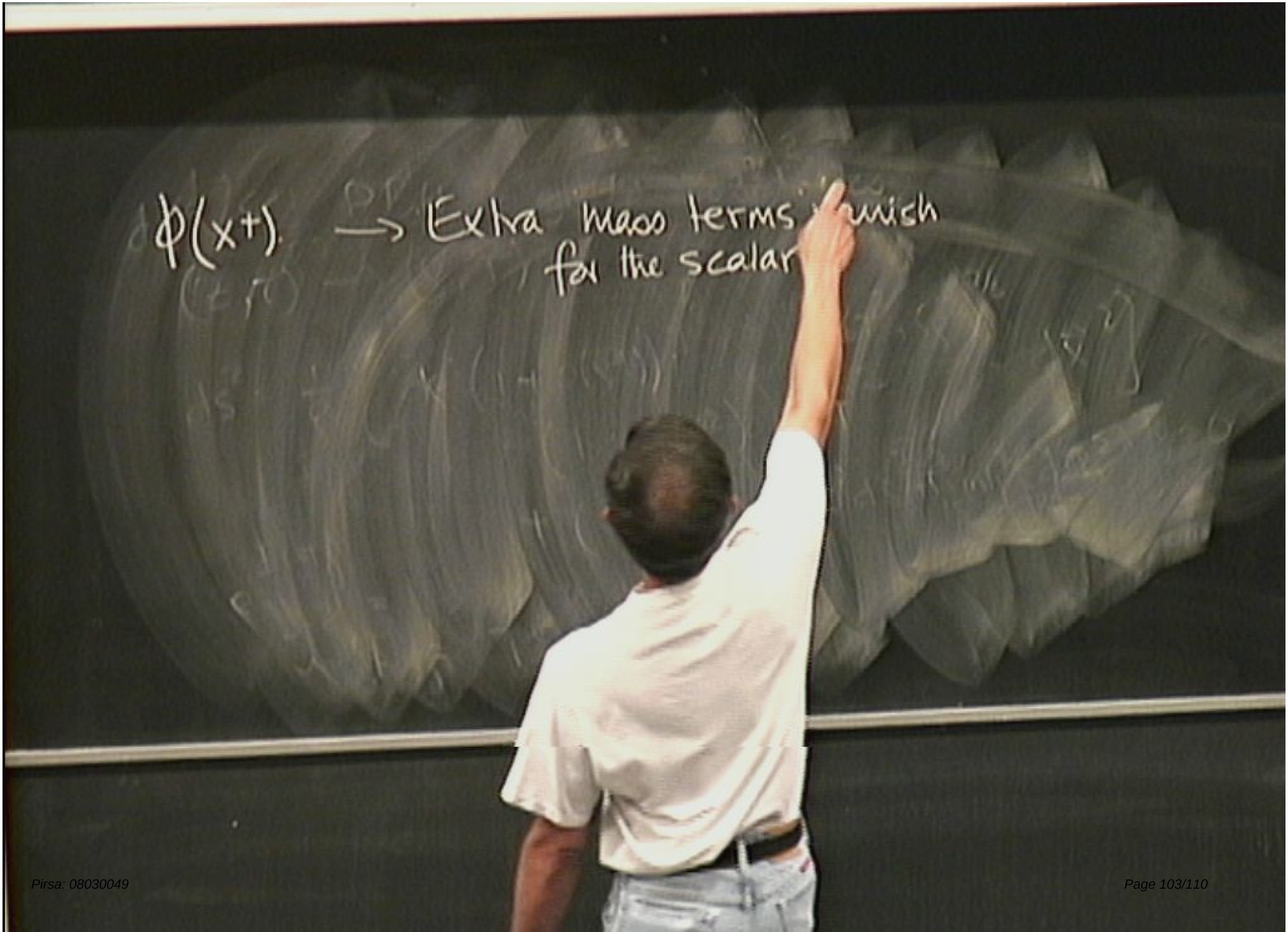
Bosonic $\quad \Phi^i \to e^{\phi/2}\Phi^i$

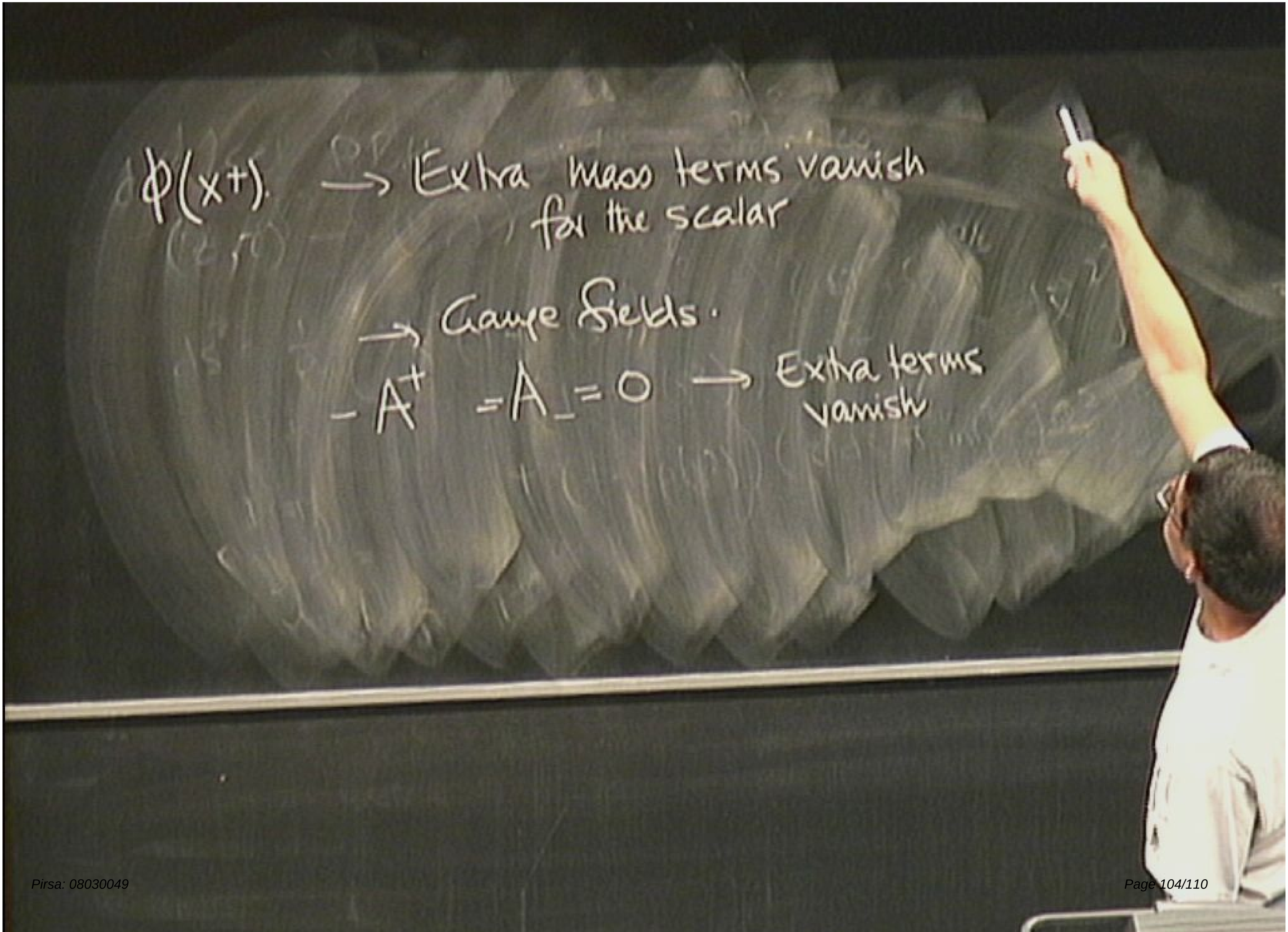
$$e^{-\phi}(\partial\Phi^i)^2 \longrightarrow -(\partial\Phi^i)^2 - \left(\frac{1}{4}(\nabla\phi)^2 - \frac{1}{2}\nabla^2\phi\right)\Phi_i\Phi_i$$

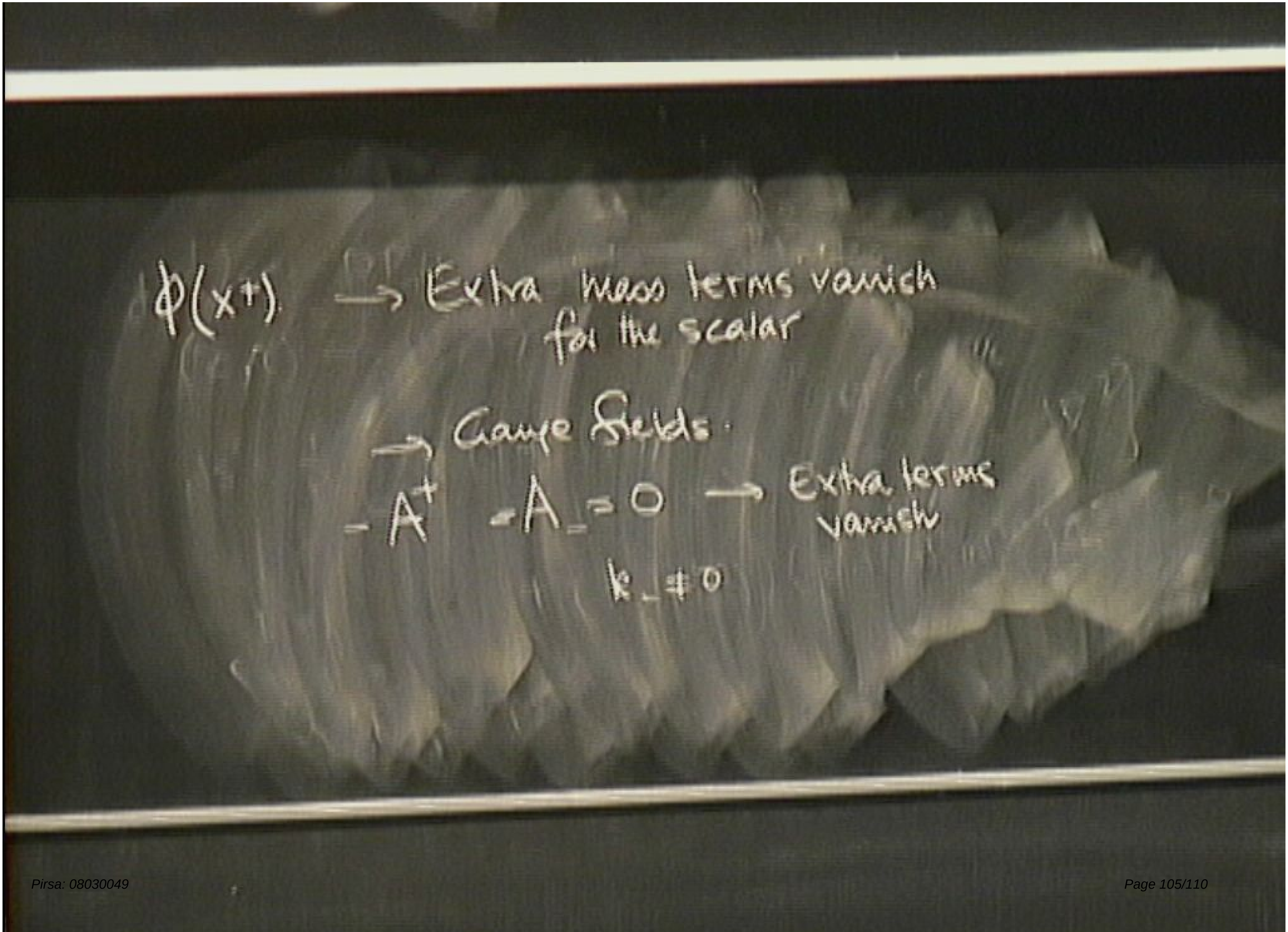
Gauge field

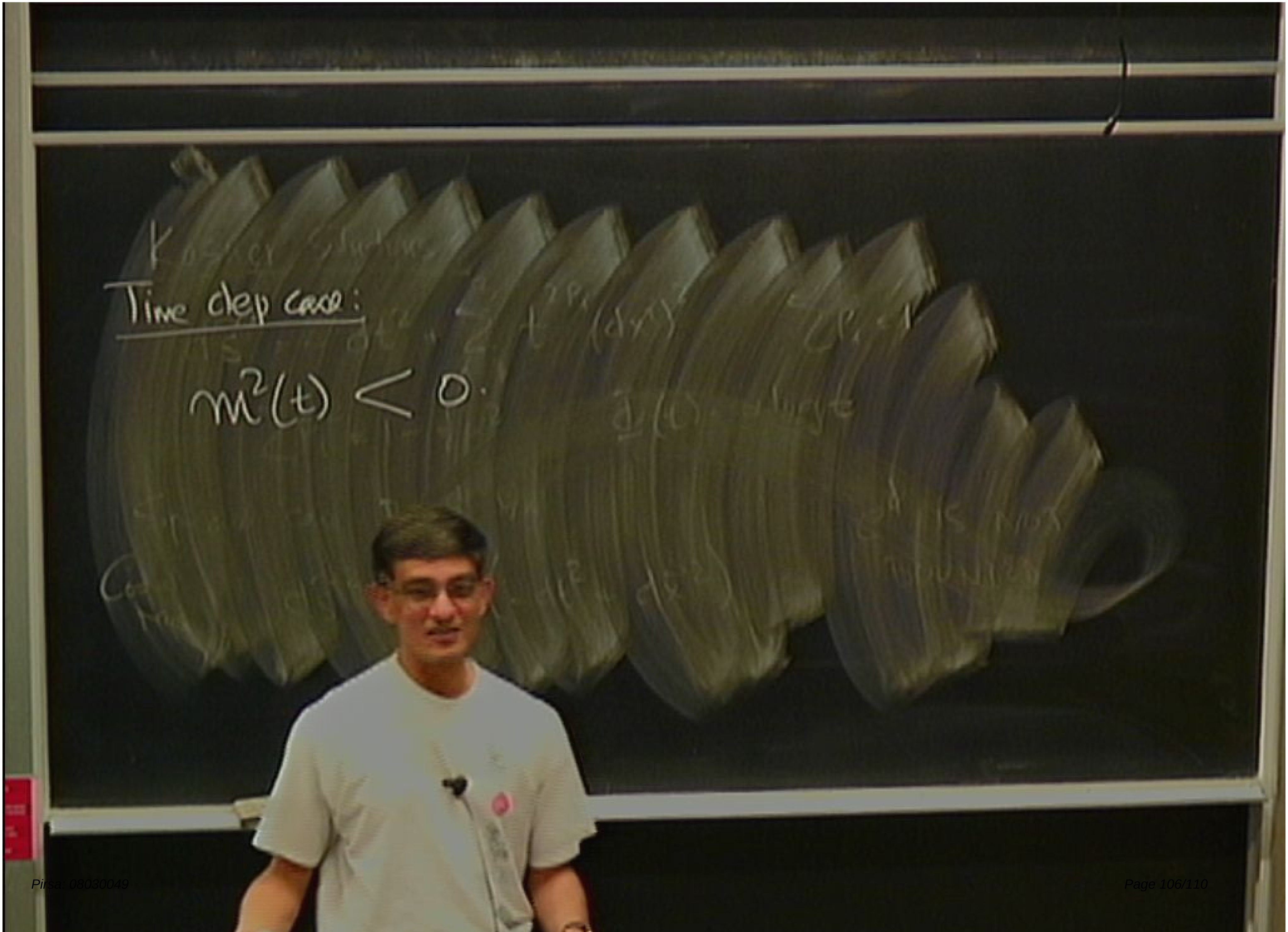
$$-F_{\mu\nu}^2 - \frac{1}{2}\left[(\nabla\phi)^2 A_\nu A^\nu - \partial_\mu\phi\,\partial_\nu\phi\,A^\mu A^\nu\right. + (\nabla^2\phi)A_\nu A^\nu + 2\,\partial_\nu\phi\,A^\mu\partial_\mu A^\nu$$

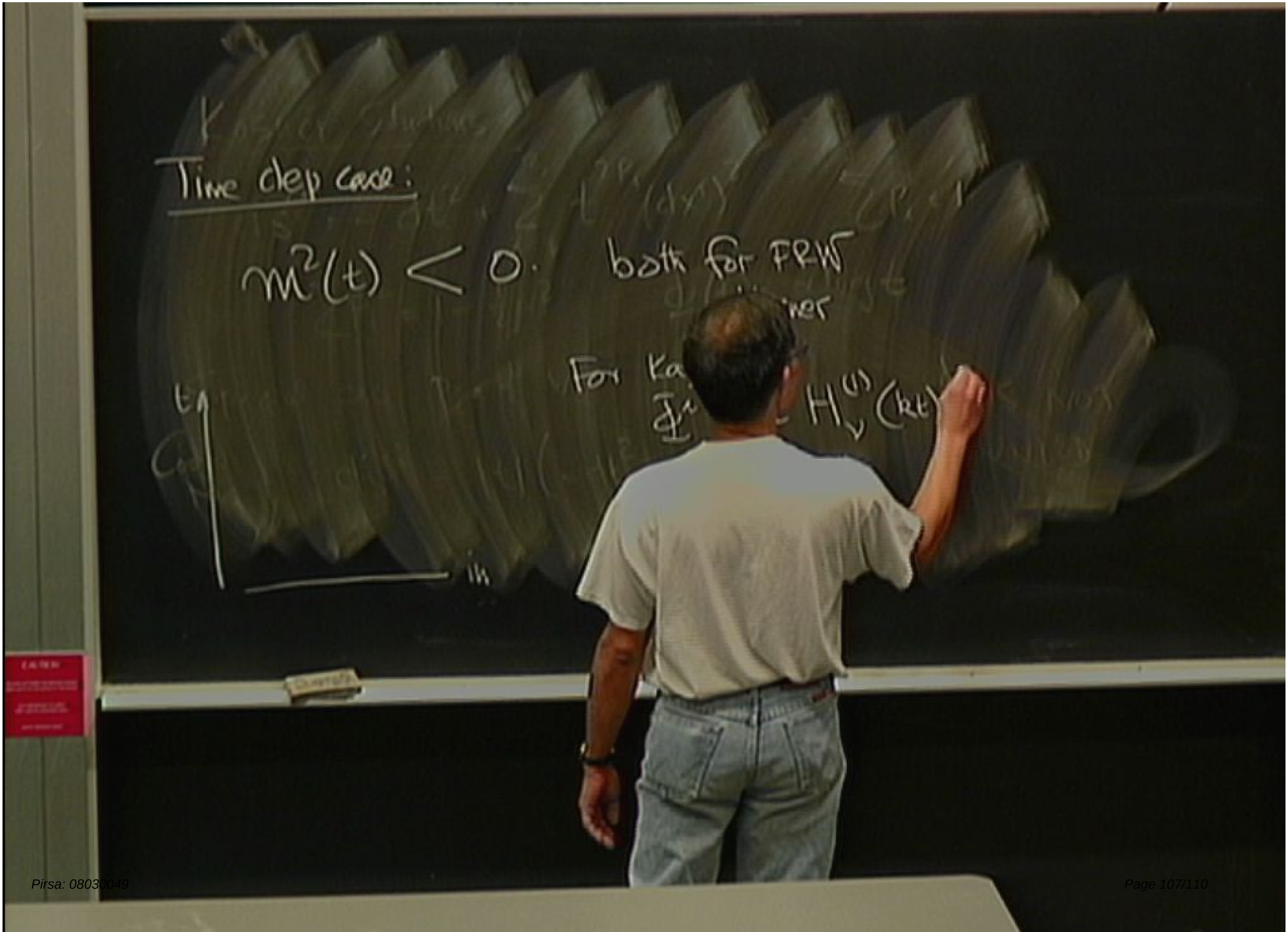

ϕ(x+) → Extra mass terms vanish

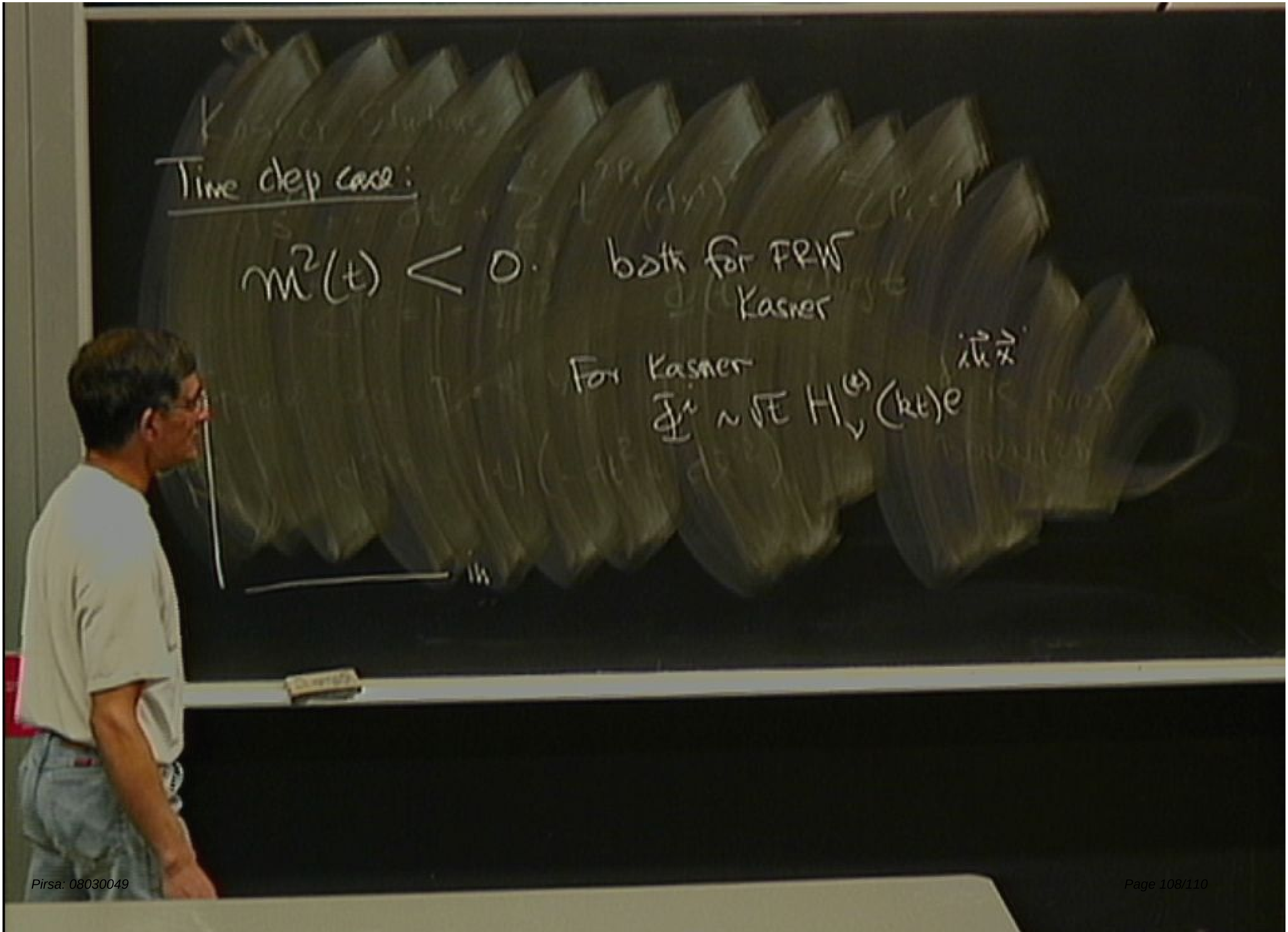

φ(x+)
→ Extra mass terms
for the scalar

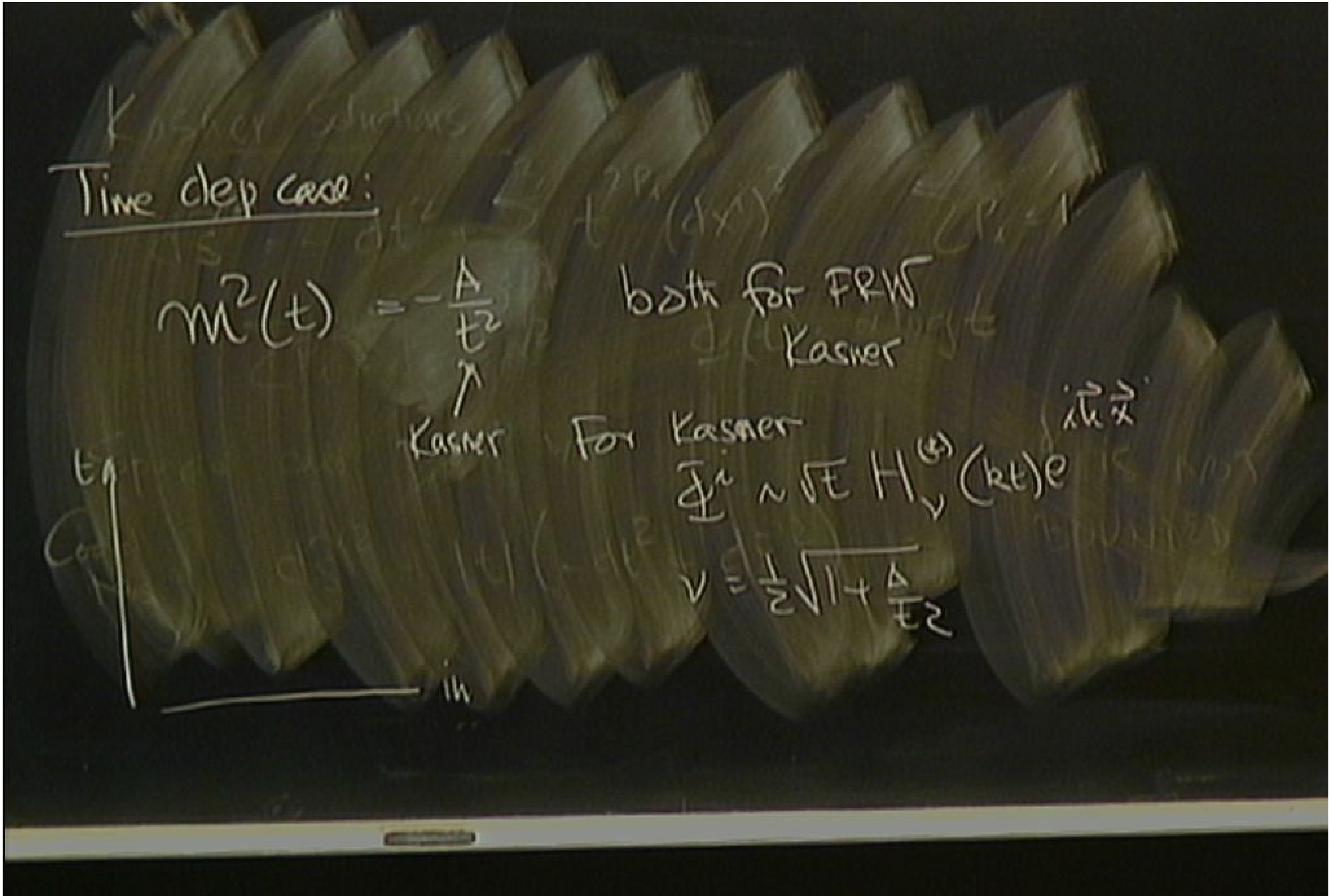

φ(x+). → Extra mass terms vanish
for the scalar
→ Gauge fields.
- A+ = A_ = 0 → Extra terms
vanish

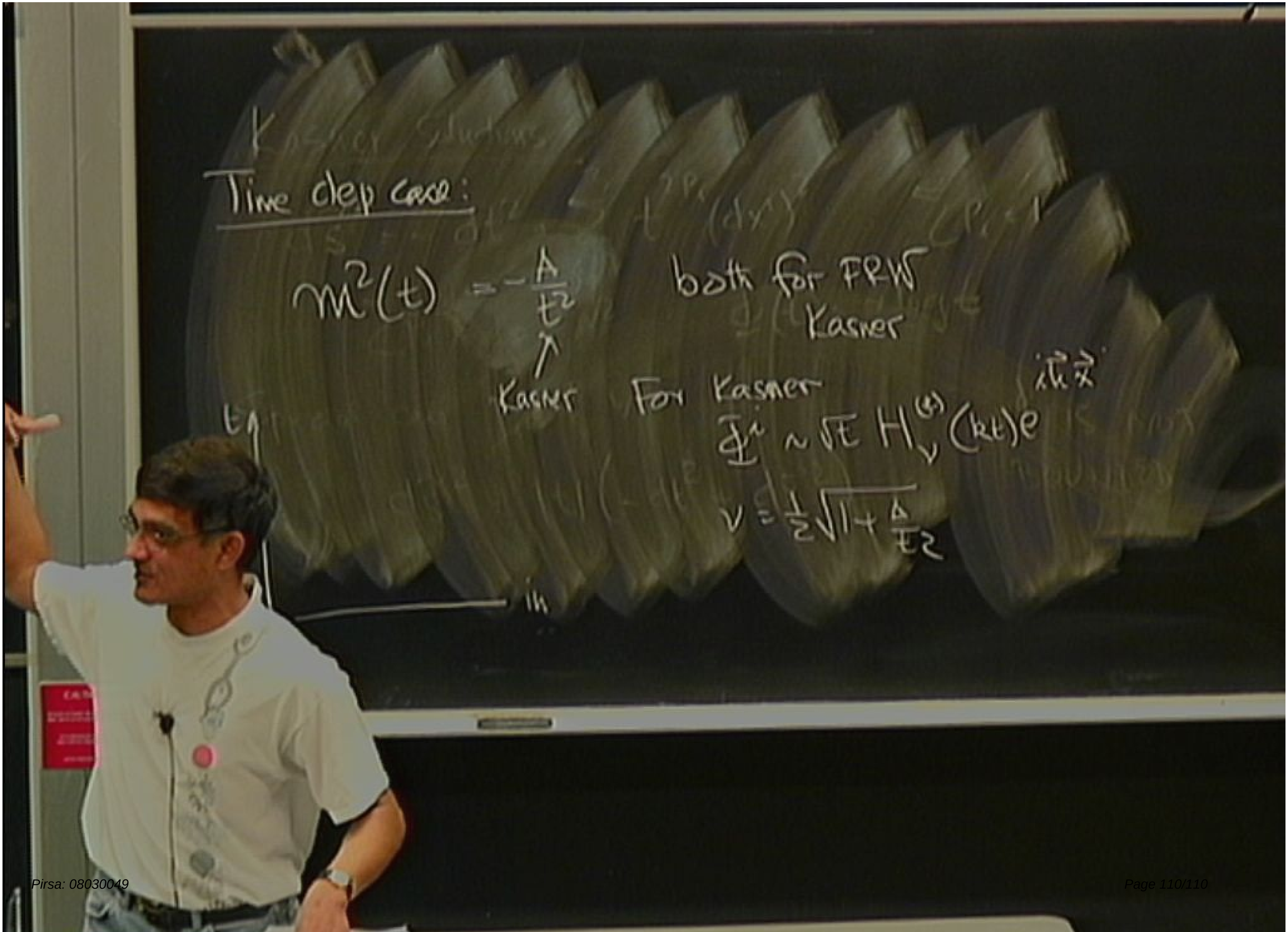

φ(x+) → Extra mass terms vanish
for the scalar
→ Gauge fields.
A+ A_ = 0 → Extra terms vanish
k_ ≠ 0


Time dep case:
m²(t) < 0.


Time dep case:
m²(t) < 0.
both for FRW


Time dep case:
m²(t) < 0. both for FRW
Kasner
For Kasner
Φᵢ ~ √t Hν(1)(kt)e^(ik·x)


Time dep case:
m²(t) = − A/t²
Kasner
both for FRW
Kasner
For Kasner
Φ ~ √t H_ν^(1)(kt) e^(ik·x)
ν = ½ √(1 + A/t²)
t
in


Time dep case:
$m^2(t) = -\frac{A}{t^2}$
both for FRW
Kasner
Kasner
For Kasner
$\Phi^i \sim \sqrt{t}\, H^{(1)}_\nu(kt) e^{i\vec{k}\cdot\vec{x}}$
$\nu = \frac{1}{2}\sqrt{1 + \frac{A}{t^2}}$