Title: New no-go theorems and the costs of cosmic acceleration

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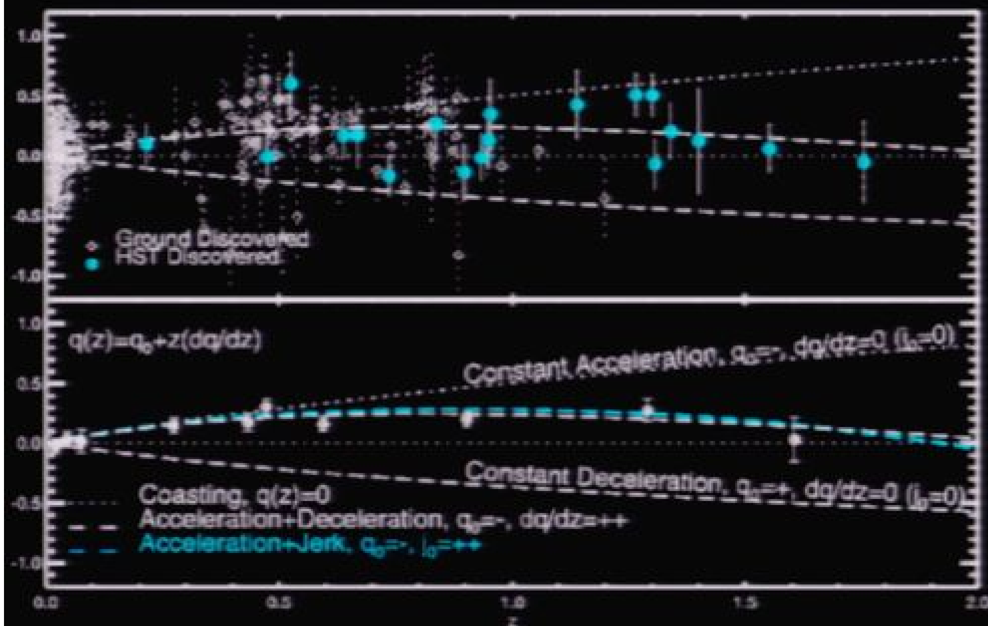
Abstract:

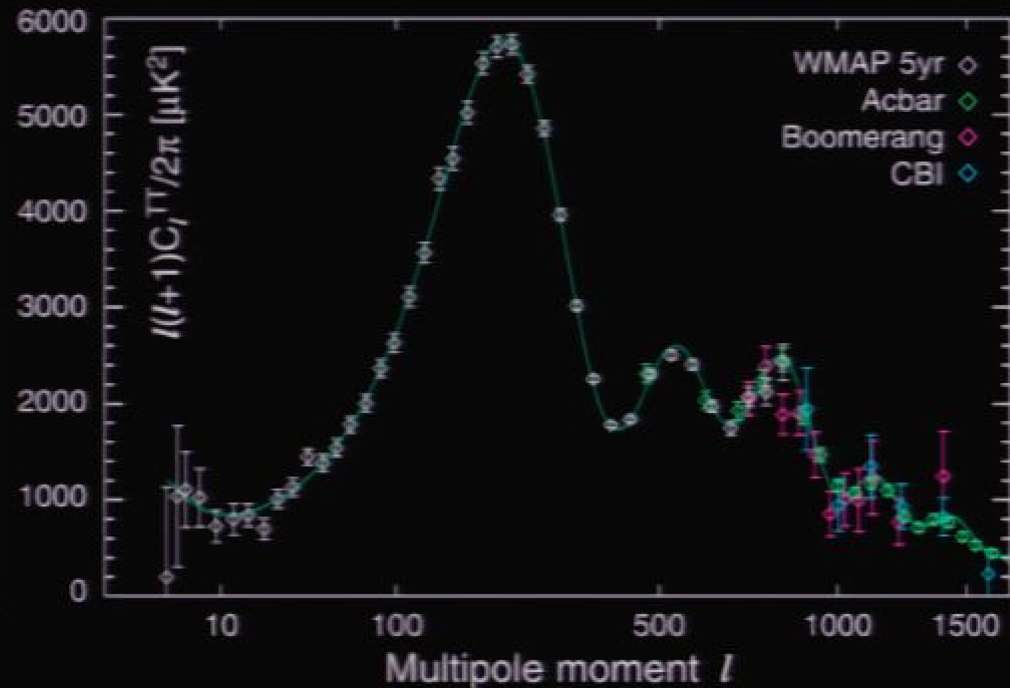
New no-go theorems and the costs of cosmic acceleration

Daniel Wesley
(Cambridge DAMTP)

0802.2106 and 0802.3214

Two data sets





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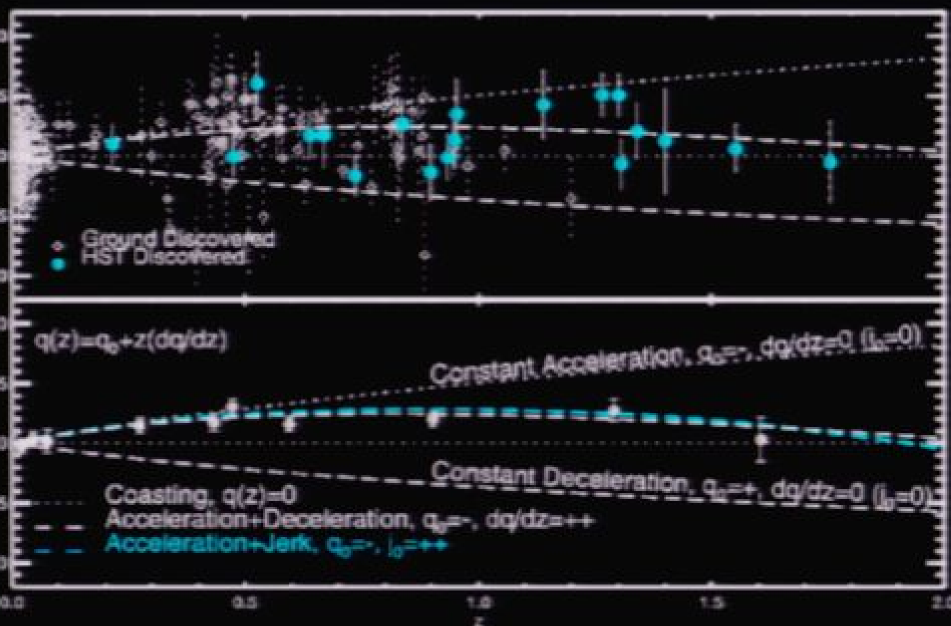

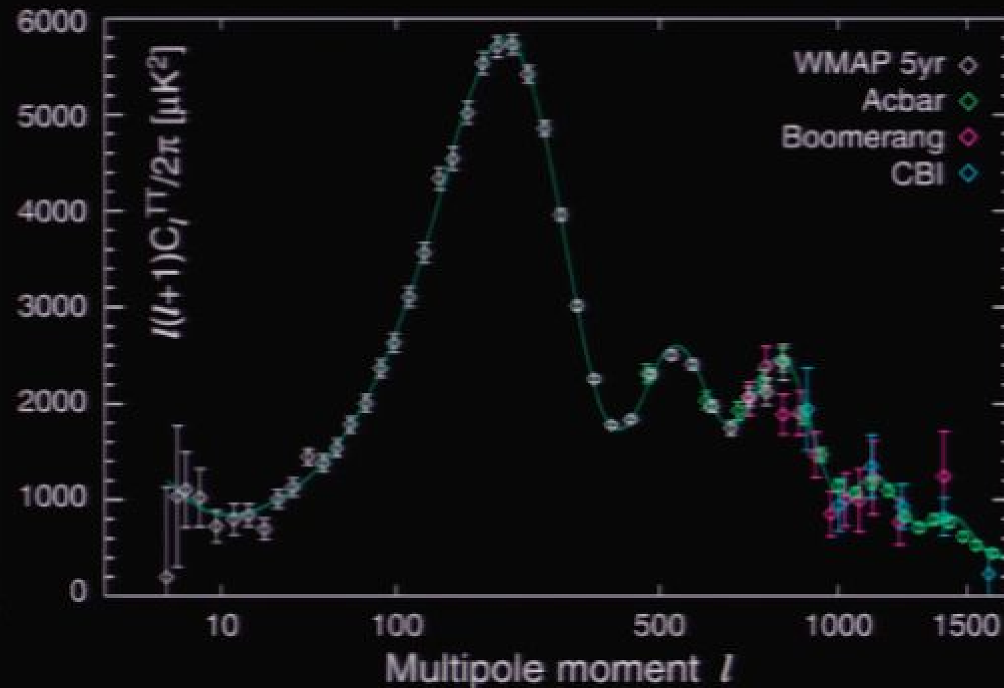

ä > 0

Late universe

(dark energy)

ä > 0 (?)

Early universe

(inflation?)

Old no-go theorems

Theorem *(Gibbons '84, Maldacena and Nunez '01)*

To obtain a four-dimensional de Sitter universe from a static warped reduction on closed compact manifold $\mathcal{M}$, one must violate the Strong Energy Condition (SEC).

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Evasive maneuvers

(a) Time dependent $\mathcal{M}$
(b) Non-de Sitter expansion
(c) Transient de Sitter
(d) Accept SEC violation **
(e) Non-compact $\mathcal{M}$ (finite G_N?)

** not always an option, depending on the theory

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New improvements

(1) Weaken energy condition
(2) Include non-de Sitter (w>-1)
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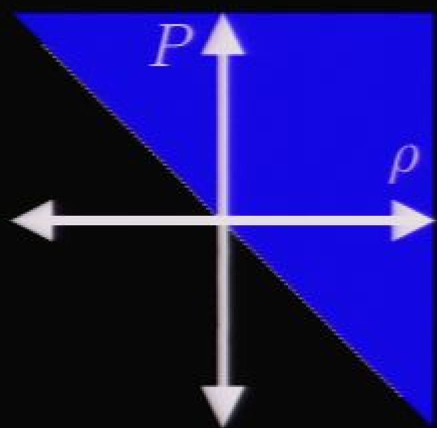
Some energy conditions

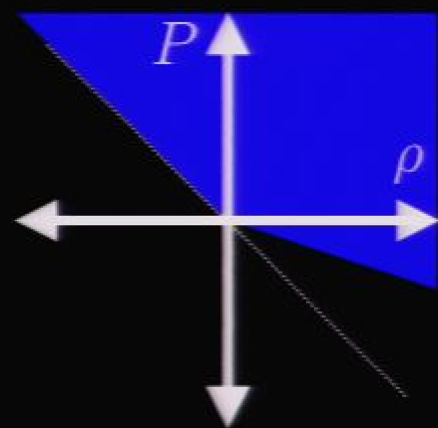
Null $T_{MN}n^M n^N \geq 0$

"gravity is stable"



Strong $R_{MN}t^M t^N \geq 0$

"geodesics converge" ≈ "gravity is attractive"

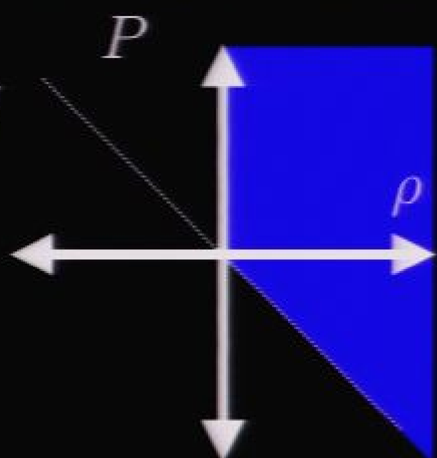


Weak $T_{MN}t^M t^N \geq 0$

"energy density positive for all observers"

$\rho \geq 0$

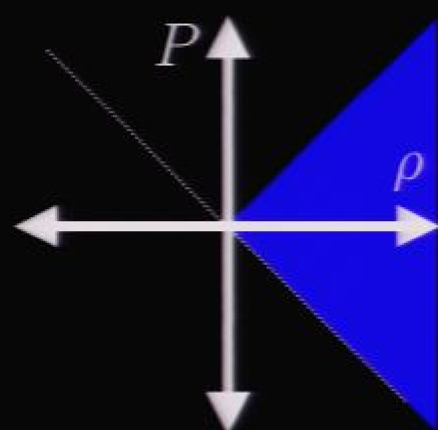
$\rho + P \geq 0$



Dominant $T_{MN}t^M$ *not S.L.*

"subluminal phase velocity"

$\rho \geq |P|$



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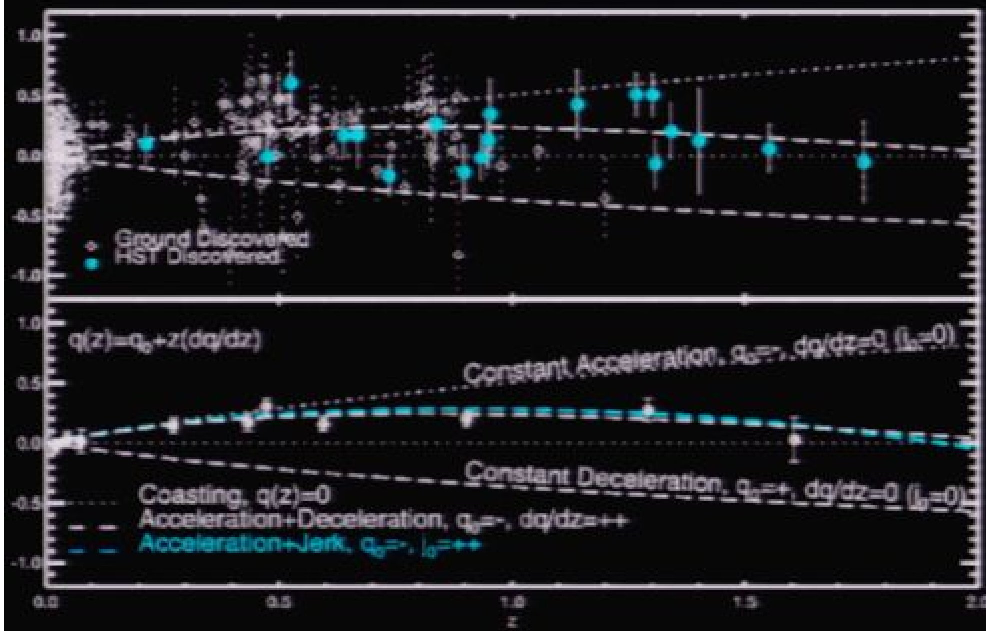

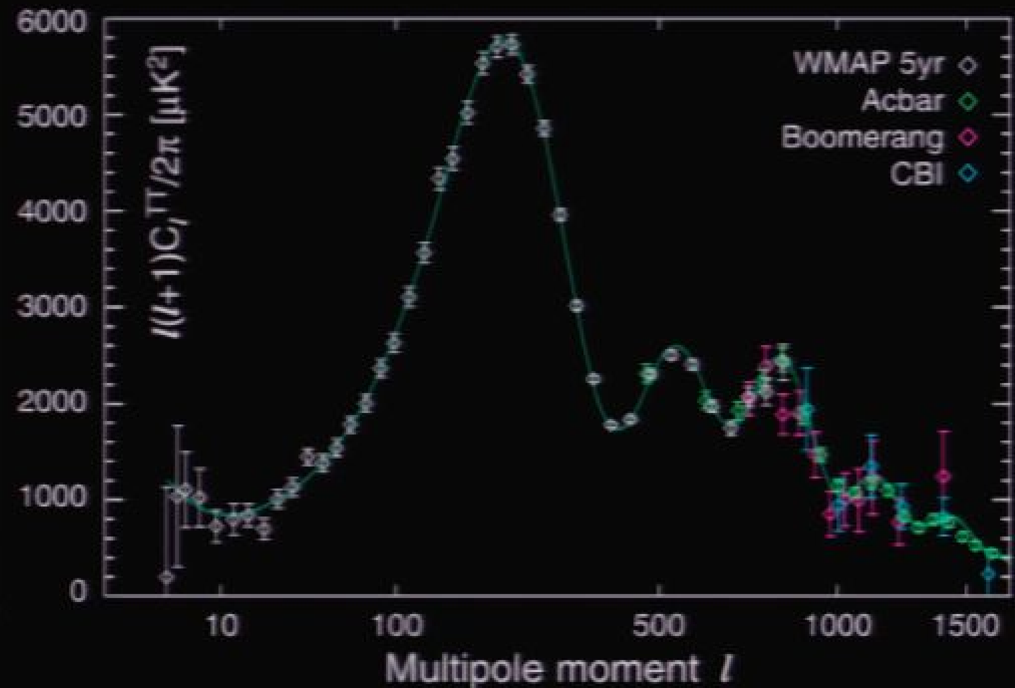

$\ddot{a} > 0$	**$\ddot{a} > 0$ (?)**
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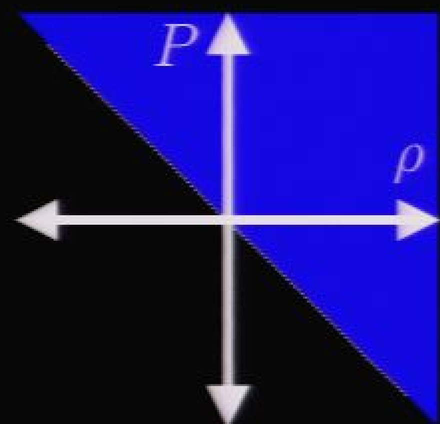

$R_{MN}t^M t^N \geq 0$ **Strong**

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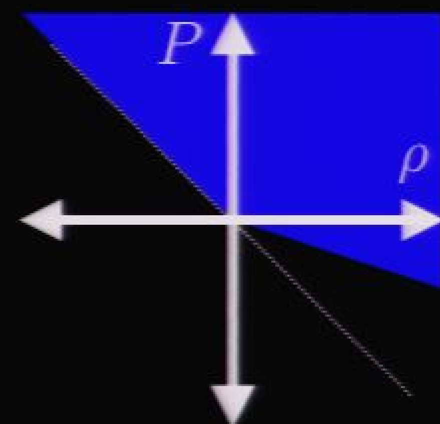

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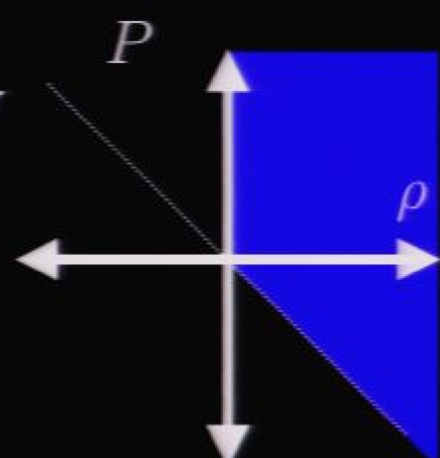

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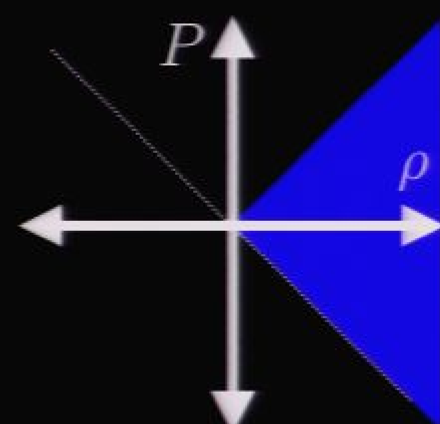

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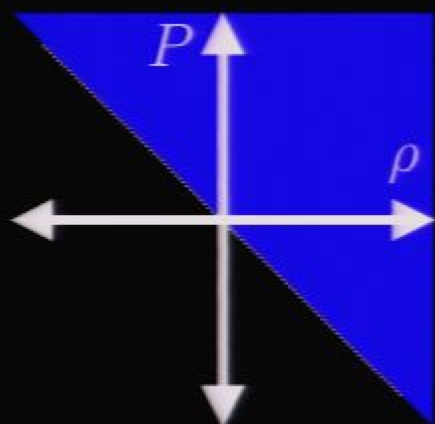

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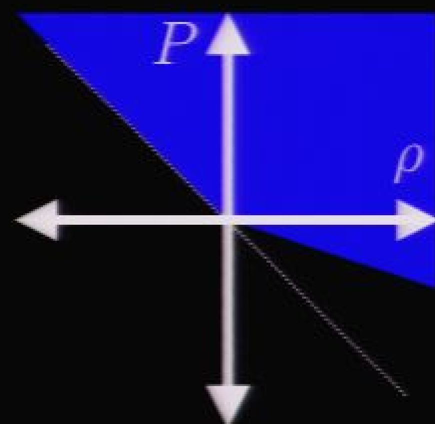

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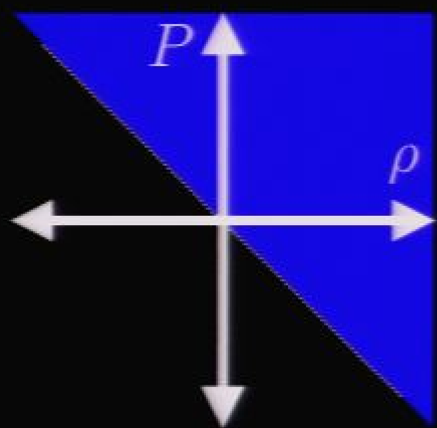

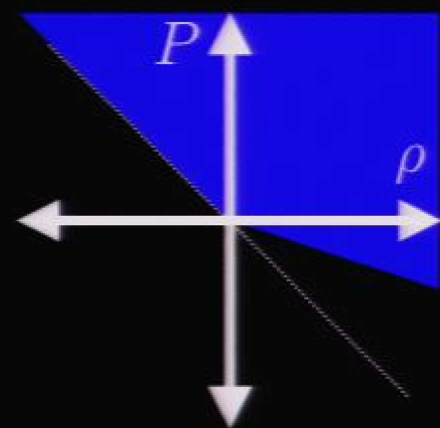

"geodesics converge" ≈ "gravity is attractive"

	Null	Strong
Satisfied by	• $\rho + P \geq 0$ • Two-derivative actions with positive definite kinetic terms, any $V(\varphi)$ • D-branes & positive tension objects • Implied by all other energy conditions	• $\rho + P \geq 0$ and $\rho + 3P \geq 0$ • Scalars with $V(\varphi) \leq 0$ • ... classical 11D SUGRA + others • anti-de Sitter $\Lambda < 0$ • dust and radiation
Violated by	• Casimir energy (...unless averaged) • Negative tension (orientifold planes) • ghost condensates	• Scalars with $V(\varphi) > 0$ anywhere • de Sitter $\Lambda > 0$ • D-branes

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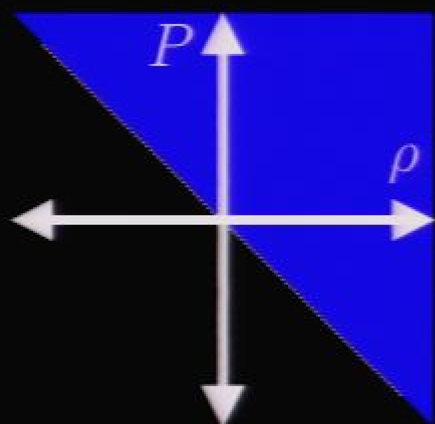

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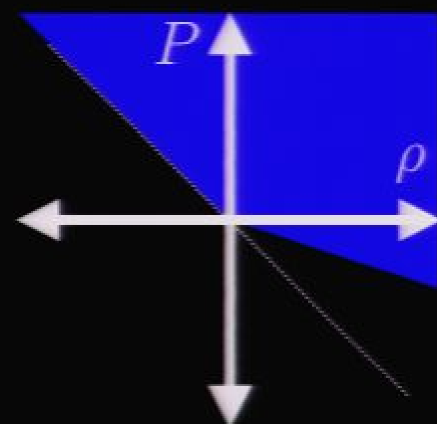

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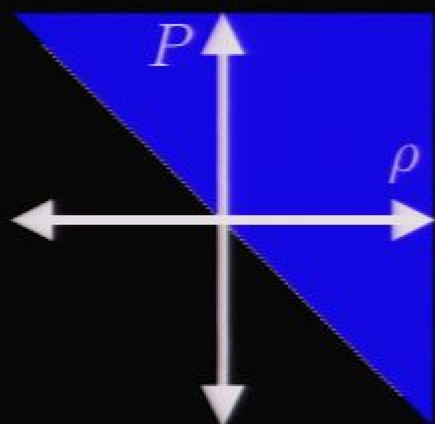

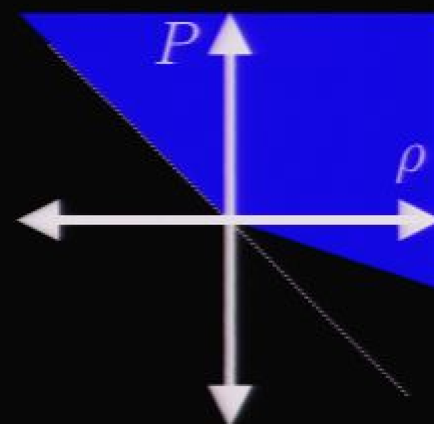

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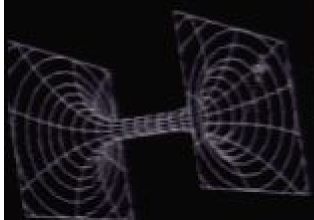
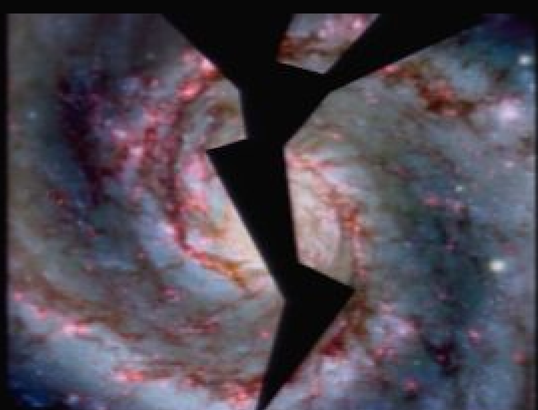
The Null Energy Condition

...keeps us safe from...





Causality violations

Instabilities & other pathologies

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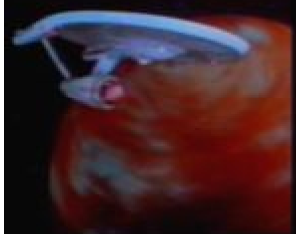

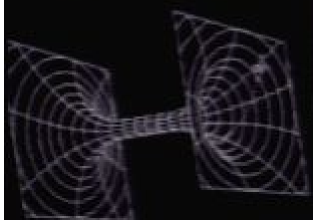

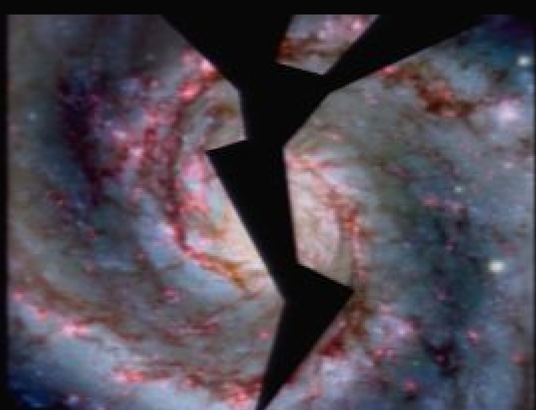

Causality violations

Superluminal travel, "warp drives," traversable wormholes, time machines, CTCs, chronology (non-)protection ...

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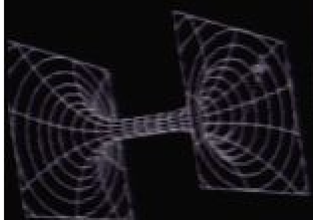

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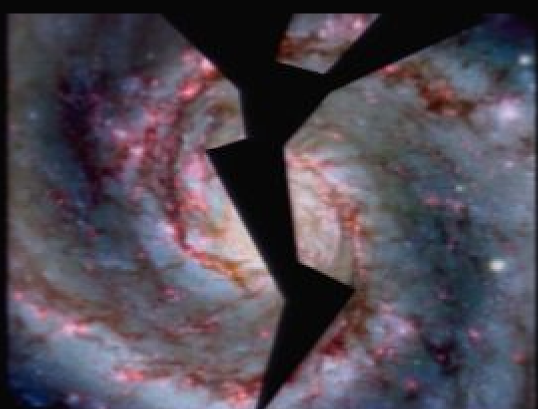

Classical: Big Rips, Big Bounces, gradient instabilities ...
Quantum: unitarity violation, rapid vacuum decay, *perpetuum mobile* ...

Cline, Jeon, Moore *Phys. Rev. D* **70** (2004) 043543 ; Hsu, Jenkins, Wise *Phys. Lett. B* **597** (2004) 270 ; Dubovsky, Gregoire, Nicolis, Rattazzi *JHEP* **0603** (2006) 025 ; Buniy, Hsu, Murray *Phys. Rev. D* **74** (2006) 063518 ; Caldwell *Phys. Lett. B* **545** (2002) 23 ; Caldwell, Kamionkowski, Weinberg *Phys. Rev. Lett.* **91** (2003) 071301, Arkani-Hamed, Dubovsky, Nicolis, Trincherini, Villadoro *JHEP* **05** (2007) 055 ; Dubovsky, Sibiryakov *Phys. Lett. B* **638** (2006) 509

Types of new theorems

de Sitter theorems

Non-de Sitter (w>-1) theorems

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de Sitter theorems

- concerned with eternal de Sitter expansion
- must violate null energy condition under broad circumstances
- previous theorems used R_{00} because it does not involve the intrinsic curvature $\mathcal{R}$ of $\mathcal{M}$. To prove the new theorems, you look at the other components and engage with the complexity of dealing with $\mathcal{R}$.

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  - Varying w (1): for any $w(t)$ the bound $N[w(t)]$ obtained by quadrature.
  - Varying w (2): if $w < w_*$ then $N[w(t)] < N(w_*)$.

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Types of $\mathcal{M}$

Curvature-free	Curved
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• One-dimensional (KK & RS)
• Tori --
  • realised by periodic identification of R^n
  • with $\mathcal{R} \geq 0$ everywhere
• Special holonomy --
  • $Sp(n)$
  • $Spin(7)$
  • $SU(n)$ (Calabi-Yau)
  • G_2 (M theory)
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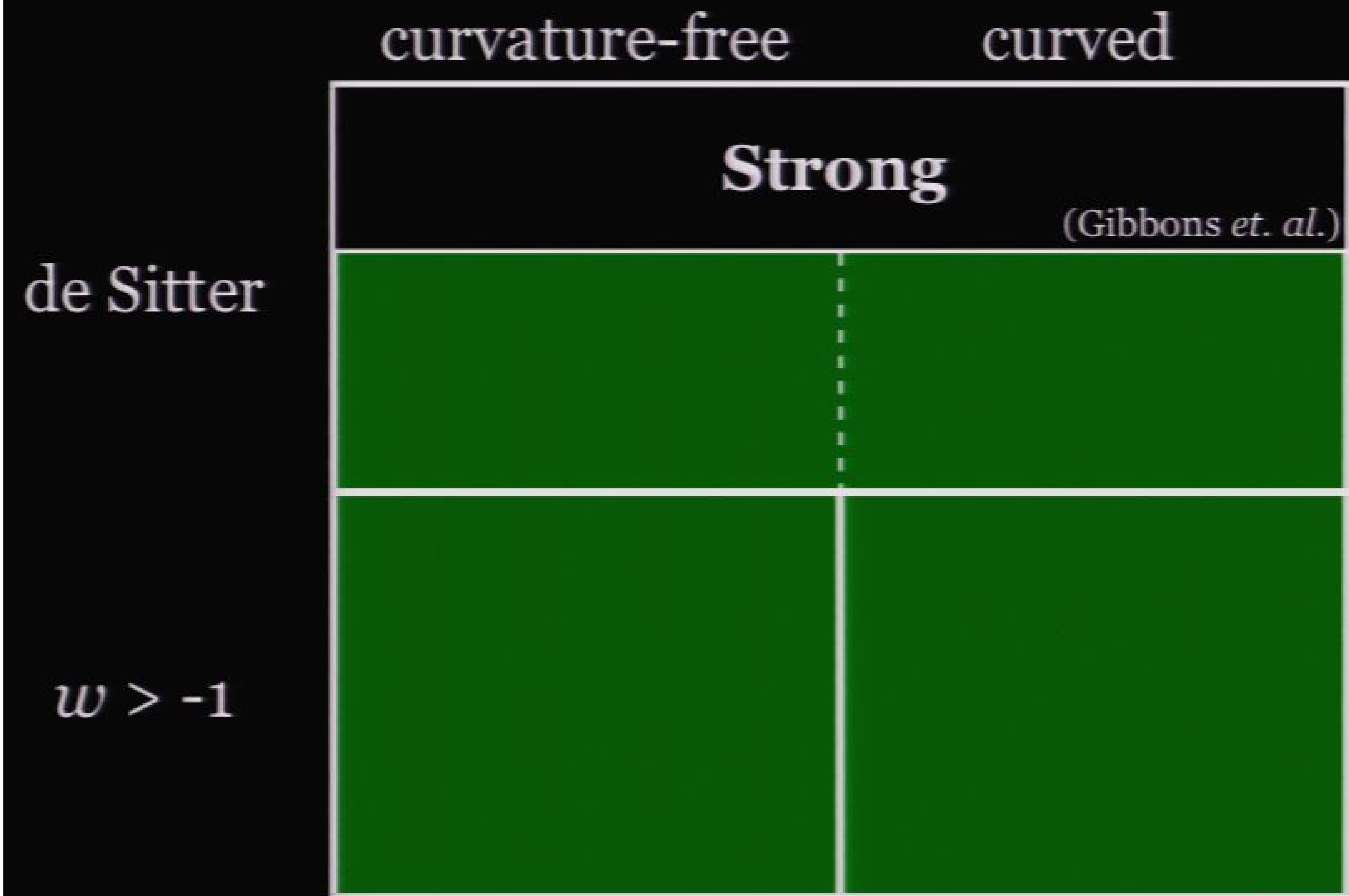
Comparison with previous theorems



Comparison with previous theorems

	curvature-free	curved
de Sitter	**Strong** (Gibbons *et. al.*)	
	NEW **Null**	*NEW* **Null*** **bounded avg. condition*
$w > -1$		

Comparison with previous theorems

<table>
<tr><th></th><th>curvature-free</th><th>curved</th></tr>
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<tr><td>$w > -1$</td><td>NEW Null (transient)</td><td>NEW Strong (transient)</td></tr>
</table>

A simple example (I)

Ricci-flat extra dimensions with breathing-mode dynamics

$$\mathrm{d}s^2_{4+k} = A(\eta)^2\left(-\mathrm{d}\eta^2 + \mathrm{d}x_3^2\right) + \exp\left[\frac{2c}{k}\psi(\eta)\right]\mathrm{d}s_k^2$$

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$$c = \sqrt{\frac{2k}{k+2}} \qquad a(\eta) = e^{c\psi/2}A(\eta)$$

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$$a(\eta) \sim \eta^{2/(1+3w)} \qquad \rho + P = \left[\frac{1+w}{3}\right]H^2 = \left[\frac{\mathrm{d}\psi}{\mathrm{d}\eta}\right]^2$$

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Yields η-dependence of ψ and *A*.

$$\psi(\eta) = \pm\left[\frac{1+w}{1+3w}\right] \ln \eta^6 + \psi_0$$

A simple example (II)

Reconstruct (4+k)D metric and use Einstein equations to compute stress-energy tensor

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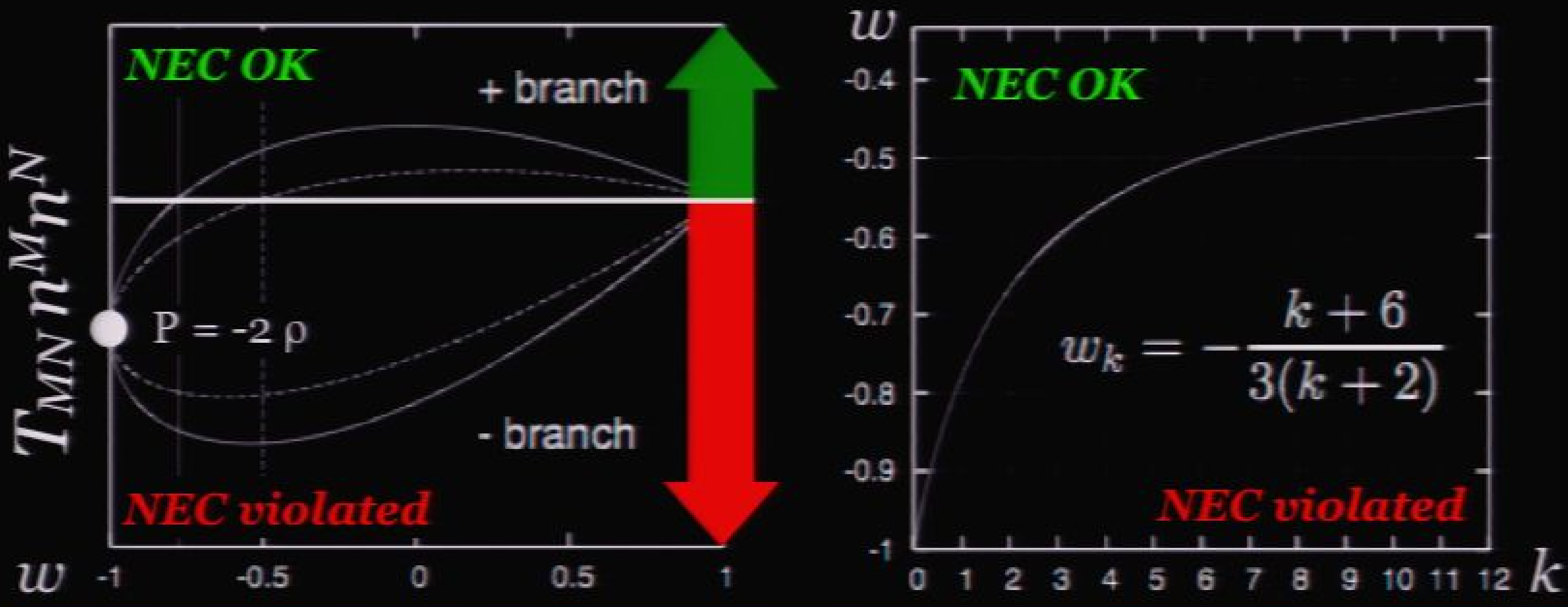

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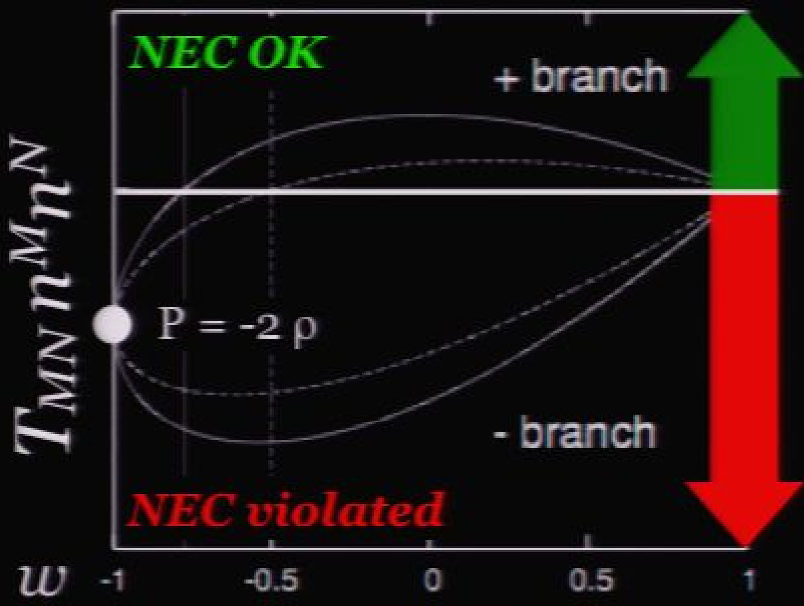
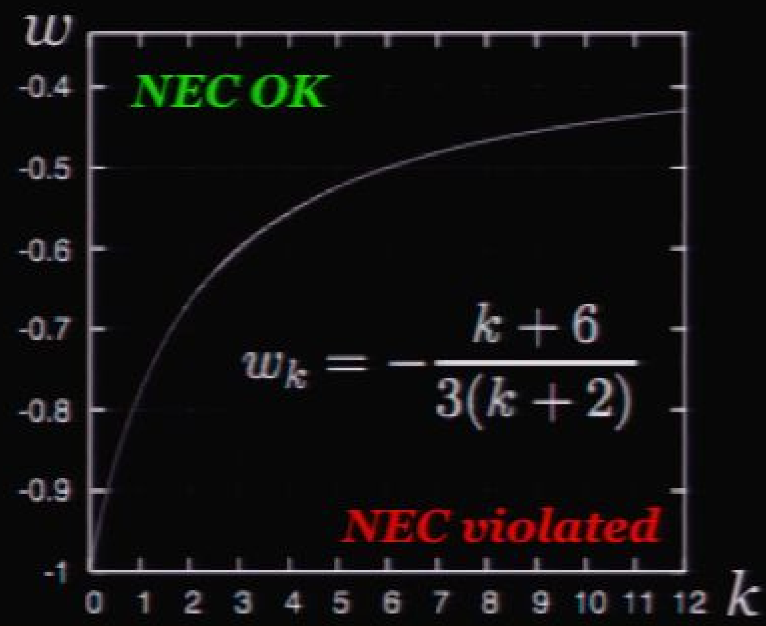
Lesson: potentials that look perfectly reasonable in 4D require exotic physics in higher-dimensional context.

Reconstruct (4+k)D metric and use Einstein equations to compute stress-energy tensor





$$\psi(\eta) = \pm\left[\frac{1+w}{1+3w}\right]\ln\eta^6 + \psi_0 \qquad V(\psi) = V_0 e^{\mp\sqrt{3(1+w)}\psi}$$

Assumptions

- $\mathcal{M}$ closed and compact, or a quotient of c.c. $\mathcal{M}/G$

$$S_{(4+k)} = \frac{1}{2\kappa^{2+k}} \int \left[R(g) + \mathcal{L}_m^{(4+k)} \right] \sqrt{-g}\, \mathrm{d}^{4+k}X$$

$$g_{MN}^{(4+k)} \mathrm{d}X^M \mathrm{d}X^N = e^{2\Omega(t,y)} h_{\mu\nu}^{(4)}(t) \mathrm{d}x^\mu \mathrm{d}x^\nu + g_{\alpha\beta}^{(k)}(t,y) \mathrm{d}y^\alpha \mathrm{d}y^\beta$$

- Higher-dimensional action has Einstein-Hilbert form ...includes $g(\varphi)\mathcal{R}$ and $F(\mathcal{R})$ models
- *Arbitrary other matter fields present*

$$S_{\mathrm{4D}} = \frac{1}{2\ell_4^2} \int R(g) \sqrt{-g}\, \mathrm{d}^4x + \text{other terms}$$

- All four-dimensional statements refer to the Einstein frame metric and its associated cosmology.

Denote by ✷ claim proven in the "long" paper 0802.3214

Three tools...

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1. Decomposition of metric time derivative.
(Proxy for KK scalars and/or metric moduli)

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}g^{(k)}_{\alpha\beta} = \frac{1}{k}\xi g^{(k)}_{\alpha\beta} + \sigma_{\alpha\beta}$$

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2. One-parameter family of averages on the manifold

$$\langle Q(t,y^\alpha)\rangle_A = \left(\int e^{A\Omega}\, Q\, \det(e_\mathcal{M})\, \mathrm{d}^k y\right)\left(\int e^{A\Omega} \det(e_\mathcal{M})\, \mathrm{d}^k y\right)^{-1}$$

$$Q(t)_{0|A} = \langle Q(t,y^\alpha)\rangle_A, \quad Q(t,y^\alpha)_{\perp|A} = Q(t,y^\alpha) - Q(t)_{0|A}$$

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3. "NEC probes." If negative the NEC must be violated.

$${}^3N = -g^{00}T_{00} + (1/3)g^{MN}\,{}^3\Pi^P_M\,{}^3\Pi^Q_N T_{PQ}$$
$${}^kN = -g^{00}T_{00} + (1/k)g^{MN}\,{}^k\Pi^P_M\,{}^k\Pi^Q_N T_{PQ}$$

...many challenges

- non-uniqueness of KK inversion
- (equiv) 4D theory gives only nonlocal information on $\mathcal{M}$
- lack of explicit moduli space description
- apparent ghosts from conformal deformations of $\mathcal{M}$
- “arbitrary” warp factor and deformations of $\mathcal{M}$
- nature of the physics that causes acceleration?
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Averages of NEC probes

$$\langle e^{2\Omega}\,{}^{3}N\rangle_A = n^2(\rho_T + P_T) - \frac{k+2}{2k}\xi^2_{0|A} - \frac{k+2}{2k}\langle \xi^2_{\perp|A}\rangle_A - \langle\sigma^2\rangle_A$$

$$\begin{aligned}\langle e^{2\Omega}\,{}^{k}N\rangle_A = {} & \frac{n^2}{2}(\rho_T + 3P_T) + 2\left(\frac{A}{4} - 1\right)\frac{k+2}{2k}\langle \xi^2_{\perp|A}\rangle_A - \frac{k+2}{2k}\xi^2_{0|A} - \langle\sigma^2\rangle_A \\ & + \left[\left(\frac{4}{k} - 1\right)(A+2) + \left(4 - \frac{4}{k}\right)\right]\langle e^{2\Omega}(\partial\Omega)^2\rangle_A \\ & + \frac{k+2}{2k}\frac{n}{a^3}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a^3}{n}\xi_{0|A}\right)\end{aligned}$$

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5. ${}^{k}N$ depends on $d\xi/dt$

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Minimax strategy -- choose A to minimise
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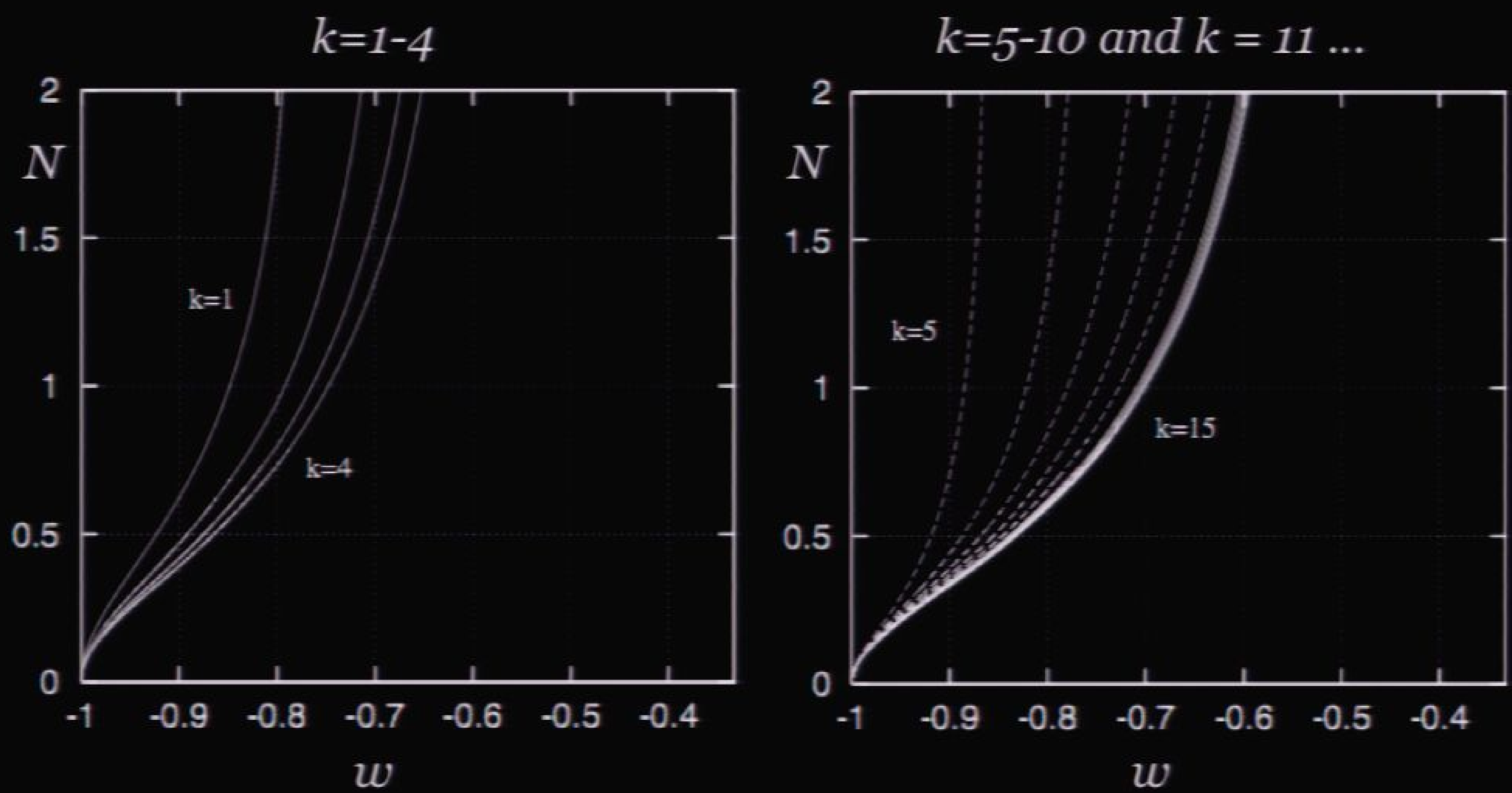
$$v(t) = t\xi_{0|A}(t)$$

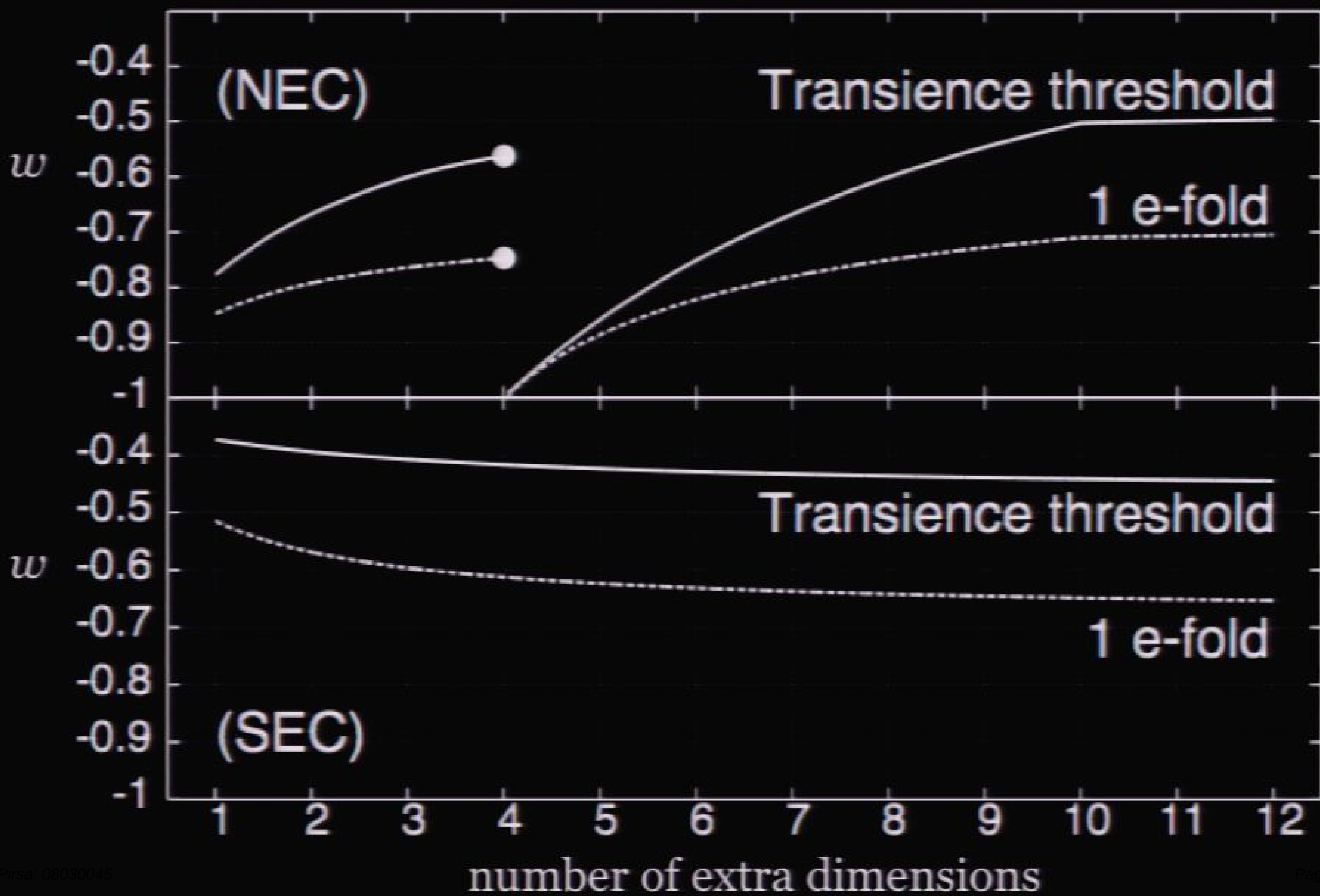
$$t\frac{\mathrm{d}v}{\mathrm{d}t} = \alpha_2 v^2 + \alpha_1 v + \alpha_0$$

This gives a differential equation and boundary conditions obeyed by the optimal solution

e-folds (I)



e-folds (II)



(NEC)
Transience threshold
1 e-fold
(SEC)
Transience threshold
1 e-fold
w
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
1 2 3 4 5 6 7 8 9 10 11 12
number of extra dimensions

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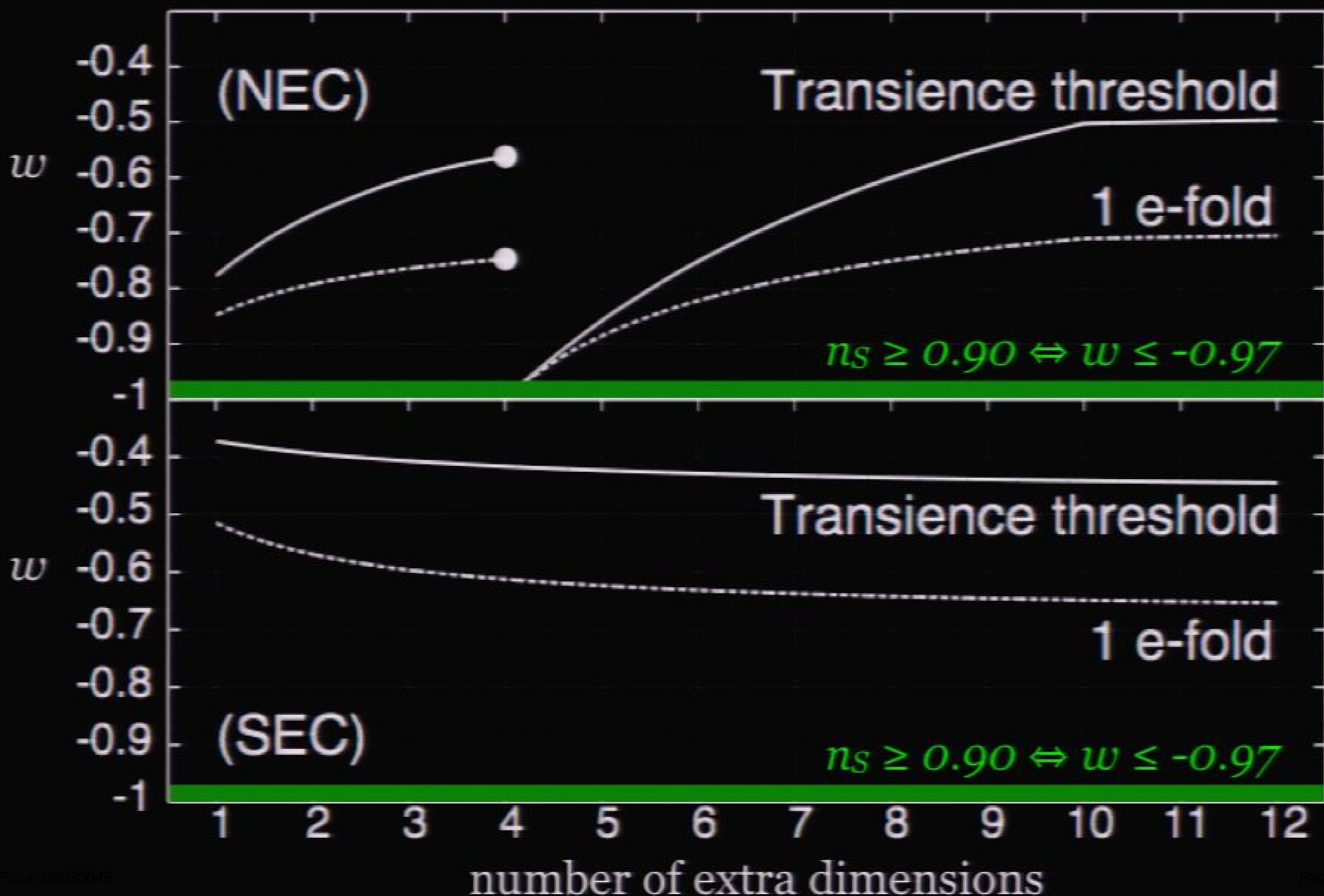

(NEC)
Transience threshold
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$n_S \geq 0.90 \Leftrightarrow w \leq -0.97$
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Conclusions

- *de Sitter case:* weakened the energy condition to the NEC in a variety of cases
- *w > -1 case:* constrained the number of allowed e-foldings of accelerated expansion consistent with energy conditions
- Instead of constructing examples of models, we make statements about broad classes.
- Challenge -- many models with acceleration must violate energy conditions consistently to be viable.