Title: Ekpyrotic Non-Gaussianity

Date: Mar 06, 2008 04:30 PM

URL: http://pirsa.org/08030040

Abstract:

Ekpyrotic Non-Gaussianity

Jean-Luc Lehners
Princeton Center for Theoretical Physics

JLL, Paul J. Steinhardt - arXiv:0712.3779

Inflation

Potential is very flat

- Inflaton has very small self-interactions
- Tiny non-gaussianities (for simple models)

$$f_{NL} \sim \epsilon \equiv \frac{3}{2}(1+w) \sim \mathcal{O}(10^{-2})$$ Maldacena

(can get larger fNL by adding fields)

Ekpyrotic models

Potential is very steep $V \sim -e^{-c\phi}$

- Fields have significant self-interactions

$$f_{NL}^{int} \sim \mathcal{O}(c) \sim \mathcal{O}(\sqrt{\epsilon_{ek}})$$ (before conversion)

where $\epsilon_{ek} \equiv \frac{3}{2}(1 + w_{ek})$

Contracting universe: curvature perturbation is not a growing mode

- Generate scale-invariant entropy perturbations
- Convert them to curvature perturbations shortly before/after big bang

Strength of conversion $\mathcal{R} \sim \frac{1}{\sqrt{\epsilon_c}} \delta s$

Hence expect $f_{NL}^{int} \sim \sqrt{\epsilon_{ek} \epsilon_c}$

Conversion mechanisms

Converting during ekpyrosis

(Buchbinder, Khoury, Ovrut; Creminelli, Senatore; Koyama, Mizuno, Vernizzi, Wands)

$$f_{NL}^{int} \sim \sqrt{\epsilon_{ek}\epsilon_c} = \epsilon_{ek} \sim \mathcal{O}(c^2)$$

Converting during kinetic phase

(JLL, Steinhardt)

$$f_{NL}^{int} \sim \sqrt{\epsilon_{ek}\epsilon_c} = \sqrt{3\epsilon_{ek}} \sim \mathcal{O}(c)$$

Converting after big bang via modulated reheating

(Battefeld)

$$f_{NL}^{int} \sim \sqrt{\epsilon_{ek}\epsilon_c} \sim \mathcal{O}(c)$$

Evolution of the entropy perturbation

$\delta s \equiv (\dot{\phi}_1 \, \delta\phi_2 - \dot{\phi}_2 \, \delta\phi_1)/\dot{\sigma}$

$\dot{\sigma} \equiv \sqrt{\dot{\phi}_1^2 + \dot{\phi}_2^2}$

$$\ddot{\delta s} + \left(V_{ss} + 3\dot{\theta}^2\right)\delta s + \left(\frac{1}{2}V_{sss} - 5\frac{\dot{\theta}}{\dot{\sigma}}V_{ss} - 9\frac{\dot{\theta}^3}{\dot{\sigma}}\right)(\delta s^{(1)})^2 = 0$$

With $\dot{\delta s} = 0$

And θ , angle of trajectory in scalar field space

Ekpyrotic phase $\dot{\theta} = 0$

Conversion $\dot{\theta} \neq 0$

Ekpyrosis $V \sim -e^{-c\phi_1} - e^{-c/\gamma\phi_2}$ $\dot{\phi}_2 \equiv \gamma\dot{\phi}_1$

then $\delta s = \delta s^{(1)} + \tilde{c}(\delta s^{(1)})^2$

$$\delta s^{(1)} \propto \frac{1}{t} \qquad \tilde{c} = \frac{\gamma^2 - 1}{4\gamma\sqrt{1+\gamma^2}} c \sim \mathcal{O}(c)$$

(heterotic M-theory $\tilde{c} = \frac{c}{4}$)

Kinetic phase: potentials unimportant at first

$$\delta s^{(1)} \sim \ln(-t) \qquad \delta s^{(2)} \sim \tilde{c}\ln(-t)$$

Time evolution of the curvature perturbation

$$\dot{\mathcal{R}} = -\sqrt{\frac{2}{\epsilon}}\dot{\theta}(\delta s^{(1)} + \delta s^{(2)}) - \frac{H}{\dot{\sigma}^2}(V_{ss} + 4\dot{\theta}^2)(\delta s^{(1)})^2$$

$\mathcal{R}_L$ $\qquad f_{NL}^{int}$ $\qquad f_{NL}^{ref}$

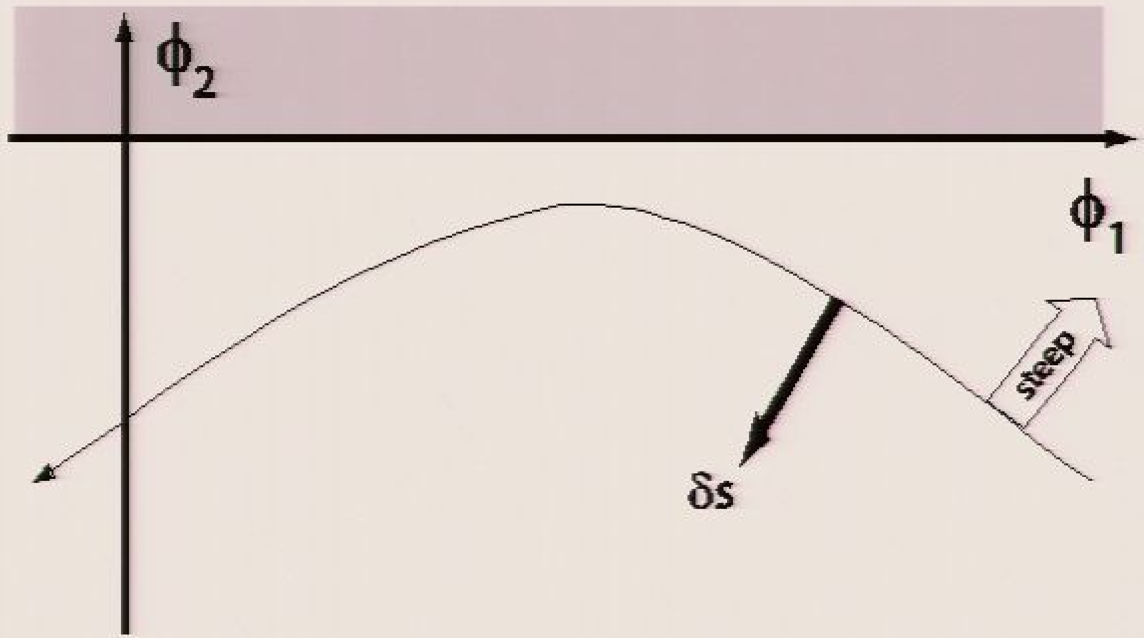
linear, gaussian

Use definition $\mathcal{R} = \mathcal{R}_L - \frac{3}{5} f_{NL} \mathcal{R}_L^2$

Comment: for sharp reflections conversion very inefficient, and hence fNL very large as $f_{NL} \propto \mathcal{R}_L^{-2}$

- sharp reflections already ruled out by observations

Conversion during kinetic phase



bend towards shallow side: subdominant $\tilde{c} > 0$

(includes heterotic M-theory example)

bend towards steep side: dominant $\tilde{c} < 0$

treat reflection as $V(\phi_2)$ only

Gradual conversion $\dot{\theta} \sim \frac{1}{|t_{ref}|}$ constant

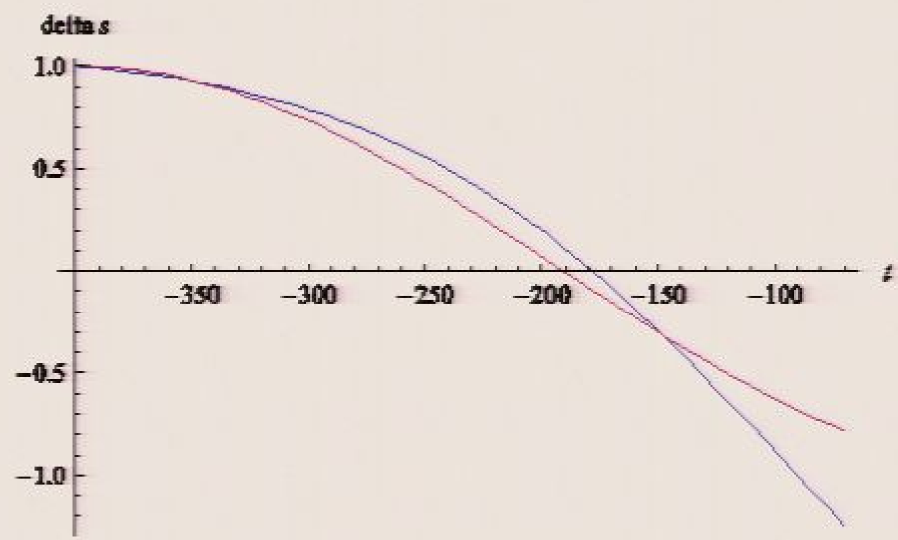
(Buchbinder, Khoury, Ovrut)

During conversion we have $$\ddot{\delta s}^{(1)} + 3H\dot{\delta s}^{(1)} + (\dot{\theta}^2 + \frac{1}{\gamma t}\dot{\theta})\delta s^{(1)} = 0$$

Approximate $\dot{\delta s}^{(1)} = 0$ and $$\omega \equiv \dot{\theta}\sqrt{1 + \frac{1}{\dot{\theta}\gamma(t_{ref} + \Delta t/2)}} \sim 3\dot{\theta}$$

Solution during conversion

$$\delta s^{(1)} = \delta s^{(1)}(t_{ref}) \cos\omega(t - t_{ref})$$



Second order perturbation behaves

Similarly (at least for large c)

$$\delta s^{(2)} = \tilde{c}(\delta s^{(1)}(t_{ref}))^2 \cos\omega(t - t_{ref})$$

Gaussian part $\mathcal{R}_L = -\int \sqrt{\frac{2}{\epsilon_c}} \dot{\theta} \delta s^{(1)} \sim \mathcal{O}(\delta s^{(1)})$

Hence the intrinsic contribution can be approximated by

$$\begin{aligned} f_{NL}^{int} &\sim \frac{1}{\mathcal{R}_L^2} \int \frac{1}{\sqrt{\epsilon_c}} \dot{\theta} \delta s^{(2)} \\ &= 10\tilde{c} \\ &= \frac{5}{2} c \end{aligned}$$

(het M-theory)

As for the reflection contribution

$$\begin{aligned} f_{NL}^{ref} &\sim \frac{1}{\mathcal{R}_L^2} \int \frac{H}{\dot{\sigma}^2} (2\dot{\theta}^2 + \frac{\dot{\theta}}{\gamma t})(\delta s^{(1)})^2 \\ &\sim -100 \end{aligned}$$

Always negative!

Conversion during kinetic phase

In total: $f_{NL} \approx 10\tilde{c} - 100$

Dominant reflections $\tilde{c} < 0$ all but ruled out by observations

Subdominant reflections $\tilde{c} > 0$ look much more promising

Example of heterotic M-theory $f_{NL} \approx \frac{5}{2}c - 100$

For $10 < c < 100$ we get $-75 < f_{NL} < 150$

Cf numerical result $-150 < f_{NL} < 200$ (JLL, Steinhardt)

Conversion during ekpyrosis (new ekpyrotic models)

Can be understood semi-analytically in same way

Again

$$f_{NL}^{int} \sim \int \dot{\theta}\delta s^{(2)}$$

But the eq of motion reduces to $\ddot{\delta s}^{(2)} = \frac{\dot{\theta}}{\dot{\sigma}} V_{ss} (\delta s^{(1)})^2$

Since $V_{ss} = \frac{-2}{t^2} < 0$, $\delta s^{(2)}$ is always driven to opposite

sign of $\dot{\theta}$ and hence f_{NL}^{int} is negative

But

$$f_{NL}^{ref} \sim -\int V_{ss}(\delta s^{(1)})^2$$

Since $V_{ss} = \frac{-2}{t^2} < 0$ we thus find $f_{NL}^{ref} > 0$

Numerical analysis gives $|f_{NL}^{ref}| > |f_{NL}^{int}|$

And f_{NL}^{total} is always positive!

Numerical results are in good agreement with

$$f_{NL} = \frac{5}{12} c_j^2$$

(Koyama, Mizuno, Vernizzi, Wands)

Conclusions

- Conversion during ekpyrosis: confirmed semi-analytically and numerically

$$f_{NL} = \frac{5}{12} c_j^2$$ large and positive

Add constraint from spectral index

$c \gtrsim 30$ to get $n_s = 0.97$

Then these models start being highly constrained by observations

- Ekpyrotic/cyclic model (embedded in heterotic M-theory) and similar subdominant conversions

If conversion gradual and $c \gtrsim 30$

Analytic $-25 < f_{NL} < 150$

Numerical $-50 < f_{NL} < 200$

WMAP results already intrude on this range.

fNL should be detected with Planck!

Conclusions

- Conversion during ekpyrosis: confirmed semi-analytically and numerically

$$f_{NL} = \frac{5}{12} c_j^2$$ large and positive

Add constraint from spectral index

$c \gtrsim 30$ to get $n_s = 0.97$

Then these models start being highly constrained by observations

- Ekpyrotic/cyclic model (embedded in heterotic M-theory) and similar subdominant conversions

If conversion gradual and $c \gtrsim 30$

Analytic $-25 < f_{NL} < 150$

Numerical $-50 < f_{NL} < 200$

WMAP results already intrude on this range.

fNL should be detected with Planck!