Title: Quantum Resolution of Cosmological Singularities using AdS/CFT

Date: Mar 07, 2008 11:00 AM

URL: http://pirsa.org/08030043

Abstract:

Quantum resolution of cosmological singularities using AdS/CFT

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Vrije Universiteit Brussel & The International Solvay Institutes

work with T. Hertog and N. Turok

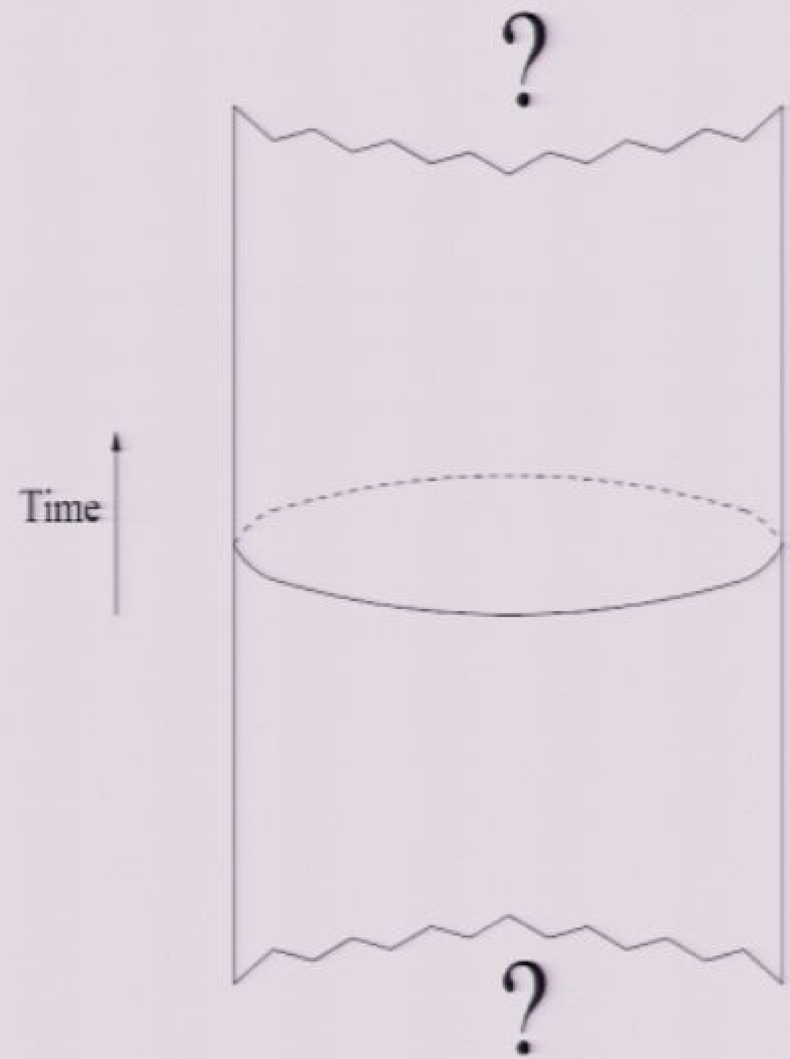
Workshop on "Novel Theories of the Early Universe"

Perimeter Institute, March 7, 2008

AdS cosmologies: basic idea

Starting point: supergravity solutions in which smooth, asymptotically AdS initial data evolve to a big crunch singularity in the future (and to a big bang singularity in the past).

Can a dual gauge theory be used to study the singularity in quantum gravity?



Plan

- Bulk theory: strings on $AdS_5 \times S^5$ with modified boundary conditions
- Boundary theory: N=4 SYM with unstable double trace deformation
- Beyond the singularity: self-adjoint extensions
- Quantum evolution of the homogeneous component
- Particle creation: does the universe bounce?
- Conclusions

AdS cosmologies: the bulk theory

Compactify 10d type IIB sugra on S^5 and truncate (consistently) to

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\nabla\phi)^2 + \left(15e^{2\gamma\phi} + 10e^{-4\gamma\phi} - e^{-10\gamma\phi}\right)\right]$$

Freedman, Gubser, Pilch, Warner

with $\gamma = \sqrt{2/15}$

This describes a scalar whose mass $m^2 = -4/R^2_{AdS}$ saturates the BF bound.

Breitenlohner, Freedman

In all solutions asymptotic to the AdS_5 metric

$$ds^2 = R^2_{AdS}\left(-(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_3^2\right)$$

the scalar field decays at large radius as

$$\phi(r) \sim \frac{\alpha\log r}{r^2} + \frac{\beta}{r^2}$$

Consider boundary conditions

$$\alpha = f\beta$$

AdS cosmologies: bulk solution

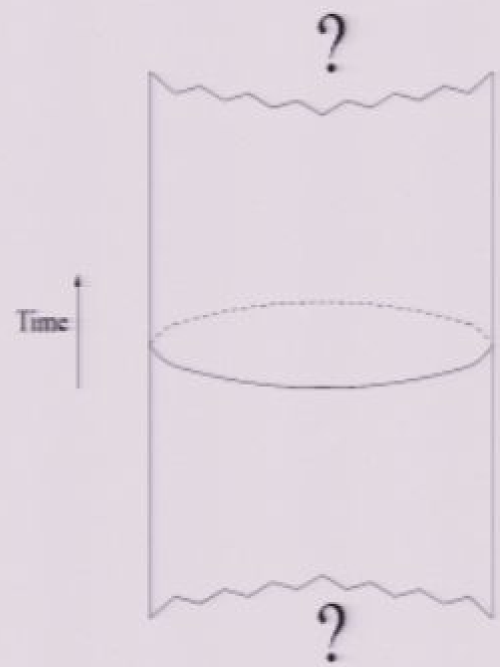
$$\phi(r) \sim \frac{\alpha \log r}{r^2} + \frac{\beta}{r^2} \qquad\qquad \alpha = f\beta$$

Standard supersymmetric boundary conditions: $f = 0$. Preserves AdS symmetry group. Pure AdS solution is stable ground state.

Gibbons, Hull, Warner; Townsend

For $f > 0$, there exist smooth asymptotically AdS initial data that evolve to a singularity that (plausibly) reaches the boundary of AdS in finite global time.

cf. Hertog, Horowitz



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The AdS/CFT correspondence

Type IIB string theory on $AdS_5 \times S^5$ with string coupling g_s and radius of curvature R	$\xleftrightarrow{\text{dual}}$	$\mathcal{N}=4$ SU(N) super-Yang-Mills theory with Yang-Mills coupling g_{YM} on the 3+1 dimensional boundary of AdS_5
g_s	$\longleftrightarrow$	$1/N$
$(R/l_s)^4$	$\longleftrightarrow$	$(g_{YM})^2 N$

The AdS/CFT correspondence gives a non-perturbative definition of string theory in (asymptotically) anti-de Sitter space.

We shall work at large N (planar limit in field theory).

Our field theory analysis will be trustworthy for small 't Hooft coupling, corresponding to a stringy bulk. Not clear yet to what extent it can be extrapolated to large 't Hooft coupling.

AdS cosmologies: dual field theory

$$\phi(r) \sim \frac{\alpha \log r}{r^2} + \frac{\beta}{r^2} \qquad\qquad \alpha = f\beta$$

- For $f = 0$ (supersymmetric) boundary conditions: dual field theory is N=4 SYM on R x S^3.

- Scalar field ϕ couples to operator $\mathcal{O} = \frac{1}{N}\mathrm{Tr}[\Phi_1^2 - \frac{1}{5}\sum_{i=2}^{6}\Phi_i^2]$

- Boundary conditions with $f > 0$ correspond to deforming the CFT by a double trace operator:

$$S \to S + \frac{f}{2}\int \mathcal{O}^2$$

Aharony, Berkooz, Silverstein; Witten; Berkooz, Sever, Shomer

This corresponds to a potential that is unbounded from below, and $\langle\mathcal{O}\rangle$ becomes infinite in finite time.

cf. Hertog, Horowitz

- Does this extend to the full quantum theory?

cf. Elitzur, Giveon, Porrati, Rabinovici

Renormalization of the boundary theory.

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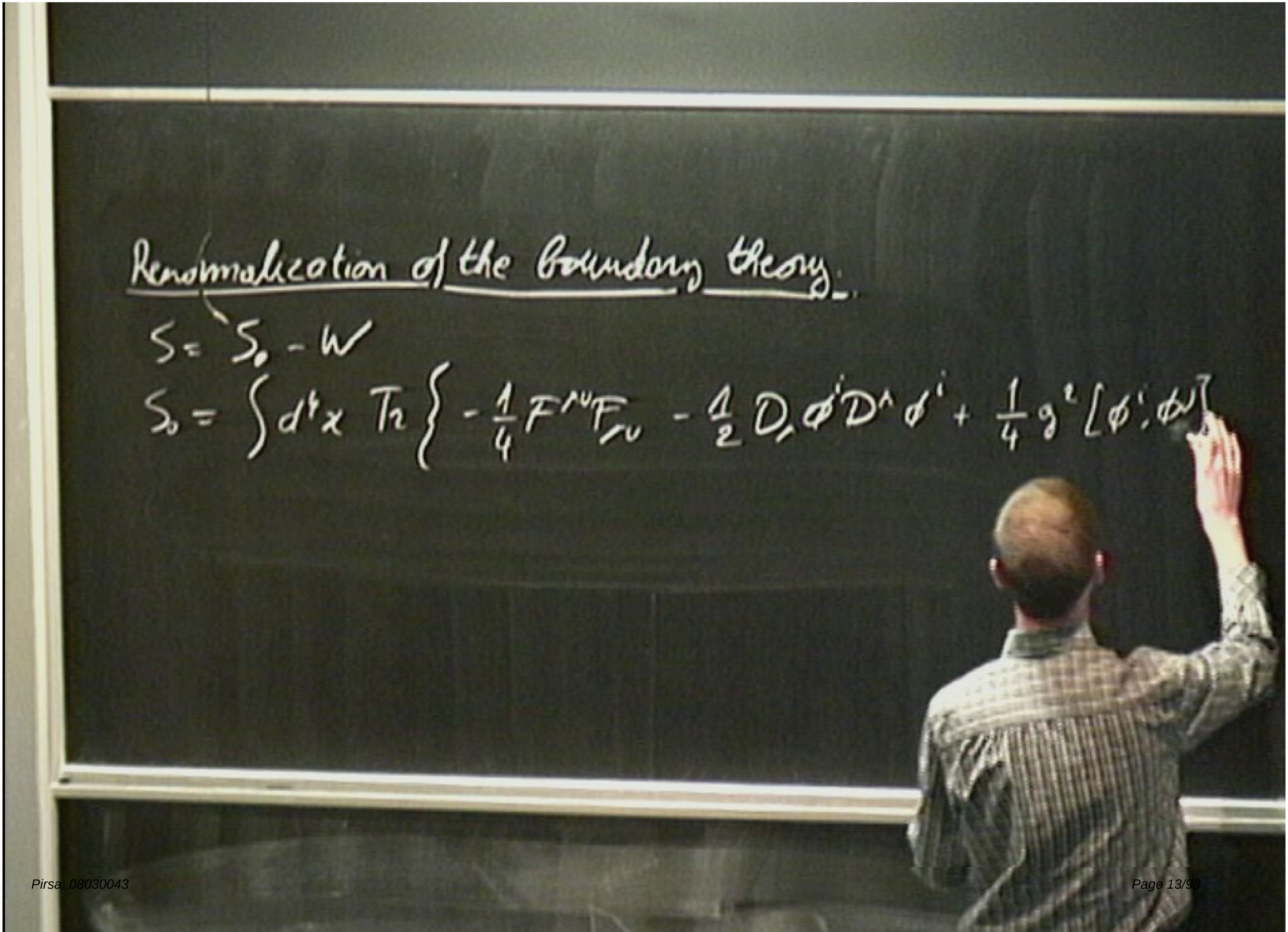
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The O^2 vertex in double line notation

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Renormali

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Renormalization of f in large N limit (with fixed f)

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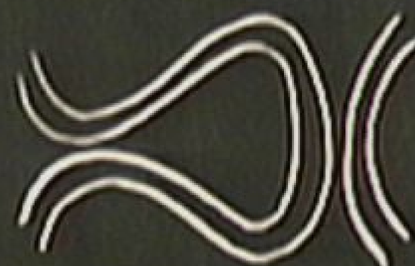

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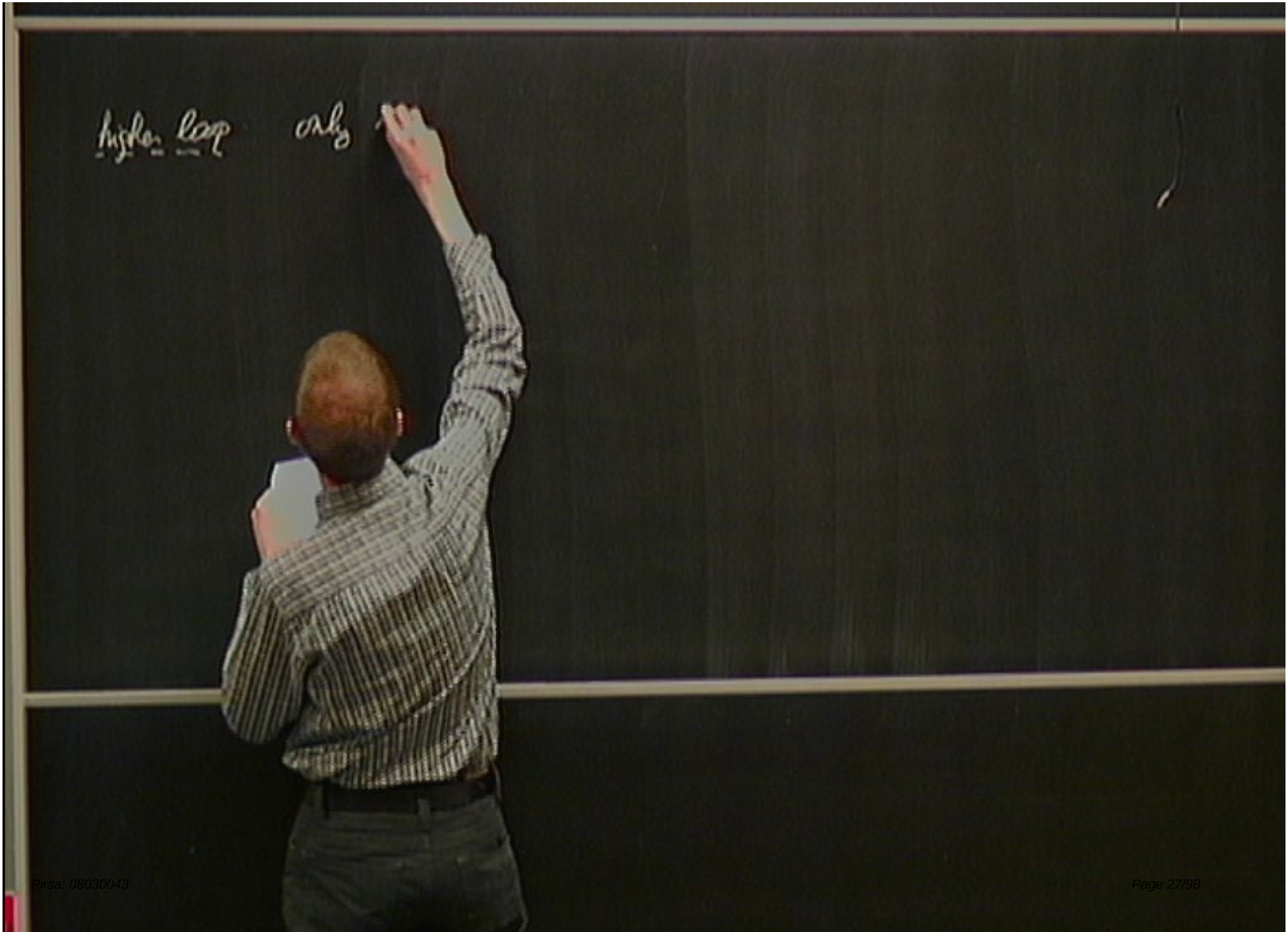
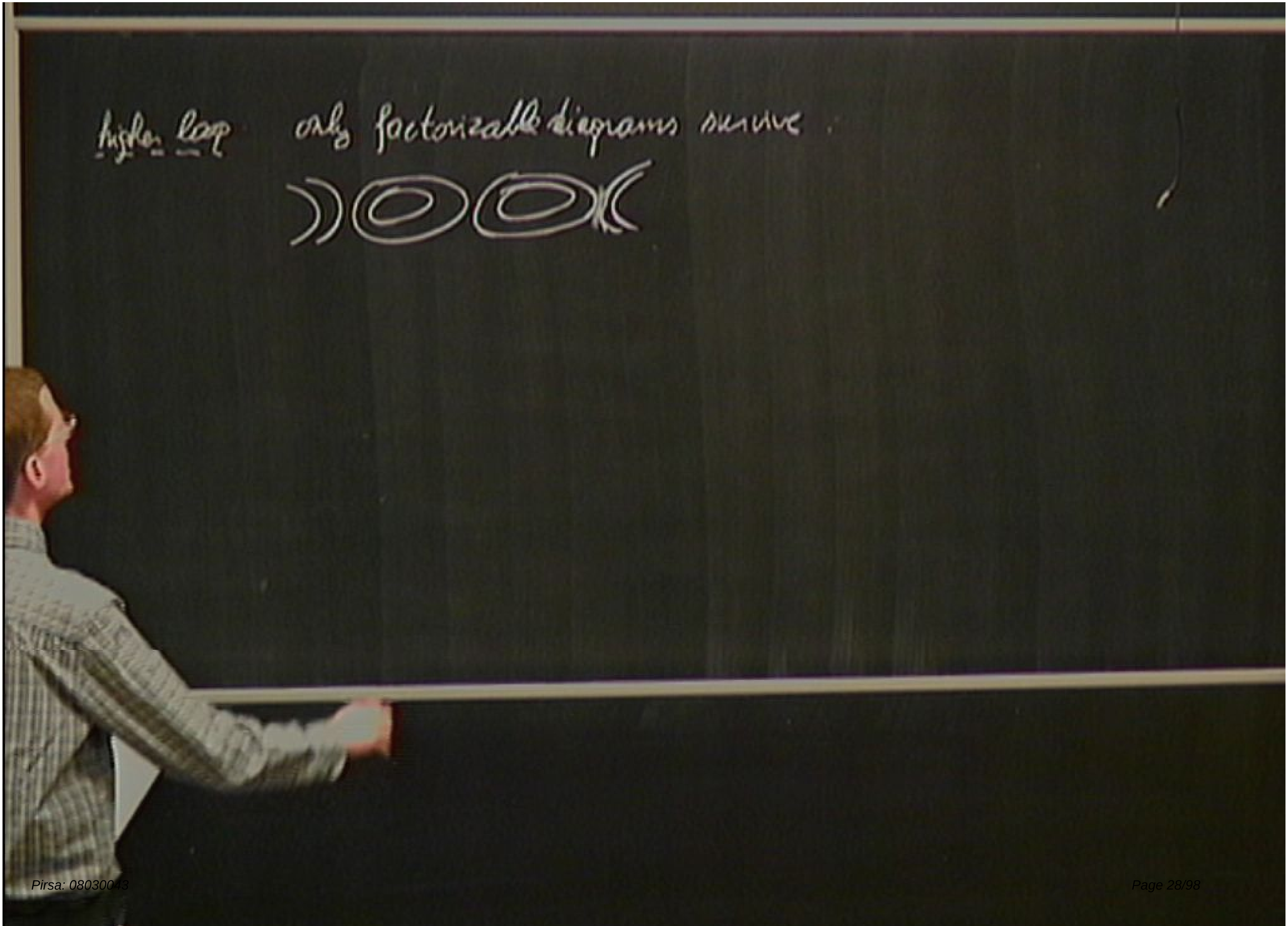

higher loop only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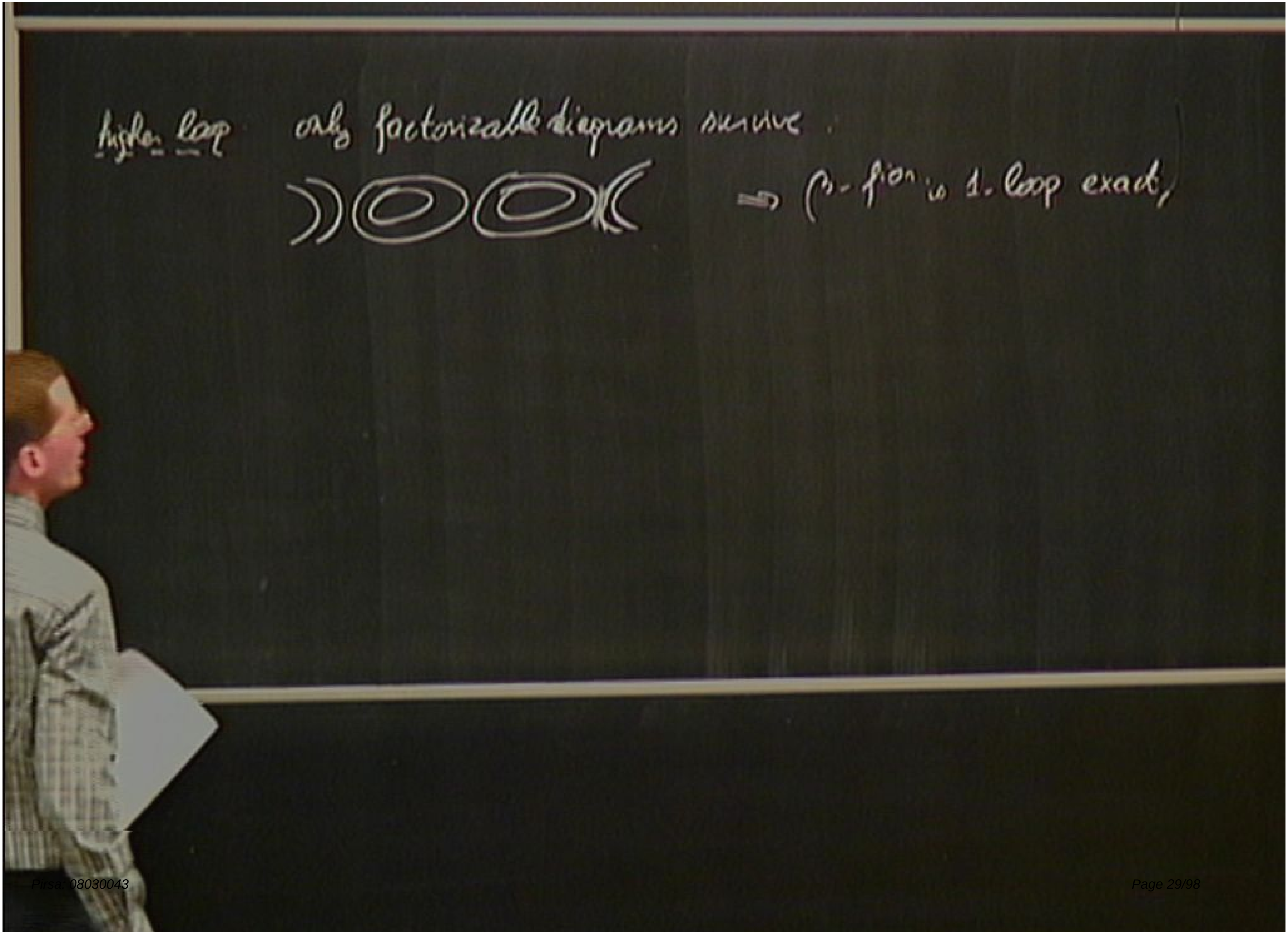
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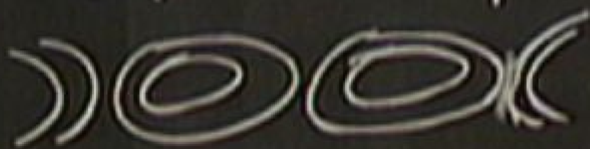
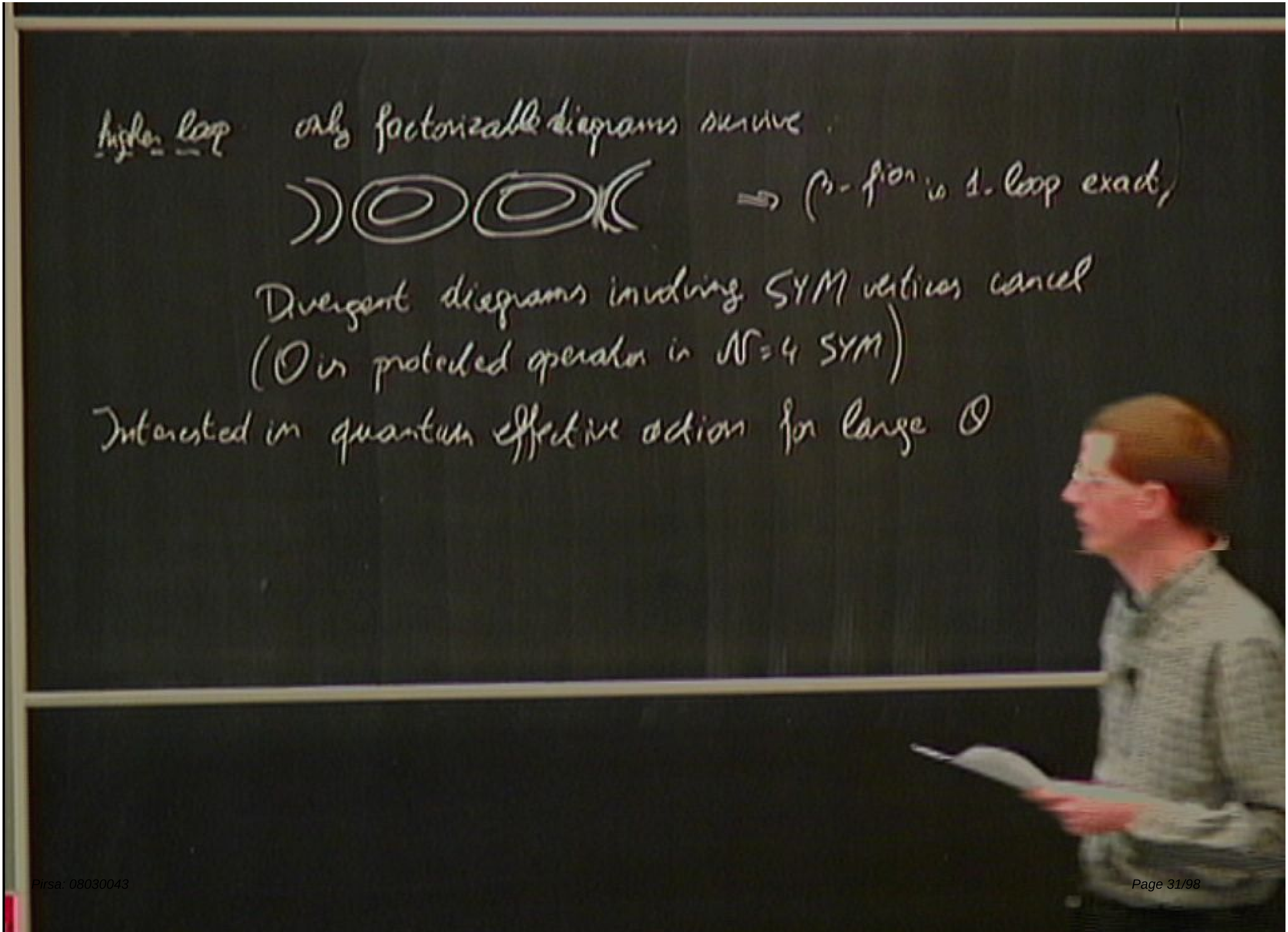


$\Rightarrow$ (β-f^{ion} is 1-loop exact)

Divergent diagrams involving SYM vertices cancel ($\mathcal{O}$ is protected operator in $\mathcal{N}=4$ SYM)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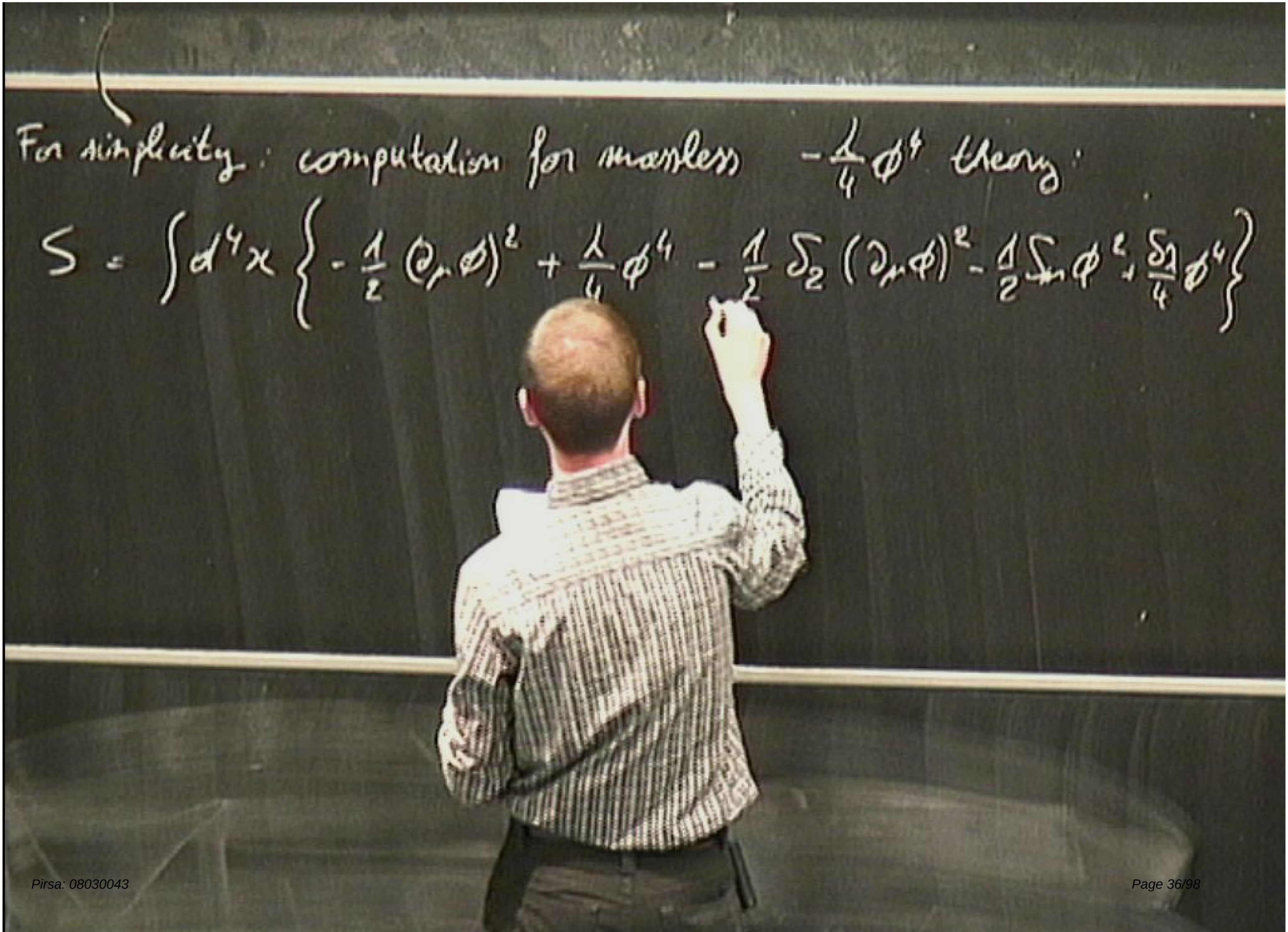
- derivative expansion

For simplicity: computation for massless $-\frac{\lambda}{4}\phi^4$ theory: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$S = \int d^4x \left\{ -\frac{1}{2}(\partial_\mu\phi)^2 + \frac{\lambda}{4}\phi^4 - \frac{1}{2}\delta_2(\partial_\mu\phi)^2 - \frac{1}{2}\delta_m\phi^2 + \frac{\delta_\lambda}{4}\phi^4 \right\}$

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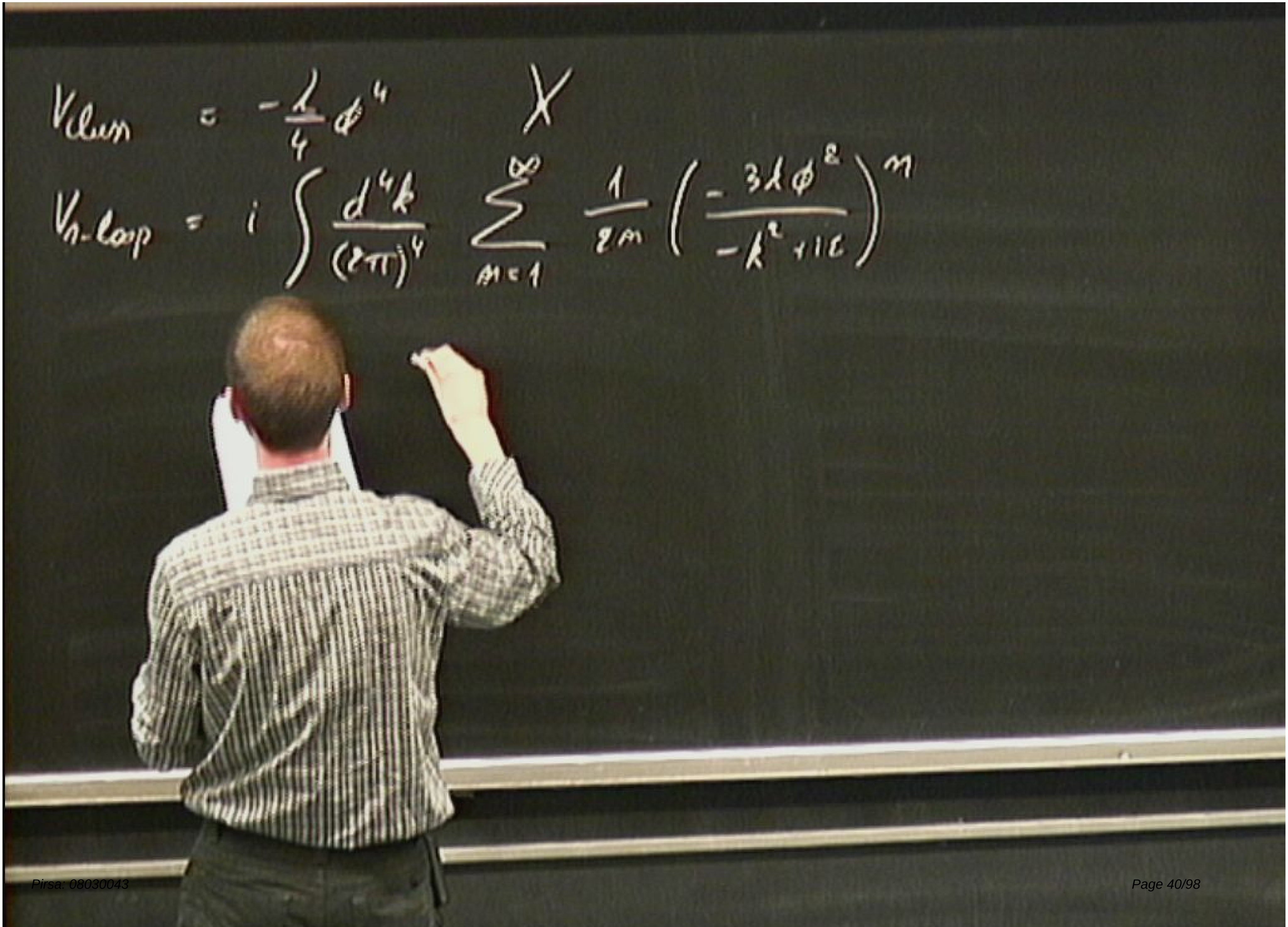
$$\Gamma = \int d^4x \left\{ -V(\phi) - \frac{1}{2}(\partial_\mu \phi)^2 Z(\phi) + \dots \right\}$$

$$V_{\text{class}} = -\frac{\lambda}{4}\phi^4 \qquad \times$$


$V_{clas} = -\frac{\lambda}{4}\phi^4$
$V_{1\text{-}loop} = i\int \frac{d^4k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{1}{2n}\left(\frac{-3\lambda\phi^2}{-k^2+i\epsilon}\right)^n$

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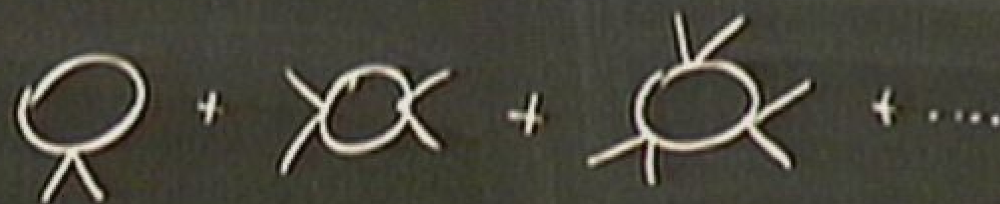

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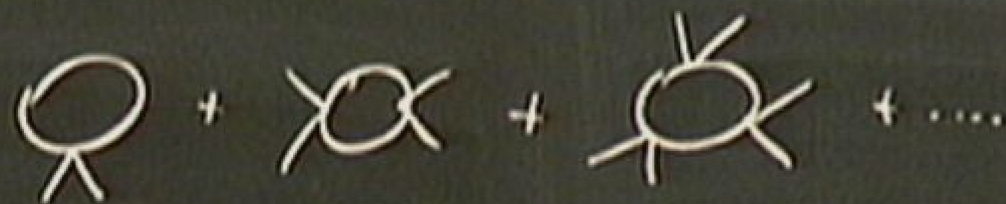

$$\underset{\substack{\text{Wick}\\ \text{rotation}}}{=} \frac{1}{32\pi^2}\int d^4k \; \ln\left(1-\frac{3\lambda\phi^2}{k^2}\right)$$

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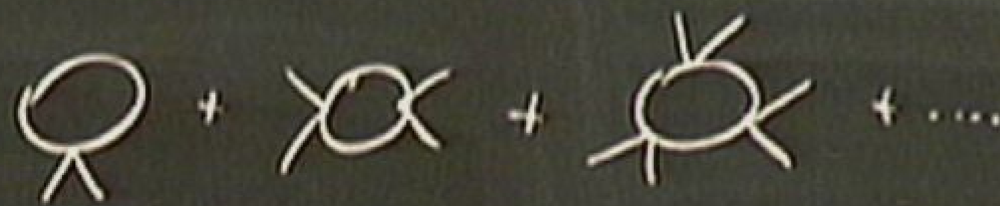

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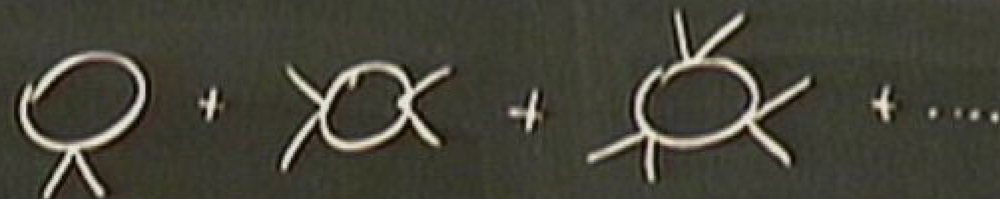

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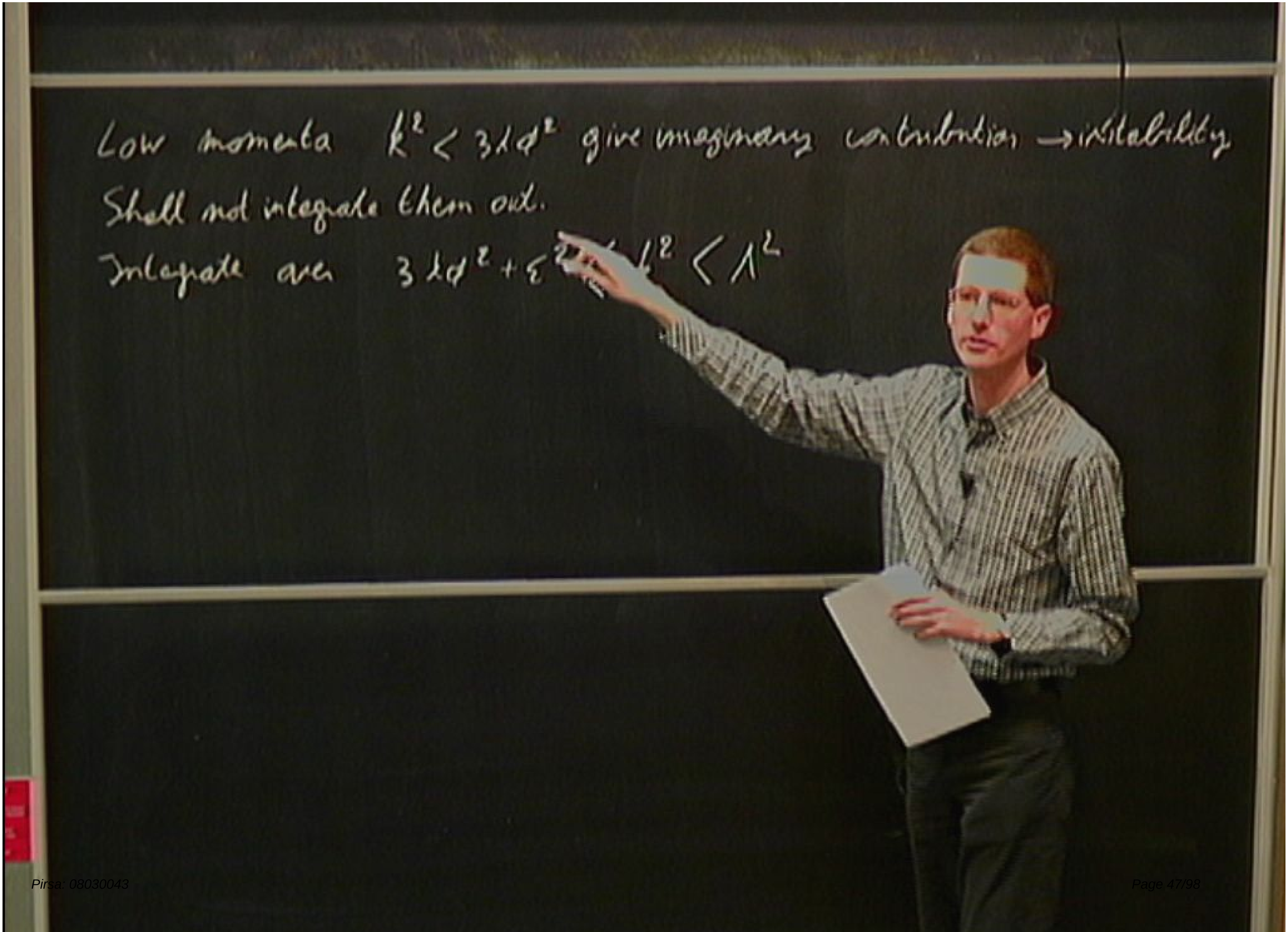
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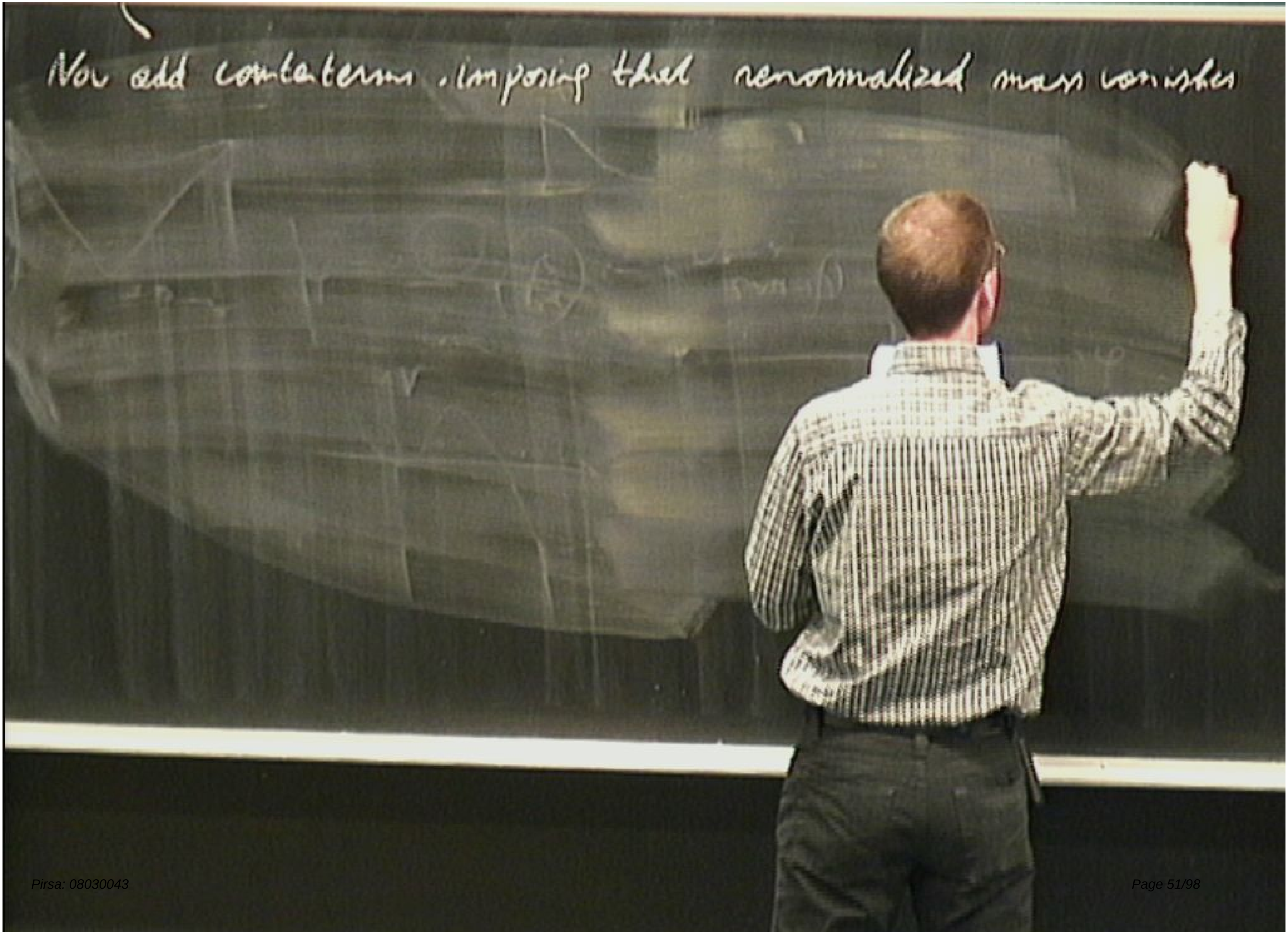
Result: $$V_{1\text{-loop}} = \frac{1}{32\pi^2}\left\{-3\lambda\Lambda^2\phi^2 + \frac{9\lambda^2\phi^4}{4} + \frac{9\lambda^2\phi^4}{2}\ln\left(\frac{3\lambda\phi^2}{\Lambda^2}\right)\right\}$$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$$\Rightarrow V(\phi) = -\frac{\lambda_\mu}{4}\phi^4 + \frac{9\lambda_\mu^2 \, \phi^4 \ln(\phi^2/\mu^2)}{64\pi^2}$$

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Demand that $V(\phi)$ be independent of μ.

$\mu \frac{d\lambda_\mu}{\ldots}$

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(ignored contribution from $\frac{d}{d\mu}$ hitting λ_μ^2, OK if $\lambda_\mu \ln(\phi/\mu) \ll 1$)

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Choose $\mu = \phi \rightarrow V(\phi) = -\dfrac{4\pi^2\phi^4}{9\ln\left(\frac{\phi^2}{M^2}\right)}$

Suppose $0 < \lambda_{\phi_0} \ll 1$, then $M < |\phi_0|$

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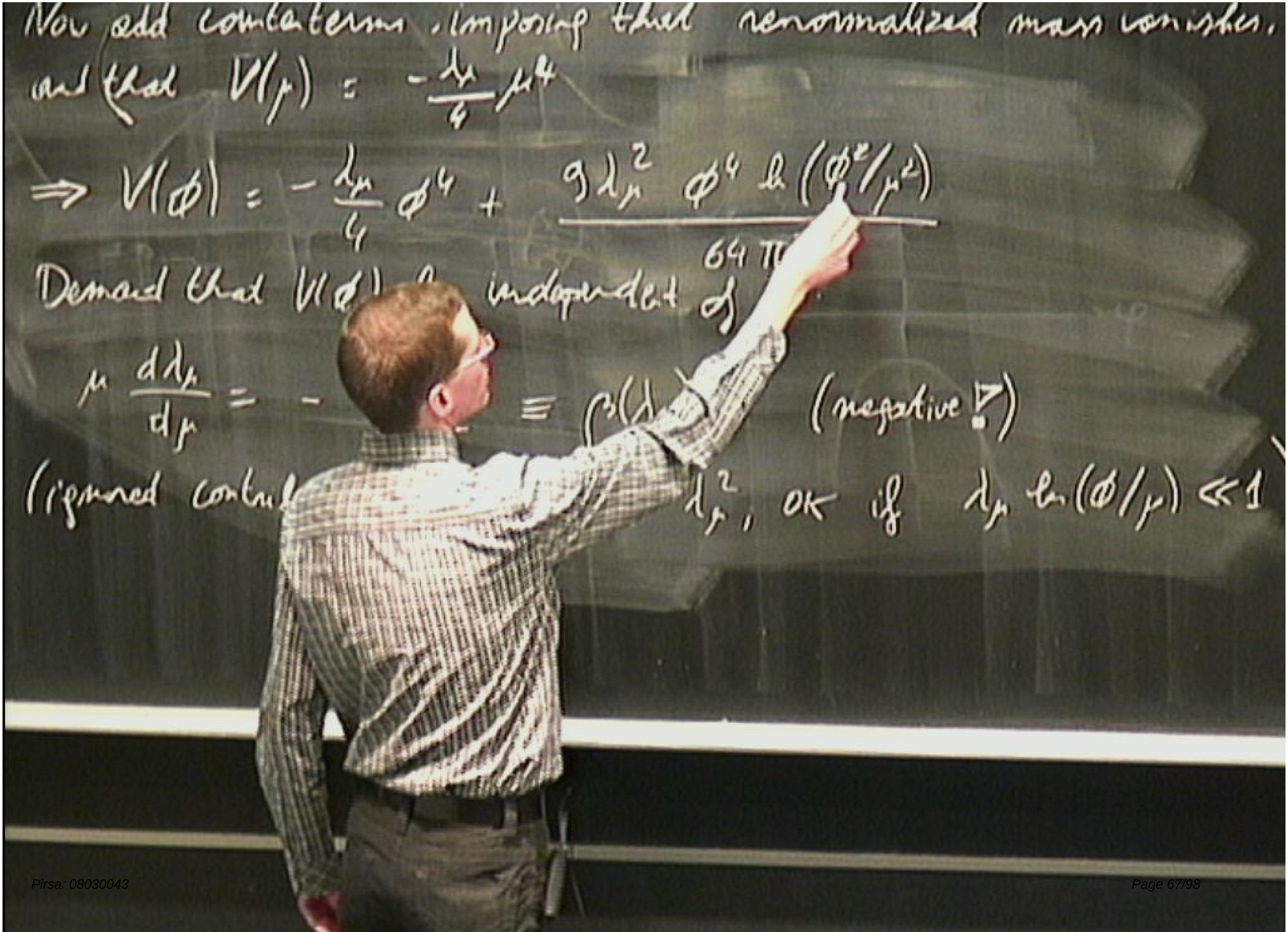
arbitrary scale (dim. transmutation)

Choose $\mu = \phi$ $\Rightarrow$ $V(\phi) = - \dfrac{4\pi^2 \phi^4}{9 \ln\left(\frac{\phi^2}{M^2}\right)}$

Suppose $0 < \lambda_{\phi_0} \ll 1$, then $M < |\phi_0|$ and 1 loop potential reliable for $|\phi| > |\phi_0|$. In particular, $V(\phi) \to -\infty$ for $|\phi| \to \infty$.

$\frac{}{M^2}$ $\frac{}{M^2}$ $\sim \frac{\lambda^2}{M^2}$ $\sim \frac{\lambda^3}{M^4}$



Now add counterterms, imposing that renormalized mass vanishes,
and that $V(\mu) = -\frac{\lambda_\mu}{4}\mu^4$
$\Rightarrow V(\phi) = -\frac{\lambda_\mu}{4}\phi^4 + \frac{9\lambda_\mu^2\,\phi^4 \ln(\phi^2/\mu^2)}{64\pi^2}$
Demand that $V(\phi$ independent of
$\mu\frac{d\lambda_\mu}{d\mu} = -$ $\equiv \beta(\lambda$ (negative!?)
(ignored contrib λ_μ^2, OK if $\lambda_\mu \ln(\phi/\mu) \ll 1$

Solution: $\lambda_\mu = \dfrac{16\pi^2}{9\ln\left(\frac{\mu^2}{M^2}\right)}$

arbitrary scale (dim. transmutation)

choose $\mu = \phi \Rightarrow V(\phi) = -\dfrac{4\pi^2\phi^4}{9\ln\left(\frac{\phi^2}{M^2}\right)}$

Suppose $0 < \lambda_{\phi_0} \ll 1$, then $M < |\phi_0|$ and 1-loop potential reliable for $|\phi| > |\phi_0|$. In particular, $V(\phi) \to -\infty$ for $|\phi| \to \infty$.

$\sim \dfrac{\lambda^2}{M^2}$ $\sim \dfrac{\lambda^2}{M^4}$

The AdS/CFT correspondence

Type IIB string theory on $AdS_5 \times S^5$ with string coupling g_s and radius of curvature R	$\xleftrightarrow{\text{dual}}$	$\mathcal{N}$=4 SU(N) super-Yang-Mills theory with Yang-Mills coupling g_{YM} on the 3+1 dimensional boundary of AdS_5

$$g_s \longleftrightarrow 1/N$$

$$(R/l_s)^4 \longleftrightarrow (g_{YM})^2 N$$

The AdS/CFT correspondence gives a non-perturbative definition of string theory in (asymptotically) anti-de Sitter space.

We shall work at large N (planar limit in field theory).

Our field theory analysis will be trustworthy for small 't Hooft coupling, corresponding to a stringy bulk. Not clear yet to what extent it can be extrapolated to large 't Hooft coupling.

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AdS cosmologies: dual field theory

$$\phi(r) \sim \frac{\alpha \log r}{r^2} + \frac{\beta}{r^2} \qquad \alpha = f\beta$$

- For $f = 0$ (supersymmetric) boundary conditions: dual field theory is N=4 SYM on R x S^3.
- Scalar field ϕ couples to operator $\mathcal{O} = \frac{1}{N}\mathrm{Tr}[\Phi_1^2 - \frac{1}{5}\sum_{i=2}^{6}\Phi_i^2]$
- Boundary conditions with $f > 0$ correspond to deforming the CFT by a double trace operator:

$$S \to S + \frac{f}{2}\int \mathcal{O}^2$$

Aharony, Berkooz, Silverstein; Witten; Berkooz, Sever, Shomer

This corresponds to a potential that is unbounded from below, and $\langle \mathcal{O} \rangle$ becomes infinite in finite time.

cf. Hertog, Horowitz

- Does this extend to the full quantum theory?

cf. Elitzur, Giveon, Porrati, Rabinovici

Plan

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Quantum mechanics with unbounded potentials

Consider $\hat{H} = -\frac{d^2}{dx^2} + V(x)$ with $V(x) = -a^2 x^p$ for $x > 0$ and $p > 2$. For such potentials, classical trajectories can reach infinity in finite time. So do quantum mechanical wavepackets, which would seem to lead to loss of probability/unitarity.

Unitarity can be restored by restricting the domain of allowed wavefunctions such that the Hamiltonian is self-adjoint ("self-adjoint extension"):

$$(\hat{H}\phi_1, \phi_2) = (\phi_1, \hat{H}\phi_2) \quad \leftrightarrow \quad \left[\frac{d\phi_1^*}{dx}\phi_2 - \phi_1^* \frac{d\phi_2}{dx}\right]_{x=\infty} = 0$$

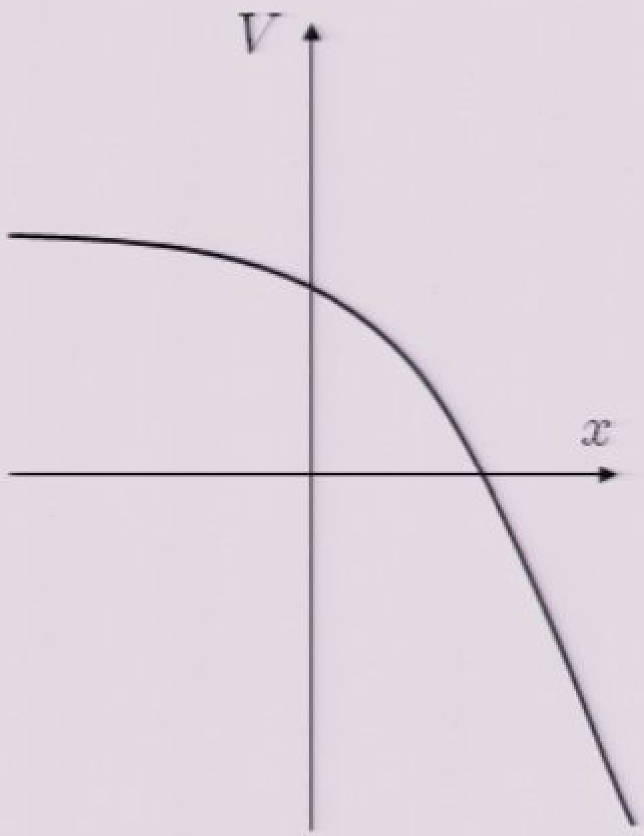
The WKB energy eigenfunctions $[2(E + a^2 x^p)]^{-1/4} \exp\left(\pm i \int_0^x \sqrt{2(E + a^2 y^p)}dy\right)$

are an increasingly good approximation for large x. Unitarity can be achieved by only allowing the linear combination that for large x behaves as

$$\psi_E^\alpha(x) \sim (2a^2 x^p)^{-1/4} \cos\left(\frac{\sqrt{2}a x^{p/2+1}}{p/2+1} + \alpha\right)$$

Reed, Simon

nterpretation of the self-adjoint extensions



Rightmoving wavepacket disappearing at infinity is always accompanied by leftmoving wavepacket appearing at infinity (think of brick wall at infinity)

Carreau, Farhi, Gutmann, Mende

Energy spectrum consists of bound states (energy levels depend on phase α) as well as scattering states (if potential is bounded from above)

Self-adjoint extensions in quantum field theory

$$V(\mathcal{O}) = -\frac{\mathcal{O}^2}{\ln(\mathcal{O}^2/\tilde{M}^2)} \qquad \mathcal{O} = \frac{1}{N}\mathrm{Tr}[\Phi_1^2 - \frac{1}{5}\sum_{i=2}^{6}\Phi_i^2]$$

Focus on steepest unstable direction:

$$\Phi_1(x) = \phi(x)U$$

canonically normalized scalar field

constant Hermitean matrix, $\mathrm{Tr}U^2 = 1$

$$V(\phi) = -\frac{\lambda_\phi}{4}\phi^4 \qquad \text{with} \qquad \lambda_\phi = \frac{1}{N^2 \ln\left(\frac{\phi}{NM}\right)}$$

For now, ignore running of coupling.

Self-adjoint extensions in quantum field theory

$$V = -\frac{\lambda}{4}\phi^4$$

Equation of motion: $\partial^2\phi = -\lambda\phi^3 + \cancel{\frac{1}{6}R(S^3)\phi}$

Ricci scalar; ignore for large ϕ

Homogeneous background solution: $\phi = \sqrt{(2/\lambda)}\,t^{-1}$. Define $\chi = (2/\lambda)^{1/2}\phi^{-1}$.

Can construct generic, spatially inhomogeneous solution to e.o.m. in expansion around (spacelike) singular surface $t = t_s(\mathrm{x})$ where ϕ is infinite:

time delay

$$\chi(t,\mathrm{x}) = -t + t_s(\mathrm{x}) + \frac{1}{6}(t_s - t)^2\nabla^2 t_s - \frac{1}{24}(t_s - t)^4(\nabla^4 t_s) + \ldots$$

$$+\frac{\lambda\rho(\mathrm{x}_s)}{10}(t_s - t)^5 + \cdots + \text{(non-linear in } \nabla t_s\text{)}$$

energy perturbation

Main observation: spatial gradients become unimportant near the singularity

→ evolution becomes ultralocal

Thus different spatial points decouple, and we can define a self-adjoint extension point by point!

How unique is the self-adjoint extension?

A priori ambiguity: one phase for every point of S^3

- It is natural to choose the theory (e.g. Lagrangian) to be symmetric
- It is unnatural to choose a state that is very symmetric

Choice of self-adjoint extension is part of definition of theory (not a choice of state)

→ Natural to choose same phase at every point

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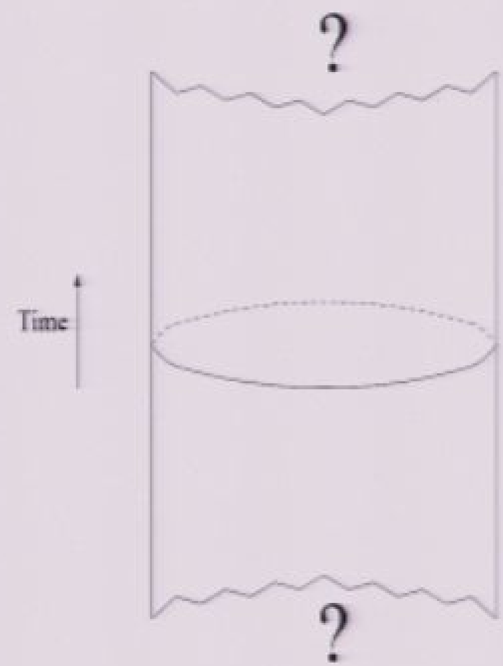
The homogeneous mode is a quantum mechanical variable

Boundary field theory lives on $\underbrace{R}_{\text{time}} \times \underbrace{S^3}_{\text{finite volume space}}$



Decompose $\phi(t, \mathrm{x}) = \bar{\phi}(t) + \delta\phi(t, \mathrm{x})$

First ignore inhomogeneous modes $\delta\phi(t, \mathrm{x})$.
which start out in ground state.

Kinetic term for homogeneous mode: $\underbrace{V_3}_{\text{finite "mass"}} \int dt \, \frac{1}{2} \dot{\bar{\phi}}^2$

Wave function will undergo quantum spreading. This will give rise to UV cutoff on creation of inhomogeneous modes.

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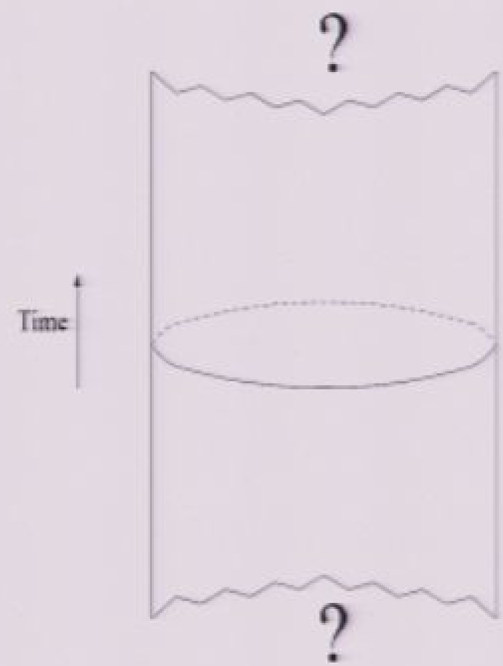
time

finite volume space



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finite “mass”

Wave function will undergo quantum spreading. This will give rise to UV cutoff on creation of inhomogeneous modes.

Complex classical solutions and quantum mechanics

- Semiclassical expansion for QM particle: $\Psi(x_f, t_f) \sim A(x_f, t_f) e^{iS(x_f, t_f)/\hbar}$
- Solved to leading order in $\hbar$ by $S = S_{Cl}(x_f, t_f)$: classical action with

  1) initial condition at $t = t_i$: $$x + 2i\frac{pL^2}{\hbar} = x_c + 2i\frac{p_c L^2}{\hbar}$$

  i.e. Gaussian wavepacket centered around (x_c, p_c) with spread L in x

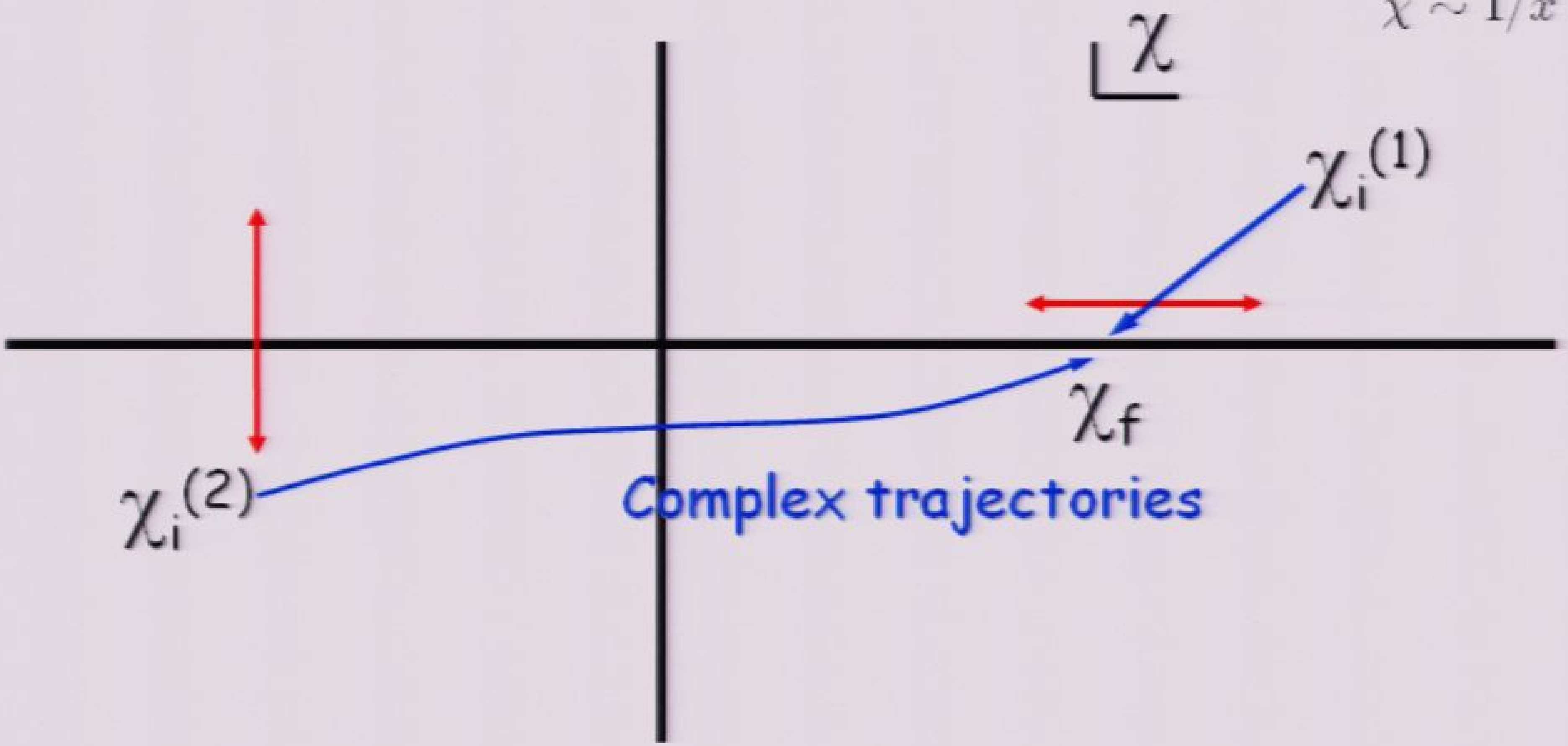
  2) final condition at $t = t_f$: $$x = x_f$$

- Classical solution with these boundary conditions is complex for nonzero spread L (unless x_f lies on classical trajectory)
- Apply to self-adjoint extension: add "mirror" wavefunction corresponding to

  initial condition at $t = t_i$: $$x + 2i\frac{pL^2}{\hbar} = -\left(x_c + 2i\frac{p_c L^2}{\hbar}\right)$$

The self-adjoint extension via complex classical solutions

$\chi \sim 1/x$



$$\Psi(x_f, t_f) = A_1(x_f, t_f) e^{iS_1/\hbar} + A_2(x_f, t_f) e^{i\theta} e^{iS_2/\hbar}$$

$\chi_i^{(1)} \to \chi_f$ $\qquad$ $\chi_i^{(2)} \to \chi_f$

Can check: belongs to domain of self-adjoint extension.

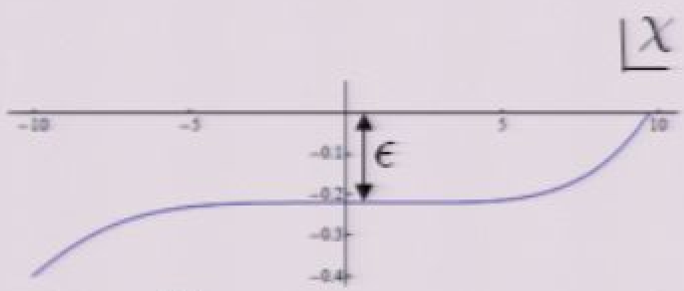
The spread of the wavepacket

Initial Gaussian wave packet with nearly zero energy, centered on ϕ_c

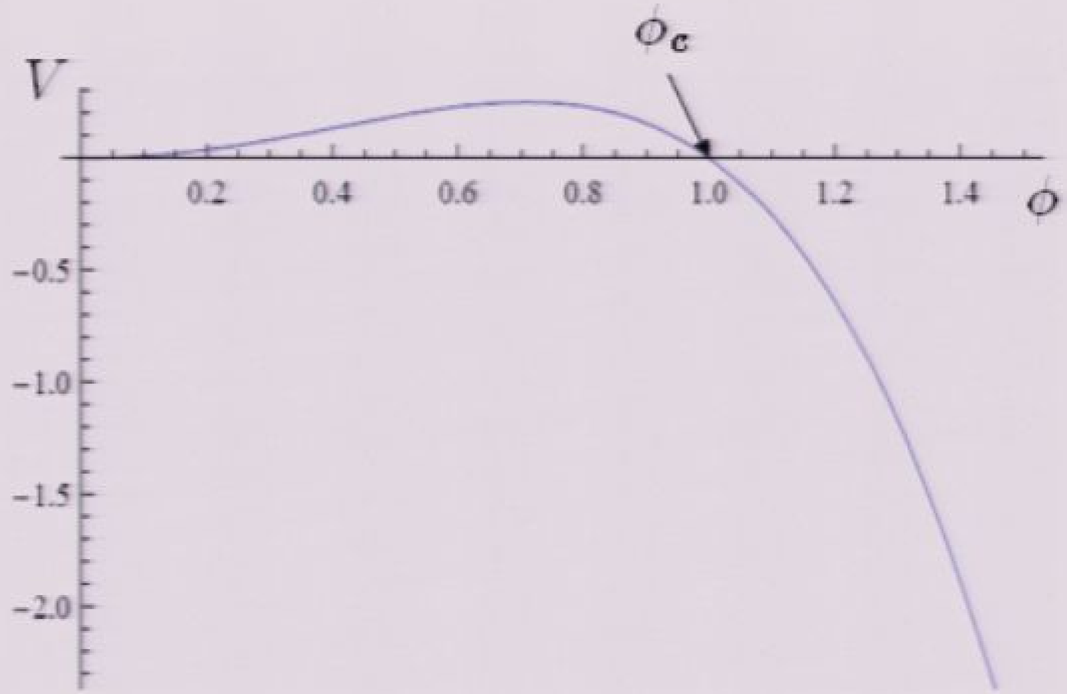
Time to roll to infinity and back is of order R_{AdS}

Introduce $\chi = \left(\frac{2}{\lambda}\right)^{1/2} \frac{1}{\phi}$

Classical solution: $\chi = |t|$



$\chi_i \approx -R_{AdS}$



For initial width $\Delta\chi_i$, final variance after bounce

$$\approx \Delta\chi_i^2 + R_{AdS}^2 (\Delta\dot{\chi}_i)^2$$

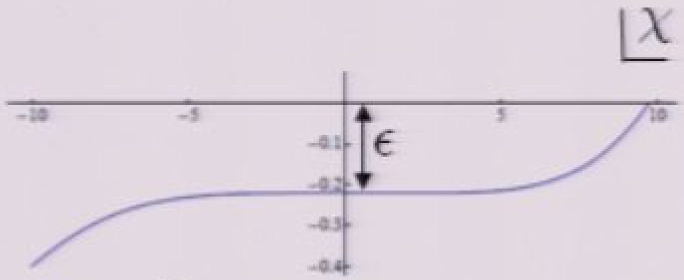
$$\approx \Delta\chi_i^2 + \frac{\lambda^2 R_{AdS}^4}{(\Delta\chi_i)^2}$$

Minimized by

$$(\Delta\chi_i)_{min} \approx \lambda^{1/2} R_{AdS} \ll R_{AdS}$$

The imaginary part of the complex classical solutions

Classical solution: $\chi = |t|$



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Minimized by

$$(\Delta\chi_i)_{min} \approx \lambda^{1/2} R_{AdS} \ll R_{AdS}$$

"Wave packet that miminizes its spread over the duration of the (putative) bounce"

Consider a more general initial wave packet with width $\Delta\chi_i = W(\Delta\chi_i)_{min}$

When discussing inhomogeneous modes and backreaction, it will be important to know how far the complex classical solutions stay away from the origin $\chi = 0$

Can show: for "most" χ_f, away from classical trajectory, $\epsilon \sim W^{-3}\lambda^{1/2} R_{AdS}$

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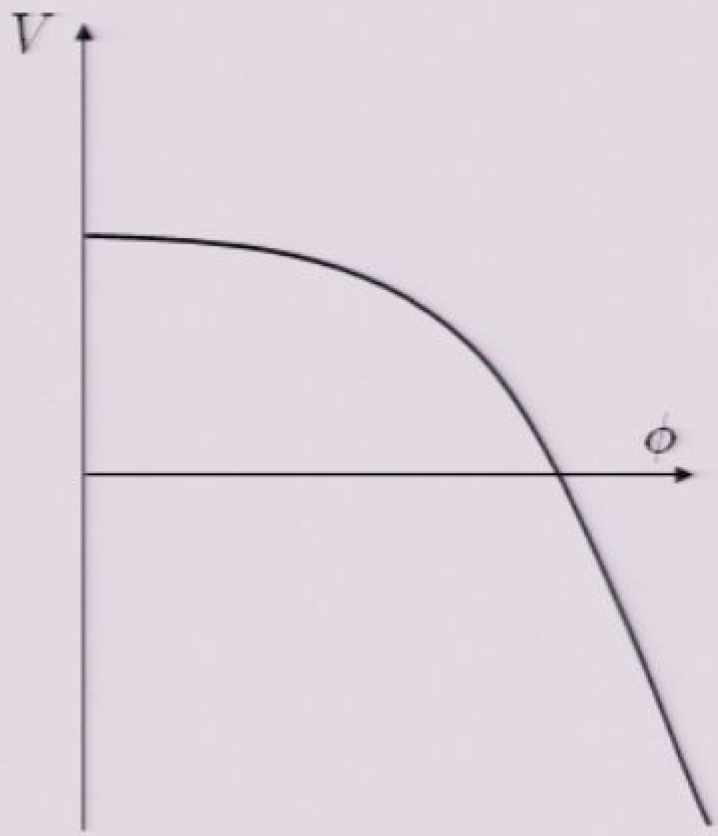
Does the universe bounce?



Consider the homogeneous mode $\bar{\phi}(t)$.

Self-adjoint extension would seem to imply that after rolling to infinity, it rolls up the hill again, returning to the original configuration → bounce in spacetime.

However, inhomogeneous modes $\delta\phi(t, \mathrm{x})$ may be created and may drain energy out of the homogeneous mode.

Thus we need to compute the energy in created particles and see how far the homogeneous mode can roll up the hill again.

Note: this is particle creation in the boundary field theory, not in spacetime!

ncluding the inhomogeneous modes

We have computed the wavefunction $\Psi\left(t_f, \bar{\phi}\right)$, ignoring $\delta\phi$, using complex classical solutions.

We want to compute the full wavefunctional $\Psi\left(t_f, \bar{\phi}, \delta\phi(\mathrm{x})\right)$ at some late time of interest.

- Semiclassical expansion: need complex classical solutions for all modes.
- Work to quadratic order in $\delta\phi$ in the action → linearized field equations around complex homogeneous background.
- Ignore backreaction on homogeneous mode.
- Check consistency: small imaginary part of complex solution for $\bar{\phi}(t)$ will be crucial since it provides a UV cutoff on particle creation.

Other particle species:

- "Heavy" modes with mass $\mathcal{M} \sim g_{YM}\phi$ behave adiabatically and can be integrated out:

$$\dot{\mathcal{M}}/\mathcal{M}^2 \sim 1/(g_t N \ln(\phi/NM))^{1/2}$$

- "Light" modes: massless gauge bosons, Higgs modes acquiring mass only from double trace deformation: similar analysis as for $\delta\phi$

Particle creation using complex classical solutions

Linearized perturbations around time-dependent, spatially homogeneous background:

$$S = \int dt \frac{1}{2}\left(\dot{q}^2 - \omega^2(t) q^2\right)$$

may be complex

Assume that $\dot{\omega}/\omega^2 \ll 1$ and that mode starts in adiabatic ground state

→ special case of previous discussion, with width of initial Gaussian wave packet determined by ω

Result: $\Psi \sim \exp\left(i\frac{S_{Cl}}{\hbar}\right)$ with $S_{Cl} = \frac{1}{2}\frac{\dot{R}_{in}^{(+)}}{R_{in}^{(+)}} q_f^2$

$$R_{in}^{(+)} \rightarrow \frac{e^{+i\int^t \omega dt}}{\sqrt{2\omega}} \quad (t \rightarrow t_i); \qquad R_{in}^{(+)} \rightarrow \frac{\alpha e^{+i\int^t \omega dt} + \beta e^{-i\int^t \omega dt}}{\sqrt{2\omega}} \quad (t \rightarrow t_f)$$

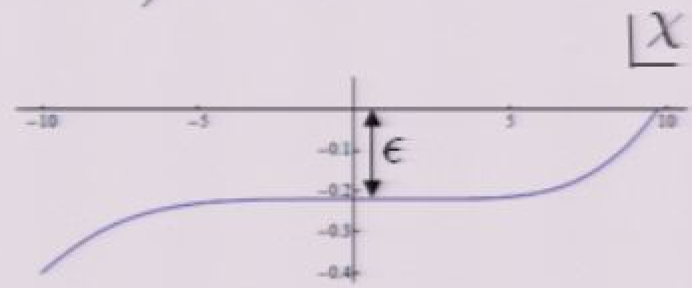
Define $a, a^\dagger$ and compute $\langle n \rangle = \langle a^\dagger a \rangle = \frac{|\beta|^2}{|\alpha|^2 - |\beta|^2}$

Equations of motion for inhomogeneous fluctuations

$V(\phi) = -\frac{\lambda_\phi}{4}\phi^4$ with $\lambda_\phi = \frac{1}{N^2 \ln\left(\frac{\phi}{NM}\right)}$; denote $l \equiv \ln(\phi/NM)$

Zero energy real scaling solution: $\phi \sim \frac{N l^{1/2}}{|t|}\left(1 + \frac{1}{2l} + \dots\right)$

Complex solution (near t=0): replace $t \to t - i\epsilon$



E.o.m. for inhomogeneous perturbations:

$$\ddot{\delta\phi} = \left(-k^2 + \frac{6}{(t-i\epsilon)^2}\left(1 + \frac{5}{12}l^{-1} - \frac{2}{3}l^{-2}\dots\right)\right)\delta\phi$$

wave number

Aim: solve for incoming positive frequency mode, determine Bogoliubov coefficients and $\langle n \rangle$

Production of inhomogeneous modes: leading order in $1/l$

$$l \equiv \ln(\phi/NM)$$

Vave equation for linearized fluctuation with wavenumber k, to leading order in $1/l$:

$$\ddot{\delta\phi} = \frac{6}{t^2}\delta\phi - k^2\delta\phi$$ (will replace $t \to t - i\epsilon$ near t=0)

Solutions (unambiguously continued through t=0 using $t \to t - i\epsilon$):

$$f^{(1)} = \cos kt\left(\frac{1}{(kt)^2} - \frac{1}{3}\right) + \frac{\sin kt}{kt}, \qquad f^{(2)} = \sin kt\left(\frac{1}{3} - \frac{1}{(kt)^2}\right) + \frac{\cos kt}{kt}$$

Asymptotics for $|kt| >> 1$:

$$f^{(1)} \sim -\frac{1}{3}\cos kt, \qquad f^{(2)} \sim \frac{1}{3}\sin kt$$

hus incoming positive frequency mode $f^{(1)} + if^{(2)}$ is also outgoing positive frequency mode!

lo particle creation for classical potential → any particle creation will be due to $1/l$ effects,
e. due to the logarithmic running of the coupling.

Production of inhomogeneous modes: next-to-leading order in $1/l$

$$l \equiv \ln(\phi/NM)$$

$$\ddot{\delta\phi} = \frac{6}{t^2}\left(1 + \frac{5}{12l} - \frac{2}{3l^2}\dots\right)\delta\phi - k^2\delta\phi$$

Solutions: $$\delta\phi^{(1)} = l^{\frac{1}{2}} f^{(1)}(kt) + l^{-\frac{1}{2}} g^{(1)}(kt) + \dots$$

$$\delta\phi^{(2)} = l^{-\frac{1}{2}} f^{(2)}(kt) + l^{-\frac{3}{2}} g^{(2)}(kt) + \dots$$

One finds: Bogoliubov coefficient vanishes to leading order in $1/l$ (cf. classical potential), but not to next order:

$$\beta_{\mathbf{k}} \approx -\frac{i\pi e^{-2k\epsilon}}{\ln(k/M)} \qquad (k \gg M) \qquad \rightarrow \qquad \langle n \rangle = \frac{\pi^2}{l_0^2} e^{-4k\epsilon}$$

Energy density in produced $\delta\phi$ particles: $$\rho_{c,\delta\phi} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} k \frac{\pi^2}{\ln^2(k/M)} e^{-4k\epsilon}$$

Production of inhomogeneous modes: energy in created particles

Energy density in

$\delta\phi$ particles: $$\rho_{c,\delta\phi} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} k \frac{\pi^2}{\ln^2(k/M)} e^{-4k\epsilon}$$

light Higgs particles: $$\rho_{c,\Phi} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} k \frac{4\pi^2 \coth^2(\pi\sqrt{3/20})}{15\ln^2(k/M)} e^{-4k\epsilon}$$

massless gauge bosons: $$\rho_{c,A} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} k \frac{\pi^2 C^2}{4\ln^4(k/M)} e^{-4k\epsilon}$$

Here, $C \sim 1/N$ and $\epsilon \sim W^3 R_{AdS}/(N|\ln(MR_{AdS})|^{1/2})$, at least for "most" ϕ, away from classical trajectory; W=1 would be minimal spread wave packet.

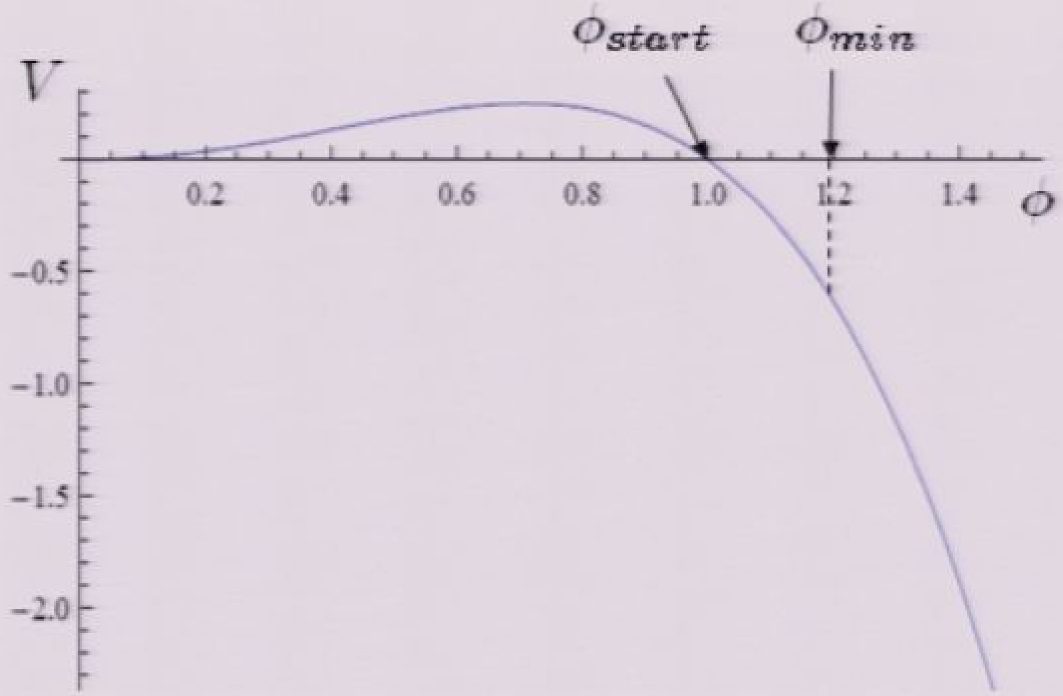
For breaking $SU(N) \to U(1)^{N-1}$: O(N) light Higgs bosons and massless gauge bosons

→ created energy density dominated by light Higgs particles

$$\rho_c \sim \frac{N}{\epsilon^4\,|\ln(MR_{AdS})|^2} \sim \frac{N^5}{W^{12}R^4_{AdS}}$$

Backreaction

$$V = \frac{\phi^2}{6R_{AdS}^2} - \frac{\lambda_0 \phi^4}{4\ln(\phi/NM)}$$



Field starts at ϕ_{start}, rolls to infinity and back to ϕ_{min}

"Small backreaction" if

$$\phi_{min} - \phi_{start} \ll \phi_{start} \quad \leftrightarrow \quad \rho_c \ll N^2 |\ln(MR_{AdS})|/R_{AdS}^4$$

$$\updownarrow$$

(W = spread/minimal spread)

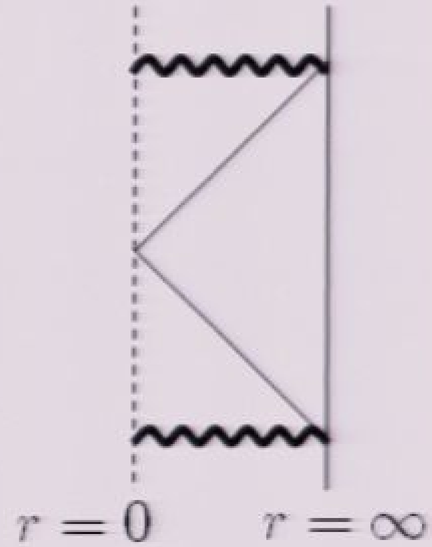
$$|\ln(MR_{AdS})| \gg \frac{N^3}{W^{12}}$$

Satisfied if, for instance, $W \sim N^{1/4}$ and $|\ln(MR_{AdS})| \gg 1$

→ spread $\Delta\chi_i \approx \dfrac{R_{AdS}}{N^{3/4}|\ln(MR_{AdS})|^{1/2}}$ much larger than minimal possible one,

but much smaller than distance R_{AdS} traveled in χ space!

Stress-energy correlators



Bulk metric couples to boundary stress tensor

Asymptotics of bulk metric determined by boundary data, such as $\langle T_{\mu\nu} \rangle$ and source, or $\langle T_{\mu\nu} \rangle$ and double trace coupling

Given asymptotics, can integrate radially inwards to determine bulk metric

Relevant modes in boundary theory are frozen in near the singularity

→ quantum fluctuations become classical

→ classical bulk metric perturbations (cosmological perturbations)

Precise correspondence yet to be worked out

Intriguingly, boundary correlators are approximately scale invariant, reflecting the slightly broken conformal invariance of the boundary theory!

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Conclusions

- String theory in $AdS_5 \times S^5$ with modified b.c.; smooth initial data evolve into big crunch.
- In dual N=4 SYM: unbounded potential, operator reaches infinity in finite time.
- Potential under excellent control near singularity. Quantum effective potential is really unbounded from below. Conformal invariance broken by logarithmic running of coupling.
- Need small 't Hooft coupling to have field theory completely under control → stringy bulk. Can we extrapolate (some of) our results to large 't Hooft coupling?
- QM with unbounded potentials: self-adjoint extensions define unitary quantum evolution.
- QFT: ultralocality → define self-adjoint extension point by point.
- If homogeneous mode were classical (i.e. in infinite volume), excessive creation of high momentum particles would seem to prevent a bounce.
- However, QM spread of the homogeneous mode (due to finite volume of 3-sphere) suppresses the effect of particle creation on "most" of the wavepacket for the homogeneous mode. Suggests large probability for bounce (in certain parameter range).
- Slightly broken conformal invariance of boundary theory might lead to nearly scale invariant spectrum of bulk fluctuations.